Prices and Contingent Prices as Incentives, with particular reference to aspects of the reward for labour

by

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Contents

List of Tables and Figures

Acknowledgements and Declaration

1. No part of this thesis has previously been submitted to this or any other university for any degree.

2. Chapters 2, 6, 7 and 8 of this thesis represent versions of papers written jointly with my colleague Peter J. Law, and are thus to be considered as 50% my own work. I am very grateful to Peter Law for his permission to use these papers in this thesis. A statement from Peter Law to this effect follows these acknowledgements.

3. Some of the chapters of this thesis have been or are to be published in some form, and have thus had the benefit of journal referees' and editorial comments. In particular:

   Chapter 6 was published in revised form in Economica as Ireland and Law (1978);
   Chapter 7 is forthcoming in the Journal of Comparative Economics as Ireland and Law (1980a);
   Chapter 8 is forthcoming in the Canadian Journal of Economics as Ireland and Law (1980b);
   Chapter 9 was published in the Review of Economic Studies as Ireland (1977).

4. Chapter 3 was presented to a seminar at the University of Manchester in December 1979 and Chapter 10 was presented to a seminar at the University of Durham, February 1980. Both chapters reflect comments of participants at these seminars.

5. Chapter 4 contains some arguments which I first discussed in a paper contributed to a summer school on property rights held at the Institut Universitaire International Luxembourg July 1979. The form of publication, if any, of the original paper has yet to be decided by I.U.I.L. who hold the copyright. A version of Chapter 3 has been accepted to be presented at the 2nd World Congress of the International Association for Economics of Self-management, in Istanbul, July 1980, and is then likely to be published by the I.A.E.S.M. in the Proceedings.

6. Chapter 2 has benefited from comments by John Bonin of Wesleyan University and participants of the Industrial Economics Workshop of Warwick University.

7. Chapters 1 and 5 have only limited originality and the scope of the contribution is indicated in the chapters.

8. Finally, I wish to gratefully acknowledge the help, discussion and suggestions that I received from my colleagues in the Department of Economics, University of Warwick; in particular my collaborator Peter Law and my chairman, Avinash Dixit.

N. J. Ireland

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I confirm the above declarations in respect of joint research that I have carried out with N.J. Ireland.

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Summary

The first five chapters investigate the incentive effects of the reward for labour being inseparably linked to that for entrepreneurship in the labour-managed firm. After an initial analysis of the standard model of such a firm in Chapter 1, Chapter 2 attempts to investigate many issues influencing productivity and efficiency by considering the work-leisure allocation of time as a choice variable. Chapter 3 argues that within the incentive structure there is a direct utility effect arising from the work environment. Chapter 4 considers the effect of different property rights structures on firm behaviour and Chapter 5 discusses the implications of price uncertainty. In these chapters we question the established pessimism concerning the "smallness" and perversity of labour-managed firms by demonstrating under reasonable assumptions (i) higher effort of workers for higher product prices (Chapter 2), (ii) the work-environment effect of reduced worker alienation (Chapter 3) and (iii) the risk-spreading behaviour caused by uncertainty (Chapter 5).

In Chapter 6 an incentive scheme to aid the mobility of labour in a labour-managed economy is described and extended to the consideration of individual labour supply. A contrast to the incentive structure of labour-managed firms is considered in Chapter 7 which analyses efficiency aspects of Soviet incentive bonuses of the Kosygin reforms of the mid 1960s with the conclusion that their early amendment was predictable. Chapter 8 investigates the incentive effects of limiting private plot size in a simple collective farm model. This question does not seem to have been considered, yet such restrictions were applied by Kruschev in the late 1950's. Chapters 9 and 10 eschew assumptions of perfect knowledge of planners and consider the relative advantages of using prices, and thus profit incentives, over quotas in a second-best world. The basic model in this area (Weitzman (1974)) is extended in a number of directions.

Introduction

In the first part of the thesis, we will consider a competitive labour market as the standard of comparison in order to investigate payments for labour in other systems, particularly one where workers fulfil a group entrepreneurship rôle, taking group decisions concerning the hire of factors of production, output levels and so forth. This system has been the subject of considerable analysis based on the early work of Ward (1958) and Domar (1966) and developed by Vanek (1970) and Meade (1972). In its simplest form, the objective function of a firm in this system- a labour-managed firm - is taken to be the maximisation of the per worker excess of revenue over non-labour cost. The payment received for a worker-member's labour is thus contingent upon the group entrepreneurial decisions, incentive effects influencing how hard workers work, and any stochastic factors such as uncertain demand conditions.

In Chapter 1, we investigate entrepreneurial decisions for such a firm. As workers take these decisions in order to maximise their individual income, it has been argued that their behaviour will be such as to equate the marginal and average net revenue products of labour by fixing membership size. This not only has the effect of making the firm smaller than its conventional profitable counterpart but also leads to some well-known perverse comparative-static results. We see in Chapter 1 that the picture becomes more complicated when entrepreneurial decisions concerning other factors of production are taken concurrently, and we attempt to offer a characterisation in terms of the changes in marginal products along a linear expansion path in input space emanating from the origin. The differences in the behaviour of the labour-managed firm from that of the conventional firm derive of course from the joint labour/entrepreneur role of a member of a labour-managed firm.

The fact that worker- members in labour-managed firms are working for themselves gives rise to incentive effects in terms of an individual's supply of labour. These effects have been investigated by Sen (1966) and a number of recent papers, for example Bonin (1977a) and Berman (1977). The size of the incentive effects is seen to depend on the interdependency of individuals' supply decisions. This can be expressed in a number of ways, which are compared in Chapter 2.

Two additional complications to the working of the incentive structure of labour-managed firms are the subjects of Chapters 3 and 4. In Chapter 3, it is argued that as well as having an incentive effect via the reward for labour, the labour-managed firm may well present a more amenable and pleasanter working environment. Less disutility of work at the margin may imply a different individual labour supply decision. The interesting question arising from both the incentive and work environment effects on individual labour supply, is how far this will effect the optimal entrepreneurial decisions as to size of membership, hire of other factors of production, level of output, etc.

A second complication to the working of the incentive structure in labour-managed firms is the likely property rights structure in such firms. There is no unique structure of property rights observed in all instances of labour-managed firms, but the Yugoslav case, which has been documented by Furubotn and Pejovich (1973), is an example where, it is claimed, the property rights structure leads to disincentives to invest in labour-managed firms. Chapter 4 contains a short discussion of some property rights issues. The object of the chapter is to emphasise the existence of not only a property rights structure for the firm's capital assets, but also for firm membership.

The assumption of no uncertainty is relaxed in Chapter 5 where uncertainty in both product price and fixed cost is analysed. A parallel is drawn between the influence of uncertainty on a labour-managed firm's membership level decision and the perverse comparative static results noted in Chapter 1.

In addition to the effects of a contingent contract, rather than a wage, for labour within a single labour-managed firm, there are further problems relating to economic systems based on such firms. One is the "Ward-Domar" problem concerning the lack of mobility of labour in such a system. Another is the absence of any short-term mechanism for full employment. Both these problems emanate from property rights of firm membership and are touched on in Chapter 4. In Chapter 6, however, an incentive fund scheme is outlined as a partial solution to these problems. The scheme is extended in a number of ways, one of which is the case of variable individual labour supply.

A technical problem in seeking the maximum net income per worker is that the latter may not be a well-behaved function of rights observed in all instances of labour-managed firms, but the Yugoslav case, which has been documented by Furubotn and Pejovich (1973), is an example where, it is claimed, the property rights structure leads to disincentives to invest in labour-managed firms. Chapter 4 contains a short discussion of some property rights issues. The object of the chapter is to emphasise the existence of not only a property rights structure for the firm's capital assets, but also for firm membership.

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A technical problem in seeking the maximum net income per worker is that the latter may not be a well-behaved function of membership level. A similar point is picked up in Chapter 7 when also the efficiency implications of using bonus functions for decentralising planned economiesare analysed. The particular bonus function considered is that pertaining to the 1965 reform in the Soviet Union.

The Soviet collective farm is the subject of Chapter 8 where the allocation of time between leisure, work on private plots and work on collective land is the behaviour that can be influenced by planners' controls. The effect of limiting the size of private plots, either by a fixed quota or by a quota which is related to time worked on collective land, is analysed.

The final two chapters of the thesis relate to the debate concerning the relative efficiency of price and quantity planning controls in an economy with uncertainties in marginal benefit and marginal cost. Here the decentralised managers have more information concerning cost than the planners. However, if set a price and rewarded according to profits, they may well over-react, in a social welfare sense, in adjusting output level to that which equates marginal cost and price. This is because at, for instance, lower levels of output marginal expected benefit may be relatively high, if the marginal expected benefit schedule is downward sloping. On the other hand if the managers are simply set a quantity to produce, they will not react at all to additional information about costs. This problem was first solved by Weitzman (1974). However, it is shown in Chapter 9 that a simple price schedule is superior in expected welfare terms to either a pure price or quantity schedule. A further problem with both price and quantity (of output) controls is that neither ensures full employment of resources. Planning controls using relative product price to resource price levels and which do ensure full employment of resources are compared with direct resource allocation in Chapter 10.

The common thread of all the topics outlined above is that of the effect on resource allocation to productive enterprises of various incentive systems. Although most emphasis has been placed on the incentive system defined by the concept of the labour-managed firm, variations to decentralised planning controls such as managerial bonus functions and quantity restrictions have been included. Within the latter context, the analysis in Chapters 9 and 10 of the welfare effects of different decentralised planning controls in second-best situations is an important topic.

CHAPTER 1

The Illyrian labour-managed firm

An orthodox entrepreneurial profit maximising firm in competitive markets, with no uncertainty, and with access to rentier capital, maximises profit Π defined by

$$\Pi = pQ(K, N) - wN - rK \tag{1.1}$$

where

p = product price

$Q(K, N)$ = twice differentiable production function with positive marginal products and negative definite hessian in the locality of solutions

N = number of workers

w = wage per time period

K = quantity of capital

r = rental per unit of capital per time period.

An equivalent labour-managed firm (LM-firm) is assumed to maximise the income per worker-member in the firm, that is to maximise

$$y = (p\,Q(K, N) - rK)/N \tag{1.2}$$

The income per worker y is simply revenue net of all non-labour costs shared equally among the worker-members. In this chapter, we will assume, unless otherwise stated, that both capital and labour are freely and costlessly adjustable to their optimal values. Also both inputs are homogeneous and of the same quality in the two types of firm.

We can compare the LM-firm with the profit-maximising firm (PM-firm) by comparing the equilibrium conditions for maximising (1.1) and (1.2). We can also assess differences in the responses of the two firms to parametric changes in the product price and the capital rental. This comparative analysis is well-known and develops from Ward (1958), Vanek (1970) and Meade (1972). We will, however, attempt characterisations,of some of the comparisons, which do not appear to have been published, although we cannot claim originality with any conviction.

The equilibrium of the PM-firm is given by maximising (1.1) with respect to the variables K and N. First-order conditions are thus

$$pQ_K - r = 0 \tag{1.3}$$

$$pQ_N - w = 0 \tag{1.4}$$

while for the LM-firm, first-order conditions for maximising (1.2) with respect to K and N are

$$pQ_K - r = 0 \tag{1.5}$$

$$pQ_N - y = 0 \tag{1.6}$$

Second-order conditions for the two firms are the same, i.e. that the matrix

$$H = \begin{bmatrix} Q_{KK} & Q_{KN} \\ Q_{NK} & Q_{NN} \end{bmatrix}$$

is negative definite.

Suppose the wage rate w for the PM-firm equilibrium is such that it is exactly equal to the maximum income per worker in the LM-firm, i.e. $w = y$. Then the equilibrium conditions (1.3) and (1.4) are identical to (1.5) and (1.6). Now consider the behaviour of the two systems of equilibrium conditions as the wage rate w varies. Obviously the LM-firm equilibrium is unchanged. Comparative statics on the PM-firm's equilibrium yields

$$\begin{bmatrix} dK \\ dN \end{bmatrix} = \frac{1}{p} H^{-1} \begin{bmatrix} 0 \\ dw \end{bmatrix}$$

i.e.
$$\begin{bmatrix} dK \\ dN \end{bmatrix} = \frac{dw}{p \cdot |H|} \begin{bmatrix} -Q_{KN} \\ Q_{KK} \end{bmatrix} \tag{1.7}$$

As H is negative definite, $|H| > 0$ and $Q_{KK} < 0$ and $\frac{dN}{dw} < 0$. Also the sign of $\frac{dK}{dw}$ is given by the sign of $-Q_{KN}$. If $Q_{KN} > 0$, and the inputs are <u>complementary</u> then $\frac{dK}{dw} < 0$ and we also have $\frac{dQ}{dw} < 0$. Thus in the case of complementary inputs we have:

if $y < w$ (arising from $dw > 0$) then capital, labour and output are greater in the LM-firm than in the PM-firm;

if $y > w$ (arising from $dw < 0$) then capital, labour and output are less in the LM-firm than in the PM-firm.

Even if $Q_{KN} \leq 0$, the same ranking of labour inputs can be made. Also if $Q_{KN} = 0$, the capital levels are the same in the two types of firm and the labour input and the output levels are ranked as above. Furthermore the same labour input and output ranking holds if capital is fixed and not variable, provided it is fixed at the same level in the two types of firm.

If $w = y$ then $\Pi = 0$. Also $\frac{d\Pi}{dw} = -N < 0$. Thus if $w \gtrless y$, then $\Pi \lessgtr 0$, and the condition $y < w$, $y > w$ in the above can be replaced by $\Pi < 0$, $\Pi > 0$ respectively. This particular technique of changing w to yield comparative static results was used to compare advertising behaviour in Ireland and Law (1977).

As capital and labour being complementary appears a reasonable assumption and as this is a sufficient condition for the LM-firm to employ lower (higher) levels of both inputs and produce less (more) output than the PM-firm when profit is positive (negative), it has been argued that the conclusions to be drawn from this analysis are that:

(i) In long run equilibrium (when profit is zero) the two firms will be identical.

(ii) The short-run is likely to be characterised by positive profits and thus smaller LM-firms than PM-firms.

Also if capital is fixed (at the same level) then the above conclusions hold without the assumption of complementary inputs.

The LM- and PM-firms can also be compared in terms of their response to the common parameters r and p. It is well-known (see Intrilligator (1971)) that for the PM-firm, output will increase for an increase in product price. Thus at least one input must be superior (the demand for this input increases with product price). An increase in an input price will reduce optimal output if the input is superior and increase it if the input is inferior.

Now consider the response of the LM-firm. Totally differentiate (1.5) and (1.6) and we obtain

$$pQ_{KK}\,dK + pQ_{KN}\,dN = dr - Q_K\,dp \tag{1.8}$$

$$pQ_{NK}\,dK + pQ_{NN}\,dN = (\frac{Q}{N} - Q_N)dp - \frac{K}{N}\,dr \tag{1.9}$$

Now $Q_K = \frac{r}{p}$ from (1.5) and $\frac{Q}{N} - Q_N = \frac{rK}{pN}$ from (1.6). We can thus solve for dK and dN as

$$dK = \frac{1}{p|H|}\left\{Q_{NN}(dr - \frac{r}{p}\,dp) + Q_{NK}(\frac{K}{N}dr - \frac{rK}{pN}\,dp)\right\} \tag{1.10}$$

$$dN = \frac{-1}{p|H|}\left\{Q_{KN}(dr - \frac{r}{p}\,dp) + Q_{KK}(\frac{K}{N}dr - \frac{rK}{pN}\,dp)\right\} \tag{1.11}$$

Consider first the case where $dp > 0$, $dr = 0$

$$\frac{dK}{dp} = \frac{-r}{p^2|H|N}(Q_{NN}\,N + Q_{NK}\,K) \tag{1.12}$$

$$\frac{dN}{dp} = \frac{r}{p^2|H|N}(Q_{NK}N + Q_{KK}K) \tag{1.13}$$

Both (1.12) and (1.13) are ambiguous in sign. Basically inspection of (1.9) if dK is constrained to be zero yields $\frac{dN}{dp} = rK/(p^2N\,Q_{NN}) < 0$ where $dr = 0$. The introduction of capital variability will reinforce this if the inputs are substitutes ($Q_{NK} < 0$) and counteract it if they are complements. Also inspection of (1.8) if dL is constrained to be zero yields $\frac{dK}{dp} = -r/(p^2\,Q_{KK}) > 0$. The introduction of labour variability reinforces this if the inputs are substitutes and counteracts it if they are complements. Before assessing the likely signs of (1.12) and (1.13) and also of $\frac{dQ}{dp}$, it will be useful to consider the other case; that when $dp = 0$ and $dr > 0$. We then obtain from (1.10) and (1.11):

$$\frac{dK}{dr} = \frac{1}{p|H|N}(Q_{NN}N + Q_{NK}K) \tag{1.14}$$

$$\frac{dN}{dr} = \frac{-1}{p|H|N}(Q_{KN}N + Q_{KK}K) \tag{1.15}$$

Obviously $\frac{dK}{dr}$ and $\frac{dN}{dr}$ are both of opposite signs to $\frac{dK}{dp}$ and $\frac{dN}{dp}$ respectively, and their signs depend on the same bracketed terms. In fact this can also be seen by noting that K, N and indeed Q are homogeneous of degree zero in (p, r). This is because maximum income per worker is homogeneous of degree 1 in (p, r), so that the same input levels are optimal if for instance p and r both double. The implication of input demands being homogeneous of degree zero in (p, r) is that $K(p, r)$ has the property

$$\frac{\partial K}{\partial p} p + \frac{\partial K}{\partial r} r = 0$$

and similarly

$$\frac{\partial N}{\partial p} p + \frac{\partial N}{\partial r} r = 0 \text{ .}$$

Expressions such as $Q_{NN} N + Q_{NK} K$ occur in other fields of analysis. Most interestingly they appear in the analysis of the regulated firm. For instance Baumol and Klevorick (1970) focus attention on the sign of such an expression which needed to be negative for the capital-labour ratio to increase in a regulated firm when the "regulation" was tightened (maximum rate of return on capital reduced). They took the view that such negativity "is obviously not necessarily true, nor is it easily interpretable", (Baumol and Klevorick (1970) p. 179). However a simple interpretation is that if Q_N decreases for a one per cent increase in both capital and labour then $Q_{NN} N + Q_{NK} K < 0$. To see this, describe

$$Q_N = Q_N(\phi N, \phi K)$$

and differentiate with respect to ϕ , i.e. increase labour and capital inputs by the same proportion. Similarly if Q_K decreases then $Q_{KN} N + Q_{KK} K < 0$. We argue that marginal products reducing with scale increases provides a reasonable basis for assuming negativity of these expressions and thus our allocation of signs for (1.12) to (1.15) would be

$$\frac{dK}{dp} > 0$$

$$\frac{dN}{dp} < 0$$

$$\frac{dK}{dr} < 0$$

$$\frac{dN}{dr} > 0$$

Of course there exist production functions which would invalidate this allocation. Note, however, that the example of Baumol and Klevorick (1970, p. 179) is inadmissable as revenue is negative for all non-trivial input levels, as it is a negative definite quadratic form in the input levels.

A genuine counter example to the above allocation of signs would be

$$Q = K \log (N + a) + b \log (K + a)$$

$a > 0$ and r is such that locally $\frac{bK}{(K + a)^2} > 1$ (for second order conditions). Then

$$Q_{NN} N + Q_{NK} K > 0$$

and $\frac{dK}{dp} < 0$ and $\frac{dK}{dr} > 0$. Opposite signs to those allocated for $\frac{dN}{dp}$ and $\frac{dN}{dr}$ can be obtained if capital and labour are interchanged in the example. Note however that at least one pair of signs must be correct as we cannot have both of $(Q_{NN} N + Q_{NK} K)$ and $(Q_{NK} N + Q_{KK} K)$ positive as this would imply that $Q(K, N)$ was not locally concave, contrary to assumption.

In order to examine how output changes with respect to a change in product price or capital rental, consider first the cases when one of capital or labour is fixed. Then if capital is fixed, we have $\frac{dN}{dp} < 0$ and $\frac{dN}{dr} = -\frac{p}{r}\frac{dN}{dp} > 0$, so that output will decrease with product price and increase with capital rental. On the other hand, if labour is fixed and capital is variable, we have $\frac{dK}{dp} > 0$ and $\frac{dK}{dr} < 0$, and thus output will increase with product price and decrease with capital rental. Thus the picture is of a "normal" response to the parameter changes if labour is fixed and a perverse response if capital is fixed. If neither is fixed, then we can describe the change in output as

$$\frac{dQ}{dp} = Q_K \frac{dK}{dp} + Q_L \frac{dN}{dp}$$

and

$$\frac{dQ}{dr} = Q_K \frac{dK}{dr} + Q_L \frac{dN}{dr}$$

Substituting in (1.12) to (1.15) and rearranging yields

$$\frac{dQ}{dp} = Q_K Q_N \frac{r}{p^2|H|N} \left\{ \frac{Q_{KK} K + Q_{KN} N}{Q_K} - \frac{Q_{NN} N + Q_{NK} K}{Q_N} \right\}$$

and $\frac{dQ}{dr} = -\frac{p}{r}\frac{dQ}{dp}$

Writing

$$Q_K = Q_K(\phi N, \phi K)$$

$$Q_N = Q_N(\phi N, \phi K)$$

we then have

$$\frac{dQ}{dp} = Q_K\, Q_N \frac{r}{p^2|H|N}\left\{\frac{d \log Q_K}{d\phi} - \frac{d \log Q_N}{d\phi}\right\}$$

so that $\frac{dQ}{dp} \gtreqless 0$ as $\frac{d \log Q_K}{d\phi} \gtreqless \frac{d \log Q_N}{d\phi}$.

Thus if on a linear expansion path from the origin, increasing capital and labour both by one per cent decreases the marginal product of capital less than it decreases the marginal product of labour then output will increase. The output response to a change in price (and thus also to a change in capital rental) is not necessarily perverse when both factors of production are variable. An interesting special case would be if the production function were locally homothetic, i.e.

$$h(\phi)\, Q(K, N) \equiv Q(\phi K, \phi N) \qquad \text{all } \phi \text{ in } (1 - \varepsilon, 1 + \varepsilon)\varepsilon > 0$$

then differentiating with respect to K and N yields

$$\frac{h(\phi)}{\phi} Q_K = \frac{\partial Q(\phi K, \phi N)}{\partial \phi K}$$

and $$\frac{h(\phi)}{\phi} Q_N = \frac{\partial Q(\phi K, \phi N)}{\partial \phi N}$$

Thus the marginal products are homothetic and their ratio is homogeneous of degree zero. Thus $\frac{d \log(Q_K/Q_N)}{d \log \phi} = 0$ and $\frac{dQ}{dp} = \frac{dQ}{dr} = 0$.

We have seen that the possibility of perverse output responses comes from (1.6), that is the condition for optimal labour input in the LM-firm. The intuition behind this is easily seen when capital is fixed. Then if product price goes up marginal revenue is reduced less (proportionately) than the income per worker. As the marginal revenue is the benefit to the LM-firm from the marginal worker, while the income per worker is the cost, optimal membership is reduced. Similarly, an increase in capital rental reduces the cost of the marginal worker (income per worker) while not affecting the benefit (the marginal revenue). Thus in this case optimal membership is increased.

All we have done here is to survey some basic analysis which we will wish to call upon later. We have assumed the existence of an interior equilibrium for the LM-firm. Due to the inherently unstable properties of ratios such as income per worker, this is in fact a considerable assumption even granted local concavity of the production function. We will have more to say on a related point in a later chapter (Chapter 7). Here we simply note that a production function which for example is homogeneous of degree $v < 1$ will permit no interior global maximum of income per worker when both labour and capital can be varied. To see this, simply note that

$$y(\phi) = \frac{\phi^v Q}{\phi N} - r\frac{K}{N}$$

$$= \frac{Q}{\phi^{1-v} N} - r\frac{K}{N}$$

As $\phi \to 0$, $y(\phi) \to \infty$, whereas at $\phi = 0$, income per worker is not defined, and there will be no interior solution to the first-order conditions, (1.5) and (1.6).

CHAPTER 2

Efficiency, Incentives and Individual Labour Supply in the Labour-Managed Firm

Although many writers on the theory of the labour-managed or co-operative firm have followed Ward (1958) in assuming that the labour input of the firm can be varied only by varying the number of workers, papers such as those of Sen (1966), Bradley (1971), Cameron (1973), Markusen (1975; 1976), Bonin (1977) and the analysis of Vanek (1970, Chapter 12), embody the assumption that the individual worker's (or individual household's) supply of hours can be varied. The recent contributions of Berman (1977) and Berman and Berman (1978) follow the latter practice in assuming that in the short run membership of the firm is fixed but hours worked per member can be varied, whereas in a planning period longer than the short run it is assumed that membership can be varied with the proviso that there are no involuntary expulsions. This approach seems a sensible attempt to increase the relevance of the theory of the labour-managed firm by eschewing the assumption of short-run membership variability and recognising that "the ability of the firm to control and vary hours of work to adjust the labour input should be considered an integral part of the worker-managers' decision-making authority". (Berman and Berman (1978)).

The present chapter has three main objectives. First we will show that some of the results of Sen (1966), Berman (1977) and Berman and Berman (1978) can be related in a number of respects and a richer analysis produced. Secondly, with the aid of the augmented model thus derived, we will explore a number of comparative statics questions. In so doing we dispute Berman and Berman's (1978) claim that the short-run effect of a rise in output price in the labour-managed firm will be for workers to increase hours worked, given the usual assumptions on income and substitution effects. Like Bonin (1977) we find that the effect is ambiguous and, we argue, in some circumstances quite likely to be perverse. We also consider comparative statics responses to price changes in the medium term in which membership is also variable. There has been considerable interest in such questions since Ward's (1958) demonstration that where labour is the only variable factor of production the optimal number of members in a co-operative will adjust perversely to a change in output price. The general analysis of the responses of the co-operative to changes in product price is contained in Section II. In Section III, following Sen (1966), needs payments are introduced so that the income of a co-operative member consists of two parts, one related to work done by the member and one to the member's needs. The responses of the co-operative to change in fixed costs and in needs payments are analysed in this section. Some of the analysis of the preceding sections is illustrated in Section IV for a specific simple utility function by means of diagrams. The importance of membership constraints is also discussed.

Our third objective is the examination of a number of efficiency questions and allied topics. Short-run, medium-term and long-run efficiency issues are discussed in Section V.

The conclusions of our analysis are briefly presented in a final section. Some of the more tedious mathematical derivations are relegated to the Appendix .

I. The Model

Our model will follow that of Sen (1966) in assuming that the co-operative consists of N families (or individuals) with identical tastes. However, given our interest in comparative statics, the product price will not be set equal to unity and it is assumed initially that the only income of a co-operative member is work income. Each member, i, has a quasi-concave utility function $U^i(y^i, \ell^i)$ where y_i is the member's income and ℓ_i the hours worked by the member and the usual restrictions $U^i_y > 0$, $U^i_\ell < 0$ (where U^i_ℓ is the marginal utility of an additional hour's work at a given income y^i), $U^i_{yy} < 0$, $U_{\ell\ell} < 0$, $U^i_{y\ell} \leq 0$, hold. $U^i_{y\ell} \leq 0$ is assumed as it is felt that the marginal utility of <u>leisure</u> would not decrease with income. Sen (1966, p.363) explains that "families are not <u>necessarily</u> indifferent to the happiness of other families (though they might also be that) and their notion of "social welfare" takes into account the utility of other families". Thus individual j attaches the weight a_{ij} to the utility of member i and the criterion function which member j seeks to maximise is

$$W^j = \sum_{i=1}^{N} a_{ij} U^i \tag{2.1}$$

where $a_{jj} = 1$ (all j) and $0 \leq a_{ij} \leq 1$.

The i^{th} member's income is obtained from the distribution of the co-operative's surplus, R, of revenue, PQ, over non-labour costs, $\sum_{k=1}^{m} F^k P^k$, according to i's relative input ℓ^i/L.

$$y_i = \frac{R\ell^i}{L} = (PQ - \sum_{k=1}^{m} F^k P^k) \frac{\ell^i}{L} \tag{2.2}$$

The co-operative is a price-taker at output price P and output Q is determined by the well-behaved production function, $Q = Q(L, F^1, F^2, \ldots F^m)$ where $L = \sum_{i=1}^{N} \ell^i$. $Q_L > 0$ and $Q_{LL} < 0$. Much of our analysis will make the assumption that the non-labour inputs are fixed. P^k is the price per unit of F^k, the k^{th} non-labour factor input.

Berman (1977, p.313) shows that if workers collude to choose a common ℓ to maximise $U^i(y^i,\ell^i)$ or more generally, where hours worked can vary reflecting taste differences among members, if each member behaves so that the elasticity of total hours worked with respect to hours worked per member is unity then this will yield the result

$$\frac{-U_\ell^i}{Uy^i} = PQ_L \qquad \text{(all } i) \tag{2.3}$$

Berman describes (2.3) as "the marginal conditions for the short-run Pareto-optimal allocation of labour in the productive process". The same result is derived in Berman and Berman (1978) by maximising the utility of a "typical worker" in the labour-managed firm. Cameron (1973, p.18) explains that (2.3) will result under identical tastes and certainty such that "each household is aware that its every move is accompanied by similar moves of all ... other households" and Bonin (1977, pp.81-82) offers some interpretations of the same equilibrium condition in the context of the Bradley (1971)-Cameron (1973) debate on models of the Soviet collective farm. Equation(2.3)is also a condition for social welfare maximisation in the Sen (1966) model and (with modification to allow for productivity differences among workers) is an efficiency condition in the Markusen (1975; 1976) papers.

We can easily show that the important result stated in (2.3) still holds if each worker considers choosing a common value of ℓ when the criterion function is W^j rather than U^j. The first-order condition for a maximum of W^j with respect to $\ell^j = \ell$ (all j) is

$$\sum_{i=1}^{N} \left\{a_{ij}(U_y^i PQ_L + U_\ell^i)\right\} = 0 \tag{2.5}$$

As we assume that all individuals have identical tastes (2.3) satisfies (2.4) for any j. Moreover if the N-square matrix $\{a_{ij}\}$ is non-singular this is the only solution. Thus all individuals will desire the same value of the common labour input and this will satisfy (2.3) for all members irrespective of the weights a member attaches to the utilities of fellow workers.

As an alternative to this collusion analysis both Berman (1977) and Sen (1966) consider a Nash or Cournot equilibrium where each worker decides on his labour hours on the assumption that other members will not make any change in their labour input in response to that decision. Berman's worker chooses ℓ^j to maximise U^j whereas Sen's maximises W^j. Sen restricts a_{ij} so that $(1/N) \sum_{i=1}^{N} a_{ij} = S^j = S$ (all j) and $(1/N) \sum_{j=1}^{N} a_{ij} = T^i = T$ (all i). The measures S and T respectively reflect sympathy felt for others and sympathy received from others and both lie in the closed interval $[1/N, 1]$. A Nash equilibrium will be defined by

$$U_y^j \left\{ SPQ_L + (1-S)\frac{Y}{L} \right\} + U_\ell^j = 0 \quad (\text{all } j) \tag{2.5}$$

Note that if $S = 1$, the case of perfect sympathy, then as Sen (1966, p.365) indicates, (2.5) will satisfy the key optimality condition which we have stated as (2.3) above. Setting $S = 1/N$ in (2.5) yields a result which corresponds to maximising U^j as in Berman's (1977) analysis and it is easily seen that this fails to satisfy (2.3) except, trivially, if $N = 1$.

It is perhaps not surprising that the same labour supply conditions hold for the case of collusion of workers to choose a common ℓ and for the case of perfect sympathy such that each member attaches the same weight to the

the utility of every other member as he does to his own.

II. Comparative Statics : Product Price Change

We will consider three kinds of adjustment period in our model. The number of labour hours supplied by each worker is assumed variable in the short run. The number of members of the co-operative is assumed variable in the medium term and other factors of production, $F^1, F^2, \ldots F^m$, are assumed variable only in the long run. Also in the long run there is entry and exit to and from the industry of co-operative and/or entrepreneurial firms in pursuit of higher income per member or higher profits.

First let us consider the comparative statics of a change in product price in the short run. We will assume that the effect of a change in product price on the co-operative member's evaluation of money income and hours worked in terms of utility is negligible so that real income and money income do not diverge.

In the short run only the ℓ^j are variable and under the identical tastes assumption all workers will have the same U^j_y, U^j_ℓ for any product price P. Thus the comparative statics can be examined by considering equation (2.5) for just one individual. Recall that for the collusion case, as opposed to the Nash equilibrium, we can simply set $S = 1$ in which case (2.5) yields (2.3) above. Differentiation of (2.5) with respect to P yields

$$\frac{d\ell^j}{dP} = \frac{Q}{-\Delta L}\left[U_{yy}^{\ j}\left\{SPQ_L\ell^j+(1-S)\frac{R}{N}\right\} + U_{\ell y}^{\ j}\ell^j + U_y^j\left\{1-S\left(1-\frac{LQ_L}{Q}\right)\right\}\right] \qquad (2.6)$$

where $\Delta < 0$ is assumed. (see appendix)

Obviously $d\ell^j/dP$ can be positive or negative. However, some progress can be made in the analysis of (2.6) if we consider $d\ell^j/dw$ where w is the wage rate and $w\ell^j$, the income, of an identical worker employed for wage income such that $w\ell^j = R/N$. In this case when $U^j(w\ell^j, \ell^j)$ is maximised it can be shown that

$$\frac{d\ell^j}{dw} = \frac{1}{-\Delta_w}\left(U_{yy}^{\ j} y^j + U_{y\ell}^{j}\ell^j + U_y^{\ j}\right) \tag{2.7}$$

where $\Delta_w < 0$ from second-order conditions. It will be assumed that (2.7) is positive; that is the one positive component $U_y^{\ j}$ (the substitution effect) outweighs the negative components (the income effect) in the numerator.

Note that $d\ell^j/dw > 0$ in (2.7) does <u>not</u> imply $d\ell^j/dP > 0$ in (2.6) for the case where $S = 1$ contrary to Berman andBerman's (1978, p.704) claim that a rise in price causes workers to increase labour input "on the usual assumption of substitution effects outweighing income effects". When $S = 1$ sign $d\ell^j/dP$ = sign $(U_{yy}^{\ j}PQ_L\ell^j + U_{\ell y}^{\ j}\ell^j + U_y^j LQ_L/Q)$ and if $Q_L < Q/L$ this is of indeterminate sign. Bonin's (1977) results confirm this ambiguity. His model corresponds to the collusion case but is complicated by the existence of a private, as well as a collective, output. However the ambiguous response of an individual's labour input on the collective crop to changes in the price of collective output, reported by Bonin (p.82) clearly persists if there is no private output. (In any case an increase in the price of the collective crop reduces an individual's labour hours on the private plot). Vanek (1970, pp.251-2) also discusses the possibility of a "backward bending supply of effort" in the short run.

There is one obvious case of a Nash equilibrium when a positive sign for (2.7) implies the same sign for (2.6). This is the case when S is small $(S \approx 1/N)$ and N is large, that is when the co-operative has a large membership and each member has small sympathy for fellow members. In these circumstances individual j's independent decision to work an extra hour will have little effect on average income per hour. Moreover, as j is only significantly concerned with his own income he ignores any effect of his extra hour's labour on the income of other members. Under these circumstances it is to be expected that his labour supply response is determined by the same criterion as that of the individual wage earner whose hours of work have no significant influence on the market wage rate.

In other cases $d\ell^j/dP$ is ambiguous but the picture is clearer, as we show below, if we assume that the co-operative is initially in a position of medium-term equilibrium with a membership level such that income per member is maximised. While this concept of equilibrium membership is familiar from 'individualistic' models of the labour-managed firm and is due to Ward (1958) its use in an analysis incorporating sympathy deserves further discussion. Specifically we are assuming that S does not vary with N during any adjustment as the firm moves from one medium-term equilibrium to another in response to (say) a price change. This might be rationalised by arguing that membership increases are evaluated only from the point of view of the initial membership (of course once at the new equilibrium a new value of S may obtain) and membership reductions are evaluated only from the point of view of remaining members. As we rule out involuntary expulsion no sympathy need attach to those who leave. More complex treatments of the relationship between S and N are not pursued here. A related point which may be noted is that our concept of medium-term equilibrium implies that the size of membership is not an argument in the utility function, U^i, of the worker or the welfare index of the firm (for an alternative view see Furubotn (1976) and the related discussion in Berman and Berman (1978)).

If N is treated as a continuous variable then, as is well known from the work of Ward (1958) and others, maximisation of R/N implies

$$PQ_L \ell^j - \frac{R}{N} = 0 \quad \text{(all } j) \tag{2.8}$$

Thus, from (2.6) in the neighbourhood of medium-term equilibrium sign $d\ell^j/dP$ = sign $\left[U^j_{yy} y^j + U^j_{y\ell} \ell^j + U_y{}^j \left\{1-S(1-LQ_L/Q)\right\}\right]$ an expression which can easily be compared with the numerator in (2.7) which we assume to be positive. Now from (2.8) $Q_L < Q/L$ and so there is a disparity between the two expressions which makes $d\ell^j/dP$ ambiguous in sign. By solving from (2.8) to obtain

$$1-S\left(1 - \frac{LQ_L}{Q}\right) = 1 - \frac{S}{PQ} \sum_{k=1}^{m} F^k P^k \tag{2.9}$$

it can be seen that when $S = 1$, corresponding to the cases of collusion and perfect sympathy the coefficient on U_y (the only positive term) in the numerator of (2.6) is less than that in the expression in the numerator of (2.7) by a factor given by the share of revenue going to labour.

Note that we are still considering a short-term analysis as N is fixed, albeit at an initially optimal level. The disparity might be greater or less for a different N.

An example of a utility function for which $d\ell^j/dP$ is in general negative is $U = y^\alpha(1-\ell)^\beta$. In this case from (2.7) $d\ell^j/dw = 0$, and making the necessary substitutions in (2.6) and using (2.5) yields

$$\frac{d\ell^j}{dP} = \frac{\alpha SPQ}{\Delta R} Q_L y^{\alpha-1}(1-\ell)^{\beta} \sum_{k=1}^{m} F^k p^k < 0 \tag{2.10}$$

Oi and Clayton (1968, p.38) argue, in their producer co-operative model of the Soviet collective farm, that "a sufficient condition for a completely inelastic supply of labor by each worker is that the utility function (with leisure and money income as arguments) be of the Cobb-Douglas form". However it is evident from (2.10) that individual labour supply would be completely inelastic with respect to price only if $S \simeq 1/N \simeq 0$. Thus in general, for $0 < S \leq 1$, (i.e. for some sympathy or collusion) if all workers have the same Cobb-Douglas utility function individual labour hours contract as product price rises. Of course with a different utility function such that (2.7) is positive less strong results would be possible. Nevertheless there appears ample reason to believe that perverse short-run responses of hours worked to an increase in product price are possible.

Now let us consider the medium term in which membership, N, as well as hours worked per member, ℓ^j, are variable. Individuals decide their own ℓ^j and also vote on whether vacancies should be replaced or membership expanded. As all workers are still assumed to be identical and to know the ℓ^j decisions of their colleagues, all votes will be unanimous and medium-term equilibrium will be defined by (2.5) and (2.8) for all prices.

We are assuming that N can be treated as a continuous variable and that adjustment to optimal N is feasible.

The comparative statics of both (2.5) and (2.8) have to be considered and in the neighbourhood of the equilibrium defined by these conditions we derive

$$\frac{d\ell^j}{dP} = \frac{Q}{-\Delta_y L}\left(U_{yy}{}^j y^j + U_{y\ell}{}^j \ell^j + U_y{}^j \right) \qquad (2.11)$$

which is positive if and only if $d\ell^j/dw > 0$ from (2.7), ($\Delta_y = \Delta_w$ for $y = w\ell$). The following can also be derived

$$\frac{dN}{dP} = \frac{1}{\ell^j P^2 Q_{LL} L} \sum_{k=1}^{m} F^k p^k - \frac{N}{\ell^j} \frac{d\ell^j}{dP} \qquad (2.12)$$

which is negative given that (2.11) is positive. Then, in the medium term hours per member will increase but the number of members will decrease in response to a price increase.

The change in total hours supplied will be given by $dL = Nd\ell^j + \ell^j dN$ and so, from (2.11) and (2.12) we may write

$$\frac{dL}{dP} = \frac{1}{P^2 Q_{LL} L} \sum_{k=1}^{m} F^k p^k \qquad (2.13)$$

which is clearly negative. Thus total hours will respond perversely to a price change in the medium term even though individual hours responds non-perversely. Note that these medium-term comparative statics results are not influenced by the values of S (which is assumed to be independent of N as explained above) and so include the cases of perfect sympathy and collusion.

Of course, in the long run when other factors are variable, then depending on the degree of complementarity between these factors and labour, it is possible that total hours supplied, L, will eventually increase in response to an output price increase.

III. Comparative Statics : Needs Payments and Fixed Costs Changes

Sen (1966) assumes that a proportion α, $0 \leq \alpha \leq 1$ of the co-operative's surplus, R, is distributed according to needs, the remainder $(1-\alpha)R$ being distributed according to labour input. Here we choose to model needs payments as a fixed cost per member deducted from the co-operative's surplus.

Let the needs income for individual j be written Z^j then we may write

$$y^j = Z^j + \frac{\ell^j}{L}\left(R - \sum_{j=1}^{N} Z^j \right) \qquad (2.14)$$

Equation (2.14) reflects the idea that 'needs' may be exogenously determined by circumstances such as dependents per family. Admittedly payments made to cover needs may, over the long run, change in a fashion reflecting changes in the wealth of the co-operative.

If needs payments are modelled as we suggest, then for Z^j fixed, the short-run comparative statics analysis of the last section and the equilibrium conditions of Section I remain substantially unaltered (on the assumption of identical preferences and needs). The expression $\sum_{k=1}^{m} P^k F^k + \sum_{j=1}^{N} Z^j$ replaces $\sum_{k=1}^{m} P^k F^k$ and where we used $d\ell/dw$ as a yardstick we now have to define the full income of a wage-earner as $w\ell^j + Z^j$ so that an unearned component is now included and w again approximately equates the full income of a wage earner with that of a co-operative member.

The question of the comparative statics response to a change in needs payments now arises. When membership is fixed we may consider (2.5) with R/L replaced by $(R-NZ)/L$ and y^j defined as in (2.14). Then, using $N\ell^j = L$ as ℓ^j is common to all (identical) individuals, comparative statics yield

$$\frac{d\ell^j}{dZ} = \frac{N}{L\Delta_Z} U_y(1-S) \qquad (2.15)$$

where $\Delta_Z < 0$ from second-order conditions. Thus if $S=1$, the allocation of co-operative income between work income and needs income has no effect on labour supply. In this case of perfect sympathy or ℓ^j fixed by collusion to be the same for all workers an increase in needs income has no disincentive effect on individual's labour supply. If $S < 1$, however, $d\ell^j/dZ < 0$, a conclusion also reached in Sen's (1966) model.

It is instructive to compare the effect on ℓ^j of a change in Z with the effect on ℓ^j of a change in one of the non-labour (fixed) input prices say P^k. Then, again from (2.5) and using $N\ell^j = L$ we have

$$\frac{d\ell^j}{dP^k} = \frac{F^k}{L\Delta} \left[U^j_{yy} \left\{R_L S + (1-S)\frac{R}{L}\right\} \ell^j + U^j_{\ell y}\ell^j + U^j_y (1-S)\right] \qquad (2.16)$$

If $S = 1$, then $d\ell^j/dP^k > 0$. If $S < 1$, then $d\ell^j/dP^k$ could still be positive: certainly if $d\ell^j/dP < 0$, then this would be the case as can be seen by comparing (2.6) with (2.16). The different responses evident from (2.15) and (2.16) are due to the first two terms in the square bracket in the latter which are zero in (2.15) where income effects are absent. Thus the effects of an increase in needs income is much the same as that of a compensated factor price change.

In the medium term membership, N, is also variable. Treating N as a continuous variable and assuming that adjustment to optimal N is feasible we may note that (given that all members choose jointly, or independently the same labour input) (2.14) is maximised with respect to N when (2.8) holds. If (2.8) is divided through by ℓ^j then it becomes an equation in L alone, and does not involve Z. Thus, in the medium term total labour input L is independent of the level of needs payments. If the short-run response to an increase in Z is to reduce man-hours, in the medium term N will increase to bring $L = N\ell$ to its initial level. The fact that (2.8) still holds when there are needs payments implies that the medium-term response to a change in product price is essentially unaltered.

IV. Comparative Statics : A Simple Example where Income Effects are Absent

For the case of the separable utility function $U = y - \beta(\ell)$, where $\beta'(\ell) > 0$, $\beta''(\ell) > 0$ it is possible to illustrate much of the analysis of Sections II and III in simple diagrams. Some elements in the diagrams draw on the work of Domar (1966), Oi and Clayton (1968) and Stern (1980, Ch.3). For this particular utility function our condition for a Nash equilibrium if there are no needs payments may be written

$$SPQ_L + (1-S)\frac{R}{L} = \beta'(\ell) \qquad (2.5)'$$

When $S=1$ we have the cases of collusion and perfect sympathy; when S is very small and N large such that $S \approx 1/N \approx 0$ we have the case of a Nash equilibrium for a firm with a "competitive" internal labour market.

(a) Change in Product Price

In Figure I the co-operative is in an initial position of medium-term equilibrium described by point A. Condition (2.5)' which is a short-run equilibrium condition (holding for a given N) is satisfied, but the firm also has attained optimal membership such that (2.8) holds implying that the value of the marginal product of an hour's labour is equated with income per hour so that the efficiency condition

$$PQ_L = \beta'(\ell) \tag{2.3)'}$$

is satisfied. Price is initially P_0 and then increases to P_1. The change in price shifts the value marginal product schedule up to the right. The

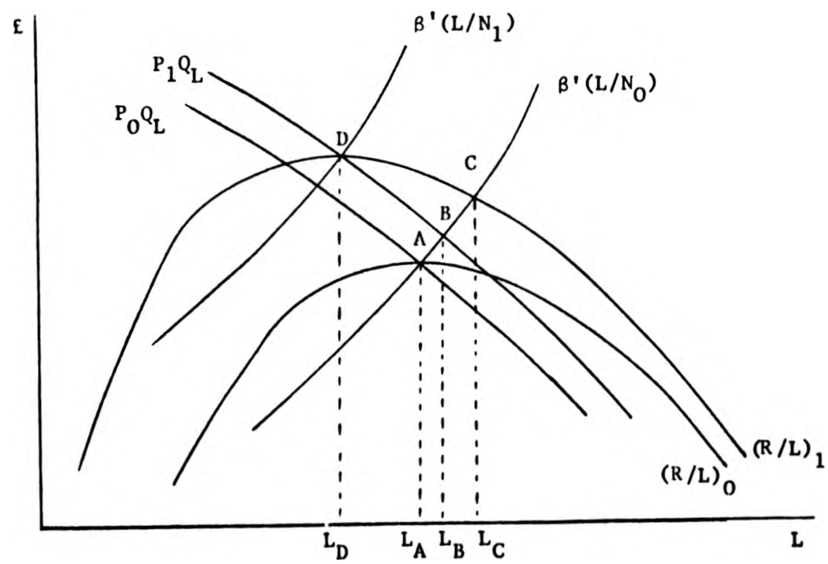


Figure I
Effects of a Price Change

R/L schedule is raised and, as there are fixed costs, the maximum of the new schedule lies to the left of that of the initial schedule. The short-run and medium-term effects of the price increase can now easily be distinguished.

If there is collusion or perfect sympathy the new short-run equilibrium will be at B where (2.3)' is satisfied and all members have increased their labour input. On the other hand, if $S \approx 0$ the new short-run equilibrium will be at C and for $0 < S < 1$ the new equilibrium will be somewhere on $\beta'(L/N_0)$ between B and C yielding a total labour input between L_B and L_C. The diagram clearly shows that workers will gain by colluding at B. No matter where the new short-run equilibrium may lie between B and C it is clear that $P_1 Q_L < (R/L)_1$, so that income per member may be raised if the firm can contract membership by failing to replace any who leave. Any reduction in N shifts the $\beta'(\ell)$ schedule to the left. The new optimal membership would be N_1, and point D depicts the new medium-term equilibrium. Note that membership has fallen and so has total hours which are now L_D. Since $\beta'(\ell)$ is an increasing function of ℓ we may also conclude that hours supplied per worker are greater at D than they were at A. Of course with this particular utility function $d\ell/dP$ is always positive in the short run as well.

(b) Change in Fixed Costs

Figure II illustrates the effects of a change in fixed costs when the firm is in an initial medium-term equilibrium at A. A fall in fixed costs raises the (R/L) schedule such that the new maximum corresponds to a smaller L. In the short run if $S=1$, ℓ does not adjust since (2.3)'

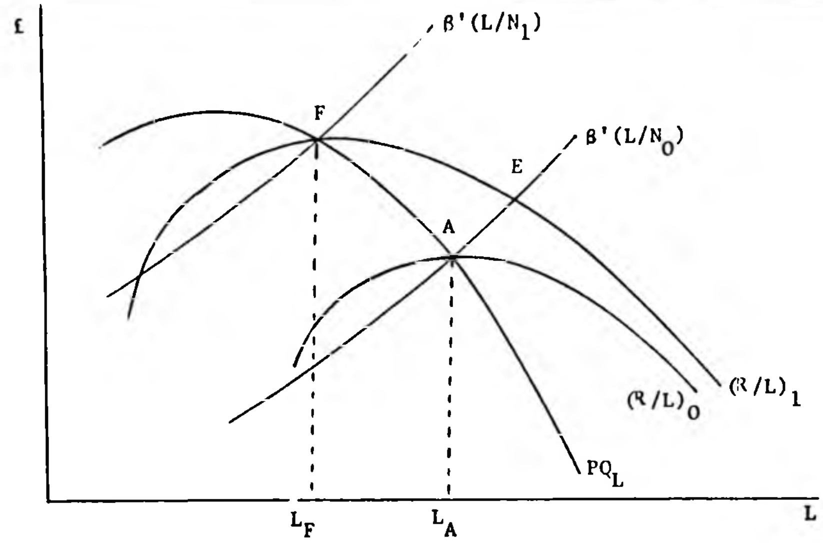


Figure II

Effects of a Change in Fixed Costs

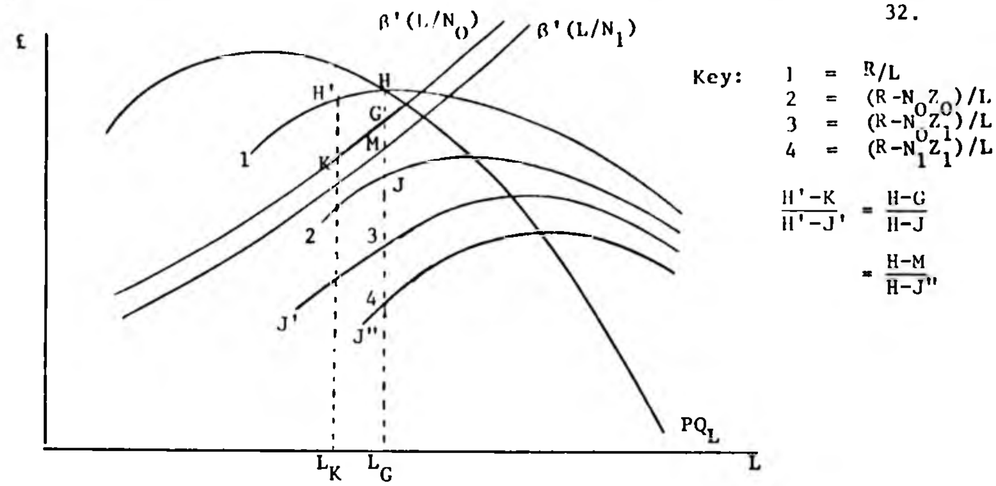
still holds. For $0 < S < 1$ when fixed costs fall a new equilibrium will be established in the short run on $\beta'(L/N_0)$ somewhere between A and E (at E if $S \approx 0$), so ℓ increases. In the medium term it will be optimal to attain a reduced membership (since at all points on AE, $PQ_L < (R/L)_1$) and establish a new medium-term equilibrium at F where total hours have fallen to L_F but hours per worker are clearly higher than at A.

(c) Change in Needs Payments

When there are needs payments, equation (2.5) must be rewritten

$$SPQ_L + (1-S)\frac{(R-NZ)}{L} = \beta'(\ell) \qquad (2.5)''$$

and the additional medium-term equlbrium condition is (2.8) as before. Figure III depicts a co-operative with $0 < S < 1$ in medium-term equilibrium at point G. If $S=1$, $\beta'(L/N_0)$ would pass through H and, if $S \approx 0$, through J. For all S, L_G is the initial medium-term equilibrium labour hours input. An increase in needs payments from Z_0 to Z_1, shifts the $(R-N_0Z_0)/L$



<u>Figure III</u>

Effects of a Change in Needs Payments

schedule to $(R-N_0Z_1)/L$ and from (2.5)'' we see that in the new short-run equilibrium $\beta'(\ell)$ and thus ℓ must be lower (unless S=1 in which case there is no change). The new short-run equilibrium is shown by K but at L_K, $PQ_L > R/L$ so it is optimal to recruit. This moves $(R-NZ)/L$ down still further and also shifts the $\beta'(\ell)$ schedule to the right until a new equilibrium is established at M. Thus the short-run reaction to an increase in needs payments is to reduce hours per worker but in the medium term the firm exactly compensates for this by increasing membership.

We may note briefly here that the analysis would be significantly different if there were discrimination between new recruits and existing membei in the sense that new recruits were available who either had no needs or did not qualify for needs payments. In Figure IV(a) and (b) below the shaded areas indicate the net gain that N_0 original members may make by taking on new recruits who only receive $(R-N_0Z)/L$ per hour). Note that in (a) the initial equilibrium is collusive and original workers cut their total hours from the initial level of L_A to L_B when they recruit, (L_C-L_B) hours being supplied by new recruits. In case (b), on the other hand there is an initial Nash equilibrium with a competitive internal labour market (N_0 being large and

$S \approx 0$), and here original workers increase their total hours

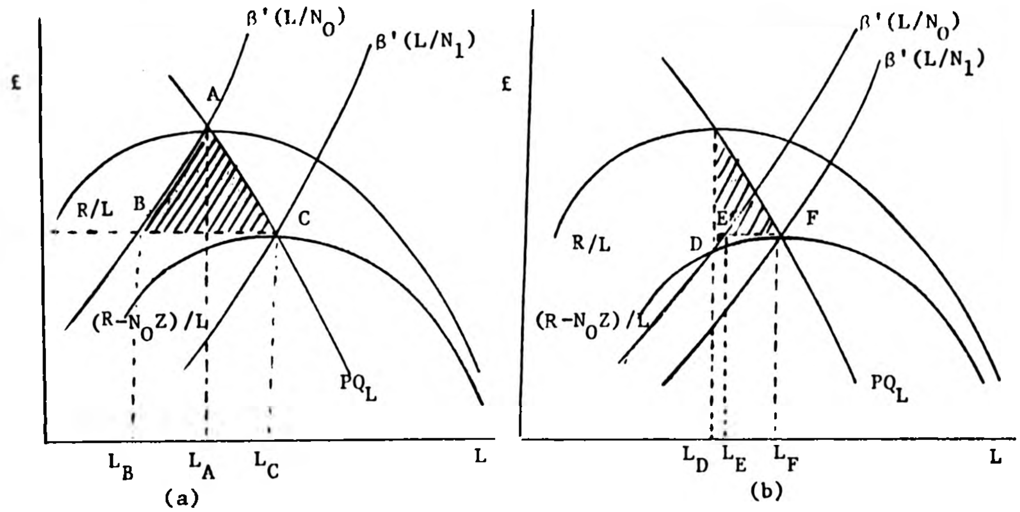


Figure IV

Equilibrium when New Recruits receive No Needs Payments

from L_D to L_E and (L_F-L_E) hours are supplied by new recruits. Such behaviour has some similarity with outside hiring as discussed by Domar (1966) and Wiles (1977, ch. 4).

(d) Membership Constraints

Throughout the paper it has been assumed that it is possible to adjust membership to the optimal level at which income per hour is equated with the value marginal product of an hour's labour. We must recognise that there may be circumstances in which this is not feasible because the co-operative is unable to offer potential members sufficiently high levels of utility to induce them to join.

Let $U = \phi(N)$ be the membership constraint. It will reflect present levels of utility of potential members and will be assumed continuous. For simplicity we will restrict the argument to the case of collusion or perfect sympathy in which case maximising U is equivalent to maximising W

and the co-operative is assumed to maximise $U = y - \beta(\ell)$ subject to $U \geq \phi(N)$. The appropriate Lagrangean function is

$$\theta = y - \beta(\ell) + \lambda\left[y - \beta(\ell) - \phi(N)\right] \qquad (2.17)$$

and maximisation with respect to ℓ and N yields the condition for an interior optimum

$$\left[PQ_L - \beta'(\ell)\right](1 + \lambda) = 0 \qquad (2.18)$$

$$PQ_L\ell - R/N = \lambda\phi'(N)/(1 + \lambda) \qquad (2.19)$$

where the multiplier λ is positive or zero according to whether the constraint is binding and $\phi'(N) > 0$.

In Figure 5(a) the membership constraint is binding and N_C is the maximum possible membership. The hatching represents the infeasible region. Note however that the optimal labour input (from (18)) is defined by L_C and not by L_y the latter being the labour input which maximises income per worker (and not utility) subject to the constraint (compare Domar (1966)). In Figure V(b) the constraint is not binding and the unconstrained optimum membership N^* is attainable. Of course if $N^* < N_0 \leq N_C$, where N_0 is the initial membership, adjustment to N^* will depend on labour turnover and deaths of members on the assumption that involuntary expulsions do not occur. (Of course if membership is immutable labour input would simply remain at L_0). Note that adjustment problems are exacerbated by the fact that when product price falls and the individual utility level offered by the co-operative also falls the co-operative will wish to expand membership and it will wish to contract membership when price and thus utility level rise.

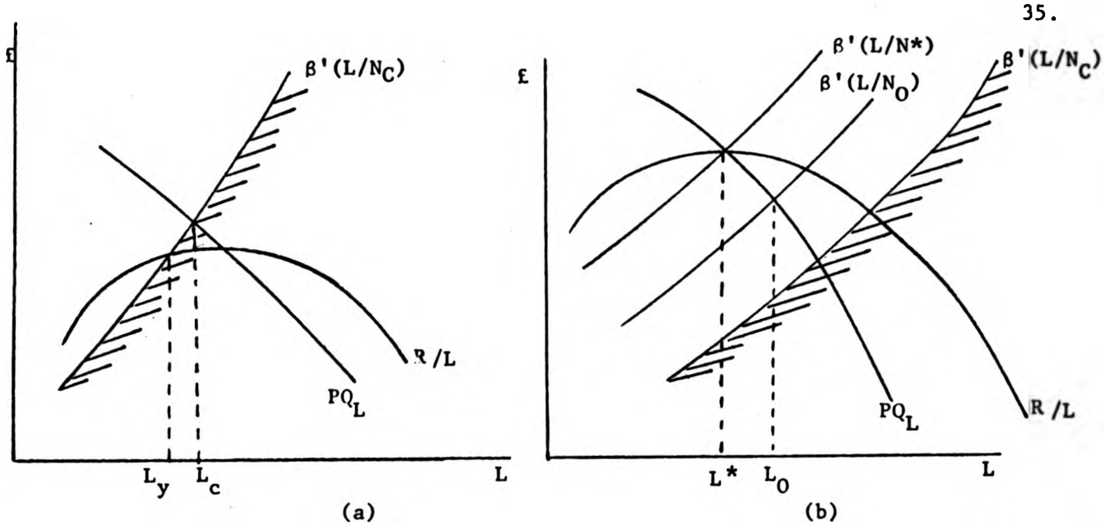


Figure V

Effects of the Membership Constraint

V. Efficiency Considerations

We will state three conditions for efficiency in the allocation of labour corresponding to the short-run, medium-term and long-run adjustment periods distinguished in Section II.

(i) In the short-run all individuals within the co-operative should have the same marginal rate of substitution (MRS) between income and hours worked which should be equal to the value marginal product of labour. This is stated as (2.3) above.

(ii) Medium-term efficiency implies that a reallocation of labour among co-operatives has taken place such that the MRS between income and work in all co-operatives is the same and equal to the (common) value of the marginal product of labour.

(iii) In the long run all inputs are variable and new entry can take place. Thus long-run efficiency requires that, at optimal levels of non-labour inputs, the value of the marginal product of labour is the same in all co-operatives.

If (i) does not hold members of the same co-operative could trade labour for income and reach a Pareto-preferred position. If, within a co-operative, the MRS and the value marginal product of labour are unequal workers can gain by (jointly) adjusting labour hours until the equality holds so that, at the margin, the gain from an extra hour's work is equated with its opportunity cost. If (ii) does not hold members of one co-operative could, in principle at least, trade labour for income with members of other co-operatives and reach a Pareto-preferred position. Finally (iii) is the familiar condition for long-run efficiency and requires that <u>all</u> inputs be adjusted to optimal levels. In relating these efficiency conditions to our earlier analysis we will continue to assume that all tastes are identical.

Although the MRS is constant for all workers in the co-operative in all cases above (see (2.3),(2.4) and (2.5)),it will only be equal to the value marginal product of an hour's labour in specific circumstances. Two cases are when $S=1$, perfect sympathy, or when workers collude to fix a common ℓ. These cases of short-run efficiency have been analysed respectively by Sen (1966) and Berman (1977). Note that for $0 < S < 1$ if N happened to be optimal so that (2.8) holds then (2.3) will also hold if there are no needs payments. (We continue to abstract from needs payments until later in this section).

Even if $S = 1$, this is not necessarily going to imply (ii) as there is no immediate reason why the value of the marginal product in one co-operative should be equal to that in another. In fact both Ward (1958) and Domar (1966) have demonstrated that the labour market may tend to be rigid in an economy of co-operative firms. Thus potential members might be attracted to a co-operative where both income per worker and the value of the marginal product are high from co-operatives where both are low. The rich co-operative may however refrain from any expansion of membership if increasing membership would reduce income per worker. In the long run, of course, this misallocation can be corrected by new entry (which equates both income per man hour and the value of the marginal product of an hour's labour across all co-operatives) but a number of solutions have been proposed which might operate during an adjustment period too short for significant new entry to take place. These include Dubravcic's (1970) suggestion that co-operatives might hire labour at a fixed wage in the short term, Meade's (1974) outside employment authority and (1972) inegalitarian co-operative, the proposal of Vanek et al for firm-specific lump sum taxes and the enterprise incentive fund scheme of Ireland and Law (1978).

The assumption of identical preferences has been retained throughout the paper but we should note that substantial differences in preferences may have efficiency implications. Furubotn (1978) suggests efficiency problems may arise where a politically dominant group of workers within the firm has preferences which differ significantly from those of other workers whereas Berman and Berman (1978) assume the utility of a 'typical worker' is maximised. The papers by Markusen (1975, 1976) discuss some efficiency aspects of differences in preferences. Preference differences would also complicate the comparative statics analysis of Section II, particularly where there are needs payments. Of course in the special case where differing needs exist among individuals but where these are exactly compensated for by the payments Z^j, in that $U^j(y^j - Z^j, \ell^j) \equiv U^j(R/N, \ell^j)$ all j, the analysis would be unchanged. This assumes that needs payments are treated by individuals as transfers to offset particular liabilities and, once these are met, individuals are identical in all respects.

Finally we should recognise that there are other aspects of efficiency in labour-mananged firms which have not been captured in our model. (See Vanek (1970, Chapter 12) and Chapter 3 for some relevant discussion).

VI. Concluding Comments

In Section I, we demonstrated that the "sympathy" approach of Sen (1966) and the equilibrium in workers' supply of hours worked discussed by Berman (1977) and Bonin (1977) may be viewed as essentially dual approaches to the problem of labour supply in a co-operative, and can be summarised by equation (2.5). In terms of our second objective of considering the comparative statics of the equilibrium we distinguished short-run, medium-run and long-run results. In the short-run, we find that there are circumstances where individual labour supply may well decrease when output price rises. In particular this is always so for a utility function of Cobb-Douglas form when $S \neq 0$, contrary to Oi and Clayton (1968), and in any case the possibility of perverse responses will not necessarily be eliminated by stating that normal assumptions on income and substitution effects hold. (Berman and Berman (1978)). However if membership is also variable so that the firm attains an equilibrium at which income per man hour is maximised by membership size, such perversity will not occur.

In the medium term, we consider that membership can always be adjusted to income maximising levels, and show the conventional result that membership contracts in response to an increase in product price. Also total hours will decrease (Equation 2.13) whatever happens to hours per member. This result may of course be modified in the long run when other factors of production are variable.

The existence of needs payments reduces the hours worked by the individual worker (unless $S = 1$, see Equation (2.15)), but in the medium-term, the membership level will adjust (for any S) so that the total hours of labour supplied to (and thus also the output from) the firm is independent of the level or existence of needs payments. We also show that the short-term effect of an increase in needs payments is analogous to that of a compensated change in the price of a fixed factor.

Many of these points are shown for the simple utility function $U = y - \beta(\ell)$ in Section IV by the construction of a diagram combining the effects of parameter changes and membership changes on hours worked.

In Section V, we specified precise conditions for what we term short-run and medium-term efficiency. We find that short-run efficiency is attained when there is perfect sympathy or collusion in hours worked ($S = 1$) (results familiar from Sen (1966) and Berman (1977)), and, in addition, in the absence of needs payments, if membership level is such that income per hour is maximised (i.e. (2.8) holds). Medium-term efficiency can be attained by a variety of alternative policies, one of which is a generalisation of an enterprise incentive fund scheme (Ireland and Law (1978), see Chapter 5).

Of course, our analysis assumes that adjustment to equilibrium is feasible and to some extent neglects the difficulties and costs of such adjustment. Also we have neglected consideration of non-identical preferences and our concern for efficiency has been limited to questions concerning allocative efficiency, whereas there are clearly other possible dimensions of enquiry. But we have attempted here to illuminate some of the results from earlier studies and derive additional results in models of the labour-managed or co-operative firm in which the individual worker's supply of hours can be varied and income and hours worked both affect the individual's utility.

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Mathematical appendix to Chapter 2

Derivation of (2.6)

From (2.1) and (2.2) we have

$$W^j = \sum_{i=1}^{N} a_{ij} U^i((pQ - \sum_{k=1}^{m} F^k p^k)\ell^i/L, \ell^i) \tag{A2.1}$$

Differentiating with respect to $\ell^i = \ell$ all i we obtain (as $\ell^i/L = N$ and is fixed) equation (2.4)

$$\sum_{i=1}^{N} a_{ij}(U^i y\, pQ_L + U^i_\ell) = 0$$

Alternatively a Nash equilibrium is obtained by A2.1 with respect to ℓ^j holding ℓ^i $(i \neq j)$ constant. We then obtain the first order conditions for all identical members:

$$\sum_{i=1}^{N} a_{ij} U_y{}^i(p\, Q_L \frac{\ell^i}{L} - R.\frac{\ell^i}{L^2}) + a_{jj}(U_y{}^j \frac{R}{L} + U^j_\ell) = 0 \quad \text{all } j \tag{A2.2}$$

As $\ell^i = \ell^j$ all i, j

and $y^i = y^j$ all i, j

We can simplify this to

$$\frac{1}{N}\sum_{i=1}^{N} a_{ij}(U_y{}^j(p\, Q_L - \frac{R}{L})) + U_y{}^j \frac{R}{L} + U_\ell{}^j = 0$$

i.e. $$U_y{}^j\left[S\ p\ Q_L + (1 - S)\ \frac{R}{L}\right] + U^j_\ell \ = \ 0 \quad \text{all} \ j \tag{2.5}$$

Differentiating (2.5) totally with respect to p and ℓ^j, $j = 1, 2 \ . \ . \ . \ N$ for fixed N is simplified by noting that all ℓ^j will react in the same way as all individuals have their equilibrium defined by an equation identical to (2.5).

We obtain

$$\Big\{U_{yy}{}^j\ p\ Q_L\ \left[S\ p\ Q_L + (1 - S)\ R/L\right]$$

$$+ U_y{}^j\left[S\ p\ Q_{LL}\ N + (1 - S)\ \left(\frac{p\ Q_L\ N.L - NR}{L^2}\right)\right]$$

$$+ U^j_{y\ell}\left[S\ p\ Q_L + (1 - S)\ R/L\right] + U^j_{\ell\ell} + U^j_{\ell y}\ p\ Q_L\Big\}\ d\ell^j$$

$$= - \Big\{U_{yy}{}^j\ \frac{Q}{L}\ \left[S\ p\ Q_L\ \ell^j + (1 - S)\ R/N\right] + U_y{}^j(S\ Q_L + (1 - S)\ \frac{Q}{L})$$

$$+ U^j_{\ell y}\ Q\ \frac{\ell^j}{L}\Big\}\ dp$$

Thus $$\frac{d\ell^j}{dp} = \frac{Q}{-\Delta.L}\ \Big[U_{yy}{}^j\{S\ p\ Q_L\ \ell^j + (1 - S)\ R/N\} + U_y{}^j(1 - S(1 - \frac{LQ_L}{Q}))$$

$$+ U^j_{\ell y}\ \ell^j\Big]$$

Now the terms in Δ are all negative apart from $U_y{}^j(1 - S)N\ \frac{(p\ Q_L\ L - R)}{L^2}$ which is ambiguous. Note that if $pQ_L \simeq R/L$ or $S = 1$ this term is negligible. We will assume that the other terms (which form the second-order condition for (2.4)) dominate this term.

Derivation of (2.7)

Maximise $U(w\ell^j, \ell^j)$

with respect to ℓ^j

First order condition, omitting superscripts, is

$$U_y w + U_\ell = 0 \tag{A2.3}$$

Totally differentiating with respect to w and ℓ we obtain

$$(U_{yy} w^2 + 2U_{y\ell} w + U_{\ell\ell})d\ell = (U_{yy} y + U_{y\ell} \ell + U_y) dw$$

Thus $$\frac{d\ell}{dw} = \frac{1}{-\Delta_w}(U_{yy} y + U_{y\ell} \ell + U_y)$$

where $$\Delta_w = U_{yy} w^2 + 2U_{y\ell} w + U_{\ell\ell} < 0$$

Derivation of (2.11) and (2.12)

Use (2.5) in (2.8) to obtain

$$U_y R/L + U_\ell = 0$$

Totally differentiate to obtain

$$\Delta_y d\ell + (0)dN + \left[U_y Q/L + U_{yy}(R/L)(Q/N) + U_{\ell y} Q/N\right]dp = 0$$

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Maximise $U(w\ell^j, \ell^j)$

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as R/L is maximised with respect to N from (2.8).

We then have

$$\frac{d\ell}{dp} = \frac{-Q}{\Delta_y L} \left| U_{yy}\, y + U_{y\ell}\, \ell + U_y \right| \tag{2.11}$$

where $$\Delta_y = \frac{U_\ell^{\,2}}{U_y^{\,2}} U_{yy} - 2 \frac{U_\ell\, U_{y\ell}}{U_y} + U_{\ell\ell}$$

Now totally differentiate (2.8) and obtain

$\ell\ p\ Q_{LL}\ N\ d\ell + \ell^2\ p\ Q_{LL}\ dN = (\Sigma\ F^K\ p^K)/(pN)dp$. Use (2.11) to obtain (2.12).

CHAPTER 3

Incentives and Work Environment

I. The theory of the labour-managed firm (LM-firm) is now well-known for predicting perverse behaviour and grave problems. The response to an output price change of a firm maximising income per worker with respect to work force size alone is an example of the former. When product price goes up, such a firm will wish to employ less labour and thus produce less output (see for example Meade (1972) and also see Chapter 1). The property rights aspects of the use of capital (see Furubotn and Pejovich (1973)) and labour market inefficiencies (see for example Ireland and Law (1978) and Chapter 6) constitute further examples of grounds for doubting the wisdom of organising production in terms of LM-firms.

However, most empirical work has been concerned with demonstrating the higher productivity achieved by LM-firms and labour-participating firms. Some of this work is summarised by Blumberg (1975). It is argued in the empirical literature that a result of moving towards labour management or participation in management is a reduction in the alienation of the labour force from the firm. A Marxist explanation has been that LM-firms may benefit from two factors: that their workers do not feel exploited by capitalists and also that their labour is not simply exchanged for a wage in the labour market as just one of many economic relations and with a corresponding lack of dignity (see Selucky (1975)).

In this chapter we will take the view that the LM-firm's advantage is not simply that incentives are such that workers are prepared to worker harder in LM-firms, a proposition well-known in the literature and well-discussed in Vanek (1970, Chapter 12), but rather that workers gain utility both from the mutual cooperation and spirit of teamwork encouraged by such incentives and from the lack of confrontation with employers in LM-firms. They then gain utility directly from an LM-firm environment as compared with a conventional firm's environment. We will take the conventional firm to be an entrepreneurial firm which we will specify more fully in Section II and denote EP-firm. We will model the shift in the utility function for identical workers in one type of firm as compared with another, and we will see that in our model it is the reduced marginal disutility from extra work effort that is important for predicting firm behaviour. We will also argue that there may exist relative economies in supervision costs in the LM-firm. As Vanek (1970, p. 238) says "if the private employer wants to produce anything, while paying a fixed contractual wage, the contract must explicitly or at least implicitly contain a provision regarding a minimum acceptable performance standard". Such a standard has to be enforced and the costs of such enforcement may be less (although of course not eliminated) in an LM-firm than in an EP-firm. The mechanism by which these assumed efficiency advantages feed through to influence productivity and other aspects of firm performance will be the primary target of our analysis, although we will also consider the distribution of gains from increasing worker participation within the context of an EP-firm.

One of the major reasons why a discussion of the direct environment effects on individuals' utilities appears desirable is the common practice of largely ignoring the alienation of labour and related questions in theoretical comparisons of LM-firms with conventional firms. Domar (1966), after comparing LM- and other firms rather that workers gain utility both from the mutual cooperation and spirit of teamwork encouraged by such incentives and from the lack of confrontation with employers in LM-firms. They then gain utility directly from an LM-firm environment as compared with a conventional firm's environment. We will take the conventional firm to be an entrepreneurial firm which we will specify more fully in Section II and denote EP-firm. We will model the shift in the utility function for identical workers in one type of firm as compared with another, and we will see that in our model it is the reduced <u>marginal</u> disutility from <u>extra</u> work effort that is important for predicting firm behaviour. We will also argue that there may exist relative economies in supervision costs in the LM-firm. As Vanek (1970, p. 238) says "if the private employer wants to produce anything, while paying a fixed contractual wage, the contract must explicitly or at least implicitly contain a provision regarding a minimum acceptable performance standard". Such a standard has to be enforced and the costs of such enforcement may be less (although of course not eliminated) in an LM-firm than in an EP-firm. The mechanism by which these assumed efficiency advantages feed through to influence productivity and other aspects of firm performance will be the primary target of our analysis, although we will also consider the distribution of gains from increasing worker participation within the context of an EP-firm.

One of the major reasons why a discussion of the direct environment effects on individuals' utilities appears desirable is the common practice of largely ignoring the alienation of labour and related questions in theoretical comparisons of LM-firms with conventional firms. Domar (1966), after comparing LM- and other firms assuming a common relationship between inputs and outputs ends with a final caveat (p. 49): "Judged by strictly economic criteria the coop has not come out well. But even on these grounds, it is quite possible that a coop may be more efficient than a capitalist or state-owned firm in societies where membership in the coop, as contrasted with hiring out for a job, has a strong positive effect on workers' incentives . . . ". Although Domar is almost certainly referring here to the incentive effects of an income schedule (rather than a given wage) on the level of effort, our point is that in addition to this there is a direct environmental effect from membership in the coop or LM-firm. When some specific attention has been paid to the alienation - firm type - productivity question it has usually been conceived of as an upward shift in the production function for the LM-firm, which blurs the direct environmental and incentive effects. For example, Carsen (1973) considers a reduction in X-inefficiency as an advantage of the LM-firm. Neither the basis for assuming such a result from alienation reduction nor the implications for firm behaviour are fully pursued. One of our aims is to consider the extent to which this approach can be justified within the context of a logical if simple model, where a given supply of labour services, supply of managerial services (by the entrepreneur in the EP-firm and the worker-managers in the LM-firm), and supply of risk-bearing services, all create disutilities which are not independent of firm type.

One interesting observation that can be made here very quickly, however, is that if the result of switching from a conventional firm to an LM-firm organisation is purely an upward shift in production by a multiple $\theta > 1$ for all input levels, then the effect on the

LM-firm's behaviour would be the same as that of an increase in product prices, referred to above, except that output may increase or decrease dependent on the amount membership is reduced and the value of θ .

It may be argued that labour-augmenting technical progress would be a more satisfactory way of viewing the effects of alienation reduction. Batra (1974) considers such technical progress in the context of collective farms but only as an exogenous rather than endogenous change. We present a specific model in Section II which links directly the productivity gain to the assumed utility function change as a result of an improved working environment. In Section III we extend the partial equilibrium analysis of Section II by considering general equilibrium aspects of the production sector. Section IV contains a summary of results and some discussion concerning gains from worker-participation within conventional entrepreneurial firms. Although this latter subject has been treated to some extent by Steinherr (1977), his work relates mostly to the optimal level of worker participation in worker-manager contracts.

In all the above analysis we take the role of the entrepreneur to be that of a manager supplying managerial services. If however the firm exists in a risky environment, then the entrepreneur (in the EP-firm) on joint entrepreneurs (in the LM-firm) supply the service of risk-bearing. We will delay a consideration of this until Chapter 5.

Mathematical derivations of equation(3.13)onwards are treated in more detail in the Appendix.

II

In this section we will assume the same given capital stock for both LM-firms and EP-firms. Also product prices and fixed costs are independent of firm type, and all parameters are known with certainty. A firm's net revenue R is defined as the given product price (p) times output (Q) minus fixed costs, and is a strictly concave function R(L) of the total supply of labour in efficiency units (L). The functional form of the net revenue function is again independent of firm type. What is not independent of firm type is firstly the firm's objective and secondly the utility function of individuals associated with the firm. We will assume for simplicity that all individuals are identical, and each individual seeks to maximise his utility which is dependent on his income (y) and on his own supply of both entrepreneurial and work effort. A particularly simple form of utility function will be used partly to avoid problems of income effects in labour supply and partly to ensure an equal ranking of the two types of firm in a utilitarian assessment in the absence of direct environmental advantages.

Thus the utility function of an individual who is employed as a worker in an EP-firm but undertakes no entrepreneurial activities, will be written.

$$U_w = w - \beta(\ell) \; ; \; \beta'(\ell) > 0 \, , \; \beta''(\ell) > 0 \qquad (3.1)$$

where w is the wage income and $\beta(\ell)$ is the disutility incurred by the individual from supplying to the firm ℓ efficiency units of labour. In general ℓ measures "effort" , while a more limited interpretation would be "hours worked".

Individuals in an economy of EP-firms (an EP-economy) can also become entrepreneurs, in which case they gain additional income, profits (Π), but incur additional costs in terms of the disutility of hiring, organising and supervising labour. We assume this disutility to be of the form $\alpha(\ell).N$, where N is the number of workers and ℓ the common number of efficiency units they each supply. Total effort supplied (L) is simply $\ell.N$. The worker-entrepreneur's utility is thus:

$$U_e = w - \beta(\ell) + \Pi - \alpha(\ell).N \quad ; \quad \alpha'(\ell) \geq 0 \quad \alpha''(\ell) \geq 0 \tag{3.2}$$

Note that because of the absence of income effects entrepreneurs will also wish to work providing $U_w > 0$ and workers will wish to become worker-entrepreneurs provided $U_e > U_w$. Also, $\beta(\ell)$ is the disutility of his work effort born by the worker and $\alpha(\ell)$ that born by the entrepreneur. The entrepreneur seeks to maximise (3.2) by choosing w, ℓ and N, but we will assume that he is faced with a competitive labour market which implies that workers will only accept employment provided $U_w \geq \bar{u}$, the competitive reservation utility, that is money wage minus disutility of work effort. Substituting $\bar{u} = U_w$ for w from (3.1)(as the entrepreneur will only wish to offer the minimum worker's utility) we can reformulate the entrepreneur's problem as

<u>maximise</u> with respect to ℓ, N

$$U_e = \bar{u} + R - (\bar{u} + \beta(\ell) + \alpha(\ell))N \tag{3.3}$$

Thus the entrepreneur maximises net revenue minus the full labour costs, incorporating a breakdown of the wage rate (into a base "wage" ($\bar{u}$) and a compensation payment $\beta(\ell)$) and also the entrepreneurial costs of employment.

In the alternative LM-firm, the entrepreneurial role is assumed to be divided equally among the worker-members. Thus each worker has a share $\frac{1}{N}$ of profit but also bears his own entrepreneurial cost $\alpha(\ell)$. Then, in the absence of direct benefits from the LM-firm environment, an LM-firm's worker has utility

$$U_m = R/N - \beta(\ell) - \alpha(\ell) \quad . \tag{3.4}$$

The particular utility functions we have used allow us to write from inspection of (3.1), (3.3) and (3.4) that for <u>given ℓ and N</u>, we have

$$(N - 1)U_w + U_e = N\, U_m \tag{3.5}$$

so that for given ℓ, N aggregate utility would be the same in the two systems, although neither the distribution of income nor utility need be. Established theory of the differences between LM-firms and profit-maximising firms lead us to suppose, however, that the choice of ℓ, N will not be the same in the two types of firm except in long-run competitive equilibrium where profits are zero. For our model of the EP-firm this long-run competitive equilibrium is interpreted as $U_e^* = \bar{u}$, where U_e^* is the maximum value of (3.3). Then no individual would be better off in terms of utility by becoming or ceasing to be an entrepreneur. Results can be found relating the behaviour of EP- and LM-firms which are analogous to the established comparisons of LM- and profit-maximising firms, and some will be noted below. However, here we will proceed to the case where less alienation of labour occurs in the LM-firm which reduces either or both of $\alpha(\ell)$ and $\beta(\ell)$ for any given ℓ. In fact we will see that we will need to be rather more specific and assume that it is the disutility of marginal work effort that is reduced. Writing $g(\ell) = \alpha(\ell) + \beta(\ell)$ for the entrepreneurial firm, we will state that a corresponding expression for the LM-firm is $g_m(\ell)$, such that the marginal disutility of work effort is less everywhere, i.e.

$$g'_m(\ell) < g'(\ell) \qquad \text{all } \ell \tag{3.6}$$

Now let us consider the conditions for optimal choice of ℓ and N in the two types of firm. For the EP-firm, first order necessary conditions for maximising (3.3) are :

$$R'(L) - g'(\ell) = 0 \tag{3.7}$$

$$R'(L).\ell = \bar{u} + g(\ell) \tag{3.8}$$

Equation (3.7) states that the level of effort (ℓ) should be chosen so as to equate the marginal net revenue product of an efficiency unit of labour with its marginal disutility. Equation (3.8) states that the marginal net revenue product of an additional worker should be equal to his full cost.

In the LM-firm, workers choose ℓ,N to maximise their utility (3.4)and necessary conditions are:

$$R'(L) - g_m'(\ell) = 0 \tag{3.9}$$

$$R'(L) = \frac{R(L)}{L} \tag{3.10}$$

If U_m^* is the maximum value of (3.4), then if $\bar{u} = U_m^*$ and $g(\ell) \equiv g_m(\ell)$ all ℓ, (3.9) and (3.10) are identical to (3.7) and (3.8). It follows then by taking comparative statics of (3.7) and (3.8) for a change in $\bar{u}$ that $\frac{d\ell}{d\bar{u}} > 0$ and $\frac{dL}{d\bar{u}}, \frac{dN}{d\bar{u}} < 0$ for the EP-firm while the LM-firm is unaffected. Thus for $\bar{u} \gtreqless U_m^*$ which implies $\bar{u} \gtreqless U_e^*$ we have $\ell \gtreqless \ell_m$, $N \lesseqgtr N_m$ and $L \lesseqgtr L_m$, where the m subscript distinguishes LM-firm optimal values. These results conform to the standard analysis of the LM-firm whenhours worked are variable, see for instance Berman (1977) and Bonin (1977).

Now suppose that (3.6) holds and $g(\ell)$, $g_m(\ell)$ are different functions. Write ℓ as L/N, and we can see that (3.9) and (3.10) are functions of L and N alone. Also if $U_e^* = \bar{u}$ and the EP-firm is in long-run competitive equilibrium then from (3.3), (3.8) can be rewritten

$$R'(L) = \frac{R(L)}{L} \tag{3.8a}$$

For a fixed N, denoted N_0, the functions $R'(L)$, $R(L)/L$, $g'(L/N_0)$ and $g_m'(L/N_0)$ can be drawn as functions of L, to form Figure 1. Note that $R(L)/L$ will always be intersected at its maximum by $R'(L)$. Also $g'(L/N)$ will shift upwards with a reduction in N. Now the EP-firm is in long-run equilibrium at an input of L_0 total labour efficiency units and a workforce of N_0, as at these values (3.7) and (3.8a) are satisfied. The LM-firm is not in equilibrium at L_0, N_0 as

(3.9) does not hold. The adjustment of the LM-firm to its equilibrium can be considered in two stages. In the short run, the number of worker-members is fixed and workers find it to their advantage to supply more efficiency units of labour: L_1 in total and L_1/N_0 per worker. At L_1, (3.9) holds but (3.10) does not. There would then be a tendency in the medium term for members who leave the LM-firm not to be replaced and the number of worker members would shrink. As this happens, $g'_m(L/N)$ would shift upwards until the number of workers reached N_1 such that (3.10) held. During this adjustment period the short-run condition (3.9) would continue to hold, and workers will supply more and more effort as the labour force contracts. Note that $g'_m(L/N_1)$ is not necessarily identical to $g'(L/N_0)$. However they both intersect with the $R(L)/L$ function at L_0. Thus after membership adjustment, the LM-firm will supply the same total work effort, earn the same net revenue and produce the same output as the entrepreneurial firm, but with less workers. A simple result which comes directly from the fact that (3.10) is independent of both the $g(\ell)$ and $g_m(\ell)$ functions and all variables other than L.

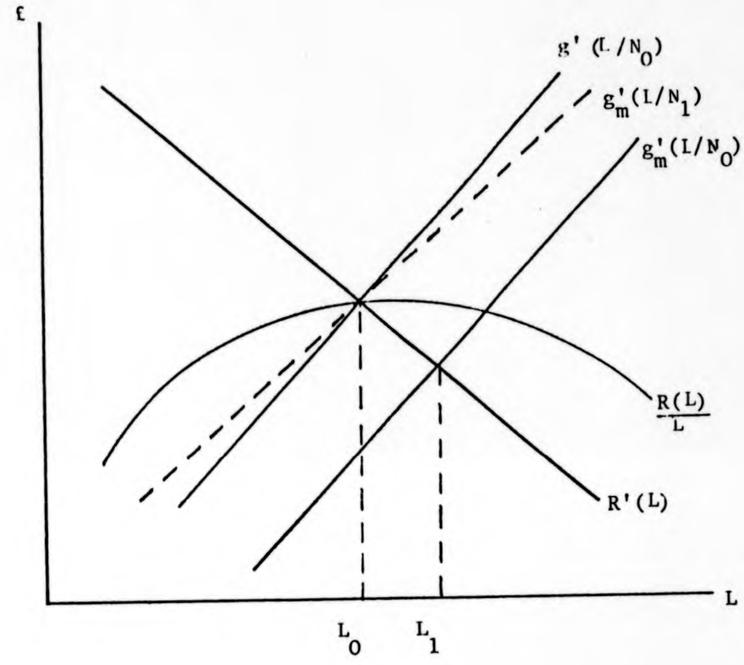

Figure 1

III

Although the analysis in the last section was of a partial equilibrium nature, some aspects carry through to a general equilibrium approach. Consider an economy where production takes place in either type of firm, but where the EP-firm sector is in long-run equilibrium ($\bar{u} = U_e^*$) , all LM-firms are in equilibrium as defined by (3.9) and (3.10) and where all prices are given constants, perhaps determined by a dominant foreign sector. Take two extreme situations: one where identical EP-firms constitute the entire economy (the EP-economy), and the other where there are no EP-firms, only identical LM-firms (the LM-economy). In both economies we also assume that full employment is achieved. In the absence of productivity differences arising from different disutilities of work the two extreme situations would give rise to exactly the same outputs supplied and inputs demanded by firms. Now suppose disutility of work is less in the LM-firms as described by (3.6).The results of Section II tell us that each LM-firm will employ less members but produce the same as an EP-firm in the EP-economy. However this would mean that more LM-firms would exist in order to satisfy full-employment. Each firm would still demand the same level of fixed inputs such as capital unless the prices of their inputs changed. Thus it is in the aggregate demand by the economy for, say, capital that general equilibrium considerations need to be taken into account, and it is this topic that will concern us here. Of course we could have focussed interest on the consumption side by not taking product prices as exogenous to the economy, but it seems reasonable to fully consider the production sector before making such extensions.

In a situation where firms buy capital with their own or their members' finance, lack of ownership rights in some versions of the LM-firm (such as the Yugoslav firm, see Furubotn and Pejovich (1973)) suggest reduced demand for capital by these firms. With such internal finance, comparison of the demand for capital goods between the LM-economy and the EP-economy is bound to be ambiguous. We will proceed, however, by assuming that in each economy there is an identical rentier institution which owns all the capital and rents this out to firms at a rental which equates supply and demand of capital. If there is a perfectly elastic supply of capital, then, as the same L holds for both types of firm and thus the marginal revenue product of capital is equated to the given rental at the same level of capital, the analysis of Section II holds. Alternatively, we might assume that the rentiers have the same fixed supply of capital in each economy and fix the rentals to clear the respective markets. Of course, the rentier may distribute the proceeds to individuals, but, providing this is done in a non-distortive way, this will not complicate the analysis. The rental on capital will be such as to distribute the available capital equally between the identical firms. Thus firms in the LM-economy will have less capital each than those in the EP-economy. This will affect both the marginal net revenue product and the average net revenue product of an efficiency unit of labour, and will feed back into the LM-firm's decisions as regards membership size and work effort. It is not a priori obvious that work effort per worker would still be higher in the LM-economy as a result of lower disutility of work, nor that aggregate output would be higher. If we can establish these points with fixed aggregate capital, however, it would seem reasonable a fortiori that they would also hold if aggregate capital could respond positively to the higher demand.

The general equilibrium is defined by the following conditions holding for each identical LM-firm, where $pQ(L, K)$ is the strictly concave revenue function and r the capital rental, so that income per member is then $(pQ - rK)/N$.

$$pQ_L = g'_m(L/N) \tag{3.9a}$$

$$pQ_L = (pQ - rK)/L \tag{3.10a}$$

$$pQ_K = r \tag{3.11}$$

$$K = hN \tag{3.12}$$

where (3.9a) and (3.10a) are just restatements of (3.9) and (3.10), (3.11) is the condition for optimal capital and (3.12) is the fixed total capital condition expressed as a fixed capital to labour ratio h . By substituting (3.12) and (3.11) into (3.9a) and (3.10a) the following comparative static results are derived in the neighbourhood of equilibrium for a small change in the parameter ϕ when

$$g'_m(L/N) \equiv \phi\, g'(L/N) \tag{3.13}$$

$$\frac{dL}{d\phi} = hg'(L/N)\left[Q_{KK}\, K + Q_{KL} L\right]/(\Delta) \tag{3.14}$$

$$\frac{dK}{d\phi} = -hg'(L/N)\left[Q_{LL}\, L + Q_{LK}\, K\right]/(\Delta) \tag{3.15}$$

where

$$\Delta = p \ h^2 N(Q_{LL} Q_{KK} - Q_{KL}^2)$$

$$- \phi \ g''(L/N)\left[Q_{KK} K^2 + 2Q_{LK} LK + Q_{LL} L^2\right]/N^2 > 0 \tag{3.16}$$

We also obtain from $\ell = L/N$ and (3.12) that

$$\frac{d\ell}{d\phi} = g'(L/N)\left[Q_{KK} K^2 + 2Q_{KL} KL + Q_{LL} L^2\right]/(L.\Delta) \tag{3.17}$$

which is negative again from (3.16) and concavity.

Note that the sign of (3.14) and (3.15) again depends on the question as to whether marginal products decrease or increase for a change in scale. If they both decrease then $\frac{dL}{d\phi} < 0$ and $\frac{dK}{d\phi} > 0$. Thus also, from (3.12) $\frac{dN}{d\phi} > 0$. $\frac{dQ}{d\phi}$ is then ambiguous as efficiency units of labour responds in the opposite direction to capital. However we can repeat the characterisation of $\frac{dQ}{d\phi}$ in terms of an increase in scale affecting the marginal products used in Chapter 1. Designate the increase in scale as an increase in λ at $(\lambda K, \lambda L)$. Then

$$\frac{dQ}{d\phi} = \frac{h \ g'(L/N)Q_K Q_L}{L.\Delta}\left\{\frac{d \ln Q_K}{d\lambda} - \frac{d \ln Q_L}{d\lambda}\right\}$$

i.e. $$\frac{dQ}{d\phi} \gtreqless 0 \text{ as } \frac{d \ln Q_K}{d\lambda} \gtreqless \frac{d \ln Q_L}{d\lambda}$$

That is, output will increase or decrease with an increase in ϕ according to whether the marginal product of capital increases proportionately more or less than the marginal product of labour for a one per cent increase in K and L. As ϕ decreases from unity, the firm's output will increase if $\frac{d \ln Q_L}{d\lambda} > \frac{d \ln Q_K}{d\lambda}$.

Finally aggregate output over all firms changes in proportion to average productivity, and

$$\frac{d(Q/N)}{d\phi} = \left[Q_L \frac{dL}{d\phi} \cdot N + (Q_K K - Q) \frac{dN}{d\phi}\right]/N^2$$

Using (3.10a) and (3.11) this simplifies to

$$\frac{d(Q/N)}{d\phi} = \frac{\partial Q}{\partial L} \cdot \frac{d\ell}{d\phi}$$

which is clearly negative.

Also we can show that $\frac{dr}{d\phi} < 0$ independent of whether factors are substitutes or complements. Using (3.14) and (3.15), we have

$$\frac{dr}{d\phi} = p \frac{d}{d\phi} \left(\frac{\partial Q}{\partial K}\right) = \frac{-g'(L/N)phL}{\Delta} \left\{Q_{LL} Q_{KK} - Q_{LK}^{\ 2}\right\} < 0$$

The final comparative static result concerns the utility of a member. We have

$$\frac{dU_m}{d\phi} = \frac{-d\ g(\ell,\ \phi)}{d\phi} - h \frac{dr}{d\phi}$$

where $\frac{dg}{d\phi}(\ell,\ \phi)$ is the change in the total disutility of supplying factor services when the marginal disutility shifts according to (3.13).

There is obviously no assurance that $\frac{dU_m}{d\phi}$ is negative, unless the extra rental is distributed back to the members of the firms by the rentier. If the extra rental is used to buy more capital goods for the next period's production then the members may eventually benefit from what is in effect forced savings. However if the rentier distributes his profit abroad or consumes it himself, then his monopoly position may be such as to allow him to appropriate all the efficiency gain and more beside.

Thus as ϕ decreases from unity, moving the LM-economy away from the long-run EP-economy equilibrium, effort per worker and average productivity per worker increases. Also if aggregate capital is fixed, and marginal products decline alone linear paths from the origin, then total efficiency units of labour per firm increase but capital and number of workers per firm falls. Also the worker-members will only be definitely better-off if all the extra rental generated by the increased demand for capital (and demand for capital will always be increased) is distributed to the worker-members.

IV

We have been concerned so far in a consideration of the effects of an improved work environment as a result of reorganising firms under collective rather than individual entrepreneurship. The results of this analysis are summarised in the first row of the Table. In the first set of columns, results from Section II are reported. These relate to responses of a single firm to such reorganisation from an initial situation of an EP-firm in long-run equilibrium. They also relate to an economy-wide reorganisation provided the supply of capital is perfectly elastic. This is of course because, in the absence of a change in capital rental, the labour input (in efficiency units) per firm is unchanged and so the same capital level solves (3.11). The right-hand set of columns constitute the results of Section III where capital in the economy is assumed fixed, so that reorganisation of the firms in the economy under collective entrepreneurship, which implies more firms with less members each, leads to an increase in the equilibrium capital rental.

An alternative way of improving work environment may be by maintaining individual entrepreneurship but involving some worker participation and self-supervision, counterbalancing this with incentives in terms of profit shares, etc., but retaining the overall objective of maximising the entrepreneur's utility. We will call such a firm a WS-firm (standing for worker participating profit sharing firm). Such systems may involve problems of the agent-principal kind (see Ross (1974)). In such an environment reduced disutility of work may occur, and this could be due to a reduction of either or both constituent parts of $g(\ell)$. A partial equilibrium analysis in such a case is

IV

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$$\bar{u} = y - \beta^*(\ell)$$

where y is worker's generalised income which may include a profit share, etc., and $\beta^*(\ell)$ is the worker's disutility of supplying ℓ efficiency units in this improved participatory environment. Obviously, the competitive labour market, if sustained, means that workers cannot improve their utility above $\bar{u}$: all gains from reduced workers' and entrepreneur's disutility of effort are available to the entrepreneur. Thus workers in an EP-firm would not be keen to initiate or agree to such a change in organisation unless it was accompanied by a measure of worker-control, which would approximate the firm to an LM-firm, or was part of a general economy-wide movement. Even in the latter case, workers will only unambiguously gain in the long run where $U_e = \bar{u}$. This is because the response of each firm to the improved disutility is ambiguous in respect to the demand for number of workers employed. If aggregate demand for labour were to fall $\bar{u}$ would be forced downwards and workers would be worse off in the short run, when the number of entrepreneurs, and thus firms, is fixed at the initial level. Furthermore, the presence of barriers to becoming entrepreneurs may make the long run heavily discounted by workers. Consider the following simple example of a single WS-firm operating in a competitive labour market.

Suppose $y = w + s\,R(L)/N$ and capital is fixed, such that w is the basic wage and s the proportion of net revenue paid as an incentive bonus to the workers. If s acted on profit rather than $R(L)/N$, w would simply be redefined net of y. Define the supervision or alienation cost in the WS-firm as

$$\phi\alpha(\ell) \quad \text{with} \quad 0 \leq \phi \leq 1 .$$

with $\beta(\ell)$ unchanged from the entrepreneurial firm. The workers maximise their utilities by colluding (see Berman (1977) and Bonin (1977)) to decide their supply function of efficiency units of labour:

$$\ell_s = \ell(s, N)$$

where ℓ_s solves

$$sR'(L) - \beta'(\ell) = 0$$

and thus $\dfrac{\partial \ell_s}{\partial s} > 0$

The entrepreneur now maximises U_e, given $\bar{u}$, with respect to N and s. Write

$$U_e = R(L) - (\bar{u} + \beta(\ell) + \phi\alpha(\ell))N + \bar{u} \tag{3a}$$

then differentiating yields first-order conditions

$$\frac{\partial u_e}{\partial N} = R'(L)\ell - (\bar{u} + \beta(\ell) + \phi\alpha(\ell)) + \frac{\partial u_e}{\partial \ell}\frac{\partial \ell_s}{\partial N} = 0$$

$$\frac{\partial u_e}{\partial s} = \frac{\partial u_e}{\partial \ell}\frac{\partial \ell_s}{\partial s} = 0$$

For the first-order conditions to hold we require $\frac{\partial u_e}{\partial \ell} = 0$ (as $\frac{\partial \ell_s}{\partial s} > 0$) . This implies that

$$(1 - s)R'(L) - \phi\,\alpha'(\ell) = 0$$

i.e. $$s = 1 - \frac{\phi\,\alpha'(\ell)}{R'(L)} \tag{3.18}$$

Equation (3.18) states that the optimal proportion of revenue should be one minus the marginal supervision cost of an extra efficiency unit of labour divided by its marginal product. Another way of putting this is that the share of revenue going to the entrepreneur should be the proportion of the marginal product of an efficiency unit of labour which is used to compensate the entrepreneur for the disutility it involved.

If $\alpha'(\ell) = 0$ then $s = 1$ and the entrepreneur has no incentive himself to fulfil his function well. In this case of course w would be negative - a "license" to work, and it is true that if the operation was to be repeated, there would be an incentive for the entrepreneur to act efficiently so as to increase the price of a license.

Models of the "sharecropping" process which visualise $s < 1$ argue that the supply of worker effort is inefficiently small (see, for instance, Markusen (1976)). By suggesting that there are always some extra entrepreneurial supervision and organisation costs

$(\phi\ \alpha'(\ell) > 0)$, albeit lower in a WS-firm, this problem is avoided and a deterministic and optimal solution forthcoming with $s < 1$, (with $R'(L) = \phi\ \alpha'(\ell) + \beta'(\ell)$). Furthermore efficiency has been improved $(\phi < 1)$ compared with the EP-firm $(\phi = 1)$, yielding the possibility of a Pareto-preferred state.

The source of the short-run ambiguity of the change in $\bar{u}$ to an improved work environment can be seen by using a parametric shift in the $\phi\ \alpha(\ell) + \beta(\ell) = g(\ell)$ function. As we have seen from the above example, it is unnecessary to spell out the exact form and operation of the contingent income function y . Let the $g(\ell)$ function shift to $\phi\ g(\ell)$, $\phi < 1$ with the improved work environment. Following from (3.7) and (3.8) we have the WS-firm's equilibrium defined by :

$$pQ_L = \phi\ g'(L/N) \tag{3.7a}$$

$$pQ_L\ L/N = \bar{u} + \phi\ g(L/N) \tag{3.8a}$$

If capital is also a variable we also have (3.11) and <u>either</u> the rental on capital is a fixed price (under the assumption of perfectly elastic supply of capital) <u>or</u> (3.12) holds under the assumption of a fixed aggregate stock of capital in the economy. Also in the short run when the number of entrepreneurs is fixed but full-employment is still required, entrepreneurs will not change their employment of workers ex post, although their demand schedules may have shifted. Thus the equilibrium system is completed by the requirement that in each firm N is constant in the short run :

$$N = \bar{N} \tag{3.19}$$

while in the long run (3.19) is replaced by $U_e{*} = U_w$. As we have argued, long-run equilibria are indistinguishable from those of the comparable (same ϕ) LM-economy. We shall therefore confine ourselves to comparative statics of the short run case, defined by (3.7a), (3.8a), (3.11), (3.19) and either r as an exogenous constant or (3.12). Consider the fixed capital stock assumption (3.12) first. From (3.12) and (3.19), both K and N are constant. (3.7a), (3.8a) and (3.11) can then be totally differentiated to find the response of L, $\bar{u}$ and r to a change in ϕ . We obtain

$$\frac{dL}{d\phi} = g'(L/N)/(pQ_{LL} - \phi\ g''(L/N)/N) < 0 \tag{3.20}$$

(and thus $\frac{d\ell}{d\phi} < 0$)

$$\frac{dr}{d\phi} = pQ_{KL}\frac{dL}{d\phi} \tag{3.21}$$

which is negative if $Q_{KL} > 0$, i.e. inputs are complements,

and $$\frac{d\bar{u}}{d\phi} = pQ_{LL}\ g'(L/N).L/(pQ_{LL}\ N - \phi\ g''(L/N)) - g(L/N) \tag{3.22}$$

Note that (3.22) is ambiguous in sign. One factor tending to depress $\bar{u}$ is the diminishing returns to labour, for as workers supply more effort, this diminishes the product of the marginal worker and thus the demand for workers. However, against this is the fact that, the bigger is $g(\ell)$, the bigger is the reduction in the cost of employing that marginal worker, caused by the reduction in ϕ.

The case where capital is in perfectly elastic supply can be considered by taking r and N as fixed and solving (3.7a), (3.8a) and (3.11) for changes in K, L and $\bar{u}$ in response to the change in ϕ. This yields :

$$\frac{dL}{d\phi} = pQ_{KK}\, g'(L/N)/D \tag{3.23}$$

$$\frac{dK}{d\phi} = -pQ_{LK}\, g'(L/N)/D \tag{3.24}$$

$$\frac{d\bar{u}}{d\phi} = -g(L/N) + L\, g'(L/N)(Q_{LL}\, Q_{KK} - Q_{KL}{}^2)p^2/N.D \tag{3.25}$$

and $$D = (pQ_{LL} - \phi\, g''(L/N)/N)pQ_{KK} - p^2\, Q_{KL}{}^2$$

As $D > 0$, (3.23) is again negative, as is (3.24) assuming input complementarity and (3.25) is ambiguous once more. The comparative static results (3.20) → (3.25) and associated results are summarised in the middle row of the Table. While these results relate to an economy-wide change in ϕ and thus change $\bar{u}$, the case of a single firm adopting a better work environment is reported in the bottom row of the Table. Here $\bar{u}$ is exogenous and fixed, as is r, and (3.7a), (3.8a) and (3.11) are used to solve for $\frac{dN}{d\phi}$, $\frac{dK}{d\phi}$ and $\frac{dL}{d\phi}$.

In the analysis of the WS-firm as opposed to the LM-firm, the shift in $g(\ell)$ rather than $g'(\ell)$ has had to be considered. This is because $g(\ell)$ appears in (3.8a) and $U_e{}^* \neq U_w$ after the change in ϕ. The parametric shift we have considered means that both $g(\ell)$ and $g'(\ell)$ change in the same proportion. This may be significant as the change in $g'(\ell)$ effects the change in ℓ and thus, through the diminishing marginal revenue product of labour, the reduction in the marginal revenue product of a worker, while $g(\ell)$ effects the amount the marginal cost of a worker is reduced, due to reduced disutility of work and hence less required compensation. It is the interplay of these two effects which determine the sign of $\frac{d\bar{u}}{d\phi}$ in the general equilibrium analysis and $\frac{dN}{d\phi}$ in the partial equilibrium analysis of the EP-firm case.

V

Although the results reported in this paper and summarised in the Table come from a rigidly structured model, incorporating many assumptions, they appear intuitively reasonable. The pricture of firms in the LM-economy employing smaller numbers of workers but with higher productivity per worker is clearly seen. Also an explanation is given for the often perceived hesitancy of labour unions to invoke worker-participation short of worker control, and the impact of different production systems on the market for capital is shown. Furthermore, the model considers the distribution of utility in addition to that of income, as well as concepts of equilibrium in an economy with two sources of disutility: that of the supply of labour services and that of the supply of entrepreneurial services. It has been our argument that only such a model is capable of yielding an analysis of the implications of the lower alienation of labour which has been claimed for LM-firms.

Finally, we must consider the extent to which the assumptions on which the analysis has been based are likely to have been either erroneous or misleading. The most obvious candidates among our

	Partial equilibrium analysis of single firm or general equilibrium of economy-wide change with perfectly elastic supply of capital							General equilibrium analysis of economy-wide change assuming fixed aggregate capital in economy.						
	$d\ell$	dN	dL	dK	dQ	$d(\frac{Q}{N})$	$d\bar{u}$	$d\ell$	dN	dL	dK	dQ	$d(\frac{Q}{N})$	$d\bar{u}$
LM-firm or WS-firm in long-run equilibrium with $U_e^* = \bar{u}$	+	−	0	0	0	+	+	+	−*	+*	−*	?	+	?
WS-economy with fixed number of firms and full employment	+	0	+	+**	+	+	?	+	0	+	0	+	+	?
WS-firm with $\bar{u}$ fixed	+	?	+	+**	+									

Table: Comparative Static results of a proportionate reduction in the disutility of work effort $(g(\ell))$.

* Assumes both marginal products decrease along straight line paths from the origin in (K, L) space

** Assumes capital and efficiency units of labour are complementary inputs.

assumptions are the form of the utility function and in particular the absence of income effects in the supply of effort, the specification of the division of disutility of effort between entrepreneurial and worker cost, the assumption of given prices, the existence of a market for rented capital, the absence of risk, and the assumption of full employment in both EP- and LM-economies. The first two of these are technical assumptions, basic to the analysis, and it is difficult to see how the analysis could proceed without either them or equally stringent alternatives.

Relaxation of the other assumptions, however, offers scope for further analysis in extending the current simple model to cover questions of monopoly, property rights in capital, risk aversion, and inefficient labour market adjustments. Such extensions, however, would all require specification of the exact forms such features were to take. We will consider the question of risk and risk aversion in Chapter 5.

Appendix to Chapter 3

Derivation of (3.14) - (3.16)

Substitute (3.12) and (3.11) into (3.9a) and (3.10a) and obtain, using (3.13) :

$$p\,Q_L(L, K) = \phi\, g'(hL/K)$$

$$Q_L(L, K) = (Q(L, K) - Q_K(L, K).K)/L$$

Totally differentiate with respect to L, K, p and ϕ :

$$\begin{bmatrix} pQ_{LL} - \phi\, g''.h/K & pQ_{LK} + \phi\, g''\, hL/K^2 \\ Q_{LL} - \left[\dfrac{(Q_L - Q_{KL}K)\, L - (Q - Q_K K)}{L^2}\right] & Q_{LK} + Q_{KK}\, K/L \end{bmatrix} \begin{bmatrix} dL \\ dK \end{bmatrix} = \begin{bmatrix} g'd\,\phi - Q_L\, dp \\ 0 \end{bmatrix}$$

which simplifies to

$$\begin{bmatrix} pQ_{LL} - \phi\, g''h/K & pQ_{LK} + \phi\, g''\, hL/K^2 \\ Q_{LL} + Q_{KL}\, K/L & QLK + Q_{KK}\, K/L \end{bmatrix} \begin{bmatrix} dL \\ dK \end{bmatrix} = \begin{bmatrix} g'd\,\phi - Q_L\, dp \\ 0 \end{bmatrix}$$

and so

$$\begin{bmatrix} dL \\ dK \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} Q_{LK} + Q_{KK}\, K/L & -[pQ_{LK} + \phi\, g''hL/K^2] \\ -[Q_{LL} + Q_{KL}K/L] & pQ_{LL} - \phi\, g''\, h/K \end{bmatrix} \begin{bmatrix} g'\, d\phi - Q_L\, dp \\ 0 \end{bmatrix}$$

where Δ is $(pQ_{LL} - \phi g''h/K)(Q_{LK} + Q_{KK} K/L) - (Q_{LL} + Q_{KL} K/L)(pQ_{LK} + \phi g''h L/K^2)$

which can be simplified to

$$\Delta = p(K/L)(Q_{LL} Q_{KK} - Q_{LK}^2) - \phi g''h(Q_{KK} K^2 + Q_{LL} L^2 + 2Q_{KL} LK)/(LK^2)$$

so that if $dp = 0$, $d\phi \neq 0$:

$$\frac{dL}{d\phi} = \frac{h(Q_{LK} L + Q_{KK} K)g'}{phK(Q_{LL} Q_{KK} - Q_{LK}^2) - \phi g''h^2(Q_{KK} K^2 + Q_{LL} L^2 + 2Q_{KL} LK)/K^2}$$

$$= \frac{h(Q_{LK} L + Q_{KK} K)g'}{pNh^2(Q_{LL} Q_{KK} - Q_{LK}^2) - \phi g''(Q_{KK} K^2 + Q_{LL} L^2 + 2Q_{KL} LK)/N^2} \quad (3.14/3.16)$$

Similarly

$$\frac{dK}{d\phi} = \frac{-h(Q_{LL} L + Q_{KL} K)g'}{pNh^2(Q_{LL} Q_{KK} - Q_{LK}^2) - \phi g''(Q_{KK} K^2 + Q_{LL} L^2 + 2Q_{KL} LK)/N^2} \quad (3.15/3.16)$$

<u>Derivation of (3.20) - (3.25)</u>.

Consider equations (3.7a), (3.8a), (3.11), (3.19) and (3.12), i.e. capital stock is fixed. Then from (3.12) and (3.19), both K and N are constants. Differentiate (3.7a), (3.8a) and (3.11) with respect to the endogenous variables L, $\bar{u}$ and r and the exogenous variable ϕ. This can be done in two separate exercises as (3.11) alone involves r.

Thus from (3.7a) and (3.8a) :

$$\begin{bmatrix} pQ_{LL} - \phi\, g''/N & 0 \\ pQ_{LL}\, L/N & -1 \end{bmatrix} \begin{bmatrix} dL \\ d\bar{u} \end{bmatrix} = \begin{bmatrix} g'\, d\phi \\ g\;\; d\phi \end{bmatrix}$$

Then
$$\frac{dL}{d\phi} = \frac{g'}{pQ_{LL} - \phi\, g''/N} \tag{3.20}$$

$$\frac{d\bar{u}}{d\phi} = -g + \frac{pQ_{LL}\, L\, g'}{pQ_{LL}\, N - \phi g''} \tag{3.22}$$

Then from (3.11)

$$\frac{dr}{d\phi} = pQ_{KL}\frac{dL}{d\phi} = \frac{pQ_{KL}\, g'}{pQ_{LL} - \phi\, g''/N} \tag{3.21}$$

When capital is in perfectly elastic supply, r and N are fixed and K, L and $\bar{u}$ are solved from (3.7a), (3.8a) and (3.11) for given ϕ, i.e. we have to consider the comparative statics of

$$pQ_L - \phi\, g' = 0 \tag{3.7a}$$

$$pQ_L\, L/N = \bar{u} + \phi\, g \tag{3.8a}$$

$$pQ_K = r \tag{3.11}$$

We then have from (3.7a) and (3.11) :

$$\begin{bmatrix} pQ_{LL} - \phi\ g''/N & pQ_{LK} \\ pQ_{KL} & pQ_{KK} \end{bmatrix} \begin{bmatrix} dL \\ dK \end{bmatrix} = \begin{bmatrix} g'\ d\phi \\ 0 \end{bmatrix}$$

so that

$$\frac{dL}{d\phi} = \frac{1}{D}\, pQ_{KK}\ g' \tag{3.23}$$

$$\frac{dK}{d\phi} = \frac{-1}{D}\, pQ_{LK}\ g' \tag{3.24}$$

where $D = (pQ_{LL} - \phi\ g''/N)pQ_{KK} - p^2 Q_{KL}{}^2$

and finally from (3.8a) :

$$\frac{d\bar{u}}{d\phi} = (pQ_{LL}\ L/N + pQ_L/N - \phi\ g'/N)\frac{dL}{d\phi}$$

$$+(pQ_{LK}\ L/N)\ \frac{dK}{d\phi} - g$$

$$= p^2 L\ g'(Q_{LL}\ Q_{KK} - Q_{LK}{}^2)/(N.D) - g \tag{3.25}$$

CHAPTER 4

Property Rights of Firm Membership and Capital

An individual in an economy may be able to decide whether to be a worker, selling the use of his labour, or an entrepreneur who buys other individuals' labour and organises it within a firm. He may be able to join with other individuals, to share the entrepreneurial burden, the labour and the rewards from both entrepreneurial and labouring activities. Another possibility is that the individual may be able to sell his labour for a claim on the income of the entrepreneur, and may in turn perform some entrepreneurial duties. These three variants on the worker-entrepreneur relationship are neither necessarily mutually exclusive nor exhaustive. They depict a conventional entrepreneurial firm, a labour-managed firm and a worker-participating, profit-sharing firm, and they can be considered as different allocations of the property rights of entrepreneurship. Property rights concerning capital employed in firms provide still further scope for discriminating between types of firm and have been prominently considered in the literature, (see Furubotn and Pejovich (1973), Pejovich (1969) and Vanek (1975)).

Property rights of firm membership are normally taken to be such that:

(i) The current members cannot be forced to leave against their will (except as a punishment for breaking rules).

(ii) Membership will only be expanded up to the point where further recruitment would not be in the interests of the current membership.

(iii) There is no market for firm membership. Thus membership is not transferable.

(iv) Income is shared between members according to set rules relating to skill, etc., and does not distinguish between income for labour and income for entrepreneurship.

These points imply that membership adjustment may be very sticky in a downward direction. However, for a poor LM-firm (low income per member) it may also be difficult to attract members. In this situation of output constrained by membership supply Domar (1966) argues that non-perverse output responses to price will occur. In general, however, the difficulties of labour mobility within an LM-economy are great and will be discussed in Chapter 6. It is however, crucial to the incentive effects within a LM-firm that some such property rights structure exists. For instance if membership could be sold, then this would be tantamount to a market for equity and speculative purchase of memberships could take place. The right to grant membership is only vested within the worker-members as a whole.

The rights of members of LM-firms are introduced here in order to investigate the crucial and interesting question of property rights in capital, when capital is purchased by the firm rather than rented from a rentier. There is no definitive allocation of property rights of membership or capital for all types and examples of LM-firm. We can however contrast two such types with a conventional entrepreneurial firm, using the notation of the last chapter, but revising the assumption of the existence of rentier capital. We will take a two-period model, which will be sufficient to discuss the argument.

The two types of LM-firms we will consider reflect opposite approaches to the question of property rights of capital. One type is exemplified by the Yugoslav LM-firm in the late 1960's (see Vanek (1975, ch. 28). Here the LM-firm is given (although at a fairly high rate of interest) sufficient initial capital to make a project viable. The firm must maintain this stock of capital at book value by investing sufficiently to offset depreciation. Increases in capital stock can be financed by ploughing back profits, implying current sacrifice by the members, or borrowing at fairly short term from the National Bank. Any increase in the book value of assets occuring as a result has to be maintained. Thus at no time can the members dispose of the capital assets of the firm, even by just allowing them to depreciate. The members have only the <u>right of use</u> not the <u>right of ownership</u> of the capital of the firm.

A very different approach to property rights in capital is shown by the Mondragon cooperative in Spain (see Oakeshott (1973)). Here members make capital contributions (a minimum of £650 in 1972). They obtain a return of 6% (in 1972) on their capital plus a wage or salary. Virtually all profit after these payments is reinvested. The most important difference between the Yugoslav firm and the Mondragon cooperative is that in the latter, members who leave receive their capital back (minus 12½% (1972)) but plus an element for capital growth.

Suppose that as well as efficiency units of labour and entrepreneurship, capital also enters into the production process. We will define the production function as $Q(K, L)$ and assume it to be strictly concave and twice differentiable. We will consider a two-period model where capital is obtained at the beginning of the first period, used during that period, and disposed of at the beginning of the second period. We will ignore depreciation of capital. In the EP-economy finance is either internal or external. If the rate of interest on savings is equal to that for borrowing then this makes no difference. We will denote the saving and borrowing interest rate as i. Then if output is sold and workers paid at the end of the period, we have in present value terms

$$U_e - \bar{u} = \frac{p\,Q(K, L) - (\bar{u} + \beta(\ell) + \alpha(\ell))N}{1 + i} - K + \frac{K}{1+i} \qquad (4.1)$$

where the last two terms indicate the payment for capital in the first period $(-K)$ and the present value of revenue from its sale at the beginning of the second period. If finance was provided externally then a flow $+D$ is received at the beginning of the first period and a repayment of $D(1 + i)$ discounted to the present is made at the beginning of the second period. The net effect is $D - D(1 + i)/(1 + i) = 0$. Then (4.1) can be written:

$$U_e - \bar{u} = (p\,Q(K, L) - (\bar{u} + \beta(\ell) + \alpha(\ell))N - iK)/1 + i \qquad (4.2)$$

If the membership of the LM-firm all believe that they will still be members of the firm in the second period and can then sell the capital, then the same approach yields (taking $\phi = 1$)

$$U_m = \frac{(p\ Q(K,\ L) - g(\ell)N - iK)}{N(1 + i)} \tag{4.3}$$

With both firms the condition

$$\frac{\partial Q}{\partial K} - i = 0 \tag{4.4}$$

is optimal, and can be added to appropriate optimality conditions with respect to ℓ and N as well as any relevant long-run equilibrium condition on the number of firms. If investment is self-financed, that is the end of the story as savings is equal to investment by definition. If external finance is used then i must be the equilibrium market-clearing rate of interest and must reflect savings behaviour.

In the post 1965 Yugoslav LM-firm, Furubotn and Pejovich (1973) claimed that internal investment is penalised because members cannot dispose of the capital assets of the firm. The members have the right of use of the capital but not of ownership. They can sell capital assets only if they replace them sufficiently to maintain the book value of capital. The fact that the length of membership is finite and may in some cases be quite short means that if <u>either</u> form of finance is available the LM-firm's maximand may differ from (4.3). The utility of members who leave at the end of the first period is, for internal finance alone,

$$U_m = \frac{(p\ Q(K,\ L) - g(\ell).N)}{N\ (1 + i)} - \frac{K}{N} \tag{4.5}$$

and this is maximised when $\frac{\partial Q}{\partial K} - (1 + i) = 0$, while for external finance alone, we have

$$U_m = \left(\frac{pQ(K, L) - g(\ell).N}{1 + i} - K + D\right)/N \qquad (4.6)$$

which is maximised for $D = K$ when $\frac{\partial Q}{\partial K} = 0$. The former case is the hypothesis of Furubotn and Pejovich (1973) who contrast this with the case illustrated by (4.3) which would also relate to the Mondragon type of LM-firm. The point that paying back of at least some debt is also avoided by leaving the LM-firm is made in Stephen (1978). The latter paper also makes the point that if repayment of debt is made earlier than when the member leaves the firm then external finance produces (4.5) rather than (4.6), as this is in effect just internal finance with some temporal rearrangement. Some differences do arise if the lending and borrowing rates of interest are different.

Note that it is the property rights of capital that is the key factor in this. If capital could be liquidated within the membership period then the maximand would be (4.3). This is true even if with internal finance each member believes that the probability of his leaving the firm at the end of the first period is positive. Suppose $N_1 \leq N$ leave at this time. Then those that leave obtain (4.5) and those who remain obtain :

$$U_m = \frac{p\ Q(K, L) - g(\ell)N}{N(1 + i)} - \frac{K}{N} + \frac{K}{N-N_1}\ \frac{1}{(1+i)} \qquad (4.7)$$

If leaving is random, then the expected value of U_m is (4.3). (See Appendix) Under external finance, in our example, it becomes preferable to leave the firm at the end of the first period, as the capital is only collateral for the principal of the debt and not the interest.

Our example of a capital project here has been simplified, perhaps too far, in order to make a complex set of arguments clear. We will now turn to consider some of the implications. In a situation where bank finance (i.e. a rentier) is available, it has other advantages over internal finance than those portrayed above. Risk spreading is an obvious and important role of external finance. Another factor is the added liquidity that external finance brings: individuals are borrowing "long" through the LM-firm and lending "short" in their private savings accounts in banks. It has been argued (Furubotn and Pejovich (1973)) that the 1965 Yugoslavian reforms brought about the use of more external finance and that this was an important cause of inflation. However a tighter monetary policy, raising interest rates and limiting credit expansion would presumably have prevented this.

The problem of default on debt is also significant, although perhaps not in the Yugoslavian context. It is likely that debt would be limited to much less than the value of the capital, and so some internal investment would be necessary. This reflects the fact that although moving to an LM-economy may reduce the alienation of labour, it may also open up an alienation of the providers of capital from the objectives of the LM-firm. One possible way out of this puzzle is the risk-sharing bond (see McCain (1977)) where the provision of capital earns the provider a share in the LM-firm's income. Another is the short-term pay-back period which reduces risks of default, and implies a higher cost of finance for members who expect to leave the firm before the full value of the capital is realised.

Finally, we will undertake some comparative-statics to see how the LM-firm reacts to changes in the effective user cost of capital which we will call q . q is i in (4.3), $(1 + i)$ in (4.5) and 0 in (4.6). When $q = i = r$, the equilibrium of the LM firm is the same as that of a counterpart PM-firm where $U_e = \bar{u}$. This is a partial equilibrium model, as otherwise q would be determined endogenously. We define the LM-firm's equilibrium as

$$p \frac{\partial Q}{\partial K} - q = 0 \tag{4.8}$$

$$p \frac{\partial Q}{\partial L} - g'(\ell) = 0 \tag{4.9}$$

$$p \frac{\partial Q}{\partial L} - \frac{pQ - qK}{L} = 0 \tag{4.10}$$

Differentiating (4.8) and (4.10) yields (see Appendix) the by now familiar results (see Chapter 1, p. 7):

$$\frac{dL}{dq} = -\frac{1}{\Delta \cdot L} (Q_{KK} K + Q_{KL} L) \tag{4.11}$$

$$\frac{dK}{dq} = \frac{1}{\Delta \cdot L} (Q_{KL} K + Q_{LL} L) \tag{4.12}$$

Thus if both marginal products decreased with scale, labour efficiency units would increase and capital would decrease as q increased. Also from (4.9), (4.11) and (4.12) $\frac{d\ell}{dq} = \frac{-K}{L.g''(\ell)} < 0$. Whether output will increase or decrease as the property rights situation (summarised by q) changes depends again (see Chapter 1) on the relative elasticity of marginal products with respect to scale.

Appendix to Chapter 4

The expected value of (4.3) when membership termination is random is found as follows. The value of U_m for those who remain is (4.7), i.e.

$$U_m^1 = \frac{pQ(K, L) - g(\ell).N}{N(1 + i)} - \frac{K}{N} + \frac{K}{(N - N_1)(1 + i)} \tag{4.7}$$

Those members who leave at the end of the first period obtain (4.5), i.e. :

$$U_m^2 = \frac{pQ(K, L) - g(\ell)N}{N(1 + i)} - \frac{K}{N}$$

The probability of obtaining U_m^1 is $\frac{N - N_1}{N}$ and that of obtaining U_m^2 is $\frac{N_1}{N}$. If U_m is a Von Neumann-Morgenstern utility function then expected utility is

$$E(U_m) = U_m^1(N - N_1)/N + U_m^2 N_1/N = \frac{pQ(K, L) - g(\ell)N - iK}{N(1 + i)}$$

To obtain the comparative static results (4.11) and (4.12), totally differentiate (4.8) and (4.10) with respect to the endogenous variables L and K and the exogenous variable q :

$$\begin{bmatrix} pQ_{KL} & pQ_{KK} \\ pQ_{LL} & pQ_{LK} \end{bmatrix} \begin{bmatrix} dL \\ dK \end{bmatrix} = \begin{bmatrix} dq \\ -\frac{K}{L}\,dq \end{bmatrix}$$

Then $$\frac{dL}{dq} = -(1/(\Delta.L))p(Q_{LK}\,L + Q_{KK}\,K) \tag{4.11}$$

$$\frac{dK}{dq} = (1/\Delta L)p(Q_{KL}\,K + Q_{LL}\,L) \tag{4.12}$$

Now from (4.9)

$$g''(\ell)\,\frac{d\ell}{dq} = p(Q_{LL}\,\frac{dL}{dq} + Q_{LK}\,\frac{dK}{dq})$$

$$= p^2(-Q_{LL}\,Q_{KK}\,K + Q_{LK}{}^2\,K)/(\Delta.L)$$

and therefore

$$\frac{d\ell}{dq} = -p^2.K(Q_{KK}\,Q_{LL} - Q_{LK}{}^2)/(\Delta.L.g''(\ell))$$

As $\Delta = p^2(Q_{KK}\,Q_{LL} - Q_{KL}{}^2) > 0$, we have $\frac{dL}{dq}$, $\frac{dK}{dq}$ ambiguous but $\frac{d\ell}{dq} = \frac{-K}{L.g''(\ell)} < 0$.

CHAPTER 5

The Labour-Managed Firm under Uncertainty

Recent papers by Muzondo (1979), Hey and Suckling (1979), Ramachandran, Russell and Seo (1979), and Bonin (1979), have attempted to provide an analysis for the LM-firm under uncertainty which paralleled the analysis of Sandmo (1971) and Ishii (1977) concerning the profit-maximising competitive firm. Hey and Suckling in particular (i) compare the behaviour of the LM-firm under price uncertainty with that under certainty and (ii) examine the effect on the firm of an increase in uncertainty, in the form of a Sandmo-type mean-preserving spread of the distribution of the product price.

Muzondo (1979) shows that the LM-firm under uncertainty will, if its members are risk averse, produce <u>more</u> under uncertainty if the number of members is the only variable factor. He also produces various comparative static results, which Bonin (1979) shows are due to an algebraic error, and in fact comparative static results are, as one would expect, much the same as in the certainty case. Ramachandran, Russell and Seo (1979) discuss the institutional setting of the Yugoslav firm in the uncertainty case.

Hey and Suckling (1979) also derive the result concerning production by the LM-firm being greater under uncertainty in this situation. Hey and Suckling (1979) alone, however, show that membership and thus production increases for a (Sandmo-type) mean-preserving spread of the product price distribution. However they do so using a rather odd and confusing transformation of variables. We will give a parallel proof without such a transformation below, before considering some important problems in commenting on the behaviour of comparative systems under uncertainty. We will then produce a very simple model embodying both decisions concerning the supply of effort and concepts of long-run equilibrium discussed in Chapter 3. The behaviour of a parallel competitive firm under uncertainty is described following Ishii (1977) in the Appendix.

I. Fixed effort and capital

In this section we will hold capital and effort per worker fixed and allow only membership to be decided by the firm. The LM-firm seeks to maximise the utility gained by the individual member from his contract of membership in the firm. In previous analysis in the thesis, if the individual's effort level (ℓ) were fixed, this was equivalent to just maximising income per member. Under uncertainty, however, the exact outcome of decisions is not known, although we will assume that subjective probabilities can be attached to all possible outcomes. The maximand is then the expected value of the appropriate Von Neumann-Morgenstern utility function; i.e. maximise with respect to N

$$E\,\{U_i(y_i)\} \tag{5.1}$$

We will again assume that y_i is the same for all members and assume that all members have the same utility function. This is sensible as the rule of equal division of entrepreneurial risk (same y_i all members) implies that there are no advantages to diversifying the membership according to risk attitudes. Similarly there exists an advantage for those individuals who wish to adopt the same attitudes and policy concerning risk to join together in an LM-firm.

Thus the assumption of similar utility functions for individuals within a particular LM-firm is rational given the contingent contract defining the LM-firm.

We will confine ourselves to uncertainty in the product price p. Membership is fixed ex ante of the price being revealed. The price has a density function $f(p)$ and a mean $\bar{p}$ and variance σ_p^2. As capital and effort are fixed, income per member is just

$$y_i = (p\,Q(N) - FC)/N \tag{5.2}$$

where FC is fixed costs.

Thus the first-order condition for N to maximise (5.1) is

$$E\,U'(y)(p\,Q'(N) - y) = 0 \tag{5.3}$$

and the appropriate second-order condition which we will assume to hold is

$$D = E\left\{U''(y)(p\,Q'(N) - y)^2 + U'(y)(p\,Q''(N) - (p\,Q'(N) - y)/N)\right\} < 0 \tag{5.4}$$

We can rewrite (5.3) as

$$-E\,U'(y)(pB - c) = 0 \tag{5.5}$$

where $B = Q/N - Q'(N)$

$c = FC/N$

With diminishing marginal product of labour (we still have Q locally concave and twice differentiable), we have $B > 0$. From (5.5) we have

$$- E\left\{U'(y)\ B\ (p - \bar{p})\right\} = - E\ U'(y)\ (c - \bar{p}B)$$

i.e. $$\text{covariance}\ (-U'(y).B,\ p) = -(c - \bar{p}B)\ E\ (U'(y)) \qquad (5.6)$$

Now $U'(y)\frac{Q}{N}$ is the derivative of utility with respect to price. We will define risk aversion to be when utility is a strictly concave function of price, i.e. when $\frac{\partial^2 U(y)}{\partial p^2} < 0$. We will similarly define risk neutrality as when $\frac{\partial^2 U(y)}{\partial p^2} = 0$ and risk loving as $\frac{\partial^2 U(y)}{\partial p^2} > 0$. As $- U'(y)B$ is negatively related to $U'(y)$ and price does not appear in B , the sign of the covariance term in (5.6) is the opposite to the sign of $\frac{\partial^2 U(y)}{\partial p^2}$. Thus as $E\ U'(y) > 0$, we have from (5.6) :

$$-(c-\bar{p}B) \gtreqless 0 \text{ as members are risk } \left.\begin{matrix}\text{averse}\\ \text{neutral}\\ \text{loving}\end{matrix}\right\} .$$ Now

$$-(c - \bar{p}B) = - \bar{p}\ Q'(N) + \bar{y}$$

where $\bar{y}$ is the income per member at price $\bar{p}$. Therefore we can state that if $\bar{N}$ is the optimal membership when price is certain and equal to $\bar{p}$, we have

$$N \gtreqless \bar{N} \text{ as members are risk } \left.\begin{matrix}\text{averse}\\ \text{neutral}\\ \text{loving}\end{matrix}\right\} .$$ In particular $N > \bar{N}$ and thus output is greater in the presence of uncertainty when members are risk averse.

We can state this as

<u>Proposition 5.1</u> Membership will be higher, the same, or lower than in the certainty case as members are risk averse, neutral, or loving.

We can proceed from this result to the fact that the more uncertainty the larger will be the difference between the membership level under certainty and that under uncertainty in the risk aversion case. We will define this increase in uncertainty as a mean preserving spread of the price distribution such that a risk averse individual becomes worse off, i.e. we have $\frac{\partial^2 U(y)}{\partial p^2} < 0$. We will confine ourselves to a Sandmo-type mean preserving spread of the form

$$p' = \gamma p + \theta(\gamma) \tag{5.7}$$

where $E(p') = \gamma\bar{p} + \theta(\gamma)$

and $\theta(1) = 0 , \frac{d\theta}{d\gamma} = -\bar{p}$

Thus for all (γ, θ) pairs $E(p') = \bar{p}$. But for $\gamma > 1$ and $\theta < 0$, p' has a wider spread than p . We can thus consider the effect of a mean-preserving spread of price on the optimal membership by carrying out a comparative static analysis using (5.3) and see how optimal membership responds to an increase in γ from an initial value of 1.

We have

$$\frac{dN}{d\gamma} = -E\left[\frac{\partial}{\partial p}\{U'(y)\left[p\, Q'(N) - y\right]\}(p - \bar{p})\right]/D \tag{5.8}$$

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$$\frac{dN}{d\gamma} = -E\left[\frac{\partial}{\partial p}\{U'(y)\left[p\, Q'(N) - y\right]\}(p - \bar{p})\right]/D \tag{5.8}$$

Thus as $D < 0$ from (5.4), (5.8) has the sign of

$$E\left[\{-U'(y)B + U''(y)\left[pQ'(N) - y\right]Q/N\}(p - \bar{p})\right]$$

which can be rewritten as

$$E\left[(-U'(y)B - U''(y)(Bp - c)Q/N)(p - \bar{p})\right] \tag{5.9}$$

Now it is sufficient for (5.9) to be positive that both

$$-E\left[U'(y)B\ (p - \bar{p})\right] > 0 \tag{5.10}$$

and

$$-E\left[(U''(y)\left[Bp - c\right]Q/N)(p - \bar{p})\right] > 0 \tag{5.11}$$

(5.10) is the left-hand side of (5.6) and is thus positive given risk aversion $(\frac{\partial^2 U(y)}{\partial p^2} < 0)$. Now (5.11) can be rewritten as

$$-E\left[U''(y)B\ (p - \bar{p})^2 \quad - U''(y)B\ (c/B - \bar{p})\ (p - \bar{p})\right]Q/N \tag{5.12}$$

The first part of (5.12), $-E\left[U''(y)B\ (p - \bar{p})^2 Q/N\right]$, is positive by concavity of the utility function $(U''(y) < 0)$. It remains therefore to show that

$$E\left[U''(y)\ (c - \bar{p}B)\ (p - \bar{p})\right] \geq 0$$

i.e. $(c - \bar{p}B)E[u''(y)(p - \bar{p})] \geq 0$

As $c - \bar{p}B > 0$ from (5.6), we require to show that

$$E[u''(y)(p - \bar{p})] \geq 0 .$$

We can only demonstrate this final sufficient condition for the case when the Arrow-Pratt coefficient of absolute risk aversion (see Arrow (1970) and Pratt (1964)) is non-increasing in income. An interpretation of this would be that, when faced with a portfolio choice of a risky asset and a non-risky asset, the individual would <u>not</u> elect to buy <u>more</u> of the risky asset, the <u>lower</u> his income.

We will define the coefficient of absolute risk aversion A as

$$A = \frac{-U''(y)}{U'(y)}$$

Let $A(p)$ represent the functional dependency of A on the product price and then with non-increasing absolute risk aversion we have :

$$(A(p) - A(\bar{p}))(p - \bar{p}) \leq 0 \quad \text{all } p \tag{5.13}$$

(5.13) holds as if $p > \bar{p}$, we have $A(p) \leq A(\bar{p})$ and if $p < \bar{p}$, $A(p) \geq A(\bar{p})$. Thus

$$A(p)\ (p - \bar{p}) \leq A(\bar{p})\ (p - \bar{p})$$
$$-U''(y)\ (p - \bar{p}) \leq U'(y)\ A(\bar{p})\ (p - \bar{p})$$

and $$U''(y)\,(p - \bar{p}) \gtrless -\,U'(y)\,A(\bar{p})\,(p - \bar{p})$$

Thus $$E\left[U''(y)\,(p - \bar{p})\right] \gtrless -\,A(\bar{p})\,E\left[U'(y)\,(p - \bar{p})\right]$$

i.e. $$E\left[U''(y)\,(p - \bar{p})\right] \gtrless 0 \tag{5.14}$$

using (5.10).

As (5.14) completes our sufficient conditions, we can state

<u>Proposition 5.2</u> If individuals are risk averse and if the Arrow-Pratt coefficient of absolute risk aversion is non-increasing in income then a mean-preserving spread in the distribution of price will increase the optimal membership. Thus output will also increase.

A further possible source of uncertainty in the LM-firm is the level of fixed cost (FC) . Even with given capital the interest, rental or forced depreciation charges may change in the short-run whereas membership decisions may be relevant only to the medium term. It is interesting to see if the association between "perverse" behaviour to price and fixed costs changes noted in Chapter 1 extends to a similar association for increased uncertainty in these parameters: will the risk-averse LM-firm employ a higher membership if fixed costs are uncertain, and will the membership be greater the greater the degree of uncertainty as indicated by a mean-preserving spread of the distribution of fixed costs ?

Write

$$G = -(pQ'(N).N - pQ) > 0 \tag{5.15}$$

Then (5.3) can be rewritten

$$-\frac{1}{N} E\left[U'(y)\ (G - FC)\right] = 0$$

i.e. $$E\left[U'(y)\ (G - FC)\right] = 0 \tag{5.16}$$

Thus $$-E\left[U'(y)\ (FC - \overline{FC})\right] = E(U'(y)\ (\overline{FC} - G))$$

$$= (\overline{FC} - G)\ E(U'(y)) \tag{5.17}$$

where $\overline{FC}$ is the expected value of fixed cost. Given risk aversion, risk neutrality, risk loving, we have $\frac{\partial^2 U(y)}{\partial FC^2} < 0$, $= 0$, > 0, and the covariance term in the left-hand-side of (5.16) can be signed as

$$\text{covariance } (-U'(y), FC) \lesseqgtr 0 \quad \text{as} \quad U''(y) \lesseqgtr 0 .$$

Thus $\overline{FC} - G = pQ'(N) - \bar{y} \lesseqgtr 0$ as members are risk $\left.\begin{array}{l}\text{averse}\\ \text{neutral}\\ \text{loving}\end{array}\right\}$, $\bar{y}$ is income per member at expected fixed cost. In particular, if risk aversion exists then membership would be greater than that in the certainty case with given fixed costs of $\overline{FC}$. We will state this as

<u>Proposition 5.3</u> If fixed cost charges are uncertain then membership will be higher (the same) (lower), than in the certainty case, if the members are risk averse, (risk neutral), (risk loving).

Now consider a mean preserving spread of the distribution of fixed costs for a risk averse membership. Again we wish to show that

$$-\frac{1}{N} E\left\{\frac{\partial}{\partial FC}\left[U'(y)\ (G - FC)\right]\ (FC - \bar{FC})\right\} > 0 \tag{5.18}$$

for the optimal membership to increase with a mean-preserving spread of the fixed cost distribution. Rewrite (5.18) as

$$\frac{1}{N} E\left[\{U''(y)y - p\ Q'(N)\ U''(y) + u'(y)\}(FC - \bar{FC})\right] \tag{5.19}$$

First take the middle term of (5.19), i.e.

$$-\ pQ'(N)E\left[U''(y)\ (FC - \bar{FC})\right]$$

We will show this to be positive if the coefficient of absolute risk aversion, which we will write $A(FC)$, is non-increasing in income, that is non-decreasing in fixed costs. Thus

$$\left[A(FC) - A(\bar{FC})\right]\ (FC - \bar{FC}) \geq 0 \qquad \text{all} \quad FC$$

Then

$$-U''(y)\ (FC - \bar{FC}) \geq U'\ \ A(\bar{FC})\ (FC - \bar{FC})$$

and thus

$$-E\ U''(y)\ (FC - \bar{FC}) \geq\ A(\bar{FC})\ E\ U'(y)\ (FC - \bar{FC})$$

which in turn is non-negative from (5.17) and $\overline{FC} - G < 0$ with risk aversion, so that

$$-pQ'(N)E\left[U''(y)\ (FC - \bar{FC})\right] > 0 \tag{5.20}$$

Now take the rest of the terms in (5.19). Write these as

$$\frac{1}{N}E\left[U'(y)\ (1 - \mathcal{R}(FC))\ (FC - \bar{FC})\right] \tag{5.21}$$

when $\mathcal{R}(FC)$ is the coefficient of relative risk aversion. If $\mathcal{R}$ is non-decreasing in income (non-increasing in fixed costs) then the implication is that the <u>share</u> of wealth used to purchase a risky asset in a portfolio decision does not increase as wealth increases. Then

$$(\mathcal{R}(FC) - \mathcal{R}(\bar{FC}))\ (FC - \bar{FC}) \leq 0 \quad \text{all FC}$$

and $$-U'(y)\,\mathcal{R}(FC)\ (FC - \bar{FC}) \geq -U'(y)\,\mathcal{R}(\bar{FC})\ (FC - \bar{FC}) \quad \text{all FC}$$

$$U'(y)\ (1 - \mathcal{R}(FC))\ (FC - \bar{FC}) \geq U'(y)\ (1 - \mathcal{R}(\bar{FC}))\ (FC - \bar{FC}) \quad \text{all FC}$$

Thus $$E\left[u'(y)\ (1 - \mathcal{R}(FC))\ (FC - \bar{FC})\right] \geq (1 - \mathcal{R}(\bar{FC}))E\left[U'(y)(FC - \bar{FC})\right]$$

which in turn is non-negative from (5.17) and risk aversion, if and only if $\mathcal{R}(\bar{FC}) < 1$. We can thus state:

<u>Proposition 5.4</u> The optimal membership will increase for a mean preserving spread in fixed costs if :

(i) The coefficient of absolute risk aversion is positive and non-increasing in income;

(ii) The coefficient of relative risk aversion is non-decreasing in income;

(iii) The coefficient of relative risk aversion is less than or equal to one at the income associated with expected fixed costs $(\bar{FC})$;

With at least one of (i), (ii) and (iii) holding strictly.

Although these are only sufficient conditions, and of course from Proposition 5.3 we know that membership must increase with a spread of the fixed cost density function at <u>some</u> point from $\gamma = 0$, it is still interesting to note that the parallel with the perverse results under certainty (that membership increases with decreases in price and increases in fixed cost) is not exactly continued here. A strict parallel would be if we could state the same sufficient conditions for an increased spread in product price being "like" a reduction in price (given risk aversion) and implying higher membership and for an increased spread in fixed cost being "like" an increase in fixed cost (given risk aversion) and implying higher membership. In fact sufficient conditions for the latter are both more restrictive and specific.

II. Variable Effort

It is of course conceivable that membership could be considered as fixed in the long-run and capital variable in the medium term. This however would imply the LM-firm making its capital decision to maximise expected utility of income per worker for given membership which would in turn lead to an analysis qualitatively similar to that of a competitive entrepreneurial firm. Results for such a firm are thus as reported in the Appendix for the competitive firm under uncertainty.

A more interesting extension occurs when although capital is fixed, both membership and effort are decided prior to the stochastic parameter being revealed. Let us take this case when the source of the uncertainty is product price, a more likely scenario than the fixed cost uncertainty in these circumstances.

Describe utility as $U(y, \ell)$ which is concave in y and $-\ell$. Maximising the expected value of utility with respect to effort ℓ and membership N yields first-order conditions of the form :

$$E\{U_1 \, pQ'(L) + U_2\} = 0 \tag{5.22}$$

$$E\{U_1(pQ'(L) - y)\} = 0 \tag{5.23}$$

where U_1, U_2 are the partial derivatives of $U(y, \ell)$ with respect to y and ℓ. We can show that the total efficiency units of labour are greater under uncertainty than under certainty. Write (5.23) as

$$E\{U_1[(Q'(L).\ell - Q/N)p + FC/N]\} = 0 \tag{5.24}$$

$$(Q'(L) - Q/L)\ell\, E\{U_1[p_1 - FC/(Q - Q'(L).L)]\} = 0 \tag{5.25}$$

and
$$E\{U_1\ (p - \bar{p})\} = \left(\frac{FC}{Q-Q'(L).L} - \bar{p}\right)E(U_1) \tag{5.26}$$

As $E\{U_1\ (p - \bar{p})\} < 0$ and $E(U_1) > 0$, we have

$$FC < \bar{p}Q - \bar{p}Q'(L).L$$

i.e.
$$\bar{p}Q'(L) < \frac{\bar{p}Q - FC}{L} \tag{5.27}$$

If price was equal to $\bar{p}$ with certainty, L would solve (5.27) with equality. Thus as $\bar{p}Q'(L)$ cuts $(\bar{p}Q - FC)/L$ from above we have L greater under uncertainty than under certainty. Thus we have demonstrated

<u>Proposition 5.5</u> If both membership and effort are ex ante decision variables then the optimal level of total efficiency units of labour, and thus optimal output, is higher under uncertainty than under certainty, when the source of the uncertainty is the product price.

A general proposition concerning the level of effort per worker under certainty compared with under uncertainty is not possible. We can however adapt the model of Chapter 3 to consider a specific example using the tool of risk premiums developed by Arrow (1970) and Pratt (1964).

Assume the variance of price is known σ^2. We will further assume that capital is fixed in both EP- and LM-firms at the same level and ignore capital market considerations. However both ℓ and N are decided before the price is revealed, and it is the joint decisions concerning these variables that is the subject of our attention.

We will build risk aversion into the model by assuming that all individuals wish to maximise the expected value of a strictly concave monotonic-increasing transformation T of U, that is each individual wishes to maximise

$$E\,T(U)\ ,\ T'(.) > 0\ ,\ T''(.) < 0 \tag{5.28}$$

We will approximate (5.28) with $T(E(U) - \eta)$ where $E(U)$ is the expected value of U, i.e. expected income minus disutility of effort, and η is the Arrow-Pratt risk premium given by

$$\eta_e = -\tfrac{1}{2}\,\frac{T''(E(U))}{T'(E(U))}\,Q^2\,\sigma^2 = \tfrac{1}{2}\,A_e\,Q^2\,\sigma^2 \tag{5.29}$$

for the EP-firm's entrepreneur and

$$\eta_m = \tfrac{1}{2}\,A_m(Q/N)^2\,\sigma^2 \tag{5.30}$$

for the LM-firm's member where A_e, A_m are their respective coefficients of absolute risk aversion evaluated at expected income minus disutility of effort. The EP-firm's workers are not faced with any risk, so that $\eta_w = 0$.

Provided that $\eta_e > N\eta_m$ there is an efficiency gain for the LM-firm. However little should be read into this, as apart from the mechanisms for risk-sharing which may be available for EP-firms but not for LM-firms, there is also the question of income and wealth distributions. The EP-firm may occur when the entrepreneur has wealth such that he is much less risk averse than the typical LM-firm member.

It is of interest to see, however, how the existence of risk changes the decisions of the two types of firm. The objective function of the LM-firm is

$$E(U_m) - \eta_m = (pQ - FC)/N - g_m(\ell) - \tfrac{1}{2}A_m(Q/N)^2 \sigma^2 \tag{5.31}$$

Optimal ℓ and N are given by

$$pQ'(L) - g_m'(\ell) = BQ'(L) \tag{5.32}$$

$$pQ'(L) - (pQ - FC)/L = B(Q'(L) - Q/L) \tag{5.33}$$

where $$B = A_m(Q/N)\sigma^2/(1 - \tfrac{1}{2} A'_m(E(U)) (Q/N)^2 \sigma^2) \tag{5.34}$$

and $B > 0$ if $A_m'(E(U)) \leq 0$, i.e. non-increasing absolute risk aversion.

Thus here the optimal supply of efficiency units of labour is greater than at L_0 in figure 1, Chapter 3, as $Q'(L) < Q/L$ from concavity. Also $(pQ - FC)/L > pQ'(L) > g_m'(\ell)$ implies that ℓ is lower under uncertainty and thus N must be higher. The result concerning the number of workers mirrors that of Muzondo (1979), and Hey and Suckling (1979) and our previous analysis. The proposition that members will work less hard under uncertainty has some intuitive appeal given risk aversion as members are opting for the non-risky consumption of leisure.

Finally consider the EP-firm. The same approach applied to the entrepreneur's objective function of

$$E(U_e) - \eta_e = \bar{u} + pQ - FC - (\bar{u} + g(\ell))N - \tfrac{1}{2} A_e(E(U))Q^2 \sigma^2 \quad (5.35)$$

yields, if $A_e(.)$ is non-increasing in its argument :

$$pQ'(L) - g'(\ell) = pQ'(L) - \frac{\bar{u} + g(\ell)}{\ell} > 0 \quad (5.36)$$

Thus $pQ'(L) > g'(\ell) = \frac{\bar{u} + g(\ell)}{\ell}$, and in the long-run equilibrium where $E\, U_e - \eta_e = \bar{u}$, we have

$$\frac{\bar{u} + g(\ell)}{\ell} = (pQ - FC - \tfrac{1}{2}A_e\, Q^2 \sigma^2)/L \quad (5.37)$$

so that $pQ'(L) > g'(\ell) > (pQ - FC)/L$

Inspection of figure 1, Chapter 3, shows us that the equilibrium labour input will be less than L_0, and the number of workers will be less than in the certainty case as the $g'(\ell)$ function has shifted to the left. However, we cannot say whether the level of effort per worker has increased or decreased. The smaller number of workers and smaller output per firm is to be expected given the results of Sandmo (1971).

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Appendix to Chapter 5

The competitive firm under uncertainty

Consider a competitive firm with one variable input, x, and producing a single product. Let the product price be p and the input price be r. The competitive firm maximises the expected value of $U(\Pi)$, where

$$\Pi = pQ - rx$$

and $\quad Q = Q(x)$ with $Q' > 0$ and $Q'' < 0$.

1. Under uncertainty concerning either p or r, less, (the same), (more), is produced as the firm exhibits risk averse, (risk neutral), (risk loving) behaviour.

Proof (i) r uncertain with $E(r) = \bar{r}$.

$$E[U'(pQ' - r)] = 0$$

i.e. $\quad E[U'(pQ' - \bar{r})] = E\,U'(r - \bar{r})$

but $\quad E\,U'(r - \bar{r}) \gtreqless 0$ as the firm is risk $\left.\begin{array}{l}\text{averse}\\ \text{neutral}\\ \text{loving}\end{array}\right\}$

$\therefore$ if the firm is risk averse for instance

$$E[U'(pQ' - \bar{r})] = (pQ' - \bar{r})\,E(U') > 0\,.$$

Thus $\quad pQ' > \bar{r}$.

(ii) p uncertain with $E(p) = \bar{p}$.

$$Q'E\left[U'(p - r/Q')\right] = 0$$

i.e. $\quad Q'E\left[U'(\bar{p} - r/Q')\right] = Q' \; E \; U'(\bar{p} - p) \gtreqless 0$ as the firm is risk $\left.\begin{matrix}\text{averse}\\ \text{neutral}\\ \text{loving}\end{matrix}\right\}$. Thus if the firm is risk averse

$$\bar{p}\,Q' - r > 0$$

So with both kinds of uncertainty, a smaller amount of the single input is used and thus a smaller output is produced that if prices were certain at their expected values $(\bar{r}, \bar{p})$.

2. Consider the firm to be risk averse and let the spread of product price be increased in a mean preserving way of the Sandmo type.

Optimal x will adjust and the sign of the adjustment is given by the sign of

$$E\left\{\frac{\partial}{\partial p}\left[U' \; Q'(p - r/Q')\right](p - \bar{p})\right\}$$

$$= E\left\{U' \; Q'(p - \bar{p}) + U'' \; Q'(p - r/Q')Q(p - \bar{p})\right\}$$

Now $Q' \; E \; U'(p - \bar{p}) < 0$ from (1) above and the second term can be rewritten :

$$Q'\ Q\ E\left[U''(p - r/Q')(p - \bar{p})\right] = Q'Q\ E\left[U''(p - r/Q')^2\right]$$

$$+ Q'\ Q\ E\left[U''(p - r/Q')(r/Q' - \bar{p})\right]$$

The first part of the right hand side above is negative as $U'' < 0$. The second part can be investigated for the case where the coefficient of absolute risk aversion $A(p)$ is decreasing in p . Then

$$(A(p) - A(\tfrac{r}{Q'}))\ (p - \tfrac{r}{Q'}) < 0 \qquad \text{all } p$$

and $$-U''(p - \tfrac{r}{Q'}) < A(\tfrac{r}{Q'})\ U'(p - \tfrac{r}{Q'})$$

$$U''(p - \tfrac{r}{Q'}) > -A(\tfrac{r}{Q'})U'(p - \tfrac{r}{Q'})$$

$$E\left[U''(p - \tfrac{r}{Q'})\right] > -A(\tfrac{r}{Q'})\ E\left[U'(p - \tfrac{r}{Q'})\right] = 0$$

$\therefore$ as $\frac{r}{Q} - \bar{p} < 0$ (from (1)) the second part is negative also. Thus all the terms are negative and a mean preserving spread in the distribution of p decreases the size of the firm.

Finally consider a mean preserving spread in the distribution of the input price r . The response of optimal x is given by the sign of

$$E\left\{\frac{\partial}{\partial r}\ U'(pQ' - r)(r - \bar{r})\right\}$$

$$= E\left[\left\{-U''(pQ' - rx) - U''(pQ'.x - pQ) - U'\right\}(r - \bar{r})\right]$$

Now $E\left[U''(r - \bar{r})\right]$ is negative if $A(r)$ is increasing in r (risk aversion as an increase in r implies lower utility). This can be seen from :

$$(A(r) - A(\bar{r}))(r - \bar{r}) > 0 \qquad \text{all } r$$

$$-U''(r - \bar{r}) > A(\bar{r})U'(r - \bar{r})$$

and $$EU''(r - \bar{r}) < -A(\bar{r})E\left[U'(r - \bar{r})\right] < 0$$

With diminishing marginal product this ensures that

$$(pQ - pQ'.x)E\, U''(r - \bar{r}) < 0$$

The rest of the expression is

$$E\left\{\left[-U''(pQ - rx)-U'\right](r - \bar{r})\right\}$$

which can be written as

$$E\left[U'(\mathcal{R} - 1)(r - \bar{r})\right]$$

where $\mathcal{R} \equiv \frac{-U''}{U'}(pQ - rx)$ is the coefficient of relative risk aversion.

Let $\mathcal{R}$ be non-increasing in income, then

$$(\mathcal{R}(r) - \mathcal{R}(\bar{r}))(r - \bar{r}) < 0 \quad \text{all } r$$

Now $E\left[U''(r - \bar{r})\right]$ is negative if $A(r)$ is increasing in r (risk aversion as an increase in r implies lower utility). This can be seen from :

$$(A(r) - A(\bar{r}))(r - \bar{r}) > 0 \qquad \text{all } r$$

$$-U''(r - \bar{r}) > A(\bar{r})U'(r - \bar{r})$$

and $$EU''(r - \bar{r}) < -A(\bar{r})E\left[U'(r - \bar{r})\right] < 0$$

With diminishing marginal product this ensures that

$$(pQ - pQ'.x)E\,U''(r - \bar{r}) < 0$$

The rest of the expression is

$$E\left\{\left[-U''(pQ - rx) - U'\right](r - \bar{r})\right\}$$

which can be written as

$$E\left[U'(\mathcal{R} - 1)(r - \bar{r})\right]$$

where $\mathcal{R} \equiv \frac{-U''}{U'}(pQ - rx)$ is the coefficient of relative risk aversion.

Let $\mathcal{R}$ be non-increasing in income, then

$$(\mathcal{R}(r) - \mathcal{R}(\bar{r}))(r - \bar{r}) < 0 \quad \text{all } r$$

$$U'(\Re(r) - 1)\,(r - \bar{r}) < U'(\Re(\bar{r}) - 1)(r - \bar{r})$$

$$E\left[U'(\Re(r) - 1)\,(r - \bar{r})\right] < (\Re(\bar{r}) - 1)\; E\, U'(r - \bar{r})$$

$$\therefore \qquad E\left[U'\;(\Re(r) - r)(r - \bar{r})\right] < 0 \quad \text{if } \Re(\bar{r}) < 1$$

Thus sufficient conditions for input and output levels to be less with a mean-preserving spread in the distribution of r are that:

(i) The coefficient of absolute risk aversion is non-increasing in income (profit);

(ii) The coefficient of relative risk aversion is non-decreasing in income (profit);

(iii) The coefficient of relative risk aversion is less than or equal to one

and at least one of (i), (ii) and (iii) above must hold strictly.

CHAPTER 6

An Enterprise Incentive Fund for Labour Mobility in the Cooperative Economy

The purpose of this chapter is to develop, within a decentralized planning framework, a system of incentive payments to firms as a means of correcting labour misallocation in an economy of labour-managed or cooperative firms. This scheme, we believe, may offer certain advantages over existing proposals.

If the labour-managed firm is assumed to adjust its labour force and other inputs to maximize income per worker, then at the firm's optimum (assuming that inputs are homogeneous and that the firm is a price-taker) the value of the marginal product of labour will be equal to income per worker; and the level of any non-labour variable input will be defined, _ceteris paribus_, by the equality of its price per unit with the value of its marginal product.

Ward (1958; 1967, Chapter 9) demonstrates that the labour market tends to be rigid in an economy of such firms. Thus, for example, at prevailing levels of income per worker there might be excess supply of labour, yet existing cooperatives will not expand membership if to do so would reduce income per worker. Moreover, consider two cooperatives each at its respective equilibrium at which income per worker is maximized, but let income per worker differ between them. Domar (1966) and others have argued it is not possible for labour to move from the "poor" to the "rich" cooperative because any such movement would reduce income per worker in both firms. Since income per worker in each cooperative is equal to the value of the marginal product of labour, it follows that there are differences in the latter between the cooperatives. Thus there is a misallocation of labour and of other resources among cooperatives in equilibrium at different levels of income per worker. Ward (1958) has also shown, for a single-product cooperative maximizing income per worker with labour the only variable input, that the typical reaction to an increase in output price is a contraction of employment and output levels. The cooperative economy thus fails in the short run to attract labour to its most highly valued uses.

Two points should be made before we consider solutions to this misallocation problem. First, as McCain (1973) and Furubotn and Pejovich (1973) suggest, contrary to the view taken in the simple income per worker maximizing model, the labour force of the cooperative firm may not easily be adjusted in the short run, especially when membership reduction is considered. Some approaches to labour force adjustment are indicated in the next section. Secondly, we would not wish to deny the importance of other problems of the cooperative economy not considered here, such as those relating to the investment decision discussed by Vanek (1975, Chapter 28) and Furubotn (1976). It has also been argued by Vanek (1970, Part III; 1975) and others that the labour-managed form of organization may exhibit certain advantages over that of the capitalist firms. These points and others have been considered in earlier chapters. They are omitted here for simplicity.

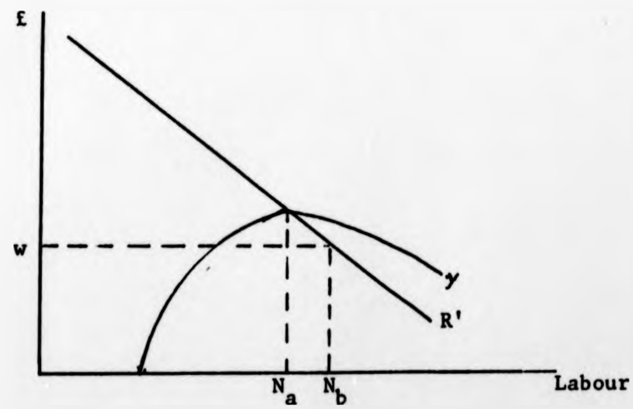
I. Some Existing Solutions

Vanek (1970) and Meade (1972) show that with free entry of new cooperatives allocative efficiency can be attained. However, this may be a lengthy process, and the establishment of appropriate conditions and institutions for the formation of new cooperatives may be a demanding task. Mechanisms that reduce inefficiency in the short-run in which no new entry takes place may therefore be worthy of serious consideration.

Meade (1974), in a discussion relating to imperfect competition, suggests that hiring and firing decisions might be taken by an outside authority which would instruct the cooperative to recruit labour so long as the value of the marginal product of labour in the cooperative exceeds its productivity in the rest of the economy. A problem with this solution, as Meade recognizes, is that it involves a severe dilution of the sovereignty of the worker-managers on which the concept of the cooperative firm is based. Others have suggested that inefficiency may be reduced to the extent that the cooperative can be induced to operate partly like a profit-maximizing firm. Thus Dubravcic (1970) proposes that cooperatives be permitted to hire new workers at a fixed wage without initially granting them a share in net income and full cooperative membership. Of course, this approach attempts to solve the problem by significantly altering the nature of the firm under consideration. Can the efficiency problem be solved while retaining intact the essence of the cooperative firm as an income-sharing institution with labour-management sovereignty?

Meade's (1972) answer is the Inegalitarian Cooperative, in which income per share is maximized but individual workers may hold different numbers of shares and therefore will not all receive identical incomes. This reflects Meade's requirement that membership adjustment can be made only if both existing (remaining) members and those who join (leave) the cooperative gain. A new recruit is attracted by an offer of shares but membership reduction is more complex and may involve partners bribing one of their number to withdraw, or a partner who leaves may have to compensate remaining members. Meade goes on to argue that the short-run adjustment process in an economy of Inegalitarian Cooperatives is Pareto-optimal.

It is interesting to contrast Meade's model with what we propose might be called the Egalitarian Cooperative. In this institution we conceive of new recruits as being treated on equal terms with existing members in income-sharing; and of members discarded through labour force adjustment as being fully compensated for any difference between what they earn outside the cooperative and what they would have earned had they remained within it. In Figure 1, which illustrates the determination of the equilibrium labour force, N , for the Egalitarian Cooperative, w is the money income that can be earned outside. R is the maximum value of the total product net of all non-labour costs, and so R' is thus the curve depicting the value of the marginal product of labour and y the net value of the average product (R/L). R' and y are drawn assuming that all non-labour variable inputs are adjusted to R-maximizing levels at any given labour input level; effort per worker is a given constant and it is also assumed that the firm has some fixed costs. Prices of output and non-labour inputs are exogenously fixed and constant and the underlying production function is assumed to be strictly concave.



y reaches max at N_a

Figure 1 Membership determination in the Egalitarian Cooperative

If the original labour force is less than N_a the firm will recruit workers until it reaches N_a the labour force that maximizes income per worker. If, on the other hand, the initial labour force exceeds N_b the firm will discard workers (thus raising y) until N_b is reached. At this point the marginal worker contributes R' ($= w$) to the total income of the cooperative and receives y. Thus the "cost" of the marginal worker to his fellows is $(y - R')$, which is just equal to the compensation, $(y - w)$, he would have to be paid if he were dismissed. By extension of the arguments it is easily seen that if the original labour force is between N_a and N_b no recruitment or dismissals will take place. The Egalitarian Cooperative, as our discussion of the possible equilibria indicates, will not in general yield allocative efficiency.

II. Incentives for Labour Mobility: A Proposal

An alternative approach to those discussed in Section I is to define an incentive scheme that will produce an efficient allocation of labour via the self interest of individual cooperatives. We will outline here such a scheme which involves a (positive or negative) transfer payment P_i from a central incentive fund to each firm i. Thus income per head in the i^{th} firm is given in (6.1) :

$$y_i = (R_i + P_i)/N_i \tag{6.1}$$

Here R_i is again the maximum income net of all non-labour costs for labour input N_i the number of members of the cooperative. Thus $R_i = R_i(N_i)$ and we will define

$$P_i = (R_i - \omega N_i)\ (N_i - \bar{N}_i)/\bar{N}_i \tag{6.2}$$

where ω and $\bar{N}_i$ are parameters of P_i, which is dependent on the decision concerning N_i. $\bar{N}_i$ is the number of members of the cooperative before the introduction of the incentive scheme and ω is chosen by the incentive fund directorate to equate planned demand and supply of labour. If $R_i - \omega N_i > 0$ and labour increases over the base or original level $\bar{N}_i$, or if $R_i - \omega N_i < 0$ and labour falls from the original level, the incentive fund pays the firm; otherwise, the firm pays the incentive fund. Substituting (6.2) into (6.1) we obtain

$$y_i = \omega + (R_i - \omega N_i)/\bar{N}_i \tag{6.3}$$

The cooperative will choose that labour input N_i which maximizes (6.3). The necessary and sufficient conditions for a unique optimum are

$$R_i' - \omega = 0 \tag{6.4}$$

and

$$R_i'' < 0 \tag{6.5}$$

We will assume that (6.5) is satisfied for all firms. Of course, there may be firms for which optimal N is zero and their entire labour force will be recruited by other expanding cooperatives. From (6.4) holding for all remaining firms we have that the marginal revenue product of labour is the same in all firms. This typifies an economy composed of profit-maximizing firms, and, if firms are assumed to be competitive (i.e. price-takers), constitutes the optimal allocation of labour. Note also that, given (6.5), an increase in ω, *ceteris paribus*, will cause all firms to reduce the level of N_i that they choose. Thus ω can be found such that planned supply and demand of labour can be equated.

An intuitive explanation for the working of the incentive scheme is based on the interpretation of ω as the shadow wage rate in the economy (from (6.4)). $R_i - \omega N_i$ is then the profit of the i^{th} firm at that valuation of labour - the shadow profit. From (6.3), income per worker in the i^{th} cooperative is then equal to the shadow wage rate plus the shadow profit per *original* unit of labour. The original workers can choose a "no-change" policy, $N_i = \bar{N}_i$ and have income R_i/N_i. If however $R_i' > \omega$, then the original workers will do better to recruit additional workers for which they will receive payment from the incentive fund if the shadow profit that results from operating at the expanded scale is positive (i.e. if $y_i > \omega$). Note that a negative shadow profit $(y_i < \omega)$ at the chosen level of operation will still imply recruitment as long as $R_i' > \omega$ even though this in turn implies a payment to the incentive fund, as shadow profit in (6.3) becomes less negative. The incentive fund thus pays "profitable" recruiting firms (where $y_i > \omega$), and receives payment from "unprofitable" recruiting firms. The situations with regard to firms that adjust their numbers of members downwards are analogous but reversed. However we can add the following interpretation. From (6.3),

$$R_i - \omega N_i = (y_i - \omega)\bar{N}_i \qquad (6.6)$$

Substituting (6.6) into (6.2) and then into (6.1) we obtain

$$y_i = \{R_i + (y_i - \omega)(N_i - \bar{N}_i)\}/N_i \tag{6.7}$$

We can interpret (6.7) as follows. Income per worker after membership adjustment is equal to the firm's operating income per worker R_i/N_i plus a compensation payment (positive or negative), $(y_i - \omega)$, per member made redundant spread over the remaining members of the cooperative. If $y_i > \omega$ then this payment reduces, and if $y_i < \omega$ it increases, income per remaining member. The incentive fund might, in the case where $y_i > \omega$, embody a characteristic of the Egalitarian Cooperative described earlier. If those made redundant could find work at the shadow wage ω , then the payment could be distributed to those made redundant and would thus act as a means of equating the incomes of those remaining in the cooperative with those made redundant. Indeed, where movement of labour from a "rich" to a "poor" cooperative is required, workers would not move voluntarily in the absence of such relocation payments to compensate them for the difference in income levels. Such compensation can of course be financed by the payments to the fund made by both cooperatives. The worker who moves to a cooperative paying a lower income per worker will share equally with other workers in the cooperative's income, but he will also be the recipient of a relocation payment from the state. Thus, as in Meade's economy of Inegalitarian Cooperatives, income differentials among homogeneous workers in the same cooperative may arise. However, in contrast to Meade's scheme, the cooperative itself does not discriminate between new members and old members, and members can leave the firm at will without ever having to compensate those who remain.

Of course, relocation payments are only pertinent where, in the absence of compensation, workers who move would be worse off, and they would not be relevant if it were assumed, following Ward (1958) and contrary to the behaviour of the Egalitarian Cooperative, that membership is freely variable so that workers may be expelled against their wishes if that raises the income of those who remain. The payments P_i effectively make it in the cooperatives' own interests in terms of maximizing incomes per worker to act like shadow profit-maximizing firms; i.e., for given ω, $\bar{N}_i$ to maximize $R_i - \omega N_i$ so that the value of the marginal product of labour is ω in all firms. Moreover, as each cooperative is now a shadow profit-maximizer, the possibility of perverse reaction to changes in output price, which we mentioned above, is removed.

In general P_i can be either positive or negative and the magnitude of ΣP_i will be determined by the distribution of all the $\bar{N}_i$ relative to N_i and the forms of the functions R_i.

The question arises as to whether the total net payments made from the incentive fund can be recouped by the increase in the yield from reasonable existing tax rates on personal incomes, which would increase in aggregate due to the gain in efficiency. Failing this, increases in tax parameters would be necessary to finance the fund. If there is initial unemployment, of course, this question is likely to be largely determined by the extent of saving from making payments to the unemployed. Otherwise examples can be constructed with very different implications for the financing of the incentive fund.

Some light can be shed by looking at three particular cases of the working of the scheme in an economy composed of two price-taking firms, both facing the same $R(N)$ function (apart from different fixed cost in one case) and with R'' a negative constant. The first two cases we will consider are described below, and illustrated in Figure II, where α, β, and γ are areas of rectangles.

<u>Case (i)</u>: Both firms have fixed cost of α. Firm A has an original number of members $\bar{N}_A$, and this number maximises income per worker (equal to $\bar{y}_A = \bar{R}'_A$) in the initial no incentive situation. Firm B has a larger original number of members $\bar{N}_B$ and income per worker ($\bar{y}_B = R'_B + 2(\beta + \gamma)/\bar{N}_B$) in the initial no incentive situation could have been higher if some members could have been persuaded to leave.

<u>Case (ii)</u>: The firms have $\bar{N}_A$ and $\bar{N}_B$ as above but have different fixed costs, that of firm A being α but that of firm B being $\alpha + 2(\beta + \gamma)$. $\bar{N}_B$ is thus optimal for firm B in the original no incentive situation and yields income per member $\bar{y}_B = \bar{R}'_B$. Thus in this case both firms have maximised income per member in the no incentive situation.

The socially optimal labour input in each firm is N in both cases above and this is achieved by applying the incentive scheme with $\omega = (\bar{R}'_A + \bar{R}'_B)/2$. The efficiency gain from the reallocation is γ in both cases, but the payments made from the incentive fund differ.

Consider first case (i). Here $R_A(N) - \omega N = R_B(N) - \omega N = \beta + \gamma/2$. Also $(N - \bar{N}_A)/\bar{N}_A = \frac{\gamma}{\beta}$ and $(N - \bar{N}_B)/\bar{N}_B = \frac{-\gamma}{\beta + 2\gamma}$. Thus the payments are from (6.2):

$$P_A = (\beta + \gamma/2)\frac{\gamma}{\beta} \tag{6.8}$$

$$P_B = -\gamma\frac{(\beta + \gamma/2)}{\beta + 2\gamma} \tag{6.9}$$

and the sum of (6.8) and (6.9) yields (6.10) :

$$P_A + P_B = \frac{\gamma^2(2\beta + \gamma)}{\beta^2 + 2\gamma\beta)} \tag{6.10}$$

It is interesting to note that the income of a original member of firm A increases by $\Delta y_A = \frac{\gamma(\beta + \gamma)}{2\beta N}$, while that of an original member of firm B increases by $\Delta y_B = \frac{\gamma(\beta + \gamma)}{2(\beta + 2\gamma)N}$ if the member remains at B and $\Delta y_B^* = ((\beta + \gamma/2)(1 + \gamma/\beta) - \beta(\beta + \gamma)/(\beta + 2\gamma))/N$ if the member moves to A . Then $\Delta y_B^* > \Delta y_A > \Delta y_B$, which suggests that an incentive would be provided for individuals to move from B to A , and that the income per member differentials between the two firms would be increased.

Now consider case (ii) . Here (6.8) still holds, but now $R_B(N) - \omega N = -(\beta + \frac{3}{2}\gamma)$ because of the extra fixed cost.

Thus

$$P_B = (\beta + \frac{3}{2}\gamma)\frac{\gamma}{\beta + 2\gamma} > 0 \tag{6.11}$$

and

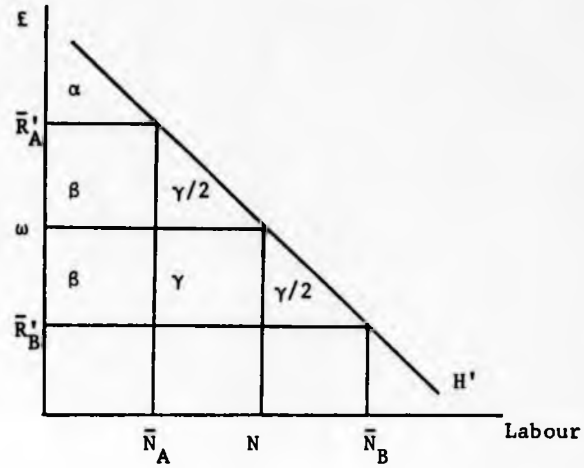


Figure II. Welfare gain from labour reallocation.

$$PA + P_B = \gamma\left(1 + \gamma/2\beta + \frac{(\beta + \frac{3}{2}\gamma)}{\beta + 2\gamma}\right) \tag{6.12}$$

Again it can be shown that $\Delta y_B^* > \Delta y_A > \Delta y_B$.

Finally we will consider case (iii) where we will ignore firm B and instead consider $N - \bar{N}_A = U$ is the amount of unemployed labour in the economy. The efficiency gain from using the unemployed labour in firm A is $\omega U + \gamma/2$. P_A is still as in (6.8) , and $\Delta y_A > 0$ as in the previous cases.

The first two cases, demonstrate very different implications for financing the incentive fund. If the marginal tax rate is t and the extra aggregate personal income is $P_A + P_B + \gamma$ then the fund will be exactly self-financing if

$$t_i = \frac{\gamma(2\beta + \gamma)}{\beta^2 + 4\gamma\beta + \gamma^2} \qquad \text{(case (i))} \tag{6.13}$$

and

$$t_{ii} = \frac{1 + \gamma/2\beta + \dfrac{\beta + \frac{3}{2}\gamma}{\beta + 2\gamma}}{2 + \gamma/2\beta + \dfrac{\beta + \frac{3}{2}\gamma}{\beta + 2\gamma}} \qquad \text{(case (ii))} \tag{6.14}$$

Choosing units such that $\beta = 1$, we find that as γ ranges from 0.1 to 1 , t_i would range from 0.15 to 0.5 while t_{ii} would range from 0.66 to 0.70 . It is thus much more likely that the incentive fund would be self-financing in case (i) than in case (ii).

It should be noted that the three cases of reallocation we have considered here are only examples and do not constitute an exhaustive analysis of all possible cases. For instance with reference to Figure II fixed costs may be such that both firms wish to be at $\bar{N}_B$ in the original situation; then firm A would like to recruit $\bar{N}_B - \bar{N}_A$ members but is prevented from doing so as there is no unemployment and B is not releasing labour.

Throughout the discussion our concept of efficiency gain is simply the increase in income that results from a reallocation of labour to equate the value of the marginal product of labour in all cooperatives. Two caveats are in order. First, this measure may be an overstatement where workers derive utility from sources other than income and there are adjustment costs associated with labour mobility. Second, it has been assumed throughout that a worker would move to another cooperative if offered employment there at a higher income. However, it must be recognized that this abstracts from impediments to mobility such as difference in housing conditions.

III. Some Further Issues

In this section we review our proposed incentive scheme in the light of some of the efficiency solutions discussed in Section I. This raises the issues of sovereignty, information requirements and long-run equilibrium. We also briefly consider the implications of extending our proposal to the multi-period case and to the case of the monopolistic cooperative economy, and finally consider incorporating heterogeneneous labour into our analysis. We continue however to abstract from problems arising from uncertainty, and to assume that workers derive utility from no other sources than income and that there are no adjustment costs associated with labour mobility.

The proposals referred to in Section I involved either dilution of the sovereignty of the worker-managers or a distortion of the cooperative firm from its pure form. Thus hiring and firing might be controlled by an employment authority (Meade, 1974), or the cooperative might be allowed to hire labour without in the short run granting full membership and entrepreneurial rights (Dubravcic, 1970); or it might be permitted to discriminate between new members and old members (Meade, 1972). The incentive scheme defined by (6.2) involves neither any inherent change in the cooperative as an institution nor any dilution of its sovereignty.

Of course the incentive scheme proposed here is not unique in this respect. For instance, lump sum taxes or subsidies, specific in quantity to each firm, could be found to reallocate labour optimally be changing the fixed costs of individual firms. Such a policy is adapted to the monopoly context by Vanek et al (1977). The problem with this approach, however, is that it requires that planners have information concerning the $R(N)$ functions of all firms. It is appropriate, at this stage, to consider the information requirements of our proposed incentive scheme.

Our scheme, like most decentralized planning instruments, would have considerable informational advantages over a policy of firm-specific lump sum taxes. It might be operated as follows. The fund directorate observes $\bar{N}_i$, states a value of ω and asks all firms to state their planned membership adjustments. If planned demand for members exceeds planned supply then ω is increased until ω equates demand and supply plans. The firms then adjust their membership and begin operation at that new membership level. At the end of the planning period they report the value of R_i they have achieved to the incentive fund directorate, which then makes or receives the payment defined in (6.2). At no point in this process does the fund directorate require any knowledge of the functions $R_i(N_i)$ (apart possibly in order to estimate the approximate cost of financing the fund for budgetary purposes). It is assumed, of course, that all firms are price-takers as far as ω is concerned. If a pure tâtonnement process is not possible, then of course the scheme may suffer from imperfections owing to attempts to operate the scheme at non-equilibrium values of ω.

We may now consider the extension of our proposal to the multi-period case and to the case of monopolistic economy. We will also outline a possible long-run adjustment mechanism.

As far as the multi-period case is concerned the incentive scheme generalizes immediately with the proviso that the $\bar{N}_i$ are <u>not</u> updated from period to period. The size of the incentive fund payments will thus vary over time as shadow profits move to zero as a result of new entry in the long run. Other proposals cannot be so easily generalized. Meade's (1972) Inegalitarian Cooperative is faced each period with a different supply price of labour in terms of number of shares. Now members' shares cannot be removed from them after they have joined the cooperative. Thus, if the supply price of labour is decreasing, the Inegalitarian Cooperative may postpone recruitment in order to take advantage of cheaper future labour markets. If, on the other hand, the supply price of labour is rising, the firm may over-recruit initially in order to prevent paying a higher supply price of labour later. Both cases involve actions that may infringe the efficiency conditions, although the second situation will not occur if members can resign and reapply.

In the long run, new cooperatives would be formed and others disbanded. Furthermore existing cooperatives may be reconstituted to undertake different activities at different intensities. The long run equilibrium that we envisage would be the result of the state forming or reconstituting cooperatives. It would do so by both choosing the area of the cooperative activity and defining $\bar{N}_i$ such as to maximize $R_i(\bar{N}_i)/\bar{N}_i$. The formation of such new cooperatives would increase productivity in the economy such that ω would have an upward trend. The payments made from or to the incentive fund by cooperatives where $y > \omega$ would diminish, and as the new cooperatives would offer $y_i(N_i) \geq R_i(\bar{N}_i)/\bar{N}_i \geq \omega$ cooperatives with $y < \omega$ would gradually be disbanded. Thus a long run equilibrium with $R_i(N_i)/(N_i) = \omega = R_i(N_i)$

for all i is obtained and is characterized by (i) Pareto efficiency and (ii) $P_i = 0$ for all i.

Note that ω would equate the supply and demand for labour both in terms of short-run adjustments via the incentive scheme and the underlying long-run adjustments via the formation of reconstitution of cooperatives. There appears little reason to suppose that progress towards long-run equilibrium would be inhibited by the short-run adjustment mechanism of the incentive fund.

If the cooperative is not a price-taker but faces a downward-sloping demand curve, the incentive function P is insufficient to maximize social welfare, as indeed are all the policy instruments considered so far. A further policy instrument is required to ensure that the firm produces the socially optimal output level. Vanek <u>et al</u> (1977) suggest coupling an administered price with lump sum taxation. This, as with all solutions to the monopoly problem, involves knowledge of the demand and cost functions facing each firm in the economy. A system of administered prices could, of course, be coupled with our incentive fund proposal. An alternative would be to use a variation of (6.2) such as (6.15) and then to approximate the consumer surplus term by readily observable magnitudes:

$$P_i^* = (R_i - \omega N_i)(N_i - \bar{N}_i)/\bar{N}_i + (S_i - \bar{S}_i)N_i/\bar{N}_i \qquad (6.15)$$

In equation (6.15) S_i is the consumer surplus associated with input decisions of labour N_i and other inputs x_i, and $\bar{S}_i$ is that associated with the original input levels $\bar{N}_i$ and $\bar{x}_i$. In this case R_i is better considered as the difference between revenue and cost other than labour costs and $R_i = R_i(N_i, x_i)$. Given (6.15), income per head is y_i^* where

$$y_i^* = \omega + (R_i - \omega N_i + S_i - \bar{S}_i)/\bar{N}_i \tag{6.16}$$

and x_i , N_i , which maximizes y_i^* , maximizes profit plus consumer surplus, i.e., $R_i + S_i - \omega N_i$. Approximating $S_i - \bar{S}_i$ by $\frac{1}{2}(\bar{p} - p)(\bar{q} + q)$ where $\bar{p}$, $\bar{q}$ are the original price and quantity demanded and p , q those relating to input levels (x_i , N_i) , P_i^* can be expressed in terms of easily observable and measurable magnitudes. The closeness of this approximation depends, of course, on how well the demand function can be approximated by a linear function.

The scheme can be extended to the case of heterogeneous labour, if we make the assumption that the occupational wage structure is fixed by the state to equate demand and supply for all classes of labour, and that all firms have to make the same relative income payments – that is, in the case of just two classes of labour, although members of each class of labour will generally have different incomes in different firms, the ratio of incomes of the two classes of labour will be constant for all firms. We define k classes or grades of labour and let $a_j (j = 1, 2, . . . k)$ represent the ratio of the income of a member of the j^{th} class of labour to that of a member of the first class. Thus $a_1 = 1$. All firms have to make the same relative income payments, that is accept given values for the a_j . Enterprise autonomy is thus somewhat reduced. We can then use the a_j to aggregate the labour input of the i^{th} firm and define :

$$N_i = \sum_{j=1}^{k} a_j N_i^j \qquad (6.17)$$

$$\bar{N}_i = \sum_{j=1}^{k} a_j \bar{N}_i^j \qquad (6.18)$$

where $\bar{N}_i^j$, N_i^j are respectively the numbers of grade j workers previously employed and to be determined in the i^{th} firm .

Substitution of (6.17) and (6.18) into (6.2) and then the result into (6.1) yields :

$$y_i = \omega + (R_i - \omega \sum_{j=1}^{k} a_j N_i^j) \sum_{j=1}^{k} a_j \bar{N}_i^j \qquad (6.19)$$

Necessary and sufficient conditions for a unique maximum of (6.19) with respect to N_i^j are :

$$\frac{\partial R_i}{\partial N_i^j} - \omega a_j = 0 \qquad j = 1, 2, \ldots, k \qquad (6.20)$$

and that the Hessian of R is negative definite.

The directorate of the incentive fund have to determine ω and a_j $(j = 2, 3, \ldots, k)$ from information concerning firms' planned supplies of and demand for different grades of labour in order to equilibrate the k labour markets. Given this, ωa_j is the shadow value of a unit of the j^{th} grade of labour.

Finally, consider the case where effort is a variable ℓ and labour input can be changed for given N . The scheme can easily be adapted to the present context provided a common ℓ is fixed by

<u>collusion of co-operative members.</u>

Each co-operative is paid from (or has to pay to) a central incentive fund an amount B relating to the values of R , ℓ and N . The incentive payment, B , is defined as follows, where R = R(L):

$$B = (R - \omega L)(\frac{N-\bar{N}}{\bar{N}}) \tag{6.21}$$

where ω and $\bar{N}$ are parameters; ω being interpreted as the shadow price of a man-hour and being common to the function (6.21) for all firms, $\bar{N}$ denoting the initial membership of the firm which will therefore vary from firm to firm. The payment B is treated as a revenue (or cost) of the firm to be shared out in relation to work-time supplied, thus

$$y = (R + B)\frac{\ell}{L} = \omega\ell + \frac{(R - \omega L)}{\bar{N}} \tag{6.21}$$

Maximisation of utility $U(\ell, y)$ with respect to ℓ and N yields respectively (6.22) and (6.23)

$$-\frac{U_\ell}{U_y} = \omega + (R_L - \omega)\frac{N}{\bar{N}} \tag{6.22}$$

$$0 = (R_L - \omega)\ell \tag{6.23}$$

which may be combined to produce the medium-term efficiency condition

$$-\frac{U_\ell}{U_y} = R_L = \omega \qquad \text{(all co-operatives)} \tag{6.24}$$

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Of course if there was an entrepreneurial sector ω would simply be the competitive hourly wage rate, otherwise ω is set by the incentive fund directorate.

In the long run cooperative mobility across industries and the formation of new cooperatives equates income per man hour in all cooperatives and B in (6.21) converges to zero as R/N converges to ω . Thus this incentive scheme produces medium-term efficiency as described in Chapter 2, and new entry and mobility of all factors ensures that Pareto efficiency exists in the long run. However needs payments if made would again present added problems.

IV. Conclusion

This chapter has been mostly concerned with presenting an incentive scheme to remove the inefficiency owing to labour market imperfections in the cooperative economy which has long been recognized in the literature. The significance of the incentive scheme can be seen in two ways. First, it can be considered as a decentralized planning analogue to the other proposed solutions to the problem, and thus as a contribution to the theoretical analysis of the inefficiency question. Second, it can be viewed as a policy prescription. However, if this latter viewpoint is taken a number of qualifications have to be made owing to the implicit and explicit assumptions of the model we have been using.

In this context, the critical assumptions include the simple income per member maximand (other maximands are discussed by Vanek, 1975, pp. 30-31, and Furubotn, 1976) and the absence of adjustment costs. Variations on either of these assumptions will almost certainly imply that revisions of the incentive scheme are required. In particular, if monetary valuation of significant differential non-pecuniary benefits and conditions of work is not possible, difficulties may be created for the operation of the scheme. Finally, short-run distribution of income consideration have been neglected as it is assumed that the welfare function is such that efficiency is preferred whatever the consequences for the distribution of income in the short run.

In conclusion, we must stress that the incentive scheme we have proposed here is one in which the income per member maximizing behaviour of the cooperative becomes identical to profit-maximizing behaviour at a given shadow wage rate ω . In so far as the individual cooperatives are price-takers, this constitutes in essence a similarity between the objectives of the cooperatives and the efficiency objective of the state. The scheme thus allows the state to achieve its aims without recourse to direct controls over the behaviour of the individual cooperative, and thus without requiring a large amount of firm-specific information.

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CHAPTER 7

Incentives and Efficiency in the Kosygin Reforms

An important feature of the 1965 reforms in Soviet industry was the establishment of a new system of enterprise incentive funds. Given that the reform measures, in the words of a translation of Premier Kosygin's (1970) famous speech, were intended "to expand the economic independence of individual enterprises" the material incentive fund (a source of bonuses) might be expected to influence enterprise behaviour. Two possible effects have been distinguished. Ellman (1971, 1977) has discussed the extent to which the incentive system induces the adoption of taut plans by the enterprise. A taut plan, as Ellman (1977, p.34) indicates, may nevertheless be inefficient in input use, scale of output, technology or product mix. Domar (1974), Bonin (1976) and Martin (1976) have analysed the possible influence of the system on input and output choices of the enterprise assuming maximisation of incentive funds or bonuses. The concern of the present chapter is also with this latter influence and we abstract from the issue of taut versus slack plans. It need hardly be said that, although this is a convenient simplification, both effects may operate together and may interact.

Several variants of the incentive scheme have been studied. In the typical formulation with which the present paper is concerned the incentive fund is determined by enterprise performance as reflected in sales growth and profitability thus,

$$\frac{B}{wN} = a\frac{\Pi}{K} + c\,\frac{(PQ-\overline{PQ})}{\overline{PQ}} \tag{7.1}$$

where B is the incentive fund or total enterprise bonuses, wN the wage bill (wage rate multiplied by numbers of workers employed), Π

profits, K capital employed and PQ revenue, $\overline{PQ}$ being the initial level of revenue. Positive coefficients a and c are set by central planners and regulate the relative importance of the two performance indices, sales growth and profitability, in bonus determination.

As w and $\overline{PQ}$ will be assumed fixed we may define Z and Z' ,

$$Z = a\frac{\Pi}{K} + bPQ \ , \qquad b \equiv \frac{c}{\overline{PQ}} \tag{7.2}$$

$$Z' = (Z - c)N \tag{7.3}$$

so that maximising (7.2) implies identical decisions to maximising B/N and maximising (7.3) implies identical decisions to maximising B . There has been some discussion on whether bonuses per worker or total bonuses is the more appropriate objective for the enterprise, (see for example Horwitz (1970), Fukada (1977), and Law (1974). Bonuses per worker of course already has a parallel in the theory of the socialist enterprise in that Ward (1958) assumes maximisation of income per worker in his seminal model of the labour-managed firm. Bonin (1976) views a formulation like (7.2) as a relevant maximand for the Soviet manager and the present analysis will concentrate on (7.2) rather than (7.3). However, an alternative view is taken in Martin's (1976) recent paper where a managerial maximand similar to Z' is employed.

According to Ellman (1971, p.133) the intention of (7.1) was to induce the enterprise to increase consumer satisfaction (proxied by sales) and efficiency (proxied by profitability). A similar interpretation is offered by Adam (1973) who states that the idea was to stimulate an expansion of output while inducing enterprises to seek increases in productivity. In the present paper we postulate one or both of the following as possible objectives of the incentive scheme.

1. To achieve technical efficiency in that:

   (i) input mixes in all enterprises are such that the marginal rate of technical substitution between the two factors of production, labour and capital, are equal in all enterprises.

   (ii) The scale of output in each enterprise is such that transferring inputs from one enterprise to another would reduce welfare.

2. To increase output without increasing inputs of factors of production by increasing effort or reducing "slack" or x-inefficiency in an enterprise.

We will argue that the bonus scheme (7.1) is unlikely to be successful in achieving either of these objectives, and that the reasons for this are at least part of the explanation for the early amendment of the scheme.

I. Technical Efficiency

Bonin (1976) has demonstrated that 1 (i) is not met by the scheme, for if Z is maximised then

$$\left(\frac{\frac{\partial Q}{\partial N}}{\frac{\partial Q}{\partial K}}\right)_i = \frac{w}{r + \left(\frac{\Pi}{K}\right)_i} \tag{7.4}$$

where w is the wage rate, r the capital rental rate and $(\Pi/K)_i$ is the profit rate on capital of the i^{th} enterprise and the left-hand side of (7.4) is the marginal rate of technical substitution of labour for capital in the i^{th} enterprise with production function $Q_i = Q_i(K_i, N_i)$. As $(\Pi/K)_i$ may be expected to differ between enterprises technical efficiency in terms of input mix is not achieved. (Marginal rates of technical substitution between labour and capital would not differ among enterprises in Domar's (1974) formulation, where the manager's maximand contains sales and profits, but not the profit rate. Bonin (1977) employs the same maximand in his recent analysis of quantity targets) Note also that the parameters a and b of (7.2) do not appear directly in (7.4) and would be irrelevant to input decisions if the enterprise simply maximised Z subject to producing a particular level of output predetermined by planners. If, on the other hand, output is variable the coefficients a and b might be expected to influence Π/K via the enterprise's output decision.

Of course although (7.4) would be valid if output levels were exogenously determined by quantity controls, it would not necessarily be valid if one or more inputs were in fixed supply.

If output levels are not exogenously determined and there are no constraints on inputs then a further unfortunate property of the bonus scheme is that given fixed output and input prices and the assumption that, for some capital/labour ratio, input levels exist which enable any output level to be produced, then the level of bonuses is unbounded from above provided $b > 0$. Equation (7.2) may be rewritten as

$$z = a\frac{PQ}{K} - a\left(\frac{wN}{K} + r\right) + bPQ \qquad (7.2)$$

Holding $\frac{K}{N}$ fixed, the limit of Z as $K, N \to \infty$ can be examined.

The limit of $a\frac{PQ}{K}$ is greater or equal to zero, and $wN/K + r$ is a constant. By the assumption above bPQ becomes arbitrarily large and thus there exists no interior global maximum for Z. Also $Z' = (Z-c)N$ is unbounded as $K, N \to \infty$ while $\frac{K}{N}$ is fixed as both $(Z-c)$ and N are unbounded. If the capital-labour ratio is also variable as the scale is increased then the managers can do at least as well as when it is fixed, and so a fortiori Z and Z' are unbounded.

The real significance of this argument is not that infinite bonuses could result - constraints on input availability or some other system reponse would occur long before this - but rather that some quantity controls on inputs or output would have to be in existence in parallel to the bonus scheme. The likelihood that the bonus scheme would have little part to play in achieving technical efficiency is increased. Of course an interior local maximum may exist but this would only be optimal in an input- or output- constrained situation and then only when the interior solution is better than all corner solutions where constraints are binding. Note that only when an interior local maximum exists and is optimal or when output is exogenously constrained does (7.4) hold.

The assumptions made to prove unboundedness of Z and Z' are of course only sufficient but not necessary to make the above points. However, if a local maximum does exist and is optimal, the question remains as to whether (7.4) and the related output decision can be reconciled with technical efficiency as we have defined it. Also if output is fixed exogenously then (7.4) is certainly an optimal condition for input mix, but again can it be reconciled with technical efficiency (i.e. 1(i))? In order to examine this further we will consider the former case as the more general, and remark that w and r need not be the social costs of the factor inputs. Now (7.4) can equate the marginal rate of technical substitution in each enterprise if it is possible to use an enterprise specific policy parameter, such as the price paid to the i^{th} producer (P_i), in order to equate optimal profit per unit of capital across enterprises (given a_i, b_i, the parameters of (7.2) for the i^{th} enterprise). Furthermore, the higher the parameter b_i is relative to a_i, the higher the optimal level of output in the i^{th} enterprise. The absolute values of a_i and b_i determine income distribution across enterprises.

Thus a set of triplets (a_i, b_i, P_i) can be found which induces technical efficiency. The method is described in detail in the Appendix . Of course, this implies that the Central Planning Board (CPB) must intervene between the producer's price and the price paid by consumers which would reflect their valuation, but such intervention already exists in principle, although not for this purpose, in the form of turnover taxes. The problem with accepting this argument is the large amount of information required by the CPB in order to find the socially optimal producer prices, taxes and bonus fund parameters : a similar scale of problem to that of the application of lump-sum taxes. Nevertheless, there is a possibility that some of the technical inefficiency embodied in (7.4) may be mitigated by CPB pricing and investment policy to reduce variance in profit rates on capital. In fact, in the context of the 1973 reorganisation, Bonin (1976, p.494) suggests that planners might allocate investment across associations according to profitability.

II. Increasing Effort

In investigating the second postulated objective of the bonus scheme we will make the simplifying assumption that K_i and N_i are fixed for all enterprises. By means of a simple model we will argue that the best form of bonus scheme (7.2) for the CPB to use is one where b_i is zero; that is one where only profit (which we assume to be both positive and less than revenue) is rewarded by the bonus scheme.

We will assume that managers of the i^{th} enterprise act so as to maximise a quasi-concave function $V_i(Z_i, \ell_i)$ being the utility of the "typical" worker. ℓ_i is the average effort per worker and $\frac{\partial V_i}{\partial Z_i} > 0$, $\frac{\partial V_i}{\partial \ell_i} < 0$. With K_i and N_i fixed output is a function $Q_i(\ell_i)$ with $\frac{dQ_i}{d\ell_i} > 0$ and $\frac{d^2Q_i}{d\ell_i^2} < 0$. Optimal effort levels ℓ_i^* $(i = 1, 2, \ldots n)$ satisfy

$$\frac{\partial V_i}{\partial Z_i}\left(a_i \frac{P_i}{K_i} + b_i P_i\right)\frac{dQ_i}{d\ell_i} + \frac{\partial V_i}{\partial \ell_i} = 0 \qquad i = 1, 2, \ldots, n \tag{7.5}$$

Consider P_i fixed and so ℓ_i^* is a function of just the two parameters of the bonus scheme a_i and b_i. The CPB are assumed to maximise a quasi-concave welfare function $U(Q_1(\ell_1), Q_2(\ell_2), \ldots, Q_n(\ell_n))$ subject to distribution of income constraints (in utility terms as a typical worker may be required to worker harder in one enterprise as compared to another) of the form

$$V_i(Z_i, \ell_i) = \bar{V}_i \qquad i = 1, 2, \ldots, n \tag{7.6}$$

and also to the response $\ell_i = \ell_i^*(a_i, b_i)$ from the comparative statics of (7.5). The CPB's problem is thus to maximize the following Lagrangean with respect to $a_i \geq 0$ and $b_i \geq 0$ $(i = 1, 2, \ldots, n)$

$$M = U(Q_1(\ell_1^*), Q_2(\ell_2^*), \ldots, Q_n(\ell_n^*)) - \sum_{i=1}^{n} \lambda_i\left[V_i(Z_i, \ell_i^*) - \bar{V}_i\right] \tag{7.7}$$

First order conditions, using (7.2) and (7.5) can be written:

$$\frac{\partial U}{\partial Q_i}\frac{dQ_i}{d\ell_i}\frac{\partial \ell_i^*}{\partial a_i} - \lambda_i \frac{\partial V_i}{\partial Z_i}\frac{\Pi_i}{K_i} \leq 0 \quad \text{and either} = 0 \text{ or } a_i = 0 \quad i = 1,2,\ldots,n \tag{7.8}$$

$$\frac{\partial U}{\partial Q_i}\frac{dQ_i}{d\ell_i}\frac{\partial \ell_i^*}{\partial b_i} - \lambda_i \frac{\partial V_i}{\partial Z_i}P_iQ_i \leq 0 \quad \text{and either} = 0 \text{ or } b_i = 0 \quad i = 1,2,\ldots,n \tag{7.9}$$

Note that $\lambda_i > 0$ provided at least one of $\dfrac{\partial \ell_i^*}{\partial a_i}$, $\dfrac{\partial \ell_i^*}{\partial b_i} > 0$.

Now from comparative statics of (7.5) we obtain :

$$\frac{\partial \ell_i^*}{\partial a_i} = \frac{1}{H_i}\left\{ S_i \frac{\Pi_i}{K_i} + \frac{\partial V_i}{\partial Z_i}\frac{dQ_i}{d\ell_i}\frac{P_i}{K_i}\right\} \quad i = 1,2,\ldots,n \tag{7.10}$$

$$\frac{\partial \ell_i^*}{db_i} = \frac{1}{H_i}\left\{ S_iP_iQ_i + \frac{\partial V_i}{\partial Z_i}\frac{dQ_i}{d\ell_i}P_i\right\} \quad i = 1,2,\ldots,n \tag{7.11}$$

where $$S_i = \frac{\partial^2 V_i}{\partial Z_i^2}\left(a_i\frac{P_i}{K_i} + b_iP_i\right)\frac{dQ_i}{d\ell_i} + \frac{\partial^2 V_i}{\partial \ell_i \partial Z_i}$$

and $$H_i = -\left\{ \frac{\partial^2 V_i}{\partial Z_i^2}\left(a_i\frac{P_i}{K_i} + b_iP_i\right)^2\left(\frac{dQ_i}{d\ell_i}\right)^2 + \frac{\partial V_i}{\partial Z_i}\left(a_i\frac{P_i}{K_i} + b_iP_i\right)\frac{d^2Q_i}{d\ell_i^2} + 2\frac{\partial^2 V_i}{\partial Z_i \partial \ell_i}\left(a_i\frac{P_i}{K_i} + b_iP_i\right)\frac{dQ_i}{d\ell_i} + \frac{\partial^2 V_i}{\partial \ell_i^2}\right\}$$

H is positive from the second order conditions for maximising V_i, which we assume to hold. Substituting (7.10) and (7.11) into (7.8) and (7.9) and multiplying (7.8) by K_i yields :

$$T_i \quad \Pi_i + \frac{\partial U}{\partial Q_i}\left(\frac{dQ_i}{d\ell_i}\right)^2 \frac{\partial V_i}{\partial Z_i} \frac{P_i}{H_i} \leq 0 \text{ and either } = 0 \text{ or } a_i = 0 \qquad (7.8a)$$

$$T_i \quad P_iQ_i + \frac{\partial U}{\partial Q_i}\left(\frac{dQ_i}{d\ell_i}\right)^2 \frac{\partial V_i}{\partial Z_i} \frac{P_i}{H_i} \leq 0 \text{ and either } = 0 \text{ or } b_i = 0 \qquad (7.9a)$$

$$\text{where } T_i = \left\{\frac{\partial U}{\partial Q_i} \frac{dQ_i}{d\ell_i} \frac{S_i}{H_i} - \lambda_i \frac{\partial V_i}{\partial Z_i}\right\}$$

Given our assumptions, any solution to the conditions (7.8a) and (7.9a) must have $T_i < 0$. Thus if $P_iQ_i > \Pi_i > 0$ and (7.8a) holds as a strict inequality then a fortiori (7.9a) must hold as a strict inequality, implying $a_i = b_i = 0$. On the other hand if (7.8a) holds as an equality (7.9a) must hold as a strict inequality implying $a_i > 0$, $b_i = 0$. Both a_i and b_i being zero could only constitute a solution in the unlikely event that $V_i(0, \ell_i^*(0, 0)) = \overline{V}_i$. Thus to increase effort the incentive scheme should typically be a function of the profit rate alone. Bonuses based on the profit rate (subject to a minimum output constraint) were a central feature of Liberman's reform proposals of the early sixties.

III. Conclusion

We have argued that as well as the problem of input mix, a further problem involved in using the bonus scheme (7.2) is that of the unboundedness of the bonus payments at large scale. This appears to suggest that the bonus scheme is an inefficient control of the enterprise managers' decisions if used alone. Quantity controls are thus likely to be needed, and the existence of such controls raises questions concerning the significance of the input mix problem (equation (7.4)) particularly as the extent of the inefficiency involved in this may be mitigated by the setting of prices paid to producers which are distinct from consumers' prices.

The bonus scheme does not appear to be a satisfactory tool for increasing labour productivity by rewarding effort. In terms of a simple model with fixed labour and capital, it is found to be dominated by a bonus scheme which relates to profits alone. Of course it is still possible that a justification of a positive b_i in (7.2) is as an incentive for the enterprise to accept larger allocations of controlled factor inputs from the CPB.

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Appendix to Chapter 7 The planners choice of bonus parameters.

To investigate a possible pricing strategy for planners it will be assumed that the relevant welfare function can be written

$$U = U(Q_1, Q_2 \ldots, Q_n) \tag{A7.1}$$

where Q_i is the quantity of the one good produced by the i^{th} enterprise. Enterprise j may of course be producing the same or a different good. Now consider the well-known exercise of maximising (A7.1) subject to production functions and labour and capital availability $\bar{N}$ and $\bar{K}$. The Lagrangean may be written

$$\Lambda = U(\cdot) + \sum_{i=1}^{n} \lambda_i \left\{ Q_i - Q_i(N_i, K_i) \right\} - \mu_N(\bar{N} - \sum_{i=1}^{n} N_i) - \mu_K(\bar{K} - \sum_{i=1}^{n} K_i) \tag{A7.2}$$

whence the following first-order conditions can be derived.

$$\frac{\partial U}{\partial Q_i} - \lambda_i = 0 \quad i = 1, 2, \ldots, n. \tag{A7.3}$$

$$\lambda_i \cdot \frac{\partial Q_i}{\partial N_i} - \mu_N = 0 \quad i = 1, 2, \ldots, n. \tag{A7.4}$$

$$\lambda_i \frac{\partial Q_i}{\partial K_i} - \mu_K = 0 \quad i = 1, 2, \ldots, n. \tag{A7.5}$$

Suppose decisions taken by enterprises are such that Z is maximised. Two questions then arise. First, can enterprise behaviour be induced to satisfy conditions (A7.3), (A7.4) and (A7.5)? Secondly. if such a correspondence is possible, can the bonus scheme be used to provide an arbitrary income distribution such that the output produced can be allocated among individuals to satisfy planners distributional preferences? (Our second question interprets Z maximisation as maximising income per worker). Maximisation of Z by the enterprise by choice of K and N implies the following first-order conditions

$$\frac{\partial Q_i}{\partial N_i} = \frac{w}{P_i(1 + \frac{b_i}{a_i} K_i)} \qquad i = 1, 2, \ldots, n \tag{A7.6}$$

$$\frac{\partial Q_i}{\partial K_i} = \frac{r + (\frac{\Pi}{K})_i}{P_i(1 + \frac{b_i}{a_i} K_i)} \qquad i = 1, 2, \ldots, n \tag{A7.7}$$

The solutions for N_i, K_i in (A7.6) and (A7.7) will be identical with those in (A7.4) and (A7.5) for some λ_i if and only if the following hold

$$w = \phi \mu_N \tag{A7.8}$$

$$P_i(1 + \frac{b_i}{a_i} K_i) = \phi \lambda_i \tag{A7.9}$$

$$r + (\frac{\Pi}{K})_i \equiv \frac{P_i Q_i - wN_i}{K_i} = \phi \mu_K \tag{A7.10}$$

Price P_i in (A7.9) and (A7.10) is now to be interpreted as the producer price received by the i^{th} enterprise and, as we will see, this will differ from that paid by consumers. Note that conditions (A7.8) and (A7.9) can be satisfied by setting w and P_i $(i = 1, 2, \ldots, n)$ for any $\phi > 0$ defining the money unit to utility unit relationship. Thus a socially optimal input mix for any output level and bonus scheme parameters can be attained. Price P_i will, of course be enterprise-specific unless all enterprises in the particular industry have the same technology. Condition (A7.9) may be redundant as the enterprise may be constrained by planners to produce a given output such that (A7.3) is satisfied. If there is no such constraint the following tax strategy can be adopted by planners. Set $P_i + t_i$ equal to $\phi\lambda_i$ where t_i is an enterprise-specific rate of sales tax or turnover tax such that the price paid by consumers $(P_i + t_i)$ for any particular commodity has the same proportional relationship to λ_i. Thus (A7.9) together with (A7.8) and (A7.10) determine welfare-maximising choices of input mix and output by the enterprise. Substituting P_i from (A7.10) and using (A7.8) an optimal tax rate t_i^* is easily derived,

$$\begin{aligned} t_i^* &= \phi\lambda_i - P_i \\ &= \frac{\phi}{Q_i}(Q_i\lambda_i - \mu_K K - \mu_N N) \end{aligned} \tag{A7.11}$$

Given (A7.3), (A7.4) and (A7.5) it is easily confirmed that $t_i^* > 0$ providing there are diminishing returns to scale. The state's total tax take will thus be

$$\sum_{i=1}^{n} Q_i t_i^* = \phi\left\{\sum_{i=1}^{n}(Q_i\lambda_i) - \mu_K\bar{K} - \mu_N\bar{L}\right\} \tag{A7.12}$$

It is assumed that government revenue is composed of enterprise profits and capital rental payments as well as taxation, and that part of these receipts is disbursed in the form of payments to enterprise incentive funds. Finally, from (A7.9) the following expression can be obtained for optimal parameters in the incentive scheme

$$\left(\frac{b_i}{a_i}\right)^* = \frac{Q_i \lambda_i - \mu_K K - \mu_N N}{\mu_K K + \mu_N N} \qquad \text{(A7.13)}$$

Note that (A7.13) only requires that particular enterprise-specific b/a ratios be set for efficiency. As long as these optimal ratios are maintained planners have some scope to pursue other objectives through choice of absolute values of a and b. The total size of incentive funds and inter-enterprise variation in incentive payments per worker can obviously be affected by choice of absolute values of a and b.

CHAPTER 8

Private Plot Restrictions in a Collective Farm Model

Following Domar (1966) a number of authors have presented theoretical analyses which portray the Soviet collective farm as a producer cooperative. They have drawn on general theory of the producer cooperative as advanced by Ward (1958), Sen (1966) and Vanek (1970) but have developed models which reflect particular characteristics of the Soviet collective farm such as the existence of members' private plots. The effects of crop quotas and price changes have been analysed by Oi and Clayton (1968); Bradley (1971, 1973) and Cameron (1973a, b) have discussed incentives and labour supply - a topic which has also been explored by Bonin (1977) in an examination of the impact of uncertainty within a producer cooperative model of the collective farm.

State policies have been studied by these writers within the framework of the producer cooperative model of the collective farm by determining the effects of price, rent and tax changes and work and crop quotas on factor allocation and crop outputs.

The objective of the present chapter is to examine the effects of restrictions on the size of private plots. This question does not seem to have been considered in the theoretical literature on collective farm models yet such restrictions have been applied in practice. Thus in the late fifties, under Khrushchev, there was pressure on collective farms to reduce the size of private plots which was reflected in 1959 in a 7 per cent reduction in private plot area. Brief accounts are given by Conquest (1968, 65-6) and Karcz (1970, 238-9) and it seems that one intention of such a policy was to induce an increase in the labour input devoted to collective crops.

Although private plots only constitute some 3 per cent of the sown area and their production has fallen in relative importance, Nove (1977, Chs. 1,5) reports that they are responsible for over 25 per cent of gross agricultural output, with collective farms accounting for 39 per cent and state farms around 30 per cent of gross output in 1973. Moreover the private plot continues to provide a significant proportion of average income of collective farm households - 27 per cent in 1972 and more in earlier years according to Wädekin (1975). It should be noted that some state-employed persons and pensioners also hold private plots.

A producer cooperative model of the collective farm is outlined in the first section of this chapter. The second section is concerned with an examination of the effects of restrictions in the size of private plots. Our analysis reveals that important distinctions may be drawn between short-run and long-run responses to changes in the area of private plots. We assume that in the short run membership of the collective farm is fixed but members may vary hours worked and the allocation of their labour time between work on the private plot and on collective land, whereas in the long run the level of membership is also variable. Of course membership reduction and recruitment might not be accomplished with equal ease and much will depend on the nature of alternative employment opportunities. Factor intensity as reflected in labour-land input ratios is shown to play a key role in the explanation of the nature of the response of the farm to the change in private plot size. In the third section we consider some implications of a system under which the area of private plots is determined by the member's contribution to communal work. Some concluding comments are presented in the final section of the chapter.

I. A Collective Farm Model

As the model we will use combines features encompassed by the Oi-Clayton (1968) and Bonin (1977) (certainty) models its structure may be outlined fairly briefly.

There are N identical collective farm members (or households) and they jointly choose the number of hours, ℓ_x , each member has to work on the collective crop(s). We thus assume that ℓ_x is determined by collusion of all members rather than, say, by individual myopic decision-taking with any one household behaving as if changes in its value of ℓ_x leave every other household's ℓ_x decision unaffected. In Cameron's model (1973, 18) under certainty and identical tastes it is argued that "each household is aware that its every move is accompanied by similar moves of all the . . . other households" so that the decision taken in this case will be the same as under collusion. Bonin (1977, 81-2) discusses a number of hypotheses which yield this outcome. Of course in any labour allocation scheme a check may have to be made on each member by the rest to ensure that labour obligations are fulfilled but we will abstract here from such supervisory costs. As well as working on the collective land each member has a plot of k hectares of land for private cultivation and chooses the number of hours, ℓ_z , that he works on the private plot. The member's concave, twice differentiable utility function, $U(y, \ell)$, is maximised by choice of ℓ_x and ℓ_z; ℓ is the sum of ℓ_x and ℓ_z , y is the member's total net income and $\partial U/\partial y > 0$, $\partial U/\partial \ell < 0$ and $\partial^2 U/\partial \ell \partial y \leq 0$ reflecting the assumption that the marginal utility of leisure $(-\partial U/\partial \ell)$ does not decrease with income. The member's net income is given by:

$$y = \left[X(\ell_x N,\ t_x N) - FC\right]/N + z(\ell_z,\ k) \tag{8.1}$$

Two interpretations of the model are possible. In the first $X(\cdot)$ is the total revenue net of all variable non-labour costs from the collective land, t_x is the collective land per member, FC represents fixed costs of the farm such as rent and $z(\cdot)$ is total revenue of the member net of all variable non-labour costs from his private plot. The collective land net revenue and the fixed costs are shared equally among members. The $z(\cdot)$ function is an envelope function, representing maximum net revenue with respect to selection of crops, fertilizer and other inputs at given input and output prices. The $X(\cdot)$ function is also an envelope function implying optimal input selection but there are restrictions on the selection of collective crops which we note below. We also make the critical assumption that $X(\cdot)$ and $z(\cdot)$ are homogeneous of degree one and that marginal net revenue products are decreasing.

The alternative interpretation follows the technological specifiction of Oi and Clayton (1968). The farm produces one collective crop and one private crop using only two inputs, land and labour. Output prices are fixed and the value of these outputs are $X(\cdot)$ and $z(\cdot)$ (per member). The corresponding critical assumption is that both crops have production functions exhibiting constant returns to scale and diminishing marginal products for both factors. We will occasionally draw on this interpretation in later discussion but it does seem worth emphasising that the earlier somewhat more general interpretation is possible.

Collective land per member, denoted by t_x , is given by

$$t_x = T/N - k \tag{8.2}$$

where T is total farm size in hectares. Now by the homogeneity assumption and using (8.2) we may rewrite (8.1) as :

$$y = x(\ell_x, T/N - k) + z(\ell_z, k) - FC/N \tag{8.1a}$$

where $x(\cdot) \equiv X(\cdot)/N$.

In the short run utility is maximised, subject to (8.1a), with respect to ℓ_x and ℓ_z. In the long run membership, N , is also variable. Membership can be increased by recruitment and reduced by choosing not to replace those who leave or die. We will assume that, in the long run, membership adjusts to the level which, jointly with ℓ_x and ℓ_z, maximises(8.1a)and thus utility. (Of course it must be recognised that in some circumstances alternative employment opportunities may be such that full adjustment of membership to a level which maximises income per member is not feasible.)

First-order conditions for maximising utility with respect to ℓ_x , ℓ_z and N , assuming an interior solution, are :

$$\frac{\partial U}{\partial y}\frac{\partial x}{\partial \ell_x} + \frac{\partial U}{\partial \ell} = 0 \tag{8.3}$$

$$\frac{\partial U}{\partial y}\frac{\partial z}{\partial \ell_z} + \frac{\partial U}{\partial \ell} = 0 \tag{8.4}$$

$$FC/T - \partial x/\partial t_x = 0 \tag{8.5}$$

Note that (8.3) and (8.4) together imply

$$\frac{\partial x}{\partial \ell_x} = \frac{\partial z}{\partial \ell_z} = -\frac{\partial U/\partial \ell}{\partial U/\partial y} \tag{8.6}$$

that is, each member allocates his time so that the marginal (net) revenue product of labour on both types of land is equal to his marginal rate of substitution between income and work. This condition is familiar from Bonin's (1977) analysis in which membership is assumed fixed whereas (8.5) which states that membership is adjusted until the marginal (net) revenue product of collective land is equal to fixed costs per hectare and the equality of the marginal (net) revenue product of labour in the two types of land is to be found in Oi and Clayton (1968).

An interpretation of (8.5) is assisted by multiplying through by T/N. Then it is seen that optimal membership equates the marginal member's contribution to fixed costs of the farm (FC/N) with the marginal (net) revenue product of collective land ($\partial x/\partial t_x$) multiplied by the overall number of hectares per member (k hectares of private land plus a member's share equivalent to $(T - Nk)/N$ hectares of collective land). Thus $(\partial x/\partial t_x)T/N$ is the marginal loss to the current members of admitting an additional member, as their individual private crop areas are unchanged, but land allocated to collective crops is reduced.

In the next section the effects of restrictions in the size of private plots, k, are discussed. Our exposition of the model is completed by stating two important assumptions relating to these plots. First, we assume that members would prefer larger to smaller private plots, that is

$$\frac{\partial U}{\partial k} = \frac{\partial U}{\partial y}\frac{\partial y}{\partial k} = \frac{\partial U}{\partial y}\left(\frac{\partial z}{\partial k} - \frac{\partial x}{\partial t_x}\right) > 0 \tag{8.7}$$

at optimal ℓ_x and ℓ_z.

The second assumption is that private plots are more intensively cultivated than the collective land,

$$\ell_z/k > \ell_x/t_x \tag{8.8}$$

As x and z are different functions, (8.8) does not follow from (8.7) and homogeneity. It is a separate assumption which reflects the argument that private crops offer a more profitable outlet (because of state controls on the collective crops which may be produced and their prices) and, as a result, members choose a higher labour input per hectare on the private plot. The vector of collective crops, production of which is permitted (ordered) by planners, is unlikely to be income-maximising from the viewpoint of the collective farm member. Indeed, if farms could freely select both the vector of communal and that of private crops, land-labour ratios would be equal in both sectors in the absence of price differences. Such freedom of crop choice cannot be assumed. On institutional grounds (8.8) appears to be a reasonable assumption in the Soviet context.

We must point out here that changing the prices of collective or private crops or changing constraints on what crops may be produced in each sector may remove the excess demand for private plot land which is reflected in our treatment of k as a permitted maximum. We are thus assuming that prices and permitted crop assortment are of such an order that excess demand for private plot land exists in the sense that $\partial U/\partial k > 0$. Of course a particular k may be optimal from a social point of view (or from the point of view of planners' preferences) even though $\partial U/\partial k \neq 0$ because of differences in pricing and crop assortment between private and collective sectors. For instance the state-determined prices of some collective crops may be relatively low for political, income-distribution, or other reasons whereas the output of private plots is sold on a 'free market'.

II. Response to Private Plot Size

We will examine the effects in the long run and in the short run of a change in k, the size of each members' private plot.

(a) Long-Run Response

The long-run response is particularly simple to analyse in this model because of the assumption of homogeneity of degree one of the net revenue functions. The latter implies that marginal net revenue products are homogeneous of degree zero. Thus as FC and T are fixed ℓ_x/t_x is constant (from (8.5)) and ℓ_z/k and $(-\partial U/\partial \ell)/(\partial U/\partial y)$ are constant (from (8.6)). This means that land-labour ratios in both types of plot are invariant with respect to the value of k and for suitable positive constants a_1, a_2 and a_3 we may write,

$$\ell_z/k = a_1 \tag{8.9}$$

$$\ell_x/t_x = a_2 \tag{8.10}$$

$$-\frac{\partial U/\partial \ell}{\partial U/\partial y} = a_3 \tag{8.11}$$

The effect of a small change in k may be determined by total differentiation of (8.11) which yields (using (8.3), (8.4) and (8.5)):

$$\frac{\partial \ell}{\partial k} = \frac{G}{H}\frac{\partial U}{\partial y}\left(\frac{\partial z}{\partial k} - \frac{\partial x}{\partial t_x}\right) \tag{8.12}$$

where $H = \frac{\partial^2 U}{\partial y^2}\left(\frac{\partial U}{\partial \ell}\right)^2 - 2\frac{\partial^2 U}{\partial y \partial \ell}\frac{\partial U}{\partial y}\frac{\partial U}{\partial \ell} + \frac{\partial^2 U}{\partial \ell^2}\left(\frac{\partial U}{\partial y}\right)^2 < 0$

and $G = \frac{\partial U}{\partial \ell}\frac{\partial^2 U}{\partial y^2} - \frac{\partial U}{\partial y}\frac{\partial^2 U}{\partial \ell \partial y} > 0$

Thus, from (8.7), $\partial \ell/\partial k < 0$. Total differentiation of (8.9) yields

$$\partial \ell_z/\partial k = \ell_z/k > 0 \tag{8.13}$$

and so we have:

$$\partial \ell_x/\partial k = \partial \ell/\partial k - \partial \ell_z/\partial k < 0 \tag{8.14}$$

The analysis is completed by differentiation of (8.10) which yields

$$\partial N/\partial k = \left[(\ell_z/k - \ell_x/t_x) - \partial \ell/\partial k\right] N^2 t_x/T\ell_x \tag{8.15}$$

whence sufficient conditions for $\partial N/\partial k > 0$ are (8.7) and (8.8).

As membership contracts in response to a fall in k this reinforces the move towards greater cooperative plot size and, as the land-labour ratio is constant total labour hours on the cooperative land increase. In the simple two-crop interpretation of the model the output of the collective crop increases and that of the privately-produced crop falls. Finally if ℓ is fixed at say $\bar{\ell}$, as in the Oi-Clayton (1968) model and income-per member alone is maximised the

$$\frac{\partial \ell}{\partial k} = \frac{G}{H}\frac{\partial U}{\partial y}\left(\frac{\partial z}{\partial k} - \frac{\partial x}{\partial t_x}\right) \tag{8.12}$$

where $\quad H = \dfrac{\partial^2 U}{\partial y^2}\left(\dfrac{\partial U}{\partial \ell}\right)^2 - 2\,\dfrac{\partial^2 U}{\partial y \partial \ell}\,\dfrac{\partial U}{\partial y}\,\dfrac{\partial U}{\partial \ell} + \dfrac{\partial^2 U}{\partial \ell^2}\left(\dfrac{\partial U}{\partial y}\right)^2 < 0$

and $\quad G = \dfrac{\partial U}{\partial \ell}\,\dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial U}{\partial y}\,\dfrac{\partial^2 U}{\partial \ell \partial y} > 0$

Thus, from (8.7), $\partial\ell/\partial k < 0$. Total differentiation of (8.9) yields

$$\partial\ell_z/\partial k = \ell_z/k > 0 \tag{8.13}$$

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A number of influences underlie these results. First it is clear that if at some initial ℓ_x , ℓ_z private plot size is reduced the marginal (net) revenue product of labour on private plots will fall relative to that on collective land. Thus at a given ℓ a reallocation of labour from private plots to collective land will occur. Secondly at a given labour input ℓ the transfer of land from private to collective use implies a sufficiently large transfer of labour such that the collective labour to land ratio will rise. This, in turn implies a rise in the marginal (net) revenue product of collective land. Gains can now be made by contracting membership since the net value of the output that can be produced on additional collective land (obtained from the private plot of a member who leaves) and remaining members' enlarged share of the net output from collective land exceed the additional fixed costs which remaining members would have to bear as a result of a member's departure. Finally of course ℓ will change because the fall in k induces a fall in income and a response in individual labour supply. It might be surmised that both a rise or a fall in individual hours are possible - and we show below that this is indeed the case in the short run. However in the long run the marginal rate of substitution between income and leisure remains constant and this plays a key rôle in determining that individual labour supply increases when k falls.

(b) Short-Run Response

In the short run the number of members, N , is fixed so that

(8.5)does not hold both before and after a change in k . Thus, in this case, the land-labour ratios on the two types of land are not invariant with respect to a change in k and the simplifications (8.9), (8.10) and (8.11) are no longer relevant.

Comparative statics of (8.3) and (8.4) yield:

$$\frac{\partial \ell_x}{\partial k} = \frac{1}{\Delta}\left[\left(\frac{\partial z}{\partial k} - \frac{\partial x}{\partial t_x}\right) G \frac{\partial^2 z}{\partial \ell_z^2} + \left(\frac{\partial U}{\partial y}\right)^2 \frac{\partial^2 x}{\partial \ell_x \partial t_x} \frac{\partial^2 z}{\partial \ell_z^2} + \frac{H}{\partial U/\partial y}\left(\frac{\partial^2 x}{\partial \ell_x \partial t_x} + \frac{\partial^2 z}{\partial \ell_z \partial t_z}\right)\right] \tag{8.16}$$

where $$\Delta = \left[\frac{\partial U}{\partial y}\frac{\partial^2 x}{\partial \ell_x^2} + \frac{H}{(\partial U/\partial y)^2}\right]\left[\frac{\partial U}{\partial y}\frac{\partial^2 z}{\partial \ell_z^2} + \frac{H}{(\partial U/\partial y)^2}\right] - \frac{H^2}{(\partial U/\partial y)^4} > 0.$$

As $G > 0$, $H < 0$ and as the cross derivatives of the revenue functions are positive (because the revenue functions are homogeneous of degree one and marginal revenue products are diminishing), a sufficient condition for (8.16)to be negative is our assumption stated in (8.7)above. Thus a reduction in k increases both land and labour inputs on the cooperative plot. However, both $\partial \ell_z/\partial k$ and $\partial \ell/\partial k$ are ambiguous in sign. If k is reduced then income effects may imply that the extra cooperative plot labour is found from leisure time rather than private plot working hours.

If the production process also involves a capital input (e.g. tractors), the level and allocation of which may be optimally determined by members, the long-run analysis is easily extended. Capital will be allocated such that its marginal net revenue product in both sectors is equated with its rental per unit. If the first-order conditions yield a unique solution for levels of the three inputs then homogeneity of degree one of the net revenue functions in land, labour and capital implies that the ratios of labour to land and capital to land in each sector are unchanged when, k, the size of a private plot changes. Thus the results of this section carry over with the ratio of capital in the collective to capital in the private sector changing in proportion to the corresponding ratio of labour inputs. The long-run effect of a cut in private plot area will be an overall reduction in the level of the capital input reflecting not only the fact that private plots would (because of their greater profitability) be cultivated in a more capital-intensive fashion but also the long-run reduction in membership. However, a caveat is in order here. If the state or management imposes limitations on the use of capital or if there are significant indivisibilities such that capital is not variable in the long run in the same sense and within the same time period as membership our analysis would be seriously affected. In particular if there is a third input, capital, which is fixed at exogenously determined levels $x(\cdot)$ and $z(\cdot)$ are no longer homogeneous of degree one in labour hours and land and the analysis will not hold.

III. Private Plots as Incentives

The 1969 Model Collective Farm Charter, reproduced in R. Stuart (1972, pp 222-223), states that labour participation of a household in the communal sector is taken into consideration in establishing the size of its private plot. Thus far we have viewed the size of the private plot as exogenously determined. In this section we will allow the area of each private plot to be endogenously determined by labour participation on the communal plot and we will consider the effects of increasing private plot area entitlement per labour hour of communal work. Thus, to be specific, let

$$k = B_o + g\ell_x \tag{8.17}$$

and assume that, optimising under this scheme, (i.e. for given B_0 and g) workers choose $\ell_x{}^*$ hours communal work implying $B_o = k_o - g\ell_x{}^*$ where k_o is the value of k at $\ell_x = \ell_x{}^*$. We will examine the response of a long-run equilibrium $(\ell_x{}^*, \ell_z{}^*, N^*)$ to a small increase in g, dg, which changes B_o and g such that $k = k_o$ at $\ell_x = \ell_x{}^*$ thus :

$$k = k_o + g(\ell_x - \ell_x{}^*) \tag{8.18}$$

First-order conditions for an interior utility maximum with respect to ℓ_x, ℓ_z and N yield

$$\frac{\partial U}{\partial y}\left[\frac{\partial x}{\partial \ell_x} + g\left(\frac{\partial z}{\partial k} - \frac{\partial x}{\partial t_x}\right)\right] + \frac{\partial U}{\partial \ell} = 0 \tag{8.19}$$

$$\frac{\partial U}{\partial y}\frac{\partial z}{\partial \ell_z} + \frac{\partial U}{\partial \ell} = 0 \tag{8.20}$$

$$\partial x/\partial t_x = FC/T \tag{8.21}$$

Condition (8.21) implies $\partial x/\partial t_x$ and $\partial x/\partial \ell_x$ are constant with respect to g and from (8.19) and (8.20) we may write the private plot as exogenously determined. In this section we will allow the area of each private plot to be endogenously determined by labour participation on the communal plot and we will consider the effects of increasing private plot area entitlement per labour hour of communal work. Thus, to be specific, let

$$k = B_o + g\ell_x \tag{8.17}$$

and assume that, optimising under this scheme, (i.e. for given B_O and g) workers choose ℓ_x* hours communal work implying $B_o = k_o - g\ell_x*$ where k_o is the value of k at $\ell_x = \ell_x*$. We will examine the response of a long-run equilibrium $(\ell_x*, \ell_z*, N*)$ to a small increase in g, dg, which changes B_o and g such that $k = k_o$ at $\ell_x = \ell_x*$ thus :

$$k = k_o + g(\ell_x - \ell_x*) \tag{8.18}$$

First-order conditions for an interior utility maximum with respect to ℓ_x, ℓ_z and N yield

$$\frac{\partial U}{\partial y}\left[\frac{\partial x}{\partial \ell_x} + g\left(\frac{\partial z}{\partial k} - \frac{\partial x}{\partial t_x}\right)\right] + \frac{\partial U}{\partial \ell} = 0 \tag{8.19}$$

$$\frac{\partial U}{\partial y}\frac{\partial z}{\partial \ell_z} + \frac{\partial U}{\partial \ell} = 0 \tag{8.20}$$

$$\partial x/\partial t_x = FC/T \tag{8.21}$$

Condition (8.21) implies $\partial x/\partial t_x$ and $\partial x/\partial \ell_x$ are constant with respect to g and from (8.19) and (8.20) we may write

$$\partial x/\partial \ell_x + (\partial z/\partial k - \partial x/\partial t_x)g = \partial z/\partial \ell_z \tag{8.22}$$

Note that the net value of the marginal revenue product of labour is no longer equal on private and collective plots. Now, from (8.22) a small increase in g implies

$$g\,d(\partial z/\partial k) - d(\partial z/\partial \ell_z) = -(\partial z/\partial k - \partial x/\partial t_x)dg \tag{8.23}$$

The right-hand side of (8.23) is negative by (8.7) and $d(\partial z/\partial k)$ and $d(\partial z/\partial \ell_z)$ must have opposite signs as they are both homogeneous of degree zero in their arguments so that we may write $\partial z/\partial k = \phi(k/\ell_z)$ and $\partial z/\partial \ell_z = \psi(k/\ell_z)$. Obviously $\phi'(\cdot) < 0$ and $\psi'(\cdot) > 0$ from the assumption of diminishing marginal (net) revenue products. Because they are of opposite signs and since the left hand side of (8.23) is negative the only configuration of signs that is possible is

$$\frac{\partial(\partial z/\partial k)}{\partial g} < 0 , \quad \frac{\partial(\partial z/\partial \ell_z)}{\partial g} > 0 \tag{8.24}$$

so that we must have $\partial(\ell_z/k)/\partial g < 0$.

Now from (8.20)

$$\frac{\partial(\partial z/\partial \ell_z)}{\partial g} = \frac{-H}{(\partial U/\partial y)^3}\frac{\partial \ell}{\partial g} + \frac{G}{(\partial U/\partial y)^2}\left(\frac{\partial z}{\partial k} - \frac{\partial x}{\partial t_x}\right)(\ell_x - \ell_x{}^*)$$

$$= \frac{-H}{(\partial U/\partial y)^3}\frac{\partial \ell}{\partial g} \tag{8.25}$$

for a small displacement from ℓ_x^* and, since $H < 0$, it follows that $\partial\ell/\partial g > 0$. The sign of $\partial\ell_x/\partial g$ may now be determined. Condition (8.24) implies that the private sector labour to land ratio, ℓ_z/k, falls when g rises. From $\partial(\ell_z/k)/\partial g < 0$ we derive

$$k\ \partial\ell/\partial g - (k + \ell_z g)\ \partial\ell_x/\partial g - \ell_z(\ell_x - \ell_x^*) < 0 \tag{8.26}$$

and since $\partial\ell/\partial g > 0$ from (8.25), $\partial\ell_x/\partial g > 0$. Also from $\partial(\ell_z/k)/\partial g < 0$ we may write

$$k\ \partial\ell_z/\partial g - \ell_z\ g\ \partial\ell_x/\partial g - \ell_z(\ell_x - \ell_x^*) < 0 \tag{8.27}$$

so that in general $\partial\ell_z/\partial g$ is ambiguous in sign at positive g due to income effects but is certainly negative at $g = 0$.

Implications for membership are derived as follows. From (8.21) $\ell_x/(T/N - k)$ is constant with respect to g whence

$$(t_x + \ell_x g)\partial\ell_x/\partial g + (\ell_x T/N^2)\ \partial N/\partial g + \ell_x(\ell_x - \ell_x^*) = 0 \tag{8.28}$$

and so $\partial N/\partial g < 0$ since, from (8.26), $\partial\ell_x/\partial g > 0$.

In general the direction of change in the total labour supply on the collective plot, and thus from (8.21) that of the total area of the collective plot, is indeterminate. However it is easily shown that

$$\partial(N\ell_x)/\partial g = \left[(k - \ell_x g)N^2/T\right]\ \partial\ell_x/\partial g \tag{8.29}$$

which is certainly positive at $g = 0$ but is, in general, ambiguous in sign. Three cases may be distinguished. First if some land is given for private use independently of ℓ_x, then $k > \ell_x g$ and $N\ell_x$ (total communal labour) and Nt_x (total collective land) will both increase with g. Secondly if $k = g\ell_x$ then total collective land and total communal labour will both be unchanged, the effect of the rise in ℓ_x being exactly offset by the fall in N. Lastly if some positive amount of communal labour has to be provided before there is any entitlement to private plots the increase in g reduces both communal labour and collective land.

IV Concluding Comments

We have shown that in a simple model of the collective farm reducing the size of private plots results in increases in land and labour inputs for collective crops both in the short run when membership is fixed and in the long run when it is variable. Presumably this has been one of the intentions of such a policy in practice. In the long run the amounts of both inputs devoted to private crops would decline; but in the short run the effects of plot area reduction on private plot labour are indeterminate, so it is possible that extra collective labour input is provided by reduction in leisure rather than private plot working hours. We have also explored some implications of a system under which private plot size depends on the member's contribution to communal work.

In addition to ignoring the complications of a capital input which we discussed earlier our theoretical analysis must be qualified which is certainly positive at $g = 0$ but is, in general, ambiguous in sign. Three cases may be distinguished. First if some land is given for private use independently of ℓ_x, then $k > \ell_x g$ and $N\ell_x$ (total communal labour) and Nt_x (total collective land) will both increase with g. Secondly if $k = g\ell_x$ then total collective land and total communal labour will both be unchanged, the effect of the rise in ℓ_x being exactly offset by the fall in N. Lastly if some positive amount of communal labour has to be provided before there is any entitlement to private plots the increase in g reduces both communal labour and collective land.

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In addition to ignoring the complications of a capital input which we discussed earlier our theoretical analysis must be qualified by recognising that income changes may have an effect on the efficiency or the quality of the labour force. Thus, in the face of severe restrictions on private plots, which significantly reduce income from given effort, the younger and more mobile workers may leave the farm or at low income levels labour efficiency might be affected. (The productivity-income-nutrition link has been extensively discussed by Bliss and Stern (1978)). Also income effects of a reduction in private plot size may reduce the supply of new members so that desired membership increases may be infeasible.

Although uncertainty of parameters other than k do not seem likely to affect the broad conclusions from the comparative statics analysis uncertainty concerning future permitted levels of k may well do so. If k were fixed until after the harvest then only the long-run membership adjustment would be affected whereas in the, perhaps less likely, case in which members believe there is a chance that some of their private plot area may be cut before harvest then the supply and allocation of individual labour would also be affected. Of course in a dynamic context expected cuts in private plots would influence activities like fertiliser application on the private plot by affecting expected future returns.

Finally, although we cannot review alternative theories here, it should be noted that Stuart (1972 Chs. 6, 9) has questioned the suitability of the "producer cooperative" model for the Soviet collective farm and it seems likely that the debate on alternative models of the Soviet collective farm will continue.This chapter, however, has been concerned with providing an analysis of the implication of private plot restrictions within a producer cooperative model.

CHAPTER 9

Ideal Prices vs. Prices vs. Quantities

I. A recent paper by Weitzman (1974) considered the choice between two planning strategies for maximising the expected value of benefit over cost. At the planning stage, benefits (B) and costs (C) were functions of output q and random variables θ and η, i.e.

$$B = B(q, \eta)$$

and

$$C = C(q, \theta)$$

η and θ were assumed independent for much of the analysis - an assumption which will be retained here. The production decision concerned finding that production level to maximise profit <u>given the planning strategy and given θ</u>. That is the planning decision is taken before θ is known; the production decision - in so far as the plan allows freedom of decision - takes place when θ is known. The derivatives of the functions with respect to the i^{th} argument is denoted by subscript i. Following Weitzman, the sign $\overset{o}{=}$ indicates "an accurate local approximation". Equation (9.1) is from Weitzman (1974), equations 2 and 16. The two strategies of the planners were:

(P) Set price equal to expected marginal benefit where expected marginal benefit equals expected marginal cost, i.e. find $\tilde{p}$ such that the output level, chosen by the producer, $h(\tilde{p}, \theta)$ solves

$$\tilde{p} = C_1(h(\tilde{p}, \theta), \theta) \overset{o}{=} E[B_1(h(\tilde{p}, \theta), \eta)] . \qquad (9.1)$$

(Q) Set output equal to that level where expected marginal benefit equals expected marginal cost, i.e. find $\hat{q}$ such that

$$E\left[B_1(\hat{q}, \eta)\right] = E\left[C_1(\hat{q}, \theta)\right] \tag{9.2}$$

Planning strategies P and Q will generally provoke different production decisions due to P allowing a response in terms of output level to θ, whereas Q constrains output level to $\hat{q}$.

Following Weitzman (1974) and locally approximating $B(q, \eta)$ and $C(q, \theta)$ by:

$$C(q, \theta) \cong C(\hat{q}, \theta) + (C' + \alpha(\theta))(q - \hat{q}) + C''(q - \hat{q})^2/2 \tag{9.3}$$

$$B(q, \eta) \cong B(\hat{q}, \eta) + (B' + \beta(\eta))(q - \hat{q}) + B''(q - \hat{q})^2/2 \tag{9.4}$$

where $E(\alpha(\theta)) = E(\beta(\eta)) = 0$ and B', C', B'', C'' are fixed coefficients, the comparative advantage of strategy P over strategy Q is found to be $\Delta(P - Q) \cong \sigma^2(B'' + C'')/(2C''^2)$ (9.5)

and where $B'' \cong B_{11}(\hat{q}, \eta)$ and $C'' \cong C_{11}(\hat{q}, \theta)$, with B'' assumed negative and C'' assumed positive. Obviously the sign $\Delta(P - Q)$ depends on whether $|B''| \gtrless |C''|$. If $|B''| > |C''|$ then $\Delta(P - Q)$ is negative;

if $|B''| < |C''|$ then $\Delta(P - Q)$ is positive. σ^2 is defined below (p. 173).

This result is of obvious theoretical and practical importance. However both strategies P and Q are inferior to a strategy based upon what Weitzman calls "ideal" prices, and which he defines by:

> "Now an ideal instrument of central control would be a contingency message whose instructions depend on which state of the world is revealed by θ and η ." (Weitzman (1974, p. 481))

Such messages can ensure that $B - C$ is maximised for any θ and η .

He then dismisses the possibility of such a strategy being used by arguing:

> "It should be readily apparent that it is infeasible for the centre to transmit an entire schedule of ideal prices or quantities. A contingency message is a complicated, specialised contract which is expensive to draw up and hard to understand. The random variables are difficult to quantify. A problem of differentiated information or even of moral hazard may be involved since the exact value of θ will frequently be known only by the producer." (Weitzman (1974, p. 481))

I will argue here that the case against ideal prices is overstated by Weitzman. For this purpose, in Section II a contingent payment function which is linear in costs and benefits per unit output is shown to have "ideal" properties. Two variations are considered. One where both θ and η are observed prior to the production decision and the other when just θ is observed. This second variation may appear more appropriate in many situations. In Section III, the comparative advantages of such ideal price strategies over P and Q are calculated using the techniques of Weitzman's paper and $\Delta(P - Q)$ related to them. Finally the implications of the analysis in terms of the relevance of ideal prices are discussed in a concluding section. In particular the information requirements of the various strategies are compared.

II. Consider the contingent price p^* where

$$p^* = (a + bB(q, \eta) + (1 - b)\,C(q, \theta))/q \tag{9.6}$$

where $0 < b \leq 1$. This price is contingent upon q, η and θ, but is only dependent upon η and θ via the functions $B(q, \eta)$ and $C(q, \theta)$. Let us call strategy IP the adoption of a contingent price p^* as in (9.6) where the production level decision is determined by profit maximisation by the producer given p^*, η and θ. Strategy IP' will differ only to the extent that the producer maximises expected profit, given p^* and θ.

With strategy IP, the producer maximises profit (Π) where

$$\Pi = p^*q - C(q, \theta)$$

$$= a + b\,(B\,(q, \eta) - C(q, \theta)) \tag{9.7}$$

i.e. he will choose q^* such that $B_1(q^*, \eta) = C_1(q^*, \theta)$, thus maximising the excess of benefit over cost in all situations, i.e.

given any η and θ . With strategy IP' , the producer maximises expected profit where

$$E(\Pi) = a + b(E\left[B(q, \eta)\right] - C(q, \theta)) \tag{9.8}$$

i.e. he will choose q' such that $E\left[B_1(q', \eta)\right] = C_1(q', \theta)$, thus maximising the excess of expected benefit over cost, given θ .

The output levels $\hat{q}$, q* and q' can be related by realising that they all maximise expected profit: $\hat{q}$ before θ or η are known, q' before η is known but after θ is known and q* after both θ and η are known.

For strategy IP' , p* could be reformulated as

$$p' = (a + b\, E\left[B(q, \eta)\right] + (1 - b)\, C(q, \theta))/q \tag{9.9}$$

It is easily seen that with risk neutrality on the part of both the producer and the planners, an identical result (q') emerges.

III. The expected net benefits of planning with ideal prices are calculated below. Such calculations are of interest for two reasons. First, as we will argue in the next Section, planning strategies IP or IP' may have advantages rather than disadvantages, in terms of informational requirements, over strategies P or Q . Secondly, even if they do not, they offer a yardstick with which to judge the comparative advantage $\Delta(P - Q)$ and to measure the incentive for using a contingent price strategy. Even if IP' is completely infeasible, it is of interest to assess the magnitude of $\Delta(P - Q)$ in terms of the theoretical welfare loss $\Delta(IP' - Q)$.

Differentiate equations (9.3) and (9.4) with respect to q and obtain :

$$C_1(q, \theta) \overset{o}{=} C' + \alpha(\theta) + C''(q - \hat{q}) \tag{9.10}$$

$$B_1(q, \eta) \overset{o}{=} B' + \beta(\eta) + B''(q - \hat{q}) \tag{9.11}$$

With strategy IP the LHS of equations (9.10) and (9.11) are equal: Thus we can solve for $q^* - \hat{q}$ as :

$$q^* - \hat{q} \overset{o}{=} (B' + \beta(\eta) - C' - \alpha(\theta))/(C'' - B'') \tag{9.12}$$

But also from (9.10) and (9.11) we know that

$$E[C_1\ (\hat{q}, \theta)] \overset{o}{=} C'$$

$$E[B_1\ (\hat{q}, \eta)] \overset{o}{=} B'$$

and thus by (9.2)

$$C' \overset{o}{=} B' \tag{9.13}$$

Substituting (9.13) into (9.12) and then substituting the resulting expression for $q^* - \hat{q}$ into (9.3) and (9.4), we obtain after taking expectations:

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and thus by (9.2)

$$C' \overset{o}{=} B' \tag{9.13}$$

Substituting (9.13) into (9.12) and then substituting the resulting expression for $q^* - \hat{q}$ into (9.3) and (9.4), we obtain after taking expectations:

$$E\left[C(q^*, \theta)\right] \stackrel{\circ}{=} E\left[C(\hat{q}, \theta)\right] - 2\gamma\, \sigma^2 + 2\gamma^2\, C''(\sigma^2 + \psi^2) \tag{9.14}$$

$$E\left[B(q^*, \eta)\right] \stackrel{\circ}{=} E\left[B(\hat{q}, \eta)\right] + 2\gamma\, \psi^2 + 2\gamma^2\, B''(\sigma^2 + \psi^2) \tag{9.15}$$

where $E\left[(\alpha(\theta))^2\right] = \sigma^2$, $E\left[(B(\eta))^2\right] = \psi^2$ and $\gamma = 1/(2(C'' - B''))$
Note that $E\left[(\alpha(\theta))^2\right]$ $(E\left[(B(\eta))^2\right])$ is an approximation for the variance of marginal cost (benefit) when $q = \hat{q}$. γ is positive as $C'' > 0$ and $B'' < 0$.

Now $$\Delta(IP - Q) = E\left[B(q^*, \eta) - C(q^*, \theta) - B(\hat{q}, \eta) + C(\hat{q}, \theta)\right] \tag{9.16}$$

$$\therefore \quad \Delta(IP - Q) \stackrel{\circ}{=} \gamma(\psi^2 + \sigma^2) > 0 \tag{9.17}$$

With strategy IP' we have, from (9.10) and (9.11):

$$C_1(q', \theta) \stackrel{\circ}{=} B' + B''(q' - \hat{q}) \stackrel{\circ}{=} \left[E\ B_1(q', \eta)\right]$$

and $$C_1(q', \theta) \stackrel{\circ}{=} C' + \alpha(\theta) + C''(q' - \hat{q})$$

$$\therefore \quad q' - \hat{q} \stackrel{\circ}{=} 2\gamma\ \alpha(\theta) \tag{9.18}$$

Substituting in (9.3) and (9.4) and forming an appropriate expression for $\Delta(IP' - Q)$, we obtain

$$\Delta(IP' - Q) \stackrel{\circ}{=} \gamma\ \sigma^2 > 0 \tag{9.19}$$

By using the fact that $\Delta(i - j) = \Delta(i - k) + \Delta(k - j)$, all the elements in the Table are found from (9.5), (9.17) and (9.19).

Table: Comparison of Four Planning Strategies

$(i, j) = \Delta(i - j)$

	j = IP'	Q	P
i = IP	$\gamma\ \psi^2$	$\gamma(\psi^2 + \sigma^2)$	$\gamma(\psi^2 + \sigma^2\ B''^2/C''^2)$
IP'		$\gamma\ \sigma^2$	$\gamma\ \sigma^2\ B''^2/C''^2$
Q			$-\ \gamma\sigma^2(1 - B''^2/C''^2)$

The following remarks can be made from consideration of the Table.

1. If $|C''| \approx |B''|$ then $\Delta(P - Q)$ is very small, whereas other elements in the Table are not in general small.

2. The smaller the absolute values of C'' and B'', the greater is the advantage of using a strategy of type IP or IP' over using Q or P .

3. No variations associated with η appear in any elements other than in the top row. This implies that such variations affect P, Q and IP' strategies equally adversely.

4. The proportion of the advantage of using IP' rather than Q gained by using P rather than Q (assuming $|B''| < |C''|$) is z_1 where :

$$z_1 = 1 - B''^2/C''^2$$

The proportion of the advantage of using IP' rather than P gained by using Q rather than P (assuming $|B''| > |C''|$) is

z_2 where :

$$z_2 = 1 - C''^2/B''^2$$

If C'' is near zero, then z_2 is near one, $\Delta(Q - P)$ is very large and dominates $\Delta(IP' - Q)$ unless B'' is likewise very small. The advantage of using IP' may still be considerable. However if $|B''|$ is near zero, then z_1 is near one and $\Delta(IP' - P)$ is near zero, and there is little advantage from using strategy IP' .

IV. We have shown that an "ideal" price can be found as a simple schedule of θ and η , (equation (9.6)) and have calculated the extent of the comparative advantages of such schedules over planning strategies P and Q . It is now necessary to consider the relative information requirements and the extent of other problems associated with the implementation of the various strategies.

Weitzman's result incorporating a criterion for preferring P to Q ($\Delta(P - Q)$) relies simply on the relative magnitudes of C'' and B'' . However the implementation of either P or Q requires much more. $\Delta(P - Q)$ is a comparison of the 'optimal' price strategy $\tilde{p}$ and the 'optimal' output strategy $\hat{q}$. To find either $\tilde{p}$ or $\hat{q}$ requires full knowledge by the planners of the functions $B(q, \eta)$ and $C(q, \theta)$ <u>ex ante</u>, together with the distribution of η and θ . In addition $\tilde{p}$ requires knowledge by the producer of the function $C(q, \theta)$ <u>ex post</u>. There is ample opportunity for differential information or even moral hazard to prevent $\tilde{p}$ or $\hat{q}$ from actually being chosen.

In order to implement the IP or IP' strategies, rather different information is required. Only ex post information on $B(q, \eta)$ and $C(q, \theta)$ is required with strategy IP . The producer requires knowledge of the functions given θ and η (in order to find q^*), and the planner requires knowledge of the actual values of the benefits and costs in order to find p^* . In respect of measuring costs, honest cost accounting combined with an inspection of profits made should suffice. The measurement of benefits would in general be more difficult requiring some unbiased referee. However, it should be noted that the objectives of both planners and producers is the same - to maximise the excess of benefits over costs. If both have to make some declaration as to performance, their interests may largely coincide. An example of the principle of similarity. See Ross (1973).

Strategy IP' requires in addition that the producer can estimate expected benefits for each output level if the price is p^* or that the planner can if the price is p' .

An additional advantage of IP or IP' strategies is that choice of the parameters a and b in (9.6) determines the allocation of risk between the central planning authority and the producer. Many different allocations of risk can be achieved while retaining the "ideal" property. However the P and Q strategies each uniquely define an allocation of risk. The distributional aspects of the planner-producer problem do not lend themselves well to strategies of type P and Q . It is by no means certain that such strategies will imply a 'desirable' allocation of both expected profit and variance of profit to the producer.

The analysis here and Weitzman's model are both probably too normative to allow a totally convincing argument for the implementation of ideal price strategies as a practical and efficient planning policy. Nevertheless, a valid conclusion would appear to be that there is no justification for dismissing ideal price strategies while considering particular non-ideal strategies such as P and Q.

CHAPTER 10

Resource Allocation and Prices vs. Quantities

I. Introduction

The use of tâtonnement processes to arrive at optimal plans has long been a subject of analysis, and recently the comparison of planning by setting prices with that of setting quantities has attracted particular interest (see Heal (1969)). On the other hand, criticism has been levelled at the tâtonnement process itself as requiring much too much time and transmission of information to be a practical proposition. Weitzman (1974) considers the comparison of price and quantity planning procedures in a world without tâtonnement and when planners act with incomplete information. In his simplest model he assumes that planners maximise the expected excess of uncertain benefits over uncertain costs of a firm's activity by controlling output, either directly by using a quota instruction or indirectly by setting a price and allowing the firm's profit maximising behaviour to determine output. The advantage of the price setting policy is that the firm chooses output with full knowledge of the cost function, while the disadvantage is that the firm ignores the effects on benefits which its decision is likely to have. Broadly speaking, his result is that if the marginal benefit curve is flatter (steeper) than the marginal cost curve then price planning is on average superior (inferior) to quantity planning, see Chapter 9.

The Weitzman model has been extended by, among others, Yohe (1978), Ireland (1977)(see Chapter 9), Laffont (1977), Karp and Yohe (1979) and Malcolmson (1978). As far as I am aware, however, the analysis has always been in the form of assessing benefits and costs from the production of one or more firms in the economy, whereas one of the major forces for studying optimal planning was to ensure both full employment and feasibility in resource allocation. There is no way of imposing this in the benefits minus costs formulations and maintaining

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a cost function which is independent of other firms' decisions: with fixed factor endowments, one firm's cost function is a result of all firm's factor demand decisions. Indeed, one of the key problems of using prices as planning instruments in a second-best world with incomplete information is the lack of assurance of the feasibility of the joint output decisions of the firms. It is the purpose of this chapter to suggest just one possible way out of this dilemma, which develops naturally from Weitzman's model and which yields planning strategies susceptible to the same kind of comparative analysis.

Price and quantity controls in this chapter will refer to prices or quantities of one (or more)resources or factors of production. The quantity control is simply an optimal allocation of the available supply by direct instruction. A price control will be factor prices offered to a subset of firms. These firms will then determine their demands and the planning authority will allocate the remainder of the factor supplies to the other firms. Two questions thus arise. First, what is the optimal price to offer, and second which firms should be in the "preferred" subset of firms which are permitted to make factor demand decisions?

A further interpretation is possible in the context of labour-managed firms in conjunction with the incentive payment for labour mobility, described in Chapter 6. Here the resource price would be the shadow price ω .

We will find in Section II that our conclusions, but not our analysis, differ markedly from that of Weitzman (1974) in that a price planning procedure of this kind for the allocation of a single resource between two firms can be found which dominates quantity controls unless the uncertainties in firm production processes are sufficiently positively correlated. We extend the analysis to any number of resources in

Section III with rather weaker results. Section IV contains an analysis of optimal factor cost schedules and draws comparisons with the work of Ireland (1977) and Karp and Yohe (1979). Section V contains some final remarks concerning the generalisation of the model to more than two firms.

II. Allocating a single resource between two firms

We will assume that the planning authority wish to maximise the expected value of a social welfare function.

$$V = b_1Q_1 + b_2Q_2 \tag{10.1}$$

where b_1 and b_2 are positive and are expected values of random variables which are independent of all arguments of Q_1 and Q_2 , which are designated :

$$Q_1 = Q_1(L_1, \Theta) \tag{10.2}$$

$$Q_2 = Q_2(L_2, \eta) \tag{10.3}$$

The resource is available in a total supply of $\bar{L}$ efficiency units, so that for feasibility and full employment we require:

$$L_1 + L_2 = \bar{L} \tag{10.4}$$

where L_1, L_2 are the resource allocations to two firms with production functions (10.2) and (10.3) respectively. The units of measurement of outputs are chosen so that the product prices are both unity. Thus Q_1 and Q_2 are the respective firms' revenue functions. The coefficients b_1 and b_2 may diverge from unity due to income distribution or other considerations.

Θ and η are collections of random variables and we will assume that (10.2) and (10.3) can be approximated by

$$Q_1 = A(\Theta) + (Q_1' + \alpha(\Theta))(L_1 - \hat{L}_1) + \tfrac{1}{2}Q_1''(L_1 - \hat{L}_1)^2 \qquad (10.5)$$

$$Q_2 = B(\eta) + (Q_2' + \beta(\eta))(L_2 - \hat{L}_2) + \tfrac{1}{2}Q_2''(L_2 - \hat{L}_2)^2 \qquad (10.6)$$

where Q_1', Q_2' are positive constants and Q_1'', Q_2'' are negative constants. Also

$$E\,\alpha(\Theta) = E\,\beta(\eta) = 0\ , \text{ and } E(\alpha(\Theta))^2 = \sigma^2\ , \ E(\beta(\eta))^2 = \Psi^2$$

$$\text{and } E(\alpha(\Theta).\beta(\eta)) = \omega$$

In (10.5) and (10.6) $\hat{L}_1$, $\hat{L}_2$ are the direct quantity allocations of the resources that maximise the expected values of (10.1) subject to (10.4), i.e. they satisfy

$$b_1\, E\,\frac{\partial Q_1}{\partial L_1} - b_2\, E\,\frac{\partial Q_2}{\partial L_2} = 0 \qquad (10.7)$$

which implies from (10.5) and (10.6):

$$b_1 Q_1' - b_2 Q_2' = 0 \qquad (10.8)$$

Note also that

$$\begin{aligned} Q_1(\hat{L}_1, \Theta) &= A(\Theta) \\ Q_2(\hat{L}_2, \eta) &= B(\eta) \end{aligned} \qquad (10.9)$$

so that (10.1) is equal to

$$b_1 A(\Theta) + b_2 B(\eta) \qquad (10.10)$$

with the quantity allocation, which we will call strategy $(\hat{L})$.

The use of quadratic approximations such as (10.5) and (10.6) have been criticised by Malcolmson (1978). However, the technical reasons for such use are powerful as it is important in what follows to have marginal products which are separable functions of decision variables and random variables.

Now we will define two alternative price planning strategies. We will assume that each firm, if offered the resource at a relative resource to product price, w, could choose a profit maximising level of use of the resource given full ex post knowledge of Θ or η as appropriate. Thus the firm has more knowledge about its own production function than the planners, although it has no knowledge concerning the other firm's production function. If prices were set for both firms, then resource use decisions would not in general satisfy (10.4). Suppose rather that we have:

Strategy (w_1): the relative resource to produce price w_1 is set for firm 1, which solves $\tilde{L}_1$, given Θ, from

$$\frac{\partial Q_L}{\partial L_1} - w_1 = 0 \tag{10.11}$$

so that $\tilde{L}_1 = \tilde{L}_1(w_1, \Theta)$, reports this back to the planning authority and then proceeds to hire the resource and produce the output. Then the planners allocate $\bar{L} - \tilde{L}_1$ units of the resource to firm 2, and require firm 2 to employ them. We will limit our analysis by assuming $0 \leq \tilde{L}_1 \leq \bar{L}$. The allocation of resources and thus expected social welfare will depend on the numerical value of w_1. This is chosen by the planners to maximise the expected value of (10.1), given the ex ante stochastic demand function $\tilde{L}_1$. Thus it satisfies

$$E\left\{\left[b_1 \frac{\partial Q_1}{\partial L_1} - b_2 \frac{\partial Q_2}{\partial L_2}\right] \frac{\partial \tilde{L}_1}{\partial w_1}\right\} = 0 \tag{10.12}$$

From (10.5) and (10.11) we have

$$\tilde{L}_1(w_1,\theta) = (w_1 - Q_1' - \alpha(\theta) + Q_1'' \hat{L}_1)/Q_1'' \tag{10.13}$$

and so

$$\frac{\partial \tilde{L}_1}{\partial w_1} = \frac{1}{Q_1''} \tag{10.14}$$

which is negative and non-stochastic. Substituting (10.14) into (10.12) we see that (10.7) holds, and using (10.5), (10.6), (10.8) and (10.13) in (10.7) yields

$$E(\tilde{L}_1 - \hat{L}_1) = 0 \tag{10.15}$$

and using (10.15) and (10.13) we obtain

$$w_1 = Q_1' \tag{10.16}$$

Then from (10.13) and (10.16)

$$\tilde{L}_1(w_1, \theta) - \hat{L}_1 = -\alpha(\theta)/Q_1'' \tag{10.17}$$

and from (10.4)

$$L_2 - \hat{L}_2 = \alpha(\theta)/Q_1'' \tag{10.18}$$

In an exactly similar way we can state

Strategy (w_2): the relative resource to product price is set for firm 2, which solves for $\tilde{L}_2(w_2, \eta)$ to maximise its profits given w_2 and η. We assume $0 \leq \tilde{L}_2 \leq \bar{L}$. The remaining $\bar{L} - \tilde{L}_2$ units of the resource are allocated to firm 1. By a symmetric argument to that for strategy (w_1), we have

$$\tilde{L}_2 - \hat{L}_2 = -\beta(\eta)/Q_2'' \tag{10.19}$$

$$L_1 - \hat{L}_1 = \beta(\eta)/Q_2'' \tag{10.20}$$

The comparative advantage, $\Delta(w_1 - \hat{L})$ of using strategy (w_1) over strategy ($\hat{L}$) is given by the expected value of (10.1) using strategy (w_1), minus (10.10). Substituting (10.17) and (10.18) into (10.5) and (10.6) and thus into (10.1) and using (10.10), we have

$$\Delta(w_1 - \hat{L}) = \tfrac{1}{2}\sigma^2(b_2 Q_2'' - b_1 Q_1'')/Q_1''^2 + b_2\omega/Q_1'' \tag{10.21}$$

and when $\omega = 0$ and θ and η are independent,

$$\Delta(w_1 - \hat{L}) \gtrless 0 \text{ as } b_2|Q_2''| \lesseqgtr b_1|Q_1''|$$

so that strategy (w_1) is better on average if firm 1's marginal benefit product curve is steeper than firm 2's, i.e. if firm 1's marginal benefit product of the resource is more sensitive to resource allocation. If $\omega > 0$ (< 0) then this decreases (increases) the relative advantage of strategy ($\hat{L}$), as when marginal product is high in firm 1 (high $\alpha(\theta)$) leading to high $\tilde{L}_1$, the marginal product is also, on average, high in firm 2 (high $\beta(n)$), and the resource is likely to be badly misallocated.

We may also derive

$$\Delta(w_2-\hat{L}) = \tfrac{1}{2}\,\Psi^2(b_1Q_1'' - b_2Q_2'')/Q_2''^2 + b_1\omega/Q_2'' \qquad (10.22)$$

so that if $\omega \leq 0$, at least one of (10.21) and (10.22) must be non-negative. (Also if $\omega \geq 0$ at least one of (10.21) and (10.22) must be non-positive). Finally, using (10.21) and (10.22) yields

$$\Delta(w_1-w_2) = (b_2Q_2'' - b_1Q_1'')H/(Q_1''^2\,Q_2''^2) \qquad (10.23)$$

where $$H = \tfrac{1}{2}\,E\left[(\alpha(\Theta)Q_2'' + \beta(n)Q_1'')^2\right] > 0$$

The sign of (10.23) is again determined by the relative slopes of the marginal product curves, weighted by the benefit coefficients.

The conclusions in this simple model are significantly different from those of Weitzman (1974), but have similarity with arguments in Laffont (1977) who considered the Weitzman quantity strategy as similar to a consumption prices strategy. We find that if $\omega = 0$ there is always a price strategy (w_1 or w_2) which is to be preferred to strategy ($\hat{L}$). Furthermore, the planners do better to allocate a resource price and the associated decentralised decision as to resource use to the firm with the steeper marginal benefit product curve, and then clear the market by allocating the rest of the resource to the other firm. Only if there is strong positive correlation between marginal products of the two firms are these results weakened.

III. Any number of resources

We can extend the analysis to the case of m resources by treating most of the notation as referring to vectors and specifying approximations to the production functions as

$$Q_1 = A(\Theta) + (Q_1' + \alpha(\Theta)(L_1-\hat{L}_1) + \tfrac{1}{2}(L_1-\hat{L}_1)^T M_1 (L_1-\hat{L}_1) \tag{10.5a}$$

$$Q_2 = B(\eta) + (Q_2' + \beta(\eta))(L_2-\hat{L}_2) + \tfrac{1}{2}(L_2-\hat{L}_2)^T M_2 (L_2-\hat{L}_2) \tag{10.6a}$$

where Q_1', $\alpha(\Theta)$, Q_2', $\beta(\eta)$ are row vectors, L_1, $\hat{L}_1$, L_2, $\hat{L}_2$ are column vectors and M_1, M_2 are $m \times m$ non-stochastic matrices of second-order derivatives of the production functions, and are thus symmetric and negative definite. The superscript T indicates transpose. Strategy (w_1) now relates to choosing a (column) vector of optimal resource prices, w_1, yielding firm 1's demand vector $\tilde{L}_1 = \tilde{L}_1(w_1, 0)$. Proceeding as in Section II we obtain from $E\,\alpha(\Theta) = E\,\beta(\eta) = 0$

$$w_1 = Q_1'^T \tag{10.16a}$$

$$\tilde{L}_1 - \hat{L}_1 = - M_1^{-1}\,\alpha(\Theta)^T \tag{10.17a}$$

$$\Delta(w_1-\hat{L}) = -\tfrac{1}{2}\,E\left[\alpha(\Theta)M_1^{-1}(b_1M_1 - b_2M_2)M_1^{-1}\,\alpha(\Theta)^T\right] + b_2\,E\left[\beta(\eta)M_1^{-1}\,\alpha(\Theta)^T\right] \tag{10.21a}$$

Similarly, for strategy (w_2), we have

$$\tilde{L}_2 - \hat{L}_2 = - M_2^{-1}\,\beta(\eta)^T \tag{10.19a}$$

and

$$\Delta(w_2-\hat{L}) = -\tfrac{1}{2}\, E\left[\beta(n)M_2^{-1}(b_2M_2-b_1M_1)M_2^{-1}\,\beta(n)^T\right]$$

$$+\ b_1\ E\left[\beta(n)M_2^{-1}\,\alpha(\Theta)^T\right] \qquad (10.22a)$$

Even if $\beta(n)$ and $\alpha(\Theta)$ are independent and the last terms in both (10.21a) and (10.22a) are zero, it is no longer true that their signs will be opposite if $(b_2M_2 - b_1M_1) \neq 0$. A sufficient condition would be that $(b_2M_2 - b_1M_1)$ were negative or positive definite.

Subtracting (10.22a) from (10.21a) yields

$$\Delta(w_1-w_2) = -\tfrac{1}{2}\, E\Big[(\alpha(\Theta)M_1^{-1} + \beta(n)M_2^{-1})\ (b_1M_1 - b_2M_2)$$

$$(M_1^{-1}\,\alpha(\Theta)^T + M_2^{-1}\,\beta(n)^T)\Big] \qquad (10.23a)$$

and again a sufficient condition for strategy w_1 to be preferred to strategy w_2 is that $b_1M_1 - b_2M_2$ is negative definite, and here the criterion is independent of any correlation between $\alpha(\Theta)$ and $\beta(n)$. Obviously if $m = 1$, M_1, M_2 are scalars and definiteness of some kind is assured.

Here, then, a rather weaker result is obtained than in the single resource case. This was to be expected as the welfare advantages of using strategy (w_1) over strategy $(\hat{L})$ may be a mixture of positives and negatives for different resources and thus the total comparison may depend upon weights derived from the levels of uncertainty of marginal products. Note however, that if M_1 and M_2 are diagonal matrices (10.21a)→(10.23a) are much simplified and the comparative advantage is additively separable over all resources. This, of course, would allow for a mixed w_1/w_2 strategy, each resource being treated independently as in Section II.

IV. Cost functions

An alternative strategy to those considered so far is to set a cost function rather than factor prices to one of the firms, again allocating the residual factor supplies to the other firm. The questions arise as to the form of the function and the criteria for choosing the "preferred" firm to make the allocation decisions. Karp and Yohe (1979), extending the analysis of Ireland (1977) (see Chapter 2) consider the optimal linear marginal revenue schedule. We will consider the optimal linear marginal factor cost schedules, set for firm 1 and defined by

$$w_1^* = w_1 + X_1(L_1 - \hat{L}_1) \tag{10.24}$$

where w_1 is the optimal price vector from strategy (w_1) and $\hat{L}$ the optimal quantity vector from strategy $(\hat{L})$ and X_1 is an $m \times m$ matrix. Note that the total cost function is

$$w_1^{*T} L_1 = C(\hat{L}_1) + w_1^T(L_1 - \hat{L}_1) + \tfrac{1}{2}(L_1 - \hat{L}_1)^T X_1(L_1 - \hat{L}_1) \tag{10.25}$$

where $C(\hat{L}_1)$ is total cost at $L_1 = \hat{L}_1$.

Profit maximisation by firm 1 implies setting (10.24) equal to marginal factor products, by choosing L_1^* such that

$$w_1 + X_1(L_1^* - \hat{L}_1) = Q_1'^T + \alpha(0)^T + M_1(L_1^* - \hat{L}_1) \tag{10.26}$$

i.e. $$L_1^* - \hat{L}_1 = (X_1 - M_1)^{-1}\alpha(0)^T \tag{10.27}$$

The vector L_1^* of demands for resources is dependent in general of all elements of $\alpha(\Theta)$. Of course if M_1 were diagonal then the demand functions are independent provided X_1 is also diagonal.

Maximising (10.1) given (10.25) with respect to the $(i, j)^{th}$ element of X_1 yields

$$E\left[\left\{b_1(Q_1' + \alpha(\Theta)) - b_2(Q_2' + \beta(n)) + \alpha(\Theta)(X_1-M_1)^{-1}(b_1M_1 + b_2M_2)\right\}\frac{d}{dx_{ij}}\left[(X_1-M_1)^{-1}\right]\alpha(\Theta)^T\right] = 0 \qquad (10.28)$$

Now consider the choice of

$$X_1^* = -b_2M_2/b_1 \qquad (10.29)$$

then (10.28) reduces to

$$E\left[b_2\beta(n)\frac{d(X_1-M_1)^{-1}}{dx_{ij}}\alpha(\Theta)^T\right] = 0 \qquad (10.30)$$

and (10.30) holds if $\beta(n)$ and $\alpha(\Theta)$ are independent. Then X_1^* satisfies the first-order conditions for an optimal linear contingent price vector. We have, substituting (10.29) back into (10.27)

$$L_1^* - \hat{L}_1 = -b_1(b_1M_1 + b_2M_2)^{-1}\alpha(\Theta)^T \qquad (10.31)$$

and the comparative advantage of using strategy (w_1^*) as defined by (10.24) over strategy $(\hat{L})$ is

$$\Delta(w_1{}^* - \hat{L}) = -\tfrac{1}{2}\, b_1{}^2\, E\left[\alpha(\Theta)(b_1M_1 + b_2M_2)^{-1}\, \alpha(\Theta)^T\right] \geq 0 \tag{10.32}$$

and (10.32) is non-negative as $(b_1M_1 + b_2M_2)$ is negative definite. This strategy is in effect the resource allocation analogue of the IP' strategy in Ireland (1977)(see Chapter 9) extended to m resources. The analysis is very general except for the key assumption of independence of the $\alpha(\Theta)$ and $\beta(\eta)$. One other situation where progress can be made in the m resource case is where $\alpha(\Theta)$ and $\beta(\eta)$ are constrained such that $\beta(\eta) = G\,\alpha(\Theta)$ where G is a non-stochastic square matrix. Then stochastic variations in one firm's marginal products can be completely explained by those of the other firm. Then

$$X_1 = M_1 - (b_1M_1 + b_2M_2)(b_1I - b_2G)^{-1} \tag{10.33}$$

satisfies the first-order conditions (10.28), and in the special case where $G = -I$, such that a gain for one firm is a loss for the other, $X_1 = (M_1 - M_2)b_2/(b_1 + b_2)$.

Finally, if there is just one resource we can find the optimal value for the scalar X_1 for the case where

$$E(\alpha(\Theta)\ \beta(\eta)) = \omega \neq 0$$

With $M_1 = Q_1''$, $M_2 = Q_2''$, we have provided $b_1\sigma^2 - b_2\omega > 0$:

$$X_1^* = \frac{b_2Q_2''\sigma^2 + b_1Q_1''\,\omega}{b_2\omega - b_1\sigma^2} \tag{10.34}$$

$$L_1^* - \hat{L}_1 = \frac{\alpha(\Theta)}{\sigma^2} \frac{(b_2\omega - b_1\sigma^2)}{(b_1Q_1'' + b_2Q_2'')} \tag{10.35}$$

and
$$\Delta(w_1^* - \hat{L}) = \frac{-1}{2\sigma^2} \frac{(b_2\omega - b_1\sigma^2)^2}{b_1Q_1'' + b_2Q_q''} \geq 0 \tag{10.36}$$

The criterion for selection of the "preferred" firm which is to decide the resource allocation is shown by $\Delta(w_1^* - w_2^*)$ where strategy (w_2^*) is that of setting an analogous cost function for firm 2. We have

$$\Delta(w_1^* - w_2^*) = - \frac{(b_1^2\sigma^2 - b_2^2\psi^2)(\sigma^2\psi^2 - \omega^2)}{2\sigma^2\psi^2(b_1Q_1'' + b_2Q_2'')} \tag{10.37}$$

in the case of one resource and

$$\Delta(w_1^* - w_2^*) = -\tfrac{1}{2} E \Big[(b_1\alpha(\Theta) - b_2\beta(\eta)(b_1M_1 + b_2M_2)^{-1}$$

$$(b_1\alpha(\Theta) + b_2\beta(\eta))\Big] \tag{10.38}$$

in the case of m resources when $\alpha(\Theta)$ and $\beta(\eta)$ are independent. Obviously (10.37) and (10.38) are identical in the case where $\omega = 0$ and $m = 1$.

V. Conclusions

The analysis in this chapter has been conducted for a two-firm economy. We have assumed that the decisions made by the preferred firm are feasible, in that the firm's demands are not more than total supplies. This latter assumption is not particularly worrying in a two-firm economy - the preferred firm could be supplied with the minimum of its demands and the available supplies - but it is a complication in extending the model to n firms. One possible way forward would be to rank the firms in terms of the criteria for judging the "preferred" firm and then ask each in turn (in the "pecking" order given by the rankings) for their demand responses to strategies of setting factor prices or cost functions until the supply is exhausted or until the last firm is reached. A further refinement would be the recomputation of w or w^* in response to demand levels from highly ranked firms already received.

An alternative to the above kind of generalisation is to reinterpret the two-firm model in the following way. Let Q_1 be the maximum revenue from given inputs in one sector of the economy, and Q_2 in the other. The coefficients b_1, b_2 again allow deviations in social value and revenue. The functions Q_1, Q_2 are envelope functions reflecting the decisions of many individual production units all equating marginal revenue products to marginal factor cost. When the latter is a given number (w_1 say) no further internal factor market is required and planning strategies can be chosen in accordance with the results of Sections II and III. Of course, when a cost function rather than a factor price is used as the planning strategy, a further internal factor market for arriving at a sector-specific market-clearing marginal cost of factors is required.

The analysis in this chapter has demonstrated an important difference in the criteria for selection of the preferred firm or sector between setting factor prices and setting factor cost functions. In the former case, and taking $m = 1$ and $\omega = 0$ for simplicity, it is the relative slope of the revenue functions of the two firms or sectors. In the cost function case, however, the criterion is the relative uncertainty in the marginal revenue products. An explanation of this key difference appears straightforward. With the cost function strategies, the deciding firm takes full account, via the cost function, of the slope of the other firm's marginal revenue product schedule. Thus all that is left is to allocate the decision to the firm or sector with most uncertainty. With strategies (w_1) and (w_2), however, no account at all of the other firm's marginal revenue product's sensitivity is taken by the deciding firm, and this relative sensitivity takes precedence over the levels of uncertainty.

In either case, the conclusion is the justification of a "pecking order" in granting firm's demands for resources in a situation of planning with incomplete or uncertain information.

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