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Fundamental Quantum Limits of Multicarrier Optomechanical Sensors

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Optomechanical sensors involving multiple optical carriers can experience mechanically mediated interactions causing multimode correlations across the optical fields. One instance is laser-interferometric gravitational wave detectors which introduce multiple carrier frequencies for classical sensing and control purposes. An outstanding question is whether such multicarrier optomechanical sensors outperform their single-carrier counterpart in terms of quantum-limited sensitivity. We show that the best precision is achieved by a single-carrier instance of the sensor. For the current LIGO detection system this precision is already reachable.

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Introduction.—The use of quantum-mechanical systems and nonclassical properties for high-precision estimation tasks has attracted interest in a number of sensing schemes, including in laser-interferometric gravitational wave (GW) detectors [1–5] and related problems [6,7], magnetometry [8,9], and atomic clocks [10,11]. Direct detection of GWs was one of the earliest problems to demand such analysis [12], suggesting use of nonclassical light—squeezed vacuum states—to improve precision [1,13].

Sensing mechanical displacements optically, such as in laser-interferometric GW detectors [14,15], relies on interactions between optical and mechanical degrees of freedom which is the domain of optomechanical [16,17] sensors. Light incident on a mechanical oscillator causes the mechanical oscillator to act as an active element which produces squeezing of optical modes [2,18]—the so-called ponderomotive squeezing. Such squeezing acts as a noise source constraining the current generation of laser-interferometric GW detectors [2,14] due to anticancelling of the quadrature in which the signal is encoded which manifests as a measurement backaction, with techniques to avoid such backaction drawing significant interest [19–21]. The same effect has been demonstrated as a squeezed light source [22–24], which can potentially improve sensors’ precision [1,2,13].

The extension to multimode optomechanical systems has proven fruitful in both the many mechanical [25] and optical [26,27] mode scenarios, as well as for optical frequency conversion [28,29]. This includes sensors such as laser-interferometric GW detectors, particularly those encompassing modifications which utilize multiple laser frequencies: so-called multicarrier interferometers. Originally implemented in Advanced LIGO for classical sensing and control purposes [30,31], a second carrier can improve the low-frequency sensitivity by partially canceling the strong backaction of the main carrier [32,33]. Multiple carriers can provide a means to enhance the sensitivity and surpass the standard quantum limit (SQL) [14] by using the optical spring effect, while not suffering from the instabilities associated with the single-carrier case and allowing for some shaping of the sensitivity curves [34,35]. The value of multiple carriers in improving the sensors’ fundamental quantum limit, which is more stringent than the SQL, remains open.

In this Letter we provide the fundamental quantum limits on the precision of multicarrier optomechanical sensors, including laser-interferometric GW detectors, using quantum metrology techniques. These limits are imposed by the classical and quantum Fisher information—via the Cramér-Rao bound (CRB) on precision of an estimator—from quantum estimation theory [36–41]. Our multimode analysis includes optical loss at the output and squeezed light injection, as well as the optomechanical interaction—the ponderomotive squeezing effect.

Multimode quantum states have been studied in quantum metrology [42–46]. By including a noise source which itself introduces multimode correlations, ponderomotive squeezing, for the first time we show that for a large class of optomechanical sensors multiple carriers are no better than single carriers. Hitherto neglected in estimation-theoretic quantum metrology studies of GW detectors [4,7], ponderomotive squeezing dominates the low-frequency quantum noise of GW detectors [2] as well as smaller optomechanical systems [47–49]. We bridge this gap, providing analytical expressions for the fundamental quantum limits of multimode optomechanical sensors featuring ponderomotive squeezing. This should guide the development of novel optomechanical sensors and the improvement of existing ones. Our large complement of results can be navigated using Table I.
TABLE I. Expressions for precision of various interferometer limits. The unsqueezed and lossless case can be most readily recognized from the lossy and unsqueezed case with limit $\eta = 1$. We provide some discussion of these results in the context of LIGO detector in Sec. VIII of the Supplemental Material [50].

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$^a$Attainable through the homodyne angle given by Eq.(17); otherwise, for general homodyne angles these are found as limits of Eq.(13) or in Sec. VI of the Supplemental Material [50].

$^b$Attainable through the homodyne angle given by Eq. (23); otherwise, for general homodyne angles these are given in Sec. VI of the Supplemental Material [50].

Framework.—We describe the optical part of our optomechanical sensor with a linear input-output relation,

$$\hat{b}(\Omega) = \mathcal{M}(\Omega)\hat{a}(\Omega) + h(\Omega)\bar{\mathcal{V}}(\Omega),$$  

(1)

where $\mathcal{M}(\Omega)$ is a complex matrix which determines a Bogoliubov transformation between the incoming and outgoing fields, and $h(\Omega)\bar{\mathcal{V}}(\Omega)$ is a displacement vector which encodes the parameter $h(\Omega)$. Such input-output relations are typically expressed in terms of the two-photon formalism [55,56], using the two operators $\hat{a}_1^{(a)} = (\hat{a}_{\omega + \Omega} + \hat{a}_{\omega - \Omega})/\sqrt{2}$ and $\hat{a}_2^{(a)} = -i(\hat{a}_{\omega + \Omega} - \hat{a}_{\omega - \Omega})/\sqrt{2}$. We introduce $\hat{d}$ pairs of such operators $\{\hat{d}_1^{(a)}, \hat{d}_2^{(a)}, \ldots, \hat{d}_2^{(a)}\}$ to describe the electromagnetic fields in an interferometer driven by light of multiple carrier frequencies $\{\omega_1, \omega_2, \ldots, \omega_d\}$. From these creation and annihilation operators, we can form Hermitian position ($\bar{\hat{x}}_{1,2}$) and momentum ($\bar{\hat{p}}_{1,2}$) operators, spanning the same phase space and obeying suitable commutation relations [50].

Suppressing the $\Omega$ argument for brevity, we focus on the case where we wish to estimate the size of the displacement $\delta$, with $\mathcal{M}$ and $\mathcal{V}$ consisting of the $2 \times 2$ and $2 \times 1$ blocks [33,34], see also Sec. II of the Supplemental Material [50],

$$\mathcal{M}_{jk} = e^{i(\beta_j + \beta_k)} \begin{pmatrix} \delta_{jk} & 0 \\ 0 & \delta_{jk} \end{pmatrix},$$

$$\mathcal{V}_j = \frac{\delta_{jk}}{\hbar_{\text{SQL}}} \begin{pmatrix} 0 \\ \chi \sqrt{2\kappa_j} \end{pmatrix},$$  

(2)

where $\delta_{jk}$ is the Kronecker delta, $\beta_j$ are phases, $\kappa_j \geq 0, \chi \in \{-1,1\}$ is the sign of the mechanical response and can be taken to be positive, since one with a negative response is identical to one with a positive $\chi$ with a fixed phase shift preceding and succeeding it, which can be captured by rotating input squeezing and output homodyne angles, respectively. The attainable precisions are thus directly related; see Sec. II of the Supplemental Material [50]. The presence of the $\sqrt{\kappa_j \kappa_k}$ term on the off diagonals produces a multimode squeezing across all the optical modes, which is ponderomotive in origin. The ponderomotive squeezing introduced with a single optical mode—with frequency $\omega$—is itself multimode with correlations between the $\omega + \Omega$ and $\omega - \Omega$. When multiple optical fields are used they each affect the mechanical motion and in turn the mechanical motion causes squeezing of each optical mode leading to entanglement between $\omega_j + \Omega$ and $\omega_k + \Omega$ optical modes.

In the case of a multcarrier laser-interferometric GW detector, as in Fig. 1 in the tuned configuration, $\kappa_i$ is the normalized intensity of the $i$th carrier,

$$\kappa_i = \frac{16I_i \omega_i \gamma_i}{mcL\Omega^2(\gamma_i^2 + \Omega^2)}, \quad \hbar_{\text{SQL}} = \frac{8h}{m\Omega^2L^2},$$  

(3)

where $I_i$ is the arm cavity power of the $i$th mode, $\omega_i$ the frequency of the $i$th mode, $\gamma_i$ the arm cavity half-bandwidth of the $i$th mode, $m$ the test mass, and $L$ the interferometer arm length [15]. The signal-recycling mirror [57–60]
introduces more involved input-output relations, but at low frequencies where radiation-pressure dominates the quantum noise, they can be approximated with the same form as Eq. (2) [61]. Interferometer modifications such as the quantum speed meter [15,60,62] also have the same form of input-output relations as Eq. (2), and our results can be applied directly with appropriate definition of input-output relations as Eq. (2), and our results can be applied directly with appropriate definition of $\kappa_i$.

As Eq. (1) is a linear mapping between creation operators, the optical fields through the sensor evolve under a Gaussian unitary [63]. Common input states such as (squeezed) vacuum are themselves Gaussian [1–3]; therefore the output state can be taken as Gaussian for relevant cases. From the evolution of the quadrature operators, 

\[
\begin{align*}
\hat{q}' &= \frac{\hat{M}\hat{a} + \hat{M}^\dagger \hat{a}^\dagger + \hbar \hat{V} + \hbar^* \hat{V}^\dagger}{\sqrt{2}}, \\
\hat{p}' &= \frac{\hat{M}\hat{a} - \hat{M}^\dagger \hat{a}^\dagger + \hbar \hat{V} - \hbar^* \hat{V}^\dagger}{i\sqrt{2}},
\end{align*}
\]

we can extract the displacement and symplectic operators

\[
\begin{align*}
\vec{d}_v &= \sqrt{2} \begin{pmatrix} \text{Re}[\hat{h}\hat{V}] \\ \text{Im}[\hat{h}\hat{V}] \end{pmatrix}, \\
\mathcal{S}_\hat{M} &= \begin{pmatrix} \text{Re}\hat{M} & -\text{Im}\hat{M} \\ \text{Im}\hat{M} & \text{Re}\hat{M} \end{pmatrix},
\end{align*}
\]

where $\text{Re}$ and $\text{Im}$ denote the real and imaginary parts. The first- and second-order moments $\vec{d}_{in}$ and $\sigma_{in}$ of a Gaussian input evolve through the sensor as

\[
\begin{align*}
\vec{d}_{out} &= \vec{d}_{in} + \vec{d}_v, \\
\sigma_{out} &= \mathcal{S}_\hat{M} \sigma_{in} \mathcal{S}_{\hat{M}}^T.
\end{align*}
\]

**Quantum estimation.**—The CRB and quantum Cramér-Rao bound are successive lower bounds on the variance $(\Delta \hat{h})^2 = \mathbb{E}[\hat{h}^2] - \mathbb{E}[\hat{h}]^2$ of an unbiased estimator $\hat{h}$ for a parameter $h$ which parametrizes some probability distribution $P(\vec{x}|h)$ and in turn some state $\rho(h)$ which is given by

\[
(\Delta \hat{h})^2 \geq \frac{1}{F(h)} \geq \frac{1}{H(h)},
\]

where $F(h)$ is the classical Fisher information (CFI) and $H(h)$ the quantum Fisher information (QFI). The CFI depends on the sampled probability distribution as [37–41]

\[
F(h) = \sum_i \frac{1}{P(\vec{x}|h)} \left( \frac{\partial P(\vec{x}|h)}{\partial h} \right)^2,
\]

and the QFI can be derived from the fidelity as [37–41]

\[
H(h) = -4 \lim_{\hbar h \to 0} \left( \frac{\partial^2}{\partial h^2} \mathcal{F}(\rho_h, \rho_{h+d\hbar}) \right),
\]

where the fidelity is $\mathcal{F}(\rho_1, \rho_2) = \text{Tr}[\sqrt{\sqrt{\rho_1} \rho_2 \sqrt{\rho_1}}]$. For single-parameter estimation there always exists some positive operator valued measurement for which the second inequality of Eq. (7) is saturated [37,39].

A parameter encoded only in the displacements of a Gaussian state, the QFI is [42,64–66]

\[
H(h) = 2(\partial_h \vec{d})^T \sigma^{-1}(\partial_h \vec{d}),
\]

where $\vec{d}$ and $\sigma$ are the displacement vector and covariance matrix of the Gaussian state, respectively.

$\mathcal{M}$ and $\tilde{\mathcal{V}}$ can be expressed as $\mathcal{M} = BB^{-1}$ and $\tilde{\mathcal{V}} = \mathcal{V}^{\text{SQL}}$, where $B = \text{diag}(e^{\phi_{12}}1_{2\times2}, \ldots, e^{\phi_{i2}}1_{2\times2})$, and $M$ and $\mathcal{V}$ are real for all cases given by Eq. (2). With an input state that can be written as $\sigma_{in} = \sigma_0 \otimes \sigma_0$, the QFI for the parameter $|h|$ is then (see Sec. III of the Supplemental Material [50]) given by

\[
H(|h|) = 4\mathcal{V}^T (M_0 M^T)^{-1} \mathcal{V}.
\]

As Eq. (11) is independent of $\arg(h)$, we henceforth take $h$ to be real and positive.

To compare with the spectral noise density which is typically used to describe the sensitivity of sensors [2,14,33], the CRBs should be multiplied by 4 as $S_{\eta}(\Omega) = 4/F(h)$; see Sec. IV of the Supplemental Material [50]. Our bounds therefore have a prefactor $h^{2}\text{SQL}/8$ in comparison to results using the single-sided spectral density where the equivalent prefactor is $h^{2}\text{SQL}/2$ [2,33].

**Sensor scheme.**—From Eq. (2) the optical modes are coupled through a multimode squeezing, which are weighted through the optical intensities of each mode. We model optical loss at the detector by mixing the outgoing modes $\hat{B}$ with a (Gaussian) environment at a beam splitter with transmissivity $\eta$ as $\hat{B} \to \sqrt{\eta} \hat{c} + \sqrt{1 - \eta} \hat{\gamma}$, with reflected light dumped in a set of modes $\hat{n}$ which are traced out from the final state, leaving only the measurable modes $\hat{c}$ accessible. The effect on the final state is

\[
\hat{d} \to \sqrt{\eta} \hat{d}, \quad \sigma \to \eta \sigma + (1 - \eta) \sigma_{\text{loss}},
\]

where we will take the input from the environment to be pure vacuum, namely, $\sigma_{\text{loss}} = 1$.

Externally squeezed light inputs can enhance precision [1–3] and has already been demonstrated in current GW detectors [3,67]. With multimode interferometers one feasible generalization is to have parallel squeezing for the sidebands of each carrier frequency, with some squeezing $\xi_j = r_j e^{\phi_j}$ in the $\hat{x}_j^{(i)}$ and $\hat{x}_j^{(j)}$ modes.

Our main result is the fundamental quantum limit to the precision of the interferometer scheme described— with arbitrary intensity and external squeezing in each mode—which is

\[
(\Delta \hat{h})^2 \geq \frac{\hbar^{2}\text{SQL}}{8} \left[ \frac{|1 - (1 - \eta)\eta(\Omega)|^2}{\eta(1 - \eta)(\Gamma) + (1 - \eta)(\Omega \Gamma)} \right],
\]

where $\eta(1 - \eta)(\Gamma)$ and $\eta(1 - \eta)(\Omega \Gamma)$ are the partial coherence and the product of partial coherence between the input and output states. The term $\frac{|1 - (1 - \eta)\eta(\Omega)|^2}{\eta(1 - \eta)(\Gamma) + (1 - \eta)(\Omega \Gamma)}$ takes on the value 1 for a Gaussian input state with no external squeezing.
where we define the diagonal matrices $Q_{ii} = (\cosh 2r_i + \sinh 2r_i \cos 2\phi_i)$, $S_{ii} = \sinh 2r_i \sin 2\phi_i$, $\Gamma_{ii} = \left\{ [(1-\eta)^2 + \eta^2] + 2\eta(1-\eta) \cosh 2r_i \right\}^{-1}$, $P = \eta \Gamma + (1-\eta)Q$, and $\langle A \rangle$ is defined as $\sum_{i=1}^{d} \sqrt{\kappa_i} A_{ii}$. The dependency on carrier-mode intensity is a function of summations over $\kappa$ weighted by various functions of the squeezing magnitude and angle in the respective mode.

Attainability of quantum-limited precision requires the application of specific measurement schemes on the quantum system. Homodyne detection covers both measurement of the signal quadrature which is in active use [2,15,68,69] and the more general frequency-dependent homodyne angle of $\kappa$.[2,15,33,60] That measures along a different quadrature for each frequency mode $\Omega$ of the signal. Both of these can be modeled by performing homodyne detection on some quadrature $\sin \theta \hat{x}_{1i} + \cos \theta \hat{x}_{2i}$ for each carrier mode, in which $\theta$ can be frequency dependent. This provides a precision of

$$ (\Delta h)^2 \geq \frac{\hbar^2}{8} \left( \frac{1-\eta \langle G^2 Y^{-1} S \rangle + \langle FG Y^{-1} Q \rangle}{\eta \langle G^2 Y^{-1} \rangle} + (1-\eta) \langle G Y^{-1} \rangle + \eta \langle G^2 Y^{-1} \rangle \right), $$

(13)

where we further define the diagonal matrices $F_{ii} = \sin \theta_i$, $G_{ii} = \cos \theta_i$, and $Y_{ii} = 1 - \eta + \eta \langle \cosh 2r_i - \sinh 2r_i \cos (2\phi_i + 2\theta_i) \rangle$. For measurements along the signal quadrature, $F = 0$, $G = 1$, $Y = T$ in Eq. (13) and the precision reduces to

$$ (\Delta h)^2 \geq \frac{\hbar^2}{8} \left( \frac{(1-\eta)\langle ST^{-1} \rangle^2}{\eta \langle T^{-1} \rangle} + \langle PT^{-1} \rangle \right), $$

(14)

where $T$ is the diagonal matrix $T_{ii} = 1 - \eta + \eta \langle \cosh 2r_i - \sinh 2r_i \cos (2\phi_i) \rangle$.

The bounds in Eqs. (12)–(14) all take the form

$$ \left( \frac{1 - \sum_i c_{ij}^i \kappa_i}{\sum_i c_{ij}^i} \right)^2 + \sum_i c_{ij}^i \kappa_j, $$

(15)

for any given input squeezing configuration, with $c_{ij}^i > 0$ and $c_{ij}^i \geq 0$, with the equality $c_{ij}^i = 0$ only holding if $\eta = 1$, which we consider explicitly as a special case later. When $c_{ij}^i > 0$ and $c_{ij}^i > 0$, namely for $\eta < 1$, Eq. (15) is always minimized (though not necessarily uniquely) over $\{ \kappa_i \} \in [0, \infty)$ by some $\kappa_{\text{tot}} = 1/\sqrt{c_{ij}^i + c_{ij}^j}$ and $\kappa_k = 0$, $\forall \ k \neq j$. See Sec. VII of the Supplemental Material [50] for the complete proof. This establishes our main conclusion that multicarrier optomechanical sensors are fundamentally no better than their single-carrier counterparts.

Special cases.—With an identical external squeezing of $re^{i\phi}$ in each mode, the fundamental quantum limit in Eq. (12) becomes

$$ (\Delta h)^2 \geq \frac{\hbar^2}{8} \left( \frac{\eta^2 + (1-\eta)(1-\eta) + 2\eta \cosh 2r - 2\eta \kappa_{\text{tot}} \sinh 2r \sin 2\phi + \eta \kappa_{\text{tot}}^2 \langle \cos 2r + \sinh 2r \sin 2\phi \rangle}{\eta \kappa_{\text{tot}}^2 \langle 1-\eta \rangle + \eta \langle \cosh 2r + \sinh 2r \sin 2\phi \rangle \rangle \right), $$

(16)

where $\kappa_{\text{tot}} = \sum_i \kappa_i$ is the sole $\kappa$-dependent term. In this case the fundamental quantum limit given in Eq. (16) can be saturated with frequency-dependent homodyne using a homodyne angle of

$$ \theta_i = \arctan \left( \frac{\kappa_{\text{tot}} \langle \cos 2r + \sinh 2r \sin 2\phi \rangle}{1 - \eta + \eta \langle \cosh 2r + \sinh 2r \sin 2\phi \rangle} \right), \ \forall \ i. $$

(17)

Measurement along the signal quadrature in this identical squeezing regime yields a precision of

$$ (\Delta h)^2 \geq \frac{\hbar^2}{8} \left( \frac{1 - \eta + \eta \langle \cos 2r - \sin 2r \cos 2\phi \rangle}{\eta \kappa_{\text{tot}}} + \kappa_{\text{tot}} \langle \cos 2r + \sinh 2r \cos 2\phi \rangle - 2 \sinh 2r \sin 2\phi \right), $$

(18)

which can be optimized by a frequency-dependent squeezing angle $\phi = \arctan \kappa_{\text{tot}}$, to give a precision

$$ (\Delta h)^2 \geq \frac{\hbar^2}{8} \left( 1 - \eta + \eta e^{-2r} + e^{-2r} \kappa_{\text{tot}} \right). $$

(19)

In the limit of zero squeezing, the fundamental quantum limit reduces to

$$ (\Delta h)^2 \geq \frac{\hbar^2}{8} \left( \frac{1}{\eta \kappa_{\text{tot}}} + (1-\eta) \kappa_{\text{tot}} \right), $$

(20)

This takes the same form as the single-mode limit [2,33], with $\kappa_{\text{tot}}$ taking the place of the single carrier $\kappa$.

Using the frequency-dependent homodyne angle given by Eq. (17), this precision can be attained with the homodyne angle $\arctan(\eta \kappa_{\text{tot}})$. Considering homodyne
along the signal quadrature instead, the precision given by Eq. (14) reduces to

$$
(\Delta h)^2 \geq \frac{\eta^2}{8} \left( \frac{1}{\eta \kappa_{tot} + \kappa_{tot}} \right).
$$

In the lossless limit $\eta = 1$ with squeezings not necessarily identical across the carriers, the fundamental quantum limit is

$$
(\Delta h)^2 \geq \frac{\eta^2}{8} \frac{1}{K_{tot}},
$$

where we define $K_{tot}$ as $K_{tot} = \sum_{i=1}^{d} \kappa_i (\cosh 2r_i + \sinh 2r_i \cos 2\phi_i)$, and the bound displays shot-noise behavior, being minimized as $\kappa_i \to \infty$. This bound is attained by the frequency-dependent homodyne angle

$$\theta_i = \arctan \left( \frac{K_{tot} - \sinh 2r_i \sin 2\phi_i}{\cosh 2r_i + \sinh 2r_i \cos 2\phi_i} \right), \quad \forall \ i,
$$

while a squeezing angle $\phi_i = 0$, $\forall \ i$ optimizes the precision.

Conclusions and discussions.—We have shown that no improvement is afforded in the fundamental sensitivity bound in a large class of optomechanical sensors by simultaneous use of multiple carrier modes, including under the effect of optical loss. With identical squeezing in each mode the precision is determined solely by $\kappa_{tot}$ and no other properties of the distribution of $\{\kappa_i\}$. Introducing squeezing with different magnitudes of angles breaks this symmetry, but the optimum interferometer configuration is not enhanced by the presence of multiple carriers.

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