ONLINE AUCTIONS - EXAMINATION OF BIDDERS' STRATEGIES: THEORY AND DATA ANALYSIS

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Thesis Submitted for the Degree of Doctor of Philosophy

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April 2018

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Acknowledgements

First and foremost I would like to thank my supervisors Prof. Michael Waterson and Prof. Dan Bernhardt for their guidance, and helpful comments.

Special thanks are due to other researchers at the Department of Economics who attended my presentations and gave me guidance at the early stages of some of the chapters.

In addition I would like to thank my family and friends for their support and help through the PhD. I would not be able to do this without them.

Declaration

I declare that the material contained in this thesis has not been used or published before. This thesis is my own work and it has not been submitted for another degree or at another university.

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Summary

This PhD thesis is concerned with buyers' strategies in sequential and concurrent auctions. It deals with both the theoretical viewpoint and data analysis of online consumer auctions. The first chapter contains a newly developed model of sequential auctions with overlapping generations of bidders. The emphasis is on the existence of learning from observed past prices. With the addition of overlapping generations the learning happens through two channels: updating on valuations and expectation of composition of bidders with different horizons lengths. The model shows how this happens on the micro level, where expected distributions of bids are updated. In the following chapter, the predictions of theoretical models of sequential auctions together with learning are tested empirically. It is shown that bidders adjust their bids as a consequence of learning as predicted by the model. Bid discounting is also observed in the data.

The following empirical chapter uses the bids data from online auctions to perform multinomial logit estimations. Individual choice model allows to analyze the aspects that attract bidders to particular auctions out of many very similar ones available. A unique dataset that contains data from many auctions for the same product is used in this new way. Dynamic aspects of auctions such as the number of bidders and bids are shown to play a role in auction choice.

Overall, there are three approaches to the empirical analysis of bidders strategies, based on the same dataset. It is shown that with appropriate adjustments the data collected from online auctions can be used in different formats to answer various questions.

Introduction

Auctions are used as a selling mechanism across wide range of industries and applications. They can be often found online for selling consumer goods on various websites accessible to everyone. Other applications range from procurement auctions, through treasury securities auctions, to charity and art. Online auctions are advantageous for analysis of bidder behavior due to a large number of participants and the presence of multiple auctions for identical products. It is of interest, from the point of view of both bidders, as well as auctioneers to know more about the relevance of theoretical frameworks, which often contain predictions for behavior of bidders, in the real world auctions. Knowing more about the bidder behavior allows, in turn, to develop theory for improved relevance. This thesis offers various approaches to the investigation of online auctions data. Based on one dataset analysis of the sequential auctions aspect, the choice between concurrent auctions and time until the next bid is made. The literature search revealed that even though the data is rich in information the researchers are very often limited to the analysis that is implied by the format of data determined at the collection phase. It is demonstrated in this thesis, that the initial format of data does not need to be restrictive. In fact one can perform appropriate formating to show, for example, analysis of auction choice, even though the choice sets were not initially available, and they do not need to be generated at the time of data collection. Various datasets can therefore be reused with appropriate formatting in order to answer different questions that may arise.

The data analyzed contains bids on all auctions for the same product across almost two month period. The fact that the products sold in auctions are almost completely homo-

geneous is explored in order to identify the impact of variables of interest on bidders' behavior.

One aspect of behavior that is analyzed is learning by bidders. The theory invoked predicts that the bidders will adjust their bids in the auction following that follows the reference auction with price was first observed. The first chapter presents a new model of sequential auctions, with the addition of overlapping generations of bidders, which shows why learning by bidders is expected to take place. This model extends the scope of learning from past prices, in relation to the basic sequential auctions model. In the new model the condition that the same bidders follow from the reference auction, which price is observed, to the future auction, which is relevant for bid discounting, is no longer a necessary condition for learning. There is a different channel through which learning happens, not related to the revealed information about valuations of bidders in the reference auction. The mixture distributions of bidders with different horizon lengths are also relevant for learning, and the future composition of old and young bidders can be partially revealed by price of finished auctions. The second chapter contains empirical investigation of learning. Crucially, learning from past auctions is identified to take place, although after dividing the data into the subsets it is shown that learning persists only for bidders who bid on the more expensive version of the object. Our intuition is that the bidders with higher valuation, who bid on the more expensive product version, have more room to adjust their bids, and therefore they are more prone to displaying learning in their bids.

Another aspect that is analyzed is the impact of current price, number of bids and bidders on auction choice. For that purpose the choice model is estimated, where the alternatives are homogeneous in their characteristics. Dynamic aspects of auctions are identified to be an important determinant for the choice of the auction in which ultimately bidders decide to place their first bid. Bidders are deterred by high current price in the auction, and number of bids, but are attracted to auctions with larger number of current bidders. The analysis of the behavior through choice model is advantageous because it gets rid of

the possible biases relating to the timing of the bid - the effect of the variables of interest are estimated within the choice sets not across the choice sets, which means alternative times of the bid placement are not influencing the results.

Chapter 1

Overlapping generations in sequential auctions - theoretical model

Abstract

This paper adds overlapping generations of bidders to the model of sequential auctions. It is capturing situations where each bidder can enter the market starting at a different auction. This stylized model shows micro-foundation behind learning from past prices. The important implications resulting from this extension are that impact of past prices is on learning not only about bidders' valuations but also about the distribution of bids that is a consequence of composition of bidders with different horizon lengths. Although learning that relates to valuations of other bidders lasts only until the bidders present in the information acquiry period exit, learning relating to composition of bidders of different generations (with different horizon lengths) persist further into the future, affecting the beliefs about all future periods.

1.1 Introduction

In the past years there has been an increasing popularity in the use of auctions across many fields. It is common for many similar products to be sold in a sequence, and it seems that it is possible for the price in past auctions to have an effect on the bidding and outcomes of future auctions. According to existing models in auction theory past prices should not have an effect on bidding strategies in sequential auctions, both first, and second price. The achieved price is an indication about the second highest bid though, and it could be indicative of what is going to be the distribution faced in the following auction. Sequential auctions are characterized by a symmetric equilibrium including the bid shading in earlier auctions. Bidders with future opportunities to bid reduce their bid by their expected surplus from winning in the future. This means that when they learn something about the future distribution of valuations or number of bidders, they have space to change their bid upwards (or downwards) in order to adjust to the new information. Auction theory, to date, has ignored the fact that in the same auction bidders may be in different periods, which affects their discounting. For example, one

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bidder could be willing to bid in seven auctions, while another one is short of time and will bid in two auctions. The fact that bidders vary in how many auctions they may participate in, means that they have different strategies (levels of discounting), even though they participate in the same auction. The bids are therefore best represented as arriving from a mixture distribution of different 'age' bidders. There can be different states, which are defined by the composition of bidders of different ages. This has implication for updating: without abandoning the assumption that the distribution of valuations is public knowledge, there is a new unknown in the model: which state we're in, and what is the most likely future state. The price from past auctions provides some information about what was the 'age' of the winning bidder, and, as an implication : what was the most likely state, and what is going to be the next most likely state in the future. Therefore, the winning bid gives an information about how many old versus young bidders there were in the last period, as well as what was the most likely age of the winner. The implications of the overlapping generation model outlined below are that a high winning bid leads to less discounting in the following period, and, therefore, to an expected higher price paid in the next period.

1.2 Literature

It is an established result that in sequential auctions, both for the first and second price, bid-shading in the earlier periods is optimal in a symmetric equilibrium. Furthermore, price announcements in a sequential first price auction have no effects on the strategies of bidders. The strategy of bidding in sequential auctions is to discount the bid in a period by the option value of winning in the next period. For identically-valued objects, in the first price auction the last period bid will be the expected second highest bid, while in the second price auction the final period bid will be the valuation for the object. Milgrom and Weber (2000)[27] prove that the price sequence will be a martingale when bidders have independent private values. There are two effects in opposite directions, that

exactly offset each other: On the one hand the symmetric equilibrium strategy of bidding is increasing for subsequent auctions (since there are less goods left with each auction completed. On the other hand, since the bidding strategy at each period is increasing in private valuation, the bidders with highest values win earlier auctions, while those with lower values stay until the following auctions. The two effects offset each other, and the price sequence is a martingale.

The revenue equivalence result between simultaneous and sequential auctions can also be used to show the martingale property of prices (a summary of literature can be found in Klemperer, 1999 [23]). When bidders have affiliated values[27], the price sequence is a sub-martingale, since the signal revealed after each auction reduce the winner's curse, and bidding is more aggressive in subsequent periods. As authors point out, the price sequence is never a decreasing sequence, which stands in contrast to the decreasing prices found in empirical research (for example Ashenfelter, 1989 [3]). The sequence of expected prices in both first and second price auctions are the same in the case of private values. The different explanations in literature related to declining prices can be found in Trifunović (2014)[34]. One of the first models to explain the declining prices is based on bidders' risk aversion. McAfee and Vincent (1993)[25] provide a model where in the case of bidders' non-decreasing risk aversion prices are falling in equilibrium. Bernhardt and Scoones (1994)[10] consider sequential auctions of two objects, where the bidders learn their valuation of the second object after the first auction is completed. Therefore the expected surplus from the second auction is the same for all bidders. The bid discounting in the first round for individuals with higher values is therefore relatively lower than that of those with lower values, and as a result the price sequence is declining.

Price announcement have no effect in the case of sequential first price auction and single-unit demand, as they do not reveal any information about the remaining bidders. Only for second price auctions do price announcements reveal the bid of one of the remaining bidders, which created information asymmetry ([27]). Kittsteiner et al (2004)[22] prove that bidding strategies for both first and second price sequential auctions are not

dependent on price announcements. They also analyze a case where the valuations for the objects are decreasing for later periods (due to time preference) and show that the resulting price sequence will be a super-martingale for both first and second price auctions.

This paper adds overlapping generations of bidders to the literature on sequential auctions outlined above. The focus of the paper is bidders' strategies, where sellers are taken as exogenous to the model. A related model introducing a degenerate case of overlapping generation is Zeithammer (2007)[38], where only two types of valuation L or H are considered, and the low-valuation bidders do not shade their bid in the first period. The author's focus in that paper is on the seller's strategy dependent on the observed prices, where he seller uses their strategic auctioning to reduce bid-shading by the high valuation bidders. On the other hand, the current paper is trying to answer the question related to bidders' learning from past prices, given continuous support of valuations for the object.

1.3 Preliminaries

1.3.1 Equilibrium

Bayesian Nash Equilibrium

The Bayesian Nash Equilibrium, as introduced by Harsanyi(1967) treats a game of incomplete information as the one with imperfect information. If a buyer does not have complete information about other buyers' valuations, it is taken as if he has uncertainty about their types. That uncertainty is expressed through probability distribution over types of buyers. Nature, as an additional player, chooses the types for players before the game starts. Each player observes their type, but not the types of other players. The initial distribution from which the types are drawn is known to all the players, and the strategies chosen are taking this information into account. The standard notation

needed to describe this equilibrium formally is as follows. The set of players is denoted as $I = \{1, 2, ..., n\}$. The set of possible types of each player $i \in I$ is denoted by X_i, this set can be an interval, $[\underline{v}, \overline{v}]$, as in later sections. $f(\cdot)$ is the probability distribution over $X = X_1 \times X_2 \times ... \times X_n$, reflecting the probabilities attached to each combination of types occurring. The set of strategies for player $i \in I$ is denoted as S_i and $s_i : X_i \to S_i$ is the decision function for player i. It is a mapping from the set of possible types to the set of possible strategies (in particular, if all players' types are derived from the same distribution over types, then $X_i = X_j$, for all i, j). The probability distribution of types x_{-i} of the players $j \neq i$ given that i knows his type is x_i, is denoted by $\hat{f}_i(x_{-i}|x_i)$. Player i updates his prior information about the distribution of other types after learning his own type x_i. Let's denote $W_i(s_i, s_{-i}, x_i, x_{-i})$ i's profit given that his type is x_i, that he chooses s_i and that other players follow strategies $s_{-i}(x_{-i}) = (s_j(x_j))_{j \neq i}$, where $s_j : X_j \to S_j$ is j's decision function, and their types are x_{-i}.

Definition 1. *A Bayesian Game is defined as a five-tuple*

$$G = [I, \{S_i\}_{i \in I}, \{W_i(\cdot)\}_{i \in I}, X_1 \times X_2 \times \cdots \times X_n, f(\cdot)] \tag{1.3.1}$$

Definition 2. *A Bayesian Nash equilibrium is a list of decision functions $(s_1^*(\cdot), \cdots, s_n^*(\cdot))$ such that $\forall i \in I, \forall x_i \in X_i$ and $\forall s_i \in S_i$:*

$$\int_{x_{-i} \in X_{-i}} W_i(s_i^*, s_{-i}^*, x_i, x_{-i}) d\hat{f}_i(x_{-i}|x_i) \geq \int_{x_{-i} \in X_{-i}} W_i(s_i, s_{-i}^*, x_i, x_{-i}) d\hat{f}_i(x_{-i}|x_i) \tag{1.3.2}$$

Definition 3. *A symmetric Bayesian Nash equilibrium is such that all players choose the same decision function.*

Dominant Strategy

Definition 4. *A strategy s_i is a dominant strategy for player i if*

$$W_i(x_i, s_i, s_{-i}) \geq W_i(x_i, \hat{s}_i, s_{-i}) \tag{1.3.3}$$

for all $\hat{s}_i \in S$ and for all $s_{-i} \in S^{n-1}$

The dominant strategy for player i is therefore a strategy which maximizes i's payoff for any possible strategies of other players. An equilibrium in dominant strategies occurs if every player plays their dominant strategy.

Definition 5. *An outcome $(s_1^*, ..., s_n^*)$ is an equilibrium in dominant strategies if s_i^* is a dominant strategy for each player $i, i = 1, ..., n$.*

An equilibrium in dominant strategies is always also a Bayesian Nash equilibrium, but the opposite is not always true.

1.3.1.1 Extensive Form Games

Games in which players' moves happen in specified order, are a particular type of games called Extensive form games. In this case the game lasts for a number of periods and at each period certain moves of players take place. Nash and Bayesian Nash equilibrium are defined for the Normal-form games, which do not have a time sequence of moves. For the extensive form games the most relevant equilibrium concept is **Perfect Bayesian Equilibrium**. In extensive form games there is history of the game, which relates to all the past periods as well as a continuation game which refers to the periods (information set and nodes) that follow. The player has beliefs about the current node and all the nodes that have been reached before, as well as all the future nodes. The belief relates to the probability of a particular node in the game tree. A sequence of nodes is the history path, and each history path has an attached probability according to the beliefs. Beliefs about the probability of continuation game paths allow the players to calculate

the expected payoff from continuation games. Before the move, the conditional beliefs from each strategy can be calculated which allows for calculation of conditional expected payoffs and strategy choice. Beliefs are derived through Bayes rule wherever possible.

Remark. *At each information set a player who gets to move must have a **belief** about which node in the information set has been reached by the game*

It is required that at the equilibrium a player's strategies must be **sequentially rational**. That is, at each information set the actions by the players must form a Nash equilibrium of the continuation game.

Remark. *A strategy is sequentially rational for player i at the information set h if player i would want to choose the action prescribed by the strategy if h is reached.*

The beliefs that players have must be **consistent**. Consistent beliefs are such that for $i, j \in I$, where I is a set of players, player i's updated beliefs are consistent with the probability distribution induced by any chance events and player j's strategy. PBE requires weak consistency of beliefs:

Remark. *Beliefs must be **weakly consistent** with strategies. That is, they are obtained from strategies and observed actions via Bayes rule whenever possible.*

Weak consistency and sequential rationality are linked through the following theorem:

Theorem 1. *Suppose σ is an equilibrium in behavior strategies in an extensive form game with perfect recall. Let $h \in H$ be an information set that occurs with positive probability under σ, and let d be a vector of beliefs that is weakly consistent with σ. Then σ is sequentially rational for player i at information set h with beliefs d.*

Perfect Bayesian Equilibrium

Sequential rationality and weak consistency are all the necessary components that allow for the definition of PBE. Perfect Bayesian Equilibrium is a solution concept that is stronger than Nash Equilibrium, eliminating the strategies that fail under additional

requirements The equilibrium is defined by the pair of strategy profile and a belief vector, an assessment (σ, d).

Definition 6. *An assesment (σ^*,d^*) is a perfect Bayesian equilibrium (PBE) if the strategies specified by the profile σ^* are sequentially rational given beliefs d^*, and the beliefs d^* are weakly consistent with σ^*.*

This equilibrium concept requires not only reasonable strategies, but also reasonable beliefs. The equilibrium consists of both, the beliefs, and the strategies. The equilibrium is somewhat circular in the sense that strategies must be optimal given beliefs and beliefs are derived from strategies. It is also certain that for any finite game with perfect recall a Perfect Bayesian Equilibrium exists.

Theorem 2. *If (σ, d) is a perfect Bayesian equilibrium of an extensive-form game with perfect recall, then σ is a mixed-strategy Nash equilibrium. For any finite extensive-form game, a perfect Bayesian equilibrium exists.*

For games of imperfect information, there is an additional move by Nature at the beginning of the game. Given that it is assumed that the distribution from which the Nature randomly chooses is known to the players, the perfect Bayesian equilibrium beliefs are also consistent with strategies of other players, but they are in a form of a probability distribution function that may be continuous distribution over all the possible types of other players in the game.

1.3.2   Sequential auctions equilibrium

Auctions can be defined in a similar way as any game of imperfect information, above. Let $i \in I$, where $I = \{1, 2, 3, ...\}$ be the numbering for individuals. Each individual has their type. The set of possible types is V, and elements in this set, $v \in [\underline{v}, \overline{v}]$, which defines the upper and lower bounds of the set of valuations. Of course the set of valuations could be unbounded, but this is the case I want to focus on for simplicity. I am considering

the case of independent and identically distributed valuations, and therefore each v_i is a random draw from a probability distribution with CDF $F_v(v)$, and PDF $f_v(v)$. The types for each player are drawn before the game starts, and each player learns their type. The term v_i is player i's valuation for the object (and also their type). Let $W_{i,n}(v_i)$ be the expected surplus of player type v_i in nth auction, in which the player participates in. n belongs to a bounded set of Natural numbers. Suppose the final period for each player is period $j = max\{n\}$. $W_{i,n}(v_i)$ is the expected surplus for player type v_i in period n. Since for player's strategy it is only important what is the period they are in, I will call period n the player's nth period. The symmetric Perfect Bayesian Equilibrium is such that each player has a sequentially rational strategy defined by a period-specific function $b_n(v_i)$, and beliefs used for calculation of continuation game surplus are consistent. Standard sequential auctions model considers the case where all players are in the same period. In this case, the game to consider is a finite game with the number of periods equal to the lifespan of bidders.

Suppose that the distribution of bids that player i faces in period j is $F_j(.)$. The PBE strategy for player i is such that he has no intention to deviate no matter what are the strategies of other players in the given period. This condition can be reiterated as: player ith chosen strategy is independent of the distribution of bids he faces, $F_j(.)$ in the current node.

Remark. *The equilibrium bidding strategy of player i in period n is characterized by the fact that it is independent of the distribution of bids in period n: $F_n(.)$*

The strategy is calculated using the expected payoff of the continuation game, which is based on the Nature's distribution of types of other bidders.Therefore, given distribution of valuations $F_v(v)$, the optimal bidding strategy can be found by maximization at each period and backward induction. Subsequently, checking that the nth period strategy,$b_n(v)$, is independent of $F_n(.)$ and solving by backwards induction will be necessary and sufficient for the strategies to be sequentially rational, and to confirm that this is the PBE strategy. The equilibrium strategy as a function of valuations - $b_n(v)$, is the

equilibrium strategy in the symmetric equilibrium.

Theorem 3. *The necessary and sufficient conditions for finding equilibrium strategy $b_n(v)$ in the sequential auctions game:*

1. $b_n(v)$ *is the solution to maximization*
2. $b_n(v)$ *is derived through backward induction, using consistent beliefs for the bids in continuation game*
2. $b_n(v)$ *is independent of $F_n(v)$.*

First, the optimization problem needs to be solved. Let's consider the second price auction. The model solved below is the equilibrium strategy of bidder i with valuation v_i, where the distribution of the second highest bid is defined by a CDF of $F_{2,n}(.)$ and pdf of $f_{2,n}(.)$. The solutions are derived for the last period, j, and the previous period, $j-1$ through backward induction. The notation is that $b_{i,j} = b_j(v_i)$ is the bid of bidder i in period j, and $W_{i,j} = W_j(v_i)$ is the surplus of bidder i in period j. The star marks the optimal solution (arrived at through maximization), where $W_{i,j}^*$ is the expected surplus(profit) of bidder i in period j and $b_{i,j}^*$ is the optimal bid of bidder i in period j. Analogically, $W_{i,j-1}^*$ is the expected surplus of bidder i in period $j-1$ and $b_{i,j-1}^*$ is the optimal bid of bidder i in period $j-1$.

Starting from the below formula for expected profit, the optimal solution, which is $b_{i,j} = v_i$ can be arrived at through differentiation (to satisfy the First Order Condition), as it is done in any standard maximization problem:

$$max_{b_i} W_{i,j} = \int_0^{b_{i,j}} (v_i - b_2) f_{2,j}(b_2) db_2 \tag{1.3.4}$$

$$\frac{\partial W_{i,j}}{\partial b_{i,j}} = (v_i - b_{i,j}) f_{2,j}(b_{i,j}) = 0$$

$$b_{i,j} = v_i$$

In the case that the surplus functions is continuous and differentiable in the whole domain, it is enough to check for the second order condition for the maximum, since the first order condition has shown only single stationary point.

Remark. *If $W_{i,j}^*(b_{i,j})$ is continuous and differentiable, then the maximum is found by solving the is the First Order Condition $\frac{\partial W_{i,j}(b_{i,j})}{\partial b_{i,j}} = 0$ and Second Order Condition $\frac{\partial^2 W_{i,j}}{\partial^2 b_{i,j}} < 0$.*

In order to find out if this is the maximum, the Second order condition needs to be satisfied, and therefore:

$$\frac{\partial^2 W_{i,j}}{\partial^2 b_{i,j}} < 0 \tag{1.3.5}$$

And this is satisfied.

So, the optimal bidding strategy is a function of valuation, $b_j^*(v_i) = v_i$. The strategy is independent of $f_{2,j}(.)$, and therefore it is the symmetric equilibrium strategy:

Theorem 4. *The Symmetric Equilibrium Bidding strategy for the last period second price auction is $b_j^*(v_i) = v_i$*

Now, substituting that back to get the last period expected profit, when bidder i bids according to their equilibrium strategy:

$$W_{i,j}^* = \int_0^{v_i} (v_i - b_2) f_{2,j}(b_2) db_2 \tag{1.3.6}$$

This can be re-arranged in order to arrive at an expression in terms of "probability of winning × Surplus in the case of winning":

$$W_{i,j}^* = v_i \int_0^{v_i} f_{2,j}(b_2) db_2 - \int_0^{v_i} b_2 f_{2,j}(b_2) db_2 = \tag{1.3.7}$$

$$= v_i F_{2,j}(v_i) - [v_i F_{2,j}(v_i) - \int_0^{v_i} F_{2,j}(b_2) db_2] = \qquad (1.3.8)$$

$$= F_{2,j}(v_i) v_i - F_{2,j}(v_i)(v_i - \int_0^{v_i} \frac{F_{2,j}(b_2)}{F_{2,j}(v_i)} db_2) \qquad (1.3.9)$$

$$= F_{2,j}(v_i)[v_i - \int_0^{v_i} (1 - \frac{F_{2,j}(b_2)}{F_{2,j}(v_i)}) db_2] = \qquad (1.3.10)$$

$$= Pr(b_2 \le v_i)[v_i - E[b_2|b_2 \le v_i]] \qquad (1.3.11)$$

[1]

Now, the steps need to be repeated for period $j-1$. We already know what is the expected jth period surplus, and the surplus in $j-1$ is expected surplus in $j-1$ plus the expected surplus in j, but of course only in the case of losing.

The surplus in period $j-1$:

$$W_{i,j-1} = \int_0^{b_{i,j-1}} (v_i - b_2) f_{2,j-1}(b_2) + \int_{b_{i,j-1}}^{\infty} W_{i,j}^* f_{2,j-1}(b_2) db_2$$

Period $j-1$ maximization problem is therefore:

$$max_{b_{i,j-1}} W_{i,j-1} = \int_0^{b_{i,j-1}} (v_i - b_2) f_{2,j-1}(b_2) + \int_{b_{i,j-1}}^{\infty} W_{i,j}^* f_{2,j-1}(b_2) db_2$$

First Order Condition:

$$(v_i - b_{i,j-1} - W_{i,j}^*) f_{2,j-1}(b_{i,j-1}) = 0$$

On the condition that $f_{2,j-1}$ is positive in the whole domain, there is only one sta-

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[1]The second term from line (3) is expanded by integration by parts to line (4) and then gets the form as in line (5) - there $F_2(v_i) = Pr(b_2|b_2 \le v_i)$ as it is the cumulative of the second highest bid (the highest of N-1 bidders), and the integral of the survival function is equal to $E[b_2|b_2 \le v_i]$, as $\frac{F_2(b_2)}{F_2(v_i)}$ is the cumulative function of the second highest bid divided by the probability that the second highest bid is below v_i.

tionary point:

$$b_{i,j-1} = v_i - W_{i,j}^*$$

The bid is discounted by the expected profit in period j. Since there is only one stationary point, and the function $W_{i,j-1}(b_{i,j})$ is continuous and differentiable in the whole domain,in order to confirm that the solution is a maximum, it is sufficient to show that the second derivative at this point is negative. The Second Order Condition:

$$\frac{\partial^2 W_{i,j-1}}{\partial^2 b_{i,j-1}} < 0 \qquad (1.3.12)$$

,at $b_{i,j-1} = v_i - W_{i,j}^*$. And this is satisfied.

This shows that the solution $b_{j-1}(v_i) = v_i - W_{i,j}^*$ is the optimal bidding strategy in period $j - 1$ for bidder with valuation v_i. It is easy to see that this expression is independent of the distribution of other bids in period $j - 1$, and as a result it is independent from strategies of other bidders in $j - 1$, and uses consistent beliefs for calculation of expected surplus in j. Above proves that $b_{i,j-1} = v_i - W_{i,j}^*$ is the PBE strategy for period $j - 1$, and the same is true for every i, which means that we have found the equilibrium bidding function for a symmetric perfect Bayesian equilibrium:

Theorem 5. *The bidding function $b_{j-1}^*(v_i) = v_i - W_j^*(v_i)$ is the equilibrium strategy for symmetric Perfect Bayesian equilibrium of period $j - 1$ second price auction.*

As it has just been pointed out, $b_{j-1}^*(v_i)$ is independent from distribution of other bids in $j - 1$. It is, though, dependent on distribution of other bids in period j, since $W_{i,j}^*$ is dependent on $f_{2,j}()$. No deviation from equilibrium bidding strategy is ever profitable, and therefore as long as bidders are rational, all bidders will bid according to $b_j^*(v)$ (sequential rationality). The only possible $f_{2,j}(v)$ is therefore calculated under the assumption that all bidders bid according to $b_j^*(v)$. This distribution is marked with asterisk, since in the same way as $W_j^*(v)$, it is the result derived from equilibrium bidding.

Remark. $f_{2,j}^*(v)$ *is the distribution of the second highest bid in period j calculated under assumption that all bidders bid according to their equilibrium strategy in period j.*

Remark. *The equilibrium strategy in period $j-1$ is independent of other bidders strategies in period $j-1$ ($b_{j-1}(v)$ indep. $f_{2,j-1}()$), but it is calculated with the assumption that all bidders bid according to their equilibrium strategies in period j. The distribution $f_{2,j}() = f_{2,j}^*()$, which allows, in period $j-1$, to treat $W_j^*(v_i)$ as a known constant for each type v_i.*

The solution can be further extended to earlier periods, and the expected surplus from $j-1$ is going to be used in strategy calculation for period $j-2$. This expected surplus is calculated below.

By substituting the equilibrium strategy in the expected surplus, the formula for expected surplus in PBE is arrived at below:

$$W_{i,j-1}^* = \int_0^{v_i - W_{i,j}^*} (v_i - b_2) f_{2,j-1}(b_2) db_2 + \int_{v_i - W_{i,j}^*}^\infty W_{i,j}^* f_{2,j-1}(b_2) db_2$$

This can be rearranged, in the same way as with jth period profit in the case other forms are easier to interpret:

$$= v_i \int_0^{v_i - W_{i,j}^*} f_2(b_2) db_2 - \int_0^{v_i - W_{i,j}^*} b_2 f_2(b_2) db_2 + W_{i,j}^* - W_{i,j}^* F_2(v_i - W_{i,j}^*) =$$

$$= F_2(v_i - W_{i,j}^*)v - F_2(v_i - W_{i,j}^*) \int_0^{v_i - W_{i,j}^*} (1 - \frac{F_2(b_2)}{F_2(v)}) db_2 + W_{i,j}^* - W_{i,j}^* F_2(v_i - W_{i,j}^*) =$$

$$= F_2(v_i - W_{i,j}^*)(v - W_{i,j}^*) - F_2(v - W_{i,j}^*) \int_0^{v_i - W_{i,j}^*} (1 - \frac{F_2(b_2)}{F_2(v_i - W_{i,j}^*)}) db_2 + W_{i,j}^* =$$

$$= F_2(v - W_{i,j}^*)[(v_i - W_{i,j}^*) - \int_0^{v_i - W_{i,j}^*} (1 - \frac{F_2(b_2)}{F_2(v_i - W_{i,j}^*)})db_2] + W_{i,j}^* = \quad (1.3.13)$$

$$= Pr(b_2 \leq v_i - W_{i,j}^*)(v_i - W_{i,j}^* - E[b_2|b_2 \leq v_i - W_{i,j}^*]) + W_{i,j}^*$$

In the equation above we can see how the current valuation is discounted by the expected profit from future auction.

Alternatively, re-writing differently line numbered (8) above:

$$= F_2(v_i - W_{i,j}^*)[v_i - \int_0^{v_i - W_{i,j}^*} (1 - \frac{F_2(b_2)}{F_2(v_i - W_{i,j}^*)})db_2] + (1 - F_2(v_i - W_{i,j}^*))W_{i,j}^* = \quad (1.3.14)$$

$$= Pr(b_2 \leq v_i - W_{i,j}^*)(v_i - E[b_2|b_2 \leq v_i - W_{i,j}^*]) + (1 - Pr(b_2 \leq v_i - W_{i,j}^*))W_{i,j}^*$$

A general take-away is that in the case of sequential auctions, in period $j - 1$ bidders discount their bid by the expected surplus from period j. This discounting is lower for bidders with low valuations - since their expectation of possibility of winning in the final period is lower. Discounting increases with valuation, and high-valuation bidders are discounting their bids by the highest amount. The discounting depends on the expected distribution of the second highest bid in the next period which is calculated using consistent beliefs. Rationality implies that the distribution of the second highest bid has to be calculated under the assumption of equilibrium strategies (consistent beliefs), and that can be used for calculations of discounting in earlier periods.

In order to make the definitions more clear I will introduce some basic notation from set theory to distinguish between the sets of valuations and bids. The set of valuations has been defined at the beginning of this section, it is the set V s.t. each element of the set

$v_i \in [\underline{v}, \overline{v}]$. Now, let's introduce the sets of bids, which are important for the definitions of bidding functions:

The set of expected bids in period n is defined as B_n, and it has upper and lower bound so that the elements in set B_n are $z_{i,n} \in [\underline{z}_n, \overline{z}_n]$

Function b_n^* is a mapping from the set of valuations V to the set of expected bids in period n, B_n.

Theorem 6. *The symmetric perfect Bayesian equilibrium of sequential second price auction game gives as solution:*

- A vector of bidding functions for each age bidder $B^*(v) = \{b_j^*(v), b_{j-1}^*(v), b_{j-2}^*(v), ..., b_1^*(v)\}$

-A vector of beliefs about the distributions of bids in each period $\{f_j^*(z_j), f_{j-1}^*(z_{j-1}), ..., f_1^*(z_1)\}$

Where:

For each n, the equilibrium bidding function takes the form $b_n^*(v_i) = v_i - W_{n+1}^*(v_i)$, and it is a mapping from set of valuations to the set of period n bids.

The next period surplus, $W_{n+1}^*(v_i)$ is calculated under rationality assumption.

Rationality implies that in expectation:

1.Bidders bid according to $b_n^*(v_i)$, which implies that

2.The distribution of bids is $f_n^*(z_n)$, and is calculated with the assumption that bidders bid according to $b_n^*(v)$

3.The expected surplus is $W_n^*(v_i)$ and is calculated using both above points 1. and 2.

Since the equilibrium bidding strategy implies discounting of valuations, as mentioned before, it can be easily seen that the upper bound of the set of bids in each period has to be lower or equal to the upper bound of the set of valuations, and that with earlier periods the upper bound of the set of bids has to be decreasing.

Remark. *The discounting of bidding for earlier periods implies that*

$$\overline{v} \geq \overline{z}_j \geq \overline{z}_{j-1} \geq \overline{z}_{j-2} \geq ... \tag{1.3.15}$$

The distributions of bids, mentioned before are defined on the sets of equilibrium bids in each period.

Remark. *The support of distribution $f_n^*()$, and all distributions of order statistics based on it, such as $f_{2,n}^*()$, is the set B_n, with bounds $[\underline{z}_n, \overline{z}_n]$.*

1.3.3 Mixture Distributions

Distributions of mixtures of random variables are called mixture distributions. Mixture distributions are used the following model, which is why current section introduces this concept to the reader. Mixture distributions relate to cases where multiple random variables with given weights are combined together, and the issue is to know the overall distribution. The easiest way to show what mixture distributions are is to use a very general example. Suppose there are two different types of random number generators. Suppose further that these random number generators are some type of objects, for example red cubes, which only have one button and a display on top. After pressing the button a random number appears on the display. Both types of the device look exactly the same, but they differ in the probability distributions from which the random numbers are drawn. The first type, $T1$, uses a probability density $f_{T1}(.)$, while the second type, $T2$ uses a probability density $f_{T2}()$. There are 5 of these generators on the table, and it is common knowledge that 2 of them are of type $T1$, while 3 of them are of type $T2$. A person chooses one generator at random. In expectation the probability distribution from which the generated number will be derived is the mixture between $f_{T1}(.)$ and $f_{T2}()$ with weights $\frac{2}{5}$ and $\frac{3}{5}$. Algebraically, the mixture distribution called $f_{M,T1,T2}()$ is equal to: $f_{M,T1,T2}() = \frac{2}{5}f_{T1}() + \frac{3}{5}f_{T2}()$. This closely relates to the use of basic rules for union of conditional probabilities. The density function $f_{M,T1,T2}()$ shows the probability densities attached to each number $x \in \Re$. Let's denote the probability density by small letter p, while discrete probability by a large letter P. Then,

$$p(x) = (P(T1) \cap p(x|T1)) \cup (P(T2) \cap p(x|T2))$$

, which exactly implies the formula for mixture distribution:

$$f_{M,T1,T2}() = \frac{2}{5}f_{T1}() + \frac{3}{5}f_{T2}()$$

, since $P(T1) = \frac{2}{5}$, $P(T2) = \frac{3}{5}$, $p(x|T1) = f_{T1}(x)$, $p(x|T2) = f_{T2}(x)$, and $P(T1)$ indep. $p(x|T1)$, $P(T2)$ indep. $p(x|T2)$, as well as $(P(T1) \cap p(x|T1)) \cap (P(T2) \cap p(x|T2)) = \emptyset$. For the following chapter it is also valuable to note, that mixture distributions are very often multiple-peaked. Combining two single-peaked distributions in a mixture, will most likely result in a two-peaked mixture distribution, unless the peaks of both elementary distributions are in exactly the same place.

1.3.4   Distributions of Order Statistics

In the following sections distributions of first and second highest order statistic are often referred to. Due to the fact that in literature there are different conventions, the current introduction provides the definitions and naming used throughout the document. The highest order statistic out of $\mathcal{Z}$ draws from a distribution with probability density function $f_{\mathcal{A}}(x)$, and cumulative density function $F_{\mathcal{A}}(x)$, is denoted $f_{1\mathcal{A}}(x)$, where $\mathcal{A}$ refers to the original distribution, and 1 to the number of the order statistic, counting from the top. As it can be noticed the number of draws are not used in the notation for order statistic distributions used throughout the chapter. It is assumed that the number of draws used for calculations can be easily found from definitions, and that these do not have to be reminded in the notation used for order statistic distributions. For $\mathcal{Z}$ draws the formula for the highest order statistic is:

$$f_{1\mathcal{A}}(x) = \mathcal{Z}F_{\mathcal{A}}(x)^{\mathcal{Z}-1}f_{\mathcal{A}}(x)$$

For the second highest order statistic of $\mathcal{Z}$ draws from distribution $f_{\mathcal{A}}()$ the formula is:

$$f_{2\mathcal{A}}(x) = \frac{\mathcal{Z}^2 - \mathcal{Z}}{2} F_{\mathcal{A}}(x)^{\mathcal{Z}-2}(1 - F_{\mathcal{A}}(x))f_{\mathcal{A}}(x)$$

Probability density functions are most commonly used throughout the chapter, but analogous naming convention is used for the cumulative distribution functions. The CDF of the highest order statistic of $\mathcal{Z}$ draws from distribution with cumulative distribution function $F_{\mathcal{A}}(x)$ is $F_{1\mathcal{A}}(x)$, and of the second highest order statistic: $F_{2\mathcal{A}}(x)$.

The general definition for the density function of the Kth highest order statistic from $\mathcal{Z}$ draws from $f_{\mathcal{A}}(x)$ is:

$$f_{K\mathcal{A}}(x) = \frac{\mathcal{Z}!}{(\mathcal{Z} - K)!(K - 1)!}(F_{\mathcal{A}}(x))^{\mathcal{Z}-K}(1 - F_{\mathcal{A}}(x))^{K-1}f_{\mathcal{A}}(x) \qquad (1.3.16)$$

1.4 Model of Sequential Auctions with Overlapping Generations of Bidders

The model of sequential auctions relates to the case of auctioning several items in a sequence of periods. I will keep to the notation that each period one auction is taking place. The attention in the previous section has been purposefully drawn to several aspects of this model. In particular, in order to calculate the amount of discounting, the expectation of the distribution of bids in the following period need to be known. As mentioned above, this distribution is calculated under the assumption of rationality of other bidders. In most of the literature, the expected surplus, used for discounting is calculated with either the assumption of exogenous, constant distribution of other bids, or with the assumption that all bidders enter the bidding in the same auction, bid in a sequence, and later all leave together after the final auction in the sequence. In short, that second assumption could be equivalently phrased as: all bidders are assumed to be in the same period, or the same "age"- expected to leave after the same sequence of

auctions. According to y knowledge it has never been studied what impact on bidding could an introduction of bidders entering at different time periods in the sequence of auctions have. Clearly, bidders entering at different auctions would imply coexistence of bidders with different lengths of horizons, and as a result with different period-specific strategies in the same auction. There is no reason to take for granted that this would not have any impact on the equilibrium strategies, which is why this paper extends the model of sequential auctions by this crucial element. The current paper proposes a model, where bidders enter the sequence of auctions at different periods. The model has been named "overlapping generations model", because the groups of bidders entering the sequence of auctions at the same period are called generations, and the numbering of auctions in which each bidder participate is the age of the bidder. The total number of auctions the bidder participates in is called the life-span of bidder. The model I am proposing is a cyclical model, in the sense that bidder's lifespan is defined a priori, and new generations of bidders enter in each period until the infinite future. In most cases, bidders who enter the game have an unknown, infinite number of auctions behind them, as well as in front of them, and therefore the equilibrium for such case is considered. It is not as common that the bidder enters, for example, in the first auction or at an auction of a known order. The arrival at equilibrium happens in an evolutionary way beginning with the first auction, and if the beginning of a sequence of auctions were analyzed, no stationary equilibrium could be found. The model focuses on some middle auction in the infinite sequence, where the rational assumption is that all bidders are in the stable equilibrium. Analyzing the beginning of the sequence o auctions is also interesting, but it does not give the possibility of finding a symmetric equilibrium, and does not, therefore, allow to get as much insight into the model. The following sections of this paper are detailing the model. In the first place, the Stationary Perfect Bayesian Equilibrium of the game is considered. Later, price announcements are introduced, and as a result the impact of the revealed information on learning is analyzed. Simulation results for the 2-period model are presented in the following section. Furthermore possible extensions to the model, and

their limitations, are discussed, and the last section contains conclusions containing the most important implications to be derived from the model.

1.4.1 Rules Of The Game

The game considered is a sequential auction game, where players are bidders in the auctions. The players choose their optimal strategies based on the assumption of rationality of other players. The basic rules of the game are as follows:

A1 Seller is taken as exogenous to the model, and an infinite series of identical objects are sold in a sequence of auctions. Each period one auction takes place, and periods are numbered $t \in \{1, 2, 3, ..., \infty\}$

A2 N new single-unit demand buyers enter in each period t. Each individual bidder is uniquely identified with their index $i \in I$, where the set of index is a set of Natural numbers $I = N$.

A3 Bidders life-span determines how many periods they participate in. 2-period-lived buyers take part in two auctions in a sequence, while 3-period-lived buyers take part in 3 auctions in a sequence. Bidder i's age is τ_i, and for two period lifespan bidders $\tau \in y, o$, while for 3-period lifespan bidders $\tau \in y, m, o$. The letters y, m, o are used in order to refer to the age as "young, medium or old", instead of numbering, since the numbers are reserved for time periods t. With longer lifespans more medium ages would be distinguished, so that $\tau \in y, m_1, ..., m_N, o$.

A4 Possible valuations for the object, v's, belong to the set $V \equiv \{v \in \Re | \underline{v} \leq v \leq \overline{v}\}$. $\underline{v}$ and $\overline{v}$ are the lower and upper bounds of the set V and in the simplest case it is assumed that $\underline{v} \equiv 0$ and $\overline{v} \equiv 1$. The distribution over these valuations has a cumulative distribution function (CDF) $F_v(v)$ with continuous and strictly positive probability density function $f_v(v)$. The support of the distribution $F_v(v)$ is the set V. Before the game starts each buyer gets their valuation v_i, which is

sometimes called their type. The valuations, v_i, are drawn independently from $f_v(v)$ (independent private values).

A5 All auctions follow immediately after each other, and there is no time discounting between periods

A6 Bidder i learns their valuation before the game starts, and knows that all other bidders' valuations are drawn independently from $F_v(v)$.

A7 Bidders are rational.

The simplest version of the model, where bidders have 2-period life span, is the main focus of the paper. The model is solved for the Symmetric Perfect Bayesian Equilibrium (PBE). The impact of price announcements is analyzed through introduction of an external observer and one bidder with a 3-period lifespan in the following sections.

1.4.2 Two Period Lifespan

Time periods are numbering auctions taking place in an infinite sequence. The numbering of periods is defined as $t \in \{1, 2, 3, ..., \infty\}$. Each period t, N new buyers arrive. First I will consider the case of some period t, where the number of auctions before and after t is an unknown λ_- and λ_+, and $\lambda_{\{-,+\}} \to \infty$. Current period will be often related to as period t, and therefore previous period as $t - 1$, and the future period as $t + 1$. Due to the cyclical nature of the model, any period where $\lambda_{\{-,+\}} \to \infty$ could be period t as it is exactly the same in terms of bidders equilibrium strategies, this is why t, $t - 1$ and $t + 1$ are just used to distinguish the sequence of periods in relative not absolute terms. The buyers, who take part in the auctions, live two periods each, and therefore, from their point of view there are only two periods, which are defined as their age $\tau \in \{y, o\}$. Period y is the first auction when the buyer arrives at the auctioning place, while period o is the second (and last) auction in which the buyer participates. The letters y and o relate to the bidder's young and old age. There is no information revealed about the

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price achieved in the previous auctions, and therefore the information available to bidder i is their own valuation v_i, the distribution from which valuations are drawn $f_v()$, as well as the rules of the game consisting of the number of bidders entering each period, the bidders' lifespan, time period t at which they enter the sequence, as well as $\lambda_{\{-,+\}}$, and the fact that the objects are being sold in a sequence of second-price auctions. For all the auctions far from the start of the auctioning, where $\lambda_{\{-,+\}} \to \infty$ there exists a symmetric equilibrium strategy $S^*(v_i) = \{b_y^*(v_i), b_o^*(v_i)\}$ which consists of decision bidding functions for age y and o of bidders as a function of the bidder's type. The equilibrium bidding strategies will be defined for the two periods τ for the buyer.

Since there are N new buyers with age $\tau = y$ at each time period t, the system can be in one of the two possible states $S \in \{A, B\}$, where the states are defined by the number of bidders in their period y, and o. Transition between states depends on the τ of the winning bidder in the previous auction, in $t-1$, which makes the state diagram a closed graph. Equilibrium needs to be solved for the strategies of the buyers, but, due to the game's cyclicality, other elements such as probabilities of states A and B , and distributions of bids in state A and B, will be part of the solution as well. What will become clear later, the equilibrium to this game is a fixed point, which consists of buyers strategies $S^*(v_i) = \{b_y^*(v_i), b_o^*(v_i)\}$, probabilities of each of the states $\{P^*(A), P^*(B)\}$ as well as the probability densities of bids for each generation $\{f_y^*(), f_o^*()\}$ and probability densities of bids in each of the states $\{f_A^*(), f_B^*()\}$. Other elements which are part of the equilibrium solution are probabilities of each age winning in each of the states $\{Pr_S^*(\tau)\}_{\{S \in \{A,B\}; \tau \in \{y,o\}\}}$.

Definition 7. $S(n_y, n_o)$ is state S with n_y bidders with $\tau = y$ and n_o bidders with $\tau = o$.

The system can be in one of two states at each point in time. These states are defined as $A(N, N-1)$ and $B(N, N)$. Figure 1 shows the transition diagram between the two states.

Definition 8. $Pr_S(\tau)$ is the probability that a bidder with age τ wins in state S

Figure 1.1: Two-period-lived Bidders Model, State Diagram
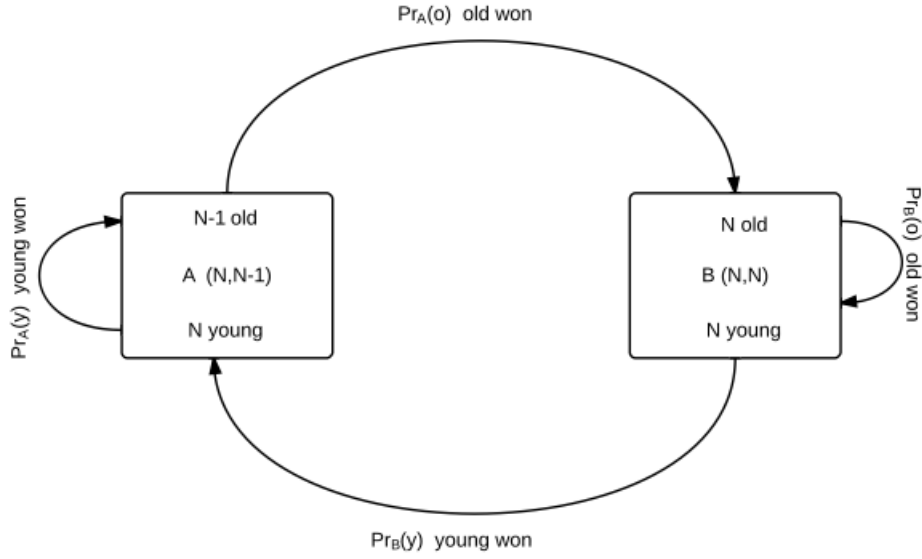
Given no price announcements, the bidders cannot infer in which state the system is. Therefore the young and old bidder's strategy has to be found without this information. Young bidder (at $\tau = y$) will have an intention to reduce their bid by the expected surplus from his $\tau = o$.

Definition 9. $Pr(S)$ *is the probability that the system is in state* S.

A general definition for order statistics from distributions, used throughout:

Definition 10. $f_{i,D}(.)$- *PDF of the i-th highest bid from distribution* D. *For example* $f_{2,A}$ *is the PDF of the second highest bid in state* A.

Definition 11. *All individual valuations are elements of the set of valuations* V, *between* $[\underline{v}, \overline{v}]$

Definition 12. *The set of young bids is* Y, *between* $[\underline{v}, \overline{z}]$, *where* $\overline{z} < \overline{v}$, *as* $\overline{z} = b_y^*(\overline{v})$

Definition 13. *The bidding function for old bidders* $f_o^*(v) : V \to V$, *and the bidding function for young bidders* $f_y^*(v) : V \to Y$

Bidder in their period $\tau = y$ will bid according to $b_y(v)$, while in period $\tau = o$ according to $b_o(v)$. These functions are the symmetric Bayes Nash equilibrium strategies

for the buyer's maximization problem. The densities of the distributions of old and young bidders' bids combined together as a mixture distribution form the distribution of bid in states A and B. The weights for mixture distribution in each of the states depend on the proportion of period $\tau = y$ and $\tau = o$ bidders.

Definition 14. $f_v(v)$ - *PDF of bidders valuations, for which support is the set V*

Young bidders have a symmetric strategy in equilibrium to shade their bid.

Definition 15. $f_y(z)$ - *PDF of young bidder's bids, for which support is the set Y.*

Distributions of old bidders' bids are different in each of the two states. In state B, with N old bidders, all of the bidders participating in the first period as young go on to the second period as old, and this transition is conditional on young not winning in $t - 1$ in any of the states. In state A, there are N-1 old bidders, and that is due to the fact that in any of the states in $t - 1$ one of the young bidders have won, so conditional on a young winning, the remaining bidders are present as old in state A. I will define these distributions below, but first a general definition for the remaining bidders:

Definition 16. $f_{R,(D)} = \sum_{n=1}^{N-1} \frac{1}{N-1} f_{n,(D)} = \frac{1}{N-1}(N f_D - f_{1,D})$. *Distribution of all but the highest order statistic from the original distribution D with N draws. That is also equal to $\frac{1}{N-1}(N f_D - f_{1,D})$.*

In the definitions of old bidders distributions some terms, which more detailed definition will follow later need to be used, so for now I will just briefly introduce them: $b(v)$ - is the bidding function of young bidders, and it maps valuations to young bidders' bids; $f_A(s)$ - is the distribution of bids in state A; $f_B(s)$ - is the distribution of bids in state B; $P(A)$ - the probability of state A; $P(B)$ - the probability of state B.

In order to find the distribution of old bidders in state A and B, first the distribution of bidders who continued to the next state needs to be found. In expectation slightly different distribution of young bidders continue to state A than to state B. First, let's

look at state B. Only young bidders who haven't won continue as old to state B, but additionally, it is the case that in states before state B an old bidder has won. Both these information together mean that the valuations of young bidders who continue to state B are on average higher than the valuations of bidders who continue to state A.

Definition 17. *The young bidders who continue to state B are distributed with probability density* $:f_{y_{t-1},B}(z) = P(A)(f_{R,A}(z)f_y(z) * C_1) + P(B)(f_{R,B}f_y(z) * C_2)$

Definition 18. $f_{o,B}(v) = f_{y_{t-1},B}(b_y^{-1}(z))$. *The distribution of old bidders' bids in state B is not exactly the distribution of valuations, there is additional information that is now revealed, the old bidders in state B have not won as young bidders, so as young, they are the remaining bidders (all but the highest bidder from the state distribution), but they are also all the young bidders (so their distribution is conditional on young not winning). To recover this distribution for the old bidders, the inverse of the bidding function needs to be applied.*

In state A there are N-1 old bidders, due to the fact that state A follows after a young person has won in the previous period. Below I will define the distribution of old bidders in state A - the state after a young bidder has won. It is not the distribution of all but the highest order statistic from distribution f_v. The additional information that is known, is that the old bidder not present any more in state A must have won in $t-1$ either in state A or in state B as a young bidder. The bidder who was a winning bidder was from the highest order statistic distribution (was the highest bidder). The remaining young bidders in $t-1$ are the remaining of the N young bidders (so N-1 young bidders), but are also among the remaining bidders in the given state (as the winner is not present), so they are remaining young bidders conditional that the young bidder who left was also the winning bidder in the state.

In order to recover the distributions of valuations of old bidders in state A, first the distribution of young bidders in period before who continue to state A needs to be revealed.

Definition 19. *The distributions of young bidders in period $t-1$ who continue to state* A *is:* $f_{y_{t-1},A}(z) = P(A)(f_{R,A}(z)f_{R,y,A}(z) * C) + P(B)(f_{R,B}(z)f_{R,y,B}(z) * C_4)$

Definition 20. $f_{o,A}(v) = f_{y_{t-1},A}(b_y^{-1}(z))$ *- the distribution of old bidders in state A is the inverse of a bidding function of the distribution of young bidders who have not won in $t-1$. Young bidders who have not won in $t-1$ consist of those from state A and those from state B with mixture probabilities being the equilibrium probabilities of state A and B. In each of the states the winning young bidder as a young must have been the highest order statistic from the young bidders distribution (with N young bidders)- that is $f_{1,(A,y)}$ and $f_{1,(B,y)}$. These are not the same as just the highest expected bidder from the Young distribution in both states, because the winning bidder won in the particular known circumstances (that is one of the states). Therefore instead of just subtracting the highest order statistic, the subtracted bid density is different, and that is:*

$$f_{R,y,A}(z) = \frac{1}{N-1}(Nf_y(z) - (f_{1,y}(z)f_{1,A}(z) * C_5)) * C_6$$

$$f_{R,y,B}(z) = \frac{1}{N-1}(Nf_y(z) - (f_{1,y}(z)f_{1,B}(z) * C_7)) * C_8,$$

and these are derived using the identity from definition 7. The highest order statistic subtracted from f_y is updated as it is known that the winning bidder has won given the distribution in state A or B. Bayesian updating requires normalizing constant.

The young winning bidder must have been the highest bidder of all the bids in the given state, therefore at the same time the winning bidder was from the distribution $f_{1,A}$ or $f_{1,B}$. The loosing young bidders conditional on the fact that a young bidder has won are from the distribution of the reminding order statistics from the young bidders as well as overall in each state, so that is the intersection of these: $f_{R,y} \cap f_{R,S}$, and the intersection of these probabilities is their multiplication. Because of the fact that everything is expressed in terms of probability density, the expressions need to be multiplied by a normalizing

constant, so that the resulting formula is a probability density function as well (C_i above).

As state A follows after a young bidder has won, we get new information about the distribution of old bidders in this state. There could be different reasons why a young bidder has won: either the distribution of old bidders was unusually low, or that young bidder's bid was very high (it was a bidder in the very top section of valuations). The young bidder won despite the fact that he has faced some old bidders in their period - bids were either from mixture distribution of state A or state B at the time. This means that the highest order statistic (representing the person who has won) was even more skewed to the right in comparison to the standard highest order statistic distribution. The winning bidder was the highest of young bidders, but also the highest of all bidders, and this is why the formula for $f_{R,y,A}$ and $f_{R,y,B}$ is different than what would be the formula for $f_{R,y} = \frac{1}{N-1}(Nf_y - f_{1,y})$. There is updating on the highest bidder who left the distribution. This doesn't happen for state B, after an old bidder has won. There, all the previously young bidders follow through to the next period. In both cases there is updating on those who follow through, as we know they were not the highest bidders in $t-1$. The main driver for the differences in the distributions of state A and B, though, is the young bidder who has won, and left the competition in state A.

The distributions from which the bids arise in each of the states are mixture distributions. In state A (the state after a young bidder has won) there are N young and N-1 old bidders, so the mixture weights are $\frac{N-1}{2N-1}$ for $f_{o,A}$ and $\frac{N}{2N-1}$ for f_y. In state B (the state after an old bidder has won) there are N young and N old bidders, so the mixture weights are $\frac{1}{2}$ for each $f_{\tau,B}$.

Definition 21. $F_A = \frac{N-1}{2N-1}F_{o,A} + \frac{N}{2N-1}F_y$ *is the CDF and* $f_A = \frac{N-1}{2N-1}f_{o,A} + \frac{N}{2N-1}f_y$ *is the PDF of bids in state A.*

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state after an old bidder has won) there are N young and N old bidders, so the mixture weights are $\frac{1}{2}$ for each $f_{\tau,B}$.

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Then the CDF of the highest bid in state A is $F_{1,A} = F_A^{2N-1} = (\frac{N-1}{2N-1}F_{o,A} + \frac{N}{2N-1}F_y)^{2N-1}$ and the CDF of the highest bid in state B is $F_{1,B} = F_B^{2N} = (\frac{1}{2}F_{o,B} + \frac{1}{2}F_y)^{2N}$. Accordingly, the respective PDFs of the highest bids are $f_{1,A}$ and $f_{1,B}$.

Given the mixture probabilities distributions of bids in state A, and state B as above, the probability that a given bid comes from a young bidder in state A is

$$Pr_A(b = young) = \frac{\frac{N}{2N-1}f_y(b)}{\frac{N-1}{2N-1}f_{o,A}(b) + \frac{N}{2N-1}f_y(b)} \tag{1.4.17}$$

, and the probability that it is from an old bidder is

$$Pr_A(b = old) = \frac{\frac{N-1}{2N-1}f_{o,A}(b)}{\frac{N-1}{2N-1}f_{o,A}(b) + \frac{N}{2N-1}f_y(b)} \tag{1.4.18}$$

. These two probabilities sum to 1, as they should, and the probability that a given bid is a winning bid is the sum of the probabilities that this bid is a young winning bid and an old winning bid:

$$Pr_A(b = b_1) \quad = f_{1,A}(b) \quad = Pr_A(b = b_1 \cap b = young) + Pr_A(b = b_1 \cap b = old) =$$
$$= f_{1,A}(b)Pr_A(b = young) + f_{1,A}(b)Pr_A(b = old)$$

The winning bid in state A can be divided between a young person's winning bid and an old person's winning bid. Somebody always wins, so the sum of these two is 1 (the CDF $F_{1,A}(\bar{v}) = 1$).

$$1 \;=\; \int_0^{\overline{v}} Pr_A(b = b_1)db = \int_0^{\overline{v}} f_{1,A}(b)db =$$

$$=\; \int_0^{\overline{v}} f_{1A}(b)Pr_A(b = young)db + \int_0^{\overline{v}} f_{1A}(b)Pr_A(b = old)db =$$

$$=\; Pr_A(y) + Pr_A(o)$$

So the probability that in state A a young one has won is

$$Pr_A(y) = \int_0^{V_H} f_{1,A}(b)Pr_A(b = young)db \qquad (1.4.19)$$

, while the probability that an old person has won in state A is

$$Pr_A(o) = \int_0^{V_H} f_{1,A}(b)Pr_A(b = old)db \qquad (1.4.20)$$

In state B
$$Pr_B(b = young) = \frac{\frac{1}{2}f_y(b)}{\frac{1}{2}f_{o,B}(b) + \frac{1}{2}f_y(b)} \qquad (1.4.21)$$

,and

$$Pr_B(b = old) = \frac{\frac{1}{2}f_{o,B}(b)}{\frac{1}{2}f_{o,B}(b) + \frac{1}{2}f_y(b)} \qquad (1.4.22)$$

. Analogically,
$$Pr_B(y) = \int_0^{V_H} f_{1B}(b)Pr_B(b = young)db \qquad (1.4.23)$$

and

$$Pr_B(o) = \int_0^{V_H} f_{1B}(b)Pr_B(b = old)db \qquad (1.4.24)$$

.

In order to calculate probability of state A, let us notice that:

$$P(A) = P(A)Pr_A(y) + P(B)Pr_B(y)$$

Also, $P(A) + P(B) = 1$, so

$$P(A) = P(A)Pr_A(y) + (1 - P(A))Pr_B(y)$$

And solving for A:

$$P(A) = \frac{Pr_B(y)}{1 - Pr_A(y) + Pr_B(y)}$$

In the same way from

$$P(B) = P(A)Pr_A(o) + P(B)Pr_B(o)$$

We can solve for B:

$$P(B) = \frac{Pr_A(o)}{1 - Pr_B(o) + Pr_A(o)}$$

[2]

The above equations determine how different parts of the model are interrelated: the probabilities of states, distributions of bids, and bidding function. The last element is to derive the bidding function in equilibrium. In the same way, as in the previous section, which derived the basic model without overlapping generations, the PBE equilibrium strategies are going to be derived sequentially from the last auction, and it is going to be defined according to Theorem 1 - the equilibrium bidding function for bidder of age τ_j needs to be the solution to maximization of bidder's surplus at the age τ. If the maximization provides a solution that is independent of the distribution of bids in the same period, then that is the equilibrium bidding strategy for bidder i. Beliefs are consistent and are used for calculation of expected surplus from the continuation game.

---

[2]A check that $P(A)$ and $P(B)$ sum to 1:

$$
\begin{aligned}
P(A) + P(B) &= \frac{Pr_B(y)}{1 - Pr_A(y) + Pr_B(y)} + \frac{Pr_A(o)}{1 - Pr_B(o) + Pr_A(o)} = \\
&= \frac{1 - Pr_B(o)}{1 - (1 - Pr_A(o)) + (1 - Pr_B(o))} + \frac{Pr_A(o)}{1 - Pr_B(o) + Pr_A(o)} = \\
&= \frac{1 - Pr_B(o) + Pr_A(o)}{1 - Pr_B(o) + Pr_A(o)} = 1
\end{aligned}
$$

The equilibrium bidding as a function of type v, along with the consistent beliefs, is the symmetric equilibrium.

In the case of two-period-lived buyers their equilibrium bidding strategy has to be defined for two ages of the bidder, the last period in which the bidder takes part, age o for him (when the bidder is 'old'), and the first period in which the bidder participates: age y for the bidder (when the bidder is 'young').

Theorem 7. *The symmetric equilibrium bidding strategy for a type v-bidder in period o and y of a sequential second-price auction with overlapping generations of two-period-lived bidders is given by b_o^* and b_y^* defined as:*

$$
\begin{aligned}
b_o(v_i) &= v_i \\
b_y(v_i) &= v_i - W_o^*(v_i) \\
&= v_i - \left(P(A) \int_0^{v_i} (v_i - b_2) f_{2,A}(b_2) db_2 + P(B) \int_0^{v_i} (v_i - b_2) f_{2,B}(b_2) db_2 \right)
\end{aligned}
$$

Proof. All young bidders will have a symmetric optimal bidding strategy as a function of their valuations. The optimal bid for a young bidder i, $b_y(v_i)$, is the solution to first period maximization problem of the young bidder with valuation v_i.

$W_o(v_i)$ - Expected surplus of bidder of type v_i and age $\tau = o$. Below, notation $W_{v_i,o}$ is used :

$$
W_{v_i,o} = P(A) \int_0^{b_{v_i,o}} (v_i - b_2) f_{2,A}(b_2) db_2 + P(B) \int_0^{b_{v_i,o}} (v_i - b_2) f_{2,B}(b_2) db_2
$$

For a continuous and differentiable function $W_o(v_i)$ if it enough now to find the first and second order condition. If function $W_o(v)$ is not continuous, then in addition to stationary points, the limits at discontinuity need to be analyzed. In the first case, let us assume that the solution to maximization is enough. The First order condition. Differentiating

w.r.t. $b_{v_i,o}$ to find the maximum:

$$\frac{\partial W_{v,o}}{\partial b_{v,o}} = P(A)(v - b_{v,o})f_{2A}(b_{v,o}) + P(B)(v - b_{v,o})f_{2B}(b_{v,o}) = 0$$

$$(v - b_{v,o})[P(A)f_{2A}(b_{v,o}) + P(B)f_{2B}(b_{v,o})] = 0$$

$$b_{v,o} = v$$

Substituting the solution back in the surplus in period o gives the expected surplus in $\tau = o$:

$$
\begin{aligned}
W_{v,o}^* &= P(A)\int_0^v (v - b_2)f_{2,A}(b_2)db_2 + P(B)\int_0^v (v - b_2)f_{2,B}(b_2)db_2 \\
&= P(A)Pr(b_{2A} \leq v)(v - E[b_{2A}|b_{2A} \leq v]) \\
&+ P(B)Pr(b_{2B} \leq v)(v_i - E[b_{2B}|b_{2B} \leq v])
\end{aligned}
$$

$b_{v,y} = b_y(v)$ - bid of type v when they are young.

Surplus in $\tau = y$:

$$
\begin{aligned}
W_{v,y} &= P(A)(\int_0^{b_{v,y}}(v - b_2)f_{2,A}(b_2)db_2 + \int_{b_{v,y}}^\infty W_{v,o}^*f_{2,A}(b_2)db_2) + \\
&\quad + P(B)(\int_0^{b_{v,y}}(v - b_2)f_{2,B}(b_2)db_2 + \int_{b_{v,y}}^\infty W_{v,o}^*f_{2,B}(b_2)db_2)
\end{aligned}
$$

Differentiating w.r.t. b_y to find the maximum:

$$\frac{\partial W_{v,y}}{\partial b_{v,y}} = P(A)(v - b_{v,y} - W_{v,o}^*)f_{2A}(b_{v,y}) + P(B)(v - b_{v,y} - W_{v,o}^*)f_{2B}(b_{v,y}) = 0$$

$$(v - b_{v,y} - W_{v,o}^*)(P(A)f_{2A}(b_{v,y}) + P(B)f_{2B}(b_{v,y})) = 0$$

$$b_{v,y} = v - W_{v,o}^*$$

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The bidding strategy of young bidder is a function of the bidder's type, v_i, and the bidder's expected surplus in $t+1$, which is $W_o^*(v_i)$. As we know, the equilibrium bidding function for old bidders has been found, and it is independent of the distribution of other bids in the same period, $b_o(v_i) = v_i$. The proposed solution for optimal bid of young bidder is $b_y^*(v_i) = v_i - W_o^*(v_i)$. All young bidders will find the same function as a solution to their maximization problem, independently of what the others decide to do in the same period. The expected surplus from $\tau = 0$ is calculated using the beliefs about distributions of bids in continuation game which are consistent with the equilibrium strategies. The equilibrium strategies are sequentially rational and use consistent beliefs.

Remark. *The young bidders bidding function $b_y^*(v_i) = v_i - W_o^*(v_i)$ is the solution to maximization, and is independent of distribution of other bids in the same period t.*

In the next period $t+1$, when the bidder i is old, the expected distribution of other bids needs to be calculated under the assumption that everyone bids according to their equilibrium strategy. That is, for all bidders $b_o^*(v_i)$, and $b_y^*(v_i)$. The game is cyclical, and therefore the solution to equilibrium bidding function does not change with t. The same function determines the solution to equilibrium strategy for each bidder i in every period t considered.

Remark. *Due to the game's cyclicality, for all periods t, where $\lambda_{+,-} \to \infty$, the solution to symmetric equilibrium bidding function of young and old bidders is the same independently of period t.*

$\square$

We can also find out what is the expected surplus of each bidder i. Substituting equilibrium strategies back to the surplus equation to get the total expected surplus for

the young bidder::

$$
\begin{aligned}
W_{v,y}^* \;=\; & P(A)\!\left(\int_0^{v-W_{v,o}^*}(v-b_2)f_{2,A}(b_2)db_2 + \int_{v-W_{v,o}^*}^{\infty} W_{v,o}^* f_{2,A}(b_2)db_2\right) + \\
& +P(B)\!\left(\int_0^{v-W_{v,o}^*}(v-b_2)f_{2,B}(b_2)db_2 + \int_{v-W_o^*}^{\infty} W_{v,o}^* f_{2,B}(b_2)db_2\right)
\end{aligned}
$$

Given this bidding strategy as a function of v_i no bidder has any intention to deviate as it would reduce their expected surplus. Buyers do not know which state they are in, and they are using rational expectations of probabilities for each of the states, and the distribution of bids in each of the states to infer their optimal strategy. All those elements together form the equilibrium bidding. In order to solve for the equilibrium, all the equations describing the relations between probabilities, density functions of bids, and equilibrium strategies need to be solved simultaneously to find a fixed point. The fixed point for all the equations describing the model in equilibrium is going to be the solution to the symmetric PBE.

Definition 23. *The **symmetric stationary Perfect Bayesian Equilibrium** of an infinite horizon overlapping generations auction game is the PBE for the case where the horizon of past and future auctions is infinite, $\lambda_{+,-} \to \infty$, and, as a result each generation has the same equilibrium strategy, independently of time period t. Such a game is cyclic, and the strategies need to be solved for each τ only. The beliefs need to be consistent with the strategies, and the strategies sequentially rational.*

Remark. *The stationary equilibrium bidding function for the cyclical auction game described above is a fixed point for the system of equations describing equilibrium strategies $b_o^*(v_i)$, $b_y^*(v_i)$, probabilities of states $P^*(A)$, $P^*(B)$, and probability densities $f_{o,A}^*()$, $f_{o,B}^*()$, $f_y^*()$, $f_A^*()$, $f_B^*()$.*

The last element of the puzzle is to prove that the surplus function $W_{y,v}(b_{y,v})$ as well as $W_{o,v}(b_{o,v})$ are continuous and differentiable in the whole domain, and therefore the way

the optimization is solved is justified.

Remark. *The surplus functions $W_{y,v}(b_{y,v})$ and $W_{o,v}(b_{o,v})$ need to be continuous and differentiable in the whole domain.*

Theorem 8. *The surplus function $W_{o,v_i}(b_{o,v_i}) = P(A) \int_0^{b_{o,v_i}} (v_i - b_2) f_{2,A}(b_2) db_2 + P(B) \int_0^{b_{o,v_i}} (v_i - b_2) f_{2,B}(b_2) db_2$ is continuous and differentiable in the whole domain.*

Proof. The domain for the surplus function is $b_{o,v_i} \in [\underline{v}; \overline{v}]$. $P(A)$ and $P(B)$ are probabilities of states A and B, and are constants. Since the sum of two continuous and differentiable functions is also continuous, it is enough to show that each of the components $P(A) \int_0^{b_{o,v_i}} (v_i - b_2) f_{2,A}(b_2) db_2$ and $P(B) \int_0^{b_{o,v_i}} (v_i - b_2) f_{2,B}(b_2) db_2$ are continuous and differentiable functions of b_{o,v_i}. Differentiability implies continuity, and the first and second derivative of this functions exist, which proves that $W_{o,v_i}(b_{o,v_i})$ is continuous and differentiable. $\qquad\square$

Theorem 9. *The surplus function $W_{y,v_i}(b_{y,v_i}) = P(A)(\int_0^{b_{y,v_i}} (v_i - b_2) f_{2,A}(b_2) db_2 + \int_{b_{y,v_i}}^{\infty} W_{v_i,o}^* f_{2,A}(b_2) db_2) + P(B)(\int_0^{b_{y,v_i}} (v_i - b_2) f_{2,B}(b_2) db_2 + \int_{b_{y,v_i}}^{\infty} W_{v_i,o}^* f_{2,B}(b_2) db_2)$ is continuous and differentiable in the whole domain.*

Proof. Analogically to the proof above. $W_{y,v_i}(b_{y,v_i})$ is a sum of continuous and differentiable functions of b_{y,v_i}, which proves that $W_{y,v_i}(b_{y,v_i})$ is also continuous and differentiable. $\qquad\square$

Continuity and differentiability are the necessary conditions for finding the optimum bidding function through maximization, and these are satisfied. As a conclusion the PBE equilibrium strategies for young and old bidders has been found in this section. The conclusion for this section is the theorem below:

Theorem 10. *For a sequential second price auction game described as above, there exist a stationary Perfect Bayesian Equilibrium, and it is described by strategies for each age of the bidders $b_o^*(v_i)$, $b_y^*(v_i)$, probabilities of states $P^*(A)$, $P^*(B)$, and probability densities*

$f_{o,A}^*(z)$, $f_{o,B}^*(z)$, $f_y^*(z)$, and $f_A^*(z)$, $f_B^*(z)$. *The equilibrium is a fixed point where all the equations are satisfied simultaneously.*

This means that for any distribution $f_v()$, and number N of new bidders entering each period t, the equilibrium can be found for $b_o^*(v_i)$, $b_y^*(v_i)$, $P^*(A)$, $P^*(B)$, $f_A^*()$, $f_B^*()$, $f_{o,A}^*()$, $f_{o,B}^*()$, and f_y^*. The equilibrium is also where the expected outcome of the game can be calculated.

The fixed point for any combination of inputs can be calculated through my Mathematica program olg2_fixed_point. The program, and it's outputs are described in the section called "Numerical Approach".

1.5   Price announcements

Price announcements considered here are seen only by an external observer, not by the bidders taking part in the auctions. Of interest here is to demonstrate what information price announcements carry, without disruption of the cyclic equilibrium of the game. An external observer is aware of the periods in the game and this section aims at showing how the observer's expectations about distributions of bids in the future periods are affected by price observation. The case where bidders could update their strategies as young bidders based on price announcements would have heavy implications for their bidding strategies and it would not be possible to arrive at an equilibrium. On the other hand, an introduction of an external observer to the environment where the 2-period model is in equilibrium allows to investigate the impact of price announcements, including the depth of the impact for future periods.

Links between States on the state diagram are important for learning. Some generalizations and vocabulary related to links between states are given below, and after that the analysis of price announcements continues.

1.5.1 Links Between States

Remark. *All the states and links between them can be represented using a directed graph. States of the world are the nodes in the graph, and links between states are edges. Each edge, $e_{i,j,a} \in E$ is directed from origin node, i to destination node, j. There are N outgoing edges from each of the nodes in the graph of states. Each edge has also another attribute, which is the age of the winning bidder, $a \in \tau$, as well as weight, which is the probability of this edge being used conditional on the origin state.*

Definition 24. *The weight of the directed link with origin at state S_1 and an end at state S_2, so the edge $e_{S_1,S_2,a}$, is denoted $g(e_{S_1,S_2,a})$. The weight is the probability of this edge conditional on the origin state: $g(e_{S_1,S_2,a}) = P_{S_1}(a)$.*

All the possible combinations of origin state, end state, and winning generation gives the set of all the possible edges.

Definition 25. *The set of all possible edges contains $e_{S_1,S_2,a}$ with all the combinations of origin state $S_1 \in S$, end state $S_2 \in S$, and winning generation $a \in \tau$.*

As it is clear, some of the edges have zero probability, and therefore these are not drawn on the graph diagram and have no effect on calculations. In order to generalize the definition it is convenient to introduce the definition for the non-existent edges:

Definition 26. *The non-existent edges are the edges $e_{S_1,S_2,a}$, s.t. $g(e_{S_1,S_2,a}) = 0$.*

The probability of a given state x in time t, $S_{x,t}$ is the sum of the probabilities of all the incoming edges to this state, multiplied by the probability of the origin states in $t-1$. This is a very general definition, and extends towards models with more generations, and larger state diagrams. Due to the fact that at the equilibrium the model is the same each period, including all the probabilities, the time notation does not need to be included, if nothing in the game conditions change. The time notation becomes useful if there is price observation after the bid in $t-1$ has been placed. The probability of any state conditional on another state in $t-1$ is defined as follows:

Definition 27. *The relation between the probability of state $S_{x_1,t}$ conditional on state $S_{x_2,t-1}$ is:*

$$P(S_{x_1}, t | S_{x_2,t-1}) = \sum_{m=1}^{M} (g(e_{S_{x_2}, S_{x_1}, a_m})) \qquad (1.5.25)$$

Where, it is clear that only one such edge has a non-zero probability (as there can be at most one directed link from state x_2 to x_1). All other edges from x_2 to x_1 are non-existent.

Definition 28. *The relation between the probability of state $S_{x_1,t}$ and the probabilities of states in $t-1$ is:*

$$P(S_{x_1,t} | t-1) = \sum_{i=1}^{Q} P(S_{x_i,t-1}) \sum_{m=1}^{M} (g(e_{S_{x_i}, S_{x_1}, a_m})) \qquad (1.5.26)$$

And here we know that there is at most one directed edge from state x_i to x_1 with non-zero probability, but in some cases there may be zero such edges between some x_i and x_1, and then the whole sum $\sum_{m=1}^{M} (g(e_{S_{x_i}, S_{x_1}, a_m}))$ is equal to zero for this i.

The above definitions are very general and allow for extending the model to more generations, and states. Since only the two-generations model is considered here, it is clear from the state diagram which edges exist with non-zero probability.

The probabilities of the two states in t conditional on state B in $t-1$ are the weights of the edges with origin at state B:

$$P(A_t | B_{t-1}) = g(e_{B,A,y}) = Pr_B(y)$$
$$P(B_t | B_{t-1}) = g(e_{B,B,o}) = Pr_B(o)$$

The probabilities of the two states in t conditional on state A in $t-1$ are the weights of the edges with origin at state A:

$$P(A_t|A_{t-1}) = Pr_A(y)$$

$$P(B_t|A_{t-1}) = Pr_A(o)$$

1.5.2 Learning By An External Observer

Two of the most important differences of this model to the standard sequential auctions model is that there are two groups with different equilibrium strategies in each auction: young and old bidders, and that there are two possible states of the world, with different number of young and old bidders in the auction. This changes the way learning happens based on price observation: the price level can be used as an indication of the age of the winning bidder. This, in turn, provides information on the more likely state - since the distributions of bids in the two states are different. Because there are links between the state in $t-1$ and the state in t, acquiring information about the price in $t-1$ does not only give information about the state in $t-1$, but, through relative probabilities of links to different states from $t-1$ to t, leads also to updating of probabilities of the state in t, and further to the future about the probabilities of states in $t+1$ and the periods that follow. The effect of price observation in $t-1$ on learning can be divided into four main categories. First one is a **direct effect** that is updating of the belief about the **probability of State A and State B in t-1**. Based on price observation one state can become more likely than other. The second one is a **direct effect** on the belief about **probability of a young or old bidder winning in t-1**. Based on price observation the probability that a bidder of a given generation won can become more likely. The first two effects result in an update on the expected probabilities of **State A and State B in t**, which can be called a **secondary effect**. The fourth effect is **indirect, through links to both states** and relates to probability of **State A and State B in t+1, t+2, etc**. The price observation is made by an external observer and therefore the equilibrium strategies of bidders are not affected. Links between t and $t+1$, $t+2$ etc. are the sum of

Figure 1.2: Learning By an External Observer Who Can Observe Price In t-1
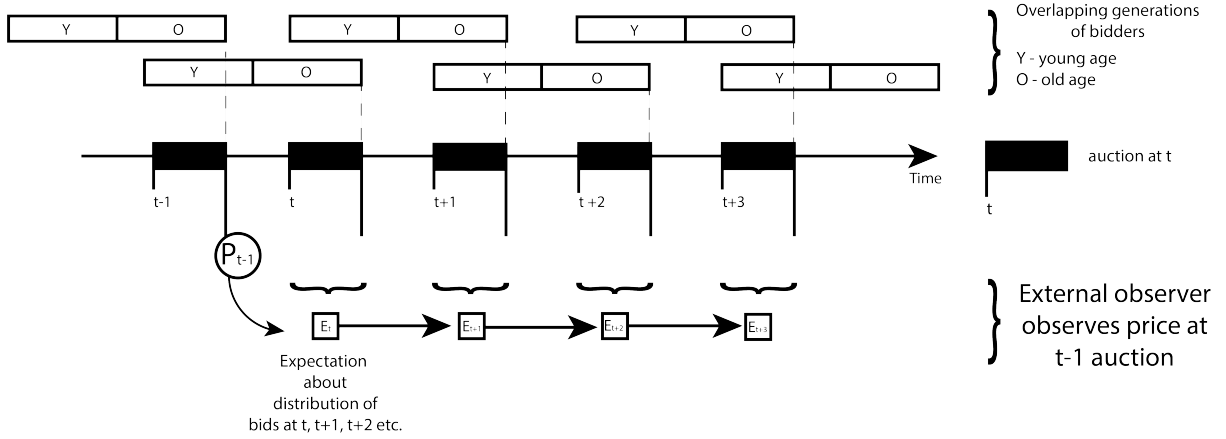
probabilities of young and old winning in each of the states multiplied by a probability of each given state. Through direct effect, the probabilities of State A and State B in t are updated based on the observed price in $t-1$. This new probabilities are then multiplied by the weight of edges between states to get the **probability of State A and State B in t+1**. Based on that the new probabilities of states in $t+1$ are used, and multiplied by the weight of edges between states give the probabilities of State A and State B in $t+2$. This continues further. The **learning effect of price p_{t-1} observation is highest in t, and diminishes with time**.

The notation used is that the acquisition of information happens in $t-1$: after the $t-1$ auction is finished, a resulting price is observed. Based on that, the observer is able to learn about the distribution of bids in $t-1$, and therefore also a little bit about the valuations of bidders. More importantly learning relates additionally to the composition of the young and old bidders - the probabilities of states A and B. Learning about the distribution in t is possible, as implied by the conclusions from the updating of beliefs about what was the state and who won in $t-1$. Consequently, the updated beliefs about distribution and composition of bidders in t leads to updating of beliefs about the expected distribution of bids in $t+1$ and further into the future.

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1.5.2.1 Direct Effects on Probabilities of States in in t-1

After observing the winning price, the belief about the state of the world of $t-1$ (A or B) can be rationally updated by bidders. Bidders know the probability densities of bids in state A and B under equilibrium, and they also know the expected probability of each of the states. Using Bayes' rule for conditional probabilities, the posterior probability after price observation can be derived. The winning price is the second highest bid, and therefore the probability of state A in $t-1$ after observing the price p_{t-1} is derived below:

$$
\begin{aligned}
P(A_{t-1}|p_{t-1}=x) &= \frac{P(A_{t-1} \cap q_{t-1}=x)}{P(q_{t-1}=x)} \\
&= \frac{P(b_1 > x|A)P(A)}{P(b_1 > x|A)P(A) + P(b_1 > x|AB)P(B)} \\
&= \frac{(1 - F_{A,1}(x))P(A)}{(1 - F_{A,1}(x))P(A) + (1 - F_{B,1}(x))P(B)} \quad (1.5.27)
\end{aligned}
$$

It is worth noting that A and B are the only possible states, and therefore $P(p_{t-1} = x) = P(p_{t-1} = x \cap A_{t-1}) \cup P(p_{t-1} = x \cap B_{t-1})$. Moreover, there is no possibility that both states happen simultaneously, so $P(p_{t-1} = x \cap A_{t-1}) \cap P(p_{t-1} = x \cap B_{t-1}) = \emptyset$. The observed price means that the highest bid must be above, this is why a reminder of the cumulative distribution of first bid in each of the states best represents this situation, where: $F_{S,1}(.)$ - the cumulative distribution function of the highest bids in state S.

The above formula uses Bayesian updating, taking into account the prior probabilities expected to be true in equilibrium : $P(A)$ and $P(B)$. In the same way, the formula for probability of state B in $t-1$ given the price $p_{t-1} = x$ is given below:

$$
\begin{aligned}
P(B_{t-1}|p_{t-1}=x) &= \frac{P(B_{t-1} \cap p_{t-1}=x)}{P(p_{t-1}=x)} \\
&= \frac{P(b_1 > x|B)P(A)}{P(b_1 > x|A)P(A) + P(b_1 > x|AB)P(B)} \\
&= \frac{(1 - F_{B,1}(x))P(A)}{(1 - F_{A,1}(x))P(A) + (1 - F_{B,1}(x))P(B)} \quad (1.5.28)
\end{aligned}
$$

After price observation, the probabilities of both states, A and B in $t-1$ are updated as shown above. The updating will be in favor of one of the states which dominates the other one in terms of the First Order Stochastic Dominance.

Since F_{1B} dominates F_{1A} in terms of First Order Stochastic Dominance, the higher the price the more likely that the winning bid was from State B.

The next step is shows how does an increase in observed price in $t-1$ relate to the update on the probability of state B versus A in $t-1$).

Let us consider how the probabilities of state A_{t-1} and B_{t-1} change once a price observed in $t-1$ is known to be x. In the notation below, the price in $t-1$ is denoted x or p_{t-1}.

Theorem 11. *Observed price p_{t-1} leads to an updating of the belief about the probabilities of states A and B in $t-1$. The relation is monotonous: a higher observed price in $t-1$ leads to an upward updating of probability of state B in $t-1$ and a downward updating of probability of state A in $t-1$.*

Proof. Distribution of bids in **State B** dominates distribution of bids in **State A** by First Order Stochastic Dominance. This implies that the probability of State B in $t-1$ increases with the higher observed price in $t-1$. $\qquad\square$

1.5.2.2 Direct Effects On Probabilities of Old And Young Win in t-1

Another direct effect is on probabilities of Old and Young winning in each of the states in $t-1$ based on p_{t-1}. This effect, though is not monotonous For some middle values the increment of probability that a Young bidder won will take place (based on an increase in p_{t-1}), since the young bidders' bids distribution has a different range than distribution of Old bids. First order Stochastic dominance may not, therefore hold. Another aspect is that above a certain point the probability of old bidder's win is increasing with p_{t-1} up to the threshold of Young bidders' bids. Above that threshold the probability that a Young bidder has won is 0 and the probability that an Old bidder won is 1.

Theorem 12. *In State A and B, an observation of the second highest bid (observed price) above the threshold of young bids means that there probability that an old bidder won is equal to 1.*

Suppose a price is observed in $t - 1$. Conditional on this price observation the probabilities of old and young winning can be updated.

Some definitions that are needed:

The distribution density function of the highest bid coming from young bidder in state A:

$$f_{1 \cap y, A}(.) = f_{1,A}(.) * f_{y,A}(.) * C_{n1}$$

The distribution function of the highest bid coming from old bidder in state A:

$$f_{1 \cap o, A}(.) = f_{1,A}(.) * f_{o,A}(.) * C_{n2}$$

The distribution function of the highest bid coming from young bidder in state B:

$$f_{1 \cap y, B}(.) = f_{1,B}(.) * f_{y,B}(.) * C_{n3}$$

The distribution function of the highest bid coming from old bidder in state B:

$$f_{1 \cap o, B}(.) = f_{1,B}(.) * f_{o,B}(.) * C_{n4}$$

, where C_{n41}, C_{n2}, C_{n3}, C_{n4} are normalizing constants. Each of the above density functions have, of course their corresponding cumulative distribution functions: $F_{1 \cap y, A}(.)$, $F_{1 \cap o, A}(.)$, $F_{1 \cap y, B}(.)$, $F_{1 \cap o, B}(.)$. It is worth noting that the distribution of young bids has a threshold which is the bid of the highest valuation $b(V_H)$. This means above that threshold, $f_{y,A}(.)$ as well as $f_{y,B}(.)$ is equal to 0.

The probability of young winning in state A is the probability that the highest bid b_1 is from a young bidder and that it is above the observed price. The probability of young winning in state A is updated as below:

$$Pr_A(y|p_{t-1} = x) = P(b_1 \in y|A, p_{t-1}) =$$

$$\frac{\frac{N}{2N-1}(1 - F_{1 \cap y, A}(x))}{\frac{N}{2N-1}(1 - F_{1 \cap y, A}(x)) + \frac{N-1}{2N-1}(1 - F_{1 \cap o, A}(x))} \quad (1.5.29)$$

Above the threshold, of the maximum young bid $F_{1 \cap y, A}(x))$ is equal to 0, and therefore the probability that the highest bid comes from a young bidder is also 0 above the threshold.

The probability of old winning in state A is the probability that the highest bid b_1 is from an old bidder and that it is above the observed price. The probability of old winning in state A is updated as below:

$$Pr_A(o|p_{t-1} = x) = P(b_1 \in o|A, p_{t-1}) =$$

$$\frac{\frac{N-1}{2N-1}(1 - F_{1 \cap o, A}(x))}{\frac{N}{2N-1}(1 - F_{1 \cap y, A}(x)) + \frac{N-1}{2N-1}(1 - F_{1 \cap o, A}(x))} \quad (1.5.30)$$

Above the threshold, of the maximum young bid $F_{1 \cap y, A}(x))$ is equal to 0, and therefore the probability that the highest bid comes from an old bidder is 1 above the threshold.

The probability of young winning in state B is the probability that the highest bid b_1 is from a young bidder and that it is above the observed price. The probability of young winning in state B is updated as below:

$$Pr_B(y|p_{t-1} = x) = P(b_1 \in y|B, p_{t-1}) =$$

$$\frac{\frac{N}{2N}(1 - F_{1 \cap y, B}(x))}{\frac{N}{2N}(1 - F_{1 \cap y, B}(x)) + \frac{N}{2N}(1 - F_{1 \cap o, B}(x))} \quad (1.5.31)$$

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Above the threshold of the maximum young bid $F_{1 \cap y,B}(x))$ is equal to 0, and therefore the probability that the highest bid comes from a young bidder is also 0.

The probability of old winning in state B is the probability that the highest bid b_1 is from an old bidder and that it is above the observed price. The probability of old winning in state B is updated as below:

$$Pr_B(o|p_{t-1} = x) = P(b_1 \in o|B, p_{t-1}) =$$
$$\frac{\frac{N}{2N}(1 - F_{1 \cap o,B}(x))}{\frac{N}{2N}(1 - F_{1 \cap y,B}(x)) + \frac{N}{2N}(1 - F_{1 \cap o,B}(x))} \quad (1.5.32)$$

Above the threshold of the maximum young bid $F_{1 \cap y,A}(x)$ is equal to 0, and therefore the probability that the highest bid comes from an old bidder is 1 above the threshold.

Both cumulative functions have the same weights in State B and the distribution of bids from old bidders dominates the distribution of bids from young bidders, but the two distributions have a different support. The distribution of Young bidders bids has a sharp increase not far from the cutoff where it ends. The distribution of old bidders bids is naturally more spread-out and more equally distributed in the middle section. This means that the First-Order Stochastic dominance will most likely not hold. It is not clear that with an increase in p_{t-1} there will be an increasing probability that the bid was from an old bidder. The distribution of Young Bids is increasing faster than the distribution of Old Bids, but the sharp increase starts later.

This means that it is likely that for some values an increase in x is not equivalent with an increase in $Pr_A(o|p_{t-1} = x)$ or $Pr_B(o|p_{t-1} = x)$. Above the threshold, though the probability that a young bidder won decreases to 0. Just before the threshold of Young bids the Similarly to the Likelihood Ratio Dominance is likely to be satisfied, which is represented in the theorem below:

Theorem 13. *Probability of old bidder winning in any period t is monotonous with the*

price observation in that period for prices in some upper-α range until the threshold. After
the threshold of young bidders' bids, the probability that an old bidder won is 1.

This is just a guess, and of course exact distributions of young and old bidders bids
will have an impact on that. The certain fact is what happens above the threshold.

1.5.2.3 Secondary Effect of Update on The Probability of State A and B in t

After price observation the overall probability that an old bidder won is the weighted
sum with the updated probabilities:

$$P_t(B) = Pr(o|p_{t-1} = x)_{t-1} =$$

$$Pr_B(o|p_{t-1} = x)P(B|p_{t-1} = x) + Pr_A(o|p_{t-1} = x)P(A|p_{t-1} = x) \quad (1.5.33)$$

And the overall probability that a young bidder won is analogically:

$$P_t(A) = Pr(y|p_{t-1} = x)_{t-1} =$$

$$Pr_B(y|p_{t-1} = x)P(B|p_{t-1} = x) + Pr_A(y|p_{t-1} = x)P(A|p_{t-1} = x) \quad (1.5.34)$$

This secondary effect of course depends on what are the direct effects of price increase
on probability of old winning as well as the probability of each of the states. The effect
on the probability of each of the states in $t - 1$ is monotonous, and unambiguous, but
the probability that an old or young bidder won in $t - 1$ may be a function of p_{t-1} of a
different shape.

Based on the fact that there is a threshold to young bidders' bids, one definite con-
clusion is :

Theorem 14. *Prices observed above the threshold of young bidders' bids in* $t - 1$ *lead to*
an update in probability of State B in t to 1 *and of State A in t to* 0.

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It can also be concluded that:

Theorem 15. *Depending on whether the observed price increases or decreases the probability that an old bidder won, the probability of State B in $t + 1$ will follow the same direction.*

Proof. This is directly implied by 1.5.33 □

1.5.2.4 Indirect Effects Through Links Between States

The probabilities of states in $t - 1$ can change in one direction or another, and this further triggers updating of probabilities for states in t. The expectation of future State probabilities in t and $t + 1$ are affected through links between states (edges on the state diagram). We can measure the strength of the link between states through the expected probability of that link conditional on state of the origin node.

All of the periods that follow after t will have updated probabilities of states A and B. In addition this updating will be in the same direction as in state t:

Theorem 16. *If the probability of State B in t is increased following a price observation in $t - 1$, then the probability of State B will also increase for all the following periods:*

$$\forall_{i \in N_+} \frac{dP(B_t|p_{t-1})}{dp_{t-1}} > 0 \implies \frac{dP(B_{t+i}|p_{t-1})}{dp_{t-1}} > 0 \tag{1.5.35}$$

The change in probabilities of the two States has the same direction for all future periods.

Proof. Implied by theorem 17. □

In state B there are more old bidders than in state A. Looking at the ratio of probabilities of state B following state A, and probability of state B following state B allows to see what impact would an increase in probability of one of the states in $t - 1$ (and decrease in the other) have. If the weight of the edge pointing to State B from

state B is higher than the weight of the edge pointing to State B from State A, then the following is true:

Theorem 17. *Increased probability of state B in any time period t leads to an increased probability of state B in the following period $t+1$.*

This is proved below:

Proof. The probability of state B in t conditional on state B in $t-1$ or A in $t-1$ is shown as below:

$$P(B_t|B_{t-1}) = Pr_B(o) = \int_0^{V_H} f_{1B}(b) Pr_B(b = old)db = \int_0^{V_H} f_{1B}(b) \frac{\frac{1}{2}f_{o,B}(b)}{\frac{1}{2}f_{o,B}(b) + \frac{1}{2}f_y(b)} db$$

$$P(B_t|A_{t-1}) = Pr_A(o) = \int_0^{V_H} f_{1,A}(b) Pr_A(b = old)db = \int_0^{V_H} f_{1,A}(b) \frac{\frac{N-1}{2N-1}f_{o,A}(b)}{\frac{N-1}{2N-1}f_{o,A}(b) + \frac{N}{2N-1}f_y(b)} db$$

The proof will follow stepwise argumentation. In the first step it needs to be determined which of the states A or B imply higher probability of old vs young winning. The second step is the fact that an increase in probability of old winning in t leads to an increase in the probability of state B, while an increase of the probability of young winning in $t-1$ implies an increase in the probability of state A in t. Old bidders in both states, A and B bid higher than young bidders (who discount bids). State B has one more of old bidders than state A. The number of young bidders is exactly the same in both states. In addition to the reduced number of old bidders remaining in state A, their distribution is also changed - they are the remaining bidders who in their young period did not win, even though one young bidder won in $t-1$. In state B, the remaining bidders have not won as young, but in the case where an old age bidder has won. Comparison of these two distributions of old bidders (or young in $t-1$, but after the winner has been removed) gives information about which distribution is dominating the other one. As old bidders, their distribution is transformed by a monotonous function, and therefore the same relation is sustained. If distribution of bidders during their young period dominates another distribution of bidders in their young period, then the first distribution of bidders

will also be dominating the second one in the old period. A distribution that dominates is the one that gives a higher probability of old winning. The distributions to compare are below:

The distribution of State A old bidders when they're young (in $t-1$):

$$f_{y_{t-1},A}(z) = P(A)(f_{R,A}(z)f_{R,y,A}(z) * C) + P(B)(f_{R,B}(z)f_{R,y,B}(z) * C_4) \qquad (1.5.36)$$

The distribution of State B old bidders when they're young (in $t-1$):

$$f_{y_{t-1},B}(z) = P(A)(f_{R,A}(z)f_y(z) * C_1) + P(B)(f_{R,B}f_y(z) * C_2) \qquad (1.5.37)$$

We can compare the pairs of $f_{R,y,A}(z)$ and $f_y(z)$ as well as $f_{R,y,B}(z)$ and $f_y(z)$ in terms of likelihood ratio dominance, since this is where the two equations differ. First of all:

$$f_y(z) \qquad (1.5.38)$$

is the same in each state, A or B, since there are the same number of young bidders entering each period, and the valuations are drawn randomly. Next, for the first pair, we need to compare it to:

$$f_{R,y,A}(z) = \frac{1}{N-1}(Nf_y(z) - (f_{1,y}(z)f_{1,A}(z) * C_5)) * C_6$$

Both, $f_{1,y}(z)$ and $f_{1,A}(z)$, are dominating $f_y(z)$ in terms of likelihood ratio dominance. This implies that the distribution after subtraction and rescaling will be lower in terms of likelihood ratio dominance. In summary, this means that $f_y(z)$ dominates $f_{R,y,A}(z)$ in terms of likelihood ratio dominance.

The distribution used in the second pair:

$$f_{R,y,B}(z) = \frac{1}{N-1}(Nf_y(z) - (f_{1,y}(z)f_{1,B}(z) * C_7)) * C_8,$$

Analogically, it is also dominated by $f_y(z)$ in terms of likelihood ratio dominance.

Remark. *Both, $f_{R,y,B}(z)$ and $f_{R,y,A}(z)$ are dominated by $f_y(z)$ in terms of likelihood ratio dominance.*

The remark above implies that $f_{y_{t-1},B}(z)$ dominates $f_{y_{t-1},A}(z)$ since a mixture of two dominating distributions has to be dominating the mixture of two dominated distributions. Since the old bidders bids are just a transformation (by $b^{-1}()$) of young bidders bids, the fact that the distribution of young bidders bids in $t-1$ of those old bidders who are in state B is dominating the distribution of young bidders who follow to be old in state A implies that the old bidders distribution in state B is dominating the old bidders distribution in state A.

The distribution of young bidders is the same in both cases, so the fact that in state B the distribution of old bidders is dominating in terms of likelihood ratio dominance the distribution of old bidders in state A implies that the probability of old winning in state B is higher than in state A.

All two of the edges pointing to state B have weights associated with the probability of old winning. If state B became more likely, then, out of the two edges, the one with origin at state B will become more important. The fact that this edge has a higher weight than the other one (with origin at state A) means that it would result in an increase in expected probability of state B occurring in period t, following $t-1$. State A, on the other hand will become less likely in t if the probability of state B increases in $t-1$. $\quad\square$

The above explains that if observed price leads to an update towards an increase in probability of state B in any state t, then in expectation the probability of state B increases in $t+1$ as well (and the probability of state A in $t+1$ decreases). If the opposite happens, so state A becomes more likely in t, then in period $t+1$ the probability of state B decreases, while the probability of state A increases.

Another observation that can be made is in the two theorems below:

Theorem 18. *The update in probabilities of future states based on p_{t-1} is diminishing with time.*

$$\forall_{i \in N_+, i>1} \frac{d}{di} \left| \frac{dP(B_i|p_{t-1})}{dp_{t-1}} \right| < 0 \tag{1.5.39}$$

Theorem 19. *The effect of update of future probabilities is decaying at a diminishing rate:*

$$\forall_{i \in N_+, i>1} \frac{d}{dt} \left| \frac{dP(B_i|p_{t-1})}{dp_{t-1}} - \frac{dP(B_{i+1}|p_{t-1})}{dp_{t-1}} \right| < 0 \tag{1.5.40}$$

The convergence continues towards the equilibrium of no additional information:

Theorem 20.

$$lim_{i \to \infty} P(B_i) = P^*(B) \tag{1.5.41}$$

,where $i > t$

Of course the above theorems about $P(B_t)$ relate also to $P(A_t)$, but it follows directly, since $P(A) = 1 - P(B)$

The prevailing assumption here is that the observations are made only by an external observer, not by the bidders who take part in the auctions. In the case whereby young bidders could observe price in previous period, after updating their probability of states A and B, they would update their strategies accordingly. In the end the expected price path could not be increasing. The young bidders, by reducing their bids would most probably lead to correction of the price path. The strategies become very complicated, and include infinite dependencies to all the future and past periods.

From the point of view of the hypothetical external observer, the expected price path follow a proportional path to the probability of State B. The expected highest bid in State B is higher than in State A and the expected price is the weighted sum of probabilities and prices in both states.

Theorem 21. *An observation of price gives information about expected price path in the following period. The effect of learning about future prices is proportional to the effect on*

probabilities of states. The expected price path will follow proportionally and monotonously the same path as the expected probability of State B: initially the change will be the highest in t and after that it will start converging to equilibrium at a diminishing rate with time.

1.5.2.5 Parallel Revealing Effect On Valuations Of Bidders

Additionally to the main effects discussed above, in parallel there is an effect of partially revealing the valuations of bidders present. The price observation in t leads to the conclusion that the remaining bidder's bids are up to the revealed price. That reveals some additional information about the valuations of the young bidders who have not won in that period and continue to the next period as the old generation bidders. Price observation leads to revealing a cutoff to valuations of bidders who lost. This is always a deduction in comparison to the equilibrium with no information. The deduction may be lower or higher. The deduction is lower with higher price observed. So the effect on $t+1$ is monotonous with the observed price: the higher the observed price, the higher is the cutoff to the valuations of remaining bidders who continue to $t+1$.

This has a further effect on the probability of old versus young bidder winning in $t+1$, and the direction of the effect is the same: the higher the price was in $t-1$ the higher the additional increase in probability of an old winning in t. The effect of learning about the valuations of bidders persists into the future periods, diminishing with time. The effect is monotonous with the observed price.

Theorem 22. *The effect of learning about the valuations of bidders from price at $t-1$ is monotonous to the observed price. The effect strengthens the effects of learning about the States described before if these are also monotonous with price. The effect of learning about the valuations is the highest in the following period t (largest deduction) and diminishes with time (towards the case of no limits to valuations).*

Following price observation the range of valuations of bidders who were young in $t-1$ needs to be updated. These bidders are in the old period in t. This leads to repercussions

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for the probabilities of young and old winning, and the adjustments due to this fact are monotonous with the observed price. This means that the valuations of young bidders who do not win in t are also monotonously updated due to this effect and the same continues further to future periods. This effect is unambiguously monotonous in relation to observed price - the ranges of all valuations of bidders in future periods are increased upwards with higher p_{t-1}. The absolute value of the effect is diminishing with time and converging to 0 - the expectation about valuations converges to the case with no limits.

1.5.3   Summary

The above discussion has the aimed at showing that revealed price in one auction is a source of information about the distribution of bids in all future periods. The price revealed in $t-1$ carries information about both the distribution of bids in $t-1$, and the probabilities of states in $t-1$. The secondary effect if that the period t States are partially revealed. The effect of learning continues into all future periods, decaying with time at a diminishing rate. The convergence leads back to equilibrium from before the price observation.

If price in $t-1$ was higher then it was more likely that an old bidder won the auction, so the state with more old bidders was more likely. That, in turn, means that in the following period the state with more old bidders will be again more likely, so again the expected price is higher. That leads to the expectation that in the future period, $t+1$ again the state with more old bidders will be more likely, and the expected price will be higher. The effects are present in all future periods, long after all the bidders from the reference period $(t-1)$ are gone.

1.6 Introduction of One Bidder with Longer Lifespan to the 2-period Model

In order to picture what the 2-period model implies in terms of learning, an introduction of one additional bidder, otherwise external to the model, with longer lifespan is proposed. This hypothetical situation allows to see the 2-period model through the eyes of an introduced external observer, who considers bidding in the same auctions as the other bidders present in the model. The added bidder is able to observe the price after he bids in their first period. The periods can be numbered $t - 1$ - for the first period when the additional bidder enters, t as the second period for the 3-period lifespan bidder, and $t + 1$ as the last period for that bidder, where the dominant strategy in the second price auction is to bid their valuation. For simplicity, let us assume that the existence of this bidder is not known to the other bidders who behave according to their cyclic equilibrium strategies defined earlier. Two period bidders do not have possibility to update their bids in their last period, even if they could observe prices after the first period they bid in. Allowing for price observation after the first period of activity for each bidder would not affect the behavior of 2-period bidders, who would remain in the cyclic equilibrium. In order to keep simplicity, let us assume that these bidders do not observe the outcomes from previous auctions. The bidder with longer lifespan, on the other hand, does benefit from price announcements and their effects on this bidder's strategy is analyzed below. Considering just the 2-period horizon bidders for now, and their equilibrium strategies, the following observations can be made:

Theorem 23. *The distribution of bids and the probability of **State B** in any period t is updated based on the observed price in period $t - 1$.*

The updating of the probabilities of States in period $t - 1$ and t following price observation in $t - 1$ is described on the example of an external observer above. Other periods after t are updated monotonously to the update in t.

Theorem 24. *Probability of **State B** in any period $t + 1$ is higher conditional on **State B** in t than conditional on **State A** in t.*

For the 3-period bidder the probabilities of States in $t + 1$ are important for their updating of bid in t. The bidder prefers higher probability of State A. In the case the probability of State B is increased due to updating the expected surplus in that period is reduced.

Theorem 25. *Distribution of bids in **State B** in any period t dominates the distribution of bids in **State A** in t in terms of **Absolute Ratio Dominance** .*

Theorem 26. *For each bidder with valuation in V the probability of wining, conditional on **State B** in any period t, is lower than the probability of winning, conditional on **State A** in t.*

Additionally, an observation can be made that:

Theorem 27. *For each bidder with valuation in V the expected surplus, conditional on **State B** in their last period, is lower than the expected surplus conditional on **State A** in their last period.*

This is due to the fact that the distribution of bids in **State B** dominates the distribution of bids in **State A** in terms of Absolute Ratio Dominance. The 3-period bidder will expect lower surplus if the probability of State B is increased. As a result the bidder will reduce their discounting of bids in that case.

Considering the one bidder with 3-period lifespan that is introduced, the following is true:

Theorem 28. *If, after updating beliefs, the expected distribution of bids in $t + 1$ is dominating the distribution of bids in $t + 1$ before updating, the 3-period lived bidder who enters their middle period in t updates their bid in period t upwards.*

If a higher observed price in $t - 1$ leads to an upward updating of probability of **State B** in $t + 1$, the 3-period lived bidder will update their bid in the middle period t in a monotonous relation to the observed price in $t - 1$.

Figure 1.3: Learning By a 3-period Bidder Who Can Observe Price In t-1
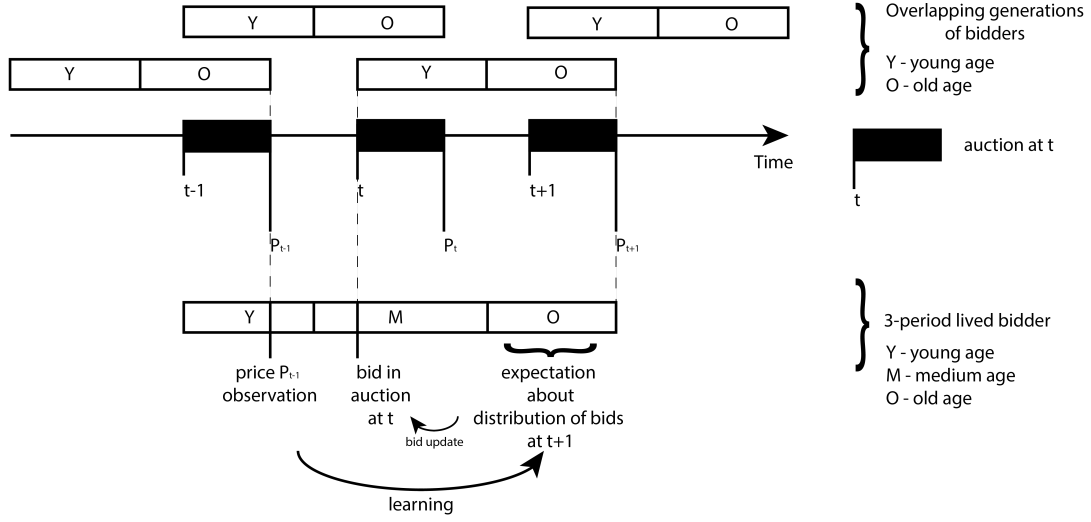
Theorem 29. *Overlapping generations model with 2-period lived bidders and one 3-period lived bidder, where bidders can only acquire information about the price after their first period of activity, implies that the bidder with 3-period lifespan will update their middle period bid as a result of observed price in $t-1$.*

And another important statement to be made is the following:

Theorem 30. *In the overlapping generations model with bidders of different lifespan, the condition that the same bidders are present at the information acquisition period and the period about which learning occurs is not a necessary condition for learning.*

In this example it is shown to be true for OLG model with 2-period lived bidders and one 3-period lived bidder.

1.7   Simulation Results

In order to get a better feel of the impact that the observation of prices has in the OLG model, a simulations of the model particular numerical examples for some distributions

were run. A program for generating the simulations was written in Mathematica. All the equations of the equilibrium were inputted to the program, and in order to simplify computations the inverse of the bidding function was approximated based on Monte Carlo simulation.

For the underlying valuations I am considering Beta distributions with different parameters - the choice of Beta distribution is due to the fact that it has a bounded support, as well as different parameters allow for almost any possible shape of the distribution. In order to numerically find fixed point for the examples, as well as generate graphs, I have written a Mathematica script which finds a fixed point for a Beta distribution with any parameter. The number of bidders in each period can also be altered, although I am going to focus on the results for the case of three new bidders in each period. In addition to facilitating estimations of magnitudes for particular underlying distributions, the numerical solution also has the benefit of being expendable to larger number of states, and I have also written a version for the three-period lived bidders model, with 5 states. It is clear, especially if one wants to consider larger number of states, that the numerical approach is the only feasible way to find solution, and gain insight into the model. In order to see that OLG of bidders has an effect on updating once the price (second or first highest bid) is revealed, the two period model is sufficient, and therefore I will focus here on this version for now.

First of all, in order to understand why revealing price from previous period would have an impact on updating beliefs and bidding strategy of young bidders, it is important to take a note of the differences between the two states, A and B. Understanding the differences in the distributions of bids in the two states makes it clear why a better idea which state will follow is important for bidding strategy. The second step is understanding the differences in distributions between old and young bidders, as well as the transitions between states, which depend on whether a young or an old bidder has won. Transitions between states are the arrows on the state diagrams above, and they

Figure 1.4: Different distributions of Old bidders in State A and State B

(a) Valuations from Beta(3,2)　　　　　(b) Valuations from Beta(1,1)
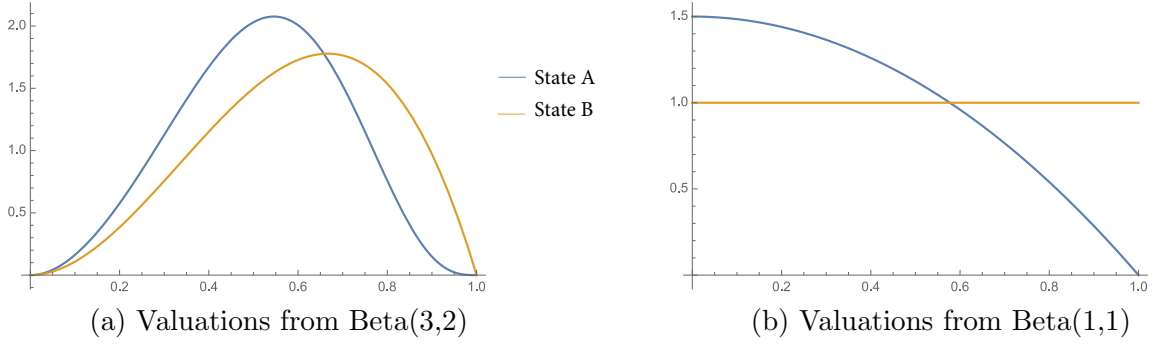
are showing the only possible states in $t + 1$ following after each of the states in t. In the two-period lived bidders the situation is very simple because after each states, the next period there can also be one of the only two states. In the three period model, though, the arrows point to only three states from each one in t, and therefore they are constraining the possible states following to a 3-element subset from the 5-element set of all states. Transitions are therefore important, but what triggers transitions, is the age of the winning bidder, and therefore the probability that a given age bidder wins (and how this probability changes upon observing a given level of the winning bid) is the third important component of the dynamics in the model.

Therefore, in conclusion, the three main elements of the model which have an influence on the outcomes are:

1. The mixture distributions in each of the states, and the differences between these distributions.

2. The probability of old versus young (versus middle in the 3-period model) winning in each of the states as a function of observed highest bid, and also the overall probability of each age winning as a whole in the system.

3. The transitions between the states.

Again, even though I have included the middle age bidder in the above list, I would like to resort now to the two-period model with two states. I will present two examples

that illustrate how the above factors result in an increased importance of past prices for expectations about the future state, and learning. The two examples are Beta(3,2) and Beta(1,1) distributions of valuations. These are mainly arbitrary choices, just to show that for different distributions of valuations, there there is a clear pattern in the results. Beta(1,1) is the most often used distribution in examples, as it is the uniform distribution, while Beta(3,2) is not a central, but a little bit skewed to the right distribution, which is my best guess of what could the distribution of valuation for an object for sale (for example on eBay) look like. The fixed point for equilibrium without price announcements has been derived through my FixedPoint program in Mathematica. The approximated fixed point solution includes the probabilities for each of the states, maximum of discounted (Young) bids, and beta parameters for the distribution of Young bids. A more detailed description of the program used for finding the fixed point can be found in Appendix. The example solutions found using the program can be found in Table 1. In all cases, the probabilities of states A and B are approximately 0.3 and 0.7, where state B is the more likely state, as expected (it is the state with a higher proportion of Old bidders and a dominating distribution).

Figure 3 shows the different distributions of Old bidders in states A and B for the two examples. The fact that the bidder with the highest valuation is not present as the Old bidder in state A any more, means that the state A Old bidders distribution has less density in the upper section than in state B. The distributions in both states consist of Old as well as Young bids, and these mixture distributions can be found in Figure 4. These graphs show on examples, that the distributions of Bids in both states will be different, which is the reason why knowing a future state a bit more precisely would make a difference for the discounting function of Young bidders. The difference in the distributions is definitely non-negligible, and when considering the order statistic distribution of highest or second highest bid (which is what matters for the discounting function), these difference are amplified. Figure 5 shows the distribution of the highest bid in both states, clearly the higher density in the upper section in state B makes even

Table 1.1: Fixed Point Simulation Examples

Inputs:

Beta distribution parameters for valuations	(3,2)	(1,1)
Number of young bidders each period	3	3

Results of Simulation:

maximum of Young bids	0.66	0.59
Beta param. of Young bids	(2.26, 0.52)	(0.92, 0.41)
Probabilities of state A and state B	(0.305, 0.695)	(0.33, 0.67)

more of a difference when highest order statistic distributions are considered. It can be seen that these distributions are two-peaked, as was expected in the theoretical section above. It is clear that state B distribution is dominating state A distribution in terms of α, β Ratio Dominance (it is not clear whether there is First Order Stochastic Dominance, because, especially when looking at the full picture, it seems to be possible that the distributions cross twice). In any case, for the highest order statistic, the upper-α covers around 30%, where PDF in state B is higher than in state A.

The above considerations explain point 1 on the list, which is the differences of distributions in both of the states. Distribution in state B is dominating the distribution in state A. That clearly means that the expected outcome from the distribution in state B will be higher than that in state A. The second point is concerning the probability of each age winning in each of the state as a function of the observed highest bid. The relative probabilities that the observed highest bid is from old or young bidder can be seen on

Figure 1.5: Distributions of bids in state A and B

(a) Valuations from Beta(3,2)

(b) Valuations from Beta(1,1)
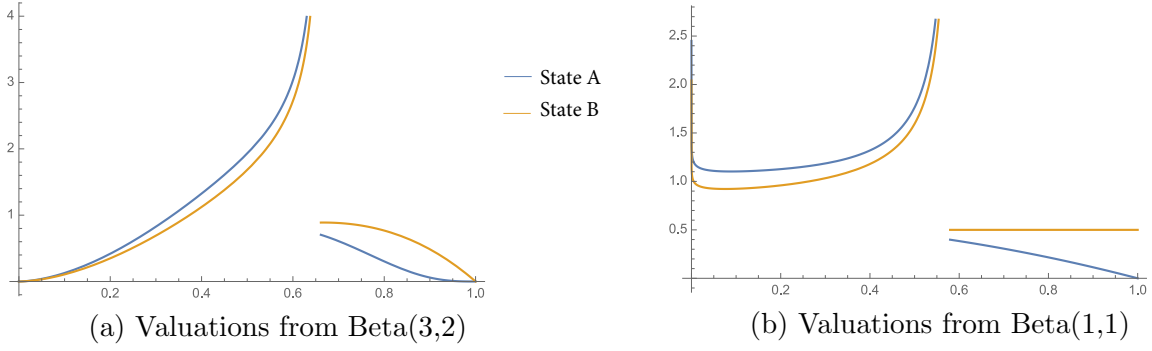
Figure 1.6: Distributions of highest bids in state A and B

(a) Valuations from Beta(3,2)

(b) Valuations from Beta(1,1)
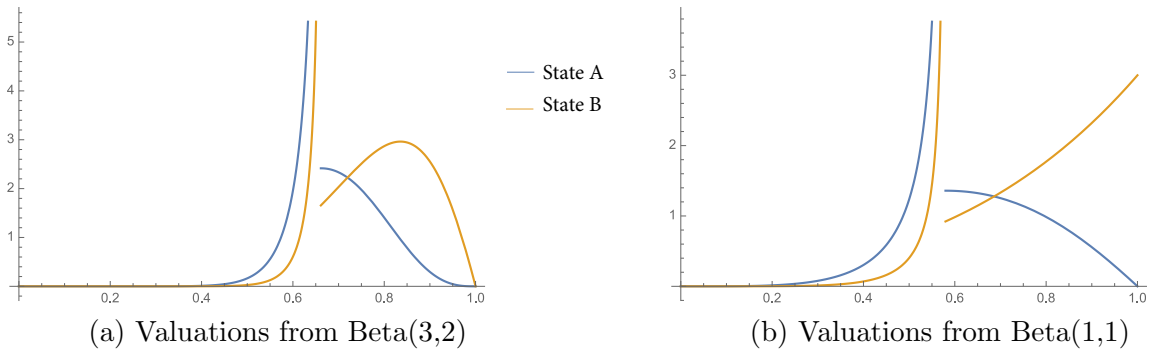
Figure 6, and the relative probabilities that the observed highest bid comes from state A or B in Figure 7. Figure 6 is especially important, because it is the fact that an Old or a Young one has won that makes a difference for the transition between the states going to the next period. As can be seen the probability that the winning bid is from the Young one is increasing until the cut-off of the Young PDF, after which the probability that the Old one has won goes to 1 (and Young to 0). This is due to the fact that there is a very high density of Young bids near the cut-off value. Point 3, which is transitions between states is very simple in the two-period model, where an Old winning bid always points to state B, while a Young winning bid always points to state A (this is not the case in the 3-period model, where the origin state plays a role as well). In the two-period model the transition story is only reliant on the relative probability that the winning bid is Old or Young, and this is what Figure 6 is showing.

If the observed winning bid is above the cut-off point, it is without doubt coming from

Figure 1.7: Probability that the observed highest bid is from Old or Young bidder

(a) Valuations from Beta(3,2)

(b) Valuations from Beta(1,1)
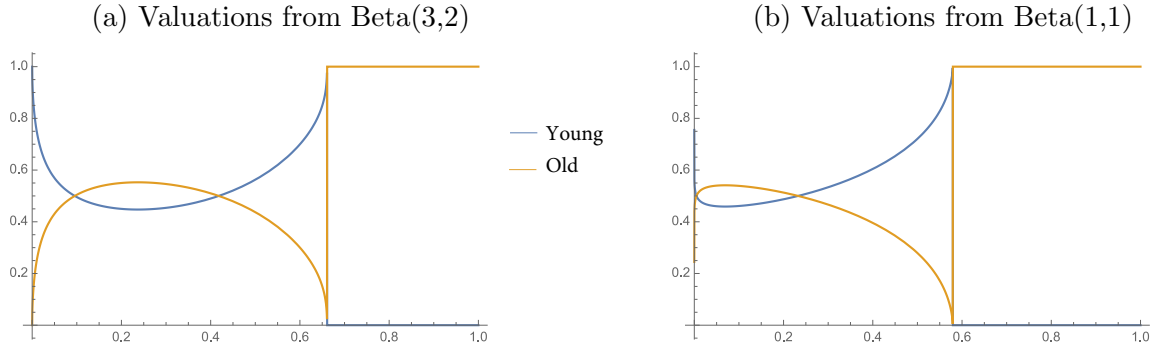
Figure 1.8: Probability that the observed highest bid is from state A or B

(a) Valuations from Beta(3,2)

(b) Valuations from Beta(1,1)
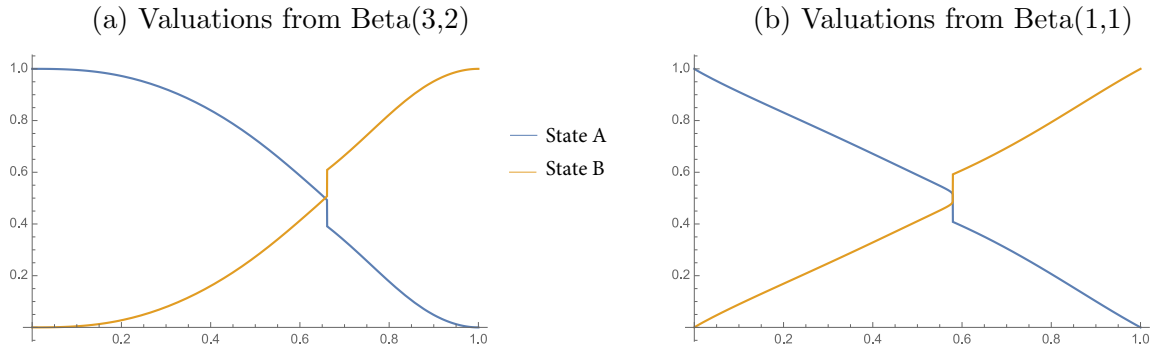
an Old bidder, and therefore the probability of state in the next period is 1 for state B and 0 for state A. The dominance of the distribution of bids in state B means that this also implies that the next period expected winning bid will be higher. In the no price announcements Equilibrium the bid is seen as coming from the Mixture distribution of State A and State B bids, with mixture weights being the equilibrium probabilities of state A and B. The Expected highest bid in $t+1$ is the expectation from that distribution. Observing a highest bid directly, means that the probabilities of the states in the next period change, and therefore the Expected highest bid in $t+1$ is changing with the observed bid in t. This can be seen on Figure 8. If the highest bid in t is above the threshold (maximum of Young bids), the expected highest bid in $t+1$ increases to the expected bid in State B. The difference after observing a high winning bid in t and updating of the expected highest price is 5.6% in the Beta(1,1) model, and 2.7% in the Beta(3,2) model.

Suppose that the observed price is in the high section (above the threshold of Young

Figure 1.9: Expected Highest Bid in next period

(a) Valuations from Beta(3,2)   (b) Valuations from Beta(1,1)
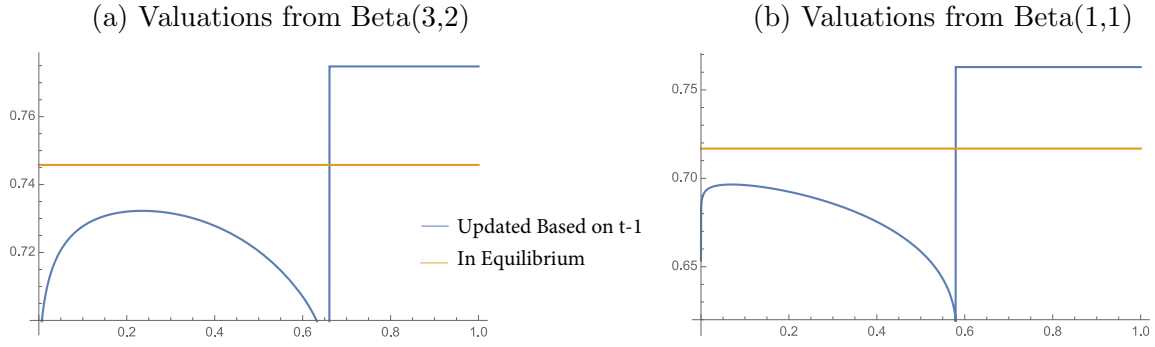
Table 1.2: Expected Highest Bid in Simulated Equilibrium. Division By States.

| | model | |
	$f_v = \text{Beta}(3,2)$, $N = 3$	$f_v = \text{Beta}(1,1)$, $N = 3$
Expected highest bid in state A	0.68	0.62
Expected highest bid in state B	0.77	0.76
Exp. highest bid in Equilibrium (mixture of states)	0.75	0.72

bidders). On the example of the Beta(1,1) model the effects of updating can be observed in Figure 9. The largest effect is for the highest bidders, but percentage-wise a decrease in discounting is larger for the low valuation bidders, since their already low chances of winning in the next period are even more decreased after learning. The minimum decrease in discounting is 7%. The increase in bid function goes up to 0.03, which is 5% of the Young bidder's bid before updating. The effect depends on the underlying distribution of valuations, and it is higher for more dispersed distributions.

1.7.1   Summary

In summary this section presented a model of auctions with overlapping generations of buyers, where past prices are informative to the bidders, and are influencing Young bidder's discounting function. This result is derived from the fact that 'younger' bidders

Figure 1.10: Effect of updating on discounting and Young bidders bidding function

(a) Effect on bid discounting

(b) Effect on bidding function
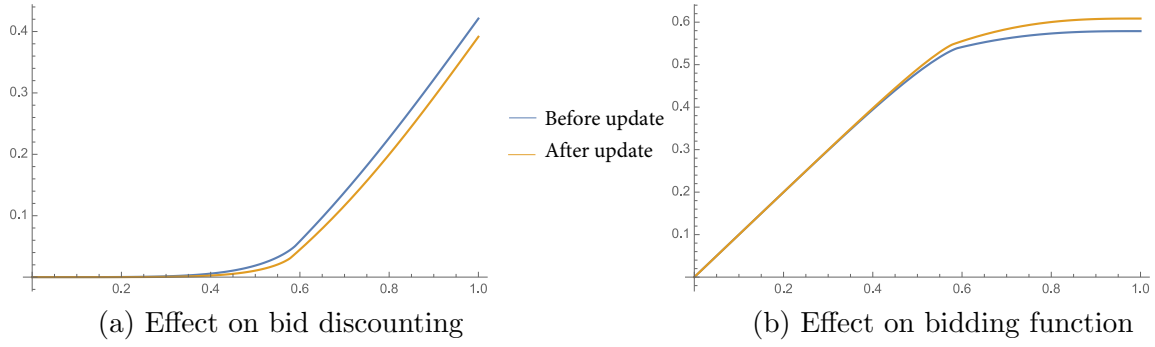
(of earlier period) discount their bids, while 'old' bidders (in their last period) bid the full valuation. Young and Old bidders are present in the same auction, and the composition of bidders with different age is defining the current state of the world. This has an effect on Young bidders' strategy. Different composition of young and old bidders leads to a different expectation of the composition in the following period. Winning price is also an indication of the valuation of the winning bidder, and, indirectly, of the possible range of valuations of the remaining bidders. As last period's price is a signal of the composition of bidders ages and valuations, it is also an indication for the expected future period composition. Due to the overlapping generations, new Young bidders, who are able to observe last period's winning price will use that information for updating their strategy (the amount they shade their bid).

This model implies that observing a winning bid in the top section is an indication that the next period bids will be coming from a higher distribution. As a general conclusion for empirical research, the model implies that past prices are not indifferent for the bidding strategy of bidders, and it is expected that prices in the top section should have a clearly positive effect on future bids (and expected prices in the following period).

As a stylized model, the conclusions that need to be drawn are more general. Of course it is unlikely that it would be possible to identify exactly whether a bidder has an intention to bid in the future (a Young bidder) or not (an Old bidder). The model shows that once the bid is high enough, it can be inferred that the person does not want to bid any more (it is the Old bidder) which leads to updating of the beliefs about next

period distribution of bids upwards. The same implications extend towards models with more period-lived bidders, although there additionally medium bidders are present. This is the first attempt at identifying the implications of overlapping generations on bidding and auction outcomes.

1.8 Extensions

1.8.1 Price announcements known before each auction

In this version of the model, the bidder in period t learns what was the price for the last period $t - 1$, before placing his bid in period t. Unlike in sequential auctions without overlapping generations, the price announcements are informative to the bidder, who can now better predict which state the system was in the last period. In the case of price announcements before each auction, the young generation bidder can use this information to update their beliefs about the following state and change their discounting strategy. Nevertheless, in this situation, the price announcements in the period $t - 2$ had also an effect on young bidders strategy in that period, while the price announcements in $t - 3$ had an effect on bidders in that period. Given that all the bidders adjusted their strategy, the equilibrium is impossible to solve for, as there will be infinite dependencies for all previous periods.

1.8.2 Comparison with sequential auctions basic model

The question which can arise is whether the seller, who has two items for sale, is better off in the case the buyers are in overlapping generations, or whether the seller is better off selling the two items in sequential auctions to $2N$ bidders, where all bidders are $\tau = y$ in the first period and $\tau = o$ in the second period.

As Krishna (2010)[24] points out, the revenue equivalence in sequential auctions (with independent private values) holds not only for the overall expected revenue to the seller

but also for each auction in the sequence. The expected revenue from each auction is the same in the case of first-price and second price rules. In sequential auctions the second price auction equilibrium bid in the k-th auction is the same as the equilibrium bid in the $(k+1)$-st first price auction:

$$b_K^{(II)}(v) = v$$

$$b_K^{(I)}(v) = E(v_2|v_2 \leq v)$$

For all $k < K$,

In second price auction the the solution to the maximization for the first period bid is $b_{T-1}^{(II)}(v) = v - E[W_T^*(v)] = v - \int_{\underline{v}}^{v} F_2(b_2)db_2 = E[v_2|v_2 \leq v]$, and this is also the solution to the first price auction maximization.

On the other hand, in the overlapping generations model, the distribution of the bids is a mixture between the distribution of discounted bids (young generation) and the non-discounted bids (old generation). As a result the young person's bid is $b_y^{(II)}(v) = v - W_o^*(v) = v - \int_{\underline{v}}^{v} (P(A)F_{2,A}(b_2) + P(B)F_{2,B}(b_2))db_2$. The mixture distribution between $F_{2,A}$ and $F_{2,B}$ with defined by probabilities of states A and B is the distribution of the second highest bid in the system. Let's call this distribution $F_{2,S}(.) = P(A)F_{2,A}(.) + P(B)F_{2,B}(.)$. The optimal bid of the young bidder is the expected second highest bid, conditional that it's lower than v, where the distribution of that second highest bid is $F_{2,s}$.

$$
\begin{aligned}
b_y^{(II)}(v) &= v - W_o^*(v) = v - \int_{\underline{v}}^{v} (P(A)F_{2,A}(b_2) + P(B)F_{2,B}(b_2))db_2 \\
&= \int_{\underline{v}}^{v} (1 - F_{2,S}(b_2))db_2 = E[b_2|b_2 \leq v]
\end{aligned}
$$

It is clear that this bid is not the same as in the case of standard sequential auctions

model.

Lemma 1. *The mixture distribution F_S is stochastically dominated by the distribution of valuations F_v (First Order Stochastic Dominance). For all $z \in [0, \overline{v}]$: $F_S(z) \geq F_v(z)$. The order statistic distribution $F_{2,S}$ is stochastically dominated by the order statistic distribution of valuations $F_{2,v}$. For all $z \in [0, \overline{v}]$: $F_{2,S}(z) \geq F_{2,v}(z)$*

Lemma 1 is implied by the fact that some bidders in $F_{2,S}$ are discounting their bid. As for all $v \in [0, \overline{v}]$: $b_y(v) \leq v$, $F_{2,S}$ will have higher relative probabilities for lower values in comparison to $F_{2,v}$.

Theorem 31. *The optimal bid in symmetric equilibrium strategy for young bidder in overlapping generations model is $\leq$ than the optimal bid in the first auction in a sequence of two sequential auctions. (this comes from Lemma 1).*

Proof. The bidder's strategy in the first out of two sequential auctions is to discount their bid by their expected payoff in the second auction. The discounting factor is $W_T^* = \int_{\underline{v}}^{v} F_2(b_2) db_2$ In the case of two sequential auctions the $F_2(b) = F_{2,v}(b)$ here, while in the case of sequential auctions with overlapping generations $F_2(b) = F_{2,S}(b)$, and therefore, by Lemma 1, the discounting in the case of overlapping generations is higher or equal to the sequential auctions case. □

What is the expected seller's revenue from two auctions in the case of overlapping generations of bidders, and in the case of sequential auctions without overlapping generations?

Theorem 32. *The second period pay-off is higher for the seller in the case of sequential auctions without overlapping generations with $2N$ bidders. (Proof by Lemma 1)*

1.8.3 Extending beyond two-period lifespan

An extension to the above model is the case of bidders living for more than two periods. Once the bidders are allowed to live for more than 2 periods, more possible states of

the world arise in the model. For a three-period model N new bidders arrive each period, but this time the bidders leave the auction sequence after three periods,not two. This means that there are buyers of three different ages, and it increases the number of possible combinations of numbers of bidders of each age. The number of different possible combinations increases with the lifespan of bidders i the model, and it results from the fact that the bidder who wins leaves the system and all the bidders become older with each time period. In the case of three-period-lived buyers there are 5 possible states of the world in the model. The complexity of the state diagram is increased with every additional generation. Knowing the possible states of the model is important for calculating the equilibrium strategies. It becomes difficult to derive with other than computational methods though. In order to find the state diagram for each lifespan of bidders, I have written a script in Java, which is included in the attachments. Object-oriented programming has been of great help in this case, as it has allowed to make the problem easily solvable. In the table below, the number of states for each life-span length up to 10 periods is shown.

Length of Lifespan	Number of States	Number of Links
2	2	4
3	5	15
4	14	56
5	42	210
6	132	729
7	429	3003
8	1430	11440
9	4862	43758
10	16796	167960

Table 1.3: Number of States and Links for lifespans of lengths 2-10 in the model.

It can be seen that an increase in the number of states with lifespan is faster than exponential, and after trying to derive a formula, it has become clear that there is no easy formula for number of states. Number of links in the system is the number of states multiplied by the lifespan (which is also the number of coexisting generations). The

state diagram for a three-period lifespan is shown on the figure below. With 5 states, and 15 links the diagram looks much more complex from the 2-period lifespan case. An increase to 4-period lifespan results in 14 states and 56 links, while the next increase leads to 42 states and 210 links. This shows that graphical representation becomes very complex with increments in lifespan. The diagram for a 4-period lifespan is already very complicated, but a further increase towards 5 periods leads to a very large graph which would definitely not fit on one A4 page.

Figure 1.11: Three-period-lived Bidders Model, State Diagram
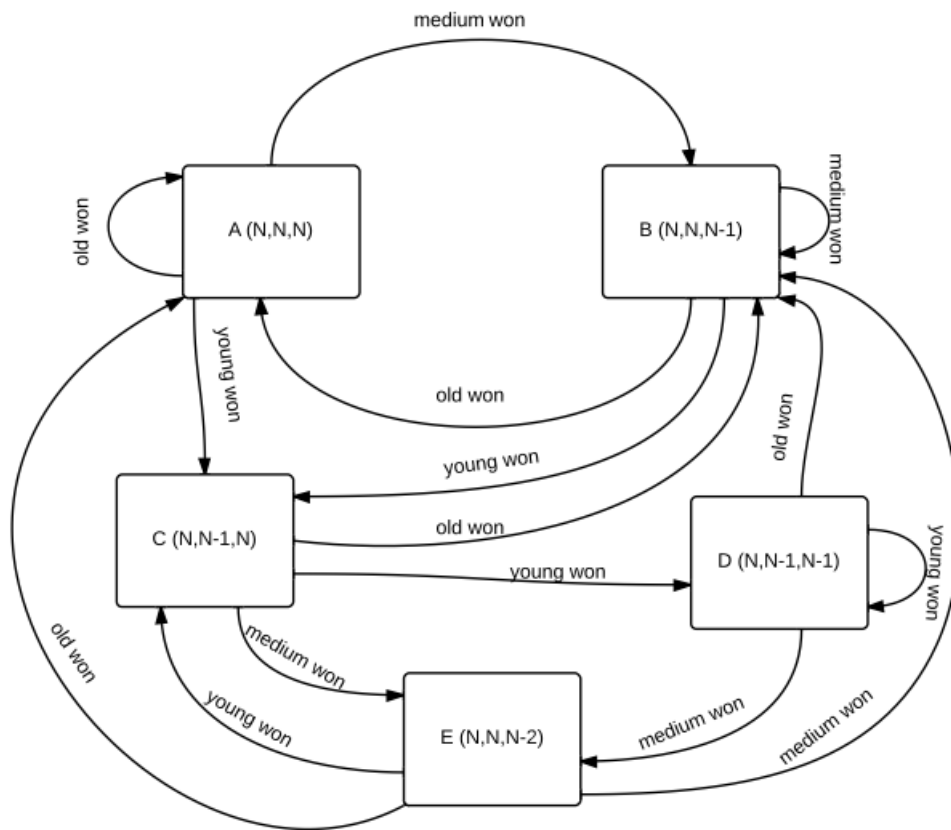
Such fast increase in complexity means that analyzing the state diagram visually is not of much help once the lifespan increases beyond 3 generations. The equilibrium formulas derived for the two-period lifespan, though, can be easily extended towards the more complicated cases. In the case of no revealed information about prices, the stable

equilibrium exists in the case of any lifespan length. Analogically to the two-period case, the equilibrium describes the stable point in the strategies of bidders.

Theorem 33. *There exists a symmetric Bayes-Nash equilibrium for any lifespan length (and number of τ periods) in the model without price announcements.*

In the case of three-period lived bidders the number of different periods τ is equal to 3, and in equilibrium each generation will have single strategy.

With more ages in the model, using letters such as y and o for distinguishing the age is no longer sufficient. The ages τ will be numbered $\tau \in 1, 2, 3, 4, ...$ where the generation with the age $\tau = 1$ is the youngest, and the age increases with τ. The maximum age is the total number of generations in the model, and this can be denoted as $T = max\{\tau\}$. For the model with T generations the solution to the maximization is:

$$b_T(v) = v$$

$$b_{T-1}(v) = v - W_T^*(v)$$

$$...$$

$$b_1(v) = v - W_2^*(v)$$

The 3-period model created much more states and links. As we can see from the diagram the distribution of old bidders ($N - 1$ old bidders) in state D and old bidders in state B (also $N - 1$) are different, because the old bidder who is not present in state D has won in state A as a young bidder, while the old bidder not present in state B could have also been a young bidder winning in state A two periods earlier, or a medium bidder winning in A, B, or E. The system of equations becomes so complicated and large in the case of 3-periods, that it makes no sense in writing it down here, as it can only be solved using computer program in any case. There are more equations to write down, but essentially the numerical solution for the case of no price announcements is just an extension to the solution of the 2-period model. In the further section I will discus the

numerical solution to the two period model, and I have also written a program to solve the 3-period model. The key point is to see the pattern in the solution, which can be projected to a model with any number of periods, and the universal conclusions which the overlapping generations model implies.

1.8.3.1 3-period model with price announcements

In the three period model both versions of price announcements considered above will have an effect on updating bidders' strategies. The winning price from $t-1$ is informative about the relative likelihood of states and winning bidder's age, just like the in the two-period model. Also, like in the two-period model that gives insight into future distribution. Learning about the future period distributions will be reflected in the bidding of young and medium bidders. In the case of price announcements before each auction the effect will be on both young and medium bidders, while in the case of price announcements after each auction seen only to the participating bidders, the effect will be on bidding of medium bidders, while the young bidders will not change their bid. In both cases, the price in $t-1$ influences bidding in t, but the price in $t-1$ is a result of bidding in $t-1$, which was influenced by the price in $t-2$. In this way, the dependencies will follow all periods back, and therefore it is impossible to solve for the equilibrium.

1.8.4 Evolution towards stable equilibrium

The case considered here is an infinite sequence of auctions. Nevertheless this sequence had to start somewhere and the equilibrium considered here is arrived at as a stable point once enough auctions have finished already. If, instead one wants to consider the beginning of the sequence of auctions, then it has to be noted that such a stable , cyclic equilibrium does not exist there. At the beginning of the sequence, at $t = 1$ there are only N bidders, and all of them are of the same generation (young). In the second period, $t = 2$, there are N young and $N - 1$ old bidders. Since then onwards the cycle from the first State Diagram in terms of number of bidders of each generation begins to apply. The

bidding strategies, and the distributions of bids are not, though, exactly the same as in equilibrium, but they are evolving towards this stable point. Each young bidder consider the future probability of winning, and since the distributions of bids are evolving, there will be an infinite sequence of small changes in future distributions that the young bidder would have to consider in their strategy choice. This makes the derivation of the strategy for the young bidder very difficult, although this could be a topic for future research in this area. The stable equilibrium considered is interesting because it has a closed form and can be more easily analyzed as opposed to the cases of evolution towards equilibrium, which is left for future research.

1.9 Conclusion

In conclusion, this chapter presents a new model of sequential auctions, where bidders enter bidding at different auctions, and their lifespans overlap. The model is used to show that such a case has implications for learning by the bidders. Previous prices are a source of information about the expected distribution of bidders valuations, as in the standard sequential auctions model, and additionally they carry information about the expected composition of bidders with different horizon lengths in future periods. It is shown that from the point of view of an external observer, the informativeness relating to bidders valuations lasts only until the last bidders present in the period of information acquisition exit. On the other hand, learning relating to the composition of young and old bidders extends over all the future periods. An introduction of one bidder with a 3-period lifespan, who is able to observe a price after their first bid, into the equilibrium environment of overlapping generations model with 2-period bidders makes the bidder update their expectation of composition of young and old bidders in $t+1$ (the last period of the 3-period bidder), even though none of the other bidders from $t-1$ (the period of price observation) are present in $t+1$. Based on that, the 3-period bidder updates their bid in their middle period, expecting a different composition of young and old 2-

period lived bidders in $t+1$. This is a novel result, in comparison to the standard model of sequential auctions, where the only channel for learning is through updating on the expected valuations of bidders in future periods. Unlike in the standard model, it is not a necessary condition that the same bidders should be present in the period of information acquisition and the period for which learning is applied.

Chapter 2

Empirical investigation based on sequential auctions theory

2.1   Introduction

The current chapter is the analysis of auction data with respect to the sequential auctions theory outlined previously. Two main hypotheses are tested. First of all the data is analyzed with respect to the expectation of bid discounting by bidders of younger generations. The theory predicts that bidders should be increasing their bid in sequential auctions, by reducing discounting of their valuation of the product. Forward-looking bidders that plan to bid in future auctions should take that into account in their bidding strategy. Secondly, the overlapping generations model predicts correlation between price in previous auction on bidding in the following auction. Higher prices in previous auctions have an effect on beliefs about the competition present in the following auctions one step further to the future, which induce bidders to increase their bids. Literature search revealed no existing studies testing those predictions. Previous empirical tests of sequential auctions usually focused on a dissonance between theoretical prediction about increasing prices and empirical observations. The empirical analyses found in literature were focused mostly on price patterns in sequential auctions, while the current chapter is trying to answer questions about bidding strategies and its links to previous prices. The model of sequential auctions considered predicts linkage between observation of past prices and bidding strategy. It is expected that an effect of prices on bidding in the following auctions is monotonous. Online auction marketplaces for consumer goods are a good place to test the predictions of the OLG model because of participation of many bidders and a tendency among them to take part in more than one auction for the same product. On the other hand, bidders sometimes place bids in parallel auctions, and therefore, in order to identify the impact of learning from past prices, one has to ensure that sequentiality is satisfied.

2.2 Literature

The most influential empirical paper discussing the theory of sequential auctions was the investigation of wine auctions in Ashenfelter(1983)[3].The authors found that the price in sequential auctions was declining, in contrast to predictions of the earlier theory - increasing price. The declining price was explained by risk-aversion in McAfee and Vincent (1993) [25], and by the deviation from Perfect Equilibrium strategy, as discussed in Milgrom and Weber (2000)[27], but also by the fact that high-valuation bidders win at the beginning, which creates a declining distribution of bidders left in later auctions. Until the boom in popularity of online auctions the availability of data on sequential auctions of identical objects was limited. Empirical investigation of price in sequences of auctions has given the researchers the grounds to add modifications to the theory of Common Values component and risk-aversion, as well as deviations from the PBE strategy. Interestingly, the empirical studies have focused on investigation of only the price in the auctions, while the most fundamental prediction of sequential auctions theory - that bidders increase their bid in following auctions has not been tested, possibly due to the lack of availability of data for individual bidders. In my investigation I am looking at strategies of individuals in the sequence of second price auctions. The main prediction of the theory presented herein is the increase of bids in sequential auctions. The empirical results confirming that are obtained using the advantage of individual-bidder data. The increases of bids is a common component in all versions of private values sequential auctions models - not depending on the horizon, number of future auctions or risk-aversion. The fundamental question addressed is whether Game Theory provides us with good predictions for the individual strategies in sequential auctions. The addition of overlapping generations of buyers to the model does not change the prediction about bid increases. Another prediction of the model tested is the influence of past prices on bidding in the next auction, which, if found to be present, would confirm the hypothesis that bidders learn by observation of results of previous auctions.

eBay auctions have been studied before, whereby most of the literature about buyer's behavior is focused on the investigation of features which deviate from basic theoretical model, such as bidding multiple times during one auction or last minute bidding. A summary of literature dealing with eBay can be found in the review paper by Hasker and Sickles (2010)[18]. As authors summarize, there are several main tactics of buyers, which include: Sniping (Last Second Bidding), Incremental Bidding, and Squatting (large early bids) or Jumping. The availability of proxy bidding enables the bidder to enter the highest price they are willing to pay for the item, allowing them to save time incurred to increase their bid later.However, this strategy is not always used and in many cases multiple bidding occurs.

Despite these additional aspects of behavior at online auctions, the theory about sequential auctions is still relevant when looking at final bids of bidders in each auction. Put aside multiple bidding and the timing of the bid placement, the final bids should be representative of bidder's valuation for the auctioned item. Ability to observe how individual bidders bid in a sequence of online auctions and availability of such data provides an opportunity to test the fundamental theory of sequential auctions. The addition of overlapping generations, as introduced in this thesis, is relevant to this data, since the generations of bidders are seen to clearly overlap in the studied auction setting.

Online auctions data typically contain anonymized user names. Literature search have not shown previous attempts at dealing with this problem, and the method presented herein could be shown to be of benefit in future research in this field.

2.3 Theoretical Predictions

The two main theoretical predictions from sequential auctions theory are the following:

1. Bidders increase their bids in subsequent auctions - this is because the amount that they discount their bids is decreasing. The discounting by which the valuation is

reduced is the expected value of the option to bid in the future. The more auctions that the bidder is still looking forward to bid in, the larger the amount of discounting.

2. Past prices influence bidding because they make the bidders better informed about the distribution of valuations of future competing bidders in the following auction - in turn it gives indication about the distribution of bids that they can expect in the following auction. In the case of the simplest model, where all bidders have the same horizon of future auctions at the same point in time, the distribution of valuation is transformed by a monotonously increasing bidding function to create a distribution of bids. In the case the model with overlapping generations, the bids distribution is a mixture distribution of bids belonging to bidders with different horizon. The operation that needs to be made on distribution of valuation to get the the distribution of bids is more complicated. Yet, what is common in these two cases, is that the general prediction of a higher observed price leads to the expectation of higher competition in the future, and therefore also leads bidders to adjust their bids upwards.

More generally, let us assume that valuations of bidders are $v \in V$ and are randomly distributied with distribution function: $g(v)$ and CDF $G(v)$, and bids are denoted $b \in B$, where the transformation that maps valuation to bids is a function $f : R \to R$, and then also $B := f(V)$. The distribution of bids with PDF $\psi(b)$ and CDF $\Psi(b)$ can be derived from the distribution of valuation and it is $R \to R$ mapping:

$$\psi(b) = \sum_{v, f(v)=b} \frac{g(v)}{|g'(v)|}$$

Generalising still further, let us define an operator ξ that creates the distribution of bids from the distribution of valuations:

$$\psi(b) = \xi(g(v))$$

The $\beta(v)$ is the bidding function before any price is observed, and this function is defined for each period $\tau \in [1, 2, 3, ..\Upsilon]$, where Υ is the final period for the bidder. Therefore, $\beta_\tau(v)$ is the bidding function for period τ. At each period other than the first, the bidder can observe the price at which the auction had closed in the previous period. Observing the price $p_{\tau-1}$ leads to an update on the information about the other bidders that took part in auction at $\tau-1$ available to the bidder. This can lead to three outcomes:

1. An update on the set of valuations present in $\tau-1$ so that the resulting distribution of bids is updated:

$$\psi_{|p_{\tau-1}}(b) = \xi(g_{v|p_{\tau-1}}(v|p_{\tau-1}))$$

2. An update on the function $f(v)$ that maps valuations to bids. As a result there is an update on the operator which transforms the distribution of valuations to the distribution of bids:

$$\psi_{|p_{\tau-1}}(b) = \xi_{|p_{\tau-1}}(g(v))$$

3. An update on both the operator ξ and the valuations of bidders present:

$$\psi_{|p_{\tau-1}}(b) = \xi_{|p_{\tau-1}}(g_{v|p_{\tau-1}}(v|p_{\tau-1}))$$

Example 1

For example, in the simplest model, where all bidders have the same horizon, the function that maps valuations to bids is exactly the bidding function, i.e. $f(v) == \beta(v)$. The observed price in τ, which is the second highest bid, reveals directly the valuation of the bidder issuing the second highest bid. It is easy to find the valuation of the second highest bidder as it is

$$v_2 = \beta^{-1}(p_{\tau-1}).$$

The distribution of valuations of remaining bidders is updated, such that all remaining bidders have valuation lower or equal to v_2. Now the set of remaining bidders have

valuations:

$$v|p_{\tau-1} \in V|p_{\tau-1}$$

and the updated distribution of valuations is

$$g_{v|p_{\tau-1}}(v|p_{\tau-1}).$$

Despite that, the shape of the bidding function remains the same and it is still $\beta : R \to R$, although now we know their underlying set of valuations was modified and it is applied to the new set $\beta(v|p_{t-1})$.

The bidding function remains the same as do the function f and operator ξ. Therefore, this example shows that the simplest model of sequential second price auctions with price announcements results in updating of the distribution of bids based on the updated set of valuations of the remaining bidders, which was outcome number 1.

Example 2

The second example is the overlapping generations model. Here, the function that maps the valuations to bids is different from the bidding function of each individual bidder. Therefore we deal with the case such that $f(v) \neq \beta(v)$. Price from period $\tau - 1$ does not directly reveal the bidder with the second highest valuation who will be present in τ, as was the case in Example 1. The price in $\tau - 1$ results in updating of both, the expected set of bidders' valuations and the composition of bidders from different generations. The density of valuations is updated based on $p_{\tau-1}$:

$$g_{v|p_{\tau-1}}(v|p_{\tau-1}).$$

Moreover, as there is more information revealed about the expected composition of bid-

ders from different generations, the function that maps valuations to bids is also updated:

$$f_{|p_{\tau-1}}(v|p_{\tau-1}).$$

This means that the operator that translates valuations into bids is itself also updated:

$$\xi_{|p_{\tau-1}}().$$

The overlapping generations are therefore an example of outcome number 3 for the impact of price $p_{\tau-1}$ on the expected distribution of bids:

$$\psi_{|p_{\tau-1}}(b) = \xi_{|p_{\tau-1}}(g_{v|p_{\tau-1}}(v|p_{\tau-1})).$$

The formula for the bidding function is not changed, although it still depends on the expected distribution of bids in future periods.

end of Example 2

The bid of the bidder i in the period τ in sequential auctions is defined by the following equation:

$$b_{i,\tau} = v_i - (\int_0^{b^*_{i,\tau+1}} (v_i - x)\psi_{2,\tau+1}(x)dx + \int_{b^*_{i,\tau+1}}^{\infty} (v_i -$$
$$(\int_0^{b^*_{i,\tau+2}} (v_i - z)\psi_{2,\tau+2}(z)dz + \int_{b_{i,\tau+2}}^{\infty} (...)\psi_{2,\tau+2}(z)dz))\psi_{2,\tau+1}(x)dx) \quad (2.3.1)$$

The meaning of the above equation is that the bid in period τ is the valuation dis-

counted by the expectation the option value to bid in the future periods. It is dependent on the distributions of bids in period $\tau+1$ and further future. After the price from period $\tau-1$ is observed the distributions of bids in all future periods are updated through one of the three channels outlined above, and the bidding for each of the future periods is updated:

$$
b_{i,\tau|p_{\tau-1}} = v_i - \left(\int_0^{b_{i,\tau+1|p_{\tau-1}}^*} (v_i - x)\psi_{2,\tau+1|p_{\tau-1}}(x)dx + \int_{b_{i,\tau+1|p_{\tau-1}}^*}^{\infty} (v_i - \right.
$$
$$
\left. \left(\int_0^{b_{i,\tau+2|p_{\tau-1}}^*} (v_i - z)\psi_{2,\tau+2|p_{\tau-1}}(z)dz + \int_{b_{i,\tau+2|p_{\tau-1}}}^{\infty} (...)\psi_{2,\tau+2|p_{\tau-1}}(z)dz \right) \right)\psi_{2,\tau+1|p_{\tau-1}}(x)dx.
$$

$$(2.3.2)$$

The higher the expected distribution in each period, $\psi_{2,\tau|p_{\tau-1}}()$, the lower the expected surplus possible to gain due to future options to bid. Subscript 2 indicates the distribution of the second order statistic from $\psi_{\tau|p_{\tau-1}}()$.

Depending on how the observed price influences updating of the distribution $\psi_{\tau|p_{\tau-1}}()$, the bids will be updated either upwards or downwards.

Definition 29. *Price $p_{\tau-1}$ is **directly representative of competition** in $\tau+1 \iff$ an increase in price $p_{\tau-1}$ leads to an update of distribution function of bids in $\tau+1$: $\psi_{\tau+1|p_{\tau-1}}$ such that $\psi_{\tau+1|p_{\tau-1}}$ dominates $\psi_{\tau+1}$ in terms of First Order Stochastic Dominance.*

In this case the higher expected price will lead to an upward update on the distribution and as a result the expectation of the other bids and the auction in $\tau+1$. The case of Example 1 of the simpleast model is clearly an example where price $p_{\tau-1}$ is **directly representative of competition** in $\tau+1$ as defined above. It has been shown on in 2-period overlapping generations model example that once the price $\tau-1$ is observed,

the distribution in $\tau + 1$ will be updated.

Theorem 34. *In every model, where the price in $\tau - 1$ is **directly representative of competition** in $\tau + 1$, bidders will update their bids in τ in monotonous relation to observed prices in $p_{\tau-1}$. Therefore, higher observed price in $\tau - 1$ would lead to an increase in bids in τ.*

The proof of the theorem above is trivial, as it is implied by Definition 29. The empirical investigation below aims at testing whether learning from past prices is present in online auctions. If so, then is updating of bids in monotonous or opposite relation to prices. The finding learning from past prices would suggest that bidders think of the prices in $\tau - 1$ as **directly representative of competition** in $\tau + 1$ according to definition and the theorem above. Additional question tackled here is what can be defined as period $\tau - 1$ for bidders in real life setting? Do bidders learn more from the auctions they have participated in actively, or from observation of past prices from most recently closed auctions?

2.4   Questions

The empirical investigation below aims at analyzing of the impact of the sequential aspect of online auctions on bidding. The questions of interest are:

1. Do bidders discount their bids in anticipation of future periods?

2. Do bidders update their bids based on observed past prices?

3. Is the updating of bids in monotonous relation to past prices?

4. What can be defined as period $\tau - 1$ in online auctions? Is the most recent auction in which the bidder participated the most important for learning? Do bidders learn from observing other auctions, in which they did not take part, as much as from the auctions in which they have actively participated?

5. If learning occurs, than is it rational? - are there other bidders who take part in both $\tau - 1$, which is used for learning, and the future periods?

2.5 Empirical Approach

Above questions are answered using a linear model estimated using OLS as well as the t-test for bid discounting with the use of smaller sample size, with selected bidders who bid only sequentially (allowing each time for the previous auction to finish before placing a bid in the new auction). In short, the ways the above list of questions is answered are listed below:

1. Do bidders discount their bids in anticipation of future periods?

   This question is answered with the use of small, pre-filtered subset of data. Users who bid sequentially are defined as those that wait until the end of the previous auction, in which they took part, before placing their final bid in a new auction. Only such bidders who satisfy this definition in each period, so each time they bid in a new auction, are selected. The bidders are then grouped by the total number of periods. The groups are also divided between bidders who win in the final period and those who do not win any object. The t-tests (matched pairs) are performed on the differences between bids in periods in a sequence. Matched pair t-tests ensure that the unobservable bidder characteristics do not influence the results. High degree of selection means that there is a small number of observations in each group, and the tests have to be limited to small number of periods. It also means this type of selected dataset could not be used for the regression with more controls.

   Regression analysis is preformed on a different type of dataset, and user fixed effects could not be included in the regression analysis. Too many bidders have bid on a small number of auctions, or have bid concurrently. In order to minimize the impact of unobservable bidder's characteristics, bidder's valuation is accounted for using

their final bid in the last auction they have participated in. The question of bid discounting, is partly confirmed in the regression results, which shows a positive coefficient next to bidder's auction number in the regressions on bid amount.

2. Do bidders update their bids based on observed past prices?

The approach to answer this question is to use regression analysis and identify the impact that the price of previous bid has on the bid amount. The analysis is limited to final bids by bidders in each auction. Period $\tau - 1$ is defined as the last auction in which the bidder participated, that has already finished before the bid in τ. Bidder could, therefore, already see the price of the finished auction. As mentioned above, bidder's valuation is treated as the most important unobservable that influence bidding. It is accounted for by the inclusion of the final bid by each bidder as a control in the regression. All the controls relating to the time of the bid and the dynamic aspects of the auction at the time of the bid as well as interaction terms between them are included. The fact that the coefficient next to previous period price is not significantly affected by the inclusion of all the interaction terms, strengthens the conviction that all important unobservables are appropriately accounted for and the impact of learning from past prices is identified. The same regressions are performed on different subsets of the data, selected based on auction characteristics. The strict rules used for the selection means that subsets contain almost identical auctions, allowing, therefore, for the identification of the effect of past prices on bidding.

3. Is the updating of bids in monotonous relation to past prices?

See above point 2.

4. What can be defined as period $\tau - 1$ in online auctions? Is the most recent auction in which the bidder participated the most important for learning? Do bidders learn from observing other auctions, in which they did not take part, as much as from the auctions in which they have actively participated?

The impact of differently defined period $\tau - 1$ on bids is tested. Alternative definitions include:

(a) The most recently finished auction in which the bidder participated.

(b) The most recently finished auction before the bid (without the necessity that the bidder placing the bid participated in that auction).

(c) The average price from the most recently finished auctions. The alternatives include 5, 10, 15 and 20 most recently finished auctions.

The past prices from period $\tau - 1$, as defined above, are included as alternatives in the regressions. The coefficients are compared which allows for building conclusions.

5. If learning occurs, then is it rational? - are there other bidders who take part in both $\tau - 1$, which is used for learning, and the future periods?
   The analysis of common bidders between auction at τ and the auction at $\tau - 1$, as defined by alternative definitions above, is performed. Percentages of common bidders (with exclusion of the bidder placing the bid in τ) are calculated.

Before the above analyses were performed, the data needed to be prepared. The dataset used contained only a small number of variables, namely the standard information on bids that can be collected from the eBay website. Some aspects of the auctions, such as live prices, current number of bidders and bids, were not readily available, and needed to be generated. One aspect of the dataset that was limiting for the analysis, was the fact that the usernames of the bidders were partially encoded, which is done for the anonymization purposes. Fortunately, the anonymization is done in a universal way in the whole dataset, and part of the username information, such as first and last letter as well as their total number of eBay wins is kept. The identification of users was made with the use of the remaining information, and it is shown, that this information, together with bid timing provides enough data to get a high certainty of the correct user identification.

This means that there was no need to acquire and process sensitive information, such as real usernames.

2.6 Data

The data includes iPhone 4 sales on eBay between 17th June 2010 and 7th August 2010. It is a unique source for the analysis of bidder behavior in auctions for a number of reasons. Most importantly it contains data on almost 2000 auctions of the same product. Secondly, it contains not only the highest bid but also all other bids in each auction, which means the analysis does not have to be limited to final auction prices. Thirdly, the data was collected at the time of the shortage of iPhone 4, shortly after its first introduction to the market. There is a lot of interest in these organic auctions, which also separates it from the low participation in auctions purposefully set up for experimental research papers. Due to the fact that there was an uncertainty about fast availability of the smartphone in physical retailers, the prices achieved were very often higher than the later fixed price after the supply shortage was resolved, reflecting high valuation for the object among bidders. The temporary supply shortage is also one of the reasons for the exceptionally high interest in the online auctions for this device at the time. eBay data can be collected through different methods, but most common are either paid data sourced directly from the provider or free collection through API or manually by observation of auctions. The data used here has been collected through two different methods, API, as well as manual collection from the eBay website. The data was originally used in Waterson and Doyle, 2009[37]. The proportions were such that about 1/3 of data was collected manually, while 2/3 has was collected by a web crawler. The data constitutes a large and unique dataset. The usernames of bidders collected from the website are partially encoded for privacy reasons, so the full usernames cannot be unambiguously identified. This issue has been addressed in order to identify the same bidders taking part in more than one auction. The fact that the usernames were matched for the first time makes the dataset unique

source for analyzing individual behavior in sequences of auctions over time. It is possible to match the usernames, since the first and last letters as well as number of current wins are available, and details of the methods is discussed in the following section.

2.6.1   Usernames

The dataset contains partially encoded usernames of buyers. The format of buyers' usernames is a String containing the first letter of the username followed by number of stars (*), which cover the middle part, and followed by the last letter, then the number of current wins in brackets. As an example *user* entry "a***s(19)" means that the first letter of the username was "a", the last "s" and that the person has a total of 19 wins on eBay to date. In the case that the users do not buy any new product over the duration of data collection, this encoding would give in fact almost 100% certainty that each distinct entry related to a different person. The data collection took place over 44 days, so it is possible that additional purchases were made over that period. Moreover, some buyers can win more than one product over the dataset duration, or continue bidding in other auctions after winning a product, therefore reasonable increases in the number of won auctions are possible. This information is used in the algorithm to identify unique users.

The choice of usernames on eBay allows using any letter, capital letters, numbers, as well as special characters, which include: full stops, asterisks, underscores, or dashes [1]. Usernames need to have a length of at least 6 characters. A username is a unique identifier of a person and one unique username is assigned automatically once a person registers on

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[1]Additional restrictions include (citation from eBay website): " User IDs can't contain: Any characters except letters, numbers, full stops, asterisks, underscores or dashes Elements that imply an email address or web address - including but not limited to .com, .net, .org, .edu or any variation (for example, _com or -com). However, your user ID can contain an element of an email address or web address that identifies you or your brand. For example, if your web address is xyz.co.uk you can use xyz as an element of your user ID Consecutive underscores An underscore, hyphen or full stop at the beginning or end of a user ID (for example, -cardcollector) The word 'eBay' The letter 'e' followed by numbers Obscene or profane words that breach our profanity policy The same user ID as another member A user ID that is similar to the name of an eBay Shop A term that could be confused with someone else's trademark or brand (for example, 'CocaColaSeller') A term that may reasonably mislead another user into thinking that the account is held by a law enforcement agency or other regulatory authority (for example, Trading Standards UK)"

the website. It is later possible to change it to a preferred one. Given that most of buyers stay with their randomly assigned username, which typically includes a mixture of letters, special characters and numbers, the first and last letter of the usernames are likely to be a unique combination. The calculation of the upper bound on the number of combinations is with the assumption that the characters forming a username are assigned at random. No matter what the total length of the username, just by knowing the first and last letter gives the number of permutation with repetition: $66^2 = 4356$ (66 is the total number of possible characters used), which means that the probability of randomly picking two identical pairs of characters is $1/66^2 = 2.296 * 10^{-4}$. In the dataset there are multiple observations for each person (multiple bids). Additionally, it is possible that a person buys as many objects as they wish on eBay. Taking into account that the probability that any other person's username is the same as the previous one is very low, of magnitude 10^{-4} and the existence of multiple bids by the same user, the probability that any two entries with the same first and last letter of the username is the same person is high (upper bound being $1 - 2.296 * 10^{-4}$). The first and last letters might not be random, but nevertheless $2.296 * 10^{-4}$ shows the lower bound on the magnitude of finding two identical usernames, which, even if in fact it is higher, is a very small number, close to zero. This shows that it is very unlikely that there will be two people with the same first and last character of the username in the dataset.

The additional information that is given is the number of total wins on eBay. This gives additional way to distinguish the users, in cases of more than one username with the same first and last character. Two extreme cases are: 1) treating the users as the same whenever the first and last letters are the same, or 2) only when both first and last letter as well as the number of wins is the same. Alternative approach is accepting the same user when first and last letters are the same, and with some restrictions on reasonable change in the number of total wins: for example if the total wins are decreasing with time, or increase too fast to not be possible.

In order to compare these different approaches I have split the variable *user* to *user1*

Figure 2.1: Two methods of distinguishing increases in number of wins.

(containing only the username part) and *numbEbayWins* (containing the number of total eBay wins). The total number of users identified through different methods are shown in table 2.1. In the table, the description of the method, the variable which corresponds to it, and the number of resulting distinct usernames can be found. *user* is the default variable containing both the encoded username and the number of total wins, *user1* is the variable created by splitting the *user* variable, containing only the username part of it. The other variables are created by imposing additional constraints on the increases/decreases of number of total wins in time. It was done by first sorting the users in a way to create an increasing ranking in the number of wins over time for each pair of first and last letter. There are two ways to approach this ranking, and the results are slightly different (Figure **??**). The observations for each user can be first sorted by time and then by number of wins, or first by number of wins and then by time. I have decided on the second approach reflecting the belief that smaller number of win increases are more likely to refer to the same person. The difference between two methods is presented on an example in Figure **??**.

I am using *Method 2* , so sorting the users by the total number of wins first, and then by time. Then, variable *user1NonD* is created by requiring that for each username, if the number of wins increase, but the time decreases, then the following entry starts a new user - that is applying *Method 2* from figure **??** without any additional restrictions on the increases in number of wins. Variable *userCoded1* is created such that the number of wins difference is at most 10 if time difference is below 24 hours and that the number

Table 2.1: Number of users when different rules of user identification applied

Total users					
username	username +number of wins match	username + number of wins not decreasing	username +n. of wins not decreasing +$\leq$10 w/h	username +n. of wins not decreasing +$\leq$5 w/h	username +$\leq$1 win in the first 1.2 min +$\leq$ 5 w/h
user1	*user*	*user1NonD*	*userCoded1*	*user15*	*user1M*
1156	6123	3994	4075	3995	3997

of wins is at most 10 wins per hour for time difference grater than 24 hours, in addition to what is required for *user1NonD*. Next example of an identifying variable is *user15*, which allows for an increase of no more than 10 wins for any time difference less than 5 hours, and and increase of no more than 5 per hour for a time difference larger than 5 hours. The restrictions on the increases used are arbitrary. Nevertheless it is more often the case that the wins for the same user might be close to each other (for example within 5 or 10 hour period), which is the reason for an initial period allowing for faster increases, set up to be a given number of hours. Otherwise, if just a simple ration per hour was used, an increase of 1 in one minute difference would not be allowed, and this can easily be the case if someone wins and then immediately places another bid which wins again. The last method, with variable *user1M*, is created by restricting that there is an increase no larger than one in the first 1.2 minute, and the other conditions remain the same as for *user15*. The difficulty is in choosing the best way for user identification, and as we can see the very upper bound on the number of users is 6123, while the lower bound is 1156, when allowing any increases in number of winning bids. Different ways of restricting these increases lead to a different number of resulting users in the dataset.

It is not clear which method should be used, although identification by *user* can be ruled out since some increases in the number of total wins should be allowed. The decreases in number of total wins cannot take place, so conditional on no mistakes in recording the data, *user1NonD* is the lower bound on the number of bidders in the dataset. Given that some of the data were inserted manually by research assistants, there can be some small

typos, which could slightly influence the decreasing number of wins in time, but this is certainly a marginal problem. As can be seen, applying some restrictions on increases in bids, as in *user15* or *user1M* does not influence the number of bidders by much (only +1 or +3 difference from *user1NonD*), so using this variable seems to be reasonable and practically equivalent to the other two methods. The *user1NonD* method has been chosen, and the variable which distinguished between bidders has been named **user**. This is the one mentioned in table 4.1 under this name. This variable has been used for creating all other variables statistics and data analysis based on user identification.

2.6.2 Live Bids, Live Prices and Bids

The data contains bids made by users. The bid data, that can be collected from the website does not reflect, though, what information is available to the bidders at the time of auction participation, because it is only available to inspect after the auction has closed. This means that in order to retrieve the information available to the bidders at the time they decide to place their bids, the live prices need to be retrieved. During the auction, when the bidder places their bid, the bid itself is not visible to other users, only so called "live bid", which is the second highest bid plus an increment. The magnitude of the increment, which changes depending on bid amount, is publicly known and available to be found on auctioneer website. It is, therefore, possible to retrieve live bids from bids data, and it has been done for example in Jank, 2010 [20]. The R script used in Jank, 2010 [20] is available on the book's website, and it has been utilized here in order to retrieve the live bids (with necessary adjustments to the dataset). Auction bids and live bids can be quite different. Most importantly, while it is possible that a lower bid is placed after a higher one by another user, live bids are always monotone increasing. Live bids show, in fact, what the highest bidder would pay if the auction ended at the given time, but do not reveal what is the current highest bid. It has to be also noted that while the code provided by Jank [20] recreates the live bids, information available to the bidder at the time of bidding is the current live price - which is the live bid just before

the new bid is placed - precisely the live bid of previous bid.

In eBay terminology the ability to place high bids in advance is called "proxy bids", and live bids are related to as "bids" - even though these "bids" are automatically made by the system on behalf of the bidder. Interestingly at the time the bidder places their bid they are not able to know what their live bid will be or whether they will even become the highest bidder.If we use the terminology conventionally used in auction theory, the proxy bids should be called bids, while eBay - a type of second price auction, where the highest bidder pays the second highest bid with an added fixed increment specified by the rules. Over the auction duration eBay keeps track of the second highest bid (+ the increment) and gives this information to auction participants and observers. This information may be partially revealing about the valuation of some of the bidders in the auction, and it can influence the bidding strategy. Due to the possibility of multiple bidding, as well as the fact the auctions are quite long (one or more days), the actual information about the valuations revealed by these live bids is very limited. It does, though, have a significant impact on the bids placed. Jank and Shmueli (2010) in fact show, that the information on live prices and time alone can be used for prediction of final price in an auction, since there are patterns of how these prices evolve over the auction duration.

The variables $bLiveBid$ and $bLivePrice$ have been generated from the bid data to represent live bids and live prices. The prefix "b" in front of variable names means that they are specific to each bid, not for example auction variables, or bidder - specific variable. Before changes made the dataset contained "MaxBid", which referred to the highest live bid in the auction, and at the same time auction price. This variable has been renamed to $aPrice$, since it relates to an auction, and represents the final price.

2.6.3 Other variables

In addition to live bids, and live prices, some other informative variables have been generated from the data. In particular, these were variables relating to the position of bids in auction, the numbering of bid sequences for each user, as well as all user-related variables.

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After differentiating between distinct users as described in the previous sections, it was possible to recover new information about number and sequencing of auctions joined by the bidders, or number of bids placed by bidders in each auction. All the variable names with corresponding definitions can be found in the Appendix table 4.1. The naming of variables have been unified so that it is clear, by looking at the prefix, at which level the variable changes (or is constant). Variables that change at each bid start with prefix "b": these are $bLiveBids$, $bLivePrice$, but also $bAmount$ - which represents the bid amount. $bAuctionBid$ tells which bid it is in a sequence of bids starting from the beginning of the auction. There are several variables relating to the timing of the bid: $bTime$ is a date-time variable which tells both the date and time of the bid; $bPercWithin$ takes auction duration as 100% and specify for each auction bid at which percentage of auction time the bid has been placed; $bTimeTillEnd$ is a time variable that tells the amount of time left to the end of an auction; $bDayOfWeek$, $bHourOfDay$ are other variables relating to time of the bid - day of week and hour of the day at which the bid has been placed. The various time variables have been created at the beginning of data cleaning process in order to have more flexibility in choosing relevant variables to control for the time of the bid in later analysis. Some indicative 0-1 variables have been created as well: $bFirstAuctionBid$, $bLastAuctionBid$, $bIsWinning$, $bLastMaxBid$. $bBIN$ is another categorical variable, set to "Yes" if the bid was placed using "Buy it now" option to purchase the object at a fixed price, not using auction mechanism. User-level variables have "u" as a prefix, and all of those variables needed to be generated after users were identified. Some user level variables are also auction level, but these have prefix "u" as well. Some variables which relate to the current bid by the user are $uAuctionBid$ - which tells which user's bid it is in that particular auction; $uBid$ - tells which user's bid it is counting from the first bid made by this bidder in the dataset; $uWinsSoFar$ tells how many items the bidder has won already, while $uTotalWins$ tells how many wins the bidder has won in total; $uAuctionNumber$ tells which auction it is in the sequence, counting from the first auction the bidder has joined, while $uReverseAuctionN$ counts

the auctions from the last one, and it has been generated in order to have uniform numbering counting from the last auction for bidders with different number of total auctions joined (the reverse counting is based on how bidder's strategies are solved from using backward induction); $uTotalAuctions$ gives the total number of auctions joined by the bidder; $uLastBid$ is the amount of user's last bid placed in the final auction before leaving the auction marketplace - according to theory the bid placed in the final auction is expected to be the highest bid, and also equivalent to bidder's valuation for the object, which is why this variable is very important for further analysis; in addition $uMaxBid$ represents user's maximum bid; $uTotalAuctionBids$ tells how many bids the bidder has placed in the particular auction; $uTotalBids$ tells the total number of bids (including all auctions joined) by the bidder; There are also some boolean (indicative) 0-1 variables include $uNewAuction$ which is 1 for a first user's bid in an auction, $uLastAuctionBid$ which indicates the last bid by the user in each auction (useful for selecting a subset including only final bids by users in auctions), $uNLR$, which indicates the users that were marked as unregistered from eBay at the time of data collection; and other user-level aggregate variables such as $uAvAmount$ which takes an average amount bid by the user, $uAvAuctionBids$ which is the average number of bids per auction for each user. Another user-level variable is the $uNumbEbayWins$ which has been used for user identification as described before and contains the total number of eBay wins by the user, including the wins outside the collected dataset (includes the whole history of the bidder since their registration).

All the auction-level information in the dataset was present in the collected data, and the variables relating to auctions were renamed to start with prefix "a". The auction level variables are: $auction$ - categorical variable representing auction number; $aCondition$ - is a categorical variable containing information about condition of the phone such as "New" or "Used"; $aDuration$ equal number of days the auction has lasted, and there are auctions lasting 1, 3, 5 or 10 days in the dataset; $aEnded$ which is a date-time format

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variable representing the date and time of the when the auction has ended; $aStartDate$ is another date-time variable for the start date of the auction; $aExtras$ a categorical variable which states whether there are any extras included with the phone, and this is none for almost all of the dataset, so this variable is not of much use; $aMaxBid$ is the maximum bid in the auction; $aPrice$ is the price of the auction; $aMaxLiveBid$ is the maximum live bid (also equal to the price of the auction but generated from live bids); $aMaxLivePrice$ equal to the last live price which the final bidder could see before placing their bid; $aModel$ is a categorical variable, which tells the different models from the point of view of data storage space - most common categories are "16" and "32"; $aNetwork$ - a categorical variable stating the network to which the phone is locked or whether the phone is unlocked (possible to use with any network); $aTotalPhotos$ tells the number of photos in the auction; $aNonStock$ is a categorical variable telling whether there are some non-stock photos (so real photos taken by the seller, not from stock photos of the phone available on the Internet); $aNonStockPhotos$ tells the number of these "non stock photos"; $aPositiveFeed$, which tells the number representing the positive feed score given by customers of the seller (certain levels of positive feed mean certain levels of stars given by eBay for the sellers to represent their reputation levels); $aStarLevel$ which is a categorical variable has been generated from the $aPositiveFeed$ variable so that it can be recorder which star levels were the sellers given (it is important because the stars of different color are visible next to the seller's name); $aStarL$ represents star levels as before, where 0 is no star, 1 is the lowest star level possible, 2 - second level, 3 - third level, etc.; $aPostage$ represents the amount to be paid for the postage or 0 is postage is free of charge; $aPostto$ shows how the "post to" field has been filled by the seller - these are all auctions on UK eBay and therefore most auctions have "UK" in this field although there are also other options present such as "Worldwide" here; $aReturns$ is a categorical variable, which can be "Yes" if returns are accepted and "No" if returns are not accepted; $aSeller$ gives the seller name (categorical variable); $aStartPrice$ is a variable representing the start price for the auction set by the seller; $aTotalBidders$,

aTotalBids are the variables telling the total number of bidders and bids in the auction; Some additional variables generated from the data are *aAvPercWithin*, which tells the average time (in therms of percentage of auction duration) at which bids have been placed in that auction, and *aAvUserAuctionBids* which is the average number of bids the users have placed in that auction. Of course not all of these variables are necessary for the following analysis, but the variables needed to be generated before the data was further refined, and the aim was to generate the most possibly relevant information for flexibility later. The descriptive table for most numerical variables can be seen in table 4.2: the full dataset contains 27648 bid, and the maximum number of user auctions is 99. The statistics related to the main variables in the dataset can be found in the Appendix in table 4.2. Some bids had no recorded corresponding username - that field was "." in the data, and therefore these bids are not included in the user-related variables (which is why the total number of bids for these variables is 21377). Of course the bids without recorded users have been removed from further analysis, as distinguishing between users is key for sequential auctions analysis.

For the analysis of bidding in sequential auctions only the final bid of each bidder in each auction is relevant. Multiple bidding can be seen as irrelevant from the theory's point of view, since only the last bid by each bidder in an auction represents the full amount that one is willing to pay for the item. The dataset was limited to final bids by each bider in auctions. After multiple bids were removed, the dataset contained 12063 bids in total.

2.7   Bid discounting in sequential auctions

Theory of sequential auctions predicts that a bidder with positive horizon of auctions to bid in the future will be discounting their bid, taking into account the present value of future option to bid. The theory predicts that bid discounting does not depend on total number of auctions joined, if we count the auctions from the last one downwards.

This a consequence of the fact that the strategies in periods are normally solved for by backward induction. I am checking the hypothesis of bid discounting for bidders bidding in different number of periods (number of auctions joined) until their first auction won (or until they stop bidding in the case they do not win any auction). The bidding behavior of bidders is analyzed separately for groups of bidders bidding in different number of periods. Those who never win, are also isolated and analyzed separately. The findings confirm the hypothesis of bid discounting for those who do win an item in the final period. On the other hand, bid discounting is not found to take place for bidders who do not win any item in the final auction. Another prediction that is confirmed is that bid discounting is higher for higher valuation bidders. A positive relationship is found between the final bid (valuation according to theory) and bid discounting in earlier periods.

2.7.1 Data used for analysis

For the analysis of bid discounting there is no interest in bidders winning multiple times. There are two ways of addressing this issue. First one is to take into account. If we used such a conservative approach only those bidders who won once and in their final period would be considered. There are many cases where bidders continue to bid after the final win, which suggests that they could be interested in an additional item. It is not possible to fully distinguish those who have intention of buying only one item from those who have an intention to buy more than one item, therefore it is futile to try isolating single-demand bidders for the analysis. There is also no grounds to think that multiple demand bidders bidding until their first win is any different than single demand bidders, or that it would not follow bid discounting strategy. Another approach is therefore proposed, and that is keeping the bids of bidders who won at least once until (and including) their first win, and all the bids of bidders who have not won at all. The periods are counted in reverse order starting from the last period (that is the winning period for those who have won, and the final period for those who have not won at all) downwards until the first period joined. This reverse counting is stored in the data as $uReverseAuctN$ variable. Total

number of periods is the total number of auctions joined until the first win or until the end of bidding if no win registered at all. Another restriction that is needed to consider sequential auctions bidding is that the previous auction should be finished before the bid in following auction is placed. Only those bidders whose full sequences of bidding contain sequential bidding (each following period bid is placed after the previous auction has finished) are considered. This restriction is very important, because otherwise the bids of bidders who bid in multiple auctions simultaneously, not knowing how many they would win, would be included, which results in a completely different strategy of bidding than this described by a model of sequential auctions.

The data contains variations of the phone - different models of the phone and different networks assigned (also unlocked phones), but all those variations are included in the analysis, since the focus is on sequences of bids on the same item by each bidder. It needs to be kept in mind that if a bidder bids on different variations of the phone in a sequence, then that could affect the amount they bid in the particular period. On the other hand, there are more aspects that would affect their bid such as the current live price of the auction, the time until the end of the auction, number of other bidders bidding in the auction and possibly more. By restricting the bids considered in analysis of sequences of bids further - for example by network and memory of the phone, the number of bidders who bid always on the same variation of the phone would be very limited, and the statistical analysis could only be performed on bidders who bid in two auctions (since this is the largest group). The interest is in comparing strategies across more than two periods of bidding, and this can be achieved when all variations are included. It is expected that bid increases can be recognized despite variations in the circumstances and auction characteristics at the time of each bid. Essentially, all auctions are for the same item, and bidders are believed to have valuations for that particular item. Variations can affect bid amounts up to certain degree only. The results show that despite the differences across auctions, the main prediction of sequential auctions theory is confirmed in this real-world data.

The data used for analysis contains sequences of bids of bidders until their first win (or no win at all), with the restriction that a previous auction need to be finished before the next bid is placed at each period. The result is dataset containing 765 bidders with two auctions joined out of which 244 have won in the final period, 230 bidders with 3 auctions joined out of which 71 have won their last auction, 65 bidders with sequence of 4 auctions joined out of which 23 have won in their last period, and 21 bidders with a sequence of 5 auctions out of which 4 have won in their last period. The restriction that only bidders who each time before joining new auction have waited until the previous one has finished is very limiting which is the reason for relatively small number of resulting bidders included.

2.7.2 Results

The first prediction tested is bid discounting. The following hypothesis is stated:

Theorem 35. *Hypothesis 1: Bids are increasing with auction number in sequential auctions.*

The sequences of bids for the same bidders were used, which is why a matched pairs t-tests can be performed. The t-tests are performed between each pair of consecutive periods with division between bidders joining different number of auctions as well as for bidders who have won and those who have not won any item. Groups with 2,3, 4 and 5 periods are considered. The tests are a one-sided matched pair t-tests as shown below:

$H_0 : b_{t,i} - b_{t-1,i} \leq 0$

$H_1 : b_{t,i} - b_{t-1,i} > 0$

The test results are presented in table 2.2.

There is a clear increasing trend between consecutive periods in all groups of bidders who won at least once. All, but one mean of difference between t and t-1 are positive. it has to be noted that the negative difference between the first and second period for 5-auction bidders with 1 win is driven by an outlier - one bidder have placed a very low

bid (50 pounds) at the beginning of an auction in their second period (that's 4th with reverse numbering). In addition to that, 5-period group with only 4 bidders does not give enough statistical power to make any conclusions about differences between means, especially that in the first period all bidders bid on the more expensive variant (model 32), while in the later periods almost all bids are in auctions for model 16. If we look at the 2 - to 4-period bidders it is clear that bids in sequence of auctions are increasing. H_0 is rejected for the differences between almost all periods at 1% or 5% significance level. The differences between period 2-3 and 3-4 for the 4-period group is not significant at 10% level, but the mean difference is positive and t-statistic is above 1.07.

On the other hand, if we look at bidders who have not won in any of the auctions, there is no increasing trend in their bidding. It gives an impression that those bidders for not follow the same bidding strategy across sequence of auctions. Bid discounting for groups of bidders who won in the final period are also presented in figure 2.2. Mean amounts for each period are marked by red dashed lines.

Given the results above, it can be concluded that indeed there are bid increases observed in sequences of auctions, but only for bidders who bought at least one item.

Theorem 36. *Hypothesis 2: Bidders with higher valuation discount their bids by more than bidders with lower valuation.*

The second hypothesis to check whether bidders who have higher valuation discount their bid by more.Figure 2.4 in the Appendix show scatter plot between discounting amount and the final bid and the fitted line. As can be seen there is a positive correlation between these two. Bid discounting is increasing with the final bid, and therefore with valuation - as predicted by the theory. The results are of course relying on the assumption that the final bid is a good representation of bidder's valuation.

Figure 2.2: Bid amounts, groupped by periods

Bidders who have won in the final period
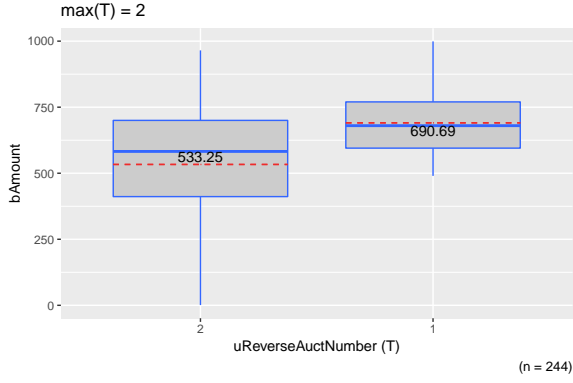
(a) Bidders with 2 auctions joined
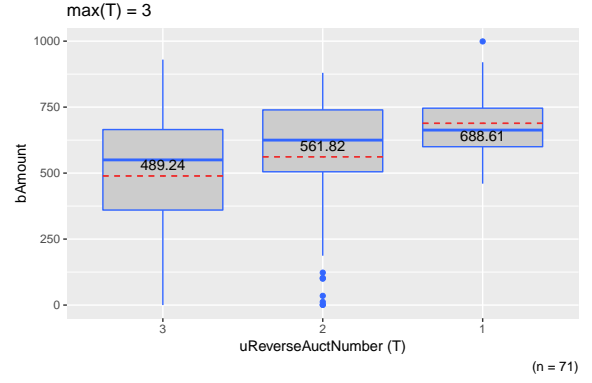
(b) Bidders with 3 auctions joined
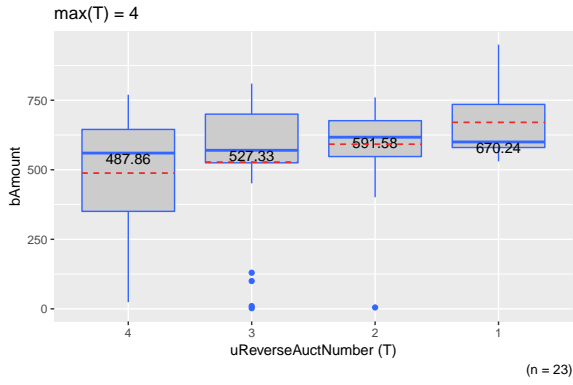
(c) Bidders with 4 auctions joined

2.7.3 Conclusion

In conclusion, the analysis of sequences of final bids in auctions for bidders who bid in different number of auctions confirms the hypothesis that there is bid discounting present. Despite different versions of the item and time and auction related circumstances of each bid, the t-tests show significant results for increases in bidding through most of the periods tested. Additionally, the discounting amount is positively correlated with valuation. Theory is matched with these non-experimental real life data. The results confirm relevance of theoretical predictions to online auctions.

2.8 Learning from past prices - Regression Analysis

The Overlapping Generations Model shows that learning about prices in previous auctions influences the implemented bidding strategy, whenever there are overlapping generations of bidders in sequences of auctions. In reality, the auctions which we can observe in online auctions marketplaces provide a much more complicated environment than the stylized model can. The addition of overlapping generations of bidders to the model creates a feedback loop between prices of finished auctions and strategies of bidders without the necessary condition that the same bidders are present in the information acquisition period and the period about which learning happens that may influence the strategies of forward-looking bidders. In the stylized auctions model each period t describes a single auction, and the auctions end before the next ones start. In real online auctions there can be a situation where a few auctions can be taking place simultaneously. How can we recognize that we are dealing with overlapping generations? It is necessary that some of the bidders from period $t-1$ continue to bid in period t, while there are also some new bidders at t. Some bidders do not continue to period t, and in the model these bidders leave the auctions altogether.The winning prices from $t-1$ are a source of information about the age and distribution of the other bidders. As such one can see that we are dealing with a highly self-consistent model when prices are observable. Observed prices in the higher end of the price distribution mean that the competition in $t-1$ was fiercer and also that competition will be fiercer during the period t as well since more older and higher distribution bidders are expected. Additionally, the fact that more old bidders are expected in period t implies an increase in the probability of the state with fircer competition for period $t+1$.This, in turn makes the bidders who observe the prices move their bids upwards (bidders that are not in their final period). Considering only bidders who are not in their final period should show that the bidding strategy is adjusted. The first step is therefore to determine the periods. In the model the period $t-1$ is the most recent period in which the bidder took part before t. Due to the complex nature of data

another restriction was applied on period $t - 1$: the auction of period $t - 1$ was required to be finished before the bid is placed in period t. This way the bidder was able to see the result of that auction before bidding.

On the other hand, I am also analyzing the impact of prices in the most recently finished auctions on the bidding strategy. The summary statistics of 20, 15, 10 and 5 most recently finished auctions (mainly the average prices in these auctions) can also have an influence on bidding. If the impact of results of most recently finished auctions on bidding is stronger than the impact of the last auction in which the particular bidder took part, it could mean that bidders are able learn more from looking at recent auctions results than from the results of auctions in which they have actively participated.

Let us consider how a bidder learns about the period $t - 1$ auction results in practice. When the auction has finished, the bidders get a notification sent to their phone or e-mail that lets them see the auction result. Another way in which a bidder learns about results from past auctions is to look at a list of recently finished auctions, which is displayed just below the product search. Typically up to 20 finished auctions are displayed together with their prices. Without any further action anyone searching for a product on eBay is able to see the finished auctions list, which is sorted by auction end time, starting with the most recent on top of the list. In order to justify that the bidders indeed derive any information from past auctions in which they took part, it should be the case that some proportion of bidders in period t have also been present in $t - 1$. As a first element of the analysis I will check, therefore, what proportion of bidders from period $t - 1$ and of recently finished auctions are also participating in period t, and the second element will be to check the influence of auction $t - 1$ result or prices or recently finished auctions on bidding strategy in t.

2.8.1 Data used for analysis

The dataset contains final bids in each auction by bidders, and to some auction categories, which are most populated in the data (this is described below).

The dataset contains 12063 final bids, which are divided between 2536 auctions and made by 3779 bidders. There are many auction characteristics present in the dataset. Categorical variables are shown in the table 2.3 . In order to reduce variability between auction characteristics, which could influence and bias the results relating to bids, less populated categories are removed from the analysis. In particular, auctions included in the analysis have duration of 1 or 3 days; model (which relates to GB in storage) is "16" or "32"; network (relating to the network on which the phone can be used or if on any - unlocked) is either "Unlocked" or "O2"; $StarL$ relates to the rating of the auctioneer - level 0 means too little seller rating points for any star level, level 1 is the lowest recognized level called "Yellow Star", level 2 is "Blue Star", level 3 -"Turquoise Star" etc. - all levels are represented by stars of a given color present on auctioneers profile and are found next to their displayed name. The variations kept are levels 1,2 and 3. 0 (no level) is excluded because auctioneers which do not have enough rating points to have given a star level are likely to be treated differently by bidders : for example with too much uncertainty about quality of product or postage, which could bias the results. Auctions with no photos, or free postage are also excluded. As a result analysis is based on a chosen subset of the dataset that contains the most populated group of auctions, with most similarity, so that on the the results are derived from data on very similar auctions.This is important because it is undesirable to distort the results by analyzing outliers or auctions where there is increased uncertainty about seller's reputation.

The main differences between bidders, besides their valuation of the object, are the number of auctions they have joined and number of objects bought throughout the period of data collection. $uTotalWins$ variable shows that a significant part of data contains bids of bidders who have won more than one auction - that is 2773 bids, while 9290 bids belong to bidders who have bought at most 1 item. Bidders who bid in only one auction are not interesting from the point of view of analysis of sequential auctions, and therefore the data for bidders with $uTotalAuctions$ equal to 1 is naturally excluded. There are

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different number of auctions in which multiple auction bidders take part, ranging from 2 to as much as 76. The most populated group is 2 auctions, but bidders with 3, 4, or 5 auctions joined constitute a large part of dataset as well. This variable is key in the investigation of sequential auctions bidding, and it is used in the following analysis. $uReverseAuctN$ variable - the so called reverse auction number, is another key variable here - it counts the auctions joined by one bidder in reverse order : 1 is the final auction the bidder has joined, 2 is the second last auction, 3 the one before etc. The theory generally predicts that, if all bidders come from the same distribution of valuations, the discounting of bids for earlier auctions depends on reverse counting, rather than numbering starting from the first auction joined by the bidder. Some bidders were recognized as "no longer registered" at the time of data collection - the $uNLR$ variable, and these are also excluded from the data used for the analysis. In some cases the "Buy it now" option was used, which is the option to buy the object at a fixed price, this however sno longer complies with the auction rules, therefore these data were also excluded (with $bBIN$ as "Yes").

2.8.2 Bidders moving between auctions - Common bidders with period $\tau - 1$

The focus of this chapter is on analysis of learning from past prices. According to theoretical predictions learning may happen if bidders bid sequentially and have a positive horizon length of number of future auctions they expect or plan to bid in. Additionally, a crucial element in every model with learning from past prices is that a proportion of bidders present in $\tau - 1$ have to be also present in τ, and a proportion of bidders in τ have to be present also in $\tau + 1$, but these do not need to be the same bidders in the OLG model. If learning happens, then price in period $\tau - 1$ affect bidding in period τ. Empirically, period $\tau - 1$, that is relevant from the point of view of learning, is not pre-defined, and different approaches are used in the current paper, and their discussion can be found below.

There are several different ways in which period $\tau - 1$ can be defined. One way is to define period $\tau - 1$ as the most recently finished auction in which the bidder participated. Then, the focus is on the sequences of auctions in which the bidder has actively participated, not on the observation of other auctions. Analysis of the number of common bidders between auction in $\tau - 1$ and τ defined in this way shows how relevant is the information derived from auction in which the bidder participated for the prediction about the following period. The expectation about the future period bidders , $\tau + 1$ will be derived by taking the same prediction one step further, from τ to $\tau + 1$. If the percentage of bidders that are in common between $\tau - 1$ and τ is found to be γ, then the percentage of bidders in common between τ and $\tau + 1$ is, in expectation, also γ. Bidders who bid sequentially can even expect that a certain number of other bidders will be present in the same sequence of auctions in which they expect to participate, as these sets may overlap Another way to define period $\tau - 1$ is the average price from a number of most recently finished auctions. Since for learning to exist, a crucial element is whether some proportion of bidders can be found in both periods $\tau - 1$ and $\tau + 1$, maybe observation of prices in the most recently finished auctions potentially carry more information than bidder's own experience in previous auction. If we consider learning from prices of recently finished auctions, then it is not as easy to take the expectations one step further from $\tau - 1$ to τ and then from τ to $\tau + 1$. Auction in period $\tau + 1$ will be further away from the learning point: $\tau - 1$, but it is not clear how much further away, as well as how one more step is added. If $\tau - 1$ is defined as an average of prices of N most recently finished auctions, and the number of bidders in common between period τ and $\tau - 1$ is ζ, then these may also in part overlap with bidders that are in common between period τ and $\tau + 1$. Alternatively, the number of common bidders with previous auctions that are further away can be considered for learning, not directly before the bid.

Although the standard sequential auctions model requires that the same bidders present in $\tau - 1$ and $\tau + 1$ is necessary condition, the OLG model relaxes this requirement and makes learning from past prices more plausible in real-life auctions.

2.8.2.1 Common bidders with 20, 15, 10 and 5 most recently finished auctions

In this analysis a number of most recently finished auctions before each bid are treated as period $t-1$. These auctions can be observed and learned from by the bidder who is about to place their bid. This is not the same as period $T-1$ in the model, because there is no requirement that the bidder from t took part in $t-1$ defined in this way. First question to answer is what proportion of bidders continue from the most recent finished auctions to period t. Each bid is considered as period t, and, relative to that period, $t-1$ encompasses a number of 5,10,15 or 20 most recently finished auctions.For each bid, the bidder who have placed the bid is removed from the set of bidders. The set of bidders in period t is compared to that in 5,10,15 or 20 most recently finished auctions. Statistics on that are shown in table 2.4. The percentage that is the common part out of the set of bidders in auction at t and the set of bidders in N most recent auctions is different because the sizes of these sets differ. It can be seen, that for 5 most recent auctions, the percentage of bidders who participate in auction t consisting of common bidders with the set of bidders in 5 most recently finished auctions is very high - 7.96%, on the other hand only 1.95% of bidders in the 5 most recent auctions are the common bidders who are also taking part in auction at t. The percentages of bidders in an auction who have also participated in at least one of N most recent auctions increases with the number of auctions considered, so that if 20 most recent auctions are considered this statistic rises to above 16% On the other hand, the percentage of common bidders out of the total number of bidders participating in N previously finished auction is decreasing with number of past auctions considered to 1.33% when $N=20$.

On average, the bidders who continued from recently finished auctions are a very large proportion of all bidders in any auction. There are, on average, about 8.7 bidders in an auction, out of which the percentage of bidders continuing from $t-1$ is between 7.96% and 16.25%. Because t-1 consists of many auctions (5- 20), the continuing bidder(s) make only between 1.33%-1.95% of the total number of bidders in N auctions. To conclude,

we get a large proportion of bidders in t who have also been present in $t-1$, but out of $t-1$ bidders the proportion of continuing bidders is between 1.33% - 1.95%. Still, this shows that a person taking part in an auction can expect as much as over 16% of other bidders in that auction to be coming from 20 most recently finished auctions. Therefore, most recent auctions results are a potential source of information about competition the bidder is likely to face, but this information may not be easy to recover since it is in a way diluted between many auctions.

This means that on the one hand, many bidders take part in auctions that are close to each other. On the other hand, if most recent auctions are considered as an indication of the level of competition the bidder is likely to face, then the relevance of information contained in the averages or any other summary statistics of these past auctions prices is low. The percentage of relevant bidders who took part in previous auctions is below 2%.The fact that bidders take part in auctions that are close to each other can also be observed in table 2.18. This table shows the percentage of common bidders with auctions that finished earlier - first interval contains 11th to 20th most recent auctions, second 21st to 30th, then 31st to 40th and 41st to 50th. The data shows that the longer the time that separates the auctions the less likely it is that there will be many common bidders between these auctions.

2.8.2.2 Common bidders with the most recently finished auction

Instead of averages of prices from N most recently finished auctions, the last finished auction could potentially carry more, and less "diluted", information about the competition. Table 2.20 shows the statistics on common bidders between auction of each bid and the most recently finished auction. Similarly, the bidder who have placed the bid is each time removed from the set of bidders, and the statistics relate to other bidders that are in common between auction at t and the most recently finished auction. As expected, the percentage of common bidders in t who also participated in the most recently finished

Figure 2.3: Graphical representation of common bidders between T and T-1
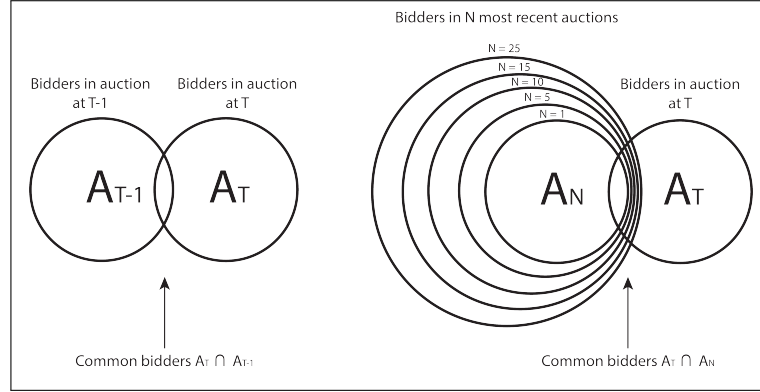
Explanation for terms used in table 2.4, 2.5:

Percentage bidders in T who also participated in $T-1$: $\frac{A_T \cap A_{T-1}}{A_T}$

Percentage of bidders in $T-1$ who also participated in T : $\frac{A_T \cap A_{T-1}}{A_{T-1}}$

Percentage bidders in T who also participated in N most recent auctions : $\frac{A_T \cap A_N}{A_T}$

Percentage of bidders in N most recent auctions who also participated in T : $\frac{A_T \cap A_N}{A_N}$

auction is much lower than in the case of larger number of auctions considered - on average 2.18%. On the other hand, the percentage of bidders in the most recently finished auction who also participated in the auction of the bid (period t) is higher - 2.3%. The two percentages are much closer to each other than in the case of N auctions, where $N \in 5, 10, 15, 20$. 2.3% is higher than a number of bidders in common between any two randomly selected auctions from the data. This shows that the price of the most recently finished auction can potentially carry useful information about competition which the bidder is likely to face.

2.8.2.3 Common bidders with previous most recently finished auction in which the bidder participated

Previous period auction, $T-1$ can be defined as the most recently finished auction in which the bidder has placed their bid. The OLG model suggests that the price from that previous auction would affect bidding strategy. A bidder, who takes part in more than one auction in a sequence, benefit from the full information about the past auction, in which they have participated. It is rational choice for the bidder to change their strategy after observing the final price in the previous auction, if there are other bidders who also

participate in both of these auctions. Table 2.5 shows statistics on other bidders (other than the bidder who placed a given bid) participating in both periods T and $T-1$ specified in this way. It can be seen that the average percentage of other bidders continuing from $T-1$ is 2.62% of bidders in auction at T. The percentage of other bidders at $T-1$ who also participate in T is 2.44%. These percentages are higher than the percentage of bidders from five most recent auction who bid in T.

In table 2.19 I have also included statistics derived from partial data containing either 32GB model or the 16GB model of iPhone 4- which were the most common variants in the data. For these partial datasets, the percentage of bidders continuing from previous auction is even higher - around 3%.

On the other hand, these datasets do not include the same information about the most recent auction in which the bidder participated. If a bidder took part in several auctions in a sequence of which some were for model 32 and others for model 16, then the information about the price of the most recent auction is not the same if only partial datasets are considered.

For comparison, table 2.20 shows the statistics on common bidders with the most recently finished auction. It shows that in fact, the percentages of bidders in common with the most recently finished auction are lower than the percentages of common bidders with the auction in $T-1$ as defined above (t = 1.9486, df = 13200, p-value = 0.02568).

2.8.2.4   Conclusion about bidders moving between auctions

In conclusion, bidders move between auctions close to each other in time. There is also a higher probability that bidders who meet in the past will also meet in the future. In other words, different bidders choose auctions in a correlated manner - that is given bidder A bids in auction X and later Z, it is more probable that in auction Z there will be another bidder who has also bid in auction X than in an auction R chosen at random. It is shown by the fact, that a higher proportion of bidders in an auction consists of bidders who have been together with a given bidder in previous auction than of bidders who have been in

the most recently finished auction.

A very large proportion of bidders in a current auction consists of bidders who have bid in the most recent 20 finished auctions (that is about 16%).

In the estimations, firstly, the period $T-1$ is defined as the most recent finished auction in which the bidder took part. The impact of price at $T-1$ defined in this way on bid amount is tested. The impact of averages of N most recently finished auctions, as well as the last most recently finished auction on the bids amounts is also analyzed in further regressions.

2.8.3 Estimation of the effect the price in T-1 has on bids in T

The proposed estimation is a linear regression, where the predicted variable is the bid, while the treatment variable is the price in period $T-1$. The hypothesis posed is that the amount of the bid is not independent from the results of previous auction in which the bidder participated. The model is as follows:

$$Y_{iat} = C_a + C_t + V_i + P_{t-1} + \epsilon_{iat} \tag{2.8.3}$$

where the dependent variable, Y_{iat} is the final bid of bidder i in auction a, and time t. The independent variables include controls relating to the particular auction in which the bid was placed, C_a, - for example the seller rating, model etc. Also the controls relating to the time at which the bid has been placed, C_t ,for example current number of bids or bidders in the auction, the time in the auction - is the bid close to beginning or end of the auction. Of course the bid depends largely on the valuation of the bidder, and ,therefore, the proxy for valuation needs to be included - V_i. The valuation should be equal to the final bid by the bidders in the last auction they have participated in. The treatment variable, effect of which is going to be estimated, is the price of an auction in $t-1$, denoted P_{t-1} in the above equation. It is expected that such specification will allow to identify the impact of learning from past prices on bids. The inclusion of valuation proxy is to

ensure that the unobservable that could affect the results is controlled for. Additionally, the regressions are performed on highly selected samples of not only the same product, but also the same model and almost identical auction characteristics. These are further discussed below.

There is a lot of variability in the dataset relating to auction characteristics, such as seller rating, photos and postage price. Because these variables could bias the results of the regression, due to unobservables such as the quality of photos or seller information, the data chosen for the regression need to have a limited variability of auction and seller characteristics. The way the data has been selected was described in the section "Data used for analysis In addition to these restrictions, the data used contains only the bidders who took part in at least 3 auctions. By necessity, the final auction bid is the valuation in the regression, and therefore the final bid itself cannot be included as the dependent variable either. The data used contains only the bids, where the period $T-1$ auction was also recorded - that is a finished most recent auction in which the bidder has participated. The bids from the first auction for each bidder cannot be, therefore, included in the regression either.

For example, if the bidder participated in 3 auctions, and the first auction in which he took part had finished before the second one has started, then only the middle bid amount is in the data as the dependent variable Y, while the price of the first auction is included as the treatment variable P_{t-1}, and the final bid in the third auction is included as the control for valuation V_i. Therefore the data used in the regressions is limited to bids in the middle periods, whenever $t-1$ can be determined. All of the steps of data collation taken are summarized as a flow chart in Figure 4.8 in the Appendix. EBay rules and short description of iPhone 4 can be found in Appendix section 4.1.

The results of the regressions can be seen in the tables 2.6 and 2.7. The naming of the variables is the same as in the dataset. P_{t-1} is the dataset variable called $nLastAuctPrice$, V_i is $uLastBid$, and other variable names were described before. $uAuctionNumber$ is the auction number, in which the bidder participated, and it is positively correlated with the

dependent variable $bAmount$. This confirms that the bids are higher for later auctions in a sequence. The most fitting relation found was logarithmic curve, based on adjusted R-squared, which shows that the impact is diminishing. The auction characteristics that showed to be significant is the phone memory (the two models considered are 32gb and 16gb), with 32Gb model bids being higher by about 33 pounds, unlocked phones having higher bids by about 30 pounds compared to O2 network phones. Star-rating of the sellers considered are 1st, 2nd, and 3rd level, where they do not show to have a significant impact on the bid amount. All auctions included have at least one photo and the number of photos above 1 do not have any significant effect on bid amount. Other characteristics, such as the starting price of the auction, or the postage fare (while all auctions have postage price above 0) are also insignificant. User's bid number in the auction ($uAuctionBid$) has a negative impact on the bid amount, which means that the more times the bidder was bidding in the auction the lower was their bid. There are two functional forms to fit the shape of the relation of user's bid number to bid amount - a logarithmic curve or a second degree polynomial. The second degree polynomial is a slightly better fit based on adjusted R squared. On the other had, the overall bid number in the auction ($bAuctionBid$) has a positive impact on the bid amount - the more bids were placed before, the higher the bid amount. The auction bid number is highly correlated with time at which the bid has been placed, so in fact it can be treated as a different time measure. All auctions included lasted one day, and the variable $bTimeWithinA$ measures the time from the start of the auction until the end - it is a percentage measure, as the total auction length is set to 100, while any other time during the auction is represented relative to the total auction length. This measure was chosen, since in case different auctions lengths were considered, they could all be easily compared. It can be seen in the table 2.6 that time at which the bid was placed is positively related to bid amount (with logarithmic shape), and once it is included in the regression the impact of $bAuctionBid$ loses it's significance. That is because both of these measures relate to the time counted from the beginning of the auction until it's end. Therefore, there are two alternative ways

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to include a measure time within the auction. f there was a trend in bids amounts over time, then $bTime$ would be expected to be significant. The absolute measure of time (as a date - time format) does not show to have a significant coefficient. Of course an important aspect that has an influence on the bid amount, in addition to time at which the bid was placed is the current price. There is a direct relationship between $bLivePrice$ and bid amount, such that an increase 1 pound in live price results in an increase in the bid amount by 0.68 pounds.

The impact of the auction number of the bidder is such that an increase in auction number by 1 results in an increase in bid amount by 11.7 pounds. Besides that, the most important variable influencing the bid amount is of course bidder's valuation of the object. That is represented by the last bid of the user - $uLastBid$ variable in the regressions. Best fit was achieved using the logarithmic curve, which suggests diminishing returns. An increase in logarithm of valuation measure by 1 has an impact of increase in bid amount by 20.2 pounds. Finally, the price of previous, most recent, auction in which the bidder took part ($nLastAuctPrice$) also has a significant impact on bid amount. An increase in price of previous auction by 1 pound results in an increase in bid amount by 0.07 pounds - that is a 7% impact of previous auction price. Regression results conducted based on the data including just one of the models with respect to memory size show very similar results, although for the cheaper 16gb model, the impact of previous auction price is not significant (table 2.12). This can be explained by the fact that those bidding on the cheaper version have lower valuation, and their discounting is overall lower than the discounting of higher valuation bidders - therefore the impact of past prices is also much lower, because there is not much room for adjusting the bid upwards in the case of low discounting in general.

On the other hand, there could be a concern that, since the effect of $T-1$ auction price is not significant for the subset of cheaper model auctions.

There might be a bias relating to the fact that people who have bid on a more expensive model are more likely to bid more in the following auction. In table 2.15 , 2.14 and 2.13

the results include a variable $nLastAuctPriceModelNetw$. This long-named variable is a different definition for price $T-1$, particularly prices of $T-1$ auction are included in the regression only if at $T-1$ the bidder has bid in an auction of the same model and the same network. On the full dataset the effect of past prices is significant. On the subset of 32Gb model the effect of past prices is again positive and significant, while on the subset of 16Gb model auctions the effect is not significant as before. This suggests that there is a persistent difference between the effect of past prices on bidders bidding on 16Gb model (cheaper) and 32Gb model (more expensive).

Other regression analysis included the averages of most recently finished auctions. The results show that the average price of 5 most recently finished auctions has a larger effect than the average of prices from 10, 15 or 20 most recent auctions. When the $T-1$ auction prices are included as well the effect becomes not significant, which suggests that the effect of price $T-1$ is the main driver when it comes to recent auctions. Tables 2.8 and 2.9 show that the effects of the price of most recently finished auction is higher than the averages of a number of recent auctions. The effects of the price of the last most recent auction, as well as effects of the average prices of 5 and 10 auctions are significant when included in the regression. On the other hand, the results in tables 2.10 and 2.11 show that once the price at $T-1$ is included as well, these effects become insignificant. The price of the auction in which the bidder actively participated has more impact on their bidding than the prices of a most recently finished auctions. It could be explained by a stylized overlapping generations model presented in previous chapter. It is, though possible that there are more possible explanations for this fact, and different possibilities need to be carefully examined. Nevertheless the results of this empirical investigations show that learning from past prices has an impact on bidding strategies.

2.8.4 Conclusion

The above empirical results show that the price of the most recently finished $T-1$ auction has a positive effect on bidding strategy. The results are supported by examining the

subset of the more expensive model of the phone, but not on the subset of the cheaper model. The most recently finished auction prices have lower effect than the most recently finished auction in which the bidder has actively participated $(T - 1)$. The regressions include a measure of valuation (final bid in the last auction the bidder participated), which is also significant. The belief that the impact of learning is correctly identified is strengthened by the fact that the coefficients are not much affected with the inclusion of fixed effects and interaction terms. The estimated effect of price at $T - 1$ on bid in T is positive and of approximately 5% to 7%.

The analysis of how bidders move between auction shown that they participate in auction in a correlated manner, and bidders who have met in one auction are more likely to meet each other again, than bidders who separately participated in the most recently finished auction.This also shows that it is reasonable to think of $T - 1$ as the most recently finished auction in which the bidder has participated, and that it has the main characteristics of the overlapping generations model whereby a fraction of bidders who participate in both T and $T - 1$.

Table 2.7: Regressions With Interaction Terms And Fixed Effects including the price of auction at T-1

	Dependent variable:			
	bAmount			
Auction Duration:	1 & 3		1	
	(1)	(2)	(3)	(4)
aModel32	37.591***	37.482***	38.354***	35.085***
	(3.802)	(4.678)	(4.495)	(4.695)
aNetworkUnlocked	32.510***	31.319***	28.606***	16.719***
	(4.015)	(5.034)	(4.800)	(4.927)
log(aPositiveFeed)	-24.013	-6.603		
	(45.015)	(51.937)		
aDurationLevels3	-27.900			
	(20.556)			

Table 2.7: Regressions With Interaction Terms And Fixed Effects including the price of auction at T-1 -cont.

	(1)	(2)	(3)	(4)
log(bPercWithin)	−11.422*	-9.159	−4.970*	0.100
	(6.616)	(6.798)	(2.773)	(2.871)
log(bAuctionBid)	16.172**	17.564***	44.551***	11.639***
	(6.304)	(6.496)	(3.615)	(2.668)
nOtherBiddersInAuct	−11.317***	−11.909***	−10.313***	
	(2.016)	(2.126)	(0.803)	
bLivePrice	0.656***	0.644***	0.653***	0.729***
	(0.013)	(0.016)	(0.014)	(0.014)
log(uAuctionBid)	−43.760***	−43.680***	−46.396***	−35.578***
	(3.544)	(4.494)	(4.323)	(4.436)
log(uLastBid)	17.435***	17.633***	23.370***	25.610***
	(1.707)	(2.108)	(1.802)	(1.876)
log(uAuctionNumber)	-15.823	-19.678	14.234***	15.528***
	(42.929)	(44.207)	(2.243)	(2.344)
nLastAuctPrice	0.040**	0.051***	0.058***	0.049**
	(0.016)	(0.019)	(0.019)	(0.020)
log(bPercWithin) x log(bAuctionBid)	−18.385***	−17.490***		
	(5.571)	(5.707)		
aDurationLevels3 x nOtherBiddersInAuct	3.517			
	(3.339)			
log(bPercWithin) x nOtherBiddersInAuct	1.218	1.062		
	(0.768)	(0.784)		
log(bAuctionBid) x nOtherBiddersInAuct	2.296***	2.398***		
	(0.739)	(0.766)		
aDurationLevels3 x log(bPercWithin) x log(bAuctionBid)	28.084**			
	(11.553)			
aDurationLevels3 x log(bPercWithin) x nOtherBiddersInAuct	0.923			
	(1.335)			
aDurationLevels3 x	-2.037			

Table 2.7: Regressions With Interaction Terms And Fixed Effects including the price of auction at T-1 -cont.

	(1)	(2)	(3)	(4)
log(bAuctionBid) x nOtherBiddersInAuct	(1.300)			
log(bPercWithin) x log(bAuctionBid) x nOtherBiddersInAuct	1.538*** (0.535)	1.542*** (0.547)		
aDurationLevels3 x log(bPercWithin) x log(bAuctionBid) x nOtherBiddersInAuct	−2.271** (1.094)			
Fixed effects:				
aStarL	YES	YES	YES	YES
uTotalAuctions	YES	YES		
aStarL x aPositiveFeed	YES	YES		
ADurationL x log(bPercWithin)	YES			
ADurationL x log(bAuctionBid)	YES			
log(uAuctionNumber) x uTotalAuctionL	YES	YES		
Constant	207.746 (209.623)	125.853 (241.396)	12.040 (18.790)	−47.915** (19.039)
Observations	2,515	1,753	1,753	1,753
R²	0.871	0.871	0.851	0.837
Adjusted R²	0.865	0.862	0.850	0.835
Residual Std. Error	81.649 (df = 2400)	82.756 (df = 1646)	86.544 (df = 1739)	90.529 (df = 1740)
F Statistic	142.606*** (df = 114; 2400)	104.647*** (df = 106; 1646)	762.230*** (df = 13; 1739)	742.083*** (df = 12; 1740)
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 2.8: Regressions Results Including most recently finished auction prices. Auction duration 1 and 3 days

	Dependent variable:			
	bAmount			
Auction Duration:	1 & 3			
	(1)	(2)	(3)	(4)
aModel32	33.857***	33.826***	33.796***	33.877***

Table 2.8: Regressions Results Including most recently finished auction prices. Auction duration 1 and 3 days - cont.

	(1)	(2)	(3)	(4)
	(3.094)	(3.091)	(3.093)	(3.098)
aNetworkUnlocked	27.041***	27.023***	27.043***	27.039***
	(3.218)	(3.216)	(3.219)	(3.220)
bLivePrice	0.710***	0.711***	0.711***	0.711***
	(0.010)	(0.010)	(0.010)	(0.010)
log(uAuctionBid)	−41.093***	−41.028***	−41.189***	−41.232***
	(2.778)	(2.777)	(2.778)	(2.778)
log(uLastBid)	14.171***	14.152***	14.165***	14.181***
	(1.361)	(1.360)	(1.361)	(1.361)
nMean_1	0.025**			
	(0.012)			
nMean_5		0.075***		
		(0.026)		
nMean_10			0.069*	
			(0.036)	
nMean_15				0.050
				(0.043)
Constant	52.051	17.105	16.181	26.401
	(149.854)	(150.412)	(151.445)	(152.347)

Interaction Terms and Fixed Effects like in specification (1) in Table 2.7:
aStarL*log(aPositiveFeed)
aDurationLevels*log(bPercWithin)*log(bAuctionBid)*nOtherBiddersInAuct
log(uAuctionNumber)*uTotalAuctionsLevels)

log(bPercWithin)	−22.303***	−22.146***	−22.635***	−22.831***
	(5.587)	(5.581)	(5.580)	(5.581)
log(bAuctionBid)	14.087***	13.849**	14.219***	14.382***
	(5.448)	(5.446)	(5.447)	(5.448)
nOtherBiddersInAuct	−11.432***	−11.479***	−11.301***	−11.348***
	(1.679)	(1.678)	(1.678)	(1.679)

Table 2.8: Regressions Results Including most recently finished auction prices. Auction duration 1 and 3 days - cont.

	(1)	(2)	(3)	(4)
Observations	3,867	3,867	3,867	3,867
R^2	0.874	0.874	0.874	0.874
Adjusted R^2	0.870	0.870	0.870	0.870
Residual Std. Error (df = 3750)	84.185	84.137	84.191	84.216
F Statistic (df = 116; 3750)	224.606***	224.898***	224.573***	224.417***
Note:				*p<0.1; **p<0.05; ***p<0.01

2.9 Tables

Table 2.2: Results of t-tests

H_1: bAmount$_{t,i}$ - bAmount$_{t-1,i} > 0$

matched pairs one-sided t-test

Total auctions joined	periods t,t-1 (reverse numbering)	mean of difference	conf. interval lower bound	t statistic	p value	number of individuals
		Bidders with a win in the final period				
2	1,2	157.44	130.29	9.58***	0.00	244
3	1,2	126.79	75.61	4.13***	0.00	71
3	2,3	72.58	13.41	2.04**	0.02	71
4	1,2	78.66	13.68	2.08**	0.02	23
4	2,3	64.25	-35.76	1.10	0.14	23
4	3,4	39.47	-23.95	1.07	0.15	23
5	1,2	33.75	-69.45	0.77	0.25	4
5	2,3	26.50	1.44	2.49 **	0.04	4
5	3,4	124.00	-197.01	0.91	0.22	4
5	4,5	-248.75	-547.45	-1.96	0.93	4
		Bidders without any wins				
2	1,2	1.31	-20.33	0.10	0.46	521
3	1,2	-22.92	-54.78	-1.19	0.88	159
3	2,3	6.10	-29.12	0.29	0.39	159
4	1,2	21.04	-54.19	0.47	0.32	42
4	2,3	-2.08	-64.11	-0.06	0.52	42
4	3,4	0.75	-64.61	0.02	0.49	42
5	1,2	17.56	-86.79	0.29	0.39	17
5	2,3	-68.41	-185.44	-1.02	0.84	17
5	3,4	107.71	14.65	2.02**	0.03	17
5	4,5	38.65	-76.69	0.59	0.28	17
Note:				*p<0.1; **p<0.05; ***p<0.01		

Table 2.3: Key Differencing Auction Characteristics in The Dataset Of Final Bids

aDuration	1 :7176	3 :3556	5 : 835	7 : 368	10: 128
aModel	16 :7030	32 :4908	(Other): 125		
aNetwork	Unlocked:6279	O2 :3298	Vodafone:1050	Orange : 859	(Other) : 577
aStarL	0 :1308	1 :2979	2 :2288	3 :4450	(Other): 1038
aStarLevel	NoStar :1308	Yellow :2979	Blue :2288	Turquoise:4450	(Other) : 1038
aReturns	No :10750	Yes: 1276	NA's : 37		
aPostageFree	No:10292	Yes: 1599	NA's : 172		
aPhotosPresent	Yes :11890	No: 173			
uTotalAuctions	1 :1255	2 :1817	3 :1430	4 :1215	(More):6346
uReverseAuctN	1 :3508	2 :2304	3 :1457	4 : 950	(Other): 3844
uTotalWins	0 :6621	1 :2669	2 :1132	3 : 514	(More): 1127
uNLR	No :11388	Yes: 675			
bBIN	No :11755	Yes: 308			

Table 2.4: Statistics on bidders moving from most recent auctions

	mean	median	sd	min	max	n. obs.
Number of bidders in auction						
	8.71	9.00	3.90	1.00	21.00	10798.00
Common bidders with N most recently finished auctions						
$N = 5$	0.62	0.00	0.95	0.00	8.00	10798.00
$N = 10$	0.95	1.00	1.18	0.00	9.00	10798.00
$N = 15$	1.18	1.00	1.32	0.00	9.00	10798.00
$N = 20$	1.31	1.00	1.44	0.00	10.00	10798.00
Total number of other bidders in N most recently finished auctions						
$N = 5$	32.57	32.00	9.72	0.00	59.00	10798.00
$N = 10$	61.42	62.00	15.01	0.00	99.00	10798.00
$N = 15$	88.06	89.00	20.57	0.00	137.00	10798.00
$N = 20$	100.01	101.00	19.47	35.00	151.00	10798.00
Percentage of bidders in N most recently finished auctions who also participate in auction that follows						
$N = 5$	1.95	0.00	3.09	0.00	27.27	10798.00
$N = 10$	1.61	1.25	2.07	0.00	16.67	10798.00
$N = 15$	1.41	1.05	1.64	0.00	16.67	10798.00
$N = 20$	1.33	0.99	1.47	0.00	10.53	10798.00
Percentage of bidders in an auction who also participated in N most recently finished auctions						
$N = 5$	7.96	0.00	13.63	0.00	100.00	10798.00
$N = 10$	12.07	7.69	16.37	0.00	100.00	10798.00
$N = 15$	14.99	10.00	18.05	0.00	100.00	10798.00
$N = 20$	16.25	11.11	18.72	0.00	100.00	10798.00

Table 2.5: Statistics on bidders participating in T and $T-1$

	mean	median	sd	min	max	n
Dataset including all auctions						
Number of other bidders in auction						
	8.54	9.00	3.92	1.00	21.00	6641.00
Common bidders with auction in $T-1$						
	0.21	0.00	0.55	0.00	5.00	6641.00
Number of other bidders in $T-1$						
	8.81	9.00	3.92	1.00	21.00	6641.00
Percentage of bidders in $T-1$ who also participated in T						
	2.44	0.00	7.21	0.00	100.00	6641.00
Percentage of bidders in T who also participated in $T-1$						
	2.62	0.00	8.08	0.00	100.00	6641.00
Price in auction at $T-1$						
	703.84	700.00	113.51	130.00	999.00	6641.00

Table 2.6: Regression Results Without Fixed Effects and Interaction Terms Including The Price Of Auction a T-1

	Dependent variable:			
	bAmount			
	(1)	(2)	(3)	(4)
aModel32	33.039***	32.957***	30.379***	28.964***
	(4.876)	(4.894)	(5.124)	(5.144)
aNetworkUnlocked	29.311***	29.424***	20.813***	22.929***
	(5.179)	(5.210)	(5.347)	(5.358)
aStarL2	4.512	4.545	5.472	5.544
	(6.237)	(6.241)	(6.435)	(6.477)
aStarL3	−0.841	−0.842	−1.163	0.566
	(5.229)	(5.230)	(5.386)	(5.419)
aTotalPhotos				2.602
				(2.386)
bMsTimeWithin			2.490	6.827**
			(3.145)	(2.978)
log(uAuctionBid)			−34.553***	−29.892***
			(4.537)	(4.405)
bLivePrice	0.678***	0.678***	0.749***	0.746***
	(0.014)	(0.014)	(0.015)	(0.015)
log(bAuctionBid)	37.086***	37.057***	10.783***	
	(3.673)	(3.677)	(2.814)	
nOtherBiddersInAuct	−8.690***	−8.687***		
	(0.875)	(0.875)		
poly(uAuctionBid, 2)1	−688.622***	−688.154***		
	(88.984)	(89.043)		
poly(uAuctionBid, 2)2	438.611***	439.017***		
	(85.658)	(85.710)		
log(uLastBid)	20.208***	20.145***	21.478***	21.669***
	(1.878)	(1.904)	(1.979)	(1.989)
uTotalAuctions		−0.032	−0.064	−0.083
		(0.155)	(0.160)	(0.161)
log(uAuctionNumber)	11.660***	12.242***	13.653***	13.358***
	(2.487)	(3.761)	(3.874)	(3.892)
nLastAuctPrice	0.070***	0.070***	0.061***	0.062***
	(0.022)	(0.022)	(0.023)	(0.023)
Constant	4.031	3.888	−82.164	−143.017***
	(20.396)	(20.414)	(54.143)	(53.131)
Observations	1,420	1,420	1,420	1,420
R^2	0.857	0.857	0.848	0.846
Adjusted R^2	0.856	0.856	0.846	0.845
Residual Std. Error	84.689	84.718	87.361	87.779
	(df = 1407)	(df = 1406)	(df = 1407)	(df = 1407)
F Statistic	701.756***	647.337***	652.416***	645.113***
	(df = 12; 1407)	(df = 13; 1406)	(df = 12; 1407)	(df = 12; 1407)

Note:		*p<0.1; **p<0.05; ***p<0.01

Table 2.9: Regression Results Including Most Recently Finished Auction Prices

	Dependent variable:			
	bAmount			
Auction Duration:	1			
	(1)	(2)	(3)	(4)
aModel32	33.781***	33.884***	34.065***	33.939***
	(3.772)	(3.772)	(3.774)	(3.783)
aNetworkUnlocked	25.370***	25.173***	25.232***	25.305***
	(3.962)	(3.962)	(3.960)	(3.964)
bLivePrice	0.705***	0.706***	0.706***	0.705***
	(0.013)	(0.013)	(0.013)	(0.013)
log(uAuctionBid)	−40.443***	−40.402***	−40.577***	−40.601***
	(3.459)	(3.458)	(3.456)	(3.459)
log(uLastBid)	14.300***	14.303***	14.363***	14.327***
	(1.677)	(1.676)	(1.676)	(1.677)
nMean_1	0.024			
	(0.015)			
nMean_5		0.065**		
		(0.032)		
nMean_10			0.103**	
			(0.045)	
nMean_15				0.064
				(0.053)
Constant	22.255	-8.449	-39.830	-13.887
	(169.783)	(170.676)	(172.288)	(173.493)

Interaction Terms and Fixed Effects like in specification (2) in Table 2.7:
aStarL*log(aPositiveFeed)
log(bPercWithin)*log(bAuctionBid)*nOtherBiddersInAuct
log(uAuctionNumber)*uTotalAuctionsLevels)

Observations	2,709	2,709	2,709	2,709
R^2	0.874	0.874	0.874	0.874
Adjusted R^2	0.869	0.869	0.869	0.869
Residual Std. Error (df = 2600)	84.960	84.933	84.915	84.978
F Statistic (df = 108; 2600)	166.901***	167.021***	167.101***	166.820***

Note:				*p<0.1; **p<0.05; ***p<0.01

Table 2.10: Regression Results Including Most Recently Finished Auction Prices and Price of T-1. Auction Duration 1 and 3 Days.

	Dependent variable:			
	bAmount			
Auction Duration:	1 & 3			
	(1)	(2)	(3)	(4)
aModel32	37.721***	37.789***	37.683***	37.674***
	(3.806)	(3.808)	(3.808)	(3.814)
aNetworkUnlocked	32.449***	32.494***	32.457***	32.439***
	(4.021)	(4.021)	(4.022)	(4.023)
bLivePrice	0.656***	0.657***	0.657***	0.656***
	(0.013)	(0.013)	(0.013)	(0.013)
log(uAuctionBid)	−43.785***	−43.689***	−43.769***	−43.771***
	(3.546)	(3.547)	(3.547)	(3.548)
log(uLastBid)	17.435***	17.479***	17.458***	17.449***
	(1.707)	(1.707)	(1.708)	(1.708)
nMean_1	0.021			
	(0.015)			
nMean_5		0.043		
		(0.031)		
nMean_10			0.034	
			(0.042)	
nMean_15				0.022
				(0.051)
nLastAuctPrice	0.039**	0.038**	0.039**	0.039**
	(0.016)	(0.016)	(0.016)	(0.016)
Constant	204.670	184.625	186.724	192.732
	(209.685)	(210.435)	(211.582)	(213.022)

Interaction Terms and Fixed Effects like in specification (1) in Table 2.7:
aStarL*log(aPositiveFeed)
laDurationLevels*og(bPercWithin)*log(bAuctionBid)*nOtherBiddersInAuct
log(uAuctionNumber)*uTotalAuctionsLevels)

Observations	2,512	2,512	2,512	2,512
R^2	0.871	0.871	0.871	0.871
Adjusted R^2	0.865	0.865	0.865	0.865
Residual Std. Error (df = 2396)	81.660	81.663	81.684	81.692
F Statistic (df = 115; 2396)	141.288***	141.276***	141.193***	141.162***

Note:	*p<0.1; **p<0.05; ***p<0.01

Table 2.11: Regression Results Including Most Recently Finished Auction Prices and Price of T-1. Auction Duration 1 Day.

	Dependent variable:			
	bAmount			
Auction Duration:	1			
	(1)	(2)	(3)	(4)
aModel32	37.612***	37.538***	37.728***	37.459***
	(4.686)	(4.696)	(4.698)	(4.706)
aNetworkUnlocked	31.293***	31.275***	31.236***	31.325***
	(5.047)	(5.052)	(5.049)	(5.051)
bLivePrice	0.644***	0.644***	0.644***	0.644***
	(0.016)	(0.016)	(0.017)	(0.017)
log(uAuctionBid)	−43.635***	−43.630***	−43.718***	−43.632***
	(4.500)	(4.501)	(4.502)	(4.502)
log(uLastBid)	17.682***	17.663***	17.707***	17.646***
	(2.110)	(2.111)	(2.111)	(2.111)
nMean_1	0.018			
	(0.018)			
nMean_5		0.009		
		(0.039)		
nMean_10			0.038	
			(0.053)	
nMean_15				-0.001
				(0.063)
nLastAuctPrice	0.050**	0.051***	0.050**	0.051***
	(0.019)	(0.019)	(0.019)	(0.020)
Constant	124.426	122.117	97.883	128.373
	(241.558)	(242.775)	(245.034)	(246.482)

Interaction Terms and Fixed Effects like in specification (2) in Table 2.7:
aStarL*log(aPositiveFeed)
log(bPercWithin)*log(bAuctionBid)*nOtherBiddersInAuct
log(uAuctionNumber)*uTotalAuctionsLevels)

Observations	1,750	1,750	1,750	1,750
R^2	0.871	0.871	0.871	0.871
Adjusted R^2	0.862	0.862	0.862	0.862
Residual Std. Error (df = 1642)	82.802	82.827	82.815	82.828
F Statistic (df = 107; 1642)	103.506***	103.436***	103.470***	103.432***

Note:	*p<0.1; **p<0.05; ***p<0.01

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Table 2.12: Regression Results - Effects of Price at T-1 On Model 32 and 16 separately

	Dependent variable:			
	bAmount			
Model:	32 Gb	32 Gb	16 Gb	16 Gb
Auction Duration:	1 & 3	1	1 & 3	1
	(1)	(2)	(3)	(4)
aNetworkUnlocked	37.764***	33.661***	31.294***	34.849***
	(6.984)	(8.570)	(5.022)	(6.485)
bLivePrice	0.672***	0.654***	0.629***	0.600***
	(0.021)	(0.026)	(0.018)	(0.023)
log(uAuctionBid)	−52.613***	−52.782***	−39.837***	−40.208***
	(6.256)	(7.973)	(4.176)	(5.335)
log(uLastBid)	17.563***	16.822***	18.828***	21.685***
	(2.834)	(3.384)	(2.150)	(2.723)
nMean_5	0.021	-0.009	0.055	0.007
	(0.050)	(0.061)	(0.040)	(0.052)
nLastAuctPrice	0.090***	0.106***	-0.019	-0.009
	(0.025)	(0.032)	(0.020)	(0.025)
Constant	-81.504	-101.590	362.562	444.698
	(356.663)	(366.487)	(255.205)	(327.311)

Interaction Terms and Fixed Effects like in Table 2.7 in specification:

	(1)	(2)	(1)	(2)
Observations	1,123	798	1,389	952
R^2	0.868	0.868	0.879	0.884
Adjusted R^2	0.854	0.850	0.869	0.870
Residual Std. Error	89.297	90.050	73.047	73.716
	(df = 1014)	(df = 699)	(df = 1279)	(df = 850)
F Statistic	61.622***	46.990***	85.402***	63.902***
	(df = 108; 1014)	(df = 98; 699)	(df = 109; 1279)	(df = 101; 850)

Note:	*p<0.1; **p<0.05; ***p<0.01

Table 2.13: Regression Results with T-1 Auction Price Included Only For The Same Model and Network Auctions in T and T-1

	Dependent variable:		
	bAmount		
Restricted to:			
model:	32	16	Both
network:	Unlocked	Unlocked	Unlocked & O2
duration:	1 &3	1 &3	1 &3
	(1)	(2)	(3)
bLivePrice	0.494***	0.604***	0.522***
	(0.034)	(0.028)	(0.018)
log(uAuctionBid)	−57.911***	−37.739***	−44.528***
	(13.027)	(6.474)	(5.159)
log(uLastBid)	16.620***	12.849***	15.142***
	(5.161)	(3.416)	(2.489)
nLastAuctPriceModelNetw	0.151***	0.030	0.034
	(0.050)	(0.048)	(0.028)
aModel32			65.886***
			(6.509)
aNetworkUnlocked			43.625***
			(7.029)
Constant	-628.652	630.094**	367.653
	(748.298)	(296.206)	(270.801)
Interaction terms and fixed effects:			
aStarL*log(aPositiveFeed)			
laDurationLevels*og(bPercWithin)*log(bAuctionBid)*nOtherBiddersInAuct			
log(uAuctionNumber)*uTotalAuctionsLevels)			
Observations	368	581	1,255
R^2	0.860	0.881	0.860
Adjusted R^2	0.813	0.858	0.847
Residual Std. Error	86.626	66.954	77.741
Residual Std. Error	(df = 274)	(df = 483)	(df = 1141)
F Statistic	18.152***	37.009***	62.264***
F Statistic	(df = 93; 274)	(df = 97; 483)	(df = 113; 1141)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 2.14: Regression Results with T-1 Auction Price Included Only For The Same Model and Network Auctions in T and T-1

	Dependent variable:		
	bAmount		
Restricted to:			
model:	32	16	Both
network:	Unlocked	Unlocked	Unlocked & O2
duration:	1 &3	1 &3	1 &3
	(1)	(2)	(3)
bLivePrice	0.532***	0.637***	0.616***
	(0.030)	(0.022)	(0.016)
log(uAuctionBid)	−61.662***	−34.672***	−43.463***
	(11.933)	(5.771)	(5.091)
log(uLastBid)	22.674***	21.666***	22.604***
	(4.801)	(2.684)	(2.242)
nLastAuctPriceModelNetw	0.124***	0.039	0.171***
	(0.048)	(0.042)	(0.023)
log(bPercWithin)	17.396**	-2.208	4.792
	(7.847)	(4.278)	(3.509)
nOtherBiddersInAuct	−11.738***	−7.656***	−9.048***
	(1.904)	(1.178)	(0.945)
log(bAuctionBid)	40.622***	36.298***	32.428***
	(8.985)	(5.326)	(4.197)
log(uAuctionNumber)	17.270***	14.240***	12.861***
	(5.504)	(3.309)	(2.669)
aDurationLevels3	3.043	-6.184	-5.098
	(11.893)	(7.013)	(5.515)
aStarL2	2.575	1.602	8.001
	(14.315)	(8.416)	(6.740)
aStarL3	4.871	2.515	7.408
	(13.725)	(7.346)	(5.920)
aStarL4	3.108	10.631	18.529*
	(22.864)	(12.092)	(10.553)
Constant	140.092***	63.723*	13.307
	(52.635)	(33.643)	(21.753)
Observations	368	581	1,256
R^2	0.780	0.845	0.819
Adjusted R^2	0.772	0.841	0.817
Residual Std. Error	95.564	70.645	84.834
	(df = 355)	(df = 568)	(df = 1243)
F Statistic	104.772***	257.528***	468.641***
	(df = 12; 355)	(df = 12; 568)	(df = 12; 1243)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 2.15: Regression Results with T-1 Auction Price Included Only For The Same Model and Network Auctions in T and T-1

	Dependent variable:		
	bAmount		
Restricted to:			
model:	32	16	Both
network:	Unlocked	Unlocked	Unlocked & O2
duration:	1	1	1
	(1)	(2)	(3)
bLivePrice	0.607***	0.597***	0.625***
	(0.031)	(0.022)	(0.016)
log(uAuctionBid)	−59.127***	−35.290***	−39.024***
	(13.965)	(6.683)	(5.810)
log(uLastBid)	20.540***	22.918***	21.095***
	(5.041)	(3.014)	(2.477)
nLastAuctPriceModelNetw	0.206***	0.020	0.183***
	(0.061)	(0.047)	(0.025)
nOtherBiddersInAuct	−11.694***	−8.533***	−9.196***
	(2.030)	(1.362)	(1.042)
log(bAuctionBid)	46.286***	40.422***	34.304***
	(9.412)	(6.068)	(4.527)
log(uAuctionNumber)	22.641***	13.725***	13.969***
	(6.415)	(3.891)	(3.031)
aStarL2	14.254	2.404	8.921
	(15.822)	(9.628)	(7.565)
aStarL3	16.977	6.306	8.383
	(15.529)	(8.300)	(6.618)
aStarL4	8.348	10.777	14.127
	(25.085)	(14.179)	(11.623)
Constant	-2.436	89.400**	0.561
	(62.716)	(37.501)	(23.560)
Observations	274	437	919
R^2	0.818	0.843	0.828
Adjusted R^2	0.810	0.840	0.827
Residual Std. Error	93.737	70.977	82.393
	(df = 262)	(df = 426)	(df = 908)
F Statistic	106.925***	229.165***	438.450***
F Statistic	(df = 11; 262)	(df = 10; 426)	(df = 10; 908)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 2.16: Regression Results for Dataset of Model 32Gb. Effects of Price at T-1.

	Dependent variable:			
	bAmount			
	(1)	(2)	(3)	(4)
aNetworkUnlocked	26.753***	29.993***	13.252	13.760
	(10.261)	(10.380)	(10.256)	(10.222)
aStarL2	20.482*	20.846*	21.335*	21.129*
	(11.711)	(11.707)	(12.055)	(12.050)
aStarL3	3.955	2.830	4.623	4.509
	(10.150)	(10.157)	(10.436)	(10.430)
aTotalPhotos				4.264
				(4.061)
log(bTimeWithinA)		3.391	10.503*	12.411**
		(5.633)	(5.606)	(5.205)
log(bTime)		3,109.365		
		(7,802.944)		
log(uAuctionBid)			−53.468***	−51.454***
			(9.982)	(9.694)
bLivePrice	0.647***	0.638***	0.697***	0.693***
	(0.024)	(0.028)	(0.026)	(0.026)
log(bAuctionBid)	36.226***	33.646***	3.793	
	(6.928)	(7.669)	(5.327)	
nOtherBiddersInAuct	−9.957***	−9.680***		
	(1.775)	(1.827)		
poly(uAuctionBid, 2)1	−623.355***	−612.479***		
	(101.404)	(103.015)		
poly(uAuctionBid, 2)2	350.331***	360.084***		
	(95.799)	(96.260)		
log(uLastBid)	20.417***	19.462***	20.676***	20.755***
	(3.613)	(3.645)	(3.736)	(3.729)
log(uAuctionNumber)	11.356**	22.705***	23.565***	23.933***
	(5.025)	(8.430)	(8.315)	(8.310)
uTotalAuctions		−0.641*	−0.649*	−0.661*
		(0.372)	(0.369)	(0.368)
nLastAuctPrice	0.160***	0.161***	0.176***	0.172***
	(0.049)	(0.050)	(0.051)	(0.051)
Constant	−4.428	−65,262.760	−235.046**	−264.613***
	(46.218)	(163,621.600)	(101.247)	(97.529)
Observations	499	499	499	499
R^2	0.825	0.826	0.814	0.814
Adjusted R^2	0.821	0.821	0.810	0.810
Residual Std. Error	94.399	94.304	97.126	97.067
	(df = 487)	(df = 484)	(df = 487)	(df = 487)
F Statistic	208.012***	164.049***	194.042***	194.333***
	(df = 11; 487)	(df = 14; 484)	(df = 11; 487)	(df = 11; 487)
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 2.17: Regression Results dataset of model 16Gb. Effects of Price at T-1.

	Dependent variable:			
	bAmount			
	(1)	(2)	(3)	(4)
aNetworkUnlocked	37.058***	36.170***	30.773***	34.237***
	(6.765)	(6.791)	(7.140)	(7.176)
aStarL2	3.565	3.077	5.446	5.736
	(8.094)	(8.096)	(8.395)	(8.515)
aStarL3	9.222	9.224	9.999	13.001*
	(6.593)	(6.589)	(6.846)	(6.902)
aTotalPhotos				3.319
				(3.407)
log(bMsTimeWithin)			0.118	5.160
			(4.142)	(3.900)
log(uAuctionBid)			−34.794***	−29.985***
			(5.771)	(5.632)
bLivePrice	0.633***	0.634***	0.710***	0.706***
	(0.018)	(0.018)	(0.020)	(0.020)
log(bAuctionBid)	37.418***	37.637***	11.365***	
	(4.731)	(4.731)	(3.640)	
nOtherBiddersInAuct	−8.679***	−8.708***		
	(1.100)	(1.100)		
poly(uAuctionBid, 2)1	−470.699***	−471.000***		
	(81.545)	(81.493)		
poly(uAuctionBid, 2)2	325.458***	319.741***		
	(78.986)	(79.043)		
log(uLastBid)	26.924***	27.652***	30.659***	30.822***
	(2.399)	(2.455)	(2.569)	(2.588)
uTotalAuctions		0.258	0.218	0.232
		(0.187)	(0.194)	(0.196)
log(uAuctionNumber)	7.957**	2.957	5.220	3.896
	(3.154)	(4.797)	(4.975)	(5.002)
nLastAuctPrice	0.007	0.008	-0.014	-0.001
	(0.038)	(0.038)	(0.040)	(0.040)
Constant	24.332	23.398	-30.154	-110.244
	(28.729)	(28.718)	(72.529)	(71.138)
Observations	721	721	721	721
R^2	0.854	0.855	0.843	0.841
Adjusted R^2	0.852	0.852	0.841	0.839
Residual Std. Error	77.957	77.907	80.851	81.350
	(df = 709)	(df = 708)	(df = 709)	(df = 709)
F Statistic	377.397***	346.552***	346.330***	341.300***
F Statistic	(df = 11; 709)	(df = 12; 708)	(df = 11; 709)	(df = 11; 709)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 2.18: Percentage of Common Bidders With Earlier Than 10th Recently Finished Auction

	mean	median	sd	min	max	n. obs.
Percentage of bidders in $I = (N_T, N_B)$ most recently finished auctions who also participate in auction that follows						
$I = (11, 20)$	1.31	0.00	1.81	0.00	16.67	10718.00
$I = (21, 30)$	1.12	0.00	1.64	0.00	12.50	10672.00
$I = (31, 40)$	1.03	0.00	1.53	0.00	13.33	10621.00
$I = (41, 50)$	0.93	0.00	1.48	0.00	25.00	10566.00
Percentage of bidders in an auction who also participated in $I = (N_T, N_B)$ most recently finished auctions						
$I = (11, 20)$	9.73	0.00	14.40	0.00	100.00	10788.00
$I = (21, 30)$	8.35	0.00	12.85	0.00	100.00	10788.00
$I = (31, 40)$	7.59	0.00	12.22	0.00	100.00	10788.00
$I = (41, 50)$	6.57	0.00	11.51	0.00	100.00	10788.00

Table 2.19: Statistics on Bidders Participating in T and $T-1$. Division To Subsets Based On Model.

	mean	median	sd	min	max	n
Dataset including M32 only						
Number of other bidders in auction						
	8.32	8.00	3.88	1.00	19.00	2023.00
Common bidders with auction in $T-1$						
	0.23	0.00	0.57	0.00	4.00	2023.00
Number of other bidders in $T-1$						
	8.49	9.00	3.82	1.00	18.00	2023.00
Percentage of bidders in $T-1$ who also participated in T						
	3.03	0.00	8.32	0.00	100.00	2023.00
Percentage of bidders in T who also participated in $T-1$						
	3.24	0.00	9.11	0.00	100.00	2023.00
Price in auction at $T-1$						
	789.19	775.00	99.38	480.00	999.00	2023.00
Dataset including M16 only						
Number of other bidders in auction						
	8.62	9.00	3.93	1.00	21.00	3309.00
Common bidders with auction in $T-1$						
	0.26	0.00	0.63	0.00	5.00	3309.00
Number of other bidders in $T-1$						
	8.91	9.00	3.98	1.00	21.00	3309.00
Percentage of bidders in $T-1$ who also participated in T						
	2.90	0.00	7.56	0.00	80.00	3309.00
Percentage of bidders in T who also participated in $T-1$						
	3.10	0.00	8.47	0.00	100.00	3309.00
Price in auction at $T-1$						
	649.74	640.00	83.93	130.00	999.00	3309.00

Table 2.20: Statistics On Common Bidders Participating In T And Most Recently Finished Auction

	mean	median	sd	min	max	n
Common bidders with the most recently finished auction						
Number bidders in most recently finished auction						
	6.95	6.00	3.76	0.00	20.00	10798.00
Number common bidders with most recently finished auction						
	0.16	0.00	0.48	0.00	6.00	10798.00
Percentage of bidders in most recently finished auction who also participated in T						
	2.30	0.00	7.49	0.00	100.00	10777.00
Percentage of bidders in T who also participated in most recently finished auction						
	2.18	0.00	7.49	0.00	100.00	10798.00
Price of most recently finished auction						
	702.68	700.00	118.06	367.00	999.00	10777.00

Figure 2.4: Scatterplot Between Bid Discounting And Final Bid Amount
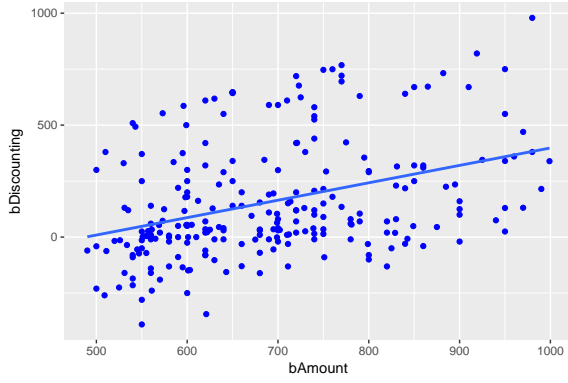
(a) Bidders with 2 auctions joined
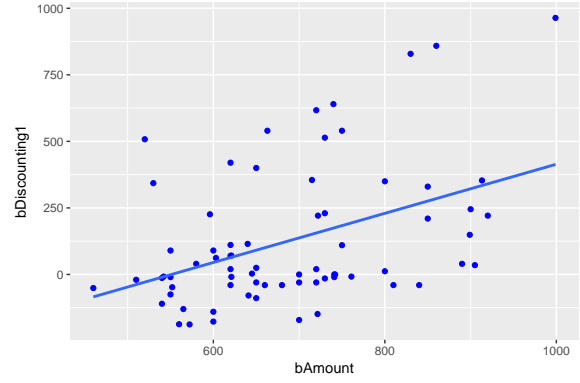
(b) Bidders with 3 auctions joined
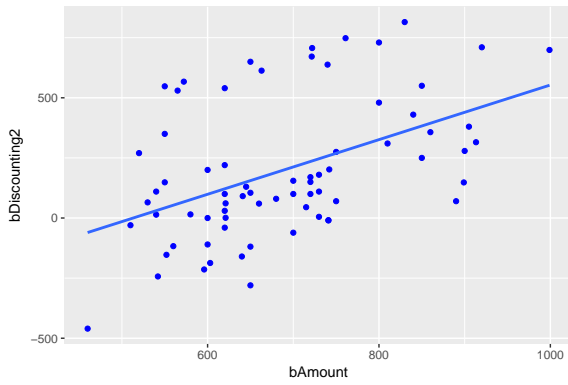
(c) Bidders with 3 auctions joined
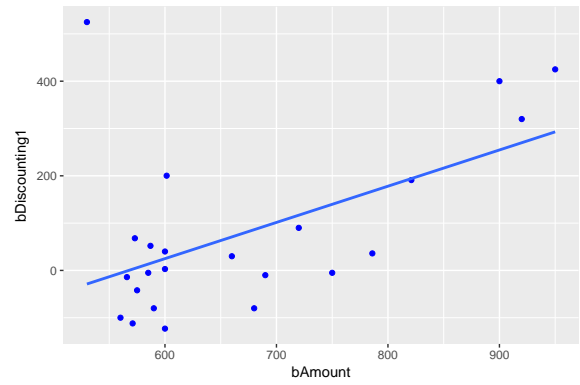
(d) Bidders with4 auctions joined
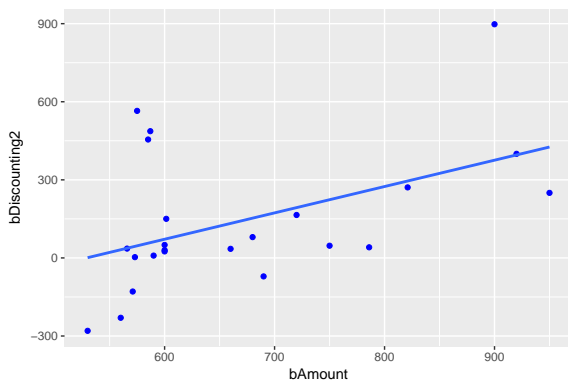
(e) Bidders with 4 auctions joined
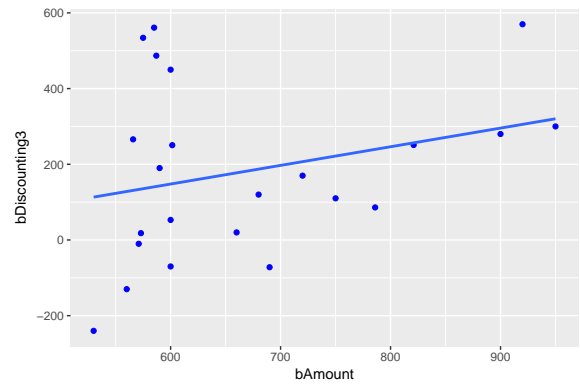
(f) Bidders with 4 auctions joined

Chapter 3

Multinomial Logit: Application to choice between identical products

3.1 Introduction

The purpose of this paper is to use multinomial logit estimation, which is commonly used in demand estimations, in the context of online concurrent auctions for identical products. The choice between auctions for identical products can be modeled in a similar way to choice between goods. The difference is that the essential variability between available options are not constant characteristics of a product, but rather the time-specific dynamic auction variables: these are current price, current number of bids and bidders in the auction. Estimation of choice model in this context allows to determine how these aspects influence decisions of bidders. The bidder at a given time t faces a choice between a number of auctions for the same product. The main aspects that enter the choice model are the time until the auction end, current price, current number of bids and bidders. The results show that some of the aspects are interdependent on each other, in particular price and time as well as number of bidders with time and number of bids. The identification of the effects of interest is ensured by choosing the most homogeneous subsets based on product, auction and seller characteristics. Various versions of the model are estimated. The models are tested on two subsets and the ones that are a good fit in both subsets are selected for interpretation. Additionally the choice model is estimated only on the first bid of each bidder to eliminate the effects that the bidding history could have. The dataset used is a good source for this analysis since the auctions recorded are for a new product, not sold anywhere before. There are many concurrent auctions which allows to limit seller and auction characteristics in the subsets.

3.2 Literature

Despite the fact that concurrent auctions for similar items are already very common, especially in the growing online auctions marketplace, the literature relating to the analysis of competing auctions is very modest [17]. Haruvy, Popkowski Leszczyc et al (2008)[17] outline different types of competition present in the online environment of consumer auc-

tions, and related literature. They divide the competition present into three categories: competition between items and between sellers, competition between formats, and competition between auction hosts. They conclude that " ... The most important dependent variable in our opinion is auction choice by consumers given the choice set, and very little is known about the determinants of such choice...". Exactly this choice between the auctions is the subject of this paper. Until the 1990s, consumer auctions were mainly isolated events, which is why the earlier auction literature has not given much attention to competition between auctions. Since the sudden increase in popularity of auctions with the opening of eBay, Amazon and other online platforms, still most of the empirical as well as theoretical analysis of auctions tried to apply the theory relating to isolated auctions, to this environment with high degree of concurrency. The first model of second price auctions was introduced by Vickrey (1961) [36] and it is still used as the basis for analysis of auctions in all type of environments. Despite that, many of the predictions from the simple early models, such as bidding own valuation once, do not prove to be useful in this case.

Literature search revealed three papers in economic literature, that are most closely related to concurrent auctions and the topic of this chapter. These are Haruvy and Popkowski Leszczyc (2010) [16], Anwar, McMillan and Zheng (2006)[1], and the theory paper by Peters and Severinov (2006) [30]. The latter one is the first paper to incorporate the existence of competing auctions on bidding behavior in a theoretical framework. Peters and Severinov show that in a multi-unit environment, cross-bidding, and incremental bidding are optimal in equilibrium. In the case of many concurrent auctions bidding own valuation in one bid is not advantageous any more, because of an opportunity to bid in a different auction with possibly lower second highest bidder. According to Peters and Severinov (2006), the bidder should choose the auction with the lowest standing bid and bid by an increment only. There shouldn't be cross-bidding during the time the bidder is the highest bidder in any auction (which is implied by single-unit demand), but in other case, cross-bidding should be observed. The environment that they analyze is a multi-

unit, second price auction environment, which is very similar to eBay, as they also point out. They focus on a single-unit-demand bidder's behavior, who is facing multiple simultaneous auctions with homogeneous items on sale. In conclusion they have expressed hope that empirical researchers will turn to the analysis of competing auctions.

Since that publication only two economics papers so far have turned into analysis of competing auctions. Anwar et al (2006)[2] are testing the cross-bidding theory implied by Peters and Severinov model, by analyzing groups of eBay auctions for CPUs with similar items and close ending times. They are examining groups of auctions of the same seller and almost identical items (which, despite the ending times, are indistinguishable for the buyers), and calculate the percentage of cross bidders in such groups. Their estimate of the extent of cross-bidding is likely to be understated, as they do not include other similar items that the bidders might consider as substitutes. Each group consists 2 or more auctions, and they analyze the extent of cross-bidding with different variation of inclusion in the competing auctions groups, with the ending times as close as within a minute apart, within an hour apart, and within a day apart. The respective samples of groups are of size 328, 1021, and 1943. and the average number of competing auction in each group varies from 2.55 in the minute sample to 2.77 in the daily sample. They find that there is more cross-bidding across closer substitutes (where the closer substitutes are those with closer ending times). The proportion of cross bidders in the minute sample is 0.20, excluding the multi-unit bidders, and 0.32 not excluding the multi-unit bidders. In the daily sample they find this proportion to be 0.14 and 0.19 respectively. The multi-unit demand bidders are here defined as those who either are winner of more than one auction, or are observed to be the highest bidder in more than one auction at a time during the last day of the auction. The hypothesis of no cross-bidding is rejected with very high t-values, and it is clear that there is a significant proportion of cross-bidders. Another theory implication they test is that bidders tend to bid on the auction where that standing bid is the lowest. They calculate the proportion of bids submitted on an auction with the lowest standing bid, during the times when more than one auction was

in progress. They find this proportion to be very high varying from 62 percent in the daily sample to 76 percent in the minute sample. They test these against the null of random bidding, where the proportion would be 50 percent, and find that this can be rejected. The observed modest monotonic increase in groups with closer ending time may simply be due to the fact that these groups contain smaller number of competing auctions. They note that this lack of apparent monotonicity (increasing proportion for auctions which compete more closely) is a surprising finding. The last question that they tackle is whether cross-bidders pay lower prices, by testing the hypothesis that the ratio between the price paid by average cross-bidders and non-cross bidders in each group is 1. They find this ratio to be from 0.91 with standard deviation of 0.7 for the minute sample to 0.94 with a standard deviation of 0.18 in the daily sample. The sample sizes are reduced in each case, since now only groups containing both cross bidders and non cross bidders are considered. The hypothesis tests show that in each sample the ratio is significantly different from 1. As a conclusion, the paper provides evidence for cross-bidding behavior as well as for the fact that cross-bidders pay lower prices, both implied by the model in Peters and Severinov (2006).

Another related paper is Haruvy and Popkowski Leszczyc (2010)[16], which is based on a field experiment on eBay. In this paper, the authors, through analyzing concurrent auctions, measure the impact of consumer search on price dispersion. They estimate a choice model of bidding between competing auctions. They consider pairs of simultaneous auctions in their estimations. The assumption that a bidder chooses to bid on an auction which gives them a higher expected utility, taking into account the current highest bid in that auction, is implied by Peters and Severinov model. As they note, estimating a choice model allows to ignore the specifics of strategies taken by the bidder, by focusing on estimating the impact that different variables have on expected utility of choosing that auction as opposed to the other one. In their model they focus on the effect of *Inertia*, which is the tendency to choose an auction one has chosen in the past. Inertia is measured in a similar way, as in the context of brand choice in Dube, Hitchi and Rossi

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(2010)[15] who estimate a multinomial logit model on brand choices in repeated supermarket visits. Haruvy and Popkowski Leszczyc (2010) use two specifications of inertia: one through a dummy variable indicating whether a bidder has placed his previous bid in that particular auctions - which they call state dependence, and another one through a percentage measure of bids by this bidder placed in that auction (out of all bids by the bidder) - which they call loyalty. Time left in the auction reduces search costs relative to it's benefits - they therefore investigate the coefficient at the interaction between time left and loyalty measure - which is predicted to have a different sign with endowment effect explanation, and search costs explanation. In order to vary the search costs, and see how this influences search, the authors influence it directly in their experimental design. The incentive to search is increased by running two identical auctions offering the winner to waive shipping costs in the case that the auction ended with lower final price, than another concurrent auction for identical item. In their analysis the authors focus on the pairs of experimental auctions, and in order to control for other competing auctions they use a covariate indicating the number of other concurrent auctions for similar items. The findings show that there is a price dispersion among the identical concurrent auctions. They find a significant average price difference of 15.25% of the average price ($2.87). They find that search incentive does have an effect on reduction of price dispersion. They also find that 19% of bidders ever switch between auctions, which is a similar figure to Anwar et al (2006) findings discussed earlier. 78.8% of all switches were to the auction with lower price, while 21.2% were switches to the auction with higher current price. Price dispersion is also found to vary over time of the auction, being lower at the beginning, suggesting initial search, and higher later on in the auctions to decrease again at the end. Despite that, as much as 26.8% of first bids are placed on a higher-priced auction. They estimate a random coefficients logit regressions to account for heterogeneity between individuals. Loyalty and state dependence are significant in their estimations, what is more intensity (which they measure as the total number of bids in the auction divided but the auction length) is also significant, which is interpreted as indicating some

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type of bidder frenzy. Remaining time is found to moderate stickiness of bidders (to the auction). In summary, their findings are that despite the seemingly easy search, still a large proportion of buyers failed to choose the lower priced option between two identical items, and tended to stay with the auction they have already entered. It also appeared to be a costly decision, since those who did switch between auctions (and therefore demonstrated more search activity) have paid lower prices, by on average $1.22. Bidders in the higher priced auction have paid an average premium of 15.25%, which was reduced by the search incentives provided in the experiment design, by 33%. As the authors summarize, their pair-auction design may not be representative of a setting with more alternatives - so often encountered in the digital world.

Other related, statistical studies focus mainly on resulting price, or price formation and it's dynamics. In chapter 4.3 of *Modeling Online Auctions* (2010) [20] Junk and Shmueli discuss a statistical technique of accounting for competition from other auctions in a regression, where the dependent variable is auction price (the related paper is [19]). They implement a semiparametric model, including three different aspects : auction component, spacial component and temporal component, which they show that fits the data better than the linear model. This suggests that there are some nonlinearities in the relationship of item features (spacial component in their estimations) and auction prices, which can be difficult to model with only linear regression. Their auction component includes transaction details (e.g. buy-it-now option, starting price, or reserve price) and is a parametric part; their spacial component is non-parametric mixed model, which measures the distances between item features (in competing auctions) in a non-parametric way; they suggest that information from past auctions should be used in an aggregated manner - which they do in their temporal component. As they argue, eBay participants are exposed to the data on past auctions in bulk, rather than at the time each auctions finishes, and therefore the difficult, irregularly spaced data can be included in a summary statistic from a given period (for example last day). Their model still doesn't address the influence of current competition on price formation (which is influenced by single

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bids) before the auctions ends. In Chapter 4.4., of the same book, the authors model bidder arrival process, and they show that it is a self-similar process (which has the same shape when considering smaller and smaller intervals towards the auctions end), which is an interesting feature. Bapna, Jank, and Shmueli (2008) [6] introduce functional data analysis to model price dynamics. They recover a functional object of price path (with the use of smoothing splines), and their first (velocity) and second derivative (acceleration). The speed of price change, as well as acceleration, are new factors which they use as dependent variables in their analysis. Explanatory variables they use include, in addition to all the static auction and bidder characteristics, the dynamic features such as current number of bidders, and the average rating of participating bidders. Some of their findings are that number of bidders is (as expected) positively correlated with current price, and also that, towards the end of the auction, higher price is correlated with faster price increase (velocity), and faster rate of increase (acceleration). This is in some way puzzling, since the heterogeneity of products is accounted for - it could be an indication that auctions with higher price (and therefore more bids already) attract more bidders, which again points to frenzy, or competitive arousal explanation.

3.3 Data

The dataset contains 2,384 eBay auctions for iPhone 4 during a two month period in June-July 2010. At any given time there were on average 234.85 concurrent auctions for the same product. The main differences between these items are their memory size - variable $aModel$, with two main categories: 16Gb and 32Gb capacity, nework ($aNetwork$), and condition ($aCondition$). All the variety of product and auction categorical variables can be found in table 3.1. The explanations of the variable names for the whole dataset can be found in table 4.1 and the statistics in table 4.2. Due to the fact that the time period of data collection was immidiately after the introduction of the iPhone 4 in the UK, there couldn't be much variability in terms of the used phones (by the fact that none of the

phones could not have been used for a long time before the sale), and most of the phones have been described as "New". Some phones were locked to a given network, while others were unlocked, and available to be used with any cellular network.

Given high homogeneity of these products, the expectation is that the bid will be placed on the item with the lowest current price, in accordance with Peters and Severinov model [30]. The purpose of the paper is to empirically estimate the influences on the choice made by bidders between the concurrent auctions. Individual choice model allows to abstract from the bidders' sequential strategies, while focusing on the aspects that influence the choice of auction at the time of each bid.

Table 3.1: Auction categorical variables in the whole dataset of bids for iPhone 4

aDurationLevels	1 :16031	3 : 8394	5 : 1685	7,10 or 6 : 1538	
aModel	16 :16237	32 :11085	. : 310		(Other): 16
aNetwork	Unlocked:14244	O2 : 7521	Vodafone: 2559	Orange : 2047	(Other) : 1227
aCondition	New :26674	Used : 510			(Other) : 464
aStarL	3 :11136	1 : 6378	2 : 5265	0 : 2557	(Other): 2312
aStarLevel	Turquoise:11136	Yellow : 6378	Blue : 5265	NoStar : 2557	(Other): 2312
aReturns	No : 24740	Yes : 2834			(Other) : 74
aPostageFree	No: : 23268	Yes : 3939			(Other) : 441
aPhotosPresent	No : 349	Yes : 27299			
aTotalPhotosLevels	1 :18913	2 : 4219	3 : 2273	4 : 1320	(Other): 923
aNonStock	No : 9666	Yes : 17982			
aPostto	UK :21108	Worldwide: 5247	EU : 522	UK,US : 24	(Other) : 747
aExtras	No : 27604	Yes : 44			

Number of Auctions:	2393
Number of Sellers:	1704
Number of bidders:	3730
Total number of bids:	27648

After generation of choice sets for for each bid

Total number of generated alternatives added to the dataset:	6465401
Total number of rows:	6493049
Average number of choices available at each bid :	234.85

3.4 Generating not-chosen alternatives

For the purpose of choice model estimation, the alternatives available for the bidders at the time they make a choice to bid needed to be known. These are not in the data collected from the eBay website. In order to find the choice sets that are present at the time of each bid, the not-chosen alternatives need to be recreated from the available data. This was possible, because all the information relating to auctions for the same product were collected : all the bids made are recorded in the dataset. The recreation of the not-chosen alternatives was made using a program written in R, which loops through all the 27648 observations in the dataset, and finds the choice set for each of these from the set of currently open auctions (after the start date but before the end date) at the time the bid was placed. Then, these not-chosen alternatives were added to the new dataset.

The original dataset contained 27648 observations of just the chosen alternatives. After running the code and generating the not-chosen alternatives for each bid, the final dataset contained 6 493 049 observations. The summary of the resulting dataset can be found at the bottom of Table 3.1. If a person opened the eBay website at some point in time during the dataset collection, and entered iPhone 4 into the search, they would see all the present open auctions sorted from the soonest to end to the last to end. These options were recreated for each bid placed in order to represent the situation described above. The variable $bClosingSequence$ is a record of the order in which the auctions are sorted at the time of each choice set. The soonest to end auction has $bClosingSequence$ equal to 1, and the number is increasing for auctions that have more time left to end. This variable relates to ordering among all auctions for iPhone 4 open at the time (including all memory sizes, networks and other variability). Each choice set has an assigned unique number, which is needed for distinction between the sets.

The dataset including all choices is not, though, the best for the estimation of multinomial logit, and the data needed to be limited. All the data collation steps can be found in Figure 4.9 in the Appendix, and are further explained below. EBay rules and information

visible to bidders are described in Appendix section 4.1.

3.5 Empirical Strategy

The dataset is rich in similar auctions and there are many concurrent alternatives at each point in time (on average 234.85). The data represented in this way allows to focus on aspects that influence choice of auction made by the bidders. First, though, the data needed to be further filtered so that the estimation was to be made on the most homogeneous sample to achieve robustness. There are several obvious aspects that may determine the choice of an auction by the individual : these are the characteristics of the object or auction - the heterogeneity between them. Real life data obviously contains heterogeneity in auction characteristics -as seen in table 3.1, even if all the auctions refer to the same product sold. All the remaining controls relating to these characteristics have to be included in the regression. Reduction of heterogeneity between auction by selecting only those with the same product and limited auction characteristics reduces the possible biases relating to unobservables, such as for example uncertainty about seller reputation and the actual quality of the product. In order to ensure identification of the variables of interest auctions chosen need to be as similar as possible.Table 3.1 shows the differences in characteristics across auctions in the whole dataset. For estimation of choice between auctions, the irrelevant alternatives should not make a difference. Number of auctions should also be reduced in order to run a multinomial logit estimation. The auctions and object characteristics are limited to create groups with almost identical products.

The variables of interest are not the constant differences between auctions, but the dynamic aspects such as time until the end of the auctions, and those that are an effect of choices made by others: current number of bidders and current number of bids in the auction, as well as live price.

The main question which the current analysis tries to answer are:

1. Which dynamic variabels affect choice and how?

The more specific questions are:

1. How does current price affect bids? Do bidders take into account both the current price and time remaining when making a bid?

2. Can we observe herding? Are auctions with larger number of current bidders preffered ober those with less current bidders?

3. Are auctions with more or with less active bidders preferred?

The multinomial logit model is used to estimate which aspects of alternatives present influence choice between them.

The probability of choosing x over other available alternatives $z \in Z$ is $P_t(x)$:

$$P_t(x) = \frac{exp(U_{xt})}{\sum_z exp(U_{zt})} \tag{3.5.1}$$

A possible equation defining utility U_{xt} is below:

$$U_{xt} = \beta_1 A_x + \beta_2 bLivePrice_{xt} +$$
$$\beta_3 bTimeWithinA_{xt} + \beta_4 bCurrentAuBidders_{xt} +$$
$$\beta_5 bAuctionBid_{xt} + ... + \epsilon_{xt} \tag{3.5.2}$$

where xt relate in this case to auction x and time t, and:

A_x - auction x characteristics

$bLivePrice_{xt}$ - live price of auction x at time t

$bTimeWithinA_{xt}$ - time within auction x at time t counted from start to the end of the auction

$bCurrentAuBidders_{xt}$ - number of current bidder at auction x and time t

$bAuctionBid_{xt}$ - number of current bids in auction x at time t[1].

Possibly more explanatory variables enter the estimation, especially interaction terms

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[1]Note: names of the explanatory variables have the same format as variable names in the dataset to make understanding easier. Variable names explanation can be found in Table 4.1

between the dynamic variables, which is why the "..." is included.

Multinomial logit model assumes independence of irrelevant alternatives - that means that if some alternatives are removed, the relative probabilities of choosing between the remaining alternatives is the same. This means we can limit the number of alternatives considered.

The above choice model allows to find the utility associated with each alternative auction A. This type of specification does not, though, reflect the situation which the bidders are facing.The problem with estimating a choice model with utility as stated in equation 3.5.2 is that each alternative is a unique auction and the same alternatives should be included in each choice set for estimation. The alternative-specific characteristics, A_x, are defined for each auction x. In the online auctions environment there are many auctions, and the choice sets with respect to alternative auctions differ widely in both: size of the choice sets and, most importantly, the composition of alternatives in each of the choice sets. An estimation, where each alternative is a different auction is not possible. The problems are the large number of alternatives and extremely high variety in choice sets. A small number of alternatives and repeated alternatives available at each choice set are desirable from the technical point of view. A different definition of alternatives is also better representing the situation the bidders are in. Especially, once homogeneous auctions are considered. A person is facing a choice between several auctions that are close to identical in A_x (auction characteristics). Each person is also facing a similar choice set.

The impact of constant auction characteristics is minimal once appropriate subset of almost identical auctions is selected, which is used for the re-definition of alternatives, and make it possible to estimate a choice model in this situation.

Auctions included all contain the same constant characteristics, so that the term A_x is removed completely, and the utility derived from each alternative does not depend on it's characteristics that globally differ it from other alternatives. All the terms that enter

the choice model relate to variables with subscript xt - variables that differ at each time-alternative level and therefore are in fact never exactly the same for two choices (given that two bids can not be made in the same auction at exactly the same time). Constant, independent of choice situation, characteristics A_x disappear completely from the model. The identical auctions can be sorted with respect to end time, and this is in fact a way they are sorted and displayed on eBay website - the top alternative being the soonest to end (besides paid advertisements). Once the auction-specific factors are at large eliminated, the alternatives can be defined in different way, making use of the display order existing on the website. The alternatives are defined as different places on the list of auctions sorted by end time. Soonest to end auction is alternative number 1, second to end - alternative number 2, third to end - alternative number 3, and fourth to end - alternative number 4. The number of alternatives included is chosen to be 4, but this is an arbitrary number that could be different. Given the assumption that irrelevant alternatives do not influence the ratios of the estimated coefficients for existing alternatives (Independence of Irrelevant Alternatives), including only up to alternative 4 is not affecting the results. It is expected that soonest to end auction has a higher probability of bid, and therefore there is an alternative-specific constant included in the estimation, which does not refer to the specific product characteristic, but rather to the place on the list that is displayed on the website. All other variables are alternative-time specific, and it's effect is not impacted by the way the alternatives are defined.

The utility gained from bidding on alternative xt is therefore[2]:

$$U_{xt} = C_x + \beta_2 bLivePrice_{xt}+$$

$$\beta_3 bTimeWithinA_{xt} + \beta_4 bCurrentAuBidders_{xt}+$$

$$\beta_5 bAuctionBid_{xt} + ... + \epsilon_{xt} \quad (3.5.3)$$

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[2]The names of the covariates reflect the naming used in the dataset and the description of variable names canbe found in table 4.1 in hte Appendix. "..." is used to show that possibly more covariates will be used.

, where C_x ist he position-specific constant (alternative -specific), and all other variables are defined as in equation 3.5.1. The probability of choosing auction at position x overall is defined in equation 3.5.1.

The multinomial logit estimations are made using mlogit package in R developed by Yves Croissant [13], by maximum likelihood. The multinomial logit model was first introduced by McFadden, 1974 [26]. The assumptions of the model are as follows:

1. Independence of Irrelevant Alternatives

2. Independence of Error Terms

3. Error Terms are identically Distributed

4. Error terms have Gumbel distribution

The estimation strategy is to make the above multinomial logit estimations on a homogeneous sample of auctions. The chosen nuber of alternatives is 4 alternatives in each choice set.

3.6   Estimation Without Interaction Terms

The first subsets considered are separate subsets of Model 32Gb and 16Gb New, Unlocked phones with the restriction that the sellers rating level can be 1st, 2nd or 3rd only, Returns not accepted, paid postage, photos present, no Extras and postage to UK only. The description of these subsets can be found in tables 3.7 and 3.10. These are large subsets with 713 and 732 choice sets. The final estimation specification for the U_{xt} was found based on the factors that were important, including the remaining auction variability-which is auction duration, amount paid for postage and number of photos. The equation

estimated is:

$$U_{xt} = C_x + \beta_1 bHourTimeTilEnd + \beta_2 aDuration +$$

$$\beta_3 aTotalPhotos + \beta_4 aNonStock + \beta_5 aPostage +$$

$$\beta_6 bAuctionBid + \beta_7 bLivePrice + \beta_8 bCurrentAuBidders +$$

$$\beta_9 bClosingSequence + \beta_{10} aTotalBids + \beta_{11} aTotalBidders + \epsilon_{xt} \quad (3.6.4)$$

The $bTimeWithinA$ was exchanged for $bTimeTilEnd$ and $aDuration$. It is the case, as suspected, that time until the end of the auction is important for the decision to bid. The negative coefficient β_1 shows that the auctions that have less time left until the end are more likely to be chosen by bidders. In addition to that the intercepts for 2nd, 3rd, and 4th choice are negative and generally increasing (more negative with higher choice number), which additionally shows that division into the alternatives by sorting the auctions with respect to time till end is well grounded. The later positions on the list are less likely to be chosen by bidders. Another variable that is a result of sorting by time till end is $bClosigSequence$ - and that is the number on the list of closing auctions among all iPhone 4 auctions, not only in the selected subset. The coefficient next to this variable is also negative and significant which suggests that bidders often choose among wider range of alternatives, not restricted to the most identical version of the phone. Again, the further the auction is on the list, the less likely that it is chosen. In addition to the more subtle $bHourTimeTilEnd$, which counts the time until the end of the auction in a continuous manner, the coarse division into positions on the list of closing auctions show to be an important factor in choice. The other variables of interest are $bAuctionBid$, $bLivePrice$ and $bCurrentAuBidders$. The coefficients next to these variables are: negative for current number of bids in the auction $bAuctionBid$, negative for current live price in the auction: $bLivePrice$, and positive for current number of bidders in the auction: $bCurrentAuBidders$. Current price and higher number of bids are deterring from entering the auction, but a higher number of bidders is in fact an encouraging factor. This is an

interesting result, that suggests there may be a type of herding, where other bidders are encouraging. The controls that relate to the remaining auction variety: $aTotalPhotos$, $aNonStock$ and $aDuration$ and $aPostage$ are not significant in most of the regressions, which confirms that the remaining variety is not important for the attractiveness of each auction. The main effects that are of interest in the regressions are the effects of the time-varying variable in auctions, and these are: $bLivePrice$, $bCurrentAuBidders$ and $bAuctionBid$.

Other controls included in specification (1) and (2) are $aTotalBiders$ and $aTotalBids$ which to the fact that there is a remaining variety between auctions with respect to the total number of bids and bidders present - it is likely that the more bids in the auction the more likely that the auction is going to be listed as a chosen auction in the dataset, additionally there is a concern that total number of bids, although endogenous, could be a reflection of some unobserved characteristics of an auction, and therefore this control is included in regression (1) and (2). As can be seen including these variables do not influence the sign of the $bAuctionBid$ and $bCurrentAuBidders$, but the inclusion of these strengthen the coefficients next to these variables, and also increase the R-squared score, which suggests that they do indeed capture unobserved effects which should be controlled for.

Table 3.2: Description of data: Model 16Gb New Unlocked 3rd Star, first bid

A. Auction-level description

aDurationLevels	1 :227	3 : 97	5 : 7		7 or 10 : 33
aModel	16 :364				(Other): 0
aNetwork	Unlocked:364				(Other): 0
aCondition	New :364				(Other): 0
aStarL	3 :364				(Other): 0
aStarLevel	Turquoise:364				(Other): 0
aReturns	No :364				(Other): 0
aPostageFree	No :364				(Other): 0
aPhotosPresent	Yes :364				(Other): 0
aTotalPhotosLevels	1:257	2: 58	3: 20		4 or 7 : 29
aNonStock	No : 74	Yes :290			(Other): 0
aPostto	UK :364				(Other): 0
aExtras	No :364				(Other): 0

Number of choice sets:	91
Number of auctions:	95
Number of sellers:	77
Number of bidders:	91
Auctions per choice set:	4

B. Non-categorical variables

	mean	median	sd	min	max	n
bAmount	536.35	601.11	198.07	0.99	760.00	364.00
bClosingSequence	14.48	9.00	14.98	1.00	82.00	364.00
bLivePrice	440.58	549.00	236.45	0.01	723.00	364.00
bCurrentAuBidders	2.87	0.00	4.07	0.00	16.00	364.00
bMinTimeTilEnd	675.04	143.48	1389.95	0.02	13170.92	364.00
bHourTimeTilEnd	11.25	2.39	23.17	0.00	219.52	364.00
bAuctionBid	7.95	3.00	9.03	1.00	36.00	364.00

C. User variability

uBidL	1:364						
uTotalAuctionsL	1 :100	2 : 88	3 : 44	4 : 44	5 : 16	10 : 16	(Other): 56
uTotalBidsL	1 : 68	3 : 60	2 : 44	5 : 32	4 : 28	6 : 16	(Other):116
uTotalWinsL	0:196	1: 92	2: 44	3: 16	4: 8	5: 4	7: 4
uWinsSoFarL	0:364						
uNumbEbayWinsL	0 : 32	1 : 16	2 : 16	7 : 16	4 : 12	6 : 12	(Other):260

D. Auction bids and bidders

aTotalBidsLevels	5 : 47	3 : 27	22 : 18	17 : 17	16 : 16	(Other):239
aTotalBiddersLevels	5 : 42	4 : 36	3 : 34	12 : 34	9 : 23	(Other):195

The subsets discussed above had some shortcomings, which led to narrowing the spectrum of selected data further. The main concern was that, in the case of no restriction on which user's bid is included, there can be unobservable effects on the choice. For example, considering first bid by a given user, it is clear that the other considerations in respect to auction loyalty or in general influences of bidding history do not influence the choice the bidder is making. On the other hand, in the case of a 2nd, 3rd or a later bid, the previous bidding history may influence the choice the bidder is making, which could be biasing the results. For that reason, the next subsets selected are subsets of first users' bids with the same restrictions on auction characteristics as before. The model 16Gb dataset description can be found in table 3.12, and the 32Gb model dataset can be found in table 3.11. Furthermore, the seller rating levels are allowed to be 1st, 2nd or 3rd Star rating, and this allows for a range of seller heterogeneity. In order to further limit the heterogeneity among sellers, the datasets were reduced further to contain only New Unlocked auctions with 3rd start seller rating, divided into two groups: model 16Gb and model 32Gb. The description of these datasets can be found in table 3.2 for the 16Gb phone and table 3.13 for the 32Gb phone. As can be seen these two further limitations ensure that there is very little auction heterogeneity left, and the effects of users' history are eliminated. The 16Gb subset contains 91 choice sets, while the 32Gb subset contains 68 choice sets.

In the 16Gb subset there are 91 bids, and 4 alternatives in each choice set. The subset contains highly homogeneous group of auctions which is confirmed by the fact that there are only 95 auctions in total and 77 sellers. The auctions are on average repeated in more than one choice set and there is even less sellers with several sellers offering multiple auctions. Of course, there is still some heterogeneity left in the constant auction-level characteristics, and that is number of photos ($aTotalPhotos$), and "non stock photos" ($aNonStock$). Additionally there are different auction duration levels ($aDuration$). The sub-setting was based on auction characteristics, not on bidder characteristics, and therefore there is variety among bidders, as seen in table 3.2. User

Table 3.3: Multinomial Logit Results: M16 Unlocked 3rd Star, 1st bid , 4 choices

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	0.980	1.023	0.693
	(0.712)	(0.705)	(0.621)
3:(intercept)	0.320	0.348	-0.070
	(0.800)	(0.797)	(0.691)
4:(intercept)	-0.296	-0.349	-0.492
	(0.901)	(0.900)	(0.800)
log(bHourTimeTilEnd)	-0.428^{**}	-0.449^{**}	-0.037
	(0.197)	(0.195)	(0.142)
bCurrentAuBidders	0.602^{***}	0.543^{***}	0.538^{***}
	(0.163)	(0.120)	(0.106)
bAuctionBid	-0.408^{***}	-0.393^{***}	-0.151^{***}
	(0.089)	(0.082)	(0.045)
bLivePrice	-0.001	-0.0004	-0.005^{***}
	(0.002)	(0.002)	(0.001)
aDuration	0.215	0.241	0.104
	(0.189)	(0.186)	(0.162)
aTotalPhotos	0.331	0.352	0.232
	(0.265)	(0.261)	(0.223)
aNonStock	-0.591	-0.594	-0.836^{*}
	(0.559)	(0.558)	(0.471)
aPostage	0.918^{*}	0.981^{**}	0.411
	(0.493)	(0.485)	(0.416)
bClosingSequence	-0.124^{***}	-0.119^{***}	-0.097^{***}
	(0.037)	(0.036)	(0.030)
aTotalBids	0.268^{***}	0.246^{***}	
	(0.074)	(0.058)	
aTotalBidders	-0.086		
	(0.153)		
Observations	91	91	91
R^2	0.664	0.662	0.555
Log Likelihood	-41.587	-41.750	-55.074
LR Test	164.213^{***} (df = 14)	163.887^{***} (df = 13)	137.238^{***} (df = 12)
Note:			*p<0.1; **p<0.05; ***p<0.01

characteristics are not expected to influence the choice the bidder makes between 1,2,3, and 4th auction. On the other hand, despite the fact that the selected auctions are very similar, there is a large variety among them with respect to total number of bids and bidders in these auctions. Final number of total bids and bidders is endogenous, and is a result of the choices of bidders made rather than an exogenous variable on which auctions can be selected.

The numbers of observations are substantially diminished, but the aim was to achieve situation where choice is made between almost identical auctions. The fact that 4 alternatives are considered meant that only those choice sets were included which contained 4 or more products in the category selected.

As a first step the regression from equation 3.6.4 is repeated on these datasets. The results are almost identical. The change introduced is that logarithmic function of $bTimeTillEnd$ is considered, and it shows to improve fit in some cases, but not necessarily in others (32Gb version - see comparison between results in tables 3.15 and 3.14). The logarithmic function of time variable $bTimeTillEnd$ is selected as the chosen form, due to the conviction that the function of time is more likely to be non-linear. Other variables reveal to have the best fit with the linear function. The equation that determines the regression (1) in table 3.3 is as below[3]:

$$U_{xt} = C_x + \beta_1 log(bHourTimeTilEnd) + \beta_2 bCurrentAuBidders+$$
$$\beta_3 bAuctionBid + \beta_4 bLivePrice + \beta_5 aDuration+$$
$$\beta_6 aTotalPhotos + \beta_7 aNonStock + \beta_8 aPostage+$$
$$\beta_9 bClosingSequence + \beta_{10} aTotalBids + \beta_{11} aTotalBidders + \epsilon_{xt} \quad (3.6.5)$$

The results of above estimation can be found in column (1) in table 3.3 for 16Gb subset, and in column (1) in table 3.14 for 32Gb subset. As before, current bids, bidders and

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[3]the ordering is changed to match the regressions in tables 3.3, and 3.14

price show to have a significant influence with the same sign as in the previous regressions. The best subset to discuss is the 16Gb New Unlocked with 3rd Star seller rating and 1st bid by the bidders, because it contains 91 choice sets, while the alternative subset with 32Gb phones contain only 61 choice sets, which may mean that with the larger number of covariates the significance in all the important factors may be difficult to reach.

3.6.1 Marginal Effects

The marginal effects on probability of choosing a given alternative are not directly interpretable from the results of the regressions, due to the fact that multinomial logit has is a non-linear estimation. Nevertheless, the signs of the coefficients are interpretable, because the estimated coefficients are not alternative-specific. Therefore, the positive coefficient next to covariate x_1 means that the effect on probability of choice is positive, while a negative coefficients means that the effect on probability of choosing given alternative is negative. The marginal effects, on the other hand hive an interpretation relating to the magnitude of the effects. The marginal effects are the derivatives of the probabilities with respect to the explanatory variables, and in the case the these variables are alternative specific with constant coefficients the formula to calculate the marginal effects is reduced to the following:

$$\frac{dP_{ij}}{dx_{ij}} = \beta_x P_{ij}(1 - P_{ij}) \tag{3.6.6}$$

So the marginal effect on probability is the coefficient β times multiplication of two probabilities, which is at most 0.25. In order to have an upper bound on the effect, the coefficient needs to be divided by 4.

The marginal effects are calculated based on the estimation in column (1) in table 3.3 specified by equation 3.6.5. The upper bound marginal effects for β_1, β_2, β_3, β_4, and β_5

are as follows:

- The marginal effect of an increase in $log(bHourTimeTilEnd)$ by 1 on probability of choosing a given alternative is a reduction in probability by 0.107. Logarithm is a monotonous transformation, but it is non-linear, and therefore an increase in time until the end of an auction by 1 hour will have a larger negative effect for small values of $bHourUntilEnd$ and a smaller negative effect for larger values of $bHourUntilEnd$. It has to be also kept in mind that a 1 hour increase in $bHourTimeTilEnd$ is not possible in the case when there is less than 1 hour left. Logarithmic transformation used for $bLogHourUntilEnd$ scales the effect on small numbers so that it is better represented. For example an increase by 1 of $log(bHourTimeTilEnd)$ is equivalent to an increase by 41 minutes from 15 minutes of time remaining, or an increase by 82 minutes from 30 minutes of time remaining, or an increase by 5 hours and 24 minutes from 2 hours of time remaining. Equivalently, that is equal to an increase by about 2 minutes from 1 minute of time remaining.

- The marginal effect of an increase in $bCurrentAuBidders$ by 1 on probability of choosing a given alternative is an increase in probability by 0.15

- The marginal effect of an increase in $bAuctionBid$ by 1 on probability of choosing a given alternative is a reduction in probability by 0.102

- $bLivePrice$ is not significant in this regression Using estimation from column(3): The marginal effect of an increase in $bLivePrice$ by 1 on probability of choosing a given alternative is a reduction in probability by 0.00125

- The marginal effect of an increase in $bClosingSequence$ by 1 on probability of choosing a given alternative is a reduction in probability by 0.031

3.6.2 Marginal Rates of Substitution

Coefficients are marginal utilities, and are not interpretable since utility is ordinal. Nevertheless, ratios of coefficients are marginal rates of substitution. For example, if the observable part of utility is : $U = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$, joint variations of x_1 and x_2 which ensure the same level of utility are such that : $dU = \beta_1 dx_1 + \beta_2 dx_2 = 0$ so that :

$$MRS_{xy} = -\frac{dx_2}{dx_1}\Big|_{dU=0} = \frac{\beta_1}{\beta_2} = \frac{M_x}{M_y} \qquad (3.6.7)$$

The marginal rates of substitution calculated based on the estimation in column (1) in table 3.3 specified by equation 3.6.5 are:

- The marginal rate of substitution of the number of bids in an auction in terms of the number of bidders:

$$MRS_{bCAuBi,bAuctBid} = \frac{0.15}{-0.102} = -1.47 \qquad (3.6.8)$$

  Addition of one current bidder is equivalent to a reduction of bids by 1.47. The user's utility would be unchanged if there was 1 more bidder and 1.47 more bids in the auction.

- The marginal rate of substitution of the position number on the list of auctions in terms of the number of bidders:

$$MRS_{bCAuBi,bClosingSeq} = \frac{0.15}{-0.031} = -4.83 \qquad (3.6.9)$$

  Addition of one bidder is equivalent to an auction moving 4.83 places up the list of auctions sorted by closing time. The bidder's utility would be unchanged if at the same time the auction moved 4.83 places down the list and one more bidder was present in the auction (with other parameters such as number of bids and price unchanged).

- The marginal rate of substitution of the time until the end of an auction in terms of the number of bidders:

$$MRS_{bCAuBi,log(bHT)} = \frac{0.15}{-0.107} = -1.4 \qquad (3.6.10)$$

The bidder is indifferent between a decrease of $log(bHourTimeTillEnd)$ by 1.4 and an increase in number of bidders in an auction by 1.

- The marginal rate of substitution of the time until the end of an auction in terms of the number of bids:

$$MRS_{bAuctBid,log(bHT)} = \frac{-0.102}{-0.107} = 0.95 \qquad (3.6.11)$$

An increase in number of bids in an auction by 1 is equally bad to an increase of $log(bHourTimeTillEnd)$ by 0.95. In order to keep the same utility level, an increase in number of bids in an auction by 1 would have to be accompanied by a reduction in $log(bHourTimeTillEnd)$ by 0.95.

- The marginal rate of substitution of the time until the end of an auction in terms of the position on the list of all auctions sorted by time until the end:

$$MRS_{bClosingSeq,log(bHT)} = \frac{-0.031}{-0.107} = 0.29 \qquad (3.6.12)$$

A bidder is indifferent between a drop by 1 place on the list of auctions sorted by the time untill the end and an increase in $log(bHourTimeTillEnd)$ by 0.29 - both are equally bad. To stay on the same indifference curve, if the position on the list was increased by 1 (a reduction in $bClosingSequence$ by 1), it would have to be accompanied by an increase in $log(bHourTimeTillEnd)$ by 0.29.

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Table 3.4: Multinomial Logit Results: M16 New Unlocked 3rd Star,first bid, 4choices; Results of bottom-up approach

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	1.816**	2.078**	2.071**
	(0.907)	(0.956)	(0.949)
3:(intercept)	1.263	1.519	1.516
	(0.925)	(1.036)	(1.034)
4:(intercept)	0.110	0.116	0.118
	(1.043)	(1.133)	(1.130)
bCurrentAuBidders	1.095***	1.361***	1.351***
	(0.298)	(0.375)	(0.345)
bLivePrice	-0.002	-0.001	-0.001
	(0.003)	(0.003)	(0.003)
bAuctionBid	-0.466^{***}	-0.322^{**}	-0.314^{***}
	(0.141)	(0.156)	(0.117)
log(bHourTimeTilEnd)	0.216	0.205	0.209
	(0.347)	(0.390)	(0.387)
aDuration	0.421*	0.341	0.339
	(0.223)	(0.237)	(0.236)
aNonStock	0.045	0.372	0.391
	(0.724)	(0.760)	(0.714)
aPostage	1.639**	2.059**	2.052**
	(0.708)	(0.856)	(0.848)
bClosingSequence	-0.092^{**}	-0.067^{*}	-0.067^{*}
	(0.044)	(0.040)	(0.038)
aTotalBids	0.266***	0.261***	0.259***
	(0.089)	(0.091)	(0.087)
aTotalBidders	-0.290^{*}	-0.291	-0.288
	(0.175)	(0.189)	(0.185)
bLivePrice:log(bHourTimeTilEnd)	-0.002^{***}	-0.002^{***}	-0.002^{***}
	(0.001)	(0.001)	(0.001)
bCurrentAuBidders:log(bHourTimeTilEnd)	-0.103	-0.112	-0.107^{*}
	(0.085)	(0.090)	(0.056)
bAuctionBid:log(bHourTimeTilEnd)	0.014	0.002	
	(0.032)	(0.033)	
bCurrentAuBidders:bAuctionBid		-0.023^{*}	-0.023^{*}
		(0.013)	(0.013)
Observations	91	91	91
R^2	0.748	0.762	0.762
Log Likelihood	-31.195	-29.433	-29.435
LR Test	184.997*** (df = 16)	188.521*** (df = 17)	188.516*** (df = 16)
Note:			*p<0.1; **p<0.05; ***p<0.01

3.7 Estimation With Interaction Terms

Furthermore, the regression specification was extended by interaction terms between the dynamic variables. The two-way interactions were added to the regressions using bottom-up approach, and those that were found to have a significant, and prevailing effect were found to be $bLivePrice : log(bHourTimeTilEnd)$, $bCurrentAuBidders : log(bHourTimeTilEnd)$, and $bCurrentAuBidders : bAuctionbid$. At the first step all two-way interactions with log of the time variable were considered, and later other two-way interactions were tested and the insignificant ones were dropped along the way. Other two-way and the three-way interaction terms were added later, in the further section. The results of the bottom-up approach on model 16Gb subset can be found in table 3.4, and on the 32Gb subset in table 3.17. The last column, (3), contains the specification that was considered a best fit. This extended regression with interaction terms is defined by the equation 3.7.13 below:

$$
\begin{aligned}
U_{xt} = C_x &+ \beta_1 log(bHourTimeTilEnd) + \beta_2 bCurrentAuBidders+ \\
&\beta_3 bAuctionBid + \beta_4 bLivePrice + \beta_5 aDuration+ \\
&\beta_6 aTotalPhotos + \beta_7 aNonStock + \beta_8 aPostage+ \\
&\beta_9 bClosingSequence + \beta_{10} aTotalBids + \beta_{11} aTotalBidders+ \\
&\beta_{12}(bLivePrice * log(bHourTimeTilEnd))+ \\
&\beta_{13}(bCurrentAuBidders * log(bHourTimeTilEnd))+ \\
&\beta_{14}(bCurrentAuBidders * bAuctionBid) + \epsilon_{xt} \quad (3.7.13)
\end{aligned}
$$

The interpretation of these two-way interaction terms are important. The first one, $bLivePrice : log(bHourTimeTilEnd)$, shows that the live price and time until the end of the auction are very intertwined together. In these specifications $bLivePrice$ and $log(bHourTimeTilEnd)$ is becoming insignificant, while the interaction term is signifi-

cant at the 0.01 level. While, it has been discussed earlier that both time remaining in the auction and price have a negative effect on choice, the two variables are connected and the same price at different times in auction is interpreted differently. More specifically, the price of 60 with 30 minutes remaining is higher than the price of 60 with 5 minutes remaining. The bidders are expecting the price to rise more in the case more time is remaining, and are less likely to choose such an auction. An increase in one of these variables: time until the end of the auction, or price, leads to a different interpretation of the other one. The coefficient at the $bLivePrice : log(bHourTimeTilEnd)$ is found to be -0.002. The next interaction term found to be negative and significant is $bCurrentAuBidders : log(bHourTimeTilEnd)$. The interpretation is the same as before, 4 bidders in the auction with 30 minutes remaining is worse than 4 bidders in the auction with 5 minutes remaining. The coefficient is -0.107 with 0.1 significance level. The last interaction term is $bCurrentAuBidders : log(bAuctionBid)$, and there is also a negative coefficient next to this variable. This shows that in general, auctions preferred are those with bidders that are less active. Number of bids is a measure of how engaged the bidders are in an auction. The more the other bidders are engaged in competitive bidding, the worse prospects for wining an auction for a potential new bidder. The auction with 5 bidders who placed 1 bid each is better than an auction with 4 bidders and 10 bids placed. The coefficient next to this variable varies between -0.023 in (3) in table 3.4 to -0.022 in (1) in table 3.16. In table 3.16 specification (1) additionally, a control of the exact seller rating (number of positive votes, which later result in division to Star levels) is included. Even though the subset contains only auctions with 3rd Star rating of sellers, this control is added. The result is that the overall R-squared is increased, but the $aStar$ variable is not significant at the 0.1 level. Other extension to regression (3) in table 3.4 is the addition of $maxCountFromTop$ as an alternative-specific factor. $maxCountFromTop$ is the total number of auctions of the same type - so in this case model 16Gb New Unlocked with Seller rating 3 star - at the time the choice is made. This is a way to control for the size of auction competition. It is a constant factor in

each choice set, but can be included as random effect to see whether the total number of alternatives impacts the choice between 1st, 2nd, 3rd, or 4th auction differently. This inclusion can be found in specification (2) and (3) in table 3.16, and it shows that, while the overall R-squares is increased, none of the factors are significant. The same extension can be found in table 3.18 for model 32Gb subset. There, the total number of alternatives is consistently negative and significant for the 4th choice. This suggests that is has a higher impact on the smaller group of 32Gb phones, and that with an increase of available alternatives, the 4th option was less likely to be chosen (coefficient -0.497 at the 0.1 significance level). In the regressions, where interaction terms are added the auction-level variable, $aTotalBidders$, which was suspected to catch some unobservables, stops being significant. The $aTotalBids$ is significant in all the regressions, which is catching the fact that the auctions with more total bids are more often recorded as the chosen ones in the data. It is therefore important to control of this linear effect.

3.7.1 Marginal Effects

The marginal effects were calculated based on estimation with interaction terms of equation 3.7.13 in column (3) of Table 3.4. The regressions extended by interaction terms mean that the marginal effects of one variable will vary with another one, if their interaction term is present. In the case the utility is such that $U = \beta_0 + \beta_1 x_1 + \beta_{12}(x_1 * x_2)$, the marginal effect of an increase in x_1 on probability of choosing alternative ij is:

$$\frac{dP_{ij}}{dx_{ij}} = (\beta_1 + \beta_2 x_2)P_{ij}(1 - P_{ij}) \tag{3.7.14}$$

The ranges of the variables can be found in the data description in table 3.2. The range of $bHourTimeTilEnd$ is between 0.00028 which is 10 seconds, and 219.52 which is 9 days and 3.5 hours [4], with an average of 11.25. The range of $bAuctionBid$ is between 1 and

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[4]The minimum of time until the end in hours reported in table 3.2 is 0 due to the fact that up to 2 decimal places are reported.

36, with an average of 7.95. The range of $bLivePrice$ is between 0.01 and 723 with an average of 440.58, and the range of $bCurrentAuBidders$ is between 0 and 16 with an average of 2.87. The upper bounds of the size of marginal effects are calculated, where $P_{ij}(1 - P_{ij}) = 0.25$. The marginal effects are estimated based on results from column (3) in table 3.4, which is the estimation of equation 3.7.13 on the 16Gb subset.

- The marginal effect of $bLivePrice$ on probability of choosing an alternative is :

$$M_{bLivePrice} = \beta_{12} * log(bHourTimeTilEnd) * 0.25 \qquad (3.7.15)$$

$\beta_{12} = -0.002$, and therefore the marginal effect of price on the probability is decreasing with more time remaining. The range of the marginal effect is between 0.0021 for 10 seconds of time remaining to -0.0047 for 9 days and 3.5 hours of time remaining. The longer the time until auction end, the worse the increase in price is. For time remaining lower than 1 hour the effect of price increase is positive, while for the time remaining above 1 hour, the effect of price increase is negative. It is worth noting that an average time remaining in the dataset is 11.25 hours, at which a 1 pound increase in live price lead to 0.00122 decrease in probability of choosing that auction. A price increase by 10 pounds meant that the auction was less likely to be chosen by 1.2%.

- The marginal effect of $bAuctionBid$ on probability of choosing an alternative is :

$$M_{bAuctionBid} = (\beta_3 + \beta_{14} * bCurrentAuBidders) * 0.25 \qquad (3.7.16)$$

$\beta_3 = -0.314$ and $\beta_{14} = -0.023$, and the current number of bidders is a positive number, which means that the negative effect of the number of bids is further strengthened with more bidders in the auction.The number of bidders vary between 0 and 16. The marginal effect of additional bid makes sense only in the case there is at least 1 bidder, since with 0 bidders there can only be 0 bids in an auction.

The effect ranges from -0.084 in the case of 1 bidder in the auction to -0.17 in the case of 16 bidders. On average, the number of current bidders is 2.87, where the effect is that an increase in 1 bid results in a decrease in probability of choosing the auction by 0.095, so almost 10%.

- The marginal effect of $bCurrentAuBidders$ on probability of choosing an alternative is :

$$M_{bCAuBi} = (\beta_1 + \beta_{13} * log(bHourTimeTilEnd) + \beta_{14} * bAuctionBid) * 0.25 \quad (3.7.17)$$

$\beta_1 = 1.351$, $\beta_{13} = -0.107$, and $\beta_{14} = -0.023$, so the marginal effect of an additional bidder is decreased with more time remaining as well as with more bids in the auction. $bHourTimeTilEnd$ is between 0.00028 which is 10 seconds, and 219.52 which is 9 days and 3.5 hours with an average of 11.25, while $bAuctionBid$ is between 1 and 36, with an average of 7.95. The effect of current bidders depends on two other dimensions and the maximum effect is with the lowest number of bids and time remaining, while the minimum effect is in the case of the highest number of bids an time remaining. The base effect of current bidders is positive, the fact that there are bids in the auction is reducing the effect and the reduction is higher with higher number of bids. The time remaining has a positive sign in the case of time below 1 hour, and a negative sign in the case of time above 1 hour. An increase in time remaining is reducing the effect of an additional bidder, and the reduction is the fastest for small numbers of $bHourTimeTilEnd$ (due to a logarithmic function). The range of the marginal effect of $bCurrentAuBidders$ on probability of choosing an auction is from 0.55 in the case of 1 bid and 10 seconds time remaining to -0.013 for 36 bids and 219.52 hours remaining. On average, in the case of 7.95 bids and 11.25 hours remaining, the marginal effect of one additional bidder is an increase in the probability of auction choice by 0.23, so 23%.

- The marginal effect of $log(bHourTimeTilEnd)$ on probability of choosing an alter-

native is :

$$M_{log(bHT)} = (\beta_{12} * bLivePrice + \beta_{13} * bCurrentAuBidders) * 0.25 \qquad (3.7.18)$$

$\beta_{12} = -0.002$, and the live price $bLivePrice$ is positive and varies from 0.01 to 723 with an average of 440.58. $\beta_{13} = -0.107$ and the current number of bidders $bCurrentAuBidders$ is positive with range between 0 and 16 and average 2.87. The marginal effect of an increase by 1 in $log(bHourTimeTilEnd)$ starts with almost 0 for the live price close to 0 and 0 current bidders. For 1 bidder and price of 10 the marginal effect is -0.112. At maximum: 16 bidders and price of 723, the auction would never be chosen - and it is only a hypothetical situation . An increase in price level and number of bidders leads to higher negative effect in choice with an increase in time remaining in the auction. With the average price level of 440.58 and 2.87 bidders an increase in $log(bHourTimeTilEnd)$ by 1 results in a decrease in probability of choosing that alternative by almost 0.53. In terms of percentage reduction 53%. The increase in $log(bHourTimeTilEnd)$ by 1 is equivalent to an increase in minutes, hours, or even days of time remaining depending on whether it is evaluated close or far from the auction end. Smaller time differences matter closer to the auction end.

- The marginal effect of $bClosingSequence$ on probability of choosing an alternative is $\beta_9 * 0.25$ which is -0.017. A drop by 1 place on the list of all auctions for iPhone 4 sorted by time until the end resulted in an average decrease in probability of choosing that auction by 1.7%.

3.7.2 Marginal Rates of Substitution

The Marginal Rates of Substitution based on the model that includes interaction terms are not constant, but dependent on the interacted variables, like in the case of Marginal Effects. In order to have an approximations to the Marginal Rates of Substitution the

marginal rates of substitution can be evaluated at the averages of the variables on which they depend. In the case the utility is such that $U = \beta_0 + \beta_1 x_1 + \beta_{12}(x_1 * x_2)$, the marginal effect of an increase in x_1 on probability of choosing alternative ij is:

$$MRS_{x,y} = \frac{M_x}{M_y} = \frac{\beta_1 + \beta_{12}x_2}{\beta_{12}x_1} \tag{3.7.19}$$

- The marginal rate of substitution of the number of bids in an auction in terms of the number of bidders:

$$MRS_{bCAuBi,bAuctBid} = \frac{M_{bCAuBi}}{M_{bAuctionBid}} =$$

$$\frac{(\beta_1 + \beta_{13}log(bHourTimeTilEnd) + \beta_{14}bAuctionBid)}{(\beta_3 + \beta_{14}bCurrentAuBidders)} =$$

$$\frac{1.351 - 0.107(log(bHourTimeTilEnd)) - 0.023(bAuctionBid)}{-0.314 - 0.023(bCurrentAuBidders)} \tag{3.7.20}$$

Evaluated at the averages for each of the variables, that is equal to -2.39. In the case there are 2.87 bidders and 7.95 an addition of one more bidder is equivalent to a reduction of bids by 2.39. So if in an auction there was one more bidder added and less than 2.39 bids (so for example 2 bid) then that would result in an increase of utility for that auction, and it would be proffered now. On the other hand an addition of one more bidder and 3 or more bids would make this auction less preferred.

- The marginal rate of substitution of the position number on the list of auctions in

terms of the number of bidders:

$$MRS_{bCAuBi,bClosingSeq} = \frac{M_{bCAuBi}}{M_{bClosingSeq}} =$$

$$\frac{(\beta_1 + \beta_{13}log(bHourTimeTilEnd) + \beta_{14}bAuctionBid)}{\beta_9} =$$

$$\frac{1.351 - 0.107(log(bHourTimeTilEnd)) - 0.023(bAuctionBid)}{-0.067} \quad (3.7.21)$$

Evaluated at the averages for each of the variables, that is equal to -13.569. In this case an addition of one more bidder is equivalent to a rise by 12.57 places up the list of auctions.

- The marginal rate of substitution of the time until the end of an auction in terms of the number of bidders:

$$MRS_{bCAuBi,log(bHT)} = \frac{M_{bCAuBi}}{M_{log(bHT)}} =$$

$$\frac{(\beta_1 + \beta_{13}log(bHourTimeTilEnd) + \beta_{14}bAuctionBid)}{(\beta_{12}bLivePrice + \beta_{13}bCurrentAuBidders)} =$$

$$\frac{1.351 - 0.107(log(bHourTimeTilEnd)) - 0.023(bAuctionBid)}{-0.002(bLivePrice) - 0.107(bCurrentAuBidders)} \quad (3.7.22)$$

Evaluated at the averages for each of the variables, that is equal to -0.765. In this case an increase by one more bidder is equally good as a decrease in $log(bHourTimeTilEnd)$ by 0.765.

- The marginal rate of substitution of current pricen in terms of the number of bid-

ders:

$$MRS_{bCAuBi,bLivePrice} = \frac{M_{bCAuBi}}{M_{bLivePrice}} =$$

$$\frac{(\beta_1 + \beta_{13}log(bHourTimeTilEnd) + \beta_{14}bAuctionBid)}{\beta_{12}log(bHourTimeTilEnd)} =$$

$$\frac{1.351 - 0.107(log(bHourTimeTilEnd)) - 0.023(bAuctionBid)}{-0.002(log(bHourTimeTilEnd)} \quad (3.7.23)$$

Evaluated at the averages for each of the variables, that is equal to -40.41. A decrease by one bidder could be compensated by simultaneous drop in price by 40.41 pounds.

- The marginal rate of substitution of the time until the end of an auction in terms of the live price:

$$MRS_{bLivePrice,log(bHT)} = \frac{M_{bLivePrice}}{M_{log(bHT)}} =$$

$$\frac{\beta_{12}log(bHourTimeTilEnd)}{(\beta_{12}bLivePrice + \beta_{13}bCurrentAuBidders)} =$$

$$\frac{-0.002(log(bHourTimeTilEnd))}{-0.002(bLivePrice) - 0.107(bCurrentAuBidders)} \quad (3.7.24)$$

Evaluated at the averages for each of the variables, that is equal to 0.0189. An increase in price by one pound is equivalent to an increase in $log(bHourTimeTilEnd)$ by 0.0189.

- The marginal rate of substitution of live price in terms of the the position on the

list of auctions :

$$MRS_{bClosingSeq,bLivePrice} = \frac{M_{bClosingSeq}}{M_{bLivePrice}} =$$

$$\frac{\beta_9}{\beta_{12}log(bHourTimeTilEnd)} =$$

$$\frac{-0.067}{-0.002(log(bHourTimeTilEnd))} \quad (3.7.25)$$

Evaluated at the averages for each of the variables, that is equal to 13.84. A rise up the display list by 1 is equivalent to a decrease in current price by almost 14 pounds.

- The marginal rate of substitution of the live price in terms of the current number of bids in an auction:

$$MRS_{bAuctBid,bLivePrice} = \frac{M_{bAuctBid}}{M_{bLivePrice}} =$$

$$\frac{(\beta_3 + \beta_{14}bCurrentAuBidders)}{\beta_{12}log(bHourTimeTilEnd)} =$$

$$\frac{-0.314 - 0.023(bCurrentAuBidders)}{-0.002(log(bHourTimeTilEnd))} \quad (3.7.26)$$

Evaluated at the averages for each of the variables, that is equal to 78.50. An increase by one bid is equivalently bad as an increase in price by 78 pounds.

- The marginal rate of substitution of the time until the end of an auction in terms

of the number of bids:

$$MRS_{bAuctBid,log(bHT)} = \frac{M_{bAuctionBid}}{M_{log(bHT)}} =$$

$$\frac{(\beta_3 + \beta_{14}bCurrentAuBidders)}{(\beta_{12}bLivePrice + \beta_{13}bCurrentAuBidders)} =$$

$$\frac{-0.314 - 0.023(bCurrentAuBidders)}{-0.002(bLivePrice) - 0.107(bCurrentAuBidders)} \quad (3.7.27)$$

Evaluated at the averages for each of the variables, that is equal to 0.319. An increase by one bid is similarly bas as an increase in $log(bHourTimeTilEnd)$ by 0.319.

- The marginal rate of substitution of the time until the end of an auction in terms of the position on the list of all auctions sorted by time until the end:

$$MRS_{bClosingSeq,log(bHT)} = \frac{M_{bClosingSeq}}{M_{log(bHT)}} =$$

$$\frac{\beta_9}{(\beta_{12}bLivePrice + \beta_{13}bCurrentAuBidders)} =$$

$$\frac{-0.067}{-0.002(bLivePrice) - 0.107(bCurrentAuBidders)} \quad (3.7.28)$$

Evaluated at the averages for each of the variables, that is equal to 0.056. A drop by one on the list of displayed auctions is as bad as an increase in $log(bHourTimeTilEnd)$ by 0.056.

3.8   Extending The Model By More Interaction Terms

Although the model with interaction terms described above is already a good fit to the data, there was still a possibility that not all the meaningful interaction terms are included. Another possibility is that price in interaction with number of bidders could be an important addition - since higher price with given number of bidders could be a signal that the bidders present have higher valuation for the product and deter newcomers. The previous section has already shown that number of bids in interaction with the number of bidders is an important factor - newcomers are deterred by auctions with more active bidders. The previous model had an R-squared measure of fit of 0.762 in the subset of 16Gb model and 0.634 in the subset of 32Gb model. Most importantly, in addition to a good R-squared measure the significant effects of variables and their interactions were in the same direction and with similar magnitude in both subsets. Extension of the model by one term - $bCurrentAuBidders : bLivePrice$ increases that R-squared measure in the 32Gb subset to 0.658, and the new term is significant at the 0.05 level without harming the significance of the other terms discussed earlier. This can be seen in table 3.5 column(2). The same done on the 16Gb subset has an effect of an increase of the R-squared measure to 0.765,but the new term is not significant at the 0.1 level and reduces the significance of $bCurrentAuBidders : bAuctionBid$. On this subset a better effect is achieved by adding a 3-way interaction term $bCurrentAuBidders : bLivePrice : bAuctionBid$ instead of two terms $bCurrentAuBidders : bAuctionBid$ and $bCurrentAuBidders : bLivePrice$. In this case the R-squared measure is increased to 0.767 and the new term as well as the previous ones are significant as before - on at least 0.05 level, which can be seen in table 3.6 column (3). This shows that there are slight differences as to a model with better fit in the two subsets, but the interaction of price and number of bidders is an important one. Of course the sign of the coefficient is negative, the fact that the current price is higher means that the positive effect of number of bidders is reduced. The effect presented is that auctions with more bidders but less active and preferably with lower valuation or

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just placing lower bids are preferred. On the one hand some more bidders present in the auction give a positive signal, on the other hand from the point of view of a bidder who wants to win it is a better situation if those other bidders are either not very active or are not willing to place very high bids.

Table 3.5: Multinomial Logit Results: M32 New Unlocked 3rd Star, 4 choices; Interaction terms II

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	−1.937**	−2.273**	−2.403***
	(0.908)	(1.003)	(0.932)
3:(intercept)	−1.918*	−2.287**	−2.617**
	(1.031)	(1.128)	(1.083)
4:(intercept)	-1.500	-1.573	−1.887*
	(1.001)	(1.021)	(0.991)
bCurrentAuBidders	1.689***	2.263***	1.550***
	(0.419)	(0.570)	(0.380)
bLivePrice	-0.001	0.0002	0.0002
	(0.003)	(0.003)	(0.003)
bAuctionBid	−0.272***	−0.339***	−0.340***
	(0.099)	(0.110)	(0.097)
log(bHourTimeTilEnd)	-0.658	-0.943	-0.667
	(0.571)	(0.608)	(0.572)
aDuration	0.572**	0.755**	0.764**
	(0.289)	(0.323)	(0.297)
aNonStock1	0.685	0.598	0.548
	(0.933)	(0.904)	(0.809)
aPostage	0.776	0.674	0.606
	(0.854)	(0.788)	(0.760)
bClosingSequence	-0.057	-0.051	-0.053
	(0.052)	(0.053)	(0.051)
aTotalBids	0.339***	0.383***	0.317***
	(0.103)	(0.106)	(0.090)
aTotalBidders	-0.117	-0.154	-0.092
	(0.175)	(0.179)	(0.168)
bLivePrice:log(bHourTimeTilEnd)	0.0005	0.001	0.001
	(0.001)	(0.001)	(0.001)
bCurrentAuBidders:log(bHourTimeTilEnd)	−0.160**	−0.196***	−0.163***
	(0.065)	(0.072)	(0.060)
bCurrentAuBidders:bAuctionBid	−0.050***	−0.049***	
	(0.014)	(0.014)	
bCurrentAuBidders:bLivePrice		−0.001*	
		(0.0005)	
bCurrentAuBidders:bLivePrice:bAuctionBid			−0.0001***
			(0.00002)
Observations	68	68	68
R²	0.634	0.658	0.628
Log Likelihood	-28.837	-26.880	-29.286
LR Test	99.691*** (df = 16)	103.604*** (df = 17)	98.792*** (df = 16)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.6: Multinomial Logit Results: M16 New Unlocked 3rd Star, 4 choices; Interaction terms II

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	2.071**	1.935**	1.937**
	(0.949)	(0.946)	(0.941)
3:(intercept)	1.516	1.377	1.444
	(1.034)	(1.057)	(1.049)
4:(intercept)	0.118	-0.024	0.024
	(1.130)	(1.145)	(1.152)
bCurrentAuBidders	1.351***	1.514***	1.319***
	(0.345)	(0.409)	(0.333)
bLivePrice	-0.001	-0.0003	-0.0004
	(0.003)	(0.004)	(0.004)
bAuctionBid	−0.314***	−0.346***	−0.383***
	(0.117)	(0.128)	(0.112)
log(bHourTimeTilEnd)	0.209	0.281	0.335
	(0.387)	(0.519)	(0.550)
aDuration	0.339	0.379	0.395
	(0.236)	(0.244)	(0.241)
aNonStock	0.391	0.483	0.401
	(0.714)	(0.734)	(0.726)
aPostage	2.052**	1.979**	2.056**
	(0.848)	(0.856)	(0.849)
bClosingSequence	−0.067*	−0.071*	−0.078*
	(0.038)	(0.040)	(0.041)
aTotalBids	0.259***	0.253***	0.265***
	(0.087)	(0.090)	(0.090)
aTotalBidders	-0.288	-0.262	-0.299
	(0.185)	(0.192)	(0.192)
bLivePrice:log(bHourTimeTilEnd)	−0.002***	−0.002**	−0.002**
	(0.001)	(0.001)	(0.001)
bCurrentAuBidders:log(bHourTimeTilEnd)	−0.107*	−0.106**	−0.103**
	(0.056)	(0.053)	(0.052)
bCurrentAuBidders:bAuctionBid	−0.023*	-0.018	
	(0.013)	(0.014)	
bCurrentAuBidders:bLivePrice		-0.0005	
		(0.001)	
bCurrentAuBidders:bLivePrice:bAuctionBid			−0.00003*
			(0.00002)
Observations	91	91	91
R²	0.762	0.765	0.767
Log Likelihood	-29.435	-29.019	-28.851
LR Test	188.516*** (df = 16)	189.349*** (df = 17)	189.685*** (df = 16)
Note:			*p<0.1; **p<0.05; ***p<0.01

Other additions include an interaction term between auction duration and the logarithm of time until the end of the auction. This extension can be seen in table 3.19 for the 16Gb model and table 3.20 for the 32Gb model. This addition increases the R-squared measures, although the added term is not significant at the 0.1 level. Further extension include additions of more interaction terms, and column (3) in table 3.21 seems like a good fit - the R-squared measure is increased to 0.799 and almost all of the terms are significant at at least 0.05 level. The advantage is that there is a second dataset to check the model and it turned out that the same estimation on the 32Gb subset lead to a decrease in R-squared measure to 0.652, and only 4 terms remained significant, which can be seen in table 3.22. The risk is that by adding combinations of terms to the regression there is always a possibility to arrive at a better fit to the particular dataset, but it can lead to over-fitting, so that the model is not good at representing universal relationships that can also be observed in another dataset. The best model should be versatile and be a good fit to both subsets.

In order to find the best fit a top-down approach was performed on both subsets. This can be seen in tables 3.23 and 3.24 for M16 (Model 16Gb) and in table 3.27 for M32 (Model 32Gb). At the beginning all the interaction terms between the dynamic variables were added, and later the terms with the lowest t-score were removed one-by one. As a result the R-squared measure in M16 subset was increased to 0.815 with 16 significant terms (as seen in clumn (3) of table 3.24) and in the M32 subset to 0.770 with 11 significant terms (column (3) of table 3.27). Additionally, all the possible interaction terms with $aDuration$ that have resulted in an increase of the overall fit of the model were added, or exchanged with other terms. The resulting best-fit regression for M16 subset can be found in column (3) of table 3.26. The R-squared of the regression was 0.890 with 15 significant terms (and one almost significant). The resulting best-fit regression for the M32 subset can be found in column (3) of table 3.28. The R-squared for that regression was 0.878 with 15 significant terms. The problem is that the models found in the two cases are completely different, and they include a range of different terms, with

only 5 having the same coefficient sign in both cases. This clearly shows the problem is over-fitting and these models are not useful to understand real effects taking place.

The above exercise, that shows the over-fitted models, increases the confidence that the models presented before: with interaction terms in tables 3.6, 3.5, 3.4 and 3.17, represent the real effects that influence choice between auctions. Even the basic model, without the interaction terms in table 3.2 and 3.14 capture the main average effects of the dynamic aspects of auctions on choice. In addition, even in the over-fitted models, that contained a lot of spurious correlations, the number of current bidders in an auction remained with positive and significant coefficient, which additionally confirms that it has an important effect.

Summing up the results show that the bidders who face a choice between concurrent auctions in on-line marketplaces base their decision, in large part, based on the observed activity of other bidders. First of all, auctions with more bidders are generally preferred, but the presence of more active bidders or high current price is discouraging. Generally, bidders prefer to place their bid close to the end of the auction when more information about other participants is revealed. Close to the end of the auction, even an increase in price is not deterring bidders. The response to an increased number of other bidders in the auction is also time-varying, and close to the beginning of an auction the effect of more bidders is negative.

3.9 Conclusion

In conclusion, the paper applies multinomial logit regression to a new setting: choice between identical products. Unlike in standard demand models, where the main variability between alternatives comes from characteristics of each object, here the objects are homogeneous, and the variability comes from the dynamic aspects of each auction, at the time the choice is made. Multinomial logit allows to identify the factors which influence the probability of choosing to bid on an auction in an online marketplace, where many

identical products are available. The sorting of the products by the time until the end of the auction is a way to divide auctions into available alternatives : alternative number 1 being the soonest to end, number 2 second soonest to end and others accordingly. The estimations made include 4 alternatives for each choice set. In addition to current price, other dynamic aspects, such current number of bids and bidders is also found to have a significant effect. The effects found show that auction choice is based in a large part on activity of other bidders in an on-line auctions marketplace.

At the time the bidder decides to place the bid they are facing a choice between almost identical auctions that differ by the time they have started, time until the end of the auction, live price, number od bids and bidders in the auction, as well as position on the list of auctions that is displayed as a result of search. This situation is mirrored in the choice model estimated. The auctions are chosen based on utilities for available alternatives. Utility for each of the available auctions depends on the same aspects. The utility function found to be the best fit contains the dynamic auction aspects and some interaction terms between them. In particular it is found that the time until the end of the auction is best fitted with a logarithmic curve and the effect of an increase in time until the end is connected to live price - the live price and logarithm of time until the auction end are significant as an interaction term, but not independently. More time left in the auction means that an increase in price has a higher negative effect - and this negative effect is decreasing with time, while close to auction end the increase in price is no longer acting as a deterrent, but can even be encouraging. Perhaps the most important impact has the number of current bidders as well as their displayed activity in the auction. The base effect of the number of bidders on auction choice is positive, so on average more bidders are a good thing. On the other hand, further away from auction end the more bidders are not encouraging to enter the auction. If bidders display large activity by placing multiple bids, that is also a deterrent for a new bidder to enter. If the bidders are perceived as high valuation due to high current price, that is also increasing

the deterrent effect. With an increase in the number of bids as well as price the positive effect of number of bidders is decreased and can be turned to negative. The current number of bids has an overall negative effect on auction choice. The way the auctions are sorted on the list of the auctions on the website also influences auction choice. Auctions that are higher on the list are preferred to those further down.

3.10   Tables

Table 3.7: Description of data: Model 16Gb and 32Gb New Unlocked auctions

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Model 32Gb Unlocked auctions only

aDurationLevels	1 :1665	3 : 929	5 : 162	6,7 or 10 : 90
aModel	32 :2852			(Other): 0
aNetwork	Unlocked:2852			(Other) : 0
aCondition	New :2852			(Other): 0
aStarL	3 :1577	1 : 699	2 : 576	(Other): 0
aStarLevel	Turquoise:1577	Yellow : 699	Blue : 576	(Other) : 0
aReturns	No :2852			(Other): 0
aPostageFree	Yes :2852			(Other): 0
aPhotosPresent	Yes :2852			(Other): 0
aTotalPhotosLevels	1:1864	2: 445	3: 237	4,5 or 6 : 306
aNonStock	No : 948	Yes :1904		
aPostto	UK :2852			(Other): 0
aExtras	No :2852			(Other): 0

Number of choice sets:	713
Number of auctions:	160
Number of sellers:	130
Number of bidders:	345
Auctions per choice set	4

Model 16Gb Unlocked auctions only

aDurationLevels	1 : 1227	3 : 685	5 : 258	7 or 10 : 758
aModel	16 : 2928			(Other): 0
aNetwork	Unlocked : 2928			(Other) : 0
aCondition	New : 2928			(Other): 0
aStarL	3 : 1344	1 : 931	2 : 653	(Other): 0
aStarLevel	Turquoise : 1344	Yellow : 931	Blue : 653	(Other) : 0
aReturns	No : 2928			(Other): 0
aPostageFree	No : 2928			(Other): 0
aPhotosPresent	Yes : 2928			(Other): 0
aTotalPhotosLevels	1 : 2228	2 : 341	3 : 207	4 or 7 : 152
aNonStock	No : 824	Yes : 2104		
aPostto	UK : 2928			(Other): 0
aExtras	No : 2928			(Other): 0

Number of choice sets:	732
Number of auctions:	201
Number of sellers:	168
Number of bidders:	341
Auctions per choice set	4

Table 3.8: Multinomial Logit Results: Model 32Gb New Unlocked

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	-0.034	-0.151	-0.192
	(0.128)	(0.125)	(0.122)
3:(intercept)	-0.733^{***}	-0.895^{***}	-0.892^{***}
	(0.161)	(0.159)	(0.154)
4:(intercept)	-0.415^{**}	-0.706^{***}	-0.672^{***}
	(0.183)	(0.177)	(0.170)
bHourTimeTilEnd	-0.009	0.001	0.006
	(0.007)	(0.007)	(0.006)
aDuration	-0.125^{***}	-0.108^{**}	-0.185^{***}
	(0.045)	(0.043)	(0.042)
aTotalPhotos	-0.079	0.002	0.001
	(0.054)	(0.053)	(0.052)
aNonStock1	0.174	-0.191	-0.285^{**}
	(0.141)	(0.130)	(0.126)
aPostage	-0.026	-0.123	0.105
	(0.133)	(0.127)	(0.120)
bAuctionBid	-0.284^{***}	-0.204^{***}	-0.098^{***}
	(0.027)	(0.021)	(0.015)
bLivePrice	-0.002^{***}	-0.0002	-0.002^{***}
	(0.0004)	(0.0004)	(0.0003)
bCurrentAuBidders	0.663^{***}	0.354^{***}	0.361^{***}
	(0.058)	(0.032)	(0.031)
bClosingSequence	-0.110^{***}	-0.107^{***}	-0.103^{***}
	(0.015)	(0.015)	(0.014)
aTotalBids	0.224^{***}	0.122^{***}	
	(0.022)	(0.015)	
aTotalBidders	-0.352^{***}		
	(0.047)		
Observations	713	713	713
R^2	0.351	0.317	0.281
Log Likelihood	-612.293	-644.742	-678.820
LR Test	663.162^{***} (df = 14)	598.264^{***} (df = 13)	530.108^{***} (df = 12)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.9: Multinomial Logit Results: Model 16GbNew Unlocked

| | *Dependent variable:* | | |
| | choice | | |
	(1)	(2)	(3)
2:(intercept)	-0.241	−0.407***	-0.234
	(0.164)	(0.157)	(0.148)
3:(intercept)	−0.356**	−0.586***	−0.519***
	(0.172)	(0.165)	(0.154)
4:(intercept)	-0.318	−0.581***	−0.666***
	(0.198)	(0.188)	(0.177)
bHourTimeTilEnd	−0.015***	−0.011***	0.003
	(0.004)	(0.004)	(0.004)
aDuration	−0.092***	−0.059*	−0.092***
	(0.033)	(0.033)	(0.032)
aTotalPhotos	0.165**	0.149**	0.125*
	(0.067)	(0.067)	(0.065)
aNonStock1	−0.361**	−0.244*	−0.239*
	(0.148)	(0.142)	(0.134)
aPostage	0.086	0.156**	0.092
	(0.073)	(0.069)	(0.067)
bAuctionBid	−0.314***	−0.243***	−0.084***
	(0.028)	(0.022)	(0.011)
bLivePrice	−0.001***	−0.0002	−0.002***
	(0.0005)	(0.0005)	(0.0003)
bCurrentAuBidders	0.640***	0.404***	0.395***
	(0.057)	(0.036)	(0.033)
bClosingSequence	−0.151***	−0.123***	−0.118***
	(0.017)	(0.015)	(0.013)
aTotalBids	0.258***	0.159***	
	(0.025)	(0.017)	
aTotalBidders	−0.331***		
	(0.051)		
Observations	732	732	732
R^2	0.532	0.507	0.456
Log Likelihood	-468.589	-493.524	-545.316
LR Test	1,066.299*** (df = 14)	1,016.429*** (df = 13)	912.845*** (df = 12)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.10: Description of data: Model 16Gb and 32Gb New Unlocked auctions

A. Description of user vairety in the dataset of similar auctions

Model 16Gb Unlocked auctions only, 4 choices dataset

uBidL	1 : 384	2 : 336	3 : 296	4 : 256	5 : 228	6 : 196	(Other):1232
uTotalAuctionsL	2 : 448	3 : 352	1 : 300	4 : 268	5 : 212	8 : 208	(Other):1140
uTotalBidsL	8 : 188	5 : 176	9 : 176	3 : 172	12 : 172	11 : 164	(Other):1880
uTotalWinsL	0 :1076	1 : 716	2 : 440	3 : 244	8 : 120	4 : 92	(Other): 240
uWinsSoFarL	0 :1588	1 : 736	2 : 300	3 : 96	4 : 80	5 : 28	(Other): 100
uNumbEbayWinsL	0 : 320	6 : 104	1 : 100	2 : 100	9 : 84	(Other):2212	NA's : 8

Model 32Gb Unlocked auctions only

uBidL	1 : 480	2 : 356	3 : 300	5 : 216	4 : 208	6 : 160	(Other):1132
uTotalAuctionsL	3 : 396	2 : 360	4 : 356	1 : 292	6 : 164	5 : 128	(Other):1156
uTotalBidsL	4 : 248	3 : 164	5 : 164	2 : 156	9 : 152	12 : 152	(Other):1816
uTotalWinsL	0 :1268	1 : 740	2 : 232	3 : 204	4 : 80	5 : 72	(Other): 256
uWinsSoFarL	0 :1796	1 : 552	2 : 248	3 : 64	4 : 44	6 : 36	(Other): 112
uNumbEbayWinsL	0 : 148	2 : 92	13 : 88	6 : 80	19 : 80	(Other):2340	NA's : 24

B. Vairety in total bids and bidders in the dataset of similar auctions

Model 16Gb Unlocked auctions only

aTotalBidsLevels	1 : 594	16 : 219	5 : 208	6 : 206	34 : 189	20 : 135	(Other):1377
aTotalBiddersLevels	1 :597	9 :356	3 :333	13 :251	11 :218	8 :212	(Other):961

Model 32Gb Unlocked auctions only

aTotalBidsLevels	14 : 257	1 : 217	8 : 208	2 : 183	4 : 169	11 : 152	(Other):1666
aTotalBiddersLevels	10 : 358	6 : 305	1 : 280	7 : 253	8 : 219	2 : 204	(Other):1233

C. Non-categorical variables

Model 16 Unlocked auctions, 4 choices dataset

	mean	median	sd	min	max	n
bAmount	584.36	620.00	176.85	0.99	999.00	2928.00
bClosingSequence	13.38	6.00	16.71	1.00	89.00	2928.00
bLivePrice	474.14	560.00	222.07	0.01	960.00	2928.00
bCurrentAuBidders	2.75	0.00	3.80	0.00	17.00	2928.00
bMinTimeTilEnd	660.46	134.00	1457.16	0.02	11413.77	2928.00
bHourTimeTilEnd	11.01	2.23	24.29	0.00	190.23	2928.00
bAuctionBid	8.84	4.00	9.89	1.00	48.00	2928.00

Model 32 Unlocked auctions, 4 choices dataset

	mean	median	sd	min	max	n
bAmount	677.70	740.00	208.67	0.01	999.00	2852.00
bClosingSequence	7.78	5.00	7.91	1.00	52.00	2852.00
bLivePrice	569.67	650.00	246.24	0.01	933.00	2852.00
bCurrentAuBidders	2.91	1.00	3.62	0.00	17.00	2852.00
bMinTimeTilEnd	656.76	253.15	957.50	0.02	7120.33	2852.00
bHourTimeTilEnd	10.95	4.22	15.96	0.00	118.67	2852.00
bAuctionBid	8.07	6.00	7.19	1.00	35.00	2852.00

Table 3.11: Description of data: Model 32Gb New Unlocked, first bid

A. Auction-level variables

aDurationLevels	1 :264	3 :180	5 : 19	6,7 or 10 : 17
aModel	32 : 480			(Other): 0
aNetwork	Unlocked : 480			(Other) : 0
aCondition	New : 480			(Other): 0
aStarL	3 :278	2 :108	1 : 94	(Other): 0
aStarLevel	Turquoise:278	Blue :108	Yellow : 94	(Other) : 0
aReturns	No : 480			(Other): 0
aPostageFree	No : 480			(Other): 0
aPhotosPresent	Yes : 480			(Other): 0
aTotalPhotosLevels	1 : 324	2: 63	3: 41	4, 5 or 6 : 52
aNonStock	No : 155	Yes : 325		
aPostto	UK : 480			(Other): 0
aExtras	No : 480			(Other): 0

Number of choice sets:	120
Number of auctions:	129
Number of sellers:	111
Number of bidders:	120
Auctions per choice set	4

B. Non-categorical variables

	mean	median	sd	min	max	n
bAmount	673.49	730.03	202.41	0.01	999.00	480.00
bClosingSequence	8.47	5.00	9.43	1.00	50.00	480.00
bLivePrice	567.05	660.00	244.69	0.01	931.00	480.00
bCurrentAuBidders	2.77	0.50	3.68	0.00	15.00	480.00
bMinTimeTilEnd	594.92	108.45	968.93	0.02	5699.15	480.00
bHourTimeTilEnd	9.92	1.81	16.15	0.00	94.99	480.00
bAuctionBid	8.05	6.00	7.45	1.00	31.00	480.00

C. User variability

uAuctionBidL	1 :119	2 : 1	NA's:360				
uAuctionNumberL	1 :120	NA's:360					
uBidL	1:480						
uTotalAuctionsL	1 :184	2 :140	3 : 64	4 : 24	6 : 20	5 : 8	(Other): 40
uTotalBidsL	1 :124	2 : 92	3 : 60	4 : 52	5 : 28	9 : 28	(Other): 96
uTotalWinsL	0 :276	1 :160	3 : 12	2 : 8	4 : 8	5 : 4	(Other): 12
uWinsSoFarL	0:400	1: 80					
uNLR	0:472	1: 8					
uNumbEbayWinsL	0 : 28	1 : 24	38 : 20	2 : 12	6 : 12	19 : 12	(Other):372

D. Total bids and bidders in auctions

aTotalBidsLevels	14 : 51	2 : 41	1 : 38	8 : 38	4 : 29	5 : 29	(Other):254
aTotalBiddersLevels	10 : 71	2 : 50	4 : 41	1 : 40	7 : 39	8 : 37	(Other):202

Table 3.12: Description of data: Model 16Gb New Unlocked, first bid

A. Auction-level variables

aDurationLevels	1 :163	3 : 73	5 : 33		7 or 9 : 118
aModel	16 :384				(Other): 0
aNetwork	Unlocked:384				(Other): 0
aCondition	New :384				(Other): 0
aStarL	3 :167	1 :148	2 : 69		(Other): 0
aStarLevel	Turquoise:167	Yellow :148	Blue : 69		(Other) : 0
aReturns	No :384				(Other): 0
aPostageFree	FALSE:384				(Other): 0
aPhotosPresent	TRUE:384				(Other): 0
aTotalPhotosLevels	1:289	2: 43	3: 32		4 or 7 : 20
aNonStock	0:104	1:280			
aPostto	UK :384				(Other): 0
aExtras	0:384				(Other): 0

Number of choice sets:	96
Number of auctions:	120
Number of sellers:	107
Number of bidders:	96
Auctions per choice set	4

B. Non-categorical variables

	mean	median	sd	min	max	n
bAmount	603.53	640.01	180.44	2.00	850.00	384.00
bClosingSequence	16.17	6.00	20.28	1.00	89.00	384.00
bLivePrice	491.74	599.00	223.59	0.01	750.02	384.00
bCurrentAuBidders	2.60	0.00	3.83	0.00	16.00	384.00
bMinTimeTilEnd	620.19	24.06	1635.00	0.02	11413.77	384.00
bHourTimeTilEnd	10.34	0.40	27.25	0.00	190.23	384.00
bAuctionBid	7.72	1.50	9.71	1.00	48.00	384.00

C. User variability

uAuctionBidL	1 : 96	NA's:288					
uAuctionNumberL	1 : 96	NA's:288					
uBidL	1:384						
uTotalAuctionsL	1 :152	2 :104	3 : 52	4 : 24	8 : 12	10 : 12	(Other): 28
uTotalBidsL	1 :108	2 : 88	3 : 52	5 : 28	4 : 20	7 : 12	(Other): 76
uTotalWinsL	0:148	1:156	2: 48	3: 16	4: 8	7: 8	
uWinsSoFarL	0:276	1:108					
uNLR	0:376	1: 8					
uNumbEbayWinsL	0 : 36	2 : 20	32 : 16	1 : 12	8 : 12	17 : 12	(Other):276

D. Total bids and bidders in auctions

aTotalBidsLevels	1 :114	5 : 33	16 : 25	20 : 19	22 : 18	6 : 17	(Other):158
aTotalBiddersLevels	1 :116	9 : 48	3 : 41	13 : 34	8 : 27	2 : 26	(Other): 92

Table 3.13: Description of data: Model 32Gb New Unlocked 3rd Star, first bid

A. Auction-level variables

aDurationLevels	1 :128	3 : 118	5 : 13	6,7 or 10 : 13
aModel	32 :272			(Other): 0
aNetwork	Unlocked : 272	O2 :126		(Other): 0
aCondition	New : 272			(Other): 0
aStarL	3 : 272			(Other): 0
aStarLevel	Turquoise : 272			(Other): 0
aReturns	No : 272			(Other): 0
aPostageFree	No : 272			(Other): 0
aPhotosPresent	Yes : 272			(Other): 0
aTotalPhotosLevels	1 : 188	2 : 30	3: 19	4,5 or 6 : 35
aNonStock	No : 89	Yes : 183		(Other): 0
aPostto	UK : 272			(Other): 0
aExtras	No : 272			(Other): 0

Number of choice sets:	68
Number of auctions:	58
Number of sellers:	51
Number of bidders:	68
Auctions per choice set	4

B. Non-categorical variables

	mean	median	sd	min	max	n
bAmount	630.93	700.00	210.32	2.00	966.22	272.00
bClosingSequence	11.70	9.00	9.72	1.00	44.00	272.00
bLivePrice	520.75	615.00	249.48	0.99	930.00	272.00
bCurrentAuBidders	2.93	2.00	3.53	0.00	15.00	272.00
bMinTimeTilEnd	841.59	429.39	971.01	0.02	4330.52	272.00
bHourTimeTilEnd	14.03	7.16	16.18	0.00	72.18	272.00
bAuctionBid	9.21	8.50	7.51	1.00	29.00	272.00

C. User variability

uBidL	1:272						
uTotalAuctionsL	1 :108	2 : 64	3 : 56	4 : 16	6 : 12	9 : 4	(Other): 12
uTotalBidsL	1 :80	2 :44	4 :40	3 :28	5 :16	6 :16	(Other):48
uTotalWinsL	0:192	1: 64	2: 8	3: 8			
uWinsSoFarL	0:272						
uNumbEbayWinsL	0 : 20	1 : 12	6 : 12	2 : 8	15 : 8	20 : 8	(Other):204

D. Total bids and bidders in auctions

aTotalBidsLevels	14 : 29	8 : 23	15 : 21	23 : 17	4 : 15	6 : 15	(Other):152
aTotalBiddersLevels	5 :49	10 :49	11 :35	8 :16	3 :15	6 :15	(Other):93

Table 3.14: Multinomial Logit Results: M32 Unlocked 3rd Star, 1stbid

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	−1.683**	−1.677**	−1.848***
	(0.717)	(0.720)	(0.698)
3:(intercept)	−2.430***	−2.432***	−2.989***
	(0.855)	(0.861)	(0.857)
4:(intercept)	−1.570*	−1.538*	−1.838**
	(0.850)	(0.839)	(0.826)
log(bHourTimeTilEnd)	-0.176	-0.161	-0.016
	(0.168)	(0.155)	(0.141)
bCurrentAuBidders	0.467**	0.497***	0.552***
	(0.183)	(0.146)	(0.146)
bAuctionBid	−0.310***	−0.307***	−0.209***
	(0.083)	(0.082)	(0.068)
bLivePrice	-0.001	-0.001	−0.003***
	(0.002)	(0.001)	(0.001)
aDuration	0.216	0.202	0.105
	(0.210)	(0.203)	(0.204)
aTotalPhotos	-0.020	-0.031	0.043
	(0.205)	(0.200)	(0.188)
aNonStock1	-0.111	-0.053	-0.492
	(0.619)	(0.574)	(0.520)
aPostage	0.554	0.521	0.702
	(0.628)	(0.614)	(0.629)
bClosingSequence	−0.102**	−0.102**	−0.100**
	(0.046)	(0.046)	(0.040)
aTotalBids	0.166**	0.167**	
	(0.066)	(0.066)	
aTotalBidders	0.040		
	(0.156)		
Observations	68	68	68
R^2	0.479	0.478	0.429
Log Likelihood	-41.019	-41.052	-44.892
LR Test	75.326*** (df = 14)	75.261*** (df = 13)	67.581*** (df = 12)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.15: Multinomial Logit Results: Model 32Gb New Unlocked auctions 3rd Star, first bid

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	−2.460***	−2.414***	−2.236***
	(0.746)	(0.698)	(0.643)
3:(intercept)	−3.332***	−3.282***	−3.487***
	(0.888)	(0.841)	(0.855)
4:(intercept)	−2.681***	−2.594***	−2.570***
	(0.975)	(0.839)	(0.829)
bHourTimeTilEnd	0.042	0.040	0.047*
	(0.029)	(0.028)	(0.026)
aDuration	0.076	0.070	-0.007
	(0.224)	(0.222)	(0.220)
aTotalPhotos	-0.020	-0.029	-0.013
	(0.209)	(0.204)	(0.197)
aNonStock1	-0.147	-0.103	-0.514
	(0.623)	(0.573)	(0.516)
aPostage	0.232	0.225	0.505
	(0.506)	(0.508)	(0.532)
bAuctionBid	−0.314***	−0.312***	−0.222***
	(0.085)	(0.084)	(0.069)
bLivePrice	-0.001	-0.001	−0.003***
	(0.002)	(0.002)	(0.001)
bCurrentAuBidders	0.563***	0.580***	0.609***
	(0.181)	(0.156)	(0.158)
bClosingSequence	−0.131**	−0.131**	−0.132***
	(0.052)	(0.053)	(0.048)
aTotalBids	0.130**	0.133**	
	(0.064)	(0.061)	
aTotalBidders	0.029		
	(0.158)		
Observations	68	68	68
R^2	0.484	0.484	0.450
Log Likelihood	-40.581	-40.598	-43.301
LR Test	76.202*** (df = 14)	76.169*** (df = 13)	70.763*** (df = 12)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.16: Multinomial Logit Results: M16 New Unlocked 3rd Star, fisrt bid, 4 choices; Extensions

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	2.088**	1.000	1.135
	(0.944)	(1.440)	(1.452)
3:(intercept)	1.468	0.506	0.410
	(1.040)	(1.749)	(1.766)
4:(intercept)	0.143	-1.927	-2.053
	(1.138)	(1.847)	(1.897)
bCurrentAuBidders	1.301***	1.427***	1.346***
	(0.362)	(0.369)	(0.386)
bLivePrice	-0.001	-0.001	-0.001
	(0.003)	(0.003)	(0.003)
bAuctionBid	−0.306**	−0.361***	−0.355***
	(0.120)	(0.135)	(0.137)
log(bHourTimeTilEnd)	0.223	0.249	0.270
	(0.382)	(0.400)	(0.394)
aDuration	0.327	0.419	0.412
	(0.239)	(0.267)	(0.268)
aNonStock	0.332	0.779	0.672
	(0.725)	(0.803)	(0.811)
aPostage	1.982**	2.594**	2.477**
	(0.845)	(1.060)	(1.053)
aStar	0.002		0.002
	(0.004)		(0.004)
bClosingSequence	−0.064*	−0.084**	−0.080*
	(0.038)	(0.041)	(0.041)
aTotalBids	0.251***	0.290***	0.277***
	(0.089)	(0.093)	(0.095)
aTotalBidders	-0.255	−0.371*	-0.321
	(0.200)	(0.206)	(0.223)
bLivePrice:log(bHourTimeTilEnd)	−0.002***	−0.003***	−0.003***
	(0.001)	(0.001)	(0.001)
bCurrentAuBidders:log(bHourTimeTilEnd)	−0.108*	−0.096*	−0.093*
	(0.057)	(0.054)	(0.055)
bCurrentAuBidders:bAuctionBid	−0.022*	-0.023	-0.021
	(0.013)	(0.014)	(0.014)
2:maxCountFromTop		0.086	0.080
		(0.084)	(0.086)
3:maxCountFromTop		0.084	0.090
		(0.119)	(0.122)
4:maxCountFromTop		0.178	0.193
		(0.122)	(0.129)
Observations	91	91	91
R^2	0.763	0.771	0.773
Log Likelihood	-29.351	-28.264	-28.119
LR Test	188.685*** (df = 17)	190.858*** (df = 19)	191.150*** (df = 20)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.17: Multinomial Logit Results: M32 New Unlocked 3rd Star,first bid, 4 choices; Results of bottom-up approach

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	−1.479*	−2.013**	−1.937**
	(0.770)	(0.948)	(0.908)
3:(intercept)	−1.850**	-1.751	−1.918*
	(0.897)	(1.066)	(1.031)
4:(intercept)	-1.387	-1.479	-1.500
	(0.960)	(1.035)	(1.001)
bCurrentAuBidders	0.541***	1.538***	1.689***
	(0.181)	(0.423)	(0.419)
bLivePrice	0.001	-0.0002	-0.001
	(0.003)	(0.003)	(0.003)
bAuctionBid	−0.184*	−0.216**	−0.272***
	(0.094)	(0.107)	(0.099)
log(bHourTimeTilEnd)	0.449	-0.321	-0.658
	(0.591)	(0.638)	(0.571)
aDuration	0.322	0.570*	0.572**
	(0.265)	(0.314)	(0.289)
aNonStock1	0.452	0.830	0.685
	(0.905)	(0.970)	(0.933)
aPostage	0.872	0.762	0.776
	(0.780)	(0.841)	(0.854)
aStar	0.003		
	(0.004)		
bClosingSequence	−0.083*	-0.069	-0.057
	(0.049)	(0.054)	(0.052)
aTotalBids	0.229**	0.369***	0.339***
	(0.093)	(0.113)	(0.103)
aTotalBidders	-0.087	-0.174	-0.117
	(0.161)	(0.182)	(0.175)
bLivePrice:log(bHourTimeTilEnd)	-0.001	0.0001	0.0005
	(0.001)	(0.001)	(0.001)
bCurrentAuBidders:log(bHourTimeTilEnd)	0.124	-0.031	−0.160**
	(0.099)	(0.121)	(0.065)
bAuctionBid:log(bHourTimeTilEnd)	−0.129**	-0.065	
	(0.055)	(0.052)	
bCurrentAuBidders:bAuctionBid		−0.047***	−0.050***
		(0.014)	(0.014)
Observations	68	68	68
R^2	0.553	0.645	0.634
Log Likelihood	-35.140	-27.964	-28.837
LR Test	87.084*** (df = 17)	101.436*** (df = 17)	99.691*** (df = 16)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.18: Multinomial Logit Results: M32 New Unlocked 3rd Star, first bid, 4 choices; Extensions

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	−1.967**	−5.965**	−5.944**
	(0.961)	(2.751)	(2.891)
3:(intercept)	−1.937*	1.005	1.017
	(1.053)	(2.432)	(2.485)
4:(intercept)	-1.538	2.777	2.793
	(1.075)	(2.754)	(2.838)
bCurrentAuBidders	1.699***	2.427***	2.425***
	(0.435)	(0.644)	(0.647)
bLivePrice	-0.001	-0.0002	-0.0002
	(0.003)	(0.004)	(0.004)
bAuctionBid	−0.272***	−0.327***	−0.327***
	(0.099)	(0.115)	(0.115)
log(bHourTimeTilEnd)	-0.665	-1.063	-1.060
	(0.575)	(0.694)	(0.706)
aDuration	0.567*	0.724*	0.727*
	(0.295)	(0.407)	(0.424)
aNonStock1	0.734	-0.403	-0.417
	(1.066)	(1.191)	(1.346)
aPostage	0.757	1.207	1.216
	(0.867)	(1.263)	(1.322)
aStar	-0.0004		0.0001
	(0.004)		(0.005)
bClosingSequence	-0.057	-0.052	-0.052
	(0.052)	(0.063)	(0.063)
aTotalBids	0.340***	0.413***	0.413***
	(0.104)	(0.122)	(0.122)
aTotalBidders	-0.120	-0.019	-0.018
	(0.179)	(0.199)	(0.206)
bLivePrice:log(bHourTimeTilEnd)	0.0005	0.001	0.001
	(0.001)	(0.001)	(0.001)
bCurrentAuBidders:log(bHourTimeTilEnd)	−0.161**	−0.202*	−0.202*
	(0.066)	(0.110)	(0.110)
bCurrentAuBidders:bAuctionBid	−0.051***	−0.070***	−0.070***
	(0.015)	(0.021)	(0.021)
2:maxCountFromTop		0.395*	0.394
		(0.237)	(0.243)
3:maxCountFromTop		-0.395	-0.395
		(0.284)	(0.286)
4:maxCountFromTop		−0.497*	−0.497*
		(0.291)	(0.291)
Observations	68	68	68
R²	0.634	0.709	0.709
Log Likelihood	-28.832	-22.858	-22.858
LR Test	99.701*** (df = 17)	111.648*** (df = 19)	111.648*** (df = 20)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.19: Multinomial Logit Results: M16 New Unlocked 3rd Star, 4 choices; Interaction terms III

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	1.936**	1.787**	1.826**
	(0.907)	(0.903)	(0.901)
3:(intercept)	1.415	1.273	1.405
	(1.025)	(1.054)	(1.049)
4:(intercept)	0.066	-0.039	0.057
	(1.117)	(1.132)	(1.153)
bCurrentAuBidders	1.374***	1.572***	1.408***
	(0.344)	(0.421)	(0.352)
bLivePrice	0.001	0.002	0.003
	(0.004)	(0.004)	(0.004)
bAuctionBid	−0.305**	−0.340**	−0.367***
	(0.123)	(0.135)	(0.114)
log(bHourTimeTilEnd)	1.154	1.270	1.438
	(0.831)	(0.889)	(0.916)
aDuration	0.583*	0.636*	0.676*
	(0.335)	(0.348)	(0.356)
aNonStock	0.460	0.598	0.563
	(0.748)	(0.780)	(0.778)
aPostage	1.936**	1.835**	1.949**
	(0.839)	(0.850)	(0.855)
bClosingSequence	-0.058	−0.062*	−0.064*
	(0.036)	(0.037)	(0.037)
aTotalBids	0.237***	0.233**	0.245***
	(0.088)	(0.092)	(0.093)
aTotalBidders	-0.283	-0.262	-0.305
	(0.191)	(0.200)	(0.204)
log(bHourTimeTilEnd):aDuration	-0.141	-0.150	-0.169
	(0.117)	(0.122)	(0.127)
bLivePrice:log(bHourTimeTilEnd)	−0.003***	−0.003***	−0.003***
	(0.001)	(0.001)	(0.001)
bCurrentAuBidders:log(bHourTimeTilEnd)	−0.102*	−0.103*	−0.103*
	(0.056)	(0.054)	(0.053)
bCurrentAuBidders:bAuctionBid	-0.022	-0.017	
	(0.013)	(0.015)	
bCurrentAuBidders:bLivePrice		-0.001	
		(0.001)	
bCurrentAuBidders:bLivePrice:bAuctionBid			−0.00003*
			(0.00002)
Observations	91	91	91
R²	0.771	0.775	0.779
Log Likelihood	-28.296	-27.780	-27.311
LR Test	190.795*** (df = 17)	191.827*** (df = 18)	192.766*** (df = 17)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.20: Multinomial Logit Results: M32 New Unlocked 3rd Star, 4 choices; Interaction terms III

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	−1.927**	−2.273**	−2.376**
	(0.911)	(1.012)	(0.944)
3:(intercept)	−1.874*	−2.237**	−2.499**
	(1.040)	(1.132)	(1.088)
4:(intercept)	-1.483	-1.545	−1.816*
	(1.008)	(1.039)	(1.022)
bCurrentAuBidders	1.686***	2.278***	1.563***
	(0.421)	(0.573)	(0.382)
bLivePrice	-0.001	0.0004	0.0005
	(0.003)	(0.003)	(0.003)
bAuctionBid	−0.277***	−0.350***	−0.349***
	(0.102)	(0.114)	(0.100)
log(bHourTimeTilEnd)	-0.619	-0.874	-0.547
	(0.592)	(0.633)	(0.611)
aDuration	0.647	0.895**	0.949**
	(0.400)	(0.431)	(0.401)
aNonStock1	0.742	0.669	0.644
	(0.961)	(0.917)	(0.824)
aPostage	0.810	0.750	0.708
	(0.889)	(0.838)	(0.825)
bClosingSequence	-0.052	-0.041	-0.039
	(0.055)	(0.056)	(0.053)
aTotalBids	0.342***	0.387***	0.320***
	(0.104)	(0.107)	(0.091)
aTotalBidders	-0.126	-0.168	-0.112
	(0.177)	(0.180)	(0.170)
log(bHourTimeTilEnd):aDuration	-0.031	-0.054	-0.076
	(0.112)	(0.110)	(0.107)
bLivePrice:log(bHourTimeTilEnd)	0.0004	0.001	0.001
	(0.001)	(0.001)	(0.001)
bCurrentAuBidders:log(bHourTimeTilEnd)	−0.158**	−0.196***	−0.162***
	(0.067)	(0.074)	(0.063)
bCurrentAuBidders:bAuctionBid	−0.050***	−0.048***	
	(0.014)	(0.014)	
bCurrentAuBidders:bLivePrice		−0.001*	
		(0.0005)	
bCurrentAuBidders:bLivePrice:bAuctionBid			−0.0001***
			(0.00002)
Observations	68	68	68
R^2	0.634	0.660	0.631
Log Likelihood	-28.798	-26.755	-29.011
LR Test	99.769*** (df = 17)	103.854*** (df = 18)	99.342*** (df = 17)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.21: Multinomial Logit Results: M16 New Unlocked 3rd Star, 4 choices; Interaction terms IV

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	1.712*	1.733*	2.004**
	(0.958)	(0.984)	(0.990)
3:(intercept)	0.704	0.789	1.376
	(1.218)	(1.262)	(1.168)
4:(intercept)	-0.658	-0.594	-0.283
	(1.318)	(1.342)	(1.361)
bCurrentAuBidders	3.167***	2.941***	2.289***
	(0.961)	(0.953)	(0.655)
bLivePrice	0.009	0.008	0.005
	(0.006)	(0.007)	(0.005)
bAuctionBid	−1.061***	−1.055***	−0.813***
	(0.393)	(0.376)	(0.270)
log(bHourTimeTilEnd)	2.937**	2.875**	2.414**
	(1.396)	(1.408)	(1.203)
aDuration	1.336**	1.268**	1.048**
	(0.587)	(0.592)	(0.487)
aNonStock	-0.099	-0.027	-0.021
	(0.927)	(0.931)	(0.896)
aPostage	1.651*	1.815**	2.103**
	(0.862)	(0.899)	(0.915)
bClosingSequence	−0.087*	−0.081*	−0.070*
	(0.050)	(0.048)	(0.041)
aTotalBids	0.298**	0.296**	0.300***
	(0.125)	(0.120)	(0.109)
aTotalBidders	−0.388*	−0.415*	−0.452**
	(0.225)	(0.229)	(0.225)
bLivePrice:bAuctionBid	0.0005	0.001	0.0004
	(0.0005)	(0.001)	(0.0004)
bAuctionBid:log(bHourTimeTilEnd)	0.190**	0.176**	0.131*
	(0.085)	(0.083)	(0.067)
bCurrentAuBidders:bLivePrice	−0.002*	−0.001	
	(0.001)	(0.001)	
log(bHourTimeTilEnd):aDuration	−0.433**	−0.417*	−0.341*
	(0.216)	(0.220)	(0.184)
bLivePrice:log(bHourTimeTilEnd)	−0.005***	−0.005***	−0.005***
	(0.002)	(0.002)	(0.002)
bCurrentAuBidders:log(bHourTimeTilEnd)	−0.522**	−0.497**	−0.411**
	(0.206)	(0.202)	(0.180)
bCurrentAuBidders:bAuctionBid	-0.013		
	(0.019)		
bCurrentAuBidders:bLivePrice:bAuctionBid		-0.00004	−0.0001*
		(0.00004)	(0.00004)
Observations	91	91	91
R²	0.801	0.804	0.799
Log Likelihood	-24.599	-24.219	-24.890
LR Test	198.189*** (df = 20)	198.949*** (df = 20)	197.607*** (df = 19)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.22: Multinomial Logit Results: M32 New Unlocked 3rd Star, 4 choices; Interaction terms IV

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	−2.329**	−2.373**	−2.373**
	(1.051)	(1.010)	(1.009)
3:(intercept)	−2.153*	−2.250*	−2.245*
	(1.184)	(1.180)	(1.171)
4:(intercept)	-1.483	-1.702	-1.702
	(1.053)	(1.048)	(1.047)
bCurrentAuBidders	2.160***	1.546**	1.533***
	(0.766)	(0.645)	(0.469)
bLivePrice	0.001	-0.0003	-0.0003
	(0.005)	(0.005)	(0.004)
bAuctionBid	-0.285	-0.329	-0.323
	(0.301)	(0.288)	(0.222)
log(bHourTimeTilEnd)	-0.599	-0.397	-0.385
	(0.884)	(0.841)	(0.745)
aDuration	0.799	0.825*	0.820*
	(0.488)	(0.474)	(0.453)
aNonStock1	0.824	0.833	0.835
	(0.950)	(0.886)	(0.885)
aPostage	0.729	0.628	0.631
	(0.818)	(0.805)	(0.799)
bClosingSequence	-0.061	-0.061	-0.061
	(0.061)	(0.058)	(0.058)
aTotalBids	0.419***	0.364***	0.363***
	(0.120)	(0.107)	(0.104)
aTotalBidders	-0.238	-0.220	-0.219
	(0.194)	(0.190)	(0.188)
bLivePrice:bAuctionBid	-0.00000	0.0001	0.0001
	(0.0004)	(0.0003)	(0.0003)
bAuctionBid:log(bHourTimeTilEnd)	-0.074	-0.081	-0.081
	(0.064)	(0.064)	(0.064)
bCurrentAuBidders:bLivePrice	-0.001	-0.00002	
	(0.001)	(0.001)	
log(bHourTimeTilEnd):aDuration	-0.003	-0.013	-0.012
	(0.117)	(0.115)	(0.113)
bLivePrice:log(bHourTimeTilEnd)	0.001	0.0004	0.0004
	(0.001)	(0.001)	(0.001)
bCurrentAuBidders:log(bHourTimeTilEnd)	-0.063	-0.013	-0.012
	(0.139)	(0.135)	(0.132)
bCurrentAuBidders:bAuctionBid	−0.044***		
	(0.015)		
bCurrentAuBidders:bLivePrice:bAuctionBid		−0.0001***	−0.0001***
		(0.00003)	(0.00002)
Observations	68	68	68
R²	0.672	0.652	0.652
Log Likelihood	-25.803	-27.354	-27.354
LR Test	105.759*** (df = 20)	102.657*** (df = 20)	102.656*** (df = 19)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.23: Multinomial Logit Results: M16 New Unlocked 3rd Star, 4 choices; Interaction terms top-down approach I

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	1.769*	1.844*	1.841*
	(1.022)	(1.008)	(1.007)
3:(intercept)	0.902	1.073	1.057
	(1.313)	(1.188)	(1.183)
4:(intercept)	-0.421	-0.303	-0.296
	(1.357)	(1.304)	(1.300)
bCurrentAuBidders	2.594**	2.351***	2.345***
	(1.071)	(0.661)	(0.660)
bLivePrice	0.008	0.008	0.007
	(0.007)	(0.007)	(0.006)
bAuctionBid	−1.135***	−1.104***	−1.093***
	(0.404)	(0.380)	(0.366)
log(bHourTimeTilEnd)	3.198**	3.183**	3.124**
	(1.573)	(1.560)	(1.480)
aDuration	1.375**	1.355**	1.325**
	(0.658)	(0.650)	(0.592)
aNonStock	-0.102	-0.118	
	(0.957)	(0.958)	
aPostage	2.058**	2.193**	2.198**
	(0.981)	(0.887)	(0.889)
bClosingSequence	−0.088*	−0.087*	−0.086*
	(0.053)	(0.053)	(0.051)
aTotalBids	0.309**	0.313***	0.312***
	(0.124)	(0.121)	(0.120)
aTotalBidders	−0.467*	−0.488**	−0.488**
	(0.248)	(0.238)	(0.240)
bCurrentAuBidders:bLivePrice	-0.001		
	(0.002)		
bCurrentAuBidders:bAuctionBid	0.033	0.042	0.042
	(0.046)	(0.034)	(0.033)
bLivePrice:bAuctionBid	0.001	0.001	0.001
	(0.001)	(0.001)	(0.001)
bAuctionBid:log(bHourTimeTilEnd)	0.172**	0.162**	0.157**
	(0.085)	(0.075)	(0.067)
bCurrentAuBidders:log(bHourTimeTilEnd)	−0.482**	−0.458**	−0.450***
	(0.205)	(0.186)	(0.174)
log(bHourTimeTilEnd):aDuration	−0.458*	−0.452*	−0.442*
	(0.244)	(0.242)	(0.228)
bLivePrice:log(bHourTimeTilEnd)	−0.005***	−0.005***	−0.005***
	(0.002)	(0.002)	(0.002)
bCurrentAuBidders:bLivePrice:bAuctionBid	-0.0001	−0.0001*	−0.0001*
	(0.0001)	(0.0001)	(0.0001)
Observations	91	91	91
R²	0.806	0.806	0.806
Log Likelihood	-23.954	-23.998	-24.006
LR Test	199.478*** (df = 21)	199.390*** (df = 20)	199.375*** (df = 19)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.24: Multinomial Logit Results: M16 New Unlocked 3rd Star, 4 choices; Interaction terms top-down approach II

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	1.963*	1.896*	1.799*
	(1.136)	(1.056)	(1.043)
3:(intercept)	0.630	0.692	1.076
	(1.433)	(1.378)	(1.215)
4:(intercept)	-0.871	-0.876	-0.379
	(1.502)	(1.481)	(1.301)
bCurrentAuBidders	5.126	4.436**	2.400***
	(3.560)	(2.067)	(0.686)
bLivePrice	0.0001		
	(0.008)		
bAuctionBid	−3.045**	−3.035**	−2.152***
	(1.304)	(1.227)	(0.825)
log(bHourTimeTilEnd)	2.233	2.176**	2.093**
	(1.460)	(0.988)	(0.934)
aDuration	0.981	0.940**	0.923**
	(0.751)	(0.475)	(0.412)
aNonStock	0.240		
	(1.126)		
aPostage	2.103*	2.071*	1.692*
	(1.084)	(1.074)	(0.867)
bClosingSequence	-0.078	-0.079	−0.077*
	(0.050)	(0.050)	(0.047)
aTotalBids	0.304*	0.310**	0.233**
	(0.162)	(0.149)	(0.108)
aTotalBidders	-0.506	−0.513*	−0.433*
	(0.316)	(0.293)	(0.250)
bCurrentAuBidders:bLivePrice	-0.004	-0.003	
	(0.006)	(0.003)	
bAuctionBid:log(bHourTimeTilEnd):bLivePrice		-0.001	−0.0004*
		(0.001)	(0.0003)
bCurrentAuBidders:log(bHourTimeTilEnd):bLivePrice		0.001	
		(0.001)	
bCurrentAuBidders:bAuctionBid:bLivePrice		−0.0003*	−0.0003**
		(0.0001)	(0.0001)
bCurrentAuBidders:bAuctionBid:log(bHourTimeTilEnd):bLivePrice		-0.00001	
		(0.00002)	
bCurrentAuBidders:bAuctionBid	0.097	0.125*	0.121*
	(0.153)	(0.075)	(0.064)
bLivePrice:bAuctionBid	0.004*		
	(0.002)		
bCurrentAuBidders:log(bHourTimeTilEnd)	-1.281	−1.096*	−0.436***
	(1.019)	(0.601)	(0.168)
bLivePrice:log(bHourTimeTilEnd)	−0.004**		
	(0.002)		
bAuctionBid:bLivePrice		0.004**	0.002**
		(0.002)	(0.001)
bAuctionBid:log(bHourTimeTilEnd)	0.642**	0.647**	0.407**
	(0.306)	(0.295)	(0.177)
log(bHourTimeTilEnd):bLivePrice		−0.004***	−0.004***
		(0.001)	(0.001)
log(bHourTimeTilEnd):aDuration	-0.360	−0.359**	−0.311**
	(0.220)	(0.156)	(0.135)
bCurrentAuBidders:bLivePrice:bAuctionBid	-0.0002		
	(0.0003)		
bCurrentAuBidders:bLivePrice:log(bHourTimeTilEnd)	0.001		
	(0.002)		
bCurrentAuBidders:bAuctionBid:log(bHourTimeTilEnd)	0.009		
	(0.045)		
bLivePrice:bAuctionBid:log(bHourTimeTilEnd)	-0.001		
	(0.001)		
bCurrentAuBidders:bLivePrice:bAuctionBid:log(bHourTimeTilEnd)	-0.00002		
	(0.0001)		
Observations	91	91	91
R²	0.821	0.821	0.815
Log Likelihood	-22.106	-22.150	-22.935
LR Test	203.175*** (df = 25)	203.087*** (df = 22)	201.517*** (df = 19)

Note:	*p<0.1; **p<0.05; ***p<0.01

Table 3.25: Multinomial Logit Results: M16 New Unlocked 3rd Star, 4 choices; Interaction terms. Current Bidders and Duration

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	1.543	2.800**	2.936**
	(1.066)	(1.353)	(1.366)
3:(intercept)	0.486	1.174	1.118
	(1.331)	(1.359)	(1.375)
4:(intercept)	-0.085	-0.286	-0.302
	(1.385)	(1.605)	(1.652)
bCurrentAuBidders	2.538***	3.165***	3.252***
	(0.800)	(1.082)	(1.070)
bAuctionBid	-1.980^{**}	-2.499^{**}	-2.188^{***}
	(0.903)	(1.120)	(0.847)
log(bHourTimeTilEnd)	1.667	1.162	1.138
	(1.055)	(0.834)	(0.841)
aDuration	-0.076		
	(0.848)		
aPostage	2.153**	2.947**	3.221***
	(1.027)	(1.316)	(1.242)
bClosingSequence	-0.059	-0.077^{*}	-0.078^{*}
	(0.044)	(0.046)	(0.047)
aTotalBids	0.429**	0.462**	0.481**
	(0.215)	(0.205)	(0.202)
aTotalBidders	-0.786^{*}	-0.582^{*}	-0.628^{**}
	(0.419)	(0.311)	(0.290)
log(bHourTimeTilEnd):aDuration	-0.296^{*}		
	(0.167)		
bAuctionBid:log(bHourTimeTilEnd)	0.366^{*}	0.426**	0.362**
	(0.196)	(0.208)	(0.152)
log(bHourTimeTilEnd):bLivePrice	-0.003^{**}	-0.004^{**}	-0.004^{**}
	(0.001)	(0.002)	(0.002)
bCurrentAuBidders:log(bHourTimeTilEnd)	-0.528^{***}	-0.602^{**}	-0.643^{**}
	(0.204)	(0.268)	(0.260)
bAuctionBid:bLivePrice	0.002	0.002	0.001
	(0.001)	(0.001)	(0.001)
bCurrentAuBidders:bAuctionBid	0.086	0.102	0.073
	(0.075)	(0.084)	(0.056)
bCurrentAuBidders:aDuration	0.218	0.346**	0.376**
	(0.162)	(0.171)	(0.162)
bAuctionBid:log(bHourTimeTilEnd):bLivePrice	-0.0003	-0.0001	
	(0.0003)	(0.0003)	
bCurrentAuBidders:bAuctionBid:bLivePrice	-0.0002	-0.0002	-0.0002^{*}
	(0.0001)	(0.0001)	(0.0001)
bCurrentAuBidders:log(bHourTimeTilEnd):aDuration		-0.134^{**}	-0.141^{**}
		(0.063)	(0.061)
Observations	91	91	91
R^2	0.824	0.836	0.835
Log Likelihood	-21.771	-20.229	-20.354
LR Test	203.845*** (df = 20)	206.929*** (df = 19)	206.679*** (df = 18)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3.26: Multinomial Logit Results: M16 New Unlocked 3rd Star, 4 choices; Interaction terms. Duration Interaction Terms.

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	3.437**	3.493**	3.251**
	(1.657)	(1.629)	(1.628)
3:(intercept)	2.355	2.432	1.851
	(1.787)	(1.713)	(1.726)
4:(intercept)	0.959	1.153	0.189
	(2.075)	(1.705)	(1.843)
bCurrentAuBidders	4.713***	4.766***	4.714***
	(1.513)	(1.496)	(1.558)
bAuctionBid	−2.645***	−2.681***	−2.727***
	(0.930)	(0.804)	(0.847)
log(bHourTimeTilEnd)	0.134		
	(0.840)		
aPostage	3.100*	3.207**	3.028*
	(1.587)	(1.464)	(1.652)
bClosingSequence	−0.113*	−0.114*	−0.113*
	(0.063)	(0.063)	(0.065)
aTotalBids	0.729**	0.768***	0.813**
	(0.365)	(0.290)	(0.331)
aTotalBidders	−1.051*	−1.108**	−1.113*
	(0.635)	(0.552)	(0.617)
bAuctionBid:log(bHourTimeTilEnd)	0.576***	0.579***	0.511***
	(0.183)	(0.179)	(0.175)
log(bHourTimeTilEnd):bLivePrice	0.015**	0.015**	0.018**
	(0.007)	(0.006)	(0.008)
bCurrentAuBidders:log(bHourTimeTilEnd)	−2.085***	−2.098***	−2.091***
	(0.659)	(0.660)	(0.679)
bAuctionBid:bLivePrice	0.002**	0.002**	0.002**
	(0.001)	(0.001)	(0.001)
bCurrentAuBidders:bAuctionBid	0.002		
	(0.059)		
bCurrentAuBidders:aDuration	0.940**	0.967***	1.087***
	(0.378)	(0.334)	(0.392)
bCurrentAuBidders:bAuctionBid:bLivePrice	−0.0002*	−0.0002**	−0.0002**
	(0.0001)	(0.0001)	(0.0001)
bCurrentAuBidders:log(bHourTimeTilEnd):aDuration	0.741**	0.743**	0.794**
	(0.328)	(0.324)	(0.355)
log(bHourTimeTilEnd):bLivePrice:aDuration	−0.017**	−0.017***	−0.020**
	(0.007)	(0.007)	(0.008)
bAuctionBid:log(bHourTimeTilEnd):aDuration			0.036
			(0.030)
Observations	91	91	91
R^2	0.886	0.886	0.890
Log Likelihood	-14.081	-14.094	-13.603
LR Test	219.225*** (df = 19)	219.199*** (df = 17)	220.181*** (df = 18)

Note:	*p<0.1; **p<0.05; ***p<0.01

Table 3.27: Multinomial Logit Results: M32 New Unlocked 3rd Star, 4 choices; Interaction terms. Top-down approach.

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	−3.413**	−3.580**	−3.618**
	(1.660)	(1.596)	(1.576)
3:(intercept)	-2.193	-2.304	-2.330
	(1.692)	(1.471)	(1.465)
4:(intercept)	-1.493	-2.256	-2.328
	(1.739)	(1.476)	(1.454)
bCurrentAuBidders	1.602	2.762	2.152***
	(3.386)	(2.130)	(0.778)
bLivePrice	0.032*	0.035**	0.034**
	(0.019)	(0.016)	(0.016)
bAuctionBid	0.458	0.581	0.617
	(0.690)	(0.663)	(0.655)
log(bHourTimeTilEnd)	2.388	3.361	3.510
	(3.259)	(2.949)	(2.902)
aDuration	0.652	0.593	0.511
	(0.753)	(0.713)	(0.670)
aNonStock1	-0.079		
	(1.242)		
aPostage	3.057	3.372*	3.337
	(1.912)	(2.017)	(2.048)
bClosingSequence	-0.090	-0.077	-0.088
	(0.095)	(0.085)	(0.078)
aTotalBids	1.056***	0.956***	0.928***
	(0.369)	(0.296)	(0.276)
aTotalBidders	-0.221		
	(0.332)		
bCurrentAuBidders:bLivePrice	0.001	-0.001	
	(0.005)	(0.003)	
bAuctionBid:log(bHourTimeTilEnd):bLivePrice		0.001**	0.001**
		(0.0004)	(0.0004)
bCurrentAuBidders:log(bHourTimeTilEnd):bLivePrice		−0.003**	−0.003***
		(0.001)	(0.001)
bCurrentAuBidders:bAuctionBid:bLivePrice		0.0002	0.0001
		(0.0001)	(0.0001)
bCurrentAuBidders:bAuctionBid:log(bHourTimeTilEnd):bLivePrice		0.0001**	0.0001**
		(0.00002)	(0.00002)
bCurrentAuBidders:bAuctionBid	-0.123	−0.184**	−0.164**
	(0.140)	(0.093)	(0.069)
bLivePrice:bAuctionBid	-0.002		
	(0.001)		
bCurrentAuBidders:log(bHourTimeTilEnd)	1.815	1.115*	1.256**
	(1.266)	(0.662)	(0.503)
bLivePrice:log(bHourTimeTilEnd)	-0.003		
	(0.004)		
bAuctionBid:bLivePrice		−0.002*	−0.002*
		(0.001)	(0.001)
bAuctionBid:log(bHourTimeTilEnd)	−0.762**	−0.793**	−0.816***
	(0.347)	(0.320)	(0.313)
log(bHourTimeTilEnd):bLivePrice		-0.005	-0.005
		(0.004)	(0.004)
log(bHourTimeTilEnd):aDuration	0.176	0.188	0.208
	(0.211)	(0.190)	(0.182)
bCurrentAuBidders:bLivePrice:bAuctionBid	0.0001		
	(0.0002)		
bCurrentAuBidders:bLivePrice:log(bHourTimeTilEnd)	−0.004*		
	(0.002)		
bCurrentAuBidders:bAuctionBid:log(bHourTimeTilEnd)	-0.035		
	(0.047)		
bLivePrice:bAuctionBid:log(bHourTimeTilEnd)	0.001*		
	(0.0004)		
bCurrentAuBidders:bLivePrice:bAuctionBid:log(bHourTimeTilEnd)	0.0001		
	(0.0001)		
Observations	68	68	68
R²	0.777	0.770	0.770
Log Likelihood	-17.555	-18.077	-18.127
LR Test	122.255*** (df = 25)	121.210*** (df = 22)	121.111*** (df = 21)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.28: Multinomial Logit Results: M32 New Unlocked 3rd Star, 4 choices; Interaction terms. Duration Interaction Terms.

	Dependent variable:		
	choice		
	(1)	(2)	(3)
2:(intercept)	−8.004**	−10.322**	−10.302**
	(3.710)	(4.916)	(4.858)
3:(intercept)	-1.084	-0.710	-0.819
	(1.587)	(1.844)	(1.828)
4:(intercept)	-2.160	-2.965	-3.228
	(2.165)	(2.205)	(2.203)
bCurrentAuBidders	6.173**	7.605**	7.514**
	(2.928)	(3.494)	(3.412)
bAuctionBid	1.674	1.371	1.153
	(1.548)	(1.189)	(1.074)
log(bHourTimeTilEnd)	12.668	14.305*	12.846*
	(8.893)	(8.253)	(7.328)
bLivePrice	0.094*	0.109**	0.103**
	(0.051)	(0.052)	(0.048)
aDuration	1.920	2.672	2.308
	(1.951)	(1.910)	(1.744)
aPostage	15.851*	18.431*	17.446**
	(8.955)	(9.441)	(8.875)
bClosingSequence	-0.110	-0.184	-0.190
	(0.126)	(0.155)	(0.148)
aTotalBids	2.220**	2.930**	2.916**
	(0.919)	(1.341)	(1.310)
log(bHourTimeTilEnd):aDuration	-0.785	-0.279	
	(0.769)	(0.646)	
bAuctionBid:log(bHourTimeTilEnd)	−2.154**	−2.598**	−2.522**
	(1.026)	(1.323)	(1.267)
log(bHourTimeTilEnd):bLivePrice	-0.018	−0.022*	−0.021*
	(0.012)	(0.012)	(0.011)
bCurrentAuBidders:log(bHourTimeTilEnd)	1.874**	2.469*	2.482*
	(0.859)	(1.344)	(1.310)
bAuctionBid:bLivePrice	−0.005*	−0.005**	−0.004**
	(0.003)	(0.002)	(0.002)
bCurrentAuBidders:bAuctionBid	−0.267**	−0.297**	−0.290**
	(0.119)	(0.132)	(0.127)
bAuctionBid:log(bHourTimeTilEnd):bLivePrice	0.003**	0.004*	0.004*
	(0.001)	(0.002)	(0.002)
bCurrentAuBidders:log(bHourTimeTilEnd):bLivePrice	−0.007**	−0.009**	−0.009**
	(0.003)	(0.004)	(0.004)
bCurrentAuBidders:bAuctionBid:bLivePrice	0.00003		
	(0.0001)		
bCurrentAuBidders:log(bHourTimeTilEnd):aDuration	0.315**	0.529**	0.514**
	(0.152)	(0.251)	(0.244)
bAuctionBid:log(bHourTimeTilEnd):aDuration		-0.120	-0.125
		(0.086)	(0.086)
bCurrentAuBidders:bAuctionBid:log(bHourTimeTilEnd):bLivePrice	0.0001*	0.0001**	0.0001**
	(0.00004)	(0.00005)	(0.00004)
Observations	68	68	68
R²	0.862	0.880	0.878
Log Likelihood	-10.872	-9.467	-9.563
LR Test	135.620*** (df = 22)	138.430*** (df = 22)	138.238*** (df = 21)
Note:			*p<0.1; **p<0.05; ***p<0.01

3.11 Herding in Concurrent Auctions - Discussion

The analysis of choice between concurrent auctions shows that the auctions with more bidders present are more likely to be chosen by new bidders. This means that either the bidders facing the choice think that there is additional information revealed about the product based on number of other bidders in the auction, or that there are other, non-rational reasons that lead to this result. Herding is a topic known in economic and psychological literature for a long time. It has been observed in many situations, from finance to buyer's decision. In auction setting herding is additionally discouraged, since more bidders in an auction can lead to a higher auction outcome. The revealed signal about the product needs to, therefore out weight the negative effects associated with more crowded auctions. This topic it not yet closed in view of current theoretical literature. The section below aims at an overview of the literature related to herding behavior more generally, as well as in auctions in particular.

3.11.1 Literature Overview

There is a large literature relating to the phenomena of herding, or following the crowd, in economic situations. The area where it is most often observed is financial markets, where observed spikes in interest in buys of a given asset drive the prices up, sometimes leading to pricing bubbles. There have been numerous attempts in explaining these phenomenon in the economic literature, which, while assuming rationality, has been trying to explain such behavior through asymmetric information and uncertainty combined with the assumption of common values, which is undoubtedly appropriate in the context of financial goods such as shares or bonds. One of the first, and most general models of herding behavior, which doesn't invoke the assumption of strong complementarity (choosing one asset is better when more people choose it) (Banerjee, 1992[5]), relates to a sequential decision in the context of common value and value uncertainty, where agents can observe actions taken by their predecessors. The fact that the decisions made by others are influencing followers

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information set leads in this model to herding behavior after several identical decisions observed in a row. Starting at some moment in the sequence, all subsequent individuals will make the same decision, regardless of their private signal. The notion of informational cascade is closely related, as defined by Bikhchandani, Hirshleifer and Welch, 1992[11] "An informational cascade occurs if an individual's action does not depend on his private information signal." Both of these models are closely related, and, as well as most others in subsequent literature, assume two types of individuals, informed and uninformed, while the individuals themselves do not know which type they are. In [11] the authors give also several different areas, where herding can be observed, from politics, to finance. They also invoke the notion of *conformity,* which is very often seen as an easy option, when faced with *uncertainty.* In the context of doctors, who are not well informed with the cutting edge research, they suggest that the chosen strategy is to imitate other doctor's treatment practices, whenever in doubt. Application of their model, which assumes that individuals do not know their type, is not perfectly well fit to this particular example, because, while doctors may differ in their knowledge about newest research, those who are knowledgeable, will know their type and therefore should not choose to follow the crowd in the wrong treatment. This example, therefore can actually be used to show that the theory relying on the agents not knowing their type, cannot be used in every context. Better informed doctors will be immune to informational cascades, and therefore the decision to herd is more closely related to the level of uncertainty, or level of knowledge. Those with lower level of knowledge will be more likely to choose conformity as opposed to their own signal.

Another approach, for explanation of herding behavior in certain situations, is provided in Scharfstein and Stein, 2001 [33]. There, in the context of manager's investment decision, the concern about reputation leads to mimicking behavior. Here also, the managers can be of two types, smart or dumb, and the assumption that they do not know their own type is necessary for such results. This model, while can explain certain behaviors, is only applicable in situations where the reputational concerns are likely to play a

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role in the decision-making. Earlier Becker, 1991[8] have reported a situation were out of two virtually identical restaurants, one has an excess demand and a queue of customers, while other is empty. Despite that, the more popular restaurant does not increase prices in order to take advantage of the possible rents they can earn, while reducing demand. Becker attributes that to the fact that the demand function for some goods is higher when more people are also consuming it, and this can be applied to restaurants or concerts, where there is higher utility derived from the good in the presence of others. Other, more numerous in papers, strand of literature relates to herding in financial markets. In addition to neoclassical utility theory, there is a separate literature, which attributes herding in financial markets to behavioral factors.

In financial markets, when faced with a choice of different assets, following the herd leads to an increase the price of the chosen asset. Avery and Zemsky, 1998 [4], as first have shown that a simple model based on Bikhchandani, Hirshleifer and Welch, 1992 [11], with an addition of asset prices, leads to elimination of herding. They introduce other levels of uncertainty (in addition to value uncertainty), which are event uncertainty, and uncertainty of the composition of the market, which in some low probability situations can generate herding (or contrarian behavior). Park and Sabourian, 2011 [29] , on the other hand, prove that U-shaped (hill shaped) private signals are an enough condition to generate herding (contrarianism) in financial markets, without the introduction of different levels of uncertainty. Existing empirical and experimental evidence on this topic, gives mixed conclusions. In an empirical study Bernhardt, Campello and Kutsoati, 2006 [9] show that, on the contrary to most of the empirical literature, there is evidence that contrarian behaviour, in the direction of private information, is more prevalent in financial markets. On the other hand, Derhmann, Oechssler and Roider, 2005 [14] conduct an internet experiment, in which they test the level of rationality of the decision makers, as well as the predictions of Avery and Zemsky, 1998 [4] model. While in Avery and Zemsky, 1998 theoretical set-up, rational agents should follow their signal exactly, and disregard the history, Derhman, Oechssler and Roider, 2005, have observed that in the experiment

not all the participants have followed their signal, which they have attributed to bounded rationality. While they have not found evidence of herding, they concluded that there was some contrarian behavior present. They found, as well, that different groups of respondents had different levels of rationality, where rational behavior was defined as following the best strategy according to theoretical predictions. On two extremes, those with Physics degree had higher level of rationality, while those with Psychology degree had significantly lower level of rationality. Surprisingly, psychologists had performed better in the game, implying that prediction of possibly irrational behavior of others have given better results than a high ability of correctly applying Bayes' rule in forming expectations.

The other side of literature does not try to explain herding behavior through neoclassical economics lens. Kirman, 1993 [21] argues that a Markov chain model may be better suited at explaining dynamic behavior of multiple individuals and group as a whole, in situations where herding or mimicking occurs. The model, based on behavior of ants can be applied to financial markets, or Becker's restaurant example (Becker, 1991). The model of ants in Kirman, 1993 is argued to have an advantage over other contemporaneous models of financial bubbles, because it does not assume the existence of a steady state as a solution to an optimization, but rather endogenously incorporates switching between different steady states, without the need of an introduction of an external shock. While it is a simple and appealing model due to it's flexibility to be readily transferred to any situation, it is not clearly explained in the paper why such behavior would occur in the first place. Kirman's view is that such observed behavior may be due to maximization, but may also be a result of not fully rational behavior. In the meantime, developments in scientific literature have given grounds to to the opinion that decisions under uncertainty are not made in the same conscious way, as those between certain outcomes. Tversky and Kahneman, 1974 [35] expose the existence of biases in judgments, which include violation of Bayes' rule, ignoring prior probability when faced with additional useless information, and ignorance of fundamental rules of statistics revealed in exper-

iments. Bechara, Damasio, Tranel and Damasio, 1997 [7], on the other hand, conduct an experiment, which shows that different parts of the brain, responsible for conscious and non-conscious responses play a role in the decision-making. The unconscious bias directing the behavior towards the advantageous strategy is a faster response than the one following explicit reasoning. Prechter Jr. and Parker , 2007 [32]argue that financial markets are fundamentally different from markets for utilitarian consumption goods. Firstly, because all the objects available in financial markets are perfect substitutes, and only differ by the unknown future valuation. Secondly, the realized valuation is characterized by high degree of uncertainty and therefore the unconscious impulses play a role in the decision-making. Prechter Jr. and Parker , 2007 [32], as well as Prechter Jr, 2001 [31] recognize herding as the main force driving transactions in financial markets, and attribute it to the instinctual human behavior, which was developed through evolution, as an advantageous strategy for many life-threatening situations that could be faced in real life by homo sapiens. As Prechter Jr argues, this unconscious impulse is not advantageous in the context of financial markets, it is though one of the cognitive biases stemming from reliance on judgmental heuristics which cannot be explained through rational choice theory. Simultaneous auctions for identical goods, although are different form financial in that they relate to consumption goods, with single-unit demands, are in numerous ways also similar to financial markets. One of the aspects is the almost absolute homogeneity of the products available, as well as high degree of uncertainty about the resulting price and any future auctions that may be available later (the value of the outside option).

One model, which directly relates to herding in auctions, and more generally in markets with sequential bids characterized by the winner's curse is the one by Neeman and Orosel, 1999 [28]. Their model of herding does not relate to the choice of a particular object, but rather to herding in the context of a single auction. It is also a specific model, where individuals are addressed sequentially by the auctioneer to place their bids. In the presence of common values, the fact that previous bids (or choices of not bidding up) are observed by the individuals approached later in the sequence leads to herding of no

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bidding up the price despite a positive private signal, due to the existence of winner's curse. Bulow and Klemperer, 1994 [12]show that in the case of sequential sales of more than one unit, rational frenzies and crashes, which can look similarly to herding, can occur because the willingness to pay (dependent on the number of remaining bidders and items) is different from the valuation. In their model of ascending auction, they show that willingness to pay is a very flat function and sensitive to a sell of even single unit. As a result, after even a single sale, the willingness to pay curve moves upwards, and a large range of buyer's valuations suddenly have WTP above the current price. Many more sales will follow at the same price, which will look like a sudden buyer frenzy.

3.11.2 Conclusion

In conclusion, there are two main reasons that herding can occur in auction choice. One is information Asymmetry or Common Values element. If an object's value is a function of number of other bidders choosing the object, then clearly this could lead to herding behavior. Another way to view this is that the bidders have uncertainty about some features of the objects and belief that valuations of others are correlated, and therefore the fact that one object is more often chosen creates a possibility that some features are not perfectly observed by the decision maker who would not have chosen this object relying only on their own information set.

The second explanation for herding behavior stems from psychology studies. Herding is treated as a very basic part of human nature, often referred to as one of so called 'animal instincts', or limited rationality. Existing literature linked to psychology suggests that some simplifying behaviors have been developed through evolution in order to be able to make faster decisions [35, 7]. Advantageous decision can be made in shorter time through brain short-cuts, inducing behavior such as following the herd, instead of trying to analyze the correct solution with limited information available.

Herding is not a new phenomenon, although it has not been observed in auction choice to date with respect to my best knowledge. Current chapter therefore shows yet another

example where this type of behavior plays a role.

3.12 Survival Analysis

3.12.1 Introduction

The previous section analyses choices of bidders by estimating an individual choice model. On the other hand, at the auction level the question asked can be: what are the aspects that decrease waiting time for bids in auctions? We have seen that the dynamic aspects of auctions, such as number of bids and bidders influence choice between auctions by bidders. Do the same factors have a significant impact on waiting time for a new bids? Is such relationship observable in the data? The topic of this chapter is survival analysis and therefore the time until the next bid is the central point of focus here.

3.12.2 Survival Analysis Using Bids Data

In order to analyze the impact on waiting time, auctions of the same length are selected - in this case 1 day auctions, since these are most common. The auctions are cut at an arbitrary time - and there are two chosen times: 12th hour (the middle of the auction) and 23rd hour (close to the end of an auction). The data is divided into subsets of similar auctions by the auction characteristics. The subsets are described in tables 3.30, 3.29, 3.32, and 3.31.

The datasets contain 1 observation per each auction. The hazard function relates to the time until the next bid counting from the cutting point - so either from 12th or 23rd auction hour. After the 23rd hour some auctions had no more bids, and these cases were also included appropriately in the analysis.

The explanatory variables for the hazard model include overall density of bids recorded in the data at the cutting time, any auction characteristics that may influence the density of bids in the particular auction, as well as live bid of the most recent bid and number of bids and bidders before the cutting point.

3.12.3 Model

The regression to estimate is Cox Proportional Hazards Model expressed by the hazard function $h(t)$:

$$h(t) = h_0(t) * exp(\beta_1 BidsDensity_t + \beta_2 A_x +$$

$$\beta_3 LiveBid_{xt} +$$

$$\beta_5 CurrentBidders_{xt} + \beta_6 CurrentBids_{xt}) \quad (3.12.29)$$

The coefficients, β's, measure the impact of covariates on hazard function. If $\beta_x > 0$, this means that the covariate x has a positive impact on hazard, and therefore on decrease of time until the next bid. On the other hand, if $\beta_x < 0$ the covariate x reduces the hazard function, and therefore has a positive impact on time until the next bid.

In order to get an appropriate format of the data, several functions in R were written. The variables which relate to the most recent bid, before the time of the cut, are prefixed with "$p\_$", while the variables relating to the future, soonest, bid are prefixed with "$f\_$" (although these were not used in the estimations).

$timeBidsDensityF$ was generated from the whole dataset, before the sub-setting was done. There is a substantial variability in bids density on iPhone 4 auctions over the time of data collection. The plot of bid density can be found in figure 3.1. In the variable $timeBidsDensityF$ the density, as seen on the y-axis, is scaled through multiplying by 1000000, so that the numbers included in the regressions are of a higher magnitude.

Bid density was the highest in the first days since the beginning of data collection, which suggests that the interest in iPhone 4 was extremely high at the point when the supply shortage in physical shops occurred first. The red line shows the fitted density function, which after multiplying by 1000000 is used in the $timeBidsDensityF$ variable. In the regressions, $timeBidsDensityF$ is the density at the time of the cut-off - so wither

Figure 3.1: Bid Density Over the Time of Data Collection
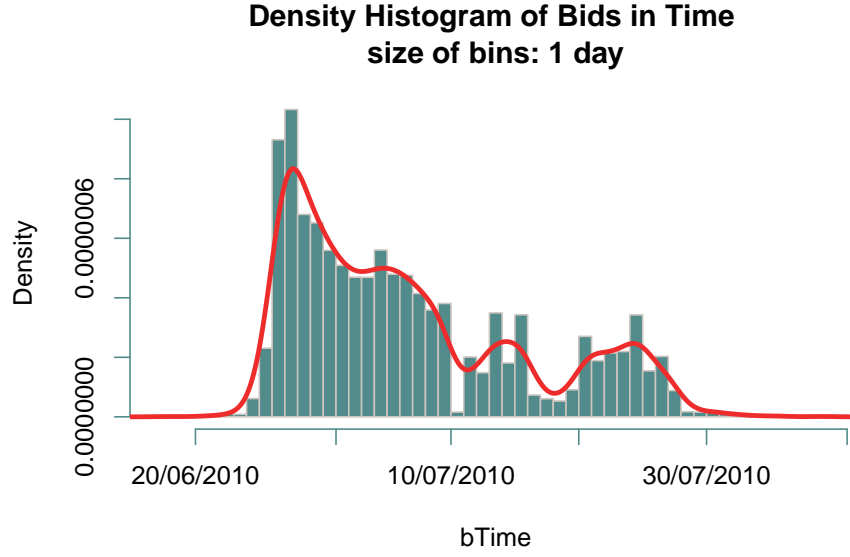
at the 12th or the 23rd hour of each auction.

The variable $timeHourUntilNext$, was constructed by subtracting the cut-off time, which is 12 hours or 23 hours, from time of the soonest future bid in the auction. Given that each auction is cut in the same place the expected time to the next bid is the same for identical auctions. Conditional on the small differences between the auctions, and the overall bid density, which are controlled for, the influence of dynamic variables on time until the next bid is being tested.

3.12.4 Estimation

The hazard function is constructed from $timeHourUntilNext$, the time until the next bid. And for calculating Cox Proportional Hazards Model R package "survival" is used. Controls include live bid at the time of the most recent bid, as well as number of current bids and bidders.

The results from estimations on the subset of 16Gb Unlocked Phones in table 3.36 show that the dynamic variables such as current number of bidders and bids have a posi-

tive impact on the hazard function - and therefore decrease the waiting time until the next bid in auctions. In other subsets - tables 3.35, 3.33, and 3.34, the impact of current bidders, although positive, is not significant. Results in all subsets show the negative impact od current price on the hazard function. Inclusion of variables like number of concurrent auctions, $cNumberOfConcAll$, $cNumberOfConcM32$, $cNumberOfConcM16$ and the umber in the sequence of auctions, $closingSeqAll$, $closingSeqM32$, $closingSeqM16$, show that the concurrent auctions influence bidding. In tables 3.37 different models including interaction terms are used. Number of current bidders has a positive impact on hazard, while current price has a negative impact. There are no prevailing effects repeated in all the subsets in spite of the negative impact of price level found.

The results found show that in this case there are shortcomings of the survival analysis. Inclusion of the start date in the auction had a positive impact which suggests that time trend could be important. Additionally, in survival analysis it is not possible to distinguish between the effect of loyalty to the auction as well as other effects stemming from bid history from the effect of auction dynamic characteristics to new bidders. Everything works together and therefore no particular pattern can be found.

3.12.5 Conclusion

In conclusion, the survival analysis does not give clear answers as to the impact of dynamic variables on bidding. The discrete choice model is more appropriate in the case the interest is in the impact of dynamic auction aspects on decisions of new bidders. The results from the survival analysis, though, show that concurrent auctions influence bids in a given auction, and that the most important variable predicting the time until the next bid is current price. Current number of bidders are found to have a significantly positive impact on reduction of time until the next bid only in the subset of Unlocked 16 Gb Phones.

3.12.6 Tables

Table 3.29: Survival Analysis Data: 1 day Auctions for 16Gb Phones

p_aModel	16:299	32: 0	
p_aNetwork	O2 :114	Unlocked:185	
p_aCondition	New :299		
p_aStarL	1: 99	2: 57	3:143
p_aStarLevel	Turquoise:143	Yellow : 99	Blue : 57
p_aReturns	No :299		
p_aPostageFree	No :299		
p_aPhotosPresent	Yes :299		
p_aTotalPhotosLevels	1:227	2: 36	3: 20 , 4: 13 , 5: 2 , 6: 1
p_aNonStock	No : 77	Yes :222	
p_aPostto	UK :299		
p_aExtras	No :299		

At 12th hour :

p_bCurrentAuBidderLevels	0 :62	1 :27	5 :23 , 2 :22 , 3 :22 , (Other):71 , NA's :72
p_bAuctionBidLevels	1 : 29	3 : 18	4 : 18 , 5 : 17 , 6 : 16 , (Other):123 , NA's : 78

	mean	median	sd	min	max	n
p_bLiveBids	406.18	465.00	191.79	0.99	770.00	299.00
timeBidsDensityF	0.52	0.52	0.22	0.02	0.83	299.00
timeHourUntilNext	4.98	4.21	3.87	0.04	12.00	298.00
p_bHourTimeWithin	7.68	8.92	3.74	0.15	12.00	221.00
p_bMinTimeWithin	461.04	534.95	224.41	9.30	719.82	221.00
f_bHourTimeWithin	16.98	16.21	3.87	12.04	24.00	298.00
f_bMinTimeWithin	1018.81	972.34	232.38	722.10	1440.00	298.00
p_bCurrentAuBidder	3.24	3.00	3.12	0.00	12.00	227.00
p_bAuctionBid	8.43	6.00	7.07	1.00	42.00	221.00

At 23rd hour :

p_bCurrentAuBidderLevels	3 : 25	2 : 21	0 : 20 , 5 : 18 , 8 : 18 , (Other):125 , NA's : 72
p_bAuctionBidLevels	2 : 21	3 : 17	4 : 17 , 5 : 14 , 8 : 13 , (Other):188 , NA's : 29

	mean	median	sd	min	max	n
p_bLiveBids	569.30	575.01	104.92	0.99	770.00	299.00
timeBidsDensityF	0.52	0.50	0.23	0.02	0.83	299.00
timeHourUntilNext	0.56	0.60	0.34	0.01	1.00	252.00
p_bHourTimeWithin	20.48	21.76	3.40	1.73	23.00	270.00
p_bMinTimeWithin	1228.54	1305.51	203.84	103.80	1379.93	270.00
f_bHourTimeWithin	23.56	23.60	0.34	23.01	24.00	252.00
f_bMinTimeWithin	1413.64	1415.84	20.17	1380.43	1440.00	252.00
p_bCurrentAuBidder	6.01	6.00	4.16	0.00	18.00	227.00
p_bAuctionBid	12.19	11.00	8.67	1.00	47.00	270.00

Number of auctions:	299
Number of Sellers:	239

Number of auctions with no more bids after 12H	1
Number of auctions with no more bids after 23H	47

Table 3.30: Survival Analysis Data: 1 day auctions for 32Gb Phones

p_aModel	16: 0	32:202		
p_aNetwork	O2 : 88	Unlocked:114		
p_aCondition	New :202			
p_aStarL	1:53	2:54	3:95	
p_aStarLevel	Turquoise:95	Blue :54	Yellow :53	
p_aReturns	No :202			
p_aPostageFree	No :202			
p_aPhotosPresent	Yes :202			
p_aTotalPhotosLevels	1:121	2: 38		3: 27 , 4: 13 , 5: 2 , 6: 1
p_aNonStock	No : 54	Yes :148		
p_aPostto	UK :202			
p_aExtras	No :202	Yes : 0		

At 12th hour :

p_bCurrentAuBidderLevels	0 :47	1 :21	4 :21 , 3 :15 , 2 :14 , (Other):41 , NA's :43
p_bAuctionBidLevels	1 :22	2 :15	3 :15 , 8 :13 , 7 :12 , (Other):71 , NA's :54

	mean	median	sd	min	max	n
p_bLiveBids	513.23	599.00	210.19	0.99	900.00	202.00
timeBidsDensityF	0.49	0.50	0.21	0.01	0.83	202.00
timeHourUntilNext	5.11	4.37	3.83	0.00	12.00	198.00
p_bHourTimeWithin	7.62	8.52	3.48	0.07	11.98	148.00
p_bMinTimeWithin	456.90	510.92	208.87	4.05	719.02	148.00
f_bHourTimeWithin	17.11	16.37	3.83	12.00	24.00	198.00
f_bMinTimeWithin	1026.62	982.24	229.60	720.18	1440.00	198.00
p_bCurrentAuBidder	2.96	2.00	2.98	0.00	13.00	159.00
p_bAuctionBid	7.21	6.00	6.05	1.00	29.00	148.00

At 23rd hour :

p_bCurrentAuBidderLevels	6 :22	2 :20	3 :16 , 1 :15 , 8 :13 , (Other):73 , NA's :43
p_bAuctionBidLevels	1 : 17	3 : 16	9 : 15 , 5 : 13 , 7 : 13 , (Other):118 , NA's : 10

	mean	median	sd	min	max	n
p_bLiveBids	680.19	690.00	111.97	91.00	983.99	202.00
timeBidsDensityF	0.49	0.49	0.23	0.01	0.83	202.00
timeHourUntilNext	0.56	0.58	0.33	0.02	1.00	163.00
p_bHourTimeWithin	20.14	21.62	4.14	1.52	23.00	192.00
p_bMinTimeWithin	1208.29	1297.41	248.37	91.20	1379.73	192.00
f_bHourTimeWithin	23.56	23.58	0.33	23.02	24.00	163.00
f_bMinTimeWithin	1413.64	1414.80	20.09	1381.33	1440.00	163.00
p_bCurrentAuBidder	5.56	5.00	3.80	0.00	16.00	159.00
p_bAuctionBid	9.81	8.00	7.39	1.00	34.00	192.00

Number of auctions: 202
Number of Sellers: 150

Number of auctions with no more bids after 12H 4
Number of auctions with no more bids after 23H 39

Table 3.31: Survival Analysis Data: 1 day Unlocked 16 Gb Phones auctions

p_aModel	16:185	32: 0	
p_aNetwork	O2 : 0	Unlocked:185	
p_aCondition	New :185		
p_aStarL	1:58	2:38	3:89
p_aStarLevel	Turquoise:89	Yellow :58	Blue :38
p_aReturns	No :185		
p_aPostageFree	No :185		
p_aPhotosPresent	Yes :185		
p_aTotalPhotosLevels	1:140	2: 26	3: 13 , 4: 5 , 6: 1
p_aNonStock	No : 45	Yes :140	
p_aPostto	UK :185	(Other): 0	
p_aExtras	No :185		

At 12th hour :

p_bCurrentAuBidderLevels	0 :31	2 :17	5 :17 , 1 :15 , 4 :15 , (Other):46 , NA's :44
p_bAuctionBidLevels	1 :17	3 :13	6 :13 , 7 :12 , 4 :11 , (Other):78 NA's :41

	mean	median	sd	min	max	n
p_bLiveBids	423.20	499.00	203.43	0.99	725.00	185.00
timeBidsDensityF	0.59	0.64	0.22	0.02	0.83	185.00
timeHourUntilNext	4.68	3.49	3.77	0.04	12.00	185.00
p_bHourTimeWithin	7.73	8.89	3.65	0.15	12.00	144.00
p_bMinTimeWithin	463.72	533.50	219.17	9.30	719.82	144.00
f_bHourTimeWithin	16.68	15.49	3.77	12.04	24.00	185.00
f_bMinTimeWithin	1001.00	929.27	226.24	722.10	1439.80	185.00
p_bCurrentAuBidder	3.58	3.00	3.19	0.00	12.00	141.00
p_bAuctionBid	8.87	7.00	7.55	1.00	42.00	144.00

At 23rd hour :

p_bCurrentAuBidderLevels	3 :17	2 :13	6 :12 , 1 :11 , 9 :11 , (Other):77 , NA's :44
p_bAuctionBidLevels	2 : 14	3 : 14	4 : 11 , 13 : 10 , 1 : 9 , (Other):117 , NA's : 10

	mean	median	sd	min	max	n
p_bLiveBids	594.07	603.00	112.33	0.99	770.00	185.00
timeBidsDensityF	0.60	0.64	0.22	0.02	0.83	185.00
timeHourUntilNext	0.52	0.53	0.33	0.01	1.00	153.00
p_bHourTimeWithin	20.25	21.56	3.74	1.73	23.00	175.00
p_bMinTimeWithin	1215.29	1293.80	224.60	103.80	1379.93	175.00
f_bHourTimeWithin	23.52	23.53	0.33	23.01	24.00	153.00
f_bMinTimeWithin	1411.47	1411.77	20.00	1380.75	1439.97	153.00
p_bCurrentAuBidder	6.40	6.00	4.15	0.00	18.00	141.00
p_bAuctionBid	12.50	11.00	9.22	1.00	47.00	175.00

Number of auctions: 185
Number of Sellers: 146

Number of auctions with no more bids after 12H 0
Number of auctions with no more bids after 23H 32

Table 3.32: Survival Analysis Data: 1 day Unlocked 32 Gb Phones auctions

p_aModel	16: 0	32:114				
p_aNetwork	O2 : 0	Unlocked:114				
p_aCondition	New :114			(Other): 0		
p_aStarL	1:22	2:36	3:56			
p_aStarLevel	Turquoise:56	Blue :36	Yellow :22			
p_aReturns	No :114					
p_aPostageFree	No : 114					
p_aPhotosPresent	Yes : 114					
p_aTotalPhotosLevels	1:68	2:24		3,4,5 or 6 : 22		
p_aNonStock	No : 28	Yes : 86				
p_aPostto	UK :114			(Other): 0		
p_aExtras	No : 114					

At 12th hour :

p_bCurrentAuBidderLevels	0 :29	1 :13	3 :10 , 4 : 8 , 5 : 7 , (Other):20 , NA's :27
p_bAuctionBidLevels	1 :14	3 :10	7 : 8 , 2 : 7 , 4 : 7 , (Other):36 , NA's :32

	mean	median	sd	min	max	n
p_bLiveBids	531.70	600.00	222.92	0.99	900.00	114.00
timeBidsDensityF	0.54	0.62	0.22	0.01	0.83	114.00
timeHourUntilNext	4.76	3.50	3.72	0.06	12.00	114.00
p_bHourTimeWithin	7.93	8.57	3.11	0.07	11.98	82.00
p_bMinTimeWithin	475.85	514.16	186.86	4.05	719.02	82.00
f_bHourTimeWithin	16.76	15.50	3.72	12.06	24.00	114.00
f_bMinTimeWithin	1005.61	929.94	223.32	723.37	1439.93	114.00
p_bCurrentAuBidder	2.70	2.00	2.93	0.00	11.00	87.00
p_bAuctionBid	6.90	5.00	5.65	1.00	22.00	82.00

At 23rd hour :

p_bCurrentAuBidderLevels	6 :14	2 :12	3 :12 , 1 : 8 ,5 : 6 , (Other):35 , NA's :27
p_bAuctionBidLevels	3 :11	1 : 9	5 : 8 , 9 : 8 , 2 : 7 , (Other):67 , NA's : 4

	mean	median	sd	min	max	n
p_bLiveBids	711.12	710.00	127.36	91.00	983.99	114.00
timeBidsDensityF	0.55	0.63	0.24	0.01	0.83	114.00
timeHourUntilNext	0.58	0.58	0.34	0.04	1.00	97.00
p_bHourTimeWithin	20.75	22.03	3.41	2.27	22.99	110.00
p_bMinTimeWithin	1245.03	1321.51	204.38	136.50	1379.43	110.00
f_bHourTimeWithin	23.58	23.58	0.34	23.04	24.00	97.00
f_bMinTimeWithin	1414.69	1415.05	20.15	1382.45	1439.93	97.00
p_bCurrentAuBidder	5.62	5.00	3.98	0.00	16.00	87.00
p_bAuctionBid	9.65	8.00	7.22	1.00	33.00	110.00

Number of auctions: 114
Number of Sellers: 87

Number of auctions with no more bids after 12H   0
Number of auctions with no more bids after 23H   17

Table 3.33: Survival Analysis at 12 and 23 hours for 32Mb Phones

Cox Proportional Hazards Model

	Dependent variable:					
	survivalAnCut.1day12H_selectM32Surv			survivalAnCut.1day23H_selectM32Surv		
	(1)	(2)	(3)	(4)	(5)	(6)
timeBidsDensityF	-1.382 (1.364)	-1.444** (0.650)	-0.964 (1.375)	0.770 (1.273)	-0.321 (0.574)	1.610 (1.267)
p_aNetworkUnlocked	0.550** (0.221)	0.545** (0.219)	0.566** (0.225)	-0.001 (0.249)	-0.010 (0.244)	0.062 (0.245)
p_aStarL2	0.015 (0.305)	0.011 (0.301)	0.041 (0.309)	0.372 (0.302)	0.347 (0.303)	0.372 (0.299)
p_aStarL3	-0.265 (0.254)	-0.254 (0.253)	-0.246 (0.253)	0.633** (0.270)	0.617** (0.269)	0.651** (0.269)
p_aNonStock1	0.452* (0.272)	0.468* (0.272)	0.459* (0.274)	-0.012 (0.252)	0.033 (0.248)	-0.024 (0.251)
p_aTotalPhotos	0.073 (0.124)	0.093 (0.124)	0.094 (0.124)	0.110 (0.110)	0.100 (0.108)	0.126 (0.109)
p_bCurrentAuBidders	0.079 (0.087)	0.078 (0.087)	0.075 (0.088)	0.019 (0.055)	0.020 (0.054)	0.015 (0.055)
p_bLiveBids	-0.002*** (0.001)	-0.002*** (0.001)	-0.002*** (0.001)	-0.004*** (0.002)	-0.004*** (0.002)	-0.005*** (0.002)
p_bAuctionBid	-0.003 (0.035)	-0.001 (0.035)	0.0001 (0.035)	-0.008 (0.027)	-0.008 (0.027)	-0.008 (0.027)
cNumberOfConcM32	-0.0002 (0.007)			-0.006 (0.006)		
closingSeqM32	0.019* (0.010)			0.012 (0.033)		
cNumberOfConcAll			-0.001 (0.003)			-0.004* (0.002)
closingSeqAll		0.006* (0.004)	0.006* (0.004)		0.003 (0.014)	0.001 (0.014)
Observations	108	108	108	123	123	123
R^2	0.262	0.256	0.257	0.179	0.172	0.191
Max. Possible R^2	0.999	0.999	0.999	1.000	1.000	1.000
Log Likelihood	-384.513	-384.948	-384.871	-460.124	-460.604	-459.204
Wald Test	33.030*** (df = 11)	32.340*** (df = 10)	22.830*** (df = 11)	21.930** (df = 10)	21.930** (df = 11)	24.550** (df = 11)
LR Test	32.836*** (df = 11)	31.966*** (df = 10)	32.121*** (df = 11)	24.200** (df = 10)	23.241*** (df = 11)	26.040*** (df = 11)
Score (Logrank) Test	35.278*** (df = 11)	34.409*** (df = 10)	34.410*** (df = 11)	23.421** (df = 10)	22.644*** (df = 11)	25.233*** (df = 11)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3.34: Survival Analysis at 12 and 23 hours for 16Mb Phones

Cox Proportional Hazards Model

	Dependent variable:					
	survivalAnCut_1day12H_selectM16Surv			survivalAnCut_1day23H_selectM16Surv		
	(1)	(2)	(3)	(4)	(5)	(6)
timeBidsDensityF	-0.146 (0.551)	-0.097 (0.543)	-0.760 (1.175)	0.621 (0.433)	0.506 (0.425)	1.445 (1.134)
p.aNetworkUnlocked	0.050 (0.203)	0.054 (0.203)	0.037 (0.204)	0.460** (0.194)	0.443** (0.194)	0.437** (0.194)
p.aStarL2	-0.455* (0.232)	-0.438* (0.231)	-0.447* (0.231)	0.652*** (0.236)	0.651*** (0.237)	0.645*** (0.238)
p.aStarL3	-0.045 (0.196)	-0.032 (0.194)	-0.035 (0.194)	0.244 (0.199)	0.231 (0.199)	0.213 (0.201)
p.aNonStock1	-0.199 (0.210)	-0.202 (0.210)	-0.185 (0.212)	0.021 (0.195)	0.041 (0.195)	0.032 (0.195)
p.aTotalPhotos	0.043 (0.100)	0.043 (0.100)	0.039 (0.100)	0.115 (0.100)	0.114 (0.100)	0.118 (0.100)
p.bCurrentAuBidders	0.073 (0.049)	0.072 (0.049)	0.063 (0.050)	-0.015 (0.042)	-0.015 (0.042)	-0.009 (0.043)
p.bLiveBids	-0.003*** (0.001)	-0.003*** (0.001)	-0.003*** (0.001)	-0.007*** (0.001)	-0.006*** (0.001)	-0.007*** (0.001)
p.bAuctionBid	0.003 (0.019)	0.003 (0.019)	0.006 (0.019)	0.031 (0.019)	0.030 (0.019)	0.030 (0.019)
closingSeqM16	0.005 (0.004)			-0.016 (0.019)		
cNumberOfConcAll			0.001 (0.002)			-0.002 (0.002)
closingSeqAll		0.003 (0.003)	0.003 (0.003)		-0.004 (0.012)	-0.004 (0.012)
Observations	164	164	164	169	169	169
R^2	0.283	0.282	0.283	0.228	0.225	0.229
Max. Possible R^2	1.000	1.000	1.000	1.000	1.000	1.000
Log Likelihood	-648.619	-648.731	-648.522	-679.609	-679.904	-679.506
Wald Test	51.540*** (df = 10)	51.460*** (df = 10)	51.950*** (df = 11)	43.770*** (df = 10)	43.620*** (df = 10)	43.620*** (df = 11)
LR Test	54.457*** (df = 10)	54.233*** (df = 10)	54.651*** (df = 11)	43.657*** (df = 10)	43.066*** (df = 10)	43.863*** (df = 11)
Score (Logrank) Test	54.230*** (df = 10)	54.172*** (df = 10)	54.783*** (df = 11)	44.400*** (df = 10)	44.263*** (df = 10)	44.478*** (df = 11)
Note:						*p<0.1; **p<0.05; ***p<0.01

Table 3.35: Survival Analysis Model 32 Unlocked

Cox Proportional Hazards Model

	Dependent variable:			
	survivalAnCut_1day12H_selectM32UnlockedSurv	survivalAnCut_1day12H_selectM32UnlockedSurv	survivalAnCut_1day23H_selectM32UnlockedSurv	survivalAnCut_1day23H_selectM32UnlockedSurv
	(1)	(2)	(3)	(4)
timeBidsDensityF	-0.744 (1.783)	0.140 (1.830)	2.995* (1.626)	3.478** (1.575)
p-aStarL2	0.581 (0.424)	0.673 (0.431)	1.519*** (0.540)	1.486*** (0.537)
p-aStarL3	-0.297 (0.374)	-0.278 (0.374)	1.889*** (0.553)	1.879*** (0.543)
p-aNonStock1	0.281 (0.421)	0.254 (0.415)	-0.162 (0.339)	-0.225 (0.341)
p-aTotalPhotos	0.171 (0.209)	0.158 (0.215)	0.115 (0.159)	0.122 (0.153)
p-bCurrentAuBidders	0.026 (0.121)	0.018 (0.124)	0.009 (0.068)	0.006 (0.067)
p-bLiveBids	-0.002*** (0.001)	-0.002*** (0.001)	0.001 (0.002)	0.001 (0.002)
p-bAuctionBid	0.050 (0.053)	0.054 (0.054)	0.034 (0.038)	0.032 (0.037)
cNumberOfConcM32	-0.008 (0.010)		-0.013* (0.008)	
closingSeqM32	0.013 (0.012)		-0.017 (0.050)	
cNumberOfConcAll		-0.005 (0.004)		-0.006** (0.003)
closingSeqAll		0.004 (0.005)		-0.005 (0.018)
Observations	58	58	71	71
R^2	0.306	0.316	0.254	0.271
Max. Possible R^2	0.998	0.998	0.999	0.999
Log Likelihood	-169.853	-169.448	-224.302	-223.501
Wald Test (df = 10)	19.540**	19.600**	18.160*	19.690**
LR Test (df = 10)	21.207**	22.017**	20.799**	22.401**
Score (Logrank) Test (df = 10)	20.495**	20.819**	19.074**	20.773**

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3.36: Survival Analysis Model 16 Unlocked

Cox Proportional Hazards Model

	Dependent variable:			
	survivalAnCut_1day12H_selectM16UnlockedSurv	survivalAnCut_1day12H_selectM16UnlockedSurv	survivalAnCut_1day23H_selectM16UnlockedSurv	survivalAnCut_1day23H_selectM16UnlockedSurv
	(1)	(2)	(3)	(4)
timeBidsDensityF	-0.340	-0.662	2.265*	2.131*
	(1.237)	(1.329)	(1.202)	(1.288)
p-aStarL2	-0.489*	-0.485*	0.276	0.275
	(0.294)	(0.291)	(0.275)	(0.277)
p-aStarL3	-0.114	-0.100	-0.414	-0.421
	(0.260)	(0.258)	(0.280)	(0.281)
p-aNonStock1	-0.429	-0.409	-0.127	-0.090
	(0.265)	(0.267)	(0.272)	(0.271)
p-aTotalPhotos	0.186	0.181	0.306**	0.309**
	(0.157)	(0.157)	(0.142)	(0.141)
p-bCurrentAuBidders	0.114*	0.103*	0.006	0.002
	(0.062)	(0.061)	(0.054)	(0.054)
p-bLiveBids	-0.004***	-0.004***	-0.006***	-0.005***
	(0.001)	(0.001)	(0.001)	(0.001)
p-bAuctionBid	0.002	0.004	0.040*	0.041*
	(0.023)	(0.022)	(0.023)	(0.023)
cNumberOfConcM16	-0.001		-0.004	
	(0.004)		(0.004)	
closingSeqM16	0.008*		-0.014	
	(0.005)		(0.026)	
cNumberOfConcAll		0.0001		-0.003
		(0.003)		(0.003)
closingSeqAll		0.005*		-0.004
		(0.003)		(0.017)
Observations	110	110	109	109
R^2	0.330	0.329	0.270	0.267
Max. Possible R^2	0.999	0.999	0.999	0.999
Log Likelihood	-388.291	-388.365	-388.464	-388.718
Wald Test (df = 10)	40.230***	40.260***	31.970***	31.700***
LR Test (df = 10)	44.064***	43.916***	34.317***	33.809***
Score (Logrank) Test (df = 10)	43.306***	43.432***	33.369***	32.990***

Note: *p<0.1; **p<0.05; ***p<0.01

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Table 3.37: Survival Analysis: M16 Unlocked, 12th Hour

Cox Proportional Hazards Model

	(1)	(2)	(3)	(4)	(5)
		Dependent variable:			
		survivalAnCut.1day12H.selectM16UnlockedSurv			
timeBidsDensityF	-0.340 (1.237)		-0.558 (1.263)	-0.790 (1.536)	-0.790 (1.536)
log(timeBidsDensityF)		0.411 (0.469)			
p.aStarL2	-0.489* (0.294)	-0.453 (0.292)	-0.514* (0.296)	-0.597** (0.304)	-0.597** (0.304)
p.aStarL3	-0.114 (0.260)	-0.064 (0.254)	-0.119 (0.258)	-0.124 (0.249)	-0.124 (0.249)
p.aNonStock1	-0.429 (0.265)	-0.345 (0.264)	-0.428 (0.264)	-0.135 (0.262)	-0.135 (0.262)
p.aTotalPhotos	0.186 (0.157)	0.083 (0.159)	0.178 (0.157)		
p.bCurrentAuBidders	0.114* (0.062)	0.498*** (0.143)	0.144** (0.071)	0.747*** (0.203)	0.747*** (0.203)
p.bLiveBids	-0.004*** (0.001)	-0.001 (0.001)	-0.004*** (0.001)	-0.001 (0.001)	-0.001 (0.001)
log(cNumberOfConcM16)				0.576 (0.399)	0.576 (0.399)
p.bAuctionBid	0.002 (0.023)	-0.022 (0.043)	0.031 (0.040)	0.005 (0.041)	0.005 (0.041)
cNumberOfConcM16	-0.001 (0.004)	-0.006 (0.004)	-0.001 (0.004)		
closingSeqM16	0.008* (0.005)	0.008* (0.005)	0.008* (0.005)	0.007 (0.005)	0.007 (0.005)
timeBidsDensityF:p.bCurrentAuBidders				-0.349** (0.177)	-0.349** (0.177)
p.bCurrentAuBidders:p.bAuctionBid		0.004 (0.005)	-0.004 (0.005)	-0.001 (0.005)	-0.001 (0.005)
p.bCurrentAuBidders:p.bLiveBids		-0.001*** (0.0003)		-0.001*** (0.0003)	-0.001*** (0.0003)
Observations	110	110	110	110	110
R^2	0.330	0.380	0.334	0.411	0.411
Max. Possible R^2	0.999	0.999	0.999	0.999	0.999
Log Likelihood	-388.291	-384.073	-387.934	-381.252	-381.252
Wald Test	40.230*** (df = 10)	51.190*** (df = 12)	40.600*** (df = 11)	56.500*** (df = 12)	56.500*** (df = 12)
LR Test	44.064*** (df = 10)	52.500*** (df = 12)	44.778*** (df = 11)	58.141*** (df = 12)	58.141*** (df = 12)
Score (Logrank) Test	43.306*** (df = 10)	55.700*** (df = 12)	43.828*** (df = 11)	62.967*** (df = 12)	62.967*** (df = 12)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3.38: Survival Analysis: M16 Unlocked, 23rd Hour

Cox Proportional Hazards Model

	Dependent variable:				
	survivalAnCut.1day23H.selectM16UnlockedSurv				
	(1)	(2)	(3)	(4)	(5)
timeBidsDensityF	2.265*		2.388*	2.186	2.186
	(1.202)		(1.226)	(1.569)	(1.569)
log(timeBidsDensityF)		0.859*			
		(0.490)			
p.aStarL2	0.276	0.218	0.274	0.186	0.186
	(0.275)	(0.283)	(0.275)	(0.282)	(0.282)
p.aStarL3	-0.414	-0.418	-0.406	-0.313	-0.313
	(0.280)	(0.279)	(0.280)	(0.272)	(0.272)
p.aNonStock1	-0.127	-0.026	-0.143	-0.002	-0.002
	(0.272)	(0.272)	(0.274)	(0.266)	(0.266)
p.aTotalPhotos	0.306**	0.266*	0.298**		
	(0.142)	(0.145)	(0.143)		
p.bCurrentAuBidders	0.006	0.176	-0.016	0.205	0.205
	(0.054)	(0.166)	(0.067)	(0.218)	(0.218)
p.bLiveBids	-0.006***	-0.003	-0.006***	-0.003	-0.003
	(0.001)	(0.002)	(0.001)	(0.002)	(0.002)
log(cNumberOfConcM16)				-0.241	-0.241
				(0.354)	(0.354)
p.bAuctionBid	0.040*	0.015	0.025	0.016	0.016
	(0.023)	(0.036)	(0.036)	(0.035)	(0.035)
cNumberOfConcM16	-0.004	-0.004	-0.004		
	(0.004)	(0.004)	(0.004)		
closingSeqM16	-0.014	-0.005	-0.013	-0.022	-0.022
	(0.026)	(0.026)	(0.026)	(0.026)	(0.026)
timeBidsDensityF:p.bCurrentAuBidders				-0.052	-0.052
				(0.128)	(0.128)
p.bCurrentAuBidders:p.bAuctionBid		0.003	0.002	0.003	0.003
		(0.003)	(0.003)	(0.003)	(0.003)
p.bCurrentAuBidders:p.bLiveBids		-0.0003		-0.0004	-0.0004
		(0.0003)		(0.0003)	(0.0003)
Observations	109	109	109	109	109
R^2	0.270	0.274	0.272	0.254	0.254
Max. Possible R^2	0.999	0.999	0.999	0.999	0.999
Log Likelihood	-388.464	-388.176	-388.302	-389.628	-389.628
Wald Test	31.970*** (df = 10)	34.490*** (df = 12)	32.090*** (df = 12)	31.370*** (df = 12)	31.370*** (df = 12)
LR Test	34.317*** (df = 10)	34.892*** (df = 12)	34.641*** (df = 11)	31.989*** (df = 12)	31.989*** (df = 12)
Score (Logrank) Test	33.369*** (df = 10)	35.947*** (df = 12)	33.546*** (df = 11)	32.920*** (df = 12)	32.920*** (df = 12)
Note:					*p<0.1; **p<0.05; ***p<0.01

Chapter 4

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Appendix

Table 4.1: List of Variables

variable name	short description
$aCondition$	Condition of the auctioned object: e.g. "New", "Used"
$aDuration$	Duration of the auction
$aEnded$	Date and time the auction has ended
$aExtras$	Indicator variable: 1 = extras included with the auctioned object, 0 otherwise
$aMaxBid$	The amount of the highest bid in the auction
$aModel$	Model of iPhone 4 in the auction: e.g. "16", "32"
$aMsDuration$	Auction duration in milliseconds
$aNetwork$	Network of the auctioned phone: e.g. "Unlocked", "O2"
$aNonStock$	Indicator variable : 1 = there are non-stock photos in the auction, 0 otherwise
$aNonstockPhotos$	Number of non-stock photos
$aPositiveFeed$	The number of points resulting from the feed that the buyers have given to the given seller
$aPostage$	Price of postage
$aPostto$	Where to is the postage possible for the item: e.g. "Uk", "EU"
$aReturns$	Indicator variable: 1 = returns are accepted, 0 otherwise

Table 4.1: List of Variables - cont.

variable name	short description
aSeller	Seller identifier
aStar	The seller rating level, which is defined for positive feed points, e.g. "Blue", "Yellow"
aStarL	The seller rating level, represented numerically, where 0 = lowest possible rating, 1 = first rating level, etc.
aStarLevel	The seller rating level (factor variable), which is defined for positive feed points, e.g. "Blue", "Yellow"
aStartDate	The date and time the auction has started
aStartPrice	The starting price for the auction
aTotalBidders	Total number of bidders in the auction
aTotalBids	Total number of bids in the auction
aTotalPhotos	Total number of photos in the auction
auction	Auction identifier
aAvPercWithin	Average percentage time of bids within the auction
aAvTimeTilEnd	Average time until the end of an auction of bids in the auction
aAvUserAuctionBids	Average number of bids per user in the auction
aTotalPhotosLevels	Total number of photos in an auction (factor variable)
aTotalBiddersLevels	Total number of bidders in an auction (factor variable)
aTotalBidsLevels	Total number of bid in an auction (factor variable)
aDurationLevels	Auction duration (factor variable)
aPostageFree	Indicator variable: 1 = postage is free, 0 otherwise
aPhotosPresent	Indicator variable : 1 = there are photos present for the auction, 0 otherwise

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Table 4.1: List of Variables - cont.

variable name	short description
$aPrice$	The highest bLiveBid in an auction
$bAmount$	Bid amount in pounds
$bAuctionBid$	Bid number in an auction (counting of bids)
$bBIN$	Identifier variable : 1 = the bid was placed with the used of "Buy it Now" option which results in auction termination, 0 otherwise
$bDayOW$	Day of week
$bDayOfWeek$	Day of week
$bFirstAuctionBid$	Indicator variable: 1 = the bid is the first bid in the auction, 0 otherwise
$bHourOfDay$	Hour of day
$bIsMaxBid$	Indicator variable : 1 = the bid is a maximum bid in the auction, 0 otherwise
$bIsWinning$	Indicator variable : 1 = the bid is the winning bid, 0 otherwise
$bLastAuctionBid$	Indicator variable: 1 = the bid is the last bid in the auction, 0 otherwise
$bMsTimeWithin$	Time within the auction counted in milliseconds
$bPercWithin$	Percentage of time of the auction that has passed already
$bTime$	Date and time of the bid
$bTimeTilEnd$	Time left until the end of the auction (in milliseconds)
$bLiveBids$	Live Bids
$bLivePrice$	Live Price (previous most recent live bid in the auction)

Table 4.1: List of Variables - cont.

variable name	short description
bSecTimeWithin	Time in seconds counted from the start of the auction
bMinTimeWithin	Time in minutes counted from the start of the auction
bHourTimeWithin	Time in hours counted from the start of the auction
bCurrentAuBidders	Number of bidders currently in an auction
uAuctionBid	User's bid in an auction count
uAuctionNumber	User's auction number - count of auctions joined by the user sorted increasingly by time
uAvAmount	Average bid amount by user
uAvAuctionBids	User's average number of bids in an auction
uAvTimeTilEnd	Average time until the end of the auction of bids made by the user
uBid	User's bid number, count of user's bids sorted increasingly by time
uLastAuctionBid	Indicator variable: 1 = last bid in an auction by user
uNLR	Indicator variable: 1 = the user was later found to be unregistered on eBay, after the end of data collecting period - this was found during the data collection process
uNewAuction	Indicator variable: 1 = first bid in an auction by a user
uNumbEbayWins	Total number of eBay wins of the user
uTotalAuctionBids	Total number of bids by the user in the auction
uTotalAuctions	Total auctions joined by the user
uTotalBids	Total bids recorded by the user
uTotalWins	Total auction wins in the dataset by the user

Table 4.1: List of Variables - cont.

variable name	short description
uWinsSoFar	Number of wins so far for a user: win count variable
user	User (bidder) identifier
uMaxBid	User's maximum bid amount recorded
uLastBid	Amount of user's last bid recorded
uReverseAuctNumber	Reverse auction count variable (counting starting from the last auction joined by the user)
uNewAuctionBid	Indicator variable: 1 = the bid is made by the user in a new auction, 0 otherwise
timeBidsDensityF	Variable representing density of bids made on all iPhone 4 auctions
nLastAuctPrice	The price of the most recently finished auction, before the bid placement, in which the same bidder participated
nLastAuctPriceModelNetw	The price of the most recently finished auction, before the bid placement, in which the same bidder participated conditional on that it was an auction for the same model and network of the phone
nMean_1	The price of the most recently finished auction before the bid placement
nMean_5	The average price of 5 most recently finished auctions before the bid placement
nMean_10	The average price of 10 most recently finished auctions before the bid placement

Table 4.1: List of Variables - cont.

variable name	short description
$nMean\_15$	The average price of 15 most recently finished auctions before the bid placement

Table 4.2: Full Dataset Statistics

	mean	sd	min	max	n
aAvPercWithin	0.57	0.28	0	1.04	27648.00
aAvTimeTilEnd	90.42	99.58	0	865.84	27648.00
aAvUserAuctionBids	1.99	0.78	1.00	6.90	21377.00
aDuration	2.25	1.89	1.00	10.00	27648.00
aMaxBid	705.36	117.77	130.00	999.99	27648.00
aNonStock	0.65	0.48	0.00	1.00	27648.00
aNonstockPhotos	1.16	1.24	0.00	12.00	27108.00
aPositiveFeed	99.36	23.31	7.00	1650.00	27097.00
aPostage	5.77	2.46	0.00	9.00	27207.00
aStar	526.31	7027.80	0.00	155274.00	27648.00
aStarL	3.19	1.23	1.00	9.00	27648.00
aStartPrice	131.57	225.96	0.01	900.00	27648.00
aTotalBidders	9.99	4.00	1.00	28.00	27648.00
aTotalBids	19.82	10.01	1.00	69.00	27648.00
aTotalPhotos	1.55	1.06	0.00	12.00	27648.00
bAmount	435.01	254.73	0.01	999.99	27648.00
bLiveBids	395.05	257.90	0.01	999.99	27648.00

Table 4.2: Full Dataset Statistics - cont.

	mean	sd	min	max	n
bLivePrice	363.50	254.49	0.01	962.00	27648.00
bAuctionBid	10.31	8.09	1.00	69.00	27648.00
bLastMaxBid	0.09	0.28	0.00	1.00	27648.00
bPercWithin	0.57	0.43	0	1.04	27648.00
uAuctionBid	2.30	3.13	1.00	59.00	21377.00
uAuctionNumber	5.33	9.63	0.00	99.00	21377.00
uAvAmount	436.67	209.15	0.60	999.00	21377.00
uAvAuctionBids	2.58	2.54	1.00	30.50	21377.00
uAvTimeTilEnd	85.80	86.43	0.00	908.17	21377.00
uBid	11.20	26.81	1.00	346.00	21377.00
uNumbEbayWins	162.44	685.25	0	32311.00	21202.00
uTotalAuctionBids	3.61	5.15	1.00	59.00	21377.00
uTotalAuctions	10.11	16.22	1.00	100.00	21377.00
uTotalBids	22.21	46.09	1.00	354.00	21377.00
uTotalWins	1.26	3.39	0.00	38.00	21377.00
uWinsSoFar	0.56	1.78	0.00	28.00	21377.00
uMaxBid	704.49	251.44	0.99	999.99	27648.00
uLastBid	514.15	223.39	0.99	999.00	27648.00
uReverseAuctNumber	5.78	10.00	1.00	100.00	21377.00

Figure 4.1: Bid placement.

4.1 Description of Ebay

The following section is aimed at explaining eBay rules for how the bids are placed as well as how the information about current bids is displayed to those looking at open and finished auctions or searching for an item.

4.1.1 Display of Bids during And After the Auction

EBay requires users to enter their bid, although the bidding is done on behalf of the bidders in an automatic way. Only in the case the bidder places a small increment above the current highest bid, eBay is not placing automatic bids before showing the real bid.

Bidders place the maximum they wish their bid to be increased to. The way the bids are entered can be seen on Figure 4.1. On behalf of them, eBay makes automatic bids, which can reach up to the maximum specified. Only once a bidder is outbid by someone else, their real bid is displayed. After that, their bid plus a minimum increment on behalf of the highest bidder is added. The highest bidder's real bid is not displayed until they are outbid (or their bid is not above the minimum increment).

The minimum increment amounts are fixed amounts by which the highest bidder's bid is increased from the second highest bidder's bid. These amounts differ for certain intervals, and are given on the eBay's website.

After the auction is finished all the real bids together with their time of placement are

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Figure 4.2: Display of bids in a finished auction.

Figure 4.3: Display of bids in a finished auction. Automatic bids included.

displayed. An example auction bids as seen after the auction has finished can be seen on Figure 4.2. The only bid that is not fully revealed is the highest bidder's bid amount but it can be certain that it is above their final bid displayed. The highest bidder's bid is displayed as the second highest bidder's bid plus the increment, which is the amount the winner needs to pay.

The times next to the bids displayed are the actual times when the bid was placed, not when the automatic bid is placed on behalf of the bidder, which is why the bid times are not sorted increasingly.

It is also possible to see the automatic bids made by eBay together with the real bids, by pressing "Show automatic bids" option. The automatic bids for the same example auction can be seen on Figure 4.3.

The dataset included only the real bids (only the highest bid was not be fully revealed) for iPhone 4 between 17th June 2010 and 7th August 2010.

During the auction, the bids are displayed in the same way as after the finished auction. After clicking on an active auction all the bids are shown, and there is an option to display automatic bids as well. Such information as current number of bids and bidders is visible and displayed above the bids history. Current price is also an automatic bid on behalf of the current highest bidder. An example of how this information is visible can be seen on Figure 4.4. By default, bids are not sorted by the time of their placement, but increasingly by amount - this is the result of auction rules, since the bid becomes visible not at the time it is placed but at the time it becomes outbid by someone else.

4.1.2 Display of auctions

EBay website is a very popular in the UK place for buying new and used products which can be sold by anyone - professionals and complete amateurs. Ebay was already very popular in 2010, and the number of listings increased year by year since it's introduction. There are numerous categories of items on sale.

In order to buy a specific item the buyers may enter the item name in the search, or they

Figure 4.4: Bids as displayed in an open auction.

Figure 4.5: List of auctions as displayed on eBay.

may look for the item by selecting categories and subcategories until finding the particular object. In the case the search is narrowed to one particular item, auctions for that item are displayed in a form of a list together with basic information about each of the auctions, as seen on Figure 4.5. It needs to be noted that thtere is a possibility to sponsor an auction so that it appears on top of the search list in a form of an advertisement. All the non-sponsored auctions are listed either by "Best Match" key which is mainly based on time remaining, or by one of the categories which can be "Ending soonest", "Lowest Price", "Highest Price", "Fewest bids", "Most bids"(Figure 4.6). The "Best Match" sorting was introduced in 2008, in order to create different default sorting for auctioned objects as well as fixed price objects. For auctioned objects "Best Match" is very similar to "Ending soonest", although in some cases it may result in slightly different ordering since some other aspects, in addition to time remaining, are added with lower weights to this ordering key.

Each auction on the list is shortly summarized in a form of miniature page with a photo. The information visible directly on this summary are current auction price, current number of bids, time left and postage amount as well as whether the item is marked as "New" or "Pre-owned". Other information such as seller name and rating level as well as current number of bidders are displayed after clicking on a listing. It is possible to customize this thumbnail to show seller information here as well (as seen on Figure 4.7)

4.1.3 iPhone 4

iPhone 4 was a long awaited release of an upgraded version of iPhone 3. It's release date was June 24th 2010 [1]. This version was only available in Black color, but there were two possible memory sizes available: 16 and 32 Gb. There were many new features to this smartphone in relation to the previous version, such as an additional camera facing to the front, which allowed for the new introduced FaceTime video calls. Other improvements

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[1] The dataset used in chapters 2 and 3 contained 8 auctions which started before the introduction date of 24th June 2010, but all of them ended after that date (between 26th and 29th June 2010). There were in total 22 bids placed before 24th June in these auctions.
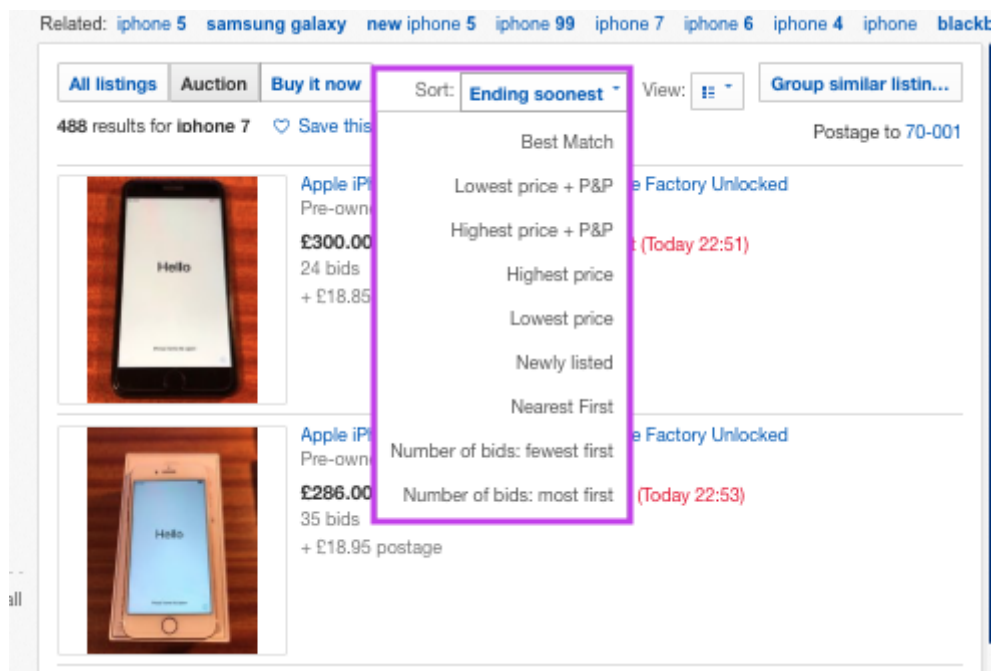
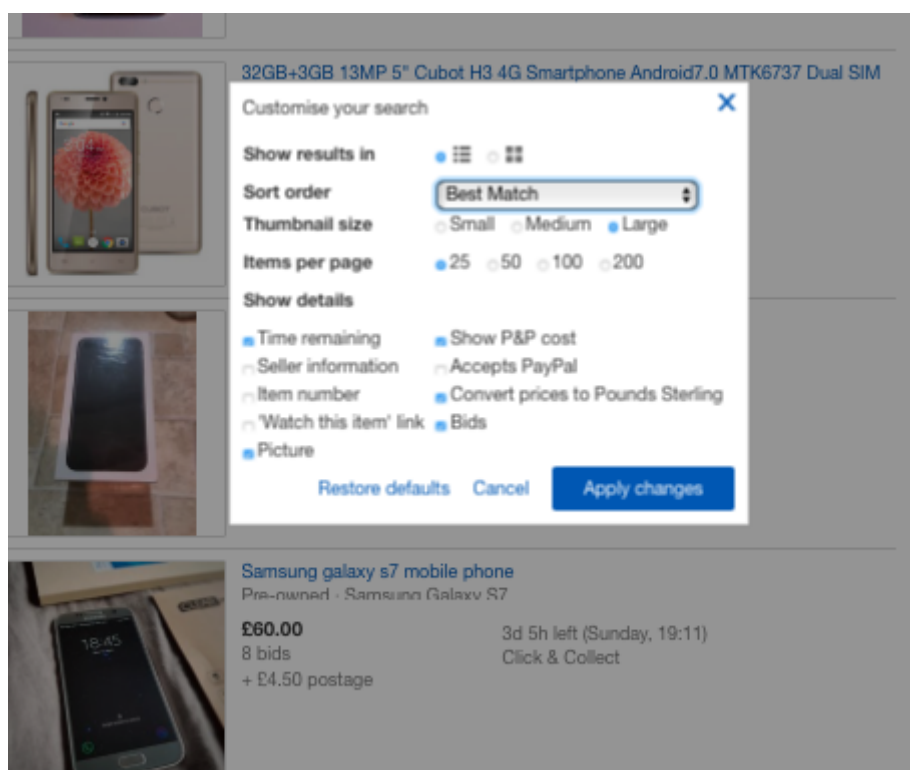Figure 4.6: Options for sorting on the list of auctions.

Figure 4.7: Options for Information displayed on item thumbnail.

were higher screen resolution that improved how all graphics were displayed. The release was preceded with many pre-orders, which resulted in a temporary supply shortage in the UK as well as in some other countries. In April 2011 a white version as well as other memory sizes of iPhone 4 were introduced.

4.2   Table with main Theorems in Chapter 1

Table 4.3: Main Theorems in Chapter 1

Theorem, page	Summary
7, p. 52	The symmetric equilibrium bidding strategy for a type v-bidder in period o and y of a sequential second-price auction with overlapping generations of two-period-lived bidders
10, p. 56	For a sequential second price auction game described as above, there exist a stationary Perfect Bayesian Equilibrium, and it is described by strategies for each age of the bidders
11, p.63	Observed price p_{t-1} leads to an updating of the belief about the probabilities of states A and B in $t-1$. The relation is monotonous
12, p. 63	n State A and B, an observation of the second highest bid (observed price) above the threshold of young bids means that there probability that an old bidder won is equal to 1.
15, p. 68	Depending on whether the observed price increases or decreases the probability that an old bidder won, the probability of State B in $t+1$ will follow the same direction.

Table 4.3: Main Theorems in Chapter 1 - cont.

Theorem, page	Summary
16, p. 68	If the probability of State B in t is increased following a price observation in $t-1$, then the probability of State B will also increase for all the following periods
17, p. 69	Increased probability of state B in any time period t leads to an increased probability of state B in the following period $t+1$.
18, p. 72	The update in probabilities of future states based on p_{t-1} is diminishing with time.
19, p. 72	The effect of update of future probabilities is decaying at a diminishing rate
21, p. 72	The effect of learning about future prices is proportional to the effect on probabilities of states. The expected price path will follow proportionally and monotonously the same path as the expected probability of State B: initially the change will be the highest in t and after that it will start converging to equilibrium at a diminishing rate with time.
29, p. 77	Overlapping generations model with 2-period lived bidders and one 3-period lived bidder, where bidders can only acquire information about the price after their first period of activity, implies that the bidder with 3-period lifespan will update their middle period bid as a result of observed price in $t-1$.

Table 4.3: Main Theorems in Chapter 1 - cont.

Theorem, page	Summary
30, p. 77	In the overlapping generations model with bidders of different lifespan, the condition that the same bidders are present at the information acquisition period and the period about which learning occurs is not a necessary condition for learning.

4.3 Diagrams showing data transformations

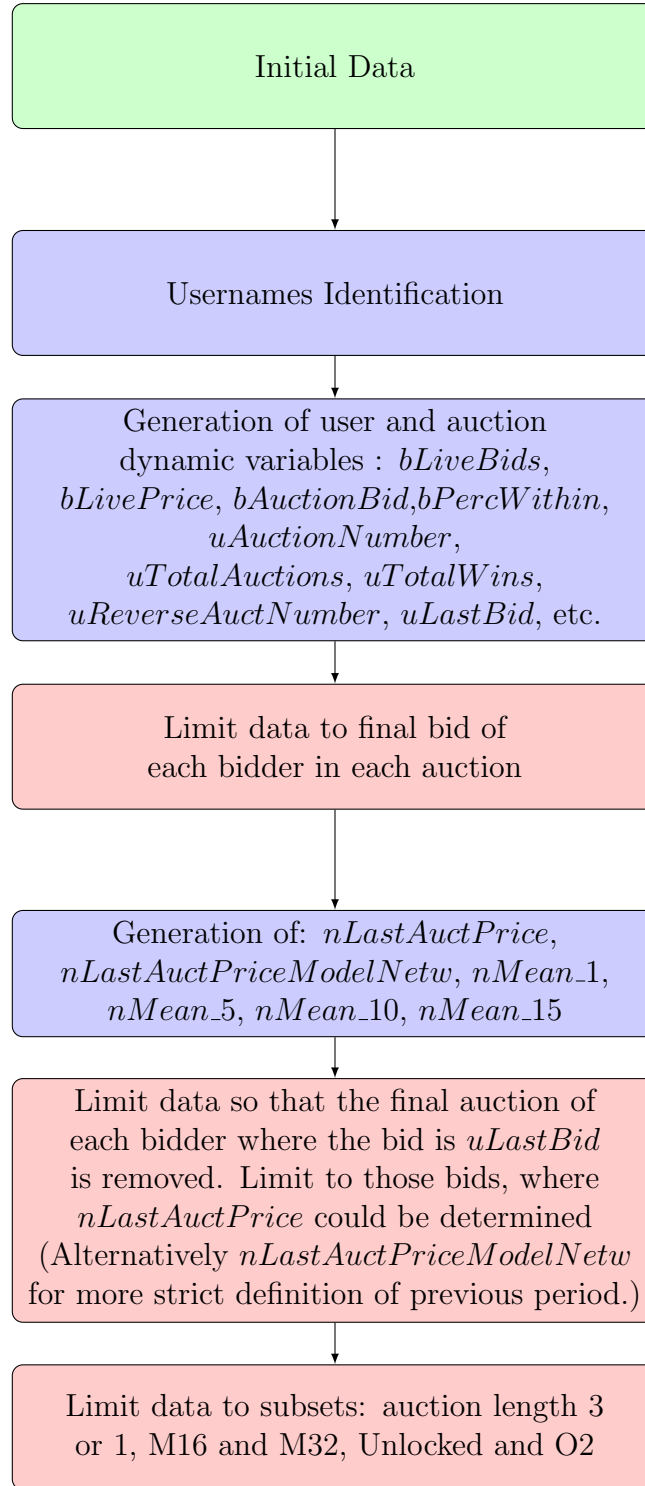Figure 4.8: Data Transformations for Estimations in Chapter II.

Figure 4.9: Data Transformations for Estimations in Chapter III.