Reflections upon Measurement and Uncertainty in Measurement

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Abstract. Measurements of physical quantities are the corner stone upon which we humans have built the scientific perception of the world. They characterise the scientific system of beliefs: measurements are the distinctive means to tell the scientific truth apart from any other kind of approach to knowledge. Moreover, measurements have been having a central part in the development of any sort of machine, instrument and artefact that humans have invented. Yet in industry, especially in small-medium size enterprises (SME's), time and money spent in measurements is often seen as a necessary evil or, worst, as a waste of valuable resources. To contribute in contrasting this negative perception, a clarification of the fundamental concept of measurement is presented. The emphasis is in particular placed on uncertainty in measurement. The need for the introduction of the concept of uncertainty is justified. The theoretical implications attached to uncertainty of measurement are analysed.

Keywords: Measurement uncertainty, International vocabulary of metrology (VIM), Guide to the expression of uncertainty in measurements (GUM)

1 Introduction

Soft-computing may be described as the collection of those computational methods robust to indeterminacy (vagueness) and capable of giving likely-suboptimal yet *sufficiently good* solutions in a simple and reasonably time-inexpensive way [8]. For example, among these techniques there is fuzzy logic, a conceptual framework developed by Lofty Zadeh in 1965 within his theory of fuzzy sets. To account for vague, qualitative, indeterminate concepts, notions such as degree of membership of an element to a set and degree of truth of a proposition were introduced and developed in a consistent formal theory.

The fact that soft-computing methods have been specifically introduced to overcome the difficulties in handling vagueness and qualitative knowledge in computational environments has generated a quite widespread misconception: once a quantity has been measured, then all the vagueness has vanished, because the

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quantity is described by a number. In truth, instead, any measurement result is inherently uncertain. In this sense therefore measurements pertain in their own right to the domain of knowledge where soft-computing techniques may be applied. In this piece of writing frequent reference is made to two of the most authoritative sources of reference in metrology, namely: the International Vocabulary of Metrology (VIM) [10] and the SI Brochure [2]. In the next section the concept of measurement is explored. In section 3 the indeterminacy inherent into the comparison of a quantity with a unit of measurement is presented. The impossibility of obtaining certainty in the realisation of unit of measurements is highlighted in section 4. The impossibility of defining completely and unambiguously any measurand is then described in section 5. A discussion follows and conclusions are drawn thereafter.

2 Measurements

Measurement is any experimental process aimed at obtaining one or more numbers and a reference that can be attributed to a property of a body, a phenomenon or a substance (cf sections 1.1, 1.19, 2.1 in [10]). This property is called a quantity and its magnitude is defined as the number and the reference considered together. The reference typically is a measurement unit (e.g. the kilogram, when measuring a mass), but it can be a measurement procedure (e.g. Rockwell C, when measuring hardness) or a reference material (e.g. the concentration of luteinizing hormone in a specimen of human blood plasma. Cf sections 1.1 and 1.19 in [10]). The Measurement unit is a quantity selected conventionally to which any other quantity of the same kind can be compared. The result of this comparison is called the ratio of the two quantities and is expressed as a number (cf section 1.9 in [10]).

From a logic perspective, it then follows that three conditions are necessary for a measurement result not to be intrinsically uncertain:

(a) It should always be possible to compare the measurand (i.e. the quantity intended to be measured, cf section 2.3 in [10]) and the measurement unit so that no indeterminacy is present in the numerical quantity value (cf section 1.20 in [10]);
(b) The unit of measurement should have an unambiguous magnitude;
(c) The measurand should be defined without any indeterminacy.

Unfortunately, none of these conditions holds, as it is described in sections 3, 4 and 5 for (a), (b)and (c), respectively.

3 Comparisons of the Measurand to the Unit

The comparison of the measurand and the measurement unit is achieved by the interaction of the body, phenomenon or substance under study and a measuring system that produces an *indication* sensitive to the measurand. A measuring system is any set of devices that is designed to generate measured quantity values (cf sections 3.2 and 2.10 in [10]). Typically, the nature of the interaction measurand-measuring system cannot be isolated from other quantities characterising the conditions in which the measurement takes place. For example, when measuring the height of a table with a tape measure the measured height value is not only a function of the height of the table, but also of a number of other quantities. Among these, there are for instance the field of air temperature and air humidity affecting the wood of the table, the temperature of the hands of the person holding the tape measure, the resolution of the tape measure, the discretion of the person reading the indication of the tape measure when judging the alignment of the tape with the table and the alignment of the scale on the tape with the extremes delimiting the table height. This interdependence between quantities is ideally captured by 'a mathematical relation among all quantities known to be involved in a measurement' which is called the measurement model (cf section 2.48 in [10]). Namely, it holds:

$$h\left(Y, X_1, X_2, \ldots, X_n\right) = 0. \tag{1}$$

In equation 1, Y is the measurand or output quantity of the model whereas the other quantities $X_1, X_2, \ldots, X_n$ are called the input quantities in the measurement model. The value of the output Y is to be calculated from that of the input quantities. Often equation 1 can be explicitly defined as follows:

$$Y = f\left(X_1, X_2, \ldots, X_n\right). \tag{2}$$

In equation 2, the function $f\left(X_1, X_2, \ldots, X_n\right)$ is referred to as measurement function (cf section 2.49 in [10]). This situation generates indeterminacy of the measurement in at least two different ways. First, the input variables in a measurement are not uniquely known. The input variables included in a model depend on the expected use of the measurement result. For example, a measurement result having implications on the life of many would justify the investigation of a large number of potential input variables for inclusion in the measurement model. The extra cost incurred for the broad array of instruments needed to measure these input variables would be justified. Second, the measuring model $h(\ldots)$ (or function $f(\ldots)$) is typically not known analytically, barring the cases when some physical model of the measurement is available. Hence the effect of an input variable upon the output must be estimated. Figure 1 is inspired to Figure 4 in ISO 14253-2:2011 [1]. It displays a grouping of candidate input variables. The figure suggests a possible systematic procedure for selecting input variables for analysis without omitting some relevant group of them. In this context, the term uncertainty used in the figure can be interpreted in its generic meaning of vagueness or indeterminacy. The figure has been produced using a free software package [12] (free as freedom not as gratis).

4 Units

The units of measurement do not have an unambiguous unique magnitude. To support this statement, the concept of 'definition of a unit' must be distinguished

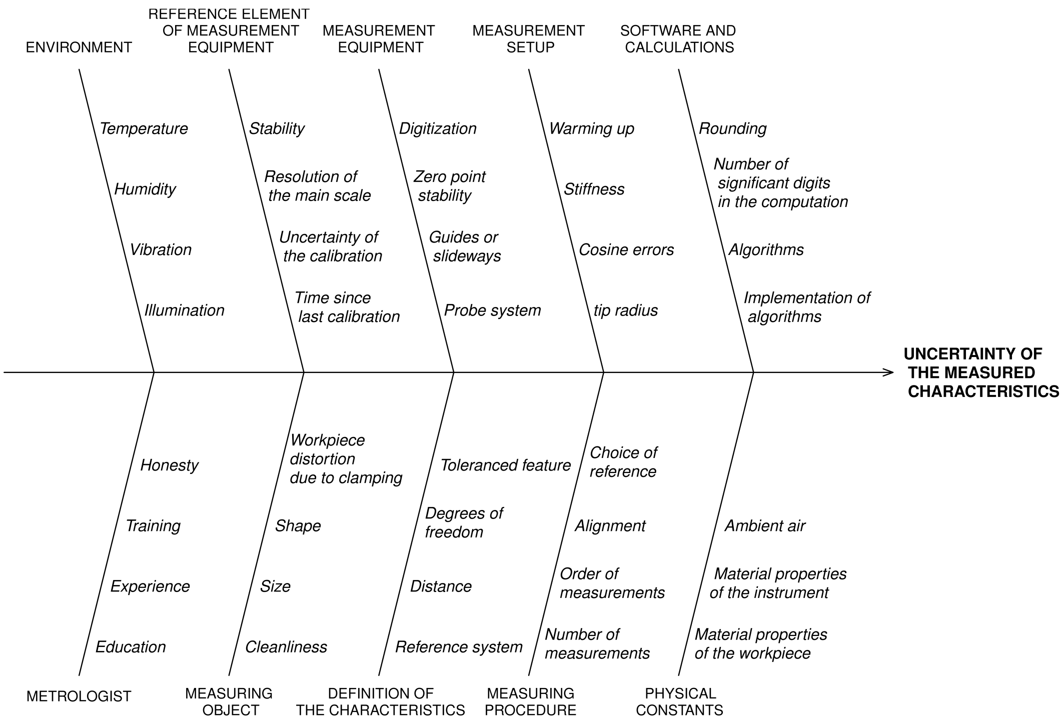

Fig. 1. An Ishikawa (fishbone) diagram with a few potential sources of uncertainty that are typically investigated for dimensional measurement results. The sources are grouped according to ISO 14253-2:2011 [1]

and kept apart from the concept of ‘realisation of a unit’, as explained in the SI Brochure (cf page 111, section 2.1.1 in [2]). A measurement unit is a quantity that is conventionally defined so as it has solid theoretical foundations and it enables measurements as reproducible as possible. The realisation of a unit is instead a procedure where a quantity value is associated to a quantity of the same kind as the unit and that fulfils the definition given for the unit. In other words, the realisation of a unit is a measurement assigning a magnitude to a realised unit, i.e. to a quantity existing in the sensory world and not just on the paper as in the unit definition. The indeterminacy and vagueness discussed above for the concept of measurement does then generate the indeterminacy in the magnitude of the realised unit. For example, the unit of length in the International System of Units (SI – Le Système International d’Unités) is the metre which is defined in the SI Brochure as ‘the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second’ (cf page 112, section 2.1.1.1 in [2]). Guidelines for the realisation of the metre are instead presented in what are referred to as the *mises en pratique*, which are published on the web to facilitate frequent revision [3, 4].

5 Measurands

To define unambiguously a measurand an infinite amount of information is needed. For example, to define the height of a table, a point P_1 on the top surface of the table could be identified. A straight line l_1 orthogonal to that surface and passing through P_1 could be constructed. Let P_2 be the point of intersection between l_1 and and a second surface representing the floor. The height of the table can be defined as the distance between P_1 and P_2. The example is illustrated in Figure 2 where two different distances satisfying the measurand definition above are shown (l_1 (A) and l_1 (B). In fact, in this definition of height of a table, there are infinite possible choices for the starting point on the top surface. The height of the table is deeply affected by such a choice. In Figure 3, two measurement results of the two distances satisfying the definition of height of a table as proposed above are displayed. The measurement results have been obtained using an articulated arm coordinate measurement machine driven by a proprietary software widely used in industry. Moreover, a large variety of different types of surfaces can be selected to represent the top surface of the table and the floor: from the natural choice of a plane shown above, to that of some more complex non-uniform rational basis spline surfaces (NURBS surfaces). In addition, a range of different choices can be made when associating the chosen type(s) of surface, which is an abstract entity of the human rationality, to a physical table or floor, which are sensory entities perceived by humans using their senses (sight and touch in this case). This unavoidable intrinsic vagueness in the definition of a measurand is called definitional uncertainty (cf section 2.27 in [10]).

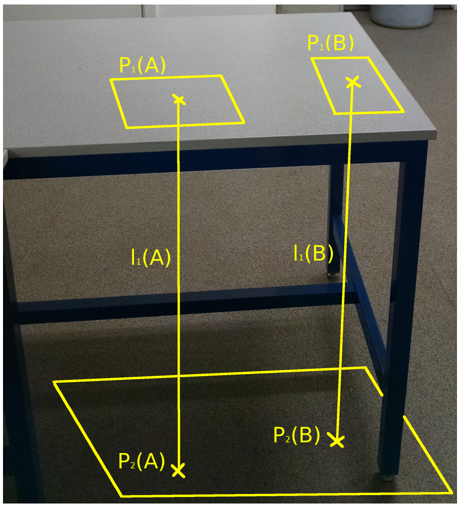

Fig. 2. Two different distances both compliant with the definition of the measurand height of a table introduced above

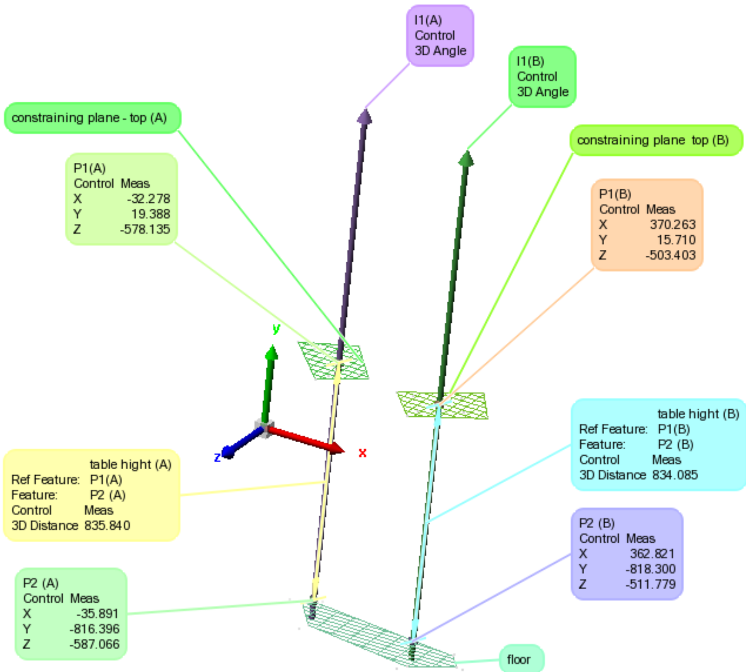

Fig. 3. Two measurement results of the two heights of a table displayed in figure 2

6 Discussion

The argumentation presented so far is aimed to make the intrinsic indeterminacy of measurement results apparent. In the Guide to the Expression of Uncertainty in Measurement (GUM), this indeterminacy or vagueness that expresses a doubt about the result of a measurement is referred to as uncertainty (cf section 2.2.1 in [9]). In the same document, the term uncertainty is however also used in a more specific way to designate a parameter providing a quantitative measurement of this generic concept of doubt. Namely, in the GUM uncertainty is defined as a 'parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand' (cf section 2.2.3 in [9]). In the VIM instead, measurement uncertainty is defined as a 'non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used' (cf section 2.26 in [10]). Typically this parameter is the standard deviation of a probability density function that models the incomplete or partial state of knowledge of the measurand achievable with measurements.

Acknowledging that measurements are always unavoidably uncertain has some profound implications on how humans construct their knowledge about the physical world in science and technology. Measurements are the ultimate source of knowledge in science: any statement to be scientifically accepted must be substantiate by experiments or observations that are expressed in terms of measurement results. If measurement results are inherently uncertain, all what can be inferred from them can be only uncertain. In other words, talking of 'exact science' when referring to Physics, for example, can be quite prone to misinterpretations. Science may be considered exact only in its methods of dealing with approximations and uncertain or partial knowledge. Stretching this view to its extreme may lead to consider science as an activity with very useful practical effects but with little use in the unambiguous identification of the truth.

Recognising that measurements are uncertain is a fact that humans can exploit in acquiring new knowledge about the physical world. Investigating the uncertainty structure in designed experimental conditions may enable experimenters to add new contributions to knowledge. Examples of how the characterisation of uncertainty fosters the acquisition of new knowledge have been directly experienced by the author in the investigation two different machining processes (micro electric discharge machining and contamination-free turning [6, 7]).

The practical importance of uncertainty in measurement is perhaps epitomised by all those cases where decisions have to be made on the basis of measurement results. For example, how could a plaster cast of a sculpture be distinguished from the original work of art on the basis of its form alone? In Figure 4 the digitisation process and the resulting digital model of a 19^{th} century plaster cast of the head of David by Michelangelo is displayed. The original David is a masterpiece of Renaissance sculpture created by Michelangelo between 1501 and 1504 currently housed in the Gallery of the Academy of Florence (Firenze, Italy). Even if the plaster cast were identical to the original, the uncertainty in measurement inherent in the digitisation process would prevent the virtual model to be identical to the plaster cast. Likewise, if a digital model of the original David were created in the same way, also that digital model would be affected by uncertainty in measurement. Then a comparison between the two digital models can only happen in probabilistic terms. Likewise, any statement regarding the identity between the plaster cast and the original sculpture made on the basis of these measurement results would necessary be of a probabilistic nature.

Fig. 4. Scanning of a 19^{th} century plaster cast of David by Michelangelo (left). The plaster cast is housed in the 'Michelangiolesco Museum' of Caprese Michelangelo (Italy). The digital model (right) is made of $37,035,104$ points (courtesy of CAM2 *s.r.l.*, Grugliasco, Italy)

Soft-computing methods with their ability to account for indeterminate and partial knowledge may provide alternative approaches to account for uncertainty in measurements. The envisaged possible benefit is that these alternative perspectives may in turn promote and facilitate the acquisition of new knowledge in science and technology. From an industrial point of view, a number of software applications have been developed and commercialised to assist practitioners in the evaluation of the uncertainty in their measurements. In Figure 5 is displayed the graphical output of a Monte Carlo simulation of the measurement of coordinates of a field of constructed points in space. The measurements being simulated refer to a laser tracker that has been set up in two different locations. In the simulation, each point has been measured 1000 time from each of the two

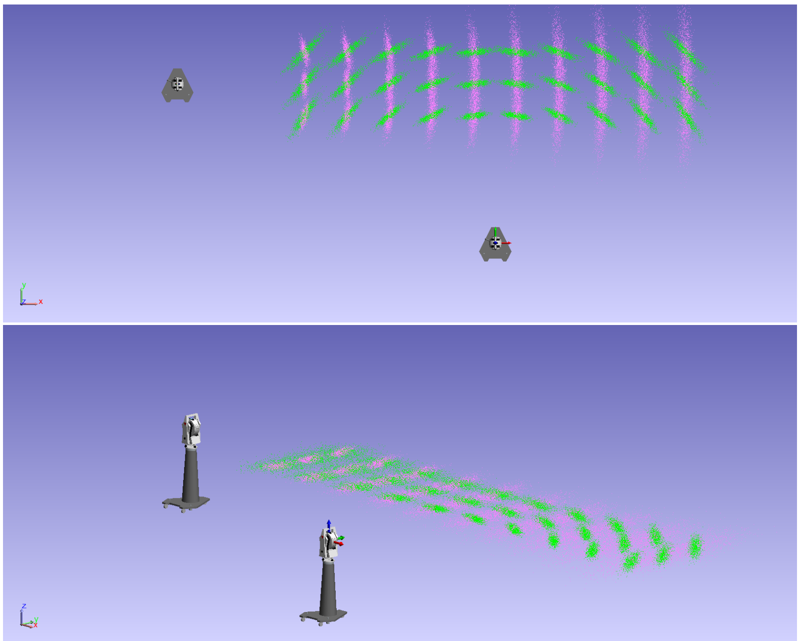

Fig. 5. Graphical representations of Monte Carlo simulation of point coordinate measurements taken with the same laser trackers placed in two different positions (starting from above: top view and axonometric projection, respectively)

tracker positions. In the figure, the two sets of 1000 measured points for each constructed nominal point are clearly different for the two tracker positions. Each of these sets of 1000 points is sometime referred to as point uncertainty field. The simulation allows a practitioner to assess visually the effect that the location of the instrument has on the reliability of the measurement taken. This kind of simulations are quite widespread in industrial portable metrology, most typically in the aerospace industry.

From a research perspective, the main issue with commercially-available software applications as the application used for figure 5 is that the large majority of the software houses active in this field operate a business model centred around restrictive licenses. They tend not to make the information regarding the models and algorithms available to their customer base. To contrast this difficulty, some researchers have endeavoured to build their own tools making them available under one of the licences from the Free Software Foundation. For example, within the free software environment for statistical computing and graphics called R [11], a couple of packages regarding measurements and uncertainty are *MetRol-*

ogy [5] and *propagate* [13]. They enable the users to make Monte Carlo and Bayesian evaluations of uncertainty within R. Some consideration of the benefits and limitations of applying soft-computing techniques in measurements problems and in uncertainty evaluations in particular has been given in the past [14]. However, the relationship between soft-computing and metrology still appears a promising open field of investigation.

7 Conclusions

An analysis of the concept of measurement and measurement result has been presented. This analysis supports the idea that even when a quantity is measured, it is not completely known. This partial state of knowledge has been ascribed to three sources of indeterminacy unavoidably-attached to any measurement result: the measurement process itself, the unit of measurement and the definition of the measurand. Uncertainty as a technical term introduced by authoritative international bodies as a means of representing this intrinsic indeterminacy in measurements has been discussed. Soft-computing methods may offer alternative models of measurement uncertainty facilitating the acquisition of new knowledge of the physical world we all live in.

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