Resource Allocation When Planning for Simultaneous Disasters

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Abstract

This paper uses stochastic optimisation techniques to allocate scarce national resource across eight cities to best respond to three simultaneous disasters happening across these locations. Our first model analyses the risk of not being able to achieve performance targets given resource constraints while our second model analyses the resources needed to meet target performance levels. A third hybrid model (constructed from the first two models) analyses the implications of different financial budgets. Additional sensitivity analysis is performed by looking into different settings of location importance, number of simultaneous disasters, and resource requirements. We reflect on the use of such modelling techniques for these problems and discuss the influence of political aspects of resource allocation which such models cannot address. We also reflect on the need for advanced modelling to recognise the abilities of the users and the availability of realistic assumptions if they are to influence the practices of disaster managers.

Keywords: OR in disaster relief, simultaneous disasters, resource allocation, stochastic optimisation

1. Introduction

Disasters are exceptional events for a country, not least because of the usually long periods when none occurs but also because responding to them consumes vast resources. These resources are often specific to a particular type of disaster and provide capabilities to save life, protect people and their possessions from harm, and lessen the effects of the impact - for example, equipment to decontaminate a population following chemical release. As disasters seldom happen, these resources are usually scarce as it is infeasible for cities to have large quantities of specialist equipment lying idle between disasters. Consequently, some resources are often held by national government to be shared across cities when the need arises [55]. Thus, an affected city can accumulate needed resources through requesting mutual aid from unaffected cities [49], regional/national assets [18], and international sources [16]. The allocation of specialist resources is made more difficult when multiple disasters occur simultaneously, each demanding the shared resource, as "The size, complexity, and

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number of simultaneous disasters influence the ability of all ... [organizations] to respond. Events may occur, particularly in multi-state disasters that exceed FEMA's planning targets." [2].

The real case that our paper considers is the UK Government's New Dimension Programme [36] which was the UK's £330m investment in emergency response capabilities following the 11th September 2001 terrorist attacks in New York. The programme focused on new capabilities to tackle three simultaneous events happening anywhere in the country including those associated with a major flood (specifically, water rescue and high volume pumping) and those associated with a terrorist attack (specifically, urban search and rescue, and mass decontamination). Our scenarios, assumptions, models and analyses mirror the New Dimension Programme to answer the question - How do we allocate scarce emergency resources (people and equipment) to respond to three large-scale simultaneous floods or terrorist attacks happening in a country? To answer this question we worked closely with New Dimension documentation and the UK's Chief Fire Officers Association's national lead officer for command and control aspects of the New Dimension Programme. The novelty of this paper is the development of Operational Research models which focus on a major problem that governments are grappling with - How to prepare for multiple simultaneous large-scale emergencies happening at the same time, needing the same resources.

Contributions and paper outline

Our paper presents two models to allocate multiple types of scarce resource in preparing for, and responding to, three simultaneous disasters. The first is a two-stage stochastic optimisation model that allocates existing resources to achieve the best performance i.e. addressing how targets can be met. The second model determines the optimal resource capacity to manage all possible scenarios i.e. addressing what additional resources are needed to meet targets. In addition to these two main models, we use a third (hybrid) model to analyse the impact of financial budget on how to allocate additional resources.

Our models reflect how simultaneous disasters thinly stretch shared resources across geographical areas and have more complex demands because, for example: affected areas have different time-dependent demand for resources which only become apparent at different times during the response; resources are shared across different disaster types; and some resources are constrained by deployment regulations. The models analyse the importance of such aspects of emergency response, culminate in sensitivity analyses of not being able to meet the demands created by simultaneous emergencies. We consider the influence of political factors on future investment decisions as well as compare the effect of the strategic importance of cities across a country. To explain the models, the paper begins with a literature review of resource allocation for responding to simultaneous disasters. The models are then introduced, followed by their application to a case study. Discussion and implications conclude the paper.

2. Literature Review

'Everyday emergencies' (such as a car crash or medical incident) are foreseen events that require a general emergency response and, for these, analysts can use historic data to plan [24]. Disasters are different as they can be unforeseen, rare events with extreme consequences ('black swan' events) or a confluence of foreseen events in ways that create unanticipated, rare consequences ('perfect storm' events) [40]. Disasters also create overwhelming demand, require a more substantial response from scarce specialist resources, and may quickly exhaust local resources thereby requiring a wider resource pool. Thus, disaster planning is difficult, in part, because of the need to model exceptions for which there is often minimal or imperfect data and because of the high risks involved in using scarce of resources [27] to protect nationally important assets [10] and help vulnerable people [50].

Our focus is on resource allocation in disasters and there is substantial research on this topic. For example, resource allocation in each of the four phases of the disaster life cycle [16] has been the topic of much study: in the *preparedness* phase (e.g. estimating loss before a disaster happens to identify important infrastructure to protect [50]) and in the *response* phase (e.g. where to move equipment to during a disaster to meet demand [14]). The disaster life cycle also includes phases for after a disaster (*recovery*) [12] and to avoid the effects of a disaster (*mitigation*) [30] but these are not the focus of our paper. The breadth of research on resource allocation in disasters is described in review papers by Özdamar and Ertem [39] who review mathematical models developed for humanitarian logistics, Anaya-Arenas et al [6] who review research on relief distribution networks, and de la Torre et al [17] who review the use of disaster relief logistics.

The published research on resource allocation exploits a breadth of model types but shows a preference towards optimisation - Galindo and Batta [21] and Altay and Green [5] found that optimisation is the most heavily exploited analytical approach in disaster analytics with 23.1% and 32.1% of the published research (respectively) - also see Tables A.10, A.11, and A.12 in Appendix A. This preference includes stochastic optimisation models (e.g. for prepositioning emergency logistics [48, 46]) and dynamic optimisation models (e.g. for adapting response strategies [19, 29]). There is also a growing interest in using game theory to understand disasters, addressing issues such as protecting citizens and infrastructure from aggressors who target system weaknesses [50] and attracting donations based on perceived effectiveness in delivering relief to disaster victims [34]. Similarly, robust optimisation models are becoming popular as they "immunize against uncertainty" so avoid the need to provide some probabilistic information which can be difficult to estimate for disasters [9] (e.g. likelihood of disaster scenarios). Robust optimisation has been applied to evacuation planning to handle scenarios with significant infeasible cost [65], planning emergency logistics to handle the problem of time-dependent uncertainty [8], organizational resilience to handle the problem of unknown loss potential [29], and relief distribution plans to handle the uncertainties following an earthquake [35]. The objectives of disaster optimisation models are varied but include, from Tables A.10, A.11, and A.12, to minimize expected costs [4], minimize casualties [48], minimize completion times [63], maximize equity of those affected [45], maximize recovery coverage [12], and maximize the fairness of relief distribution [56].

Common across many disaster optimisation studies is resource allocation based on the value or importance of elements being modelled, for example, the value of a target to a defender [10, 50, 27], the value of a business function to a plan [12], the importance of an item in a relief pack [46], the importance of products that satisfy recipients [56], the priority of helping wounded people [66], or the priority of transporting evacuees compared to products [38, 35]. For good reason, not all studies optimise importance e.g. Salmerón and Apte [48] assume "both groups [of people] are equally important in the sense that failing to meet either demand results in persons to perish" (p566). Importance is an aspect considered in our models and is a focus of our discussion.

While there is much research on everyday emergencies occurring at the same time, there is less on simultaneous disasters. We define simultaneous disasters as two or more disasters that temporally overlap, may be geographically distant, yet make demands of the same pool of resources. As this pool is finite, the resourcing challenge is the competition from the disasters for scarce specialist resources combined with the need to transport these across an area, efficiently, to satisfy competing demands. We distinguish these from 'multiple disasters' (incidents occuring in the same place but with no temporal overlap [47]) and from 'secondary disasters' [67] or 'serial disaster chains' ("a series of major disasters that occur as an offshoot of a major disaster" [57, p510], such as an earthquake causing a landslide [64]). These distinctions are important because unrelated multiple incidents do not make resource allocations more challenging than for single disasters, and the physical co-location of serial disaster chains means the need for resource is geographically concentrated thereby avoiding extensive travel and easing the challenge of mutual aid (discussed below). On the number of simultaneous incidents, the UK government's "planning assumptions [were] ... based on three simultaneous major incidents" [58], while the USA government sought to "develop, acquire, and coordinate a national operational capability, and the resources and assets to simultaneously respond to four catastrophic plus twelve non-catastrophic incidents, anywhere in the country" [2, p11].

This focus on simultaneous disasters is novel as "there are few methods to model the practical situations of multiple-resource, multiple-response and multiple-point" [67, p11067] because "the traditional emergency resource allocation in the literature considers only a single incident at a time" [52, p200]. When the literature considers more than one disaster, it is typically a single type of disaster in more than one place, for example from Table A.10, simultaneous failures disrupting communication networks [1, 44], more than one area being affected by a storm [48], more than one wildfire [33], or secondary disasters such as earthquake aftershocks [67]. Some research considers simultaneous events but assumes that each rescue agency can only respond to a single event at any time [25], while others limit the potential of applying their model to more than one incident to future work [27]. Wex et al [63, p697] deconstructed disasters into "a large number of geographically-dispersed incidents, such as fires and collapsed buildings" but they focus on efficient allocation of response at a local level rather than taking a national perspective. We found only two papers that consider more than one type of disaster happening simultaneously: Albores and Shaw [3] who evaluate three simultaneous events of different types, requiring different sorts of responses, happening in different locations; Su et al [52, p200] who "allocate multiple emergency resources of multiple rescue agencies to multiple concurrent incidents in a parallel manner".

Another aspect that is important to our paper but seldom found in the literature is the borrowing of emergency resources from across cities to address the scale of the disaster, so-called mutual aid. Mutual aid is a facility that "authorizes a state [or region] to enter into a bilateral or multilateral agreements with its neighbors. A mutual aid agreement allows one government agency to come to the aid of another" [49, p102]. Su et al [52] do not consider mutual aid as a way of coping with simultaneous disasters, but Albores and Shaw [3] do through a prioritised system of borrowing resources from regions in a pre-specified order. The complication that mutual aid brings in simultaneous incidents is the conflicting priorities for which regions should satisfy which requests.

The final aspect not often found in the disaster planning resource allocation literature which this paper considers is the impact of deployment regulations on the ability of personnel to work safely in hazardous situations. In existing studies often the number of personnel are modelled as a homogeneous type [52] or not treated as a constraint [7]. However, deployment regulations ensure safe working especially in hazardous environments, for example, the amount of aircraft flying time for aircrew [7] or, as in our case, the working time in firefighting breathing apparatus [3]. Deployment regulations are time-dependent resource constraints which influence the capacity of staff to complete tasks and so effect the number of staff needed to accomplish the activity on time.

The issues reviewed above are central to our development of stochastic optimisation models that consider: three simultaneous disasters of different types requiring different types of response; the demand for scarce, shared, local and national, specialist resources; responses needed at different cities that may be geographically spread; prioritising some cities based on their national importance including the receiving of mutual aid; deployment regulations to ensure safe working. In our first model, we handle the risk of not being able to fully respond to simultaneous disasters with existing capabilities using a stochastic optimisation approach. The second optimisation model addresses how to eliminate that risk by considering additional resource capabilities. The third (hybrid) model considers both aspects given a limited financial budget. The details of the models to analyse resource allocations are described next.

3. Mathematical Formulations

3.1. Problem Description

Reserves of national resources are strategically placed around a country to handle potential (simultaneous) disasters at several locations. We focus on this strategic resource allocation problem in the preparedness phased while considering its effects in the response phase when actual simultaneous disaster scenarios happen. Both *equipment* and *personnel* resources are needed in response to disasters and different operations require different types of equipment. For example, search and rescue units are needed for urban search and rescue (USAR) operations while response units are needed for mass decontamination (MD) operations. In addition to equipment, we need *specialists*, the personnel qualified to operate these equipment. We also need *generalists* who support specialists and handle generic operations. Generalists and specialists are firefighters and we assume there are enough firefighters who can be trained as generalists and specialists. Specialist equipment units include vehicles to transport the equipment as well as personnel needed to operate the unit. For example, a high volume pump (HVP) unit for flood pumping operations consists of two vehicles to carry pump and hose equipment and required personnel. Additional specialists and generalists (if required) travel using fire engines and we assume there are enough fire engines to transport these additional personnel when needed.

The requirements for resources are different for different types of disasters. For example, a radioactive bomb attack requires specialist equipment for MD and USAR while a natural flood would need HVP and USAR but may not need mass decontamination. Common resources such as USARs are needed in response to several types of disasters, which makes the resource allocation difficult given that more than one type of disasters will be considered in our models.

Resource allocation decisions made in the preparedness phase affect decisions made in the response phase when an actual disaster happens. In general, the first few hours after a disaster happens are the most important to save life and protect assets, thus our models focus on these first hours as the timespan of the response phase. In addition to specialist equipment and personnel, additional generalists are required to support specialists and handle general tasks. Most requirements of specialist equipment (and personnel) are *non-cumulative*, i.e., they only specify the required number of specialist equipment (or personnel) at a specific time without accounting for how long those specialist equipment (and personnel) are available before that. For example, a requirement of 10 generalists on-site within 3 hours is satisfied if there are 10 generalists available right after the third hour. These generalists can arrive on-site any time within the first 3 hours. For some equipment, the requirement is throughput-related and *cumulative*, i.e., represented by the amount of work done over time. For example, a decontamination target of 3000 causalities within 3 hours is 3 unit-hours if the decontamination capability of the units is 1000 casualties per hour. This requirement is satisfied if there is one response unit available right after the first hour and one more unit is available right after the second hour. Figure 1 shows the differences between non-cumulative and cumulative requirements. Additional specialists are required after the first deployment period to make sure that specialist equipment can be operated without stopping given that *rest breaks between deployment shifts* are required for personnel. For generalists, we assume that initial requirements are sufficient to cover breaks between deployments given the flexibility of generalists in handling different operations.

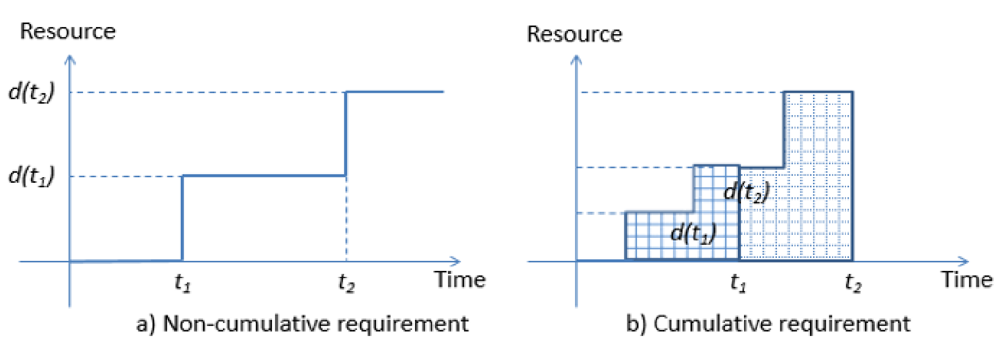

Figure 1: Non-cumulative vs. cumulative requirement

Given these time-dependent resource requirements, requested specialist equipmentand personal need to be transported from multiple locations to the disaster site. These *resource request decisions*, i.e., when, where, which resource, and how much to request, need to be made throughout the response phase. A *simultaneous disaster scenario* describes the details of where these simultaneous disasters happen and their resource requirements. Together with resource allocation decisions, these resource request decisions determine how effective the response is in satisfying resource requirements for simultaneous disaster scenarios. We aim to make resource allocation decisions which allow effective responds, i.e., resource request decisions, to be made later in anticipation of potential simultaneous disaster scenarios. Given our focus is on resource allocation decisions in the preparedness phase, not dynamic resource request decisions in the response phase, we consider this as a *two-stage problem* with resource allocation decisions in the preparedness phase as first-stage decisions and resource request decisions in the response phase are second-stage decisions. The scenario-based representation of uncertainty of simultaneous disasters, i.e., when and where these disasters might happen and what resource will be needed, is relevant for the two-stage models. Given a simultaneous disaster scenario where all time-dependent resource requirements are known, resource request decisions can be determined using a *multi-period setting* in which the timespan of the response phase is discretized appropriately. Figure 2 shows the overall two-stage decision framework with respect to two phases of the disaster life cycle, preparedness and response phase.

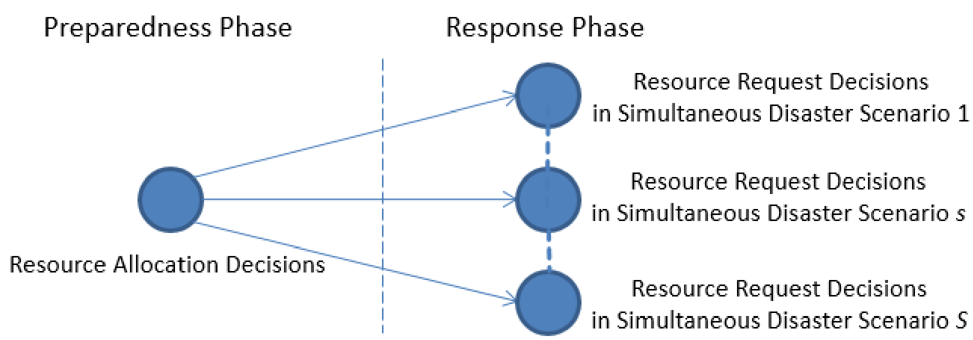

Figure 2: Two-stage decision framework with respect to preparedness and response phase

3.2. Notation and Model Settings

For clarity of exposition, Table 1 summarises all relevant parameters used to develop our mathematical optimisation models. We now describe these parameters.

The set of locations where resources can be placed (and disasters might happen) is denoted by $\mathcal{N}$. We let $\mathcal{E}_i \subseteq \mathcal{N}$ be the list of locations from where equipment and personnel can be requested in response to a disaster happening at location i, $i \in \mathcal{N}$. Resources should be requested first from the locations where a disaster happens, which implies $i \in \mathcal{E}_i$ for all $i \in \mathcal{N}$. To emphasize the importance of utilizing resources from the location where disasters happen, p_{ij}, $j \in \mathcal{E}_i$, denotes the preference of requesting resources from location j for a disaster happening at location i with $p_{ii} = 1$ and $p_{ij} > 1$ if $j \neq i$ for all $i \in \mathcal{N}$. The preference depends on travel times between pairs of locations among other factors. Finally, the response time needed to transport specialist equipment and personnel from location j to location i is denoted by r_{ij} for all $i \in \mathcal{N}$ and $j \in \mathcal{E}_i$.

Table 1: Parameters for the proposed optimisation models

$\mathcal{N}$	set of locations
$\mathcal{E}_i$	lists of locations equipment and personnel can be requested when a disaster happening at location $i \in \mathcal{N}$
p_{ij}	request preference/priority of location j, $j \in \mathcal{E}_i$
r_{ij}	response time needed to bring equipment and personnel from location j to location i
$\mathcal{R}$	set of equipment types
α_r	number of specialists required to operate a unit of type-r equipment, $r \in \mathcal{R}$
β_r	number of generalists required to support a unit of type-r equipment
o_r	on-site setup time of type-r equipment
$\mathcal{K}$	set of disaster types
$\mathcal{R}_k$	set of equipment types need in response to a type-k disaster, $k \in \mathcal{K}$
$\mathcal{T}_k$	set of time points when resource requirements need to be specified in response to a type-k disaster
$c_{k,r}$	indicator whether type-r equipment requirements in response to a type-k disasters are cumulative or not
$\mathcal{S}$	set of scenarios
$\mathcal{L}_s$	set of simultaneous disasters happening in scenario s, $s \in \mathcal{S}$
i_l	location where disaster l happens, $l \in \mathcal{L}_s$
k_l	type of disaster l, $l \in \mathcal{L}_s$
$d_r^l(t)$	requirement for type-r equipment, $r \in \mathcal{R}_{k_l}$, at time t, $t \in \mathcal{T}_{k_l}$, in response to disaster l, $l \in \mathcal{L}_s$
$d_0^l(t)$	requirement for generalist at time t, $t \in \mathcal{T}_{k_l}$, in response to disaster l, $l \in \mathcal{L}_s$
q_s	probability of scenario s
C_r	capacity of type-r equipment
T_s	time horizon of resource requirements in response to all disasters in scenario s
D	duration of deployment shifts (and deployment breaks)
ι_i	importance indicator of location i
γ_r	cost per unit of type-r equipment
B	total budget

There are different types of specialist equipment and $\mathcal{R}$ denotes the set of all equipment types. α_r and β_r denote the number of specialists and supporting generalists needed to operate a unit of type-r equipment, $r \in \mathcal{R}$, respectively. These personnel are transported together with equipment units when requested. Finally, the setup time for type-r equipment before they can be operated is denoted by o_r for all $r \in \mathcal{R}$.

More than one type of disasters is considered. We use $\mathcal{K}$ to denote the set of disaster types and let $\mathcal{R}_k \subseteq \mathcal{R}$ be the set of equipment types needed in response to type-k disasters, $k \in \mathcal{K}$. To model time-dependent resource requirements throughout the timespan of the response phase, we use $\mathcal{T}_k$ to denote the set of time points when requirements for all types of specialist equipment in $\mathcal{R}_k$ and those for generalists need to be specified in response to type-k disasters, $k \in \mathcal{K}$. In addition, $c_{k,r} \in \{0, 1\}$ is used to indicate whether requirements for type-r equipment in response to type-k disasters are cumulative or not for all $k \in \mathcal{K}$ and $r \in \mathcal{R}_k$.

Potential simultaneous disaster scenarios are given in a set $\mathcal{S}$. For a scenario s, $s \in \mathcal{S}$, $\mathcal{L}_s$ denotes the set of simultaneous disasters happening in that scenario. For each disaster l, $l \in \mathcal{L}_s$, happening in scenario s, i_l and k_l denote its location and its disaster type, respectively. In addition, the requirement for type-r specialist equipment, $r \in \mathcal{R}_{k_l}$ at time t, $t \in \mathcal{T}_{k_l}$, in response to the disaster l is denoted by $d_r^l(t)$ for all $l \in \mathcal{L}_s$. Similarly, $d_0^l(t)$ denotes the requirement for generalists at time t, $t \in \mathcal{T}_{k_l}$, in response to the disaster l, $l \in \mathcal{L}_s$. To handle time-dependent resource request decisions, we define T_s such that $[0, T_s]$ is a discretized time horizon that covers all the time points when resource requirements are specified for all simultaneous disasters happening in scenarios s. Note that T_s can be different for different scenario $s \in \mathcal{S}$ depending on types and scales of simultaneous disasters in those scenarios. Finally, for each scenario s, $s \in \mathcal{S}$ is assigned with a probability of $q_s \geq 0$ such that $\sum_{s \in \mathcal{S}} q_s = 1$. Given the set $\mathcal{S}$ of simultaneous disaster scenarios and current total capacities C_r of type-r equipment, $r \in \mathcal{R}$, we are ready to develop mathematical models to determine how to allocate available resources so that whatever disaster scenario happens, it can be handled as effectively as possible.

3.3. Penalty-Based Model

The first optimisation model aims to determine if there are feasible allocations of equipment among all locations that can respond satisfactorily to all scenarios $s \in \mathcal{S}$ given the current capacities of all resources, i.e., resource requirements of all simultaneous disasters happening in each scenario s are satisfied at all times during the timespan of the response phase. We impose a penalty if a resource requirement is not satisfied in any scenario and formulate the problem as a *total expected weighted penalty minimization* problem. It is obvious that if there is no penalty (i.e., zero minimum expected penalty), the current capacities of all resources are enough to handle effectively all given simultaneous disaster scenarios. Before discussing how to formulate the objective of this penalty-based model, we describe all decision variables (see Table 2) and constraints of the model.

Resource allocation decisions in the preparedness phases are the main first-stage decision variables. Let $w_r^i \in \mathbb{Z}_+$ be the number of type-r equipment units allocated at location i, $i \in \mathcal{N}$. Given the capacity C_r of type-r equipment, $r \in \mathcal{R}$, these decision variables need to satisfy the following capacity constraints:

$$\sum_{i \in \mathcal{N}} w_r^i = C_r, \quad \forall\, r \in \mathcal{R}. \tag{1}$$

Given a scenario $s \in \mathcal{S}$, time-dependent resource request decisions in the response phase are the main second-stage decision variables. Let $x_{r,s}^{i,j}(t)$ be the number of type-r equipment units, $r \in \mathcal{R}$, to be requested in response to disasters happening at location i, $i \in \mathcal{N}$, from nearby locations $j \in \mathcal{E}_i$ at the beginning of each time period t, $t = 1, \ldots, T_s$. Together with the equipment requested, $\alpha_r \cdot x_{r,s}^{i,j}(t)$ type-r specialists will be transported from location j to location i so that the equipment can be operated. Similarly, $\beta_r \cdot x_{r,s}^{i,j}(t)$ generalists will also travel together with the equipment requested. In addition to these generalists who help with specialist equipment, we take into account the requests for additional generalists to support the operation. Let $x_{0,s}^{i,j}(t)$ be the number of

Table 2: Decision variables of the penalty-based model (P)

First-stage decision variables:	
w_i^r	number of type-r equipment units, $r \in \mathcal{R}$, allocated at location i, $i \in \mathcal{N}$
$\bar{w}_i^r$	number of specialists who can operate type-r equipment, $r \in \mathcal{R}$, allocated at location i, $i \in \mathcal{N}$
Second-stage decision variables:	
$x_{r,s}^{i,j}(t)$	number of type-r equipment units, $r \in \mathcal{R}$, requested in responses to disasters happening at location i, $i \in \mathcal{N}$, from nearby location j, $j \in \mathcal{E}_i$, at the beginning of time period t, $t = 1, \dots, T_s$, in scenario s, $s \in \mathcal{S}$
$x_{0,s}^{i,j}(t)$	number of additional generalists requested in responses to disasters happening at location i, $i \in \mathcal{N}$, from nearby location j, $j \in \mathcal{E}_i$, at the beginning of time period t, $t = 1, \dots, T_s$, in scenario s, $s \in \mathcal{S}$
$\bar{x}_{r,s}^{i,j}(t)$	number of additional specialists who can operate type-r equipment, $r \in \mathcal{R}$, requested in responses to disasters happening at location i, $i \in \mathcal{N}$, from nearby location j, $j \in \mathcal{E}_i$, at the beginning of time period t, $t = 1, \dots, T_s$, in scenario s, $s \in \mathcal{S}$
$y_{r,s}^i(t)$	number of type-r equipment units, $r \in \mathcal{R}$, in operation at location i, $i \in \mathcal{N}$, at the beginning of time period t, $t = 1, \dots, T_s$, in scenario s, $s \in \mathcal{S}$
$y_{0,s}^i(t)$	number of generalists presenting at location i, $i \in \mathcal{N}$, at the beginning of time period t, $t = 1, \dots, T_s$, in scenario s, $s \in \mathcal{S}$
$\bar{y}_{r,s}^i(t)$	number of specialists who can operate type-r equipment, $r \in \mathcal{R}$, presenting at location i, $i \in \mathcal{N}$, at the beginning of time period t, $t = 1, \dots, T_s$, in scenario s, $s \in \mathcal{S}$
$z_{r,s}^i(t)$	number of type-r equipment units, $r \in \mathcal{R}$, ready to be operated at location i, $i \in \mathcal{N}$, at the beginning of time period t, $t = 1, \dots, T_s$, in scenario s, $s \in \mathcal{S}$
$v_{r,s}^i(t)$	actual requirement of specialists who can operate type-r equipment, $r \in \mathcal{R}$, at location i, $i \in \mathcal{N}$, at the beginning of time period t, $t = 1, \dots, T_s$, in scenario s, $s \in \mathcal{S}$
$u_{r,s}^l(t)$	penalty value if the requirement for type-r equipment, $r \in \mathcal{R}$, in response to disaster l, $l \in \mathcal{L}_s$, is not satisfied at the beginning of time period t, $t = 1, \dots, T_s$, in scenario s, $s \in \mathcal{S}$

additional generalists that need to be requested in response to disasters happening at location i, $i \in \mathcal{N}$, from nearby locations $j \in \mathcal{E}_i$ at the beginning of each time period t, $t = 1, \dots, T_s$. Finally, given the requirement of rest breaks between deployment shifts, we need to request additional specialists to arrive later to ensure the continuity of the operation of specialist equipment once it starts. Let $\bar{x}_{r,s}^{i,j}(t)$ be the number of additional specialists who can operate type-r equipment, or type-r specialists for short, $r \in \mathcal{R}$, that need to be requested in response to disasters happening at location i, $i \in \mathcal{N}$, from nearby locations $j \in \mathcal{E}_i$ at the beginning of each time period t, $t = 1, \dots, T_s$.

In order to check whether resource requirements of all simultaneous disasters happening in scenario s are satisfied, we compute how many specialist equipment units are in operation at each location within the time horizon $[0, T_s]$. Given a scenario $s \in \mathcal{S}$, let $y_{r,s}^i(t)$ be the number of type-r equipment units, $r \in \mathcal{R}$, which are in operation at location i, $i \in \mathcal{N}$, at the beginning of each time period t, $t = 1, \dots, T_s$. Similarly, let $y_{0,s}^i(t)$ be the number of generalists, and $\bar{y}_{r,s}^i(t)$ be the number of type-r specialists, $r \in \mathcal{R}$, which are currently at location i, $i \in \mathcal{N}$, at the beginning of each time period t, $t = 1, \dots, T_s$. Clearly, $y_{0,s}^i(1) = y_{r,s}^i(1) = \bar{y}_{r,s}^i(1) = 0$ for all $i \in \mathcal{N}$ and $r \in \mathcal{R}$. We can compute these numbers for other time periods as follows:

$$
\begin{aligned}
y^i_{r,s}(t+1) &= y^i_{r,s}(t) + \sum_{j\in\mathcal{E}_i:t+1>o_r+r_{ij}} x^{i,j}_{r,s}(t+1-o_r-r_{ij}), && (2)\\
y^i_{0,s}(t+1) &= y^i_{0,s}(t) + \sum_{j\in\mathcal{E}_i:t+1>t_{ij}} \left[x^{i,j}_{0,s}(t+1-t_{ij}) + \sum_{r\in\mathcal{R}} \beta_r \cdot x^{i,j}_{r,s}(t+1-t_{ij})\right], && (3)\\
\bar{y}^i_{r,s}(t+1) &= \bar{y}^i_{r,s}(t) + \sum_{j\in\mathcal{E}_i:t+1>t_{ij}} \left[\bar{x}^{i,j}_{r,s}(t+1-t_{ij}) + \alpha_r \cdot x^{i,j}_{r,s}(t+1-t_{ij})\right], && (4)
\end{aligned}
$$

for all $r \in \mathcal{R}$ and $t = 1, \ldots, T_s - 1$. (2) takes into account the travel time and the setup time for equipment to be ready at the disaster site. (3) and (4) state that additional numbers of generalists and specialists to be requested to fulfill the demand.

To handle the requirement of additional specialists to cover deployment breaks, we compute $z^i_{r,s}(t)$, the number of type-r equipment units, $r \in \mathcal{R}$, at location i, $i \in \mathcal{N}$, which are ready to be operated at the beginning of period t, $t = 1, \ldots, T_s$. We have:

$$z^i_{r,s}(t) = \sum_{j\in\mathcal{E}_i:t>o_r+r_{ij}} x^{i,j}_{r,s}(t-o_r-r_{ij}), \quad \forall\, r \in \mathcal{R},\, t = 1, \ldots, T_s. \tag{5}$$

Supposing specialists can work for D time periods, i.e., the duration of each deployment shift, before taking a break of the same interval, the actual requirement of type-r specialists is as follows:

$$v^i_{r,s}(t) = \sum_{\tau=0}^{\min\{t,D\}-1} z^i_{r,s}(t-\tau) + 2 \cdot \sum_{\tau=\min\{t,D\}}^{t-1} z^i_{r,s}(t-\tau),\ \forall\, r \in \mathcal{R},\, t = 1, \ldots, T_s. \tag{6}$$

This formulation indicates that we need to request more specialists to handle deployment breaks after the first deployment. Similar to the case of generalists, these requirements of type-r specialists are not cumulative.

Finally, to formulate the objective of this penalty-based model, for each scenario $s \in \mathcal{S}$, let $u^l_{r,s}(t) \geq 0$ be the penalty if the requirement for type-r equipment, $r \in \mathcal{R}$, in response to the disaster $l \in \mathcal{L}_s$ in a simultaneous disaster scenario $s \in \mathcal{S}$ is not satisfied at the beginning of the time period t, $t \in \mathcal{T}_{k_l}$. For non-cumulative requirements, the penalty is defined as

$$u^l_{r,s}(t) = \max\left\{d^l_r(t) - y^{i_l}_{r,s}(t), 0\right\}. \tag{7}$$

For cumulative requirements, the penalty is defined as the cumulative shortfall,

$$u^l_{r,s}(t) = \max\left\{d^l_r(t) - \sum_{\tau=1}^{t} y^{i_l}_{r,s}(\tau), 0\right\}. \tag{8}$$

Given that it is more pressing to satisfy earlier requirements in the context of disaster response, we weight the penalties with higher weights for smaller t. In this model, we choose the weights $T-t+1$ for all $t=1,\ldots,T_s$, where $T=\max_{s\in\mathcal{S}} T_s$. The total expected weighted penalty can be computed as

$$\sum_{s\in\mathcal{S}} q_s \sum_{l\in\mathcal{L}_s} \iota_{i_l} \sum_{r\in\mathcal{R}_{k_l}} \sum_{t\in\mathcal{T}_{k_l}} (T-t+1)\cdot u^l_{r,s}(t), \tag{9}$$

where ι_i is an importance indicator of location i. The higher the value of ι_i is, the more important it is to satisfy the resource requirements of disasters happening at location i. This penalty represents the risk of not being able to handle uncertain simultaneous disaster scenarios effectively and minimising it will be the main objective in this model.

Given that requirements for specialists are not the same as requirements of specialist equipment due to the need of covering deployment breaks, we introduce a secondary objective into the model to determine how many specialists should be allocated at different locations. Let $\bar{w}^i_r \in \mathbb{Z}_+$ be the number of type-r specialists, $r\in\mathcal{R}$, that should be allocated at location i, $i\in\mathcal{N}$. These are additional first-stage decision variables of the model and, in practice, the consideration of these would help disaster managers determine and plan specialist training based on the specialist requirement at each location. A secondary objective of minimizing the total number of specialist equipment and specialists, $\sum_{i\in\mathcal{N}}\sum_{r\in\mathcal{R}}\left(w^i_r+\bar{w}^i_r\right)$ allows us to determine the numbers of specialist equipment and specialists needed at each location. Finally, to enforce the request orders and timings for equipment and personnel, we impose another secondary objective of minimizing $\sum_{s\in\mathcal{S}}\sum_{t=1}^{T_s}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{E}_i} p_{ij}(T-t+1)\cdot\left(x^{i,j}_{0,s}(t)+\sum_{r\in\mathcal{R}} x^{i,j}_{r,s}(t)\right)$. These two secondary objectives will be scaled appropriately to reflect their secondary nature. We can now formulate our first model (P) (P for penalty) as a two-stage mixed-integer linear stochastic optimisation model as follows.

$$\begin{aligned}
(P):\min\quad & \sum_{s\in\mathcal{S}} q_s \sum_{l\in\mathcal{L}_s} \iota_{i_l} \sum_{r\in\mathcal{R}_{k_l}} \sum_{t\in\mathcal{T}_{k_l}} (T-t+1)\cdot u^l_{r,s}(t) + \theta\cdot\left[\sum_{i=1}^{N}\sum_{r\in\mathcal{R}}\left(w^i_r+\bar{w}^i_r\right)\right]+\cdots\\
& \mu\cdot\left[\sum_{s\in\mathcal{S}}\sum_{t=1}^{T_s}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{E}_i} p_{ij}(T-t+1)\cdot\left(x^{i,j}_{0,s}(t)+\sum_{r\in\mathcal{R}} x^{i,j}_{r,s}(t)\right)\right]
\end{aligned}$$

s.t.

$$y^i_{0,s}(1)=y^i_{r,s}(1)=\bar{y}^i_{r,s}(1)=0,\quad \forall\, i\in\mathcal{N},\, r\in\mathcal{R},\, s\in\mathcal{S}, \tag{10}$$

$$y^i_{r,s}(t+1)=y^i_{r,s}(t)+\sum_{j\in\mathcal{E}_i:t+1>o_r+r_{ij}} x^{i,j}_{r,s}(t+1-o_r-r_{ij}), \tag{11}$$

$$y^i_{0,s}(t+1)=y^i_{0,s}(t)+\sum_{j\in\mathcal{E}_i:t+1>t_{ij}}\left(x^{i,j}_{0,s}(t+1-t_{ij})+\sum_{r\in\mathcal{R}}\beta_r\cdot x^{i,j}_{r,s}(t+1-t_{ij})\right) \tag{12}$$

$$\bar{y}^i_{r,s}(t+1)=\bar{y}^i_{r,s}(t)+\sum_{j\in\mathcal{E}_i:t+1>t_{ij}}\left(\bar{x}^{i,j}_{r,s}(t+1-t_{ij})+\alpha_r\cdot x^{i,j}_{r,s}(t+1-t_{ij})\right), \tag{13}$$

$$\forall\, i\in\mathcal{N},\, r\in\mathcal{R},\, s\in\mathcal{S},\, t=1,\ldots,T_s-1,$$

$$z^i_{r,s}(t) = \sum_{j \in \mathcal{E}_i : t > o_r + r_{ij}} x^{i,j}_{r,s}(t - o_r - r_{ij}), \tag{14}$$

$$v^i_{r,s}(t) = \sum_{\tau=0}^{\min\{t,D\}-1} z^i_{r,s}(t-\tau) + 2 \cdot \sum_{\tau=\min\{t,D\}}^{t-1} z^i_{r,s}(t-\tau), \tag{15}$$

$$\forall\, i \in \mathcal{N},\, r \in \mathcal{R},\, s \in \mathcal{S},\, t = 1, \dots, T_s,$$

$$y^{i_l}_{0,s}(t) \geq d^l_0(t), \quad \forall\, l \in \mathcal{L}_s,\, t \in \mathcal{T}_{k_l},\, s \in \mathcal{S}, \tag{16}$$

$$\bar{y}^{i_l}_{r,s}(t) \geq v^{i_l}_{r,s}(t), \quad \forall\, l \in \mathcal{L}_s,\, r \in \mathcal{R}_{k_l},\, t = 1, \dots, T_s,\, s \in \mathcal{S}, \tag{17}$$

$$u^l_{r,s}(t) \geq d^l_r(t) - y^{i_l}_{r,s}(t), \quad \forall\, l \in \mathcal{L}_s,\, r \in \mathcal{R}_{k_l} : c_{l,r} = 0,\, t \in \mathcal{T}_{k_l},\, s \in \mathcal{S}, \tag{18}$$

$$u^l_{r,s}(t) \geq \left(d^l_r(t) - \sum_{\tau=1}^{t} y^{i_l}_{r,s}(\tau) \right), \quad \forall\, l \in \mathcal{L}_s,\, r \in \mathcal{R}_{k_l} : c_{l,r} = 1,\, t \in \mathcal{T}_{k_l},\, s \in \mathcal{S}, \tag{19}$$

$$u^l_{r,s}(t) \geq 0, \quad l \in \mathcal{L}_s,\, t \in \mathcal{T}_{k_l},\, r \in \mathcal{R}_{k_l},\, s \in \mathcal{S}, \tag{20}$$

$$\sum_{i=1}^{N} w^i_r \leq C_r, \quad \forall\, r \in \mathcal{R}, \tag{21}$$

$$\sum_{t=1}^{T_s} \sum_{j : i \in \mathcal{E}_j} x^{j,i}_{r,s}(t) \leq w^i_r, \quad \forall\, i \in \mathcal{N},\, r \in \mathcal{R},\, s \in \mathcal{S}, \tag{22}$$

$$\sum_{t=1}^{T_s} \sum_{j : i \in \mathcal{E}_j} \left(\bar{x}^{j,i}_{r,s}(t) + \alpha_r \cdot x^{j,i}_{r,s}(t) \right) \leq \bar{w}^i_r, \quad \forall\, i \in \mathcal{N},\, r \in \mathcal{R},\, s \in \mathcal{S}, \tag{23}$$

$$x^{i,j}_{0,s}(t), x^{i,j}_{r,s}(t), \bar{x}^{i,j}_{r,s}(t) \in \mathbb{Z}_+, \quad \forall\, i \in \mathcal{N},\, j \in \mathcal{E}_i,\, r \in \mathcal{R},\, s \in \mathcal{S},\, t = 1, \dots, T_s, \tag{24}$$

$$w^i_r, \bar{w}^i_r \in \mathbb{Z}_+, \quad \forall\, i \in \mathcal{N},\, r \in \mathcal{R}, \tag{25}$$

where $0 < \theta, \mu < 1$ are small enough scaling parameters. The first four constraints, (10), (11), (12), and (13), provide calculations of the state variables $y^i_{0,s}(t)$, $y^i_{r,s}(t)$, and $\bar{y}^i_{r,s}(t)$ for $i \in \mathcal{N}$, $r \in \mathcal{R}$, $s \in \mathcal{S}$, and $t = 1, \dots, T_s$. The next two constraints, (14) and (15), handle the calculations of specialist requirements given the need of covering deployment breaks. The next two constraints, (16) and (17), make sure personnel demands are satisfied. The next three constraints, (18), (19), and (20), handle the penalties of unsatisfied equipment requirements defined in (7) and (8). The next three constraints, (21), (22), and (23), make sure that capacity limits are taken into account properly. Finally, the last two constraints, (24) and (25), indicates that personnel and equipment requests as well as resource allocations need to be non-negative integers.

The proposed model allows us to make strategic first-stage decisions (resource allocation decisions in the preparedness phase) as well operational second-stage decisions (resource request decisions in the response phase) in response to potential simultaneous disaster scenarios as effectively as possible given the current resource capabilities. It is natural to follow up with the question of whether current resource configurations could be changed to provide better responses to potential simultaneous disaster scenarios. In the next section, we propose another model to address this issue.

3.4. Resource-Based Model

The model proposed in the previous section deals with the current resource capabilities. It is important to check whether the current resource capabilities are enough to satisfactorily respond to all potential simultaneous disaster scenarios provided. The main objective of this second model is to determine the minimum resource capabilities which can be used to handle all simultaneous disaster scenarios without any penalty, i.e., all resource requirements are satisfied throughout the timespan of the response phase. In order to model this objective, let $\Delta C_r \in \mathbb{Z}_+$ be the required number of additional type-r equipment units, $r \in \mathcal{R}$, to guarantee that all resource requirements are satisfied in each simultaneous disaster scenario. These are first-stage decision variables, i.e., how much additional resource capabilities are needed, are determined in the preparedness phase.

Similar to the previous model, we again need to determine how to allocate resources at each location and how to request specialist equipment and personnel when a simultaneous disaster scenario happens. However, instead of aiming to minimize the risk of not satisfying requirements in response to disasters in each scenario, we impose the condition that resource requirements need to be *always satisfied no matter which simultaneous disaster scenario happens* given the new resource capabilities. This condition removes the risk of not being able to respond satisfactorily in potential simultaneous disaster scenarios. More concretely, the three penalty-related constraints (18), (19), and (20) are now replaced by the following two new constraints, which make sure resources requirements are satisfied no matter which simultaneous disaster scenario happens:

$$y^{i_l}_{r,s}(t) \geq d^l_r(t), \quad \forall\, l \in \mathcal{L}_s,\, r \in \mathcal{R}_{k_l} : c_{l,r} = 0,\, t \in \mathcal{T}_{k_l},\, s \in \mathcal{S}, \tag{26}$$

and

$$\sum_{\tau=1}^{t} y^{i_l}_{r,s}(\tau) \geq d^l_r(t), \quad \forall\, l \in \mathcal{L}_s,\, r \in \mathcal{R}_{k_l} : c_{l,r} = 1,\, t \in \mathcal{T}_{k_l},\, s \in \mathcal{S}. \tag{27}$$

The main objective of this model is to minimize the total cost of changes made to the current resource capabilities, which is defined as $\sum_{r\in\mathcal{R}} \gamma_r \cdot \Delta C_r$, where γ_r is cost per unit of type-r equipment, $r \in \mathcal{R}$. Similar to the previous model, we keep the two secondary objectives with their appropriate scaling parameters. The second model (R) (R for resource) can be formulated as follows:

$$\begin{aligned}
(R) : \min \quad & \sum_{r\in\mathcal{R}} \gamma_r \cdot \Delta C_r + \theta \cdot \left[\sum_{i\in\mathcal{N}} \sum_{r\in\mathcal{R}} \left(w^i_r + \bar{w}^i_r\right)\right] + \cdots \\
& \mu \cdot \left[\sum_{s\in\mathcal{S}} \sum_{t=1}^{T_s} \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{E}_i} p_{ij}(T-t+1) \cdot \left(x^{i,j}_{0,s}(t) + \sum_{r\in\mathcal{R}} x^{i,j}_{r,s}(t)\right)\right] \\
\text{s.t.} \quad & y^i_{0,s}(1) = y^i_{r,s}(1) = \bar{y}^i_{r,s}(1) = 0, \quad \forall\, i \in \mathcal{N},\, r \in \mathcal{R},\, s \in \mathcal{S}, && (28) \\
& y^i_{r,s}(t+1) = y^i_{r,s}(t) + \sum_{j\in\mathcal{E}_i : t+1 > o_r + r_{ij}} x^{i,j}_{r,s}(t+1-o_r-r_{ij}), && (29)
\end{aligned}$$

$$y_{0,s}^i(t+1) = y_{0,s}^i(t) + \sum_{j\in\mathcal{E}_i:t+1>t_{ij}} \left(x_{0,s}^{i,j}(t+1-t_{ij}) + \sum_{r\in\mathcal{R}} \beta_r \cdot x_{r,s}^{i,j}(t+1-t_{ij}) \right) \tag{30}$$

$$\bar{y}_{r,s}^i(t+1) = \bar{y}_{r,s}^i(t) + \sum_{j\in\mathcal{E}_i:t+1>t_{ij}} \left(\bar{x}_{r,s}^{i,j}(t+1-t_{ij}) + \alpha_r \cdot x_{r,s}^{i,j}(t+1-t_{ij}) \right), \tag{31}$$

$$\forall\, i \in \mathcal{N},\, r \in \mathcal{R},\, s \in \mathcal{S},\, t = 1, \ldots, T_s - 1,$$

$$z_{r,s}^i(t) = \sum_{j\in\mathcal{E}_i:t>o_r+r_{ij}} x_{r,s}^{i,j}(t - o_r - r_{ij}), \tag{32}$$

$$v_{r,s}^i(t) = \sum_{\tau=0}^{\min\{t,D\}-1} z_{r,s}^i(t-\tau) + 2 \cdot \sum_{\tau=\min\{t,D\}}^{t-1} z_{r,s}^i(t-\tau), \tag{33}$$

$$\forall\, i \in \mathcal{N},\, r \in \mathcal{R},\, s \in \mathcal{S},\, t = 1, \ldots, T_s,$$

$$y_{0,s}^{i_l}(t) \geq d_0^l(t), \quad \forall\, l \in \mathcal{L}_s,\, t \in \mathcal{T}_{k_l},\, s \in \mathcal{S}, \tag{34}$$

$$\bar{y}_{r,s}^{i_l}(t) \geq v_{r,s}^{i_l}(t), \quad \forall\, l \in \mathcal{L}_s,\, r \in \mathcal{R}_{k_l},\, t = 1, \ldots, T_s,\, s \in \mathcal{S}, \tag{35}$$

$$y_{r,s}^{i_l}(t) \geq d_r^l(t), \quad \forall\, l \in \mathcal{L}_s,\, r \in \mathcal{R}_{k_l} : c_{l,r} = 0,\, t \in \mathcal{T}_{k_l},\, s \in \mathcal{S}, \tag{36}$$

$$\sum_{\tau=1}^{t} y_{r,s}^{i_l}(\tau) \geq d_r^l(t), \quad \forall\, l \in \mathcal{L}_s,\, r \in \mathcal{R}_{k_l} : c_{l,r} = 1,\, t \in \mathcal{T}_{k_l},\, s \in \mathcal{S}, \tag{37}$$

$$\sum_{i\in\mathcal{N}} w_r^i \leq C_r + \Delta C_r, \quad \forall\, r \in \mathcal{R}, \tag{38}$$

$$\sum_{t=1}^{T_s} \sum_{j:i\in\mathcal{E}_j} x_{r,s}^{j,i}(t) \leq w_r^i, \quad \forall\, i \in \mathcal{N},\, r \in \mathcal{R},\, s \in \mathcal{S}, \tag{39}$$

$$\sum_{t=1}^{T_s} \sum_{j:i\in\mathcal{E}_j} \left(\bar{x}_{r,s}^{j,i}(t) + \alpha_r \cdot x_{r,s}^{j,i}(t) \right) \leq \bar{w}_r^i, \quad \forall\, i \in \mathcal{N},\, r \in \mathcal{R},\, s \in \mathcal{S}, \tag{40}$$

$$x_{0,s}^{i,j}(t), x_{r,s}^{i,j}(t), \bar{x}_{r,s}^{i,j}(t) \in \mathbb{Z}_+, \quad \forall\; i \in \mathcal{N},\, j \in \mathcal{E}_i,\, r \in \mathcal{R},\, s \in \mathcal{S},\, t = 1, \ldots, T_s, \tag{41}$$

$$w_r^i, \bar{w}_r^i \in \mathbb{Z}_+, \quad \forall\, i \in \mathcal{N},\, r \in \mathcal{R},, \tag{42}$$

$$\Delta C_r \in \mathbb{Z}_+, \forall\, r \in \mathcal{R}. \tag{43}$$

In this model, two constraints (36) and (37) make sure that all equipment requirements are satisfied. The constraint (21) in the previous model is replaced by (38). Note that this model does not need probabilistic information of potential simultaneous disaster scenarios, which can be an advantage given that, in reality, it is usually difficult to determine the probability of simultaneous disaster scenarios occurring. To conclude this section, we provide a comparison between the two proposed models in Table 3. Note that an additional hybrid model, which we call *budget-based model* (see Appendix D), is proposed later to incorporate decisions on buying and allocating additional resources given a limited budget while minimizing the risk of not satisfying all resource requirements in potential simultaneous disaster scenarios.

Table 3: Comparison between the penalty-based and resource-based model

	Penalty-base model (P)	Resource-based model (R)
Main objective	To minimize the risk of not satisfying all resource requirements	To minimize the cost of additional resource capabilities
Distinct decision variables	Penalty decision variables $u^l_{r,s}(t)$	Additional resource capability decision variables ΔC_r
Resource requirement constraints	Penalty-related constraints: (18), (19), and (20)	Resource requirements always satisfied: (36) and (37)
Capacity constraints	$\sum_{i \in \mathcal{N}} w^i_r \leq C_r, \forall\, r \in \mathcal{R}$	$\sum_{i \in \mathcal{N}} w^i_r \leq C_r + \Delta C_r, \forall\, r \in \mathcal{R}$

4. Case Study

4.1. Case Study Development

To build the models we first wrote a case study of UK planning for a disaster. The case study was developed using Critical Incident Case Study method [60] which is used to explore the relationships between variables in a significant critical situation, so is particularly useful for understanding disaster response [61]. Our critical incident was the deployment of UK New Dimension resources [53, 3]. We collected data to understand the deployment context and factors to be modelled.

There were three aspects to developing the case study. First, a document review [11] identified publicly available details about the New Dimension Programme such as: preparing for three simultaneous incidents [58] of different types [53], small number of core regions/cities [3], throughputs [13], protocols for requesting additional resources [3], financial details [36] and regular travel times. Documents were mostly from the UK Government plus limited academic literature. While useful in giving official government information, the documents did not contain sufficient detail on factors to model - hence we sought detailed information from collaboration with experts.

Second, we collaborated with a recently retired strategic commander who had 40 years of experience working in and commanding UK emergency response operations [1]. From this expert we enhanced our understanding of the factors from the document review and identified new factors to better align the models with real world practices. Initially we conducted two unstructured interviews with the expert which followed the exploratory method of critical incident technique [60, p2] which is "used for the study of factors, variables, or behaviours that are critical to the success or failure of an activity". Thus, in these interviews the expert talked about critical incidents and past experiences of managing New Dimension incidents response from which we extracted the structure of the emergency response context, practical insights, variables and relationships between variables. For example, we found additional details about appropriate scenarios (flooding and terrorist at-

[1]Shortly after the 9/11 attacks, the expert was appointed as the UK Chief Fire Officers Association's national lead officer for command and control aspects of the New Dimension Programme and, in an associated role, designed the national course to train the UK's strategic commanders to prepare them for command of disaster response operations, including New Dimension, so was a nationally recognised authority.

tack), transportation of equipment, equipment set up times, manpower and equipment availability, staff requirements for safe working, and shift breaks. After the first interview we wrote a detailed 7 page case study of New Dimension response to an incident to create a case study scenario (on which Section 4.2 is based), with timings and resources requests (Figure 3), capacities and requirements (Table 4) and targets (Tables B.13 and B.14). The case was enhanced with details from the second interview. The expert provided feedback on each version of the written case study and provided feedback to correct information and add nuances to enhance its realism. Our initial models were built based on this case study but many remaining questions led to three follow-up interviews with the expert. As the expert was not a modeller, we did not present our actual models during follow-up interviews but we did present (in a written and narrative, non-technical manner) their variables, assumptions and logic. To consider how they were treated in the model, each interview took the expert through new developments of the model to gain feedback on how to more closely align it with operational realities - after which the written case was updated and checked by the expert. Also, by understanding additional nuances we identified variables appropriate for sensitivity analysis. All interviews were held across five half-day sessions.

While the bases for our case were New Dimension practices, on advice of the expert to avoid political and security considerations we changed or generated some details which had no significance on the process of analysis. For example, we changed country information (layout, population information and priority of cities (Fig 3)) and we generated details to respect political sensitivities (such as location importance - ι_i in Tables 6a and 6b), and security considerations (such as risk likelihoods - q_s).

Lastly, once mature we gave the written case to two other UK commanders who exercised to receive New Dimension resources for real emergencies. We interviewed these commanders with a view to using their knowledge of our critical incident to check its variables, assumptions and logic to ensure it was as realistic as possible without compromising sensitivities. Interviews were structured by the written case as we traced through its contents to gain feedback. The interviews increased our confidence in the case as no major changes were identified. The product was a case which experts gave us confidence accurately reflected the New Dimension Programme realities and which is provided in Sections 4.2 and 4.3. We now explain the case.

4.2. Case Study Description

We situate our study in a country which has 54 million inhabitants, 27 million of whom live in its eight major cities (A-H). Each city has a large population with a similar demography. These cities are considered as network locations where we need to prepare resource allocation for responding to simultaneous disasters. The estimated travel times between cities are displayed in Figure 3. In addition, given that the specialist equipment is seldom used and so not stored in the city centre, we need to consider the time to move equipment to the scene from its storage location within each city (displayed in bracket next to each city).

When disasters happen, cities can request the help of equipment/personnel from other cities to augment their own resources. When a city requests such help, there is an established protocol for

Figure 3: City map, estimated travel times, and resource request orders

the order in which they ask for help. If another city is not able to supply assistance when asked (or not able to supply sufficient assistance to meet the response targets), the next city on the list is asked to provide assistance i.e. A will ask B first, C second, and so on until it has sufficient resource. The list is displayed next to each city in Figure 3.

The national government in this country has a policy that each of the eight cities must plan sufficient emergency resources to address two types of disaster: coordinated terrorist attacks and major flooding. Declaring a disaster means that city officials can ask neighbouring cities for assistance in the form of emergency response personnel and equipment to help tackle the disaster. Furthermore, the national government has set performance targets that emergency responders should plan to meet. However, even during a disaster, specialists and generalists need 5 hours break between deployments to allow for recuperation and ensure safe working.

The first disaster for which cities must plan is a series of coordinated terrorist attacks involving radioactive bombs (so-called 'dirty bombs') which contaminate an area. Two main tasks needed are mass decontamination of affected people and urban search and rescue to find victims. To conduct mass decontamination (MD), responders require a vehicle called an IRU (Incident Response Unit) to transport the equipment. Each IRU can transport two MD1 structures (to decontaminate members of public) and one MD4 structure (to decontaminate firefighters and their equipment). Throughput of each MD1 structure is 150 people processed each hour of operation and there are 72 IRUs within the country. Urban search and rescue (USAR) operations are conducted using USAR units. There are 20 USAR units in the country.

The second disaster is a major flood (either coastal or from nearby reservoirs). The locations of cities that are more vulnerable to a flood have been identified. The potential for the collapse of city structures from flood waters bring the need for two main operations: flood water pumping and USAR. USAR operations use USAR units as in a terrorist attack disaster. Flood water pumping operation needs High Volume Pump (HVP) units, each of which consists of one pump vehicle unit and one hose unit vehicle. There are 46 HVP units available in the country. Requirements for all three specialist equipment are in Table 4. Note that the demand requirement for IRU is cumulative while those for other specialist equipment are non-cumulative.

Table 4: Capacity and requirements for specialist equipment

Unit type	IRU	USAR	HVP
Number of units available	72	20	46
Specialists per unit	10	10	2
Generalists per unit	20	25	5
Setup time (mins)	60	30	60

Research has been conducted to estimate the maximum target response levels (e.g. amount of equipment/personnel) needed to deliver an appropriate weight of response to a disaster taking into account the population size of each city. The detailed target response levels for terrorist attack disaster are in Table B.13 while those for flooding disaster are in Table B.14 in Appendix B.

The objective is to allocate these equipment around the country so that response targets can be met when simultaneous disasters happen. The national policy requires response to up to three simultaneous disasters. Therefore, we generate all potential scenarios in which three simultaneous disasters of a same type happen in three different cities, either terrorist bomb attacks or major flooding. For example, a scenario is three simultaneous disasters happening in any three cities A-H, e.g., A, B and H. Given our focus is on resource allocation, we will not consider scenarios with less than three simultaneous disasters. Also, we only need to consider maximum target response levels in the scenarios. If scenarios with maximum target response levels can be handled efficiently, other scenarios with lower target response levels can be handled appropriately too. Given eight cities and two disaster types, a total 112 scenarios of three simultaneous disasters can be constructed. Given that there are limited data that can be used to estimate the actual probabilities of these scenarios, we use equal probabilities for all scenarios in the first model (P) as suggested by the experts. Arguably, equal scenario probabilities in this model indicate that inadequate response levels (due to lack of resources) in any disaster scenarios are equally important. In the next section, we show how the case study is solved and report the results.

4.3. Computational Results

Both proposed models, (P) and (R), are mixed-integer optimisation problems with a scenario-based structure. In this case study, the demand requirements indicate that the time horizon we need to consider for these disaster scenarios is 10 hours. Given other time-related inputs, it is reasonable to set the time unit to be 20 minutes, i.e., 30 time periods will be considered over the time horizon. We heuristically set $\theta = 10^{-3}$ and $\mu = 10^{-5}$ for the secondary objectives and the detailed heuristics is described in Appendix C.1. The models are solved using IBM CPLEX 12.7 coded in C++ on a Linux computer with 3.00 gigahertz CPU and 32 gigabyte RAM. Before reporting the results obtained from the models, we generate random instances of the models with different numbers of cities to test the computational aspect. The random instances are generated with random locations chosen uniformly within a square box of 100 by 100. The travel times are computed based on their Euclidean distances. The request preferences are determined based on distances between locations. Simultaneous disaster scenarios are constructed as in the case study with three disasters of a same type (either flooding or terrorist attack) happening at three different cities. The setup times are also randomly generated but the maximum setup time is set to be 3 time periods. The periods when resource requirements are provided are generated randomly within the planning horizon with the total number of these periods is kept the same as in the case study. The first of these periods is generated such that it is always possible to fulfil the requests if there are enough resources by taking into consideration the setup times. The amounts of resources requested are randomly generated using a uniform distribution with the maximum variation of 20% from the amount requested of the same resource in the case study. Finally, we keep the deployment duration the same as in the case study. Using this data generation procedure, we generate 10 instances for each value of $|\mathcal{N}|$, the number of locations, from 5 to 12. Figure C.13 in Appendix C.2 shows the city locations in a 9-city random instance while Figure C.14 shows different time-dependent IRU resource requirements in 10 different random instances.

We solve generated random instances using IBM CPLEX with MIP relative gap of 0.01% and time limit of 3600 seconds. Figure 4 shows sizes of these instances in terms of number of decision variables and number of constraints for the model (P). The average computational times as well as the worst-case and best-case computational times among 10 instances are plotted in Figure 5 given different numbers of locations. We focus mainly on results of the model (P) here. For the model (R), similar results of computational times are obtained - reported in Figure C.15 in Appendix C.3.

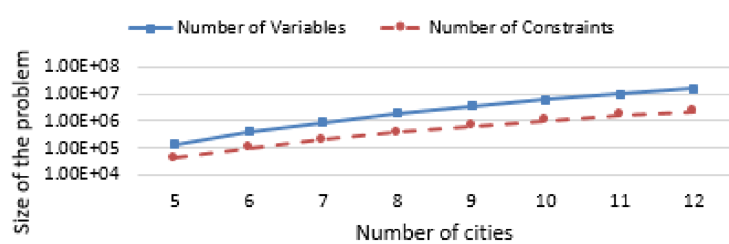

Figure 4: Problem sizes of model (P) given different numbers of locations

The results show that the size of the problem increases almost exponentially in terms of the number of locations. For $|\mathcal{N}| = 11$, there are almost 10^7 decision variables and 1.5×10^6 constraints. Computational times also increase exponentially in terms of the number of locations. Among 10 randomly generated instances for $|\mathcal{N}| = 11$, there is one instance that cannot be solved within the time limit (returning zero first-stage solutions which are feasible). The remaining instances can be solved with the average computational time of approximately 2800 seconds. For $|\mathcal{N}| = 12$, all generated instances cannot be solved within the time limit. Given their large sizes, approximately

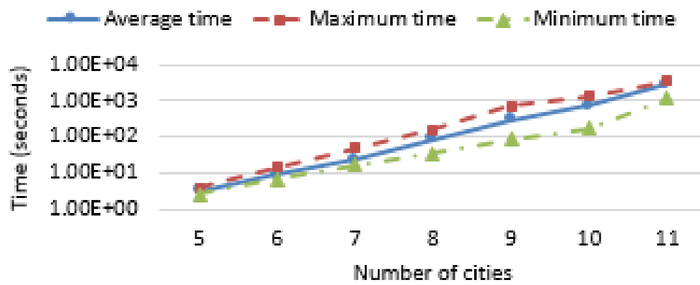

Figure 5: Computational times of model (P) given different numbers of locations

1.5×10^7 decision variables and 2×10^6 constraints, the physical memory required to handle these instances exceeds 20 gigabytes. We solve one of the instances again without time limit and an optimal solution is found (with the given MIP optimality gap of 0.01%) after approximately 9000 seconds during which the physical memory required increases to almost 30 gigabytes. There are few heuristics to solve large-scale mixed-integer stochastic optimisation problems such as integer L-shaped algorithms [28] which rely on optimality cuts or progressive hedging algorithms (PHA) (see, e.g.,[59, 20]) which use scenario decomposition. For the two proposed stochastic optimisation models, scenario decomposition is more effective than the introduction of optimality cuts to provide approximation of the second-stage objective. Given that the main case study, which is a focus of the paper, can be solved by IBM CPLEX, we only report some results of the PHA heuristic in Appendix C.4. Results obtained for the case study are reported next.

4.3.1. Penalty-Based Model and Location Importance

The first model (P) provides us with resource allocation given the existing capabilities of the country. For these experiments, we set the importance of all locations to be the same, i.e., $\iota_i = 1$ for all $i \in \mathcal{N}$. The results show in every scenario considered with three simultaneous disasters, there are not enough USAR units to meet the demand for USAR operations. Given 20 USAR units are available in the country, the model recommends to place 10 units in A and 10 units in C. This is reasonable given that the resource requirements in A are the highest while the location of C acts as a transit hub, which allows us to move equipment and personnel around efficiently. For example, when three simultaneous terrorist attacks happen in A, B, and D, the optimal solution is to use 10 USAR units located in A for the attack in A while moving 10 USAR units located in C to D. Both A and C need 5 more USAR units to fulfill the requirements while B received no USAR units at all. A similar situation arises for scenarios with three simultaneous flooding disasters. For example, when there are floods happening in F, G, and H at the same time, certain USAR units are moved from A and C to H and F to partially satisfy the demand requirements while G received no USAR units. Figure 6 shows two cases of unsatisfied demands happening in these examples.

In contrast to USAR resource, the results show that there are enough IRUs and HVP units. The allocation of all resources are displayed in Table 5a. It also shows that we only need to use 48 (out of 72) IRUs and 35 (out 46) HVP units without causing any issue in handling the given scenarios of three simultaneous disasters, either terrorist attacks or major flooding. For HVP, the

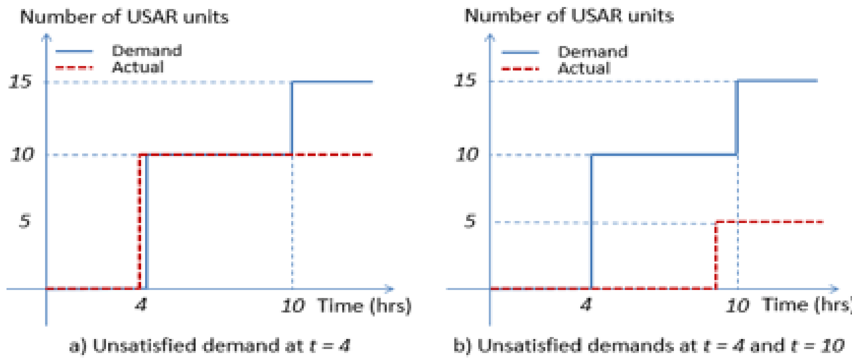

Figure 6: Unsatisfied demands due to insufficient number of USAR units

results indicate that D and G are also important given that they can act as transit hubs, from where equipment and resources can be delivered to disaster sites efficiently when requested.

Given that we need extra specialists to handle deployment breaks, the number of specialists based in each city can be higher than the number required to handle the specialist equipment. For example, the results show 30 HVP specialists are required in A (instead of 20 to handle 10 HVP units). These extra specialists will be transfered to disaster locations to replace the first group of specialists during their deployment breaks if needed when disasters happen. Similarly, even though there is no USAR units located in B and D, the model recommends to keep some specialists at these two locations so that deployment breaks can be handle efficiently.

Table 5: Resource allocation results from different models

(a) Stochastic optimisation model with all scenarios

	A	B	C	D	E	F	G	H
IRU	6	6	6	6	6	6	6	6
USAR	10	0	10	0	0	0	0	0
HVP	10	3	3	5	3	3	5	3

(b) Single scenario of three bomb attacks at A, B, and D

	A	B	C	D	E	F	G	H
IRU	6	6	0	6	0	0	0	0
USAR	10	10	0	0	0	0	0	0
HVP	0	0	0	0	0	0	0	0

To evaluate the stochastic optimisation model, we compute two measures, the Expected Value of Perfect Information (EVPI) and the Value of Stochastic Solution (VSS). The EVPI considers the penalties obtained from wait-and-see solutions for individual scenarios, i.e. with perfect information, and compares the expected value (WS) of these penalties with the expected penalty (RP) obtained from the stochastic optimisation model (P). On the other hand, the VSS compares the performance of solutions obtained from the deterministic model when data are assumed to be deterministic with that of the solution obtained from the stochastic optimisation model in terms of expected penalty. For this case study, the expected value of the penalties obtained with wait-and-see solutions is WS = 215.01 while the minimal expected penalty of the stochastic optimisation model is RP = 289.66. The EVPI is computed as EVPI = RP - WS = 74.65. It shows that the expected penalty is decreased by 25.77% with perfect information. In order to compute VSS, we need to select data for the deterministic model as a reference. Given that there is no "average" scenario to represent all scenarios, we use individual disaster scenarios as reference data for the deterministic model. We compare the expected penalties (EEVs) obtained from these 112 deterministic solutions with the expected penalty obtained from the stochastic solution and the VSS measure is computed as VSS = EEV - RP. The results show that the average expected penalty from deterministic solutions is 2028.66 (7 times increase), the minimum is 1489.30 (5 times increase) while the maximum is 2545.10 (8 times increase). Under this setting of reference data for the deterministic model, wait-and-see solutions for individual scenarios are solutions of deterministic models. These solutions focused only on three locations specified in each scenario. For example, when there are three simultaneous bomb attacks at A, B, and D, the optimal solution is to allocate 6 IRUs at each of these locations, 10 USARs each at A and B, and there is no need to allocate any HVPs (see Table 5b).

Clearly, if this scenario happens, the wait-and-see solution mentioned will results in the minimum penalty, which shows the value of perfect information. However, if a different scenario happens, no matter whether it is another bomb attack scenario at different locations or a flooding scenario, the provided solution will not be able to cope. Indded, it performs much worse with the expected penalty of 1646.30 than the stochastic solution (RP=289.66) given the stochastic solution considers all scenarios in which disasters can happen at any three locations and it reflects how resources are allocated as shown in Table 5a. The analysis of VSS therefore indicates the importance of the stochastic solution compared to deterministic solutions in this case study.

The initial setting of location importance assumes that the risk of not satisfying resource requirements at any location is equivalently significant. In reality, these parameters can be set by disaster managers based on different criteria. We present here two potential approaches to set the importance ι_i of all locations. The first approach is based on the level of resource requirements at each location. Given the resource requirements in Tables B.13 and B.14, we run model (P) with two additional settings, I1 and I2, for location importance, which splits the locations into three groups based on population size, (A), (D,G), and (B,C,E,F,H). These settings are shown in Table 6a.

Table 6: Location importance settings based on different criteria

(a) Population size

	A	D	G	B	C	E	F	H
I1	1	0.5	0.5	0.2	0.2	0.2	0.2	0.2
I2	1	0.5	0.5	0.1	0.1	0.1	0.1	0.1

(b) Geopolitics

	A	B	C	D	H	E	F	G
I3	0.2	0.5	0.5	0.5	0.5	1	1	1
I4	0.1	0.5	0.5	0.5	0.5	1	1	1

With these two settings of location importance, the results show that the allocation of equipment remains the same as the original setting. The changes appear in the allocation of specialists, especially the specialists for USAR units. Figure 7a shows these changes in detail with I0 denoting the original setting of location importance. The results show that when we reduce the importance of other locations as compared to that of A, the allocation of USAR specialists concentrates more and more on A (due to its importance and size). On the other hand, the allocation of USAR specialists in C reflects the significance of its transit hub nature.

The second approach of how to set the importance parameters is based on a hypothetical political assumption under which the three northern locations E, F, and G are the most important

	A	B	C	D	E	F	G	H
I0	135	35	170	30	0	0	0	35
I1	130	100	170	0	0	0	0	0
I2	300	0	100	0	0	0	0	0

(a) Population size

	A	B	C	D	E	F	G	H
I0	135	35	170	30	0	0	0	35
I3	60	60	40	100	0	0	120	20
I4	0	0	20	100	0	0	280	0

(b) Geopolitics

Figure 7: USAR specialist allocation given different settings of location importance

whereas the southern location A is the least important. The two additional settings of importance parameters, I3 and I4, are shown in Table 6b.

With these two new settings of location importance, the allocation of USAR units is completely changed. One now should allocate 10 USAR units in D and the remaining 10 units in G, which shows the importance of G as well as the transit hub nature of both D and G. The allocation of USAR specialists is also changed with the results shown in Figure 7b. Similar to the allocation of USAR units, the allocation of USAR specialists emphasize the importance of D and G. When the importance of A is reduced further, the allocation of USAR specialist concentrates more on D and G. These results show that it is critical to calibrate these importance settings appropriately.

4.3.2. Resource-Based Model and Budget Analysis

Given the need more equipment to satisfy resource requirements when the given simultaneous disaster scenarios happen, we now run the second model (R) to figure out the needed additional amounts of resources. For this model (R), we use the costs per unit shown in Table 7.

Table 7: Cost per unit (in million pounds) for specialist equipment

	IRU	USAR	HVP
γ_r	0.77	4.4	1.17

The results show that one would need 25 more USAR units (or 45 units in total) to efficiently handle all given simultaneous disaster scenarios and no additional IRUs or HVP units are required. The allocation of USAR units does not concentrate on A and C any more. The additional units are allocated in various locations and it shows that G is an important location given its transit hub nature. The detailed allocation is shown in Table 8.

Table 8: Allocation of USAR units given additional resources

A	B	C	D	E	F	G	H
10	0	10	5	5	5	9	1

We now consider the effect of number of simultaneous disasters in each scenario on required resources. We vary the number of simultaneous disasters in each scenario from 1 to 7 and solve the model (R) using all scenarios in each case. Table 9 shows the total number of scenarios, $|\mathcal{S}_k|$, given k simultaneous disasters in each scenario, for $k = 1, \ldots, 7$.

The results show that allocations for HVPs remain the same in all cases while those for IRUs only reduce slightly, by 4 IRUs when $k = 1$ and 1 IRUs when $k = 2$). It implies there are enough

Table 9: Total number of scenarios given different number of simultaneous disatsers

k	1	2	3	4	5	6	7
$\lvert\mathcal{S}_k\rvert$	16	56	112	140	112	56	16

IRUs and HVPs and their allocations allows us to handle effectively multiple disasters at different locations simultaneously with minimum requests from other locations. On the other hand, there are not enough USAR units and even for scenarios with single disasters, we still need 10 more USAR units. Figure 8 shows the total numbers of additional USAR units needed to handle scenarios of k simultaneous disasters, $k = 1, \ldots, 7$.

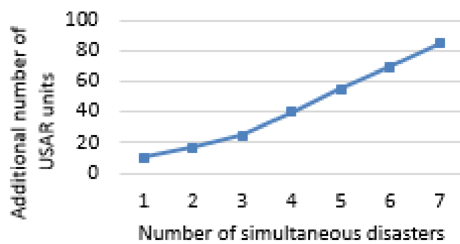

Figure 8: Number of additional USAR units needed given different number of simultaneous disasters

The number of additional USAR units needed is increased when the number of simultaneous disasters in each scenario increases, showing more USAR units are essential since they are needed in both types of disasters. Similar to the original case of three simultanous disasters, in addition to A and C being important locations, G also becomes important with a high number of allocated USAR units. For example, when there are $k = 7$ simultaneous disasters in each scenarios, 15 USAR units are allocated at G, which is the same allocated at A and C.

The need of additional resources depends on resource requirements in each scenario. In the next experiment, we vary the resource requirements with a scaling factor σ, i.e., $d_r^l(t) \leftarrow \sigma \cdot d_r^l(t)$ and $d_0^l(t) \leftarrow \sigma \cdot d_0^l(t)$ for all r, l, and t, where $\sigma \in [0.4, 1.6]$. Figure 9 shows the total amounts of resources needed to handle three simultaneous disasters with different levels of resource requirements. We can see that when $\sigma = 0.4$, no additional resources are needed and when $\sigma = 1.6$, all three resources requires additional units. The results show that the current capacity of HVPs is appropriate for a slight increase (less than 20%) in the level of resource requirements while that of IRUs can handle three simultaneous disasters if the resource requirements are increased by 50%. The constraint is the USAR resource, whose current capacity can only handle less than 50% of the current level of resource requirements.

Next, we analyse the effect of the budget for additional resources. To effectively handle three simultanous disasters, the total amount of money needed for additional resources is £110m. In reality, the budget could be smaller and we now analyse the impact of budget on how to buy and allocate additional resources. In order to do so, we construct a new model (B) (B for budget) which incorporates decisions buying and allocating additional resources with a given budget while aiming to minimize the risk of not satisfying all resource requirements when simultaneous disaster

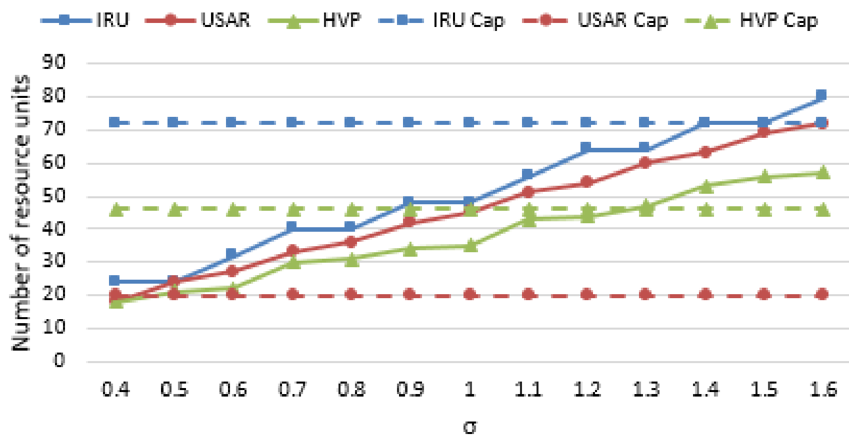

Figure 9: Total mumbers of resource units needed given different resource requirement levels

scenarios happen. The budget-based model (B) can be constructed based on the two proposed models (P) and (R) with an additional budget parameter B. The details of the model (B) are presented in Appendix D. We run the model (B) with different levels of budget B while considering two importance settings, I1 and I3. It is obvious that the budget will be used for additional USAR units given that other resources are already sufficient. Figure 10 shows the allocation results of additional USAR units under those two settings for location importance. The total number of USAR units increases from 20 to 42 when the budget increases from 0 to £100m. When the budget is small, the location importance plays a significant role in deciding where to allocate USAR units. If the budget is increased, additional USAR units are spread to various locations, especially those which can act as transit hubs (C, D and G). When the budget is close to sufficient ($B = 100$), both allocation results are exactly the same, which shows that the diminishing influence of location importance setting. The results are close to the ones obtained from the model (R) of unlimited budget without any location importance setting. In the next section, we discuss the implications of these results for our case study as well as disaster planning applications in general.

Budget	A	B	C	D	E	F	G	H
0	10	0	10	0	0	0	0	0
25	10	0	5	5	0	0	5	0
50	10	0	5	6	1	3	6	0
75	10	0	8	6	1	5	7	0
100	10	0	12	5	3	2	10	0

(a) Location importance setting I1

Budget	A	B	C	D	E	F	G	H
0	0	0	0	10	0	0	10	0
25	0	0	5	5	5	0	10	0
50	1	1	9	9	0	1	10	0
75	7	3	7	6	1	6	7	0
100	10	0	12	5	3	2	10	0

(b) Location importance setting I3

Figure 10: Allocation of USAR units given different budgets under different location importance settings

5. Discussions and Conclusions

The proposed models are built to address two issues faced by decision makers in disaster planning. The first issue is how to minimize the risk of not being able to meet the response targets for all disaster scenarios given existing capabilities. The second issue is how to reduce or eliminate that risk by purchasing additional resources. We have demonstrated that these issues can be addressed using the proposed stochastic optimisation models. Real constraints such as requirements on deployment breaks can be handled effectively in these mathematical optimisation models. For the first model, we argue that equal scenario probabilities can be considered if there are limited data. However, if we gather experts' opinions on the risks (or probabilities) of these simultaneous disaster scenarios and estimate them with some confidence intervals, the stochastic optimisation model (P) could be extended to a robust stochastic optimisation model to handle the ambiguity in scenario probabilities (see, e.g., [54]). In practice, it could be difficult for experts to estimate such risks [40]. This difficulty is reflected in the UK where the national government's Cabinet Office only publishes estimates of single events occurring in the next five years e.g. coastal and inland flooding are estimate to have a probability of "between 1 in 200 and 1 in 20"; while "catastrophic terrorist attacks" is "medium low" [37]. Similar to scenario probabilities, it is also difficult to estimate the (maximum) target response levels for disaster scenarios, i.e., the scales of the simultaneous disasters, especially when they are time dependent. The proposed models could benefit from an accurate estimation model of these disaster scales or one could attempt to model the uncertainty of these target respone levels in a relevant robust stochastic optimisation model. Computationally, the proposed models have a large number of second-stage decision variables and constraints due to the multi-period structure of its second stage. Even though the focus of the paper is the case study, further development of more efficient heuristics could be useful for larger applications.

Models such as the ones described in this paper present new ways of analysing the issues to identify efficient configurations of resources for disaster response. One practical consideration often missing from these models, however, is the political nature of decision making which is especially notable in disaster management [50, 41]. In the preparedness phase, resource allocations should recognize the characteristics of an area/city, i.e., that different cities have different hazards, risks, vulnerabilities and defenses [50]. Thus, citizens will experience disasters differently because of how they have prepared their personal resilience [51] and because of exogenous factors, for example: the ease of access to physical upgrades to better disaster-proof houses [32]; societal deprivation that reduces local resource availability to attenuate negative effects; operational limitations that cause response systems to operate sub-optimally [42]. Partly because disasters are evocative, many characteristics (including those mentioned here) are politically charged so resource allocations can be politically influenced especially when inequalities across a population are exposed and make headlines. Here, the equality of citizens concept [62] highlights the need for government to establish a consistency of response for citizens irrespective of where they live to ensure that local inequalities of preparedness do not put some people at a disproportionate risk. In both preparation and response there exists a tension between the need for equality across a country and enhanced protection of areas of special importance, for example, those that contain critical national infrastructure or create national economic wellbeing. Consequently, some areas may be perceived to receive priority when resource allocations are decided. Thus, while our results may indicate rational allocations, these are not necessarily politically feasible [31]. Ultimately, the intervention of political will could inform solutions because disasters are political spaces [55]. Although a limitation, this is also the strength of analytical models in their power to bring transparency and evidence to such decisions - cutting through political biases. In our proposed models, we introduce the local importance parameters which decision makers can set to partially balance out the political dimensions of implementation when the final decisions are made.

Another practical consideration made in this paper is that the decision makers are expert risk managers, potentially with knowledge and know-how relevant to the models [41] but not with an analytical training. This is problematic as Gonçalves [22] found that humanitarian decision makers often rely on experience and simple decision heuristics which lead to non-optimal decisions. Analytical models offer different ways of thinking but the skills required to decipher their intricacies (and their results) may not be possessed by potential users. Thus, we concentrate on aligning our model as closely as possible to information actually available in the context - to allow for maximum translation by emergency responders. Aligned with this is our assertion that our models are more useful in the planning phase rather than in the heat of emergency response, especially the budget model to consider the purchase of additional resource. In trainings, potential users can benefit from the analytical capabilities offered by such models e.g. to stress test their knowledge of the relationship between variables; experiment with resource configurations to explore the dynamics of the system they seek to understand; and, tailor the inputs, processes and outputs to their particular context. Once confidence has been built in training sessions, testing their ability to calculate optimality during emergency exercises (and thereby produce output to be integrated into the decision making process) is a step towards operational deployment during real incidents.

In conclusion, we began this paper by noting that the allocation of scarce resources is a challenge for disaster preparation and response and this is especially apparent given the political nature of the contexts. The models we provide can inform decisions by giving an evidence base to balance the politics when allocating scarce national resource so that all citizens are equally protected from the effects of simultaneous disasters. While there may be a temptation to procure as much equipment as funds allow, the balance of different types of equipment requires understanding of the tradeoffs in achieving performance targets. Such models are institutionalised in other fields and so their propagation in disaster management is worthwhile to inform investment and allocation decisions. On future work, robust stochastic optimisation models could be considered to handle the ambiguity in scenario probabilities and the uncertainty in target response levels. Computationally, efficient heuristics for larger instances, perhaps with additional practical constraints, are also relevant as future work.

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Appendix A. Reviews of Papers in Resource Allocation in Disaster Management

Tables A.10, A.11, and A.12 list a variety of papers in resource allocation in disasters management with different models, objectives, and settings.

Table A.10: Reviews of papers in resource allocation disaster management

Citation	Model Type	Objective(s)	Characteristic(s)	Types of event	Disaster stage
This paper	Two-stage stochastic optimisation	Minimise the penalty of not being able to satisfy demand and minimize the total cost of additional capacities to meet demand	Response capacity to simultaneous incidents happening across a country based on, demand/availability/targets, deployment regulations, protocols for requesting assistance from other cities, transportation	Terrorist and flooding	Preparedness / Response
Agarwal et al, 2010 [1]	Combinatorial optimisation	Analyse vulnerable points by measuring impact of attacks on networks	Probabilistic network failure model including: nodes of communications hubs, links between nodes, location nodes, intersection of links, distance links, cuts to links, probability of failure	Telecommunication collapse from unspecified disaster	Mitigation
Albores & Shaw, 2008 [3]	Discrete event simulation	Analyse the time taken for adequate response to arrive at scene	Three concurrent incidents across nine locations, travel times, crew and equipment availability, priority to request mutual aid	Terrorist and flooding	Preparedness
Alem et al, 2016 [4]	Mixed-integer two stage stochastic optimisation	Minimise costs of prepositioning stock and vehicles, and excess inventory and unmet demand	Risk-averse model of budget allocation based on fleet of vehicle types, procurement, lead times, demand in relief centres, and demand in warehouses	Flooding and landslide	Preparedness / Response
Bastian et al, 2016 [7]	Mixed-integer stochastic weighted goal programming optimisation	Minimise the response time, target budget, and the total amount of aid shortage	Trade-off between supply chain efficiency and supply chain responsiveness, trade-off between cost, response time and amount of demand satisfied	Humanitarian aid	Response
Bier et al, 2008 [10]	Non-linear optimisation	Minimise expected loss from attack	Optimize budget allocation for defending multiple terrorist targets based on probability of attack, probability of success, cost effectiveness of defence, value of target	Terrorist	Mitigation / Preparedness
Bryson et al, 2002 [12]	Mixed integer linear optimisation	Maximise the total value of recovery coverage provided by selected subplans	Develop a disaster recovery plan by selecting between alternative subplans according to disaster types, recovery effects, resources	Unspecified	Recovery
Chang et al, 2007 [14]	Two-stage ttochastic optimisation	Model 1: Minimise expected shipping distance of rescue equipment. Model 2: Minimise costs of setup, equipment, transportation, supply-shortage and demand-shortage penalty	Resource allocation based on rescue areas and level of emergency as well as organizational structure, store locations, resource availability, travel distances	Flooding	Preparedness
Chiu & Zheng, 2007 [15]	Linear optimisaton of node-arc network	Minimise travel time of evacuees and emergency response groups	The flow of evacuees and emergency response resources to destinations using a traffic network based on an evacuation zone, safe destinations, and prioritised routing strategies	No-notice disasters	Response
Fiedrich et al, 2000 [19]	Dynamic combinatorial optimisation	Minimise the total number of fatalities	Search and rescue performance based on the classifaction of affected areas, facilities in those areas, resources, and time factors (e.g. survival rates, probability of secondary event)	Earthquake	Response
Grabowski et al, 2016 [23]	Dynamic resource allocation	Resource allocation and effectiveness of response	Assess the effectiveness and allocation of resources for remote environmental disasters through capabilities, equipment, time period, incidents	Oil spill	Response

Table A.11: Reviews of papers in resource allocation in disaster management (cont.)

Citation	Model Type	Objective(s)	Characteristic(s)	Types of event	Disaster stage
This paper	Two-stage stochastic optimisation	Minimise the penalty of not being able to satisfy demand and minimize the total cost of additional capacities to meet demand	Response capacity to simultaneous incidents happening across a country based on, demand/availability/targets, deployment regulations, protocols for requesting assistance from other cities, transportation	Terrorist and flooding	Preparedness / Response
Iannoni & Morabito, 2007 [26]	Hypercube, spatially distributed queuing	Analyse the expected coverage	Configuration and operation of dispatch policies based on call types, response capabilities, location-based response protocols, multiple dispatches	Medical response	Response
Kroshl et al, 2015 [27]	Integer programming and evolutionary agent based optimisation	Minimise damage on spatially distributed node/arc network	Use Stackelberg 'leader follower' game to allocate defensive / attacker resources to achieve stable strategies	Terrorist	Mitigation / Response
MacKenzie & Zobel, 2016 [30]	Robust optimisation	Maximise system resilience by reducing loss and improving recovery time	Allocate fixed resources to reducing loss and improving recovery assuming the effect of allocating resources to those are known to be linear, exponential, quadratic, or logarithmic	Storm	Mitigation
MacKenzie et al, 2016 [29]	Static and dynamic optimisation	Minimise (economic direct and indirect) production loss due to disruption	Determine static and dynamic resource allocations to create regional economic recovery from disruption assuming consumers stop purchasing and producers stop producing	Oil spill	Recovery
Michel-Kerjan et al, 2013 [32]	Probabilistic catastrophe cost benefit model	Analyse risk, costs and benefits of disaster reduction strategies	Evaluate the effectiveness of strategies to prepare buildings such as roof upgrades and strengthened windows / doors and the impact of this on cost of the event	Storm and earthquake	Preparedness
Minciardi et al, 2009 [33]	Mathematical optimisation	Minimise potential unmet demand, inappropriate assignment of resource, and costs	Manage disasters in (pre-)operational stages by analysing resource availability/location, demand, cost of moving resources and of storing resources	Wildfire	Preparedness / Response
Nagurney et al, 2016 [34]	Generalised Nash equilibrium network model	Maximise the financial gains from donors relative to the visibility of their effectiveness at disaster sites	Non-governmental organization attracting financial donations based their visibility through media of the relief item flow allocations and perceived effectiveness at disaster sites	Hurricane	Response / Recovery
Najafi et al, 2013 [35]	Multi-objective robust optimisation	Minimise the unserved injured people, unsatisfied demands, and vehicles utilized in the response	Manage logistics of supplies and injured people in a multi-objective, multi-mode, multi-commodity, multi-period stochastic model	Earthquake	Response
Ozdamar and Demir, 2012 [38]	Mixed-integer linear optimisation	Minimise travel time of vehicles and promote efficient resource utilisation	Efficient network flow model using a hierarchical "cluster first, route second" approach based on vehicle and supply availability, importance of injured people and commodities, travel times	Earthquake	Response
Rachaniotis et al, 2013 [43]	Stochastic optimisation	Minimise travel time of aid to beneficiaries	Last mile aid distribution fleet management through analysing vehicle allocation and routing, hubs, and demand for aid	Humanitarian aid	Response

Table A.12: Reviews of papers in resource allocation in disaster management (cont.)

Citation	Model Type	Objective(s)	Characteristic(s)	Types of event	Disaster stage
This paper	Two-stage stochastic optimisation	Minimise the penalty of not being able to satisfy demand and minimize the total cost of additional capacities to meet demand	Response capacity to simultaneous incidents happening across a country based on, demand/availability/targets, deployment regulations, protocols for requesting assistance from other cities, transportation	Terrorist and flooding	Preparedness / Response
Ransikarbum and Mason, 2016 [45]	Multi-objective integer optimisation	Maximise the equity through satisfied demand and minimise the unsatisfied demand and cost	Equity-based strategic decisions in supply, distribution and network distribution based on a network of disrupted supply ports, demand warehouses, transportation links	Earthquake	Response / Recovery
Rahnamay-Naeini et al, 2011 [44]	Stochastic geographic stress simulation	Analyse the reliability and efficiency of networks to simulataneous failures	Analyse centres of stress which create correlated failures in communications network reliability to understand network vulnerability including node and link failure	Telecommunication collapse from unspecified disaster	Mitigation
Rawls & Turnquist, 2012 [46]	Stochastic mixed-integer optimisation	Minimise expected cost from selecting pre-position locations, stocking and storage factors, transportation of resource, unmet demand	Determine the location and amount of different types of resource to be pre-positioned based on multiple locations of uncertain demand, uncertain stock and transportation availability	Storm	Preparedness / Response
Salmeron and Apte, 2010 [48]	Stochastic mixed-integer optimisation	Minimise expected casualties and people not evacuated	Strategic budget allocation for the purchasing and positioning of resources for relief based on evacuation demand and supply, supplies for non-evacuees	Storm	Preparedness
Shan & Zhuang, 2013 [50]	Sequential game theory	Minimise expected loss by allocating part of resources equally and the remainder according to risk	Effect of the equity coefficient on optimal defensive allocation and the expected loss by allocating defensive resources according to the risk of potential targets while allocating a portion across all regions equally	Terrorist	Preparedness
Su et al, 2016 [52]	Integer linear optimisation	Minimise travel time of resources and and the cost of allocated resources	Allocate multiple emergency resources to multiple concurrent incidents based on response time and cost	Unspecified	Response
Tzeng et al, 2007 [56]	Fuzzy multi-objective optimisation	Maximise efficiency (by minimising cost and travel time) and maximise fairness (by maximising satisfaction) of relief distribution	Ensure difficult to reach areas receive aid based on different relief package being transported from a collection point to a relief centre to a demand point including aspects such as package volume, travel time, transport cost	Earthquake	Response
Wex et al, 2014 [63]	Binary quadratic optimisation	Minimise sum of completion times of incidents weighted by their severity	Modified multiple travelling salesman problem and parallel-machine scheduling problem to schedule rescue units and assign them to incidents in order to minimize casualties and economic loss	Earthquake	Response
Yi & Kumar, 2007 [66]	Mixed-integer multi-commodity network flow model	Minimise time delay in providing prioritized resources to affected people	Logistics of disaster relief using dynamic vehicle routing and multi-commodity dispatch to analyse different vehicle types to transport aid to affected areas and to evacuate wounded people	Humanitarian aid	Response
Zhang et al, 2012 [67]	Mixed-integer linear optimisation	Minimise the resource cost of the primary disaster and the expectation cost of secondary disaster	Analyses the priority of preference for each location where the secondary disasters will take place with certain possibilities	Earthquake	Response

Appendix B. Case Study: Target Response Levels

Table B.13 displays the target response levels for terrorist attack disaster while those for flooding disaster are shown in Table B.14.

Table B.13: Response targets of equipment being on-site and operational following a terrorist attack disaster

City/Operation	Mass decontamination
A	Process 3000 casualties within 3 hrs Remainder of 1000 casualties within 10 hrs Require 50 generalists
D, G	Process 3000 casualties within 3 hrs Remainder of 1000 casualties within 10 hrs Require 50 generalists
B, C, E, F, H	Process 3000 casualties within 3 hrs Remainder of 1000 casualties within 10 hrs Require 50 generalists
City/Operation	Urban search and rescue
A	Provide 10 USAR units within 2 hrs Remainder of 5 units within 10 hrs Require 150 generalists
D, G	Provide 10 USAR units within 4 hrs Remainder of 5 units within 10 hrs Require 100 generalists
B, C, E, F, H	Provide 10 USAR units within 4 hrs Remainder of 5 units within 10 hrs Require 80 generalists

Table B.14: Response targets of equipment being on-site and operational following a flooding disaster

City/Operation	Flood water pumping
A	Provide 10 HVP units within 2 hrs
D, G	Provide 5 HVP units within 2 hrs
B, C, E, F, H	Provide 3 HVP units within 2 hrs
City/Operation	Urban search and rescue
A	Provide 10 USAR units within 2 hrs Remainder of 5 units within 10 hrs Require 150 generalists
D, G	Provide 10 USAR units within 4 hrs Remainder of 5 units within 10 hrs Require 100 generalists
B, C, E, F, H	Provide 10 USAR units within 4 hrs Remainder of 5 units within 10 hrs Require 80 generalists

Appendix C. Computational Tests: Settings and Additional Results

Appendix C.1. Parameter Settings

We first solve the model only with the primary objective of minimising the total expected weighted penalty by setting $\theta = \mu = 0$. The results show that resource requests do not follow the given preferences. We heuristically set the value of μ to $\mu = 10^{-5}$ with which the penalty remains the same as with $\mu = 0$ while the secondary objective of $\sum_{s\in\mathcal{S}}\sum_{t=1}^{T_s}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{E}_i} p_{ij}(T-t+1)\cdot \left(x_{0,s}^{i,j}(t)+\sum_{r\in\mathcal{R}} x_{r,s}^{i,j}(t)\right)$ decreases significantly and becomes stable as show in Figure C.11. Similarly, in order to determine the actual (minimum) numbers of specialist equiment and specialists, especially when there are more than enough resources, we need to set an appropriate positive value for θ. According to Figure C.12, with $\theta = 10^{-3}$, the secondary objective of $\sum_{i\in\mathcal{N}}\sum_{r\in\mathcal{R}}\left(w_r^i+\bar{w}_r^i\right)$ achieve its minimum value given that the penalty remains the same as with $\theta = 0$. Given the analysis, we set $\theta = 10^{-3}$ and $\mu = 10^{-5}$ for all experiments.

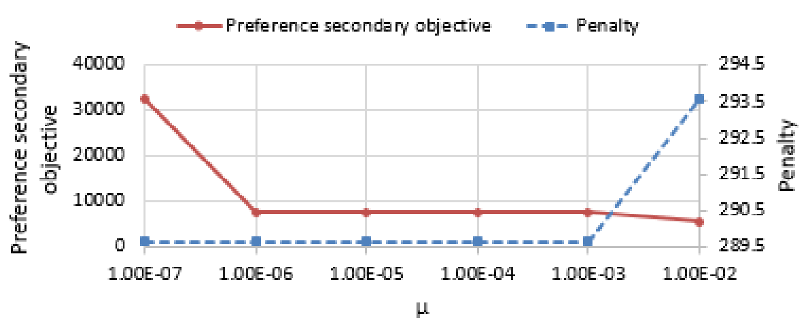

Figure C.11: Values of preference-related secondary objective with different μ

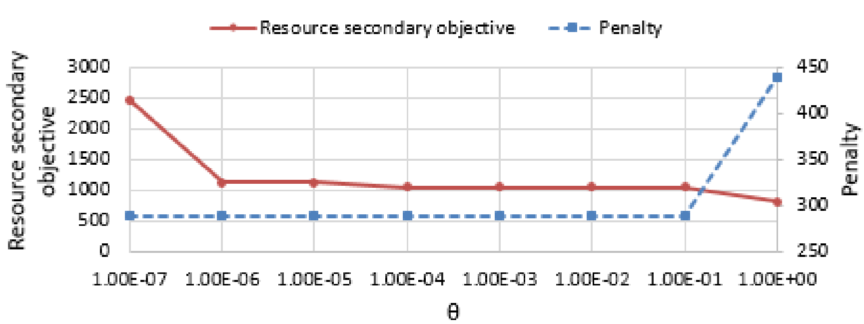

Figure C.12: Values of resource-related secondary objective with different θ

Appendix C.2. Random Instance Generation: Examples

Figure C.13 shows the city locations in a 9-city random instance while Figure C.14 shows different time-dependent IRU resource requirements in 10 different random instances.

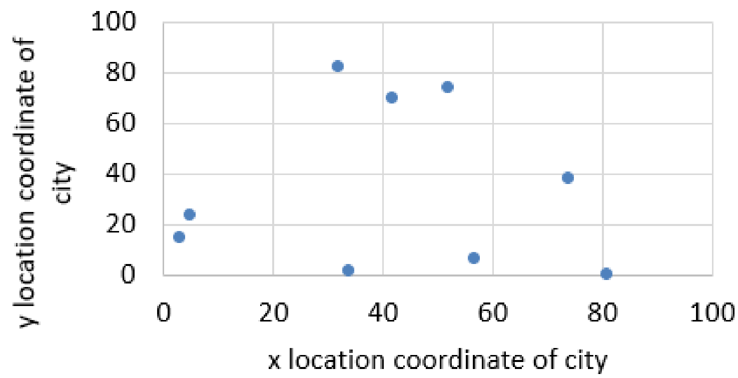

Figure C.13: City locations in a random instance with 9 cities

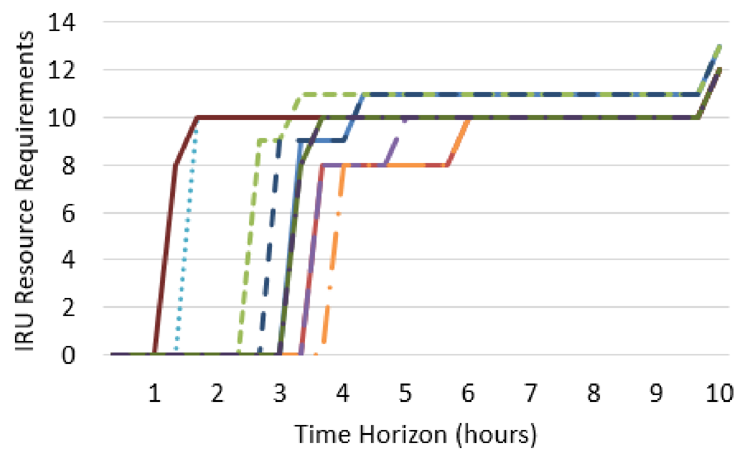

Figure C.14: Time-dependent IRU resource requirements in 10 different random instances

Appendix C.3. Computational Results for Model (R) using IBM CPLEX

Figure C.15 shows computational times of the model (R) given different numbers of cities.

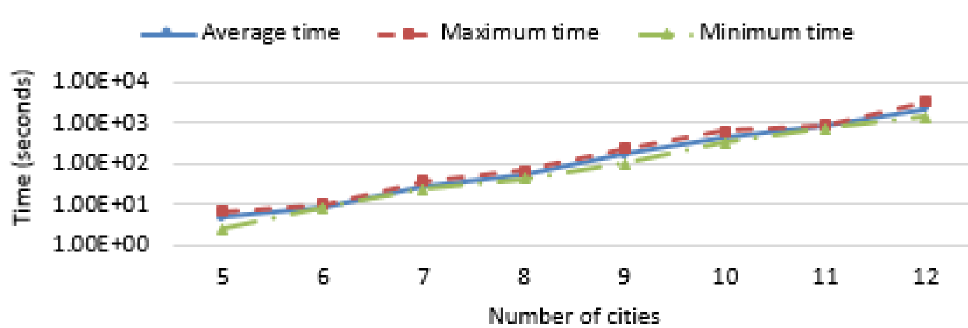

Figure C.15: Computational times of model (R) given different numbers of cities

Appendix C.4. Computational Results with PHA

We implement the PHA algorithm using a similar framework as described in [20, Algorithm 2] for two-stage mixed-integer stochastic optimisation problems with scenario bundles. For our models, the scenarios are bundled such that all locations and all types of disasters are covered in most of the bundles. The lower bound is computed from the first initialisation stage of the algorithm ([20, Proposition 1]). The original algorithm requires us to solve mixed-integer quadratic optimisation problems in the decomposition stage and it is impractical to solve given the sizes of tested instances. We modify these sub-problems by replacing the proximal term with squared two-norm by the proximal term with ℓ_1-norm, which allows us to reformulate the sub-problems as mixed-integer linear optimisation problems. These problems are solved with IBM CPLEX. We follow recommendations in [59] for parameter selection and termination criteria. The best solution (in terms of the objective) among all solutions obtained in the last iteration of the PHA algorithm is chosen as the heuristic solution. We solve instances of the model (P) with different number of cities using this PHA-based heuristic and compare the results with those with IBM CPLEX. Figure C.16 shows that with large instances of more than 9 cities, the heuristic is more efficient in terms of computational time. In addition, given the advantage of the PHA algorithm in terms of memory usage, we are able to solve larger instances with 15 and 20 cities using the heuristic whereas IBM CPLEX cannot handle them.

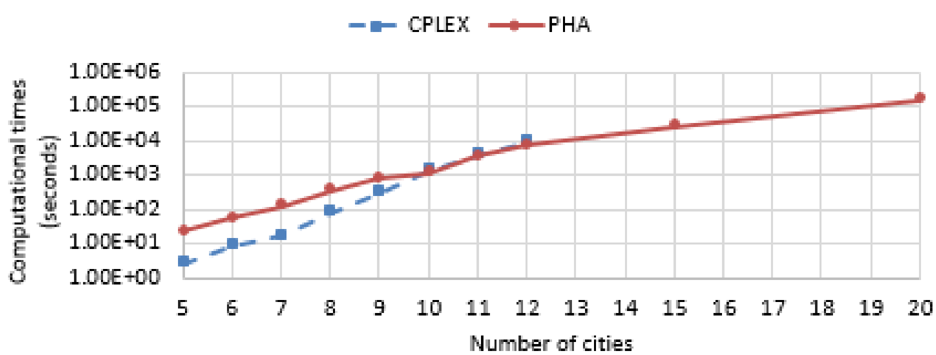

Figure C.16: PHA vs. CPLEX computational times given different numbers of locations

The trade-off is the quality of the final solution. Let define Z_H and Z_L to be the objective value of the heuristic solution and the lower bound, respectively, obtained from the PHA algorithm, and Z^* to be the optimal value obtained from IBM CPLEX. Figure C.17 shows the actual (relative) optimal gap $\rho = \dfrac{Z_H - Z^*}{Z^*}$ and the maximum optimality gap $\rho_m = \dfrac{Z^H - Z_L}{Z_L}$ for all tested instances with different numbers of cities. The results indicate that the maximum gaps can be close to 50% but the actual gaps are much smaller. The PHA algorithm achieves 7% gap in the best instance while it is approximately 25% in the worst instance. Note that the PHA algorithm is a heuristic for mixed-integer problems and these computational results show that while it can solve larger instances than IBM CPLEX, the quality of the solution is not guaranteed to be high in all instances.

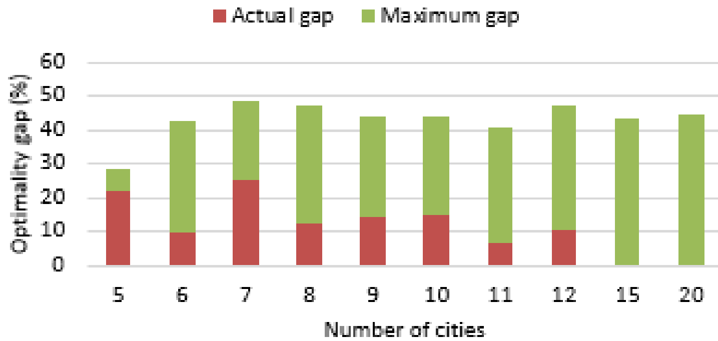

Figure C.17: Actual and maximum (relative) optimal gaps for different instances

Appendix D. Budget-Based Model (B)

The budget-based model (B) incorporates the additional resources into the capacity constraints:

$$\sum_{i=1}^{N} w_r^i \leq C_r + \Delta C_r, \quad \forall\, r \in \mathcal{R},$$

and imposes an additional budget constraint, $\sum_{r\in\mathcal{R}} \gamma_r \cdot \Delta C_r \leq B$, where B is the total budget. The detailed model (B) is written as follows:

$$\begin{aligned}
(B): \min \quad & \sum_{s\in\mathcal{S}} q_s \sum_{l\in\mathcal{L}_s} \iota_{i_l} \sum_{r\in\mathcal{R}_{k_l}} \sum_{t\in\mathcal{T}_{k_l}} (T-t+1)\cdot u_{r,s}^l(t) + \theta \cdot \left[\sum_{i=1}^{N}\sum_{r\in\mathcal{R}} (w_r^i + \bar{w}_r^i)\right] + \cdots \\
& \mu \cdot \left[\sum_{s\in\mathcal{S}}\sum_{t=1}^{T_s}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{E}_i} p_{ij}(T-t+1)\cdot \left(x_{0,s}^{i,j}(t) + \sum_{r\in\mathcal{R}} x_{r,s}^{i,j}(t)\right)\right] \\
\text{s.t.} \quad & y_{0,s}^i(1) = y_{r,s}^i(1) = \bar{y}_{r,s}^i(1) = 0, \quad \forall\, i\in\mathcal{N},\, r\in\mathcal{R},\, s\in\mathcal{S}, && \text{(D.1)} \\
& y_{r,s}^i(t+1) = y_{r,s}^i(t) + \sum_{j\in\mathcal{E}_i: t+1>o_r+r_{ij}} x_{r,s}^{i,j}(t+1-o_r-r_{ij}), && \text{(D.2)} \\
& y_{0,s}^i(t+1) = y_{0,s}^i(t) + \sum_{j\in\mathcal{E}_i: t+1>t_{ij}} \left(x_{0,s}^{i,j}(t+1-t_{ij}) + \sum_{r\in\mathcal{R}} \beta_r \cdot x_{r,s}^{i,j}(t+1-t_{ij})\right) && \text{(D.3)} \\
& \bar{y}_{r,s}^i(t+1) = \bar{y}_{r,s}^i(t) + \sum_{j\in\mathcal{E}_i: t+1>t_{ij}} \left(\bar{x}_{r,s}^{i,j}(t+1-t_{ij}) + \alpha_r \cdot x_{r,s}^{i,j}(t+1-t_{ij})\right), && \text{(D.4)} \\
& \quad \forall\, i\in\mathcal{N},\, r\in\mathcal{R},\, s\in\mathcal{S},\, t=1,\ldots,T_s-1, \\
& z_{r,s}^i(t) = \sum_{j\in\mathcal{E}_i: t>o_r+r_{ij}} x_{r,s}^{i,j}(t-o_r-r_{ij}), && \text{(D.5)} \\
& v_{r,s}^i(t) = \sum_{\tau=0}^{\min\{t,D\}-1} z_{r,s}^i(t-\tau) + 2\cdot \sum_{\tau=\min\{t,D\}}^{t-1} z_{r,s}^i(t-\tau), && \text{(D.6)} \\
& \quad \forall\, i\in\mathcal{N},\, r\in\mathcal{R},\, s\in\mathcal{S},\, t=1,\ldots,T_s,
\end{aligned}$$

$$y^{i_l}_{0,s}(t) \geq d^l_0(t), \quad \forall\, l \in \mathcal{L}_s,\, t \in \mathcal{T}_{k_l},\, s \in \mathcal{S}, \tag{D.7}$$

$$\bar{y}^{i_l}_{r,s}(t) \geq v^{i_l}_{r,s}(t), \quad \forall\, l \in \mathcal{L}_s,\, r \in \mathcal{R}_{k_l},\, t = 1, \ldots, T_s,\, s \in \mathcal{S}, \tag{D.8}$$

$$u^l_{r,s}(t) \geq d^l_r(t) - y^{i_l}_{r,s}(t), \quad \forall\, l \in \mathcal{L}_s,\, r \in \mathcal{R}_{k_l} : c_{l,r} = 0,\, t \in \mathcal{T}_{k_l},\, s \in \mathcal{S}, \tag{D.9}$$

$$u^l_{r,s}(t) \geq \left(d^l_r(t) - \sum_{\tau=1}^{t} y^{i_l}_{r,s}(\tau) \right), \forall\, l \in \mathcal{L}_s,\, r \in \mathcal{R}_{k_l} : c_{l,r} = 1,\, t \in \mathcal{T}_{k_l},\, s \in \mathcal{S} \tag{D.10}$$

$$u^l_{r,s}(t) \geq 0, \quad l \in \mathcal{L}_s,\, t \in \mathcal{T}_{k_l},\, r \in \mathcal{R}_{k_l},\, s \in \mathcal{S}, \tag{D.11}$$

$$\sum_{i=1}^{N} w^i_r \leq C_r + \Delta C_r, \quad \forall\, r \in \mathcal{R}, \tag{D.12}$$

$$\sum_{r \in \mathcal{R}} \gamma_r \cdot \Delta C_r \leq B, \tag{D.13}$$

$$\sum_{t=1}^{T_s} \sum_{j : i \in \mathcal{E}_j} x^{j,i}_{r,s}(t) \leq w^i_r, \quad \forall\, i \in \mathcal{N},\, r \in \mathcal{R},\, s \in \mathcal{S}, \tag{D.14}$$

$$\sum_{t=1}^{T_s} \sum_{j : i \in \mathcal{E}_j} \left(\bar{x}^{j,i}_{r,s}(t) + \alpha_r \cdot x^{j,i}_{r,s}(t) \right) \leq \bar{w}^i_r, \quad \forall\, i \in \mathcal{N},\, r \in \mathcal{R},\, s \in \mathcal{S}, \tag{D.15}$$

$$x^{i,j}_{0,s}(t), x^{i,j}_{r,s}(t), \bar{x}^{i,j}_{r,s}(t) \in \mathbb{Z}_+, \quad \forall\; i \in \mathcal{N},\, j \in \mathcal{E}_i,\, r \in \mathcal{R},\, s \in \mathcal{S},\, t = 1, \ldots, T_s, \tag{D.16}$$

$$w^i_r, \bar{w}^i_r \in \mathbb{Z}_+, \quad \forall\, i \in \mathcal{N},\, r \in \mathcal{R}, \tag{D.17}$$

$$\Delta C_r \in \mathbb{Z}_+, \forall\, r \in \mathcal{R}. \tag{D.18}$$