Collaborative Coding Multiple Access Communications

By

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To My Father

AL-Haj Hassan Ali Jasam AL-Mafrage

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Declarations

The following is a list of the materials which have either been published or submitted for publication, during the course of this research program, along with the sections to which they relate. These materials are the direct results of the work carried out by the author.

- "Information Capacity of Additive White Gaussian Noise Channel with Practical Constraints", IEE Proceedings, Vol. 137, Pt I, No. 5, pp295-301, Oct. 1990.

(Relevant to Chapter 3)

- "Capacity of T-user Collaborative Coding Multiple Access Scheme Operating over A Noisy Channel", IEE Electronics Letters, Vol. 25, No. 11, pp742-744, May 1989.

(Relevant to Chapter 4)

- "Collaborative Coding Multiple Access Channel", Colloquium on Radio Communication Techniques and Systems, University of Warwick, July 1989.

(Relevant to Chapter 4 & 6)

- "Performance Study of Collaborative Coding Multiple Access Over Noisy Channel", Proceedings of Fourth Joint Swedish-Soviet International Workshop on Information Theory, pp157-164, Aug.27-Sept.1 1989.

(Relevant to Chapter 4 & 6)

- "Information Transmission Capacity and Error Control Capability of Collaborative Coding Multiple Access System", Proceedings of Second Bangor Symposium on Communications, pp195-198, 23-24 May 1990.

(Relevant to Chapter 4 & 5)

- "Soft Decision Decoding Technique for Collaborative Coding Multiple Access Channels", Proceedings of Third IEE Conference on Telecommunication, Edinburgh, pp141-147, 17-20 March 1991.

(Relevant to Chapter 5)

- "Low Complexity Soft Decision Decoding Technique for T-user Collaborative Coding Multiple-Access Channels", IEE Electronics Letters, Vol. 27, No. 13, pp1167-1169, June 1991.

(Relevant to Chapter 5)

- "Collaborative Coding Multiple Access Employing Manchester and CMI Codes", Proceedings of Fourth Bangor Symposium on Communications, 27-28 May 1992.

(Relevant to Chapter 5)

- "Collaborative Coding and Decoding Techniques for Multiple Access Channels", Submitted for Publication to IEE Proceedings, Pt I, May 1992.

(Relevant to Chapter 5)

Abstract

This thesis investigates collaborative coding multiple access (CCMA) channel communication schemes. The CCMA schemes potentially permit efficient simultaneous transmission by several users sharing a common channel, without subdivision in time, frequency or orthogonal codes. The main areas of investigation include the information transmission capacity for single and multiple access channels, coding/decoding techniques and practical system design for CCMA schemes.

The information transmission capacity of a sampled and quantised single access AWGN channel is developed. It is determined and optimised when the channel input and output are limited by certain practical constraints. These investigations have led to the development and determination of the information transmission capacity of multiple access channels. The capacity of a multiple access channel is studied for two different classes of T-user channel models from both theoretical and practical points of view. It is shown, in principle, that higher transmission rates or, equivalently, more reliable communication than with time sharing is achievable employing the same signalling alphabet.

The CCMA schemes, in addition to providing the multiple access function, can also incorporate a certain degree of error control capability. Two main decoding techniques, hard decision and maximum likelihood soft decision, are presented with uniquely decodable CCMA schemes. A new low complexity maximum likelihood decoding technique is described and analysed. Reliability performance of various collaborative codes is studied by simulation employing these decoding techniques. It is shown that uniquely decodable schemes permit the multiple access function to be combined with forward error correction. It is also found that soft decision decoding can provide an energy gain over hard decision decoding.

The final area of investigation is a practical CCMA modem system design to combine collaborative coding and modulation. An M-ary frequency shift keying based modulation scheme is described for the T-user CCMA schemes. Three particular types of demodulation techniques, square-law, zerocrossing counting, and quadrature receiver, are described. These techniques are developed in software, tested and evaluated over noiseless and noisy channels.

Chapter 1

Introduction

The purpose of modern communication theory is to enable the design of systems which facilitate rapid, reliable, and efficient transfer of information through a medium which is called a communication channel. A telephone wire transmission line, or radio frequency electromagnetic propagation system are two very common examples of communication channels. Intuitively, a communication channel is any medium which supports the propagation of energy from a source to a destination with sufficient control to allow movement of some data.

The classical model of a communication system has a single transmitter sending information to a receiver through a channel which in some way corrupts the transmitted information. This is a single access communication channel (SAC). Developments in satellite communication systems, computer communication networks, mobile radio systems, and other communication systems involving multi-user have led communication systems designers to investigate the simultaneous transmission of information from several terminals over a common communication channel. An important model of multi-user communication is the multiple access communication channel (MAC).

The information theory of SAC [Shannon 1948] has shown that, noise and other disturbances on the channel do not limit the reliability by which digital data can be transmitted but rather only limits the rate at which data of arbitrarily high reliability can be transmitted. The highest rate at which such reliable data can be transmitted over a

channel is known as the capacity of the channel. The information theory of MAC has shown that, multiple users can communicate data with arbitrarily small probability of error over a common channel provided that the rates of the individual data streams lie within the capacity region for the channel. The set of rates at which simultaneous reliable transmission is possible is called the capacity region of MAC. Shannons capacity theorem gives us a theoretical value for the error free capacity of a channel, given that the time taken to evaluate the data is infinite. However, in most practical circumstances, this is of little use as any practical demodulation/decoding scheme must give a result in a finite period of time. Therefore, the practical system designer must keep in mind the theoretical channel capacity, and probably more importantly, certain practical restrictions which must also be satisfied.

One of the basic ways of increasing the throughput of a communications resource is to make the allocation of the communications resource more efficient. This approach is the domain of multiple access communications. The problem is to efficiently allocate portions of the fixed communications resource to a large number of users who seek to communicate digital information to each other. There are many ways of distributing the communications resource among the active users. However, the purposes here are to rule out conventional channel sharing techniques such as TDMA, FDMA and CDMA. More intuitively, the rationale behind this is to investigate collaborative coding multiple access (CCMA) techniques by which a single transmission medium can be shared efficiently and in a distributed fashion among many users.

Collaborative coding constructions have found short codes which are easy to decode such that, the active users which transmit independently (i.e., without prior reservations and without feedback during transmission) through the same channel can

be decoded uniquely at the receiver. In particular, there exist collaborative codes which allow two or more users to share the same transmission bandwidth and able to communicate at a combined information rate which is greater than unity. That is, the summary rate of the users is greater than the ideal rate (unity) which can be achieved by means of time sharing or TDMA. Since the transmission channel is always susceptible to external noise, a collaborative coding design needs, not only to be uniquely decodable but also must be able to correct transmission errors.

In practical systems, the transfer of collaborative coded messages involves the utilisation of various modules, i.e. modulator, demodulator and the communication medium. The modulator, which is employed at the transmitter side, translates the coded message stream into a suitable format for transmission over the multiple access communication medium. On the other hand, the demodulator, is situated at the receiver and performs the reverse operation to that of the modulator. The demodulation process involves the detection of the received composite signal and the subsequent mapping of these signals into the format of the original message stream.

Investigations of information transmission capacity of both SAC and MAC, coding/decoding, and practical system design for theses CCMA schemes are of considerable importance to provide the overall efficient multiple access system. The work of this thesis is a contribution towards these objectives.

The second Chapter of this thesis reviews the principles and various techniques of multiple access communications. A perspective, classification and discussion of multiple access communications are given. Following this, the most common techniques of MAC are discussed. A description of CCMA schemes is also presented in the same section. The MAC models describing the form of a signals interaction over a channel

are presented with some examples. The main achievements of the information theory and coding/decoding approaches to the MAC are also briefly reviewed and discussed in this Chapter.

In Chapter three, the information transmission capacity of a sampled and digitised single access AWGN channel limited by input and output practical constraints is described. The channel input is characterised by the input signal amplitude and average power, and the channel output is characterised by the output signal clipping due to quantisation applied at the receiver. The input signal amplitude/output signal clipping (ISA/OSC) constrained capacity, and the input signal amplitude and average power/output signal clipping (ISAP/OSC) constrained capacity are determined and optimised separately. The optimum input signal amplitude distributions and the optimum output signal clipping that maximises the capacity are also determined. These two capacities are developed by software and simulation results and discussions are presented at the end of this Chapter. The work described in Chapter three gave insight and more understanding to the more complicated case of MAC capacity which is the subject of investigation in Chapter four.

Chapter four is concerned with theoretical investigations of the information transmission capacity of MACs. Two particular types of T-user M-ary MAC models, interesting from their theoretical and practical applications, are introduced and described. The T-user M-ary frequency MAC is presented in two forms, with and without intensity information of the received signal energy level. The information transmission capacity is developed, theoretically, in bits per channel use, for these models in noiseless and noisy channel conditions. This capacity is simulated in software for various transmission systems employing coherent and noncoherent combining of

signals at the channel and for various number of users, T, and input signal levels, M. The practical implications of the MAC capacity is also discussed.

Chapter five investigates the capability of CCMA schemes to provide the multiple access function as well as the channel error control. Uniquely decodable coding schemes are given to provide these functions. Hard decision CCMA (HD_CCMA) and maximum likelihood soft decision CCMA (MLSD_CCMA) decoding techniques are presented. These techniques are described together with symbol-by-symbol HD (SBS_HD) decoding. A new low complexity maximum likelihood decoding technique is presented to utilise the error control capability. The decoding procedure and algorithm for this technique are given. A particular 2-user uniquely decodable scheme is analysed with this new technique. The error probability is derived for the T-user binary channel model employing HD_CCMA and MLSD_CCMA decoding techniques. The theoretical calculations are presented for a particular 2-user uniquely decodable case. Finally this Chapter ends with simulation of various 2-user uniquely decodable schemes. The reliability performance of these codes employing HD_CCMA and MLSD_CCMA is evaluated in the presence of AWGN conditions. The results are presented in the form of symbol and codeword error rates as a function of signal to noise ratios.

Practical design of CCMA modem is investigated in Chapter six. The design of CCMA modulation scheme based on the M-ary frequency shift keying (MFSK) modulation scheme is described. Three particular types of demodulation techniques are investigated with combined collaborative coding and modulation signals. These techniques are square-law, zerocrossing counting, and quadrature demodulators. The quadrature demodulator is presented and described for two models, with and without intensity information. The overall designed CCMA systems are developed in software,

and verified in a noiseless channel. An assessment and comparison of the relative reliability performance of these schemes is carried out by simulation over AWGN channel. The relative merits of the schemes are discussed, particularly with respect to their implementation complexity.

The thesis ends with a conclusions and further future works on each Chapter of this thesis.

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Chapter 2

Introduction to Multiple Access Communications

2.1 Introduction

In this chapter, we briefly review the principles and techniques of multiple access communications. Since a general introductions to the multiple access communications channel have been given in details elsewhere e.g. [Meulen 1977, Meulen 1986, El Gamal and Cover 1980, Farrell 1981, Wolf 1981, and Gallager 1985], the reader is referred to these works for more thorough treatment of the introductory material. We shall point out these works whenever is required throughout this chapter.

2.2 Multiple Access Channel

Competition for the use of existing communications resource leads to the question of simultaneous channel usage by more than one user. This kind of communication situation is known as MAC and illustrated in Figure 2.1, in which there are several users communicating with one receiver over a common channel. Examples of multiple access communications include several mini-computers sending data simultaneously to a large computer [Abramson 1970, and Schwartz 1977], several ground stations accessing a satellite repeater [Stiglitz 1973, Ince 1978, Nirenberg and Rubin 1978, and Sommer 1968], several mobile radio to base station [Steele 1988, Farrell 1985, and Farrell, et al., 1986], etc. or simply several students questioning a

professor at the same time.

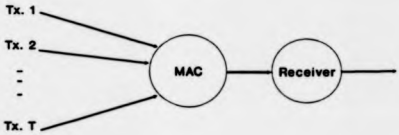

Figure 2.1
Multiple Access Communication Channel

In the MAC communication situation, each transmitter is fed by an information source, and each information source generates a sequence of messages. The generated successive messages arrive at random instants of time to be transmitted. The received signal is corrupted by noise and mutual interference between the transmitters during transmission over the channel. Therefore, the main issues in multiple access communication systems are interference between users, noise, and the random, or "bursty", arrivals of messages. Classification of the main research work on multiple access communication according to Gallager [Gallager 1985] is shown in Figure 2.2.

Figure 2.2
Classification of Multiple Access Communication

This classification shows that there have been mainly three bodies of research on multiple access communications, each using totally different models. These main areas are multiple access information theory, collision resolution and spread spectrum.

The multiple access information theoretic approach was initiated in 1961 by Shannon in his fundamental paper [Shannon 1961], and established in 1971 with a coding theorem developed by [Ahlswede 1971, and Liao 1972]. This approach appropriately models the noise and interference of the MAC but ignores the random arrival of messages. It is tacitly assumed by information theoretics that the sources have a reservoir of data to send which is never empty. Thus the theoretical results in this area do not address the question of the delay that arises in multiple access systems because of the random arrival times of data to be transmitted. This assumption is adequate from the theoretical point of view, since the random arrivals of messages can be smoothed out by appropriate source coding [Gallager 1985]. From a more practical point of view this model is not adequate because the long time intervals required for the source arrivals to be smoothed out are typically far greater than the tolerable delays. For more information refer to [Meulen 1977, Meulen 1986, Wyner 1974, and El Gamal and Cover 1980].

The collision resolution approach [Massey 1985, Massey and Mathys 1985, Abramson 1985, and Massey 1986], has always concentrated on the random arrival of messages and on the transmission delays which are due to the interference between users, but has generally ignored all other aspects of the underlying communication process (e.g. noise). This approach to the multiple access communication came in 1970 with Abramson's ALOHA network [Abramson 1970]. The basic idea of this system was that whenever a message (or packet) arrived at a transmitter, it would simply be

transmitted, ignoring all other transmitters in the network. If another transmitter was transmitting in an overlapping interval, interference would prevent the message from being correctly received, no acknowledgment would be sent, and the transmitter would try again later. The later time would be pseudorandomly chosen to avoid the certainty of another collision if both transmitters waited the same time. Over the years, this basic strategy has been improved, generalised, and analysed in many ways. For a more detailed exposition of the collision resolution approach refer to the special issue on random access which consist of many papers [Massey 1985, Massey and Mathys 1985, Abramson 1985, and Massey 1986].

Spread spectrum [Cook, et al., 1982, Scholtz 1982, and Pickholtz, et al., 1982], is a mode of communication originally developed to protect against jamming in a military environment. For multiple access communication using spread spectrum several sources can transmit at once using different modulating sequences, and each will look like broadband noise to the others. Therefore, in the multiple access spread spectrum approach the interference from other users is treated as additional, potentially intelligent, noise. For more detailed discussions and work carried in this area refer to the special issue on spread spectrum which consist of many papers [Cook, et al., 1982, Scholtz 1982, and Pickholtz, et al., 1982].

2.3 Multiple Access Techniques

In multi user systems, there are many ways of sharing the communications resource among the active users [Sklar 1988 pp476-531]. The main objective of all these multiple access techniques is that various signals share a communications resource

without creating unmanageable interference to each other in the detection process. The allowable limit of such interference is that signals on one communications resource channel should not significantly increase the probability of error in another channel. Classification of the most common multiple access techniques together with the CCMA under investigation is given in Figure 2.3.

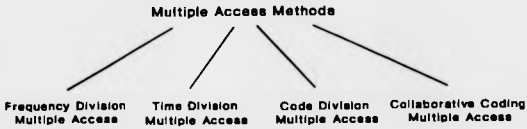

Figure 2.3
Classification of Multiple Access Methods

A brief description of these techniques is given here.

2.3.1 Frequency Division Multiple Access (FDMA)

In this technique [Sklar 1988 pp476-491], the communications resource sharing is accomplished by allocating certain frequency bands as shown in Figure 2.4, in which each user is assigned one of these bands to communicate. The assignment of a user to a frequency band is long term or permanent, the communications resource can simultaneously contain several spectrally separate signals. In its simplest form, each subscriber operates within a separate operating frequency band. Mutual interference between subscribers is kept to a minimum by using nonoverlapping frequency bands. For example, the first frequency band contains signals that operate between frequencies,

f_0 and f_1, the second between frequencies f_2 and f_3, and so on. The spectral regions between assignments, called guard bands, act as buffer zones to reduce interference between adjacent frequency channels.

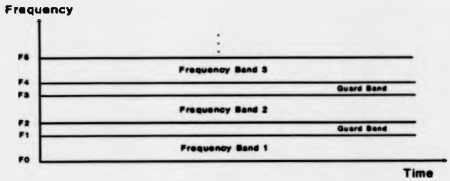

Figure 2.4
Frequency Division Multiple Access

The FDMA channels require no synchronisation or central timing, in which each channel is almost independent of all other channels. The main advantages of FDMA is its simplicity and the low cost of the equipment required. However, one of the problems with FDMA is the cross talk between different channels that can result in some performance degradation [Nirenberg and Rubin 1978, and Sklar 1988 pp476-491].

2.3.2 Time Division Multiple Access (TDMA)

Multiple access by time division [Sklar 1988 pp476-491] is accomplished by assigning each of the users the full spectral occupancy of the system for a short duration called a time slot, selected to eliminate signal overlap at the intended receiver(s) as shown in Figure 2.5. The unused time regions between slot assignments, called the guard time, allow for some time uncertainty between signals in adjacent time

slots, and thus act as buffer zones to reduce interference.

Figure 2.5
Time Division Multiple Access

In TDMA, time is segmented into intervals called frames. Each frame is further partitioned into assignable user time slots. The frame structure repeats, so that a fixed TDMA assignment constitutes one or more slots that periodically appear during each frame time. Each station transmits its data in bursts, timed so as to arrive coincident with its designated time slot(s). When the bursts are received, they are retransmitted together with the bursts from other stations. A receiving station detects and demultiplexes the appropriate bursts and feeds the information to the intended user.

TDMA has found wide application because of its ability to permit many subscribers to access a common channel without causing mutual interference. TDMA systems may suffer from other problems. Predominant among these are, strict inter-user synchronisation or the extra channel time required to ensure the TDMA channel allocations, excess hardware required to participate in a structured TDMA network, and the delay in accessing the channel [Dill 1977, Rubin 1979, Nirenberg and Rubin 1978, Stiglitz 1973, and Sklar 1988 pp476-491].

2.3.3 Code Division Multiple Access (CDMA)

Figure 2.6, illustrates the communications resource being partitioned by the use of a hybrid combination of FDMA and TDMA known as CDMA.

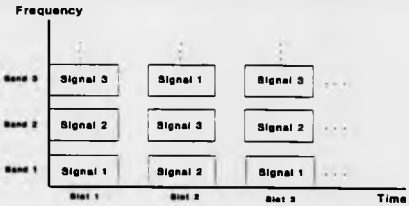

Figure 2.6
Code Division Multiple Access

CDMA is an application of spread spectrum techniques [Cook, et al., Scholtz 1982, Pickholtz, et al., 1982, and Sklar 1988 pp491-493]. A spread spectrum communication system can be defined as one in which: (a) the transmitted signal bandwidth is much greater than the minimum bandwidth necessary to send the message information, (b) all users have access to the whole time frequency space of the channel, (c) some function other than the message is used to determine the bandwidth of the transmitted signal, and (d) the signals, codes, of each user are all mutually orthogonal in some sense so that the signals, codes, may be unscrambled at the receiver by correlation [Ince 1978].

Spread-spectrum techniques can be classified into two major categories: direct-sequence and frequency hopping. In direct-sequence schemes, the data signal is modulated onto a digital, pseudo-random code sequence which has a digit rate much

higher than that of the data. Each of the user groups is given its own code. The user codes are approximately orthogonal, so that the cross-correlation of two different codes is nearly zero. The signals to be transmitted are modulated by these nearly orthogonal sequences over much broader frequency band than necessary. By using appropriate sequences, each transmitted signal will look like broadband noise to the others. The receiver can use the same sequence to despread the received signal to recover the transmitted messages. Frequency hopping schemes, can be easily viewed as the short-term assignment of a frequency band to various signal sources. The data signal is modulated onto a sinusoidal carrier, the frequency of which is caused to change in discrete increments, according to a pattern determined by a pseudo-random code sequence. Each user is given a set of hopping patterns such that each pattern of a given set is nearly orthogonal to all patterns of other sets.

The most important advantage of CDMA schemes, compared to TDMA, is that all the participants can share the full spectrum of the resource asynchronously. There is no need to precise time coordination among the various simultaneous transmitters, i.e., the transmission times of the different users' symbols do not have to coincide. The orthogonality between user transmissions on different codes is not affected by transmission time variations. This will become clear upon closer examination of the auto-correlation and cross-correlation properties of the codes. These schemes are advantageous under certain circumstances in that they permit flexibility as to the number and activity of the users, they degrade gracefully as the number of users increases, and there is an automatic trade-off between the number of users and the degree of error protection.

The TDMA, FDMA, and CDMA techniques may also be used in either fixed or

demand assigned multiple access modes. In the fixed assigned multiple access mode the transmission formats of the techniques does not change, even though the traffic load varies from time to time. In the demand assigned multiple access mode, the formats of transmission of the techniques are changed as needed, depending on the traffic demand. Consequently, the demand assigned mode is more efficient, but it usually costs more to implement and maintain [Sklar 1988 pp476-497].

2.3.4 Collaborative Coding Multiple Access (CCMA)

In situations where the bandwidth is a very restricted resource, conservation of the spectrum, which is a valuable and finite resource, is very important. For example, the radio frequency bands represent an inflexible resource and it is unlikely that significantly larger frequency bands will become available. Therefore, it is necessary to investigate efficient ways of sharing the available spectrum channels between as many users as possible. It is also of considerable importance to use a simple and effective multiple access coding technique capable of error control as well as the multiple access function. The CCMA schemes permits potentially efficient simultaneous communications by two or more users in the same bandwidth without subdivision in time, frequency or orthogonal code, though this scheme may be used in a mixed format with TDMA, FDMA, CDMA. It allows a substantial increase in the number of users that can access the system simulateneously, leading to a higher combined information rate and hence a potentially more efficient system. In addition to providing the multiple access function these schemes can incorporate certain degree of error protection against noise [Farrell 1981].

In collaborative coding, digital modulation and coding are intimately related, and the simultaneous signals from various users are demodulated together, as a combined

multi-level signal. This permits the use of relatively short and simple codes in contrast to the spread spectrum case. It can also be used with the single access modulation techniques (e.g. amplitude shift keying, phase shift keying and frequency shift keying), and applies to binary as well as multi-level signals, though at the cost of an increase in complexity.

These techniques exist which lie between the two extreme cases of TDMA and CDMA, and offer in certain circumstances the possibility of rate sums higher than unity with modest synchronisation requirements [Farrell 1981]. In TDMA, either strict inter-user synchronisation, or potentially wasteful time slot allocation is required. Where in CDMA, simultaneous transmission without inter-user synchronisation can be achieved, but this can be wasteful of bandwidth because of the relatively low number of users that can operate simultaneously.

Collaborative codes exists for MAC and broadcast channel (BC) [Farrell, et al., 1986]. In the MAC case, each user is provided with a code which enables the receiver to decode the individual information streams, by detecting the resulting combined signal. In the BC case, a combined coded signal is transmitted, and each receiver is able to detect and decode the information destined for it. The BC is the inverse of the MAC; the sources and their common encoder are all at the same locations, whereas the decoders and associated sinks are in different locations. The information broadcast may be private to each sink, or may have common elements.

These techniques have many applications, for example, digital mobile radio communication systems [Farrell 1983, Farrell 1985, and Farrell, et al., 1986], in which they can be applied to both mobile-to-base and base-to-mobile transmission. These techniques have also been proposed for optical fibre communication systems [Bridge

1986] and types of random access MAC, such as a satellite asynchronous multiple access system [Wolf 1978, and Weldon 1978].

2.4 Multiple Access Channel Models

The purpose of MAC models is to describe how the input signals interact in the channel to produce the channel output. Many MAC models have been proposed and used by various researchers. Classification of the discrete memoryless MAC models according to the input combining function in the noiseless case, is shown in Figure 2.7.

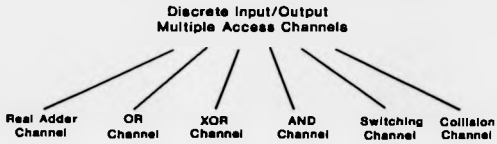

Figure 2.7
Classification of Discrete Input MACs

Each model is described very briefly here.

(i) Adder Channel: This is the most popular MAC model and has been considered for the information theory and coding aspects by various authors [Kasami, et al., 1975, Kasami and Lin 1976, Kasami, et al., 1983, Weldon 1978, Deatt and Wolf 1978, Chang and Weldon 1979, Ferguson 1982, Chang 1984, Braak and Tilborg 1985, Khachatrian

1982, Khachatrian 1984, and Wilson 1988]. The channel output symbol value is the arithmetic sum of the input symbol values, in the absence of noise. The T-user noiseless adder MAC is defined as a channel with T-input, and one output given by:

$$Y=\sum_{i=1}^{T} X_i \tag{2.1}$$

where X_i is the i-th channel input, Y is the channel output and the "Σ" sign denotes real addition. For example, consider the 2-user noiseless binary adder MAC shown in Figure 2.8.

Figure 2.8
2-user Noiseless Binary Adder MAC Model

It has two inputs, X_1 and $X_2 \varepsilon \{0,1\}$, and one output Y, which is the ordinary arithmetic sum of the inputs, $Y=X_1+X_2$, $Y \varepsilon \{0,1,2\}$. The arrowed lines represent the channel conditional probabilities $p(Y \mid X_1,X_2)$. The adder channel is also known as the binary input erasure MAC [Wolf 1975], because the output symbol "1" cannot be unambiguously decoded, even in the noiseless case.

(ii) OR Channel: This model is used by various researchers in different communication situations [Sommer 1968, Cohen, et al., 1971, Viterbi 1978, Gyorfi and Kerekes 1981, and Wolf 1981]. The output of the channel can be written for the T-user noiseless channel as;

$$Y = \bigvee_{i=1}^{T} X_i \qquad (2.2)$$

where the "V" sign is logical OR and $X_i \epsilon \{0\ 1\}$. That is, if X_i denotes the binary input of the i-th user, then the output of the channel is zero if and only if $X_1=X_2=...X_T=0$. For example, the 2-user noiseless binary OR MAC is shown in Figure 2.9, where the channel output is "0" if $X_1=X_2=0$ and "1" other wise.

Figure 2.9
2-user Noiseless Binary OR MAC Model

(iii) Exclusive-OR (Modulo-2) Channel: This channel [Wolf 1975, and Farrell 1981], output is modulo-2 sum (exclusive-OR function) of two or more input values. Thus all inputs and the output have the same alphabet {0,1}. This channel is also known as modulo-2 addition channel. The T-user noiseless channel output can be written as;

$$Y=\sum_{i=1}^{T} X_i \tag{2.3}$$

where the summation sign "Σ" is over GF(2) and $X_i \varepsilon \{0,1\}$. For example, the 2-user noiseless binary exclusive-OR MAC is shown in Figure 2.10, where the channel output is the modulo-2 sum of two binary inputs.

Figure 2.10
2-user Noiseless Binary Exclusive-OR MAC Model

(iv) AND Channel: This channel is also called binary multiplying channel as mentioned in [Meulen 1977]. The capacity region and coding strategy for this channel model is considered by [Schalkwijk 1982, and Schalkwijk 1983]. The noiseless T-user channel output can be written as:

$$Y=\prod_{i=1}^{T} X_i \tag{2.4}$$

where the multiplying sign "Π" is over GF(2) and $X_i \varepsilon \{0,1\}$. For example, the 2-user binary multiplying MAC is shown in Figure 2.11. The inputs and output are binary, and

the channel operation is defined by $Y=X_1X_2$.

Figure 2.11
2-user Noiseless Binary AND MAC Model

(v) Switching Channel: The switching channel model was originally introduced because it is in some sense similar to the binary input real adder channel but exhibits quite a different behaviour in terms of its capacity region [Vanroose and Meulen 1987, and Vanroose 1988]. For example, the 2-user noiseless binary switching MAC is shown in Figure 2.12. The channel accepts two binary inputs and outputs a ternary symbols according to the bit wise deterministic transitions. Therefore, the channel output for the 2-user noiseless case can be written as;

$$Y=\frac{X_1}{X_2} \tag{2.5}$$

where the $X_1/0$ is infinity and $X_2\varepsilon\{0,1\}$. This is similar to the 2-user binary adder channel which has two binary input and ternary output.

Figure 2.12
2-user Noiseless Binary Switching MAC Model

(vi) Collision Channel: This channel model [Gallager 1985, Massey 1985, Massey and Mathys 1985, and Massey 1986], is related to the collision resolution approach discussed in section 2.2. It is based on the assumption that whenever two or more users transmit simultaneously, the receiver can only detect that a collision took place. This is can be written as follows:

$$Y = X_i \text{; if only user i transmits}$$
$$= C \text{ (collision); if two or more users transmits} \qquad (2.6)$$

2.5 Multiple Access Information Theory

The information theory of MAC is fundamentally concerned with the simultaneous information transmission of several users through a common channel, as effectively as possible, in the presence of arbitrary interference and noise. The main objective of this information theory is to characterise the capacity region of MAC for

certain communication situations. That is, the determination of a set of simultaneously achievable rates which allow each user to communicate with the receiver with arbitrarily small error probability in the decoder output sequences. A rate point $R=(R_1,R_2,...,R_T)$, for a T-user MAC, is said to be achievable if and only if there exist encoders and decoder such that the probability of error in the information streams supplied to the sinks can be made as small as possible [Wolf 1978]. The information rate of the i-th user (R_i) can be described as a point in an T-dimensional rate space denoted by the coordinates $(R_1,R_2,...,R_T)$. The set of achievable coordinates for a particular channel is known as the channel capacity, C_{MAC}. One aspect of this capacity region is that the sum of the rates, $R_{\_\_}$ is upper bounded by the joint mutual information [Wolf 1981];

$$R_{\_\_} \leq \textit{maximum } I(X_1,X_2,...,X_T;Y) = C_{MAC} \tag{2.7}$$

where the maximum is taken over all product distributions on the input random variables $X_1,X_2,...,X_T$. The information theory of various multiple access communication situations have been investigated over many years by many researchers. Here, we briefly review the most important results achieved for the capacity region of MAC, for more details refer to [Meulen 1977, Meulen 1986, El Gamal and Cover 1980, Wolf 1980, Wolf 1981, Gallager 1985, and Wyner 1974].

The information theory of MAC communication was first mentioned in the basic paper of Shannon [Shannon 1961] in connection with his study of two-way communication channels. The capacity regions for the 2-user and 3-user discrete memoryless (DM) MAC with independent sources have been determined by [Ahlswede 1971]. Meulen [Meulen 1971] put forward a limiting expression and simple inner and

outer bounds on the sets of simultaneously achievable rates. Liao [Liao 1972] studied the general T-user DM-MAC with independent sources. He determined a set of rates which allow each transmitter to communicate with the receiver with an arbitrarily small probability of error. He also showed that for any set of rates outside the capacity region, the probability of error cannot be made arbitrarily small. The capacity region for the 2-user adder MAC with binary input is shown in Figure 2.13, as derived by [Ahlswede 1971, and Liao 1972].

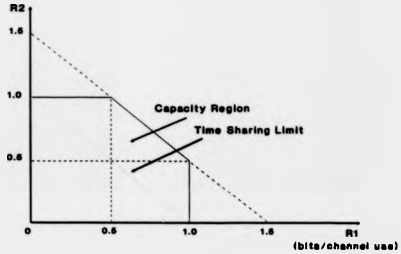

Figure 2.13
Capacity Region of 2-user MAC

The basic assumption is that the encoders are to operate independently of each other. Yet in the first derivation of the capacity region for this channel, it was assumed that the encoders utilised block codes and that the encoders produced codewords that were in block and bit synchronism. Furthermore it was assumed that the decoder was in block and bit synchronism with the encoders. Ahlswede [Ahlswede 1974] extended the two-input one-output multiple access case to two-input and two-output. Ulrey

[Ulrey 1975], generalised the multiple access case with two-input and two-output results by Ahlswede to arbitrary-input and arbitrary-output. The information capacity of Gaussian MAC have been determined by [Wyner 1974, and Cover 1975], for two independent users and also for various channel models [El Gamal and Cover 1980].

For the single access channel, it is known that feedback will not increase the capacity, but [Gaarder and Wolf 1975] have proved that the capacity of the 2-user binary adder MAC increases when adding noiseless feedback links from the output of the channel to the encoders. The capacity of an additive white Gaussian noise MAC with feedback link is also determined by [Thomas 1987] and shown that feedback can at most double the capacity. He also showed that for any T-user MAC, feedback cannot increase the total capacity by more that a factor of T. It is also noted in [Farrell 1981], that the occurrence of errors reduces the size of capacity region of the binary adder MAC, so that it lies within the capacity region shown in Figure 2.13.

All the aforementioned work was done under the assumption of block synchronism. The synchronisation techniques may take many forms [Meulen 1986, Kasami, et al., 1983, and McEliece and Posner 1977]. A MAC is said to be synchronous, if the encoders and the decoder are in block synchronism. If no block synchronism exists between the encoders and decoder, then the MAC is said to be asynchronous. However, it is quasi-synchronous, if the encoders are not in block synchronism with each other, but the decoder knows the block position of each encoder in the received sequence of symbols, and bit synchronism is maintained amongst the encoders and decoder. Various communication situations of asynchronous MAC and the problems arising when there is no time synchronisation have been investigated and discussed by number of authors [Meulen 1986, Cohen, et al., 1971, McEliece and

Posner 1977, Deatt and Wolf 1978, Grigor'ev 1979, Cover, et al., 1981, Gyorfi and Kerekes 1981, Poltyrev 1983, Kasami, et al., 1983, and Hui and Humblet 1985]. It is shown that the capacity of the quasi-synchronous MAC is the same as that of its synchronous counterpart. It has also been proved that the capacity of the binary adder MAC without block synchronisation but with symbol synchronisation is the same as for the fully (block and symbol) synchronised MAC [McEliece and Rubin 1976, and McEliece and Posner 1977].

2.6 Multiple Access Coding/Decoding Techniques

Various block and convolutional codes have been constructed over the years for synchronous and asynchronous MACs. These codes guarantee unique decipherability and can also incorporate a certain degree of error correction. Generally, block codes appear to perform better than convolutional codes [Farrell 1981]. Convolutional codes are not capable of achieving a rate sum greater than unity. In addition, block codes have the advantage that in a multi-user scenario various ways of simplifying the decoding process can be found, this does not seem to be true for convolutional codes. In this brief review, we mainly concentrate on the block coding techniques, however, many other convolutional and trellis codes exist [Chevillat 1981, Peterson and Costello 1979, Ohkubo 1977, Cohen, et al., 1971, Raychaudhuri and Rappaport 1979, and Lin and Wei 1986]. Block coding techniques have been constructed for noiseless and noisy channels. These codes provide the unscrambling function for the noiseless channels and the unscrambling and error control functions for the noisy channels.

2.6.1 Code Constructions for Noiseless Channel

The code constructions for noiseless synchronous MAC model have followed two main approaches [Mathys 1988]. The first one focused on achieving the bounds promised by multiple access information theory for the 2-user binary input adder MAC. Code constructions which belong to this class are the ones given in [Kasami, et al., 1975, Kasami and Lin 1976, Kasami, et al., 1983, Weldon 1978, Khachatrian 1982, Khachatrian 1983, Braak and Tilborg 1985, and Lin and Wei 1986]. The second approach with the same philosophy is to construct codes for the T-user noiseless binary input adder MAC shown in Figure 2.14, with the goal of achieving channel capacity asymptotically as T goes to infinity. Code constructions for this class of codes are the ones given in [Chang and Weldon 1979, Ferguson 1982, Chang 1984, and Wilson 1988].

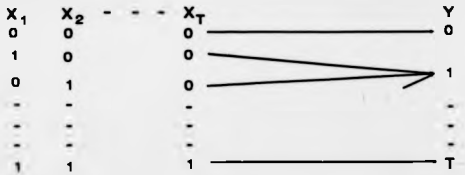

Figure 2.14
T-user Noiseless Binary Adder MAC Model

Uniquely decodable coding schemes have been extensively studied by [Kasami and Lin 1976, Kasami, et al., 1978a, and Kasami, et al., 1983] for the 2-user adder MAC. A code is said to be uniquely decodable if and only if all the received composite

codewords, which result from the users' codewords transmission, are distinct. A simple coding scheme for a 2-user uniquely decodable block code of length N=2, is constructed by [Kasami, et al., 1975, and Kasami and Lin 1976]. That is, user 1 has the codewords C_1=(00,11) and user 2 the codewords C_2=(00,01,10). Then (C_1,C_2) is a uniquely decodable code pair, in which all the received composite codewords all unique. Therefore, the decoder can unscramble the two messages without ambiguity. The overall rate sum achieved by this scheme is $R_{sum}=R_1+R_2=1.292$ bits per channel use, which is higher than time-sharing. The capacity of the 2-user binary adder MAC is shown in Figure 2.13, from which it is seen that the maximum value of R_{sum} is 1.5 bits per channel use.

This simple coding scheme is extended by the same author [Kasami, et al., 1975, and Kasami and Lin 1976] to block length N. The rates are $(R_1,R_2)=(1/N,\{\log_2(2N-1)/N)\})$, and the rate sum decreases with increase in N, tending to unity. Thus N=2 is both the simplest and the most efficient cases. Uniquely decodable code pairs have also been constructed by [Braak and Tilborg 1985], with higher rate pair using a computer search. It is noted by [Farrell 1981] that if each user is allowed three binary 2-tuples, and the channel is, in effect, capable of mapping the nine distinct pairs of binary 2-tuples presented to it into the nine distinct ternary 2-tuples, then the rate sum achieved is $R_{sum}=\log_2(3)= 1.585$. This is the highest rate sum that can be achieved with ternary channel symbols [Meulen 1977]. Other class of 2-user adder coding schemes and extension to them are also considered by the same authors [Kasami, et al., 1975, and Kasami and Lin 1976].

Weldon [Weldon 1978] point out that additional rate pairs can be achieved by using the time-sharing technique. That is, the users agree to use codes C_1 and C_2 for

certain time t_1, and then C_1' and C_2' for a certain time t_2, and so on repeatedly. For example, if $C_1=C_2'=(00,11)$, $C_1'=C_2=(00,01,10)$, and $t_1=t_2$, then $R_1=R_2=0.645$ and $R_{sum}=1.29$, which is the best equal rate pair found [Weldon 1978]. Wolf [Wolf 1978] has shown that, with number of users greater than two, the capacity is approximately, $0.5\log_2(\pi eT/2)$, and T-user binary codes with two codewords per user may be found with rate sums asymptotically close to capacity. Class of T-user uniquely decipherable codes for the noiseless binary MAC, with rates asymptotically equal to the maximal achievable value are also constructed by [Chang and Weldon 1979]. In this coding scheme each user code C_i $i=1,2,\ldots,T$, is given two codewords of length N. The overall rate achieved by this coding is thus T/N. Generalisation to these T-user codes for binary adder MAC is given in [Ferguson 1982]. A class of uniquely decodable codes of arbitrary length and asymptotically achieving the maximal capacity sum is given by [Chang 1984]. Uniquely decodable coding technique is also constructed for a T-active users out of T' (where T'>T) by [Mathys 1987, and Mathys 1988]. A set of T' codes have been constructed such that any T codes which are used at the same time yield a uniquely decodable code combination.

A simple block coding scheme has been constructed for the asynchronous 2-user binary adder MAC by [Wolf 1978]. This coding scheme does not require block synchronisation, though symbol synchronisation is still required. That is, user one uses the two codewords (00,11) and user two uses a code such that in any concatenation of the codewords two successive ONES never occur. For example, with $N_1=2$ and $N_2=3$, $C_1=(00,11)$ and $C_2=(000,001,010)$ gives $R_{sum}=1.028$. The decoder can synchronise with encoder one because the pattern ..00110.. could only have been caused by encoder one. By making the block length of C_2 longer and increasing the number of codewords, the

rate sum can be made to approach 1.169 asymptotically. By using a variable rate code for C_2 with the codewords (0,01), a rate sum of 1.167 may be achieved [Wolf 1978].

These schemes are extended in [Deaett and Wolf 1978] by permitting C_1 to consist of two codewords with K consecutive ZEROS or ONES. The results show that the best rate sum of 1.21 is obtained when K=3. Lower bounds on the achievable rates of uniquely decodable codes for the asynchronous adder channel have been derived in [Kasami, et al., 1983]. The asynchronous binary adder MAC can be reduced to the quasi-synchronous binary adder MAC by a scheme given in [Kasami, et al., 1976, and Kasami, et al., 1983]. That is, at the beginning of data transmission, each encoder sends a synchronising sequence to the decoder. The synchronising sequence has the property that, upon reception of a synchronising sequence from one encoder, the decoder can establish block synchronism with it, regardless of what codeword the other encoder is transmitting during the same period. After the reception of synchronising sequences from both encoders, the decoder has established synchronism with each individual encoder and the asynchronous binary adder MAC is reduced to the quasi-synchronous binary adder MAC.

In principle, all the T-user uniquely decodable coding schemes can be decoded with a look-up table using an exhaustive searching nearest-neighbour decoding scheme, since there is a one-to-one correspondence between each received N-tuple and the only possible set of T transmitted codewords. Therefore, in noiseless conditions the decoder is capable of decoding every possible received vector, without ambiguity, into the T codewords that were transmitted by the T encoders. An iterative decoding procedure has been constructed for the noiseless T-user uniquely decodable codes [Chang and Weldon 1979], which is simpler than look-up table for large T and N.

2.6.2 Code Constructions for Noisy Channel

Codes designed for noisy MAC not only must be uniquely decipherable but also must be able to correct transmission errors. Block codes which both unscramble symbols and correct errors exist for this channel. The general T-user noisy MAC model is shown in Figure 2.15.

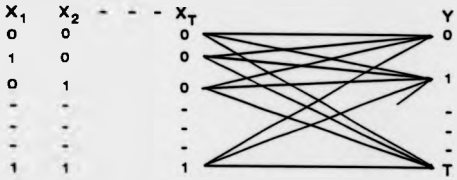

Figure 2.15
T-user Noisy Binary Adder MAC Model

Coding scheme known as δ-decodable code has been derived by [Kasami, et al., 1975, Kasami and Lin 1976, Kasami and Lin 1978a, and Kasami and Lin 1978b, and Tilborg 1978] for the noisy 2-user adder MAC, capable of correcting $t=\lfloor(\delta-1)/2\rfloor$ or fewer errors, where $\lfloor x\rfloor$ is the largest integer less than or equal to x. For example, a 2-user code (C_1,C_2) is said to be δ-decodable (δ>0) if and only if, for any two distinct pairs (u,v) and (u',v') in (C_1,C_2), the distance between the vector (u+v) and (u'+v') is greater or equal to δ. It has been proved that the constituent codes of a δ-decodable code pair have minimum distance at least δ [Kasami and Lin 1976]. If a code has minimum distance d_{min} and distance is a metric, its error correcting capability is $\lfloor(d_{min}-1)/2\rfloor$ [Peterson and Weldon 1972].

A method of error correction is reported in [Wolf 1975] for the two user case. C_1 is binary t-error-correction code with every digit repeated to bring the block length up to N. C_2 is a ternary t-error-correcting code with one ternary to two binary translation which again increases the block length to N. Decoding consists of a first stage which creates erasures when ambiguous pairs of digits are received, followed by separate binary and ternary error-and-erasure decoders for each user. A coding scheme for the noisy T-user adder MAC is constructed by [Chang and Weldon 1979]. Code construction which allows error correction for the T-user binary adder channel with two codewords per user is also given in [Wilson 1988]. Concatenated coding schemes are described by [Weldon 1978, and Ohkubo 1980] for the 2-user case. Also, concatenated code construction for a noisy binary adder MAC is given in [Mathys 1989] for only T-active users out of T' at a given time.

In noisy channel, the N-tuple which is received may differ from the transmitted codeword. In this case, the decoder must ask for a re-transmission of the message or a choice as which codeword was most probable sent has to be made. This kind of decoding where the received vector r is decoded into vector closest to r, guarantees correct decoding in the noiseless synchronous uniquely decodable coding scheme case. A decoding procedure for 2-user δ-decodable codes, that can correctly decode $\lfloor(\delta-1)/2\rfloor$ or fewer transmission errors is also given in [Kasami and Lin 1978b]. The decoding procedures utilise the structure of the codes to reduce the number of computations and the storage size for the two user case. However, the decoding complexity of this scheme still grows exponentially in both computation time and storage size as the code length increases for fixed rates. As in the single access communication situation the decoding table becomes unmanageably large as N and T

increases. Therefore, what is needed is a simple and systematic means of calculating the transmitted vectors from the received vector.

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Chapter 3

Information Transmission Capacity of Single Access Channel

3.1 Introduction

The channel capacity is a fundamental concept in the mathematical theory of communications. It was introduced by Shannon [Shannon 1948] to specify the asymptotic limit on the maximum rate at which information can be conveyed reliably over a channel. The information theory of SAC has been concerned with the reliable transmission of information from a single information source to a single information sink. Shannon showed that, there exists a capacity, C, for a given channel, and that communication can be achieved with an arbitrarily small probability of error for any rate R, smaller than C. The basic principles and concepts of constrained and unconstrained channel capacity can be found in [Gallager 1968, Blahut 1972, and Blahut 1987].

This theory gives the theoretical channel capacity value for maximum amount of information of a channel, given that the time taken to evaluate the data is infinite. In most practical situations the demodulation/decoding must give a result in a finite period of time. For example, in practical systems which uses digital signal processing (DSP) integrated circuitry, an analogue signal coming over a channel is sampled and digitised by means of an analogue-to-digital convertor (ADC) before any information can be extracted. The sampled signal must then be processed by the DSP in real time in order to recover the transmitted data. There are a number of parameters involved in this

process which influence the possible throughput and reliability of the data delivered to the final user (e.g. the number of quantisation steps in the ADC, the degree of signal clipping at the ADC input etc.)

The determination and optimisation of sampled and digitised SAC capacity under channel input and output practical constraints are considered in this chapter. The channel input constraints are those of signal amplitude, or signal amplitude and average power. The channel output constraint is that of signal clipping (SC) due to quantisation applied at the receiver. The input signal amplitude/output signal clipping (ISA/OSC) constrained capacity, and the input signal amplitude and average power/output signal clipping (ISAP/OSC) constrained capacity of an AWGN channel are determined separately.

3.2 Theoretical Basis and Development of Channel Capacity

In the general model of a digital communication system if we assume that the modulator and demodulator are considered to be parts of the channel, then we have a composite channel which can be characterised [Gallager 1968 pp71-72] by; (i) a set of possible inputs available at the input terminal, (ii) a set of possible outputs available at the output terminal, (iii) a set of conditional (transition) probabilities relating the possible outputs to the possible inputs. Therefore, a SAC consists of two sets X and Y, and a collection of conditional probabilities, $p(Y|X)$. The set X is called the "input alphabet" and the set Y is called the "output alphabet".

Consider discrete memoryless channel (DMC) [Gallager 1968 pp73-74] with finite input and output alphabets. Each output letter depends probabilistically only on

the corresponding input and this probabilistic dependence is independent of time. A single user consist of a source producing symbols from a finite alphabet according to a stationary probability, and an encoder mapping source symbol sequences to channel symbol sequences. The source produces a symbol every T_s seconds, and the channel transmits a symbol every T_a seconds. Therefore, the number of source sequences of length, k, for an alphabet of M elements is (M^k), and the information (I) contained in one such sequence is,

$$I = \log_2(M^k); \quad \text{bits} \tag{3.1}$$

Since k source symbol intervals translate into N channel symbol intervals, the condition $kT_s=NT_a$ is imposed. Therefore, the information transmission rate, R, in bits per second is,

$$\begin{aligned} R &= \log_2(M^k)/NT_a \\ &= \log_2(M)/T_s; \quad \text{bits per second} \end{aligned} \tag{3.2}$$

or

$$R = k\log_2(M)/N; \quad \text{bits per channel use} \tag{3.3}$$

where the limiting value to this maximum rate is the channel capacity.

3.2.1 Unconstrained Channel

The capacity of unconstrained channel can be written in terms of the average mutual information, I(X;Y), between channel input alphabet of M elements and output

alphabet of J elements [Gallager 1968 pp74] as follows;

$$I(X;Y) = \sum_{i}^{M} \sum_{j}^{J} p(i)\ p(j \mid i)\ \log(p(j \mid i)/p(j)) \tag{3.4}$$

where p(i) is the probability of input symbol i,

$$\sum_{i}^{M} p(i) = 1 \tag{3.5}$$

p(j) is the probability of receiving the output symbol j, which can be written as;

$$p(j) = \sum_{i=1}^{M} p(i)\ p(j \mid i) \tag{3.6}$$

and $p(j \mid i)$ is the transition probability. The i and j can take values between 1 and M, and $p(j \mid i)$ values will be a function of M. The largest average mutual information that can be transmitted over the channel in one use, maximised over all input probability assignments, is called the capacity of the channel and is denoted by C. That is,

$$C = \max_{p(i)}\ I(X;Y); \qquad i=1,2,\ldots,M \tag{3.7}$$

The units of C are bits per input symbol into the channel (bits per channel use) when the logarithm is base 2. If a symbol enters the channel every T_s seconds then the channel capacity in bits per second is C/T_s.

In the noiseless channel, the capacity is reduced to being the entropy of the channel output i.e.,

$$C = -\sum_{j=1}^{M} p(j) \log_2(p(j)) \tag{3.8}$$

If all the source symbol probabilities are assumed to be equal,

$$p(i) = 1/M; \quad \text{for all } i \tag{3.9}$$

then the unconstrained SAC capacity can be written as;

$$C = \log_2(M); \quad \text{bits per channel use} \tag{3.10}$$

The assumption of equiprobable occurrence of symbols is not always true for certain channel constraints, as will be seen later.

3.2.2 Constrained Channel

In practical systems, the information transmission capacity of a SAC is limited by channel input and output constraints, which is termed here as "practical" constraints [Gallager 1968, Smith 1971, Blahut 1987, and Honary, Ali and Darnell 1990]. The channel input is characterised by the input signal amplitude, A, and average signal power, σ_s^2, whilst the channel output is characterised by the number and separation of quantisation levels, QL, at the receiver. The input signal amplitude and average power

constraints are defined by restricting the channel input to have values within finite interval [-A,+A] and also to have average power equal to some specified value.

The channel output takes discrete values in the range [-SC,+SC], where SC is the signal clipping level at the receiver. The SC at the channel output is defined here as the output signal amplitude level beyond which the signal will be clipped, that is the maximum allowed output signal level. This signal level will effect the determination of quantiser step size. The range [-SC,+SC] is segmented according to the value of QL specified by the number of bits, b, at the output of the ADC. Since a word of length b can only represent 2^b distinct signal levels, i.e.,

$$QL = 2^b \tag{3.11}$$

then each sample of the signal output is quantised to one of 2^b signal levels. In the present case, the output signal is sampled by an 8-bit ADC, i.e. 256 distinct signal levels, and the step size of the quantiser is taken to be,

$$\delta q = 2SC/(2^b-1) \tag{3.12}$$

Hence, the constrained SAC capacity is defined as the maximum mutual information between input and output over a certain range of channel input probability distribution and certain range of signal clipping at the output.

Consider, a scalar AWGN channel characterised [Smith 1971, and Proakis 1989 pp128] by the expression,

$$Y = X + N \tag{3.13}$$

where Y, X, and N are all real-valued scalars; N is an additive Gaussian noise random variable with zero mean and variance σ_N^2; X is the channel input random variable assumed to take only values within a finite interval [-A,+A], where A is an arbitrary positive number; and Y is the channel output random variable, with different sample value each time it is received. In practice Y and N are samples of continuous random processes. Since the amplitude probability density function (PDF) of a Gaussian variable z is given by;

$$p(z)= \exp(-(z-m)^2/2\sigma_N^2)/\sqrt{(2\pi)}\sigma_N \tag{3.14}$$

where m and σ_N^2 are the mean and variance respectively, then it follows that, for a given sample value of $X=x_i$, the channel output Y is Gaussian with mean x_i and variance σ_N^2. That is,

$$p(Y \mid X=x_i)= \exp(-(Y-x_i)^2/2\sigma_N^2)/\sqrt{(2\pi)}\sigma_N \tag{3.15}$$

The average mutual information I(X;Y) for SAC has been shown to be [Proakis 1989 pp132, and Blahut 1987 pp273];

$$I(X;Y) = \sum_{i=1}^{M} \int_{-\infty}^{\infty} p(Y|x_i)\,p(x_i)\log_2(p(Y|x_i)/p(Y))\,dY \tag{3.16}$$

where p(Y) is the probability density function of the output given by;

$$p(Y) = \sum_{i=1}^{M} p(Y \mid x_i)\, p(x_i) \tag{3.17}$$

and $p(x_i)$ is the probability of occurrence of the input level x_i. Equation (3.16), modified in response to the quantisation process described by the output constraint, becomes;

$$I(X;Y) = \sum_{i=1}^{M} \sum_{-SC}^{SC} p(Y \mid x_i)\, p(x_i)\, \log_2(p(Y \mid x_i) \,/\, \sum_{j=1}^{M} p(x_j)\, p(Y \mid x_j)) \tag{3.18}$$

Since the channel output is quantised, the conditional probability function $p(Y \mid x_i)$ can be written for the AWGN as in Appendix A;

$$\begin{aligned} p(Y \mid x_i) &= \mathrm{Erf}(\,(y-x_i+\delta q/2)/\sigma_N\,) - \mathrm{Erf}(\,(y-x_i-\delta q/2)/\sigma_N\,), \text{ where } -SC<y<SC \\ &= \mathrm{Erf}(\,(y-x_i+\delta q/2)/\sigma_N\,), \quad \text{where } y=-SC \\ &= \mathrm{Erfc}(\,(y-x_i-\delta q/2)/\sigma_N\,), \quad \text{where } y=+SC \end{aligned} \tag{3.19}$$

where Erf(x) and Erfc(x) are the error function and the complementary error function of x respectively, given in Appendix A [Stremler 1982].

<u>Example:</u> Consider a binary input AWGN channel with two possible inputs X={A,-A}. Assume that the average mutual information is maximised when the input probabilities are p(A)=p(A)=1/2, i.e. the input symbols are equiprobable and SC=8. Hence, from equation (3.18), the capacity of the channel, in bits per channel use, can be written as;

$$C = 1/2 \sum_{-SC}^{SC} p(Y \mid A) \log_2(p(Y \mid A)/p(Y)) + 1/2 \sum_{-SC}^{SC} p(Y \mid -A) \log_2(p(Y \mid -A)/p(Y)) \tag{3.20}$$

where $p(Y \mid A)$ and $p(Y \mid -A)$ are defined by equation (3.19). Hence,

$$p(Y) = 0.5(p(Y \mid A)+p(Y \mid -A)) \tag{3.21}$$

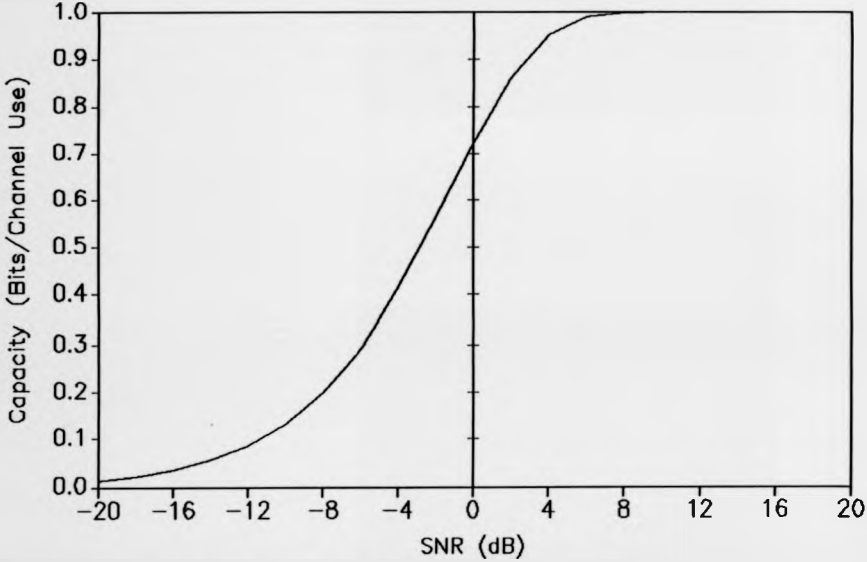
This capacity, C, is computed and presented graphically in Figure 3.1 as a function of signal to noise ratio, SNR, which is equal to $A^2/2\sigma_N^2$. It can be seen that C increases monotonically from 0 to 1 bit per channel use as the SNR increases. In this example, the capacity of the channel is obtained when the input symbols are equiprobable. However, the choice of equiprobable input symbols to maximise the average mutual information is not always the optimum solution [Proakis 1989 pp127-136] for the capacity formula given by expression (3.18). In general, a set of necessary and sufficient conditions on an input probability, $p(x_i)$, to maximise $I(X;Y)$, and thus to achieve the capacity of the AWGN memoryless channel with transition probabilities $p(Y \mid x_i)$, is given in [Gallager 1968 pp91, and Proakis 1989 pp133] as:

$$I(x_i;Y) = C \qquad \text{for all i with } p(x_i) > 0 \tag{3.22a}$$

$$I(x_i;Y) <= C \qquad \text{for all i with } p(x_i) = 0 \tag{3.22b}$$

in which $I(x_i;Y)$ is the mutual information for input x_i averaged over the outputs,

Figure 3.1 Channel Capacity versus SNR
for Binary Input AWGN Channel
(with Two Equiprobable Mass Points)



$$I(x_i;Y)= \sum_{-SC}^{SC} p(Y \mid x_i) \log_2(p(Y \mid x_i) / \sum_{j=1}^{M} p(x_j)\, p(Y \mid x_j)) \tag{3.23}$$

It is possible to check if the equiprobable set of input symbols satisfies the conditions in (3.22a) and (3.22b). If not, then the set of unequal probabilities, $p(x_i)$, that satisfy expressions (3.22a) and (3.22b) must be determined.

Therefore, for a given signal input and output signal clipping, the calculation of the capacity of an AWGN channel involves maximising a nonlinear function I(X;Y) of many variables, M, with both the inequality and equality constraints [Gallager 1968 pp82-97].

$$p(x_i) >= 0 \quad \text{and} \quad \sum_{i=1}^{M} p(x_i) = 1 \tag{3.24}$$

3.3 ISA/OSC Constrained Capacity

For an arbitrary fixed, but positive, finite real number A, let F_A denote the corresponding class of all distribution functions F having all the mass points positions on [-A,+A]. The mass points positions represent here the channel input amplitude levels which lie between -A and +A. Also, for certain fixed values of SC, let the output random variable Y take values between -SC, +SC, in steps of $\delta q=2SC/(2^b-1)$, assuming an b-bit quantiser. Therefore, for a particular SC value, the average mutual information I(X;Y) can be treated as a functional in the space F_A, of probability distributions F of the input random variable X, and written as;

$$I(F;SC)= \sum_{i=1}^{M} \sum_{-SC}^{SC} p(x_i) \; p(Y \mid x_i) \; \log_2(p(Y \mid x_i)/p_F(Y)) \tag{3.25}$$

where

$$p_F(Y)= \sum_{i=1}^{M} p(Y \mid x_i) \; p(x_i) \tag{3.26}$$

Hence the capacity of the ISA/OSC constrained channel [Smith 1971] can be written as;

$$C(A;SC)= \max_{F \in F_A} I(F;SC) \tag{3.27}$$

Now the capacity limits, for a fixed A and SC, can be defined as the maximum of a function of a finite dimensional vector, the components of which are the mass point positions (input amplitude levels) and the mass points values (the probability of occurrence of each level).

Suppose the correct number of mass points is known (say n) for particular values of A and SC; let $(x_1,x_2,...,x_n)$ denote the mass point positions of an arbitrary input distribution F, and let $(q_1,q_2,...,q_n)$ denote the corresponding mass point values. Then the cumulative distribution function F(x) can be written as;

$$F(x)= \sum_{i=1}^{n} q_i \; u(x-x_i) \tag{3.28}$$

where $u(x-x_i)$ denotes the unit step function at x_i.

Let $Z=(Z_1,\ldots,Z_{2n})$ be a vector comprising the components,

$$Z_i = q_i \qquad \text{for all } i=1,2,\ldots,n \tag{3.29a}$$

and

$$Z_{n+i} = x_i \qquad \text{for all } i=1,2,\ldots,n \tag{3.29b}$$

Then the output probability density function can be defined as;

$$p_Z(Y) = \sum_{i=1}^{n} Z_i \, p(Y \mid Z_{n+i}) \tag{3.30}$$

Hence the average mutual information can be treated as a function of the vector Z, and written as;

$$I(Z;SC) = \sum_{i=1}^{n} \sum_{-SC}^{SC} Z_i \, p(Y \mid Z_{n+i}) \log_2(p(Y \mid Z_{n+i})/p_Z(Y)) \tag{3.31}$$

Let G, denote the region of n-dimensional Euclidean space in which the vector Z must lie; let the following restrictions be imposed on the region G;

(i) all mass point values are non-negative,

(ii) all mass point positions lie in [-A,+A],

(iii) the sum of all the mass point values is unity.

Thus, G is the intersection of all the restriction sets within which the constraints are satisfied. Then, the ISA/OSC constrained capacity, C(A;SC), can be defined as;

$$C(A;SC) = \max_{Z \text{ in } G} I(Z;SC) \tag{3.32}$$

An optimisation algorithm from the NAG computer library routines [NAG 1984] has been used to solve the problem of maximising a known function I(Z;SC) over all vectors $Z=(Z_1,...,Z_n)$ which lie in a well defined restriction region G. The optimisation theorem used guarantees the existence of a unique maximising input distribution and provides necessary and sufficient conditions for achieving the maximum [Gallager 1968 pp82-97].

For a particular value of SC, and any arbitrary value of amplitude limit A, let n denote the number of elements in the vector Z. If n is known, then the determination of the capacity C(A;SC) is the well defined optimisation problem as discussed above. In general, if n is not known, the following steps are necessary:

(i) Start with a very small value of A, assuming the optimum number of mass points M is two, and then find the optimum capacity.

(ii) Increment A by a small amount, check as A increases whether the number of mass points M already used is sufficient or not. If not, increment M by one and apply the optimisation algorithm.

The programming procedure used to test whether M is sufficient or not is based on whether the optimisation program output forces the extraneous mass point values (if a larger value of M is used) to zero or not.

Since the Gaussian noise has a symmetric probability density function, the set of mass points is also symmetric i.e. $q_i=q_{-i}$ and $x_i=x_{-i}$. With this result, the optimisation problems can be formulated as the determination of the optimal pairs of mass points.

Hence, the optimal set of mass point pairs is characterised by some $(q_1,\ldots,q_n, x_1,\ldots,x_n)$, where n now denotes the number of mass point pairs (a mass point at the origin is also treated as a pair) and restricted by;

$$0 \le q_i \le 1/2, \tag{3.33a}$$

$$\sum_{i=1}^{n} q_i = 1/2, \tag{3.33b}$$

and

$$-A \le x_i \le 0 \qquad \text{for all } i=1,2,\ldots,n \tag{3.33c}$$

Since $\Sigma\, q_i=1/2$, then the number of independent variables is further reduced as follows:

(i) if n is odd (mass point pairs at the origin), then the optimal set of mass point pairs is characterised as $(q_1,\ldots,q_{n-1}, x_1,\ldots,x_{n-1})$, where $q_n=1-2\Sigma\, q_i$ for $i=1,\ldots,n-1$ and $x_n=0$;

(ii) if n is even, then the optimal set of mass point pairs is characterised as $(q_1,\ldots,q_{n-1}, x_1,\ldots,x_n)$ where $q_n=0.5-\Sigma\, q_i$ for $i=1,\ldots,n-1$.

The analysis program employs the above arrangement, which simplifies the problem further, and reduces the number of variables over which the function must be optimised. Consequently, the computation time to find the optimum input distribution and the capacity for each fixed amplitude limit is reduced. For a particular value of amplitude limit, A, the optimisation program is tested with different values of SC. The values of the SC that gives the largest information capacity value, and hence the optimum input distribution, is chosen to represent the optimum level of signal clipping.

3.4 ISAP/OSC Constrained Capacity

The ISAP/OSC constrained capacity problem is similar to the ISA/OSC shown above, with the added constraint of the average signal power, σ_s^2, being chosen to give the fixed ratio $A^2/\sigma_s^2=2$. For any A with a fixed σ_s^2 and SC limit, the ISAP/OSC capacity can be defined as;

$$C(A,\sigma_s^2;SC)= \max_{\substack{F \text{ in } F_{A\sigma} \\ \sigma_F^2 \le \sigma_s^2}} I(F;SC) \tag{3.34}$$

where $F_{A\sigma}$ is the class of all distribution functions, F, with the extra constraint that;

$$\sigma_F^2 = \sum_i q_i x_i^2, \qquad \text{for all } i \tag{3.35}$$

and σ_s^2 is defined to be equal to the ratio $A^2/2$. Thus, the capacity of an ISAP/OSC constrained channel can similarly be formulated as the maximum of a function of finite dimensional vectors. The components and restrictions are the same as before, except for an added restriction to include the variance constraint defined above.

The simulation procedure for the ISAP/OSC constrained channel is also similar to the ISA/OSC, with the following extra restriction imposed on the region G;

$$\sigma_F^2 = \sum_{i=1}^{n} q_i x_i^2 \le \sigma_s^2 \tag{3.36}$$

Then the ISAP/OSC constrained capacity is defined as;

$$C(A,\sigma_s^2;SC) = \max_{Z \text{ in } G} I(Z;SC) \tag{3.37}$$

The same optimisation routines from the NAG computer library [NAG 1984] were then used to solve this problem.

3.5 Simulation Results and Discussions

The simulation analyses are carried out with the following assumptions;

(i) The input to the channel is limited by the input signal amplitude A; i.e. it is allowed to take values between -A and +A.

(ii) The channel is AWGN with zero mean and unit variance.

(iii) The normalised SNR is $10\log_{10}(\sigma_s^2)$ dB.

(iv) The quantisation level QL is 256.

(v) The optimisation routine used is "E04UAF" from the NAG computer library routines and described in detail in [NAG 1984 pp1-23].

(vi) The ISAP/OSC constrained capacity is obtained with the extra assumption of A^2/σ_s^2 is equal to 2.

The optimum capacity of ISA/OSC and ISAP/OSC constrained channels are shown in Figures 3.2 and 3.3 as functions of the input signal amplitude A and normalised SNR, respectively. These capacities are achieved by unique and discrete input distributions taking a finite number of values. The optimum mass points

Figure 3.2 Constrained Capacities As a Function of Input Signal Amplitude

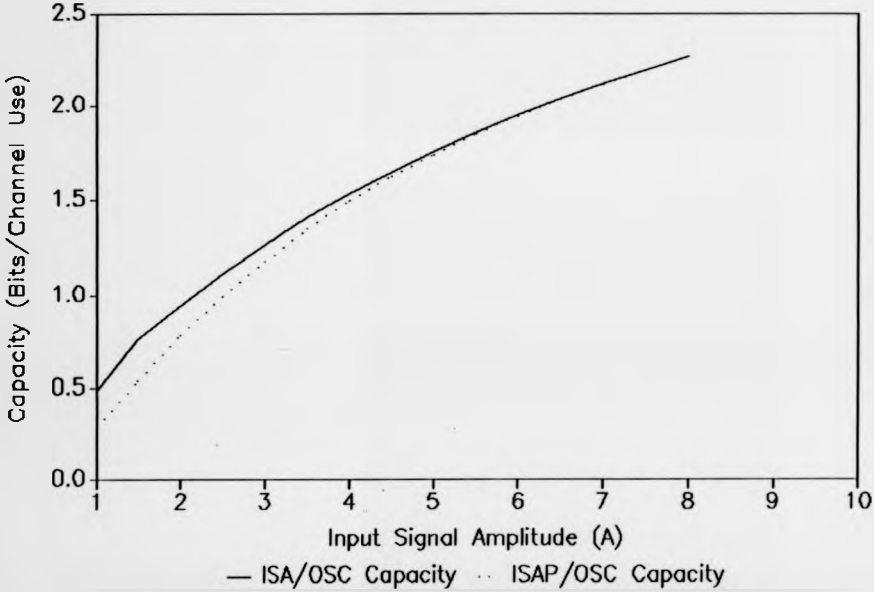
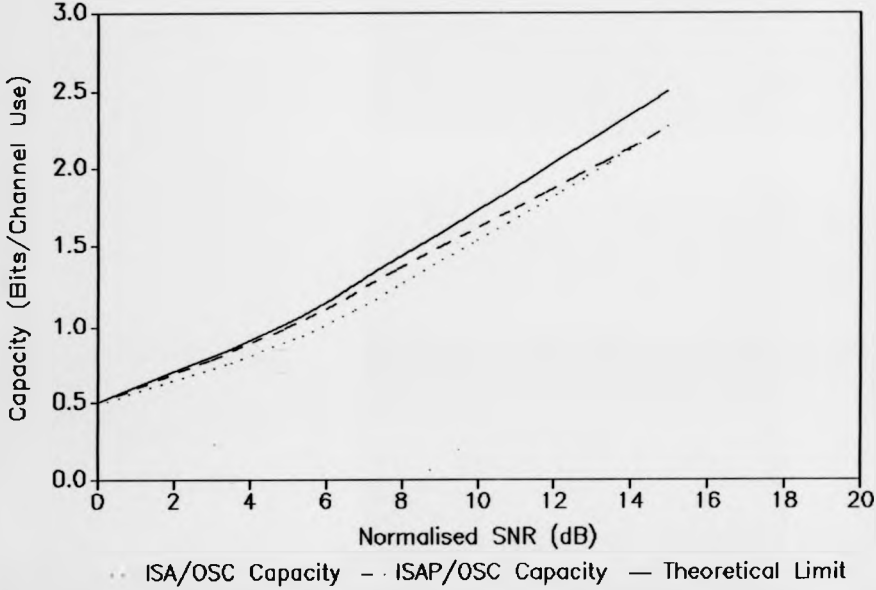


Figure 3.3 Constrained Capacities As a Function of Normalised SNR (dB)



distributions are shown in Figures 3.4a-3.4h and 3.5a-3.5h, for the two cases respectively, at selected values of A. It can be seen from Figure 3.2, that the capacity of ISAP/OSC is less than the ISA/OSC, due to the extra constraint imposed. However, as A increases, the peak power A^2 of the ISA/OSC case becomes closer to $2\sigma_s^2$, and hence the two capacity curves close together. The limiting theoretical curve of the AWGN channel capacity $0.5\log_2(1+SNR)$, is also calculated and included in Figure 3.3. This limiting curve represents the capacity as the ratio A^2/σ_s^2 tends to infinity. Comparing this limiting curve with the ISAP/OSC case, we can see that there is a loss in capacity, for example, about 6.5% at about 10dB normalised SNR which is the penalty for the peak power limitation of $2\sigma_s^2$. Also, it is found that there is a loss of about 11% in the ISA/OSC capacity at the same point of SNR and with peak power limitation of $1.6\sigma_s^2$.

However, the ISA/OSC capacity curve closes to ISAP/OSC at higher SNR as the ratio A^2/σ_s^2 tends to 2. It is also found that any ratio of A^2/σ_s^2 greater than 2 would yield capacity curves between the ISAP/OSC and the limiting curve in Figure 3.3. For example, for A=1.0, QL=256, and $A^2/\sigma_s^2=4$, the following is found;

(a) the capacity with ISAP/OSC constraints is between the two curve values on Figure 3.3,

(b) four mass points are needed rather than three for the input distribution,

(c) the optimum output signal clipping is found to be of the same value as when using the ratio $A^2/\sigma_s^2=2$.

Figure 3.4a Optimum Mass Points
Distribution of ISA/OSC Constraints
With A=1.0 and SC=6.0

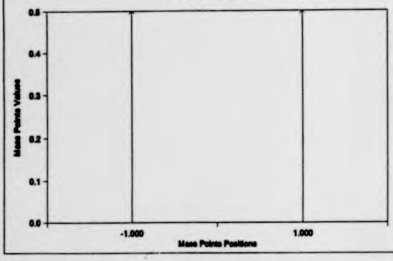


Figure 3.4b Optimum Mass Points
Distribution of ISA/OSC Constraints
With A=1.5 and SC=6.0

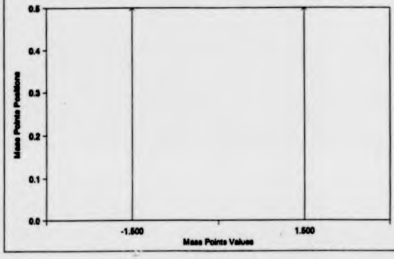


Figure 3.4c Optimum Mass Points
Distribution of ISA/OSC Constraints
With A=2.0 and SC=6.0

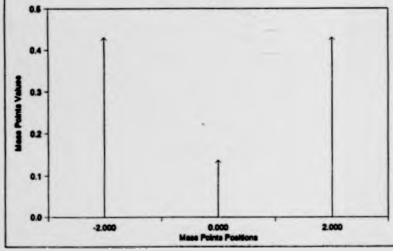


Figure 3.4d Optimum Mass Points
Distribution of ISA/OSC Constraints
With A=2.5 and SC=7.0

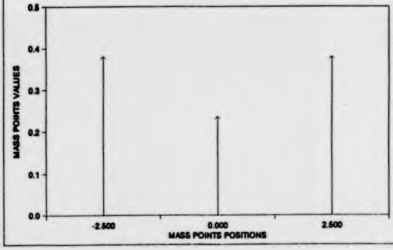


Figure 3.4e Optimum Mass Points
Distribution of ISA/OSC Constraints
With A=3.0 and SC=7.0

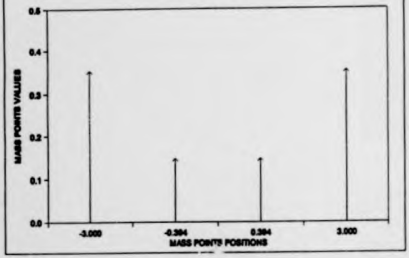


Figure 3.4f Optimum Mass Points
Distribution of ISA/OSC Constraints
With A=4.0 and SC=8.0

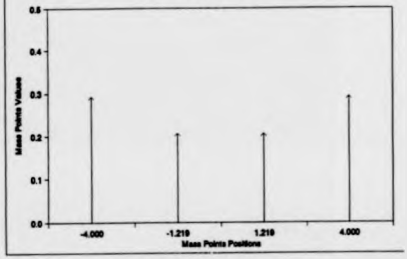


Figure 3.4g Optimum Mass Points
Distribution of ISA/OSC Constraints
With A=5.0 and SC=9.0

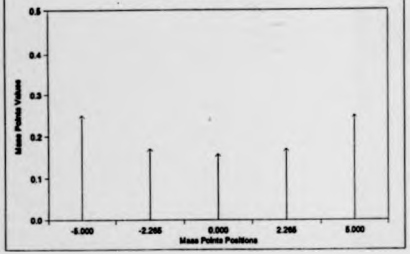


Figure 3.4h Optimum Mass Points
Distribution of ISA/OSC Constraints
With A=6.0 and SC=10.0

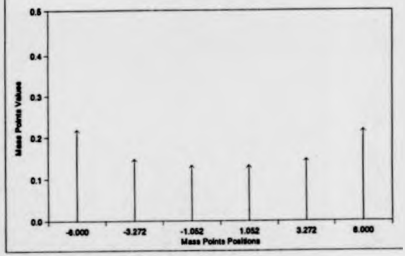


Figure 3.5a Optimum Mass Points Distribution of ISAP/OSC Constraints With A=1.0 and SC=6.0

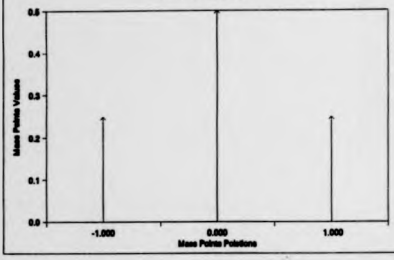


Figure 3.5b Optimum Mass Points Distribution of ISAP/OSC Constraints With A=1.5 and SC=6.0

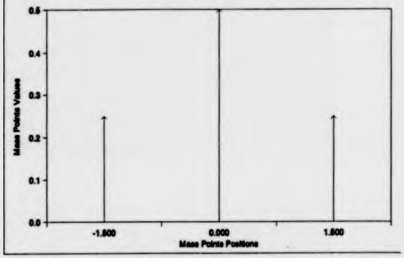


Figure 3.5c Optimum Mass Points Distribution of ISAP/OSC Constraints With A=2.0 and SC=6.0

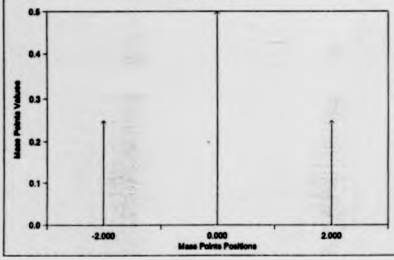


Figure 3.5d Optimum Mass Points Distribution of ISAP/OSC Constraints With A=2.5 and SC=7.0

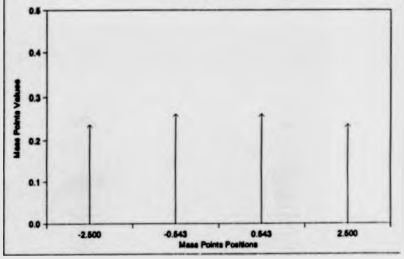


Figure 3.5e Optimum Mass Points
Distribution of ISAF/OSC Constraints
With A=5.0 and SC=7.0

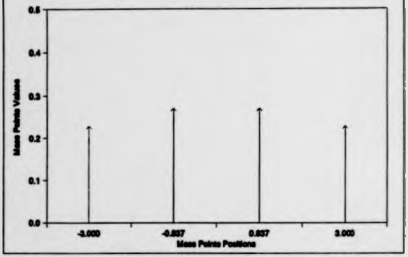


Figure 3.5f Optimum Mass Points
Distribution of ISAF/OSC Constraints
With A=4.0 and SC=6.0

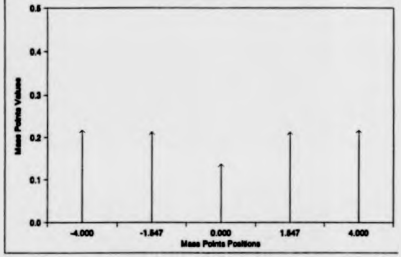


Figure 3.5g Optimum Mass Points
Distribution of ISAF/OSC Constraints
With A=6.0 and SC=8.0

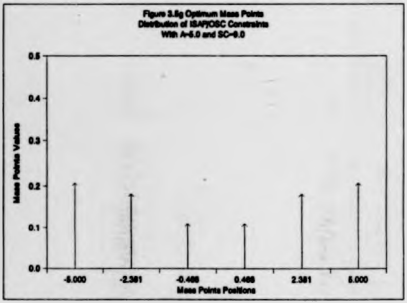
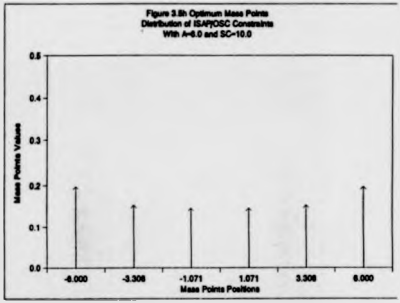


Figure 3.5h Optimum Mass Points
Distribution of ISAF/OSC Constraints
With A=6.0 and SC=10.0



The optimum output signal clippings that give the optimum capacity for the two cases are shown in Figure 3.6 as a function of the normalised SNR. The optimum PDF of the channel output, before the quantiser, are also calculated using the corresponding input distributions. The results of these calculations are shown in Figures 3.7a-3.7h, and 3.8a-3.8h for the two cases respectively, at selected values of A. It can be seen from Figure 3.6, that higher values of SC, are needed to compensate for the extra fixed ratio imposed for the ISAP/OSC case. It is also found that, as the OSC level increases (quantisation levels spacing increases), the capacity increases and then decreases as the OSC gets very large (due to very large quantisation level spacing). Therefore, the optimum OSC for large values of signal amplitude is greater than the OSC for those with smaller amplitude. This suggests that the quantiser should have more closely spaced levels at the low signal amplitudes and more widely spaced levels at the large signal amplitudes, as is the case with companding systems.

Arising from the constraints imposed on the input and output of the channel, the results obtained can be used to determine the number of amplitude levels at the input needed to maximise the capacity at different SNRs. For example, for the ISA/OSC constrained channel it is found by simulation that at SNR of about 10dB, we need four amplitude levels to achieve ISA/OSC capacity, whereas five levels are required to achieve ISAP/OSC capacity. Hence, these results can be used in signal design. Also, the very wide range of signal levels which may be input to, say, a radio communication receiver makes it essential that some form of automatic gain control or amplitude signal clipping system, should be incorporated. Therefore, the optimum OSC achieved also becomes of interest in such applications employing digital signal processing systems.

Figure 3.6 Optimum OSC As a Function of Normalised SNR (dB)

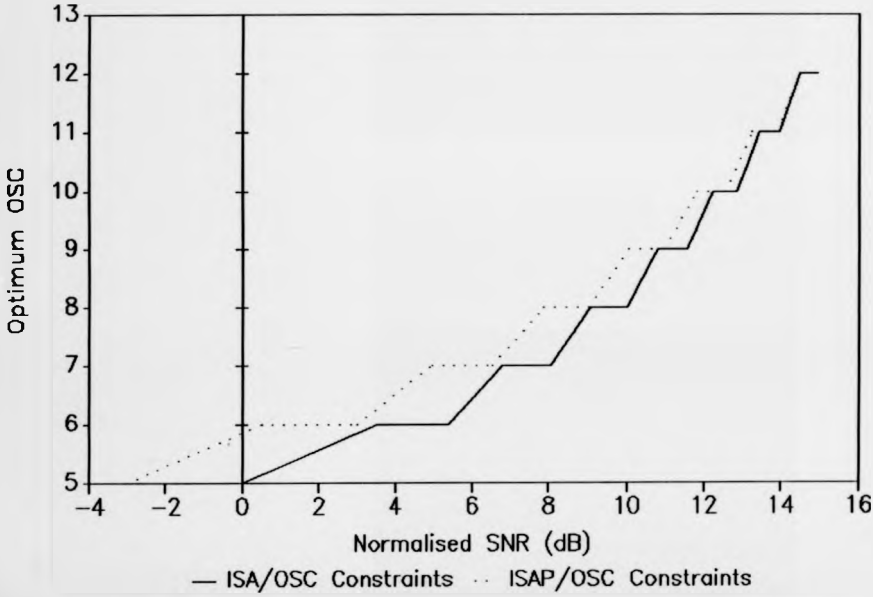


Figure 3.7a Output PDF
of ISA/OSC Constraints With
A=1.0, SC=6.0, M=2, and SNR=0dB

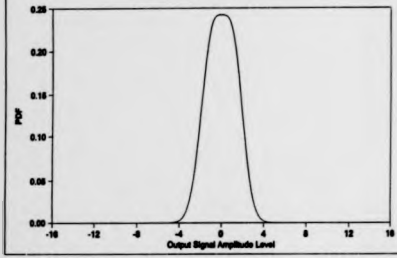


Figure 3.7b Output PDF
of ISA/OSC Constraints With
A=1.5, SC=6.0, M=2, and SNR=3.522dB

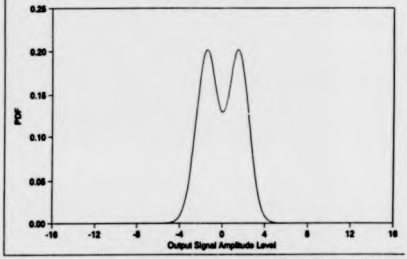


Figure 3.7c Output PDF
of ISA/OSC Constraints With
A=2.0, SC=6.0, M=3, and SNR=6.377dB

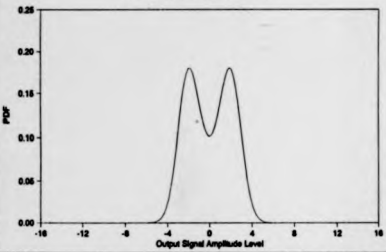


Figure 3.7d Output PDF
of ISA/OSC Constraints With
A=2.5, SC=7.0, M=3, and SNR=6.783dB

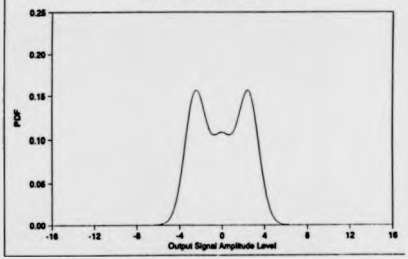


Figure 3.7e Output PDF
of ISA/OSC Constraints With
A=5.0, SC=7.0, M=4, and SNR=9.002dB

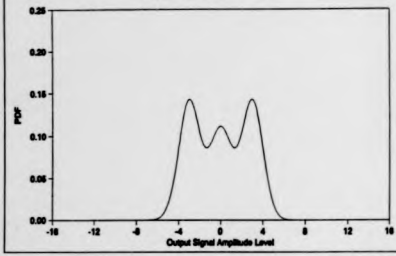


Figure 3.7f Output PDF
of ISA/OSC Constraints With
A=4.0, SC=8.0, M=4, and SNR=9.995dB

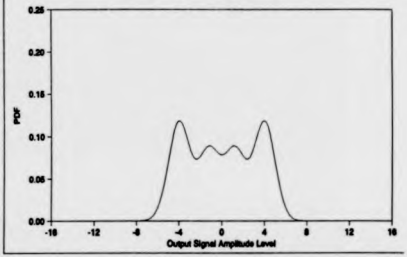


Figure 3.7g Output PDF
of ISA/OSC Constraints With
A=6.0, SC=9.0, M=6, and SNR=11.543dB

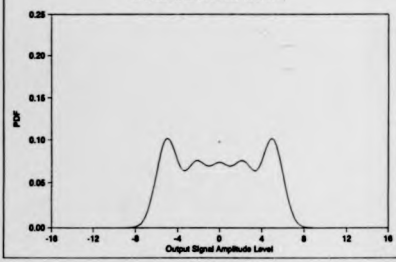


Figure 3.7h Output PDF
of ISA/OSC Constraints With
A=6.0, SC=10.0, M=6, and SNR=12.84dB

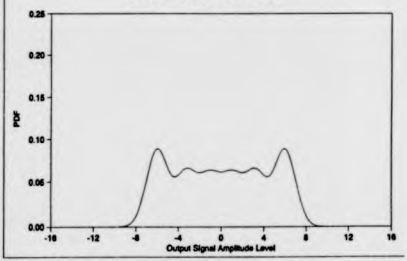


Figure 3.8a Output PDF
of ISAP/OSC Constraints With
A=1.0, SC=6.0, M=0, SNR=3.01dB

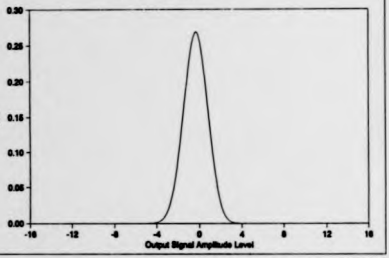


Figure 3.8b Output PDF
of ISAP/OSC Constraints With
A=1.5, SC=6.0, M=0, SNR=0.512dB

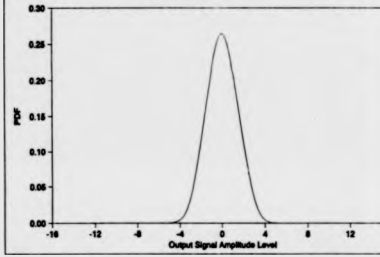


Figure 3.8c Output PDF
of ISAP/OSC Constraints With
A=2.0, SC=6.0, M=0, SNR=3.01dB

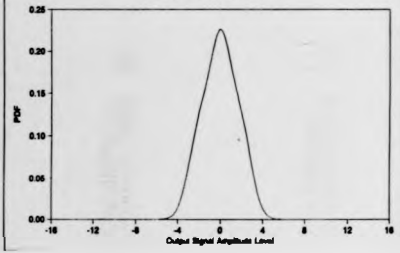


Figure 3.8d Output PDF
of ISAP/OSC Constraints With
A=2.5, SC=7.0, M=4, SNR=4.94dB

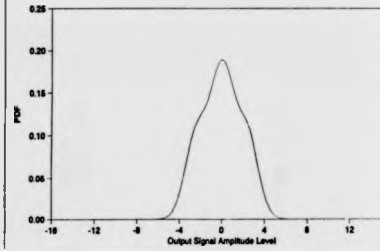


Figure 3.6e Output PDF
of ISAF/OSC Constraints With
A=3.0, SC=7.0, M=4, SNR=8.532dB

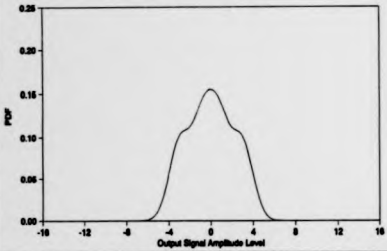


Figure 3.6f Output PDF
of ISAF/OSC Constraints With
A=4.0, SC=8.0, M=5, SNR=9.031dB

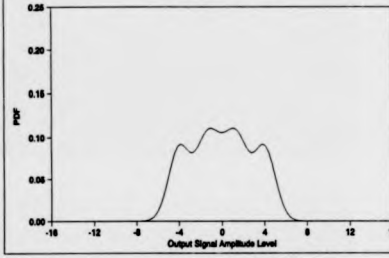


Figure 3.6g Output PDF
of ISAF/OSC Constraints With
A=5.0, SC=9.0, M=6, SNR=10.97dB

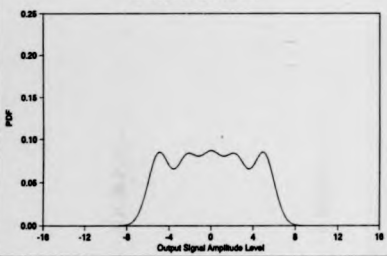
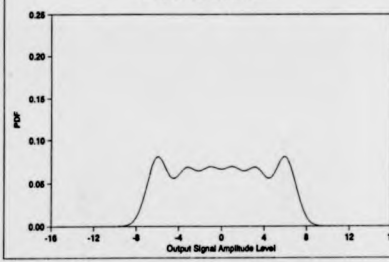


Figure 3.6h Output PDF
of ISAF/OSC Constraints With
A=6.0, SC=10.0, M=8, SNR=12.6dB



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Chapter 4

Information Transmission Capacity of Multiple Access Channels

4.1 Introduction

The information theory study of MACs [Wyner 1974, Meulen 1977, El Gamal and Cover 1980, Meulen 1986 and Gallager 1985] showed that the capacity region of a T-user MAC, allows the T users to communicate using a single receiver with an arbitrarily small probability of error. The maximum achievable rate sum of all the T users is called the T-user MAC capacity (C_{MAC}). This chapter is mainly concerned with the theoretical calculation of the information transmission capacity of certain MAC models. These channel models have theoretical and practical applications; these are described and developed for the capacity calculations in different communication conditions.

4.2 T-user Multiple Access Communication System

The general MAC communication system is depicted in Figure 4.1. There are T independent sources transmitting data to T separate destinations over a common discrete channel with one decoder serving T sinks. The inputs and their associated sources and encoders may be in different physical locations; for example different rooms in a building or different mobiles in an area. The signals over the channel will interfere, superimpose or combine in some way. The single decoder at the receiver is required to

unscramble and deliver the messages to their corresponding sinks, and if possible, without errors. The T messages generated from the T sources are encoded independently, and transmitted simultaneously over the common channel. Such that, each user is provided with a code which enables the receiver to unscramble the individual information streams, from the received combined composite signal.

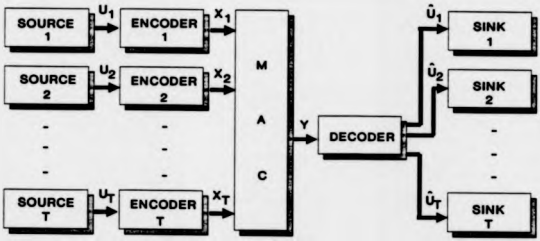


Figure 4.1
Block Diagram of T-user Multiple Access Communication System

(i) Information Sources: There are T independent active user's source information. The i-th user source output a symbol, U_i i=1,2,...,T, which is chosen from a finite alphabet. The symbols chosen from the respective alphabets are assumed to be equiprobable, and the different user's symbols are assumed to be statistically independent. The sources are in time synchronous, in which each source always sends a symbol to its encoder at the same instant of time.

(ii) Encoders: The function of the encoders is to map the source symbols $U_1, U_2, ..., U_T$ into transmitting sequences $X_1, X_2, ..., X_T$ each with block length, N. The encoders are

independent, i.e. the output sequence $X_i=(x_1,x_2,\ldots,x_N)$ from the i-th encoder is a function of U_i only. Each symbol of the sequence X_i takes values from a finite alphabet. The encoders are in time synchronous, in which each encoder sends one symbol each unit of time. Thus, if the i-th user's encoder has a code book containing CW_i codewords of length N and each codeword is equally likely, the transmission rate, in bits per channel use, of the i-th user is,

$$R_i = \log_2(CW_i)/N \tag{4.1}$$

and the rate sum, R_{sum}, of all the users is given by;

$$\begin{aligned} R_{sum} &= R_1+R_2+ \ldots +R_T \\ &= \sum_{i=1}^{T} \log_2(CW_i)/N \end{aligned} \tag{4.2}$$

<u>(iii) Discrete Memoryless Channel:</u> Discrete memoryless MAC (DM-MAC) is a channel which operates in a discrete time. The T-input takes values from finite alphabets, and the output depends on the T-input which also takes values from a finite alphabet. The output symbol depends only on the corresponding T-input and not on preceding or following inputs. The DM-MAC can be characterised by (a) T-input, $X_1,X_2,\ldots,X_T$, (b) single output, Y, and (c) transition probabilities, $p(Y \mid X_1,X_2,\ldots,X_T)$, of an output Y given the T-input. The channel operates synchronously, in which during each unit of time there are T-input sent to the channel and one output received from the channel.

<u>(iv) Decoder:</u> After the sequence Y is received, the decoder attempts to reliably

reproduce the users data streams in order to supply this information to the corresponding sinks. If we assume maximum likelihood decoding is used, the decoder would decode the sequence Y into $\hat{U}_1, \hat{U}_2, \ldots, \hat{U}_T$ estimated symbols.

4.3 Modelling of T-user M-ary Adder Channel

The T-user M-ary adder MAC model has been used by many researchers to characterise it's capacity region and construct a coding schemes to achieve this capacity [Liao 1972, Kasami and Lin 1976, Meulen 1977, Meulen 1986, El Gamal and Cover 1980, and Farrell 1981]. This channel has T-input, X_i i=1,2,...,T, and one output, Y. The channel output is given by the sum of the T-input symbols i.e.,

$$Y = \sum_{i=1}^{T} X_i \tag{4.3}$$

Where the "Σ" sign denotes real addition. Since the channel superposes the T-input signals in additive fashion, the T-user MAC is called T-user multiple access adder channel. In each symbol time interval, a combination of T-input symbols $(X_1, X_2, \ldots, X_T)$ is mapped into one of the output symbols, Y.

For the binary channel model where M=2, each user's input symbol is integer from the set {0,1} and the channel output symbol is integer from the set {0,1,...,T}. However, for the general case of M>=2, each user's input symbol is integer from the set {0,1,...,M-1}, and the channel output symbol is integer from the set {0,1,...,L-1}, where L is given by;

$$L = T(M-1)+1 \tag{4.4}$$

Therefore, the corresponding T-user M-ary adder channel maps K-size input alphabet, into L-size output alphabet, where K is given by;

$$K = M^T \tag{4.5}$$

For example, for binary channel we have $K=2^T$ and $L=T+1$. Examples of the noiseless T-input M-ary adder MAC model are shown in Figures 4.2 and 4.3, for the binary and the general M-ary channels. The number of output signal levels is calculated as a function of M and T, and shown in Figure 4.4.

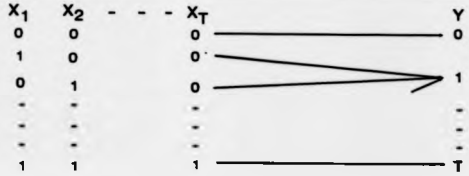


Figure 4.2
Noiseless T-user Binary Adder MAC Model

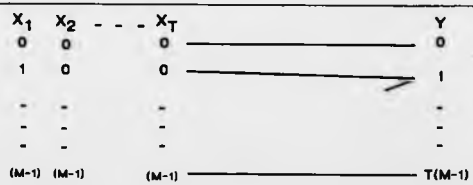
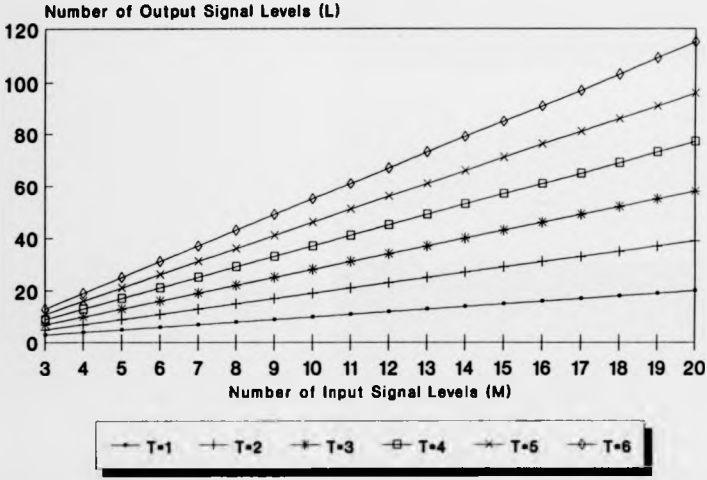


Figure 4.3
Noiseless T-user M-ary Adder MAC Model

Figure 4.4 Number of O/P Signal Levels versus I/P Signal Levels for T-user M-ary Adder MAC



(i) Noiseless Channel: Consider the noiseless T-user binary adder MAC. If we assume that the inputs of the channel $X_i \varepsilon \{0,1\}$ are statistically independent and identically distributed random variables i.e.,

$$p(1)=p=0.5 \quad \text{and} \quad p(0)=q=1-p=0.5 \tag{4.6}$$

The probability distribution of Y can be found as follows; from equation (4.3) the range of Y is the set of integers from 0 to T. The probability that Y=0 is simply the probability that all the X_i=0. Since the X_i are statistically independent, thus;

$$p(Y=0) = q^T \tag{4.7}$$

The probability that Y=1 is the probability that one X_i=1 and the rest of the, X_i=0. Since this event can occur in T different ways;

$$p(Y=1) = Tpq^{T-1} \tag{4.8}$$

In general, the probability that Y=j is the probability that j of the X_i are equal to one and T-j are equal to zero. Since there are,

$$d(j) = T!/j!(T-j)! \tag{4.9}$$

different combinations which result in the event Y=j, it follows that;

$$p(Y=j) = d(j)p^j q^{T-j} \tag{4.10}$$

Substituting for p=q=0.5 we get;

$$p(Y=j) = d(j)/2^T; \quad j=0,1,....,T \tag{4.11}$$

Therefore, substituting for d(j), the channel output probability distribution for the noiseless T-user binary adder MAC model can be derived as;

$$p(j) = T!/j!(T-j)!2^T; \quad j=0,1,...,T \tag{4.12}$$

In general for the case of M>=2, if the number of combinations that the M-ary input symbols superpose to one of the output symbols is d(j) j=0,1,...,L-1, i.e. the number of ways which result in the event Y=j. Then it follows that, the probability of Y=j is given by;

$$p(j) = d(j)/M^T; \quad j=0,1,...,L-1 \tag{4.13}$$

This is the generalised equation for the output probability distribution of noiseless T-user M-ary adder channel, where d(j) is given in equation (4.9) for the binary case. However, for M>2, d(j) is calculated using a computer search program. That is, for a given M and T, all the possible inputs (M^T) are generated, then d(j) is calculated by searching and counting the number of possible inputs which result in the j-th output symbol.

(ii) Noisy Channel: The noisy T-user M-ary adder MAC model is shown in Figure 4.5. It shows that there is a transition from any of the $K=M^T$ inputs to any of the $L=T(M-1)+1$ outputs. The transition probabilities $p(Y \mid X_1, X_2, \ldots, X_T)$ are defined by the channel noise, which represent the probability that the output symbol Y is received, given that the input symbols $X_1, X_2, \ldots, X_T$ are transmitted.

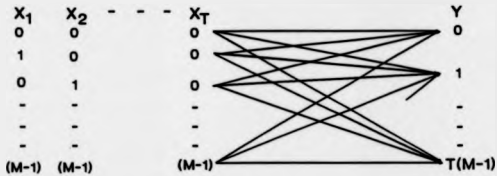


Figure 4.5
Noisy T-user M-ary Adder MAC Model

For the purposes of theoretical investigations of MAC in a noisy environment, a more appropriate noisy channel model is introduced [Wyner 1974, Chevillat 1981, Gallager 1985, and Honary, Ali and Darnell 1989]. This model consist of a cascade of two stages. The channel is considered equivalently as a noiseless adder MAC, adding all the inputs, in tandem with a single input noisy channel, as shown in Figure 4.6. Therefore, the noisy T-user M-ary adder MAC is characterised as a noiseless T-input MAC followed by a noisy L-input L-output single access channel. The inputs to the noisy single access channel are defined by the symbol S, $i=0,1,\ldots,L-1$.

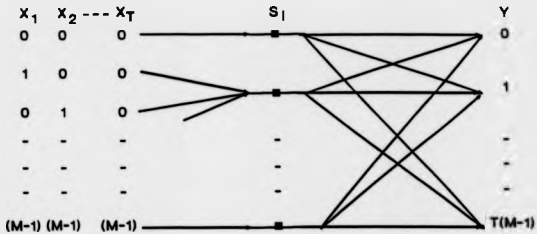


Figure 4.6
Equivalent Noisy T-user M-ary Adder MAC Model

Examples for the probability distribution for the noiseless T-user binary adder channel are shown in Figures 4.7a-d for various values of T. The PDFs at the noisy channel output are also calculated and shown in Figures 4.8a-d, for various values of channel signal to noise ratios (SNR). The SNR values are given by the ratio of average signal power per user to noise power for the binary transmission system.

4.4 Modelling of T-user M-ary frequency Channel

The T-user M-ary frequency model [Omura 1979, and Chang and Wolf 1981] is mainly motivated by its practical application. The T-user have available M-sinusoidal carriers, each at a different frequency $(f_1, f_2, ..., f_M)$. Every T_s seconds, each user selects one of these common frequencies to transmit. The receiver observe the composite signal and decide which frequencies have been transmitted during each period of T_s seconds. A receiver using M signal energy detectors tuned to the M signalling frequencies then

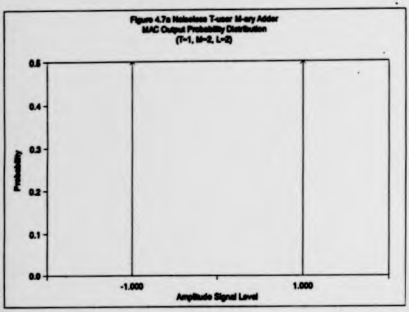


Figure 4.7a Noiseless T-user M-ary Adder
MAC Output Probability Distribution
(T=1, M=2, L=2)
Probability
0.0
0.1
0.2
0.3
0.4
0.5
-1.000
1.000
Amplitude Signal Level

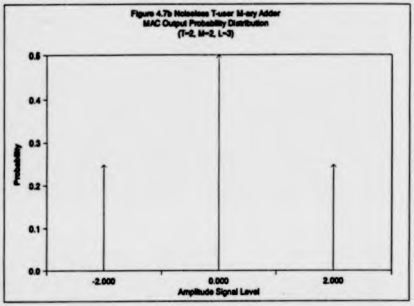


Figure 4.7b Noiseless T-user M-ary Adder
MAC Output Probability Distribution
(T=2, M=2, L=3)
Probability
0.0
0.1
0.2
0.3
0.4
0.5
-2.000
0.000
2.000
Amplitude Signal Level

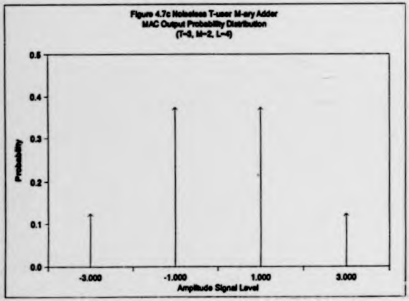


Figure 4.7c Noiseless T-user M-ary Adder
MAC Output Probability Distribution
(T=3, M=2, L=4)
Probability
0.0
0.1
0.2
0.3
0.4
0.5
-3.000
-1.000
1.000
3.000
Amplitude Signal Level

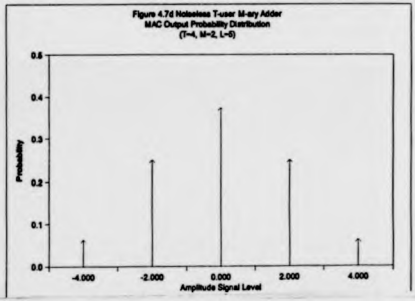


Figure 4.7d Noiseless T-user M-ary Adder
MAC Output Probability Distribution
(T=4, M=2, L=5)
Probability
0.0
0.1
0.2
0.3
0.4
0.5
-4.000
-2.000
0.000
2.000
4.000
Amplitude Signal Level

Figure 4.8a Noisy T-user M-ary Adder MAC
Output Probability Density Function
(T=2, M=2, L=3, SNR=20dB)

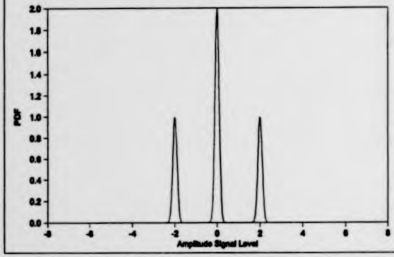


Figure 4.8b Noisy T-user M-ary Adder MAC
Output Probability Density Function
(T=2, M=2, L=3, SNR=10dB)

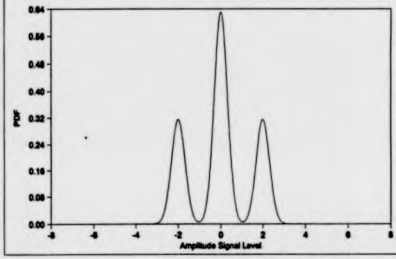


Figure 4.8c Noisy T-user M-ary Adder MAC
Output Probability Density Function
(T=2, M=2, L=3, SNR=-10dB)

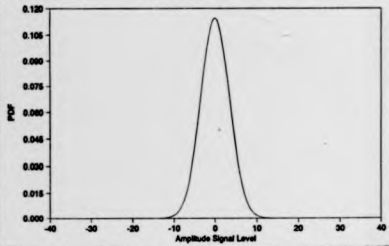
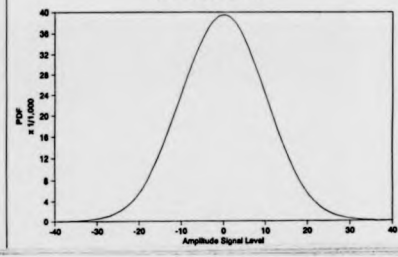


Figure 4.8d Noisy T-user M-ary Adder MAC
Output Probability Density Function
(T=2, M=2, L=3, SNR=-20dB)



obtains M samples whose values are proportional to the number of users transmitting at frequency f_i. The channel output, at each symbol interval T_s, can generally be represented by a vector Z;

$$Z = (z_1, z_2, \ldots, z_M) \tag{4.14}$$

where z_i is the measured energy at the frequency f_i during the T_s interval. Since the tones are assumed to be orthogonal, then we assume there is no energy component in z_i due to tones of frequency f_j where $j \neq i$. Two channel models are considered here.

<u>(i) Without Intensity Information:</u> During each symbol interval the i-th energy detector measure the presence of the frequency regardless of the number of tones at each frequency. That is;

$$z_i \in \{0,1\}; \quad i=1,2,\ldots,M \tag{4.15}$$

where "1" and "0" indicates the presence or absence of the i-th frequency f_i, respectively. That is, the presence of a frequency is always indicated by "1" without its intensity information.

The number of possible channel input for this model is given by $K=M^T$ and the number of possible channel output is given by;

$$L = \sum_{j=1}^{\Omega} d(j) \tag{4.16}$$

where Ω=min(T,M) and d(j) is given by;

$$d(j) = M!/j!(M-j)! \tag{4.17}$$

Examples of this type of channel model, for various values of T and M, are shown in Figure 4.9. It can be seen that when T=M=2, this channel model is equivalent to the 2-user binary adder MAC. The number of output signal levels is calculated as a function of T and M, and shown in Figure 4.10. The output probability distribution of these output signals can be written as;

$$p(j) = d(j)/M^T; \quad j=0,1,\ldots,L-1 \tag{4.18}$$

Substituting for d(j) we get;

$$p(j) = M!/j!(M-j)!M^T; \quad j=0,1,\ldots,L-1 \tag{4.19}$$

<u>(ii) With Intensity Information</u>: Here, during each symbol interval the i-th energy detector measure the presence of the frequency and its intensity information, i.e., an indication of how many tones of the same frequency are received. That is;

$$z_i \in \{0,1,\ldots,T\}; \quad i=1,2,\ldots,M \tag{4.20}$$

and

$$z_1+z_2+ \ldots +z_M = T \tag{4.21}$$

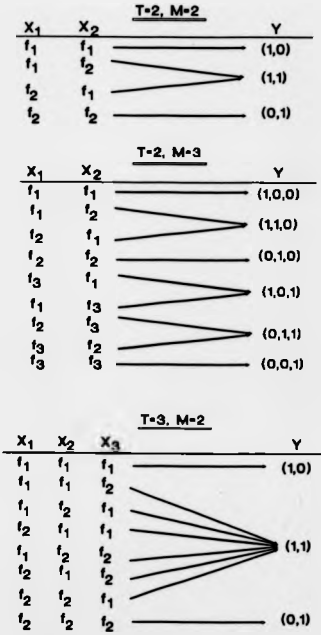
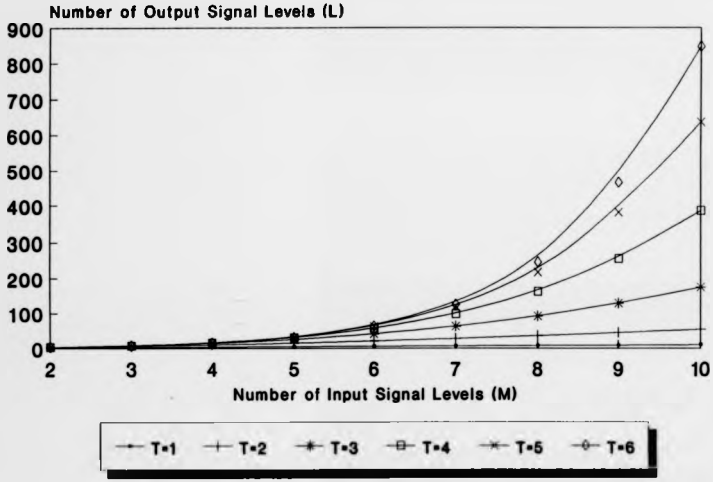


Figure 4.9 T-user M-ary Frequency MAC Without Intensity Information

Figure 4.10 Number of O/P Signal Levels vs I/P Signal Levels for T-user M-ary Freq. MAC Without Intensity Information



The number of possible channel inputs for this model is also given by $K=M^T$ and the number of possible channel output, L, is calculated by computer search. An examples of this channel model are shown in Figure 4.11, for various values of T and M. It can also be seen that when T=M=2, this channel model is equivalent to the 2-user binary adder MAC. The number of output signal levels is also shown in Figure 4.12, for various numbers of T and M. The output probability distribution, p(j), of this model is similarly given as in equation (4.18). However, d(j) in this case is calculated using computer search, i.e., for a given values of T and M, all the possible inputs are generated and then d(j) is calculated by counting the number of possible inputs which result in the j-th output symbol.

4.5 Multiple Access Capacity

The information capacity of MAC, C_{MAC}, can be written in terms of the average mutual information I(X;Y) between the input and output maximised over all possible input probabilities, p(i), that is;

$$C_{MAC} = \max_{p(i),\ i=1,2,\ldots,T} I(X;Y) \tag{4.22}$$

where, as indicated, the maximum is taken over all p(i) which describe statistically independent random variables.

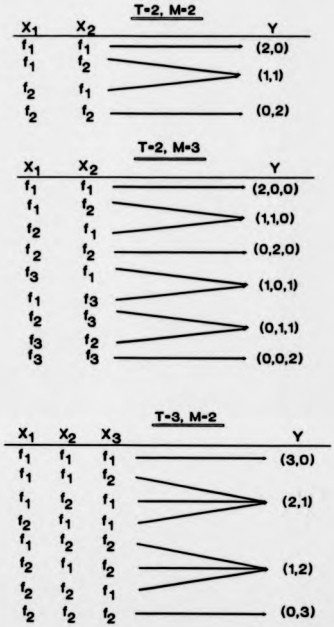
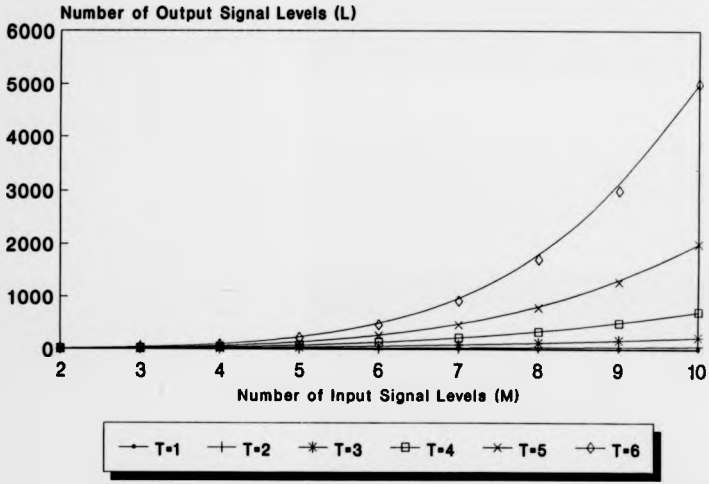


Figure 4.11 T-user M-ary Frequency MAC With Intensity Information

Figure 4.12 Number of O/P Signal Levels vs I/P Signal Levels for T-user M-ary Frequency MAC With Intensity Information



4.5.1 Noiseless Channel

Generally, the average mutual information between the input and output alphabets can be written [Gallager 1968 pp74], in bits per channel use, as follows;

$$I(X;Y) = \sum_{i=1}^{K} \sum_{j=1}^{L} p(i)\, p(j \mid i)\, \log_2(p(j \mid i)/p(j)) \tag{4.23}$$

where i and j are the i-th input and the j-th output, p(i) is the i-th input probability, p(j | i) is the conditional probability, and p(j) is the channel output distribution, which is given by;

$$p(j) = \sum_{i=1}^{K} p(i)\, p(j \mid i) \tag{4.24}$$

For the noiseless MAC, the average mutual information of MAC, reduces to the channel output entropy and can be written as;

$$I(X;Y) = -\sum_{j=1}^{L} p(j)\, \log_2(p(j)) \tag{4.25}$$

If we assume that the channel outputs are equiprobable, then the channel output probability can be written as;

$$p(j) = 1/L; \quad \text{for } j=1,2,\ldots,L \tag{4.26}$$

Thus equation (4.25) is reduced to

$$I(X;Y) = \log_2(L); \quad \text{bits/channel use} \tag{4.27}$$

This MAC capacity with the assumption of uniform distribution of the output signals is called the unconstrained capacity. However, the assumption of uniform distribution of output signals is quite general and dose not exhibit the actual distribution of output signals for this channel models. Consider the MAC capacity using the actual output probability distributions calculated previously for each channel model. If it is assumed that during each symbol interval, each user transmit statistically independent symbols with equal probability. Then, substituting for the actual output distribution p(j), into equation (4.25), the maximum average mutual information for the noiseless channel models can be written as;

$$I(X;Y) = -1/M^T \sum_{j=0}^{L-1} d(j) \log_2(d(j)/M^T) \tag{4.28}$$

where d(j) is the probability distribution for the j-th output symbol given previously for each channel model. This is the noiseless MAC capacity constrained by the actual output distribution of signals and will be referred to, some times, as the constrained capacity.

4.5.2 Noisy Channel

Consider more practical situation where, a number of transmitters attempting to

communication with a single receiver in the presence of AWGN [Wyner 1974, El Gamal and Cover 1980, Chevillat 1981, Gallager 1985, and Honary, Ali and Darnell 1989]. The channel output can be written as;

$$Y = \sum_{i=1}^{T} X_i + N \tag{4.29}$$

where N is Gaussian noise random variable with zero mean and variance σ_N^2, independent of the inputs X_i. In the calculation of this noisy channel capacity, we use the cascaded noisy channel model, in which the noisy MAC can be characterised as, a noiseless T-user M-ary adder MAC followed by an L-input, L-output noisy single user channel. The noisy stage channel input symbols are, S_i, where i=0,1,...,L-1. Therefore, the average mutual information is, at most, the capacity of single input channel with it's input constrained to the average power level. The capacity of T-user adder MAC over AWGN channel is computed as the average mutual information between the input and output of the noisy single user channel with the input symbols, S_i. That is,

$$I(S;Y) = \sum_{i=0}^{L-1} \int_{-\infty}^{\infty} p(S_i)\, p(Y|S_i) \log_2 (p(Y|S_i)/p(Y))\, dY \tag{4.30}$$

where $p(Y|S_i)$ is the channel transition probability, $p(S_i)$ is the i-th input symbol probability, and p(Y) is the probability density function of the output Y, which can be written as;

$$p(Y) = \sum_{i=0}^{L-1} p(S_i)\ p(Y \mid S_i) \tag{4.31}$$

For practical system, the signals which can be distinguished by the receiver is limited by the number of quantisation levels at the receiver [Honary, Ali and Darnell 1989, and Honary, Ali and Darnell 1990]. Therefore, if we assume that the channel output Y takes values between -SC and +SC in steps of δq given by;

$$\delta q = 2SC/(QL-1) \tag{4.32}$$

where SC and QL are the signal clipping and quantisation levels at the receiver, respectively. The quantisation levels can be written in terms of the number of bits, b, in the quantiser as $QL=2^b$. Therefore, the channel output is quantised and equation (4.30) can be modified in response to this as;

$$I(S;Y) = \sum_{i=0}^{L-1} \sum_{-SC}^{SC} p(S_i)\ p(Y \mid S_i) \log_2(Y \mid S_i)/p(Y)) \tag{4.33}$$

$p(Y \mid S_i)$ is the conditional probability given for Gaussian distribution as;

$$p(Y \mid S_i) = \exp(-(y-S_i)^2/2\sigma_N^2)/\sqrt{(2\pi)}\sigma_N \tag{4.34}$$

and since the channel output is quantised, the $p(Y \mid S_i)$ can be written as shown in Appendix A;

$$p(Y \mid S_i) = Erf((Y-S_i+\delta q/2)/\sigma_N) - Erf((Y-S_i-\delta q/2)/\sigma_N);\ \text{for } -SC<Y<SC, \tag{4.35a}$$

$$= Erf((Y-S_i+\delta q/2)/\sigma_N);\quad \text{for } Y=-SC, \tag{4.35b}$$

$$= Erfc((Y-S_i-\delta q/2)/\sigma_N);\quad \text{for } Y=+SC, \tag{4.35c}$$

4.6 Simulation Results and Discussions

Three T-user transmission systems are considered here and simulated for the calculation of the information capacity of MAC models.

(i) Binary Signalling: In this transmission scheme, the binary "0" and "1" are transmitted directly as signal levels "0" and "A", respectively, where A is the signal amplitude. The signals from the T-user are assumed to be superposed coherently by amplitude, giving composite signal symbols S_i at the noiseless MAC output. Therefore, S_i for M=2 can be written as;

$$S_i = i\sqrt{(2E)}; \quad i=0,1,\ldots,L-1 \tag{4.36}$$

where $E=A^2/2$ is the average signal energy per user. For example, for T=M=2, $S_i \varepsilon \{0,A,2A\}$ as shown in Table 4.1 below;

X_1	X_2	X_1'	X_2'	S_i
0	0	0	0	0
1	0	A	0	A
0	1	0	A	A
1	1	A	A	2A

Table 4.1 Composite Signal Symbols for Binary Signalling (T=M=2)

where X_i' is the i-th user transmitted signal.

(ii) Antipodal Signalling: In the binary antipodal signalling scheme, the binary "0" and "1" are transmitted as signal levels "-A" and "A", respectively. The signals from the T-

users are assumed to be superposed by amplitude at the channel. The composite signal symbols S_i for M=2 can be written as;

$$S_i = (2i-T)\sqrt{E}; \quad i=0,1,\ldots,L-1 \tag{4.37}$$

where $E=A^2$ is the average signal energy per user. For example, for T=M=2, $S_i \varepsilon \{-2A,0,2A\}$ as shown in Table 4.2 below;

X_1	X_2		X_1'	X_2'		S_i
0	0	⟶	-A	-A	⟶	-2A
1	0	⟶	A	-A	⟶	0
0	1	⟶	-A	A	⟶	0
1	1	⟶	A	A	⟶	2A

Table 4.2 Composite Signal Symbols for Antipodal Signalling (T=M=2)

(iii) On-Off Keying: In this case, the binary "0" and "1" are transmitted as signals "0" and "Asin(wt)" respectively. The signals from the T-users are assumed to be superposed noncoherently by power at the channel. The composite signal symbols S_i for the T-user binary channel can be written as;

$$S_i = (2iE); \quad i=0,1,\ldots,L-1 \tag{4.38}$$

where $E=A^2/4$ is the average signal energy per user. For example, for T=M=2, $S_i \varepsilon \{0,A^2/2,A^2\}$ as shown in Table 4.3 below;

X_1	X_2		X_1'	X_2'	S_1
0	0	⟶	0	0	0
1	0	⟶	$A^2/2$	0	$A^2/2$
0	1	⟶	0	$A^2/2$	
1	1	⟶	$A^2/2$	$A^2/2$	A^2

Table 4.3 Composite Signal Symbols for On-Off Keying (T=M=2)

4.6.1 Capacity of T-user M-ary Adder Channel

The unconstrained and constrained capacities are computed and shown graphically in Figures 4.13 and 4.14 respectively, as a function of M and T. It can be seen from these Figures that there is a reduction in the channel capacity when the actual output signal distribution is imposed. For example, the channel capacity decreases from 1.584 to 1.5 bit/channel use, for T=M=2. This reduction increases as T and M increases, for example, the capacity decreases from 3.7 to 3.2 bits/channel use for T=M=4.

The capacity of the noisy T-user M-ary adder AWGN channel is calculated as a function of E/N_o and shown in Figures 4.15-4.20, for various values of T and M, and various transmission systems. The signals are assumed to be superposed either by amplitude or by power over the channel giving the composite signal symbols S_i, as indicated above in section 4.6. The E/N_o, is the ratio of average signal energy per user (E) to the noise power spectral density ($N_o=2\sigma_N^2$). The number of bits in the quantiser is assumed to be equal to 8, and hence the quantisation levels QL=256. The SC values are chosen sufficiently large without causing any reduction in the capacity [Honary, Ali, and Darnell 1989, and Honary and Ali 1990].

Figure 4.13 Capacity of T-user M-ary
Adder MAC with Uniform O/P Distribution

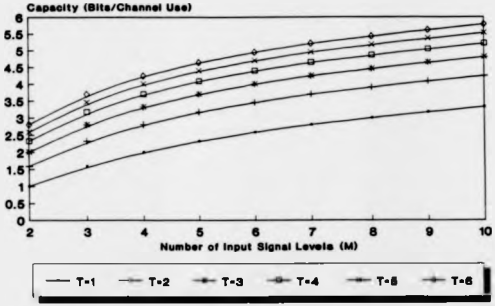


Figure 4.14 Capacity of T-user M-ary
Adder MAC with Actual O/P Distribution

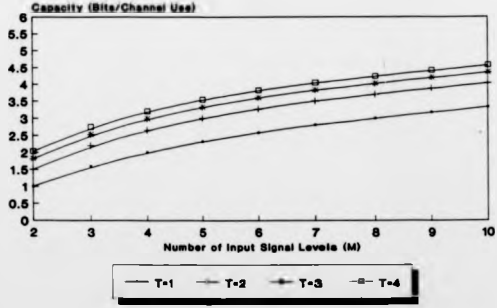
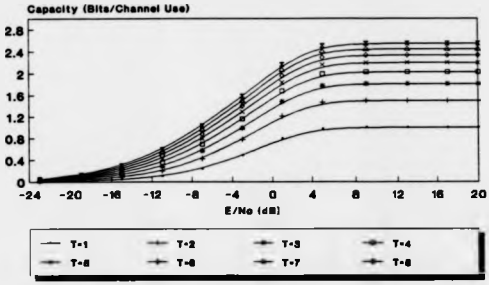
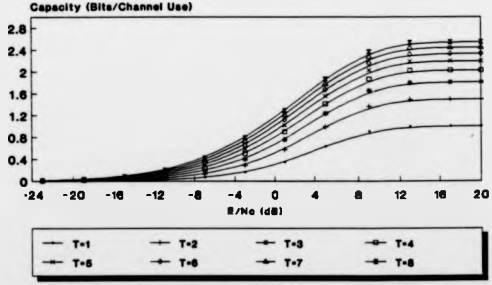


Figure 4.15 Capacity of T-user M-ary Adder MAC versus E/No (dB)
(M=2, Antipodal Signalling Scheme)



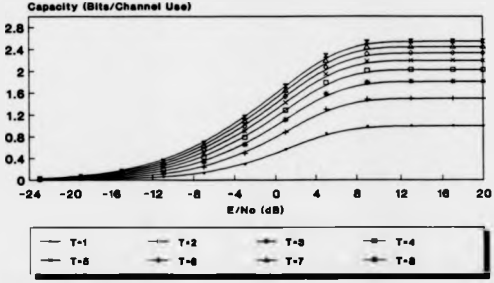
Additive superposition of Signals

Figure 4.16 Capacity of T-user M-ary Adder MAC versus E/No (dB)
(M=2, On-Off Keying Scheme)



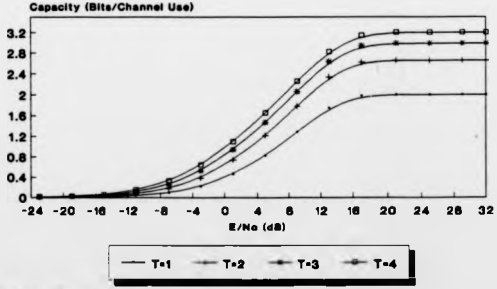
Power Superposition of Signals

Figure 4.17 Capacity of T-user M-ary
Adder MAC versus E/No (dB)
(M=2, Binary Signalling Scheme)



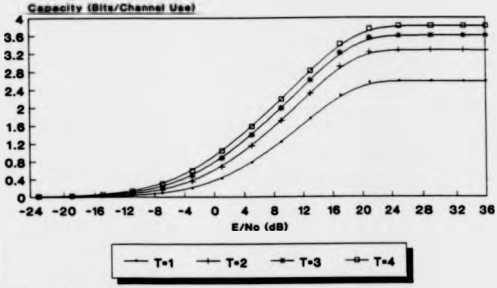
Amplitude Superposition of Signals

Figure 4.18 Capacity of T-user M-ary
Adder MAC versus E/No (dB)
(M=4)



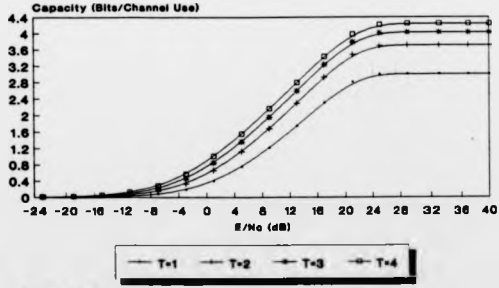
Binary Signalling Scheme
Amplitude Superposition of Signals

Figure 4.19 Capacity of T-user M-ary
Adder MAC versus E/No (dB)
(M=6)



Binary Signalling Scheme
Amplitude Superposition of Signals

Figure 4.20 Capacity of T-user M-ary
Adder MAC versus E/No (dB)
(M=8)



Binary Signalling Scheme
Amplitude Superposition of Signals

The curve of T=1, in Figures 4.15-4.20, corresponds to T-time sharing for any number of users, T>=1. It can be seen that the capacity of T-user M-ary adder MAC exceeds that of time sharing substantially without increase in the size of the M-ary signalling alphabets, i.e. employing the same number of input signal levels. That is, for time sharing to achieve the same rates, a higher number of M-ary levels must be used.

For example, in Figure 4.15 where coherent combining of signals are used, for a rate of 1 bits/channel use, the E/N_o is about 9dB for T=1, and about -1dB for T=2. Also, for a rate of 0.5 bits/channel use, the E/N_o is about -3dB for T=1, and about -6dB for T=2. This suggests that gains of about 10dB and 3dB may be achieved by T-user MAC schemes over uncoded and coded time sharing T=1, respectively. Higher gains appear to be achievable for T>2. For noncoherent combining of signals, Figure 4.16 shows that for a rate of 1 bits/channel use, the E/N_o is about 15dB for T=1, and about 5dB for T=2. Also, for a rate of 0.5 bits/channel use, the E/N_o is about 3dB for T=1 and about 0dB for T=2. This suggests that at high E/N_o, noncoherenet superposition of signals gives the same gain as the coherent case. However, only coherent combining promises substantial coding gain at low E/N_o.

It can also be seen from Figure 4.17-4.20, that a higher transmission rate, and hence higher gain, is possible to achieve employing M-ary signalling within a given bandwidth. However, this increase in information rate comes at the expense of added transmitter power. For example, for T=2, the capacities are equal to 1.5, 2.655, and 3.7 bits/channel use, for M=2, 4, and 8, respectively. However, the E/N_o needed to achieve this increase in capacity are about 12dB, 20dB, and 28dB, respectively, for binary transmission system and coherent combining of signals.

4.6.2 Capacity of T-user M-ary Frequency Channel

The capacities of constrained and unconstrained channel models are calculated and shown in Figures 4.21-4.24, as a function of M and T. It can be seen from these figures that there is a reduction in the channel capacity when the actual output signal distribution is imposed. It can also be seen from Figure 4.22, that for a fixed number of M, the capacity of the T-user M-ary frequency without intensity information does not always increase as T increases. For example, the capacity values for M=2 are 1.5, 1.06, and 0.66 for T=2, 3, and 4 respectively. This suggests that the number of frequencies M, should be chosen such as to optimise the capacity for a given number of users, T.

Generally, the capacity of T-user MAC with intensity information is higher than that without intensity information. This is due to the extra information gained from the intensity information of the received signal energy level, which results in a higher number of output signal levels to accommodate this extra information. It should also be noted here that, generally, the number of output signal levels at the receiver increases rapidly as M and T increases, as shown in Figures 4.4, 4.10, and 4.12. However, this increase in the number of output signal levels, from a practical point of view, may limit the number of active users, due to practical constraints that may be imposed at the receiver.

Figure 4.21 Capacity of T-user M-ary
Freq. MAC Without Intensity Information
(with Uniform O/P Distribution)

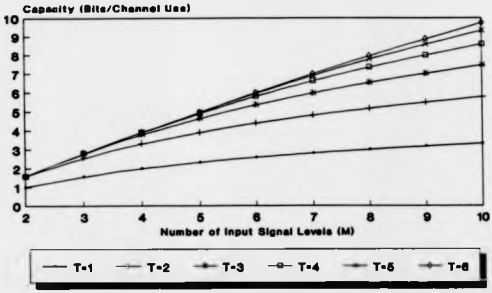


Figure 4.22 Capacity of T-user M-ary
Freq. MAC Without Intensity Information
(with Actual O/P Distribution)

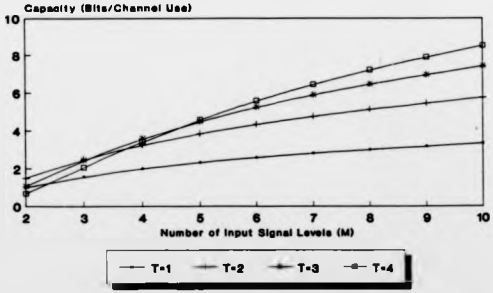


Figure 4.23 Capacity of T-user M-ary Frequency MAC With Intensity Information (with Uniform O/P Distribution)

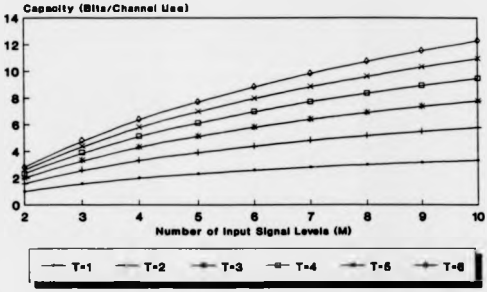
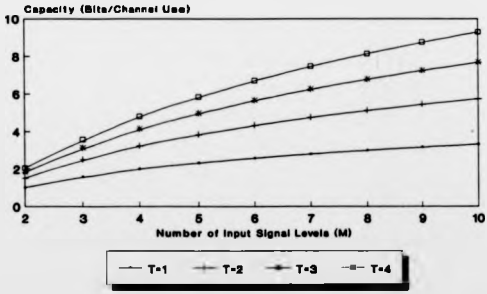


Figure 4.24 Capacity of T-user M-ary Frequency MAC With Intensity Information (with Actual O/P Distribution)



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Chapter 5

Collaborative Coding/Decoding Multiple Access Techniques

5.1 Introduction

It is highly desirable to use simple and effective multiple access coding/decoding techniques which are capable of multiple access function and error control. The collaborative coding schemes are constructed to allow the simultaneous transmission by several users over a common channel and can also be extended to incorporate a certain degree of error protection capability. In this chapter, the multiple access coding/decoding schemes for the T-user binary adder MACs are investigated to utilise these above functions.

5.2 T-user Encoding Techniques

In the collaborative coding schemes, the T messages generated from the T sources are encoded independently such that they are interference free during simultaneous transmission over a common channel (refer to Figure 4.1). Each user is provided with a code which enables the receiver to unscramble the individual information streams, by detecting the resulting combined signal. The data from the i-th source, U_i, where i=1,2,...,T, is encoded by the i-th encoder according to a uniquely assigned block code C_i of length N. The resulting codeword vector X_i is then transmitted over the channel where it combines with the other (T-1) codeword vectors

to produce a composite codeword vector Y of length N. The transmitters are assumed to operate in perfect symbol and block synchronisation over a common discrete T-user MAC. At the receiving end the single decoder decodes Y into estimates of the original data streams $U_1, U_2, \ldots, U_T$, if possible without errors.

The T codes $C_1, C_2, \ldots, C_T$ together are called a "T-user collaborative code", where each component is termed a "constituent code". If all the constituent codes are binary block codes then the codeword vector X_i is an N-symbol binary vector. The rate of the i-th constituent code C_i containing CW_i codewords each of length N is given in equation (4.1) and also the rate sum, R_{sum}, of all the T-user code is given in equation (4.2). Various collaborative codes have been constructed to allow several users access to a common channel simultaneously and unscramble the individual users' information without any ambiguity. Uniquely decodable coding schemes are the most common multiple access techniques [Kasami, et al., 1975, Kasami and Lin 1976, Kasami, et al., 1978a, Weldon 1978, Chang and Weldon 1979, Khachatrian 1982, Ferguson 1982, Kasami, et al., 1983, Khachatrian 1983, Chang 1984, Braak and Tilborg 1985, and Wilson 1988].

Consider a T-user code $(C_1, C_2, \ldots, C_T)$. Let $(Z_1, Z_2, \ldots, Z_T)$ and $(Z'_1, Z_2', \ldots, Z'_T)$ be two distinct sets of vectors with Z_i and $Z_i' \epsilon C_i$ for $1 \leq i \leq T$. Then, the T-user code $(C_1, C_2, \ldots, C_T)$ is said to be "uniquely decodable" if and only if, for every such distinct pair $(Z_1, Z_2, \ldots, Z_T)$ and $(Z_1', Z_2', \ldots, Z'_T)$,

$$Z_1+Z_2+\ldots+Z_T \neq Z_1'+Z_2'+\ldots+Z_T' \tag{5.1}$$

where, the plus sign denotes real addition and the addition operation is performed

component wise.

As an example, a simple coding scheme for a 2-user uniquely decodable code with block length of N=2, is given here [Kasami, et al., 1975, and Kasami and Lin 1976]. The codewords for user 1 and 2 are, C_1=(00,11) and C_2=(00,01,10), respectively. This 2-user code (C_1,C_2) is uniquely decodable because all the received composite codewords are distinct as shown in Table 5.1;

		(C_1)	
	C_1+C_2	(0 0)	(1 1)
(C_2)	(0 0)	0 0	1 1
	(0 1)	0 1	1 2
	(1 0)	1 0	2 1

Table 5.1 2-user Uniquely Decodable Code

Therefore, the decoder can unscramble the two messages without any ambiguity.

Uniquely decodable coding schemes can also have some error protection capability [Kasami, et al., 1975, Kasami and Lin 1976, Kasami and Lin 1978a, Weldon 1978, Chang and Weldon 1979, Farrell 1981 and Wilson 1988]. In particular, it has been found codes for the 2-user binary adder MAC with rates up to 1.292 bits per channel use which achieve the MAC function and offer some error protection capability. The error protection degree depends on the distance of the code. The distance between two codewords $Z_i=z_1,z_2,\ldots,z_N$ and $Z'_i=z'_1,z'_2,\ldots,z'_N$ of the i-th user code C_i is the number of places where they differ and denoted by $d_i(Z_i,Z'_i)$. The minimum distance of the i-th user code C_i is given by;

$$d_{min} = \min d_i(Z_i,Z_i'); \quad Z_i \varepsilon C_i,\ Z_i' \varepsilon C_i,\ Z_i \neq Z_i' \tag{5.2}$$

If Z and Z' are defined as N-symbol composite codewords given by;

$$Z = Z_1+Z_2+...+Z_T, \quad Z'=Z'_1+Z'_2+...+Z'_T, \quad \text{and } Z \neq Z' \tag{5.3}$$

Then, the L-distance between Z and Z' can be defined as follows;

$$d_L(Z,Z') = \sum_{i=1}^{N} | \; z_i - z'_i \; | \; = \; | \, | Z-Z' | \, | \tag{5.4}$$

where, the minus sign denotes real subtraction, $| z_i - z'_i |$ denotes the absolute value of $z_i - z_i'$, and the symbol $| \, | Z-Z' | \, |$ means that the L-distance is a metric. The L-weight of Z is simply the sum of the absolute values of its coordinates.

The minimum L-distance, d_{min}, of a T-user collaborative code is the smallest value of $d_L(Z,Z')$ over all $Z \neq Z'$ [Chang and Weldon 1979]. This distance d_{min} can also be defined in terms of the minimum distances of the constituent codes as;

$$d_{min} = \min \, (d_{1min}, d_{2min}, ..., d_{Tmin}) \tag{5.5}$$

where d_{imin} is the minimum distance of the i-th user constituent code. The number of transmission errors is defined as the L-distance between the transmitted composite codeword, say, Z and a received codeword $\hat{Z}$, i.e.,

$$e(Z,\hat{Z}) = | \, | Z-\hat{Z} | \, | \tag{5.6}$$

and the error vector is,

$$e(Z,\hat{Z}) = e_1,e_2,\ldots,e_N \tag{5.7}$$

Consider the 2-user code given in Table 5.1, the symbol by symbol error conditions can be defined as follows;

(a) Single error conditions occurs if,

(i) user 1 and 2 transmits the symbols 0 and 0, or 1 and 1, respectively, and the received symbol is 1,

(ii) user 1 and 2 transmits the symbols 0 and 1, or 1 and 0, respectively, and the received symbol is 0 or 2.

(b) Double error conditions occurs if,

(i) user 1 and 2 transmits the symbols 0 and 0, respectively, and the received symbol is 2,

(ii) user 1 and 2 transmits the symbols 1 and 1, respectively, and the received symbol is 0.

5.3 T-user Decoding Techniques

In the noiseless channel conditions, the decoder is capable of decoding every received composite codeword vector, without ambiguity, into T-codeword that were transmitted by the T-encoder. However, if the channel is noisy, the received composite codeword may differ from the transmitted codeword. In this case, the decoder chooses the codeword which is closest to the received as measured by some metric distance. The

general decoding process involves searching amongst all the possible composite codewords and choosing the codeword that satisfies certain decoding strategy. The metric distance values are the decoding strategy measure of the received codeword with respect to all the codewords. The decoding of T-user collaborative coding schemes is based here on two techniques, hard and soft decision decoding. These two decoding techniques are employed to decode collaborative coding schemes [Ali and Honary 1992].

5.3.1 Hard Decision (HD) Decoding

In HD decoding of CCMA schemes, the demodulator set (T) decision thresholds to detect the (T+1) possible signal levels transmitted by the T-user. Here, each received symbol is detected independently over N received symbols. This process is called symbol-by-symbol HD (SBS_HD) decoding technique [Ali and Honary 1990]. However, this decoding technique cannot be used on its own to perform full decoding process to deliver the individual users information to their intended destinations. This is due to the fact that, some times in noisy conditions, the SBS_HD decoding will result in a codeword which is not admissible. In this case the decoder will fail to deliver the individual users information. Therefore, L-distance HD decoding is used in conjunction with SBS_HD to complete the decoding process and resolve this ambiguity. This complete process is referred to as HD_CCMA decoding technique [Ali and Honary 1991a, and Ali and Honary 1991b].

The HD_CCMA decoder, calculates all the L-distances between the SBS_HD codeword and all the possible admissible codewords. Then, the codeword with least L-distance is chosen. This kind of decoding, guarantees correct decoding in the noiseless

uniquely decodable coding scheme. However, in the noisy case, the number of errors which can be corrected under this decoding is $t=\lfloor (d_{min}-1)/2 \rfloor$, where $\lfloor x \rfloor$ means integer number less than or equal to x, [Peterson and Weldon 1972]. The generalised HD_CCMA decoding algorithm can be summarised in the following steps:

Step 1: Perform SBS_HD decoding on the received N-symbol codeword.

Step 2: Calculate the L-distance metric values between the SBS_HD codeword and all the admissible codewords.

Step 3: Choose the codeword with the least L-distance metric value to represent the decoded codeword.

Step 4: A look up table is used to decode the individual users' codewords and hence their original sink information.

5.3.2 Maximum Likliehood Soft Decision (MLSD) Decoding

It is seen in the previous section that the HD_CCMA decoder operates on the demodulator hard symbol decisions. This, however, neglected the fact that there is an additional information in the received signal which can be made available by the demodulator and fed forward to the decoder. The technique which make use of this extra information in the received signal is called soft decision (SD) decoding [Sklar 1988 pp329-331, and Cattermole 1986 pp180]. Therefore, when the demodulator sends a hard symbol decision to the decoder, it sends a single symbol. However, when the demodulator sends a soft symbol decision, it effectively sends a word in place of a single symbol which is equivalent to sending the decoder a measure of confidence along with the symbol. In such a case, the demodulator can be configured to have a number of quantisation levels greater than (T+1). Optimum SD decoding is obtained by having

infinite number of quantisation levels. The SD decoding is most readily understood as a discrete approximation to maximum likelihood (ML) detection.

The SD scheme is also used here in conjunction with SBS_HD decoding as in the previous case of HD_CCMA decoding. The demodulator sets a number of decision thresholds to decide which of the possible signal levels have been transmitted. In addition, the demodulator assigns each symbol a confidence level which is extracted from the received signal quantisation. The SD distance between the SBS_HD codeword with it's symbols confidence level and all the admissible codewords is calculated. The admissible codewords are stored with the highest confidence level of each symbol. The codeword with least SD distance is chosen to represent the SD decoding. This SD decoding technique can be summarised in the following steps:

Step 1: Perform SBS_HD decoding on the received N-symbol codeword.

Step 2: Calculate the SD metric values between the SBS_HD codeword and all the admissible codewords.

Step 3: Choose the codeword with the least SD metric value to represent the SD decoding.

Step 4: A look up table is used to decode the individual users' codewords and hence their original sink data messages.

Consider a set of composite codewords each comprising N-symbol. The received signal is $W=(w_1,w_2,...,w_N)$, where w_i is the magnitude of the element representing the i-th symbol. In principle, joint ML decision carried out on the complete word is a very powerful detection technique [Cattermole 1986 pp180]. Therefore, if the actual signal magnitude of N-symbol codeword is made available to the decoder, then a ML decoding for CCMA schemes can be performed. This is achieved by calculating the

Euclidean distances between the received codeword and all the admissible codewords. The codeword with minimum Euclidean distance (MED) is chosen to represent the MLSD decoding codeword. Provided the codewords are all equally likely, this strategy is optimum in the sense that it minimises the probability of error in the decoder. A generalised MLSD decoding technique algorithm steps can be summarised here as follows:

Step 1: Calculate the Euclidean distances between the received soft information codeword and all the possible codewords.

Step 2: The codeword with the MED is chosen to represent the MLSD decoding.

Step 3: A look up table is used to decode the individual users' codewords and their original sink information.

However, this technique is difficult to implement in practice, because this would require the storage of the precise amplitudes of all symbols as received. In addition, the decoding table becomes unmanageably large as the length of the code and the number of active users increases. Therefore, what is needed is a simple means of calculating the possible transmitted codewords from the received codeword with least number of operations possible.

5.4 Low Complexity MLSD CCMA Decoding

This technique has the reliability of ML decoding with less implementation complexity and also reduces the number of computations required to decode a given codeword [Ali and Honary 1991a, and Ali and Honary 1991b].

5.4.1 Decoding Procedure Description

The decoding problem at the receiver can be defined as follows "Given $W=(w_1,w_2,...,w_N)$, where w_i is real valued scalar, it is necessary to decode the transmitted codeword in such a way that the total probability of codeword error is minimised". For a given T-user CCMA code structure, two sets of "admissible" and "forbidden" codewords are defined. Assume $A_i=(a_{i1},a_{i2},...,a_{iN})$ is the i-th admissible codeword where $i=1,2,...,N_a$; N_a is the number of admissible codewords given by;

$$N_a=\prod_{l=1}^{T} CW_l \tag{5.8}$$

CW_l is the number of codewords in the l-th user, and a_{ij} is the j-th symbol value of the i-th admissible codeword where $j=1,2,...,N$. A set of forbidden codewords, for certain error conditions, is defined as, $F_i=(f_{i1},f_{i2},...,f_{iN})$, where $i=1,2,...,N_f$; N_f is the number of forbidden codewords, and f_{ij} is the j-th symbol value of the i-th forbidden codeword where $j=1,2,...,N$.

If we assume the transmitted codeword is $A_k=(a_{k1},a_{k2},...,a_{kN})$, and the received codeword is $W=(w_1,w_2,...,w_N)$. Then, in order to construct the decision decoding table to decode the received codeword, the following procedure is required;

(i) Define the subset of admissible codewords nearest to each forbidden codeword for certain error conditions.

(ii) Calculate all the Euclidean distances between the received codeword $W=(w_1,w_2,...,w_N)$ and all the codewords from the admissible codewords subset nearest to the i-th forbidden codeword.

(iii) Choose the codeword with the MED. That is, if the generalised distance between

W and the admissible codeword, say, $A_p \varepsilon (A_i)$ is minimum, then $A_p=(a_{p1},a_{p2},...,a_{pN})$ is accepted as the transmitted codeword.

(iv) Comparison thresholds are then found for each forbidden codeword to form the decoding table for this decoding technique.

This decoding technique is also used in conjunction with SBS_HD decoding. This arrangement allows to correct some errors which can not be corrected under SBS_HD and HD_CCMA decoding techniques. In addition, this decoding scheme may perform both error detection and correction at the same time.

5.4.2 Decoding Algorithm

The generalised decoding algorithm steps for this MLSD_CCMA decoding technique can be summarised as follows:

Step 1: Perform SBS_HD decoding on the received codeword $W=(w_1,w_2,...,w_N)$.

Step 2: "Error Detection": perform error detection by checking the SBS_HD decoded codeword, whether it is admissible or forbidden; if it is admissible goto step 4, else continue.

Step 3: "Error Correction": perform error correction by checking certain decision thresholds for the current detected forbidden codeword (according to decoding decision table);

Step 4: Individual users' information is then decoded by using the normal decoding procedure used in the noiseless case.

It can be seen from the above steps that, it is not required to calculate the Euclidean distances between the received codeword and all the admissible codewords every time a codeword is received. It is only needed to check certain conditions

according to a decision decoding table. Therefore, in addition to performing MLSD decoding, the total number of operations are reduced compared to conventional techniques.

5.4.3 2-user MLSD_CCMA Decoding Scheme

As an example and analysis of this MLSD_CCMA decoding technique, the 2-user code given in Table 5.1 is considered and referred to as code 1. That is, user one and two codewords are C_1=(00,11), C_2=(00,01,10), respectively. The set of admissible composite codewords is (00,10,01,11,12,21). The single error conditions for this coding scheme is defined previously in section (5.2). That is, if the transmitted symbols from each user are (0,0) or (1,1) and the received composite symbol is 1; and also if (0,1) or (1,0) are transmitted and either 0 or 2 is received, then single error has occurred during transmission. Therefore, for this single error conditions, the set of forbidden composite codewords is (02,20,22) and the subset of admissible composite codewords nearest to each forbidden codeword are defined as shown in Table 5.2;

Forbidden Codewords		Nearest Admissible Codewords
02	——————>	(12,01)
20	——————>	(21,10)
22	——————>	(21,12)

Table 5.2 Forbidden and Nearest Admissible Codewords for 2-user Code 1

The construction of decoding decision table can be obtained by calculating the Euclidean distances between the received codeword $W=(w_1,w_2)$ of a forbidden codeword and the nearest admissible codewords, defined above in Table 5.2, as shown below.

(i) The Euclidean distances for the forbidden codeword 02:

$d_1^2=(w_1-1)^2+(w_2-2)^2$; between $W=(w_1,w_2)$ and 12,

$d_2^2=(w_1-0)^2+(w_2-1)^2$; between $W=(w_1,w_2)$ and 01,

$d_1^2-d_2^2=-2w_1-2w_2+4$;

then

$d_1^2<d_2^2$ if $w_1+w_2>2$; therefore d_1^2 is the minimum distance.

(ii) The Euclidean distances for the Forbidden codeword 20:

$d_3^2=(w_1-2)^2+(w_2-1)^2$; between $W=(w_1,w_2)$ and 21,

$d_4^2=(w_1-1)^2+(w_2-0)^2$; between $W=(w_1,w_2)$ and 10,

$d_3^2-d_4^2=-2w_1-2w_2+4$;

then

$d_4^2>d_3^2$ if $w_1+w_2>2$; therefore d_3^2 is the minimum distance.

(iii) The Euclidean distances for the Forbidden codeword 22:

$d_5^2=(w_1-2)^2+(w_2-2)^2$; between $W=(w_1,w_2)$ and 21,

$d_6^2=(w_1-1)^2+(w_2-2)^2$; between $W=(w_1,w_2)$ and 12,

$d_5^2-d_6^2=-2w_1+2w_2$;

then

$d_5^2<d_6^2$; if $w_1>w_2$; therefore d_5^2 is the minimum distance.

Therefore, a decision table is constructed based on these calculations to be used for the decoding purposes as shown in Table 5.3;

Forbidden Codewords	Comparison Thresholds	Decoding Decision
02	$w_1+w_2>2$	12
	$w_1+w_2<2$	01
20	$w_1+w_2>2$	21
	$w_1+w_2<2$	10
22	$w_1>w_2$	21
	$w_1<w_2$	12

Table 5.3 Decoding Decision Table for 2-user Code 1

Example: say, the transmitted codeword is (10) and the received soft information codeword is W=(1.6,0.3). Then, performing SBS_HD decoding, the decoded codeword will be (20), this means there is a single error from the transmitted composite codeword. Employing HD_CCMA decoding technique, will result in two possible codewords (10) and (21) which have the same minimum L-distance from the codeword (20). Therefore, the HD_CCMA decoder chooses either codeword with equal probability. If (10) is chosen then a single error has been corrected, however, if the codeword (21) is chosen then a double error has been introduced.

Now, employing MLSD_CCMA decoding technique, since an error is detected by the forbidden codeword (20) and $(w_1+w_2)<2$, then the output of the decoder is the codeword (10) as can be seen from Table 5.3. Thus, a single error has been corrected from SBS_HD decoder or corrected double error from the HD_CCMA decoder (if the HD_CCMA decoder had chosen the codeword (21)). Therefore, employing MLSD_CCMA decoding improvement in the decoding is possible, which allows some of the detected errors to be corrected. This decoding technique will give higher

improvement gain if the code used has some error protection capability.

5.5 Error Probability Analysis

The theoretical performance of T-user CCMA decoding schemes is evaluated here in terms of both the probability of composite symbol and codeword error. The reliability of individual users' sink information depends on how accurate the composite codeword is decoded. These analyses are carried out over AWGN channel of zero mean and variance σ_N^2.

5.5.1 Symbol Error Probability

The total probability of symbol error, $P_{se}(to)$, or some times referred to as symbol error rate (SER), for a given T-user CCMA scheme can be written as;

$$P_{se}(to) = 1 - P_{sc}(to) \tag{5.9}$$

where $P_{sc}(to)$ is the total probability of correct symbol decision and can be written as;

$$P_{sc}(to) = \sum_{i=1}^{L} p(i)\, P_{sc}(i) \tag{5.10}$$

where L is the total number of output symbols and p(i) is the i-th symbol probability of occurrence,

$$\sum_{i=1}^{L} p(i) = 1 \tag{5.11}$$

and $P_{sc}(i)$ is the i-th output symbol probability of correct decision which can be written as;

$$P_{sc}(i) = \int_{\{G_i\}} f_i(w)\, dw \tag{5.12}$$

where $\{G_i\}$ is the region of correct decision of the i-th symbol and $f_i(w)$ is the PDF of the i-th symbol given by;

$$f_i(w) = \left(\frac{1}{\sqrt{(2\pi)}\sigma_N}\right) \exp\left(\frac{-(w-S_i)^2}{2\sigma_N^2}\right) \tag{5.13}$$

where w is real valued scalar in the region $\{G_i\}$ and S_i is the i-th symbol real value. Therefore, the total probability of correct decision is,

$$P_{sc}(to) = \sum_{i=1}^{L} \left(p(i) \left(\frac{1}{\sqrt{(2\pi)}\sigma_N}\right) \int_{\{G_i\}} \exp\left(\frac{-(w-S_i)^2}{2\sigma_N^2}\right) dw \right) \tag{5.14}$$

Substituting in equation (5.9) we get the total probability of symbol error,

$$P_{se}(to) = 1 - \sum_{i=1}^{L} \left(p(i) \left(\frac{1}{\sqrt{(2\pi)}\sigma_N}\right) \int_{\{G_i\}} \exp\left(\frac{-(w-S_i)^2}{2\sigma_N^2}\right) dw \right) \tag{5.15}$$

This procedure is used to calculate the composite SER for the T-user CCMA schemes and the derivation is shown in Appendix B. For the T-user binary schemes L=T+1 and the decoder set T detection thresholds to detect the L symbols at the receiver. If the admissible symbols for this case are (S_0,S_1,S_2) then the probability of error for each symbol is calculated in Appendix B and shown that;

$$P_{se}(S_0) = \mathrm{Erfc}(u_0-S_0/\sigma_N) \tag{5.16}$$

$$P_{se}(S_1) = \mathrm{Erf}(u_0-S_1/\sigma_N)+\mathrm{Erfc}(u_1-S_1/\sigma_N) \tag{5.17}$$

$$P_{se}(S_2) = \mathrm{Erf}(u_1-S_2/\sigma_N) \tag{5.18}$$

The total probability of symbol error can be written as;

$$P_{se}(to) = p(S_0)\mathrm{Erfc}(u_0-S_0/\sigma_N)+p(S_1)(\mathrm{Erf}(u_0-S_1/\sigma_N)+\mathrm{Erfc}(u_1-S_1/\sigma_N)) \\ +p(S_2)\mathrm{Erf}(u_1-S_2/\sigma_N) \tag{5.19}$$

where Erf, Erfc are the normalised and complementary error functions respectively, given in Appendix A; u_0, u_1 are the decision thresholds; σ_N is the standard deviation of the noise; $p(S_i)$ is the probability of occurrence of S_i. Substituting for S_0=0, S_1=1, S_2=2, and chosen the decision thresholds to be half way between any two given signal levels i.e., u_0=0.5 and u_1=1.5 we get;

$$P_{se}(to) = \mathrm{Erfc}(1/2\sigma_N)(p(0)+2p(1)+p(2)) \tag{5.20}$$

Similarly, the SER is derived for T>2 and a generalised form is found and can be written as follows;

$$P_m(to) = Erfc(1/2\sigma_N)(p(0)+2(p(1)+...+p(T-1))+p(T)) \tag{5.21}$$

This equation is used to evaluate the SER for any given T-user binary CCMA scheme. The channel output SER is shown graphically in Figure 5.1 as a function of T and E/N_o, employing binary signalling scheme, where E is the average signal energy per user and N_o is the noise spectral density.

5.5.2 Symbol Error Probability Minimisation

The probability of error minimisation is initiated by the fact that the occurrence of the MAC output signals is not equiprobable. The decision thresholds chosen half way between the above signal levels is not an optimum way of detection [Schwartz 1990 pp429-432, and Cattermole 1986 pp168-177]. An optimum decision thresholds for the T-user signals are derived using the knowledge of their probability of occurrence. Therefore, the total probability of error is differentiated with respect to all the decision thresholds associated with a particular T-user scheme to find the optimum levels.

For example, consider the case of T=M=2, the total probability of symbol error given in equation (5.19), is differentiated with respect to the decision thresholds u_0 and u_1, and found that;

$$u_0 = (2\sigma_N{}^2 \log(p(S_0)/p(S_1))-S_0{}^2+S_1{}^2))/2(S_1-S_0) \tag{5.22}$$

$$u_1 = (2\sigma_N^2 \log(p(S_1)/p(S_2)) - S_1^2 + S_2^2))/2(S_2 - S_1) \tag{5.23}$$

It is thus apparent that the optimum u_0 and u_1 (in the sense of minimum error probability) depends on the signal amplitude, probability of occurrence, and noise variance. It is found as expected that the decision thresholds are biassed towards the least probable signal level. Also, in the noiseless conditions, the optimum decision thresholds values, is very near to the suboptimum case, due to the actual probability distribution of output signal levels. However, the derived optimum decision levels are used to calculate the channel output SER and shown graphically in Figure 5.2, for various values of T and E/No, employing binary signalling scheme. This suggests that there is no significant improvement in system performance by varying the threshold levels.

5.5.3 Codeword Error Probability

If the total number of admissible composite codewords for a given T-user code is N_a, then the total probability of correct decision can be written as;

$$P_{cc}(to) = \sum_{i=1}^{N_a} p(i)\, P_{cc}(i) \tag{5.24}$$

where p(i) is the probability of the i-th admissible codeword,

$$\sum_{i=1}^{N_a} p(i) = 1 \tag{5.25}$$

Figure 5.1 T-user CCMA Channel O/P SER
(with midpoint Decision Thresholds)

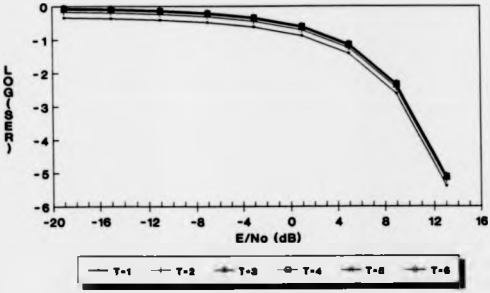
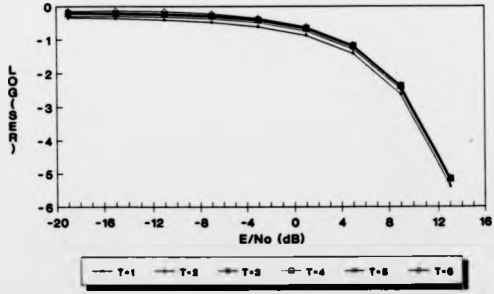


Figure 5.2 T-user CCMA Channel O/P SER
(with Optimum Decision Thresholds)



and $P_{cc}(i)$ is the probability of correct decision of the i-th admissible codeword,

$$P_{cc}(i)=\int\int_{\{G_i\}}\cdots\int f_i(w_1w_2\ldots w_N)\,dw_1dw_2\ldots dw_N \tag{5.26}$$

where $\{G_i\}$ is the region of correct decision of the i-th composite codeword, and $f_i(w_1w_2\ldots w_N)$ is the joint pdf of the i-th admissible codeword, which can be written for the AWGN as;

$$f_i(w_1w_2\ldots w_N)=\left(\frac{1}{\sqrt{(2\pi)}\sigma_N}\right)^N \exp\left(\sum_{j=1}^{N}\left(\frac{-(w_j-a_{ij})^2}{2\sigma_N^2}\right)\right) \tag{5.27}$$

where a_{ij} is the j-th symbol in the i-th admissible codeword, where j=1,2,....,N and i=1,2,...,N_a. Substituting back into equation (5.26) and (5.24), the total probability of correct decision is,

$$\begin{aligned}P_{cc}(to)=&\sum_{i=1}^{N_a}\Big(p(i)\left(\frac{1}{\sqrt{(2\pi)}\sigma_N}\right)^N\int\int_{\{G_i\}}\cdots\int\\&\exp\left(\sum_{j=1}^{N}\left(\frac{-(w_j-a_{ij})^2}{2\sigma_N^2}\right)\right)\Big)\,dw_1dw_2\ldots dw_N\end{aligned} \tag{5.28}$$

and the total probability of error can be written as;

$$\begin{aligned}P_{cc}(to)=1-&\sum_{i=1}^{N_a}\Big(p(i)\left(\frac{1}{\sqrt{(2\pi)}\sigma_N}\right)^N\int\int_{\{G_i\}}\cdots\int\\&\exp\left(\sum_{j=1}^{N}\left(\frac{-(w_j-a_{ij})^2}{2\sigma_N^2}\right)\right)\Big)\,dw_1dw_2\ldots dw_N\end{aligned} \tag{5.29}$$

For MLSD_CCMA decoding, the region of correct decision $\{G_i\}$ represent the set of points for which the Euclidean distance to the i-th admissible codeword is smaller than for all other admissible codewords. That is;

$$\{G_i\} = \{ \{w_1 w_2 ... w_N\} : \sum_{j=1}^{N} (w_j - a_{ij})^2 < \sum_{j=1}^{N} (w_j - a_{kj})^2 \tag{5.30}$$

where $(w_1 w_2 ... w_N)$ is the set of points in $\{G_i\}$; the colon sign (:) means defined as; w_j is real valued scalars, which represents the coordinates for each point in the region $\{G_i\}$; a_{ij} and a_{kj} are the j-th symbols in the i-th and k-th admissible codewords respectively, where $i=1,2,...,N_a$, $j=1,2,...,N$, and $i \neq k$.

5.5.4 2-user CCMA Decoding Schemes Error Probability

The probability of error for the 2-user binary CCMA scheme is considered and analysed here. The 2-user binary collaborative code is given in Table 5.1. The probability of receiving a given admissible codeword correctly, $P_{cc}()$, is derived in Appendix C, employing hard decision decoding. It is found that;

$$P_{cc}(00) = Erf^2(0.5u/\sigma_N) \tag{5.31}$$

$$P_{cc}(01)=P_{cc}(10) = (Erf(0.5u/\sigma_N))\ (2Erf(0.5u/\sigma_N)-1) \tag{5.32}$$

$$P_{cc}(12)=P_{cc}(21) = (2Erf(0.5u/\sigma_N)-1)\ (Erf(0.5u/\sigma_N)) \tag{5.33}$$

$$P_{cc}(11) = (2\mathrm{Erf}(0.5u/\sigma_N)-1)^2 \tag{5.34}$$

Substituting these probabilities in the total probability of correct decision we get:

$$P_{cc}(to) = (P_{cc}(00)+2P_{cc}(10)+2P_{cc}(12)+P_{cc}(11))/6 \tag{5.35}$$

and the total probability of error can be calculated by substituting in the following equation,

$$P_{ce}(to) = 1 - P_{cc}(to) \tag{5.36}$$

Similarly, the derivation of the codeword error rate employing MLSD_CCMA decoding is given in Appendix D. The probability of correct decision of each admissible codeword is,

$$P_{cc}(00) = \mathrm{Erf}^2(0.5u/\sigma_N) \tag{5.37}$$

$$P_{cc}(01) = (Erf(0.5u/\sigma_N) - 1)\,Erf(0.5u/\sigma_N) + \int_{-\infty}^{0.5u/\sigma_N} (\exp(-t^2/2)/\sqrt{2\pi})\; Erf(-t+u/\sigma_N)\; dt \tag{5.38}$$

$$P_{cc}(21) = \int_{-0.5u/\sigma_N}^{\infty} (\exp(-t^2/2)/\sqrt{2\pi})\,(Erf(t+u/\sigma_N) - Erf(-t-u/\sigma_N)) \tag{5.39}$$

$$P_{cc}(11) = (2Erf(0.5u/\sigma_N)-1)^2 \tag{5.40}$$

Then, equation (5.35) and (5.36) is used to get the total probability of error.

5.6 Simulation Results and Discussions

The simulation is carried out to evaluate the reliability performance of CCMA schemes employing various coding and decoding schemes. Various 2-user CCMA schemes [Kasami, et al., 1975, and Kasami and Lin 1976] are introduced first and used throughout the simulation analysis. These collaborative codes are chosen to be simple short codes with summary rate, in most cases, higher than one bits/channel use. In addition, they are chosen to have different error protection capability of the overall 2-user code and it's constituent codes. These codes are;

(a) Code 1: C_1=(00,11), C_2=(00,01,10),

CW_1=2, N_1=2, R_1=0.5, d_{1min}=2,

CW_2=3, N_2=2, R_2=0.792, d_{2min}=1,

R_{sum}=1.292, d_{min}=1.

(b) Code 2: C_1=(000,111), C_2=(000,001,010,011,100,101,110),

CW_1=2, N_1=3, R_1=0.333, d_{1min}=3,

CW_2=7, N_2=3, R_2=0.935, d_{2min}=1,

R_{sum}=1.269, d_{min}=1.

(c) Code 3: C_1=(0000,0011,1100,1111),

C_2=(0000,0001,0010,0100,0101,0110,1000,1001,1010),

CW_1=4, N_1=4, R_1=0.5, d_{1min}=2,

CW_2=9, N_2=4, R_2=0.792, d_{2min}=1,

R_{sum}=1.292, d_{min}=1.

(d) Code 4: C_1=(0000,0001,0011,1100,1110,1111),

C_2=(0000,0101,0110,1001,1010,1101),

CW_1=6, N_1=4, R_1=0.646, d_{1min}=1,

CW_2=6, N_2=4, R_2=0.646, d_{2min}=1,

R_{sum}=1.292, d_{min}=1.

(e) Code 5: C_1=(00,11), C2=(10,01),

CW_1=2, N1=$_2$, R_1=0.5, d_{1min}=2,

CW_2=2, N2=$_2$, R_2=0.5, d_{2min}=2,

R_{sum}=1.0, d_{min}=2.

It is assumed that the 2-user CCMA communication system is in perfect synchronisation. In addition, the modulation and demodulation are assumed to be available for these codes and considered to be part of the discrete channel. The simulation performance analysis results are presented graphically in terms of the probability of error. The composite codeword error rate (CER) and the constituent users sink SER are calculated for each 2-user collaborative code. The composite CER is defined here as the total number of composite codewords in error over the total

transmitted. The individual constituent user's sink SER is defined as the total number of user's sink symbols in error over the total transmitted. The channel is assumed to be AWGN of zero mean and variance σ_N^2. The ratio E/N_o, is also defined here as the average signal energy per user to noise power spectral density given by $\sigma_N^2 = N_o/2$.

The composite CER versus E/N_o, employing the HD_CCMA decoding is shown in Figure 5.3, for all the five codes. It can be seen from this figure that the reliability of these codes are very similar, since their correction capability is the same under HD_CCMA decoding. The small difference is due to the variation in the number of admissible and forbidden codewords from one code to another. The composite CER versus E/N_o, employing the MLSD_CCMA decoding is also shown in Figure 5.4, for all the five codes. It can be seen clearly that code 5 gives the best performance because its $d_{min}=2$, which means that under this decoding a single error can be corrected.

For comparison purposes and calculating the energy gain achieved by employing MLSD_CCMA decoding, the CER for each code is presented separately in Figures 5.5-5.9, employing HD_CCMA and MLSD_CCMA decoding techniques. Also included with these Figures is the CER of each code when SBS_HD decoding is employed. It can be seen for the first four codes, Figures 5.5-5.8, that the MLSD_CCMA decoding gives the best performance with some detection gain. However, when the code employed has some error protection capability, as the case in Figure 5.9, this gain is much higher, as can be seen very clearly at high E/N_o. The gain achieved is more than 2.5dB at an error probability of 10^{-4}.

The effect of employing these coding and decoding techniques is also investigated on the constituent codes and hence their user's sink data. The sink SER for each user is presented in Figures 5.10-5.14 for all codes. It can be seen, for example,

Figure 5.3 HD_CCMA Decoder CER

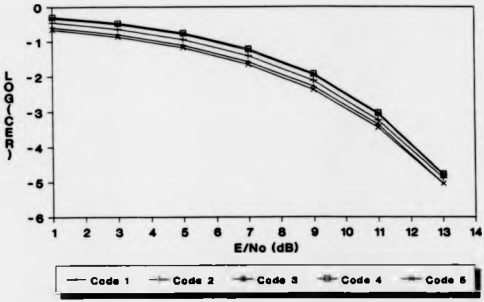


Figure 5.4 MLSD_CCMA Decoder CER

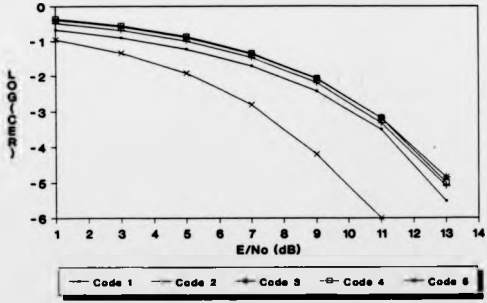


Figure 5.5 CCMA Decoding Schemes CER
(Code 1)

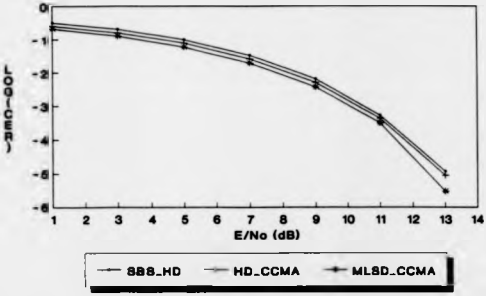


Figure 5.6 CCMA Decoding Schemes CER
(Code 2)

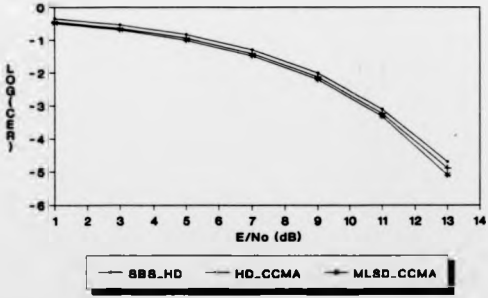


Figure 5.7 CCMA Decoding Schemes CER
(Code 3)

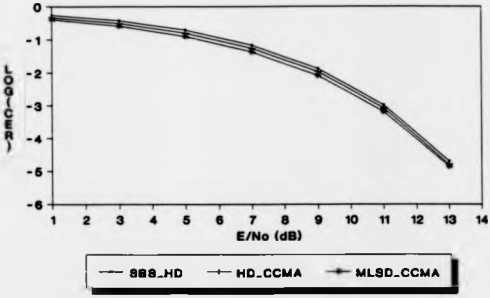


Figure 5.8 CCMA Decoding Schemes CER
(Code 4)

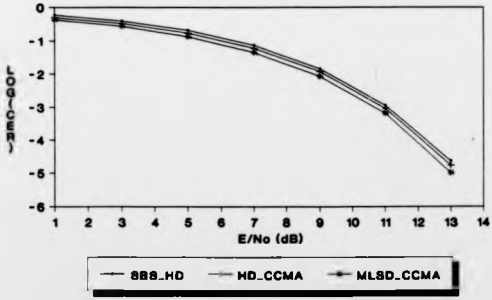


Figure 5.9 CCMA Decoding Schemes CER
(Code 5)

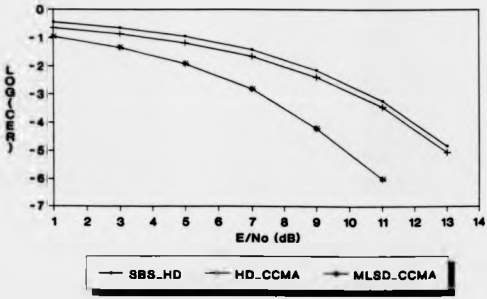


Figure 5.10 CCMA Decoding Schemes
Users Sink SER
(Code 1)

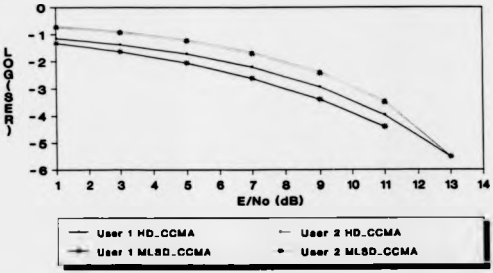


Figure 5.11 CCMA Decoding Schemes
Users Sink SER
(Code 2)

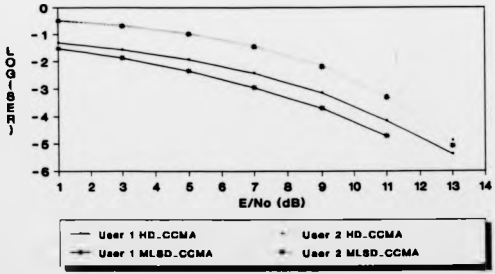


Figure 5.12 CCMA Decoding Schemes
Users Sink SER
(Code 3)

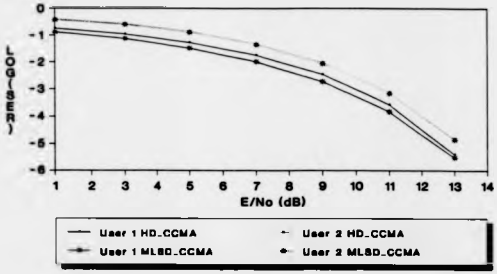


Figure 5.13 CCMA Decoding Schemes
Users Sink SER
(Code 4)

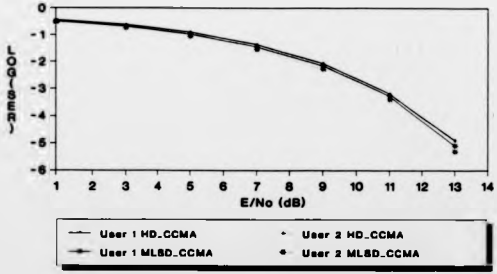
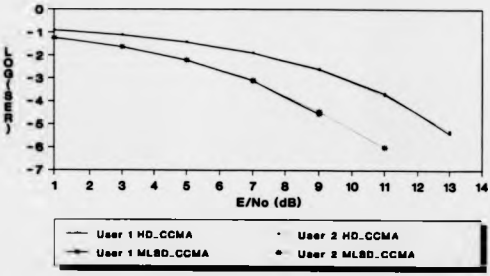


Figure 5.14 CCMA Decoding Schemes
Users Sink SER
(Code 5)



in Figure 5.10, user 2 sink SER is very close for both cases of MLSD_CCMA and HD_CCMA decoding techniques. However, user 1 reliability employing MLSD_CCMA decoding is better than HD_CCMA decoding because $d_{1,\_\_}=2$. This gain is also shown in Figures 5.11 and 5.12 for user 1 of code 2 and code 3, respectively. Since code 4 is a balanced code, the reliability of each user is very close as shown in Figure 5.13. Code 5 is also balanced code with $d_{1,\_\_}=d_{2,\_\_}=d_{\_\_}=2$. Therefore, the sink SER is the same for each user as shown in Figure 5.14. It can also be seen from Figure 5.14 that a coding gain of more than 2.5dB at 10^{-4} error probability is achievable employing MLSD_CCMA over the HD_CCMA decoding technique.

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Chapter 6

Practical CCMA System Design

6.1 Introduction

Practical system design for CCMA schemes is of considerable importance to maintain the efficient simultaneous communication of several users sharing the same bandwidth allocation, with a combined throughput which is higher than that which can be achieved by other multiple access methods. In this chapter certain modulation/demodulation (modem) techniques to provide the practical combining and unscrambling of the users' collaborative coded data signals are investigated. The performance analyses of these techniques are carried out in the noiseless and noisy channel conditions.

6.2 Modem Techniques Considerations

Generally, the data information to be transmitted can be sent over a communication channel by varying the parameters of a sinusoidal carrier signal: amplitude, frequency, and phase in some recognisable format. This leads directly to the basic techniques of modulation which are amplitude shift keying (ASK), frequency shift keying (FSK), and phase shift keying (PSK). In ASK technique the amplitude of a fixed carrier is varied according to the data to be transmitted. For the binary case, the carrier is switched on and off for 1's and 0's respectively. If we wish to transmit more bits per

symbol, we must use more levels, normally using a power of two so that there will be a whole number of bits per symbol. However, as the number of levels increases, the amplitude increment between adjacent carrier amplitudes decreases and so does immunity to noise. Since the amplitude is used to convey the information, the decision boundaries in the receiver must be constantly varied and accurately tracked with changes in the received signal levels for reliable system performance. This problem is worsened in the case of multi-level ASK, since more boundaries exist which must be tracked with greater accuracies.

In the PSK technique, the carrier phase is changed according to the data to be transmitted. The most simple example of PSK modulation is that of phase reversal keying (PRK) [Stremler 1982 pp582] whereby the carrier or the inverse of the carrier is transmitted, depending on the data. More complex systems switch the phase to one of a larger number of possible values, usually evenly spaced between 0 and 2π. As the number of phases increases, the distances between the constellation points decreases, which results in reduction in the immunity to noise. The demodulation of these signals becomes progressively more difficult as the number of phases is increased [Lindsey and Simon 1972]. Also, one of the major problems associated with PSK systems is that high channel phase stability is required. Otherwise, these phase instabilities must be accurately tracked.

The principle of FSK modulation technique is that, the carrier signal frequency is changed according to the data to be transmitted. For example, binary FSK consists of simply transmitting one tone for 0's and another for 1's. In M-ary FSK (MFSK) signalling format, the M tones are usually evenly spaced through the signalling bandwidth. Unlike the other two modulation schemes, the distance between the

constellation points can be kept constant as the number of levels or tones increases, by having orthogonal frequency spacing. This may be achieved by making the symbols longer, since the frequency spacing for orthogonality is equal to the reciprocal of the symbol period [Ralphs 1977, and Ralphs 1985]. It is suggested that MFSK is a more realisable method of modulation for time varying channels (e.g. HF channel) due to its long symbol period [Ralphs 1977, Ralphs 1985, and Shaw 1989].

The task of the demodulator or detector is to retrieve the data symbol streams from the received composite waveform, as nearly error free as possible, in spite of the distortion to which the users' signals may have been subjected. The detection process can be performed by using one of the two basic techniques [Clark 1983 pp49-93], i.e. coherent or noncoherent detection. In coherent detection, the receiver makes use of a prior knowledge of the phase of the signal carrier in an element detection process. Therefore, phase estimation at the receiver is required. The noncoherent demodulation refers to systems employing demodulators that are designed to operate without knowledge of the absolute value of the incoming signal's phase, therefore, phase estimation is not required. Generally, the advantage of noncoherent systems over coherent systems is reduced complexity, and the price paid is increased probability of error under AWGN conditions [Arthurs and Dym 1962, Edwards 1973, Stremler 1982, and Clark 1983].

6.3 MFSK_CCMA Modem Model

The modulation channel model considered here is MFSK_CCMA which is a much more realistic form of MAC, interesting from both the theoretical and application

point of view [Honary and Ali 1989, and Ali 1989]. In this MFSK_CCMA signalling scheme, the T-user have available M sinusoidal carriers, each at a different frequency $(F_1,F_2,...,F_M)$. The general block diagram of the practical transmission system is shown in Figure 6.1.

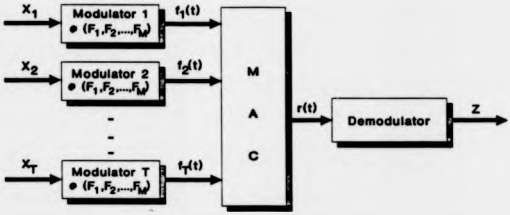


Figure 6.1
Block Diagram of MFSK_CCMA Transmission System

During any symbol interval T_s, each user will selects one of these common frequencies to transmit, the receiver must then decide which frequencies have been transmitted during each period of T_s seconds. The T-user are synchronised so that each user transmits a frequency at the same fixed time T_s seconds intervals. Each tone frequency would need to be separated from the frequencies above and below it by an amount sufficient to allow the receiver to reject these frequencies. The minimum frequency separation, δF, that would guarantee a fixed energy per received symbol is $\delta F=1/T_s$ [Ralphs 1985].

This frequency assignment scheme can be expressed mathematically as follows, the i-th user is assigned the carrier signal, $f_i(t)$, according to its data codeword

symbols;

$$f_i(t) = A_i \sin(w_{ij} t + \phi_i); \quad i=1,2,..,T, \quad j=1,2,...,M \tag{6.1}$$

where w_{ij} is the j-th frequency of the i-th user signal, A_i is i-th user signal amplitude, and ϕ_i is the i-th user arbitrary phase angle. During any symbol interval T_s, the i-th user transmits the MFSK carrier signal according to its data symbols, where it combines with the other users' carrier signals at the channel. The resulting channel output or demodulator input, r(t), assuming noiseless channel is;

$$r(t) = \sum_{i=1}^{T} f_i(t)$$

$$= \sum_{i=1}^{T} A_i \sin(w_{ij} t + \phi_i) \tag{6.2}$$

Hence, the received signal at any instant of time is a composite of T sinusoidal signals, from which the transmitted frequencies have to be recovered. The users' carrier have to be phase coherent at the receiver to provide the MAC output, without carrier coherence, cancellation fading occurs. In MFSK signalling scheme, the phase of the tones is presumed to carry no information and the magnitude of the signal at each frequency is only considered. In addition, for coherent detection of these MFSK signal, considerable equipment complexity is required for the extraction of the reference carrier at the receiver [Clark 1983 pp79], where noncoherent demodulation provide less complexity. Three particular techniques of noncoherent demodulation are investigated with this MFSK signalling scheme.

6.4 Square-Law Demodulation Technique

The frequency assignment scheme used with this technique [Brine and Farrell 1985] is somehow different from than the MFSK signalling format described in the previous section. That is, during every symbol interval, each user will transmit on-off keyed carrier signal according to it's coded data symbols. Also, the transmitted frequencies from each user are orthogonal to each other. That is, the i-th user is assigned the carrier;

$$f_i(t) = X_i A_i \sin((w_0+i\delta w)t+\phi_i); \quad i=0,1,\ldots,T-1 \tag{6.3}$$

where w_0 is the 0-th user carrier frequency, δw is fixed frequency separation, chosen to be $\delta w=2\pi/T_s$, to give the minimum carrier frequency separation, and $X_i \varepsilon \{0,1\}$ is the i-th user constituent codeword symbol. The resulting demodulator input r(t), assuming noiseless channel, is;

$$r(t) = \sum_{i=0}^{T-1} X_i A_i \sin((w_0+i\delta w)t+\phi_i) \tag{6.4}$$

From this equation (6.4), the collaborative constituent codeword symbols have to be recovered. The process used to recover these symbols is called square-law demodulation technique and shown in Figure 6.2. That is the received signal r(t) is squared, integrated and dumped over the symbol interval T_s. Consider the 2-user case, the resulting demodulator input is;

$$r(t) = X_0A_0\sin(w_0t+\phi_0) + X_1A_1\sin((w_0+\delta w)t+\phi_1) \tag{6.5}$$

Substituting for $\delta w=2\pi/T_s$ we get;

$$r(t) = X_0A_0\sin(w_0t+\phi_0) + X_1A_1\sin((w_0+(2\pi/T_s))t+\phi_1) \tag{6.6}$$

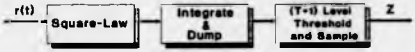


Figure 6.2
Block Diagram of Square-Law Demodulator

The demodulation process to recover the 2-user symbols X_0 and X_1 from equation (6.6) can be expressed mathematically as follows; the demodulator output is;

$$Z=\int_0^{T_s} r^2(t)\ dt \tag{6.7}$$

$$Z=\int_0^{T_s} (X_0A_0\sin(w_0t+\phi_0)+X_1A_1\sin((w_0+(2\pi/T_s))t+\phi_1))^2\ dt \tag{6.8}$$

$$Z = (X_0A_0)^2\int_0^{T_s}\sin^2(w_0t+\phi_0)\,dt$$
$$+(X_1A_1)^2\int_0^{T_s}\sin^2((w_0+(2\pi/T_s))t+\phi_1)\,dt \tag{6.9}$$
$$+2X_0X_1A_0A_1\int_0^{T_s}\sin(w_0t+\phi_0)\ \sin((w_0+(2\pi/T_s))t+\phi_1)\ dt$$

By virtue of the mutual orthognality of the users' transmissions achieved by the specified carrier separation, all cross product resulting from the squaring process integrate to zero. Hence, equation (6.9) can be closely approximated to;

$$Z = 0.5T_s(X_0A_0)^2 + 0.5T_s(X_1A_1)^2 \tag{6.10}$$

The approximation accuracy increases as wT_s increases, i.e. assuming $wT_s>>1$, that is many carrier cycles per symbol. Assuming $A_0=A_1=1$, equation (6.10) becomes;

$$Z = 0.5T_s(X_0+X_1) \tag{6.11}$$

Then since T_s is constant for a given system, sampling the integrator output after each symbol interval will provides us with the channel output symbol required. Therefore, during each symbol interval T_s, the output of the demodulator is represented by one dimensional vector Z which is the measure of the received composite signal energy during the T_s seconds interval. The channel input and output for the 2-user square-law demodulation technique are shown in Table 6.1;

(X_0, X_1)	Channel input	Z
(0,0) ——>	(0,0) ——>	(0)
(0,1) ——>	(0,F_1) ——>	(1)
(1,0) ——>	(F_0,0) ——>	(1)
(1,1) ——>	(F_0,F_1) ——>	(2)

Table 6.1 2-user Square-Law Demodulator Output

It can be seen from this table that all the possible transmission of the 2-user are recovered by the symbols (0,1,2). The ambiguity of the symbol "1" is resolved by employing CCMA uniquely decodable schemes to decode the individual users' data symbols.

6.5 Zerocrossing Demodulation Technique

The method investigated here, uses the number of zerocrossing (ZC) counts as a means of demodulating the received composite signal. This investigation has been initiated as a result of studies carried out by [Nelson 1975, Kedem 1986a, Kedem 1986b, He and Kedem 1989, and Shaw, Honary and Darnell 1989]. In these papers, they discusses and relates the counts of ZCs to the frequency components of a signal.

Consider the demodulator input signal r(t) which comprises T sinusoids;

$$r(t) = \sum_{i=1}^{T} A_i \sin(2\pi F_{ij} t + \phi_i); \quad j=1,2,\ldots,M \tag{6.12}$$

where A_i is the i-th user signal amplitude, F_{ij} is the j-th frequency of the i-th user

signal, and ϕ_i is the i-th user arbitrary phase. From this received signal, we wish to predict the average number of ordinary ZCs (OZC) and higher ZCs (HZC) counts. The probability of a OZC of a sampled waveform r(t), is defined simply as the probability of two successive samples having opposite sign. The HZC are defined in a similar way, except that the original series is first differenced or summed before the crossings are counted [Kedem 1986a, Kedem 1986b, He and Kedem 1989, and Shaw, Honary and Darnell 1989]. The effect that a differencing or summing procedure has on the series, is to change the magnitudes of the sinusoidal components. The differencing procedure is essentially differentiating or high pass filtering, causing the number of ZC to increase landing at or near the highest frequency. Summing is like integrating or low pass filtering and so causes the number of crossings to decrease landing at or near the lowest frequency.

The average number of ZC counts is denoted by, D_l, where l=0,1,...,J, is the order of ZC, i.e. the number of times the differencing or summing operator applied to the original series. Here, a useful normalization is used, that of scaling the time series such that the sampling interval becomes one second, and hence average ZC per sample. This normalization is quite useful as it makes the calculation independent of the system under consideration and simplifies notation. The normalized average ZC, μ_l, can be written as;

$$\mu_l = \pi D_l / (N'-1); \quad l=0,1,...,J \tag{6.13}$$

where N' is the number of samples taken per symbol.

Therefore, when a certain frequency band is dominant, it attracts μ_0 to admit

values in this band, at or near this frequency. In other words, a dominant frequency when it exists in the spectrum, can be quickly detected by the 0-th order of ZC only. However, when a differencing or summing operator is applied to the original series, it modifies the spectral weight emphasizing some bands while attenuating others. Consequently, when a discrete frequency exists in the spectrum it can be enhanced and then estimated by μ_i. This tendency of ZC (after proper scaling) to admit values in a neighbourhood of a significantly dominant frequency is called the dominant frequency principle [Nelson 1975, Kedem 1986a, and Kedem 1986b].

The demodulation process used here is shown in Figure 6.3, which consist of counting the number of ZCs D_i and calculating the normalised average μ_i for all orders of the received signal. Then μ_i is used to estimate the individual frequencies in the received composite signal. Therefore, during each symbol interval T_s, the output of ZC demodulator can be represented by the vector Z;

$$Z = (z_1, z_2, \ldots, z_M) \tag{6.14}$$

where z_i is the measure of the normalised average of ZCs counts of frequency F_i during the T_s second interval.

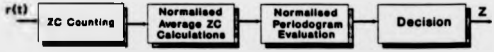


Figure 6.3
Block Diagram of Zerocrossing Counting Demodulator

As an illustrative example, consider the 2-user case with following parameters; M=2, F_1=1.25KHz, F_2=2KHz, and T_s=4msec. Since the number of cycles of F_1 and F_2 is 5 and 8, respectively, then the expected average number of ZCs counts would be 10 and 16, respectively. Therefore, during each symbol interval D_i and μ_i are calculated to detect the presence of these two frequencies in the received composite signal. Assume both frequencies are present in the received signal, then the following results are obtained for the original and differenced signals as shown in Table 6.2;

i	D_i	μ_i
0	15	1.963
1	16	2.094
2	16	2.094

Table 6.2 Normalised ZC Counts with Difference Procedure

Since the highest frequency in the composite signal is F_2, therefore it is estimated by D_0. It can also be seen from Table 6.2 that D_1=D_2=16, which indicates that there is strong components at F_2. This means that, when the highest frequency is dominant it can be detected by D_0 and more accurately by D_1 and D_2. The lowest frequency F_1 can be made dominant by applying the summing operator on the original signal and the following results are obtained as shown in Table 6.3;

i	D_i	μ_i
0	15	1.963
1	10	1.308
2	10	1.308

Table 6.3 Normalised ZC Counts with Sum Procedure

It can also be seen from Table 6.3 that D_0 is too high an estimation of F_1 because F_2 is still the dominant frequency. Therefore, as the summing operator applied, F_1 is made more dominant and estimated more accurately by D_1 and D_2. However, when l>0, a technique is needed in order to decide which of the μ_l for l=0,1,..,J, represent the true transmitted frequency. The periodogram estimation method [Kedem 1986a, Kedem 1986b,and Oppenheim and Schafer 1975 pp541-548] is used here. The periodogram is given by;

$$I(\mu_l) = 2/N' \left| \sum_{n=0}^{N'-1} r(n) \exp(-c\mu_l n) \right|^2 \tag{6.15}$$

where $c=\sqrt{-1}$. When μ_l is close to discrete frequency, the periodogram will in general be inflated. In this way we can tell which of the μ_l landed at or near a discrete frequency. Actually, the normalised periodogram, $I^*(\mu_l)$, is used here and given by;

$$I^*(\mu_l) = I(\mu_l) \Big/ \sum_{l=0}^{J} (\mu_l) \tag{6.16}$$

Therefore, the normalised relative periodograms are calculated for each order and the highest is chosen to indicate the transmitted frequency. For the above example with T=M=2, the resulting channel input and output for this demodulation process are shown in Table 6.4;

Channel input	Z
(F_1, F_1)	(1.308,0.000)
(F_1, F_2)	(1.308,2.094)
(F_2, F_1)	
(F_2, F_2)	(0.000,2.094)

Table 6.4 2-user 2FSK ZC Demodulator Output

6.6 Quadrature Demodulation Technique

It is well known that quadrature receiver is optimum method of orthogonal signals detection [Whalen 1971 pp200-201, and Clark 1983 pp65-67]. This can be achieved by using a bank of noncoherent correlators matched uniquely to the M assigned tone frequencies. The noncoherent correlators operate by evaluating the inphase and quadrature components of the received incoming signal as shown in Figure 6.4. These components are calculated by multiplying the incoming signal by sine and cosine functions generated locally at the M possible tone frequencies. The output of the multipliers are then integrated over the preceding interval corresponding to the symbol duration. The inphase and quadrature components obtained in this manner are squared, summed and finally square-rooted to produce the desired output of the correlator. The maximum correlator output is obtained when the frequency of the input signal to the correlator is equal to the frequency of the correlator reference signal. This process can be expressed mathematically as follows; the output of the l-th correlator, z_l, where l=1,2,...,M, is;

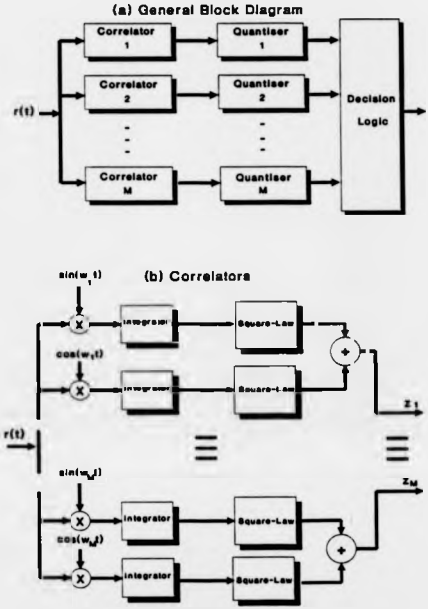


Figure 6.4
Block Diagram of Quadrature Demodulator

$$z_i = \sqrt{\left(\int_0^{T_s} r(t)\sin(w_i t)\,dt\right)^2 + \left(\int_0^{T_s} r(t)\cos(w_i t)\,dt\right)^2} \tag{6.17}$$

where r(t) is the received composite signal and is given by;

$$r(t) = \sum_{i=1}^{T} A_i \sin(2\pi F_{ij} t + \phi_i); \quad j=1,2,..,M \tag{6.18}$$

where A_i is the i-th user signal amplitude, F_{ij} is the j-th frequency of the i-th user signal, and ϕ_i is the i-th user arbitrary phase. The inphase and the quadrature components of r(t), I_l and Q_l are obtained by the following relationships, respectively;

$$I_l = \int_0^{T_s} \left(\sum_{i=1}^{T} A_i \sin(2\pi F_{ij} t + \phi_i) \right) B\sin(2\pi F_l t)\, dt; \quad l=1,2,\ldots,M \tag{6.19}$$

$$Q_l = \int_0^{T_s} \left(\sum_{i=1}^{T} A_i \sin(2\pi F_{ij} t + \phi_i) \right) B\cos(2\pi F_l t)\, dt; \quad l=1,2,\ldots,M \tag{6.20}$$

where $B\sin(2\pi F_l t)$ and $B\cos(2\pi F_l t)$ are the inphase and quadrature correlating signals at the receiver, A_i and B are the received and correlating signals amplitudes, and F_{ij} and F_l are the received and correlating tone frequencies, respectively.

As an example, consider the noiseless 2-user case with M=2. Here, all the possible transmitted tone frequencies are $(F_{11}, F_{12}, F_{21}, F_{22})$ and the correlating tone frequencies are F_1 and F_2. For this particular case the following hypothesis can be defined;

(i) Hyp0: this corresponds to the condition when each user transmits the same frequency $(F_{1j}=F_{2j})$ and this frequency is equal to the correlator reference frequency signal $(F_{ij}=F_l)$.

(ii) Hyp1: this corresponds to the condition when each user transmits different frequencies and only F_{1j} is equal to the correlator reference signal frequency ($F_{1j}=F_l$).

(iii) Hyp2: this is the same as Hyp1 except that only $F_{2j}=F_l$.

(iv) Hyp3: this corresponds to the condition when F_l is not equal to F_{1j} or F_{2j} ($F_{1j}\neq F_l$ and $F_{2j}\neq F_l$).

Evaluating the l-th correlator inphase I_l and quadrature Q_l components with respect to these hypotheses, (assuming noiseless channel conditions and $A_1=A_2=A$), we get the following:

$$I_l = 0.5ABT_s(\cos\phi_1+\cos\phi_2);\quad F_{1j}=F_{2j}=F_l \quad \text{(Hyp0 True)} \tag{6.21a}$$

$$I_l = 0.5ABT_s(\cos\phi_1);\quad F_{1j}=F_l \quad \text{(Hyp1 True)} \tag{6.21b}$$

$$I_l = 0.5ABT_s(\cos\phi_2);\quad F_{2j}=F_l \quad \text{(Hyp2 True)} \tag{6.21c}$$

$$I_l = 0;\quad F_{1j}\neq F_l \text{ and } F_{2j}\neq F_l \quad \text{(Hyp3 True)} \tag{6.21d}$$

and

$$Q_l = 0.5ABT_s(\sin\phi_1+\sin\phi_2);\quad F_{1j}=F_{2j}=F_l \quad \text{(Hyp0 True)} \tag{6.22a}$$

$$Q_l = 0.5ABT_s(\sin\phi_1);\quad F_{1j}=F_l \quad \text{(Hyp1 True)} \tag{6.22b}$$

$$Q_l = 0.5ABT_s(\sin\phi_2);\quad F_{2j}=F_l \quad \text{(Hyp2 True)} \tag{6.22c}$$

$$Q_l = 0;\quad F_{1j}\neq F_l \text{ and } F_{2j}\neq F_l \quad \text{(Hyp3 True)} \tag{6.22d}$$

The output of the correlator z_l, is given by;

$$z_l = \sqrt{(I_l^2+Q_l^2)} \tag{6.23}$$

Substituting for I_l and Q_l, we get;

$$z_1 = \sqrt{(0.5(ABT_s)^2(1+\cos(\phi_1-\phi_2)))}; \quad \text{(Hyp0 True)} \tag{6.24a}$$

$$z_1 = 0.5ABT_s; \quad \text{(Hyp1 True)} \tag{6.24b}$$

$$z_1 = 0.5ABT_s; \quad \text{(Hyp2 True)} \tag{6.24c}$$

$$z_1 = 0; \quad \text{(Hyp3 True)} \tag{6.24d}$$

The same results will hold for all the remaining correlators, since the transmitted frequencies are all orthogonal. During each symbol interval T_s, the output of all the correlators can be represented by the vector $Z=(z_1,z_2,...,z_M)$, where z_i is the measured energy at the frequency F_i during the T_s second interval. The correlators can be followed by quantisers to indicate the intensity information or confidence level of each received symbol.

(i) With Intensity Information: Here, the demodulator output at each instant of time, indicates which frequencies were transmitted at that instant and howmany of each frequency were transmitted. This assumes that the carriers are frequency and phase locked. In this case, $z_i \varepsilon \{0,1,2,...,T\}$ and $z_1+z_2+...+z_M=T$. That is, each quantiser output indicates the number of signals received at that particular frequency. Examples of 2-user MFSK system with intensity information are shown in Table 6.5 and Table 6.6 for M=2 and 4;

Channel input	Z
(F_1, F_1)	(2,0)
(F_1, F_2)	(1,1)
(F_2, F_1)	
(F_2, F_2)	(0,2)

Table 6.5 2-user 2FSK With Intensity Information Quadrature Demodulator Output

Channel input	z
(F_1,F_1)	(2,0,0,0)
(F_1,F_2)	(1,1,0,0)
(F_2,F_1)	
(F_2,F_2)	(0,2,0,0)
(F_1,F_3)	(1,0,1,0)
(F_3,F_1)	
(F_2,F_3)	(0,1,1,0)
(F_3,F_2)	
(F_3,F_3)	(0,0,2,0)
(F_1,F_4)	(1,0,0,1)
(F_4,F_1)	
(F_2,F_4)	(0,1,0,1)
(F_4,F_2)	
(F_4,F_4)	(0,0,0,2)

Table 6.6 2-user 4FSK With Intensity Information Quadrature Demodulator Output

(ii) Without Intensity Information: The channel output at each instant of time is defined as a symbol which identifies which subset of frequencies occurred as inputs to the channel at that instant of time without the intensity of each frequency occurrence. This results in the demodulator that measure the presence of one or more tones at each frequency without regard for the number of tones at each frequency, i.e. $z_j \epsilon \{0,1\}$. Examples of 2-user MFSK system without intensity information are shown in Table 6.7 and Table 6.8 for M=2 and 4;

Channel input	z
(F_1,F_1)	(1,0)
(F_1,F_2)	(1,1)
(F_2,F_1)	
(F_2,F_2)	(0,1)

Table 6.7 2-user 2FSK Without Intensity Information Quadrature Demodulator Output

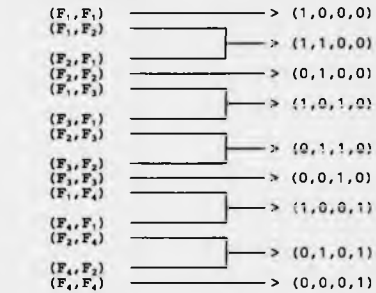


Table 6.8 2-user 4FSK Without Intensity Information Quadrature Demodulator Output

6.7 2-user MFSK_CCMA System Development

A complete practical CCMA communication system has been developed and simulated in a software. This consisted of two main sections; the users' encoders/decoder, and modulators/demodulator. The design procedure of MFSK_CCMA system modulator is critical in terms of the choice of the frequency allocation. This choice is constrained by the following considerations [Shaw 1989, and Zolghadr 1989]; (a) system bandwidth, (b) orthogonality, (c) bandwidth power containment, and (d) implementation limitations.

The tones frequency spacing is primarily specified by the system bandwidth available, which should be divided appropriately such that all the system requirements are satisfied. The frequency separation is chosen such that to satisfy the orthogonality constraint and to guarantee a fixed energy per received symbol. The frequency spacing

and more importantly, the frequency guard spaces at the boundaries of the system bandwidth, specify the power containment within that bandwidth. A frequency guard space of one tone spacing at each edge of the system passband is sufficient to meet 99% power containment [Shaw 1989, and Zolghadr 1989]. The other limitations are specified by the practical implementation constraints. Since we are considering a sampled system, we must expect that the sampling frequency, FS, will have some effect also. The value of FS should be set at the minimum value possible in order to have the minimum amount of processing to do i.e. avoiding redundancy. In addition, to avoid phase discontinuity T_s is taken to be an integer multiple of $\delta t=1/FS$ and the tones must have an integer number of whole cycles. Therefore, a 2FSK/4FSK CCMA modem is designed to satisfy these requirements. The designed tone frequencies are, F_1=1.25kHz, F_2=1.5kHz, F_3=1.75kHz, and F_4=2.0kHz. These choices correspond to the following parameters, T_s=4msec, δF=250Hz, and FS=6.25kHz.

The demodulation techniques described in this chapter are used and simulated in software. Generally, the demodulator output vector is compared with each of the stored reference vectors in turn and a measure of the distance between them is calculated. The distance is taken by finding the difference between the vector elements. The sum of the squares of the differences is then taken as the distance measure from the received vector to the reference vector. The reference vector with the smallest distance from the received vector is then selected to be the one demodulated. The 2-user encoders employed Code 1, given in chapter 5, where user one and two codewords are (00,11) and (00,01,10), respectively. The single decoder uses maximum likelihood decoding and a look-up table to decode the users' constituent codewords and hence the original data symbols.

6.8 Simulation Results and Modems Testing

The simulations are carried out for the following synchronous CCMA communication systems;

System 1: 2-user 2-frequency square-law demodulator,

System 2: 2-user 2FSK ZC demodulator,

System 3a: 2-user 2FSK/4FSK quadrature demodulator with intensity information,

System 3b: 2-user 2FSK/4FSK quadrature demodulator without intensity information.

Also the following tests are carried out for these systems. The first test is to verify the systems correct operation; secondly, tests are carried out by simulation techniques under AWGN channel conditions.

6.8.1 Modems Operation Verification

This test is carried out to verify the complete systems operation under noise free conditions. The operation of the modem is verified by examining its time domain outputs. Figures 6.5-6.6, shows the overall modulator output, demodulator input, and demodulator output waveforms for system 1 and system 3, respectively. The system 2 modulator is the same as in system 3, and the demodulation process is counting the number of ZCs.

6.8.2 AWGN Channel Tests

Although many practical channels do not introduce significant levels of Gaussian noise, the relative tolerances of different data transmission systems to AWGN is a good measure of their relative tolerances to most practical types of additive noise [Clark 1983 pp27]. Therefore, these systems are tested under AWGN conditions. The noise level is

(a) Transmitted Constituent Codeword Symbols

X_1	1	1	0	0	0	0	1	1	0	0	1	1	1	1	0	0
X_2	1	0	0	0	0	1	1	0	0	1	1	0	0	1	0	0

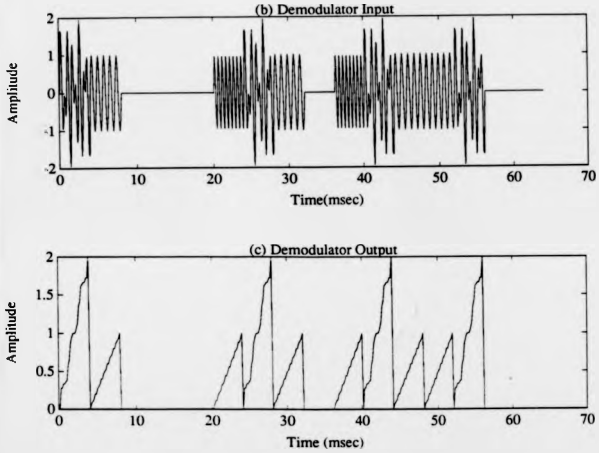


Figure 6.5 System 1 Demodulator Input and Output Waveforms

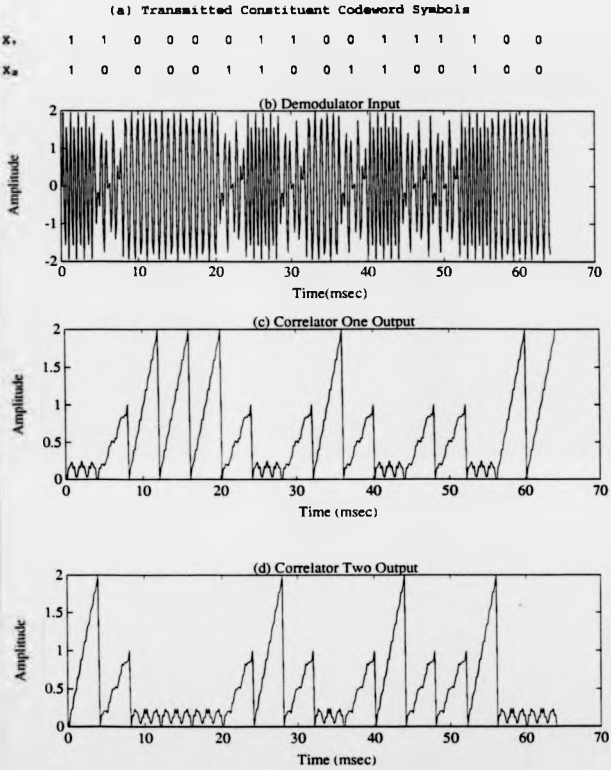


Figure 6.6 System 3 Demodulator Input and Output Waveforms

specified by the ratio E/N_o, where E is the average signal energy per user and N_o is the power spectral density. For the purpose of simulation, E/N_o is given by;

$$E/N_o = 10\log_{10}(PN'/2\sigma_N^2) \tag{6.25}$$

where P is the average signal power per user, σ_N^2 is the noise variance, and N' is the number of samples per symbol. The simulation results are obtained and presented in terms of the demodulator output symbol error rates versus E/N_o, as shown in Figure 6.7. It is shows that system 3 with intensity information has the best reliability. However, these systems vary in terms of their complexity. A comparison of quadrature receivers employing 2FSK and 4FSK transmission systems is also shown in Figure 6.8. The 2FSK signalling is seen to perform better than the 4FSK, due to the increase in the number of signals at the receiver.

6.9 Discussions

The principles of practical design and performance of CCMA systems were described. It is shown that the square-law demodulation technique performs reliably in noise free conditions, but it's performance degrades in noisy conditions, in particular at low E/N_o. However, its performance may be improved by preceeding the demodulator by a bandpass filter to limit the amount of noise entering the demodulator. It is also seen that the ZC demodulator reliability is worse than that of the quadrature receiver. However, the ZC demodulator requires easily implementable functions for its operation. Also, this technique may improve in wideband systems where the M frequencies can

Figure 6.7 Demodulator
SER versus E/No (dB)

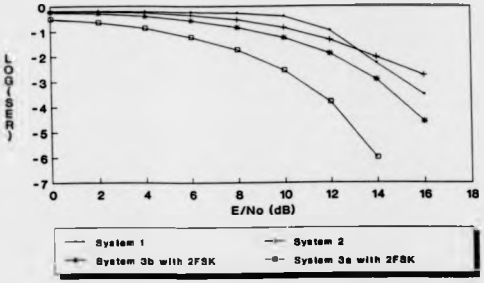
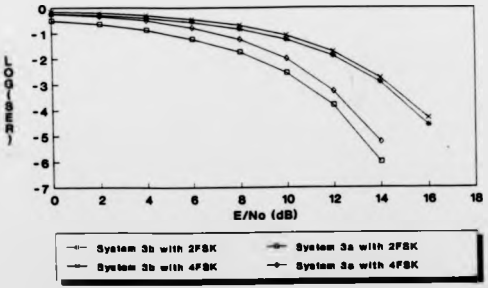


Figure 6.8 System 3 Demodulator
SER versus E/No (dB)



be separated as much as possible, resulting in a larger difference in the number of ZCs. The ZC demodulation technique may also improve with some form of adaptive filtering. In such a case, (i) the highest frequency can be determined first by counting the number of ZC of the received signal, (ii) filter out the highest frequency and count the number of ZC, and (iii) repeat (i) and (ii) until M frequencies are detected. It can be seen here that the cutoff frequency of the filter is adapted accordingly every time a certain frequency is detected.

The simulation results analyses have shown that the quadrature receiver model with intensity information gives an energy gain over the model without intensity information. The model without the intensity information, is similar to a hard decision demodulator which decides on the received symbol without the indication of the decision confidence level. However, the demodulation technique which takes into account the intensity information or the confidence level of the received symbol, is comparable to soft decision demodulation. Therefore, this demodulation can be easily adjusted to take into account these different techniques. This may be achieved by employing a quantiser, after the energy detectors, of a specified number of quantisation levels. This technique has many other advantages and applications, such as (i) the demodulator output can be easily modified to convey the hard or soft decision information and (ii) the ability to be adapted to extract the symbol synchronisation which is assumed in the present investigation.

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Chapter 7

Conclusions and Further Work

7.1 Conclusions

The major research reported in this thesis has been carried out in order to investigate various theoretical and practical aspects of CCMA communication systems. These investigations involved the following main areas; the information transmission capacity, coding/decoding and practical design. The information capacity of SAC when subjected to practical constraints has also been investigated as a first stage to the investigative study of CCMA schemes. In the following sections of this Chapter we give the conclusions and achievements obtained from this study. Extensions and suggestions for further work are also given.

7.1.1 Information Capacity of Constrained SAC

In Chapter three, the principles, determination and optimisation of a sampled, and quantised SAC information transmission capacity limited by practical constraints have been described. The ISA/OSC and ISAP/OSC constrained capacities of an AWGN channel have been determined separately. The capacity analysis has been carried out from theoretical and simulation points of view. The results obtained verify the theoretical results given in [Smith 1971], when a sampled and quantised channel is employed.

It has been found that the constrained capacities can be achieved by unique and

optimal input distributions and finite output signal clipping values. These optimal distributions have been found to be discrete, taking a finite number of values, with the quantisation levels being relatively closely spaced at the low signal amplitudes and more widely spaced at the large signal amplitudes. In addition, the results obtained can be used to determine the number of input amplitude levels needed to maximise the capacity at different SNRs. Hence, these results may be used in signal design and channel evaluation as discussed further in section (7.2.1).

7.1.2 Information Capacity of MACs

The principles of MACs information transmission capacity have been described in Chapter four. The capacity of T-user M-ary adder MAC and T-user M-ary frequency MAC with and without intensity information models have been described. The capacity calculation has been carried out theoretically and by simulations in both noiseless and noisy channels. It has been shown that, in principle, T-user CCMA permits higher transmission rates than TDMA employing the same signal alphabet or, equivalently, achieve a coding gain over time sharing. This enables efficient simultaneous transmission by several users sharing the same channel, and provides a degree of error protection to protect the messages from channel disturbances.

It has also been shown that T-user M-ary signalling models provide a means of increasing the rate of information transmission within a given bandwidth, however, this increase was at the expense of an additional transmitter power. In addition, it has been shown that the practical model of T-user M-ary frequency MAC with intensity information achieved higher capacity than that without intensity information. This is due to the extra information provided by this model to indicate the intensity or confidence

level of the received signal. These two models are comparable with soft and hard decision demodulation techniques, in which higher gain is achievable employing a soft decision model.

7.1.3 Collaborative Coding/Decoding Multiple Access Techniques

Collaborative coding and decoding techniques to utilise the MAC function and error control capability of collaborative codes have been described in Chapter five. In particular, HD_CCMA and MLSD_CCMA decoding techniques were described in conjunction with SBS_HD decoding. A new low complexity MLSD decoding technique has been described. This decoding algorithm is attractive because it can be very easily realised. The reliability performance have been carried out with various collaborative codes.

It has been shown that uniquely decodable CCMA schemes permit the multiple access function to be combined with that of forward error correction, provided the users maintain symbol and block synchronisations. The MLSD_CCMA decoding technique decreases the overall probability of error with some energy gain. The energy gain achieved is higher when the codes used have some error protection capability. A coding gain of more than 2.5dB has been achieved employing MLSD_CCMA over HD_CCMA decoding technique.

7.1.4 Practical CCMA modem Design

In Chapter six, MFSK modulation technique has been described with three particular demodulation techniques. A combined collaborative coding and modulation system has been developed to provide a realistic and practical way of combining the

users' signals at the channel and data recovery at the receiver. These techniques have been developed and simulated for their reliability performance over AWGN channels.

The MFSK quadrature receiver provided the best reliability performance. The T-user M-ary frequency channel model with intensity information gave an energy gain over the model without intensity information. These models can be used to represent the soft and hard decision demodulation techniques with simple adjustment of the demodulator output. In addition, this demodulation technique has other advantages, for example symbol derived synchronisation may be extracted from the received waveform out of the demodulation process, as described later in section (7.2.6).

7.2 Further Work

The following sections present extensions and suggestions for further work that have come about during the course of this research. Some of these ideas stem from my own research interests.

7.2.1 Optimisation of Channel Capacity

It has been shown in Chapter three that the optimisation of channel capacity was achieved by the use of optimum input signal distributions. These optimum input signal distributions may be used in upgrading the signalling format for certain channel conditions. As far as the author is aware, a coding scheme that can make use of the uneven probabilities is not available. Therefore an area of considerable research would be the design of codes sequences with specified rates and optimum symbol probabilities.

In order to optimise the system performance adaptively in response to channel

conditions, an estimate of the channel SNR or the receiver's error rate is required to initiate control action. Real time channel evaluation (RTCE) techniques [Darnell 1978, 1983] are useful tools for obtaining on-line estimates of the channel state. Therefore, using some form of channel evaluation technique, the designed signals can be used for transmission according to various channel states in a certain time. A statistical RTCE technique [Zolghadr, et al., 1989] may be used which is based upon the statistical analysis of both the transmitted and the received data. The transmitter data model can be based on the signal levels and their distribution already found. The received stream of data can then be monitored and its statistical properties determined in order to formulate the receiver model. If the channel is noiseless then the received data stream would have the same statistical structure as the transmitted data. However, when the channel is noisy, any variation in the received data statistical structure can potentially provide information on the channel state. Therefore, combining signal design and channel evaluation techniques can be viewed as a form of multi-function coding design philosophy.

In the work of Chapter four, it has been assumed in the modelling and development of MAC capacity that the capacity is optimised when the input signal probabilities are uniformly distributed. This assumption is not always true [Chang and Wolf 1981]. Therefore, analytical investigation of the capacity optimisation for a certain MAC model, and given values of T and M, is very important. In addition, the optimisation of MAC capacity under practical constraints also need investigation. For example, at the channel output the number of quantisation levels and signal clipping level may be imposed, while the mean signal power for each user may be imposed at the channel input.

7.2.2 Improved Collaborative Coding/Decoding Designs

In Chapter five, it has been found that although, in some cases, the overall T-user code has no error protection capability, the constituent codes have some protection. Therefore, utilisation of the individual user's code protection capability may improve the reliability performance of each user. This may be viewed as some form of inner and outer decoding process. That is, inner decoding may be performed on the overall received composite codeword and outer decoding on the constituent codes. This process may be used with concatenated CCMA coding scheme [Weldon 1978, Ohkubo 1980, and Mathys 1989], in which the transmitted block of data consist of inner and outer code. In this case, the inner code performs the multiple access function (possibly with some protection capability), and the outer code performs error correction where the existing single access coding schemes can be used. Also of very considerable importance is to find simple and efficient decoding schemes; firstly to unscramble the received composite signal and secondly to perform error correction if possible.

It has been found in the investigation of CCMA schemes that, symbol and block synchronisation must be maintained for efficient functioning of these schemes. Therefore, a reduction in the degree of synchronisation required is very important area of investigation. Ideally, the CCMA system would be completely asynchronous. Some work in this area is under investigation [Ni and Honary 1992], which is based on the information extracted from the channel sensed signals and used for both symbol and block synchronisation. That is, before transmission, every user senses the channel in advance. If the channel is idle, transmission will start immediately. Otherwise, if the channel is busy, the ready user try to synchronise its transmission with the existing transmission both in symbol and block.

7.2.3 Adaptive CCMA Transmission System

From the investigation study of this thesis, there is a need for more combined approach to multiple access communication. One approach is an adaptive T-user CCMA MFSK transmission system in which the information throughput of the system can be optimised by an appropriate choice of (i) T, the number of active users; (ii) M, the number of tones; (iii) R, the user's code rate. Also by using some form of RTCE technique [Darnell 1978], an adaptive system may be investigated to match the transmission format to the state of the channel.

Two of the important parameters which need to be varied in the present coding schemes are the number of users accessing the channel and the code rate of the individual users so that users can have equal or different rates. Most of the work on MAC has concerned the situation when all T users in the system are simultaneously active. The area of T-active users out of N was studied by [Wolf 1981, Mathys 1987, and Mathys 1988]. Ideally, an appropriate model for multiple access communication is to incorporate a large community of users, some fraction (small or large) being active simultaneously. In these situations, there are also two fundamental tasks to be done by the receiver; (i) identify the active users, and (ii) decode the messages of the active users without errors if possible.

Optimisation of the number of tones used in the T-user MFSK transmission system also requires investigation. For example, it has been found in the work of Chapter four that the capacity of the T-user M-ary frequency without intensity information decreases for a certain fixed value of M and a certain increase in the number of users T. This suggests that the number of tones used with this particular number of users is not sufficient to achieve the capacity, hence, M should be optimised

with respect to the active number of users.

7.2.4 Multi-Functional Signal Design Format

Conventional communication systems design philosophy view the design of the modem and the error control subsystems as two separate issues. This has lead to the division of these two fields. However, it is suggested that combining the various subsystems of a communication system can provide additional gain; this has been broadly referred to as multi-functional coding [Darnell and Honary 1986]. The potential of multi-functional coding may be used here, combining collaborative coding and modulation. Also the integration of the demodulation and channel decoding processes together with symbol and block synchronisation may lead to more reliable system. One of these ideas is explained here.

Imperfect symbol synchronisation is one of the problems in the CCMA schemes. Here, we propose a scheme called modulation derived synchronisation (MDS) technique to be investigated for the CCMA. This is a symbol synchronisation method with a multi-functional capability for synchronisation acquisition, demodulation and RTCE. It can be applied in modems that employ digital signal processing techniques to operate on the received waveform. This was originally developed for MFSK transmission systems [Darnell and Honary 1988]. The technique has the advantage that it requires no specific synchronisation "overhead" to be incorporated into the transmitted signal, but uses the normal traffic signal formats.

This technique can be used with the noncoherent quadrature MFSK transmission system already investigated as outlined very briefly here. At the receiver, at any arbitrary sample time, T_s, the summation of the product of the last n samples with (a)

sine components and (b) cosine components of the M tones (in sampled form) is computed. The number of samples, n, corresponds to a symbol interval for the transmission. Thus what is effectively being implemented is a matched filter for the system at each sampling instant. The procedure yields a vector with 2M orthogonal components, if the tone frequencies are selected appropriately. Taking the modulus of corresponding sine and cosine components gives an M-dimensional vector at each sampling instant. The magnitude of the vector associated with a particular tone will be a maximum when that tone has been transmitted and n samples correspond exactly to the symbol interval [Shaw 1989]. The main problem with a digital demodulation process is to derive the optimum time instant for sampling the correlator outputs. It is evident from the waveform Figure 6.6 in Chapter six, that the sampling instant corresponds to the end of a given symbol interval; however, in the presence of noise this position becomes less obvious.

In addition to these investigations, if carrier phase coherence is not maintained the effects of fading caused by phase cancellation should be reduced to the minimum, ideally the system would be completely independent of these effects.

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Symbols and Abbreviations

A	Input Signal Amplitude
ADC	Analogue to Digital Convertor
ASK	Amplitude Shift Keying
AWGN	Additive White Gaussian Noise
BW	Bandwidth
BER	Bit Error Rate
C	Capacity
C_{SAC}	Capacity of Single Access Channel
C_{MAC}	Capacity of Multiple Access Channel
C(x)	Capacity as a Function of x
C_i	Code of i-th User
CCMA	Collaborative Coding Multiple Access
CDMA	Code Division Multiple Access
CER	Codeword Error Rate
CW_i	Number of Codewords of i-th User
$d_i(Z,Z')$	Distance Between the Vectors Z and Z' in the i-th User Code
d_L	L-distance
d_{min}	Minimum Distance
d_{min_i}	Minimum distance of i-th User Code
dB	Decibel
DMC	Discrete Memoryless Channel

d(i)	Number of Combinations which result in the i-th event
E	Energy per User
E/N_o	Energy per User to Noise Energy Ratio
EDC	Error Detection and Correction
Erf	Error Function
Erfc	Complementary Error Function
FDMA	Frequency Division Multiple Access
FS	Sampling Frequency
f_i, F_i	The i-th Frequency
$f_i(t)$	The i-th User Signal
$f_i(w)$	Probability Density Function of the i-th Symbol
$f_i(w_1 w_2 ... w_N)$	Joint Probability Density Function of the i-th Admissible Codeword
FSK	Frequency Shift Keying
G_i	Region of Correct Decision
HF	High Frequency (2-30MHz)
Hz	Hertz, frequency in cycles per second
HD	Hard Decision
HD_CCMA	Hard Decision Collaborative Coding Multiple Access
I(X)	Information Content of X
I(X;Y)	Mutual Information Between X and Y
ISA/OSC	Input Signal Amplitude and Output Signal Clipping
ISAP/OSC	Input Signal Amplitude, Average Signal Power and Output Signal Clipping
k	Number of Information Digits

K	Channel Input Alphabet Size
L	Number of Channel Output Signal Levels
M	Number of Channel Input Signal Levels
MAC	Multiple Access Channel
MED	Minimum Euclidean Distance
MFSK	M-ary Frequency Shift Keying
ML	Maximum Likliehood
MLSD_CCMA	Maximum Likliehood Soft Decision Collaborative Coding Multiple Access
N	Block Length of a Code or a Vector; or Noise Power
N'	Number of Samples per Symbol
N_a	Number of Admissible Codewords
N_f	Number of Forbidden Codewords
N_o	Noise Power Spectral Density
OSC	Output Signal Clipping
PDF	Probability Density Function
p(i)	Probability of i-th Symbol
$P_{se}(i)$	Probability of Error of i-th Symbol
$P_{se}(to)$	Total Probability of Symbols Error
$P_{ce}(i)$	Probability of Error of i-th Codeword
$P_{ce}(to)$	Total Probability of Codewords Error
$P_{sc}(i)$	Probability of i-th Correct Symbol
$P_{sc}(to)$	Total Probability of Correct Symbols

$P_{cc}(i)$	Probability of i-th Correct Codeword
$P_{cc}(to)$	Total Probability of Correct Codewords
$p(Y \mid X)$	Conditional or Transition Probability
PSK	Phase Shift Keying
QL	Number of Quantisation Levels
R	Rate of Information
R_i	Rate of i-th User
R_{sum}	Rate Sum
S_i	The i-th Composite Symbol
SER	Symbol Error Rate
SNR	Signal to Noise Ratio
SAC	Single Access Channel
SC	Signal Clipping
SD	Soft Decision
SBS_HD	Symbol-by-Symbol Hard Decision
T	Number of Active Users
TDMA	Time Division Multiple Access
T_s	Symbol Period
U_i	Source Symbol of the i-th User
$\hat{U}_i$	Sink Symbol of the i-th User
u_i	i-th Decision Threshold
X	Channel Input
X_i	Channel Input of the i-th User
Y	Channel Output

ZC	Zerocrossing
δt	Sampling Interval
δq	Quantiser Step Size
σ_S^2	Signal Variance
σ_N^2	Noise Variance
$\lfloor x \rfloor$	Integer Number Less Than or Equal to x

Appendix A

Conditional Probability Density Function for Quantised AWGN Channel Output

The Gaussian probability density function with zero mean and unit variance is,

$$P(x) = \exp(-x^2/2)/\sqrt{(2\pi)} \tag{A.1}$$

The normalised error function, Erf(x), of zero mean and unit variance is defined as;

$$Erf(x) = \int_{-\infty}^{x} \exp(-t^2/2)/\sqrt{2\pi}\, dt \tag{A.2}$$

The Erf(0)=0.5 and Erf(∞)=1. Then the complementary error function Erfc() is defined as;

$$Erfc(x) = \int_{x}^{\infty} \exp(-t^2/2)/\sqrt{2\pi}\, dt \tag{A.3}$$

The conditional probability density function, for i=1,...,M,

$$\begin{aligned} p(Y \mid x_i) &= Erf((y-x_i+\delta q/2)2)/\sigma_N) - Erf((y-x_i-\delta q/2)/\sigma_N), \text{ where } -SC<y<SC \\ &= Erf((y-x_i+\delta q/2)2)/\sigma_N), \text{ where } y=-SC \\ &= Erfc((y-x_i-\delta q/2)/\sigma_N), \text{ where } y=SC \end{aligned} \tag{A.4}$$

Appendix B

Symbol Error Rate for 2-user Binary CCMA Scheme

The number of admissible symbols at the channel output is given by L=T+1. Therefore, we have three admissible symbols given by S_0="0", S_1="1", and S_2="2". If we assume that; (i) the channel is AWGN of zero mean and variance ($\sigma_N{}^2$), (ii) the decision thresholds are u_0, u_1 between [S_0,S_1] and [S_1,S_2] respectively. Then the probability of receiving a given symbol in error $P_{se}(S_i)$, can be calculated as follows;

$$P_{se}(S_0) = \int_{u_0}^{\infty} \exp(-(w-S_0)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw \tag{B.1}$$

Let $t=(w-S_0)/\sigma_N$, then $dt=dw/\sigma_N$

$$P_{se}(S_0) = \int_{(u_0-S_0)/\sigma_N}^{\infty} \exp(-t^2/2)/\sqrt{2\pi}\,dt \tag{B.2}$$

This equation can be written in terms of the error function equations given in Appendix A;

$$P_{se}(S_0) = \mathrm{Erfc}(u_0-S_0/\sigma_N) \tag{B.3}$$

Similarly the probability of Error of the second symbol (S_1) can be written as;

$$P_{se}(S_1) = \int_{-\infty}^{u_0} \exp(-(w-S_1)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw$$
$$+ \int_{u_1}^{\infty} \exp(-(w-S_1)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw \tag{B.4}$$

Let $t=(w-S_1)/\sigma_N$, then $dt=dw/\sigma_N$

$$P_{se}(S_1) = \int_{-\infty}^{(u_0-S_1)/\sigma_N} \exp(-t^2/2)/\sqrt{2\pi}\, dt$$
$$+ \int_{(u_1-S_1)/\sigma_N}^{\infty} \exp(-t^2/2)/\sqrt{2\pi}\, dt \tag{B.5}$$

Hence

$$P_{se}(S_1) = \mathrm{Erf}(u_0-S_1/\sigma_N) + \mathrm{Erfc}(u_1-S_1/\sigma_N) \tag{B.6}$$

Similarly again the probability of error of the third symbol (S_2) is given by;

$$P_{se}(S_2) = \int_{-\infty}^{u_1} \exp(-(w-S_2)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw \tag{B.7}$$

Let $t=(w-S_2)/\sigma_N$, then $dt=dw/\sigma_N$

$$P_{se}(S_2) = \int_{-\infty}^{(u_1-S_2)/\sigma_N} \exp(-t^2/2)/\sqrt{2\pi}\, dt \tag{B.8}$$

Hence

$$P_{se}(S_2) = \mathrm{Erf}(u_1-S_2/\sigma_N) \tag{B.9}$$

Then the total probability of symbol error rate for the 2-user binary MAC model can be written as;

$$P_m(to) = p(S_0)P_m(S_0)+p(S_1)P_m(S_1)+p(S_2)P_m(S_2) \tag{B.10}$$

where u_0, u_1 are the decision thresholds equal to 0.5 and 1.5 respectively for the binary case; σ_N is the standard deviation of the noise; $p(S_i)$ is the probability of occurrence of S_i; Erf, Erfc are the normalised and complementary error functions respectively, given in Appendix A.

Appendix C

Codeword Error Rate Employing HD Decoding for 2-user Binary CCMA Scheme

The composite admissible codewords for the 2-user code 1 given in Table 5.1 are (00,01,10,12,21,11). The total probability of error can be written as;

$$P_{ce}(to) = 1 - P_{cc}(to) \tag{C.1}$$

and the total probability of correct decision is,

$$P_{cc}(to) = (P_{cc}(00) + 2P_{cc}(10) + 2P_{cc}(21) + P_{cc}(11))/6 \tag{C.2}$$

where $P_{cc}(21) = P_{cc}(12)$ and $P_{cc}(10) = P_{cc}(01)$. If we assume that the channel is AWGN of zero mean and variance σ_N^2, then the probability of correct decision for each admissible codeword can be found as follows;

For the codeword "00"

$$P_{cc}(00) = \int_{-\infty}^{0.5u} \exp(-(w_1)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_1 \int_{-\infty}^{0.5u} \exp(-(w_2)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_2 \tag{C.3}$$

Take

$$I = \int_{-\infty}^{0.5u} \exp(-(w_2)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_2 \tag{C.4}$$

Let $t = w_2/\sigma_N$, $dt = dw_2/\sigma_N$

$$I = \int_{-\infty}^{0.5u/\sigma_N} \exp(-t^2/2)/\sqrt{2\pi} \; dt \tag{C.5}$$

Write in terms of the error function equations, we get;

$$I = \mathrm{Erf}(0.5u/\sigma_N) \tag{C.6}$$

Substitute back in (C.3), we get;

$$P_{cc}(00) = \mathrm{Erf}^2(0.5u/\sigma_N) \tag{C.7}$$

In similar manner the probability of correct decision for the remaining admissible codewords can be found as shown below.

<u>For the codeword "11"</u>

$$P_{cc}(11) = \int_{0.5u}^{1.5u} \exp(-(w_1-u)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_1 \int_{0.5u}^{1.5u} \exp(-(w_2-u)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_2 \tag{C.8}$$

Take

$$I=\int_{0.5u}^{1.5u} \exp(-(w_2-u)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_2 \tag{C.9}$$

Let $t=w_2-u/\sigma_N$, $dt=dw_2/\sigma_N$

$$I=\int_{(0.5u-u)/\sigma_N}^{(1.5u-u)/\sigma_N} \exp(-t^2/2)/\sqrt{2\pi}\; dt \tag{C.10}$$

$$I= \mathrm{Erf}(0.5u/\sigma_N) - \mathrm{Erf}(-0.5u/\sigma_N) \tag{C.11}$$

Since $\mathrm{Erf}(-x)=1-\mathrm{Erf}(x)$

$$I= (2\mathrm{Erf}(0.5u/\sigma_N) - 1) \tag{C.12}$$

Hence, substitute back in (C.8) we get:

$$P_{cc}(11)= (2\mathrm{Erf}(0.5u/\sigma_N) - 1)^2 \tag{C.13}$$

<u>For the codeword "10"</u>

$$P_{cc}(10) = \int_{0.5u}^{1.5u} \exp(-(w_1-u)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_1 \int_{-\infty}^{0.5u} \exp(-(w_2)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_2 \tag{C.14}$$

Take

$$I = \int_{-\infty}^{0.5u} \exp(-(w_2)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_2 \tag{C.15}$$

Let $t=w_2/\sigma_N$, $dt=dw_2/\sigma_N$

$$I = \int_{-\infty}^{(0.5u)/\sigma_N} \exp(-t^2)/\sqrt{2\pi} \; dt \tag{C.16}$$

$$I = \mathrm{Erf}(0.5u/\sigma_N) \tag{C.17}$$

Take other integral in equation (C.14);

$$I' = \int_{0.5u}^{1.5u} \exp(-(w_1-u)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_1 \tag{C.18}$$

Let $t=w_1-u/\sigma_N$, $dt=dw_1/\sigma_N$

$$I' = \int_{(0.5u-u)/\sigma_N}^{(1.5u-u)/\sigma_N} \exp(-t^2)/\sqrt{2\pi} \; dt \tag{C.19}$$

$$I' = (\mathrm{Erf}(0.5u/\sigma_N) - \mathrm{Erf}(-0.5u/\sigma_N)) \tag{C.20}$$

$$I' = (2\mathrm{Erf}(0.5u/\sigma_N) - 1) \tag{C.21}$$

Hence, substituting back for I and I', we get:

$$P_{\infty}(10) = \mathrm{Erf}(0.5u/\sigma_N)\;(2\mathrm{Erf}(0.5u/\sigma_N)-1) \tag{C.22}$$

For the codeword "12"

$$P_{cc}(12) = \int_{0.5u}^{1.5u} \exp(-(w_1-u)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_1 \int_{1.5u}^{\infty} \exp(-(w_2-2u)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_2 \qquad \text{(C.23)}$$

Take

$$I = \int_{1.5u}^{\infty} \exp(-(w_2-2u)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_2 \qquad \text{(C.24)}$$

Let $t=w_2-2u/\sigma_N$, $dt=dw_2/\sigma_N$

$$I = \int_{(1.5u-2u)/\sigma_N}^{\infty} \exp(-t^2)/\sqrt{2\pi}\,dt \qquad \text{(C.25)}$$

$$I = \mathrm{Erf}(0.5u/\sigma_N) \qquad \text{(C.26)}$$

$$I' = \int_{0.5u}^{1.5u} \exp(-(w_1-u)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_1 \qquad \text{(C.27)}$$

Let $t=w_1-u/\sigma_N$, $dt=dw_1/\sigma_N$

$$I' = \int_{(0.5u-u)/\sigma_N}^{(1.5u-u)/\sigma_N} \exp(-t^2)/\sqrt{2\pi}\;dt \qquad \text{(C.28)}$$

$$I' = (\mathrm{Erf}(0.5u/\sigma_N) - \mathrm{Erf}(-0.5u/\sigma_N)) \qquad \text{(C.29)}$$

$$I' = (2\mathrm{Erf}(0.5u/\sigma_N)-1) \qquad \text{(C.30)}$$

Hence

$$P_{cc}(12) = (2\mathrm{Erf}(0.5u/\sigma_N)-1)\;\mathrm{Erf}(0.5u/\sigma_N) \qquad \text{(C.31)}$$

Substitute these probabilities of correct decision of each admissible codeword in to equation (C.2) and then use (C.1) to get the total probability of error.

Appendix D

Codeword Error Rate Employing MLSD Decoding for 2-user Binary CCMA Scheme

The same procedure can be carried out for employing MLSD decoding. The probability of correct decision of the codewords "00" and "11" are the same as in the hard decision case for this particluar code structure. However, the probability of correct decision of the remaining codewords can be calculated as follows;

For the codeword "01"

$$P_{cc}(01) = \int_{-\infty}^{0.5u} \exp(-(w_1)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_1 \int_{0.5u}^{(2u-w_1)} \exp(-(w_2-u)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_2 \qquad \text{(D.1)}$$

Take

$$I = \int_{0.5u}^{2u-w_1} \exp(-(w_2-u)^2/2\sigma_N^2) \;/\; \sqrt{2\pi}\,\sigma_N \, dw_2 \qquad \text{(D.2)}$$

Let $t=w_2-u/\sigma_N$, $dt=dw_2/\sigma_N$

$$I = \int_{(0.5u-u)/\sigma_N}^{(2u-w_1-u)/\sigma_N} \exp(-t^2/2)/\sqrt{2\pi}\; dt \qquad \text{(D.3)}$$

$$I = \mathrm{Erf}((u-w_1)/\sigma_N) - \mathrm{Erf}(-0.5u/\sigma_N)) \qquad \text{(D.4)}$$

Substitute back in equation (D.1);

$$P_{cc}(01) = \int_{-\infty}^{0.5u} \exp(-(w_1)^2/2\sigma_N^2)/\sqrt{2\pi}\sigma_N \quad (Erf((u-w_1)/\sigma_N) - Erf(-0.5u/\sigma_N))\, dw_1 \tag{D.5}$$

Let $t=w_1/\sigma_N$, $dt=dw_1/\sigma_N$

$$P_{cc}(01) = \int_{-\infty}^{0.5u/\sigma_N} \exp(-t^2/2)/\sqrt{2\pi} \quad (Erf((u/\sigma_N)-t) + Erf(0.5u/\sigma_N) - 1)\, dt \tag{D.6}$$

Hence

$$P_{cc}(01) = (Erf(0.5u/\sigma_N) - 1)(Erf(0.5u/\sigma_N)) + \int_{-\infty}^{0.5u/\sigma_N} (\exp(-t^2/2)/\sqrt{2\pi})\ (Erf((u/\sigma_N)-t))\, dt \tag{D.7}$$

In similar manner the probability of correct decision for the remaining admissible codewords can be found as shown below.

<u>For the codeword "21"</u>

$$P_{cc}(21) = \int_{1.5u}^{\infty} \exp(-(w_1-2u)^2/2\sigma_N^2)\ /\ \sqrt{2\pi}\sigma_N\, dw_1 \int_{2u-w_1}^{w_1} \exp(-(w_2-u)^2/2\sigma_N^2)\ /\ \sqrt{2\pi}\sigma_N\, dw_2 \tag{D.8}$$

Take

$$I = \int_{2u-w_1}^{w_1} \exp(-(w_2-u)^2/2\sigma_N^2)\ /\ \sqrt{2\pi}\sigma_N\, dw_2 \tag{D.9}$$

Let $t=(w_2-u)/\sigma_N$, $dt=dw_2/\sigma_N$

$$I = \int_{(u-w_1)/\sigma_N}^{(w_1-u)/\sigma_N} \exp(-t^2/2)/\sqrt{2\pi}\ dt \tag{D.10}$$

$$I = \mathrm{Erf}((w_1-u)/\sigma_N) - \mathrm{Erf}((u-w_1)/\sigma_N) \tag{D.11}$$

Substitute back in (D.8);

$$P_{cc}(21) = \int_{1.5u}^{\infty} \exp(-(w_1-2u)^2/2\sigma_N^2)/\sqrt{2\pi}\sigma_N \\ (Erf((w_1-u)/\sigma_N) - Erf((u-w_1)/\sigma_N))\ dw_1 \tag{D.12}$$

Let $t=w_1-2u/\sigma_N$, $dt=dw_1/\sigma_N$,

$(w_1-u)/\sigma_N=t+u/\sigma_N$, and $(u-w_1)/\sigma_N=-t-u/\sigma_N$,

Hence

$$P_{cc}(21) = \int_{-0.5u/\sigma_N}^{\infty} (\exp(-t^2/2)/\sqrt{2\pi}) \\ (Erf(t+u/\sigma_N) - Erf(-t-u/\sigma_N))\ dt \tag{D.13}$$

Substitute for these probabilities of correct decision of each admissible codeword into equation (C.2) and use (C.1) to get the total probability of error employing MLSD_CCMA decoding for this particular code structure.