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The orientability of spacetime

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Abstract.

Contrary to established beliefs, spacetime may not be time-orientable. By considering an experimental test of time orientability it is shown that a failure of time-orientability of a spacetime region would be indistinguishable from a particle antiparticle annihilation event.

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1. Introduction

It is widely believed that spacetime must be both orientable and time-orientable [1]. Arguments are that there is no evidence of a lack of orientability and that a non-orientable spacetime would be incompatible with the observed violations of P (parity) and T (time reversal) invariance. These arguments are shown to be false.

This paper aims to answer the questions: *How can time-orientability be tested?* and *Is a non-time orientable spacetime consistent with observations?*. The focus of this paper is on experiments a physicist would have to do to test the orientability of a spacetime region.

The work is motivated by results suggesting that non-time orientable spacetime regions might be used to explain quantum theory [2], the existence of electric charge [3] and spin half [4]. Topological models of elementary particles within the equations and framework of general relativity require a breakdown of the causal structure to allow topology changing interactions. A non-timeorientable spacetime structure can also exhibit net electric charge from source free Maxwell equations and would naturally have the transformation properties of a spinor under rotations.

Mathematically, the orientability of a manifold is a simple and unambiguous property. A few mathematical definitions and theorems are given in the next section. This paper looks at how the mathematical definitions can be used in practice to answer questions about spacetime. Some simple experiments are postulated; they follow the mathematical definitions very closely, but the interpretation remains ambiguous.

2. Preliminary results

This paper is exclusively concerned with connected spacetimes.

Remark 2.1 *Locally spacetime is always orientable.*

Spacetime is modelled by a 3+1 dimensional semi-Riemannian manifold. Therefore mappings to flat Minkowski spacetime exist in a region of every point. Minkowski spacetime is orientable hence spacetime is locally orientable. It follows that orientability is a global rather than a local property.

Remark 2.2 *On a non-orientable spacetime there exists a closed path along which a consistent orientation cannot be defined.*

This follows from the definition of orientability and topological arguments.

Remark 2.3 *If spacetime is not orientable then through every point there exists a path on which an orientation cannot be defined.*

This follows simply from remark 2.2 above and assumed connectedness of spacetime. If A has a non-orientable path, C, through it and X is any other point. Then by connectedness there exists a path, P, from X to A and a path, P', from A to X. A non-timeorientable closed path through X is constructed by joining the paths P-C-P'.

Putting the three remarks together, spacetime could be orientable for all paths except those going through a small remote region. For all normal purposes such a spacetime would appear orientable. Only an experiment that probed the remote region \mathcal{R} could detect the non-orientability. However, Non-orientability in a connected spacetime cannot be strictly localised as there would be a curve through every spacetime point that went through the non-timeorientable region.

Mathematically, orientability is a global property of spacetime. By analogy with a möbius strip it is possible that paths comparable in size to the Universe could be non-orientable. However a non-orientable region could be microscopic in size: for example a non-orientable wormhole with dimensions comparable to the Plank length. This work is motivated by the possibility that elementary particles may be regions of non-trivial topology, but the arguments are generally applicable.

It could of course be possible for all *timelike* paths in a non-orientable spacetime to be orientable. A non-orientable region behind an event horizon would be such an example.

The observed violation of T-invariance implies that an observer can unambiguously define the direction of time. This is compatible with a localised time-reversing region, because the boundary conditions (the expanding Universe) define a time direction and because a consistent time direction could still be defined if the experimenter never probed the time-reversing region. Microscopic localised nonorientable regions as described in [3] are therefore consistent with observations.

3. Tests of space and time orientability

This section examines classical tests of the orientability of spacetime - they are based upon classical general relativity and classical physics. It cannot be overemphasised that this argument is not based on quantum field theory or even quantum mechanics, it is classical - classical mechanics and classical general relativity.

Consider a space that is flat and space-orientable everywhere except for a region \mathcal{R} . In spacetime \mathcal{R} would be a tube. The construction of spacetimes such as this, with all permutations of orientability, are described in [3]: Sorkin [5] constructs a non-orientable wormhole, which is also asymptotically flat. Gibbons and Herdeiro [6] give an asymptotically flat example for a supersymmetric rotating black hole. The arguments in this paper are independent of the exact form of the metric in \mathcal{R} .

The orientation of \mathcal{R} is tested by sending a probe into \mathcal{R} . The probe contains a triad of unit vectors \hat{i} , \hat{j} and \hat{k} . The triad defines a space-orientation continuously along the path taken by the probe. The path (timeline) of the probe and the path (timeline) of the observer together define a closed loop. An orientation can be defined in the laboratory, excluding region \mathcal{R} . If the probe that emerges from \mathcal{R} has a different orientation from that defined in the rest of the laboratory then space is not orientable (see Figure 1).

An example of a region \mathcal{R} that is not time-orientable is easily constructed (see

for example Ref. [4]): remove a ball from space - that is a world-tube from spacetime. Then identify opposite points of the sphere using the antipodal map. At the same time identify t with $-t$ in a similar manor to the creation of a Möbius strip. Like a Möbius strip this example is continuous and flat.

A similar test of the time orientability of \mathcal{R} would need a probe that defined a time direction continuously along a trajectory through \mathcal{R} and back to the observer. A clock would be a suitable probe, with the positive time direction defined by increasing times displayed on the clock. However the experiment is not so simple. Consider a clock that enters \mathcal{R} at an Observer time τ and emerges at a later observer time but counting backwards (see Figure 2). This is not a demonstration of non-time orientability, because in this experiment, the clock increases in value and then decreases. At some point in the path it attains a maximum reading and at that point it does not define a time direction.

In a true test of time orientability on a region \mathcal{R} through which time cannot be oriented, the clock readings would increase steadily along the path taken by the clock. Before entering \mathcal{R} the observer sees the time values increasing on the clock. When the clock exits at a time ρ the increasing clock times would be at ever decreasing values for the observer time. The observer would still see a backwards counting clock, but only at times before ρ (see Figure 3).

This might seem like a clear example of a successful demonstration of a failure of time orientability. The author would be sympathetic to such a view! However the observer sees the backwards (clock-time counting backwards) moving clock entering \mathcal{R} as well as the forward counting clock entering \mathcal{R} . At observer times greater than ρ the observer sees no clock at all - neither inward nor outward moving. So the observer can interpret the experiment as a clock and an anti-clock entering \mathcal{R} and annihilating each other.

It must be emphasised that this is still a description in terms of classical physics. The expression *anti-clock* is a logical one - it does not refer to anti-particles in the normal sense - indeed it is totally independent of the construction of the clock. If you expect the number of clocks to be preserved, then describing the backward counting clock as an anti-clock achieves the objective. Using the concept of an anti-clock, the experiment has zero clocks most of the time. Before τ there is a clock and anti-clock after ρ there are no clocks. The alternative is to postulate two rather different real objects (the clock and the backward counting clock) that both enter \mathcal{R} and vanish. The analogy shares with anti-particles the fact that both the clock and anti-clock must exist for this to happen. Unlike the conventional particle -antiparticle annihilation there does not appear to be any conservation of energy.

Gibbons and Herdeiro make similar observations for a particle traversing their supersymmetric rotating black hole.

In general τ and ρ would be different. In a normal particle-antiparticle annihilation event they would be equal. The difference between τ and ρ depends upon the geometry. For a similar construction to that described above the identification of $\tau - t$ with $\tau + t$ (rather than t and $-t$) would result in τ and ρ being equal. A simple would be just a

special case of this

To make matters even worse, the experiment is dependent upon the observer. Since any attempt to stop the backward counting clock from entering \mathcal{R} would be inconsistent with the clock entering and leaving a time reversing region. So not only is the anti-clock description a possible alternative description, the time reversing event *requires* the existence of anti-clocks (time reversed clocks) to exist before the experiment takes place. Since a clock is normally a macroscopic object composed of atoms, anticlocks do not exist in the sense of clocks composed of antiparticles. Therefore, an apparently simple test may be physically impossible to carry out.

This highlights a weakness in the way the problem was posed. Space orientability is a property of space. However timeorientability can only be a property of spacetime. Furthermore it is a global property of spacetime. It would appear that the existence or otherwise of a time reversing region is dependent upon the observer. A probe and an anti-probe must both be prepared for a consistent result, in which case it could be argued that the existence of the time reversing region depends upon the experiment being performed to test it.

4. Quantum field theory

Although the preceding argument referred to anti-clocks, this was a purely logical classical description. Quantum Field theory, which correctly describes both particles and antiparticles - and indeed requires anti-particles for completeness, was not being used. The similarities of these classical arguments to some quantum phenomena is not coincidental. It has already been shown [7] that quantum logic can be derived from classical general relativity with acausal spacetimes.

It is not possible to define a spinor field on a non-time orientable spacetime [8]. Since fermions exist, this well-known result has been cited as evidence that time must be orientable. However the argument relies on a realist interpretation of the wavefunction and the false assumption that a wavefunction is defined at each spacetime point. In fact a wavefunction is a function defined on a $3N$ -dimensional configuration space where, N , is the number of particles. See Ballentine [9] Chapter 4 for the clearest discussion of arguments against a realist interpretation of a wavefunction (also [10]).

Fermion fields are used to calculate the probabilities of results of experiments. The extension of a fermion field to inaccessible microscopic non-orientable regions is neither relevant nor particularly meaningful, since measurements cannot take place within these regions. While on a spacetime with accessible time-reversing regions, the inability to define a spinor field, simply means that QFT cannot be used to calculate probabilities of the results of experiments on a space time that is not timeorientable a conclusion that is hardly surprising!

5. Conclusion

Although the analysis is clear, the interpretation is by no means obvious. The following statements can all be supported by the arguments above:

- (i) Spacetime is not time orientable. Particle antiparticle annihilation events are evidence of this.
- (ii) A failure of time orientability and particle antiparticle annihilation are indistinguishable. They are alternative descriptions of the same phenomena.
- (iii) Time orientability is untestable.
- (iv) Non time orientability cannot be an objective property of spacetime because the outcome of our test would depend upon the observer.

The conventional view of the experiment in Figure 3 is that it depicts a particle antiparticle annihilation. This view has developed and now predominates for historical reasons. It is an interpretation that conforms to the classical paradigm - modelling Nature in terms of initial conditions, unique evolution and observer independent outcomes. The classical paradigm is clearly inconsistent with a failure of time-orientation.

The explanatory power of a non-timeorientable spacetime in relation to interactions in geon models, electric charge and spin-half, adds weight to the interpretation of Figure 3 as a demonstration of the non-timeorientability of spacetime.

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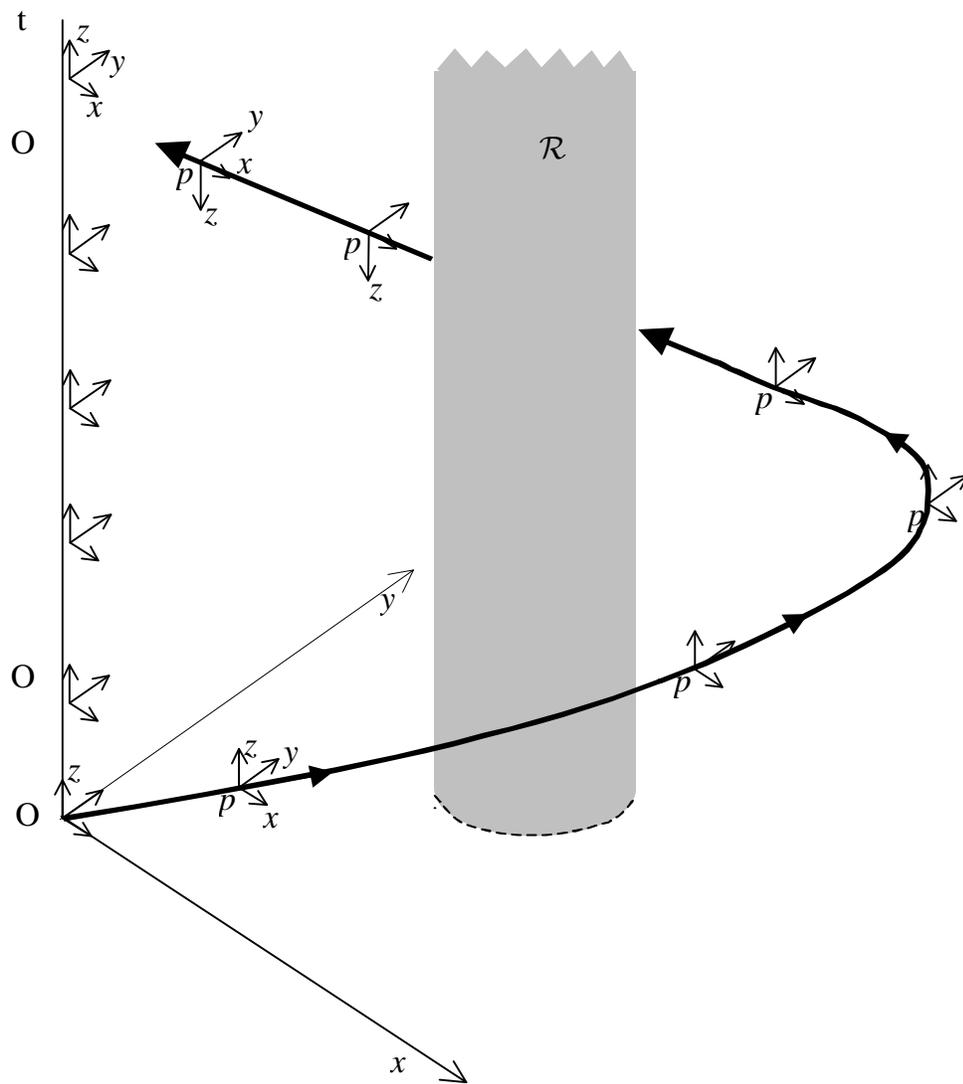


Figure 1. A spacetime diagram showing a positive test that region \mathcal{R} is not timeorientable. Observer, O , sends a probe, P , through \mathcal{R} , it exits \mathcal{R} and returns to O with opposite orientation.

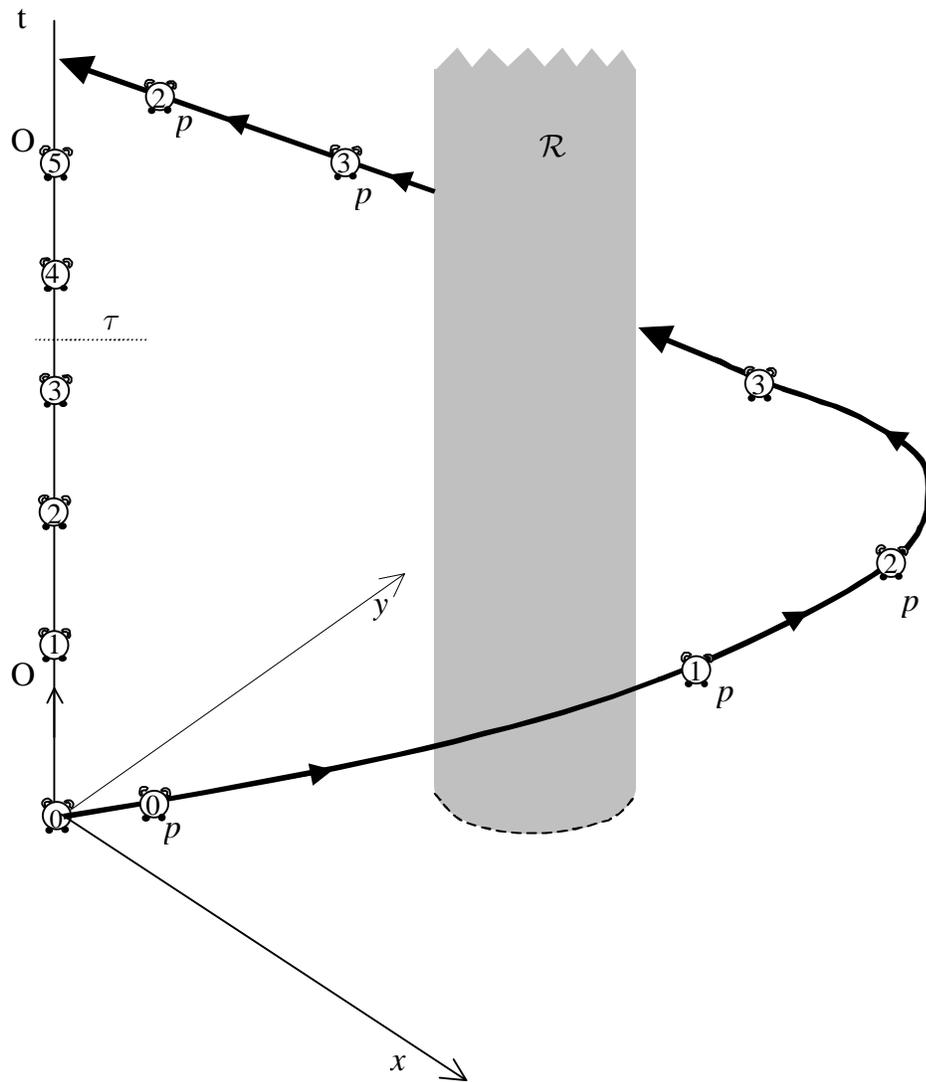


Figure 2. A flawed attempt to test time-orientability: An Observer, O , sends a clock, P , through \mathcal{R} , it exits \mathcal{R} and reappears counting backwards. This is not successful because the clock has failed to define a time direction throughout the experiment.

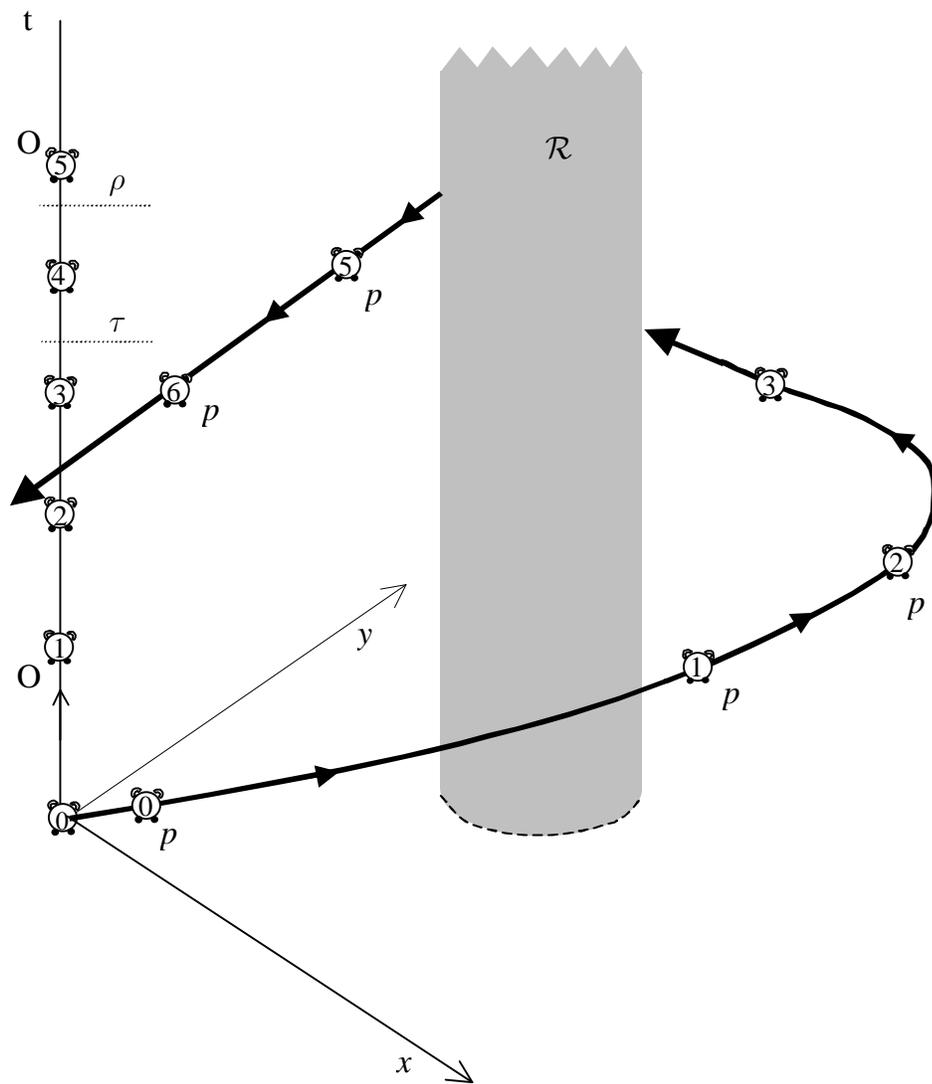


Figure 3. The clock, P , counts forward continually, it defines a time direction throughout its path. The observer, O , sees a backward counting clock and a forward counting clock enter \mathcal{R} . The observer sees no clocks after observer time ρ .