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CHAPTER 3

New Light on Early Arabic ْAfāq Literature

فإ ن علم الأعداد للوفق من لطائف العلوم العقلية التي تبسط النفس فيها بما يطلع منها بسهولة على عجائب خواصها مع ما ذكر أصحاب الطلسمات من خواص فوائدها وافقوا عليها.

The science of harmonious numbers is one of the subtleties of the rational sciences in which the soul delights at the wonders of the occult properties that emerge from them with ease, along with their useful properties mentioned and agreed upon by the talisman makers.

—MUHAMMAD B. AL-MUZAFFAR AL-TUSI

Introduction

What are magic squares? A simple formal definition will state that a magic square is an arrangement of a set of numbers (usually consecutive beginning from one) into a square grid in such a way that the sums of the numbers contained in each row, column and the two corner-to-corner diagonals are equal. But, beyond this, what are they? What did their makers think they were for, and what accounts for their wide diffusion and historical tenacity? Are they games of recreational mathematics, illustrations of number theory, magic tricks, talismans, a combination of some or all of these things or something else entirely? Of course, these questions can be answered in many ways, since magic squares were undoubtedly understood and appreciated differently by various people at various times according to their context, training, orientation and expectations. This paper seeks to explore some of these historic perspectives.

on magic squares focussing on the authors who wrote on this subject in order to take account of their contexts and legacies.

From the mathematician’s perspective, magic squares are a type of figurate numbers, arrangements of sequences of numbers in geometrical patterns. The search for and calculation of all the possible arrangements of numbers within a magic square of a certain order that fulfil a given set of conditions is within the domain of mathematical subdiscipline of combinatorics. Attempts to set out as succinctly as possible the series of mathematical operations needed to arrange the numbers in a magic square in such a way as to fulfil those conditions is part of study of algorithms. Indeed, according to the historian of mathematics Jacques Sesiano, the leading expert on Islamicate magic squares (awfāq), ‘One of the most impressive achievements in Islamic mathematics, in any case the most original one, is the development of general methods for constructing magic squares.’

2 Norman L. Biggs, “The Roots of Combinatorics,” Historia Mathematica 6 (1979): 118-24; Ahmed Djebar, “Islamic Combinatorics,” in Combinatorics: Ancient and Modern, ed. Robin Wilson and John J. Watkins (Oxford: Oxford University Press, 2013), 84. The ‘order’ of a magic square is square root of the total number of cells it contains or the number of cells it has on each side. Thus, the simplest magic square, that of 9 cells, is said to be a square of order 3, since it contains 3x3 cells. There is only one way to construct a 3x3 magic square. All apparent differences in squares of this order are simply rotations or mirror images such that the central cell will always contain the number 5 and the relative positions of the other numbers in the peripheral cells can never vary. As the order of the magic square is raised, the total number of possible arrangements of the numbers within the square increases dramatically: the square of order 3 has only one possible arrangement, that of order 4 has 880 and that of order 5 has 275,305,224 (see Clifford A. Pickover, The Zen of Magic Squares, Circles, and Stars [Princeton: Princeton University Press, 2002], 3-6).


4 When speaking of Islamicate contexts in this article, I will use the term wafq (pl. awfāq) to refer to what are usually called magic squares in European languages. I have not extended the same courtesy to Indian, Chinese or any other cultural contexts, not because I do not think it is important to do so, but because I have neither the historical nor linguistic expertise to do so in a sensitive, nuanced or even meaningful way. In Arabic contexts, a typical wafq consists of a ‘table’ (lamb) or ‘grid’ (jadwal) of ‘cells’ (bayt, pl. buyat) arranged in vertical (tali) and horizontal (‘ardi) ‘rows/columns’ (dil, pl. adla’) and diagonals (qur, dual qatrayn). The grid most often takes the form of a square (murabba’), but awfāq in the form of triangles and other polygons, stars, circles, cubes, spheres and cones are also attested in the manuscripts. The grid and the numbers and or letters within it are usually together simply referred to as the ‘figure’ (shakl). The history of the mathematics of awfāq has been explored in depth by Jacques Sesiano, most recently in Jacques Sesiano, Magic Squares in the Tenth Century. Two Arabic Treatises by Anṭākī and Būzjānī (Cham, Switzerland: Springer, 2017).

On the other hand, as a feature of occult technology, magic squares functioned as talismans in their own right, or as constituent elements of more complex talismans. Their powers or occult properties (khawāṣṣ) were often understood to derive from the numbers they contained, the letters of divine names alphanumerically encoded in the squares, the elemental natures of these letters themselves or the planets and astral spirits with which the squares were associated. The powers harnessed by the magic squares allowed their makers to perform wonders (ʿajāʾib).

The history the awfāq has most often been discussed solely from the perspective of the history of mathematics, frequently with a focus on locating scientific ‘firsts’. But history, be it intellectual, cultural or social, is not best understood as a series of eureka moments, as if only the first instance of every historical phenomenon is important, or that every repetition of an action or idea, with all its contextual variations and permutations, is derivative and therefore of limited interest. Focussing on ‘firsts’ obscures a general ignorance of the historic and cultural significance of awfāq in Islamicate societies.

Would-be students of the cultural history of the awfāq are also often hindered by an over-reliance upon a small number of works from the corpus attributed to the šūfī master of letter magic (ʿilm al-ḥurūf) Abū al-ʿAbbās Aḥmad al-Būnī (fl. 622/1225), which tend unjustifiably to be granted the archetypal status of summa of all previous literature on awfāq. This is unfair not least because Islamicate writings on awfāq were produced in a number of genres and modes most not represented by the Būnīan corpus. Or, to put it another way, the treatment of awfāq in the Būnīan corpus lacks context without an appreciation of the centuries-old Islamicate traditions of awfāq that precede it. To make matters worse in this regard, the most prolific scholar and editor of Arabic awfāq literature, Jacques Sesiano, is a historian of mathematics adept at analysing the numerical properties and construction methods of awfāq and an authority on the history and chronology of their mathematical development, but he has little interest in social questions concerning the status and contexts of the authors of awfāq treatises or of

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the cultural and practical roles played by *awfāq* in the lives of the mathematicians and non-mathematicians who dealt with them. This purely mathematical interest in *awfāq* on the part of expert mathematicians sheds light on their mathematical significance, but does not in itself produce a well-rounded history of *awfāq*.

This article is a contribution to the foundations of a cultural history of *awfāq*. It seeks to explore their prevalence in Islamicate literature and to outline the nature, scope and form of their literary appearances. It will examine the development of the cultural position of *awfāq* in the Islamicate world by contextualising these literary appearances within their intellectual landscapes and situating their authors, where possible, not just chronologically, but also within recognised locales of scientific endeavour (notably sites of astronomical activity), which were themselves nodes in networks of patronage, collaborative relationships and scholarly debate. It will also attempt, where possible, to trace these networks of patronage, collaboration and correspondence between the various authors of *awfāq* texts in order to identify hubs of *awfāq* research.

We will begin with a survey of discussions of *awfāq* in Islamicate literature from their earliest known appearance in the mid-3rd/9th century down to the huge rise in cultural prevalence of the *awfāq* in the early 7th/13th century with the occultist-sufi synthesis typified by the writings of Muḥyī al-Dīn Ibn ʿArabi (d. 638/1240) and al-Būnī. The second half of this article will focus on a newly identified manuscript of a previously unknown *awfāq* treatise from the 6th/12th century: the *Collection of Harmonious Number* (*Dīwān al-ʿadad al-wafq*, hereafter referred to as the *Dīwān*). I will use this manuscript to connect three major periods and locations often identified as ‘golden ages’ of Islamicate cultural and scientific patronage: (1) Baghdad under ʿAḍud al-Dawla (reg. 367-72/978-83) and subsequent Būyid amīrs, (2) Isfahān and Marw under the Great Saljuq sultans Malikshāh (reg. 465-85/1073-92) and Sanjar (reg. 511-52/1118-57), and (3) Delhi under the Mughal padishah Shāhjahān (reg. 1037-76/1628-66).

The historical trajectory traced by the unique manuscript of the *Dīwān* is indicative of the extensive chronological range and geographical breadth of the Islamicate interest in *awfāq*, as well as the fact that many of the greatest names in the history of the sciences in the Islamicate world (both scientists and patrons) were caught up in this fascination. But before we can

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speak of the origin of awfāq literature in the Islamicate world, we should outline the current state of knowledge concerning magic squares in the pre-Islamic world and their subsequent development in the cultures bordering on or subsumed within the growing Islamicate world.

I. Pre-Islamic magic squares and the later developments of these traditions

Much ink has been spilt in trying to determine when and where the magic squares originated and their routes of transmission into (or perhaps parallel development in) the Islamicate world and thence into Latin Europe. The well-known general outlines of this history are as follows.

China supplied the first known source of knowledge on magic squares, but this was restricted in the earliest instances to the square of the lowest order (i.e. 3x3), which may be alluded to in Chinese texts from the first two centuries BC, but is not patently discussed until the late first century AD. Squares of higher orders are not been found in Chinese literature until the latter half of the 13th century, long after their appearance in Arabic and Persian texts around the mid-4th/10th century. The earliest mention of the square of 4x4 is found in Sanskrit literature in the mid-6th century, but early appearances of magic squares in India are restricted to those of 3x3 and 4x4. By the 3rd/9th century, the magic squares had entered Islamicate literature and, within about a century, Arabic treatises devoted to awfāq began to appear. During the 5th/11th century, the Toledan astronomer al-Zarqālī wrote a treatise on the talismanic use of the awfāq, and by the 7th/13th century, the magic squares had entered Europe by means of Latin and Old Castilian translations of al-Zarqālī’s treatise. Because European scholars were introduced to the squares via a talismanic handbook and not a mathematical treatise, the term ‘magic square’ has endured in European languages.

Although the precise chronology of this history is still difficult to establish with the evidence currently available, the abundance of this evidence is remarkable. The widespread existence, cultural significance and persistence of magic squares across Eurasia indicate that far from being hidden on the edges of society, the marginal property of any one culture, time, place or group, they were a significant and enduring phenomenon of the global Middle Ages. Before we focus on magic squares in the Islamicate world, it will be helpful to have a closer look at three major cultural blocs that have influenced their development.
Greece

The feeling that magic squares would be at home in Neopythagorean number theory has led a number of scholars to suspect that their origin lies in the Hellenistic world. Attempts to confirm this hunch, however, have been largely unsuccessful. George Sarton’s claim, for example, that the Neopythagorean mathematician and astronomer Theon of Smyrna (fl. ca 100) discussed the 3x3 magic square was demonstrated to be false by N.L. Biggs, who flatly denied the existence of magic squares in classical or late-antique Greek literature, arguing that

If the Greeks or their followers had known about magic squares, then it is unthinkable that just one passing reference to them should have survived: to be sure, some Greek wisdom has been lost, but magic squares are simply too memorable to have disappeared completely.

Likewise, H.E. Stapleton’s statement that the ‘Magic Square [of 3x3] was known in Europe to Theodorus, a pupil of ... Porphyry [d. ca 305]’ remains unsubstantiated.

More recently, however, Nicolas Vinel has made a case for the existence of esoteric allusions to magic squares in late-antique Greek mathematical literature, notably in the Neoplatonic philosopher Iamblichus’ (d. 325) commentary on the Arithmetical Introduction, a treatise on Neopythagorean number theory by the philosopher and mathematician Nicomachus of Gerasa (d. ca 120), a text we shall return to below.

Although there remains no generally accepted proof of the knowledge of magic squares in Graeco-Roman antiquity, archaeological evidence has recently

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come to light that may add weight to Vinel’s theory. A Greek letter square (or rather an oblong) depicted on a fifth- or sixth-century writing tablet from Antinoupolis in Middle Egypt may be a sort of proto-magic square. While admittedly not conforming to the usual mathematical constraints of a magic square, when the letters in this oblong are assigned their positional values within the alphabetic sequence rather than their usual alphanumerical values many equal sums are discoverable in its columns and rows.\textsuperscript{12}

With this archaeological discovery and Vinel’s work, the tide of scholarly opinion shows signs of turning against Biggs’ hard-line stance. J. Sesiano, the greatest living historian of Islamicate magic squares, has recently admitted that ‘the surprising results already obtained in the science of magic squares [in the Islamicate world] by the tenth century thereby suggest [...] a much earlier, possibly Greek time for the first discoveries’ and concludes that ‘... although ... [the] ... Greek allusions [adduced by Vinel] are in themselves not convincing [that the magic square of 3x3 was known in the Graeco-Roman world], they can no longer be dismissed \textit{a priori}’.\textsuperscript{13} For the time being, however, the earliest known explicit discussion of magic squares in Greek literature is found in the fourteenth-century writings of the grammarian Manuel Moschopoulos of Constantinople, and this has been shown to derive from Islamicate sources.\textsuperscript{14}

\textbf{China}

In fact, the earliest known magic squares were constructed in China, where they were and continue to be an important part of the science of divination and geomancy. The simplest magic square, that of 3x3, appears perhaps as early as 190 BC in Xu Yue’s Memoir on Some Traditions of the Mathematical Art (\textit{Shushu jiyi}), where it is called the ‘nine-palaces diagram’ (\textit{jiugong tu}), and apparently served divinatory and cosmological/cosmographical functions.\textsuperscript{15}

\begin{thebibliography}{10}
\bibitem{12} Julia Lougovaya, “A Perfect Pangram: A Reconsideration of the Evidence,” \textit{Greek, Roman, and Byzantine Studies} 57 (2017): 165, 179-80 and 189-90. I thank Juan Acevedo for bringing this article to my attention.
Around 80 AD, the 3x3 square appears again in the more reliably dated Record of Rites by Dai the Elder (Da Dai Liji) by Dai De, where it is found in a chapter called ‘Bright Hall’ (Mingtang), a reference to a palace of nine halls in which ceremonial rites were carried out by the emperors of the Zhou dynasty (1046–771 BC). The Luo River Chart (Luoshu), an important diagram in the development of Chinese geomantic thought, may be considerably older than the writings of Xu Yue and Dai De just mentioned, but its origin is legendary, and since it was written schematically and without numerals, its identification with the ‘nine-palace diagram’ and ‘Bright Hall’ (and thus with the 3x3 square) did not take place until as late as the time of the great geomancer and commentator on the Classic of Changes (Yijing) Cai Yuanding (d. 1198).

Historically, the semantic range of the Chinese term shuxue, usually translated into English as ‘mathematics’, also touched upon philosophy, astrology and divination, and the magic square of 3x3 played important roles in all these categories. This magic square was a potent cosmological symbol embodying ‘the two forces of Yin and Yang at work, the cycles of the Four Seasons and the Five Elements, and the deployment of the Nine Directions of space, emphasis always remaining on cosmic centrality like a kind of powerhouse’. Fundamental to the compass school of geomancy (fengshui), it is also the basis for a number of astrological divinatory practices whereby the numbers in the square are each assigned a colour and circulated throughout the 3x3 square to form a total of nine different arrangements.

Despite the great antiquity of magic squares in China, squares of orders higher than three do not appear in Chinese literature until 1275, when they were included in the *Continuation of Ancient Mathematical Methods for Elucidating the Strange* (*Xugu zhaiqi suanfa*) by Yang Hui.\(^2^2\) This mathematical text discusses magic squares of 3x3 to 10x10, as well as magic circles and other mathematically ‘magic’ figures, but only explains the construction methods for the lowest two orders (3x3 and 4x4). Furthermore, Yang Hui treats magic squares solely from the point of view of mathematics and he is apparently the first to refer to a magic square without the usual Chinese legendary epithets, but simply as ‘a vertical and horizontal plan’ (*zonghengtu*), a name still used for them in Chinese today. Both the 3x3 and 9x9 magic squares played an important role in Daoist rituals during which a priest takes steps (*Yubu* or *gangbu*) following the pattern of the magic square in order to harness the power of the stars of the constellation of the Plough, and numbers derived from magic squares of higher orders were also used to justify the importance of numbers within the number theory of the *Classic of Changes* (*Yijing*).\(^2^3\)

It is tempting to contextualise the appearance of higher order squares in Chinese literature within the Mongol incursions into Central Asian and Near Eastern Islamicate realms that began in the 1220s and culminated with the fall of Baghdad in 1257 and the establishment of the Īlkhānid, Kipchak (Golden Horde) and Chagatai Khānates in and to the north and east of the Islamicate world. Although diplomatic, commercial, scientific and other cultural transactions are well-documented between China and Muslims from the Islamicate world in the first century from the death of the prophet Muḥammad (d. 11/632), clear evidence for points of transmission of magic squares knowledge between the two cultural blocs has yet to be adduced and interrogated. An interesting starting point for research in this direction could be the cast iron 6x6 square with eastern Arabic numerals, which was excavated in 1957 from the foundations of the palaces of the Prince of Anxi,

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3 km northeast of Xi’an, Shaanxi Province in northwestern China.24

Figure 3.1: Cast iron 6x6 magic square (12 x 12 x 1.5 cm), Anxi (Xi’an), early Yuan Dynasty (late 13th century), Shaanxi History Museum (photo courtesy of Marilyn Shea, Ph.D., University of Maine at Farmington)

The palaces were the residence of the Yuan Dynasty princes of Anxi, and were constructed during the reign of the first such prince, Mangqala (enthorned 1272), the third son of Qubilai Qaghan (reg. 1260–99). It is likely, however, that the magic square was buried in the foundations of the palace of the Prince of Anxi during the reign Mangqala’s son and successor Ananda (enthorned 1282), who is well known to have converted to Islam early in his life, enthusiastically followed Islamicate customs and propagated Islam within his realm.25 At any rate, the fact that the magic square was placed inside a carved stone coffer and then deposited in the foundations of a royal edifice clearly indicates that this was an official ritual act and it points firmly towards the influence of Islamicate magic squares in the east of the Mongol realm within 20 years of the fall of Baghdad and shortly after the publication of Yang Hui’s work on higher-order magic squares.26

24 The square is held in the Shaanxi History Museum and a catalogue record for it can be found in Dongshan Ji and Jianwu Han, Charm and Brilliance: An Appraisal of the National Treasures in the Shaanxi History Museum - The Jade and Other Objects. Shen yun de hui hang: Shanxi li shi bo wu guo bao jian shang - yu za qi juan (Xi’an: Sanqin Publishing House, 2006), 216. I am grateful to Han-Lin Hsieh for finding and translating this catalogue record for me, and to her and Emma Harrison for general help with Chinese. See also Dezhi Ma, “CHINESE! [Investigations of the Yuan Dynasty Palace of the Prince of Anxi in Xi’an],” Kaogu 5 (1960); Nai Xia, “Yuan Anxi Wang Fu Alabo Shuma Huanfang [The Remains of the Yuan Dynasty Palace of the Prince of Anxi and the Arabic Numerical Magic Square],” Kaogu 5 (1960); Nancy Shatzman Steinhardt, “Imperial Architecture along the Mongolian Road to Dadu,” Ars Orientalis 18 (1988): 68–69; Nancy Schatzman Steinhardt, “Towards the Definition of a Yuan Dynasty Hall,” Journal of the Society of Architectural Historians 47.1 (1988): 61; and Martzloff, A History of Chinese Mathematics, 365–66 (Fig. 20.2).


26 It has been proposed that the magic square was not an import from Islamicate Central Asia, but rather produced locally in Chang’an (mod. Xi’an; see Xia, “Yuan Anxi Wang Fu.” 23
India

The earliest known appearance of a magic square in Indian literature is the 4x4 square mentioned in the Great Compilation (Bṛhatsamhitā). This Sanskrit text mainly treating divination was written around 550 by the mathematician and astronomer Varāhamihira (d. 587) of Ujjain, an important centre of both political power and mathematical/astronomical research in Central India. The 4x4 magic square is used in that text not for divination, but for determining the correct proportions of the 16 ingredients used to make a perfume called the ‘all-auspicious’ (sarvatobhadra).

There follows a large gap in the historical record until about 900 when the physician Vṛnda included a 3x3 magic square in his Sanskrit āyurvedic medical compendium the Siddhayoga. He recommends that pregnant women cited by Steinhardt, “Imperial Architecture,” 77, n. 76). Mathematically, however, it is clear that the Anxi iron 6x6 magic square was constructed independently of Yang Hui’s Continuation of Ancient Mathematical Methods, since the numerals in the only 6x6 square to appear in that work are arranged differently to those in the Anxi magic square. From a talismanic and ritual perspective, it is not surprising that the Anxi magic square is of 6x6 cells, since in the two major Islamicate systems of associations between magic squares and the planets, the square of 6x6 is associated with the Sun and is used for ensuring long and prosperous reigns for ruler. What is unusual, though, is that the square is constructed of iron and not of gold, the metal usually associated with the Sun and prescribed for such a talisman.


28 Ujjain is taken as the base location (al-Qubba) from which all longitudes are measured in the Zīj al-Sindhind (translated shortly after 153/770 at Baghdad from a Sanskrit astronomical work [siddhānta]; see Edward S. Kennedy, “A Survey of Islamic Astronomical Tables,” Transactions of the American Philosophical Society 46.2 (1956): 129-30).


30 Vṛnda, The First Treatise of Āyurveda on Treatment. Vṛndamādhava or Siddha Yoga, ed. and
suffering a difficult labour should gaze (*drśtvā*) at this magic square in order to ease childbirth. This eutocic usage of the 3x3 square is one of the most stable and enduring features within the lore of the magic squares in Islamicate lands, and as we shall see below, it is found in Arabic literature about half a century before it appears in Sanskrit.

From the 11th century, magic squares appear on monuments related to Śaiva tantric traditions in Northern India in which they were employed as numerical ritual diagrams (*aṅkayantra*). These magic squares were a subset of the more general tantric ritual diagram (*yantra*) to be gazed at or visualised internally by the yogi in meditation in order to achieve magical or spiritual aims, much as Vṛnda prescribed gazing at the 3x3 square to obtain a eutocic benefit some five centuries earlier.\(^{31}\)

The first discussion of magic squares in an Indian mathematical treatise is found in a short section within the fourth and final chapter of the Prakrit *Gaṇitasārakaumudī* written around 1300 by Ṭhakkura Pherū (d. after 1323), a Jaina scholar employed at the Delhi mint under the Khaljī sultans and perhaps also under the early Tughluqs.\(^{32}\) Much longer is the discussion of magic squares and other figures in the fourteenth and final chapter (on ‘Auspicious Mathematics’ [*Bhadragaṇita*]) of Narāyaṇa Paṇḍita’s Sanskrit mathematical treatise, *Gaṇitakaumudī*, written in 1356.\(^{33}\) By at least the 14th century magic squares appear on monuments related to Śaiva tantric traditions in Northern India in which they were employed as numerical ritual diagrams (*aṅkayantra*). These magic squares were a subset of the more general tantric ritual diagram (*yantra*) to be gazed at or visualised internally by the yogi in meditation in order to achieve magical or spiritual aims, much as Vṛnda prescribed gazing at the 3x3 square to obtain a eutocic benefit some five centuries earlier.\(^{31}\)

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squares appear in Jaina hymns and at an as yet unconfirmed date they appear carved on Jaina temples.\textsuperscript{34}

II. \textit{Awfāq}: Islamicate magic squares

\textit{Awfāq} are ubiquitous in the material culture of Islamicate societies. They are frequently encountered scribbled on the flyleaves of manuscripts, inscribed and painted on ceramics and metalware, on ceremonial flags and military standards, protective talismanic shirts, and on engraved, inscribed and block-printed (\textit{tarsh}) talismans.\textsuperscript{35} Outside texts on mathematics, magic, medicine and natural philosophy, the cultural status of \textit{awfāq} is attested to by their appearance in literary works on such diverse subjects as chess,\textsuperscript{36} music\textsuperscript{37} and genealogy.\textsuperscript{38} Just as the Chinese and Indian traditions, \textit{awfāq} first

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\begin{itemize}
\item \textsuperscript{34} See Hayashi, “Magic Squares in Indian Mathematics,” 2601.
\item \textsuperscript{36} Muḥammad b. ʿAbd al-Raḥmān al-Sakhāwī (d. 902/1497), ʿ\textdag{	extdag{	extdag}{ Urdu al-muḥtajj fi ḥūkam al-shaṭranj}, ed. Usāma al-Harīrī and Nazīr Ka‘ka (Kuwait: Dar al-Nawādir, 2012), 152.
\item \textsuperscript{37} \textit{Treatise of the Treasure of Gifts Concerning Music} (Risāla-yi kanz al-tuhaf dar mustaqā), BL, Or. 2361, f. 267r (copy completed at Delhi, 16 Rajab 1075/2 February 1665): a 5x5 \textit{wafq} appears between an Arabic prayer to Venus (begins f. 266v, ult.) and a Persian verse prayer to Venus (ends f. 267v) attributed to Naṣīr al-Dīn al-Ṭūsī (d. 672/1274). The authorship of this treatise in Or. 2361 is unknown, but Henry G. Farmer, “Iranian Musical Instruments in the Ninth/Fifteenth Century,” \textit{Islamic Culture} 38.3 (1964): 175-76 suggests (on the authority of C. Huart, without further reference) that the author is the \textit{mawlāwī} dervish Amīr b. Khīḍr Mālī and that he composed the treatise in 838/1434. This is contradicted by a collation note after the colophon of this text (f. 269v) stating that the copy was collated against a manuscript dated Tuesday 1 Dūḥ al-Qa‘da 784/6 January 1383. Furthermore, Charles Rieu, \textit{Supplement to the Catalogue of the Arabic Manuscripts in the British Museum} (London: The British Museum, 1894), 561 (item 823) interprets a chronogram at the end of the treatise as indicating a composition date of 746/1345-6. As for the possibility that Naṣīr al-Dīn al-Ṭūsī had an interest in \textit{awfāq}, a treatise called \textit{al-Awāfāq} attributed to al-Ṭūsī’s teacher Kamāl al-Dīn Muḥammad b. Yūnus (d. 639/1242) is edited in Jacques Sesiano, “An Arabic Treatise on the Construction of Bordered Magic Squares,” \textit{Historia Scientiarum} 42 (1991).
appear in Islamicate literature in medicinal and magical contexts. Only later are entire treatises - both talismanic and mathematical in scope - dedicated to them. They are taken up as the subjects of mathematical treatises within a century of their first explicit appearance in Arabic literature. In comparison to the Chinese and Indian traditions in which they are not found in mathematical literature until some seven and a half and eleven centuries after their initial appearances respectively, the awfāq make a much swifter transition in Islamicate cultures from their occasional appearance in medical and talismanic texts to being the subject of prolonged mathematical studies. This early and intense engagement between mathematical research and talismanic practice with respect to awfāq had a profound effect on the history of the occult sciences in the Islamicate world. In the following pages, I attempt to trace the contours of an early cultural history of awfāq in order to place their historical study on more solid foundations. By bringing together a selection of passages discussing the most well-known wafq (that of 3x3) from Arabic literary works written before the end of 6th/12th century, I hope to explore the Islamicate thinking on awfāq that fed into and informed the explosion in popularity of letterism (‘ilm al-ḥurūf), and the associated massive increase in the profile, diffusion and variety of awfāq, in the 7th/13th century propelled by the likes of al-Būnī and Ibn ‘Arabī.\(^{39}\)

1. Ibn Rabban al-Ṭabarī (fl. ca. 235/850)

The earliest dateable reference to a wafq (although this designation is not used) in Islamicate literature is found in a discussion of methods for facilitating childbirth written by Abū al-Ḥasan ʿAlī b. Rabban Sahl al-Ṭabarī. Born into a family of state secretaries at Marw in Tabaristan, Ibn Rabban lived the majority of his life as a Christian (probably a Nestorian),

\(^{39}\) The theoretical underpinnings of the paradigm shift that coincided with the rise of letterism, which saw Sufi cosmology and revelation displace natural philosophy and astral causality as the interpretative framework in which magic was conceived has been studied by Liana Saif, “From Gāyat al-ḥakīm to Šams al-maʿārif: Ways of Knowing and Paths of Power in Medieval Islam,” *Arabica* 64 (2017). The following survey of Arabic texts on the 3x3 wafq written before 596/1200 is surely not exhaustive, but is intended to be broadly illustrative of the place of awfāq in Islamicate literature before this paradigm shift. I have reproduced here the original Arabic of every passage cited in order to facilitate future analysis. The Arabic text has been kept to the footnotes when a reliable critical edition is available. I have put the Arabic text in the main body of the article only when presenting an unedited text or an edited text that is being re-edited here from the manuscripts.
before converting to Islam in his later years. His father Sahl was given the title *rabban* (Syriac, ‘our master’) owing not only to his great scriptural erudition, but even more so to his medical knowledge, and if later testimony is to be believed also to his scholarly achievements in mathematics, astronomy and the translation of scientific texts. ʿAlī b. Rabban followed in his father’s footsteps with political service as a secretary first to the governor of Ṭabaristān, Māzyār b. Qārīn (d. 226/841), and then at the Abbasid court at Sāmarrā’ under the caliphs al-Muʿtasim (reg. 218–27/833–42), al-Wāthiq (reg. 227–32/842–7) and al-Mutawakkil (reg. 232–47/847–61). He was a boon companion (*nadīm*) to al-Mutawakkil, and it was likely during the reign of that caliph that Ibn Rabban converted to Islam around the age of 70.

It was also during the reign of al-Mutawakkil in 235/850, perhaps before his conversion, that Ibn Rabban wrote his most famous book: an encyclopaedic work on medicine and natural philosophy called the *Paradise of Wisdom* (*Firdaws al-ḥikma*). This work contains a summary of Indian medicine (*āyurveda*) that demonstrates Ibn Rabban’s familiarity with Sanskrit medical sources and traditions. It is thus possible that he derived his knowledge of the eutocic 3x3 *wafq* from these Indian sources. Be that as it may, Ibn Rabban claims that the immediate source of his knowledge about the eutocic powers of the 3x3 *wafq* was his father. The passage in the *Paradise of Wisdom* in which Ibn Rabban mentions the 3x3 *wafq* is found only in the earliest (7th/13th century) manuscript of the work: BL, Arundel Or. 41, f. 135r–135v (hereafter L).

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43 A digital copy of L is at https://www.qdl.qa/en/archive/81055/vdc_100023664595.0x000051. Bilingual (Hebrew/Arabic) ownership inscriptions on f. 2r of this manuscript show that it was owned by Rabbi Yosef al-Tiflīsi (fl. early 8th/14th century), son of the Exilarch, head of the Jewish community of Gagra on the Black Sea coast of western Georgia, and then by his son Emmanuel the Dayān, brother of Rabbi Isaiah of Tabriz (fl. first half of 8th/14th century), author of the cabbalistic treatise *Sefer ha-Kavod*. On this Jewish scholarly family see Michael Beizer, Michael Zand and Mordkhai Neishtat, “Georgia,” in Encyclopaedia Judaica, 2nd ed., ed. Fred Skolnik and Michael Berenbaum (Detroit - New York - San Francisco - New Haven - Waterville - London: Thomson Gale, 2007), vol. 7, 497: Walter J. Fischel, “Azarbaijan in
Because the standard edition of the *Paradise of Wisdom* contains numerous textual errors, I have re-edited this passage from L and present it here with a fresh translation.

**Text**

وقد كان أبي يكتب لعرض الولد شيئاً عجيباً، وحُسب ما حسبت أولاً وعَرَض ما نازع إلى زاوية كان خمسة عشر وتكن حولها آتين من الزيوت وتوقد بها المرأة حتى تنظر إلى ما فيها من الكتابة نعماً ثم تضعها تحت قدميها وكان يأمر أن يؤخذ من غش الخطاطيف شيء قليل من ذلك الطين ويسحق به رزقي وترخ به عانائها وحَقَيقها وأن يقلع 48 أصل كزبرة رقيقًا و يؤخذ من زعَّب زابود فكتوبن حولها وهو بالسريانية ونقشها "أخير نصي من المجس لانشتروكم 50 وليؤم آراك 51 إذا أنت كافيفتي وله أسبي كثيرة غير هذه.

**Translation**

For difficulties in childbirth, my father used to prescribe something wondrous and tested, namely that you take two pieces
of pottery from a new earthenware jug (kūz) or jar (jarra) never
touched by water and on each one you draw a figure like this [one
below] in which you write a sum such that however you add it up
vertically, horizontally, or from corner to corner, you get
fifteen. Around each one you write two verses from the Psalms,
and they are brought to the woman so that she gazes pleasantly
(? niʿmān) at the writing on them. Then you place them under her
feet.

He [sc. my father] used to order that a little clay be taken
from a swallow’s nest and ground with lily oil, that her pubes
and groin be anointed with it, and that a stem of coriander be
gently plucked and its root taken and bound to the woman’s thigh.

This [, below,] is an illustration of the figure, which is two,
nine, four, then seven, five, three, then six, one, eight. As for
the two verses from the Psalms of David, they are written around
it in Syriac and are translated [as follows]:

Bring my soul out of prison that I may give thanks unto Your
name.
Truly Your righteous ones will expect me when You have rewarded
me. ⁵²

It also had many other things beside this.

[INSERT ILLUSTRATION 2 HERE - NO CAPTION]

Commentary

The specification that the two 3x3 wafqs used in this treatment should be
‘brought to the woman so that she gazes pleasantly (?) at the writing on them’
(wa-tuʿatā bihā al-marʿatā ḥatā tanqura ilā mā fiḥimā min al-kitābati niʿman) is reminiscent of Vṛnda’s Sanskrit prescription in which the square is to
be gazed at by the woman in labour. It is thus tempting to see Ibn Rabban’s
use of the eutocic 3x3 wafq as a continuation or even (if the direction of

⁵² This is Ps. 141:8 in the Vulgate (142:7 KJV), but Ps. 142:8 in the Masoretic text and
in the Syriac Peshīṭṭa text referred to by Ibn Rabban al-Ṭabarī: ܐܦܩܢܦܫܝܡܢܒܝܬܚܒܘܫܝܐܚܒܘܫܝܬܝܕܐܘܕܐܠܫܡܟܐܠܝܢܣܟܘܢܙܕܝܩ̈ܝܟܟܕܬܦܪܥܢܝ(text from The Old Testament in Syriac according to the
Peshīṭṭa Version, Pt. ii, fasc. 3. The Book of Psalms, edited on behalf of the International
Organization for the Study of the Old Testament by the Peshīṭṭa Institute Leiden [Leiden:
E.J. Brill, 1980]). The text of the Leiden Peshīṭṭa can be consulted along with lexical tools
at http://call.cn.huc.edu/.
transmission went the other way) the origin of the Indian tradition of the ankayantra (see above, p. ??). At any rate, we are dealing with very similar cultural phenomena.

In Islamicate works on medicine, magic and the natural sciences, the eutocic power of the 3x3 wafq is frequently mentioned from the 3rd/9th century onward. Many of the details of its method of use as prescribed by Ibn Rabban’s father Sahl in late-second/eighth- or early-third/ninth-century Marw survived through the ages, such as the inscription of the wafq on pieces of pottery untouched by water, gazing upon them and their placement under the feet of the woman in labour. Other features of Ibn Rabban’s prescription such as the inscription of verses from the Psalms around the wafq and the anointment of the woman in labour with lily oil and mud from a swallows nest, however, are not found again after the Paradise of Wisdom. On the other hand, divine, angelic, or saintly names or other words and phrases from Holy Scripture are commonly found surrounding or incorporated into awfāq in talismans of later periods.

2. Ibn Akhī Ḥizām (fl. 280/893)

Within about fifty years of the composition of the Paradise of Wisdom, 3x3

53 The idea of gazing at the 3x3 wafq reappears in an important but overlooked and as yet undated anonymous treatise on awfāq preserved in a unique ninth/fifteenth-century manuscript (Chester Beatty Library, Ar. 5087, hereafter referred to as Anon. CBL). This text prescribes the use of three 3x3 awfāq ‘two of which [one should] place beneath the feet of the woman having difficulty in giving birth and one before her eyes while she gazes at it’ (وريثين منها تحت قدمي امرأة تعسر ولادته وواحدًا قبئل عينيهإ تنتظر ا ليهإ سهلت عليهإ الولادة, f. 115v, lines 3-4). A memory of the association of the 3x3 wafq with the words of the Psalm ‘Bring my soul out of prison’ may be detectable in Abū al-Walid Ismā’īl b. Ṭayyab b. Hārūn b. Abī Naṣr al-Malaqī’s Treatise on the Existence of Amicable Numbers and Square Figures with Numerical Planes, and their Properties with regard to the Arrangement of the Properties of the Heavenly Bodies (Maqāla fī wujūd ʿillat al-ʿadādiyya wa-khāvissihā min jihat awdāʾ khawāss al-ajrām al-falakiyya). Al-Malaqī says of the 3x3 wafq that ‘its properties include the releasing of a prisoner or a foetus, the easing of every difficulty, and things that are associated with the Moon’ (ومن خواصه إخراج المحبوس ولادة إخراج الجنين وتسهيل كل عسير ولادة وواع). See Tewfik Canaan, “The Decipherment of Arabic Talismans,” Berytus 4 (1937), 71-89.

number squares appeared in another 'Abbāsid medical text, this time one of the earliest Arabic hippiatric treatises, which was written by Abū Yūsuf Ya‘qūb b. Akhī Ḥizām, stable master to the caliph al-Mu‘taḍid (reg. 279-89/892-902). These 3x3 number squares are not, strictly speaking, awfāq since neither one is filled with the first nine consecutive numbers, nor are the sums of the numbers in their rows, columns and corner-to-corner diagonals equal. Nonetheless, their similarity to the 3x3 wafq is undeniable. Ibn Akhī Ḥizām’s text recommends that a horse suffering from ‘stoppage’ (inqiṭā’, presumably ‘incomplete emptying of the bladder’) be made to walk over two 3x3 number squares.

Text

Text

Translation


56 This definition of inqiṭā‘ is suggested by the appearance of a very similar prescription in the Book of Veterinary Science (Kitāb fi ‘ilm al-bayṭara), a ninth/fifteenth-century Mamluk hippiatric manual in which inqiṭā‘ is discussed alongside ‘urinary retention’ (ḥusr al-bawl), BL, Add. MS 14056, f. 73r-v, https://www.qdl.qa/en/archive/81055/vdc_100045800789.0x0000a5.


58 The letters in the squares are unpointed in the manuscript, so no attempt has been made to interpret them numerically or phonetically in the translation. Perhaps they are to be interpreted as follows.
For incomplete emptying of the bladder (inqīṭāʾ): also amongst what was written by the ancients is that these two images be drawn with the numbers they contain and the beast is made to walk upon them. It will be cured by the will of God the Exalted. This is one of the occult properties of numbers.

Commentary

Note that the theme of release is present in these earliest medical prescriptions utilising the 3x3 number square (whether true wafq or not): the words of the Psalm ‘release my soul from prison’, the release of the stuck foetus and here the release of the stopped urine.

3. The Jābirian corpus (ca. 260/874-ca. 365/975)

The 3x3 wafq is next found in the writings attributed to the semi-legendary alchemist Jābir b. Ḥayyān. This author, is reputed to have lived in the 2nd/8th century, so was perhaps a contemporary of Sahl Rabban al-Ṭabarī, but the vast corpus of texts that pass under his name are generally thought to be the work of a number of Shīʿī authors writing between ca 260/874 and ca 365/975. The authors of the Jābirian corpus deal with the 3x3 wafq using their trademark method of esoteric exposition: ‘dispersal of knowledge’ (tabdīl al-ʿilm), or dividing up the discussion of a given topic and spreading it across a number of texts in order to prevent the casual reader from grasping the whole meaning without considerable effort. Accordingly we find three separate discussions

| 60 | 8 | 70 |
| 30 | 45 | 6 |
| 60 | 70 | 31 |

| 31 | 12 |
| 17 | 17 | 12 |
| 4 | 17 | 12 |

In the lower square, ﺣ waiver may stand for 13 (١٤١ or 1+10+2), but the usual abjad notation for 13 is (١٤ or 10+3).

59 On this esoteric practice as found in the writings of Jābir and other ancient and mediaeval authors, see Paul Kraus, Jābir Ibn Ḥayyān: contribution à l’histoire des idées scientifiques dans l’Islam (Cairo, Imprimerie de l’Institut Français d’Archéologie Orientale, 1942–3), vol. 2, XXXI–XXXIII.
of 3x3 \textit{wafq} scattered across the Jabirian corpus.

3a) Into a section of the Jabirian \textit{Small Book of Balances} (\textit{Kitāb al-mawāzin \textit{al-ṣaghīr}) in which the occult properties (\textit{khawāṣṣ}) of animals, plants and minerals are discussed, the author inserts the following passage on the 3x3 \textit{wafq}.

\textbf{Translation}

This image, whose number is three vertically, horizontally and diagonally, is 15 in every direction. Apollonius claimed that it is one of the puzzles of magic [\textit{min ‘uqad al-sihr}]. It has nine cells, and this is its image:

\begin{center}
\begin{tabular}{ccc}
4 & 9 & 2 \\
3 & 5 & 7 \\
8 & 1 & 6
\end{tabular}
\end{center}

When you write this image on two pieces of pottery untouched by water and place them beneath the leg of a woman for whom childbirth has become difficult, she will give birth.\textsuperscript{60}

\textbf{Commentary}

Here the \textit{wafq} is transmitted with the approval of the Neopythagorean philosopher Apollonius of Tyana (d. ca 100). It is hardly surprising to find Apollonius mentioned in this context. Often referred to in Arabic sources simply as ‘the talisman maker’ (\textit{ṣāhib al-ṭalāsim/al-ṭilasmāt}), this wandering holy man and wonder-worker has had a large number of Islamicate pseudepigraphic texts on the occult sciences fathered upon him, and his name was already associated with talismans in Greek sources since sat least the

time of Eusebius of Caesarea (d. 339). The talismans of which he is said to be the author in these Greek texts, however, are not magic squares nor even the portable, often written talismans known from Islamicate sources, but rather monumental stone statues and stelai with apotropaic powers.

In the Jābirian presentation of the 3x3 wafq, Ibn Rabban’s appeal to the divine and healing power of the Judaeo-Christian Syriac Psalms is absent. Instead the power of the square is understood in terms of khawāṣṣ (sing. khāṣṣa), ‘occult properties’ or mysterious forces inherent within objects that allow them to act at a distance with no apparent cause of the action. In fact, it is not unlikely that Ibn Rabban also viewed the power of the eutocic wafq in relation to khawāṣṣ since one of the earliest descriptions of these occult properties is found in his Paradise of Wisdom. In the chapter ‘On the properties of things’ (Fī khawāṣṣ al-āṣhya’, 5.1.1), Ibn Rabban explains khawāṣṣ as follows:

Everything has a power (quwwa) evidenced by experience of it, and a property (khāṣṣa) whose cause is unknown and source undiscerned except by experimentation. For properties (khawāṣṣ) are concealed and hidden in things like the property of the magnet by which it attracts iron, and [that of] amber [by which it attracts] wheat chaff.

3b) In another Jābirian passage concerning occult properties (khawāṣṣ), this time in the Book of the Passage of Potentiality into Actuality (Kitāb ikhrāj mā fī al-quwwa ilā al-faʿl), we find the following fleeting reference to the eutocic 3x3 wafq without any associated image or diagram:

[Translated text]

61 For a recent discussion of the sources for the Greek tradition of Apollonius’ talismans, see Manuel Á. Martí-Aguilar, “Talismans against Tsunamis: Appolonius of Tyana and the stelai of the Herakleon in Gades (VA 5.5).” Greek, Roman, and Byzantine Studies 57 (2017): 971-81; for the Arabic tradition, see Manfred Ullmann, Die Natur- und Geheimwissenschaften im Islam (Leiden: E.J. Brill, 1972), 378-81.

62 I have been unable to find awfāq in any of the plausibly authentic Graeco-Arabic writings attributed to Apollonius.

63 See Kraus, Jābir Ibn Ḥayyān, vol. 2, 61-95 with regard to the Jābirian corpus, and more generally Lucia Raggetti, “The ‘Science of Properties’ and its Transmission,” in In the Wake of the Compendia: Infrastructural Contexts and the Licensing of Empiricism in Ancient and Medieval Mesopotamia, ed. Justin Cale Johnson (Boston: De Gruyter, 2015); and, especially important in interpreting the next few passages we shall adduce, Saif, “From Ḏayat al-ḥakīm to Šams al-maʿārif,” 299-309.

64 إن لكل شيء قوة يس تدل عليهإ بمذاقتهإ وله خإصة لا يعرف علتهإ ولا يدرك غورهإ ا لا بلتجإرب لأنهإ خواص غإمض خإصة حجر المغنإطيس الذي إجذب به الحديد والكهربء لقشور الحنطة
Translation

If we arrange in the nine cells anything other than what indicates fifteen, childbirth will not be eased. 65

Commentary

The passage is so brief and contains so few details that its subject would hardly be recognisable without comparison to other such references in the Jābirian corpus and elsewhere.

3c) The most extensive discussion of the 3x3 wafq known from the Jābirian corpus appears in the Great Book of Occult Properties (Kitab al-khwāṣṣ al-kabīr, chap. 18). 66 Here the action of the wafq is assessed and reasons for its possible failure are explained. The passage is of particular interest because it while it lacks appeals to religious or philosophical authorities, instead it frames the discussion in terms of causation based on Aristotelian natural philosophy and on astrological principles:

Text

ولقد كنا مثلنا كيك في موضع آخر من هذه الكتاب ما العلة التي لها صار المثلث الذي يجمع خمسة عشر من العدد يعمل في أمر الولادة مثلاً تلمعه نصفه على الحقيقة وهو من الخواص فإنه امتحنه فوجدته قد بطل في بعض الأحوال حتى يكاد أن يكون محللاً البيلة وذلك لزيادة العلة على مقدار قوة العمل ولهة أخرى استناد من هذه وهي لأجل على علة غير علة الولادة قد عرضت الولادة والحساب إما يبلغ الولادة وليس له في العلة الأخرى مدخل فامتنع ذلك السبب فلسبب آخر وهو غير ذلك وهو الوقت الحاضر الذي تمت فيه الكتابة أن يكون كعب وخمس طاغ فلوجب ما حدث من الامتثال من الولادة في ذلك الوقت وهو وإن تأخر وأبطأ فإنه لا يدك أن فاعل ما وصينا وما كنا قد صورنا هذه الخروج فغير ضار أن نورد ذكر هذه الأبيات السبعة فإنما ذكرها مصورة إلا هنا وعند وقوع آخر من كتب ومن كل ذلك قد ذكرنا إلى هاذا إذ هو أخص الخواص به فامتحنه وأعمال عليه فإنه من كتب النحو وقد نحب أن يكون في الوسط من ثمن حمصة وفي تصلب إلقاء من فتانا ما فيه وذلك أنها ثلاثة بيوت في الأعلى منها ثانية وفي الذي يلي الثالثة وفي الأصف أربعة فذلك خمسة عشر وكسب الوسط وقد نحب أن يكون في الأعلى وفي الأعلى ومن كتب زوجاً في الأعلى فذلك خمسة وخمسة عشر وهذه صورته:

67 BL, Or. 4041 (8th/14th cent.), f. 36r, line 13-36v, line 8, https://www.qdl.qa/en/archive/81055/vdc_100044824246.0x000051.
Elsewhere in these books, we have given you an example of the reason why, amongst numbers, the threefold [i.e. 3x3] which adds up to 15 performs the action it is renowned for with regard to the matter of childbirth. But even if in fact it does perform [this action], this is owing to its occult properties (min al-khawāṣṣ). Indeed, I have tested it and found that in some cases it can be so slow as to be almost completely disappointing. But this is due to an additional cause, which surpasses the limit of [the square’s] potential to act, and to another cause superior to this [power of the square]. This is on account of a counteracting cause other than that of the birth [itself], so although the writing and numbering [of the square] aids the birth, it has no way into [effect on?] the other cause, and for this reason it is obstructed. Another reason [the square can fail] apart from this is that the present moment in which the writing of the square is completed may be such that it is written while the ascendant is malefic (wa-nahīsţāliʿ), so this necessitates that the birth be obstructed at that time. But, even if it is late and slow, it will inevitably work. So, do what we have described and [make] these nine letters we have depicted. It will cause no harm if we mention these 9 cells. Indeed, we have not mentioned them with an illustration except here and in another place in our books, but I have mentioned them to you here since this is the most relevant (akhaṣṣ) place for it. I have tested it and worked with it, and it is one of the greatest aids. We may [?] want 5 to be at the centre of it, and in achieving [the central placement of] this 5, the rest of what is needed in it comes out, [f. 36v] namely
three cells: 8 in the upper [cell], 3 in the next one and 4 in the lower [cell], and that [makes] 15. In the middle half, 1 is above, 5 in the one beneath it, and 9 in the lowermost [cell], and that [makes] 15. On the third side, 6 is in the upper [cell], 7 in the one beneath it, and 2 in the lower [cell], and that [makes] 15. This is the image:

Six   One   Eigh

Seve Five Thre

Two Nine Four

No numbers repeat in any [of the cells] if the like already occupies another by design nor purpose (? bi-wajhi wa-lā sabab) unless there is an error in the numbering. And when the corner [diagonals] are added up, they too [make] 15, and when [the numbers] are added up horizontally they equal [the sum of the numbers added] vertically, and that also is the like solution to the numbers by means of the wondrous occult property. So, concerning that which you see in it, work what you will, God willing.

Commentary

This passage is particularly significant in that it contains the earliest known discussion of the importance of the astrologically determined propitious moment to the efficacy of the wafq. The link between awfāq and astral influences continued to develop in later centuries as awfāq became a fundamental component of the Islamicate science of talismans. In the Jābirian corpus, however, there is no suggestion that the actions of the 3x3 wafq might be modified by the practitioner through the choice of various astrologically significant moments at which to construct the talisman in order that it might be used for something other than its traditional eutocic purpose. This text simply warns that using the wafq at an inopportune astrological moment (for example, when the ascendant is malefic) can impede its eutocic power. Furthermore, there is no indication at all in the Jābirian corpus that the properties of awfāq of higher orders could be harnessed talismanically.
4. *Epistles of the Brethren of Purity* (mid-4th/10th century)

The next appearance of the eutocic 3x3 *wafq* is in the *Epistles of the Brethren of Purity* (*Rasāʾil Ikhwān al-Ṣafāʾ*) written by a secretive group of anonymous authors probably at Baghdad during the mid-4th/10th century. Here, we find a shift in the frame of reference in which the *awfāq* are conceived of. To begin with, the *Epistles of the Brethren of Purity* discuss not only the 3x3 *wafq*, but the first seven *awfāq* (3x3–9x9). While it is unlikely the Brethren of Purity were the first to write about *awfāq* of orders higher than 3, owing both to uncertainties concerning the composition date of their *Epistles* and to the patchiness of our evidence for the early study of the *awfāq*, we are not in a position to make a final judgement on this. There is, however, no indication that they associated these first seven *awfāq* with the seven planets as becomes routine in Islamicate literature roughly a century later.

4a) With the *Epistles of the Brethren of Purity*, we leave the medical context in which the 3x3 *wafq* appears in the *Paradise of Wisdom* and in which it remains to a certain extent in the Ibn Akhī Ḥizām’s hippiatric treatise and in the Jābirian corpus. The discussion of *awfāq* in the *Epistles of the Brethren of Purity* still focuses on occult properties (*khawāṣṣ*), just as in the Jābirian corpus, but it is framed within the mathematical discourse of the *Epistle On Geometry* (2.26). Here, the *awfāq* are clearly conceived of in terms of figurate numbers, and as such they are imagined to combine the occult properties inherent in both the numbers they contain and the geometrical figures into which these numbers are arranged. When the two sets of occult properties are combined, they yield new and unexpected properties. Epistle 2.26 begins with the Brethren of Purity’s conception of the combination of number and form.

**Translation**

68 For this dating of the composition of the *Epistles*, see Liana Saif, “Ikhwān al-Ṣafāʾ’s Religious Reform and Magic: Beyond the Ismaʿili Hypothesis,” n. 87.


70 I.e. the arrangement of sequences of numbers into geometrical patterns, the most famous ancient example of which being the Pythagorean tetractys.

Since we have shown something of the properties of these figures..., and, previously, something of the properties of numbers ..., we want to mention something of the properties of their combination. That is to say that when some numbers are combined with some geometrical figures, other properties are manifested out of them that had been indiscernible in either of them in isolation. An example of this is when the nine units are inscribed in a ninefold figure in this form [shown below], its property within this ninefold figure is that however it is added up, the sum is fifteen, like this:

\[
\begin{array}{ccc}
6 & 7 & 2 \\
1 & 5 & 9 \\
8 & 3 & 4 \\
\end{array}
\]

Commentary

The results of the combinations of number and form are observed in changes in properties (khawāṣṣ), and specifically in the appearance of new, previously unknown properties. The 3x3 wafq is then chosen as an example of such unique properties manifested by a geometrical arrangement of numbers. The Brethren of Purity explain that the new, previously indiscernible property made manifest when the first nine numbers (i.e. 1–9) are combined in a 3x3 square is the sum (jumla) of the numbers in each row, column, and the two corner-to-corner diagonals, namely 15. 72 While the term khāṣṣiyya (pl. khawāṣṣ) can simply mean ‘property’ or ‘characteristic’, the subsequent discussion at the end of Epistle 2.26 leaves no doubt that what is meant are ‘occult properties’, that is to say an object’s active properties the cause of which is not readily apparent, but which allows the possessor of such properties the power to act from a distance. 73 In the case of the wafq the occult

\[
\begin{array}{ccc}
6 & 7 & 2 \\
1 & 5 & 9 \\
8 & 3 & 4 \\
\end{array}
\]

72 This sum is often referred to in modern literature as the ‘magic constant’ of a magic square.
73 The standard example of such khawāṣṣ is, as we saw in the quotation from the Paradise of Wisdom (see above, p. 72), the power of the magnet to attract iron from a distance without
property is equated with the ‘magic constant’ itself, which only manifests when the nine numbers are properly arranged within the square.

4b) After displaying the arrangements, stating the ‘magic constants’ of each of the first seven awfāq, and suggesting that the same example could be followed for other numbers and figures, the text uses the eutocic 3x3 wafq to exemplify the awfāq’s practical utility. Here some fascinating details are given concerning how the Brethren of Purity imagined the talismanic power of the awfāq to function.

Translation

apparent cause. The intermediary position occupied by the awfāq in the Epistles of the Brethren of Purity between the sciences of geometry, arithmetic, khawāṣṣ and talisman making is still discernible even after their wholesale assimilation into lettrism (ʿilm al-hurūf) in the division of sciences laid out by the Ottoman encyclopaedist Taşköprüzāde (d. 968/1561) in his Key of Felicity and Lamp of Sovereignty (Miftāḥ al-saʿāda wa-misbah al-siyāda, completed 948/1541): ‘This science [sc. ‘the science of the numbers of harmony’, i.e. of awfāq] is one of the branches of the science of number with respect to their numerical calculation, and one of the branches of the science of khawāṣṣ with respect to their effects and benefits’.

And this science is one of the branches of the science of the numbers of harmony, i.e. of awfāq, and number and letter permutations (taksīrāt) amongst the branches of the science of arithmetic with respect to the arrangement of numbers, and amongst the branches of the science of geometry with regard to the equilibration of those numbers or letters in the harmonious tables i.e. the physical arrangement within the grids of the awfāq. But, since one can place it amongst the [science of] the properties of letters in consideration of the laying out of the harmony with letters [i.e. letter squares], I have mentioned it [here] in [the article on] the properties of letters, which are amongst the properties of the Qurʾān, and this science is one of the branches of the science of the numbers of harmony, i.e. of awfāq, and number and letter permutations (taksīrāt) amongst the branches of the science of arithmetic with respect to the arrangement of numbers, and amongst the branches of the science of geometry with regard to the equilibration of those numbers or letters in the harmonious tables i.e. the physical arrangement within the grids of the awfāq. But, since one can place it amongst the [science of] the properties of letters in consideration of the laying out of the harmony with letters [i.e. letter squares], I have mentioned it [here] in [the article on] the properties of letters, which are amongst the properties of the Qurʾān.


It is possible to place this science [sc. ‘the science of spiritual properties from number and letter awfāq and number and letter permutations (taksīrāt)’] amongst the branches of the science of arithmetic with respect to the arrangement of numbers, and amongst the branches of the science of geometry with regard to the equilibration of those numbers or letters in the harmonious tables i.e. the physical arrangement within the grids of the awfāq. But, since one can place it amongst the [science of] the properties of letters in consideration of the laying out of the harmony with letters [i.e. letter squares], I have mentioned it [here] in [the article on] the properties of letters, which are amongst the properties of the Qurʾān.


74 Epistles of the Brethren of Purity 2.26, ed. El-Bizri, 142-4 (text), 159 (translation, adapted here).
As for their benefits and uses, we have mentioned this in the Epistle on Talismans and Incantations and presented something of them [there]. But we will give one example of them in this chapter as an indication of the truth of what we have said.

We say: The property and benefit of this ninefold figure is that it eases childbirth when inscribed on two pieces of pottery untouched by water and you suspend them from a woman suffering in labour. If the Moon happens to be in the ninth house and is joined to the lord of the ninth [house], childbirth is eased, or if it is [joined] to the lord of its house in the ninth [degree], or some similar ninefold [astral arrangement].

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Talisman makers arrange them following this method. That is to say that there is nothing in existence be it mathematical, physical or divine without some property that it shares with nothing else in existence, [nor are there] combinations [of properties] without properties they share with no individual [i.e. uncombined] number, figure, form, place, time, element, flavour, colour, smell, sound, word, letter, action or movement. So, when you combine them according to harmonious relationships, their properties and actions are manifested. The actions of theriacs, ointments and potions are evidence that what I have said is correct, [as are] the influences of musical melodies on bodies and souls alike, which do not escape the notice of every intelligent and sage philosopher, of which we have explained something in the Epistle on Music.

**Commentary**

وعلى هذا الطريق سلك أصحاب الطلسمات في مصر وذلك أن هناك من موجودات الرياضية والطبيعية والإلهية إلا وله خاصية ليست لشيء آخر من الموجودات وجمعها خواص ليست مجردية من الأعداد والأشكال والصور والمكان والزمان والغيرات والعلم والألوان والرائحة والأشكال والكلمات والحرف والأفعال والتحركات إذا جمعت بينها على النسب التأليفية ظهرت خواصها وأفعالها والدليل على صحة ما قاله أصحاب النعوات والمراهم والشربات وأغراض الموسيقى وتأثيرها في الأجسام والنفسوس جميعًا. لا خفاء به عن كل ذي حكم فيلسوف كم بتنا طفأ من ذلك في رسالة الموسيقى.
The instructions given here for using the eutocic 3x3 wafq are similar to those already seen in the writings of Ibn Rabban al-Ṭabarî, Vrnda and in the Jābirian corpus. But in contrast to those texts, the Brethren of Purity offer both a more theoretical conceptualisation of the powers of the wafq as well as further astrological instructions for its use. The wafq’s powers, they tell us, are the result of ‘harmonious relationships’ (al-nasab al-taʾlīfiyya) that exist between the various numbers it contains and between those numbers and the figure in which they are arranged. In arranging the numbers harmoniously within a wafq, the talisman maker is like the physician who effects great cures through the harmonious relationships he sets up when he skilfully mixes his many and varied materia medica, or like the musician who blends tone and rhythm in his melodies and is thus able to affect at a distance both the bodies and souls of his listeners. Although the Brethren of Purity never use the term wafq (lit. ‘harmony’) to refer to a magic square, their insistence that a ‘harmonious relationship’ (al-nisba al-taʾlīfiyya) is the source of its power is a good indication of what the name ‘wafq’ implies from a talismanic point of view.

This passage is also the first to give clear instructions on electing an astrologically propitious moment to construct the 3x3 wafq. A number of different potentially good options are presented, but they all share in common the observance of the Moon in relation with the number nine (the ninth house, the lord of the ninth house, the ninth degree etc.). It is common in later Islamicate texts on the awfāq to associate the 3x3 wafq with the Moon, and the Brethren of Purity are the first to hint at this association. If they did meant to associate the 3x3 wafq with the Moon, which is not certain as we shall see from the next passage, it might be assumed that their reason for introducing the first seven awfāq directly before this passage was to suggest that the next six awfāq (4x4-9x9) are associated with the rest of the seven planets, but they do not make this explicit. It is difficult to imagine, though, how the associated astrological numbers could work as the order of the squares increases. While the 9 of the 3x3 wafq works because there are more than 9 houses and degrees within a house, this system might be strained trying to find astrally significant 81s for the 9x9 wafq, since there are only 12 houses and only 30 degrees within each house.

But the Brethren of Purity have more to say on this matter. The beginning of this passage suggests that the present discussion of awfāq is just a small sample of more extended exploration of this subject to be found in another epistle called the Epistle on Talismans and Incantations (Risāla fī al-ṭalismāt wa-l-ʿazāʾim). The epistle referred to is the epistle on magic, the title of which begins ‘On what magic, incantations, the evil eye, taming,
imagination, and spells are; and on how talismans are used...’ Surprisingly, though, no further discussion of the awfāq appears in either of the two recensions of the Epistle on Magic (Epp. 52a and 52b). The end of the passage, however, points us in the direction of the Epistle on Music, and there we find very similar description of the use of the eutocic 3x3 wafq.

4c) Following a discussion of the numerous tetrads found in creation (e.g. the four ages of man, seasons, elements and cardinal directions) and the correspondences and oppositions between them, the potency of combinations embodying a ‘harmonious relationship’ (nisba ta’līfīyya) is illustrated by medicine. Again, the examples of theriacs, ointments and potions is given. Finally, talisman making is adduced as an example of an art that employs an underlying structure of harmonious relationship timed with corresponding terrestrial and astral phenomena to bring about a desired result. The prime example given of talismanic method is the eutocic 3x3 wafq.

Translation

Know, my brother – God aid you and us by means of a spirit from Him! – that when these corresponding things are joined in a harmonious relationship (‘alā al-nisbati al-tāʾlīfī), they are combined, doubling their potentials. Their actions are manifested, they overcome their opposites, conquering what is contrary to them. By knowing them, the sages discovered medicines that cure diseases and heal illnesses like the theriacs, ointments and potions known amongst physicians and prescribed in their books. Talisman makers work similarly in their preparations, following their knowledge of the natures, properties and correspondences of things, how they are composed
and their harmonious relationships. An example of this is the ninefold figure in its facilitation of childbirth when the nine numbers are inscribed in it during the ninth month of pregnancy, at the ninth hour of labour, when the lord of the ascendant is in the ninth house or the lord of the ninth is in the ascendant, and the Moon is in the ninth house or joined to a planet in the ninth house, or some similar ninefold [astral arrangement].

Commentary

This passage is nearly identical in content to the passage 4b, discussed above. Here, in the Epistle on Music, the action of the wafq is not explicitly likened to music as this analogy is implied by the context. Once again, though, the property of the wafq is compared to that of knowledgeably and skilfully mixed compound remedies, and especially the elaborate and wonderous antidotes known as theriacs. We are additionally told that the harmonious relationships within the wafq (as also within compound remedies and music) double the potential of their constituent elements, activating them so that they are able to overcome their opposites. Finally, further ninefold arrangements are given as options for the timing of the wafq’ s construction: three astral (the lord of the ascendant in the ninth house, the lord of the ninth house in the ascendant or the Moon is joined to a planet in the ninth house) and one terrestrial (the ninth month of pregnancy at the ninth hour of labour). The fact that not all the suggested ninefold astral arrangements involve the Moon may be taken as evidence that the Brethren of Purity did not, in fact, associate the 3x3 wafq solely with the Moon. At any rate, it is clear that a larger role in the timing of this wafq’ s construction is played by the number nine (or perhaps the Brethren of Purity would prefer we say ‘the ennead’) than by any particular astral body.

5. Maslama al-Qurṭubi (d. 353/964)

It should be noted that in the passages so far adduced, the term wafq has not been used. In fact, no standard term is used in these texts to refer to either the magic square itself or to the ‘harmonious’ arrangement of numbers within it. Rather the awfāq are generically referred to as ‘figure’ (shakl, pl. ashkāl) by Ibn Rabban and the Brethren of Purity and ‘image’ (ṣūra, pl. ṣuwar) by Ibn Akhī Ḥizām and the Jābirian corpus.

The first use of the term wafq to refer to a magic square in an extant text in the tradition of the eutocic 3x3 square is found in Maslama ibn al-Qāsim
al-Qurṭubī’s (d. 353/964) extremely influential grimoire, the Goal of the Sage (Ğhayat al-ḫakīm, Lat. Picatrix). The Goal of the Sage (6.8) contains only a single and very brief mention of the eutocic 3x3 wafq.

**Translation**

The 15 from the numbers in the three[fold] cells of the wafq is [helpful] for a difficult childbirth.

**Commentary**

Despite the brevity of the reference to the wafq, the fame and wide diffusion of the Goal of the Sage was perhaps influential in ensuring the general acceptance and continued use of wafq as the standard technical term for a magic square, although the term may already have been standard in mathematical literature (see below, p. ??). It is also noteworthy that the eutocic power of the 3x3 wafq is found in a list of substances and objects with occult properties (khawāṣṣ) and not in a section of the text devoted to talisman making. This points to the intermediary position held by the awfāq between something like a stone, a letter or a number which might have naturally inherent khawāṣṣ and a fully elaborated talisman that must be constructed out of the right substances in the right form at the right astrologically elected moment.

It is interesting that Maslama al-Qurṭubī places the number 15 itself (the jumla or ‘magic constant’ of the 3x3 wafq), rather than the nine numbers arranged within the wafq’s cells from which the 15 is derived, in his list of things with khawāṣṣ. This is very similar to what The Brethren of Purity explain in the Epistle On Geometry (2.26, our passage 4a above) that the new, previously indiscernible property (khāṣṣiya) made manifest when the numbers of the 3x3 wafq are combined harmoniously is 15. This similarity should come as no surprise, though, since it is well-known that Maslama al-Qurṭubī had read the Epistles of the Brethren of Purity and even claimed that he was himself one of the authors of these Epistles.78

6. al-Zarqālī (d. 493/1100)

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Abū Isḥāq Ibrāhīm b. Yaḥyā al-Naqqāsh al-Tujībī al-Tulayṭīlī al-Zarqālī’
’s Treatise on the Movements of the Planets (Maqāla fi ṭarākh āl-kawākib
al-sayyāra)79 marks a major turning point in the cultural history of the awfāq.
This Andalusian work outlines practices for using the first seven awfāq (i.e.
3x3–9x9) as talismans of the seven planets and is the earliest dateable
text in which correspondences between all these awfāq and the planets are
explicitly stated. It is also the first text to explain the talismanic use
of awfāq of orders higher than 3 and the first to assert that a single wafq
can have more than one talismanic function depending upon the materials used
to create it, the astrologically elected moment at which it is created, the
incense used to suffumigate it and the method by which it is employed. In
al-Zarqālī’s treatise, the choices and actions of the wafq maker become at
least as important for activating and directing the power of the wafq as the
correct arrangement of the numbers within the wafq.80 With that the awfāq
become more or less identical with other types of talismans. For purposes of
comparison, I will extract here only al-Zarqālī’s treatment of the eutocic

79 Cambridge, MS Gg. 3.19, ff. 140r–148v (ca 860/1456, hereafter A). This treatise is also
called the Epistle on Movements and Governance of the Planets (Risāla fi ṭarākh āl-kawākib
al-sayyāra wa-tadhbrīḥa, Cairo, Dār al-Kutub, MS Tāl’at majāmī 424, ff. 51v–60v [ca
1200/1785], hereafter C3), and The Operation of the Authority and Figures of the Planets
(Tadbīr amr āl-kawākib wa-ashkalihā, Vienna, Nationalbibliothek Wien, MS A.F. 162d, ff.
1v–11v [ca 963/1556], hereafter V). This treatise is also found, without title, in BL, Add.
MS 9599, ff. 128r–131r and 133r–136v (ca 1223/1808, hereafter B). On al-Zarqālī (a.k.a. Walad
al-Zarqiyāl and Ibn al-Zarqālūh), see most recently Julio Samsó, “Ibn al-Zarqālluh,” in
EF. I am grateful to Liana Saif for acquainting me with the earliest copy of this text (A),
which is not mentioned by modern scholars in relation to al-Zarqālī’s treatise on awfāq,
in which the eutocic properties of the 3x3 wafq are discussed on f. 142r. The dating of the
Cambridge copy is based on the colophon to another text in the same manuscript (f. 98v);
an erroneous reading of the date in this colophon as 767/1366 is given in Edward G.
Browne, A Hand-List of the Muhammadan Manuscripts, Including All Those Written in the
Arabic Character, Preserved in the Library of the University of Cambridge (Cambridge: The University
Press, 1900), 201. I am currently preparing a study of the tradition of al-Zarqālī’s treatise
which will include annotated editions and translations of the Arabic text as well the Latin
and Old Castilian versions (edited, translated and analysed by Rosa Comes and Emilia Calvo).
80 It may be mentioned here that there is a further Andalusian appearance of the eutocic
3x3 wafq, of the more traditional variety, in a Hebrew medical treatise attributed to Abraham
b. Ezra (d. ca 560/1165), which clearly derives from Arabic sources on the science of occult
properties (Ila al-khawaṣṣ). The prescription is a simple formulation similar to that found
in the Jabirian Small Book of Balances, but unusually it is attributed to the Graeco-Roman
physician Galen of Pergamon (d. ca 215). See Abraham ibn Ezra, Sefer hanisyonot. The
Book of Medical Experiences. Medical Theory, Rational and Magical Therapy. A Study in Medievalism,
ed. and trans. Joshua Otto Leibowitz and Shlomo Marcus (Jerusalem: Magness Press, 1984), 70–1
(commentary) and 238–41 (text and translation). In general it can be said that Jewish literary
references to magic squares (in both Hebrew and Judaeo-Arabic texts) stem from and form a
part of the larger Islamicate tradition.
power of the 3x3 wafq (which, following his atypical system, is associated not with the Moon but with Saturn) and leave out his discussion of its other powers.\textsuperscript{81}

Text\textsuperscript{82}

\begin{tabular}{|l|l|l|}
\hline
4 & 9 & 2 \\
\hline
3 & 5 & 7 \\
\hline
8 & 1 & 6 \\
\hline
\end{tabular}

If you want to work with it in any way, make it as I describe it

\textsuperscript{81} al–Zarqālī goes on to explain that, under different conditions, this \textit{wafq} can also be used to destroy and depopulate an area, to depose a governor or to win the favour of a king.

\textsuperscript{82} Abu Ishāq b. Yahyā al-Naqsh al-Zarqālī, \textit{Maqāla fī ḥarakāt al-kawākib al-sayyāra}, A f. 142r, C\textsuperscript{1} ff. 53v-54r, V f. 4r and B f. 133v.

84 If you want to work with it in any way, make it as I describe it
to you. This is an image of it:

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If you want it for someone who is having difficulty in childbirth, inscribe it on Saturday, at the hour of Saturn, when it [sc. Saturn] is in its ‘ease’, ‘increase’, or ‘fullness’, which you will know from the generous gift drawn up for it. ¹⁰⁰ Take a new rag from a cotton robe, mark the figure on it [C 54r] as it has been described, and tie it to the [woman’s] right hip. Suffumigate it with the fur of ‘the sagacious one’ (i.e. a cat) and the pregnant woman will give birth straight away.

Commentary

While this description of how to use the eutocic 3x3 wafq is in some respects similar to earlier prescriptions, there are major changes to both the materials used and their employment. For example, only one wafq is to be used, and it is not to be written on a piece of new pottery and placed under the feet, but on a rag of new white cotton and tied to the hip. ¹⁰¹ More substantial innovations are the addition to the prescription of a suffumigation and the election of an astrologically propitious moment for constructing the wafq, both of which are fundamental talismanic practices.

7. al-Ghazâlî (d. 505/1111)

¹⁰⁰ ‘Generous gift’ (makruma) is the term al-Zarqâlî uses for the tables placed at the end of his treatise, which indicate when each planet is in various states such as the ‘ease’, ‘increase’ and ‘fullness’ mentioned here.

¹⁰¹ The shift from a piece of pottery (خزف) to a rag (خرقة) can be explained by the graphical similarity between these two words in Arabic, but the other changes lack obvious textual explanations.
No survey of literary appearances of the 3x3 \textit{wafq} prior to the 7th/13th century could have any claim to completeness without mentioning Abū Ḥāmid Muḥammad b. Muḥammad al-Ṭūsī al-Ghazālī. Indeed, his association with the 3x3 \textit{wafq} is so great that it is sometimes named after him as the ‘threefold’ \textit{(muthallath)} or ‘seal’ \textit{(khitām)} of al-Ghazālī. But, in fact, he did no more than use it as a well-known example of something with occult properties \textit{(khawāṣṣ)} that are mysterious and inscrutable, the power of which is nonetheless undeniable.

In his autobiographical \textit{Deliverer from Error and Transmitter to Him of Power and Glory} (\textit{al-Munqidh min al-ḍalāl wa-mawaṣṣil ila dhī al-ʾizza wa-al-jalāl}), al-Ghazālī argues against the ‘those whose faith has been corrupted by way of philosophy’ \textit{(man fasada ʾīmānuhu bi-ṭarīq al-falsafa)} so that they have lapsed into denial of prophecy and prophetic injunctions, and in the course of his arguments, he uses examples chosen to be acceptable to students of philosophy such as ‘the occult properties of medicine and the stars, ... physics, magic and talismans’ \textit{(khawāṣṣ al-ṭibb wa-al-nujām ... wa-al-ṭabīʿa wa-al-sīhr wa-al-ṭilāsmāt)}. \textsuperscript{102} ‘Why’, asks al-Ghazālī, ‘is it not permissible that within the prescriptions of the \textit{sharīʿa} there should be occult properties for healing and purifying hearts that are incomprehensible by intellectual wisdom, but visible only to the eye of prophecy?’ \textsuperscript{103}

Al-Ghazālī goes on to forestall his opponents’ objections using the example of the inscrutable yet undeniable \textit{khawāṣṣ} of the 3x3 \textit{waqf}. If one believes that the numbers within the \textit{waqf} hold inexplicable powers, why should he deny that the numbers prescribed by the prophet (e.g. the specific numbers of inclinations and prostrations \textit{[rakaʿāt]} prescribed in each of the 5 daily prayers) might also hold inscrutable powers?

\textbf{Translation} \textsuperscript{104}


\textsuperscript{103} ʿAyyād and ʿṢalībā, \textit{Munqidh}, ed. ʿAyyād and ʿṢalībā, 126; translation in Watt, \textit{Faith and Practice}, 79 (adapted here).

\textsuperscript{104} al-Ghazālī, \textit{Munqidh}, ed. ʿAyyād and ʿṢalībā, 126-7 (some punctuation modified and orthography standardised here); translation in Watt, \textit{Faith and Practice}, 79-80 (adapted here).
Indeed, they have recognised occult properties (khawāṣṣ) more wondrous than this in what they have presented in their writings of the wondrous properties proven in the treatment of a pregnant woman for whom delivery is difficult by means of this figure inscribed on two rags untouched by water.

The pregnant woman gazes at them with her eyes, and they are placed beneath her feet, then at once the child comes out quickly. They acknowledge the possibility of this and list it amongst the wonders of properties (khawāṣṣ). It is a figure in which there are nine cells, each with a specific number written in it [so that] the sum of what is written in a single line (jadwal) is 15 [whether] you read it in the figure vertically, horizontally or diagonally (? ‘ālā al-ta’rīb).

Commentary

It is clear that al-Ghazālī’s philosophical opponents took the efficacy of

وتنظر إليها الحامل بعينها، ووضعا تحت قدميها، فيسرع الولد في الحال إلى الخروج. وقد أثروا إمكان ذلك وأوردوه في مجاهذ الخواص وهو شكل فيه تسعة بيوت يرقم فيه رقم مخصوصة يكون مجموع ما في جدول واحد خمسة عشر; فقرأ فها رقم فيها رقم مخصوصة، يكون مجموع ما في جدول واحد خمسة عشر; فقرأ فها رقم فيها رقم مخصوصة، يكون مجموع ما في جدول واحد خمسة عشر; فقرأ فها رقم فيها رقم مخصوصة، يكون مجموع ما في جدول واحد خمسة عشر; فقرأ فها رقم فيها رقم مخصوصة، يكون مجموع ما في جدول واحد خمسة عشر;
the eutocic 3x3 wafq for granted. Al-Ghazālī, for his part, says nothing to suggest that he does not also believe in the power of the wafq, and in fact uses its khawāṣṣ as an example to make the same point in a Persian tract against antinomian Sufis. By the time al-Ghazālī wrote the Deliverer from Error in Saljuq Khurasan, it seems that the perceived truth of the power of the eutocic 3x3 wafq was unassailable.

The early texts on the eutocic powers of the 3x3 wafq reviewed above do not explain the mathematical rules governing the construction of awfāq. Nor do they give instructions for algorithms by which their constructions could be carried out even if not fully understood. The texts are accompanied by images of the awfāq that must be slavishly copied by would-be practitioners. Even the Epistles of the Brethren of Purity, in which a discussion of awfāq 3x3-9x9 appears in a mathematical context, and the treatise of al-Zarqālī with its extensive treatment of the talismanic uses these same awfāq are both silent about their methods of construction. There are some apparently early awfāq treatises which deal with both the khawāṣṣ of the awfāq and their talismanic uses on the one hand and the mathematics behind their methods of construction on the other, but these texts have yet to be securely dated.


106 An important example of these dual genre awfāq treatises is Anon. CBL, the first two thirds of which deal exclusively with the mathematics of awfāq and the last third with their talismanic uses. Another such text is the anonymous Persian treatise found in manuscript BL, Add. MS 7713 (hereafter Anon. Pers. BL; see Charles Rieu, Catalogue of the Persian Manuscripts in the British Museum [London: The British Museum, 1879-83], vol. 2, 487). This latter manuscript, which appears to date from the 7th/13th century, has a colophon added by a hand perhaps later than that of the main scribe (f. 237v) dated Rajab 608/December-January 1212, although this date may have been transcribed from its archetype. The manuscript is defective at beginning and end, so the preface, part of the introduction and some folios near the end and the end of the conclusion are missing. The first folio is a replacement on European paper, containing a new beginning (f. 1) by a much later (Ottoman?) hand, which attributes the treatise to the Persian astrologer Abū Maʿṣar al-Balkhī (d. 272/886). This attribution is, however, surely as spurious as the attribution to Abū Maʿṣar of another treatise involving awfāq (see Oliver Kahl and Zeina Matar, “A Treatise on the Amicable Numbers 220 and 284 Attributed to Abū Maʿṣar al-Balkhī,” Journal of Semitic Studies 35.2 [1990]); a forthcoming article by Sāqib Bāburī and the present author will argue that the text preserved in Anon. BL Pers. is the product of sixth/twelfth-century Ghaznavid patronage. Anon. Pers. BL is of particular interest here because, as is the case with Anon. CBL, the bulk of this text is devoted solely to mathematical constructions giving no cause to suspect that the interest in these constructions is not purely mathematical, but also magical. Then, a very brief and fragmentary conclusion (f. 237v) discussing the occult properties (khawāṣṣ) and astral associations of certain awfāq demonstrates that there is no reason to assume a mathematician fully engaged in arithmetical analyses should necessarily dismiss the magical
Most treatises entirely devoted to the *awfâq*—which is what I term ‘*awfâq* literature’ proper—and written before the 7th/13th century deal almost exclusively with mathematics. This is not to say that the authors of such *awfâq* literature were not interested in employing the *khawāss* of the *awfâq* they studied, but merely that this was not the aim of this genre of text. Regardless of ultimate intentions of these authors with the *awfâq*, however, their mathematical advancements in this field allowed the construction of larger and more complex *awfâq* not just in the form of squares, but also triangles, circles, stars, cubes, cones, and spheres. Research presented by these mathematicians (e.g. methods for the inclusion of names and words into *awfâq*) prepared the way for the confluence of lettrism and *awfâq* popularised in the writings of al-Bûnî and others, and taken up enthusiastically by later authors such as 'Abd al-Rahmân al-Bîstâmî (d. 858/1454).  

### III. The rise of *awfâq* literature

The question of who wrote the earliest treatise devoted to the *awfâq* remains open. The earliest known text with a title containing the word *wafq/* *awfâq* is by al-Jâhîz (d. 255/868-9), who himself tells us that he wrote a work (sadly now lost) called the *Book of Awfâq and Mathematics* (*Kitâb al-* *awfâq wa-l-* *riyâdât*). Given the fact that al-Jâhîz is a well-known author of *adab* works and is not known to have written any mathematical or, strictly speaking, scientific treatises, it is unlikely that this text is a candidate for the earliest mathematical treatise on *awfâq*. It is all but certain that al-Jâhîz’ *Book of Awfâq and Mathematics* is an *adab* work, no more a treatise on the mathematics of *awfâq* than his *Book of Circling and Squaring* (*Kitâb al-* *tarbî‘ wa-l-* *tadwîr*) is a treatise on geometry. But just as the *Book of...* 

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Circling and Squaring touches on questions of geometry, his Book of Awfāq and Mathematics may well have dealt in some tangential way with the mathematics of awfāq. Be that as it may, the title alone is evidence that the awfāq themselves were sufficiently well known by this name and their link with mathematics sufficiently established in the mid-3rd/9th century that al-Jāḥiẓ’s use of the term would have had cultural currency.¹¹⁰

So, if al-Jāḥiẓ’s Book of Awfāq and Mathematics was not the first Islamicate treatise dedicated to the awfāq, what was? To date there has not been a satisfactory answer to this question. It is often claimed that the earliest Arabic work devoted to the awfāq was written by the Șābīan mathematician, translator and physician Abū al-Ḥasan Thābit b. Qurra al-Harrānī (d. 288/901). This claim rests on fairly good authority since Ibn al-Qiftī (d. 646/1248) lists an Epistle on the Harmonious Number (Risāla fī al-’adad al-wafq) amongst the works attributed to Thābit b. Qurra, and Ibn al-Qiftī tells us that he is merely reproducing a list he found in a genealogical document relating to Thābit b. Qurra and written by the hand of Thābit’s great-great-grandson Abū ‘Alī al-Muḥassin b. Ibrāhīm b. Hilāl al-Ṣabi’ (d. 410/1010).¹¹¹

Despite the excellent pedigree of this information, there remain some reasons to doubt whether Thābit b. Qurra did, in fact, write a treatise on awfāq. To begin with, not only does this text not appear in the earliest list of Thābit b. Qurra’s works, that of Ibn al-Nadīm’s (d. ca 385/995) Fihrist,¹¹²

¹¹⁰ The Book of Awfāq and Mathematics must have been composed before the Kitāb al-ḥayawan in which it is mentioned. On the date of the Kitāb al-ḥayawan, see Pellat, “Gāhiziana III,” 153 (item 29), who thinks it was written prior to 232/847; and Montgomery, Al-Jāḥiẓ: In Praise of Books, 237, who argues it was written after 244/858. Either way, al-Jāḥiẓ wrote his Book of Awfāq and Mathematics at roughly the time, and perhaps some years before, Ibn Rabban al-Ṭabarī wrote the Paradise of Wisdom, and about a century before the term wafq appeared in Maslama al-Qurṭubī’s Ghayat al-ḥakīm.


but it is not mentioned by any source that is not dependent upon Ibn al-Qifti. Rosenfeld and İhsanoğlu claim that there is a reference to Thābit’s *Epistle on the Harmonious Number* in al-Bīrūnī’s (d. ca 442/1050) *Treatise on the Derivation of the Chords in a Circle* (*Maqāla fī istikhrāj al-awtār fī al-dā‘ira*), 113 but a careful search of the two printed editions of this text, Suter’s German translation and the manuscripts presented by Hogendijk has failed to locate any such reference. 114 Elsewhere, Rosenfeld goes further and claims that Thābit introduced the correspondence between the seven planets and the first seven *awfāq* in his *Epistle on the Harmonious Number*, but this claim must be baseless if that text is not extant, if it ever existed. 115 It may also be suspected that a discussion of *awfāq* might be found in the treatise on talismans attributed to Thābit b. Qurra. Yet, no such thing appears in either the surviving Judaeo-Arabic fragments of that text or the Latin version, *De Imaginibus*. The talismans ascribed to Thābit in this text are similar to those of the pseudo-Aristotelian Hermetica in many regards including the fact that neither set contains any mathematical talismans. 116


115 Boris A. Rozenfel’d and Nurii G. Khafreṭdinova, *Sabit ibn Korra: 836–901 Moscow: Nauka, 1994*, 102. Although admitting the *Epistle on the Harmonious Number* (Risāla fī al-‘awtār fī al-wafq) is not extant, Rosenfeld and Khafreṭdinova silently alter the title to read *Book of the Number of the Wafq* (which presupposes *Kitāb ‘awtār al-wafq*) and assume arbitrarily that the ‘number’ in question is the sum of all the numbers in a *wafq* (rather than e.g. the root [or order number] of the square, its magic constant, middle term, highest number or any of the other significant numbers derivable from it), saying ‘... the title of his treatise ... seemingly implies the sum of its numbers that was placed in correspondence with the Sun, Moon and the planets’. From this assumption they reach the groundless conclusion that the correspondence between Saturn and the 3x3 square based on the fact that the numerical value of the Arabic for Saturn (Ζυῆλα, \([Z = 7] + [H = 8] + [L = 30] = 45\)) is equal to the sum of all the numbers in the 3x3 square \((1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45)\) ‘was already mentioned in the treatise by Thābit Ibn Qurra’. Rosenfeld and Khafreṭdinova also repeat as fact, again without evidence, that ‘the use and application of magic squares was well known in the Hellenic world’.

116 See Charles Burnett, “Ṭābit ibn Qurra the Ḥarrānian on Talismans and the Spirits of
Not only does no trace of any text by Thābit b. Qurra on awfāq survive, but no later treatises on awfāq even mention Thābit b. Qurra amongst the authors of works on this subject. Moreover, Thābit is mentioned in the introduction to an anonymous sixth/twelfth-century awfāq treatise edited and translated by Sesiano, but not as an author of an awfāq treatise himself. Rather he is said there to have followed Plato in writing on amicable numbers — a reference to Thābit’s Book of Amicable Numbers (Kitāb al-a‘dād al-mutabhābah). This same anonymous twelfth-century awfāq treatise also credits the Pre-Socratic father of natural philosophy Thales of Miletus (d. ca 545 BC) with the discovery of magic squares (lit. ‘the harmonious number’ [al-‘adad al-wafq]) and with this we are in the realm of origin legends of the awfāq. The ancient Greek philosophers are commonly made to be the originators of the awfāq in these legends. As already seen (passage 3a, above), the Jābirian Small


118 For a recent discussion of Thābit’s work on amicable numbers along with a new edition and French translation of the treatise, see Roshdi Rashed and Christian Houzel, “Théorie des nombres amiables,” in Thābit ibn Qurra. Science and Philosophy in Ninth-Century Baghdad, ed. Roshdi Rashed (Berlin - New York: De Gruyter, 2009). The earliest author to mention amicable numbers is not, in fact, Plato but Iamblichus, who attributes the discovery of the only pair of amicable numbers (philoi arithmoi) known in antiquity (220 and 284) to Pythagoras (d. ca 495 BC). Iamblichus, In Nicomachi Arithmeticam, ed. and trans. Nicholas Vinel (Pisa: Fabrizio Serra Editore, 2014), ii, 126. Whether or not Pythagoras himself knew the amicable numbers, the fact that they are two matching halves of an arithmetical whole that fit together only with each other makes them a suitably mathematical Pythagorean symbolon, on which, see Peter T. Struck, Birth of the Symbol: Ancient Readers at the Limits of their Texts (Princeton: Princeton University Press, 2004), chap. 2.

119 Sesiano, “Une compilation arabe,” 154-55 (translation) and 173-74 (text). Thales is there said to have placed a 100x100 square in a temple where it was used for swearing oaths and curing diseases. Although, the general term for magic square in Arabic is waqf (lit. ‘harmony’, ‘agreement’), the odd phrase al-‘adad al-waqf (lit. ‘harmonious number’, strange in that it uses waqf adjectivally) appears, as we shall see, in the titles of a some of the earliest Arabic awfāq literature as a designation of their subject. From its use here and in other early awfāq literature, it is clear that the phrase ‘harmonious number’ (al-‘adad al-waqf) referred to the awfāq themselves (i.e. the harmonious arrangements of the numbers within their grids [jadāwil], and most importantly their ‘magic constants’), but also to the science behind these arrangements. For the adjectival use of the word waqf, see Francis Joseph Steingass, The Student’s Arabic-English Dictionary. Companion Volume to the Author’s English-Arabic Dictionary (London: Crosby Lockwood and Son, 1884), 1224. I am grateful to Julia Bray for pointing me in the direction of this lexicographical reference.
Book of Balances claims that Apollonius of Tyana at least discussed if not actually invented the 3x3 wafq. Zakariyya b. Muḥammad al-Qazwīnī (d. 682/1283) attributes the invention of magic squares to Archimedes (d. 212 BC). 120

The legends of the awfaq also frequently link them with the prophets and figures from early Islamic history. Anon. CBL (f. 125r-v) attributes the discovery of the 100x100 wafq to the legendary Persian king Fereydun, and says that it was handed down through the line of Persian kings, kept in their treasuries to ensure their military invincibility and then found by the Caliph ʿUmar b. al-Khāṭṭāb (d. 23/644) after his defeat of the final Sassanid shāh Yazdegerd III (d. 30/651). A briefer, but very similarly worded, version of the story of Fereydun, Yazdegerd and the 100x100 wafq was presented by Luṭfullāh al-Ṭūqātī (d. 900/1494), palace librarian to the Ottoman sultan Mehmed II (reg. 848/1444-850/1446 and 855/1451-886/1481) at Istanbul, later recommended to Sultan Bāyezīd II (reg. 886/1481-918/1512) for the same position by the celebrated astronomer ʿAlī Qushjī (d. 879/1474), erstwhile head of the observatory of Ulugh Beg at Samarqand.121 In his Treatise on the Doubling of the Altar [of Delos] (Risālat tād‘if al-madhbaḥ), Luṭfī al-Ṭūqātī extended the lineage of the wafq in question through Greek astrologers and philosophers and the Abrahamic prophets all the way back to Adam.122
Legends and unverified attributions aside, scholarship—thanks largely to the editorial and analytical efforts of Jacques Sesiano—has uncovered enough texts to present a rather detailed history of early Arabic awfāq literature. Sesiano has identified nine texts written before the 7th/13th century that are either entirely devoted to the awfāq or contain substantial sections devoted to them (see Table 3.1).

<table>
<thead>
<tr>
<th>Title</th>
<th>Author/date</th>
<th>Location(s) of author</th>
<th>Patron(s) of author</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Book on the Arrangement of the Harmonious Number in Squares (Kitāb fī tartīb al-ʿadad al-wafq fī al-marabbaʿāt)124</td>
<td>Abū al-Wafāʾ al-Bozjānī (d. 388/998)</td>
<td>Bāb al-Tibn Observator y, Baghdad</td>
<td>‘Aḍud al-Dawla, Şamsām al-Dawla and...</td>
</tr>
</tbody>
</table>

16-21 (text) and 52-61 (trans.); see also Sesiano, Les Carrés magiques dans les pays islamiques, 266-68. A version of the legend of ‘Umar’s discovery of Yazdegerd’s 100x100 wafq even briefer still, the wording of which is less similar to that in the Anon. CBL text, is recounted by Ibn Khalūn (d. 808/1406) in Muqaddima vi, 27 (= Muqaddimāt Ibn Khaldūn. Prole gomènes d’ Ebn Khaldoun, ed. Éttiène Quatremère [Paris: Benjamin Duprat, 1858], vol. 3, p. 135; translated in The Muqaddimah: An Introduction to History, trans. Franz Rosenthal [London: Routledge & Kegan Paul, 1958], vol. 3, 168-69). According to a variant of the legend preserved in a gloss in a twelfth/eighteenth-century manuscript, knowledge of the awfāq originated with the prophet Hūd, who placed a 100x100 wafq in the foundations of Mecca, and the caliph ‘Alī b. Abī Ṭālib (d. 40/661) was the first to employ this wafq on the banner of Islam. On the manuscript containing this gloss, see Jacques Sesiano, Un traite médiéval sur les carrés magiques: De l’arrangement harmonieux des nombres. Édition, traduction et commentaire d’un texte arabe anonyme décrivant divers modes de construction (Lausanne: Presses Polytechniques et Universitaires Romandes, 1996), 13-14, n. 16; and text 5 in Table 3.1 below. Sesiano, Les Carrés magiques dans les pays islamiques, 267, n. 164 also mentions Saʿd Allāh b. Șadr al-Dīn’s Risāla fī al-wafq, (Süleymaniye, MS Resid Efendi 1068, ff. 49v-56r [not 53r!]) as a further source of such legends. In fact, that text is anonymous and Saʿd Allāh b. Șadr al-Dīn is the manuscript’s scribe not the text’s author. The text mentions al-Būnī, so was written after the mid-7th/13th century, and claims Archimedes studied awfāq. Thales was inspired to invent the 100x100 wafq and also discusses the problem of the doubling of the altar of Delos (ff. 49v-50r).

123 Only chapter 3 of this work is extant and only section 2 of that chapter is dedicated to awfāq. The text is edited and translated in Sesiano, Magic Squares in the Tenth Century, 120-205 (translation), 260-334 (text).

124 Excerpts edited and translated in Sesiano, Magic Squares in the Tenth Century, 207-52
Sharaf al-Dawla,
Būyid amīr (combined reg. 367-79/978-89)

<table>
<thead>
<tr>
<th></th>
<th>3 On the Numbers of Harmony (Fī ‘adād al-wafq) 126</th>
<th>Abū ‘Alī al-Ḥasan Ibn al-Haytham (d. ca 430/1040)</th>
<th>Basra (354-ca 390/965-ca 1000); Cairo (from ca 390-430/ca 1000-39, with a gap)</th>
<th>al-Ḥākim bi-Amr Allāh, Fatimid caliph (reg. 386-411/996-1021)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4 Abridgement on Guidance to the Harmony of Numbers (Mukhtaṣar fī al-‘irshād ilā wafq al-aʿdād) 126</td>
<td>Unknown (fl. early 5th/11th century?)</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
<tr>
<td></td>
<td>5 Book of Numeration on the Harmony of Numbers (Kitāb al-iʿdād fī wafq al-aʿdād) 127</td>
<td>Unknown (fl. 1st half of 5th/11th century?)</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
</tbody>
</table>

127 Edited, translation and study in Sesiano, Un traite médiéval. See also Sesiano, Les Carrés
|   | Treatise on the Movements of the Planets  
(Maqāla fī ḥarakāt al-kawākib al-sayyāra)  
(128) | Abū Iṣḥāq Ibrāhīm al-Zarqālī  
(d. 493/1100) | Toledo; Córdoba  
(from ca 482/1080) | Şaʿīd al-Andalusī, astronomer and Chief Qāḍī of Toledo  
(d. 472/1070) |
|---|---|---|---|---|
| 7 | Unknown  
(129) | Abū Ḥātim al-Muẓaffar al-Isfizarī  
(d. ca 510/1116) | Isfahan  
(from 476/1074-5); Balkh  
(ca 506/1112) | Jalāl al-Dīn Malikshāh, Saljūq sultan  
(reg. 465-85/1073-92) |
| 8 | Epitome on the Harmonious Number  
(Talkhīṣ fī ʿadad al-wafq)  
(130) | Jamāl al-Zamān al-Kharaqī  
(d. 533/1138-9) | Marw | Sanjar b. Malikshāh?, Saljūq sultan  
(reg. 511-52/1118-57) |
| 9 | Unknown  
(131) | Unknown (fl. between 515/1121 and 587/1191) | Unknown | Unknown |

Table 3.1: *Awfāq* treatises written before the end of the 6th/12th century and discussed by Jacques Sesiano.

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magiques dans les pays islamiques, 13.
129 Traces of this lost treatise may be found in the anonymous twelfth-century treatise edited in See also Sesiano, *Les Carrés magiques dans les pays islamiques*, 14-15, and below.
IV. New manuscript evidence for early awfāq literature

Despite Sesiano’s pioneering work, the majority of works of Islamicate awfāq literature remains unstudied in manuscripts, and the treatises edited by Sesiano tend to be relatively short. The largest pre-seventh/thirteenth-century text dealt with by Sesiano is the Book of Numeration on the Harmony of Numbers (Kitāb al-iʿād fī wafq al-aʿād, text 5 in Table 3.1), which despite running to only 44 folios in the manuscript (74 pp. in Sesiano’s edition), is according to Sesiano, one of the longest and oldest awfāq treatises known. 132 The anonymous early fifth/eleventh-century author tells us in the introduction that he has spent the smaller portion of the book recounting information about the awfāq from earlier sources, but he does not cite any of his sources by name. The larger portion is devoted to his own discoveries with the awfāq, which do not go higher than 10x10 and are all squares (no triangles, circles or other wafq figures). The treatise does, however, contain an early discussion of methods for inserting names and other text into a wafq using the abjad values of the letters, a technique that would become widespread in later lettrist talisman making. 133

A number of much longer early awfāq treatises, which are also more diverse in content and even more historiographically detailed do, in fact, survive in the manuscripts. The previously mentioned Anon. Pers. BL, for example, runs to 237 folios making it, to my knowledge, the single largest Islamicate treatise on the mathematical construction of (square, triangular and circular) awfāq. 134 Anon. CBL, which we have also already had a few occasions to mention, appears to be a ninth/fifteenth-century Timurid copy of a single awfāq treatise of 125 folios, dealing with constructions for magic squares, triangles, circles, cubes and spheres. 135 As with the Persian treatise just discussed, Anon. CBL begins with a long section devoted entirely

132 Sesiano, Les Carrés magiques dans les pays islamiques, 13. For a brief description of the contents of the treatise see Sesiano, Un traité médiéval, 11-12.
133 Sesiano, Un traité médiéval, 72-74 (text) and 129-32 (translation). Sesiano removes the names (Sirāj, Shihāb, ʿAlī and Aḥmad) from his translation, leaving only their numerical values, so that readers of the French translation without access to the Arabic text would have no idea that the text is discussing meaningful names and not randomly chosen numbers.
134 See above, p. ???
135 Like Anon Pers. BL, Anon. CBL is defective at the beginning and the end and we know neither its title nor author. For a very brief description of the manuscript, see Arthur J. Arberry, The Chester Beatty Library, A Handlist of the Arabic Manuscripts (Dublin: Emery Walker Ltd. and Hodges, Figgis and Co., 1955-66) vol. 7, 29.
to mathematical constructions of the *awfāq* without mention of their *khawāṣṣ*, but then concludes with information about their talismanic uses.

A newly discovered sixth/twelfth-century Arabic treatise of 92 folios called the *Collection of the Harmonious Number* (*Dīwān al-'adad al-wafq*) provides invaluable resources for the early history of the *awfāq*. Although the identity of the author of this treatise is unknown, by carefully citing the sources of the information he transmits, he substantially increases our knowledge of mathematicians engaged in the study of *awfāq*, their writings and discoveries. Before moving on to consider the contents of this newly texts we should consider the history of the unique manuscript that contains it.

**British Library, Delhi Arabic 110**

The British Library’s Delhi Collection is a rich and largely untapped resource for the literary and intellectual history of Islamicate India and Central Asia. The collection comprises over 3,700 manuscripts (mostly Arabic and Persian, but also Urdu, Panjabi and Pashto) from the Mughal Imperial Library and other collections in Delhi that were dispersed during British reprisals in the aftermath of the Indian Uprising of 1857-58 then sold to the (colonial) Government of India at a sale arranged in 1859 by the Delhi Prize Agents. An inventory of the Delhi Collection in the form of a handwritten Catalogue of the Delhi Collection was drawn up in Calcutta at some time after 1869, under the supervision of Heinrich (Henry) Ferdinand Blochmann (1838-78), Philological Secretary of the Asiatic Society of Bengal and Assistant Professor at the Calcutta Madrasa. The collection was

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138 BL, 10 Islamic 4601-4606. Blochmann’s catalogue contains basic information on each text in the collection’s manuscripts, listing, when known, the author, title, incipit, explicit, production date, number of folios and lines per page, along with basic judgements on the
subsequently shipped to London and deposited in the Library of the India Office in 1876. Some of the Arabic manuscripts from the Delhi Collection appear in published and unpublished catalogues compiled first half of the 20th century, but many have not been described since the time of Blochmann. Finally, in 1996 the India Office Library was incorporated into the British Library, where the Delhi Collection is now housed.

The Delhi Collection’s strong holdings of Indian Islamic religious and mystical texts has long been noted, but its relatively small number of scientific texts has been almost entirely ignored. One of these overlooked texts is Delhi Arabic 110, a composite volume comprising two unrelated manuscripts: a fourteenth/nineteenth-century manuscript of Ibn Ḥajar al-‘Asqalānī’s (d. 852/1449) Selection of Thoughts concerning the Traditionists’ Terminology (Nukhbat al-fikr fī muṣṭalāḥ ahl al-athar, ff. 2v-27r), and the Collection of the Harmonious Number (Dīwān al-‘adad al-wafq, ff. 28r-119v; hereafter, referred to as the Dīwān). Blochmann’s catalogue presents the following information about the Dīwān:

Dīwánul’ ad walwafq


140 An exception to this rule is Delhi Arabic 1949, https://www.qdl.qa/en/archive/81055/vdc_100028004317.0x000001. This sixth/twelfth- or seventh/thirteenth-century manuscript of Sharaf al-Zamān Tāhir al-Marwazi’s (fl. mid-5th/11th/early-6th/12th century) zoological treatise The Natures of Animals (Ṭabā’ī’ al-ḥayawān) was hailed by Arberry as a ‘tantalizing witness to the pristine splendour of the Royal Library at Delhi’ (Arberry, The Library of the India Office, 85).

141 Catalogue of the Delhi Collection, Arabic MSS, vol. 1, 165, BL, 10 Islamic 4604.

142 Whoever added this transliteration (not the same hand as added the transcriptions) was
Description, provenance and date of the manuscript of the Diwān

Sadly, the manuscript is defective at beginning and end, and the first and last remaining folios have been mutilated and repaired with the loss of some text. Since the catalogue entry above states that the manuscript comprised 96 folios, while in its current state it comprises 92 folios, it would seem that 4 folios have been lost from the Diwān since ca 1869. Historic foliation in eastern Arabic numerals begins at 3 on f. 29r—the corner bearing the foliation on f. 28r, the present first folio of the Diwān manuscript, has been mutilated—which suggests that only a single leaf has been lost from the beginning. The historic foliation is continuous and does not indicate any folios missing from the middle of the text, so we can say that 3 or more folios are missing from the end. A comparison of the extant text with with the table of contents (ff. 28r-29r) shows that the missing final leaves contained the remainder of Bāb 3 and Bābs 4 and 5 of Maqāla 8.

From the perspective of the arts of the Islamic book, as opposed to Arabic literature or the history of scientific thought, the copy of the Diwān in Delhi Arabic 110 is most notable as a masterpiece of layout and design rather than decoration and artistry. The hand used is not ornate or excessively polished, but neat, legible and uniform. It is really in the precision and clarity of line in the accompanying figures (of which there are over 325 squares and triangles), the skilful use of colour and shape to present technical material and the well-conceived and executed tables and diagrams that this manuscript achieves distinction.

apparently confused by this title since he read the second dāl in ‘adad as a wāw, changing the title to the Collection of Enumeration and Harmony.
143 This text comes not from the beginning of the manuscript, but rather from the beginning of Maqāla 1 (f. 29v).
144 It is thus probable that only a single page of text, or less, has been lost from the beginning: the text in a carefully prepared Islamic manuscript customarily begins on the verso side, and some space on that side may have been taken up by a decorative ‘unwān.
In the absence of any scribal colophon or patron statement (owing to the loss of the initial and final folios) we have no firm evidence with which to establish the manuscript’s date. On palaeographical grounds, the it appears to date from late ninth/fifteenth- or early tenth/sixteenth-century Timurid Iran or Khurāsān. A manuscript copied in a very similar hand, on very similar paper, in a similar format and indeed on a similar subject (viz. square and other figurate lettrist talismans) can be seen in MS Princeton University Library, Third Series no. 591. According to its colophon (f. 26v), this Princeton manuscript was transcribed on 17 Rajab 888/21 August 1483 by Ḥaᶜjjī b. Jamāl al-Kātib al-Kāshi. Furthermore, the abjad (alphanumeric) numeral forms employed in Delhi Arabic 110 in particular are typical of late ninth/fifteenth- or early tenth/sixteenth-century Iranian manuscripts.

Provenance information in the form of owners’ inscriptions, marginalia and seal impressions does not extend back far enough to support this production date but does prove that the manuscript was part of the Mughal imperial library collection. To begin with, the seal of Ḥaqq b. Qāsim Shirāzī (d. 1054 or 55/1644–5), dated 1037/1627–8 is found on f. 28r, the present first folio of the treatise. Ḥaqq Shirāzī, known from 1141/1632 as Amānat Khān, was a palace librarian under Shāhjahān (reg. 1037–76/1628–66) whose seal appears in a great number of the manuscripts then in the Mughal imperial library, but he is best known as the master calligrapher whose work is signed on the Taj Mahal and other eleventh/seventeenth-century Mughal monuments.

The appearance of his seal on this folio suggests that the original first folio of the manuscript was already lost and the historic foliation was already present at the beginning of Shāhjahān’s reign. Later, during the reign of Aurangzīb (ʿĀlamgīr, reg. 1068/1658–1118/1707), another librarian, Fakhr al-Dīn Muhammad left his seal impression dated 1110/1698–9 along with the following valuation note (f. 27a–r):

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145 A digital surrogate of this “Treatise on charms and talismans”, along with a catalogue record can be found on the Princeton University Library website, http://arks.princeton.edu/ark:/88435/8336h195t.
147 See the CBL Islamic Seals Database, seal no. 186. A further two illegible seal impressions are found on BL, Delhi Arabic 110, ff. 28r and 119v.
149 The seal and note are found on a detached strip of repair paper inserted between ff. 27 and 28, now bound into the volume as f. 27a. For a nearly identical seal impression, bearing the same name but dated 1126/1714-15, see CBL Islamic Seals Database, seal no. 632. I thank
Copy of the Dīwan of Harmonious Number from al-Mufaddal b. Thābit ... end

This precious and rare copy concerning the harmony of numbers ... the Abode of the Caliphate Shāhjahān[ābād, i.e. Delhi] ... The most needy and wretched Fakhr al-Dīn Muḥammad. The present value ... [semi-legible raqm numbers followed by the seal impression]

Although the value ascribed to the manuscript by Fakhr al-Dīn Muḥammad is not clearly legible, we can see that already in his day it was considered ‘precious and rare’. Also probably dating from the manuscript’s time in the Mughal library is an intriguing note in the repaired upper margin of f. 28r:

This book is the Collection of Number concerning the science of arithmetic and mathematics ... like alchemy and the elixir for the connoisseur of this science.

The text of the Dīwān

The Dīwān is a comprehensive treatise on the construction of awfāq, which focuses solely on mathematical analyses and entirely neglects the subject of
their occult properties (khawāṣṣ) or talismanic uses. Because the first folio of the unique manuscript of the Dīwān is missing, the text begins abruptly in a preface, which lists the types of awfāq that will be dealt with in the treatise. The author then briefly explains his method of composition, the meaning of the treatise’s title, states the text’s composition date (517/1123-4), and finally gives an outline of the structure of the Dīwān before it is presented visually in a table of contents that directly follows the preface. The complete text of what remains of the author’s preface is presented here (f. 28r, lines 1-10):

... the simple (sādha) [wafq], then, after that, the ringed (muḥallaq), then the strange (gharīb) wafq, then the adorned (muwashshah) and mixed (muntaziḥ), and after all of them the composite (murakkab) one with another [i.e. composed by joining multiple awfāq]. We conclude it [sc. the Dīwān] with the wondrous solid (mujassam ʿajīb) wafq and the triangular (shakl muthallath) wafq. We point out rarities and discrepancies that appear in the passages [from previous works on awfāq] in which they occur as succinctly as possible without being unfaithful to any one of their sources. Rather, we simplify what is obscure in it [i.e. in each source text] and clarify what is mysterious. We named it The Book of the Collection of Harmonious Number, and this name suits the thing named since my aim in it is to collect all that has been contributed on this [subject] up to our own day, that is the year 517 AH [1123-4 AD], mentioning its ordered parts in simple terms, we arranged it in eight books. The first deals with the required prolegomena (muqaddimāt), the second with the simple wafq that is vertically, horizontally and diagonally [harmonious], the third with the ringed [wafq] that is

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150 This is true for the extant portions of the text. Khawāṣṣ and talismans may, of course, have been discussed in the missing portion at the beginning of the authors preface or in the missing Bābs at the end of Maqāla 8.
vertically, horizontally, diagonally and concentrically (iḥāṭatan) [harmonious], the fourth with the strange wafq, the fifth with the adorned (muwashshah) wafq, the sixth with the mixed glittering (mukhtalat muhamma') [wafq'] from what I have mentioned, the seventh with the solid wafq, and the eighth with the triangular wafq. Each of these books (maqālāt) is comprised of parts (aqsām), and each of these parts of chapters (abwāb) and sections (fuṣūl) as in this table of contents.

The treatise is divided into eight books (maqālāt), each successively subdivided into parts (aqsām), chapters (abwāb) and sections (fuṣūl). Occasionally a further subdivision is used which is not mentioned in the preface or table of contents: individual methods (wujūh) of construction, each given a rubricated heading with description. The general logical structure of the Diwān is as follows.

1 Book (maqāla)
   1.1 Part (qism)
      1.1.1 Chapter (bāb)
         1.1.1.1 Section (faṣl)
            1.1.1.1.1 Method (wajh)

The Diwān' s distinctive table of contents (ff. 28r-29r) is a work of great clarity and efficiency, expertly designed to effectively visualise the intricate hierarchy of its divisions in an easy-to-read tabular fashion. While certainly unusual, this type of table of contents is not unique and, as we shall see below, may help identify or at least contextualise the Diwān' s author.

After the preface printed above and the table of contents (ff. 28r-29r), the Diwān' s eight books are as follows:

Book I: Introductions (ff. 29v-35v);
Book II: The Simple (sādha{j) Wafq (ff. 35v-58v);
Book III: The Bordered (muḥallaq) Wafq (ff. 58v-80v);
Book IV: The Unique Strange Bordered (al-fard al-gharīb al-muḥallaq) Wafq (ff. 80v-93v);
Book V: The Adorned (muwashshah) Wafq (ff. 93v-99r);
Book VI: The Mixed (mumtazi{j) Wafq (ff. 99r-109v);
Book VII: The Solid (mujassam) Wafq (ff. 109v-116v);
The most elaborate and largest wafq in the Dīwān covers a full opening (ff. 108v-109r). It is a composite (murakkab) wafq of 28x28 cells, which contains all the numbers from 1 to 784 and has a magic constant of 10,990. This wafq is composed of sixteen 7x7 awfāq each containing 49 consecutive numbers. Each of these 16 7x7 squares is a bordered (muhallaq) square, so if the outer border of cells is removed a 5x5 wafq is left, and if the outer border of that wafq is removed a 3x3 wafq is left.\(^{151}\) The middle term (i.e. the number in the central cell) of each of the 7x7 (and, thus, 5x5 and 3x3) awfāq has been circled, and these circled middle terms form a 4x4 wafq with a magic constant of 1570. Unfortunately, the accompanying text does not attribute this wafq to a particular author although it may have been extracted from an earlier work, so it is only possible to give this construction *terminus ad quem* date of (517/1123-4), the composition date of Dīwān. Even so, the Dīwān is the earliest known text to describe a wafq of such size and complexity.

![Figure 3.2](image)

As the author the Dīwān says in the preface, he bases his treatise on previous works on awfāq and names explicitly the seven authors of such works without, however, mentioning the titles of their treatises. The composition date (517/1123-4) given in the author’s preface is lent credence by the fact that of these seven authors none lived later than the 6th/12th century. These authors are presented in Table 3.2 in roughly chronological order. Asterisks indicate authors already discussed in the writings of Jacques Sesiano.

<table>
<thead>
<tr>
<th>Author/date</th>
<th>Location(s) of author</th>
<th>Patron(s) of author</th>
<th>Appearance in Dīwān(^{152})</th>
</tr>
</thead>
</table>

\(^{151}\) This property of bordered squares to maintain their harmonious (i.e. mathematically ‘magic’) qualities with the removal of each successive concentric borderer (i.e. the cells of each of the outer rows and columns) is what the author of the Dīwān means when he says in the preface that the bordered square is harmonious ‘concentrically’ (iḥāṣatan, lit. ‘surroundingly’).

\(^{152}\) Included in this column are lists of all folios in BL, Delhi Arabic 110 (a) on which the given author is referred to explicitly or (b) which comprise sections of the Dīwān indicated in their titles or elsewhere as being drawn from the writings of the given author.
<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Location</th>
<th>Title</th>
<th>Dates</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Abū al-Khaṭṭāb al-Mufaḍḍāl b. Thābit al-Ḥarrānī</td>
<td>Baghdad</td>
<td>al-Muhallabī (d. 352/963, ) wazīr to Muʿizz al-Dawla (reg. 334-56/945-67)</td>
<td>3.2 (60v-64r), 4.1 (80v-87r), 5.1 (94v-95v), 5.2 (95v-99r), 6 (102r-109v)</td>
<td>Total: 50 pages</td>
</tr>
<tr>
<td>2 *</td>
<td>Abū al-Wafāʾ al-Būzjānī (d. 388/998)</td>
<td>Bāb al-Tibn Observatory, Baghdad (ca 348/960)</td>
<td>'Aḍud al-Dawla, Șamșām al-Dawla and Sharaf al-Dawla (Būyid amīrs, combined reg. 367-79/978-89)</td>
<td>3.3 (64r-77v), 4.2 (87r-91v), 4.3.1 (91v-92r), 5.1 (94v-95v), 5.2 (95v-99r)</td>
<td>Total: 52 pages</td>
</tr>
<tr>
<td>3 *</td>
<td>Abū ‘Alī al-Ḥasan Ibn al-Haytham (d. ca 430/1039)</td>
<td>Basra (354-ca 390/965-ca 1000); Cairo (from ca 390-430/ca 1000-39, with a gap)</td>
<td>al-Ḥākim bi-Amr Allāh (Fatimid caliph, reg. 386-411/996-1021)</td>
<td>2.1 (35v-40v)</td>
<td>Total: 11 pages</td>
</tr>
<tr>
<td>5 *</td>
<td>Abū Ḥātim al-Muḥaffar b. Ismāʿīl al-Isfizārī (d. ca 510/1116)</td>
<td>Isfahan (from 476/1074-5); Balkh (ca 506/1112)</td>
<td>Jalāl al-Dīn Malikshāh (Saljūq sultan, reg. 465-85/1073-92); Abū al-Muẓaffar Barkiyāruq (Sāljūq sultan, reg. 487-98/1094-1105); Abū Saʿd Jarrāḥ (Khwārizmshāh prince at Balkh)</td>
<td>2.2 (41r-44v)</td>
<td>Total: 8 pages</td>
</tr>
<tr>
<td>6</td>
<td>‘Umar b. Ibrāhīm al-Khayyāmī</td>
<td>Malikshāh Observatory (Isfahan, from</td>
<td>Jalāl al-Dīn Malikshāh (Saljūq sultan, reg.</td>
<td>6.1.2 (107v-109v)</td>
<td>Total: 5 pages</td>
</tr>
</tbody>
</table>
Table 3.2: Authors of *awfāq* treaties mentioned in the *Dīwān al-ʿadad al-wafq* (asterisks indicate authors discussed by Sesiano).

Comparing the list of *awfāq* treatise authors in Table 3.2 with that of the authors discussed by Jacques Sesiano in Table 3.1, we can get a general view of *awfāq* authorities, observe pockets of interest in the mathematical study of *awfāq* and suggest chains of transmission of knowledge in this field. First, it can be seen, perhaps unsurprisingly, that authors of *awfāq* treatises written before the 7th/13th century tended to be astronomer-mathematicians, many of whom were engaged in the production of *zīj*es and employed at observatories under royal patronage. Furthermore, concerning the geographic distribution of these authorities, a concentration is observed in Iraq in the 4th/10th century, with some developments to the West in the 5th/11th century (e.g. the Basran Ibn al-Haytham who spent much of his career in Egypt and the Toledan al-Zarqālī in al-Andalus), followed by a subsequent Eastward movement across Iran into Khurāsān in the first half of the 6th/12th century. This geographic pattern of development may indicate little more than the fact that *awfāq* treatises were produced in the major Islamicate centres of political power and scientific patronage as these shifted to the East, a

153 Al-Zarqālī’s work on *awfāq* should, however, not be considered an isolated Iberian phenomenon any more than his astronomical work should be taken out of the context of the international cultural-scientific scene that had developed in Islamicate Spain during the so-called Orientalisation of Andalusian science in the 3rd/9th-5th/11th centuries. See Julio Samsó, *Las ciencias de los antiguos en el Al-Andalus* (Almería: Fundación Ibn Tufayl de Estudios Árabes, 2011), 45-49 and Julio Samsó, “Ibn al-Zarqālluh,” in *EF*. 

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phenomenon that mirrors the geographic development of mathematics and astronomy more generally. But this Eastward shift of *awfāq* activities in the period directly preceding the composition of the *Diwân* does allow us to speculate on the location, professional and scholarly affiliation and even the identity of *Diwân* author.

V. Defining hubs of *awfāq* activity

In the remainder of this article, we will take a closer look at the two major hubs of activity in the study of *awfāq* that have been identified by the manuscript evidence in Iraq and Iran/Khurāsān. Both of these hubs have been partially highlighted by the work of Sesiano, so we will pay particular attention to *awfāq* authorities newly made known by the *Diwân* and explore their social and scientific contexts. We will work in reverse chronological order so that we begin nearest in time to the author of the *Diwân* and move backwards.

Saljūq Isfahān and Marw

The three latest authorities drawn upon by the *Diwân*, al-Isfizārī, al-Khayyāmī (better known in Europe as ʿUmar Khayyām, although it is doubtful whether the poet and mathematician were the same person) and al-Khāzinī, were well-known astronomer-mathematicians of Saljūq-era Iran and Khurāsān, who were amongst the most brilliant scientists of their day. The few biographical details we have of these three scholars indicate close connections between them. Al-Isfizārī and al-Khayyāmī were colleagues under the patronage of Sultan Malikshāh in the Saljūq capital Isfahan, where they collaborated on astronomical observations documented in the *Zīj-i Malikshāhī* (also known as *Zīj-i Jalālī* and *Zīj-i Khayyām*), which were necessary for calendrical reforms that resulted in the establishment of the Jalālī calendar in 471/1079. Later, al-Isfizārī and al-Khayyāmī are known to have been together in Balkh in 506/1112. By 508/1114, al-Khayyāmī had moved eastward into ‘the religious, cultural, and intellectual heart of the Islamic world’ to Marw, the capital

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of the Great Saljūq sultan Sanjar where he worked under the patronage of that sultan with al-Khāzīnī as his pupil.  

It is not known when or where al-Isfizārī died, but the Saljūq historian al-Bayhaqī (d. 565/1169) relates, probably spuriously, that his death was brought about by the destruction of a complex hydrostatic balance he had constructed for the measurement of density. The balance was designed to use the density of an alloy to ascertain the relative quantities of its constituent metals for the purpose of testing the purity of gold and silver used for currency. A treasurer wishing to keep his adulteration of the currency hidden destroyed this balance, or so the story goes, and when al-Isfizārī learned of its destruction he fell ill and died.  

Regardless of the veracity of this tale, al-Isfizārī’s interest in hydrostatics and mechanics more generally is well attested as is the fact that he did construct a hydrostatic balance, and this is another interest besides astronomy and awfāq that unites al-Isfizārī, al-Khayyāmī and al-Khāzīnī.  

Al-Khāzīnī’s best known writing is the Book of the Balance of Wisdom (Kitāb mīzān al-ḥikma), a comprehensive treatise on the theory of density and hydrostatics and the practical construction and use of hydrostatic balances, which traces developments in these fields from Euclid (fl. 300 BC) and Archimedes (d. 212 BC) down to al-Khāzīnī’s own day. In this work, al-Khāzīnī summarises works by both al-Isfizārī and al-Khayyāmī concerning the hydrostatic balance. This close relationship between these three scholars who had shared interests, personal relationships of patronage and training, wrote on similar subjects and about each other’s work should be considered as something of a school (in the sense of a node in the network of scholars) in which the study of the awfāq played an important role both within the intellectual content of the school’s worldview and also within the output of scholarship.

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159 See most recently the texts edited in Abattouy and al-Hassani, The Corpus of al-Isfizārī.  

presented to their patrons. This Saljūq Isfahānī/Marwazī school specialising in (inter alia) mathematical astronomy, mechanics, and awfāq is of particular interest to us because it seems likely that the author of the Diwān was either a member of this school himself or at least drew much inspiration from it.

The Diwān author

The first indication that the author of the Diwān may have been a member of the Saljūq school and likely a resident of Marw is its composition date: 517/1123-4. When the authorities drawn upon by the Diwān author are arranged chronologically there is, as we have seen, a general Eastward shift in their locations, and the last two or three authors (i.e. those who were contemporaries of the Diwān author) were located at Marw. The Diwān author’s familiarity with the works of these contemporary authors suggests a proximity that allowed their works to be available to him soon after their publication.

Furthermore, there are notable similarities between the works of al-Khāzinī and the Diwān that suggest the strong influence of al-Khāzinī on the Diwān author. First, there is a conceptual similarity between the Diwān and al-Khāzinī’s Balance of Wisdom in that both texts are unusual in approaching theoretical and technical subjects with an eye to charting their historical development, naming previous authorities and quoting from their works. While it is true that the Diwān does not stick to the strictly chronological structure of the Balance of Wisdom, in both works contributions of previous authorities are kept distinct and clearly attributed through explicit citations and clear signposting by means of section headings and diagram captions.

The second similarity between the Diwān and the works of al-Khāzinī is stylistic and has to do with the clear signposting just mentioned. The hierarchal structure of the Diwān is identical that of al-Khāzinī’s Zīj for Sanjar (al-Zīj al-muʿtabir al-Sanjarī) composed at Marw around 532/1137 and dedicated to the sultan Sanjar ibn Malikshāh.161 Not only this, but the Zīj for Sanjar contains a table of contents with the same unusual and distinctive

161 David Pingree ( “A Preliminary Assessment of the Problems of Editing the Zīj al-Sanjarī of al-Khazini,” In Editing Islamic Manuscripts on Science, ed. Yusuf Ibish [London: al-Furqān Islamic Heritage Foundation, 1999], 107) gives the composition date as ca 513/1120. In fact, al-Khāzinī seems to have written his Zīj for Sanjar after 10 Rabīʿ I 532/26 November 1137, which is the latest date mentioned in the chronology of the Saljūq dynasty al-Khāzinī included in the Zīj for Sanjar (see BL, Or. 6669, f. 77v, at foot of table). Pingree does not mention the occasional appearance of wujūḥ in the Zīj for Sanjar, but they can be found in BL, Or. 6669, on e.g. ff. 22r and 46v, https://www.qdl.qa/en/archive/81055/vdc_100029495503.0x000001.
layout as that found in the *Diwan* mentioned above (see p. ???).

Based on this evidence, it is tempting to suggest that the *Diwan* author may have been a close associate of al-Khāzīnī, perhaps his student. An apparent anachronism in the *Diwan*, however, raises some doubts about this. The only author mentioned in the text whose name is routinely followed by the Islamic benediction formula for the deceased (*taḥīm*) is al-Khāzīnī. But al-Khāzīnī is believed to have died only after 532/1137. This means that while al-Khāzīnī was alive for at least a decade and a half following the composition of the *Diwan* in 517/1123-4, the *Diwan* author appears to have believed he had already died by that date. It is hard to square these two facts unless a later copyist in the chain of transmission between the author’s autograph of the *Diwan* and the surviving copy added the *taḥīm* following al-Khāzīnī’s death, but, then, why did other authorities mentioned in the text did not receive the same treatment?264

The *Diwan* author may well be amongst the authors studied by Sesiano and listed in Table 3.1. Since the *Diwan* author would not have listed himself as a previous authority on *awfāq*, he cannot be amongst the authors listed in Table 3.2, so if we remove these authors from the list in Table 3.1, also discarding authorities who lived too early to have written the *Diwan* in 517/1123-4, two authors remain as candidates for authorship of the *Diwan*: (a) the unknown

162 See BL, Or. 6669, ff. 2v, 1r, 1v and 3r and compare with the table of contents in the *Diwan*, Delhi Arabic 110, ff. 28r-29r, https://www.qdl.qa/en/archive/81055/vdc_100046596797.0x000043. Just as in the *Diwan* the sections of the *Zīj* for Sanjar called *wujūh* are not included in the table of contents. See also the similar table of contents in al-Khāzīnī’s *Book of the Balance of Wisdom*, University of Pennsylvania, LJS 386, pp. 17 [=15] and 18 [=16], http://hdl.library.upenn.edu/1017/d/mendren/4824919. A less complex predecessor of the type of table of contents employed in the works of al-Khāzīnī and in the *Diwan* can be seen in Abu Rayḥān Muḥammad b. Ḥabīb al-Bīrūnī’s (d. ca 440/1048) *Canon For Masʿūd (al-Qānūn al-Masʿūdī)*, see BL, Or. 1997, ff. 2r-5r, https://www.qdl.qa/en/archive/81055/vdc_100022880536.0x000001.

163 For further evidence that al-Khāzīnī was still alive in 517/1123-4, see Pingree, “A Preliminary Assessment,” 105, who states that al-Khāzīnī published a *Summary of the Zīj* for the Sultan (*Wajiz al-zīj al-muʿtabir al-suṭṭanī*), an abridgement of his own *Zīj for Sanjar*, in 525/1130-1.

164 A second apparent anachronism in the *Diwan* is that its author follows the first mention of Majd al-Dīn al-Amīr b. Abū Naṣr Maḥṣūr b. Muḥammad b. ʿAlī with benedictions that clearly indicate that the author believed this scholar to be still living (f. 51r). I have not, however, been able to positively identify this scholar, who seems to have been a son of the mathematician and astronomer Abū Naṣr Maḥṣūr b. ʿAlī b. ʿIrāq (d. ca 427/1036), a student of al-Būzjānī and teacher of al-Bīrūnī. If this identification is correct, it would be hard to imagine that Ibn ʿIrāq’s son could have outlived his father by more than 85 years, which would need to have been the case if he were still living at the time of the composition of the *Diwan*. 62
author of the anonymous text edited in Sesiano, “Un compilation arabe” (text 9 in Table 3.1), and (b) Jamāl al-Zamān 'Abd al-Jabbār b. 'Abd al-Jabbār Abū Muḥammad al-Kharaqī (d. 533/1138–9), who flourished at Marw and authored the *Epitome on the Harmonious Number* (*Talkhīṣ fī ʿadad al-wafq*; text 8 in Table 3.1). The anonymous author lists only three *awfāq* authorities in his text, one of which (Ibn al-Haytham) is discussed in the *Dīwān*, but another (al-Anṭākī, see text 1 in Table 3.1 for references) is not mentioned in the *Dīwān*, and the third author appears in the unique manuscript of that text as Abū Ḥātim Muẓaffar al-Isfarāyinī, which Sesiano is correct to emend to Abū Ḥātim Muẓaffar al-Isfarāyinī to Abū Ḥātim Muẓaffar al-Iṣfizārī. If the error in this name was made by the author of the text rather than by a later copyist, then this plus the references to al-Anṭākī, missing from the *Dīwān*, make it unlikely that the unknown author of this text is the *Dīwān* author. As for al-Kharaqī, both the title of his treatise and the *Dīwān* contain the phrase *al-ʿadad al-wafq*, so it might be hoped that the *Epitome of the Harmonious Number* is an epitome of the *Dīwān* (full title: *Collection of Harmonious Number*). Unfortunately, though, a comparison between the contents of the two treatises reveals little other similarity in terms of terminology, structure or subject matter. So, although there is some overlap in terminology and authorities used in the anonymous treatise and in the *Dīwān*, and the date and location of al-Kharaqī match those expected for the *Dīwān* author, the available evidence does not allow us to make a positive identification. It is also equally likely that the *Dīwān* author does not appear in Table 3.1.

**Awfāq in the Renaissance of Islam: Būyid Baghdad**

The other hub of *awfāq* activity is also already known through the work of Sesiano: Būyid Baghdad. There, in the latter half of the 4th/10th century, around the time or shortly after the *Epistles of the Brethren of Purity* were written, Abū al-Qāsim ‘Alī al-Anṭākī (d. 376/987) and Abū al-Wafā’ al-Būzjānī (d. 388/998) were known to have worked and written *awfāq* treatises under the patronage of Aḍud al-Dawla (reg. 338–72/949–83) and subsequent Būyid amirs (see Table 3.1). Just as with the *awfāq* hub in Saljūq Isfahān and Marw, the *Dīwān* introduces us to one more related authority and thereby gives a greater

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165 Süleymaniye, Fatihat 3439, f. 181r, line 25 and Sesiano, “Une compilation arabe,” 138, 168 (translation) and 185 (text). The error in the manuscript is likely due to confusion with the name of the theologian Abū al-Muẓaffar al-Isfarāyinī (d. 471/1078–9).

166 It should be mentioned that the anonymous treatise just discussed also uses the phrase *al-ʿadad al-wafq*, but it is not known if it appeared in the work’s lost title (see Sesiano, “Une compilation arabe,” 173).
sense of there having been a school of *awfāq* thought in Būyid Baghdad.

As mentioned above (p. ??), it is reported by Ibn al-Qiftī on the authority of Abū Ḥālid al-Muhāssin b. Ibrāhīm b. Hilāl al-Ṣābi’ that Thābit b. Qurra authored a treatise on *awfāq*. Yet, this text is not mentioned by any author independent of Ibn al-Qiftī’s reference, and no other trace of the work has been found. The *Diwan* introduces us to the previously unsuspected author of the now earliest known *awfāq* treatise, al-Mufaddal b. Thābit al-Ḥarrānī, providing new details about this author and his work as well as insights into the Harrānian Ṣābian community of Būyid Baghdad.

At first glance, the name al-Mufaddal b. Thābit al-Ḥarrānī appears to belong to one of the sons of Thābit b. Qurra. This identification even appears to be confirmed by the fact that at one point in the *Diwan* we find him mistakenly referred to as al-Mufaddal b. Thābit b. Qurra (f. 80v, title to Book 4, Part 1, see below). In fact, however, this al-Mufaddal is from another Harrānian Ṣābian family in Baghdad descended not from Qurra, but from Zahrūn (for clarification, see Table 3.3). Keeping the two families distinct is not always easy, though, owing not only to intermarriages, but also to the popularity of certain names within the Ṣābian community at Baghdad. Al-Mufaddal’s father, for example, shared not only his *ism* (Thābit) with Thābit b. Qurra, but also the *kunya* Abū al-Ḥasan and the *nisbas* al-Ḥarrānī and al-Ṣābi’. This, combined with Thābit b. Qurra’s family’s reputation as mathematicians makes accidental co-option of al-Mufaddal into the line of Thābit b. Qurra, as we find in one instance in the *Diwan*, all but inevitable.

Figure 3.3: Family tree of the Harrānian Ṣābians of Baghdad (after François C. de Blois, “Ṣābi’,” in *EI* and Muhammad Yonis Abdel All Ridwan, “Abū al-Khattāb al-Mufaddal bin Thābit al-Ṣābi’ wa-mā tabqā min nathrihi: nathr wa-dirāsa,” *Majallat Jami’at al-Malik Saʿūd* 2, al-Ādāb 1 [1990], 39)

The Harrānian Ṣābian identity of al-Mufaddal, the fact that no other mathematical texts are attributed to him and confusion with Thābit b. Qurra within the manuscript of the *Diwan* itself all suggest the possibility that al-Muhāssin b. Ibrāhīm al-Ṣābi’’s list of works by Thābit b. Qurra, which was copied by Ibn al-Qiftī, may have erroneously contained a work actually by al-Mufaddal b. Thābit, al-Muhāssin b. Ibrāhīm’s first cousin once removed (i.e. his great uncle’s son). At any rate, regardless of whether Thābit b. Qurra did indeed author a lost *awfāq* treatise or whether he has merely received credit for a work actually penned by al-Mufaddal b. Thābit, the *Diwan* provides us with fascinating details of what is now the earliest known *awfāq* treatise.
But what do we already know about its Ṣābian author?

VI. The earliest known Āwfaq author

The Ḥarrānians can easily be made into secret agents or scapegoats of intellectual history ... precisely because so little is known about them.  

Abū al-Khaṭṭāb al-Mufaddal b. Thābit b. Ibrāhīm b. Zahrūn al-Ṣābi’ al-Ḥarrānī (b. ca 315/927–8, d. before 368/978–9) is unknown to the history of science and the vast majority of references to him in pre-modern literature concern his poetry. Apart from the discussion of his treatise on āwfaq in the Dīwān, there is no indication in the historical record that he produced scientific literature or had strong interests in the sciences. His family context, however, was certainly conducive to such interests.

Abū al-Khaṭṭāb al-Mufaddal was born into an influential family of Ṣābian physicians at Baghdad, descended from a certain Zahrūn, who lived in the generation of Thābit b. Qurra. Abū al-Khaṭṭāb al-Mufaddal’s father, Abū al-Ḥasan Thābit b. Ibrāhīm b. Zahrūn (d. 365 or 369/976 or 980), was an elite physician in early Būyid Baghdad and an older friend and colleague to Thābit b. Qurra’s grand-son Abū al-Ḥasan Thābit b. Sinān (d. 365/976). Beyond


168 See above, Table 3.2, author 1. The only modern study devoted to Abū al-Khaṭṭāb al-Mufaddal is Rıldan, “Abū al-Khaṭṭāb al-Mufaddal bin Thābit al-Ṣābi’,” in which the fragments of his poetry are assembled and discussed and a list of primary sources mentioning Abū al-Khaṭṭāb al-Mufaddal is given on p. 38, n. 1. See also Daniel Abramovich Chwolson, *Die Ssabier Und Der Ssabismus* (St Petersburg: Buchdruckerei der Kaiserlichen Akademie der Wissenschaften, 1856), Band I, 586.

169 Abū al-Ḥasan Thābit b. Ibrāhīm was likely the character intended by the Mu’tazilite theologian ‘Abd al-Jabbār ibn Ahmad al-Hamadhānī (d. ca 415/1024) when he claimed that Abū l-Ḥasan ibn Zahrūn al-Ṣābi’ al-Ḥarrānī was “the figurehead and chief of medical science in Baghdad” (*wāhid al-tibb bi-Baghdād wa-raʾtsuhu; ‘Abd al-Jabbār ibn Ahmad al-Hamadhānī, *Tahbīt dala’il al-nubūwa*, ed. ʿAbd al-Karim ʿUthmān [Beirut: Dār al-ʿArabiyyah lil-Ṭībāʾah wa-al-Nashr wa-al-Tawzit, 1966], 619; on ‘Abd al-Jabbār see Margaretha T. Heemskerk, “ʿAbd al-Jabbār b. Ahmad al-Hamadhānī,” in E13). The popularity of the kunya Abū al-Ḥasan amongst the Ṣābians of Baghdad seems to have confused historic authors and modern editors alike; for example, Gregor Schwarb, who cites in translation the passage referring to Abū l-Ḥasan ibn Zahrūn al-Ṣābi’ al-Ḥarrānī can hardly be blamed for identifying this character as Thābit
simply practicing medicine, Thābit b. Ibrāhīm was a ‘sage of the falsafa tradition’ (ḥakīm mutafalsīf), who translated Greek texts into Arabic (perhaps via Syriac intermediaries), authored medical writings of his own, and perhaps studied mathematics.\textsuperscript{170}

Abū al-Khaṭṭāb al-Mufaḍḍal is generally overshadowed by his older and more famous cousin Abū Ishāq Ibrāhīm b. Hilāl al-Ṣābi’ (d. 384/994), the great-grandson of Thābit b. Qurra, and his rivalry with his cousin all but defines him in the literature. Abū al-Khaṭṭāb al-Mufaḍḍal and Abū Ishāq Ibrāhīm were amongst the first generation of Harrānian Ṣābians to serve as chancery secretaries (kuttāb diwān al-inshā’) to the ‘Abbāsid and Buyid courts. Abū Ishāq Ibrāhīm found greater political success than his younger cousin, achieving great fame first as secretary to the ‘Abbāsid caliph al-Mu’ti’ (reg. 334-63/946-74), then as Chief Secretary (ṣāhib diwān al-inshā’) to the Būyid amīrs of Iraq from Mu’izz al-Dawla to Bahā’ al-Dawla (combined reg. 324-403/945-1012).\textsuperscript{171} It was perhaps through his cousin’s intercession that

\textsuperscript{170} No mathematical work attributed to Thābit b. Ibrāhīm is known, but his interest in mathematics is suggested by his appearance in a list of the students (talāmīd) of Thābit b. Qurra in a section of al-Nadīm’ s Fihrist (ed. Flügel, Roediger and Müller, vol. 1, 272) devoted to the ‘Class of Modern Geometers and Masters of Mechanics, Numbers and the like’ (Ṭabaqat muḥaddithin min al-muḥandisin wa-ṣāḥib al-ḥiyal wa-l-a‘dād wa-ghayrihī). It is, however, chronologically difficult to imagine that Thābit b. Ibrāhīm studied with Thābit b. Qurra in person, since the former is thought to have been no older than 8 when the latter died. Thābit b. Ibrāhīm’s propensities for mathematics are reiterated by al-Bayhaqī, Ta’rikh hukamā’ al-Islām, 90 (trans. in Meyerhof, ‘Ali al-Bayhaqī’s Tatimmat Siwān al-Hikma,” 152), who also calls him a ‘sage of the falsafa tradition’ (ḥakīm mutafalsīf).

\textsuperscript{171} For a précis of Abū Ishāq Ibrāhīm’s political career and his role in the creation of Buyid dynastic propaganda, see Wilferd Madelung, “Abū Ishāq al-Ṣābi on the Alids of
Abū al-Khaṭṭāb al-Mufaddal entered the circle and patronage of Abū Muhammad al-Muhallabī (d. 352/963), vizier to Muʿizz al-Dawla (reg. 324-56/936-67), where he consorted with some of the greatest minds of his day. Although nothing is known of Abū al-Khaṭṭāb al-Mufaddal’s scientific pursuits apart from his interest in awfāq, some more can be said about those of his cousin Abū Iṣḥāq Ibrāhīm, and these may at least be indicative of Abū al-Khaṭṭāb’s own interests.

Although best remembered as the consummate secretary, master of adab, prose stylist and poet, Abū Iṣḥāq Ibrāhīm began his career, following family tradition, as a hospital physician before abandoning this path owing to a distaste for medicine. Throughout his subsequent secretarial career, however, he kept up his scientific activities in the other family specialisms of mathematics and astronomy. His correspondence concerning questions of geometry and mechanics with the mathematician and astronomer Abū Sahl al-Kūhī (d. ca 384/995) - himself an associate of the awfāq author Abū al-Wafā’ al-Būzjānī at the Baghdad observatory of Sharaf al-Dawla - are partially extant, while the book of astronomical tables (zīj) and treatise on geometry he produced for the Būyid amīr ‘Aḍud al-Dawla have not survived.


172 On the circle of al-Muhallabī, see Joel L. Kraemer, *Humanism in the Renaissance of Islam. The Cultural Revival during the Buyid Age* (Leiden - New York - Köln: E.J. Brill, 1992), 54-55. One of the only two anecdotes about Abū Khattab al-Mufaddal not relating to his poetry - both preserved by Abū Ḥayyān al-Tawḥīdī (d. 414/1023) - is set in al-Muhallabī’s majlis. In this anecdote we find a snippet of a conversation between Abū Khattab al-Mufaddal and his cousin in which Abū Khattab asks Abū Iṣḥāq about the physiological and temperamental basis for differences in thought and opinion. In the second anecdote we find Abū Khattab al-Mufaddal in discussion with Abū Sulaymān al-Sijistānī (d. 375/985), a philosopher of the so-called Baghdad school of Aristotelians around the Jacobite philosopher Yaḥyā b. ‘Adī (d. 363/974) and approving of Abū Sulaymān’s reformulations of Sufi sentiments in philosophical language. For both anecdotes, see Kraemer, *Philosophy in the Renaissance of Islam*, 75-76.


Abū Isḥāq Ibrāhīm was also a noted maker of scientific instruments. He is known to have constructed an astrolabe for Qābūs b. Wushmgīr the Ziyārid ruler of Tabaristān and Gurgān (reg. 367–71/978–81 and 387–402/997–1012), and he combined his skills in mathematical astronomy with his poetic talents in the miniature astrolabes he constructed for Ḍudud al-Dawla and his son Ṣamsām al-Dawla (reg. 372–88/983–98), both of which were presented with accompanying poems.  

Alexandre M. Roberts has recently argued convincingly that much of Abū Isḥāq Ibrāhīm’s scientific output can be viewed at least in part as a conscious attempt to emphasise the distinctive attributes of his Ṣābian religious minority status (viz. special mastery of mathematical and astronomical/astrological arts) in the hope of increasing his own cachet and cultural capital. A similar overt expression of Abū Isḥāq Ibrāhīm’s Ṣābianism can be seen in an anecdote about a dinner at the house of the vizier Abū Muḥammad al-Muhallabī, at which Abū Isḥāq refused to eat owing to the presence on the table of fava beans, which were forbidden by Ṣābian dietary laws as they were by the Pythagorean symbolon a fabis abstine (κ ναμο ὐπέχεσθαι).  

Such a desire to highlight his Ṣābian uniqueness may well have formed part of Abū al-Khaṭṭāb al-Mufād’dal’s own motivation to write his treatise on awfāq. At least since the appearance of the Epistle On Geometry in the Epistles of the Brethren of Purity (2.26) in the mid-4th/10th century, the talismanic functioning of the awfāq had been considered part of astral magic. Therefore, by their mathematical and astral characteristics awfāq could well have been expected to fall within the realm of Ṣābian expertise. The awfāq were - to put it another way - part of what Roberts has called the ‘Ṣābian brand’, regardless of the fact that they had not originated with the Ṣābians.
Abū al-Khaṭṭāb al-Mufāḍdal b. Thābit al-Ṣābi’ al-Ḥarrānī in the Diwān

From a historiographical point of view, the most valuable and unusual aspect of the Diwān is that its author explicitly names his sources and indicates clearly which sections of the Diwān are attributable to which authority. Judging by the wide range of subjects covered by the Diwān that are attributed to al-Mufāḍdal, we may assume that his lost treatise on awfāq was not a brief work. These subjects include the bordered (muḥallaq) wafq (ff. 60v-64r), the unique, strange bordered (al-fard al-gharib al-muḥallaq) wafq (ff. 80v-87r), the adorned (muwashshaḥ) wafq (ff. 94r-95v), a construction technique called the balanced mixture (al-mizāj al-muʿtadil) (ff. 95v-99r), and awfāq of mixed (mumtazji) composition (f. 102r-109v).

After mentioning al-Mufāḍdal b. Thābit’s adorned wafq and balanced mixture, the author of the Diwān claims that Abū al-Wafā’ al-Būzjānī ‘alluded to’ (āshāra ilayhi) the adorned wafq and ‘presented in his book’ (awradahu ... fī kitābihi) the balance mixture. The Diwān author does this in such a way as to suggest, without explicitly stating as much, that al-Mufāḍdal b. Thābit’s treatise on awfāq predated al-Būzjānī’s. This is not unlikely given the age difference between the two authorities (al-Būzjānī was born ca 328/940 and al-Mufāḍdal was born ca 315/927).

The preface to Maqāla 3, Qism 2 of the Diwān (f. 60v, lines 1-12) contains a long quotation from Abū al-Khaṭṭāb al-Mufāḍdal’s treatise in which he describes his own autodidactic initiation into awfāq studies and the books he consulted along the way. This passage is especially noteworthy since it preserves the earliest known awfāq author discussing his predecessors in the field. It is thus the earliest known source for the history of mathematical interests in awfāq. The fact that this passage is historical rather than legendary in character sets it apart from other known Arabic discussions of the origins of awfāq, which, as we have seen, attribute their discovery and transmission to legendary figures who had nothing to do with them such as

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179 This section contains a wafq of 19x19 cells (f. 86v), the largest in any of the sections of the Diwān attributed to Abū al-Khaṭṭāb al-Mufāḍdal, and indeed one of the largest to appear in the Diwān or in other early awfāq literature.

180 See Behnaz Hashemipour, “Būzjānī: Abū al-Wafā’ Muḥammad ibn Muḥammad ibn Yaḥyā al-Būzjānī,” in BEA, and Ulrich Rebstock, “Abū 1-Wafā’ al-Būzjānī,” in EF. Al-Būzjānī’s only surviving work on awfāq, the Book on the Arrangement of the Harmonious Number in Squares (Kitāb fī tartīb al-ʿadad al-wafq fī al-marabbaʿat), has been studied and partially edited and translated by Sesiano, Magic Squares in the Tenth Century, 207-52 (translation), 335-81 (text) and Sesiano, “Le Traité d’ Abū 1-Wafā’ sur les carrés magiques, who has not, however, given any indication that it contains references to Abū al-Khaṭṭāb al-Mufāḍdal or, for that matter, to any other previous authorities.
Fereydūn and Thales. This passage is full of details about Abū al-Khaṭṭāb al-Mufaddal’s context, resources and hermeneutic methodology, as well as a number of obscure references and phrases, so it is worth transcribing and translating in full here.

Part Two of the Bordered Wafq: Summary of what Mufaddal b. Thābit al-Harrānī presented by way of narrative, without proofs

[1] He said: ‘My knowledge of numbers in harmony began with the grid of three (jadwal al-thalātha) mentioned by Nicomachus [of Gerasa] in the Arithmetic. [2] Then Abū al-Qāsim al-Hijāzī came upon the grid of four (jadwal al-arbaʿa), which begins from one and increases (yatafāḍal) one by one to sixteen and he was amazed by it. [3] Then I found the grid of six (jadwal al-sitta) on the flyleaf (ẓahr al-kitāb)184 of Isḥāq’s text185 of the book of

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181 | 182 | 183 | 184 | 185
---|---|---|---|---
الأرض | scripsi: | cod. | الأرضية | scripsi: | cod.
الحس | scripsi: | cod. | الحسن | تصنيفه | scripsi: | cod.
184 The ẓahr al-kitāb is the ‘recto of the first folio’ of a manuscript, often used as a flyleaf or title page - a typical location for owners’ inscriptions, seal impressions and other notes. See Adam Gacek, “Ownership Statements and Seals in Arabic Manuscripts,”
Euclid. [4] Next, I came upon a book in which were three or four of the grids below [order] ten. [5] Then, in a storehouse of the books of our elders, I found two books so destroyed by termites that only a little of each of them could be made out. [6] The shorter (mukhtāṣar, lit. ‘abridgement, summary’) of the two was by the hand of al-Мāhānī, and the first folio of the longer was by the hand of al-Hasan b. Mūsā al-Nawbakhtī. [7] When I read them, I found comprehending them extremely fatiguing (fa-wajadtu istikhrājahumā mu’tiban jiddan), 186 [8] but it occurred to me that it was possible to derive some of what was damaged in each one of them from what had survived in the other and from the fact that since they deal with the same subject, the same terminology is used in the same ways so that the collation can be confirmed. [9] So I began this [task], asking for God’s assistance in it. [10] Al-Khāzinī said: ‘His [sc. Abū al-Khaṭṭāb al-Mufaddal’s] work contains the whole of what we have summarised by way of instruction [i.e. algorithm?] and tabulation (mu‘āmaratan wa-jadwalan)’.

The first thing to note is that it is not the custom in the Dīwān for qāla (‘he said’) to introduce the authorial voice, which is typically introduced in this treatise by the 1st person plural nadhkuru/dhakarnā (‘we will mention/we mentioned’). The use of qāla to introduce this passage thus indicates a quotation or paraphrase of a voice other than that of the author, namely that of Abū al-Khaṭṭāb al-Mufaddal. 187 It is also linguistically

185 Reading nushkati Išhāq (‘of Išhāq’s text’) rather than nasakhahu Išhāq (‘copied by Išhāq’). Although the points on the presumed tā’ marbūta of نسخة are not present in the manuscript, I prefer this reading for reasons mentioned below and because properly speaking the relative pronoun allādhī would be needed after the definite kitāb Ūqlīdis if ‘the Book of Euclid copied by Išhāq’ were meant.
186 Cf. the saying استخراج المفهوم من فونكت شعاع (‘the elicitng of the meaning of that which is made enigmatical is a cause of fatigue to minds’) transmitted by al-Zamakhsharī (d. 538/1144: Asās, s.v. التعب, cited in Edward William Lane, An Arabic-English Lexicon, Derived from the Best and Most Copious Eastern Sources… [Beirut: Librarie du Liban, 1863], s.v. استخراج and التعب. The translation is Lane’s.
187 Cf. The beginning of Maqāla 4, Qism 1 of the Dīwān, ‘On the unique, strange bordered
significant that al-Mufaddal uses the term ‘harmonious number’ (al-ʿadad al-wafq) to refer to the subject of his treatise: the ‘magic constant’, which stands synecdochically for the awfāq themselves (i.e. for the harmonious arrangements of the numbers within their grids [jadāwil]) and, by extension, for the science that seeks to understand these arrangements. This same term is found in the titles of the earliest known awfāq treatises such as that attributed to Thābit b. Qurra and that by al-Bûzjâni.\textsuperscript{188}

Next we should note that the passage takes the form of a somewhat stylised discovery narrative, so we must be aware that Abū al-Khaṭṭāb al-Mufaddal is not simply recording a bibliography that can be taken at face value.\textsuperscript{189} A final general observation is that, although Abū al-Khaṭṭāb al-Mufaddal was himself a Ṣābian, the authorities from which he draws his knowledge of the awfāq are not Ṣābians, but rather a Graeco-Roman Pythagorean, a Nestorian Christian, a Persian Shīʿī Muslim, and Arab Sunnis. Far from indicating that the awfāq were initially secret knowledge and the special preserve of Ṣābian scholars, Abū al-Khaṭṭāb al-Mufaddal’s sources suggest that interest in awfāq was widespread and, more generally, are illustrative of the cosmopolitan and collaborative intellectual scene at ‘Abbāsid/Būyid Baghdad and of the Ṣābian engagement with that scene.

Turning to the details of al-Mufaddal’s sources, it is curious to see that the first is the Arithmetical Introduction by the Neopythagorean philosopher and mathematician Nicomachus of Gerasa (d. ca 120), in which al-Mufaddal claims to have found the 4x4 wafq. The text of the Arithmetical Introduction survives both in the original Greek and in an Arabic translation by Thābit b. Qurra, but no mention of awfāq is found in either of these texts.\textsuperscript{190}

\footnotesize[\textsuperscript{188}On this term see \textsuperscript{n. 91}, above. For examples of titles containing this term, see Tables 3.1 and 3.2.\textsuperscript{189} The idea, for example, that Abū al-Khaṭṭāb al-Mufaddal came upon the awfāq in more or less consecutive order, learning first of the 3x3 wafq, then the 4x4 etc., is surely a literary conceit. Charles Burnett has drawn my attention to a comparable discovery narrative related by al-Kindī, TEXT (DATE), PAGE, LINE concerning the ‘ad, in which instruments with a consecutively increasing number of strings were invented in chronological order.\textsuperscript{190} The Greek text is edited in Nicomachi Geraseni Pythagorei Introductionis arithmeticae libri II, ed. Richard Gottfried Hoche (Leipzig: Teubner, 1866), and translated into English in Nicomachus of Gerasa, Introduction to Arithmetic, trans. Martin Luther D’Ooge (New York: Macmillan Co., 1920). The Arabic version is edited in Ṣābit B. Qurra’s arabische Übersetzung der ‘Arithmetik eisagōgê’ des Nikomachos von Gerasa, ed. Wilhelm Kutsch (Beyrouth: Imprimerie Catholique, 1959). Kutsch’s text, without apparatus, is available in “A Digital Corpus for Graeco-Arabic Studies,”]
Although the Arithmetical Introduction itself does not discuss awfāq, the third and only extant chapter of a Commentary on the Arithmetical [Introduction] (Kitāb tafsīr al-Arithmāṭiq) by al-Mufaddal’s contemporary Abū al-Qāsim ‘Alī al-Anṭākī (d. 376/987) is in part devoted to awfāq, and not just the 3x3 wafq, but awfāq 3x3-12x12. Even stranger is the fact that there is no discernible connection between Nicomachus’ text and al-Anṭākī’s commentary. Why both al-Mufaddal and al-Anṭākī relate the Arithmetical Introduction to awfāq is a still a mystery, but if Vinel is correct in his claim that there are veiled allusions to magic squares in Iamblichus’ commentary on the Arithmetical Introduction (see above, p. ), perhaps the tradition of that commentary had some influence on Arabic authors of the 4th/10th century.

The next authority al-Mufaddal adduces is one Abū al-Qāsim al-Hijāzī. I have not found any mathematical texts attributed to an author of this name. He may, however, be identifiable with the author of the Supplementary History (al-Tāʾrīkh al-mulḥaq), who is also known as the copyist of a copy of the Book of Internal News (Kitāb al-akhbār al-dākhila) seen by al-Mufaddal’s contemporary, the bibliographer Ibn al-Nadīm.

The next source is even more obscure. If I am correct in reading ‘Iṣḥāq’ s text of the book of Euclid’, this is likely a reference to Iṣḥāq b. Ḥunayn’s (d. ca 298/910) translation of Euclid’s (fl. 300 BC) Elements. Al-Mufaddal does not, however, specify the identity of the author of the awfāq he saw on the flyleaf of that book.

Sentences 5–9 give us our best insight into Abū al-Khaṭṭāb al-Mufaddal’s world. Sentence 5 tells us that an ancestral library at Baghdad contained at least two manuscripts on awfāq, but who did al-Mufaddal refer to as ‘our elders’ (shuyūkhunā)? With whom was he identifying; whom did he intend when he wrote ‘our’? It is tempting to think he meant the elders of the Sabian community at Baghdad, and this may be the case, but there could be other explanations. For example, he may have been referring to the elders


191 See Sesiano, Magic Squares in the Tenth Century, 15.
some other group of which he felt a part, such as the majlis around the vizier al-Muhallabī. Sentence 6 identifies the scribes, but not the authors, of the two manuscripts he found in this ancestral library, the shorter of which is said to be an abridgement (mukhtasār) of the longer. The two scribe’s are illustrious characters from late third/ninth-century intellectual history in their own rights: the abridgement was copied by the mathematician Abū 'Abd Allāh Muḥammad ibn 'Īsā al-Māhānī (d. ca 275/888), while the longer manuscript was copied by the Shī‘ī mu‘tazilite mutakallim al-Ḥasan b. Mūsā al-Nawbakhtī (d. between 300/912 and 310/922), descendant of the Persian astrologer Nawbakht who assisted in the calculations for the election of the most propitious moment for the founding of Baghdad in 145/762, and older associate of such translators of Greek texts as Isḥāq b. Hunayn and Thābit b. Qurra. 194

The description in sentences 5 and 9 of the method of collation used by al-Mufaddal in order to reconstruct the text of two badly insect damaged manuscripts shows a rigorous scholarly approach to manuscript research and textual criticism akin to that demonstrated by the risāla of the celebrated translator Hunayn b. Isḥāq (d. 264/873) on his translations of the works of Galen. 195 Finally, in sentence 10, the author of the Dīwān states that al-Mufaddal’s treatise on awfāq was discussed with approval by al-Khāzinī in his own lost awfāq treatise.

The veracity and exact details of Abū al-Khaṭṭāb al-Mufaddal’s statement about his personal initiation into the science of awfāq must await future research. For the moment, though, it appears to demonstrate that the science of awfāq already had a developed literature in Arabic and a diverse set of engaged researchers, including at least one Harrānian Ṣabian of Baghdad, by about the mid-4th/10th century and that this research and literature stretched back to the later 3rd/9th century.

in mind the numerous criticisms of Tardieu’s understanding of the Ṣabians, usefully summarised in Van Bladel, The Arabic Hermes, 69-77.


Conclusion

Awfāq were believed to have wondrous healing powers from their earliest appearance in Islamicate literature. By the time of the earliest extant mathematical treatises devoted to awfāq were written in the mid-4th/10th century, the occult properties (khawāṣṣ) of the awfāq that produced their occult properties had already been associated with astral forces.

Two major concentrations of state-sponsored research into awfāq are clearly are visible in the historical record. The first is in fourth/ninth-century Baghdad during the so-called Renaissance of Islam, especially under the patronage of the Būyid amīr ʿAuḍud al-Dawla. The second is in the late-5th/11th and early-6th/12th centuries under the patronage of the Great Saljūqs, first at Sultan Malikshāh’s capital, Isfahān, and then at Marw, the capital of his son, the Sultan Sanjar. Both of these efflorescences of awfāq literature appear at locations and times already known as sites of so-called ‘cultural revival’. In practice they are situated at the sites of state-sponsored large-scale astronomical observation and zīj production.

On the one hand, this says little, since state funded astronomer-mathematicians would probably have accounted for the majority of people at any given moment who had the requisite mathematical knowledge and training to engage significantly with the mathematics of awfāq. Indeed, most of the known Būyid and Saljūq awfāq authors were state-sponsored astronomer-mathematicians involved in both observations and zīj production. On the other hand, the case of Abū al-Khaṭṭāb al-Mufaddal offers an example of serious mathematical engagement with the awfāq by a figure not known to have been directly involved in astronomical work. His prominent place in the Būyid court, however, still shows that research into the awfāq was, like astrology and talisman making, of central importance to the ruling classes of the day. Furthermore, the fragment of Abū al-Khaṭṭāb al-Mufaddal’s lost treatise preserved in the Dīwān gives evidence that Arabic writings on the awfāq already existed by the late 3rd/9th century.

At least by the time of al-Zarqālī in the latter half of the 5th/11th century and very likely before, the first seven awfāq (3x3-9x9) had become astral talismans in their own right. By the early 6th/12th century, mathematical procedures were well understood that allowed names and other words and phrases to be entered into and manipulated within awfāq. These centuries of development prepared the necessary technology for the boom in talismanic applications of awfāq in the number/letter magic traditions grouped under the rubric of the Science of Letters (ʿilm al-ḥurūf), which rose to great
popularity in the 7th/13th century under the influence of the Sufi-occultist synthesis spearheaded by Ibn ʿArabī and al-Būnī. We must uncover the early history of the Science of Letters to see its affiliations, patrons, detractors and developments. Only then will we know what was inherited by its later masters, codifiers and popularisers in the 7th/13th century and after.

Abbreviations

Anon. CBL = Anonymous awfāq treatise [Arabic], Chester Beatty Library, Ar. 5087.
BL = London, British Library.
CBL = Chester Beatty Library
HR = History of Religions.

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**al-Ḥayyān. Kitāb al-khawāṣṣ al-kabīr.** BL, Or. 4041.


**Jābir b. Ḥayyān. Kitāb al-khawāṣṣ al-kabīr.** BL, Delhi Arabic 110, ff. 28a-r-119v.


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