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Vertical ground reaction forces on rigid and vibrating surfaces for vibration serviceability assessment of structures

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Abstract

Lightweight structures are sensitive to dynamic force generated by human walking and consequently can exhibit excessive vibration responses. The imparted forces, known as ground reaction forces (GRFs), are a key input in the vibration serviceability assessment of footbridges. Most GRF measurements have been conducted on rigid surfaces such as instrumented treadmills and force plates mounted on strong floors. However, it is thought that the vibrating surface of a footbridge might affect the imparted human force. This paper introduces a unique laboratory experimental setup to investigate vertical GRFs on both rigid surface (strong floor) and a higher frequency flexible surface (footbridge). 810 walking trials were performed by 18 test subjects walking at different pacing frequencies. For each trial, test subjects travelled a circuit of a vibrating footbridge surface followed by a rigid surface. A novel data collection setup was adopted to record the vertical component of GRFs, and the footbridge vibration response during each trial. Frequency-domain analysis of both single-step and continuous GRFs was then performed. The results show that the footbridge vibration affects GRFs, and changes GRF magnitudes for harmonics in resonance with the footbridge vibration (up to around 30% reduction in the dynamic load factor of the third harmonic). This

26 finding, and the measured GRFs, can be used for more accurate vibration serviceability
27 assessments of existing and new footbridges.

28 **Keywords:** Footbridges; Vibration; Human; GFRP; Ground reaction forces; Dynamic load
29 factors.

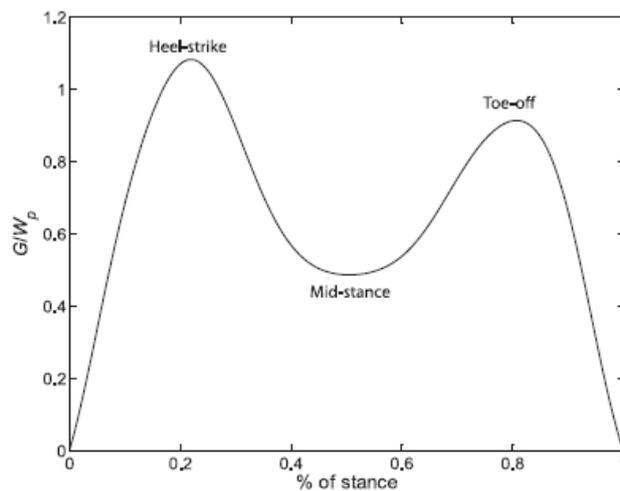
30 **1. Introduction**

31 **1.1. Background**

32 Due to their increasingly slender nature, many modern structures are prone to excitation from
33 human activity. Human activities such as walking, running, jumping, and bouncing, can
34 cause uncomfortable vibrations, potentially leading to reduced usage of the facility. Among
35 these activities, walking is a key consideration for footbridge vibration. For low-frequency
36 structures having one or more natural frequencies within range of first harmonic of walking
37 force (1.6–2.4 Hz), walking at a pacing frequency close to the natural frequency of the
38 structure might cause a vibration response that is considered uncomfortable by bridge users.
39 The vibration response of a footbridge is generally largest if the resonance is excited by the
40 first harmonic of walking force. For structures with natural frequencies within range of higher
41 harmonics of walking force (larger than about 3.2 Hz – “higher-frequency”), the resonance
42 by the second or third forcing harmonic might also be significant, even though the force
43 amplitudes are smaller. To investigate higher-frequency vibration effects, extensive walking
44 experiments were conducted on a higher-frequency footbridge for which the first frequency is
45 in resonance with the third harmonic of walking force.

46 1.2. Ground reaction forces

47 To have a good prediction of footbridge vibration response, accurate estimation of the input
 48 walking force and reliable modelling of the structure are required. The former is the focus of
 49 this study. Humans apply an approximately periodic time-dependent force with vertical,
 50 lateral, and longitudinal components, referred to as ground reaction force (GRF) [1–3]. The
 51 vertical GRF has two distinctive peaks at heelstrike and toe-off phases, and a trough at mid-
 52 stance phase for one step during walking, as shown in Fig. 1. The vertical GRF has received
 53 much attention by previous researchers [4–19].



54
 55 **Figure 1: Typical shape of a vertical GRF for a single step in walking.**

56 In the time domain, continuous walking GRFs are commonly described using a Fourier series
 57 [20–23]:

$$58 \quad G_c(t) = W_p \sum_{k=0}^r DLF_k \cos(2\pi k f_p t + \varphi_k) \quad (1)$$

59 where $W_p = m_p g$ and m_p is the pedestrian mass, g is the acceleration due to gravity; f_p is the
 60 walking pacing frequency; and DLF_k is the dynamic load factor (DLF) for the k th harmonic.
 61 The phase angle of the k th harmonic is denoted by φ_k , and r represents total number of
 62 harmonics considered. In this representation, the harmonic $k=0$ corresponds to the static
 63 pedestrian weight, and so $\varphi_0=0$ and $DLF_0=1$.

64 All GRF studies explained so far originate from GRF measurements on rigid surface. These
65 GRFs were measured by force plates and instrumented treadmills placed on rigid floors. This
66 leaves the possibility that the reported vertical GRFs could be different to those that actually
67 occur on lively footbridges, i.e. they could be affected by the vertical movement of the
68 walking surface. Only a few works in the past have considered this. Ohlsson [24] reported
69 that the spectrum of the walking force showed a drop around the natural frequency of the
70 structure where the response was significant. Baumann and Bachmann [25] similarly reported
71 DLFs of walking force, which were around 10% lower on the vibrating surface. However,
72 they measured only single footsteps by a force plate mounted on a 19m prestressed beam of
73 frequency 2.3 Hz (“low-frequency bridge”). Pimentel [26] also suggested 10% and 40%
74 reductions respectively in the first and second DLFs of the walking force by matching
75 measured vibration responses with those calculated from an updated finite element (FE)
76 model using a moving force model for two test subjects; but DLF models based on rigid
77 surface measurements were used, and no GRFs were measured on the vibrating footbridge. In
78 a unique study, Dang and Živanović [27] studied the influence of vertical vibration on
79 vertical GRFs using an instrumented treadmill on a low-frequency laboratory footbridge. The
80 results show that the footbridge vibration reduces vertical GRFs at the first harmonic of
81 resonant walking. However, only a limited number of test subjects walked on-the-spot for
82 this study, and it is limited to a footbridge with frequency at the first harmonic of the walking
83 force (“low-frequency bridge”). To conclude, the literature lacks measurements of GRFs due
84 to walking on vibrating bridge surfaces, particularly for higher-frequency footbridges for a
85 large range of test subjects. The aim of the paper is to address this gap using a novel
86 experimental set-up.

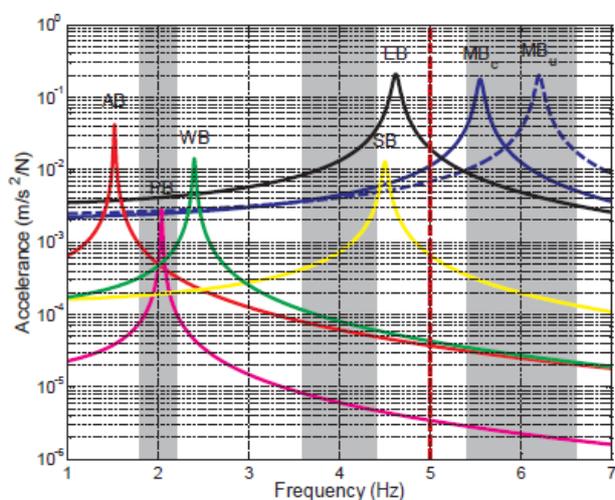
87 **1.3. Lightweight high-frequency footbridges**

88 Glass fibre reinforced polymer (GFRP) material is increasingly applied in the construction
89 industry for its desirable properties such as high strength-to-weight ratio and good durability
90 in extreme environments. These properties make GFRP well suited to modular structural
91 forms such as floors and footbridges. However, GFRP structures are lighter than equivalent
92 conventional structures, rendering them potentially more susceptible to human-induced
93 vibration due to a higher accelerance amplitude (acceleration response per unit harmonic
94 force) [28]. Therefore, a GFRP footbridge was designed and built to establish the
95 performance of such structures, and the influence of structural vibration on GRFs.

96 The vibration design rules for FRP footbridges have evolved from experience with steel and
97 concrete structural forms [29,30]. The AASHTO Design Guideline for FRP Footbridges [29]
98 states that bridges with a first natural frequency greater than 5 Hz are deemed acceptable for
99 vibration serviceability. However, this seems to neglect the altered mass-stiffness relationship
100 of FRP when compared with traditional steel and concrete structures. The altered relationship
101 affects the magnitude of the accelerance function. Živanović et al. [31] compared accelerance
102 functions of several FRP footbridges against comparable steel/concrete footbridges. The
103 accelerance functions of Monash University laboratory GFRP footbridge—uncovered and
104 covered (to be described later)—have been added to those presented by Živanović et al. [31],
105 and they are shown in Fig. 2. In addition, the frequency ranges for first three walking
106 harmonics are shown shaded, along with the 5 Hz limit [29]—shown as red dashed line in the
107 same figure.

108 Fig. 2 shows that the GFRP footbridges (AB, EB, MBu, MBc) exhibit higher accelerance
109 compared to other footbridges. Given that vibration response increases when the natural
110 frequencies lie in the harmonic ranges excitable by human normal walking, these footbridges

111 could have vibration serviceability design problems. Interestingly, the 5 Hz frequency limit,
112 developed many decades ago from experience with steel and concrete structures has been
113 adopted in AASHTO [29]. As seen in Fig. 2, the purpose-built Monash Bridge (MB) was
114 designed to meet the 5 Hz limit. The resulting bridge has a natural frequency within the range
115 excitable by the third harmonic of walking force and creates opportunity to critically evaluate
116 the suitability of the 5 Hz limit for lightweight structures.



117
118 **Figure 2: First mode acceleration frequency response functions (FRFs) of different footbridges, walking**
119 **harmonics (Shaded grey), and the 5 Hz limit. AB – Aberfeldy Footbridge (GFRP), PB – Podgoricia**
120 **Bridge (Steel), WB – Warwick Bridge (Steel-Concrete Composite), SB – Sheffield Bridge (Prestressed**
121 **Concrete), EB – EMPA Bridge (GFRP deck), MB – Monash Bridge, uncovered (GFRP), and MBc –**
122 **Monash Bridge, covered (some data from [31]).**

123 1.4. Contribution

124 Although most GRF models are based on data collected on rigid surfaces, it is the GRFs
125 imparted on the actual bridge surfaces, which are typically flexible, that are of most interest
126 for predicting the vibration response of lively structures reliably. Further, higher-frequency
127 lightweight footbridges ought to be studied, as resonance with higher harmonics of the
128 walking force might result in a large vibration response despite the bridge satisfying the 5 Hz
129 limit. To address these two goals, reliable measurement of vertical GRFs on both rigid and a
130 higher- frequency vibrating bridge surface is conducted. A higher-frequency lightweight

131 laboratory footbridge—the Monash University GFRP footbridge – is instrumented with three
132 devices to simultaneously record vertical GRFs and vibration responses. A novel
133 instrumentation set-up is used to measure full time history GRFs and single footstep GRFs on
134 both the footbridge and rigid surfaces. Finally, frequency-domain analysis of single-step
135 GRFs and continuous walking GRFs (dynamic load factors) are carried out for both surfaces
136 to infer potential effects of vibration on the harmonics of vertical walking force. The ultimate
137 goal is that these effects can then be incorporated into future vibration severability checks
138 which will not be addressed in this study.

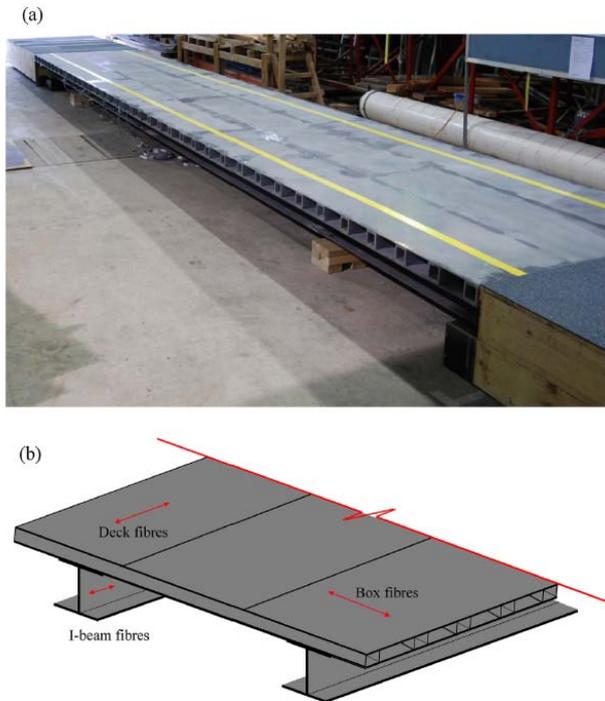
139 **2. Experimental setup**

140 **2.1. Description of Monash GFRP footbridge**

141 The deck of the Monash University GFRP footbridge is a sandwich panel made from
142 pultruded GFRP box sections placed between two GFRP flat sheets as shown in Fig. 3a. The
143 1.5m wide orthotropic deck sits on two pultruded FRP I-beam girders, spanning 8.7m
144 between supports. All components of the footbridge are joined using epoxy bonding to ensure
145 full composite action. No bolted connections or steel components were used. Bidirectional
146 fibre orientations for flat sheets, box sections, and I-beam girders were adopted to maximize
147 strength and stiffness in both transverse and longitudinal directions as shown in Fig. 3b. The
148 Monash University GFRP footbridge has a mass of 92.5 kg/m (61.6 kg/m²). This makes it
149 very lightweight compared to more traditional structures, for example, the steel-concrete
150 composite Warwick University laboratory footbridge which has a mass of 829 kg/m [27].

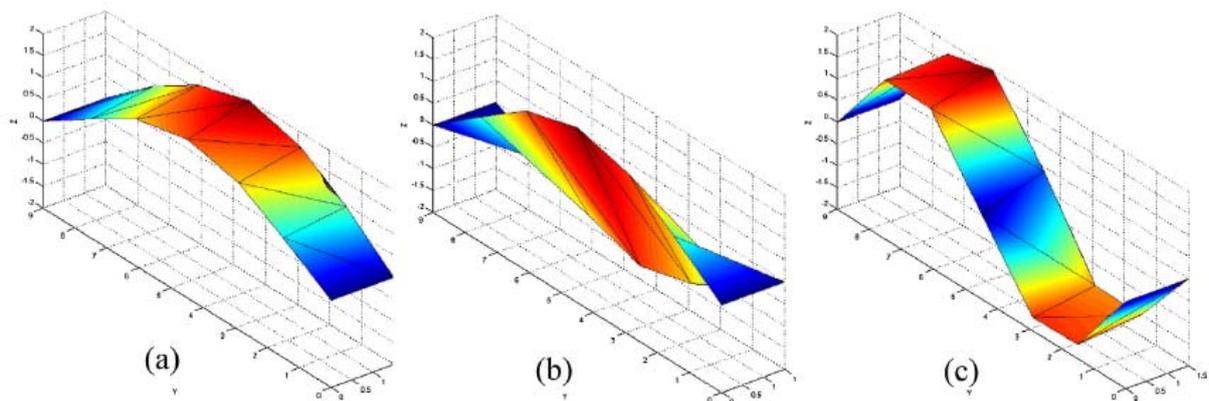
151 Fig. 4 shows the first three modes of the uncovered footbridge structure, MBu, from an
152 impact hammer test. The first mode is a flexural mode having natural frequency of 6.0 Hz
153 and damping ratio of 0.6%. The second mode is a torsional mode with frequency of 10.0 Hz

154 and damping ratio of 1.0%. The third mode is the second bending mode with frequency of
155 18.1 Hz and 0.6% damping ratio.



156

157 **Figure 3: Monash GFRP footbridge: (a) footbridge structure with end walkways, and (b) fibre direction**
158 **of different components.**



159

160 **Figure 4: Experimental modal analysis of the footbridge: (a) first bending mode, 6.0 Hz and 0.6%**
161 **damping ratio, and (b) first torsional mode, 10.0 Hz and 1.0% damping ratio; (c) second bending mode,**
162 **18.1 Hz and 0.6% damping ratio.**

163 **2.2. Experiment setup for the bridge surface**

164 Measurements of GRFs and structural vibration responses are key elements to human-
165 induced vibration studies. A unique experimental setup was designed to measure GRFs and
166 bridge acceleration. The uniqueness of the study is in measuring GRFs using three
167 independent measurement approaches: a force plate, load cells at the supports, and a state-of-
168 the-art in-shoe plantar pressure recording system (see Fig. 5).

169 A 400 wide×600 long×75mm high BERTEC FP4060-07 force plate, was placed on the
170 footbridge surface at the mid-span, 200mm off the bridge centreline, towards the left edge,
171 where the force plate is highly likely hit by test subjects' foot. Such force plates are
172 commonly used for gait analysis. They consist of force transducers that measure six force
173 components: three orthogonal forces and the moments about the three axes [32]. The force
174 plate mass, natural frequency, maximum vertical load capacity, and resolution are 38 kg, 340
175 Hz, 5 kN, and±0.5 N, respectively [32].

176 Four C10 HBM load cells were placed in the supports at the four ends of the GFRP I-beams.
177 They are capable of measuring both tensile and compressive forces up to 25 kN with
178 accuracy class of 0.04% (e.g. maximum of load cell deviations specified as percentage) and
179 have a resonant frequency higher than 5.8 kHz [33]. In the bridge walking experiments, the
180 measured reactions in the supports are used to determine the total vertical force and its
181 instantaneous location on the footbridge.

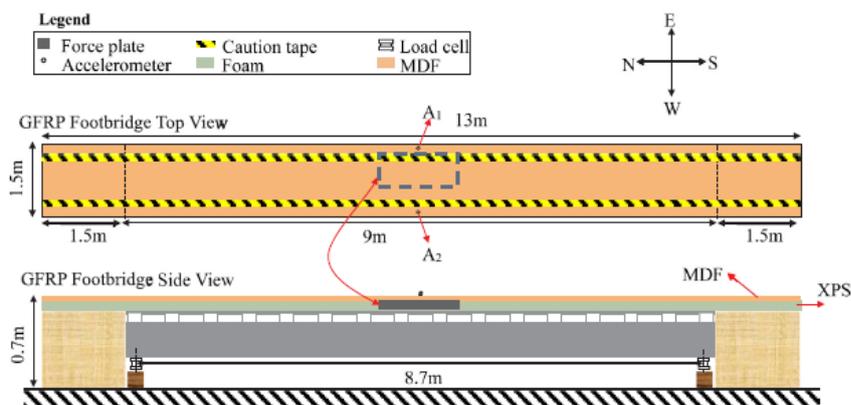
182 A state-of-the-art in-shoe pressure measurement system, the Tekscan F-scan, was used to
183 measure GRFs on both bridge and rigid surfaces [34]. These sensors consist of a grid of
184 capacitors, and each sensor measures the plantar pressure on an area of about 15mm² [34].
185 Tekscan pressure sensors are used across multiple industries such as medicine, dentistry, and

186 biomechanical research [34–37], for the measurement of contact forces, pressure distribution,
187 and centre of pressure. For walking, the plantar pressure force gives a reliable measurement
188 of the vertical walking force [38,39]. Their accuracy depends on factors such as the
189 calibration method, contact area and contact time with the sensors [40,41].

190 To measure the vibration response of the footbridge, two DYTRAN 3191A1 accelerometers
191 of nominal sensitivity of 10 V/g were placed at the mid-span on each sides of the bridge deck
192 (A1 and A2 in Fig. 5). They have capability to measure vibration in the frequency range of
193 0.08–1000Hz, with maximum acceleration of 0.5g, and have a resonant frequency above
194 8kHz.

195 Due to the additional 75mm height of the force plate on top of the structure deck, the GFRP
196 footbridge structure was covered with additional materials to provide a flush walking
197 surface—the covered footbridge, MBc, of Fig. 2. These materials were carefully selected to
198 provide a stiff walking surface while having little effect on the structure dynamic properties.
199 Consequently, 600 wide×750 long×75mm high Styroboard XPS 250 extruded polystyrene
200 sheets (a stiff foam-like material) were used (Fig. 5). These blocks have nominal density of
201 35 kg/m³ and breaking compressive strength of 375 kPa, light and stiff enough for walking
202 purposes. The blocks are not adhered to each other or the bridge, ensuring minimal influence
203 on the structure behaviour. Finally, 3mm medium-density fibreboard (MDFs) was used to
204 finish the walking surface, providing test subjects with a homogenous walking surface across
205 both the footbridge and approach lengths (see Fig. 5). The MDF was placed in 1×1.5m sheets
206 and not adhered to each other or the XPS, so as not to contribute to the longitudinal bending
207 stiffness of the footbridge. It should be noted that this covering eliminates the potential for
208 targeting of the force plate by the test subjects, since they do not know here it is located
209 beneath the MDF. The additional materials and force plate add 106 kg to the uncovered

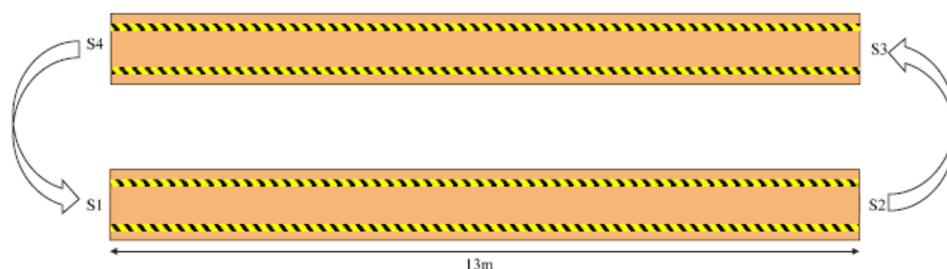
210 footbridge to give a total mass of 939 kg for the covered footbridge, MBc, hereafter referred
 211 to as the footbridge.



212
 213 **Figure 5: Experiment setup for the bridge walk (A1 and A2 are accelerometers).**

214 **2.3. Walking trials procedure**

215 Each trial consists of a bridge surface (BS) walk and a rigid surface (RS) walk, as shown in
 216 Fig. 6. Test subjects travel a complete loop to perform one trial. After being given an audio
 217 signal, each test subject starts walking from station S1 while looking straight ahead at a target
 218 sign in front, traverses the footbridge (near its middle line), and stops at station S2 (bridge
 219 surface walk). Afterwards, the test subjects are guided (down the steps) to station S3, from
 220 where they perform nominally the same test but this time over the rigid surface, and stop at
 221 station S4 (rigid surface walk). A metronome was used to provide an aural cue to assist test
 222 subjects maintain the intended pacing frequency.



223
 224 **Figure 6: Walking path during each walking trial.**

225 A wide range of 18 test subjects, 9 males and 9 females, participated in the walking trials.
 226 Their physical data are listed in Table 1. The weight of test subjects ranges from 444 N to
 227 1489 N and the height ranges from 154 cm to 190 cm. All test subjects were adults in the 20–
 228 40 years age range with no reports or indications of medical walking-related problems.

229 **Table 1: Test subjects participated in this study (M and F stand for male and female respectively).**

Test subject no.	Height (cm)	Weight (N)	Gender
1	174	865	M
2	172	718	M
3	166	654	M
4	154	444	F
5	181	678	M
6	186	862	M
7	179	717	M
8	175	970	M
9	166	522	F
10	182	1063	M
11	171	647	F
12	173	773	F
13	161	495	F
14	165	609	F
15	164	509	F
16	168	683	F
17	182	1489	F
18	190	1112	M
-	173 ± 9	767 ± 262	-

230
 231 Before each experiment, all XPS and MDF pieces for the bridge and rigid surfaces were well
 232 packed. The mid-span accelerometers were taped to the footbridge MDF surface using double
 233 sided tape, and their cables were taped to the sides of the footbridge with sufficient slack.
 234 Load cells and force plate readings were zeroed. Before the walking trials for each test
 235 subject, an APS 113 ELECTRO series electrodynamic shaker and free decay vibration tests
 236 were performed to determine the actual dynamic characteristics of the covered footbridge.
 237 This was done since different environmental temperatures and other factors could affect the
 238 dynamic properties of the footbridge.

239 A generic flat-soled canvass shoe was used by all test subjects to eliminate the influence of
240 footwear from the experiment. The trials for each test subject took around 3 h to complete.
241 Fig. 7 shows a test subject instrumented with the Tekscan equipment, consisting of Tekscan
242 sensors, ankle cuffs, data recorder, and cables. A significant effort was made to ensure that
243 the test subject felt comfortable while walking. In particular, the ankle cuffs should not be too
244 tight and the cables from the cuffs to the recorder should be loose enough to allow
245 uninhibited walking. The in-shoe sensors must be flat without any folds or creases.
246 Calibration and zeroing of sensors (explained later in more detail) were performed after each
247 set of 5 consecutive trials to eliminate the potential influence of sensor drift or degradation.
248 The test subjects completed a minimum of 15 trials for each of three pacing frequencies.



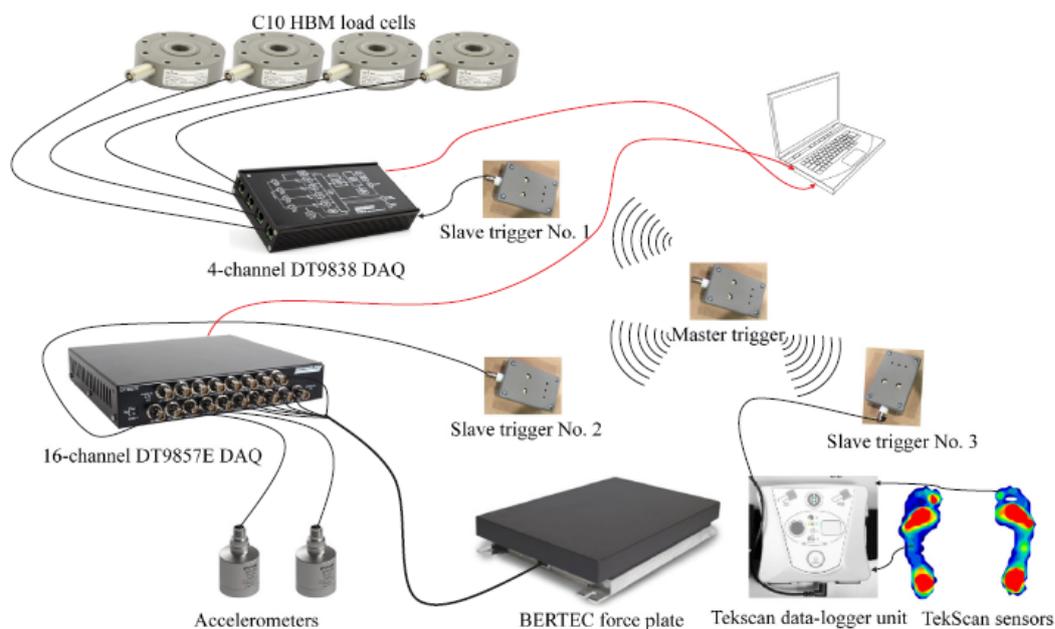
249
250 **Figure 7: Setup of the Tekscan equipment on a test subject.**

251 Before each experiment, comprehensive instructions were given to the test subject and a
252 consent form was signed. To ensure minimal influences of the laboratory environment, the
253 test procedure was followed exactly from a step-by-step workflow, so that all test subjects
254 had a consistent experience. Variations then, are natural of the test subjects, and not of

255 experimental procedure or environment insofar as is possible. Due to the involvement of
256 human subjects, the experiment was approved by the Monash University Human Research
257 Ethics Committee (Approval no. MUHREC-4455).

258 **2.4. Data collection setup for the bridge surface**

259 Fig. 8 shows the data collection setup for the bridge walk part. A 4-channel DT9838 module
260 was used to record the data from the four load cells [42]. A 16-channel DT9857E module
261 (with high resolution of 24 bits) [42] was used to collect the data from the accelerometers
262 (two channels) and force plate (six channels). Both acquisition modules were directly
263 connected to a computer to store the recorded data. A wireless data-logger unit, worn by the
264 test subjects, was used to record the data from Tekscan F-scan sensors. The data is stored in
265 the data-logger's internal micro-SD memory card for transfer to the computer at a later time
266 (done for each test subject after completing every 5 trials). All of the data was saved in a
267 format suitable for later analysis in MATLAB.



268

269 **Figure 8: Triggering, instrumentation, and data collection setup for the bridge walk.**

270 QuickDAQ and F-scan software were used to set the acquisition parameters for the DT
271 modules and Tekscan data-logger unit, respectively. Each signal was recorded for 20 s with a
272 sampling frequency of 500 Hz – far above the Nyquist frequency for the vibrations of interest.
273 This measurement period was long enough to capture the crossing event and free decay
274 vibration after the test subjects walked off the footbridge.

275 A key aspect of the experiment setup is in ensuring time synchronization between the
276 different data acquisition systems by using different triggering methods. To accomplish this,
277 a bespoke set of wireless transceivers were developed, with multiple output signal types, as
278 suited to the input trigger signal for each DAQ. A single master trigger is activated by a
279 button push, which wirelessly triggers each DAQ device simultaneously. When the pre-
280 determined measurement period finishes, the DAQs stop recording automatically. It should
281 be noted that both the DT9838 and DT9857E modules and Tekscan data-logger unit were
282 used to collect the data (Fig. 8) for the walking over the bridge (see Fig. 6). For walking over
283 the rigid surface (see Fig. 6) the Tekscan data-logger unit was used only, but it was still
284 wirelessly triggered for a consistent test subject experience.

285 **3. Preparatory measurements**

286 Preparatory experiments were conducted before the main walking trials for two reasons: (1)
287 to select suitable pacing frequencies for the main trials, and; (2) to ensure accurate
288 measurements for each of the instruments. Specifically, for (2), it was necessary to remove
289 the footbridge vibration effects from the load cells and force plate outputs.

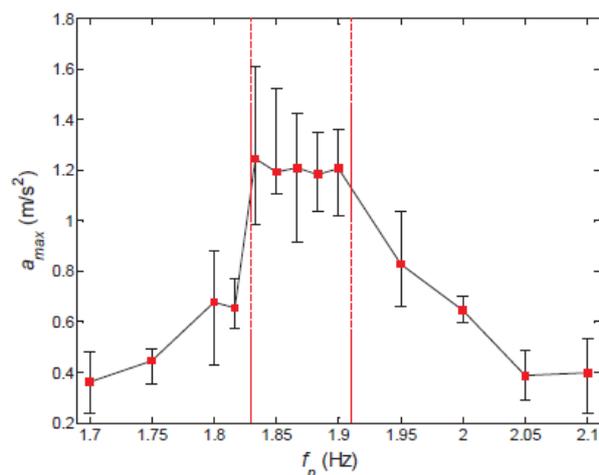
290 **3.1. Pacing frequency selection process**

291 Selection of the pacing frequencies was done empirically by examining the footbridge
292 vibration response under a wide range of pacing frequencies. The resonant pacing frequency,

293 to be targeted in the main experiments, is determined as the pacing frequency that caused the
294 highest possible vibration response. A target non-resonant pacing frequency is also
295 determined; the comparison of resonant and non-resonant responses will give insight in the
296 effects of vibration levels on GRFs. Finally, a normal (uncontrolled) pacing frequency
297 (pacing frequency at which a test subject walks naturally and unprompted by any external
298 stimulus) is used to observe the footbridge liveliness under more natural conditions.

299 For the selection of pacing frequencies, a test subject carried out five successful walking
300 trials for each pacing frequency between 1.7 Hz and 2.1 Hz with an increment of about 0.017
301 Hz (1 beat per minute of the metronome setting) around resonance and 0.05 Hz away from
302 resonance. Fig. 9 shows the variation of maximum footbridge response at the mid-span, a_{max} ,
303 with test subject pacing frequency, f_p . The vibration of the footbridge is greatest for pacing
304 frequencies between 1.83 Hz and 1.91 Hz. The target pacing frequency (whose third
305 harmonic causes the resonance of the footbridge) is then taken as 1.87 Hz. This is due to two
306 reasons: (1) the first frequency of the covered footbridge, MBC , (5.6 Hz from experimental
307 modal analysis of the covered footbridge) lies in the third harmonic range of the walking
308 force frequency (5.6 Hz/3=1.87 Hz) and (2) during the walking trial experiments, a test
309 subject is likely to walk within a small range of the target pacing frequency, and so 1.87 Hz is
310 selected as it lies within ± 0.04 Hz of the resonant range, shown by red dashed lines in Fig. 9.

311 A pacing frequency of 1.7 Hz is selected as the target non-resonant pacing frequency as, on
312 average, it gives the lowest response. Therefore, the main trials were conducted for these
313 target resonant, non-resonant, and normal pacing frequencies. For each pacing frequency, 15
314 acceptable trials were performed to allow for a reliable statistical analysis.



315

316 **Figure 9: Identification of resonant and non-resonant pacing frequency ranges using vibration responses**
 317 **from 5 trials each at 1.7–2.1 Hz pacing frequencies.**

318 **3.2. Effect of footbridge vibration on load cells output**

319 For each trial, the readings of all four load cells are summed to obtain the total force
 320 measured by the load cells, G_{lc} . Fig. 10a shows a typical G_{lc} signal (back line) for test subject
 321 no. 1 (see Table 1) and trial no. 9 at resonance. Note that this specific test subject and trial is
 322 used as an example to demonstrate the data analysis procedure and experimental results
 323 throughout the paper, and it is referred to hereafter as the “exemplar trial”. For the walk over
 324 the bridge surface, the total force induced in the load cells consists of the vertical GRFs
 325 generated by the walker, GBS, and the inertial force of the footbridge, G_I , due to its vibration
 326 (Fig. 10a):

327
$$G_{lc} = G_I + G_{BS} \quad (2)$$

328 Using frequency-domain signal processing, say, a notch filter, it is not possible to remove
 329 only the bridge inertial force from the load cells’ total force measurement because the third
 330 harmonic component of walking force would also be filtered out (see acceleration shown in
 331 Fig. 10a). Therefore, an alternative approach is developed. Theoretically, considering just the
 332 first vertical flexural mode, the total inertial force of the footbridge is:

$$G_I(t) = \int_0^L m(x)\ddot{u}(x)dx = \left[\int_0^L m(x)\phi_1(x)dx \right] \ddot{q}_1(t) = M_I \ddot{q}_1(t) \quad (3)$$

333
 334 in which $m(x)$ is the mass distribution of the covered footbridge; $\ddot{u}(x)$ is the acceleration of
 335 the footbridge at location x ; $\phi_1(x)$ is the unit normalized mode shape, and $\ddot{q}_1(t)$ is the modal
 336 acceleration. At the midspan $\phi_1(L/2) = 1$ and therefore a measured mid-span acceleration is
 337 equal to modal acceleration $\ddot{q}_1(t)$, i.e. $a_b(t) = \ddot{u}(L/2, t) = \phi_1(L/2)\ddot{q}_1(t) = \ddot{q}_1(t)$. Thus, Eq.
 338 (3) can be rearranged to determine the “inertial mass”, M_I , of the footbridge as:

$$M_I = \frac{G_I(t)}{a_b(t)} \quad (4)$$

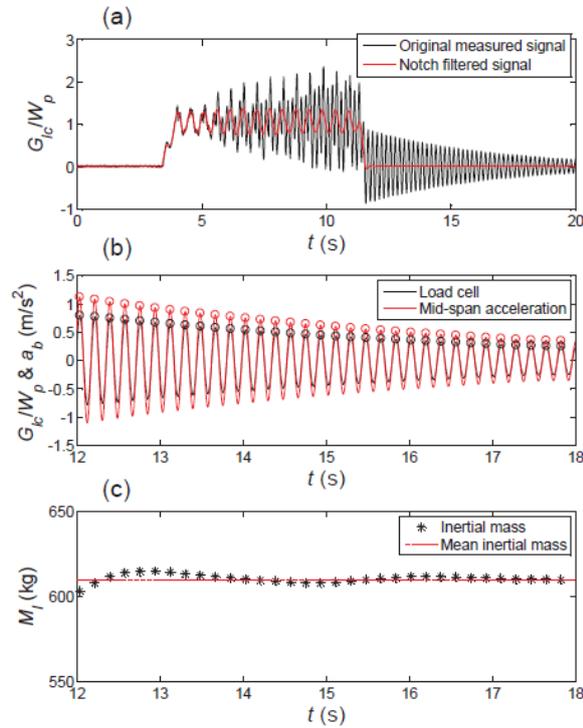
340 Based on this, the inertial mass is calculated using the free decay vibration part of the mid-
 341 span acceleration and load cells force signals. During free decay vibration, only the inertial
 342 force of the footbridge exists (the test subject has already walked off the footbridge), and thus
 343 $G_{BS} = 0$, which gives $G_{lc} = G_I$ from Eq. (4). Picking peak values of load cell force and
 344 acceleration at the mid-span (Fig. 10b) and using Eq. (4), gives a set of inertial mass
 345 measurements, shown as black stars in Fig. 10c. As seen in Fig. 10c, these inertial masses are
 346 very similar, and the mean inertial mass is found to be $M_I = 610$ kg.

347 To determine the third harmonic of the walking force (that has frequency around 5–6 Hz)
 348 from load cells, $G_{lc,3h}$, first the measured midspan acceleration of the footbridge, a_b , is filtered
 349 by a zero-phase 4th order bandpass Butterworth filter in range of 5–6 Hz to isolate the
 350 vibration of the first bending mode of the footbridge. Then, the inertial force of the footbridge
 351 during the bridge walk (while the test subject is present on the footbridge) is obtained using
 352 Eq. (4) with M_I as determined previously. The load cells force is similarly filtered, $G_{lc,f}$. This
 353 force comprises the inertial force of the footbridge, G_I , and the walking force around third
 354 harmonic, $G_{lc,3h}$ for the bridge walk part. Thus, the walking force around third harmonic,
 355 $G_{lc,3h}$ is obtained from:

356

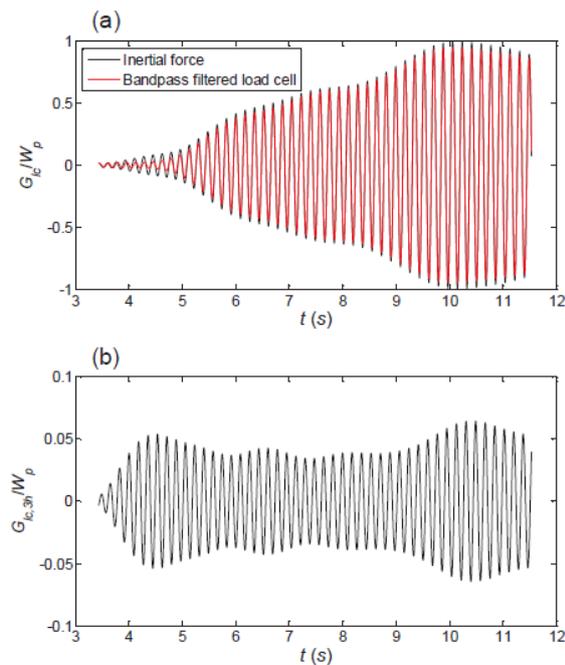
$$G_{lc,3h}(t) = G_{lc,f}(t) - G_I(t) \quad (5)$$

357 An example of the application of these steps is shown in Fig. 11 for the exemplar trial.



358

359 **Figure 10: Determination of the inertial mass of the footbridge for the exemplar test subject no 1, trial no**
 360 **9: (a) original and filtered load cells total force, illustrating that the filtered signal cannot be used, (b) load**
 361 **cells total force and mid-span acceleration for the free decay vibration part, and (c) inertial mass of the**
 362 **footbridge.**



363

364 **Figure 11: Example extraction of the walking force third harmonic from the load cells: (a) inertial force**
 365 **and filtered load cell, and (b) subtraction of inertial force from filtered load cell.**

366 3.3. Effect of footbridge vibration on force plate output

367 Fig. 12a shows the force plate reading for the exemplar trial (black line). Some low-
368 amplitude ripples in the original (raw) force plate readings, G_{fp}^o , are observed due to the
369 footbridge vibration when the test subject is not on the force plate. Similar to the load cell
370 outputs, using a filter to remove the effect of the footbridge vibration (Fig. 12a, red line)
371 would also remove the third harmonic of the force plate-measured GRFs, which is the
372 quantity of interest.

373 The inertia force component induced in the force plate due to the footbridge vibration, G_{fp}^b ,
374 is related to the moving mass of the force plate, M_{fp} , and recorded acceleration, a_{fp} , by:

$$375 \quad G_{fp}^b(t) = M_{fp} a_{fp}(t) \quad (6)$$

376 To determine this force, both M_{fp} and a_{fp} , must be measured and related to the footbridge
377 mid-span acceleration, a_b . Consequently, two accelerometers were placed, one on the force
378 plate, a_{fp} , and one on the footbridge surface beside the force plate, a_b , and the footbridge was
379 excited by the electrodynamic shaker using a swept sine signal with range of frequencies, 1–
380 100 Hz. Fig. 12b shows that the acceleration time histories for both footbridge and force plate
381 are very similar, $a_{fp}=a_b$; this means that there is little relative movement. This can be
382 expected since the force plate natural frequency (340 Hz according to the manufacturer) is far
383 higher than the footbridge natural frequency (5.6 Hz). Therefore, since $a_{fp}=a_b$, the force plate
384 moving mass, M_{fp} , is calculated as 21.3 kg using Eq. (6) (see Fig. 12c).

385 The identified force-plate moving mass, 21.3 kg, is used to remove the force component
386 induced in the force plate due to the footbridge vibration (Fig. 12c). For each bridge walk, the
387 original force plate reading, G_{fp}^o , is used to determine the force plate reading excluding the
388 footbridge vibration effects:

$$G_{fp}(t) = G_{fp}^o(t) - M_{fp} a_b(t) \quad (7)$$

389

390 where the footbridge acceleration at the mid-span, a_b , is measured for each bridge walk.

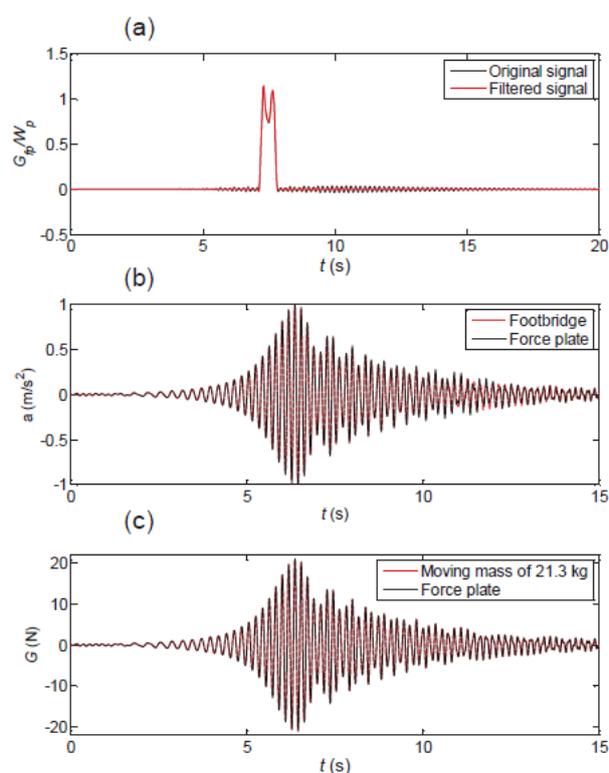
391 The accuracy of the load cells and force plate output was tested using a shaker experiment.

392 The shaker was placed on the force plate and its applied force was compared with the load

393 cells and force plate after removal of vibration effects. The results showed $\pm 3\%$ deviation

394 from the shaker's applied force, which gives confidence in the processing of the force plate

395 and load cells measurements.



396

397 **Figure 12: Experiments to remove footbridge vibration effects from force plate: (a) force plate reading**
 398 **during the exemplar trial, (b) force plate and footbridge acceleration in the shaker test, and (c) force plate**
 399 **moving mass calculation.**

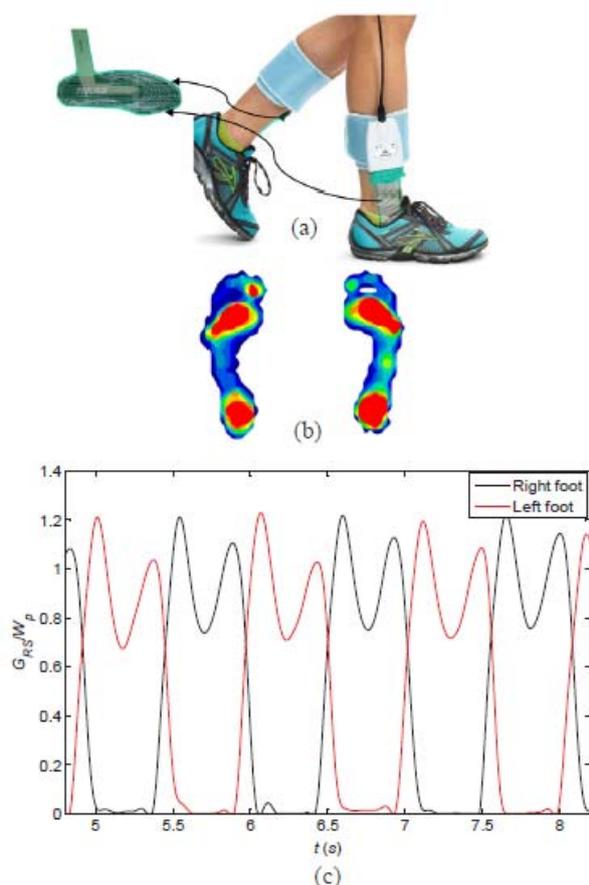
400 3.4. Tekscan F-scan force

401 To measure vertical walking force on both rigid and bridge surfaces during each walking trial

402 experiment, the Tekscan F-scan in-shoe pressure sensors [36,37] were used in this study. In

403 contrast to force plate and instrumented treadmill studies, where test subjects walk on-the-

404 spot on a rigid (vast majority – see Section 1.2) or bridge surface (e.g. [25,27]), vertical GRFs
405 during each trial on both surfaces were measured. These pressure sensors provide force-time
406 histories for each foot, allowing detailed gait analysis. Tekscan F-scan in-shoe sensors,
407 pressure distribution, and rigid surface force signals for the left and right feet for the
408 exemplar trial are shown in Fig. 13.

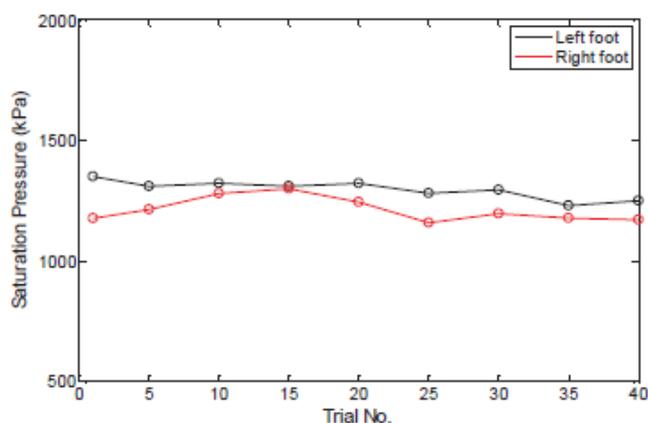


409
410 **Figure 13: Tekscan F-scan in-shoe sensors: (a) instrument, (b) example output pressure distribution, and**
411 **(c) calibrated and zeroed integrated force signals of left and right feet for the exemplary test subject on**
412 **the bridge surface (images (a) and (b) taken from [34]).**

413 The Tekscan in-shoe sensors comprise 960 individual pressure sensing capacitor cells,
414 referred to as sensels. The sensels are arranged in rows and columns on each sensor. The 8-bit
415 output of each sensel is divided into $2^8=256$ increments, and displayed as a value, (e.g. Raw
416 Sum) in the range of 0 to 255 by the associated F-scan software. The left and right feet force
417 are shown as raw sum on F-scan software. When all sensors reach a raw count of 255, the
418 corresponding pressure is termed the saturation pressure. The sensor outputs are calibrated to

419 engineering measurement units. Obviously, proper calibration of the sensors is critical to
420 obtaining accurate force readings. It is also necessary to zero the sensor output. Indeed, when
421 one foot is supporting the body weight during walking, the other foot is up in the air and its
422 force reading should be zero. However, because the foot sensors are pre-tensioned to the sole
423 of the foot by shoe-lacing, the output of sensors is not necessarily zero when the foot is not
424 touching the ground. Hence, it is necessary to zero the force output for each trial during a leg
425 swing phase of walking (Fig. 13).

426 Due to degradation of the sensors, drift of the sensors output can occur over time.
427 Additionally, the sensors can become damaged so that rows or columns of the ‘sensels’ no
428 longer export forces. Saturation pressure (described above) is closely related to the
429 calibration factor. Therefore, if some sensors become damaged during walking, the saturation
430 pressure will change, and so this was tracked throughout the trials. A step calibration, which
431 uses the test subject’s weight to adjust the calibration factor was used to convert raw sum
432 values into force measurement unit for each set of 5 consecutive trials. Fig. 14 shows a
433 sample of saturation pressure record for the exemplar test subject. It can be seen that the
434 accuracy of trials is reliable because the saturation pressures over 40 trials (a period of about
435 3 h) remain consistent.



436

437 **Figure 14: Little sensor degradation evidenced by almost constant saturation pressure across all trials for**
438 **the exemplar test subject.**

439 Due to the mentioned error involved in the Tekscan, and also the high accuracy of the force
440 plate and load cells, the force plate and load cells are taken as the benchmark to check the
441 reliability of the Tekscan results.

442 **4. Main experimental results**

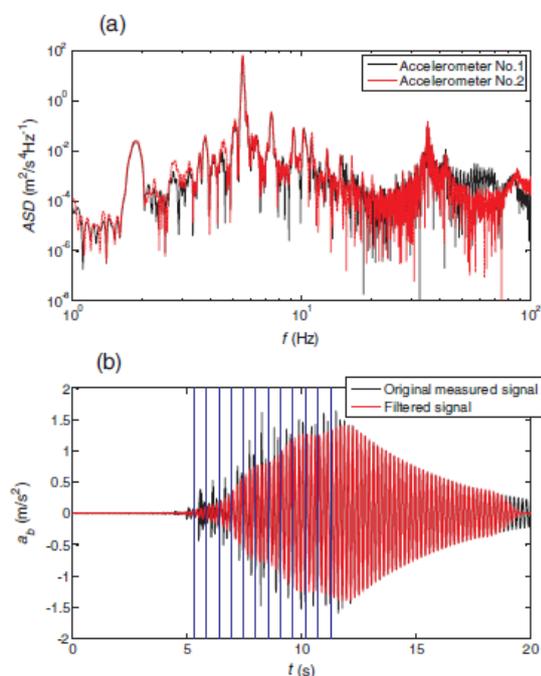
443 **4.1. Measured vibration response**

444 Fig. 15a shows auto-spectral densities (ASDs) of the two accelerometers for the exemplar
445 trial at resonance. Both ASDs can be seen to have high amplitudes at the first bending mode
446 frequency of the footbridge (5.6 Hz) and at least two order of magnitudes lower amplitudes at
447 other frequencies. The ASDs show that most of the footbridge vibration energy is distributed
448 in the 5–6 Hz frequency range and originates from the first bending mode. They also indicate
449 that there is little contribution from the first torsional mode since the magnitude of the ASDs
450 are very close to zero at its frequency (around 9–10 Hz – see Fig. 15a). Thus, the mean of the
451 two acceleration measurements is taken as the bridge vibration response at the mid-span. The
452 frequency components of the response outside range of 5–6 Hz are removed for all trials
453 using a zero-phase 4th order band-pass Butterworth filter (Fig. 15b). Zero-phase filtering
454 avoids any time shift in the filtered signal.

455 High-frequency components are observed in the original measured acceleration signal, and it
456 could be hypothesized that these come from the heel strike impulses of the pedestrian.
457 However, the occurrence of heel strikes (as identified using TekScan) for the exemplar text
458 subject are indicated as blue dashed lines in Fig. 15b, and do not coincide with the significant
459 spikes in the signal. Thus, these high-frequency components are more likely related to other
460 noise sources on the footbridge, such as the movements of the MDF boards. Humans are
461 more sensitive to low-frequency vibrations [43], and consequently the footbridge vibration
462 response outside of its first bending mode frequency range is filtered out in this work.

463 The considered footbridge response metric is the maximum value of the footbridge vibration
464 response, a_{\max} . Maximum 1-s root-mean-square (RMS) could be used instead, but is directly
465 proportional to the peak acceleration over a few cycles of vibration, and so response ratios are
466 unaffected by the measure used. Fig. 16 shows the maximum acceleration response for all test
467 subjects and trials, against the actual pacing frequency achieved. In Fig. 16a, the red dashed
468 lines specify the previously defined boundaries for the resonant frequency range. The figure
469 shows that the test subjects followed the metronome beat well since almost all actual pacing
470 frequencies fall within their relevant range. The footbridge experiences maximum
471 accelerations up to 3.3 m/s^2 .

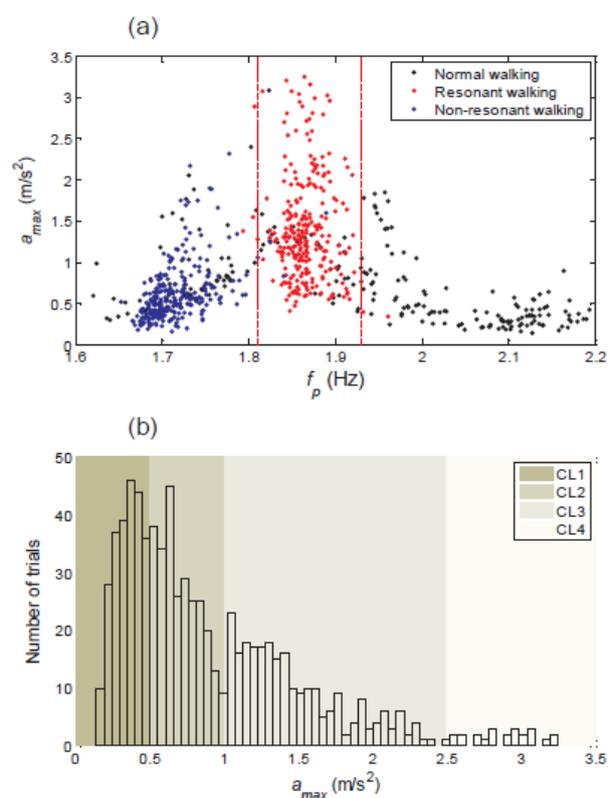
472 Acceleration levels are shown in Fig. 16b along with the limits in the Setra guideline [44],
473 reproduced in Table 2. Table 2 clearly shows that in certain cases the footbridge provides
474 “unacceptable discomfort” (CL4) to the occupants (7% of the walking trials) and, in many
475 cases a “minimum comfort” (CL3, 32% of the walking trials).



476

477 **Figure 15: For the exemplar test subject no 1, trial no 9: (a) frequency content of the vibration response,**
478 **and (b) mean acceleration response at the mid-span (the blue dashed lines show the feet location and the**
479 **tapering at the end of the filtered signal is an artefact of the filter). (For interpretation of the references to**
480 **colour in this figure legend, the reader is referred to the web version of this article.)**

481 It should be noted that even though the Setra acceleration limits were developed for
482 vibrations up to 5 Hz, they are used here to characterize vibration levels since the vibration
483 frequency of 5.6 Hz is not too far from the 5 Hz limit. In addition, the test subject's opinion
484 about the vibration levels perceived was requested following each walking trial. The test
485 subjects reported that the footbridge vibration was acceptable and occasionally affected the
486 walking style in 25% of the trials, and the vibration was strong or uncomfortable and affected
487 the walking style most of the time in 18% of the trials. This shows that the Monash
488 University GFRP footbridge is considered to be a lively structure by some people and, as
489 such, it is well-suited for studying human-induced vibration problems. In addition, it seems
490 that the 5 Hz AASHTO limit might not provide adequate guidance for lightweight higher-
491 frequency structures (Fig. 2).



492

493 **Figure 16: Footbridge vibration response for different: (a) true pacing frequencies (determined as**
494 **described in Section 5.1), and (b) perception levels according to Setra [44] (Note that the resonant and**
495 **non-resonant walking trials might not reflect natural walking situations as a metronome was used**
496 **to adjust walking frequency).**

497

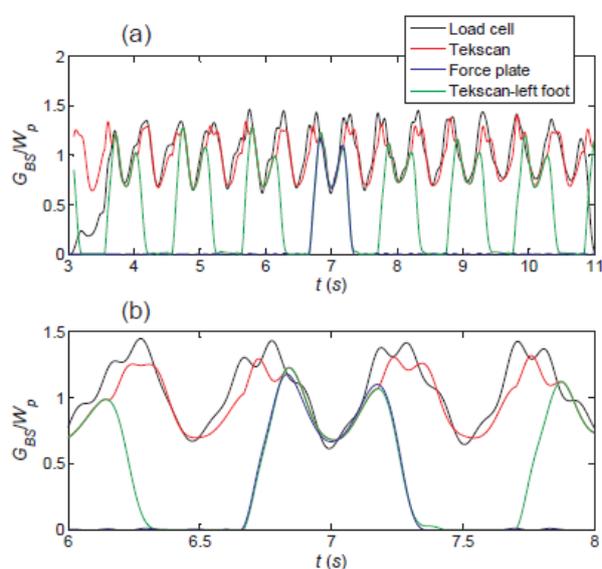
498 **Table 2: Comfort levels and acceleration ranges [44].**

Comfort level	Degree of comfort	Vertical acceleration limits (m/s ²)	Distribution of trial responses (%)
CL1	Maximum	< 0.5	29
CL2	Medium	0.5–1.0	32
CL3	Minimum	1.0–2.5	32
CL4	Unacceptable discomfort	> 2.5	7

499

500 4.2. Measured GRFs

501 Fig. 17 shows measured GRFs for the exemplar trial, from all three sets of measuring
 502 instruments. The force plate only measures one footstep due to its finite dimension on the
 503 bridge surface, while the load cells and Tekscan measure the total GRF continuously.
 504 Comparison of the three for the single step shows a good and consistent match, giving
 505 confidence in the measurements.



506

507 **Figure 17: Exemplar force measurements by the load cells, force plate, and Tekscan: (a) full bridge walk,**
 508 **and (b) zoomed around the force plate reading.**

509 Despite placing the force plate where it is highly likely to be hit by the test subjects, in some
 510 cases, the whole foot might not be on the force plate. To ensure that the force plate reading is
 511 from a full-contact footstep, three criteria were simultaneously considered numerically:

512 1. Overall shape of vertical GRFs: the GRF shape should have two distinctive peaks (heel-strike
513 and toe-off phases, G_{max1} and G_{max2}) and a trough (mid-stance phase, G_{min}), expressed as:

514
$$\frac{G_{max1}}{W_p} > 1; \frac{G_{max2}}{W_p} > 1; \frac{G_{min}}{W_p} < 1 \quad (8)$$

515 2. Step duration: the step duration from the force plate GRFs, t_{fp} , and the corresponding step
516 from the Tekscan GRFs on the bridge surface, t_{ts} , should be similar:

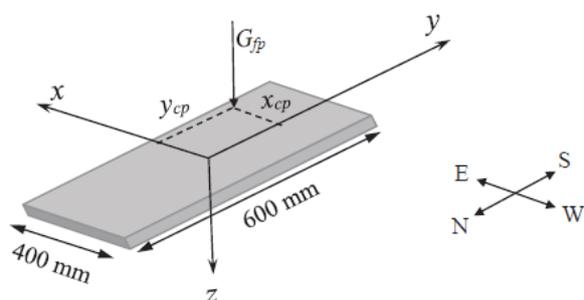
517
$$\frac{|t_{fp} - t_{ts}|}{t_{ts}} \leq 0.1 \quad (9)$$

518 3. GRFs trajectory: the centre of pressure must remain within the force plate area.

519 A footstep is not a full-contact GRF if it fails any of these criteria. The location of the centre
520 of pressure of the foot is calculated from the measured force and moment components of the
521 force plate as:

522
$$x_{cp} = \frac{-hG_x - M_y}{G_z}; \quad y_{cp} = \frac{-hG_y + M_x}{G_z} \quad (10)$$

523 where x_{cp} and y_{cp} are the coordinates of the centre of pressure relative to the coordinate axes
524 of the force plate (Fig. 18) and h is the thickness above the top surface of any material
525 covering the force plate (4 mm comprising 3mm MDF plus 1mm shim between the force
526 plate and MDF sheeting). The origin of the coordinate system is centred on the top surface of
527 the force plate (Fig. 18).



528

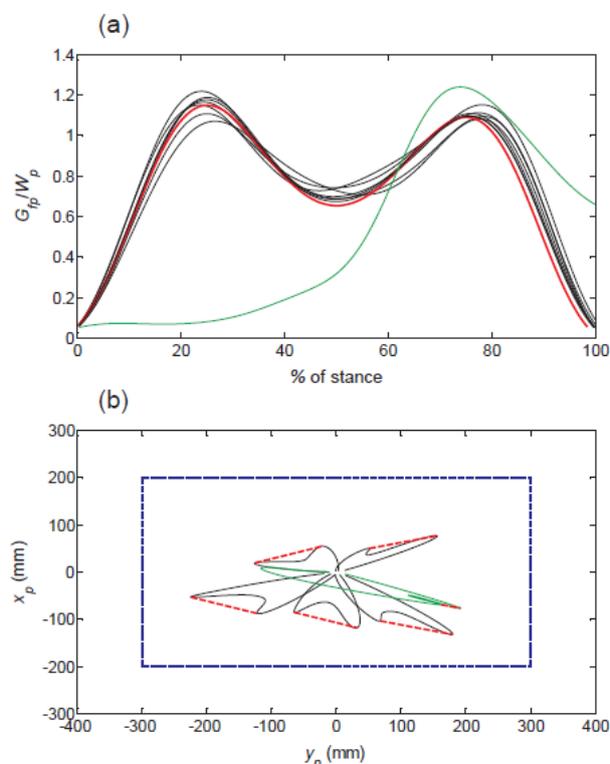
529 **Figure 18: Force plate coordinate system along with the footbridge surface.**

530 Fig. 19a shows full-step force plate GRFs and one identified incomplete GRF as a negative
531 example for the exemplar test subject at resonance. Considering criterion (1) above, the
532 complete GRFs display two distinctive peaks for heel-strike and toe-off phases and a trough,
533 mid-stance phase while the incomplete step clearly does not exhibit two peaks ($G_{\max 2}/W_p < 1$).
534 For criterion (2), the contact time of the incomplete step is shorter than the duration of the
535 same step as recorded in the GRF measured by Tekscan. Finally, for criterion (3), the GRF
536 trajectories using Eq. (10) are shown in Fig. 19b for a few complete steps. The blue dashed
537 line shows the force plate boundary. As seen, all GRF trajectories are within the force plate
538 area. Each force trajectory starts from the force plate centre and ends at the same point, and
539 the red dashed lines connect heel-strike to toe-off. However, for the incomplete GRF (shown
540 in green), despite its force trajectory being within the force plate area, the overall shape of the
541 GRF illustrates only the heel-strike phase, and the toe-off phase is outside the force plate (the
542 red dashed line is very short). Although the number of incomplete GRF steps varies between
543 different test subjects and walking frequencies, around 52% of all trials resulted in
544 measurement of full GRF steps.

545 **5. Detailed analysis of GRFs**

546 In this section, all measured single-step GRFs and continuous walking GRFs are statistically
547 analysed to examine effects of footbridge vibration on the walking force. Whenever
548 appropriate, statistical hypothesis testing is performed to quantify the statistical significance
549 of differences between variables. Two-sided independent sample Student's t-test and F-test
550 are carried out to test the statistical significance of any difference between the mean and
551 standard deviation of two sets of variables. The p -values from these tests are reported: small
552 p -values show that differences in the mean or standard deviations of the two sets of variables

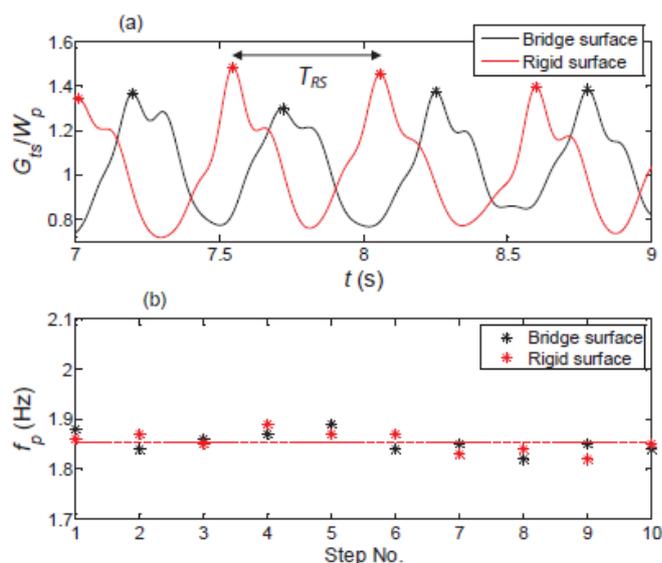
553 are statistically significant, while high p -values indicate little statistically-meaningful
554 difference.



555
556 **Figure 19: Identification of complete and incomplete steps on the force plate (the red one corresponds to**
557 **the exemplar trial no. 9): (a) sample GRFs, and (b) centre of pressure trajectory on the force plate, and its**
558 **criterion (red dashed lines connects heel-trike to toe-off and green one shows an incomplete step).**

559 5.1. Pacing frequency analysis

560 Peaks from the Tekscan total GRF are used to determine the true pacing frequencies during
561 each walking trial for the rigid and bridge surfaces (the load cells give almost identical results
562 to the Tekscan for the bridge surface)—Fig. 20a shows the normalised GRF for the exemplar
563 trial. The actual pacing periods for both surfaces, T_{BS} and T_{RS} , are determined using two
564 consecutive peaks, and from them the pacing frequencies, as shown in Fig. 20b. The
565 variability in the pacing frequencies even for just one walk for both BS and RS reflects intra-
566 subject variability. For all tests subjects and trials, an average is taken across the measured
567 pacing frequencies for the trial—the dashed lines in Fig. 20b—and is considered as the actual
568 pacing frequency for the trial.



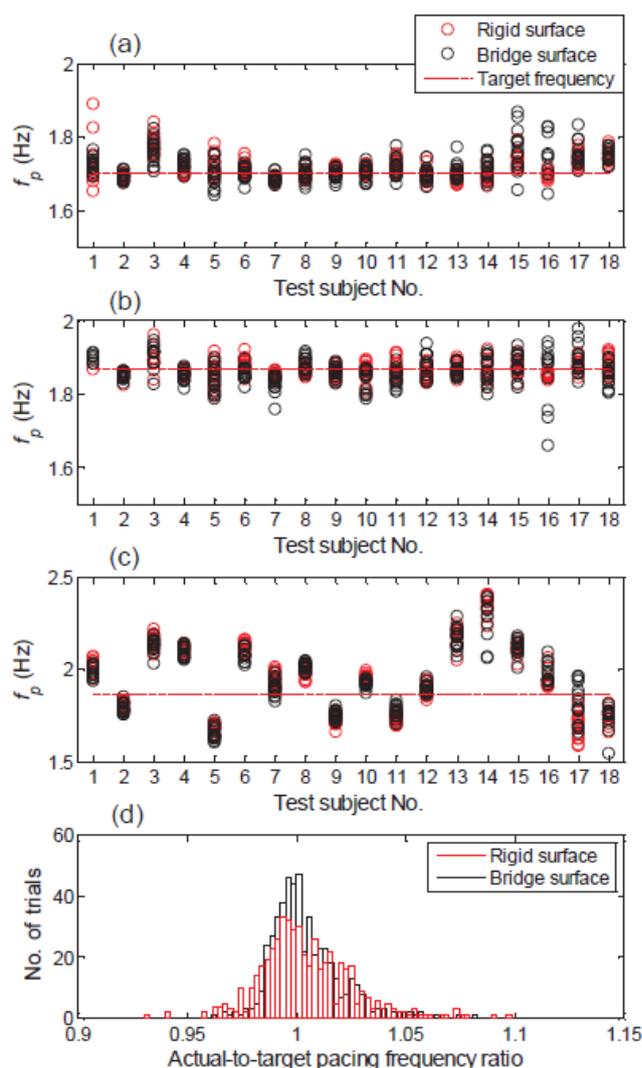
569

570 **Figure 20: For the exemplar test subject and trial: (a) actual pacing periods, and (b) actual pacing**
 571 **frequencies.**

572 Fig. 21a and 21b show actual pacing frequencies and target pacing frequencies for the rigid
 573 and bridge surfaces for all test subjects. The inter-subject variability in the data results in
 574 different level of success in matching the target frequency by different test subjects. The
 575 variability in mean actual pacing frequencies is low: the coefficient of variation, CoV (ratio
 576 of standard deviation to mean) is <0.009 for almost all test subjects. The exceptions are
 577 comparatively larger variations for test subject 1 on the rigid surface for non-resonant walk
 578 (CoV=0.038) and test subject 16 on the bridge surface for resonant walk (CoV=0.046). Small
 579 differences between the actual and target pacing frequencies is also observed typically. This
 580 means that test subjects, on average, synchronized their pacing frequencies quite well with
 581 the metronome beat (especially test subject 2). For uncontrolled normal walking pacing
 582 frequencies, the normal walking of test subjects 2, 7, 12, and 17 is close to resonance with the
 583 footbridge; for the remaining test subjects, it is out of resonance with the footbridge (see Fig.
 584 21c).

585 Fig. 21d shows histograms of actual-to-target pacing frequency ratios for the rigid and bridge
 586 surfaces. The statistical parameters of the two distributions are summarized in Table 3. As

587 seen, their mean and median are almost identical while results of the bridge surface have
 588 higher coefficients of variation.



589
 590 **Figure 21: Actual pacing frequencies for: (a) non-resonant, (b) resonant, (c) normal walking pacing**
 591 **frequencies (red dashed line shows target pacing frequency, for normal walking it shows resonance target**
 592 **pacing frequency), and (d) actual-to-target pacing frequency ratio.**

593 **Table 3: Statistical parameters of actual-to-target pacing frequency ratios.**

Surface type	Mean	Median	CoV
Rigid	1.004	1.000	0.016
Bridge	1.005	1.002	0.026

594
 595 The p -values from the rigid and bridge surface pacing frequencies are 0.98 and 0.00
 596 respectively for Student's t-test and F-test. These values show that the actual-to-target pacing
 597 frequency ratios on the bridge and rigid surfaces have no difference in their means but have a

598 statistically significant difference in their standard deviations. This difference in standard
599 deviation presumably indicates that the vibrating bridge surface makes it harder for the test
600 subjects to maintain a set pacing frequency (according to the metronome beat). The results
601 are similar to those found on a low-frequency footbridge [27] where the vibration effects on
602 the mean of pacing frequencies was small while the effects on the CoV of pacing frequencies
603 was higher.

604 **5.2. Single-step GRFs**

605 Single-step GRFs are not widely available in literature; however they are becoming of
606 interest in discreet footfall moving force models in which the footstep forces are applied at
607 the feet locations [45,46]. To inform development of single-step force models, complete
608 single footsteps identified in Section 4.2 for all test subject trials are statistically analysed. To
609 examine the footbridge vibration effects on the single-step GRFs, it is necessary to compare
610 the footsteps on the rigid and bridge surfaces. The Tekscan GRFs on the bridge surface
611 corresponding to the force plate GRFs are used (see Fig. 17). For Tekscan GRFs on the rigid
612 surface, since they are not measured simultaneously with the GRFs on the bridge surface, it is
613 not possible to find a corresponding step, and thus a representative step is randomly selected
614 from the middle third of full-trial GRFs. Hence for each trial a comparison is made between
615 randomly-selected single steps from the bridge and rigid surface measurements.

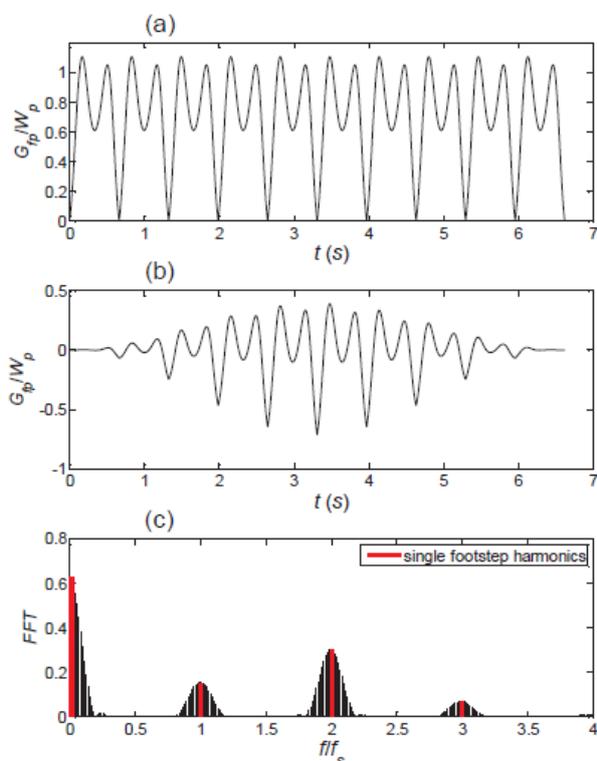
616 For time-domain analysis of single-step GRFs, the peak at heel strike, the peak at toe-off, and
617 the trough at mid-stance were considered [2]. Vibration effects of the footbridge could not be
618 clearly observed in the time-domain. Therefore, the single-step GRFs are compared in the
619 frequency domain to understand the effect of vibration on individual footstep forces.

620 For frequency-domain analysis of step GRFs, a Fourier representation of single steps is used,
621 [47]:

622

$$G_x(t) = W_p \sum_{n=0}^N A_n \cos(2\pi n f_s t + \theta_n) \quad (11)$$

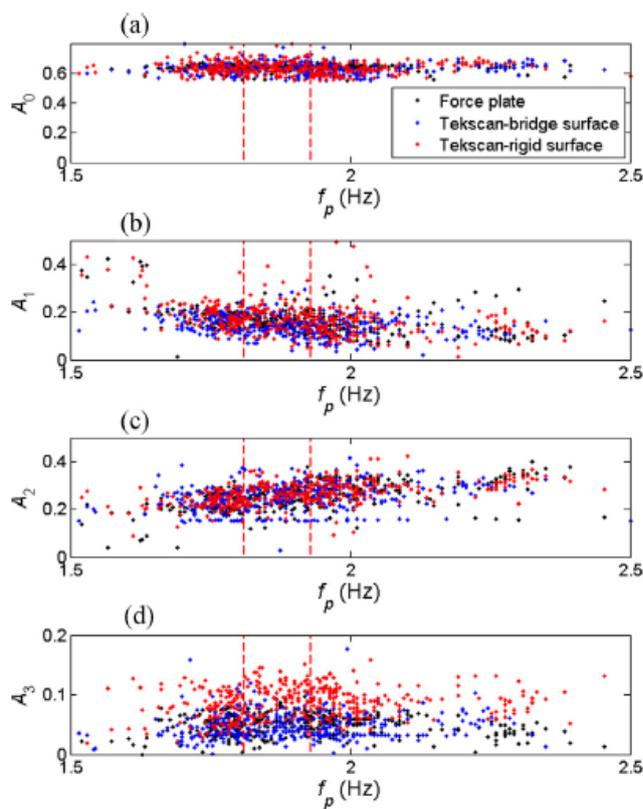
623 where A_n and θ_n are the n th harmonic and phase angle of the footstep; N is total number of
 624 harmonics considered; $f_s=1/t_s$ and t_s is the single step duration. The footstep frequency is
 625 proportional to the pacing frequency on average for all trials and test subjects, $f_s/f_p=0.82$, with
 626 the 95% confidence interval 0.82 ± 0.06 . To calculate harmonics of single-step vertical GRFs,
 627 each one is repeated 10 times to form a longer periodic signal. This periodic signal is
 628 windowed using a Hann window to suppress leakage and zero-padded to increase its
 629 frequency resolution. It is then transformed to the frequency domain using the Fast Fourier
 630 Transform (FFT), and its amplitude in the frequency domain is corrected for the side-lobe
 631 loss due to using a spectral window [48]. Fig. 22 shows the DC component (A_0) and the first
 632 three harmonics of the force plate single-step GRF for the exemplar test subject and trial.



633

634 **Figure 22: Single-step GRF harmonics: (a) repeated single-step GRF signal for the exemplary test subject,**
 635 **(b) windowed and trimmed repeated single-step GRF, and (c) Fast Fourier Transform of the trimmed**
 636 **repeated single-step GRF.**

637 The DC (constant) component and first three harmonics of all measured single-step GRFs on
638 both surfaces are shown in Fig. 23. The red dashed lines show the resonant range of the
639 footbridge. Visually, it appears the footbridge vibration reduces the third harmonic of single
640 footsteps (Fig. 23d) and that the footbridge vibration effect on other harmonics seems
641 negligible (Fig. 23a–c). This seems reasonable as the footstep frequency is proportional to
642 pacing frequency, and for the resonant walking trials, the third harmonic of the single
643 footsteps is closer to the bridge frequency compared to the other harmonics.



644

645 **Figure 23: Relationships of single footstep harmonics with pacing frequency for: (a) DC component, (b)**
646 **first harmonic, (c) second harmonic, and (d) third harmonic for all test subjects and trials.**

647 The p -values for the footsteps harmonics are calculated for all test subjects and trials and are
648 given in Table 4. Differences between the results of the load cells and Tekscan on the bridge
649 surface shows any inaccuracy of Tekscan, while differences between the results of Tekscan

650 on rigid and bridge surfaces is a relative indication of vibration effects on footstep harmonics.
 651 Note that it is assumed that any error in Tekscan measurements affects the results on both
 652 rigid and bridge surfaces in the same manner (especially in a statistical sense). This implies
 653 that the differences between the measurements on the two surfaces are solely due to influence
 654 of the surface itself. For bridge surface steps only, Table 4 shows that the Tekscan and force
 655 plate measured GRFs are consistent and do not exhibit statistically significant differences. In
 656 contrast, for the Tekscan results across the rigid and bridge surfaces, the results show an
 657 statistically-significant difference for the third harmonics of single footsteps. In this case the
 658 p -values observed are near zero ($\approx 10^{-7}$).

659 For representing vertical walking force in a single step it is useful to report the average
 660 magnitudes of each harmonic found, 0th–3rd, across all tests. As a proportion of body mass,
 661 for rigid surface walking these are 0.64, 0.18, 0.26, and 0.087 respectively, while for the
 662 bridge surface walk they are 0.63, 0.16, 0.25, and 0.047. Consequently, the mean reduction in
 663 third harmonic magnitude is about 46%.

664 **Table 4: Hypothesis testing results (p-values) for single footstep harmonics for all trials and test subjects.**
 665 **(Recall the t-test examines differences in means, while the F-test examines differences in standard**
 666 **deviations. Values near zero indicate very high statistically-significant differences).**

Harmonic no.	Bridge Surface only: Force plate and Tekscan		Tekscan only: Rigid and Bridge Surfaces	
	t-test	F-test	t-test	F-test
0	0.49	0.22	0.21	0.11
1	0.21	0.15	0.15	0.19
2	0.47	0.26	0.21	0.53
3	0.59	0.19	0.00	0.00

667

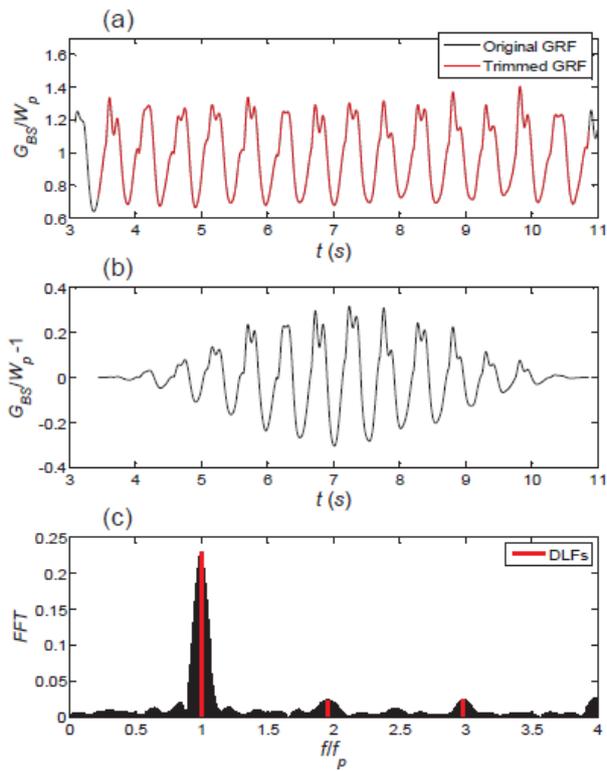
668 5.3. Continuous GRFs

669 To investigate vibration effects on continuous vertical GRFs, DLFs are selected as the metric,
 670 consistent with the literature [24–26,49]. DLFs of the first three harmonics are determined for

671 full time history force of the load cells and Tekscan on both rigid and bridge surfaces.
672 Comparison of the Tekscan and load cell differences for bridge surface walks are made to
673 assess Tekscan accuracy as before. Comparison of the Tekscan measurements between the
674 rigid and bridge surfaces are also made as before, to assess any influence of bridge vibration.

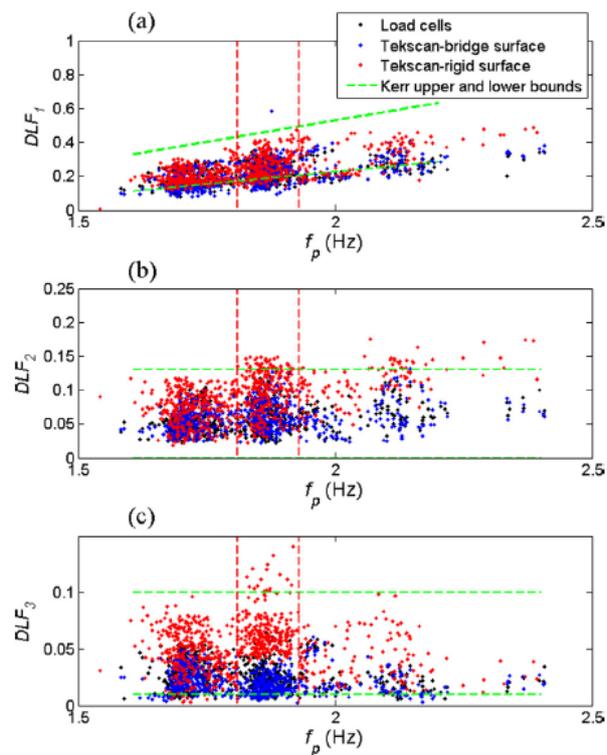
675 To calculate the DLFs from the GRF measurements, the start and end of the recorded GRF
676 signals are trimmed such that a signal consists of an even number of full steps. The DC
677 component is subtracted from the signal and it is then windowed using a Hann window to
678 suppress leakage. Similar to the single footstep analysis, the signal is then zero-padded to
679 increase its frequency resolution and transformed into the frequency domain using the FFT.
680 The signal amplitude in the frequency domain is corrected for the side-lobe loss due to using
681 a spectral window [48] as was done for the single footsteps. Fig. 24 shows the steps in
682 determining DLFs for the exemplar trial, highlighting the first three harmonics. Consistent
683 with past experiments, and as seen earlier in the intra-subject variability results, the walking
684 force is not perfectly periodic but it is a narrow band signal with some of its energy spread to
685 adjacent frequencies [17,18]. Subharmonics are also evident from Fig. 24c.

686 For each trial and surface (rigid and bridge surfaces), the first three DLFs of the continuous
687 walking GRFs are calculated and shown in Fig. 25. For comparison, Kerr's [50] upper and
688 lower bounds for each DLF are shown by green dashed lines, and the vertical red dashed
689 lines show the resonant range of the footbridge (Figs. 9 and 14a). The first DLF increases
690 with increasing pacing frequency, while the second and third DLFs do not show a discernible
691 trend as would be expected [50,51,15]. For the third harmonic of walking vertical force, the
692 bridge DLFs on the bridge are lower than those on the rigid surface. Further, there is a
693 difference between DLFs from the load cells and Tekscan measured on the bridge surface,
694 emphasizing some error in the Tekscan force measurement.



695

696 **Figure 24: GRF DLFs: (a) bridge-measured Tekscan original and trimmed GRFs for the exemplary test**
 697 **subject and trial, (b) windowed trimmed GRF, and (c) Fast Fourier Transform of the trimmed and**
 698 **windowed GRF signal in (b).**



699

700 **Figure 25: Relationships of DLFs with pacing frequency for: (a) first harmonic, (b) second harmonic, and**
 701 **(c) third harmonic for all test subjects and trials.**

702 The p -values for DLFs are calculated for all test subjects and trials and are given in Table 5.
 703 The p -values for the differences between the loads cells and Tekscan are not statistically
 704 significant but give an indication of the measurement error involved in using Tekscan. More
 705 interestingly, in the relative comparison of Tekscan results between the rigid and bridge
 706 surfaces: there are small p -values for the third DLF, indicating a statistically significant
 707 difference in both means and standard deviations. This suggests that the footbridge vibration
 708 affects the third harmonic far more than the first and second harmonics.

709 **Table 5: Hypothesis testing results (p-values) for DLFs for all trials and test subjects. (Recall the t-test**
 710 **examines differences in means, while the F-test examines differences in standard deviations).**

DLF no.	Bridge Surface only: Load cells and Tekscan		Tekscan only: Rigid and Bridge Surfaces	
	t-test	F-test	t-test	F-test
1	0.80	0.81	0.28	0.31
2	0.49	0.91	0.19	0.16
3	0.36	0.21	0.00	0.00

711
 712 To compare the effects of the vibrating footbridge for the resonant and non-resonant pacing
 713 frequencies, the p -values between the bridge and rigid surface Tekscan DLFs are obtained for
 714 all test subjects, given in Table 6. As seen from this table, for both resonant and non-resonant
 715 walking, p -values of DLF1 and DLF2 are relatively high, illustrating little statistical
 716 difference between the DLFs of rigid and bridge surfaces for both resonant and non-resonant
 717 walking. However, for DLF3, again, very small p -values result, indicating significant
 718 differences in both mean and standard deviation for both resonant and non-resonant walking.
 719 This suggests that the footbridge vibration influences the nearest harmonic of walking force
 720 for any pacing frequency. This phenomenon is explored next.

721 To analyse the DLFs in more detail, p -values of the first three DLFs are obtained for each test
 722 subject at the resonant and non-resonant pacing frequencies (see Tables 7–9). These are based

723 on the statistics of the GRFs from 15 trials at each pacing frequency for each test subject. The
 724 test subject-to-footbridge mass ratio, μ , is used to discuss the results for each test subject.
 725 Again, very small p-values are observed for the third harmonic compared to the other two
 726 harmonics for all test subjects. This is strong evidence that the effects of the footbridge
 727 vibration on the third harmonic is significant. Further, the effect roughly increases with
 728 increasing mass ratio. On the other hand, the first and second harmonics are not influenced
 729 much by vibration since their p values are high, on average.

730 **Table 6: Hypothesis testing results (p-values) for DLFs for all trials and test subjects in resonant and non-**
 731 **resonant cases. (Recall the t-test examines differences in means, while the F-test examines differences in**
 732 **standard deviations).**

DLF no.	Resonant		Non-resonant	
	t-test	F-test	t-test	F-test
1	0.28	0.14	0.55	0.05
2	0.19	0.98	0.11	0.64
3	0.00	0.00	0.00	0.01

733

734 **Table 7: The first DLFs hypothesis testing results (p-values) and increment for each test subject in**
 735 **resonant and non-resonant cases. (Recall the t-test examines differences in means, while the F-test**
 736 **examines differences in standard deviations).**

μ (%)	Resonant			Non-resonant		
	t-test	F-test	Δ_{DLF} (%)	t-test	F-test	Δ_{DLF} (%)
4.8	0.04	0.00	4.6	0.45	0.43	-0.3
5.4	0.18	0.31	4.5	0.95	0.15	-0.9
5.5	0.23	0.33	3.	0.64	0.05	3.7
5.7	0.15	0.02	5.2	0.55	0.07	2.1
6.6	0.23	0.33	4.9	0.95	0.12	3.5
7.0	0.04	0.01	3.6	0.45	0.04	4.0
7.3	0.00	0.04	6.0	0.30	0.76	2.4
7.4	0.15	0.39	3.0	0.73	0.08	1.9
7.4	0.13	0.20	5.2	0.42	0.24	2.1
7.8	0.27	0.11	5.1	0.85	0.11	2.6
7.8	0.14	0.02	6.3	0.56	0.10	2.3
8.4	0.14	0.33	4.2	0.68	0.21	1.2
9.3	0.04	0.85	3.6	0.97	0.35	0.2
9.4	0.03	0.07	3.0	0.32	0.25	-0.2
10.6	0.13	0.03	2.8	0.62	0.08	-1.2
11.6	0.12	0.02	3.3	0.43	0.05	-0.9
12.1	0.54	0.64	2.4	0.73	0.74	-1.5
16.2	0.29	0.13	2.3	0.55	0.04	-1.5

737

738 **Table 8: The second DLFs hypothesis testing results (p-values) and increment for each test subject in**
 739 **resonant and non-resonant cases. (Recall the t-test examines differences in means, while the F-test**
 740 **examines differences in standard deviations).**

μ (%)	Resonant			Non-resonant		
	t-test	F-test	Δ_{DLF} (%)	t-test	F-test	Δ_{DLF} (%)
4.8	0.19	0.00	-6.7	0.09	0.57	-3.6
5.4	0.01	0.19	-2.3	0.02	0.87	-3.7
5.5	0.17	0.72	-5.4	0.12	0.37	-4.0
5.7	0.03	0.18	-4.6	0.02	0.77	-6.7
6.6	0.10	0.95	-3.8	0.03	0.71	-5.4
7.0	0.00	0.01	-5.5	0.01	0.90	-8.1
7.3	0.11	0.00	-4.2	0.07	0.73	-5.2
7.4	0.14	0.74	-3.2	0.09	0.41	-6.4
7.4	0.12	0.00	-6.5	0.03	0.60	-5.3
7.8	0.81	0.74	-5.0	0.66	1.00	-4.3
7.8	0.02	0.00	-4.6	0.07	0.86	-3.9
8.4	0.01	0.05	-2.3	0.01	0.88	-1.8
9.3	0.45	0.06	-5.9	0.85	0.88	-2.4
9.4	0.06	0.00	-6.8	0.09	0.70	-5.2
10.6	0.01	0.00	-6.0	0.00	0.41	-6.5
11.6	0.03	0.14	-6.0	0.04	0.77	-6.9
12.1	0.10	0.30	-6.3	0.92	0.25	-6.7
16.2	0.13	0.72	-5.4	0.10	0.51	-6.4

741

742 **Table 9: The third DLFs hypothesis testing results (p-values) and increment for each test subject in**
 743 **resonant and non-resonant cases. (Recall the t-test examines differences in means, while the F-test**
 744 **examines differences in standard deviations).**

μ (%)	Resonant			Non-resonant		
	t-test	F-test	Δ_{DLF} (%)	t-test	F-test	Δ_{DLF} (%)
4.8	0.01	0.00	-24.6	0.01	0.01	-12.9
5.4	0.00	0.00	-23.8	0.00	0.01	-11.1
5.5	0.00	0.00	-21.4	0.00	0.01	-11.2
5.7	0.00	0.00	-21.3	0.01	0.00	-13.0
6.6	0.00	0.00	-25.7	0.00	0.01	-17.0
7.0	0.00	0.00	-29.6	0.00	0.00	-16.4
7.3	0.01	0.00	-30.4	0.00	0.07	-16.5
7.4	0.00	0.00	-28.7	0.00	0.02	-15.4
7.4	0.00	0.00	-28.9	0.04	0.05	-14.9
7.8	0.00	0.00	-27.5	0.00	0.30	-15.5
7.8	0.00	0.00	-26.2	0.01	0.03	-15.6
8.4	0.00	0.00	-27.7	0.00	0.00	-17.2
9.3	0.00	0.00	-28.9	0.01	0.32	-18.8
9.4	0.00	0.00	-28.5	0.02	0.03	-18.4
10.6	0.00	0.00	-25.3	0.01	0.01	-17.6
11.6	0.00	0.00	-23.6	0.01	0.00	-16.8
12.1	0.00	0.00	-28.6	0.00	0.27	-18.4
16.2	0.00	0.00	-28.8	0.00	0.01	-19.2

745

746 Tables 7–9 also present relative changes in the mean DLFs, Δ_{DLF} :

747
$$\Delta_{DLF_i}(\%) = 100 \frac{(\overline{DLF}_i^{BS} - \overline{DLF}_i^{RS})}{\overline{DLF}_i^{RS}} \quad (12)$$

748 where \overline{DLF}_i^{BS} and \overline{DLF}_i^{RS} are mean DLFs (across the 15 trials) for the i th harmonic. For
749 DLF_3 , a significant drop is seen for both resonant and non-resonant cases. Apart from this,
750 DLF_3 reductions at the resonant walking are larger than those for the non-resonant walking,
751 emphasizing high footbridge vibration effects at the resonant walking. As evident from Fig.
752 14a, the footbridge experiences high vibration response even at the non-resonant walking
753 particularly for heavy test subjects. Considering that the footbridge vibration response is
754 distributed over the range of 5–6 Hz (Section 4.1, Fig. 13a), the footbridge vibration is seen
755 to clearly affect the third harmonics of the non-resonant walking force, but not as
756 significantly as it affects the resonant walking force.

757 It is worth noting that similar reductions in DLFs close to the bridge frequency were found in
758 trials on the low-frequency Warwick footbridge [27]. A possible explanation for these
759 reductions is that similar to a stationary human [43,2], a moving human also applies an
760 interaction force to the structure, i.e., G_v , proportional to the structural acceleration [52,2].
761 With this concept, there are two components combining to give the GRF on the bridge
762 surface, G_{BS} : the rigid surface force, G_{RS} , and a vibrating surface force component, G_v :

763
$$G_{BS} = G_{RS} + G_v \quad (13)$$

764 For the higher-frequency Monash footbridge, the vibrating surface force component still
765 exists at non-resonant pacing frequencies (according to the non-resonant pacing frequency
766 results). In contrast, for the low-frequency Warwick footbridge, the vibrating surface walking
767 force is similar to that of the rigid surface (very similar DLFs) and so the vibrating surface
768 force component (G_v component in Eq. (13)) is negligible. Since other factors are accounted

769 for, this difference is most likely due to the different human-to-structure mass ratios. The
770 Monash footbridge is far lighter than the Warwick footbridge and has much higher
771 accelerance (see Fig. 2). Consequently, it seems that heavier test subjects can highly vibrate
772 the footbridge even at non-resonant pacing frequencies (Fig. 14a).

773 **6. Conclusions**

774 In this paper, a novel experimental approach is introduced to quantify the extent of human-
775 structure interaction on lightweight bridges with natural frequency above 5 Hz. A purpose-
776 built higher frequency GFRP footbridge was used for walking trials. A unique experimental
777 setup was designed to measure vertical walking forces on both rigid and flexible surfaces.
778 This setup enables measuring both single-step and continuous GRFs on both a rigid surface
779 and a vibrating bridge surface for 18 test subjects and trials. In addition, during walk over the
780 bridge, vibration of the structure is also recorded.

781 It is consistently found that the vibrating bridge surface causes a statistically significant drop
782 in the magnitude of the walking force harmonic closest to the vibration frequency. The
783 amount of the reduction depends on whether the pacing frequency is such to cause a resonant
784 or non-resonant condition between the relevant bridge frequency and walking harmonic. This
785 result is similar to some results from a study on a low-frequency bridge. The findings support
786 the hypothesis that the bridge surface vibration significantly decreases the magnitude of the
787 harmonic of walking force that is closest to the vibration frequency. Further, it is also found
788 that pacing frequencies vary more on a vibrating surface than on a rigid surface. Currently,
789 these aspects are not considered in design guidelines and could be of significance in more
790 accurately predicting vibration serviceability of lightweight structures.

791 Finally, the results of the trials conducted here show that the 5 Hz recommendation for FRP
792 bridges provided by AASHTO performs poorly. The Monash GFRP footbridge reaches
793 uncomfortable vibration levels even though the footbridge frequency is higher than 5 Hz.
794 Therefore, it is necessary to develop more suitable design criteria for FRP bridges, or indeed,
795 any lightweight bridge characterised by high magnitude of the acceleration function.

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