Optimal Security Design under Asymmetric Information and Profit Manipulation*

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Abstract

We consider a model of external financing in which entrepreneurs are privately informed about the quality of their projects and seek funds from competitive financiers. The literature restricts attention to monotonic or 'manipulation proof' securities and finds that straight debt is the uniquely optimal contract. Monotonicity is commonly justified by the argument that it would arise endogenously if the entrepreneur can window dress the realized earnings before contract maturity. We explicitly characterize the optimal contracts when entrepreneurs can engage in window dressing and/or output diversion, and derive necessary and sufficient conditions for straight debt to be optimal. Contrary to conventional wisdom, debt is often suboptimal and it is never uniquely optimal. Optimal contracts are non-monotonic and induce profit manipulation in equilibrium. They can be implemented as performance-sensitive debt.

Key words: security design, capital structure, asymmetric information, profit manipulation, window dressing.

JEL classification: D82; D86; G32; M40.

*We would like to thank Robert Akerlof, Dan Bernhardt, Peter Buisseret, Peter DeMarzo, Bhaskar Dutta, Marina Halac, Peter Hammond, Ilan Kremer, Dimitri Migrow, Motty Perry, Herakles Polemarchakis, Uday Rajan (the editor), David Ronayne and an anonymous referee. All mistakes are our own.

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1 Introduction

Asymmetric information is often used to explain the prevalence of debt as a means of external financing. Early theories focused on frictions that arise after cash flows realize, showing that debt minimizes the deadweight losses associated with monitoring and verification when cash flows are only partially or costly observable.[1] Subsequent work derived the optimality of debt under ex-ante asymmetries of information.[2] The idea is that borrowers are often better informed than lenders about the distribution of future cash flows, or that they need to exert effort to generate the cash flows. In either case, the same two conditions are sufficient for debt to be optimal: (i) cash flows distributions are hazard-rate ordered in the borrower's quality, or in the degree of effort exerted; (ii) securities' payoffs are restricted to be monotonic functions of the realized cash flows, so that performance-based bonuses or other non-monotonicities are assumed away.[3]

In this paper we explore when and why ruling out such bonuses is optimal, and we derive necessary and sufficient conditions on fundamentals under which the optimal securities are monotonic. Although we focus on adverse selection (that is, we build on Nachman and Noe (1994)), similar insights extend to the moral hazard setting of Innes (1990). When borrowers are better informed about the distribution of future cash flows relative to their lenders, high quality borrowers can either credibly signal their type, or they end up paying an information cost through the underpricing of the securities they issue. Since better borrowers are less likely to generate low cash flows, this information cost is minimized if the securities issued offer the greatest downside protection and the lowest upside gain which secure that the lenders obtain their desired return.

It follows that non-monotonic contracts challenge the optimality of straight debt. This occurs because, relative to debt, they increase the cash payed out to the lenders in low states of the world, at the expense of the cash payed out to them in high states. Namely, lenders who issued non-monotonic contracts have a greater downside protection and a lower upside gain than lenders who issued straight debt. In the security design language,

[1] See the costly-state-verification models of Townsend (1979) and Gale and Hellwig (1985).

[2] See especially Innes (1990, 1993), Nachman and Noe (1994) and DeMarzo and Duffie (1999). These models formalize intuitions already present in Myers and Majluf (1984).

[3] In a moral hazard setting, Hébert (2017) studies a model where effort moral hazard and risk shifting are simultaneously present and shows that the optimal contract is always monotonic; Antic (2014), and Lee and Rajan (2016) show that monotonic contracts are optimal when the principal is ambiguity averse. In an adverse selection setting, Malenko and Tsoy (2018) show that the optimal contract is monotonic when the principal is ambiguity averse.

the payoff of such non-monotonic securities crosses the one of straight debt from the right, reducing the mispricing of the securities issued by better quality borrowers.

So, what prevents such contracts from being used? The common explanation appeals to window dressing possibilities. A (possibly discrete) performance-based bonus is paid when the realized cash flows exceed some pre-specified benchmark. Now, suppose that a borrower can window dress the cash flows - perhaps by secretly borrowing additional funds from a friend.[4] She clearly has an incentive to window dress when the realized cash flows do not hit the benchmark, in order to cash the bonus. So, window dressing effectively transforms debt with bonuses into straight debt, with 'real' (correctly anticipated) face value equal to the contractual face value minus the bonus. The above story is sound if: (i) window dressing possibilities are unbounded (e.g., the 'friend' has a deep pocket); and (ii) borrowers may never be caught cheating. However, often some of these conditions fail empirically.

In this paper, we generalize the results of Nachman and Noe (1994) to encompass such cases. We characterize optimal securities and derive novel testable predictions relating the size of bonuses to cross-sectional differences in asymmetric information and to the efficacy of the legal system. In particular, we explicitly model window dressing by assuming that, in the interim period, the borrower has access to a credit line from a 'friend'. The credit line enables her potentially to window dress earnings up to a fixed upper bound (with respect to which we perform comparative static exercises), as well as to divert output. We allow the window dressing and diversion bounds to depend on the realized cash flow and we show that the optimal contracts is non-monotonic and it involves profit manipulation.

The key observation driving the results is as follows. Consider three possible realizations of the cash flows: \$10, \$20 and \$30. Further, suppose that cash flows can be window dressed up to \$15, perhaps by secretly borrowing from a friend. An entrepreneur with a realized cash flow of \$10 can claim to have \$20, but not \$30. In contrast, an entrepreneur with \$20 can easily claim to have \$30. Because the \$10 type cannot pretend to be the \$30 type, this example describes a situation known in the contracting literature as a failure of the *nested-range-condition*, as defined in Green and Laffont (1986). This condition is both necessary and sufficient for the revelation principle to hold – or, in our language, for concentrating attention without loss of generality on contracts that prevent any profit

[4]Importantly, a secret third-party loan is just one of the many possible examples of window dressing, which is a more general phenomenon. As it will be clear, our qualitative results do not depend on the specific source of window dressing opportunities, but rather on their extent.

manipulation on the equilibrium path.

The immediate and, in our opinion, important consequence of a failure of the nested-range-condition is that contracts involving profit manipulation in equilibrium implement allocations that cannot be achieved otherwise. In fact, we show that they strictly reduce the mispricing of the securities issued by better borrower types for some (most) parameter values. We characterize two parameter regions where the uniquely optimal security requires profit manipulation. In one region, the equilibrium is pooling and all borrowers issue non-monotonic securities, such as debt with a strictly positive performance-based bonus. In the other region, the equilibrium is separating and better borrowers issue non-monotonic securities, whereas the worst borrowers can issue any other security, e.g. debt or equity.

Furthermore, for sufficiently large window dressing possibilities the aforementioned justification for monotonicity of securities holds, and every non-monotonic contract is ex post equivalent to straight debt, with 'real' face value equal to the nominal face value minus the bonus (as in Nachman and Noe (1994)). One contribution of our paper is to explicitly characterize what 'sufficiently large' means analytically, i.e., to derive necessary and sufficient conditions for debt to be optimal. We also show that such conditions are never satisfied if the distribution of future cash flows is unbounded above - this is especially relevant for structural models, where often cash flows are assumed to be either log-normally or exponentially distributed. Moreover, debt is never uniquely optimal: there always exists a non-monotonic contract ex post equivalent to debt. We also provide numerical examples to demonstrate that the region where debt contracts belong to the set of optimal contracts is not only quite limited, but it also corresponds to the most extreme degrees of manipulation opportunities and adverse selection. In this region, there always exist optimal non-monotonic contracts that are ex post equivalent to debt.

Our model is explicitly designed to study the effect of window dressing on optimal securities. However, window dressing is only one possible type of output manipulation, the other being cash diversion. In the analysis, we allow for both window dressing and cash diversion; our results cover both cases. The main insight obtained from cash diversion is that it induces manipulation also with monotonic securities, which is not true for window dressing. As a result, it constraints the set of firms that can be successful in raising external financing.

One important question concerns the empirical implications of our exercise. Non-

monotonic contracts are claimed to be rarely observed in reality, and presumably this explains some of the literature's incentives to assume monotonicity in the first place. We would like to point out that, although not as frequently as debt, non-monotonic contracts do exist, especially in the context of managerial compensation.

Managerial compensation typically features performance-based bonuses, that may often be discrete and sizable. These bonuses induce a non-monotonicity in outside investors' compensation, and – consistent with our predictions – they introduce incentives for the managers to manipulate earnings in order to cash the bonus. Our model provides an equilibrium explanation of such manipulations, where they are priced out and used optimally as a signaling device or to reduce mispricing. We predict that *the lower the extent of feasible manipulations and the smaller the distance across entrepreneurial types in terms of net present values, the larger should be both the dispersion in bonuses across entrepreneurs, as well as the informational content of the bonus.* Indeed, larger bonuses are granted to higher quality entrepreneurs in the separating region of the model.

The paper unfolds as follows. Section 2 reviews the relevant literature. Section 3 presents the baseline model. Section 4 introduces the relevant securities, and discusses when and how they induce profit manipulation. Section 5 derives our main results. Section 6 presents some parametric examples to streamline our findings. Section 7 concludes.

2 Literature Review

Our paper is closely related to the literature on security design under asymmetric information. Myers and Majluf (1984) developed the 'pecking order' theory of debt optimality under asymmetric information in a set up where only debt and (inside or outside) equity contracts were allowed. Noe (1988) first showed that their theory required somewhat restrictive assumptions on the distributions of earnings. Innes (1993) and Nachman and Noe (1994) revisited the theoretical argument allowing for a broader set of contracts than debt and equity. These papers found that to obtain debt as the optimal security some monotonicity constraint has to be imposed both on the type space, and on the set of feasible securities. The latter constraint restricts admissible contracts to those that are 'manipulation proof'. Since then, the monotonicity constraint has been widely used. Prominent examples include DeMarzo and Duffie (1999); DeMarzo, Kremer, and Skrzypacz (2005); Inderst and Mueller (2006); Axelson (2007); Axelson, Strömberg, and

Weisbach (2009); Gorbenko and Malenko (2011); Philippon and Skreta (2012); Scheuer (2013). Our contribution is to derive necessary and sufficient conditions for monotonicity to be without loss of generality.

Furthermore, our paper is related to the literature on optimal contracting under profit manipulation. The existing papers can be separated along two dimensions: (i) whether manipulations are assumed to be bounded (and a function of types) or not; and (ii) whether repayments can be extracted via additional tools such as verification, termination or liquidation of the firm. A literature originating from Townsend (1979) and Gale and Hellwig (1985) models unbounded manipulation with the possibility of verifying the earnings at a cost (the so-called 'costly state verification' models). Bolton and Scharfstein (1990) and Hart and Moore (1998) study related models where verification is substituted with the threats of termination and liquidation. In contrast, Green and Laffont (1986) consider a set up with bounded manipulation possibilities but no verification. They provide a necessary and sufficient condition for the revelation principle to hold – the *nested range condition* – which fails naturally in financial contracting models where the set of feasible manipulations is likely to be convex and depends on the type.

Lacker and Weinberg (1989) studied a model of ex post moral hazard where profit manipulation opportunities move optimal contracts from debt toward equity-like arrangements. Recent work that explicitly models profit manipulation opportunities includes Picard (2000), Crocker and Slemrod (2007), Sun (2014), Guttman and Marinovic (2017) and Strobl and Povel (2013). To our knowledge, previous papers did not allow for profit manipulation within an adverse selection context, which is the focus of this paper.

3 The Economy

There are two dates $\{0, 1\}$, an *entrepreneur* and a competitive *financier*. Both agents are risk-neutral and maximize date one consumption. The entrepreneur has a project that generates stochastic date one earnings $x \in X$ and requires a fixed input of $I > 0$ at date zero. The financier has wealth $W > I$, and can either lend it to the entrepreneur or store it without depreciation. The set of possible earnings realizations is $X \equiv [0, K]$. When we allow for unbounded future earnings, we let K approach infinity. There are two types of projects (entrepreneurs), $t \in T \equiv \{l, h\}$. Types differ according to their distribution of earnings. The cumulative distribution function (*cdf*) over X for a type t project is

$F_t(x)$; its associated density is $f_t(x) > 0$ for every $x \in X$. The project's type is private information of the entrepreneur. Outside financiers only know that a fraction $\lambda_l \in (0,1)$ are type l projects, and a fraction $\lambda_h = (1 - \lambda_l)$ are type h projects. All projects have positive net present value, and the firm's assets in place are assumed to be zero. Denote by $\mathbb{E}_t[x] = \int_0^K x\, dF_t(x)$ the full information expected value of a type t project. We assume that all projects have positive net present value:

Assumption 1: $\mathbb{E}_t[x] \geq I > 0$ for every $t \in T$.[5] **(A1)**

In addition, we make the following standard assumptions on the distributions of earnings:

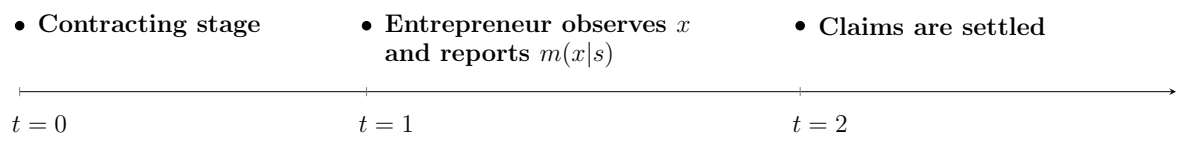
Assumption 2: **(A2)**

- *Strict Monotone Likelihood Ratio Property (MLRP):* $\frac{\partial}{\partial x}\left(\frac{f_h(x)}{f_l(x)}\right) > 0$ *for every* x.

Continuity simplifies the analysis and it prevents contracts that penalize realizations with strictly positive probability only for one type. Strict MLRP implies that $\mathbb{E}_h[x] > \mathbb{E}_l[x]$. Both assumptions are standard in the literature (see for instance DeMarzo, Kremer, and Skrzypacz (2005)). The timing of the game is as follows:

- **date 0:** The entrepreneur of type t issues publicly a *security* (financial contract) denoted by s. Each financier simultaneously quotes a price $P(s)$ at which he is willing to buy the securities. If a contract is signed (securities are sold), the entrepreneur collects $P(s)$. Subsequent investment is observable and verifiable;
- **date 1:** Realized earnings $x \in X$ are perfectly but privately observed by the entrepreneur. She can costlessly manipulate reported earnings by secretly borrowing from friends up to $\bar{\eta}(x) \geq 0$ or diverting $\underline{\eta}(x) \leq x$, and then report earnings $m \in M(x) \equiv [x - \underline{\eta}(x),\, x + \bar{\eta}(x)]$. The possibility of manipulation and its magnitude is common knowledge at date 0;
- **date 2:** Claims are settled based on borrower's reported earning; the game ends.

Timeline



[5] **A1** guarantees that investment is risky, because $I > 0$ and strict positivity of $f_t(x)$ for every x imply that $F_t(I - \epsilon) > 0$ for all $t \in T$, for $\epsilon > 0$.

The novel ingredient that differentiates our findings from existing results is the possibility of ex post profit manipulation. We summarize the restrictions imposed on the manipulation technology in the following assumption:

Assumption 3: *The set of feasible manipulated earnings for given $x \in X$ is:* **(A3a)**

$$M(x) \equiv [x - \underline{\eta}(x),\, x + \overline{\eta}(x)]$$

where $\underline{\eta}(x)$ and $\overline{\eta}(x)$ are $\mathbb{C}^1$ functions such that for every x: (i) $\underline{\eta}(x) \leq x$ and $\underline{\eta}'(x) \in [0, 1)$; (ii) $\overline{\eta}(x) \leq K - x$ and $\overline{\eta}'(x) \geq -1$.

Condition $\bar{\eta}'(x) \geq -1$ guarantees that the higher the realized output, the higher the output that can be reported. We allow for unbounded secret borrowing to nest Nachman and Noe (1994) results as a special case of our framework. We assume that $M(x)$ is a closed interval (hence, convex set) because if the entrepreneur can divert $\underline{\eta}(x)$ dollars from the project to his own accounts, we believe she should be able to divert any amount lower that $\underline{\eta}(x)$. Similarly, if she can window dress an amount equal to $\bar{\eta}(x)$, she should be able to window dress any amount lower that $\bar{\eta}(x)$.

Since profit manipulation possibilities are the key innovation of our model relative to the existing work, a few comments are due. First, we use the term window dressing to denote any *upward profit manipulation* for two main reasons: first, to be in line with most of the previous security design papers that refer to window dressing possibilities as the main problem affecting non-monotonic security designs; second, and most importantly, because our assumption is equivalent to a simple formal model of window dressing in the literal sense, that is, a model of *secret borrowing* from friends. Indeed, we could think about a third party who is directly related to the borrower (and so not subject to ex post informational asymmetries) and who provides a short-term bridge financing at the interim stage. The bounds on window dressing possibilities would naturally corresponds to the depth of this friend's pocket, hence such a model would be equivalent to ours.

In addition, although we believe that there is a potential risk of strategic default as well as other issues in raising the necessary funds from a 'friend', in this model we assume that all those impediments are absent for the following reason. The literature justifies the monotonicity constrain due to the possibility of window dressing and concludes that debt arises as the optimal contract under this assumption. We will claim that this is not the case (as it is true only in some extreme cases). To clarify this point we make

this manipulation as easy as possible for the entrepreneur (subject to our manipulation boundaries) and show that even in this hypothetical scenario manipulation does not eliminate non-monotonicity of the equilibrium contact. Surely, if one makes manipulation even more difficult the monotonicity constraint is even harder to justify.

As for the possibility of output diversion, it is important to stress that none of our qualitative results depend on it, so the reader who just wants to consider window dressing can simply set $\underline{\eta}(x) = 0$ from now on.[6] Because of earnings misreporting, a security $s(\cdot)$ cannot be a function of x as in the previous literature. Instead, it is a function of the reported earnings $m(x|s)$. Because $M(x)$ is a compact set for every $x \in X$, we know that for every security s and every x there exists a best message $m^*(x|s)$ defined so that:

$$m^*(x|s) \equiv \arg\min_{m \in M(x)} \{s(m)\}.$$

Further, because $M(x) \subseteq X$, we know that $m^*(x|s) \in X$.[7] Hence, the expected repayment of a security (or its *real payoff*) is a function $s(m^*(x|s)) : X \to \mathbb{R}$. It should be noticed that the ex post verification problem created by the possibility of profit manipulation prevents the application of the revelation principle, since $M(x)$ does not need to satisfy the nested-range-condition of Green and Laffont (1986).

The only restriction we impose on the contract space is that each security must satisfy limited liability, as appropriately redefined in terms of messages:

Assumption 4: *The set of admissible securities is given by:* **(A4)**

$$S \equiv \{s(m) \mid 0 \leq s(m) \leq m, \ \ \forall m \in X\}.$$

If the borrower declares m and cannot repay $s(m)$ to his financier, then the financier becomes the legitimate owner of borrower's assets.[8]

A possible interpretation of our limited liability assumption is that sending a message

[6] We could have modeled diversion as *output destruction*, in which case the entrepreneur could not put the diverted amount in his pocket. However, in such a scenario the entrepreneur would be indifferent between diverting and not in equilibrium, making such possibilities useless.

[7] Our formulation restricts attention to direct mechanisms where the set of messages is a subset of the set of states X, following Green and Laffont (1986).

[8] Since the limited liability constraint must be defined in terms of messages rather than realized output, we should consider the case in which the entrepreneur declares earnings that exceed true earnings, and does not have the resources to repay the contractual obligation. In this case, the fraud becomes observable and verifiable: it is revealed that he is either lying about x or refusing to make the payment he committed to make. We implicitly assume that when the fraud is revealed the agent receives a large punishment.

is equivalent to the action of *showing* the balance on the firm's account to the lenders, after any eventual window dressing or cash diversion. In this case, payments can only be weakly smaller than the total cash shown to be available in the accounts, i.e. m.[9]

Denote by V_t the profits of an entrepreneur of type t whose offered security s has been priced at P by the financier, and by V_f the financier's profits. Then we can write:

$$V_t = P - I + \mathbb{E}_t\big[x - s(m^*(x|s))\big], \tag{1}$$

$$V_f = \mathbb{E}_{\boldsymbol{\lambda}(s)}\big[s(m^*(x|s))\big] - P(s). \tag{2}$$

The expectation in (2) is given by the sum across types (weighted by the posterior belief $\lambda(t|s)$ that type t is issuing the contract s) of the final payoff of the security after manipulation takes place:

$$\mathbb{E}_{\boldsymbol{\lambda}(s)}\big[s(m^*(x|s))\big] \equiv \sum_{t\in T} \lambda(t|s)\bigg[\int_{x\in X} s(m^*(x|s))dF_t(x)\bigg].$$

Notice that we can write $m_t^*(x|s) = m^*(x|s)$ because the cost and benefits of output manipulation ex post are not type-dependent.

Here we adopt the concept of Perfect Bayesian Equilibrium (PBE) in pure strategies:

PBE: *A strategy profile* $\big(s_t^*,\, m^*(x|s),\, P^*(s)\big)$ *and a common posterior belief* $\lambda^*(t|s)$ *for* $t \in T$ *form a pure strategy PBE of the game if the following conditions are satisfied:*

1. *For every* $x \in X$ *and for* $s \in S$*:* $m^*(x|s) = \arg\min_{x\in M(x)} \big\{s(m)\big\}$;
2. *For every* $t \in T$*,* s_t^* *maximizes* $V_t(s_t, P^*(s_t), m^*)$ *subject to the limited liability constraint (*$s_t \in S$*);*
3. *The posterior belief* $\lambda^*(t|s_t)$ *is obtained from Bayes' Rule whenever possible;*
4. *Competitive Rationality: for* $s_t \in S$*,* $P^*(s_t) = \mathbb{E}_{\boldsymbol{\lambda}^*(t|s_t)}[s_t]$

As standard, a PBE is said to be *separating* if $s_h^* \neq s_l^*$, and *pooling* otherwise. Notice that, because investment is observable and verifiable, in every equilibrium it must be the case that either $P^*(s_t) = 0$ (no investment), or $P^*(s_t) \geq I$ (investment takes place). To rule out 'unreasonable' equilibria, we refine the off-equilibrium-path beliefs using the Intuitive Criterion by Cho and Kreps (1987).[10]

[9] We thank the editor for having suggested this interpretation to us.

[10] The formal definition of the Intuitive Criterion is left to the Appendix.

4 Debt and Bonus Contracts

It is useful to stress again the distinction that arises in this model (unlike the existing literature) between the *promised expected payoff* and the *real expected payoff* of a security. The *promised* expected payoff is given by $\mathbb{E}_{\boldsymbol{\lambda}^*(s)}[s(m = x)]$, where each realized x is assumed to be reported truthfully. In contrast, the *real* expected payoff is given by $\mathbb{E}_{\boldsymbol{\lambda}^*(s)}[s(m^*(x|s))]$, where $m^*(x|s)$ solves condition (1) of a PBE, i.e. it maximizes the entrepreneur's ex post payoff. The characteristic features of a debt contract are: (i) the fixed repayment in non-bankruptcy states; and (ii) seniority in bankruptcy states. If we denote the face value of debt by d, then whenever $m \geq d$, the debt security specifies $s = d$. If, instead, $m < d$, a bankruptcy state, the debt holder is a senior claimant on the assets, obtaining repayment $s(m) = m$.

In order to characterize the real payoff of a standard debt contract, we need to introduce some additional notation. In particular, consider a debt contract with a face value d and suppose that $K - \underline{\eta}(K) > d$. Define as $\delta(d)$ the highest threshold such that output diversion is profitable for entrepreneurs, i.e.:

$$\delta(d) \equiv \max_{x \in X} \{x | x - \underline{\eta}(x) < d\}.$$

Such point exists and is unique by the intermediate value theorem due to continuity and monotonicity of function $x - \underline{\eta}(x)$ (see Assumption 3) and the fact that $K - \underline{\eta}(K) > d$ and $-\underline{\eta}(0) < d$. If $K - \underline{\eta}(K) \leq d$, instead, then simply set $\delta(d) \equiv K$.

Given this definition of $\delta(\cdot)$, for any debt security s with fixed repayment d, the entrepreneur optimally reports:

$$m^*(x|s) = \begin{cases} x - \underline{\eta}(x), & \text{if } x \leq \delta(d), \\ x, & \text{otherwise.} \end{cases}$$

The dashed curve in Figure 1 depicts the real payoff of a standard debt contract.

Insert Figure 1 about here

The contract that turns out to be generically optimal (we call it a *bonus contract*)

takes the following form. For $(b, d) \in X^2$:

$$s(m) = \begin{cases} m, & \text{if } m < b \\ d, & \text{otherwise.} \end{cases} \tag{3}$$

A bonus contract can be decomposed in two parts: straight debt with face value b, and a bonus of size $b - d$ to be paid whenever the firm does not default on its debts. It is a non-monotonic contract, and the reason for its optimality (as we shall prove) lies in the fact that it crosses the payoff of straight debt from the right. In other words, it provides greater downside protection and lower upside gain to the outside lenders.

Since any admissible security satisfies limited liability ($s \in S$), we must have $d \in \big[0,\ b\big]$.[11] Moreover, if $d = 0$ the bonus contract becomes an asset-or-nothing binary option (or, equivalently, debt with a strictly positive performance-based bonus). Figure 2 depicts the *promised* payoff of contracts as defined in (3).

Insert Figure 2 about here

We next characterize the optimal amount of secret borrowing under bonus contracts. To do so we need to introduce a final piece of notation. Define a function $\bar{\bar{\eta}}(x) \equiv x + \bar{\eta}(x)$. This function is continuous and strictly increasing on $[0, K]$. Consider a bonus contract (b, d) such that $\bar{\eta}(0) < b$. Given that $\bar{\bar{\eta}}(b) \geq b$ by definition, the intermediate value theorem implies that there exists $\beta(b) \in (0, b]$ such that $\bar{\bar{\eta}}(\beta(b)) = \beta(b) + \bar{\eta}(\beta(b)) = b$. Monotonicity of the function $\bar{\bar{\eta}}(x)$ ensures the uniqueness of such a point. This defines a strictly monotonic function[12] on the interval $(\bar{\eta}(0), K)$. We extend this function on $[0, d]$ by setting $\beta(b) = 0$ for any $b \in [0, \bar{\eta}(0)]$. For a given b, the value $\beta(b)$ specifies the smallest realized earnings so that the entrepreneur can reach the cut-off point b by means of secret borrowing. Consider a bonus contract (b, d) such that $\bar{\eta}(0) < b$.

Lemma 1. (The Real Payoff of a Bonus Contract) *Given any bonus contract s with fixed repayment d and threshold b, we have two cases:*

[11] Standard debt contracts are special cases of (3) where $d = b$. For this reason we shall always make explicit whether the contracts we discuss must feature a strictly positive bonus or not.

[12] It followed from the implicit function theorem and Assumption 3.

1. *If $b - d > \underline{\eta}(b)$, then the entrepreneur optimally reports,*

$$m^*(x|s) = \begin{cases} x - \underline{\eta}(x), & \text{if } x < \max\{\beta(b), \delta(d)\}, \\ b, & \text{if } x \in [\max\{\beta(b), \delta(d)\}, b), \\ x, & \text{otherwise.} \end{cases}$$

2. *If $b - d \leq \underline{\eta}(b)$, then the real payoff of a bonus contract is equivalent to that of a debt contract with face value d.*

As a result, a bonus contract is ex post non-monotonic if and only if $\beta(b) > \delta(d)$.

Proof. See the Appendix

Figure 3 depicts the real payoff of a bonus contract for the cases of two different levels of profit manipulation. In Panel A of Figure 3, $\beta(b) > \delta(d)$ and the real payoff is *not* ex post monotonic. In Panels B and C of Figure 3, $\beta(b) < \delta(d)$ and the real payoff is ex post equivalent to that of a debt contract with face value d.

Insert Figure 3 about here

5 Optimal Security Design

In this section we solve for the optimal securities. We first consider the case in which the separation can be achieved and then turn attention to pooling equilibria.

5.1 Separating Equilibria

In this section we characterize the set of Separating Perfect Bayesian Equilibria (SPBE). Such equilibria never arise with the exogenous monotonicity constraint (Nachman and Noe, 1994). The intuition behind the SPBE is the following: the most productive type tries to distinguish himself from the less productive one by offering securities with high downside protection and low upside payoff for the financier (e.g., bonus contracts). By doing so, high types impose a relatively higher cost on low types should they try to mimic. Of course, separation can only be achieved if $\mathbb{E}_l[x - \underline{\eta}(x)] \geq I$. From now on, we assume that this inequality holds.

In a SPBE, $s_l \neq s_h$. Moreover, given the offered security s_t, the posterior belief that it is offered by type t is one, i.e. $\lambda(t|s_t) = 1$ for every $t \in T$. Incentive compatibility for

type t requires:

$$\mathbb{E}_t[x - s_t(m^*(x|s))] \geq \mathbb{E}_t[x - s_{t' \neq t}(m^*(x|s))],$$

or, equivalently: $\mathbb{E}_t[s_t(m^*(x|s))] \leq \mathbb{E}_t[s_{t' \neq t}(m^*(x|s))]$. Further, at any SPBE it must be that $\mathbb{E}_t[s_t(m^*(x|s))] = I$, for every $t \in T$, so we can rewrite the incentive constraint as:

$$\mathbb{E}_{t'}[s_{t'}(m^*(x|s))] - \mathbb{E}_t[s_{t'}(m^*(x|s))] \leq 0.$$

Finally, it is trivial to show that the only incentive constraint that may be binding is that for the l type not to mimic the h type, i.e:

$$\mathbb{E}_h[s_h(m^*(x|s))] - \mathbb{E}_l[s_h(m^*(x|s))] \leq 0. \tag{4}$$

This formulation of the incentive constraint allows us to proceed and solve for the optimal contract as it will become clear below.

Suppose that s_h is a bonus contract (b_h, d_h). After some algebraic manipulation which are left to the Appendix, inequality (4) reads:

$$\begin{aligned} \text{IC} \equiv & \underbrace{\int_0^{\beta(b_h)} \Big[F_l(x)\big(1 - F_h(\beta(b_h))\big) - F_h(x)\big(1 - F_l(\beta(b_h))\big)\Big](1 - \underline{\eta}'(x))dx}_{\geq 0 \text{ by FOSD and by Assumption 3a}} \\ & + \underbrace{\Big[F_l\big(\beta(b_h)\big) - F_h\big(\beta(b_h)\big)\Big]}_{\geq 0 \text{ by FOSD}} \underbrace{\Big[I - \beta(b_h) + \underline{\eta}(\beta(b_h))\Big]}_{\text{sign?}} \leq 0. \end{aligned} \tag{5}$$

Recall that $\beta(b_h)$ is defined as the threshold such that for every $x \leq \beta(b_h)$ diversion is weakly preferred to window dressing. Hence, inequality (5) highlights the key mechanism that underlies separation: setting a threshold b_h that makes $\beta(b_h)$ high enough so that the last bracket becomes not just negative, but low enough that the second line counterbalances the first. The key properties of (5) that are useful in the analysis are given in Lemma 2:

Lemma 2. (Incentive Compatibility) *If $\exists\, b_h \in [0, K]$ that satisfies (5), then:*

1. *There is a unique b_h at which the inequality (5) binds. We denote it by b^1;*
2. *For every $b_h < b^1$ the inequality (5) is violated, and separation fails;*
3. *For every $b_h \geq b^1$ the inequality (5) is satisfied, and separation succeeds.*

Proof. See the Appendix.

The argument to prove Lemma 2 is not immediate, as inequality (5) is a non-monotonic function of b_h. The proof relies on the fact that if the inequality is satisfied for some $b_h < K$, one can show that the set of b_h such that the inequality is binding is a singleton, and the inequality is always satisfied for values $b_h \geq b^1$, and never otherwise. If (5) is satisfied, then a contract is incentive compatible and leaves the financier at his participation constraint. However, it remains to guarantee that the underlying contract belongs to the set of admissible securities, i.e. that $d_h \geq 0$.

Denote by β^{max} the solution to the zero profit condition in a SPBE for type h when the face value of debt $d_h = 0$, and by b^{max} the corresponding contractual threshold such that $\beta^{max} = \beta(b^{max})$. We have:

$$\int_0^{\beta^{max}} \big(x - \underline{\eta}(x)\big) f_h(x) dx = I. \tag{6}$$

We obtain the following theorem:

Theorem 1. (SPBE) *If b^1 exists and $b^1 \leq b^{max}$ then:*

a. *there exists a separating equilibrium e_s^* in which a type h entrepreneur issues a bonus contract (b_h^*, d_h^*) such that the financiers make zero profits, $b_h^* \in [b^1, b^{max}]$, and $\beta(b_h^*) > \delta(d_h^*)$;*

b. *type l entrepreneurs are indifferent between any contract such that $\mathbb{E}_l(s) = I$, as long as it is not a bonus contract with $d_l^* \leq d_h^*$;*

c. *no pooling equilibrium satisfies the Intuitive Criterion.*

Proof. See the Appendix.

Intuitively, when $b^1 \leq b^{max}$ separation may be achieved because MLRP implies that the low type ($t = l$) expects to repay more relative to the high type. Thus, by choosing a sufficiently high threshold for the bonus contract (and a sufficiently low face value of debt) the high type can make the cost of mimicking for the low type excessively high, and credibly signal his type to the uninformed financiers. Separation requires that the contract issued by the high type is ex post non-monotonic, otherwise the analysis in Nachman and Noe (1994) goes through. So, a necessary condition for separation is $\beta(b) > \delta(d)$, which guarantees that the bonus contract is ex post non-monotonic (see Lemma 1).

It should be stressed that this equilibrium is not unique in terms of the securities used for achieving it. However, the equilibrium allocation is unique in the sense that in any of the separating equilibria the securities issued are fairly priced. Thus, in any

separating equilibrium each type of entrepreneur get the net present value of his project. The multiplicity of separating equilibria is the feature of all models where the signal does not imply the deadweight loss (non-dissipative signals, see Bhattacharya (1980)).

Nevertheless, our choice of focusing on bonus contracts still needs to be justified. Does a separating equilibrium exist outside the region covered by Theorem 1? And if so, what contracts support it? The answers to these questions are negative: *if separation is not implementable through bonus contracts, then credible signaling cannot happen under any other security that satisfies limited liability.* This occurs because under a bonus contract the full reported earnings are transferred to the financier if they lie between zero and the threshold b. Because the probability that the low type reports earnings below b is higher, the bonus contract maximizes the cost of mimicking for the l type. Given limited liability, no other contract can achieve a higher expected repayment for the low type in this region. In other words, the conditions in Theorem 1 are *both necessary and sufficient* for separating equilibria to exist:

Corollary 1. (Necessity of bonus contracts for separation) *If b^1 does not exist or $b^1 > b^{max}$, then any PBE of the game must be pooling.*

Proof. See the Appendix.

The intuition for this result is as follows: a higher threshold for the bonus contract (and a lower face value of debt) increases the cost of mimicking for the low type. This cost is maximized when $d_h = 0$ and the threshold is b^{max}. If the distributions are such that the incentive constraint for the low type is violated at this contract, then separation is impossible and every equilibrium must be pooling. The argument is sometimes referred to in the literature as showing that 'no security in S crosses the repayment function of a bonus contract from the left'. We characterize the pooling equilibria next.

5.2 Pooling Equilibria

Since Nachman and Noe (1994) seminal paper, the literature has adopted a stronger refinement than the intuitive criterion to deal with pooling equilibria: the D1 criterion. As is well known, the intuitive criterion does not bind in the pooling region of such models. The reason is that both types may benefit from any deviation depending on the posterior belief of the financier. D1 refines the equilibrium set and obtain a unique equilibrium because it is a condition on the range of beliefs for which a deviation is profitable.

The zero-profit condition for a bonus contract $(b, 0)$ is:

$$ZP(b) \equiv \lambda_h \int_0^{\beta(b)} \big(x - \underline{\eta}(x)\big) f_h(x)dx + (1-\lambda_h) \int_0^{\beta(b)} \big(x - \underline{\eta}(x)\big) f_l(x)dx - I. \quad (7)$$

Applying D1 yields:

Theorem 2. (PPBE, part (a)) *If $b^1 > b^{max}$ (or b^1 does not exist) and there exists $b_{\boldsymbol{\lambda}} \in (0, K]$ solving (7), then there is a unique pooling equilibrium e_p^* which satisfies D1. At e_p^*, all types issue a bonus contract with zero face value of debt $(b_{\boldsymbol{\lambda}}, 0)$.*

Proof. See the Appendix.

Theorem 2 characterizes the set of equilibria that satisfy D1 when separation is not feasible ($b^1 > b^{max}$), but the manipulation possibilities are relatively low ($b_{\boldsymbol{\lambda}} < K$). In this region, the uniquely optimal contract is a bonus contract with zero face value of debt. The intuition for the result is similar to that of Theorem 1: bonus contracts minimize the mispricing of securities issued, even though they do not reduce it to zero.

To conclude the characterization, define the zero-profit condition for a debt contract with face value d (when window dressing opportunities are shut down due to monotonicity of the contract) is:

$$\begin{aligned} ZP^-(d) \;\equiv\; & \lambda_h \int_0^{\delta(d)} \big(x - \underline{\eta}(x)\big) f_h(x)dx + (1-\lambda_h) \int_0^{\delta(d)} \big(x - \underline{\eta}(x)\big) f_l(x)dx \\ + \; & \Big[\lambda_h F_h\big(\delta(d)\big) + (1-\lambda_h) F_l\big(\delta(d)\big)\Big] d - I. \end{aligned} \quad (8)$$

The following result holds:

Theorem 3. (PPBE, part (b)) *If the solution $b_{\boldsymbol{\lambda}}$ of (7) satisfies $b_{\boldsymbol{\lambda}} > K$ and $ZP^-(K) \geq 0$ then there is a unique pooling equilibrium e_p^* satisfying D1, at which all types issue a bonus contract (b_p^*, d_p^*) with $d_p^* > 0$. If $ZP^-(K) < 0$ then there is no financing.*

Proof. See the Appendix.

Unlike the previous cases, where debt has been shown to be suboptimal, the result in Theorem 3 allows debt contracts to arise in equilibrium, since $d_p^* > 0$. To be precise: A necessary and sufficient condition for debt to be optional under output diversion is:

Corollary 2. (Optimality of straight debt) *Suppose that $b_{\boldsymbol{\lambda}} > K$ and $ZP^-(K) \geq 0$. Then the optimal contract (b_p^*, d_p^*) is ex post equivalent to straight debt if and only if*

$\beta(b_p^*) \leq \delta(d_p^*)$, which implies that any bonus contract that satisfies the pooling zero profit condition is ex post monotonic (see Lemma 1).

The Corollary follows immediately from Theorem 3 and Lemma 1. Importantly, debt is never uniquely optimal because its payoff can *always* be replicated by a bonus contract with a higher threshold and identical face value d_p^*.

Finally, observe that both Theorem 3 and its corollary rely on the distribution of earnings being bounded above. For this reason, whenever the earnings distribution is not bounded above they describe empty sets. Such result would hold, for instance, whenever earnings belong to the normal or exponential family.

Theorem 4. (Unbounded support) *If the distribution of earnings is unbounded above, i.e. $K \to \infty$, then straight debt is suboptimal regardless of parameter values.*

Proof. See the Appendix.

In this section we characterized the set of equilibria of the model. Bonus contracts are always optimal, and they provide necessary and sufficient conditions to characterize the unique equilibrium allocation of the game. Debt contracts only arise as a corner solution, when limited liability is binding and the earnings distribution is bounded above.

6 Examples

We now show how our results translate both for some families of widely used distributions that satisfy **A2**: the exponential, normal and log-normal distribution. We also present the case of the distribution with linear density function. The probability distribution functions for this family are linear and given by:

$$f_t(x) = \frac{1}{K}\left[\frac{K-2x}{(\mu_t+1)K} + 1\right] \tag{9}$$

with $\mu_{t=l} > 0$ and $\mu_{t=l} < \mu_{t=h}$. This family satisfies strict MLRP, because for any $(t, t') \in T^2$ such that $t' < t$ we have:

$$\frac{\partial}{\partial x}\left(\frac{f_t(x)}{f_{t'}(x)}\right) = \underbrace{\frac{2K(1+\mu_{t'})}{(K(2+\mu_{t'})-2x)^2(1+\mu_t)}}_{>0}[\mu_t - \mu_{t'}]$$

and $\mu_{t'} < \mu_t$ whenever $t' < t$. We solve for optimal contracts for a range of parameter values and for two cases: when output diversion is forbidden and when it is allowed

along with window dressing. In this exercise we demonstrate under which parameters we observe separating vs. pooling equilibria as well as equilibria with monotonic vs. non-monotonic contracts. Since debt can only arise only in cases of distributions with bounded support we consider truncated versions of the above mentioned distribution families. The examples are solved under the assumption that (i) $\underline{\eta}(x) = 0$ and $\overline{\eta}(x) = \eta$ for every x, for $\eta \in [0, 10]$ for the case of window dressing only and (ii) $\underline{\eta}(x) = \overline{\eta}(x) = \eta$ for every x, for $\eta \in [0, 4]$ for the case of window dressing and output diversion.[13] Table 1 summarizes the parameter values that we assume.[14]

Insert Table 1 about here

Figures 4 and 5 shows the characterization of equilibria for the examples. The black region is where a separating equilibrium exists (and it is unique, in terms of allocations). The gray regions are where a pooling equilibrium exists, and it is unique. Dark gray area corresponds to pooling equilibrium with non-monotonic contract, while light gray area represents cases with debt being the unique pooling equilibrium contract. Finally, the white region is where no financing occurs. Figure 4 corresponds to the case when entrepreneurs can engage only in window dressing and Figure 5 draws types of equilibria when both window dressing and output diversion are allowed.show that the regions described in Theorems 1 and 2 are non-empty.

Insert Figures 4 and 5 about here

7 Conclusion

We have shown that the optimal financial contract under ex ante asymmetric information, limited liability and ex post profit manipulation (window dressing) has the following features: (i) it is non-monotonic in earnings; (ii) it exhibits profit manipulation on-the-equilibrium path. That is, the standard justification for restricting attention to monotonic, manipulation-proof securities is not sound. The results suggest that ex ante asymmetric information is not sufficient to theoretically justify the optimality and the widespread use of debt contracts. We derive necessary and sufficient conditions for monotonic securities to arise in equilibrium. Monotonic securities are never uniquely optimal,

[13] More precisely, at the boundaries of the set X we assume that: (i) whenever $x - \eta < 0$, then $\underline{\eta}(x) = x$; and (ii) whenever $x + \eta > K$, then $\overline{\eta}(x) = K - x$.

[14] The examples include *all* projects such that $\$1 \approx I \leq \mathbb{E}_l(x) < \mathbb{E}_h(x) \approx \$4$.

and they may prevail only if both earnings are bounded, and feasibility is binding. Our model should be of interest in at least two empirical contexts: CEO compensation, where the presence of stock options and other performance-related bonuses is widespread, and venture capital, where valuation-based milestones might induce non-monotonicities in the payoff of both the investors and the firm's managers/owners.

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8 Figures and Tables

Figure 1: The real payoff of debt

This figure depicts real (dashed line) and promised (solid line) payoffs of a standard debt contract. The promised payoff is based on the realized output while the promised payoff is based on the declared output.

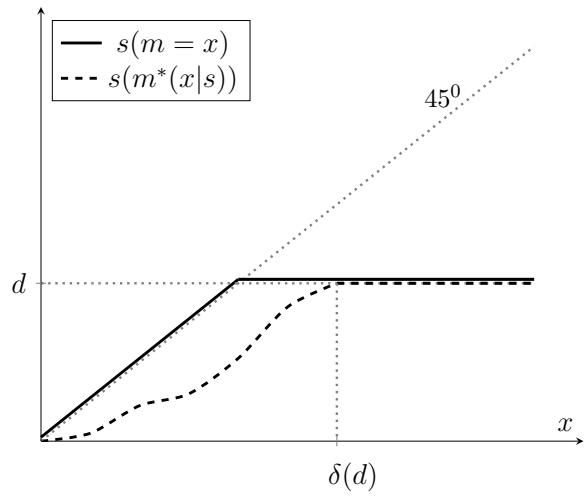

Figure 2: The promised payoff of a bonus contract

This figure depicts the promised payoff of a bonus contract as a function of realized output.

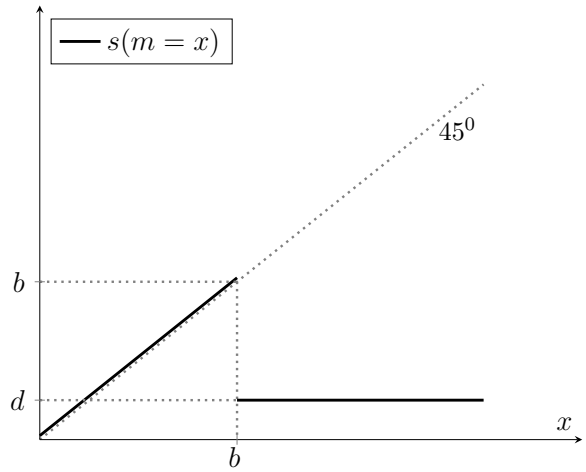

Figure 3: The real payoff of a bonus contract

This figure depicts real (dashed line) and promised (solid line) payoffs of a bonus contract for the cases of two different levels of profit manipulation. The promised payoff is based on the realized output while the promised payoff is based on the declared output. In Panel A, $\beta(b) > \delta(d)$ and the real payoff is *not* ex post monotonic. In Panels B and C, $\beta(b) < \delta(d)$ and the real payoff is ex post equivalent to that of a debt contract with face value d.

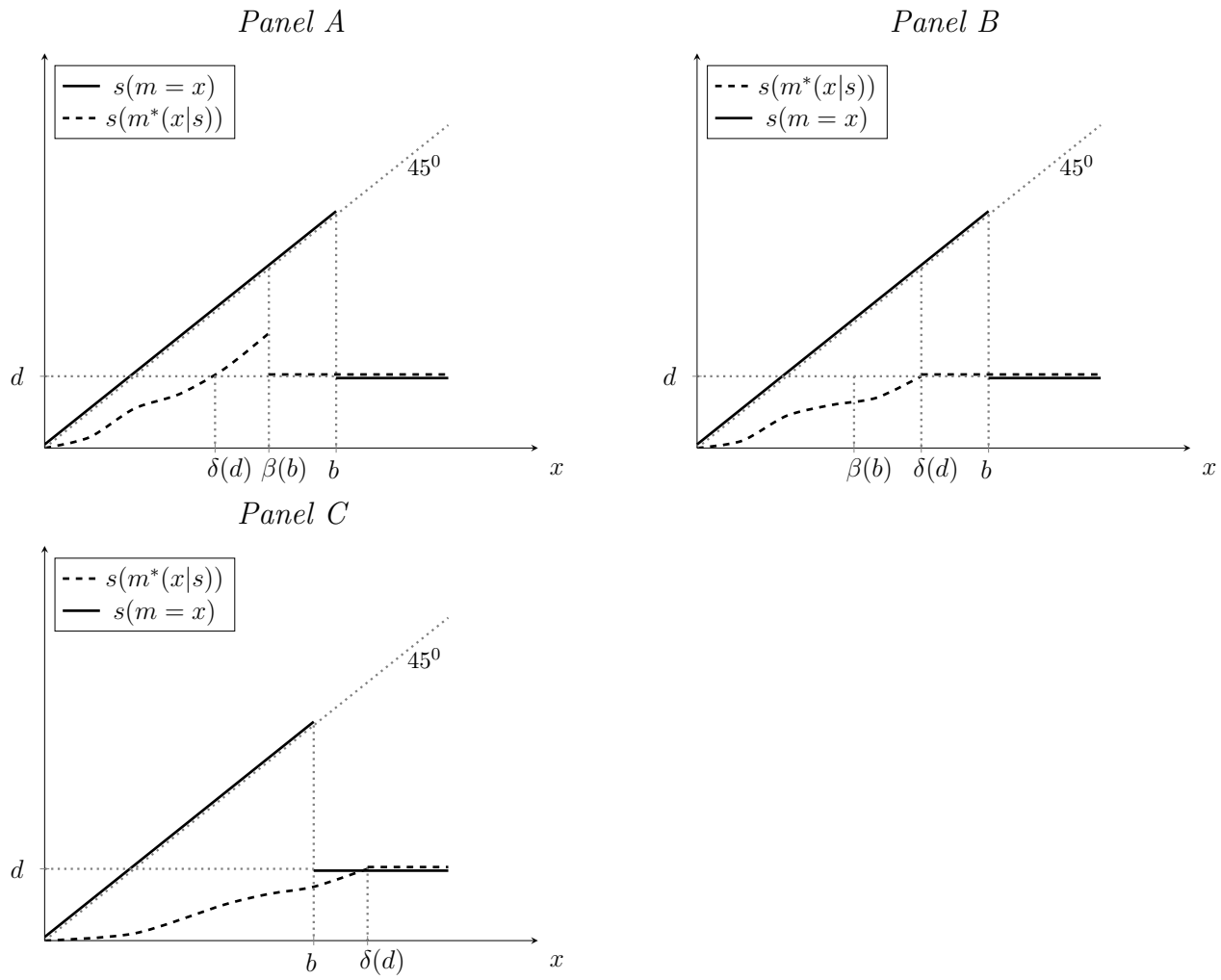

Figure 4: Numerical example: Equilibria types under window dressing only

This figure shows the characterization of equilibria for different distributions of the output. The black region corresponds to parameters values where a unique separating equilibrium exists. Recall that separation can only be implemented by means of non-monotonic contracts. The dark gray region is where a unique pooling equilibrium in bonus contracts exists, and it is unique. The light gray area corresponds to monotonic (debt contract) pooling equilibrium. Finally, the white region is where no financing occurs. Four panels corresponds to four different distributions: Exponential (truncated), Normal (truncated), Lognormal (truncated) and Linear.

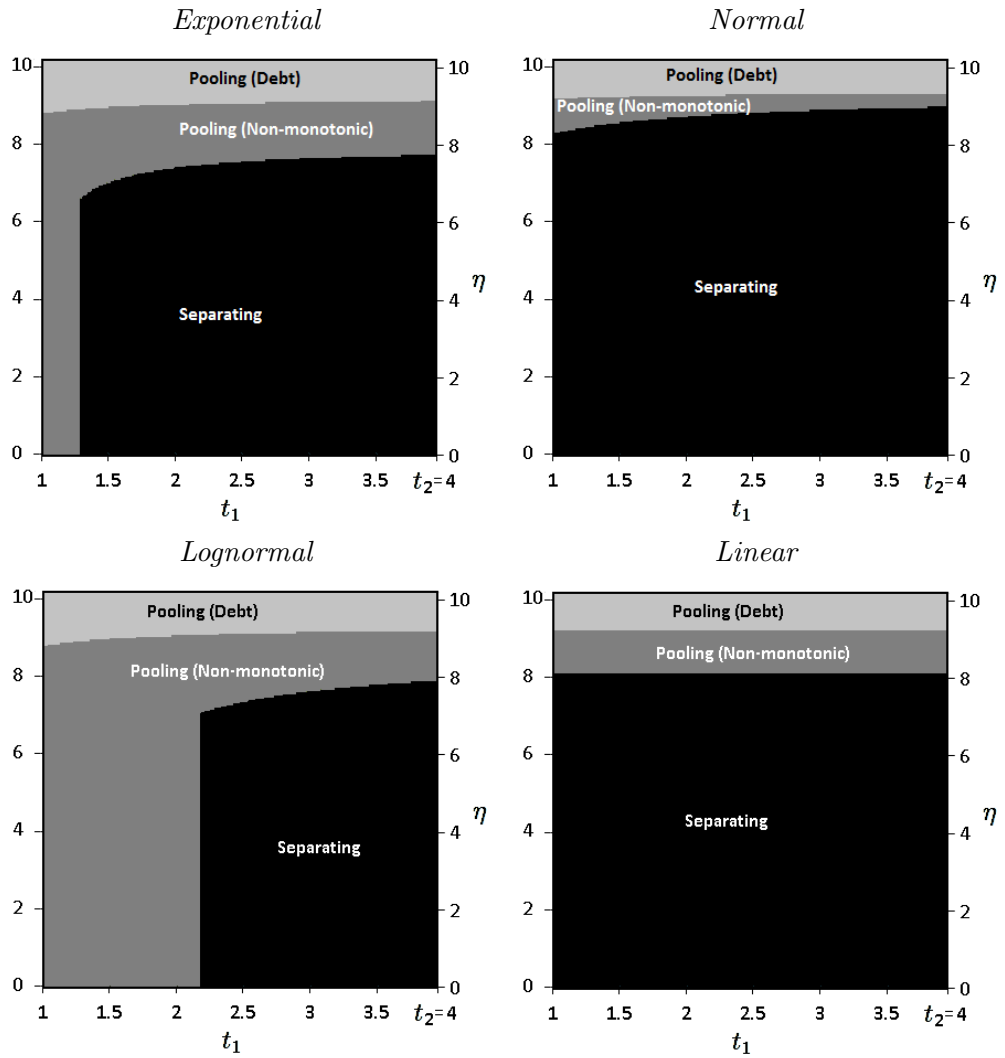

Figure 5: Numerical example: Equilibria types under window dressing and output diversion

This figure shows the characterization of equilibria for different distributions of the output. The black region corresponds to parameters values where a unique separating equilibrium exists. Recall that separation can only be implemented by means of non-monotonic contracts. The dark gray region is where a unique pooling equilibrium in bonus contracts exists, and it is unique. The light gray area corresponds to monotonic (debt contract) pooling equilibrium. Finally, the white region is where no financing occurs. Four panels corresponds to four different distributions: Exponential (truncated), Normal (truncated), Lognormal (truncated) and Linear.

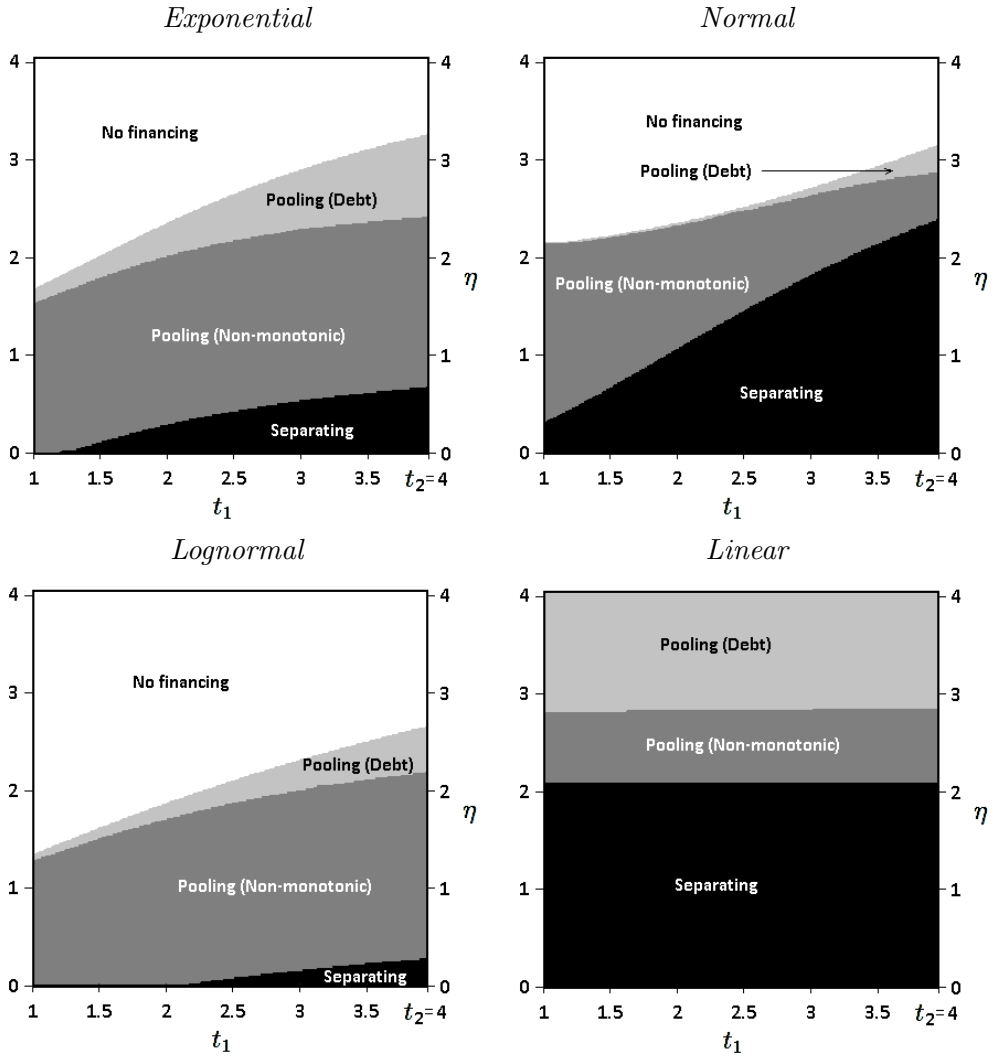

Table 1: Parameter Assumptions

This table summarizes the parameter values for a set of statistical distributions that are used for the numerical calculation of equilibria. The examples are solved under the assumption that $\underline{\eta}(x) = \overline{\eta}(x) = \eta$ for every x, for $\eta \in [0, 4]$. Examples include exponential (truncated), normal (truncated), lognormal (truncated) distributions and a distribution with linear density function. Notation: erf denotes the error function, whereas $erfc$ denotes the complementary error function.

Family	CDF	I	Parameter
Truncated Exponential	$F_t(x) = \begin{cases} 0, & x < 0, \\ \frac{1-e^{-x/t}}{1-e^{-K/t}}, & x \in [0, K], \\ 1, & x > K \end{cases}$	$K = \$10$ $I = \$1$	$t_l \in [1, 4)$ $t_h = 4$
Truncated Normal	$F_t(x) = \begin{cases} 0, & x < 0, \\ \frac{\mathrm{erf}((x-t)/\sqrt{2})-\mathrm{erf}(t/\sqrt{2})}{\mathrm{erfc}((K-t)/\sqrt{2})-\mathrm{erfc}(t/\sqrt{2})}, & x \in [0, K], \\ 1, & x > K \end{cases}$	$K = \$10$ $I = \$1$	$t_l \in [1, 4)$ $t_h = 4$
Truncated Lognormal	$F_t(x) = \begin{cases} 0, & x < 0, \\ \frac{\mathrm{erfc}((\ln(x/t)+0.5)/\sqrt{2})}{\mathrm{erfc}((\ln(K/t)+0.5)/\sqrt{2})}, & x \in [0, K], \\ 1, & x > K \end{cases}$	$K = \$10$ $I = \$1$	$t_l \in [1, 4)$ $t_h = 4$
Linear	$F_t(x) = \begin{cases} 0, & x < 0, \\ \frac{K(2+t)x-x^2}{K^2(1+t)}, & x \in [0, K], \\ 1, & x > K \end{cases}$	$K = \$10$ $I = \$1$	$t_l \in [1, 4)$ $t_h = 4$

Appendix

The Intuitive Criterion

Denote by $V_t(s_t^*, e^*)$ the expected utility of type t entrepreneur issuing s_t^* at the equilibrium e^*, and by $\Pi^*(s|T)$ the set of *all* possible PBEs of the game played by financiers given an observed $s \in S$[15]. We say that:

A PBE is does not satisfy the Intuitive Criterion *if there exist an out-of-equilibrium security $s' \in S$ such that only a subset of types $\tau \subset T$ may benefit from deviating to s'. That is, for every $t \in \tau$ and $t' \in T \setminus \tau$*

$$V_t(s_t^*, e^*) \leq \max_{P^* \in \Pi^*(s'|T)} V_t(s', e^*),$$

$$V_{t'}(s_t^*, e^*) > \max_{P^* \in \Pi^*(s'|T)} V_{t'}(s', e^*).$$

In words, suppose there are two types. Consider a pooling equilibrium, and a deviant security that could only benefit the high type (if accepted) compared to the equilibrium contract, for some off-equilibrium beliefs. The Intuitive Criterion prevents equilibria that are sustained by the off-equilibrium belief that such a security would be offered by a low type with positive probability. The next section introduces the key properties of the two contracts which are relevant in this framework: debt and bonus contracts.

D1 refinement

Denote by V_t' the utility of type t entrepreneur at the deviant contract, and by V_t^* the utility of type t entrepreneur at the equilibrium contract. Moreover, denote by $D(t|s')$ the set of responses of the financier that would deliver strictly higher utility to type t entrepreneurs than the utility he would obtain at the equilibrium contract. Formally:

$$D(t|s') \equiv \{P^*(s') \geq I : V_t' > V_t^*\}$$

where by competitive rationality, $P^*(s') = \mathbb{E}_{\boldsymbol{\lambda}^*(s')}[s']$ for *all* $\lambda^*(s') \in \Delta_T$, as beliefs off-the-equilibrium path are arbitrary. Finally, define the indifference set $D^0(t|s')$:

$$D^0(t|s') \equiv \{P^*(s') \geq I : V_t' = V_t^*\}.$$

[15] Each element of the set can be parameterized by a posterior belief $\boldsymbol{\lambda}(s) \in \Delta_T$, where we adopt the convention that bold symbols represent vectors.

The D1 restriction can be defined as follows[16]:

D1: *Suppose $s' \in S$ is observed off-the-equilibrium path. Then for all $t \in T$:*

$$\lambda_t^*(s') = \begin{cases} 0, & if \quad \exists\, t' \in T \text{ s.t. } t' \neq t, \text{ and } D(t|s') \cup D^0(t|s') \subset D(t'|s'), \\ 1, & if \quad D(t'|s') \cup D^0(t'|s') \subset D(t|s'),\ \forall t' \neq t \in T, \\ 1 - \lambda_{t' \neq t}, & \text{otherwise.} \end{cases}$$

Derivation of the Incentive Constraint – Inequality (5)

Start from inequality (4) in the main text:

$$\mathbb{E}_h[s_h(m^*(x|s))] - \mathbb{E}_l[s_h(m^*(x|s))] \leq 0.$$

Suppose that s_h is a bonus contract (b_h, d_h). Then, inequality (4) reads:

$$\int_0^{\max\{\delta(d_h),\beta(b_h)\}} \big(x - \underline{\eta}(x)\big) \Big[dF_h(x) - dF_l(x)\Big] + \Big[F_l\big(\max\{\delta(d_h), \beta(b_h)\}\big) - F_h\big(\max\{\delta(d_h)\beta(b_h)\}\big)\Big] d_h \leq 0.$$

Notice that if $\max\{\delta(d_h), \beta(b_h)\} = \delta(d_h)$, the incentive constraint can never be satisfied because the real payoff of the bonus contract is monotonic. Hence, for the rest of this section suppose that $\max\{\delta(d_h), \beta(b_h)\} = \beta(b_h)$. This condition imply that $\beta(b_h) - \underline{\eta}(b_h) \geq d_h$, which together with the fact that $\beta(b_h) = b_h - \bar{\eta}(\beta(b_h))$ lead to $b_h - d_h > \underline{\eta}(b_h)$. Rewrite the incentive compatibility constraint as:

$$\int_0^{\beta(b_h)} \big(x - \underline{\eta}(x)\big) \Big[dF_h(x) - dF_l(x)\Big] + \Big[F_l\big(\beta(b_h)\big) - F_h\big(\beta(b_h)\big)\Big] d_h \leq 0. \tag{10}$$

Substituting $\mathbb{E}_h(s_h) = I$ into (10) we get:

$$\int_0^{\beta(b_h)} \big(x - \underline{\eta}(x)\big) \Big[dF_h(x) - dF_l(x)\Big] + \Big[F_l\big(\beta(b_h)\big) - F_h\big(\beta(b_h)\big)\Big] \left(\frac{I - \int_0^{\beta(b_h)} \big(x - \underline{\eta}(x)\big) dF_h(x)}{1 - F_h(\beta(b_h))}\right) \leq 0.$$

[16] The D1 restriction is stronger than the intuitive criterion, hence Theorem 1 goes through unchanged if D1 is imposed.

Integrating by parts and rearranging yields:

$$\text{IC} \equiv \underbrace{\int_0^{\beta(b_h)} \Big[F_l(x)\big(1-F_h(\beta(b_h))\big) - F_h(x)\big(1-F_l(\beta(b_h))\big)\Big](1-\underline{\eta}'(x))dx}_{\geq 0 \text{ by FOSD and by Assumption 3a}}$$
$$+ \underbrace{\Big[F_l\big(\beta(b_h)\big) - F_h\big(\beta(b_h)\big)\Big]}_{\geq 0 \text{ by FOSD}} \underbrace{\Big[I - \beta(b_h) + \underline{\eta}(\beta(b_h))\Big]}_{\text{sign?}} \leq 0.$$

Proof of Lemma 1

1. Suppose that $b - d > \underline{\eta}(b)$ and $\beta(b) \leq \delta(d)$. In this case, for any $x < \delta(d)$ it is not optimal to window dress as the entrepreneur is better off with output diversion. For any $x \in [\delta(d), b)$ we have that $\bar{\bar{\eta}}(x) \geq \bar{\bar{\eta}}(\beta(b)) \geq b$ and therefore the entrepreneur can report b which makes him better off than diverting the output. Finally, for any $x > b$ neither output diversion nor windows dressing benefits the entrepreneur and he truthfully reports x.

Suppose now that $\beta(b) > \delta(d)$. In this case the entrepreneur diverts output for any $x < \beta(b)$ since it is impossible to reach the bonus region $\{x\colon x \geq b\}$ by means of window dressing. For any $x \in [\beta(b), b)$ window dressing is beneficial since $d = \delta(d) - \underline{\eta}(\delta(d)) < x - \underline{\eta}(x)$. Finally, for any $x > b$, as above, the entrepreneur truthfully reports x.

2. Assume now that $b - d \leq \underline{\eta}(b)$. Note that in this case $\delta(d) > b$. For any $x < \delta(d)$ it is not optimal to window dress as the entrepreneur is better off with output diversion. For any $x > \delta(d) > b$ the entrepreneur truthfully reports x. *Q.E.D.*

Proof of Lemma 2

Proof. From the definition of b^1 we know that, if b^1 exists, it must generate a threshold $\beta^1 = \beta\left(b^1\right)$ such that:

$$\int_0^{\beta^1} \Big[F_l(x)\big(1-F_h(\beta^1)\big) - F_h(x)\big(1-F_l(\beta^1)\big)\Big](1-\underline{\eta}'(x))dx$$
$$+\Big[F_l\big(\beta^1\big) - F_h\big(\beta^1\big)\Big]\Big[I - \beta^1 + \underline{\eta}(\beta^1)\Big] = 0.$$

The proof consists on showing that the derivative of the incentive constraint with respect to $\beta(b_h)$ evaluated at β^1 is strictly negative.

To simplify notations in this section, we denote $\beta_h = \beta(b_h)$. Differentiating the

incentive constraint (5) with respect to β_h yields:

$$\begin{aligned}\frac{\partial IC}{\partial \beta_h} &= \big(f_l(\beta_h) - f_h(\beta_h)\big)[I - \beta_h + \underline{\eta}(\beta_h)] + \big(F_l(\beta_h) - F_h(\beta_h)\big)(-1 + \underline{\eta}'(\beta_h)) \\ &+ \big[F_l(\beta_h)(1 - F_h(\beta_h)) - F_h(\beta_h)(1 - F_l(\beta_h))\big](1 - \underline{\eta}'(\beta_h)) \\ &- f_h(\beta_h)\left[\int_0^{\beta_h} F_l(x)(1 - \underline{\eta}'(x))dx\right] + f_l(\beta_h)\left[\int_0^{\beta_h} F_h(x)(1 - \underline{\eta}'(x))dx\right].\end{aligned}$$

Adding and subtraction $f_l(\beta_h)\left[\int_0^{\beta_h} F_l(x)(1 - \underline{\eta}'(x))dx\right]$ and rearranging terms yields

$$\begin{aligned}\frac{\partial IC}{\partial \beta_h} &= \big(f_l(\beta_h) - f_h(\beta_h)\big)\left[I - \beta_h + \underline{\eta}(\beta_h) + \int_0^{\beta_h} F_l(x)(1 - \underline{\eta}'(x))dx\right] \\ &- f_l(\beta_h)\left[\int_0^{\beta_h} \big(F_l(x) - F_h(x)\big)(1 - \underline{\eta}'(x))dx\right]\end{aligned}$$

Evaluating the derivative at β^1 yields:

$$\begin{aligned}&\left.\frac{\partial IC}{\partial \beta_h}\right|_{\beta_h=\beta^1} = \frac{f_l(\beta^1) - f_h(\beta^1)}{F_l(\beta^1) - F_h(\beta^1)} \int_0^{\beta^1} \left[F_h(x)\big(1 - F_l(\beta^1)\big) - F_l(x)\big(1 - F_h(\beta^1)\big)\right](1 - \underline{\eta}'(x))dx \\ &+ \big(f_l(\beta^1) - f_h(\beta^1)\big) \int_0^{\beta^1} F_l(x)(1 - \underline{\eta}'(x))dx - f_l(\beta^1) \int_0^{\beta^1} \big(F_l(x) - F_h(x)\big)(1 - \underline{\eta}'(x))dx \\ &= \frac{f_l(\beta^1)(1 - F_h(\beta^1))}{F_l(\beta^1) - F_h(\beta^1)} \int_0^{\beta^1} (F_h(x) - F_h(x))(1 - \underline{\eta}'(x))dx \\ &- \frac{f_h(\beta^1)(1 - F_l(\beta^1))}{F_l(\beta^1) - F_h(\beta^1)} \int_0^{\beta^1} (F_h(x) - F_h(x))(1 - \underline{\eta}'(x))dx \\ &= \underbrace{\frac{\int_0^{\beta^1} (F_l(x) - F_h(x))(1 - \underline{\eta}'(x))dx}{F_l(\beta^1) - F_h(\beta^1)}}_{>0 \text{ by FOSD and } \underline{\eta}'(x) \in [0,1)} \underbrace{\left[f_l(\beta^1)[1 - F_h(\beta^1)] - f_h(\beta^1)[1 - F_l(\beta^1)]\right]}_{<0 \text{ because MLRP implies HRO}} < 0.\end{aligned}$$

The fraction is strictly positive whenever $\beta^1 > 0$, which is clearly satisfied at every contract that implements investment. The second bracket is negative because we assumed strict MLRP, and it is well known that strict MLRP implies the strict HRO - hazard rate ordering - which in turns guarantees that the bracket is strictly negative.

An immediate consequence of the strict inequality is that if the incentive constraint crosses zero, it must do so only once. Lemma 2 follows. *Q.E.D.*

Proof of Theorem 1

Proof. **Claims a and b:** The claims follow from Lemma 2, definition of b^1 and b^{max} and the fact that if type l entrepreneur issues a bonus contract with $d_l^* \leq d_h^*$ that breaks even on his type, the h type would mimic him and the financier would end up with a rate of repayment lower than I. If $\beta(b_h^*) \leq \delta(d_h^*)$, the incentive compatibility constraints are violated because the contract is ex-post monotonic (see the discussion preceding Lemma 2).

Claim c: Suppose that all agents are in the pooling equilibrium $\hat{e}$ of the game. Type h (the better type) is certainly paying a strictly positive net rate of return to the investors, therefore she would prefer to separate if possible. Type l would prefer to pool, but she still prefers to separate than to mimic type h and issue the non-monotonic contract that satisfies (5). Hence, whenever the non-monotonic contract is observed by the lenders off-equilibrium, the Intuitive Criterion implies that they must believe that the deviation comes from type h with probability one. If this is so, the deviation is profitable and the pooling equilibrium does not satisfy the Intuitive Criterion. *Q.E.D.*

Proof of Corollary 1

Proof. To establish this result, some preliminary steps are required.

Denote the bonus contract with $b_h = b^{max}$ and $d_h = 0$ as s^*, and compare it with another generic security s such that $\mathbb{E}_h[s^*] = \mathbb{E}_h[s] = I$. Define the following sets:

$$\Pi_+(s) \equiv \{m | s^*(m = x) > s(m = x)\}$$

$$\Pi_-(s) \equiv \{m | s^*(m = x) < s(m = x)\}$$

Lemma 3. *For every pair $(m_l = x_l, m_h = x_h)$ in X^2 such that $m_l \in \Pi_+$ and $m_h \in \Pi_-$ we have $m_h > m_l$. Moreover, $m^*(x_l|s^*) \geq m^*(x_l|s)$ and $m^*(x_h|s^*) \leq m^*(x_h|s)$.*

Proof. First notice that $\Pi_+(s) = \emptyset$ if and only if $\Pi_-(s) = \emptyset$, because $f_t(x) > 0$ for every $x \in [0, K]$, for every $t \in T$. In this case the lemma is not very useful, but it is still satisfied. Suppose $\Pi_+(s)$ is non-empty. Because of limited liability, it must be the case that $m_h > b^{max}$ for every $m_h \in \Pi_-$, and $m_l < b^{max}$ for every $m_l \in \Pi_+(s)$. As for the claim about the real payoff, it follows directly from the shape of s^*. *Q.E.D.*

Lemma 4. *Denote the bonus contract with $b_h = b^{max}$ and $d_h = 0$ as s^*. For any generic security s such that $\mathbb{E}_h[s^*] = \mathbb{E}_h[s] = I$, we have that $\mathbb{E}_l[s^*] > \mathbb{E}_l[s]$.*

Proof. The only interesting case is, again, when $\Pi_+(s)$ is non-empty (else the lemma holds trivially). Suppose so. Furthermore, suppose we move from s^* toward s through a series of steps such that in each step we create a security s' such that $\mathbb{E}_h[s'] = I$, but there exists a small interval $dx_a \in \Pi_+(s)$ such that $s'(m^*(dx_a)) < s^*(m^*(dx_a))$ and this change is compensated by inducing a change in the real payoff for another small interval $dx_b \in \Pi_-(s)$ so that $s'(m^*(dx_b)) > s^*(m^*(dx_b))$[17]. Then,

$$\begin{aligned}\mathbb{E}_l[s^*] - \mathbb{E}_l[s'] &= f_l(x_a)\big[s^*(m^*(dx_a)) - s'(m^*(dx_a))\big] + \ f_l(x_b)\big[s^*(m^*(dx_b)) - s'(m^*(dx_b))\big] \\ &= \underbrace{\big[s^*(m^*(dx_b)) - s'(m^*(dx_b))\big]}_{<0 \text{ by construction}} \underbrace{\left(\frac{f_l(x_b)}{f_h(x_b)} - \frac{f_l(x_a)}{f_h(x_a)}\right)}_{<0 \text{ by MLRP}} f_t(x_b) > 0,\end{aligned}$$

where the second equality comes from $\mathbb{E}_h[s^*] = \mathbb{E}_h[s']$. The iteration of this procedure one step at a time concludes the proof. *Q.E.D.*

Because of Lemma 4 we know that $\mathbb{E}_h[s^*] - \mathbb{E}_l[s^*] < \mathbb{E}_h[s] - \mathbb{E}_l[s]$, for every $\mathbb{E}_h[s^*] = \mathbb{E}_h[s] = I$. The Corollary follows. *Q.E.D.*

Proof of Theorem 2

Proof. **Existence:** Suppose there exists an $b_{\boldsymbol{\lambda}}$ that satisfies the pooling zero profit condition. Define the security s_p so that: $d_p = 0$ and $b_{\boldsymbol{\lambda}}$ solves the pooling zero profit condition. Moreover, suppose that the market posterior is equal to the prior at s_p, and it is $\lambda_h = 0$ at any other $s' \neq s_p$ such that $s' \in S$. Then, all types issuing s_p is an equilibrium. It remains to show that it satisfies D1. In particular, we need to prove that $D(1|s') \cup D^0(1|s') \not\subset D(2|s')$ for every $s' \neq s_p$ such that $s' \in S$. There are two cases:

1. If $\mathbb{E}_l[s'] < \mathbb{E}_l[s_p]$, then $D(1|s') = [I, \infty)$. Hence $D(2|s') \subseteq D(1|s') \cup D^0(1|s')$;

2. If $\mathbb{E}_l[s'] \geq \mathbb{E}_l[s_p]$, Lemma 4 implies $\mathbb{E}_h[s'] \geq \mathbb{E}_h[s_p]$ as well. But we can say more:

   Suppose we move from s_p to s' through a series of consecutive steps (i.e. interim contracts s'') such that in each step we induce an increase in the real payoff of s_p by raising $s''(m_k = x_k)$ for some $x_k \in X$. Clearly, it must be that $x_k \geq b_{\boldsymbol{\lambda}}$. Notice that because s_p is a pooling equilibrium, it must be that it does not satisfy (5). Hence, because of MLRP, at x_k we must have $f_l(x_k) < f_h(x_k)$ - i.e. x_k must exceed

[17] In both cases, construct the interval such that it is of equal length as the pdf centered at the two points: $f(x_a)$, $f(x_b)$

the (unique) crossing point of the two densities. Therefore:

$$\begin{aligned}\mathbb{E}_l[s''] - \mathbb{E}_l[s_p] &= f_l(x_k)\big[s''(m^*(x_k|s'')) - s_p(m^*(x_k|s_p))\big] \\ &= f_l(x_k)(s''(m^*(x_k|s'')) < f_h(x_k)(s''(m^*(x_k|s'')) = \mathbb{E}_h[s''] - \mathbb{E}_h[s_p].\end{aligned}$$

Iterating the same logic we conclude that $\mathbb{E}_h[s'] - \mathbb{E}_h[s_p] > \mathbb{E}_l[s'] - \mathbb{E}_l[s_p]$. It follows that at e_p^* it must be the case that, for all $P^* \geq I$:

$$\big(V_h' - V_h^*\big) - \big(V_l' - V_l^*\big) = \big(\mathbb{E}_l[s'] - \mathbb{E}_l[s_p]\big) - \big(\mathbb{E}_h[s'] - \mathbb{E}_h[s_p]\big) < 0,$$

which implies that $D(2|s') \subseteq D(1|s') \cup D^0(1|s')$ again.

Uniqueness: From Corollary 1 we know that there can only exist other pooling equilibria if the conditions required for Theorem 1 to apply do not hold. We now show that if there exists an $b_{\boldsymbol{\lambda}} \in (b^{max}, K)$ such that (7) is satisfied, then every pooling equilibrium e' of the game such that $e' \neq e_p^*$ does not satisfy D1.

Consider a generic $e' \neq e_p^*$. The above analysis and Lemma 4 imply that there exists s' such that $\mathbb{E}_l[s'] \geq \mathbb{E}_l[s_p]$ but $\mathbb{E}_h[s'] < \mathbb{E}_h[s_p]$. Then the logic of the previous proof (point 2 above) is reversed. We conclude that such a equilibrium does not satisfy D1. *Q.E.D.*

Proof of Theorem 3

Proof. This theorem can be proved in the same fashion as Theorem 2, with a twist: now it must be the case that a bonus contract with $d = 0$ cannot satisfy the pooling zero profit condition. Hence, we start by finding the minimum $d > 0$ such that the condition can be satisfied. Then, the result follows from the logic of the previous proof.

Note that with a bonus contract (b_λ, d_p^*) with $d_p^* > 0$ we are hitting the upper bound of the distribution of earnings. If $ZP^-(K) < 0$ then such contract does not exist because of high degree of output diversion so that even issuing a debt contract with face value K does not satisfy the zero-profit condition. Hence, any other security could not break even for the financier. *Q.E.D.*

Proof of Theorem 4

Proof. When $K \to \infty$ there always exists b^{max} such that the pooling zero profit condition is satisfied for $d_p = 0$, because $f_t(x) > 0$ for every $x \in X$ and $t \in T$. Moreover, regardless of the extent profit manipulation, as long as it is bounded, the pooling contract

with $d_p = 0$ has a real payoff which is non-monotonic. As a result, any contract with a monotonic real payoff cannot be part of an equilibrium that satisfies D1. *Q.E.D.*