SUPPLEMENTARY NOTE

In these supplementary informations we propose a theoretical description of flows arising in the neurectoderm as a result of the underlying motion of ppl cells. In order to analyze the specific effect of ppl cells on neurectoderm flows, we use experimental measurements of neuroectoderm velocity field in the presence of ppl cells, denoted $\mathbf{v}^{\mathrm{tot}}$, and experimental measurements of neuroectoderm velocity field in the absence of ppl cells in MZ*oep* mutants (denoted $\mathbf{v}^{\mathrm{MZ}}$). We then calculate the difference between these velocity fields:

$$\mathbf{v} = \mathbf{v}^{\mathrm{tot}} - \mathbf{v}^{\mathrm{MZ}}. \tag{1}$$

Our theoretical description aims at reproducing this velocity field.

1. Simplified one dimensional description of neurectoderm flows

We describe the neurectoderm as a two-dimensional viscous compressible fluid, flowing with velocity vector $\mathbf{v}$. We assume that the effect of ppl cells is to exert an external force on the neuroectoderm, and we analyze the flow profile induced by this external force.

1.1. **Main equation of one-dimensional viscous flow.** We first discuss a simplified description of the flow in one dimension. We aim here at understanding flow profiles measured along the animal-vegetal axis of the embryo, with y denoting the coordinate along the animal-vegetal axis, going positively towards the vegetal axis (Fig. 5a). We consider the velocity towards the vegetal pole v_y, and the tension within the tissue is denoted σ_y. We consider the tissue to be fluid with effective one-dimensional viscosity $\bar{\eta}$, such that the stress within the tissue reads

$$\sigma_y = \bar{\eta}\partial_y v_y \tag{2}$$

We assume that neurectoderm cells flow with a friction coefficient ξ_0 relative to the surrounding tissues other than ppl cells. For instance, ξ_0 could be caused by relative friction between the neurectoderm and the EVL or by friction between the neurectoderm and the yolk. In addition, we assume that within the ppl domain, delimited by the region $y^{\mathrm{ppl}} - L_y^{\mathrm{ppl}} < y < y^{\mathrm{ppl}} + L_y^{\mathrm{ppl}}$,

ppl cells exert a uniform force density f. Force balance at low Reynolds number then reads:

$$\partial_y \sigma_y = \xi_0 v_y \quad \text{outside of ppl domain} \tag{3}$$

$$\partial_y \sigma_y = \xi_0 v_y - f \quad \text{inside the ppl domain} \tag{4}$$

Finally, we denote L^E the distance from the animal pole to the neurectoderm margin, such that $y = 0$ denotes the animal pole and $y = -L^E$ and $y = L^E$ denote the neurectoderm margin on the ventral and dorsal side. We impose that the flow vanishes at the margin, $v_y(-L^E) = v_y(L^E) = 0$. The solution for the flow reads:

$$v_y = C_1 \left(e^{\frac{y}{l_0}} - e^{-\frac{2L^E+y}{l_0}} \right) \quad , \quad y < y^{\text{ppl}} - L_y^{\text{ppl}} \tag{5}$$

$$v_y = C_2 e^{\frac{y}{l_0}} + C_3 e^{-\frac{y}{l_0}} + \frac{f}{\xi_0} \quad , \quad y^{\text{ppl}} - L_y^{\text{ppl}} < y < y^{\text{ppl}} + L_y^{\text{ppl}} \tag{6}$$

$$v_y = C_4 \left(e^{\frac{y}{l_0}} - e^{\frac{2L^E-y}{l_0}} \right) \quad , \quad y > y^{\text{ppl}} + L_y^{\text{ppl}} \tag{7}$$

with $l_0 = \sqrt{\bar{\eta}/\xi_0}$ an hydrodynamic length, and $C_i, i = 1, ..., 4$ are constants which can be determined from the conditions of continuity of the velocity and its derivative at the boundaries of the ppl domain. The flow profile induced by a localized external force decays exponentially away from the ppl region exponentially on the length l_0 (Supplementary Fig. 6b).

In the limit where friction acting on the tissue from other sources than ppl cells can be neglected, ($l_0 \gg L^E$), the solution for the flow profile reads:

$$v_y = \frac{f L_y^{\text{ppl}}}{\bar{\eta}} \left(1 - \frac{y^{\text{ppl}}}{L^E} \right) (L^E + y) \quad , \quad y < y^{\text{ppl}} - L_y^{\text{ppl}} \tag{8}$$

$$v_y = -\frac{f}{2\bar{\eta}} \left((y - y^{\text{ppl}})^2 + \frac{2L_y^{\text{ppl}} y^{\text{ppl}}}{L^E} (y - y^{\text{ppl}}) + \frac{L_y^{\text{ppl}}}{L^E} (2{y^{\text{ppl}}}^2 - 2{L^E}^2 + L^E L_y^{\text{ppl}}) \right) \quad , \quad y^{\text{ppl}} - L_y^{\text{ppl}} < y < y^{\text{ppl}} + L_y^{\text{ppl}} \tag{9}$$

$$v_y = \frac{f L_y^{\text{ppl}}}{\bar{\eta}} \left(1 + \frac{y^{\text{ppl}}}{L^E} \right) (L^E - y) \quad , \quad y > y^{\text{ppl}} + L_y^{\text{ppl}} \tag{10}$$

In that case the flow profile decays linearly towards the EVL margin.

1.2. **Comparison to experimental flow profile along the dorsal midline.**

1.2.1. *Wild-type velocity profiles.* To compare to experiment, we first aimed at matching theoretical profiles with wt velocity profiles along the animal-vegetal axis, on the dorsal line. Estimates of the lengths $L^{\rm ppl}$, L^E and $y^{\rm ppl}$ used for comparison are discussed in section 3 and reported in Table 2.

A fitting procedure then gave $f/\bar{\eta} = -4.2 \cdot 10^{-5} \mu\text{m}^{-1} \cdot \text{min}^{-1}$ and $\bar{\eta}/\xi_0 = 6.3 \cdot 10^5 \mu\text{m}^2$ (Table 1), corresponding to an hydrodynamic length $l_0 = 793\ \mu$m. Because l_0 is close to the system size L^E, we conclude that external friction arising from other tissues than ppl cells has a small effect on the flow profile in wt. Predicted profiles without external friction from surrounding tissues ($\xi_0 = 0$) indeed also match experimental profiles closely (Supplementary Fig. 6c).

Assuming that the force density f exerted by ppl cells on the neuroectoderm can be described by a dynamic friction force, we then have the following relation:

$$f = \xi(v_y^{ppl} - v_y^{\rm tot}), \tag{11}$$

where v_y^{ppl} is the velocity of ppl cells, $v_y^{\rm tot}$ is the wild-type neuroectoderm total velocity at the same point and ξ is the friction coefficient. To determine the friction coefficient, we average Eq. 11 in the domain of ppl cells, assuming here for simplicity that the force density f is homogeneous within the ppl domain:

$$f = \xi(\langle v_y^{ppl} \rangle - \langle v_y^{\rm tot} \rangle). \tag{12}$$

We then estimate the experimental values of $\langle v_y^{ppl} \rangle$ and $\langle v_y^{\rm tot} \rangle$. The corresponding values are reported in Table 1. We then obtain an estimate of f/ξ, and using the value of $f/\bar{\eta}$ obtained by comparison with experimental profiles, we then can estimate the ratio $\bar{\eta}/\xi$ (Table 1). A characteristic hydrodynamic length can be defined from this ratio by:

$$l_1 = \sqrt{\bar{\eta}/\xi}. \tag{13}$$

Note that $l_1 \neq l_0$ since ξ is associated to friction acting in between the neurectoderm and ppl cells, while ξ_0 is associated to friction in between the neurectoderm and other tissues. We find $l_1 \simeq 288\ \mu$m, corresponding to a friction coefficient about 8 times larger in between neurectoderm and ppl cells than in between neurectoderm and other tissues.

Simplified one-dimensional description							
Exp	$f/\bar{\eta}$	$\bar{\eta}/\xi_0$	$\langle v_y^{\mathrm{ppl}} \rangle$	$\langle v_y^{\mathrm{tot}} \rangle$	f/ξ	$\bar{\eta}/\xi$	l_1
Unit	μm$^{-1}\cdot$min^{-1}	μm^2	μm/min	μm/min	μm/min	μm^2	μm
wt	-4.2 $\cdot 10^{-5}$	$6.3\cdot 10^5$	-4	-0.5	-3.5	$8.4\cdot 10^4$	290
slb	-3.4 $\cdot 10^{-5}$	$6.3\cdot 10^5$	-2.5	0.4	-2.9	$8.4\cdot 10^4$	290
Two-dimensional description							
Exp	f/η_b	η_b/η	$\langle v_y^{\mathrm{ppl}} \rangle$	$\langle v_y^{\mathrm{tot}} \rangle$	f/ξ	η_b/ξ	l_2
Unit	μm$^{-1}\cdot$min^{-1}		μm/min	μm/min	μm/min	μm^2	μm
wt	-2 $\cdot 10^{-4}$	1	-4	-0.5	-3.5	$1.8\cdot 10^4$	133
slb	-1.6 $\cdot 10^{-4}$	1	-2.5	0.4	-2.9	$1.8\cdot 10^4$	133

Table 1. Values of parameters obtained by comparison of the one dimensional simplfied theory and two-dimensional theory with experiments.

1.3. ***slb* mutant velocity profiles.** ppl cells have a reduced velocity in *slb* mutants. We therefore estimate the reduction in the magnitude of ppl exerted force density f from Eq. 12, using experimentally measured average velocities in the ppl region (Table 1). We then obtain the corresponding predicted theory profile, keeping mechanical parameters other than f as in wt, and using geometrical parameters reported in Table 2. This yields a very good agreement with experimental profiles (Fig. 5-b1).

2. Two-dimensional flow

2.1. **Main equations.** We discuss here the two-dimensional flow profiles obtained in the region of experimental measurement, a square domain of size $2L$. For simplicity, we ignore the curvature of the embryo and use 2D cartesian coordinates x, y. For comparison with flows observed in the zebrafish embryo, the x axis is going on the surface of the embryo along the left-right direction, away from the dorsal midline of the embryo, and the y axis as going along the animal-vegetal direction, away from the animal pole. The corresponding geometry is represented in Fig. 5a. As before, we assume that the effect of ppl cells is to exert an external force on the neuroectoderm. We solve for the theoretical flow within the region of observation, and impose experimentally measured velocities at the boundary of the region.

Force balance on the neuroectoderm in two dimensions now reads

$$\partial_i \sigma_{ij} = -f_j^{\mathrm{ppl}}, \tag{14}$$

where σ_{ij} is the two-dimensional stress tensor within the neurectoderm, and $\mathbf{f}^{\mathrm{ppl}}$ is the two-dimensional force density exerted by ppl cells on the neurectoderm.

The stress tensor σ_{ij} in the neuroectoderm tissue reads:

$$\sigma_{ij} = 2\eta\Big[v_{ij} - \frac{1}{2}v_{kk}\delta_{ij}\Big] + \eta_b v_{kk}\delta_{ij}, \tag{15}$$

where η is the neurectoderm shear viscosity, η_b is the neurectoderm bulk viscosity, and $v_{ij} = (\partial_i v_j + \partial_j v_i)/2$ is the symmetric part of the gradient of flow.

We further choose the following form for the force density exerted by ppl cells:

$$\begin{aligned} f_j^{\mathrm{ppl}} &= f\left(\mathcal{H}[x + L_x^{\mathrm{ppl}}] - \mathcal{H}[x - L_x^{\mathrm{ppl}}]\right) \\ &\quad \times \left(\mathcal{H}[y - y^{\mathrm{ppl}} + L_y^{\mathrm{ppl}}] - \mathcal{H}[y - y^{\mathrm{ppl}} - L_y^{\mathrm{ppl}}]\right)\delta_{jy} \end{aligned} \tag{16}$$

where $\mathcal{H}$ is the Heaviside function. This choice corresponds to a uniform force density with magnitude f, acting along the y direction, and exerted on a rectangular domain of size $2L_x^{\mathrm{ppl}} \times 2L_y^{\mathrm{ppl}}$, centred around $x = 0$ and $y = y^{\mathrm{ppl}}$.

By using the constitutive equation 15, the force balance equation 14, and the force density 16, we obtain the following equation for the flow field:

$$\begin{cases} \eta\Delta v_x + \eta_b(\partial_x^2 v_x + \partial_y\partial_x v_y) = 0, \\ \eta\Delta v_y + \eta_b(\partial_y^2 v_y + \partial_y\partial_x v_x) = -f\left(\mathcal{H}[x + L_x^{\mathrm{ppl}}] - \mathcal{H}[x - L_x^{\mathrm{ppl}}]\right) \\ \qquad\qquad \times \left(\mathcal{H}[y - y^{\mathrm{ppl}} + L_y^{\mathrm{ppl}}] - \mathcal{H}[y - y^{\mathrm{ppl}} - L_y^{\mathrm{ppl}}]\right). \end{cases} \tag{17}$$

Eqs. 17 are then solved on a rectangular domain of size $2L \times 2L$. The boundary conditions are set by imposing experimentally measured velocity profiles

$$v_x(L, y) = v_x^{R,\exp}(y), \quad v_y(L, y) = v_y^{R,\exp}(y) \tag{18}$$

$$v_x(-L, y) = v_x^{L,\exp}(y), \quad v_y(-L, y) = v_y^{L,\exp}(y) \tag{19}$$

$$v_x(x, L) = v_x^{U,\exp}(x), \quad v_y(x, L) = v_y^{U,\exp}(x) \tag{20}$$

$$v_x(x, -L) = v_x^{D,\exp}(x), \quad v_y(x, -L) = v_y^{D,\exp}(x). \tag{21}$$

2.2. **Calculation of flow fields.** To solve for the velocity field and get explicit expressions for v_x and v_y, we used the method of superposition. For convenience, we used the following decomposition of the velocity field:

$$v_x(x,y) = v_x^{\text{per}}(x,y) + \sum_{n=1}^{N} \alpha_n(y) \sin\left(\frac{n\pi(x-L)}{2L}\right) + \sum_{n=1}^{N} \beta_n(x) \cos\left(\frac{n\pi(y-L)}{2L}\right) + \beta_0(x) \tag{22}$$

$$v_y(x,y) = v_y^{\text{per}}(x,y) + \sum_{n=1}^{N} \gamma_n(y) \cos\left(\frac{n\pi(x-L)}{2L}\right) + \sum_{n=1}^{N} \delta_n(x) \sin\left(\frac{n\pi(y-L)}{2L}\right) + \gamma_0(y) \tag{23}$$

In the equation above, $v_x^{\text{per}}(x,y)$ and $v_y^{\text{per}}(x,y)$ are solution of Eq. 17 which are periodic along the x axis and have vanishing velocities at the boundaries $y=-L$ and $y=L$. The additional truncated Fourier sums are introduced to verify the remaining boundary conditions.

2.2.1. *Determination of the periodic solution.* To obtain the periodic solution, we introduce the Fourier transforms of the velocity field along the x direction:

$$\begin{cases} \tilde{v}_x(k_x,y) = \frac{1}{\sqrt{2\pi}} \int dx e^{-ik_x x} v_x^{\text{per}}(x,y) \\ \tilde{v}_y(k_x,y) = \frac{1}{\sqrt{2\pi}} \int dx e^{-ik_x x} v_y^{\text{per}}(x,y). \end{cases} \tag{24}$$

where $k_x = \pi n/L$ with $n \in \mathbb{Z}$. We then obtain from Eq. 17:

$$\begin{cases} \eta \partial_y^2 \tilde{v}_x + \eta_b i k_x \partial_y \tilde{v}_y - (\eta+\eta_b) k_x^2 \tilde{v}_x = 0 \\ (\eta+\eta_b)\partial_y^2 \tilde{v}_y + \eta_b i k_x \partial_y \tilde{v}_x - \eta k_x^2 \tilde{v}_y = C(k_x)\mathcal{H}_y, \end{cases} \tag{25}$$

where we have introduced $C(k_x) = -f\sqrt{\frac{2}{\pi}} \frac{sin(k_x L_x^{\text{ppl}})}{k_x}$ for $k_x \neq 0$ and $C(0) = -f\sqrt{2/\pi} L_x^{\text{ppl}}$, and $\mathcal{H}_y = \mathcal{H}[y - y^{\text{ppl}} + L_y^{\text{ppl}}] - \mathcal{H}[y - y^{\text{ppl}} - L_y^{\text{ppl}}]$. These equations can be rewritten after rearrangement

$$\begin{cases} \partial_y^4 \tilde{v}_x - 2k_x^2 \partial_y^2 \tilde{v}_x + k_x^4 \tilde{v}_x = -\frac{C(k_x)\eta_b i k_x \mathcal{H}_y'}{\eta(\eta+\eta_b)} \\ \partial_y^4 \tilde{v}_y - 2k_x^2 \partial_y^2 \tilde{v}_y + k_x^4 \tilde{v}_y = \frac{C(k_x)\mathcal{H}_y''}{\eta+\eta_b} - \frac{C k_x^2 \mathcal{H}_y}{\eta}. \end{cases} \tag{26}$$

We solve separately these equations in the following three domains (1), $-L_y^{\rm ppl} < y < L_y^{\rm ppl}$; (2), $y > L_y^{\rm ppl}$, (3), $y < -L_y^{\rm ppl}$, and the corresponding solutions read for $k_x \neq 0$:

$$\begin{cases} \tilde{v}_x^1 = (D_1 + D_2 y)e^{k_x y} + (D_3 + D_4 y)e^{-k_x y} \\ \tilde{v}_y^1 = \left[\frac{i}{\eta_b k_x}(2\eta + \eta_b)D_2 - iD_1 - iD_2 y\right]e^{k_x y} + \left[\frac{i}{\eta_b k_x}(2\eta + \eta_b)D_4 + iD_3 + iD_4 y\right]e^{-k_x y} - \frac{C(k_x)}{\eta k_x^2} \\ \tilde{v}_x^2 = (G_1 + G_2 y)e^{k_x y} + (G_3 + G_4 y)e^{-k_x y} \\ \tilde{v}_y^2 = \left[\frac{i}{\eta_b k_x}(2\eta + \eta_b)G_2 - iG_1 - iG_2 y\right]e^{k_x y} + \left[\frac{i}{\eta_b k_x}(2\eta + \eta_b)G_4 + iG_3 + iG_4 y\right]e^{-k_x y} \\ \tilde{v}_x^3 = (M_1 + M_2 y)e^{k_x y} + (M_3 + M_4 y)e^{-k_x y} \\ \tilde{v}_y^3 = \left[\frac{i}{\eta_b k_x}(2\eta + \eta_b)M_2 - iM_1 - iM_2 y\right]e^{k_x y} + \left[\frac{i}{\eta_b k_x}(2\eta + \eta_b)M_4 + iM_3 + iM_4 y\right]e^{-k_x y}. \end{cases} \tag{27}$$

where we have introduced 12 constants D_i, G_i, M_i, that can be determined from boundary conditions. Using the boundary conditions of vanishing velocity at the boundaries $y = -L$ and $y = L$, together with the continuity of the velocity field v_x, v_y and of the stress components σ_{xy} and σ_{yy} at the interfaces $y = L_y^{\rm ppl}$ and $y = -L_y^{\rm ppl}$, we obtain a system of equations that allow to determine all the constants. For $k_x = 0$, the solution reads

$$\begin{cases} \tilde{v}_x = 0 \\ \tilde{v}_y^1 = -\sqrt{\frac{2}{\pi}} \frac{f L_x^{\rm ppl}}{2(\eta + \eta_b)} y^2 + K_1 y + K_2 \\ \tilde{v}_y^2 = W_1 y + W_2 \\ \tilde{v}_y^3 = Z_1 y + Z_2. \end{cases} \tag{28}$$

The 6 constants K_i, W_i and Z_i are determined from the boundary conditions imposing the continuity of the velocity at the ppl boundary, the continuity of the stress σ_{yy} at the ppl boundary, and zero v_y velocity at the domain boundary.

From the solution in Fourier space, the real-space solution is then obtained from

$$\begin{cases} v_x^{\rm per}(x, y) = \frac{1}{\sqrt{2\pi}} \frac{\pi}{L} \sum_{k_x} e^{ik_x x} \tilde{v}_x(k_x, y), \\ v_y^{\rm per}(x, y) = \frac{1}{\sqrt{2\pi}} \frac{\pi}{L} \sum_{k_x} e^{ik_x x} \tilde{v}_y(k_x, y). \end{cases} \tag{29}$$

where the sum is numerically performed in practice for $0 < n < N_{max}$. In the results reported here, we chose $N_{max} = 400$.

2.2.2. *Determination of the remaining solution of the homogeneous equation.* The remaining part of the solution $\bar{v}_x = v_x - v_x^{\rm per}$ and $\bar{v}_y = v_y - v_y^{\rm per}$ verifies the homogeneous equation

$$\eta\Delta\bar{v}_x + \eta_b(\partial_x^2\bar{v}_x + \partial_y\partial_x\bar{v}_y) = 0, \tag{30}$$

$$\eta\Delta\bar{v}_y + \eta_b(\partial_y^2\bar{v}_y + \partial_y\partial_x\bar{v}_x) = 0. \tag{31}$$

We find that these equations are satisfied using the following form for the undetermined functions in Eq. 22:

$$\alpha_n(y) = (A_1^n + A_2^n y)e^{-\frac{n\pi y}{2L}} + (A_3^n + A_4^n y)e^{\frac{n\pi y}{2L}} \tag{32}$$

$$\gamma_n(y) = (a_n A_2^n + A_1^n + A_2^n y)e^{-\frac{n\pi y}{2L}} + (a_n A_4^n - A_3^n - A_4^n y)e^{\frac{n\pi y}{2L}} \tag{33}$$

$$\beta_n(x) = (a_n D_2^n + D_1^n + D_2^n x)e^{-\frac{n\pi x}{2L}} + (a_n D_4^n - D_3^n - D_4^n x)e^{\frac{n\pi x}{2L}} \tag{34}$$

$$\delta_n(x) = (D_1^n + D_2^n x)e^{-\frac{n\pi x}{2L}} + (D_3^n + D_4^n x)e^{\frac{n\pi x}{2L}} \tag{35}$$

$$\beta_0(x) = D_1^0 + D_2^0 x \tag{36}$$

$$\gamma_0(y) = A_1^0 + A_2^0 y. \tag{37}$$

with $a_n = (2L(2\eta + \eta_b)/(n\pi\eta_b))$.

We then impose the boundary conditions for $\bar{v}_x, \bar{v}_y$:

$$\bar{v}_x(L,y) = \bar{v}_x^{R,\exp}(y) = v_x^{R,\exp}(y) - v_x^{\rm per}(L,y) \tag{38}$$

$$\bar{v}_y(L,y) = \bar{v}_y^{R,\exp}(y) = v_y^{R,\exp}(y) - v_y^{\rm per}(L,y) \tag{39}$$

$$\bar{v}_x(-L,y) = \bar{v}_x^{L,\exp}(y) = v_x^{L,\exp}(y) - v_x^{\rm per}(-L,y) \tag{40}$$

$$\bar{v}_y(-L,y) = \bar{v}_y^{L,\exp}(y) = v_y^{L,\exp}(y) - v_y^{\rm per}(-L,y) \tag{41}$$

$$\bar{v}_x(x,L) = \bar{v}_x^{U,\exp}(x) = v_x^{U,\exp}(x) \tag{42}$$

$$\bar{v}_y(x,L) = \bar{v}_y^{U,\exp}(x) = v_y^{U,\exp}(x) \tag{43}$$

$$\bar{v}_x(x,-L) = \bar{v}_x^{D,\exp}(x) = v_x^{D,\exp}(x) \tag{44}$$

$$\bar{v}_y(x,-L) = \bar{v}_y^{D,\exp}(x) = v_y^{D,\exp}(x). \tag{45}$$

A set of equations is then obtained by projecting the boundary conditions in Fourier sine and cosine series for $n > 0$ (Einstein convention is used for repetition of indices):

$$\bar{v}_n^{x,R} = \Big(D_1^n + (a_n + L)D_2^n\Big)e^{-\frac{n\pi}{2}} + \Big(-D_3^n + (a_n - L)D_4^n\Big)e^{\frac{n\pi}{2}} \tag{46}$$

$$\bar{v}_n^{x,L} = \Big(D_1^n + (a_n - L)D_2^n\Big)e^{\frac{n\pi}{2}} + \Big(-D_3^n + (a_n + L)D_4^n\Big)e^{-\frac{n\pi}{2}} \tag{47}$$

$$\bar{v}_n^{y,U} = \Big(A_1^n + (a_n + L)A_2^n\Big)e^{-\frac{n\pi}{2}} + \Big(-A_3^n + (a_n - L)A_4^n\Big)e^{\frac{n\pi}{2}} \tag{48}$$

$$\bar{v}_n^{y,D} = \Big(A_1^n + (a_n - L)A_2^n\Big)e^{\frac{n\pi}{2}} + \Big(-A_3^n + (a_n + L)A_4^n\Big)e^{-\frac{n\pi}{2}} \tag{49}$$

$$\begin{aligned}\bar{v}_m^{y,R} &= \Big(a_n A_2^n + A_1^n\Big)d_{nm} + A_2^n e_{nm} + \Big(a_n A_4^n - A_3^n\Big)f_{nm} - A_4^n g_{nm} + \\ &\quad + A_2^0 b_m + \Big[\Big(D_1^n + D_2^n L\Big)e^{-\frac{n\pi}{2}} + \Big(D_3^n + D_4^n L\Big)e^{\frac{n\pi}{2}}\Big]c_{nm}\end{aligned} \tag{50}$$

$$\begin{aligned}\bar{v}_m^{y,L} &= (-1)^n\Big[\Big(a_n A_2^n + A_1^n\Big)d_{nm} + A_2^n e_{nm} + \Big(a_n A_4^n - A_3^n\Big)f_{nm} - A_4^n g_{nm}\Big] + \\ &\quad + A_2^0 b_m + \Big[\Big(D_1^n - D_2^n L\Big)e^{\frac{n\pi}{2}} + \Big(D_3^n - D_4^n L\Big)e^{-\frac{n\pi}{2}}\Big]c_{nm}\end{aligned} \tag{51}$$

$$\begin{aligned}\bar{v}_m^{x,U} &= \Big[(A_1^n + A_2^n L)e^{-\frac{n\pi}{2}} + (A_3^n + A_4^n L)e^{\frac{n\pi}{2}}\Big]c_{nm} + \Big[(D_1^n + a_n D_2^n)d_{nm} + D_2^n e_{nm} + \\ &\quad (a_n D_4^n - D_3^n)f_{nm} - D_4^n g_{nm}\Big] + D_2^0 b_m\end{aligned} \tag{52}$$

$$\begin{aligned}\bar{v}_m^{x,D} &= \Big[(A_1^n - A_2^n L)e^{\frac{n\pi}{2}} + (A_3^n - A_4^n L)e^{-\frac{n\pi}{2}}\Big]c_{nm} + (-1)^n\Big[(D_1^n + a_n D_2^n)d_{nm} + D_2^n e_{nm} + \\ &\quad (a_n D_4^n - D_3^n)f_{nm} - D_4^n g_{nm}\Big] + D_2^0 b_m,\end{aligned} \tag{53}$$

and for $n = 0$:

$$\bar{v}_0^{x,R} = 2D_1^0 + 2D_2^0 L \tag{54}$$

$$\bar{v}_0^{x,L} = 2D_1^0 - 2D_2^0 L \tag{55}$$

$$\bar{v}_0^{y,U} = 2A_1^0 + 2A_2^0 L \tag{56}$$

$$\bar{v}_0^{y,D} = 2A_1^0 - 2A_2^0 L, \tag{57}$$

and the 4 additional conditions for $\bar{v}_0^{x,U}$, $\bar{v}_0^{x,D}$, $\bar{v}_0^{y,R}$, $\bar{v}_0^{y,L}$ are approximately satisfied by continuity of the velocity at the corners of the rectangle. In the equations 46 to 53, we have introduced the coefficients

$$c_{nm} = \frac{2n[-1+(-1)^{m+n}]}{\pi(n^2-m^2)} \text{ for } m \neq n, c_{nn} = 0 \quad (58)$$

$$d_{nm} = \frac{2n[-e^{-\frac{n\pi}{2}} + (-1)^m e^{\frac{n\pi}{2}}]}{(n^2+m^2)\pi} \quad (59)$$

$$e_{nm} = 2L\frac{[m^2(2-n\pi) - n^2(2+n\pi)]e^{-\frac{n\pi}{2}} - (-1)^m[n^2(n\pi-2) + m^2(2+n\pi)]e^{\frac{n\pi}{2}}}{[(n^2+m^2)\pi]^2} \quad (60)$$

$$f_{nm} = \frac{2n[(-1)^{(1+m)}e^{-\frac{n\pi}{2}} + e^{\frac{n\pi}{2}}]}{(m^2+n^2)\pi} \quad (61)$$

$$g_{nm} = 2L\frac{[n^2(n\pi-2) + m^2(2+n\pi)]e^{\frac{n\pi}{2}} + (-1)^m[m^2(n\pi-2) + n^2(2+n\pi)]e^{-\frac{n\pi}{2}}}{[(n^2+m^2)\pi]^2} \quad (62)$$

$$b_n = \frac{4L[1-(-1)^n]}{(n\pi)^2}, \quad (63)$$

and the Fourier coefficients of the boundary conditions

$$\bar{v_n}^{x,R/L} = \frac{1}{L}\int_{-L}^{L} dy \bar{v}_x^{R/L,\exp}(y) \cos\frac{n\pi(y-L)}{2L} \quad (64)$$

$$\bar{v_n}^{y,R/L} = \frac{1}{L}\int_{-L}^{L} dy \bar{v}_y^{R/L,\exp}(y) \cos\frac{n\pi(y-L)}{2L} \quad (65)$$

$$\bar{v_n}^{x,U/D} = \frac{1}{L}\int_{-L}^{L} dx \bar{v}_x^{U/D,\exp}(x) \cos\frac{n\pi(x-L)}{2L} \quad (66)$$

$$\bar{v_n}^{y,U/D} = \frac{1}{L}\int_{-L}^{L} dx \bar{v}_y^{U/D,\exp}(x) \cos\frac{n\pi(x-L)}{2L} \quad (67)$$

To obtain the unknown constants A^n, D^n the system of linear equations 46 to 57 is solved numerically for $0 \leqslant n \leqslant N$. In practice we choose $N = 100$.

2.3. **Comparison to two-dimensional experimental flow profiles.** To compare theoretical flow profiles with experimentally measured velocity profiles, we assumed $\eta/\eta_b = 1$. We then adjusted the normalised force exerted by ppl cells, f/η_b. We found that values of f/η_b reported in Table 1 accounted well for 2D experimental profiles obtained in wild-type and in *slb* mutants. In *slb* mutants, the force exerted by ppl was decreased by 20%, in accordance to the decrease in relative velocity between ppl cells and neurectoderm (Table 1).

As in section 1.2, we then can estimate the ratio η_b/ξ from the value of f/η_b and the value of f/ξ. A characteristic hydrodynamic length can be defined from this ratio by:

$$l_2 = \sqrt{\eta_b/\xi}. \tag{68}$$

Note that although we expect l_1 and l_2 to have similar order of magnitude, these two lengths are not equal in general as l_2 is defined from the 2D bulk viscosity of the fluid, while l_1 is defined from an effective one-dimensional viscosity. We report in Table 1 a value of $l_2 \simeq 133\mu$m.

To estimate a corresponding physical value of friction against the ppl cells, one would need an estimate of the neurectoderm viscosity η_b. Since we are not aware of direct measurements, we estimate it from $\eta_b \sim \eta_{3D} h$ with η_{3D} the tissue 3D shear viscosity and h the width of the neurectoderm. Using measurements of zebrafish deep-cell viscosities $\eta_{3D} \simeq 10^3$Pa.s [1] and $h \simeq 20\mu$m, we arrive at $\eta_b \simeq 2 \cdot 10^4 Pa.s.\mu$m and $\xi \simeq 1$Pa.s.μm^{-1}.

3. Determination of geometrical parameters

3.1. **Determination of geometrical parameters.** We discuss here the determination of geometrical parameters of the theoretical description. Geometrical parameters were measured at 70% epiboly.

- The initial position of the leading edge of ppl cells relative the margin, along the dorsal midline of the embryo, was measured in each experiment. In addition, ppl cells velocity was acquired over time by cell tracking. By integrating the ppl cell AV velocity over one hour time frame, we calculated the overall displacement of ppl cells over this period. By using their initial position, we were able to pinpoint the location of the leading edge at the end of the considered time frame. From this average position we then extracted the value of y^{ppl}.
- The length L^E was set according to experimental measurements of the distance from the animal pole to the EVL boundary measured at 70% epiboly. Its value in each experiment is given in Table 2.
- The lengths L_x^{ppl} and L_y^{ppl} are obtained from measurements of the size of the domain covered by ppl cells and are reported in Table 2.

Exp	R (μm)	L^E (μm)	$L_x^{\rm ppl}$ (μm)	$L_y^{\rm ppl}$ (μm)	$y^{\rm ppl}$ (μm)	L (μm)
wt	350	670	50	90	320	175
slb	350	590	50	90	373	175

Table 2. Values of the geometrical parameters used for the plots in Fig.5. Lengths are reported at 70% epiboly.

- We also report in Table 2 the size of the measurement domain L used for comparison between theory and experiment.

References

[1] H. Morita, S. Grigolon, M. Bock, G. S. Krens, G. Salbreux & C.-P. Heisenberg, Developmental Cell 40, no. 4 (2017): 354-366.