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Quasi-invariance of countable products of Cauchy measures under non-unitary dilations

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Abstract
Consider an infinite sequence $(U_n)_{n \in \mathbb{N}}$ of independent Cauchy random variables, defined by a sequence $(\delta_n)_{n \in \mathbb{N}}$ of location parameters and a sequence $(\gamma_n)_{n \in \mathbb{N}}$ of scale parameters. Let $(W_n)_{n \in \mathbb{N}}$ be another infinite sequence of independent Cauchy random variables defined by the same sequence of location parameters and the sequence $(\sigma_n \gamma_n)_{n \in \mathbb{N}}$ of scale parameters, with $\sigma_n \neq 0$ for all $n \in \mathbb{N}$. Using a result of Kakutani on equivalence of countably infinite product measures, we show that the laws of $(U_n)_{n \in \mathbb{N}}$ and $(W_n)_{n \in \mathbb{N}}$ are equivalent if and only if the sequence $|\sigma_n|^2 - 1$ is square-summable.

Keywords: Cauchy distribution; change of measure; equivalence of measure; random sequence.

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1 Introduction

Let $(U_n)_{n \in \mathbb{N}}$ be a sequence of independent, Cauchy random variables defined on a probability space $(\Omega, \mathcal{F}, P)$, where each random variable $U_n$ has the density

$$f_n(x, \delta_n, \gamma_n) = \frac{1}{\pi \gamma_n} \frac{\gamma_n^2}{(x - \delta_n)^2 + \gamma_n^2}.$$  \hspace{1cm} (1.1)

The distribution of $U_n$ is parametrised by the location and scale parameters $\delta_n$ and $\gamma_n$ respectively. We shall assume throughout that $\gamma_n$ is strictly positive for all $n \in \mathbb{N}$. For every $U_n$, we may define another Cauchy random variable $W_n$ by multiplicatively perturbing the scale parameter $\gamma_n$ by $\sigma_n \neq 0$, so that the pair $(\delta_n)_{n \in \mathbb{N}}$ and $(\sigma_n \gamma_n)_{n \in \mathbb{N}}$ determines the law of $(W_n)_{n \in \mathbb{N}}$. In this note, we prove the following result:

**Theorem 1.1.** Let $(U_n)_{n \in \mathbb{N}}$ be a sequence of independent Cauchy random variables defined by the sequences $(\delta_n)_{n \in \mathbb{N}}$ and $(\gamma_n)_{n \in \mathbb{N}}$ of location and scale parameters, and let $(W_n)_{n \in \mathbb{N}}$ be a sequence of independent Cauchy random variables with the sequences $(\delta_n)_{n \in \mathbb{N}}$ and $(\sigma_n \gamma_n)_{n \in \mathbb{N}}$ of location and scale parameters, where $\sigma_n \neq 0$ for all $n \in \mathbb{N}$. Then the laws of $(U_n)_{n \in \mathbb{N}}$ and $(W_n)_{n \in \mathbb{N}}$ are equivalent if and only if $|\sigma_n|^2 - 1$ is square-summable.

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Equivalence of laws under additive perturbations to the sequence of locations or means has been studied for Gaussian $U_n$ [1, Example 2.7.6], for sequences of random variables with finite Fisher information [5], and for stable Radon probability distributions on locally convex spaces [2, Theorem 5.2.1]. In contrast, Theorem 1.1 on multiplicative perturbations of scale parameters appears to be new.

2 Proof of Theorem 1.1

We shall use Kakutani’s theorem [4, Theorem 1], which we specialise to our context.

Theorem 2.1. Let $(U_n)_{n \in \mathbb{N}}$ and $(W_n)_{n \in \mathbb{N}}$ be given as in Theorem 1.1. Then the laws of $(U_n)_{n \in \mathbb{N}}$ and $(W_n)_{n \in \mathbb{N}}$ are either equivalent or singular. The laws of $(U_n)_{n \in \mathbb{N}}$ and $(W_n)_{n \in \mathbb{N}}$ are equivalent if and only if both the following conditions hold: the laws $\mathbb{P} \circ U_n^{-1}$ and $\mathbb{P} \circ W_n^{-1}$ of $U_n$ and $W_n$ respectively are equivalent for every $n \in \mathbb{N}$, and

$$
\sum_{n \in \mathbb{N}} - \log \mathbb{E} \left[ \sqrt{\varphi_n(U_n, |\sigma_n|)} \right] < \infty \tag{2.1}
$$

where $\varphi_n(\cdot, |\sigma_n|)$ is the Radon–Nikodym derivative of $\mathbb{P} \circ W_n^{-1}$ with respect to $\mathbb{P} \circ U_n^{-1}$.

If $\sigma_n \neq 0$, then the laws of $U_n$ and $W_n$ are equivalent for every $n \in \mathbb{N}$, with

$$
\varphi_n(x, |\sigma_n|) = |\sigma_n| \left(\frac{x - \delta_n}{(x - \delta_n)^2 + \sigma_n^2} \right). \tag{2.2}
$$

Let $y_n := (x - \delta_n)/\gamma_n$. From (1.1) and (2.2) it follows that

$$
\mathbb{E}[\sqrt{\varphi_n(U_n, |\sigma_n|)}] = \frac{\sqrt{|\sigma_n|}}{\pi} \int_{\mathbb{R}} \frac{1}{\sqrt{y_n^2 + \sigma_n^2}} \frac{1}{y_n^2 + 1} \, dy_n = \frac{1}{\pi} \int_{\mathbb{R}} \frac{1}{\sqrt{y_n^2 + |\sigma_n|}} \, dy_n. \tag{2.3}
$$

We use (2.3) in the following lemma.

Lemma 2.2. For all $|\sigma_n| > 0$, $-\log \mathbb{E}[\sqrt{\varphi_n(U_n, |\sigma_n|)}]$ is nonnegative, and is strictly positive if and only if $|\sigma_n| = 1$. Furthermore, if (2.1) holds, then $\lim_{n \to \infty} |\sigma_n| = 1$.

Proof. Using Jensen’s inequality with the concave map $x \mapsto \sqrt{x}$, and using that $\varphi_n$ is the Radon–Nikodym derivative of $\mathbb{P} \circ W_n^{-1}$ with respect to $\mathbb{P} \circ U_n^{-1}$, we obtain

$$
1 = \mathbb{E}[\sqrt{\varphi_n(U_n, |\sigma_n|)}] \geq \mathbb{E}[\sqrt{\varphi_n(U_n, 0)}],
$$

and hence $-\log \mathbb{E}[\sqrt{\varphi_n(U_n, |\sigma_n|)}]$ is nonnegative for all $|\sigma_n| > 0$. Since $x \mapsto \sqrt{x}$ is not affine, equality holds above (and hence $-\log \mathbb{E}[\sqrt{\varphi_n(U_n, |\sigma_n|)}] = 0$) if and only if $\varphi_n(U_n, |\sigma_n|)$ is $\mathbb{P}$-almost surely constant. By (2.3), $\varphi_n(U_n, |\sigma_n|)$ is $\mathbb{P}$-almost surely constant if and only if $|\sigma_n| = 1$. This proves the first statement.

To prove the second statement, observe that (2.3) implies continuity of the map $|\sigma_n| \mapsto \mathbb{E}[\sqrt{\varphi_n(U_n, |\sigma_n|)}]$ on $(0, \infty)$. The dominated convergence theorem and the second equality in (2.3) imply that, if $(|\sigma_n|)_{n \in \mathbb{N}}$ decreases to zero or increases to infinity, then $\lim_{n \to \infty} \mathbb{E}[\sqrt{\varphi_n(U_n, |\sigma_n|)}] = 0$. Summarising, we have that the map $|\sigma_n| \mapsto \mathbb{E}[\sqrt{\varphi_n(U_n, |\sigma_n|)}]$ from $(0, \infty)$ to $[0, 1]$ is continuous, attains the value 1 if and only if its argument is 1, and does not increase asymptotically to 1 at either end of its domain. It follows that $\lim_{n \to \infty} \mathbb{E}[\sqrt{\varphi_n(U_n, |\sigma_n|)}] = 1$ if and only if $\lim_{n \to \infty} |\sigma_n| = 1$. Since (2.1) implies that $\lim_{n \to \infty} \mathbb{E}[\sqrt{\varphi_n(U_n, |\sigma_n|)}] = 1$, the proof is complete.

Remark 2.3. A consequence of Lemma 2.2 that we shall use repeatedly is the following: if (2.1) holds, and if $|\sigma_n| \neq 1$ for all $n \in \mathbb{N}$, then, for any $0 < \alpha < 1$, there exists some $N(\alpha) \in \mathbb{N}$ such that

$$
0 < |\sigma_n| - 1 < \alpha, \quad \forall n \geq N(\alpha). \tag{2.4}
$$
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To simplify the subsequent presentation, we shall assume that \( \alpha \) is sufficiently small for our purposes, and that \( N(\alpha) = 1 \). If \((2.1)\) holds, then by Lemma 2.2 we may assume this without any loss of generality, since we may discard finitely many terms in the series \((2.1)\) without losing convergence.

Given \((2.3)\), we may define for \( 0 < \alpha < 1 \), a function \( I : (1 - \alpha, 1 + \alpha) \to (0, \infty) \) by

\[
I(|\sigma|) := \frac{1}{\pi} \int_\mathbb{R} \frac{1}{\sqrt{y^2 + \sigma^2}} \frac{1}{\sqrt{y^2 + 1}} \, dy.
\]  

(2.5)

Therefore, \( I(|\sigma|) = 1 \) if and only if \( |\sigma| = 1 \). Furthermore, for some \( 0 < \alpha < 1 \) (see Remark 2.3), it follows from \((2.4)\) that the integrand on the right-hand side of \((2.5)\) is dominated by an integrable function that depends on \( \alpha \) but not on \( \sigma \). Thus, by the dominated convergence theorem, we may interchange integration and differentiation to compute the derivative of \( I \) with respect to \( |\sigma| \). By computing the higher-order derivatives of \((y^2 + \sigma^2)^{-1/2}\) with respect to \( |\sigma| \) and employing the same argument, the assertion holds for higher-order derivatives of \( I \) as well. The Taylor expansion of \( I(|\sigma|) \) about \( |\sigma| = 1 \) up to second order is

\[
I(|\sigma|) = 1 + a_1(|\sigma| - 1) + a_2(|\sigma| - 1)^2 + O((|\sigma| - 1)^3),
\]

(2.6a)

\[
am_m = \frac{1}{m! \pi} \int_\mathbb{R} \frac{1}{\sqrt{y^2 + 1}} \left( \frac{\partial}{\partial |\sigma|} \right)^m \frac{1}{\sqrt{y^2 + |\sigma|^2}} \, dy_{|\sigma|=1}.
\]  

(2.6b)

Note that the \( a_m \) are independent of \( |\sigma| \).

We shall use the following lemma to calculate the values of \( a_m \) for \( m = 1, 2 \).

**Lemma 2.4.** Let \( r, s \in \mathbb{N}_0 \) with \( r \leq s - 1 \). Then

\[
\int_\mathbb{R} \frac{x^{2r}}{(x^2 + 1)^s} \, dx = \pi \frac{(2r)!}{4^{s-1} r! (s-r-1)! (s-1)!}.
\]

**Proof.** Letting \( y := x^2 \), we have \( dx = 2y^{-1/2} \, dy \), so

\[
\int_\mathbb{R} \frac{x^{2r}}{(x^2 + 1)^s} \, dx = \int_0^\infty \frac{y^{r-1/2}}{(y+1)^s} \, dy = \int_0^\infty \frac{y^{(r+1/2)-1}}{(y+1)^s} \, dy.
\]

Using [6, Equation (2)] and properties of the gamma function \( \Gamma(t) = \int_0^\infty x^{t-1} e^{-x} \, dx \) (for \( \text{Re} \, t > 0 \), we have, for \( 0 < \text{Re} \, (r+1/2) < \text{Re} \, s \),

\[
\int_0^\infty \frac{y^{(r+1/2)-1}}{(y+1)^s} \, dy = \frac{\Gamma \left( r + \frac{1}{2} \right) \Gamma \left( s - r - \frac{1}{2} \right)}{\Gamma(s)} = \frac{(2r)!}{4^r r!} \sqrt{\pi} \frac{(2(s-r-1))!}{4^{s-r-1} (s-r-1)!} \sqrt{\pi} \frac{1}{(s-1)!}.
\]

Simplifying the right-hand side yields the desired conclusion. \( \square \)

Thus we have

\[
a_1 = \frac{1}{\pi} \int_\mathbb{R} \frac{1}{\sqrt{y_n^2 + 1}} \frac{(-1)}{(y_n^2 + 1)^{3/2}} \, dy_n = -\frac{1}{\pi} \left( \frac{0!}{4 \cdot 1!} \right) = -\frac{1}{2}, \quad (2.7a)
\]

\[
a_2 = \frac{1}{2\pi} \int_\mathbb{R} \frac{1}{\sqrt{y_n^2 + 1}} \frac{2 - y_n^2}{(y_n^2 + 1)^{5/2}} \, dy_n = \frac{1}{2\pi} \left( \frac{9\pi}{8} - \frac{\pi}{2} \right) = \frac{5}{16}, \quad (2.7b)
\]

Given the function \( I \), a sequence \((\sigma_n)_n \in \mathbb{R}\) satisfying \((2.4)\), and \((2.7b)\), define

\[
\epsilon_n := \frac{1}{|\sigma_n| - 1} \left( I(|\sigma_n|) - 1 - a_1(|\sigma_n| - 1) \right), \quad \forall n \in \mathbb{N}.
\]

(2.8)
Lemma 2.5. Let $\epsilon_n$ be as in (2.8). If (2.1) holds, then for some $0 < \alpha < 1$ and some $C > 0$ that do not depend on $n$,
\[ c ||\sigma_n| - 1| \leq |\epsilon_n| \leq C ||\sigma_n| - 1|, \quad \forall n \in \mathbb{N}. \tag{2.9} \]

Proof. First, note that by Remark 2.3, we may bound the constant in the $O(||\sigma_n| - 1|)$ term in (2.5) by some $C > 0$ that may depend on $\alpha$, but not on $|\sigma|$, i.e.
\[ I(|\sigma|) = 1 + \sum_{n=1}^{2} a_n (|\sigma| - 1)^n + C(|\sigma| - 1)^3; \quad \forall |\sigma| \in (1 - \alpha, 1 + \alpha). \]

Thus by (2.8), the triangle inequality, and (2.4), we obtain
\[ |\epsilon_n| = |a_2(|\sigma_n| - 1) + C(|\sigma_n| - 1)^2| \leq \max\{C, a_2\} ||\sigma_n| - 1| (1 + ||\sigma_n| - 1|), \]

which yields the upper bound in (2.9) since $||\sigma_n| - 1| < \alpha$ by (2.4). For the lower bound, we have for the same $C > 0$ as above that
\[ |\epsilon_n| \geq |a_2(|\sigma_n| - 1)| - |C(|\sigma_n| - 1)^2| = ||\sigma_n| - 1| (|a_2| - C ||\sigma_n| - 1|). \]

and for sufficiently small $\alpha$, we can make $|a_2| - C ||\sigma_n| - 1|$ strictly positive, by (2.4). \qed

Again by making $\alpha$ small enough, we have from (2.8) and (2.7a) that
\[ \log I(|\sigma_n|) = \sum_{m \in \mathbb{N}} \frac{(-1)^{m-1}}{m} (I(|\sigma_n|) - 1)^m = \sum_{m \in \mathbb{N}} \frac{(-1)^{m-1}}{m} \left( \epsilon_n - \alpha \right)^m. \tag{2.10} \]

Proposition 2.6. If the laws of $(U_n)_{n \in \mathbb{N}}$ and $(W_n)_{n \in \mathbb{N}}$ are equivalent, then $(||\sigma_n| - 1|)_{n \in \mathbb{N}}$ is square-summable.

Proof. If the laws of $(U_n)_{n \in \mathbb{N}}$ and $(W_n)_{n \in \mathbb{N}}$ are equivalent, then by Theorem 2.1, (2.1) holds. Without loss of generality, we may assume that $|\sigma_n| \neq 1$ for all $n \in \mathbb{N}$, since otherwise the corresponding summand vanishes, by Lemma 2.2. Furthermore, (see Remark 2.3), we may also assume that (2.4) holds for some $0 < \alpha < 1$ that we can set as small as needed for (2.10) to hold and such that we can rewrite $\log |\sigma_n|$ as a Mercator series in $|\sigma_n| - 1$, for all $n \in \mathbb{N}$. Using these conditions, (2.3), (2.5), and (2.10), we obtain
\[ - \log \mathbb{E}[\sqrt{\varphi_n(U_n, |\sigma_n|)}] = - \sum_{m \in \mathbb{N}} \frac{(-1)^{m-1}}{m} \left( \frac{1}{2} + \left( \epsilon_n - \alpha \right)^m \right). \tag{2.11} \]

The summand for $m = 1$ is, by (2.8) and (2.5),
\[ -\epsilon_n(|\sigma_n| - 1) = (a_1(|\sigma_n| - 1) + 1 - I(|\sigma_n|)) = -a_2(|\sigma_n| - 1)^2 + C(|\sigma_n| - 1)^3, \]

where $C > 0$ may depend on $\alpha$ but not on $|\sigma_n|$; see the proof of Lemma 2.5. Expanding the coefficient of $(|\sigma_n| - 1)^3$ for the summand in (2.11) corresponding to $m = 2$, we obtain that its value is
\[ \frac{1}{2} \left[ \frac{1}{2} + \left( \epsilon_n^2 - \epsilon_n + \frac{1}{4} \right) \right] = \frac{3}{8} + \frac{1}{2} \left( \epsilon_n^2 - \epsilon_n \right). \]

By Lemma 2.5, $\epsilon_n^2 - \epsilon_n$ converges to zero linearly in $|\sigma_n| - 1$. Therefore, combining this observation with the preceding two equations, we obtain from (2.11) and (2.7b) that, for some $C'$ independent of $|\sigma_n|$,
\[ - \log \mathbb{E}[\sqrt{\varphi_n(U_n, |\sigma_n|)}] = \left( \frac{5}{16} + \frac{3}{8} \right) (|\sigma_n| - 1)^2 + C'((|\sigma_n| - 1)^3) = \frac{1}{16} (|\sigma_n| - 1)^2 + C'((|\sigma_n| - 1)^3). \]
Therefore, by making $\alpha$ sufficiently small, we may ensure that for any $0 < \delta < 1/16$,
\[
\frac{1}{(|\sigma_n| - 1)^2} \log E[\sqrt{\varphi_n(U_n, |\sigma_n|)}] - \frac{1}{16} (|\sigma_n| - 1)^2 = |C'| |\sigma_n| - 1| < \delta.
\]
Thus, for any $0 < \delta < 1/16$,
\[
0 < \left( \frac{1}{16} - \delta \right) (|\sigma_n| - 1)^{-1} < \frac{1}{16} (|\sigma_n| - 1)^{-1} - \log E[\sqrt{\varphi_n(U_n, |\sigma_n|)}] - \frac{1}{16} (|\sigma_n| - 1)^2.
\]
Since $-|x| < x$ if $x > 0$, and $-|x| = x$ if $x < 0$, it follows that the right-hand side of the inequality above is less than or equal to $-\log E[\sqrt{\varphi_n(U_n, |\sigma_n|)}]$. Summing over $n$ yields
\[
0 < \left( \frac{1}{16} - \delta \right) \sum_{n \in \mathbb{N}} (|\sigma_n| - 1)^{-1} < \sum_{n \in \mathbb{N}} -\log E[\sqrt{\varphi_n(U_n, |\sigma_n|)}] < \infty,
\]
and therefore the summability of Kakutani’s series implies that $(|\sigma_n| - 1)_{n \in \mathbb{N}}$ is square-summable.

**Proposition 2.7.** If $(|\sigma_n| - 1)_{n \in \mathbb{N}}$ is square-summable, then the laws of $(U_n)_{n \in \mathbb{N}}$ and $(W_n)_{n \in \mathbb{N}}$ are equivalent.

**Proof.** Without loss of generality, we may assume that the $\ell^2$ norm $\|(|\sigma_n| - 1)_{n \in \mathbb{N}}\|_2$ of $(|\sigma_n| - 1)_{n \in \mathbb{N}}$ is strictly less than 1. Then, for $1 < p \leq q < \infty$, the $\ell^p$-norm of $(|\sigma_n| - 1)_{n \in \mathbb{N}}$ is larger than or equal to the $\ell^q$-norm of $(|\sigma_n| - 1)_{n \in \mathbb{N}}$. Using this fact, the sum formula for a geometric progression, and the hypothesis on the $\ell^2$ norm of $(|\sigma_n| - 1)_{n \in \mathbb{N}}$, we have
\[
\sum_{m \geq 2} \sum_{n \in \mathbb{N}} |\sigma_n| - 1|^{-m} \leq \sum_{m \geq 2} \left( \sum_{n \in \mathbb{N}} |\sigma_n| - 1|^{-1} \right)^{m/2} \leq \frac{\|(|\sigma_n| - 1)_{n \in \mathbb{N}}\|_2^2}{1 - \|(|\sigma_n| - 1)_{n \in \mathbb{N}}\|_2^2} < \infty. \tag{2.12}
\]
Since $(|\sigma_n| - 1)_{n \in \mathbb{N}}$ converges to zero, we may choose $\alpha$ in (2.4) so small that, for $\epsilon_n$ defined in (2.4), it holds that $|\epsilon_n| < 1/8$ for all $n \in \mathbb{N}$, by Lemma 2.5. Since
\[
\left| \frac{1}{2} + (\epsilon_n - \frac{1}{2})^m \right| \leq \left( \frac{1}{2} + (|\epsilon_n| + \frac{1}{2})^m \right)^m = \frac{m}{1 + (|\epsilon_n| + \frac{1}{2})^m},
\]
the condition that $|\epsilon_n| < 1/8$ implies that we may bound the left-hand side by 1 for all $m \in \mathbb{N}$. Using this observation and the triangle inequality, we may bound the right-hand side of (2.11) according to
\[
- \sum_{m \in \mathbb{N}} \frac{(-1)^{m-1}}{m} \left( \frac{1}{2} + (\epsilon_n - \frac{1}{2})^m \right) (|\sigma_n| - 1)^{-m} \leq \sum_{m \in \mathbb{N}} |\sigma_n| - 1|^{-m},
\]
and hence from (2.11) it follows that $-\log E[\sqrt{\varphi_n(U_n, |\sigma_n|)}] \leq \Sigma_{m \in \mathbb{N}} |\sigma_n| - 1|^{-m}$. Summing this inequality over $n$ yields
\[
\sum_{n \in \mathbb{N}} -\log E[\sqrt{\varphi_n(U_n, |\sigma_n|)}] \leq \sum_{m \in \mathbb{N}} \sum_{n \geq m} |\sigma_n| - 1|^{-m},
\]
where the right-hand side is finite, by changing the order of summation in (2.12). Therefore, (2.1) holds, and by Theorem 2.1, the laws of $(U_n)_{n \in \mathbb{N}}$ and $(W_n)_{n \in \mathbb{N}}$ are equivalent.

**Remark 2.8.** An anonymous referee pointed out that one can express the function $I$ defined in (2.5) in terms of complete elliptic integrals (see, e.g., [3, Section 3.152, Formula 1] and [3, Section 8.112, Formula 1]), and that the series representations of these elliptic integrals in [3, Section 8.113] may provide an alternative method for obtaining the results that we presented above.
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References


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