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Waiting for a haircut? A bargaining perspective on sovereign debt restructuring

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November, 2016

Abstract
Recent investigation of sovereign debt negotiations finds that prompt, market-friendly ‘reprofiling’ often fails to achieve sustainability; but serious debt restructuring typically involves delay. We develop an incomplete information bargaining model to account for this, highlighting economic recovery and sustainability considerations as complementary reasons for delay. Some recent settlements are discussed along with policy implications of excessive profiling.

Keywords: Debt restructuring, delay, growth, sustainability, asymmetric information.

JEL: F34, C78

Acknowledgements
We are grateful to the ESRC under Projects RES 156-25-0032 and RES 051-27-0125: ‘Debt and Development’ for funding this research. We thank Daniel Benjamin, Javier García-Fronti, Martin Guzman, Daniel Heymann, Mark Wright and participants at seminars in the Bank of Spain, the University of San Andres, the Royal Economic Society Meetings and Glasgow University for helpful comments.

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Section 1: Introduction

Sovereigns in default are, in practice, faced with a choice between two types of debt renegotiation - reprofiling or restructuring. This is the conclusion of a recent extensive study of foreign currency debt negotiations by Mariscal et al. (2015). In the terminology of IMF (2014), reprofiling are relatively fast, they may be pre-emptive (in the sense that they are announced before the country actually enters default) and frequently have a zero 'haircut' of principal. Restructurings, on the other hand, do have face value haircuts, have deeper reductions of present value and are typically arranged ex post. For foreign currency bonds, the average haircut for "reprofiling" is about 15 percent while for "restructuring" it is some 50 percent.

Because over 40 percent of reprofilings are, in practice, followed by a second reprofiling or restructuring within 6 years, however, the authors argue that quick, market-friendly settlements "may not actually solve the underlying debt problem". To explain the attraction of prompt but inadequate settlements in their economic model of the debtor country, Mariscal et al. (2015), hereafter MPST, add exogenous output costs - which are three times greater for restructuring.

What light might bargaining theory throw on why some settlements take longer than others and embody greater debt reductions? That is the issue addressed in this paper.

It is, of course, true that prompt agreement is a key feature of the basic single-creditor, complete information bargaining models of sovereign debt restructuring often used: where preferences and the size of the pie to be divided are known, this is what the ‘alternating offers’ approach of Rubinstein (1982) delivers. But several reasons for delay have been also been considered in the literature. Merlo and Wilson (1995), hereafter MW, showed that delay may prove beneficial where it permits the debtor country's economy to recover from a crisis; and it may act as a signal between participants with asymmetric information, Rubinstein (1985). Problems of creditor coordination and creditor heterogeneity have also been explored, Pitchford and Wright (2012), Ghosal and Miller (2016); and the role of political factors, as in Trebesch (2016).

In this paper, we present an incomplete information model of debt restructuring with a representative private bondholder which combines two of these features. Delay occurs along the equilibrium path of play driven firstly by the prospect of uncertain economic recovery; and also as a means of signalling sustainability concerns by the debtor state. The length of delay is positively correlated with the size of the haircut: while one-period delay permits economic recovery, multi-period delay allows the debtor state to signal the need for a larger "haircut".

What this provides is an endogenous account of delay, based on the incentives of the participants in a process of bargaining; and a rationale for the correlation between length of delay and size of write-down noted by MPST. It also offers a reason for why there may be too many quick but inadequate settlements: when the costs of signalling are too high for the debtor, it doesn't get a "fresh start" with substantial debt reduction.

As an appropriate indicator of what restructuring might be needed for heavily indebted poor countries, for example, the Paris Club adopted progress toward the Millenium Development Goals. More generally, transparency has been identified as one of the basic principles for efficient sovereign debt restructuring by the UNCTAD (UN General Assembly, 2015). Potentially, the IMF could resolve uncertainty about future growth or sustainability concerns where these are in dispute. But its position as senior creditor may limit its ability to play that role.

In the absence of an institutional solution for overseeing debt restructuring for higher income countries, contractual alternatives offer an alternative: Brooke et al. (2013), for example, make a case
for adopting sovereign CoCos and GDP-indexed bonds ex ante so as to help align the debtor’s commitments with its capacity to pay.

Related Literature

The evidence that sovereign bond restructurings involve costly delay is extensive, see for example Cruces and Trebesch (2013), Roubini and Setser (2004b) and the monograph by Sturzenegger and Zettlemeyer (2007).

In a carefully calibrated model of sovereign default, with debt renegotiation modelled along the lines of MW, Bi (2008) finds that delay was beneficial for Argentina as it gave the economy breathing-space to recover from deep recession. While multi-period delay can arise in this framework, delay length may be negatively correlated with haircut size.

Asymmetric information as a cause of delay in sovereign debt restructuring is explored in Bai and Zhang (2012). In this case, it is the creditor's reservation value that is private information, and the debtor government knows only its distribution: delays in reaching agreements arise in equilibrium because the debtor uses costly delay to screen the creditors' reservation value. In contrast, in our paper, it is the debtor's type that is private information. For Bai and Zhang, moreover, the key fact to be explained is that sovereign debt renegotiations take an average of five years for bank loans but only one year for bonds' and the explanation advanced is that the secondary market for bonds provides information on creditors reservation value. In this paper, however, the key fact to be explained is why prolonged delay may occur even with bonds.

How lack of commitment may bedevil negotiations is explored in the comprehensive study by Benjamin and Wright (2016), where the rationale for delay in settling is that the creditor's ability to share in the future surplus is threatened by the risk that the debtor will default on what is agreed. The data set they examine (and use in quantifying their model) includes bank loans as well as bonds; and the data period (since 1970) includes the "lost decade" of the Latin American debt crisis of the 1980s. It has been argued, however, (by MW amongst others) that the a substantial reason for delay at that time was the risk that prompt write-downs of syndicated bank loans would have wiped out the equity of key banks in the US financial system.

By assuming that debt restructuring involves bargaining with a representative private bondholder, we abstract from problems of coordination among multiple private creditors. There is an extensive - and growing - literature on this topic, see, for example, Kletzer (2002), Ghosal and Miller (2003), Weinschelbaum and Wynne(2005), Pitchford and Wright (2012) and Ghosal and Thampanishvong (2013). As the factors generating multi-period delay we examine do not, in an essential way, require multiple private creditors, such considerations are omitted from the formal model; but the role of specialist hold-out creditors is discussed in considering the case of Argentina.

The remainder of the paper is structured as follows. Section 2 presents a simple three period bargaining model. In Section 3, after brief discussion of some settlements involving substantial haircuts, we outline the challenge offered by the Greek debt swap of 2011/12 and the recent decision in favour of holdouts from the Argentine restructuring; and consider various policy implications. Section 4 concludes. The appendix contains the infinite horizon extension of the three period incomplete information model studied in the main text.

Section 2: A model of recovery, sustainability and delay in bargaining

To highlight the factors on which our analysis is focussed, we begin by first reviewing the MW logic for delay – postponing a settlement so as to allow for economic recovery. Then we look at the case where the uncertainty is about the type of the debtor. Assuming that the creditor’s acceptance of
sustainability requirements shifts the bargaining position in favour of the debtor, but there is asymmetric information about the debtor’s type, we show how this may give rise to a signalling equilibrium with delay; and use parameters from MPST to check the consistency of our results with their empirical findings.

After these preliminaries, both factors are combined in a three-period model, where we find conditions for longer delay, first to allow for economic recovery and then for signalling. An infinite horizon extension of this three-period bargaining model is provided in the Appendix.

2.1 Waiting for recovery

Conditional on default, we assume there is bargaining between debtor and creditor over restructuring sovereign debt. The bargaining surplus is the additional tax revenue generated to the sovereign debtor by gaining access to the international capital market once the outstanding debt has been successfully restructured. We suppose that the debtor uses the tax revenue for one of the two reasons, either spending on a public good or debt repayment. Debt repayment requires the debtor to divert funds away from expenditure on the public good (e.g. cutting expenditure on public services (austerity) to run a primary budget surplus). The debtor's "offer" to the creditor then corresponds to the amount of tax revenue diverted from public good provision. We will denote the total tax revenue available for bargaining as $\pi > 0$, and refer to it as ‘the pie’.

The analysis begins at a point at which the debtor is in default and in a recession, i.e. we assume that the value of $\pi$ at the initial period, immediately after default, is low and is denoted by $\pi_L$. However, we allow the future value to be stochastic so that $\pi$ can continue to be low at $\pi_L$ with probability $p$ or grow to a higher level, $\pi_H$, with probability $1-p$, where the probabilities are common knowledge and the growth of $\pi$ corresponds to an increase in the tax revenue. The bargaining model formalizes the argument that in the initial period, the current surplus, $\pi_L$, can be shared between debtor or creditor, but doing so means giving up on the prospect of economic growth, i.e. this is a primitive endogenous growth model.

Assume, for convenience, that creditor and debtor share a common discount factor $\delta < 1$. If no contingent contracts can be written, we find that delay will occur when the expected increase in the pie exceeds the interest rate. Although their infinite horizon analysis makes the analysis much less straightforward, this is the principal feature of the equilibria in Merlo and Wilson (1998), who cite the delay in restructuring Latin American debt in the 1980s as an illustration of their approach.

The bargaining game is specified as follows. The debtor makes the offer in the first period; but each party has equal probability of making an offer (being the proposer) in the subsequent period. In the second (and final) period, bargaining takes the form of an ultimatum game, where the proposer takes all. Breakdown payoffs are zero for both players. (It is assumed that offers matching breakdown payoffs will be accepted.)

Table 1 illustrates, with the debtor’s payoff first, depending on the state of the economy and who makes the offer.

---

1 An assumption that is relaxed when it comes to calibration.
Table 1 Final period payoffs (probabilities shown in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>Depression (p)</th>
<th>Recovery (1-p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debtor’s Offer (1/2)</td>
<td>$\pi_L, 0$</td>
<td>$\pi_H, 0$</td>
</tr>
<tr>
<td>Creditor’s Offer (1/2)</td>
<td>$0, \pi_L$</td>
<td>$0, \pi_H$</td>
</tr>
</tbody>
</table>

Moving to the first period, we calculate the continuation values. Bearing in mind that the choice of proposer in the second period depends on the toss of a coin and that there is a common discount factor $\delta$, the continuation value is the same for each player, namely $\frac{\delta E\pi}{2}$, where $E\pi = p\pi_L + (1 - p)\pi_H$ denotes the expected size of the pie.

In the first period, with debtor as proposer, these continuation values limit the offers that can be made. The current pie can be shared between debtor or creditor; but doing so means giving up on the prospect for economic growth. Formally, whenever, the expected growth of the economy, $\frac{E\pi - \pi_L}{\pi_L} = Eg > r = \frac{1 - \delta}{\delta}$, the time rate of discount, there will be delay. If, for example, the rapid Argentine recovery of about 8% per annum had been expected, this would have exceeded the real interest rate of 5% assumed by Bi (2008), who uses detailed calibration to argue that this was indeed a case of extended ‘efficient delay’. Figure 1 depicts the situation, with debtor payoffs on the horizontal payoff and the creditor payoffs on the vertical axis. Pareto-efficient current settlements lie on the downward-sloping line labelled “current pie”, the boundary of the set of feasible settlements. Future growth prospects, summarized by the continuation values, lie on the upward-sloping 45 degree line, a reflection of the ex-ante symmetry of bargaining power in the second period. These continuation values lie outside the “current pie” when $Eg > r$ and inside if growth prospects fall short.

Figure 1: Continuation values and ‘efficient’ delay
Note that continuation values in the interior of the set of feasible settlements are Pareto-dominated by points on the line segment AB, i.e. current settlement dominates delay. At point E, for example, by offering the creditor no more than its continuation value, the debtor can achieve the payoff of A which improves on continuation.

2.2 Signalling concern for sustainability

By allowing information to differ as between creditor and debtor, delay can arise as a costly signal by the debtor concerned with sustainability, designed to secure an improvement in continuation game payoffs. Specifically, we assume the debtor may be one of two types - one concerned with sustainability, the other not- where the debtor knows his type but the creditor is not sure.

The two types of debtor, labelled Cautious and Optimistic, differ in the utility they get from consuming their part of a ‘pie’ of size $\pi$. For both, utility is linear in their payoff, but for the Cautious debtor there is a discontinuity to reflect concern about the sustainability of any settlement. (We continue to use a two-period example where there is a common discount factor $\delta < 1$.) The bargaining game is specified as follows.

Let $\pi$ denote the size of the pie that is to be divided between the debtor and creditor, with no growth expected. We assume utility is linear in (equal to) the payoff only for payoffs greater than or equal to $s \in [0, \bar{s}]$: for the Optimist debtor $s = 0$ while for the cautious debtor $s = \bar{s} > 0$. Let $\pi_i, i = O, C$ denote the share of the surplus obtained by either the optimist or the cautious debtor. For the Optimistic debtor $u_O = \pi_O$ for any allocation, but for the Cautious debtor this is only true for $\pi_C \geq \bar{s}$; for any share strictly less than $\bar{s}$, $u_C = 0$. The debtor knows his type, the creditor does not, but believes that the debtor is Optimistic with probability $q$. (This initial prior is bounded from below for reasons that will be made clear.)

The debtor makes the initial offer in the first period; but each party has equal probability of making an offer in the subsequent period. Breakdown payoffs are zero for both players in period two.

In the event that delay occurs in period one, however, the debtor enjoys an inside option, $\omega$, which is greater for the Cautious debtor than for the Optimist, i.e. $\omega_C > \omega_O$. Consider the Perfect Bayesian Equilibria of this bargaining game.

We begin with the ultimatum game in the final period, when the belief of the creditor that the debtor is an Optimist has evolved and is denoted $q$. Consider the creditor’s offers at extreme values of $q$. As indicated in Table 2a, the offer is zero unless the debtor is thought to be concerned about sustainability; in which case it is $\bar{s}$.

<table>
<thead>
<tr>
<th>Creditor’s Belief as to Debtor’s type</th>
<th>Creditor’s offer to Debtor</th>
<th>Payoff for Creditor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q = 1$ (it’s an Optimist)</td>
<td>0</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$q = 0$ (it’s not)</td>
<td>$\bar{s}$</td>
<td>$\pi - \bar{s}$</td>
</tr>
</tbody>
</table>

Table 2a Creditor’s offers with extreme beliefs

^2 We make this assumption purely for expositional convenience in order to show the existence of a signalling equilibrium in pure strategies. In the Appendix, in the infinite horizon case, we dispense with this assumption and show the existence of a signalling equilibrium in mixed strategies which retains the same qualitative features of the pure strategy separating equilibrium but where the expected payoff calculations and expressions are more complicated.
For less extreme beliefs, $0 < q_i < 1$, the creditor’s expected payoff from a high offer (of $\tilde{s}$), acceptable to either type, will be $\pi - \tilde{s}$; but the expected payoff from a low offer (of zero), acceptable only to the Optimist, will be $q_i \pi$. If $q_i \pi > \pi - \tilde{s}$, the creditor will do better by making a low offer, and conversely for $q_i \pi < \pi - \tilde{s}$. When $q_i = \pi - \tilde{s} \over \pi$, the two offers have the same expected payoff, and we assume that the debtor randomises, offering low with probability $\theta$ and high with probability $1 - \theta$.

To summarise, the creditor’s offers are shown as a function of his priors in the table.

<table>
<thead>
<tr>
<th>Creditor’s Belief as to debtor’s type</th>
<th>Creditor’s offer</th>
<th>Expected payoff for creditor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_i &gt; \pi - \frac{\tilde{s}}{\pi}$ (it’s probably an Optimist)</td>
<td>$0$</td>
<td>$\pi q_i$</td>
</tr>
<tr>
<td>$q_i = \pi - \frac{\tilde{s}}{\pi}$</td>
<td>$0$ with probability $\theta$ and $\frac{\tilde{s}}{\pi}$ with probability $1 - \theta$</td>
<td>$\theta \pi q_i + (1 - \theta)(\pi - \tilde{s})$</td>
</tr>
<tr>
<td>$q_i &lt; \pi - \frac{\tilde{s}}{\pi}$ (it’s probably Cautious)</td>
<td>$\frac{\tilde{s}}{\pi}$</td>
<td>$\pi - \tilde{s}$</td>
</tr>
</tbody>
</table>

Table 2b: Creditor’s offers in the ultimatum game for all values of beliefs

As for the debtor, his offer to the creditor is simply zero - which will be accepted in this ultimatum game.

From the perspective of the initial period, bearing in mind that each player has a 50% probability of making the next offer, the expected discounted payoffs as a function of the prior are as in Table 3:

<table>
<thead>
<tr>
<th>Creditor’s Belief as to debtor’s type</th>
<th>Expected payoff for debtor (for both types)</th>
<th>Expected Payoff for creditor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_i &gt; \pi - \frac{\tilde{s}}{\pi}$ (probably an Optimist)</td>
<td>$\frac{\delta \pi}{2}$</td>
<td>$\frac{\delta \pi q_i}{2}$</td>
</tr>
<tr>
<td>$q_i = \pi - \frac{\tilde{s}}{\pi}$</td>
<td>$\frac{[\pi + (1 - \theta)\tilde{s}]\delta}{2}$</td>
<td>$\frac{[\theta \pi q_i + (1 - \theta)(\pi - \tilde{s})]\delta}{2}$</td>
</tr>
<tr>
<td>$q_i &lt; \pi - \frac{\tilde{s}}{\pi}$</td>
<td>$\frac{(\pi + \tilde{s})\delta}{2}$</td>
<td>$\frac{(\pi - \tilde{s})\delta}{2}$</td>
</tr>
</tbody>
</table>

Table 3: Continuation values in the initial period of bargaining

Assume that the debtor makes the offer in the initial period. In a separating equilibrium, the offer made by the debtor (denoted by $x_{i,D}$) will be conditioned on one or other of the extreme priors, as follows:

<table>
<thead>
<tr>
<th>Creditor beliefs given the debtor’s offer</th>
<th>Debtor’s payoff</th>
<th>Offer to creditor</th>
<th>Outcome</th>
</tr>
</thead>
</table>

Offers consistent with separation are illustrated in Figure 2, where point B indicates the continuation values for the known Optimist and point C for the debtor known to be Cautious. The offer of \( \frac{\pi \delta}{2} \) shown by the horizontal line through B is attractive enough for the creditor to accept given that he believes he is dealing with an Optimist; but an offer of \( x_{1,D} < \frac{(\pi - s) \delta}{2} \) is too low to be accepted by a creditor who believes he is dealing with a Cautious debtor as it lies below the full information continuation values shown at point C. These are separating offers.

Consider now the conditions required to ensure that the types reveal themselves in a separating equilibrium with delay: namely (i) that Cautious doesn’t imitate Optimist; (ii) that Optimist doesn’t imitate Cautious; and (iii) that Cautious wants to delay.

To ensure that the Cautious debtor reveals himself requires that he will not do better by imitating an Optimist, i.e. that the present discounted value of continuing and being identified as Cautious exceeds
the benefits of settling quickly, so point C lies to the right of point A in the figure. Thus condition (i) requires that
\[
\frac{(\pi + s)\delta}{2} > \pi - \frac{\pi \delta}{2} \quad \text{or} \quad \frac{s}{\pi} > 2\frac{(1-\delta)}{\delta}.
\]
i.e. sustainability concerns have to be sufficiently important to outweigh the cost of delay. If this condition is satisfied, then after one period of delay, the cautious debtor will receive a payoff of \( s \) if it is the creditor who makes the offer; or the whole pie if he holds all the cards in the ultimatum game.

This could help explain the willingness of sovereigns who choose to wait for a Restructuring. But note that where, as in MPST, the discount factor for the debtor is higher than for the creditor, the sustainability factor needed for delay will be larger as impatience is costly with alternating offers. The following table reports a simple calibration showing how adding the usual assumption of an impatient debtor (using a discount rate for the debtor that is about three times that for the creditor, as in MPST) affects payoffs in our simple signalling model - with results in the middle row broadly comparable with the haircuts reported in MPST shown in the last line.

<table>
<thead>
<tr>
<th>Condition for no delay</th>
<th>Haircut with no delay</th>
<th>Condition for delay</th>
<th>Expected haircut after delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common discount rate of 7%</td>
<td>( \frac{s}{\pi} \leq 14% )</td>
<td>50%</td>
<td>( \frac{s}{\pi} &gt; 14% )</td>
</tr>
<tr>
<td>Discount rate for debtor 3 times greater</td>
<td>( \frac{s}{\pi} \leq 42% )</td>
<td>17%</td>
<td>( \frac{s}{\pi} &gt; 42% )</td>
</tr>
<tr>
<td>Average haircuts in MPST data</td>
<td>“Reprofiling” 15%</td>
<td>“Restructuring” 50%</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: ‘Haircuts’ with delay as signal: allowing for ‘impatient’ debtors

If sustainability is a smaller fraction of the pie, however, the gain from delay may not be worthwhile for the debtor. This may help to explain the early settlement of the Reprofiling cases reported by MPST, a point strengthened by the calculations reported in Table 5.

To verify the other conditions characterizing an equilibrium are satisfied, the inside option \( \omega_o \) available to the Optimist needs to be taken into account. To show (ii), that Optimist does not imitate Cautious, note that if he does, \( q_i = q_o \). If \( q_o > \frac{\pi - s}{\pi} \), and the creditor thinks the debtor is probably an Optimist, then the expected payoff for an optimist who delays will be \( \omega_o + \frac{\delta\pi}{2} \), where \( \omega_o \) is the inside option for the Optimist.

To ensure that there is no incentive to imitate Cautious in this way requires that this is less attractive than settling, i.e. \( \omega_o + \frac{\delta\pi}{2} < \pi - \frac{\delta\pi}{2} \). As can be seen from rewriting this as \( \omega_o < \pi(1-\delta) \), this gives an upper bound on the inside option for the Optimist.

Will the Cautious debtor want to delay, condition (iii) above? Observe that if the Cautious debtor makes an offer to the creditor of \( \frac{(\pi - s)\delta}{2} \), this will signal to the creditor the debtor type so that \( q_i = 0 \); and given \( q_i = 0 \), the creditor will choose to accept debtor’s offer. For the Cautious debtor to
want delay, therefore, requires that continuation value together with inside option $\omega_c$ is more attractive than settling, i.e. $\omega_c + \frac{(\pi + s)}{2} \delta > \pi - \frac{(\pi - s)}{2} \delta$. This implies a lower bound on the inside option for the Optimist, as can be seen by rewriting this as $\omega_c > \pi(1 - \delta)$.

It is worth noting that, where the sustainability condition is common knowledge, this shifts bargaining power in favour of the Cautious debtor without any need for signalling. This reallocation does not, however, affect the MW condition given in section 2.1 for delay where there is uncertainty as to the growth of the pie, a point we formally verify in the appendix.

2.3: The three period model

This section shows how recovery and sustainability can each contribute to cumulative delay bargaining. By extending the bargaining model by an extra period, we find that extended delay can occur, first due to prospects of recovery and second due to signalling of sustainability concern. A consequence of this is that prolonged delay is associated with a larger haircut in the debt swap.

In the three-period case, the timing of events is as follows:

<table>
<thead>
<tr>
<th>$t = 1$</th>
<th>Size of Pie</th>
<th>Proposer</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_L$</td>
<td>debtor makes offer</td>
<td>if offer accepted, game ends; if offer rejected, game continues, and conditional on $\varphi$, nature chooses $\pi$ is ${\pi_L, \pi_H}$ with prob. $p, 1-p$; $s$ is ${0, s}$ with prob. $q_0, 1-q_0$.</td>
<td></td>
</tr>
</tbody>
</table>

$t = 2$: $\pi_L$ or $\pi_H$ with prob $\frac{1}{2}$ debtor makes offer with prob $\frac{1}{2}$ creditor makes offer if offer accepted game ends; if offer rejected, game continues to final period (with no intervention by nature).

$t = 3$: $\pi_L$ or $\pi_H$ with probability $\frac{1}{2}$ debtor makes offer with probability $\frac{1}{2}$ creditor makes offer if offer accepted game ends; if there is no agreement, there are disagreement payoffs of $(0,0)$.

We begin with the ultimatum game in the final period. At this stage, the size of the bargaining surplus is commonly known to both the debtor and the creditor. Denote the bargaining surplus by $\pi$. Let the posterior beliefs of the creditor over the two types of the debtor be denoted by $q_2$. Then, using an argument identical to that in section 2.2, the creditor’s offer, as a function of his beliefs, is:

<table>
<thead>
<tr>
<th>Creditor’s Belief as to debtor’s type</th>
<th>Creditor’s offer</th>
<th>Expected payoff for creditor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_2 &gt; \frac{\pi - s}{\pi}$ (it’s probably an Optimist)</td>
<td>0</td>
<td>$\pi q_2$</td>
</tr>
<tr>
<td>$q_2 = \frac{\pi - s}{\pi}$</td>
<td>0 with probability $\theta$ and $\frac{s}{s}$ with probability $(1-\theta)$</td>
<td>$\theta \pi q_2 + (1-\theta)(\pi - s)$</td>
</tr>
<tr>
<td>$q_2 &lt; \frac{\pi - s}{\pi}$</td>
<td>$s$</td>
<td>$\pi - s$</td>
</tr>
</tbody>
</table>
As for the debtor, as before, his offer to the creditor is simply zero. From the perspective of t=2, the expected discounted payoffs are:

<table>
<thead>
<tr>
<th>Creditor’s Belief as to debtor’s type</th>
<th>Expected payoff for debtor (for both types)</th>
<th>Expected Payoff for creditor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_2 &gt; \frac{\pi - s}{\pi}$ (probably an Optimist)</td>
<td>$\frac{\delta \pi}{2}$</td>
<td>$\frac{\delta \pi q_2}{2}$</td>
</tr>
<tr>
<td>$q_2 = \frac{\pi - s}{\pi}$</td>
<td>$\frac{\delta (\pi + (1-\theta)s)}{2}$</td>
<td>$\frac{\delta (\pi q_2 + (1-\theta)(\pi - s))}{2}$</td>
</tr>
<tr>
<td>$q_2 &lt; \frac{\pi - s}{\pi}$</td>
<td>$\frac{\delta (\pi + s)}{2}$</td>
<td>$\frac{\delta (\pi - s)}{2}$</td>
</tr>
</tbody>
</table>

When the debtor is chosen to make the offer at t=2, again by using an argument identical to that in section 2, in a separating equilibrium, the offer will be:

<table>
<thead>
<tr>
<th>Induced posterior beliefs for the creditor given the debtor’s offer</th>
<th>Debtor’s payoff</th>
<th>Offer to creditor</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_2 = 1$ (it’s an Optimist)</td>
<td>Optimist: $\frac{\pi - \delta \pi}{2}$</td>
<td>Optimist: $\frac{\delta \pi}{2}$</td>
<td>High offer will induce creditor to accept</td>
</tr>
<tr>
<td>$q_2 = 0$ (it’s not)</td>
<td>Cautious: $\frac{\pi - x_{2,o} &gt; \pi - \frac{\delta \pi}{2} + \frac{\delta s}{2}}{2}$</td>
<td>Cautious: $\frac{\delta (\pi - s)}{2}$</td>
<td>Offer too low to be accepted</td>
</tr>
</tbody>
</table>

where $x_{2,o}$ denotes the cautious debtor’s offer at t=2.

When the creditor is chosen to make the offer at t=2, we claim that in a separating equilibrium, given that the optimist debtor accepts and the cautious debtor rejects, the creditor’s offer will be:

<table>
<thead>
<tr>
<th>Induced posterior beliefs for the creditor in a separating equilibrium</th>
<th>Expected payoff to creditor</th>
<th>Offer to debtor</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_2 = 1$, if debtor accepts; $q_2 = 0$, if debtor rejects</td>
<td>$q_0 \left( \frac{(2-\delta)\pi}{2} \right) + (1-q_0) \left( \frac{\delta (\pi - s)}{2} \right)$</td>
<td>$\frac{\delta \pi}{2}$</td>
<td>Optimist accepts but cautious rejects</td>
</tr>
</tbody>
</table>
It is easy to see that the Optimistic debtor will accept the creditor’s offer as he is indifferent between accepting and rejecting the creditor’s offer. Moreover, the Cautious debtor will also reject the creditor’s offer as \( \frac{\pi \delta}{2} < \frac{\delta (\pi + \bar{s})}{2} \).

In a separating equilibrium, the creditor makes an offer that the Optimist debtor accepts but a Cautious debtor rejects. We need to check that the creditor has no incentive to deviate by making an offer that either (i) both types of the debtor reject, and/or (ii) both types of the debtor accept.

(i) If both types of the debtor reject, \( q_z = q_o \) and as long as \( q_0 > \frac{\pi - \bar{s}}{\pi} \), the creditor’s expected continuation payoff is \( \frac{\delta \pi}{2} q_0 \). We require that
\[
\frac{\delta \pi}{2} q_0 < q_0 \left( \frac{2 - \delta}{2} \pi \right) + (1 - q_0) \left( \frac{\delta (\pi - \bar{s})}{2} \right)
\]
which is always holds as \( \frac{(2 - \delta) \pi}{2} > \frac{\delta \pi}{2} \).

(ii) If the creditor makes an offer that both types of the debtor accept, then the lowest such offer is \( \frac{\delta (\pi + \bar{s})}{2} \) which implies that the creditor’s payoff is \( \pi - \frac{\delta (\pi + \bar{s})}{2} \).

We require that
\[
\pi \left( \frac{\delta (\pi + \bar{s})}{2} \right) < q_0 \left( \frac{2 - \delta}{2} \pi \right) + (1 - q_0) \left( \frac{\delta (\pi - \bar{s})}{2} \right)
\]
which, by computation, simplifies to the inequality
\[
(2 - \delta) \pi < q_0 \left( \frac{2 - \delta}{2} \pi + \delta (\pi - \bar{s}) \right) + \delta \pi.
\]

Note that when \( q_o \) is one, the inequality always holds while if \( q_o \) is zero, the direction of the inequality is reversed. Moreover, the right hand side of the inequality is increasing in \( q_o \). It follows that there exists a critical value \( \bar{q}_o \), \( 0 < \bar{q}_o < 1 \), such that whenever \( q_0 > \bar{q}_o \), the inequality \( (2 - \delta) \pi < q_o \left( \frac{2 - \delta}{2} \pi + \delta (\pi - \bar{s}) \right) + \delta \pi \) is always satisfied.

It follows that whenever \( q_0 > \max \left\{ q_o, \frac{\pi - \bar{s}}{\pi} \right\} \), a separating equilibrium with delay always exists. By computation, it follows that the continuation payoff to the creditor from rejecting the debtor’s current offer at \( t=1 \) denoted \( \tilde{a} \) is defined as:
\[
\tilde{a} = \delta \left[ p \left\{ q_o \left( \frac{\delta \pi L}{2} \right) + (1 - q_0) \left( \delta \left( \frac{\pi - s}{2} \right) \right) \right\} + (1 - p) \left\{ q_0 \delta \pi H + (1 - q_0) \left( \delta \left( \frac{\pi - s}{2} \right) \right) \right\} \right]
\]
\[
= q_0 \frac{\delta E \pi}{2} + (1 - q_0) \frac{\delta^2 (E \pi - \bar{s})}{2} < \frac{\delta E \pi}{2}
\]

Suppose that following two inequalities are satisfied:

(a) \( \pi_L - \tilde{a} < \frac{\delta}{2} E \pi \),
so the Optimistic debtor will delay in period 1;

(b) $\pi_L - a < \frac{\delta^2 (\bar{E} + \bar{s})}{2}$,

so the Cautious debtor will delay in period one. Then, providing the conditions for delay the penultimate period derived for the pure signalling case in Section 2.2 (i.e. $\frac{s}{\pi} > 1 - \frac{1}{\delta}, \pi \in \{\pi_L, \pi_H\}$) so that $\frac{s}{\bar{E} - \pi} > 1 - \frac{1}{\delta}$ and, in addition, $\omega_o < \pi (1 - \delta) < \omega_c$ are also satisfied, there will be a positive probability of two-period delay in bargaining.

Finally, note that along the equilibrium path of play, the creditor’s payoff is lower than with one period delay. With one period delay, which occurs with probability $q_o$, the creditor’s expected payoff is $\frac{\delta}{2} E\pi > \frac{\delta^2 (\bar{E} + \bar{s})}{2}$, the creditor’s expected payoff with two period delay. It follows that prolonged delay is positively correlated with a larger haircut in our model of a sovereign debt swap, the main result of the three period model.

Section 3: Evidence of haircuts; two challenging cases; and policy implications

3.1 Evidence of haircuts

As MPST note, the distribution of haircuts in forty years of sovereign debt renegotiations is bi-modal. Using data from Cruces and Trebesch (2011), including all renegotiations with foreign banks and bondholders between 1970 and 2010, the histogram has peaks at haircuts of about 25% and 85%, as shown below.

Figure 3 Distribution of haircuts. Source: Mariscal et al.(2015, p.17)
When they “filter the debt renegotiations taking out those that are donor funded (typically the HIPIC renegotiations) and those that are bank debt renegotiations then the distribution becomes smoother. Now taking this subset of bond renegotiations and considering the two types (those with and those without face value haircuts), we find that the average haircut of the former is some 50 percent while the average haircut of the latter is some 15 percent”. MPST(p.7)

For bonded debt, Table 6 that follows illustrates this correlation between delay and haircuts. But it also indicates the heterogeneity of outcomes when sovereigns seek to restructure their debts.

<table>
<thead>
<tr>
<th>Sovereign State</th>
<th>Restructuring Negotiations</th>
<th>Default?</th>
<th>&quot;Delay&quot; - after default</th>
<th>Face Value, $b</th>
<th>Haircut</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 countries</td>
<td>08/1982-5/1994</td>
<td></td>
<td>141 months</td>
<td>30-35%</td>
<td></td>
</tr>
<tr>
<td>Bonds only</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ecuador</td>
<td>8/1999-8/2000</td>
<td>Yes</td>
<td>12 months</td>
<td>$6.5</td>
<td>60%</td>
</tr>
<tr>
<td>Ukraine</td>
<td>1/2000-4/2000</td>
<td>Yes</td>
<td>3 months</td>
<td>$2.6</td>
<td>40%</td>
</tr>
<tr>
<td>Pakistan</td>
<td>2/1999-12/1999</td>
<td>No</td>
<td>-</td>
<td>$0.6</td>
<td>30%</td>
</tr>
<tr>
<td>Uruguay</td>
<td>4/2003-5/2003</td>
<td>No</td>
<td>-</td>
<td>$3.8</td>
<td>26%</td>
</tr>
</tbody>
</table>

Table 6: Sovereign Debt Restructurings with haircuts until 2005
Sources: Table 14 and 15 in Sturzenegger and Zettlemeyer (2005)3; Table A.3 in Roubini and Setser (2004b); Table 1 in Bi (2008)

To highlight the contrast between bank and bond when it comes to restructuring, earlier experience reported by Bi (2008) for sovereigns that restructured bank debt as well as bonds is briefly summarized in the first row. The average delay of more than eleven years shown there includes the ‘lost decade’ of Latin American growth, where US banks had to accumulate substantial reserves before writing-down around a third of their loans under the Brady Plan, as discussed by Cline (1995) and Merlo and Wilson(1998).

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3 The losses that defaults have inflicted on creditors are largely based on the comparison between the (remaining) payment stream that was originally promised to investors and the payment stream associated with the restructured instruments, both discounted at a common interest (Sturzenegger and Zettlemeyer, 2007).
The sovereign debt restructurings shown in rest of the table, ordered by length of delay after default, highlight the positive correlation of delay and write-down. While the average haircut is about 50%, those with above average haircuts (Russia, Argentina and Ecuador) took the longest to resolve - suggestive evidence that negotiating a bigger haircut involves longer delay.

In his investigation of ‘efficient delay’ Bi (2008) argues that the entire delay of 40 months shown for Argentina can be accounted for in terms of ‘waiting for recovery’ as in MW. We would argue, however, that, following default in 2001, there were two separate phases leading up to the debt swap in 2005. First, from the beginning of 2002 to mid-2003, the Argentine economy was recovering strongly from deep recession and there appeared to be a consensus between debtor and creditors to await recovery (a consensus reinforced by the political difficulties faced by the regime in power pending Presidential elections). As for the second phase, one could interpret the meagre offer made by Argentina in September 2003 as driven by sustainability concerns. In the context of our analysis, see Figure 2, this low offer was designed to be rejected - leading to further delay and a reappraisal of the debtor's type, and to a debt exchange that acknowledged these sustainability concerns. This was by no means the end of the restructuring, however, as we discuss in the next section.

3.2 Two challenging cases: Argentina and Greece

Argentina and the vulture funds

As the debt exchange achieved by Argentina in 2005 involved only 76% of the bonds in default, negotiations continued - leading to a second exchange in 2010, raising creditor participation to 93%. The Republic of Argentina treated the second swap as effectively a defeat for the holdouts, and made it clear that the remaining 7% would not receive any payment. The remaining holdouts included specialist funds determined not to accept any write-down, however: and, in circumstances where the debtor appeared to be challenging the authority of the US court handling the case, these funds - NML in particular - were able to secure an injunction to prevent the payment of coupons on the bonds already exchanged unless the claims of the holdouts were also paid as claimed. As this ‘Pari Passu’ injunction threatened to undermine all that had been achieved in previous restructurings, the Republic of Argentina - after election of a new administration - finally agreed to a settlement with the principal holdouts in early 2016, Guzman (2016).

Without question, much of this extra decade-long delay was due to the activities of holdout creditors, in particular those of the specialist funds. Before concluding that it is essential to include creditor heterogeneity – holdouts in particular – as a reason to expect delay in future, it is worth reflecting on how unusual this case has proved. For the legal ingenuity exhibited by the specialist funds in relentless pursuit of their (highly profitable) claims has raised doubts as to the sustainability of the Pari Passu interpretation they obtained. It has, for example, been argued that, if holdouts can henceforth make claims for full settlement that rank equally with those of exchange bond holders who have accepted a haircut, then the process of ex post sovereign debt restructuring is doomed. One would have to wait forever to get any write-down!

Market sentiment does not, however, support this interpretation. Rather, it would seem, the Argentine case is being seen as an outlier, one where the battle between the debtor and holdouts was threatening the authority of the US courts, as discussed by Weidemaier and McCarl ( 2014). It has been suggested, therefore, that the US courts may choose to limit the applicability of Argentine precedent when some of the remaining holdouts try to use it to secure the same rights as NML. If so, the Argentine episode may have little impact.

Greece and the Troika

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4 often referred to as vulture funds
The subsequent case of Greek debt restructuring does not fit the pattern that appears in Table 6. It was settled without default, involved a write-down of 69% on debt with face value of $199b, and was achieved after only 8 months of negotiation. The institutional differences between debt problems of a member of a currency union and those of stand-alone nation states like Argentina are surely relevant here. The Eurozone may be incomplete - a currency union without fiscal and banking union - but policy-makers of the so-called Troika (the Commission, the ECB and the IMF) were determined it should not fail. This hardly fits a model of a private creditor and a debtor state involved in bargaining with asymmetric information with no third party intervention!

How relevant in such a context are the factors we have discussed as favouring delay? The first - waiting for economic recovery, as per Merlo and Wilson - was over-ruled by the need to preserve the currency union. When a member country 'loses market-access', interest rates diverge sharply and there is a risk of contagion and cumulative crisis. So either the country must change its ways; 5 or it should leave.

The second reason, that delay can act as a signal of the need for debt restructuring, had little traction either. The IMF was closely involved in monitoring the situation; indeed it published a review in mid-July 2011 concluding that Greece's outlook "does not allow the staff to deem debt to be sustainable with high probability." (Zettelmeyer et. al. (2013, p.5)). So the need for a haircut was clear enough; and the institutions involved were able to put pressure on creditors to accept the haircuts deemed necessary.

The need to allow for the impact of third party intervention is only too evident in the Greek exchange; and it proved successful in securing a large write-down in a short space of time. But in their report for the European Parliament on “The Troika and financial assistance in the euro area: successes and failures”, the authors observe drily: “In May 2010, Greece entered a stand-by agreement [with the IMF] and was therefore the first euro area country to fall under a “Troika” financial assistance programme. The performance of the programme disappointed in many respects and it can be rightly considered as the least successful one.” Sapir et al. (2015, p.29 )

3.3 Policy implications

In terms of the analysis provided earlier, sustainability concerns may be signalled by delay; but this will only happen if the benefits to the debtor are sufficient to cover the cost of delay. The fact that MPST have uncovered - that there are so many repeat ‘reprofilings’ - suggests that the costs of using delay as a signal may be substantial.

If their evidence means that delay is too costly, is there an alternative? The Greek case may have delivered a prompt restructuring but it hardly provides a tempting template. The experience of the Paris Club may be worth recalling in this context. For, as Sachs (1995) pointed, when negotiations were seen simply in terms of debt enforcement, this led to debt reductions which failed to give countries a ‘fresh start’. So the same debtor countries frequently reappeared seeking further concessions. A key idea that Sachs (2002) discussed was to use the Millenium Development Goals as a benchmark; so a restructuring would be judged inadequate if it failed to help the debtor make progress with these goals. The paper by Cheng et. al. (2016) takes a 60 year view of the operations of

5 “The Greek debt crisis began in October 2009, when the newly elected government of George Papandreou revealed that the country had understated its debt and deficit figures for years. The projected budget deficit for 2009, in particular, was revised upwards from an estimated 7 per cent to more than 12 per cent (it eventually ended up at 15.6 per cent).” (Zettelmeyer et al. 2013, p.4)

6 The report goes on to quote an earlier assessment that “ IMF and the EU bet on the materialisation of optimistic tax revenue and privatisation assumptions. Instead of formulating a robust programme capable of withstanding adverse economic, political and financial developments, they did just the opposite. It is no surprise that these optimistic assumptions were not vindicated by events.” Sapir et al. (2015, p.29 )
the Paris Club and, as the title suggests, charts a significant shift before and after 1980 - a shift “from
debt collector to relief provider”; and attempts to measure the success of this change of focus - in
terms of achieving economic growth in particular.

The sovereign debtors involved in the Paris Club negotiations are typically heavily indebted poor
countries, however. What of countries with middle or higher incomes? What criteria might signal
appropriate restructuring for them? When Argentina went into default at the end of 2001, Anne
Kreuger took the bull by the horns by advocating a form of sovereign bankruptcy procedure,
analogous to what domestic bankruptcy law provides for corporations and municipalities. Her
proposal, Kreuger (2002), gave a central role to the IMF, which would naturally have a data available
on the country\(^7\) and its problems (and has unparalleled experience of providing support – with
conditions - to countries in financial trouble). The proposal was not adopted, however, largely because
the US Treasury was not convinced: a key issue being that the IMF would face a conflict of interest
being both judge and creditor. Potentially, the IMF could resolve uncertainty about future growth or
sustainability concerns when these are in dispute. But this same conflict of interest may preclude the
IMF from acting as a neutral assessor of restructuring needs.

Consider a simple extension of the model studied above which differentiates between an official
senior creditor (such as the IMF) and private bondholders whose claims are subordinate. Assume that
bargaining surplus over which the debtor and the private bondholder bargain over, \(\pi\), is the
residual amount available after any payments to IMF are made. As a senior creditor, the IMF’s claim on
resources will reduce the bargaining surplus \(\pi\) available. As this will have the effect of exaggerating
the significance of debtor's sustainability constraint \(s\) over which the private bondholder and the
debtor can bargain, it will tend to make delay more or not less likely. Hence, given its senior creditor
status, the IMF may not be able to play the role of disinterested assessor who can reduce delay.

The alternative to institutional reform that the US Treasury pursued was to advocate renegotiable
contracts - specifically, Collective Action Clauses in sovereign debt instruments so that a
supermajority of creditors could, after default, engineer debt restructuring as they judged appropriate.
The apparent success of the contractual approach has encouraged further proposals.

Brooke et al. (2013), a paper by economists at the Bank of Canada and Bank of England, for example,
makes the case for ‘sovereign cocos’ to ease liquidity crises, where the provision of emergency
funding by the IMF would trigger a rollover by private bondholders. For issues of solvency, the
authors advocate the issuance of GDP linked bonds\(^8\). Such state-contingent contracts, long advocated
by Robert Shiller, have been positively assessed in a study byBarr et al. (2014) at the Bank of
England; and have been strongly endorsed by Blanchard and De Mauro (2016), arguing that this
would assist European countries handle the debt overhang.

Section 4: Conclusion

With the exception of Greece, the pattern of recent sovereign debt restructurings has been that
considerable delay is involved in securing a substantial haircut. Consistent with the evidence in
Mariscal et. al. (2015), we develop a bargaining model to account for this. With a stochastic
bargaining surplus and asymmetric information about the debtor's sustainability concern, we show that
multi-period delay can occur, initially reflecting recovery followed by signalling about sustainability;
and that prolonged delay is positively correlated with a large haircut.

\(^7\) Blustein (2005) provides discussion on the official sustainability assessments of Argentina made by the IMF but kept
confidential.

\(^8\) GDP linked warrants were issued by both Argentina and Greece: but this was \textit{ex post}, and the contracts suffered from a
marked novelty premium, estimated at 800 bps.
These are by no means the only factors to be considered, however. The Greece case highlights the importance of third party intervention, for example. Possible institutional development, problems of creditor coordination and prospects for contractual innovation are further issues in sovereign debt restructuring clearly worth exploring.

References

Appendix: An infinite horizon bargaining model with delay

In this section, we model bargaining between a representative private bondholder and a sovereign debtor who is already in default.

The model and key results

As in the three period model, conditional on default, there is bargaining between debtor and a representative private bondholder (referred to as the "creditor") over the terms of debt restructuring where the total tax revenue available for bargaining as $\pi$, where $\pi > 0$ and the value of $\pi$ at the initial period, is low (denoted by $\pi_L$). The future value of $\pi$ can continue to be low at $\pi_L$ with probability $p$ or grow to a higher level, $\pi_H$, with probability $1-p$. The sustainability constraint, $s$, represents the minimum fraction of the tax revenue required by the sovereign debtor consistent with economic and political stability (ensured by a minimum threshold level of public good provision) of the debtor state. We will assume that there is incomplete information about the sustainability constraint so that the sustainability constraint determines the type of the debtor: Optimistic and Cautious corresponding to $s = 0$ and $s = \bar{s} > 0$ respectively where $\pi_L - \bar{s} > 0$ and payoffs of the two debtor types (as function of the share of bargaining surplus) is the same as in the main text. We consider the case where bargaining takes place over a number of time periods $t = 1, 2, 3, \ldots$. Formally, a state of the world consists of a pair $(\pi, s)$ where $\pi \in \{\pi_L, \pi_H\}$ and $s \in \{0, \bar{s}\}$. Conditional on realization of the state of the world at the beginning of $t = 1$, the debtor and the private bondholder both attach a probability of $\{p, 1-p\}$ over $\{\pi_L, \pi_H\}$. The bondholder attaches a probability $\{q_0, 1-q_0\}$ over $\{0, \bar{s}\}$ while the debtor knows the realization of the value of the sustainability constraint. We assume that the uncertainty with respect to $\pi$ and with respect to debtor’s type are resolved at $t = 2$. The debtor’s type is revealed to the debtor but not the private creditor.

We assume that the sovereign debtor makes the offer at $t = 1$, but, in the subsequent periods, each party has an equal probability of making an offer (being the proposer). Offers are made at discrete points in time, namely, at times $1, 2, 3, \ldots$. An offer is a number greater than or equal to zero and less than or equal to $\pi$. If the offer is accepted, the game ends; otherwise, the game continues to the next period. This process continues until the offer is accepted.

For ease of exposition we will assume that both debtors and creditors have a common discount factor $\delta < 1$. In any period in which there is disagreement, the debtor is excluded from access to the international capital market. Formally, the disagreement payoffs are zero for both players.
The timing of events is summarized in Table A1 below.

<table>
<thead>
<tr>
<th>Size of $\pi$</th>
<th>Debtor’s type</th>
<th>Proposer</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 1$</td>
<td>$\pi_L$</td>
<td>Debtor makes offer</td>
<td>If offer accepted, game ends. If offer rejected, game continues.</td>
</tr>
<tr>
<td>$t = 2$</td>
<td>Uncertainty over $\pi$ is resolved.</td>
<td>Debtor makes offer with prob. $\frac{1}{2}$ Creditor makes offer with prob. $\frac{1}{2}$</td>
<td>If offer accepted, game ends. If offer rejected, game continues.</td>
</tr>
<tr>
<td>$t = 3, ...$</td>
<td></td>
<td>Debtor makes offer with prob. $\frac{1}{2}$ Creditor makes offer with prob. $\frac{1}{2}$</td>
<td>If offer accepted, game ends. If offer rejected, game continues.</td>
</tr>
</tbody>
</table>

**Table A1:** Timing of events

In order to distinguish between one and two period delay, we look at two distinct cases corresponding to different information structures: (1) the bargaining surplus, $\pi$, can change over time with some probability but the debtor’s type (i.e. the debtor’s sustainability constraint) is known and (2) $\pi$ can change over time and, in addition, there is an asymmetric information about the debtor’s type (i.e. $0 < q_0 < 1$).

Note that, formally, the two cases correspond to two distinct incomplete information models with different information structures. We solve for the *Perfect Bayesian Equilibrium* for each of the two models.

**Case 1: Stochastic Bargaining Surplus when the debtor’s sustainability constraint is common knowledge**

In this case, at $t = 1$ the debtor’s type is known: the debtor is either Optimistic or Cautious. We begin by calculating the equilibrium payoffs for the creditor and the debtor when there is complete information about debtor’s type and the value of $\pi$ is known at period $t = 2$.

First, let us consider the case where the debtor is Optimistic. Let $x_{CR}$ and $x_O$ denote the payoffs for the Optimistic debtor when the creditor and the debtor are the proposers, respectively, and let $\pi - x_{CR}$ and $\pi - x_O$ denote the share of the available bargaining surplus for the creditor. In the equilibrium, either party will agree to a debt restructuring proposal if the proposal offers the party at least as much in discounted present value as it can expect to attain by waiting until the next period, given the strategies of both parties. Therefore, to compute the payoffs for the...
creditor and the Optimistic debtor, the following two equations need to be solved:

\[ x_{CR} = \frac{\delta}{2} (x_{CR} + x_O), \quad (A1) \]

\[ \pi - x_O = \frac{\delta}{2} (\pi - x_{CR} + \pi - x_O). \quad (A2) \]

Equations (A1) and (A2) have a unique solution, given by \((x^*_O, x^*_{CR})\):

\[ x^*_O = \frac{2 - \delta}{2} \pi \text{ and } x^*_{CR} = \frac{\delta \pi}{2}. \quad (A3) \]

In equilibrium, the Optimistic debtor offers \(\pi - x^*_O = \frac{\delta \pi}{2}\) to the creditor and the creditor’s offer is \(x^*_{CR} = \frac{\delta \pi}{2}\). Given that each party has an equal probability of making an offer (being the proposer), it follows that the expected payoffs for the Optimistic debtor and the creditor are given by \((\frac{\pi}{2}, \frac{\pi}{2})\), respectively.

The Cautious debtor has concern about the sustainability of any settlement. The presence of a sustainability constraint reduces the amount of surplus that is available for bargaining from \(\pi\) to \(\pi - \bar{s}\) while the cautious debtor always keeps \(\bar{s}\). Then, using the same logic as above, an agreement is reached instantaneously and the expected payoffs for the debtor and the creditor are \((\frac{\pi + \bar{s}}{2}, \frac{\pi - \bar{s}}{2})\), respectively.

Next we calculate the condition for delay when the sustainability constraint for the debtor is common knowledge but the size of \(\pi\) is unknown.

**Optimistic Debtor:** The best offer that the Optimistic debtor can make is the excess of the available bargaining surplus over his own continuation value, given by \(\pi_L - \frac{\delta E\pi}{2}\). If this offer falls below the creditor’s continuation value, \(\frac{\delta E\pi}{2}\), this offer will not be accepted. Formally, the condition for the first-period delay for the Optimistic debtor is given by:

\[ \frac{\delta E\pi}{2} \leq \frac{E\pi}{2} \Leftrightarrow \frac{1}{\delta} \leq \frac{E\pi}{\pi L}, \]

Let \(Eg\) denote the expected growth of the economy where \(Eg = \frac{(E\pi - \pi_L)}{\pi L}\) and \(r = \frac{1 - \delta}{\delta}\) denote the discount rate. Note that

\[ \frac{1}{\delta} < \frac{E\pi}{\pi L} \Leftrightarrow r < Eg. \]

Therefore, in this case, delay occurs if expected growth of the economy, \(Eg\), exceeds the time rate of discount, defined as \(r = \frac{1 - \delta}{\delta}\).

**Cautious Debtor:** The best offer that the Cautious debtor can make is the excess of the available bargaining surplus over his own continuation value, given by \(\pi_L - \frac{\delta E\pi + \bar{s}}{2}\). If this offer falls below the creditor’s
continuation value, $\frac{\delta (E \pi - \bar{s})}{2}$, this offer will not be accepted. Formally, the condition for the first-period delay for the Cautious debtor is given by:

$$\pi_L - \frac{\delta (E \pi + \bar{s})}{2} < \frac{\delta (E \pi - \bar{s})}{2} \iff \frac{1}{\delta} < \frac{E \pi}{\pi_L} \iff r < E_{g}.$$ 

Note that for either debtor type, the condition for delay is the same: with relatively expected low growth prospects, there is immediate agreement. This condition for one-period delay is essentially the same as that in Merlo-Wilson.

Case 2: Stochastic Bargaining Surplus and Asymmetric Information About the Sustainability Constraint

Next, we study the case where, in addition to $\pi$ changing over time, there is an asymmetric information about the debtor’s type. We show that there is a mixed strategy equilibrium with two period delay along the equilibrium path of play, initially to permit for economic recovery followed by signalling about the sustainability constraint.

We begin with period $t = 3$. Let the posterior beliefs of the creditor over the two types of the debtor be denoted by $q_1$. In Table A2, we consider the creditor’s offers at extreme values of $q_1$.

<table>
<thead>
<tr>
<th>Creditor’s belief as to debtor’s type</th>
<th>Creditor’s offer to debtor</th>
<th>Payoff for creditor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1 = 1$ (Optimist)</td>
<td>$\frac{\delta \pi}{2}$</td>
<td>$\pi - \frac{\delta \pi}{2}$</td>
</tr>
<tr>
<td>$q_1 = 0$ (Cautious)</td>
<td>$\frac{\delta (\pi - \bar{s})}{2}$</td>
<td>$(\pi - \bar{s}) - \frac{\delta (\pi - \bar{s})}{2}$</td>
</tr>
</tbody>
</table>

*Table A2:* Creditor’s offers to debtor at $t = 3$ with extreme beliefs

For less extreme beliefs, i.e. $0 < q_1 < 1$, the creditor’s expected payoff from a high offer of $\frac{\delta (\pi - \bar{s})}{2}$, which is acceptable to both types of debtor, will be $\left(\frac{\delta \pi}{2}\right) (\pi - \bar{s})$, while the creditor’s expected payoff from a low offer of $\frac{\delta \pi}{2}$, which is only acceptable to the Optimistic debtor, will be $\left(\frac{\delta \pi}{2}\right) (\pi - \bar{s})$. If $q_1 \left(\pi - \frac{\delta \pi}{2}\right) > \left(\frac{2 - \delta}{2}\right) (\pi - \bar{s})$, the creditor will do better by making a low offer but otherwise for $q_1 \left(\pi - \frac{\delta \pi}{2}\right) < \left(\frac{2 - \delta}{2}\right) (\pi - \bar{s})$. When $q_1 \left(\pi - \frac{\delta \pi}{2}\right) = \left(\frac{2 - \delta}{2}\right) (\pi - \bar{s})$, the two offers give the creditor the same expected payoff. This condition implies that $q_1 = \frac{\pi - \bar{s}}{\pi}$. Since $0 < \bar{s} < \pi$, it follows that $q_1 < 1$.

The creditor’s offers as a function of his belief are shown in the table below:
As for the debtor, the Optimistic debtor’s offer to the creditor is \( q_1 > \frac{\pi - \bar{s}}{\pi} \) (Probably an Optimist), while the Cautious debtor’s offer to the creditor is \( q_1 \leq \frac{\pi - \bar{s}}{\pi} \) (Probably Cautious).

From the perspective of the second period, given the common discount factor \( \delta \) and bearing in mind that each player has a 50% probability of being a proposer, the continuation values for the debtor and the creditor are given by:

<table>
<thead>
<tr>
<th>Creditor’s belief as to debtor’s type</th>
<th>Creditors’ offer to debtor</th>
<th>Expected payoff for creditor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_1 &gt; \frac{\pi - \bar{s}}{\pi} ) (Probably an Optimist)</td>
<td>( \frac{\delta \pi}{2} )</td>
<td>( q_1 \left( \pi - \frac{\delta \pi}{2} \right) )</td>
</tr>
<tr>
<td>( q_1 \leq \frac{\pi - \bar{s}}{\pi} ) (Probably Cautious)</td>
<td>( \frac{\delta (\pi - \bar{s})}{2} )</td>
<td>( \left( \frac{2 - \delta}{2} \right) \left( \pi - \bar{s} \right) )</td>
</tr>
</tbody>
</table>

One observation that arises from the preceding table is that, when \( q_1 \to 1 \), the continuation value for the creditor approaches \( \frac{\delta \pi}{2} \) as in the case with complete information about debtor’s type.

Moving backwards, we now consider the bargaining at \( t = 2 \). There is an asymmetric information about the debtor’s type. Let us fix \( q_1 \) for which the continuation values for the creditors are as in the preceding table. The continuation values for the creditor and the debtor corresponding to such continuation belief of the creditor as to the debtor’s type are given by \( \frac{\delta (\pi - \bar{s})}{2} \) and \( \frac{\delta (\pi + \bar{s})}{2} \), respectively.

Fix \( \pi \in \{ \pi_L, \pi_H \} \). We will now construct a continuation PBE mixed strategy equilibrium where at least one of the two debtor types and the creditor randomize at \( t = 2 \). Note that for the creditor to randomize at \( t = 2 \), it must be the case that \( q_1 = \frac{\pi - \bar{s}}{\pi} \).

Suppose that the debtor is chosen to make an offer at \( t = 2 \). Let \((x_2, \pi - x_2)\) denote the offer made by the debtor. Suppose that \( \bar{x}_2 = \frac{\delta (\pi + \bar{s})}{2} \); then, by computation,

\[
\pi - \bar{x}_2 = \left( \frac{2 - \delta}{2} \right) \pi - \frac{\delta \bar{s}}{2} > \frac{\delta (\pi - \bar{s})}{2}.
\]

Let \( x'_2 \) be any positive number such that \( \pi - x'_2 < \frac{\delta (\pi - \bar{s})}{2} \).
The Optimistic debtor offers \((\tilde{x}_2, \pi - \tilde{x}_2)\) with a probability \((1 - \beta)\) and offers \((x'_2, \pi - x'_2)\) with a probability \(\beta\), while the Cautious debtor offers \((x'_2, \pi - x'_2)\) with a probability 1. The posterior belief of the creditor at \(t = 3\), which is \(q_1 = \frac{\pi - \tilde{s}}{\pi}\), can be obtained using the Bayesian updating rule: \(q_1 = \frac{\beta q_0}{\beta q_0 + (1 - q_0)}\), where \(q_0\) is the creditor’s belief at \(t = 2\) is simply her prior belief. It follows that

\[
\frac{\beta q_0}{\beta q_0 + (1 - q_0)} = \frac{\pi - \tilde{s}}{\pi}.
\] (A4)

Solving equation (A4) for \(\beta\) yields:

\[
\beta^*(\pi, \tilde{s}) = \left(\frac{\pi - \tilde{s}}{\tilde{s}}\right) \left(\frac{1 - q_0}{q_0}\right) > 0.
\]

Since we need \(\beta^*(\pi) \leq 1\), the following condition is required to be satisfied:

\[
\left(\frac{\pi - \tilde{s}}{\tilde{s}}\right) \left(\frac{1 - q_0}{q_0}\right) \leq 1,
\]

or, equivalently,

\[
\pi (1 - q_0) \leq \tilde{s} \iff q_0 \geq \frac{\pi - \tilde{s}}{\pi}.
\] (A5)

As long as \(q_0 > \frac{\pi - \tilde{s}}{\pi}\), \(\pi \in \{\pi_L, \pi_H\}\), there exists \(\beta \in (0, 1)\) which solves equation (A4). It follows that, for the creditor, by observing \(x'_2\),

\[
q_1 = \frac{\pi - \tilde{s}}{\pi}.
\]

Given \(q_1 = \frac{\pi - \tilde{s}}{\pi}\), creditor is indifferent between accepting \((\tilde{x}_2, \pi - \tilde{x}_2)\) and rejecting it and obtains his continuation value.

From condition (A5), we must have \(q_0 > \frac{\pi - \tilde{s}}{\pi}\). If the creditor is chosen to make the offer at \(t = 2\), the creditor offers \((\tilde{x}_2, \pi - \tilde{x}_2)\) and the Optimist debtor accepts the offer with probability \(\beta^*(\pi, \tilde{s})\) and Cautious Deboner rejects the offer with probability one.

Clearly, given the preceding computations, both debtor types are choosing a best-response and given the debtor’s strategy and the condition that prior probability satisfies the inequality \(q_0 > \frac{\pi - \tilde{s}}{\pi}\), the creditor cannot do better either. The condition that \(q_0 > \frac{\pi - \tilde{s}}{\pi}\) has the interpretation that the creditor attaches a sufficiently high probability to the debtor being an Optimist.

To summarize:

(i) Under the assumption that \(q_0 > \frac{\pi - \tilde{s}}{\pi}\) (as \(\frac{\pi - \tilde{s}}{\pi}\) is increasing in \(\pi\)), at \(t = 2\):

(a) If the debtor is chosen to make the offer: The Optimistic debtor offers \((\tilde{x}_2, \pi - \tilde{x}_2)\) with a probability \((1 - \beta^*(\pi, \tilde{s}))\) and offers \((x'_2, \pi - x'_2)\) with a probability \(\beta^*(\pi, \tilde{s})\). The Cautious debtor offers \((x'_2, \pi - x'_2)\) with a probability 1. The creditor accepts the offer \((\tilde{x}_2, \pi - \tilde{x}_2)\) with probability 1 and rejects the offer \((x'_2, \pi - x'_2)\) with a probability 1;
(b) If the creditor is chosen to make the offer: the creditor offers \((\bar{x}_2, \pi - \bar{x}_2)\) and the Optimist debtor accepts the offer with probability \(\beta^* (\pi, \bar{s})\) and Cautious Debtor rejects the offer with probability one.

(ii) At \(t = 3\), the creditor’s belief as to the debtor’s type is \(q_1 = \frac{\pi - \bar{s}}{\pi}\): The creditor’s payoff is \(\frac{\delta (\pi - \bar{s})}{2}\). The debtor’s payoff is \(\frac{\delta (\pi + \bar{s})}{2}\).

Moving to \(t = 1\), we calculate the continuation values for the creditor and both debtor types. The continuation payoff for the creditor is

\[
\hat{a} = q_0 \left[ (1 - \beta^* (\pi, \bar{s})) (\pi - \bar{x}_2) + \beta^* (\pi, \bar{s}) \frac{\delta (\pi - \bar{s})}{2} \right] + (1 - q_0) \frac{\delta (\pi - \bar{s})}{2} \tag{A6}
\]

\[
= \frac{\delta (\pi - \bar{s})}{2} + \delta q_0 (1 - \delta) (1 - \beta^* (\pi, \bar{s})) \pi.
\]

The continuation payoff for both debtor types is \(\frac{\delta (\pi + \bar{s})}{2}\).

In the first period, the sovereign debtor is a proposer. It follows, by computation, that the expected continuation payoff for the creditor (from rejecting the debtor’s offer at \(t = 1\)) is given by

\[
\delta E\hat{a} = \frac{\delta^2 (E\pi - \bar{s})}{2} + \delta q_0 (1 - \delta) E [(1 - \beta^* (\pi, \bar{s})) \pi]. \tag{A7}
\]

Next, we derive the conditions which make delay becomes attractive for each type of debtor. For both debtor types the best offer is the excess of the available bargaining surplus over his own continuation value, i.e. \(\pi_L - \delta \left[ \frac{\delta (E\pi + \bar{s})}{2} \right]\). If such offer falls below the expected payoff for the creditor, \(\delta E\hat{a}\), the debtor’s offer will not be accepted.

To show the correlation between delay and size of the haircut in sovereign debt, as already noted, for each \(\pi, \pi - \bar{x}_2 = \left( \frac{2}{\delta} - \frac{\bar{x}}{\pi} \right) \pi - \frac{\delta \bar{x}}{2} > \frac{\delta (\pi - \bar{s})}{2}\). Therefore, the creditor’s expected payoff from one period delay is greater than the creditor’s expected payoff from two period delay.