A Review of Revenue Management: Recent Generalizations and Advances in Industry Applications

Robert Klein\textsuperscript{a}, Sebastian Koch\textsuperscript{a}, Claudius Steinhardt\textsuperscript{b}, Arne K. Strauss\textsuperscript{c,*}

\textsuperscript{a}Universität Augsburg, 86135 Augsburg, Germany
\textsuperscript{b}Universität der Bundeswehr München, 85577 Neubiberg, Germany
\textsuperscript{c}University of Warwick, Coventry CV4 7AL, United Kingdom

Abstract

Originating from passenger air transport, revenue management has evolved into a general and indispensable methodological framework over the last decades, comprising techniques to manage demand actively and to further improve companies’ profits in many different industries. This article is the second and final part of a paper series surveying the scientific developments and achievements in revenue management over the past 15 years. The first part focused on the general methodological advances regarding choice-based theory and methods of availability control over time. In this second part, we discuss some of the most important generalizations of the standard revenue management setting: product innovations (opaque products and flexible products), upgrading, overbooking, personalization, and risk-aversion. Furthermore, to demonstrate the broad use of revenue management, we survey important industry applications beyond passenger air transportation that have received scientific attention over the years, covering air cargo, hotel, car rental, attended home delivery, and manufacturing. We work out the specific revenue management-related challenges of each industry and portray the key contributions from the literature. We conclude the paper with some directions for future research.

Keywords: Revenue Management, Industry Applications, Availability Control, Capacity Control

*Corresponding author

Email addresses: robert.klein@wiwi.uni-augsburg.de (Robert Klein), sebastian.koch@wiwi.uni-augsburg.de (Sebastian Koch), claudius.steinhardt@unibw.de (Claudius Steinhardt), arne.strauss@wbs.ac.uk (Arne K. Strauss)
1. Introduction

The original purpose of revenue management (RM) – as it evolved in passenger air transport in the 1970s after the deregulation of the US airline market – was the task of optimally selling a fixed and perishable inventory within a given time horizon. More precisely, having defined products based on a set of common services using price differentiation (e.g., standard and saver fares in case of airline tickets), the idea was to dynamically control the availability of these products over time to maximize overall obtainable revenues (or profits).

Until today, this concept lies at the heart of many RM systems. In general, RM research incorporates work on increasingly sophisticated approaches to model, estimate and forecast demand, as well as to optimize subsequent demand management decisions with a high level of automation. The introduction of such automated systems led to significant revenue improvements in various industries as illustrated by some finalists in the prestigious INFORMS Franz Edelman award competition. For example, the award in 2017 went to Holiday Retirement, a large senior housing operator in the United States. As reported by Kuyumcu et al. (2018), implementation of a RM system increased revenue from new rentals by approximately 9% (corresponding to $88m). The finalists in 2018 included Europcar with their implementation of an integrated RM system that they claim has led to $584m increase in revenue.

The paper on hand is the second and final part of a series surveying the theoretical and practical developments of RM as they are documented by scientific publications in the field. The first part of the survey, Strauss et al. (2018), is dedicated to methodological advances in choice-based RM, where we dynamically control the availability of products under consideration of substitution effects. This second part gives an overview of various extensions to the standard RM setting and of advances in industrial applications. Again, we focus on ‘quantity-based’ RM, as opposed to ‘price-based’ RM, where demand is influenced by changing prices for products, and include scientific contributions that have been made between 2004 and 2018, i.e., since the seminal book by Talluri and Van Ryzin (2004).

Using the term “revenue management” in the standard search of Scopus™ delivers 2,114 results for the time range given above. Clearly, we can only present a subset of the existing generalizations and applications. To select these, we have restricted the discussion to topics which, in our opinion, are either representative for a particular class of generalizations and applications, or which will have a considerable impact on the future development
and use of RM. Still, some further restrictions have been necessary. For example, combining the term “revenue management” with the terms “overbooking” or “hotel” leads to 105 and 303 results in Scopus™, respectively. Hence, this paper builds on the one by Strauss et al. (2018) and aims at elaborating the challenges from a modeling and methodological point of view when going beyond the traditional setting, in particular, in choice-based RM. Again, we prioritize decision making on an operational level for our presentation. The basic idea is to give researchers and practitioners a reference point concerning models and methods when they want to address new generalizations or applications instead of giving an all-encompassing overview.

The paper is structured as follows: In §2 we state the standard (network) RM framework using a stochastic dynamic program (DP) and we introduce essential notation. Furthermore, we briefly describe how the resulting control problems can be solved. §3 builds upon this foundation and is devoted to generalizations. All the generalizations have in common that they require to be adapted to the basic framework. Beyond overbooking, which is the oldest generalization, they are induced by new technological capabilities allowing more flexible services, by new business models, or by an alternative objective. For each of the generalizations, we give a detailed description and analyze in which way it generalizes the standard setting. For this purpose, we reconsider the DP and discuss how to adapt it appropriately. Furthermore, we portray the evolvement in the recent scientific literature. We then discuss today’s most prominent industrial applications of RM including traditional ones (§4) as well as emerging non-traditional ones (§5). However, as it turns out, these applications lead to control problems which are less structured than the generalizations described in §3 and cannot easily be fitted into the standard framework in all cases. Therefore, we no longer state the DP in all cases but restrict to more general descriptions. Where possible, we show how the applications presented relate to the generalizations presented in §3. Regarding the non-traditional applications, we consider two representatives of online-to-offline services (attended home delivery and manufacturing). As it turns out, those applications share the commonality that some scheduling of the services is required. For each industry application, we analyze the specific challenges and recent scientific contributions. In §6 we conclude the review with an outlook on future research opportunities.

Our paper complements other general reviews that have appeared since 2004, namely Chiang et al. (2007), Shen and Su (2007), and Weatherford and Ratliff (2010).
Chiang et al. (2007) elaborate on only three industries: cargo and freight, Internet services, and retailing. Other industries are covered by brief tabular overviews, referring to relevant literature and giving a description of similar practices “in one sentence”. Shen and Su (2007) exclusively concentrate on methodological contributions concerning different types of customer behavior. Likewise, Weatherford and Ratliff (2010) focus on models and methods under dependent demand exclusively for the airline industry. They do not consider any generalizations. Note that there exist some other surveys solely focusing on specific generalizations or industries that we also cover, such as, e.g., Göensch (2017) who focuses on risk-based RM. We will discuss such literature later on in the corresponding sections.

2. The (network) revenue management problem

Most research in the area of RM deals with controlling the sales process at an operational level. We introduce one of the most frequently used methods to frame such problems, namely a stochastic dynamic program (DP). After a general description of the corresponding control problem in 2.1, we formally describe the two basic types of control in 2.2 and 2.3 which differ concerning the assumption on the behavior of customers and introduce some essential notation. 2.4 sketches methods for solving the DPs obtained.

2.1. General description

We consider a firm that sells products to customers with heterogeneous preferences. Products usually correspond to services and are linked to some sales restrictions or other conditions to segment the market. The prices of the products are fixed. The customers' demand is stochastic and materializes over time during a selling horizon, which is also known as booking horizon.

Product provision consumes certain resources, and some products may require more than one type of resource. Resources may have both a physical and a temporal dimension. Most commonly, the service is provided after the selling horizon. For example, in passenger air transport, a single day of service is considered (temporal resource dimension). A resource refers to each compartment (physical resource dimension) on each point-to-point flight of the considered departure day, with capacity equal to the number of seats in the compartment. A product is a ticket for the desired itinerary in a particular compartment and is associated with a specific fare, a booking class, as well as a subset of resources (i.e., flights) from which one capacity unit (i.e., one seat) is used. The selling horizon typically spans a few months before the departure day.
Facing such a situation, the firm actively manages demand on an operational level by using RM techniques to maximize the overall expected profit. The term revenue management stems from the fact that, in passenger air transport, both variable and fixed costs are barely under operational control. Since in passenger air transport the itineraries offered are often combinations of several point-to-point flights, the term ‘network’ was added in the past to distinguish this case from considering a single connection only. Today, this term is used to describe the supply side for settings with several resources or for settings that consider multiple service periods, like in the hotel or car rental industry.

On the demand side, we differentiate between two classes of demand models. In the first class, we assume that each customer segment considers a set of products alternatives for purchase. The purchase decision is the outcome of a choice process among all the offered products that they consider for purchase. Over the past 15 years, the term ‘choice-based RM’ has been established to represent this context. Choice-based RM may offer better representation of demand but comes with greater challenges in estimating models of customer choice as well as in optimizing decision policies.

In the second setting, customers consider the purchase of a specific product only and, if this is not available, will not buy at all. Here, the term ‘independent demand’ is common, because the demand for a product is independent of the availability of others. The independent demand assumption is reasonable for quasi-monopolistic applications with strongly differentiated products and customers. Initially, the airline industry applied corresponding models and to some extent still does. Thus, much research on generalizations and industrial applications refers to this setting.

In the context of choice-based RM, Strauss et al. (2018) refer to ‘availability control’ as deciding on which set of products to offer. In contrast, they connect the terms ‘capacity control’ and ‘inventory control’ with the independent demand case. We follow this distinction although there is some ambiguity around how literature is using these terms.

2.2. Availability control

Availability control consists of varying the set of offered products over time and takes multi-product substitution into account. The optimal control policy for the resulting choice-based network RM problem is, in theory, obtained by solving a DP. Let $\mathcal{J} = \{1, ..., J\}$ and $\mathcal{H} = \{1, ..., H\}$ be the sets of products and resources, respectively. Products’ resource consumptions are defined in matrix $A = (a_{hj})_{h \in \mathcal{H}, j \in \mathcal{J}}$, with $a_{hj}$ denoting the number of capacity units product $j$ consumes from resource $h$. The $j$-th column vector $a_j$
describes the overall resource consumption of product \( j \). Each product \( j \in J \) is sold at a fixed revenue (or contribution margin) \( r_j \). Products are sold over a finite selling horizon, discretized into \( \mathcal{T} = \{1, \ldots, T\} \) time periods such that the probability for more than one customer arrival per period is negligible. Periods are numbered forward in time, and the service provision is planned in \( t = T + 1 \). The customers’ arrival probability is assumed to be fixed at \( \lambda \). At the beginning of period \( t \), a state equals the company’s available inventory given by the vector \( c_t = (c_{ht})_{h \in \mathcal{H}} \), where \( c_{ht} \) denotes the remaining capacity of resource \( h \). Thus, the state space is denoted by \( \mathcal{C} = \{c_t \in \mathbb{Z}_+^\mathcal{H} : c_{ht} \in \{0, 1, \ldots, c_{h1}\}\} \). In every time period \( t \) of the selling horizon, the company has to decide on the subset \( S_t \subseteq J \) of products to offer for sale. We assume that technically any set from the power set of all products can be offered. This defines the action or decision space, with the literature on DPs using both terms. Note that the action space may be constrained to immediately exclude infeasible offer sets given the current state \( c_t \) (see Strauss et al. 2018). Furthermore, for some applications additional restrictions may arise such as constraints on the cardinality of the offer set.

As stated in the previous section §2.1, we assume that customers have heterogenous preferences concerning the products. Since the company does not fully know these preferences, it will use some choice model to estimate the probability \( P_j(S_t) \) for selling product \( j \) depending on the offer set \( S_t \) (given a customer arrival). Thus, the probability that product \( j \) is sold is \( \lambda P_j(S_t) \). The probability for not selling any product covers the case that there is a customer arrival, but the customers decides to leave without purchase (i.e., \( \lambda P_0(S_t) \)), as well as the case that there is no arrival (i.e., \( 1 - \lambda \)), and it holds that \( \lambda P_0(S_t) + 1 - \lambda = 1 - \sum_{j \in S_t} \lambda P_j(S_t) \). For an overview on different choice models, we again refer to Strauss et al. (2018).

With the notation at hand, we can formulate the DP. Let the value function \( V_t(c_t) \) denote the optimal expected revenue-to-go in period \( t \) with capacity \( c_t \) and let \( \Delta_j V_{t+1}(c_t) = V_{t+1}(c_t) - V_{t+1}(c_t - a_j) \) be the opportunity cost associated with selling product \( j \). Then, \( V_t(c_t) \) satisfies the Bellman equation

\[
V_t(c_t) = \max_{S_t \subseteq J} \left\{ \sum_{j \in S_t} \lambda P_j(S_t)(r_j + V_{t+1}(c_t - a_j)) + \left(1 - \sum_{j \in S_t} \lambda P_j(S_t)\right)V_{t+1}(c_t) \right\}
\]

\[
= \max_{S_t \subseteq J} \left\{ \sum_{j \in S_t} \lambda P_j(S_t)(r_j - \Delta_j V_{t+1}(c_t)) \right\} + V_{t+1}(c_t) \quad \forall c_t, \forall t, \quad (2.1)
\]

with boundary conditions \( V_{T+1}(c_{T+1}) = -\infty \) if \( c_{T+1} \not= 0 \) and \( V_{T+1}(c_{T+1}) = 0 \) else. Equation (2.1) is the standard formulation of choice-based RM.
2.3. Capacity control

Capacity control describes the practice of accepting or denying booking requests for individual products under the assumption of independent demand. Again, a DP defines the optimal control policy. However, we assume that there is one customer segment for each product. A customer from a segment associated with product $j$ arrives with probability $\lambda_j$ in a given time period and will always purchase product $j$, if it is available; otherwise the customer leaves without purchase.

Let us express the offer set by a vector $u_t = (u_{jt})_{j \in J}$ of binary decision variables $u_{jt}$. The Bellman equation (2.1) then simplifies to

$$V_t(c_t) = \max_{u_t \in \{0,1\}^J} \left\{ \sum_{j \in J} \lambda_j u_{jt} (r_j + V_{t+1}(c_t - a_j)) + \left( 1 - \sum_{j \in J} \lambda_j u_{jt} \right) V_{t+1}(c_t) \right\}$$

$$= \max_{u_t \in \{0,1\}^J} \left\{ \sum_{j \in J} \lambda_j u_{jt} + V_{t+1}(c_t - a_j) + \left( 1 - \sum_{j \in J} \lambda_j \right) V_{t+1}(c_t) \right\}$$

$$= \sum_{j \in J} \lambda_j \max_{u_{jt} \in \{0,1\}} \left\{ r_j u_{jt} + V_{t+1}(c_t - a_j) \right\} + \left( 1 - \sum_{j \in J} \lambda_j \right) V_{t+1}(c_t)$$

$$= \sum_{j \in J} \lambda_j \max \left\{ r_j + V_{t+1}(c_t - a_j), V_{t+1}(c_t) \right\} + \left( 1 - \sum_{j \in J} \lambda_j \right) V_{t+1}(c_t)$$

$$= \sum_{j \in J} \lambda_j \max \left\{ r_j - \Delta_j V_{t+1}(c_t), 0 \right\} + V_{t+1}(c_t) \quad \forall c_t, \forall t. \quad (2.2)$$

Line 1 of (2.2) comes from replacing the probability $\lambda P_j(S_t)$ in (2.1) with $\lambda_j u_{jt}$ and line 2 from some simple algebraic manipulations. In line 3, we change the order of the expectation and maximization, which is possible due to the independence of $\lambda_j$ and $u_t$. Thus, the combinatorial availability decision decomposes by product. The formulations of the Bellman recursion in the final two lines are those most commonly found in RM literature. Compared to choice-based availability control, the action space of capacity control comprises only two decisions for each product: either it is on sale (namely when the product’s fixed revenue exceeds opportunity cost) or otherwise it is not offered. These two decisions are often interpreted as accepting and rejecting booking requests, respectively.

Although the independent demand assumption is somewhat outdated today, service providers in traditional industries such as passenger air transport or car rental often still rely on independent demand systems due to legacy issues. In this context, the so-called fare transformation allows to include customer choice behavior to some extent by modifying the input parameters, namely revenues and arrival probabilities (see Fig et al. 2010). However, as we discuss in Strauss et al. (2018), the fare transformation is only exact subject to some strong assumptions.
2.4. Problem solution

In general, the multidimensional state space prohibits an exact solution of the dynamic programs (2.1) and (2.2). Therefore, a lot of effort has been spent on developing approximations of the value function as discussed in Strauss et al. (2018).

Particularly prominent are the well-known deterministic linear program (DLP), its choice-based pendant (CDLP; see Liu and van Ryzin 2008 and Miranda Bront et al. 2009), and the randomized linear program (RLP). The idea of those static approximations is to disregard the dynamics and to replace the stochastic demand by its expected value (for the DLP and the CDLP) or by samples drawn from anticipated demand distributions (for the RLP). In a second step, dual information of the capacity constraints from those approximations is usually used to replace the opportunity costs by additive bid prices $\pi_h$, which are estimates of the marginal value of one capacity unit of a resource. In the independent demand setting, such a heuristic control policy is termed bid price control (BPC). An alternative is certainty equivalent control (CEC), going back to Bertsimas and Popescu (2003), where the objective values of the DLP or the RLP are used in a one-step look-ahead policy to approximate the values $V_{t+1}(c_t)$ and $V_{t+1}(c_t - a_j)$ in equations (2.1) and (2.2). A second group of approximations goes back to the pioneering work of Adelman (2007) and makes use of the linear programming formulation of (2.1) and (2.2). The idea of those so-called approximate linear programs (ALPs) is to assume linear approximations, for example $V_t(c_t) \approx \sum_{h \in H} \pi_{ht} c_{ht}$, where the parameters (in our case, time-dependent bid prices $\pi_{ht}$) can be directly used to replace opportunity cost after they are estimated. To this end, the approximations are plugged into the DP’s linear programming formulation, whose dual is subsequently solved by column generation. A third prominent type of approximation, called dynamic programming decomposition (DPD; for example, Liu and van Ryzin (2008)) assume that the value function is approximated by $V_t(c_t) \approx v_{th}(c_{ht}) + \sum_{k \neq h} \pi_k c_{kt}$, where the functions $v_{th}(\cdot)$ can be obtained by solving a single-resource problem.

Compared to the network problem, the single-resource problem’s state space is one-dimensional and can therefore easily be solved to optimality. Moreover, it can be shown that the value function is concave in capacity and time, i.e., the marginal value of capacity increases as capacity decreases and decreases as time passes. These structural properties do not hold for the network problem in general, but are nevertheless inherent to several approximations. For example, it can be shown that optimal time-dependent bid prices $\pi_{ht}$ of the ALP approach decrease over time.
3. Generalizations

In this section, we give an overview of the generalizations of the standard setting that have received most attention in academia over the past 15 years. For each of the considered generalizations, we first give a brief introduction and practical motivation. Then, we show how the standard model formulation as given by the DPs has to be extended, and discuss the resulting challenges as well as the corresponding scientific contributions.

The first two presented generalizations are opaque products (§3.1) and flexible products (§3.2) and result from product innovations. Thereafter, we review the literature on upgrades (§3.3) and on the probably most important generalization, namely overbooking (§3.4). The generalizations mentioned so far share the commonality that the fulfillment is not clear at the time of purchase. From a modeling perspective, the service provision and, consequently, the DPs become more complicated from subsection to subsection, in particular with respect to action space, state space, and boundary condition.

In §3.5 we consider personalization which requires knowing (or learning) customer segments and preferences on a very detailed level. We close with a review of risk-averse RM as an example of an alternative objective (§3.6). Another example of such an alternative objective results from the integration with customer relationship management. However, literature is rare in the latter field and the corresponding models hardly fit the classical RM framework. We refer the interested reader to von Martens and Hilbert (2011) as starting point.

3.1. Opaque products

3.1.1. Introduction

When selling an opaque product, the selling company hides specific properties of the product until the sale has been completed. Most common is some kind of travel roulette, where for instance the destination of an itinerary or the specific hotel in which a customer will stay is concealed. Opaque products are often sold by intermediaries like Hotwire and Priceline.com, but also by tour operators or airlines. From a marketing perspective, opaque products are basically an instrument of price discrimination in order to attract additional low value demand, but without excess cannibalization (see Granados et al. (2018) for a recent empirical investigation of those effects). As Jerath et al. (2010) show, opaque products are thus often a superior marketing instrument compared to last-minute selling. Post and Spann (2012) report on the success of variable opaque products in practice at
the case of the airline Germanwings, where customers are allowed to vary the amount of opacity. The authors report that revenues due to opaque products are (almost) exclusively noncannibalistic and contributed 4.7% to the total revenue in 2010.

3.1.2. Mathematical model

With the incorporation of opaque products, the modeling of resource consumption in equation (2.1) needs to be generalized in the sense that there is no longer a single fixed relation between product and required capacity of resources. Instead, one has to define alternatives (or modes) $M_j$ that a product $j$ can be assigned to, where alternative $m$ refers to a certain resource consumption $a_m$. Since we decide in which mode $m \in M_j$ an opaque product is delivered immediately after the purchase and, hence, we know the resources $a_m$ used by mode $m$, we can still represent the state space by the remaining capacity $c_t$.

However, opaque products slightly complicate the action space as the opportunity cost $\Delta_m V_{t+1}(c_t) = V_{t+1}(c_t) - V_{t+1}(c_t - a_m)$ depends on the assigned alternative. With the new state definition, this can be seen by the Bellman equation

$$V_t(c_t) = \max_{S_t \subseteq J} \left\{ \sum_{j \in S_t} \lambda P_j(S_t)(r_j - \min_{m \in M_j} \Delta_m V_{t+1}(c_t)) \right\} + V_{t+1}(c_t) \quad \forall c_t \forall t, \quad (3.1)$$

subject to the standard boundary conditions from Section 2.2. In case we want to sell regular products next to opaque ones, we can still use formulation (3.1), by simply defining only a single mode for each regular product $j$, i.e., $|M_j| = 1$.

Having the same state space as in the standard setting has turned out to make a straightforward adoption of standard solution approaches possible. As we will see, also several properties regarding monotonicity and approximations carry over.

3.1.3. Scientific contributions

Chen et al. (2010) and Anderson and Xie (2012) consider a single opaque product that can be assigned to a set of completely substitutable resources, namely parallel flights (e.g., during the same day at different departure times) and hotel rooms with hidden location in a city, respectively. The authors show that the well-known monotonicity properties of the DP from the the standard single-resource RM problem carry over. For the network setting, Göensch and Steinhardt (2013) integrate opaque products into the DPD approach and show that the common properties regarding the relation to the objective value of the DP as well as of the DLP carry over. They then use the approach within a comprehensive simulation study, investigating the revenue impact of introducing opaque products. They show that
potential benefits strongly depend on the specific demand parameters such as demand induction and cannibalization, as well as on the degree of opacity. Also using (3.1) as starting point, Sayah (2015) first presents the ALP as well as the corresponding reduction. He shows that – in contrast to the standard RM setting without opaque products – the reduction is not equivalent to the ALP in general as a result of the modified action space.

3.2. Flexible products

3.2.1. Introduction

Gallego and Phillips (2004) and Gallego et al. (2004) introduce a flexible product as a set of substitutable alternatives such that the seller can assign the purchaser to an alternative at a time near service provision. An example is the product “Just AIDA” offered by the cruise company AIDA, where the itinerary, the departure time and the cabin type are hidden at the time of sale and revealed later. From a marketing perspective, flexible products are thus a similar instrument for demand induction as opaque products. However, from a RM perspective, they are even better to improve capacity utilization, as the assignment decision can be delayed until a time when there is much less uncertainty regarding future demand. For example, considering instances from the scientific literature, Koch et al. (2017) have shown that delaying the assignment may increase revenues by up to 2% if the demand forecast is accurate and up to 8% under forecast errors. Gönisch (2019) has provided a recent literature review on flexible products, also including opaque products and not restricted to RM. Thus, we only portray the key concepts and contributions regarding availability control in the following.

3.2.2. Mathematical model

Since for a flexible product the final assignment of a mode remains flexible at the time of sale, it is no longer sufficient for the seller to keep track of the remaining capacity to describe the state space. Instead, we have to keep track of the reservations, using a vector \( y_t = (y_{jt})_{j \in J} \) as state variable. Selling a product \( j \) increases the reservations vector to \( y_t + 1_j \), with \( 1_j \) denoting the \( j \)th standard vector in \( \mathbb{R}^J \). With the new state definition, the optimal expected revenue satisfies the Bellman equation

\[
V_t(y_t) = \max_{S_t \subseteq J} \left\{ \lambda \sum_{j \in S_t} P_j(S_t) \left( r_j + V_{t+1}(y_t + 1_j) \right) + \left( 1 - \sum_{j \in S_t} \lambda P_j(S_t) \right) V_{t+1}(y_t) \right\} \\
= \max_{S_t \subseteq J} \left\{ \sum_{j \in S_t} \lambda P_j(S_t) \left( r_j - \Delta_j V_{t+1}(y_t) \right) \right\} + V_{t+1}(y_t) \quad \forall y_t, \forall t.
\]
However, in comparison to the previous sections, not only the state definition alters. At some point in time before service provision, the reservations have to be assigned to the alternatives, i.e., the company has to ensure that the capacity is sufficient to satisfy all reservations. In case that this assignment is at the time of service provision $T+1$, this is captured by the feasibility problem

$$\sum_{m \in M_j} x_{jm} = y_{j,T+1} \quad \forall j \in J \quad (3.3)$$

$$\sum_{j \in J} \sum_{m \in M_j} a_{hm} x_{jm} \leq c_{h1} \quad \forall h \in H \quad (3.4)$$

$$x_{jm} \in \mathbb{N}_0 \quad \forall j \in J, \forall m \in M_j, \quad (3.5)$$

where $x_{jm}$ denotes the number of reservations for product $j$ assigned to alternative $m$. If a feasible solution exists for to (3.3) - (3.5), then we will set the boundary conditions to $V_{T+1}(y_{T+1}) = 0$, and otherwise to $V_{T+1}(y_{T+1}) = -\infty$.

Several challenges come with the introduction of flexible products. First, since the number of products will usually exceed the number of resources in practical settings, the state space of (3.2) will be considerably larger than the one of (2.1). Second, it is difficult to decide whether a flexible product can be feasibly offered as well as to maintain the flexibility throughout the booking horizon, as both would require solving problem (3.3) - (3.5) for each product in each period. Third, the adoption of standard solution approaches such as BPC, DPD, or ALP is complicated as products do not uniquely correspond to a set of resources and, thus, the use of resource-based approximations is hampered.

3.2.3. Scientific contributions

For the capacity control problem, Petrick et al. (2012) show in a couple of computational experiments that flexible products are particularly useful if demand is not forecasted accurately. For this purpose, they consider a variant of (2.2) where resource allocation does not necessarily take place after the booking horizon, but at an arbitrarily chosen point in time between sale and service provision, and they computationally investigate the impact of different intervals of postponing the final resource allocation on revenue. Also for the capacity control problem, Petrick et al. (2010) and subsequently Gönsch et al. (2014) focus on the second challenge, using the DLP of (2.2) and extending a classical BPC. In particular, Petrick et al. (2010) propose different heuristic approaches to ensure feasibility in real-time without having to resolve the entire feasibility problem for each incoming request. Gönsch et al. (2014) postulate an inherent value of flexibility that they formally
define using (2.2), and argue that it is neglected by the DLP. Based on this observation, they then propose a simulation-based adoption of the DLP outcome over time, showing good revenue performance in computational experiments. Koch et al. (2017) consider the availability control problem and focus on the third challenge, in particular on DPD. They propose a generic approach based on Fourier-Motzkin-elimination to technically transform the problem into a standard network RM problem such that, under mild conditions, DPD becomes applicable. While this is done at the cost of creating additional artificial resources that pool the capacity of two or more regular resources for the joint use of several products, the authors show that for many network structures relevant to RM practice, the number of artificial resources is not exponential in the parameters and, thus, the resulting model is manageable. Sierag (2017) extends the setting by considering group bookings as well as dynamic pricing instead of availability control, and suggests two simple heuristics that operationalize the solution of an extension of the DLP.

3.3. Upgrading

3.3.1. Introduction

Upgrading allows the seller to satisfy demand for a lower-quality product with a higher-quality product from a set of hierarchically ordered substitutes. Usually, an upgrade is given at no extra charge. Thus, conventional upgrades are different from the practice of upselling (or paid upgrades), where a customer is urged to voluntarily buy the higher-quality product at a discounted price. Upgrades are particularly important for car rental companies. As the differences in costs are usually small, car rental companies tend to acquire considerably fewer economy cars but more midsize cars than required. Geraghty and Johnson (1997) estimate that around half of the fleet consists of midsize cars, which has also been confirmed by Gönsch and Steinhardt (2015) in an industry project with a major German car rental company. To overcome the resulting mismatch of demand and capacity, the companies make extensive use of upgrades along the car type hierarchy. Two basic forms of upgrading can be distinguished (see Gallego and Stefanescu 2009): Full cascading (multistep models) allow the seller to fulfill the demand for a product with any higher-quality product, while limited cascading (single-step models) allow an upgrade only to the next higher-quality product.
3.3.2. Mathematical model

If upgrades are granted at the time of sale, upgrades can be modeled as opaque products, where the alternative sets $M_j$ capture the upgrade hierarchy. If upgrades occur at the time of service provision, an upgrade is in fact a special case of a flexible product. Therefore, we do not repeat the Bellman equations here, but refer the reader to §3.1 and §3.2, respectively. Gallego and Stefanescu (2009) introduce the corresponding dynamic programs and analyze the DLP in detail for both, limited and full cascading upgrades. Consideration of fairness becomes important under full cascading since this strategy may lead to grant a customer an upgrade even though another customer has paid less. Gallego and Stefanescu (2009) show that any flexible upgrade model satisfying certain properties has a fair optimal solution, where a fair solution is defined to satisfy the condition that all customers who purchase a product have priority in upgrades relative to all other customers who purchased an inferior product.

3.3.3. Scientific contributions

Steinhardt and Gönsch (2012) and subsequently Gönsch and Steinhardt (2015) analyze the dynamic programs with full cascading upgrades in the context of car rental and passenger air transport, respectively. Steinhardt and Gönsch (2012) prove that if only a single rental day (i.e., a single leg) is considered, opportunity cost is monotonous with regard to the upgrade hierarchy. This property greatly simplifies the control policy as not all available upgrade options need to be considered, and the authors reuse this property within a decomposition approach that they propose for the multi-day (network) case. As a second important property, the authors show that in the single day setting, granting upgrades at the time of sale is equivalent to delaying the assignment. However, this result does not carry over to the multiday case, which can be shown by straightforward counterexamples. However, as Gönsch and Steinhardt (2015) show, the result does indeed carry over to the airline network context, where leg-wise upgrading is allowed. More precisely, as opposed to car rental where an upgrade has to be offered throughout the entire rental duration (product-wise upgrading), airline customers can be offered upgrades only on part of their itinerary (leg-wise upgrading). Also, Gönsch and Steinhardt (2015) propose two different DPD approaches, one based on a model reformulation that borrows ideas from production planning (see Leachman and Carmon 1992), and discuss how their individual advantages and drawbacks depend on the problem parameters, giving advice for practical application. The monotonicity property from Steinhardt and Gönsch (2012) is later
on also exploited by Gösch et al. (2013) who develop an extension of the well-known EMSR-a heuristic for the independent demand single-leg setting. Guerriero and Olivito (2014) consider limited cascading upgrades in the car rental industry and suggest a DLP together with a BPC. McCaffrey and Walczak (2016) investigate a single-leg upgrading problem with two compartments under the marginal revenue transformation that reduces a problem with dependent demand into an equivalent independent demand problem under certain assumptions. For this transformed problem, they show that the value function of the two-dimensional dynamic program is sub-modular and exploit this result to derive an exact and tractable solution algorithm.

3.4. Overbooking

3.4.1. Introduction

Overbooking describes the practice of selling more products than physically available capacity in order to hedge against the cases that customers cancel before service provision (cancellations) or do not show up at the time of service provision (no-shows). Overbooking is particularly important in passenger air transport. For example, based on data from Continental Airlines, Gorin et al. (2006) estimate that 4% of customers do not show up at departure and that 15% to 18% of the total revenue improvements gained by RM stem from overbooking. However, in case that more customers show up than physical capacity is available, the seller has to bump customers at pre-specified or negotiated penalties and additionally may incur goodwill losses. Overbooking is one of the oldest RM instruments. Early overbooking models were static, ignoring the dynamics of cancelations and arrivals over time, with the objective of determining an overbooking limit that describes the maximum number of reservations the company is willing to accept. As time progresses through the booking horizon, the overbooking limit may change due to re-optimization.

3.4.2. Mathematical model

Modern dynamic approaches integrate availability and overbooking decisions in the DP. Like in the case of flexible products, the state space and the boundary conditions have to be modified. Again, the seller has to keep track of and control the reservations such that we consider a vector $y_t = (y_{jt})_{j \in J}$ of reservations. No-shows and cancelations represent additional sources of uncertainty: cancelations can be modeled as an additional stochastic process, with state-dependent cancelation rates $\gamma_j(y_t)$ and refunds $f_j$. No-shows are usually integrated by specifying product-specific probabilities $q_j$ that a reservation shows up, so
that the number of show ups for product \( j \) follows a binomial distribution (with \( y_{j,T+1} \) being the number of trials). Therefore, also the total penalty cost \( C(y_{T+1}) \) is a random variable which is realized after the booking horizon. Using the choice-based framework, the optimal expected net revenue \( V_t(y_t) \) satisfies the Bellman equation

\[
V_t(y_t) = \max_{S_t \subseteq \mathcal{J}} \left\{ \sum_{j \in S_t} \lambda P_j(S_t)(r_j + V_{t+1}(y_t + 1_j)) + \sum_{j \in \mathcal{J}} \gamma_j(y_t)(-f_j + V_{t+1}(y_t - 1_j)) + (1 - \sum_{j \in S_t} \lambda P_j(S_t) - \sum_{j \in \mathcal{J}} \gamma_j(y_t))V_{t+1}(y_t) \right\} \quad \forall y_t \forall t \in \mathcal{T},
\]

with boundary condition \( V_{T+1}(y_{T+1}) = -E[C(y_{T+1})] \).

If the parameters \( \theta_j \) denote the penalty costs of bumping a reservation, the minimal penalty cost will be easy to compute in the single-resource case. Given realizations \( Z_j(y_{j,T+1}) \) of show ups for product \( j \), we consider the products in the order of non-decreasing penalty cost and simply bump reservations following this order until their total number equals the capacity. However, in the network case, minimizing the total cost requires solving an optimization problem, where the decision variables \( x_j \) describe the corresponding number of bumped reservations:

\[
\text{Minimize} \sum_{j \in \mathcal{J}} \theta_j \cdot x_j \quad (3.7)
\]

\[
s.t. \sum_{j \in \mathcal{J}} a_{hj} \cdot (Z_j(y_{j,T+1}) - x_j) \leq c_h \quad \forall h \in \mathcal{H} \quad (3.8)
\]

\[
x_j \leq Z_j(y_{j,T+1}) \quad \forall j \in \mathcal{J} \quad (3.9)
\]

\[
x_j \in \mathbb{N}_0 \quad \forall j \in \mathcal{J}. \quad (3.10)
\]

In comparison to the generalizations discussed so far, overbooking adds an additional level of complexity, in particular, because a stochastic optimization problem has to be solved to be able to determine the value function in the boundary.

### 3.4.3. Scientific contributions

Aydin et al. (2013) and subsequently Sierag et al. (2015) and Wang and Walczak (2016) analyze the single-resource setting under various assumptions on cancellation rates. An important result is that, if cancellation rates, no-show rates, and penalty costs are the same for all reservations, \( (3.6) \) boils down to a one-dimensional problem and can therefore be solved optimally. The intuition behind this result is that – after the purchase – all reservations can be perceived as equal by the firm and can therefore be aggregated. However,
this result does not hold in the general case, where the reservations must be distinguished also after the sale in order to determine which reservations to bump.

A series of papers has been devoted to the network setting under the capacity control framework with no-shows but without cancellations (Kunnumkal and Topaloglu 2008, Erdelyi and Topaloglu 2009, Kunnumkal and Topaloglu 2011, Erdelyi and Topaloglu 2010, and Kunnumkal et al. 2012), from which we highlight two. Erdelyi and Topaloglu (2010) focus on DPD. As in the traditional approach, the authors decompose the network by resources and end up with single-resource value functions. Interestingly, in contrast to the standard network RM problem, these single-resource value functions are still intractable because reservations have to be tracked in order to differentiate between them for bumping. The authors overcome this issue by approximating the penalty costs so that they only depend on the total number of reservations. As a result, the multidimensional state space collapses to a scalar. Kunnumkal et al. (2012) generalize the RLP from the standard network RM problem. To this end, the authors do not only generate random demand samples, but – for each demand sample – a set of random show up samples. Dai et al. (2019) study network RM with both no-shows and cancellations under both independent and choice-based demand models using a fluid model (meaning continuous time and continuous states).

Note that besides the research that can directly be related to the Bellman equation, static overbooking models are still being researched due to their high relevance in practice. Aydin et al. (2013) analyze the problem of determining the optimal overbooking limit in the single-resource setting; see Klophaus and Pölzt (2007) and Wang and Walczak (2016) for similar static models but only with no-shows, i.e., without cancellations. Topaloglu et al. (2012) consider a novel static model, called open loop policy, where each product is sold with a fixed probability. Only considering no-shows, the authors show that the optimal sales probabilities can be derived in closed form. For a network of resources, Karaesmen and van Ryzin (2004) and Gosavi et al. (2007) suggest a simulation-based optimization approach to approximate optimal overbooking limits. The setting of Karaesmen and van Ryzin (2004) is somewhat different from the standard network RM problem as the resources are substitutes and a two-stage model is considered.

3.5. Personalization

3.5.1. Introduction

Tailoring an offering to a customer based on observed or derived information on this individual is called personalization. Personalization is currently one of the most discussed
topics of RM in industry. This is mainly due to the recent and ongoing changes in airlines’ distribution technology, offering completely new possibilities with regard to making much more flexible and customized offers (see e.g. Westermann 2013). First attempts have been undertaken to propose models that dynamically decide on availability based on customer specific characteristics, e.g. the trip purpose (see Wittman and Belobaba 2017a) or making personalized fare offers (see Wittman and Belobaba 2017b). Customer data could, e.g., be obtained during the booking process and potentially linked with data on the specific customer obtained through the airline’s loyalty programs and also contain the specific historical booking etc. Therefore, a tight integration with analytics and data mining techniques is required.

3.5.2. Mathematical model

One approach consists of identifying one of several predefined segments an incoming requests originates from, and then offering a segment-specific preassembled offer set (which may also include the prebundling of seat and ancillaries, see Madireddy et al. 2017). With regard to the choice-based framework introduced in §2.2, this logic would mean that we assume a number of segments $K = \{1, \ldots, K\}$, with each customer originating exactly from one segment $k \in K$. Each segment comes along with its individual choice behavior which is expressed by the probabilities $P_j^{(k)}(S_t)$. The customers’ arrival probability decomposes into segments, i.e., $\lambda = \sum_{k \in K} \lambda^{(k)}$. Moreover, we do not assume segments to be latent, but that each incoming customer’s segment can precisely be observed before booking. Therefore, the maximization can be performed segment-wise, i.e., maximization and expectation over segments are interchanged (similar to the independent demand setting), which results in the Bellman equation

$$V_t(c_t) = \sum_{k \in K} \lambda^{(k)} \max_{S_t \subseteq J} \left\{ \sum_{j \in S_t} P_j^{(k)}(S_t)(r_j + V_{t+1}(c_t - a_j)) \right\}$$

$$+ \left(1 - \sum_{j \in S_t} P_j^{(k)}(S_t)\right)V_{t+1}(c_t), \quad (3.11)$$

with the standard boundary conditions. Note that segments containing only a single individual would correspond to full personalization (discussed as segment-of-one, 1-to-1 personalization in practice). Despite the strong interest in personalization in practice, the amount of rigorous scientific contributions is still quite limited. In the age of Big Data, the major challenge at the moment is learning the segments as well as their behavior, before personalized offers could be made.

18
3.5.3. Scientific contributions

There are some contributions regarding dynamic assortment planning, which is somehow related to the availability control problem of RM we consider here. In dynamic assortment planning, a seller offers an assortment to each customer on arrival (over a finite selling horizon), with the objective of maximizing expected total revenue. The individual assortment problems are linked, typically either due to inventory constraints, and/or due to learning aspects of demand such that we face the typical exploration/exploitation trade-off over time. However, limited capacity as the key characteristic of availability control is often not considered in assortment optimization, while limitations mostly concern the amount of products that can be offered or displayed at the same time, i.e., the cardinality of the offer set.

There are some recent exceptions that study (personalized) dynamic assortment planning problems with limited inventory and heterogeneous customers, and that partly explicitly mention the link to RM. Bernstein et al. (2015) assume that the customer population consists of a finite number of segments, each choosing according to the multinomial logit model with known preference values. They study the effect of inventory constraints, and find that it can be optimal to restrict certain segments’ choice set so as to reserve products with low inventory for future customers with higher preference for these products. Ciocan and Farias (2012) discuss a generalized class of dynamic allocation problems including the network RM and the online advertisement display problems as special cases. Their approach is based on re-optimization and forecast updates and does not require the demand rate process to be specified. The distribution of customer types (segments) is assumed to remain constant over time, although the size of the market may change. In contrast, the closely related work Golrezai et al. (2014) allows the distribution of customer types to vary. The proposed algorithms do not require any forecasting and uses an index for every product consisting of its revenue multiplied with a virtual discount factor that depends on the product’s remaining inventory. Gallego et al. (2015) consider personalized resource allocation where multiple customer types choose products from the offered assortments following a general choice model. In contrast to Golrezai et al. (2014), rewards may depend on also the customer type, as opposed to the sold products only.

So far, all works assume that customer segments are known a priori and remain static over time. Very recently, dynamic assortment planning research has started to investigate these problems from a learning perspective, that means how to dynamically optimize
personalized assortments when the choice parameters are unknown. Again, note that the setting is often not completely in line with availability control. We emphasize that, in RM, demand learning has not only been used in the context of personalization (see, e.g. Besbes and Zeevi 2012), but we limit ourselves to discussing learning in this context since this is an area that appears to have recently captured significant interest. Chen et al. (2015) present an approach that is easily extendable to fairly general choice models, including semi-parametric and non-parametric ones. Bernstein et al. (2018) present an approach for the multinomial logit model; they draw on Bayesian data analysis literature to estimate preferences through dynamic clustering and machine learning. Customer segments are dynamically updated over time as more information becomes available. Likewise for learning (personalized) MNL model parameters, Kallus and Udell (2016) develop an exploration/exploitation algorithm that requires to offer assortments to be chosen uniformly at random for multiple rounds. Cheung and Simchi-Levi (2017) criticize that this requirement can be problematic in practice when there are constraints on offering personalized assortments such as the need for the assortment to match a certain search keyword. Accordingly, they propose a policy that can be implemented under such personalization constraints. More work has been done in this area but, due to space limitations, we refer the interested reader to the given key references.

3.6. Risk-aversion

3.6.1. Introduction

The major argument for the assumption of a risk-neutral decision maker with the expected revenue as objective criterion is the large number of repetitions of similar sales processes. Thus, the law of large numbers ensures convergence of the long-term average revenue to the expected value. However, human decision makers in daily practice tend to overrule risk-neutral decisions manually with less risky decisions. Additionally, the assumption of risk-neutrality is questioned in applications with infrequent sales processes, or where the outcome is critical for economic survival. Often cited examples include a seller of real estates or concert tickets. There are two different concepts to incorporate risk-aversion: expected utility theory and risk-measures. Gönsch (2017) has provided a recent literature review on risk-aversion in RM and, thus, we only portray the key concepts and contributions regarding availability control in the following.
3.6.2. Mathematical model

The expected utility theory goes back to [von Neumann and Morgenstern (1944)]. The underlying idea is that decision makers value the same revenue differently due to individual preferences, which are captured by an utility function $U(\cdot)$. A decision maker is risk-averse if the utility function is concave. General utility functions are not time-separable, which implies that an evaluation of the total revenue is only possible after the selling horizon. To comply Bellman’s principle of optimality, one has to store a portion of the history of the selling process in the state space, namely the cumulated revenue, and convert the achieved total revenue into utility in the boundary. However, storing knowledge of history is unusual in dynamic programming. Moreover, it leads to an explosion of the state space as there may be a state for every combination of capacity and potential cumulated revenue, such that only toy examples can be solved. Additionally, the well-known monotonicity properties with regard to opportunity cost in the single resource problem are lost. An important exception is the case of an exponential utility function, e.g. $U(R) = -e^{-\gamma R}$, with risk sensitivity $\gamma$. Say that the random variable $R = \sum_t R_t$ is the total achieved revenue, with contribution $R_t$ of period $t$. Then, an exponential utility function is time-separable in the sense that it allows the multiplicative decomposition $U(R) = \prod_t e^{-\gamma R_t}$. This time-separability allows tracing back the dynamic program to a classical state space. Stated under the independent demand framework, the optimal expected utility from period $t$ in state $c_t$ satisfies the Bellman equation

$$V_t(c_t) = \sum_{j \in J} \lambda_j \max \left\{ e^{-\gamma r_j} V_{t+1}(c_t - a_j), V_{t+1}(c_t) \right\} + (1 - \sum_{j \in J} \lambda_j) V_{t+1}(c_t).$$

(3.12)

3.6.3. Scientific contributions

Barz (2007), Barz and Waldmann (2007), and Feng and Xiao (2008) were the first to include an exponential utility function into the dynamic program of availability control. Considering the single resource problem, the authors were able to show that the mononicity properties from the risk-neutral case carry over.

Not only in finance, risk measures are receiving more and more attention. Koenig and Meissner (2015a) and Koenig and Meissner (2015b) consider the optimization of the risk measures “target percentile risk” and “value-at-risk”, respectively. In the language of RM, the target percentile risk expresses the probability that a certain target value of revenue is not exceeded, while the value-at-risk evaluates the revenue that will not be exceeded at a given probability level. Similar to general utility functions, optimization requires
to store the cumulated revenue over time. The value-at-risk is often criticized because the distribution below the probability level is ignored. This drawback is resolved by the conditional value-at-risk, which is defined as the corresponding expected revenue. Pflug and Pichler (2016) show how to dynamically decompose the conditional value-at-risk in general multistage stochastic programs, albeit with convex action spaces. Gönsch et al. (2014) use this decomposition as starting point for a heuristic approach for the availability control problem with its discrete action space.

Note that most literature focuses on a certain objective criterion and theoretically analyses toy settings. However, there is also some literature without clear objective, but where risk-neutral approaches are slightly modified by calibrateable parameters such that expected utility or an arbitrary risk measure can be optimized on demand, either manually (Huang and Chang 2011 and Koenig and Meissner 2015b) or by simulation-based optimization (Koch et al. 2016).

4. Traditional industry applications

In this section, we portray important traditional industry applications beyond passenger air transport, specifically air cargo (§4.1), hotel (§4.2), and car rental (§4.3). We analyze their specific challenges, discuss their relationship to the generalizations discussed in §3, review the scientific contributions of the last 15 years, and point out avenues for future research. For the sake of brevity, we do not give a DP formulation, but discuss how the control problem relates to the generalizations discussed in §3.

Further industries that fall in this class and share strong similarities with the ones presented are railway, cruise lines, restaurants, casinos, theme parks, liner shipping, or tour operators. For some of these industries, specific reviews exist (e.g., Armstrong and Meissner 2010 for railway, Sturm and Fischer 2016 for cruise lines). For the other industries, there is only rare methodological work (e.g., Zurheide and Fischer 2015 for liner shipping) as well as only few case studies and conceptual work (e.g., Kimes 2005 for restaurants, Metters et al. 2008 for casinos, Heo and Lee 2009 for theme parks).

4.1. Air cargo

Airlines carry cargo along with passengers to best utilize the available capacity and to gain additional income. These revenue streams are becoming increasingly important: according to a study by Boeing, e-commerce sales are expected to continue drive growth of air cargo by over 4% per year for the next 20 years Crabtree et al. 2018.
Selling air cargo capacity adds some fundamental complexities to the standard network RM setting. To begin with, a large portion of air cargo capacity is sold based on long-term contracts, which are planned upfront. Only the remaining capacity is sold on the so-called spot market. The spot market handles the short-term demand approximately the last 30 days prior to service provision. Past research in RM has focused on controlling the sale on the spot market. Second, both loadable weight and volume of an aircraft represent a physical resource, such that two capacity (and demand) dimensions have to be considered. Third, volume and weight capacity available for service provision are stochastic. They may, among others, depend on the used or reserved capacity for long-term contracts, on the number of passengers having priority over cargo, or on weather conditions. Fourth, the exact weight and volume requirements of cargo bookings are not known prior to service provision and therefore represent an additional source of uncertainty. In air cargo, a product refers to a shipping class and is associated with a distribution of weight and volume capacity consumption and a contribution margin $r_j$. Let the random variables $W_j$ and $V_j$ denote the weight and volume of a class-$j$ booking. As (expected) immediate contribution in the dynamic program, one uses $E[r_j \cdot \max\{W_j, V_j/\gamma\}]$, where $\gamma$ is a constant representing the weight to volume ratio of a standard shipment, and $\max\{W_j, V_j/\gamma\}$ is called the chargeable weight. As final additional complexity, a cargo product is often not defined by a specific itinerary, but only loosely linked to an origin-destination pair as long as it arrives in time. Thus, the firm has an additional routing flexibility, and the final routing decision can be delayed until after the selling horizon.

The routing flexibility means that this application is linked to the flexible product concept as discussed in §3.2. Furthermore, the overbooking concept (§3.4) is important here since show-up rates vary widely over time as shown by Popescu et al. (2006). Bringing the literature on overbooking and flexible products together, reservations are assigned to routes at the time of service provision. The optimal penalty costs are therefore uncertain and depend on the reservations, the realizations of weight and volume capacities, and realizations of volume and weight requirements of the reservations.

For air cargo RM on a single flight leg with deterministic weight and volume capacity, the work of Amaruchkul et al. (2007) has attracted significant attention. The unique volume and weight requirements of a cargo booking facilitate customer segmentation such that the independent demand assumption can be used. However, the problem cannot be reduced to a tractable (two-dimensional) problem even if only a single-leg flight is
considered. This is because the exact weight and volume requirements only realize at the
time of service provision and, thus, reservations have to be tracked (similar to \[3.4\]). The
authors propose the DP and decomposition architectures based on the expected weight and
volume requirements, decomposing the problem into two one-dimensional weight as well
a volume problems, respectively. Subsequent work built on these theoretical foundations:
see Han et al. (2010), Huang and Chang (2010), Xiao and Yang (2010), Qin et al. (2012),
Zhuang et al. (2012), Hoffmann (2013a), Hoffmann (2013b), and Moussawi-Haidar (2014).

For the network case, Pak and Dekker (2004) were the first to propose an capacity con-
trol model for cargo RM, assuming a deterministic capacity and that each request is unique
but with a deterministic capacity consumption at the time of sale. The authors suggest
a multi-dimensional knapsack model for approximation purpose and derive a correspond-
ing BPC. Bartodziej et al. (2007) extend the setting by considering the routing flexibility,
and show that the corresponding DLP can be formulated as network flow problem on a
time-space network. More recently, Barz and Gartner (2016) have considered a generalized
setting in order to comprehensively model the spot market availability control problem.
The authors derive solution approaches based on DLP, RLP, ALP, and DPD. With the
latter approach, which turns out to be the most profitable in numerical experiments, the
authors extend the work of Amaruchkul et al. (2007) and decompose the problem by re-
sources (that is, weight and volume as well as flights). However, similar to the setting with
flexible products, the utilized resources of a reservation are unclear at the time of booking.
To overcome this issue and render a decomposition possible, a reservation is immediately
assigned to a route at the time of booking, which parallels the setting with opaque products.
Similar solution approaches as in Barz and Gartner (2016) but without the consideration
of route flexibility and individual penalty costs can be found in Hoffmann (2013a). Also
Levina et al. (2011) cover the main complexities on the spot market. Whereas the previous
literature considers service provision at a single day, Levina et al. (2011) formulate the
spot market problem as infinite horizon dynamic program in order to find a fixed periodic
schedule with respect to the routing decisions.

Interestingly, only few papers cover the integration of upfront contract planning. In
this setting, contracts are signed before the spot market selling horizon starts, and the
airline has to decide on how much capacity to guarantee for contract customers and how
much capacity to retain for the spot market. As starting point for potential research, we
refer to Levin et al. (2012) as key reference.
4.2. Hotel

The hotel industry was one of the earliest industries adopting RM techniques. Various authors have reported that RM can improve revenues by 2% to 6% (Vinod and Vinod 2004, Pekg"un et al. 2013, Pimentel et al. 2018, and Saito et al. 2019).

Hotel RM can be formulated as a standard network RM problem. A product corresponds to a multi-day stay at a certain price, and each day of the service period represents a resource with a capacity equivalent to the number of rooms. There are some structural differences to the airline RM problem: while an airline’s itineraries seldom have more than three subsequent legs, stays in a hotel for a week or longer are not uncommon. Thus, hotel RM problems do not have a clear end of the service period. A popular way to circumvent the resulting infinite horizon is to specify certain cut-off days and to use standard methods on a rolling horizon basis. In this context, Zhang and Weatherford (2017) explore DPD for dynamic pricing of hotel rooms; see also Aslani et al. (2013) for a similar approach.

Overbooking (§3.4) is an important concept in hotel RM to hedge against cancellations; see Koide and Ishii (2005), Ye et al. (2019) for static and Sierag et al. (2015) for dynamic models, respectively. In a recent publication, Aydin and Birbil (2018) additionally consider the case of so-called stay-overs, i.e., customers that request an extension of their reservation during the service period. Under the assumption that those requests occur with certain probabilities, the resulting problem is very similar to the network RM problem with overbooking and no-shows. The authors extend the approach of Birbil et al. (2014) in order to decompose the multi-day problem with stay-overs by product types (i.e., pairs of check-in and check-out days).

An interesting research topic is the interaction with online platforms such as Booking.com or Expedia. Recent work includes Anderson and Xie (2012) on opaque selling and Sierag (2017) on flexible products. More work is desirable on how to deal with competition in the sharing economy, e.g. on how the availability of rich listing data from AirBnB could be exploited in RM systems of hotel chains that compete with this platform. In the marketing community, Zervas et al. (2017) recently investigated the economic impact that AirBnB has on hotels, but the research question of whether and if so how RM systems should take the available information into account is still open.

4.3. Car rental

Car rental RM shares some characteristics with hotel RM, such as the fact that the service period spans several days. Resources refer to a certain car type at a specific day.
of the service period. The capacity of a resource is equivalent to the inventory of the car type. A product refers to a rental of a car type over a number of days, linked to a certain price, a pick-up and a return station, and some additional options such as insurances. A multi-day rental consumes a capacity unit from multiple resources, thus this application falls under the category of network RM. Car rental RM has some additional characteristics that make it quite challenging: extensive use of upgrades, one-way rentals, flexible and uncertain inventory (fleet management across stations, uncertain return stations and rental durations) and contractual versus walk-in customers. An overview is also given in Lieberman (2007) and a recent survey by Oliveira et al. (2017). Guillen et al. (2019) report on the components and success of EUROP CAR’s RM systems, which has improved both the fleet utilization by around 3% and the revenue-per-day by around 2% in the markets France, Portugal, and Spain.

Upgrading is discussed as a concept in §3.3 with a focus on car rental. One-way rentals allow customers starting and returning their rentals at different stations. Guerriero and Olivito (2014) include one-way rentals by assuming that there is a known pick-up and a corresponding return station for each product. The authors provide the DLP and use the corresponding BPC. In practice, the return station is often not certain.

Another characteristic of car rental RM is the flexible capacity. A traditional RM assumption is that capacity is fixed. However, in car rental, the fleet size remains flexible to a certain extent even on an operational level, as the firm may transfer the fleet between stations as well as acquire or register additional cars in the short term. Haensel et al. (2012) consider car transfers between stations. They formulate their problem as two-stage stochastic program. In the first stage, both a booking limit control policy and a transfer policy for the days of the service period are determined, while the second stage captures the random demand. Car transfers between stations are also considered in online optimization settings, that is, without RM decisions or even anticipation of future demand (e.g., Fink and Reiners 2006 and Conejero et al. 2014). Li and Pang (2017) consider a single station, where additional cars can be procured at pre-defined costs. Different from standard formulations, the authors model the problem as infinite horizon dynamic program and propose two heuristic approaches to solve it. Apart from the RM literature, please note that there is a large body of literature on isolated fleet planning (e.g., Pachon et al. 2006 and George and Xia 2011). Furthermore, capacity is uncertain due to the unforeseen variation of the return station or rental duration; similar issues arise in hotel RM with
stay-overs as mentioned in §4.2.

A trade-off needs to be managed between long-term contracts with business customers at lower rates and walk-in customers paying higher rates. In addition to the usual demand arriving over the selling horizon, so-called walk-in customers show up in the last minute and are typically willing to pay a higher price. Without specific industry context, Gans and Savin (2007) consider the problem with stylized contract and walk-in customer types. Contract customers pay a fixed rental fee or may be rejected against a penalty, whereas walk-in customers are dynamically priced. The structure of the optimal control policy is analyzed for an infinite planning horizon.

Similarly, as in the hotel industry, the sharing economy is also introducing new challenges for RM research in the car rental domain. For example, at Gatwick Airport in the UK, a car sharing offering ‘Car & Away’ has been launched in January 2018 that allows travelers to rent out their car throughout their absence. Airports already are using RM to manage pricing for parking. The potential to rent out cars that otherwise would be standing idle on the car park is attractive as a new income stream, and it poses new research challenges: whilst the RM problem is naturally similar to car rental RM, we have a major difference in as far as we have a constantly changing ‘fleet size’, which makes planning considerably harder.

5. Innovative industry applications

Beyond the industries discussed or at least mentioned in §4, many others have adopted RM concepts over the last decades, where the application of RM is less obvious: broadcasting and internet advertising (see Pandey et al. (2017) for a review), cloud computing (e.g., Püschel et al. 2015), apartment and office rental (e.g., Chen et al. 2014), banking, (airport) parking (e.g., Guadix et al. 2009), or e-commerce. However, taking a closer look, these applications still share many similarities with the ones presented, both from a modeling and methodological point of view. In this section, we discuss two applications that combine RM techniques with scheduling: attended home delivery (§5.1) and manufacturing (§5.2).

For the case of attended home delivery, we will present a DP formulation, because its use is quite common and researchers study essentially quite similar settings in this research area. In manufacturing, authors study a quite distinct setting and usually do not use such a framework explicitly so that we skip the DP.
5.1. Attended home delivery

Online grocery retailing is growing year-on-year: for example, in the United Kingdom, it is the quickest-growing sector in UK grocery with annual market growth of around 13% (as compared to only a 2.5% growth of the wider grocery sector in 2017), as reported by [Carrol, 2016]. Competitive pressure squeezes the profit margins such that efficient fulfilment services are essential. Home delivery of groceries usually requires the customer to be at home and therefore delivery time windows need to be offered that often are relatively narrow (such as 1 hour) to increase convenience for the customer. This may lead to expensive fulfilment operations, thus triggering research into ways on how to influence customers’ delivery slot bookings using slot availability control or slot pricing to arrive at more efficient routes. The same logic also applies to other e-commerce sectors, such as sales of large furniture items and the like. See [Agatz et al., 2008] for a general review on e-fulfilment and [Agatz et al., 2013] as well as [Snoeck et al., 2018] for recent reviews specifically dedicated to RM in last-mile delivery.

Using RM terminology, a product corresponds to one of the possible delivery time windows. Accepting an order in a time windows consumes delivery capacity which may impact a firm’s ability to serve other customers since it has a finite fleet. Therefore, the fulfillment resources (vehicles) link the products such that we can regard the control problem of dynamically deciding on time window availability and/or prices as generalized network RM problem. It is generalized in as far as it is not clear which route a vehicles will be assigned to at the time of receiving an order, and as such, the concept of ‘remaining available capacity’ to serve the demand for a product is not straight-forward. The actual routes are only finalized once no more orders can be received (in the context of next-day delivery). When considering same-day delivery, demand management and vehicle dispatch and routing decisions need to be made in conjunction rather than in succession.

We can use dynamic programming to characterize the optimal control policy for dynamic time window availability control (or dynamic pricing). Assuming unit-sized orders, the state $I_t$ of orders received until time $t$ is described by tuples $(i, j)$ (one for each time period where a customer placed an order), with $i \in I$ denoting a customer’s delivery location and $j$ the selected time window. Let $\lambda_i^{(i)}$ denote the probability of a customer arrival from location $i$ at time $t$, mirroring a segment of one as described in [3.5] As mentioned above, a customer reveals the order as well as the delivery location at the time of arrival, based upon which the service provider can decide on the offered time windows. Therefore,
\( r_i \) denotes a customer’s contribution margin due to the order. The optimal expected profit \( V_t(\mathcal{I}_t) \) satisfies the Bellman equation

\[
V_t(\mathcal{I}_t) = \sum_{i \in \mathcal{I}} \lambda_i^{(i)} \max_{S_t \subseteq \mathcal{J}} \left\{ \sum_{j \in S_t} P_j(S_t) \left( r_i + V_{t+1}(\mathcal{I}_t \cup (i,j)) \right) \right. \\
+ \left. \left( 1 - \sum_{j \in S_t} \lambda P_j(S_t) \right) V_{t+1}(\mathcal{I}_t) \right\}
\]  

(5.1)

with boundary conditions \( V_{T+1}(\mathcal{I}_{T+1}) = -\infty \) if there is no feasible solution to the vehicle routing problem with time windows (VRPTW) in the service period, and \( V_{T+1}(\mathcal{I}_{T+1}) = -C(\mathcal{I}_{T+1}) \) else. The term \( C(\mathcal{I}_{T+1}) \) is a shortcut for the optimal delivery costs of the VRPTW.

Formulation (5.1) introduces difficult challenges. The boundary condition requires checking in a fraction of seconds for each customer arrival and time window whether there is a feasible route plan if the customer is accepted. Moreover, an approximation of the value function (or, equivalently, the opportunity costs) requires anticipation of the future number of customers that can be served and their value as well as of the final delivery costs.

Regarding the feasibility problem, most approaches utilize variants of the well-known cheapest insertion heuristic, going back to Rosenkrantz et al. (1974) and Solomon (1987). The idea is to maintain at least one feasible route plan when transition from one state to the next. Given a feasible route plan, a new customer is inserted into a position that leads to the lowest increase in delivery costs.

Under the independent demand assumption, Campbell and Savelsbergh (2006) suggest to use those so-called insertion costs as opportunity cost estimates. Ehmke and Campbell (2014) additionally consider time-dependent travel times. Yang et al. (2016) use the insertion costs together with a modification that additionally accounts for historical route plans as opportunity cost estimates. Both Yang and Strauss (2017) and Klein et al. (2018) replace the VRPTW in the boundary with an approximation, namely the cluster-first-route-second delivery costs approximation of Daganzo (1987) and a model-based approximation building on the seed-based heuristic of Fisher and Jaikumar (1981), respectively. The advantage of using an approximation in the boundary is that (5.1) can be traced back to a simpler approximate dynamic program that only stores the number of reservations for the time windows and some subareas (e.g., due to zip codes) of the whole delivery region. Yang and Strauss (2017) use a simulation-based approximate dynamic programming approach to heuristically solve the resulting problem, whereas Klein et al. (2019) plug their approxi-
mate routing model into the CDLP and apply CEC. In contrast to the previous literature, Ulmer and Thomas (2019) as well as Koch and Klein (2018) derive opportunity cost estimates directly from the route plans maintained throughout the booking horizon. Their basic idea is to measure and valuate the time left in the service period for delivery of future customers. Asdemir et al. (2009) and Cleophas and Ehmke (2014) work with the limiting assumption that delivery capacity levels are committed a priori to each time window and subarea. Therefore, delivery costs are fixed, and the problem reduces to a standard network RM problem with the remaining capacity of each time window and subarea in the state space.

Attended home delivery is linked to the concept of flexible products that was discussed in §3.2: Gulpinar et al. (2018) study this link under a scenario where customers can choose either regular or flexible time windows; the latter consist of several regular (say, 1 hour) slots. A customer is allocated to one of these regular time windows (and informed of this) only shortly prior to their designated delivery day. In exchange for granting this flexibility, they receive a discount. This concept is currently being used by Tesco in the UK.

In future research, the linkage of RM in attended home delivery to other existing streams should be made more transparent and eventually exploited, e.g., to online customer selection in supply chain management (e.g., Elmachtoub and Levi 2015, Elmachtoub and Levi 2016) or to dynamic vehicle routing (e.g., Bent and Van Hentenryck 2004, Ulmer et al. 2018). For example, as a result of such an integrated study, research in same-day delivery could be pushed and streamlined. Furthermore, there are many research challenges in shipping alliances: see Allen et al. (2017), for example. In the sharing economy concept, two or more shipping companies work together to increase efficiency. Managing joint pricing and revenue splitting rules is rather difficult due to the large variety in shipping requests. Finally, RM challenges arise in crowdshipping: for instance, how should the payment rates for crowdsourced drivers be dynamically set so as to influence driver availability in the right locations and times, and how to integrate this with demand-side pricing and availability decisions?

5.2. Manufacturing

Although the major part of the RM literature is devoted to the service industry, there has been a significant interest over the past decade to expand the available-to-promise or capable-to-promise logic of manufacturing’s advanced planning systems by RM ideas; see Quante et al. (2009b) for a previous review. In the context of manufacturing, the task of
RM is to balance demand and production. As demand is usually given by independent orders, the resulting problems are also known as dynamic order acceptance and scheduling problems. To date, most research in order acceptance and scheduling is on deterministic models (see Slotnick 2011). Clearly, the manufacturing sector is vast and literature often investigates somewhat unique problems such that our discussion is necessarily generic. In the following, we first review the RM literature in make-to-order (MTO) manufacturing, before we discuss some extensions in a make-to-stock (MTS) and assemble-to-order (ATO) context.

In MTO manufacturing, production is triggered by an arrival of a customer. A bottleneck resource typically refers to a machine during a certain time interval of the service period. An order, which may refer to the production of some standardized products or to a unique production contract, is associated with a contribution margin as well as a machine usage over a certain period of time. Thereby, most of the literature represents machine capacity and orders’ capacity consumptions on an aggregate level, that is, as available and required production time during a sufficiently small time span of the service period. In the following, we denote those small time spans as planning periods.

Under this aggregated representation of machine capacity, there is some literature which considers an upfront booking horizon followed by the production in the service period. Spengler et al. (2007), Hintsches et al. (2010), and Volling et al. (2012) assume that orders have a clear capacity consumption over the service period, e.g. because only a single planning period is considered, such that the problem can be reduced to a standard network RM problem. In a similar setting, Hung and Lee (2010) and subsequently Hung et al. (2014) assume that the capacity consumption and the profit are random variables.

More commonly, in contrast to the standard network RM problem, a real-time planning horizon consisting of a largely overlapping booking horizon and service period is considered. Perry and Hartman (2004) formulate the resulting multi-period problem as dynamic stochastic knapsack model, ignoring that orders are usually associated with certain attributes, in particular with due dates. Integrating those attributes implies that the production sequence has to be planned. Clearly, the capacity consumption depends on these scheduling decisions. If preemption is not allowed, the state space comprises both the accepted orders and the remaining machine capacity over the future planning periods. The latter accounts for those orders for which production has already started. In such a setting, Barut and Sridharan (2004) as well as Barut and Sridharan (2005) consider a
single bottleneck machine and develop approaches based on protection levels, while Gallien et al. (2004), Chevalier et al. (2015), as well as Mlinar and Chevalier (2016) investigate the problem in infinite horizon. Closer to our network RM framework, Guhlich et al. (2015a) consider the production on several machines organized in a flow shop over a limited planning horizon. The authors suggest a BPC based on a RLP.

It may be the case that orders are associated with further attributes such as release dates, sequence-dependent setup times, or precedence constraints. In this case, it is not sufficient to represent machine capacity on an aggregate level. Instead, the inherent scheduling problem has to be considered in detail, similar to the attended home delivery setting. Some literature tackles such problems in an online optimization environment (e.g. Ebben et al. 2005 as well as Mandelbaum and Shabtay 2011). Only few papers integrate the anticipation of future orders. For this purpose, for each incoming order, a tentative schedule has to be built, e.g. by priority rules or a cheapest insertion heuristic, and evaluated. Arredondo and Martinez (2010) use a simulation-based approximate dynamic programming approach to evaluate the tentative schedules, while Xu et al. (2015) provide a DLP-based approximation and use CEC.

In contrast to MTO, a MTS manufacturer produces standardized products based on demand forecasts. Products are storable and, thus, a resource refers to the finished products’ inventory in a planning period. Correspondingly, the manufacturer has to keep track of the inventory in the state space. Inventory replenishment may be exogenous or endogenous. Quante et al. (2009a) and Yang and Fung (2013) consider the case of exogenous replenishments. Order sizes are stochastic and can be partially accepted. Because the authors consider the sale of a single product, the problems boil down to one-dimensional dynamic programs. Some papers study endogenous replenishments in joint MTO/MTS environments, characterizing the optimal policy in infinite horizon dynamic programs. Dfregger and Kuhn (2007) assume that, in each planning period, a request for the single product under consideration can be accepted or rejected and production to inventory can be started. Gupta and Wang (2007) as well as Iravani et al. (2012) allow two product types, namely one product that is produced to stock and one product that is produced to order.

Finally, an ATO manufacturer assembles intermediate materials to end products. If intermediate materials are held in stock and replenishment is difficult, they represent an additional resource that must be included in the state space; see Gao et al. (2012) and
As our above discussion reveals, literature at the intersection of RM and manufacturing is quite heterogeneous. In particular, it remains unclear to what extent the investigated problems could be connected and whether solution approaches could be transferred. Thus, future research should be devoted to developing an integrated research framework using DPs.

6. Conclusion and outlook

This review forms the second part of a literature review on RM, focusing on recent generalizations of the basic availability control problem in RM. The first part, Strauss et al. (2018), concentrated on methodological advances with regard to the standard problem of choice-based availability control.

For the future, we expect different generalizations discussed in this paper to be combined at least partially in some fashion to address current industry developments. Like in the past, the airline industry may set the scene. With the introduction of IATA’s New Distribution Capability (NDC), the operative task of RM will gradually move from availability control for a given set of products to some kind of offer management. Basically, NDC represents a communication standard based on the Extensible Markup Language (XML) (see e.g. Westermann 2013). In contrast to the existing global distribution systems, the new standard allows airlines to respond directly to requests from travel agents as this is also already possible via corporate web sites. The effect of allowing interactive communication during the shopping process can already be seen by the increasing incorporation of ancillaries and incremental sales into RM. For years, incremental sales have already played an important role e.g. in the context of the RM for cruise lines (e.g. onboard shopping, land excursions) or for casino hotels (see Metters et al. 2008 and Sturm and Fischer 2016). In the last decade, even the traditional major network carriers have started to unbundle their products in order to offer a more tailored service and to generate incremental sales. Among many others, the resulting ancillaries could include onboard shopping and drinks/meals, bag transportation, preferred seats, boarding priority a.s.o. Going beyond, airlines may dynamically design offers and set prices based on the specific information at the time of the request. For example, personalization may be used to determine the set of itineraries a customer is shown. Machine learning ideas can be useful in adding a learning component to a RM system as we discuss in §3.5 on personalization; we expect more research in this...
vein will emerge in the coming years, much of it driven by the current world-wide appetite for artificial intelligence approaches. This trend is also reflected by new machine learning-based commercial solutions offered by leading RM system providers such as PROS (who launched a new solution to optimize personalized offers using machine learning in 2017).

Airline practitioners and RM researchers may want to (re-)start investigating group bookings. From a methodological point of view, group bookings could be incorporated into the presented RM framework by defining one product for each possible group size. However, open research questions include how to accurately forecast group sizes (machine learning techniques may help here) and to study the impact of group bookings when relaxing the common assumption that all bookings are for individuals.

The applications in attended home delivery and in manufacturing share the commonality that products correspond to orders for which some scheduling has to be performed during and/or after the selling horizon in case they are accepted. This is necessary to determine whether the corresponding services can feasibly be delivered with the available resources and to calculate the resulting costs for service provision as a basis for maximizing the expected profit. Thus, they address the interface of demand and operations management. We believe that this interface will get increased attention in the forthcoming years due to new business models and the necessity for a better alignment of these two functional areas. For example, in healthcare, web- and mobile-based systems for patient self-scheduling like ZocDoc or Mychart are becoming popular. Ridesellers like Uber or Lyft start offering real ridesharing services like UberPool, UberExpressPool, or Lyft Line, which require some scheduling to match demand with supply. Equipment rental, which has been common in the construction and transportation industry for years, is increasingly used in the B2C context (for example, in the case of bike and car sharing). Finally, in omni-channel retailing, we see the increasing use of RM for order fulfilment to balance demand and supply over different channels.

Including a scheduling component into availability control makes the solution of the corresponding dynamic program much more difficult. As a consequence, scalable optimization approaches like (simulation-based) approximate dynamic programming (ADP) are likely to be seen more frequently for such applications in the future. A number of studies have already shown a high potential of ADP in traditional RM settings (e.g., [Huang and Liang 2011, Koch 2017]), where such approaches have been competitive to mathematical programming approaches like ALP or DPD. As simple structures such as bid prices are unlikely to
work well in integrated RM and scheduling environments, the key issue here will be how to capture the most important features of the state space, given the mutual dependence of already accepted and future orders. In general, we expect that the combination of RM approaches with other domains such as scheduling (but not limited to it) will continue to provide a fertile ground for future research.

References


Chen, J., Wang, J., Bell, P. C., 2014. Lease expiration management for a single lease term
in the apartment industry. European Journal of Operational Research 238 (1), 233–244.


Klein, R., Neugebauer, M., Ratkovitch, D., Steinhardt, C., 2019. Differentiated time slot pricing under routing considerations in attended home delivery. Transportation Science


Steinhardt, C., Gönsch, J., 2012. Integrated revenue management approaches for capacity


