Common pricing across asset classes: Empirical evidence revisited*

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ABSTRACT
Intermediary and downside-risk asset-pricing theories lay the foundations for spanning the multi-asset return space by a small number of risk factors. Recent studies document strong empirical support for such factors across major asset classes. We revisit these results and show that robust evidence for common factor pricing remains elusive. Importantly, the proposed risk factors do not seem to provide incremental information to the traditional market factor. We argue that most of the economic and statistical challenges are not specific to these analyses and, with the aid of a placebo test, offer general recommendations for improving empirical tests, thus adding to the prescriptions in Lewellen, Nagel, and Shanken (2010).

Keywords: Intermediary asset pricing; Capital risk factor; Downside risk factor; Sharpe ratio; Efficient frontier; Model misspecification and identification; Small-sample inference.

JEL classification: G12; C12; C13.

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1. Introduction

Empirical asset pricing has made important strides towards a better understanding of the determinants of pricing kernels for individual asset classes such as equities or bonds. However, identifying and characterizing the underlying common factor structure of expected returns across asset classes – arguably the asset-pricing equivalent to the quest for the Holy Grail – proves to be much more challenging. Until recently, robust empirical evidence of the sources of risk and common variation across asset classes has remained largely elusive. Two notable exceptions that offer a unified pricing framework in a multi-asset context are the intermediary and downside risk capital asset pricing models.

Intermediary asset pricing furnishes a new perspective on the role of financial intermediaries as marginal investors in the major asset markets. The intermediary asset-pricing models of He and Krishnamurthy (2012, 2013) and Brunnermeier and Sannikov (2014) provide the foundations and key determinants of the pricing kernel of financial intermediaries. On the empirical side, Adrian, Etula, and Muir (2014) propose an intermediary pricing kernel with broker-dealer leverage shocks as a single risk factor. In a similar vein, He, Kelly, and Manela (HKM, 2017) advocate the use of shocks to the equity capital ratio of primary dealers in a two-factor model along with the market factor. HKM provide extensive evidence for significant risk premia and explanatory power of this intermediary capital ratio factor across seven asset classes and contrast their findings with those in Adrian, Etula, and Muir (2014). Overall, the paper offers strong empirical support to the capital ratio of primary dealers as a priced factor across asset classes with no evidence of pricing for the widely used market factor.

Downside risk theory builds on the observation that investors tend to demand a premium for holding assets that exhibit pronounced downward price movements. While the theory of weighing differently downside losses versus upside gains dates back to Roy (1952) and Markowitz (1959), Ang, Chen, and Xing (2006) undertake a comprehensive empirical analysis of the premium for bearing downside risk in a cross-section of equity returns. Lettau, Maggiori, and Weber (LMW, 2014) extend the downside risk capital asset pricing model (DR-CAPM) to other asset classes such as currencies, commodities, sovereign bonds, options, etc. More specifically, they employ the relative difference between the unconditional and downside market betas as a proxy for downside risk and argue that a tightly parameterized empirical model based on this proxy is able to price the cross-section of expected returns across various asset classes.

In this paper, we revisit the results in HKM and LMW using popular economic metrics and a more comprehensive set of statistical methods. The main finding of HKM and LMW is striking: a simple linear two-factor model appears to price a wide range of asset classes with the capital and downside risk factors carrying large and highly significant risk premia. In HKM, this pricing – which appears to be more impressive for non-equity asset classes – is achieved in the context of highly heterogeneous and volatile test assets and an extremely low number of effective time series observations per test asset (i.e., the cross-sectional dimension is large relative to the time series dimension). In this case, standard asymptotics may not provide an accurate approximation as it often results in inflated statistical significance. In LMW, the inference is conducted using the Fama and MacBeth (1973) procedure under the assumption of a correct and fully identified model. Model uncertainty and possible identification failure further reinforce these concerns. For example, if the relative downside risk beta is close to a zero vector, the inference is likely to be spurious. (See Kan and Zhang, 1999.) Against this background, is it realistic to expect such statistical and economic significance of the proposed risk factors? A robust evaluation of the model requires that the economic and statistical analysis captures the data limitations mentioned so far. The main features of this robust framework can be summarized as follows.

First, we use some popular metrics such as the distance from the mean-variance frontier and maximum Sharpe ratios to shed light on the economic significance of the capital factor. The preliminary evidence provided by this exploratory analysis suggests that financial intermediary capital serves as another – more volatile – market proxy that does not dominate the broad market return neither in terms of investment performance nor in terms of enhancing mean-variance efficiency.

Second, the empirical analyses in HKM and LMW are conducted using the ordinary least squares (OLS) estimator. Lewellen, Nagel, and Shanken (LNS, 2010) argue that the OLS cross-sectional regression (CSR) R^2 has little economic interpretation and can be large even when the fundamental asset-pricing relation is violated. In addition, there is substantial heterogeneity (with respect to factor structure and volatility) of the returns across asset classes. LNS build a convincing case for the use of generalized least squares (GLS) metrics and downweighing the informational content of standard OLS-based goodness-of-fit measures. The GLS estimation and inference substantially weakens the initial results for a priced capital risk factor. Furthermore, we argue that assessing the incremental pricing of capital should not be performed by focusing on the price of *multivariate* beta risk but on the price of covariance risk. This simple shift of perspective renders the capital factor largely insignificant for both OLS and GLS estimators.

Several other statistical issues, that are still often ignored in empirical asset pricing, loom large in this analysis. One of them is model misspecification and uncertainty. Since economic theory provides only limited guidance on the exact structure of the common pricing kernel, it is prudent to explicitly acknowledge the model uncertainty surrounding evaluation of the pricing ability of a set of risk factors across asset classes. Because all financial models are constructed to approximate a complex reality, they are inherently misspecified. This additional source of uncertainty should be routinely incorporated in the statistical inference as misspecification adjustments for linear and nonlinear models are now readily available. The misspecification-robust inference, which continues to be valid even when the model is correctly specified, essentially reinforces the result of no incremental pricing (except possibly for equities) of the capital factor.

Finally, data limitations constrain the analysis to small samples of a relatively large cross-section of test assets. It is now well-understood that the small number of time series observations and the large cross-sectional dimension render the inference based on asymptotic approximations largely unreliable. (See, for example, Kleibergen and Zhan 2020.) For that purpose, we also develop and report bootstrap methods for inference and model evaluation. Our bootstrap procedure – which can be used for bias correction, testing individual and joint statistical significance, as well as specification testing – is robust to model misspecification, is agnostic to the true factor structure, and allows for general form of serial correlation, conditional heteroskedasticity, and cross-correlation structure of the data.

It should be noted that our remarks are empirical and methodological in nature and do not target the theoretical foundations of the intermediary asset-pricing models. In fact, we find the idea of financial intermediaries as marginal investors to be sound and compelling, at least for some asset classes. Many of the issues that we are raising in this paper represent some of the major challenges that empirical asset pricing, especially in a multi-asset class setup, is facing. We provide guidance and recipes that would ensure more reliable and less fragile inference and evaluation of asset-pricing models and factors. To highlight the relevance of our practical recommendations, we also subject HKM's and our proposed methods to a placebo test, that is, we investigate whether factor portfolios (that share some of the time series properties of HKM's traded leverage factor but do not fit HKM's theoretical narrative) pass HKM's and our battery of asset-pricing tests.

The rest of the paper is organized as follows. Section 2 reports some descriptive and statistical evidence for the capital factor as an investment vehicle and a mean-variance efficiency enhancing factor. Sections 3 and 4 motivate and detail additional inference tools and procedures that assess the robustness of the findings for the pricing ability of the capital factor in HKM. They report the main results and discuss potential issues and suggested solutions. In addition, we analyze how HKM's financial intermediary factor performs relative to alternative factors. Section 5 presents a placebo test aimed at determining the robustness of HKM's and our proposed methods in factor selection. Section 6 provides an extended analysis of the pricing performance of the downside risk factor in LMW across asset classes. It points out the violation of the main identification condition that renders the original analysis invalid. Moreover, we develop statistical methods that are robust to identification failure and permit a sound inference procedure. Section 7 takes a broader view of the intrinsic challenges that surround empirical asset pricing and offers general recommendations for empirical practice with special emphasis on economic interpretability and statistical robustness. Section 8 summarizes our main conclusions. Propositions, proofs, and additional material are provided in the Internet Appendix.

2. Summary statistics, mean-variance frontiers, and Sharpe ratios across asset classes

HKM propose a two-factor beta-pricing model which, in addition to the value-weighted market excess return (MKT), includes a financial intermediary capital risk factor. HKM consider nontraded and traded measures of this financial intermediary capital risk factor. The first measure ($CPTL$) is constructed using AR(1) innovations to the market-based capital ratio of primary dealers, scaled by the lagged capital ratio. The second measure ($CPTLT$) is the value-weighted equity return for the New York Fed's primary dealer sector and it does not include new equity issuance. HKM find significant pricing results for their financial intermediary capital factor (both in its traded and nontraded version) in CSR analyses. Specifically, they show that financial intermediary risk is priced in the cross-section of seven asset classes: equities (FF25), corporate bonds (US bonds), sovereign bonds (Sov. bonds), options (Options), credit default swaps (CDS), commodities (Commod.), and foreign exchange (FX).[1] Moreover, HKM find that the traditional market factor is crowded out by their capital risk factor in two-pass CSRs. In principle, these could be major findings and the message from HKM's paper is undoubtedly provocative and wide-ranging.[2]

In this section, we revisit the evidence provided in HKM mainly from an economic perspective and deal with the CSR analysis later on. In addition, throughout the paper we focus on quarterly data (from 1970:Q1 to 2012:Q4), as HKM do in most of their work.[3] All the results for monthly data are available in the Internet Appendix. To gain economic intuition, in this section we also focus on HKM's traded capital factor, $CPTLT$. Since $CPTLT$ is a return, we subtract the risk-free rate in order to make it an excess return. In Panel A of Table 1, we report the sample means of MKT and $CPTLT$, together with their p-values.

Table 1 about here

The sample means are close to each other (0.015 for MKT and 0.019 for $CPTLT$). Interestingly, we cannot reject the null of zero mean for the financial intermediary risk factor, that is, on average there is no return spread between HKM's capital factor and the risk-free asset. Certainly, the financial intermediary capital factor does not appear to be an attractive investment vehicle, if we are allowed to only invest in it. In contrast, the sample mean for MKT is reliably positive. The lack of significance for the capital factor is due to the high factor's standard deviation, 0.133, compared to 0.091 for the market. So, the first thing that emerges from our analysis is that HKM's capital factor has a relatively higher mean than the market but is also substantially more volatile. Moreover, the correlation between MKT and $CPTLT$ is very high, about 84%.[4] This observation raises the question of whether MKT and $CPTLT$ are really capturing different sources of economic risk or they are both proxies for the same source of underlying market risk. To make the correlation

[1] The data is from Asaf Manela's website.

[2] In a recent paper, Adrian, Moench, and Shin (2019) undertake a comprehensive empirical comparison of alternative (leverage-based and equity-based) intermediary asset-pricing theories. They find strong (little) support for leverage-based (equity-based) asset-pricing theories.

[3] The sample periods for equities, US bonds, sovereign bonds, options, CDS, commodities, and FX are 1970:Q1–2012:Q4, 1975:Q1–2011:Q4, 1995:Q1–2011:Q1, 1986:Q2–2011:Q4, 2001:Q2–2012:Q4, 1986:Q4–2012:Q4, and 1976:Q2–2009:Q4, respectively.

[4] Note that the correlation between $CPTLT$ and its nontraded counterpart, $CPTL$, is large at about 94%. Therefore, several of the arguments discussed in this section apply to a certain degree also to the nontraded capital risk factor analyzed in the later sections.

analysis more explicit, we plot *MKT* and *CPTLT* in Figure 1.

Figure 1 about here

It is clear from the graph that *CPTLT* is a more noisy version of *MKT*. However, the series are synchronized and overall exhibit similar peaks and troughs at the same dates. The higher volatility of *CPTLT* relative to *MKT* should not come as a surprise since the construction of *CPTLT* (and of *CPTL*) relies on a relatively small number of primary dealers' returns compared to the thousands of firms' returns that enter the construction of the value-weighted market factor. Smaller diversification behind the primary dealer capital factor is the primary reason why *CPTLT* is much more volatile than the market. The entry-exit mechanism for primary dealers contributes much less to the volatility of the series since returns at a quarterly frequency do not exhibit a high degree of persistence.

To further foster economic intuition for HKM's findings, it is of interest to examine the performance of their model with the Sharpe metric. In this respect, we consider, in addition to the traditional static capital asset pricing model (CAPM), HKM's two-factor model (that we also denote by HKM) and their one-factor formulation that only includes the capital factor (HKMSF, HKM single-factor). Panel B of Table 1 reports the sample squared Sharpe ratios for the three models.[5] The sample squared Sharpe ratios are bias-adjusted as in BKRS. It turns out that HKM and HKMSF have even lower bias-adjusted squared Sharpe ratios than CAPM. In addition, the p-values in square brackets indicate that all these squared Sharpe ratios are not significantly different from zero at the 5% significance level. Abstracting from the result of the pre-test that these Sharpe metrics are not reliably different from zero, Panel C of Table 1 reports formal pairwise model comparison tests based on squared Sharpe ratios. The very large p-values in Panel C indicate that the three models perform about the same.[6] In other words, the capital factor is *spanned* by the market. We reach similar conclusions in Figures 2 and 3. In the figures, we plot the mean-standard deviation frontiers for the various asset excess returns considered by HKM. Figure 2 is for FF25 while Figure 3 is for US bonds, sovereign bonds, options, CDS, commodities, and FX. Besides

[5]Gibbons, Ross, and Shanken (GRS, 1989), Barillas and Shanken (2017, 2018), Fama and French (2018), and Barillas, Kan, Robotti, and Shanken (BKRS, forthcoming) provide the economic rationale for considering this squared Sharpe ratio measure.

[6]The econometric details for the nested and non-nested model comparison tests in Panel C of Table 1 are provided in BKRS.

the mean-standard deviation frontier, we report the (red) tangency line of the frontier constructed using the test asset excess returns.[7]

Figures 2 and 3 about here

Finally, the other straight lines emanating from the origin correspond to the various models' factors. The slopes of these lines represent the Sharpe ratios (maximum Sharpe ratio in the case of HKM) attainable from investing in the models' factors. The straight lines for the various factors are on top of each other (practically indistinguishable the ones for the *MKT* and *(CPTLT, MKT)* factors), thus indicating that these models perform about the same based on the Sharpe ratio metric. Importantly, the slopes of the tangency lines (asymptotes) of the frontiers are so much steeper than the slopes of the factors' lines. This shows, in a descriptive way, that the factors in these three models are far from being mean-variance efficient. In Panel D of Table 1, we formalize the economic intuition behind Figures 2 and 3, and include the GRS test for each of the three models.[8] For six out of seven asset classes, we strongly reject CAPM, HKM, and HKMSF. We cannot reject the models only for commodities. Notice however that for commodities, none of the models (including CAPM) is rejected. The likely reason why the GRS lacks power for commodities is the very high volatility of commodity returns (the average standard deviation across commodity returns is 13.4% compared, for example, to an average standard deviation of 1.4% across CDS returns.)

The findings in this section are all consistent with each other and with the monthly data analysis in Table D.1 in the Internet Appendix. The three models perform about the same and they are all equally good (or bad!) from an economically reasonable and easy to understand mean-variance angle. In brief, do we really need a model with two market proxies, *MKT* and *CPTLT*, in it? Certainly not from an investor's perspective, and we will revisit this point when discussing HKM's

[7] For options, CDS, commodities, and FX, the multivariate Sharpe ratios are negative. These are situations in which the point of tangency between the mean-standard deviation frontier and the straight (red) line emanating from the origin lies on the inefficient part of the frontier, and the minimum instead of the maximum Sharpe ratio is attained. Therefore, since we are interested in optimal risky portfolios (maximum Sharpe ratio portfolios), we follow Maller and Turkington (2002) and report the asymptote of the minimum-variance frontier for options, CDS, commodities, and FX.

[8] The GRS statistic is proportional to a quadratic form in the intercept estimates from regressing the test asset returns on an intercept term and a model's factor(s). The test of the null hypothesis that the model's intercepts are jointly zero is performed based on a conditional heteroskedastic version of the GRS test as in BKRS. Qualitatively similar results are obtained using the traditional conditional homoskedastic version of the GRS.

CSR results in the next sections.

3. Risk premia and goodness-of-fit measures

In this section, we focus on the main specification in HKM, that is, the model with the nontraded capital risk factor, *CPTL*, at a quarterly frequency. The analyses with traded capital and monthly data are in the Internet Appendix. It is well-known that when some or all of the risk factors are nontraded, pinning down the factors' risk premia requires a model and, unless the model is correctly specified, the particular choice of weighting matrix matters. Like HKM, we focus on risk premia using the two-pass CSR methodology pioneered by Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973).

3.1. Choice of weighting matrix, betas vs. covariances, and misspecification-robust inference

We start with some definitions and notation. Let f be a K-vector of factors, r be a vector of excess returns on N test assets with mean μ_r and covariance matrix V_r, and β be the $N \times K$ matrix of *multivariate* regression betas of the N assets with respect to the K factors. The proposed K-factor beta-pricing model specifies that asset expected returns are linear in β, that is,

$$\mu_r = X\gamma, \tag{1}$$

where $X = [1_N,\ \beta]$ is assumed to be of full column rank, 1_N is an N-vector of ones, and $\gamma = [\gamma_0,\ \gamma_1']'$ is a vector consisting of the zero-beta rate (γ_0) and risk premia on the K factors (γ_1).

When the model is misspecified, the pricing-error vector, $\mu_r - X\gamma$, will be nonzero for all values of γ. In that case, it makes sense to choose γ to minimize some aggregation of pricing errors. Denoting by W an $N \times N$ symmetric positive-definite weighting matrix, we define the (pseudo) zero-beta rate and risk premia as the choice of γ that minimizes the quadratic form of pricing errors:

$$\gamma_W = \begin{bmatrix} \gamma_{W,0} \\ \gamma_{W,1} \end{bmatrix} = \operatorname{argmin}_{\gamma}(\mu_r - X\gamma)'W(\mu_r - X\gamma) = (X'WX)^{-1}X'W\mu_r. \tag{2}$$

The corresponding pricing errors of the N assets are then given by $e_W = \mu_r - X\gamma_W$ and, following Kandel and Stambaugh (1995), the centered CSR R^2 is defined as

$$\rho_W^2 = 1 - \frac{Q}{Q_0}, \tag{3}$$

where $Q = e_W' W e_W$, $Q_0 = e_0' W e_0$, and $e_0 = [I_N - 1_N(1_N' W 1_N)^{-1} 1_N' W]\mu_r$ represents the deviations of mean excess returns from their cross-sectional average.

Empirical research in asset pricing has mostly focused on the price of *multivariate* regression beta risk to infer if the underlying factors are priced. Kan, Robotti, and Shanken (KRS, 2013) argue that, if the factors are correlated and the goal is to determine if an underlying factor is incrementally useful in explaining the cross-section of asset returns, the analysis should focus on λ, the price of covariance risk (equivalently, the *univariate* betas), and not on the price of *multivariate* regression beta risk. (See also Cochrane 2005.) This suggests running the second-pass regressions with covariances instead of betas. Define the matrix $C = [1_N,\ V_{rf}]$, where V_{rf} is the covariance between the excess returns and the factors. Then,

$$\lambda_W = \begin{bmatrix} \lambda_{W,0} \\ \lambda_{W,1} \end{bmatrix} = \text{argmin}_\gamma (\mu_r - C\lambda)' W (\mu_r - C\lambda) = (C'WC)^{-1} C' W \mu_r \tag{4}$$

is the choice of coefficients that minimizes the corresponding quadratic form in the pricing errors, $\mu_r - C\lambda$. It is easy to verify that the pricing errors in the two CSRs are the same (and so are the R^2s). In addition, $\gamma_{W,0} = \lambda_{W,0}$.

When constraining the risk-free rate to equal the zero-beta-rate (a relatively common practice in the asset-pricing literature), we have

$$\gamma_W = (\beta' W \beta)^{-1} \beta' W \mu_r, \tag{5}$$

$$\lambda_W = (V_{rf}' W V_{rf})^{-1} V_{rf}' W \mu_r. \tag{6}$$

In addition, the uncentered R^2 involves (weighted) sums of squared values of the dependent variable (mean excess returns) in the denominator, not squared deviations from the cross-sectional average:

$$\rho_{W,U}^2 = 1 - \frac{Q}{Q_0}, \tag{7}$$

where $Q = (\mu_r - \beta\gamma_W)' W (\mu_r - \beta\gamma_W)$ and $Q_0 = \mu_r' W \mu_r$. This ensures that the R^2 is always between zero and one. We would like to emphasize that unless the beta-pricing model is correctly specified, the gammas, the lambdas, the pricing errors, and the R^2s depend on W. Two popular choices of W in the literature are $W = I_N$ (OLS CSR) and $W = V_r^{-1}$ (GLS CSR).

Turning to estimation of the models, with T observations on f_t and r_t, the popular two-pass method first obtains estimates $\hat{\beta}$ by running the following multivariate regression:

$$r_t = \alpha + \beta f_t + \epsilon_t. \tag{8}$$

Let 1_N be an N-vector of ones, we then run a single CSR of the sample mean vector $\hat{\mu}_r$ on $\hat{X} = [1_N,\ \hat{\beta}]$ ($\hat{C} = [1_N,\ \hat{V}_{rf}]$) to estimate γ (λ) in the second pass, where $\hat{V}_{rf}$ is the sample estimate of V_{rf}. When W is known (as in OLS CSR),

$$\hat{\gamma} = (\hat{X}'W\hat{X})^{-1}\hat{X}'W\hat{\mu}_r, \tag{9}$$

$$\hat{\lambda} = (\hat{C}'W\hat{C})^{-1}\hat{C}'W\hat{\mu}_r, \tag{10}$$

$$\hat{\rho}^2 = 1 - \frac{\hat{Q}}{\hat{Q}_0}, \tag{11}$$

where $\hat{Q} = \hat{e}'W\hat{e}$, $\hat{Q}_0 = \hat{e}_0'W\hat{e}_0$, $\hat{e} = \hat{\mu}_r - \hat{X}\hat{\gamma}$, and $\hat{e}_0 = [I_N - 1_N(1_N'W1_N)^{-1}1_N'W]\hat{\mu}_r$. For GLS, we need to substitute the inverse of the sample covariance matrix of the excess returns in the expressions above. Similarly, when the cross-sectional intercept is constrained to be zero, the population quantities in Eqs. (5)–(7) need to be replaced with their sample counterparts.

As emphasized above, the researcher needs to take a stand on the weighting matrix. This is a population issue, and not just a sampling feature of GLS being more efficient than OLS when the asset-pricing restriction holds. When the factors are (excess) returns, Kandel and Stambaugh (1995) and LNS argue that the OLS R^2 tells us little if anything about the mean-variance efficiency of a model's factors. In contrast, the GLS R^2 is completely determined by the factors' proximity to the minimum-variance boundary. If a factor is nearly mean-variance efficient, the GLS R^2 will be close to one, but the OLS R^2 can, in principle, be anything. For nontraded factors, LNS reach a similar conclusion, that is, the GLS R^2 is determined by the mimicking portfolios' proximity to the minimum-variance boundary, whereas the OLS R^2 is not. In addition, given the large values of the sample OLS R^2 even when the model is strongly rejected by the data, LNS forcefully advocate performing GLS estimation and reporting the GLS R^2. It is also important to note that HKM consider seven asset classes (FF25, US bonds, sovereign bonds, options, CDS, commodities, and FX), most of which are characterized by a strong factor structure individually. The percentage of the variance explained by the three largest eigenvalues exceeds 86% for all asset classes except for commodities. It is notable that this percentage is 99.5% for options, 96.7% for US bonds, and 94.1% for FF25. For all asset classes, except for commodities, the largest eigenvalue explains between 70% and 90% of the variance. The OLS CSR estimator does not incorporate explicitly any information about the covariance structure of the test assets and it tends to produce large R^2 values even when the factors have weak explanatory power. (See LNS and Kleibergen and Zhan 2015.) Finally, in

Figures 4 and 5, we report OLS and GLS CSR rolling window results, respectively.

Figures 4 and 5 about here

Due to lack of a sufficiently large number of time series observations at a quarterly frequency (see the last row of Table 1), the estimation is performed for monthly data. We employ rolling windows of 120 months and nontraded capital in HKM's two-factor model to display the behavior of the OLS and GLS CSR prices of beta risk for the market and capital factors for six of the seven asset classes.[9] For ease of comparison, we keep the scale of the vertical axes across the two figures the same. The OLS results in Figure 4 indicate that the two series exhibit extreme fluctuations and often assume negative values. In contrast, the plots in Figure 5 show that the GLS prices of beta risk for the market and capital factors are much more stable. This may be another reason for considering GLS in addition to or instead of OLS in the CSR analysis.

Before turning to the empirical analysis, we would like to stress the importance of accounting for potential model misspecification in two-pass CSRs. Given that the asset-pricing models should be viewed only as approximations of the true pricing kernel, it seems prudent to adjust the inference procedure for the possibility of model misspecification. We account for potential model misspecification by constructing misspecification-robust t-statistics as described in KRS. In this respect, another advantage of GLS over OLS is the robustness of the GLS standard error of the estimate not only to potential model misspecification but also to possible lack of identification. (See the Internet Appendix.)

3.2. Main empirical results

This subsection reports cross-sectional asset-pricing results for HKM's two-factor model with market and nontraded capital (Tables 2–5) as well as their corresponding single-factor specifications (Tables 6-9). The t-statistics are in round brackets while the p-values are in square brackets. Below each risk premium estimate, we report the t-statistic under correctly specified models (t-stat$_c$) and the misspecification-robust t-statistic (t-stat$_m$) proposed by KRS.[10] To be consistent with HKM,

[9] We do not report results for CDS since the time series is too short. In addition, for readability purposes, we do not include confidence intervals in the figures. The 90% confidence intervals (based on normal asymptotics with misspecification-robust standard errors) are found to be wide due to sampling and model uncertainty, and they are available from the authors upon request.

[10] The t-statistics under correctly specified models are the standard generalized method of moments (GMM) t-

we do not adjust for serial correlation in the computation of t-statistics and p-values. Doing so would render the standard errors of the estimates and the p-values of the tests even larger. We also include the OLS CSR R^2 and a test of correct model specification (SPEC TEST) based on the generalized Shanken (1985) test statistic and its corresponding asymptotic p-value. (See KRS for details.)

In Panel A of Table 2, we report OLS cross-sectional asset-pricing tests for HKM's two-factor model.

Table 2 about here

Based on t-stat$_c$ and a 5% significance level of the test, Panel A of Table 2 shows that the capital risk factor receives a nonzero price of beta risk in five out of seven asset classes. At a 10% significance level, the capital risk premia estimates are statistically significant in all asset classes. Moreover, when considering an unbalanced panel of all asset excess returns (the All column), the overall capital risk premium is 9.35% per quarter with a t-statistic under correctly specified models of 2.52. These are precisely the results that led HKM to claim that a unifying kernel that prices the most important financial asset classes indeed exists.

Note, however, that the picture starts to change when considering t-statistics that are robust to model misspecification, t-stat$_m$. In this case, at a 5% significance level, capital risk is priced only in the cross-section of FF25, options, and FX. Importantly, when bundling all assets together, the t-statistic decreases from 2.52 under correctly specified models to 1.12 under potentially misspecified models. Model misspecification (as signalled in several instances by the relatively small asymptotic p-values of the specification test) drives a wedge between the two t-ratios. Consistent with the findings of LNS and Kleibergen and Zhan (2015), the OLS CSR R^2s are found to be unrealistically large. In addition, the estimated prices of beta risk are not very sensible. For example, for options, the capital risk premium is about 22.4% per quarter when the market risk premium is typically around 1.5%. Large capital risk premia can be observed in all asset classes, and this raises the question of how meaningful it is to report the price of beta risk from two-pass CSRs. Balduzzi and Robotti (2010), in their (CSR) economic risk premium decomposition, explain why the OLS and GLS risk premia are often found to be so large. It is mainly due to the unspanned and

statistics under conditional heteroskedasticity. (See also Jagannathan and Wang 1998.)

mispricing components of the economic risk premia. They argue that the unspanned component of the economic risk premium can be easily manipulated and made arbitrarily large (in absolute value) by adding measurement error, unrelated to security returns, to the candidate pricing kernel and to the factors. Similarly, the mispricing component reflects the inability of the model to price assets and is also arbitrary. In fact, one obtains different estimates of the pricing-kernel parameters, and, hence, of an economic risk premium, depending on the moment conditions imposed in estimation. With HKM's data, the unspanned and mispricing components are found to be substantial.

Importantly, as previously noted, it is incorrect to focus on the price of beta risk (the gammas) if the factors are correlated (the correlation between MKT and $CPTL$ is about 78% in the data) and the researcher intends to perform model selection based on the parameter estimates. In this case, as strongly argued by Cochrane (2005) and KRS, one needs to consider the price of covariance risk (the lambdas). HKM (p. 12) state: "The significance of intermediary capital risk after controlling for the market indicates that our pricing kernel statistically improves on the CAPM for all sets of test assets." They reach this conclusion by looking at the gammas. As shown in Panel B of Table 2, when looking at the lambdas, the pricing ability of intermediary capital is largely compromised. Using misspecification-robust standard errors and a 5% significance level, there is only some evidence of incremental pricing for capital in the case of equities. These results suggest that there is no need for a factor model with two market factors. The analysis of single-factor models later on will indeed show that the CAPM and the single-factor version of HKM (HKMSF) perform about the same.

More generally, we think that the importance of the price of *multivariate* beta risk should be de-emphasized. As noted by Balduzzi and Robotti (2010), when the factors are far from being spanned by the security returns, the price of beta risk is very difficult to interpret. Essentially, it can be made arbitrarily large (in absolute value) without altering the properties of the underlying security returns. In this context, it is difficult to assign some economic meaning to the magnitude of the gammas as much as it is difficult to assign some economic meaning to the magnitude of the lambdas. At the same time, the t-statistics associated with the lambda estimates allow us to perform model selection in a meaningful way, while the t-statistics associated with the gamma estimates do not. A viable alternative to the lambdas would be to use *univariate* betas in the CSR. (See Kan and Robotti, 2011.)

In light of the previous discussion on the choice of weighting matrix, we report our GLS findings in Table 3.[11]

Table 3 about here

With GLS, the significance of the capital factor completely disappears when misspecification-robust t-ratios and a 5% significance level are used. Even for equities, the capital factor does not appear to be priced anymore, regardless of whether we focus on the gammas or on the lambdas. Consistent with LNS, the GLS R^2s are now much lower than in the OLS case. The highest R^2 (0.37) is for FX, and the lowest (0.04) is for equities.

As in HKM, we also consider a scenario in which the cross-sectional intercept is set equal to zero, that is, the zero-beta rate coincides with the risk-free rate. Table 4 is for constrained OLS, while Table 5 is for constrained GLS.[12]

Tables 4 and 5 about here

The results in Table 4 are largely consistent with the findings in Table 2, that is, using misspecification-robust standard errors and the usual 5% significance level, capital appears to be priced only in three asset classes when considering the price of beta risk in Panel A, and only in the cross-section of equities when focusing on the price of covariance risk in Panel B. Table 5 provides further support to the GLS results in Table 3. When focusing on the price of covariance risk, we find that the capital risk factor has no incremental explanatory power in any asset class.

Given the poor performance of HKM's two-factor model and our belief that MKT and $CPTL$ carry similar pricing implications, in Tables 6 through 9 we consider two single-factor models: CAPM and HKMSF. Since we now analyze single-factor models, focusing on gammas or lambdas is equivalent, and we choose to report results for the price of beta risk (results for the price of covariance risk are similar and available upon request).

Tables 6 and 7 about here

[11]Note that bundling assets together as in the All column in OLS is no longer feasible in GLS, which requires inverting the covariance matrix of asset excess returns. Since the effective number of time series observations in HKM's unbalanced panel is 35 and the number of test assets is 124, this covariance matrix is clearly column rank deficient and cannot be inverted.

[12]For the constrained GLS case, the covariance premia coincide with the parameters that minimize the modified Hansen and Jagannathan (1997) distance. (See Kan and Robotti 2008.)

Tables 6 and 7 indicate that the pricing performance of CAPM and HKMSF is very similar. The market and capital factors seem to do a good job for US bonds, options, CDS, and FX, and not so much for equities and commodities. The constrained OLS and GLS results for single factor models are reported in Tables 8 and 9. Once again, the market and capital factors in single-factor models perform similarly and seem to receive a nonzero price of beta risk in several asset classes.

Tables 8 and 9 about here

In summary, when considering the two-factor model of HKM, we find some evidence of incremental pricing for the capital factor in the case of equities only. For single-factor models, the pricing performance of the market and capital factors is very similar across asset classes. To assess whether HKM's two-factor model produces more statistically insignificant pricing errors on the various test assets relative to single-factor models, we developed asymptotic tests of statistical significance of individual pricing errors in two-pass OLS and GLS CSRs. (See the Internet Appendix for the methodological results.) Empirically, we find that HKM's two-factor model generates a larger number of insignificant individual pricing errors relative to the single-factor models only for equities and in the OLS case. We also performed pairwise nested model comparison tests of HKM vs. CAPM based on CSR OLS and GLS R^2s, as described in KRS. As for OLS, HKM dominates CAPM only in the case of equities. We find no instance of outperformance of HKM with GLS (the results of the analysis are available upon request). CSR results with traded capital and monthly data are in the Internet Appendix and are consistent with the analysis in the paper. We believe that the spectacular performance of HKM's two-factor model is largely due to conducting model selection based on gammas instead of lambdas, focusing on OLS only, and not accounting for potential model misspecification in the analysis. It is reassuring to see that our CSR findings largely confirm the economic analysis based on Sharpe ratios of the previous section.[13]

[13] In our analysis, we intentionally considered only HKM's single-factor and two-factor models to ensure fair comparison with their results. CSR findings for specifications with additional factors, such as the Fama and French (1993) factors for example, are available upon request. In these augmented models, the pricing ability of HKM's capital factor is even weaker and the evidence of identification failure is pervasive.

4. Additional analysis

In this section, we subject our previous empirical analysis to further robustness checks. First, we explicitly deal with the small T and large N features of the various panel datasets. Next, we investigate whether HKM's model is well-identified. Finally, we relate the capital factor of HKM to the leverage factor of Adrian, Etula, and Muir (2014).

4.1. Finite-sample inference

It is common practice in empirical work to resort to asymptotic inference in evaluating the performance of asset-pricing models. Despite the substantial advances in developing a unifying and robust asymptotic framework for analysis, there are two main features that may make the asymptotic inference unreliable. First, the number of available time series observations T is much smaller than the one that is typically required to ensure an accurate asymptotic approximation. Second, the number of test assets N is often large relative to T or, equivalently, the number of effective time series observations per moment condition (pricing error) is small. This may potentially induce size distortions and biases that could be non-trivial in samples with a small number of time series observations and a large cross-section of test assets.

Therefore, in this study, we also consider a bootstrap approach to deal with these finite-sample issues. (See the Internet Appendix for the bootstrap details.) The conditions for the validity of the bootstrap are weak and essentially require stationarity and ergodicity of the returns on the test assets and the factors. This could accommodate, in a model-free way, serial correlation and conditional heteroskedasticity of unknown form by fully preserving the cross-correlation structure of the data.[14] When possible, we bootstrap standardized (pivotal) statistics for testing statistical significance and impose the null hypothesis for specification testing.[15]

The results in Tables 2 through 9 indicate that the misspecification-robust t-ratios and the bootstrap p-values provide largely consistent answers. Not surprisingly, given the short time series and large number of assets, the bootstrap rejects the null hypothesis typically less than the asymptotics under potential model misspecification. It often becomes difficult to detect deviations from

[14] To be consistent with HKM and the previous asymptotic analysis, we report bootstrap results (based on 1,999 replications) without blocking the observations. For block sizes of more than one observation, the p-values of the specification and parameter significance tests are generally larger. (Results are available upon request.)

[15] Note that the bootstrap is not implementable in the All assets case given the highly unbalanced nature of HKM's panel of asset excess returns.

the null of exact pricing when looking at the bootstrap p-values. This is likely due to the short time series dimension and inevitable low power of the specification test in this framework. As for HKM's two-factor model, the low power of the specification test could be also due to potential lack of identification, as we will explain in the next subsection.

4.2. Model identification across asset classes

The matrix $X = [1_N,\ \beta]$ needs to be of full column rank for the risk premia in two-pass unconstrained CSRs to be identifiable. For constrained CSRs, it is the β matrix that needs to be of full column rank. A violation of these rank conditions implies that the model parameters are unidentified, and the corresponding estimates and t-statistics are characterized by highly irregular and non-standard limiting distributions. GMM inference breaks down in reduced-rank asset-pricing models. This is why it is of the utmost importance to subject the X or β matrices to a rank test.

We are concerned that HKM's two-factor model may be of reduced rank, at least for some asset classes. Simple inspection of the various β matrices suggests that for some asset classes the betas on the capital factor are close to zero. Our concern seems legitimate since in their Table 4, HKM report a cross-sectional mean for the capital betas of 0.01. (See the last column in HKM's Table 4.) Despite such small betas, HKM reject that the betas on the market and capital factors are jointly zero for all asset classes. Simulation experiments available from the authors upon request suggest that the outcomes of their joint beta test cannot be trusted. The size distortions of the asymptotic chi-squared test proposed by HKM are very large, thus making this test unreliable. Importantly, the joint beta test proposed by HKM is not a rank test and, more in general, is not a meaningful test. We now explain why. Rank deficiency can definitely occur when the betas are jointly zero. But, in the unconstrained CSR case, it can also occur when one or more columns of the beta matrix are constant. HKM's test will not capture this scenario. Moreover, it could be that the individual betas are large but that a linear combination of them is nearly zero or constant. This would arise, for example, if two or more factors are proxies of the same underlying factor and are highly correlated. Figure 1 and Panel A of Table 2 suggest that this may well be the case in the current context. HKM's test will not capture this scenario either. Finally, since there is an intercept in the CSR, the number of linear combinations of the betas and vector of ones that can give rise to spuriousness of the results increases. This latter scenario is also not considered by HKM.

In brief, a rank test is needed. Kan and Robotti (2012) strongly advocate the use of an (approximate) finite-sample F-test to determine whether X or β are rank deficient. (See the Internet Appendix for details.) Their proposed test appears to have desirable size and power properties even under strong conditional heteroskedasticity. (Simulation results are available from the authors upon request.) Panel A of Table 2 reports this (approximate) finite-sample F-test and its corresponding p-value (fs p-val) in square brackets. For five of the seven asset classes, we cannot reject that the two-factor model is of reduced rank. The rank test results for constrained CSRs in Panel A of Table 4 are similar. The only exception is for equities. In the unconstrained CSR, the rank test of X suggests that there are identification problems for equities, while in the constrained CSR the two-factor model seems to be well identified over the same asset space. The likely reason for this result is that in the unconstrained CSRs, a linear combination of the asset betas is close to the column of ones, thus causing identification failure. It is not the betas being close to zero, but it is the presence of the cross-sectional intercept to cause problems. In principle, the proposed rank test should capture all the possible scenarios that lead to identification failure. Thus, given the possible lack of power of the test for correct model specification in under-identified models (see Gospodinov, Kan, and Robotti 2017), it is recommendable to always employ misspecification-robust inference regardless of the outcome of the specification test. It is important to stress that in the presence of identification failure, the traditional inference based on the OLS CSR estimator is seriously compromised. In contrast, the misspecification-robust standard errors for GLS and the Hansen-Jagannathan distance are robust not only to potential model misspecification but also to possible identification failure. (See the Internet Appendix and Gospodinov, Kan, and Robotti 2014.) As for single factor models, the rank tests always reject the null of a deficient rank of the X and β matrices. This suggests that for CAPM and HKMSF, GMM inference is reliable once it is robustified against potential model misspecification.

4.3. Leverage factor of Adrian, Etula, and Muir (2014)

In this subsection, we explore whether the factor of Adrian, Etula, and Muir (AEM, 2014) outperforms the leverage factor of HKM. As mentioned in the introduction, AEM proposed a broker-dealer leverage risk factor. HKM and, more recently, Adrian, Moench, and Shin (2019) undertake a comprehensive comparison of alternative intermediary asset-pricing factors and report notable differences in the performance of the competing factors they consider. It is instructive to subject the AEM factor to the same analysis of the HKM factor and determine whether the differences in their pricing and investment performance continue to persist.

For the analysis in this subsection, two versions of the AEM factor are employed. The first one (*LevFac*) is a nontraded version of leverage, as discussed in AEM and extended by HKM. The second one (LMP) is a traded version of the leverage factor, and it is constructed as a mimicking portfolio from projecting the nontraded AEM factor on the market excess return, HKM's traded factor, a momentum factor, and the six Fama-French size and book-to-market portfolio excess returns.[16] Both factor versions cover the period 1970:Q1–2012:Q4. To characterize the investment performance of the AEM factor, we use AEM's mimicking portfolio, LMP.[17]

Table 10 about here

Panel A of Table 10 provides some summary statistics for the leverage mimicking portfolio. For ease of comparison, in the same table we also report the corresponding descriptive statistics for the market, MKT, and the traded version of HKM's capital factor, $CPTLT$.

Several observations are in order. First, unlike the high (84%) correlation between MKT and $CPTLT$ discussed in Section 2, the correlation between LMP and MKT or $CPTLT$ is moderate – 31% and 33%, respectively. Second, and importantly, the leverage mimicking portfolio has a larger factor mean and a smaller standard deviation than both MKT and $CPTLT$. This, in turn, results in a larger Sharpe ratio for LMP. Panel B of Table 10 shows that the sample squared Sharpe ratios of the two-factor model, with MKT and LMP (AEM), and single-factor model, with LMP only (AEMSF), are strongly statistically significant. Furthermore, Panel C of Table 10 reveals that the sample squared Sharpe ratios for AEM and AEMSF are higher than the ones for the CAPM and HKM and HKMSF, at least at the 10% significance level.[18] The documented outperformance of the traded leverage factor is consistent with the results in Adrian, Etula, and Muir (2014).[19]

[16] The six Fama-French size and book-to-market portfolio returns and the momentum factor are from Kenneth French's website, while the AEM nontraded factor is from Asaf Manela's webpage.

[17] To verify that the mimicking portfolio is well identified, we performed a rank test that strongly rejected the null hypothesis of a reduced rank.

[18] See BKRS for the required toolbox to compare asset-pricing models with mimicking portfolios.

[19] We conducted two additional sensitivity checks. First, we projected AEM's nontraded leverage factor, *LevFac*, only on MKT and $CPTLT$. In this case, the differences between the models become statistically insignificant at the 5% nominal level. Second, we constructed the same mimicking portfolio for the nontraded HKM factor, $CPTL$, and used this as a traded factor in the analysis. The use of this alternative factor leaves the results for HKM and HKMSF in Tables 1 and 10 largely unchanged.

Next, we turn to the OLS and GLS cross-sectional regression analysis for beta and covariance risks in the two- and single-factor AEM formulations using their nontraded leverage factor, *LevFac*.

Tables 11, 12, and 13 about here

Tables 11 (OLS) and 12 (GLS) are concerned with the results for the two-factor AEM model and mimic Tables 2 and 3 for the HKM two-factor model. Table 13 presents the results for the single-factor AEM model and should be compared with Panel B of Tables 6 (OLS) and 7 (GLS). Overall, the nontraded factor analysis suggests that the HKM and AEM models (in their two-factor or one-factor formulation) perform about the same. In particular, the GLS inference that we advocate in this paper does not produce evidence of pricing, neither for the AEM nor for the HKM factor at the 5% significance level.

We also performed pairwise model comparison based on cross-sectional OLS and GLS R^2 differences. Inference is carried out using the sequential test for non-nested models proposed by KRS only for those asset classes for which there is evidence of identification for both models. The results suggest that the two models cannot be convincingly differentiated on statistical grounds (at the 5% significance level) except for equities in the OLS case, where AEMSF performs better than HKMSF (with a p-value of 0.000).

In summary, given the usual caveat on the choice of the projection span in the construction of the mimicking portfolio, the AEM leverage mimicking portfolio is characterized by a better investment performance relative to the HKM factor. However, similar to HKM, any evidence of pricing for AEM's nontraded leverage factor in a wide range of asset classes remains elusive.

5. Placebo test: Industry portfolios

In the following analysis, we subject HKM's and our proposed methodology to a placebo test. In principle, factor portfolios that mimic some of the statistical properties of HKM's traded leverage factor but do not conform to HKM's narrative should not easily pass HKM's and our battery of asset-pricing tests.[20] This allows us to further put into context the results of our earlier analysis. The considered factor portfolios are the 49 value-weighted industry portfolios from Kenneth

[20]We thank an anonymous referee for suggesting this test to us.

French's webpage over the period 1970:Q1 – 2012:Q4 (the same period as in HKM and our previous analysis). We initially apply a two-way filter to these placebo industries, that is, we require the various industry excess returns (in excess of the risk-free rate) to have means and standard deviations greater than the mean and standard deviation of the market excess return (0.015 and 0.091, respectively). The idea here is to mimic the first and second moment of HKM's traded leverage factor that has a higher mean and a higher standard deviation than the market excess return (0.019 and 0.133, respectively). This leaves us with 38 industry portfolios, and we further eliminate portfolios 45 (Banking) and 48 (Trading) because they contain some of the New York Fed primary dealers that enter the construction of HKM's value-weighted intermediary equity return. Each of the 36 remaining industry excess returns serves, in turn, as an asset-pricing factor in a two-factor model that includes the excess market return. The competing model is the usual static CAPM with the market as the only pricing factor. We implement pairwise model comparisons based on squared Sharpe ratios as in Panel C of Table 1. Out of the 36 pairwise model comparisons, we cannot reject the null of equal squared Sharpe ratios at the 5% nominal level in 34 cases. Consistent with our discussion around Table 1, this indicates that the industry factor in the two-factor model does not provide substantial diversification benefits and delivers a multivariate Sharpe ratio that is very close to the one of the single-factor CAPM. The two only instances in which we reject the equality of squared Sharpe ratios occur when the two-factor model either contains portfolio 5 (Tobacco Products) or portfolio 30 (Petroleum and Natural Gas). This is easily explained by the fact that these two industry portfolios have relatively low correlations with the market (0.426 and 0.635, respectively) and thus provide likely greater diversification benefits.[21]

Given the importance of accounting for the correlation between the market and the industry factor when considering two-factor models, we next apply an additional filter to the above 36 industry portfolio returns by requiring that their correlation with the excess market return is greater than 0.838 (the correlation of HKM's factor with MKT). With the remaining 10 industry portfolios, we implement asset-pricing tests as in Tables 1 to 3.[22] Our findings in the previous paragraph imply that neither a two-factor model with the market and, in turn, each of the 10

[21]When considering comparisons of each industry factor excess return with the market excess return, we never reject the null of equal squared Sharpe ratios at any conventional significance level. The set of empirical results for this section is too large to be included even in the Internet Appendix but is available from the authors upon request.

[22]The 10 industry portfolios are, in the order, Printing and Publishing, Construction Materials, Machinery, Electrical Equipment, Electronic Equipment, Measuring and Control Equipment, Transportation, Wholesale, Retail, and Insurance.

selected industry portfolios nor a single-factor model with only an industry portfolio can produce significantly higher squared Sharpe ratios than the CAPM. Turning to the cross-sectional analysis, we consider asset classes for which HKM provide evidence of pricing using their traded leverage factor in either a single-factor or a two-factor beta-pricing model. (See HKM's Table 17.) The four asset classes are US bonds, Options, CDS, and FX. Before summarizing our results, it is important to remind the readers about the methodology employed by HKM in cross-sectional pricing. They draw their conclusions on model risk premia based on the price of beta risk, OLS weighting, and asymptotic standard errors that do not account for potential model misspecification and lack of identification. In contrast, we recommend focusing on the price of covariance risk (when the model contains correlated factors), GLS, and bootstrap inference that is robust to model misspecification as well as possible lack of identification.

Since we consider four asset classes and 10 industry portfolios, we have a total of 40 cases to examine. As for the two-factor model with industry and market factors,[23] if we were to use HKM's methodology and a 5% significance level, we would conclude that in 40% of the cases (16 times out of 40) the industry factor is priced. In contrast, when using our proposed methods, we never find evidence of pricing for the industry factor at the 5% nominal level. The drivers of this big difference in results are the extensive evidence of identification failure for the two-factor model on these asset classes, the emphasis on covariance versus beta risk, and GLS-based bootstrap inference that is robust to potential model misspecification as well as possible lack of identification. The results for the one-factor model with only an industry factor are even more striking. Based on HKM's methodology and a 5% nominal size of the tests, the industry factor is found to be priced in 39 out of 40 cases. This overwhelming evidence of pricing occurs precisely in situations where there is widespread evidence of model misspecification, some evidence of failure of the full rank condition, and the time series sample size is small relative to the number of test assets. In contrast, our proposed battery of tests only retains the industry factor one time out of 40 (for portfolio 41 (Transportation) in the case of US bonds).

To conclude, this experiment reveals that if we were to implement the traditional asset-pricing tests as in HKM, we would likely conclude that a factor is priced even if it does not fit the narrative of the intermediary asset-pricing theory. In contrast, by accounting for several important features

[23]In the cross-sectional analysis, for consistency with HKM, we do not subtract the risk-free rate from the industry portfolio returns. This has only a minor impact on the results.

of the data generating process, our proposed tests pose a serious challenge to empirical and theory-based asset-pricing factors and specifications.

6. Downside market risk

LMW claim that the downside risk capital asset pricing model (DR-CAPM) can price the cross-section of currency, equity, commodity, sovereign bond, and equity index option returns. To capture the relative importance of downside market risk, they propose that expected excess returns follow

$$\mu_r = \beta\gamma + (\beta^- - \beta)\gamma^-, \tag{12}$$

where β and β^- are the *univariate* unconditional and downside betas, respectively, γ is the market risk premium, and γ^- is the price of downside market risk (DR).[24]

Let $B = [\beta,\ \beta^-]$, $\iota = [-1,\ 1]'$, $B_1 = B\iota \equiv [B_{1,1}, B_{1,2}, \ldots, B_{1,N}]'$, $H = (B_1'B_1)^{-1}$, $A = HB_1'$, $\mu_{rc} = \mu_r - \beta\mu_f$, and $e_0 = [I_N - 1_N(1_N'1_N)^{-1}1_N']\mu_r$. The OLS second-stage risk premium on (relative) downside beta risk, γ^-, is given by

$$\gamma^- = A\mu_{rc}. \tag{13}$$

The corresponding vector of pricing errors on the N test assets is

$$e = \mu_{rc} - B_1\gamma^-, \tag{14}$$

and the centered and uncentered OLS R^2s are

$$\rho^2 = 1 - \frac{e'e}{e_0'e_0} \tag{15}$$

and

$$\rho_U^2 = 1 - \frac{e'e}{\mu_r'\mu_r}, \tag{16}$$

respectively.

LMW's empirical strategy consists of first running two univariate time series regressions, one for the entire sample and one for the downstate observations:

$$r_t = \alpha + \beta f_t + \epsilon_t, \tag{17}$$

$$r_t = \alpha^- + \beta^- f_t + \epsilon_t^- \text{ if } f_t \leq \delta, \tag{18}$$

[24] LMW constrain the zero-beta rate to be equal to the risk-free rate. In addition, they impose the restriction that the market risk premium is equal to the expected market excess return, that is, $\gamma = \mu_f$.

where f_t is the market excess return and $\delta = \hat{\mu}_f - \sqrt{\hat{V}_f}$ is an exogenous threshold for the downstate. More generally, δ could be defined as $\delta = \hat{\mu}_f - a\sqrt{\hat{V}_f}$ for $a > 0$. Based on the first-stage beta estimates, $\hat{\beta}$ and $\hat{\beta}^-$, the second-stage OLS estimate of the DR premium is then obtained by LMW as

$$\hat{\gamma}^- = \hat{A}\hat{\mu}_{rc}, \tag{19}$$

where $\hat{B} = [\hat{\beta},\ \hat{\beta}^-]$, $\hat{B}_1 = \hat{B}\iota$, $\hat{H} = (\hat{B}_1'\hat{B}_1)^{-1}$, $\hat{A} = \hat{H}\hat{B}_1'$, and $\hat{\mu}_{rc} = \hat{\mu}_r - \hat{\beta}\hat{\mu}_f$.

From the onset, it is important to realize that γ^- is well defined if and only if $\beta \neq \beta^-$, that is, $B_1 \neq 0_N$. In the Internet Appendix, we derive asymptotic tests of $H_0:\ B_1 = 0_N$ and $H_0:\ B_{1,i} = 0$ for $i = 1, \ldots, N$. We also show how to conduct asymptotically valid inference on γ^-, the DR premium. In unreported simulation experiments (available from the authors upon request), we demonstrate that our test statistics enjoy good finite-sample properties even when the effective number of time series observations is relatively small.[25] To further improve the finite-sample properties of our tests, we also implement the bootstrap as described in the Internet Appendix.[26]

The data is from LMW and consists of monthly log excess returns on currencies, equities, commodities, sovereign bonds, equity index options, and the value-weighted Center for Research in Security Prices (CRSP) US equity market index. In Table 14, we investigate whether the DR-CAPM is well-identified, that is, we conduct tests of $H_0:\ B_1 = 0_N$ and of $H_0:\ B_{1,i} = 0$ for $i = 1, \ldots, N$, based on the results in Proposition 1 of the Internet Appendix.

Table 14 about here

In the table, we present $\hat{B}_{1,i}$ as well as the asymptotic and bootstrap p-values of a t-test of H_0 : $B_{1,i} = 0$ for $i = 1, \ldots, N$. In addition, we include a joint test, JT, of $H_0:\ B_1 = 0_N$, and report its asymptotic and bootstrap p-values.[27] The p-values of the individual and joint tests are generally very large. Based on the asymptotic p-values, we can reject the null of $H_0:\ B_1 = 0_N$

[25] The effective number of time series observations is strictly smaller than T because the parameters in Eq. (18) and associated (cross) moments are estimated based on the downstate observations only. It should be noted that the effective number of observations is decreasing in a.

[26] To be consistent with the rest of the analysis, the bootstrap is performed without block resampling. The number of bootstrap replications is set equal to 1,999.

[27] In unreported results available upon request, we performed F-tests of $H_0:\ \beta = 0_N$ and of $H_0:\ \beta^- = 0_N$. Consistent with LMW's findings, we reject these null hypotheses for all asset classes. The implication of this preliminary analysis is that the potential lack of identification of the DR-CAPM is not due to small unconditional and downside betas.

only in one case out of nine, i.e., when the test assets include currencies, equities, and sovereign bonds. When considering bootstrap p-values, we never reject the null of lack of identification for the DR-CAPM at any conventional significance level. The bootstrap p-values range from 0.558 (for currencies, commodities, and options) to 0.924 (for options). This pattern is largely confirmed by the individual t-tests: except for call options, the evidence of equality of unconditional and downside betas is overwhelming. Because of the joint beta equality finding, this also implies that the DR-CAPM reduces to the CAPM. Overall, our analysis casts serious doubts on the proper identification of the downside risk model and, therefore, on the validity of the pricing results in LMW's paper.

Although these pre-tests suggest that the strong evidence for identification failure of the model will likely compromise the subsequent standard inference, we nevertheless turn to risk premium estimation and present OLS and GLS *DR* premium results in Tables 15–17. To be consistent with LMW, we include the market factor among the test assets.[28] We report the t-ratios based on the standard error of Fama and MacBeth (1973), and the t-ratios under correctly specified and potentially misspecified models based on Propositions 2 and 3 of the Internet Appendix. In addition, following LMW, we present the CSR R^2,[29] the root mean squared pricing error (RMSPE), and a chi-squared model specification test based on the Fama and MacBeth (1973) asymptotic covariance matrix of the sample pricing errors.[30]

Starting from currencies and equities as in Table 5 of LMW, Panel A of Table 15 reports the OLS *DR* premium estimate for the DR-CAPM.

Table 15 about here

The *DR* risk premium estimates are sizable and range from 1.41 for currencies and equities to 2.34 for developed currencies. The Fama and MacBeth (1973) t-ratios are high, leading LMW to conclude that downside market risk prices the cross-section of currencies and equities. However, as mentioned above, the use of the Fama and MacBeth (1973) standard error can only be theoretically

[28]In an OLS setting, including a factor among the test assets does not force the factor risk premium to be equal to the factor mean. This would only happen if GLS were to be used. Importantly, in LMW's OLS setting, the *DR* premium is the same regardless of whether the market factor is included among the test assets or not. However, the pricing errors and related R^2 measures are somewhat affected by this choice.

[29]Besides the centered R^2 reported by LMW, we include the uncentered R^2 because the latter has the nice property of being always bounded between 0 and 1.

[30]It can be shown that this test of joint significance of pricing errors is numerically identical for OLS and GLS.

justified when the betas are fixed and the model is correctly specified. In LMW's setting, the unconditional and downside betas are estimated. In addition, the model specification test that they employ (and we also report in the table) strongly rejects the DR-CAPM for all asset classes.[31] In Proposition 2 of the Internet Appendix, under general conditions, we derive an asymptotic standard error for the OLS *DR* premium estimate that accounts for the errors-in-variables (EIV) bias induced by the estimation of the betas. The t-ratio based on this standard error is denoted by t-stat$_c$. In the same proposition, we also derive an asymptotic standard error for the *DR* premium estimate that is robust to model misspecification: t-stat$_m$. It should be noted that while the OLS misspecification-robust t-ratio just described is not standard normally distributed when $B_1 = 0_N$ (or B_1 is very close to a zero vector), it still provides a great size improvement over the t-ratio based on the standard error of Fama and MacBeth (1973) used by LMW. Simulation experiments (available upon request) indicate that the t-test based on the standard error of Fama and MacBeth (1973) strongly overrejects when the betas are estimated, and the model is misspecified and poorly identified. Any evidence of pricing for the *DR* factor disappears when t-stat$_c$ or t-stat$_m$ are employed. This evidence is reinforced by inspection of the bootstrap p-values. They range from 0.242 for currencies and equities to 0.364 for currencies.

In Proposition 3 of the Internet Appendix and related discussion, we explain how a misspecification-robust standard error of the *DR* premium estimate can be derived in the GLS case. The proposed t-ratio is asymptotically standard normally distributed even when $B_1 = 0_N$. (See the Internet Appendix.) Therefore, differently from the OLS case, GLS-based risk premium inference is asymptotically justified. As for OLS, Panel B of Table 15 shows that at any conventional significance level, there is no evidence of pricing for the *DR* factor when misspecification-robust t-ratios or bootstrap p-values are considered.

In Table 16, we apply the same tools used for Table 15 to investigate whether downside market risk is priced in the cross-section of commodities and sovereign bonds. As in Table 6 of LMW, we consider four asset classes: (i) currencies and commodities; (ii) currencies, equities, and commodities; (iii) currencies and sovereign bonds; and (iv) currencies, equities, and sovereign bonds.

[31] It should be noted that the p-values of the model specification test, reported by LMW, need to be adjusted for the additional degree of rank deficiency from including the market factor in the test assets. For example, this incorrect computation of the number of degrees of freedom of the χ^2-test has a first-order impact on their results for the case of developed currencies.

Table 16 about here

Misspecification-robust inference, based on Propositions 2 and 3 and on the bootstrap of the Internet Appendix, reveals that downside market risk is not priced in these four asset classes at the standard 5% significance level. The results are robust to the choice of weighting matrix (OLS vs. GLS). Figures 6 and 7 present rolling and recursive window OLS and GLS estimates for downside market risk.

Figures 6 and 7 about here

The *DR* premium estimate in the DR-CAPM can often be negative, and more so when OLS is used as a weighting scheme. Similar to Figures 4 and 5 for HKM, the risk premium estimates appear to be more stable when GLS is considered. Finally, we consider options as test assets in the analysis, and we replicate and extend Table 9 in LMW.

Table 17 about here

The evidence of misspecification of the DR-CAPM is again pervasive. Using misspecification-robust standard errors, we find no evidence of pricing for (i) options and (ii) options, currencies, and commodities. However, for OLS in Panel A, we reject the null of a zero *DR* premium based on the bootstrap p-value. This is the only instance in which asymptotic inference and the bootstrap deliver conflicting messages. One possibility for this discrepancy is that in Table 17, the number of time series observations is small relative to the number of test assets. This is a situation in which inference based on asymptotic approximations could be somewhat unreliable, and the bootstrap, in principle, should work better. This said, and as emphasized throughout the paper, we do not recommend using OLS when the model is likely to be unidentified. For options and options, currencies, and commodities, the asymptotic (bootstrap) p-values of the test of $H_0 : B_1 = 0_N$ are 0.976 and 0.269 (0.924 and 0.558), respectively. When such strong evidence of identification failure exists, OLS-based inference (including the bootstrap) should not be trusted too much. In these scenarios, only the misspecification-robust GLS t-ratio based on Proposition 3 of the Internet Appendix (and the corresponding bootstrap p-value) will be asymptotically justified. It turns out that when GLS is considered, the misspecification-robust t-ratios are essentially zero, and the bootstrap p-values are 0.876 for options and 0.902 for options, currencies, and commodities.

In summary, we provide overwhelming evidence that the empirical validity of the DR-CAPM proposed by LMW should be interpreted with caution. The Fama and MacBeth (1973) standard error should not be used when the betas are estimated, and the model is misspecified and unidentified. Failure to do so will result in a substantial overstatement of the statistical significance of the DR premium estimate. In this setting, we show how asymptotically valid inference can be performed, at least in the GLS case. Given the results of the joint beta equality tests, we believe it would be more appropriate to conclude that there is simply not enough evidence of superiority of the DR-CAPM over the CAPM. This is essentially the point of LMW (see p. 200 in their paper), who correctly argue that when $\beta = \beta^{-}$, the DR-CAPM reduces to the CAPM.

7. Some recommendations for empirical practice

Our arguments so far point to a number of challenges that accompany empirical work in asset pricing across asset classes which are not specific to the papers by HKM and LMW. Several useful rules and recommendations emerge from our analysis that could ensure the robustness and reliability of the results. They cover some new aspects of the analysis that are largely ignored in the empirical literature and tend to complement and reinforce the general recipes in LNS. Below, we summarize these general recommendations for guiding the empirical practice.

Economic Significance. It is desirable to determine if some simple diagnostics and statistics of the proposed factors can be validated on economic grounds. In a single-factor setup, for a traded factor to represent a desirable investment strategy, its mean in excess of the risk-free rate should be reliably positive. In a multi-factor setup with traded factors, the spanning tests of BKRS will allow us to determine whether a proposed factor is Sharpe ratio improving. (See also Fama and French, 2018.) Moreover, the estimates of prices of beta risk shouldn't be too far from the factors' sample means. Large deviations of risk prices from the factors' sample means can be due, in addition to the unspanned component of the factors, to nontrivial departures from the equilibrium asset-pricing relation. (See Balduzzi and Robotti 2010.) Similarly, when a model is close to being correctly specified, the zero-beta rate should be close to the risk-free rate (with excess returns, the cross-sectional intercept should be close to zero).

At least from a mean-variance perspective, a good model should entail a maximum Sharpe ratio that is reasonably large in magnitude. Numerous papers have shown that empirically, the CAPM is not a good model. Therefore, alternative models that cannot improve on the CAPM can be hardly considered as interesting innovations. Formal model comparisons tests based on GLS CSR R^2s and maximum (squared) Sharpe ratios have been advocated by KRS, Barillas and Shanken (2017, 2018), Fama and French (2018), and BKRS. The practical implementation of these model comparison tests is straightforward and the results of the analysis should be routinely reported in empirical work.

Finally, plots of the simple mean-variance frontier with the models' maximum Sharpe ratios would help visualizing the departures of the proposed factors from mean-variance efficiency. In this respect, the GRS is a meaningful statistic to consider because it measures (in a squared Sharpe ratio sense) how far inside the ex-post frontier the factor returns are. Although a factor's proximity to the minimum-variance boundary is not the only metric for evaluating a model, it does seem to be economically reasonable.

When it comes to risk premia on nontraded factors, the analysis mentioned above can still be carried out using factor-mimicking portfolios. In this framework, the basis (test) asset span needs to be carefully chosen as so maximize the squared correlation between the nontraded factor(s) and the projection assets. In addition, the GRS cannot be used any longer because the weights of mimicking portfolio returns need to be estimated. Fortunately, BKRS provide all the necessary econometric tools for carrying out the mimicking portfolio analysis in a maximum Sharpe ratio environment.

Metrics for Goodness-of-Fit. When the test assets exhibit a strong factor structure, standard metrics for goodness-of-fit such as R^2s could be misleading and uninformative. Specifically, LNS point out that the OLS R^2 has, in general, little connection to the factor's or factor-mimicking portfolio's location in the mean-variance space. They strongly recommend reporting the GLS R^2 because the OLS R^2 often leads to a very generous assessment of the cross-sectional explanatory power of the factors of a given model. Kleibergen and Zhan (2015) also argue that the large values of the OLS R^2 are not indicative of the strength of the asset-pricing relation in the presence of an unexplained factor structure in the first pass residuals. In this respect, our analysis supports LNS and Kleibergen and Zhan's (2015) arguments. In addition, we recommend using the GRS when the model contains traded factors only and a suitably modified version of the GRS for general factors. (See BKRS.)

Covariance vs. Beta Risk. KRS document some subtle differences between the prices of beta risk and the prices of covariance risk when the risk factors are correlated. (See also Cochrane 2005.) Empirical work on multi-factor asset-pricing models typically focuses on whether factors are "priced" in the sense that coefficients on the *multivariate* regression betas are nonzero in the CSR relation. However, if the question is whether the extra factors f_2 improve the CSR R^2, then what matters is whether the prices of *covariance* risk associated with f_2 are nonzero. To be more specific, a factor can possess additional explanatory power for the cross-sectional differences in expected returns but yet have a zero risk premium in a model with multiple factors. This makes it problematic to use the price of beta risk of a factor for the purpose of model selection. All the statistical tools for implementing model selection based on the prices of covariance risk are provided in the Internet Appendix of KRS.

Alternatively, Chen, Roll, and Ross (1986) and Jagannathan and Wang (1996, 1998) define the beta of an asset with respect to a given factor as the OLS slope coefficient in a simple regression of its return on the factor. These betas are usually referred to as *univariate* regression betas. In models with univariate betas, adding or removing a factor will not change the values of the betas corresponding to the other factors and selecting models based on risk premia becomes more meaningful. Kan and Robotti (2011) derive misspecification-robust standard errors for the estimated prices of univariate betas in OLS, GLS, and weighted least squares (WLS) two-pass CSRs.

Overall, the analysis based on covariances and/or univariate betas truly addresses the issue of whether a factor adds to the explanatory power of a model. From a practical point of view, focusing on covariances and/or univariate betas instead of multivariate betas does not pose any additional challenge.

Efficient Estimation. Volatility of the test assets often overwhelms volatility of risk factors, especially macro risk factors, which hampers the sharpness of the empirical analysis. Different asset classes are typically characterized by strong within-class factor structure and vastly different across-class return volatility. The OLS estimator, that uses the identity matrix as a weighting matrix, does not take these important covariance data characteristics into account. Furthermore, the OLS estimator lacks a clear economic interpretation, as emphasized by Kandel and Stambaugh (1995). On the other hand, GLS and the Hansen and Jagannathan (1997) distance are better grounded in the economic analysis and provide a more robust estimation and inference framework. Other efficient estimators, such as maximum likelihood and continuously-updated GMM, share similar properties but appear to be hypersensitive to deviations from full identification. (See Gospodinov, Kan, and Robotti 2019.) GLS, on the other hand, is less fragile to such deviations and can be further robustified against various model deficiencies as described below. The invariant estimators are still useful to report in order to increase the confidence in the estimates or to exaggerate certain weaknesses of the analysis.

Robustness Evaluation. The stability of the risk premium estimates is often overlooked in the asset-pricing literature and researchers typically report estimates for only one sample choice. In this respect, it would be advisable to also present rolling and recursive window estimates as an additional robustness check. Furthermore, the statistical properties of the various estimators and test statistics should be subjected to greater scrutiny by means of Monte Carlo simulations. For example, Gospodinov, Kan, and Robotti (2013) provide guidance on how to simulate data in both linear and nonlinear (correctly specified and misspecified) asset pricing models by imposing different values on the true parameters. Finally, placebo tests such as the one in Section 5 (see also HKM, among others) would likely reveal whether the newly proposed factors actually add important and independent information for our understanding of financial markets and/or provide empirical support for a particular theoretical narrative.

Small-Sample Analysis. Economic theory typically develops structural asset-pricing models that are more likely to hold at lower frequency – say, quarterly or annual frequency. Evaluating the model empirically at quarterly or annual frequency also averages out the noise in the higher frequency financial data. This results in a relatively small times series sample size T. At the same time, as suggested by LNS, the pricing performance of a candidate model is assessed using a larger number of test assets N. When N is large relative to T (e.g., $N = 25$ and $T = 100$), the asymptotic theory provides a poor approximation of the finite-sample distribution of the statistic of interest and can lead to severely distorted inference. (See Ahn and Gadarowski 2004; Kleibergen and Zhan 2020, among others.) Direct finite-sample approximations are characterized by better properties (see Kan and Zhou 2002; Chen and Kan 2004, among others), but they are rarely available and are developed under restrictive conditions on the data. In this paper, we advance the use of a flexible, model-free bootstrap procedure that mimics closely the unknown structure of the data and approximates accurately the finite-sample distributions of the statistics of interest. If the interest lies in testing statistical significance, we bootstrap the relevant t-statistic. Since misspecification is a common feature of the empirical analysis, we recommend the use of the misspecification-robust t-statistic. The resampled data can also be used to construct bias-adjusted estimates. When the hypothesis of interest is testing a joint hypothesis or the correct specification of the model, we adjust the resampled data (typically the returns on the test assets or their means) to reflect the corresponding null hypothesis. The proposed bootstrap is easy to implement and exhibits excellent finite-sample properties.

Model Misspecification. Asset-pricing models are only approximations to reality and likely to be misspecified. Hou and Kimmel (2006), Shanken and Zhou (2007), Kan and Robotti (2009, 2011), KRS, and Gospodinov, Kan, and Robotti (2013, 2018) derive misspecification-robust standard errors of the parameter estimates in stochastic discount factor and expected return formulations of the underlying asset-pricing model. We expect these misspecification-robust standard errors to become an integral and permanent part of the analytical toolbox for evaluating asset-pricing models as their misspecification-robust counterparts in a maximum likelihood framework. (See White 1980; 1982.) Similarly to the White's (1980) standard errors, these misspecification-robust standard errors remain valid even when the model is correctly specified. Embracing model ambiguity and uncertainty requires a shift in the standard practice of using the Fama-MacBeth framework for inference and model evaluation.

Identification Failure. The statistical inference can be particularly fragile when model misspecification is combined with a possible identification failure. While the case of spurious inference with factors that are only weakly correlated with the test asset returns is now better studied and understood, rank failure can also occur for other reasons. For example, the inclusion of two factors that are individually highly correlated with asset returns can still be problematic – e.g., two noisy proxies of the true underlying risk factor (such as consumption, investment, market, etc.). A linear combination of these two factors will behave as a nearly spurious factor and will generate similarly misleading results as a genuine spurious factor. The recent tendency of constructing various (state-dependent) market proxies may jeopardize the standard inference. (See LMW, for example.)

Similarly, the presence of a cross-sectional intercept in the beta-pricing formulation can interact with the factors' betas and make $X = [1_N,\ \beta]$ column-rank deficient. In linear models, a rank test would likely detect the identification failure regardless of its source. In nonlinear models, testing for model identification is more involved but detecting lack of identification is still feasible. (See Wright 2003.) In general, poorly identified or under-identified models will give rise to inconsistent estimates and tests as well as highly non-standard inference. Finally, for some estimators (GLS and Hansen-Jagannathan distance, for example), misspecification-robust standard errors are immune to lack of identification of the model. (See the Internet Appendix and Gospodinov, Kan, and Robotti 2014.) It should be noted that the proposed misspecification-robust tests are not too conservative. For example, if the models are well-identified, they have similar size-corrected power to the standard tests. (See KRS.) The misspecification-robust tests become conservative when the identification of the model is seriously compromised, which suggests that the data is uninformative about the model under consideration.

Collectively, these simple rules can have overarching implications on how a robust and credible empirical analysis – guided by economic principles and accounting for salient data features of factors and test assets, data limitations, as well as model and sampling uncertainty – should be performed and validated.

8. Conclusion

A common thread in the recent empirical asset-pricing literature is the relentless search for risk factors with robust pricing performance. (See, for example, Harvey, Liu, and Zhu 2016; Feng, Giglio, and Xu forthcoming; Hou, Xue, and Zhang 2015 forthcoming, among others.) However, statistical evaluation of factor models on test assets even within a single asset class (equities, bonds, etc.) has remained challenging. Model uncertainty, poor identification, small time series sample sizes relative to the number of test assets, etc. pose a number of problems for the empirical analysis. Performing the pricing evaluation across asset classes, characterized by different volatility and factor structure, makes some of the issues more acute. We summarize these inference limitations within the context of intermediary and downside risk asset pricing and offer some general recommendations that would provide a reliable framework for assessing the economic and statistical significance of the proposed risk factors. Further work on constructing balanced panels of test assets across major asset classes will go a long way in providing empirical foundations for identifying common risk factors in expected returns.

References

Adrian, T., Etula, E., Muir, T., 2014. Financial intermediaries and the cross-section of asset returns. Journal of Finance 69, 2557–2596.

Adrian, T., Moench, E., Shin, H. S., 2019. Dynamic leverage asset pricing. Working Paper. IMF.

Ahn, S., Gadarowski, C., 2004. Small sample properties of the GMM specification test based on the Hansen-Jagannathan distance. Journal of Empirical Finance 11, 109–132.

Ang, A., Chen, J., Xing, Y., 2006. Downside risk. Review of Financial Studies 19, 1191–1239.

Balduzzi, P., Robotti, C., 2010. Asset pricing models and economic risk premia: A decomposition. Journal of Empirical Finance 17, 54–80.

Barillas, F., Shanken, J., 2017. Which alpha? Review of Financial Studies 30, 1316–1338.

Barillas, F., Shanken, J., 2018. Comparing asset pricing models. Journal of Finance 73, 715–754.

Barillas, F., Kan, R., Robotti, C., Shanken, J., forthcoming. Model comparison with Sharpe ratios. Journal of Financial and Quantitative Analysis.

Black, F., Jensen, M. C., Scholes, M., 1972. The capital asset pricing model: Some empirical tests. In Jensen, M. C. (Ed.), Studies in the Theory of Capital Markets. Praeger, New York.

Brunnermeier, M. K., Sannikov, Y., 2014. A macroeconomic model with a financial sector. American Economic Review 104, 379–421.

Chen, R., Kan, R., 2004. Finite sample analysis of two-pass cross-sectional regressions. Working Paper. University of Toronto.

Chen, N., Roll, R., Ross, S. A., 1986. Economic forces and the stock market. Journal of Business 59, 383–403.

Cochrane, J. H., 2005. Asset Pricing. Princeton University Press, Princeton and Oxford.

Fama, E. F., French, K. R., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33, 3–56.

Fama, E. F., French, K. R., 2018. Choosing factors. Journal of Financial Economics 128, 234–252.

Fama, E. F., MacBeth, J. D., 1973. Risk, return, and equilibrium: Empirical tests. Journal of Political Economy 71, 607–636.

Feng, G., Giglio, S., Xiu, D., forthcoming. Taming the factor zoo: A test of new factors. Journal of Finance.

Gibbons, M. R., Ross, S. A., Shanken, J., 1989. A test of the efficiency of a given portfolio. Econometrica 57, 1121–1152.

Gospodinov, N., Kan, R., Robotti, C., 2013. Chi-squared tests for evaluation and comparison of asset pricing models. Journal of Econometrics 173, 108–125.

Gospodinov, N., Kan, R., Robotti, C., 2014. Misspecification-robust inference in linear asset-pricing models with irrelevant risk factors. Review of Financial Studies 27, 2139–2170.

Gospodinov, N., Kan, R., Robotti, C., 2017. Spurious inference in reduced-rank asset-pricing models. Econometrica 85, 1613–1628.

Gospodinov, N., Kan, R., Robotti, C., 2018. Asymptotic variance approximations for invariant estimators in uncertain asset-pricing models. Econometric Reviews 37, 695–718.

Gospodinov, N., Kan, R., Robotti, C., 2019. Too Good to Be True? Fallacies in Evaluating Risk Factor Models. Journal of Financial Economics 132, 451–471.

Hansen, L. P., Jagannathan, R., 1997. Assessing specification errors in stochastic discount factor models. Journal of Finance 52, 557–590.

Harvey, C. R., Liu, Y., Zhu, H., 2016. ... and the cross-section of expected returns. Review of Financial Studies 29, 5–68.

He, Z., Kelly, B., Manela, A., 2017. Intermediary asset pricing: New evidence from many asset classes. Journal of Financial Economics 126, 1–35.

He, Z., Krishnamurthy, A., 2012. A model of capital and crises. Review of Economic Studies 79, 735–777.

He, Z., Krishnamurthy, A., 2013. Intermediary asset pricing. American Economic Review 103, 732–770.

Hou, K., Kimmel, R. L., 2006. On the estimation of risk premia in linear factor models. Working Paper. Ohio State University.

Hou, K., Xue, C., Zhang, L., 2015. Digesting anomalies: An investment approach. Review of Financial Studies 28, 650–705.

Hou, K., Xue, C., Zhang, L., forthcoming. Replicating anomalies. Review of Financial Studies.

Jagannathan, R., Wang, Z., 1996. The conditional CAPM and the cross-section of expected returns. Journal of Finance 51, 3–53.

Jagannathan, R., Wang, Z., 1998. An asymptotic theory for estimating beta-pricing models using cross-sectional regression. Journal of Finance 53, 1285–1309.

Kan, R., Robotti, C., 2008. Specification tests of asset pricing models using excess returns. Journal of Empirical Finance 15, 816–838.

Kan, R., Robotti, C., 2009. Model comparison using the Hansen-Jagannathan distance. Review of Financial Studies 22, 3449–3490.

Kan, R., Robotti, C., 2011. On the estimation of asset pricing models using univariate betas Economics Letters 110, 117–121.

Kan, R., Robotti, C., 2012. Evaluation of asset pricing models using two-pass cross-sectional regressions. In Duan, J.-C., Gentle, J. E., Haerdle, W. (Ed.), Handbook of Computational Finance. Springer.

Kan, R., Robotti, C., Shanken, J., 2013. Pricing model performance and the two-pass cross-sectional regression methodology. Journal of Finance 68, 2617–2649.

Kan, R., Zhang, C., 1999. Two-pass tests of asset pricing models with useless factors. Journal of Finance 54, 203–235.

Kan, R., Zhou, G., 2002. Hansen-Jagannathan distance: Geometry and exact distribution. Working Paper. University of Toronto.

Kandel, S., Stambaugh, R. F., 1995. Portfolio inefficiency and the cross-section of expected returns. Journal of Finance 50, 157–184.

Kleibergen, F., Zhan, Z., 2015. Unexplained factors and their effects on second pass R-squared's. Journal of Econometrics 189, 101–116.

Kleibergen, F., Zhan, Z., 2020. Robust inference for consumption-based asset pricing. Journal of Finance 75, 509–550.

Lettau, M., Maggiori, M., Weber, M., 2014. Conditional risk premia in currency markets and other asset classes. Journal of Financial Economics 114, 197–225.

Lewellen, J. W., Nagel, S., Shanken, J., 2010. A skeptical appraisal of asset pricing tests. Journal of Financial Economics 96, 175–194.

Maller, R. A., Turkington, D. A., 2002. New light on the portfolio allocation problem. Mathematical Methods of Operations Research 56, 501–511.

Markowitz, H., 1959. Portfolio Selection: Efficient Diversification of Investments. Yale University Press, New Haven.

Roy, A. D., 1952. Safety first and the holding of assets. Econometrica 20, 431–449.

Shanken, J., 1985. Multivariate tests of the zero-beta CAPM. Journal of Financial Economics 14, 327–348.

Shanken, J., Zhou, G., 2007. Estimating and testing beta pricing models: Alternative methods and their performance in simulations. Journal of Financial Economics 84, 40–86.

White, H., 1980. A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. Econometrica 48, 817–838.

White, H., 1982. Maximum likelihood estimation of misspecified models. Econometrica 50, 1–25.

Wright, J. H., 2003. Detecting lack of identification in GMM. Econometric Theory 19, 322–330.

Table 1

Summary statistics, Sharpe ratio analysis, and GRS.

Panel A reports factor means (Fac. Mean), standard deviations (Fac. SD), and correlation (Fac. Corr). In Panel B, we report bias-adjusted squared Sharpe ratios (Sh^2) for the CAPM, the two-factor model of HKM (HKM), and the single-factor model of HKM (HKMSF). Panel C is for differences in bias-adjusted sample squared Sharpe ratios between models. Finally, Panel D reports a conditional heteroskedastic version of the GRS test. MKT and $CPTLT$ denote the market and traded capital factors, respectively. N and T represent the number of assets and time series observations, respectively. Panels A–C are based on $T = 172$. p-values are in square brackets.

Panel A: Summary statistics

	MKT	*CPTLT*
Fac. Mean	0.015	0.019
	[0.030]	[0.062]
Fac. SD	0.091	0.133
Fac. Corr.	0.838	

Panel B: Squared Sharpe ratios

	CAPM	HKM	HKMSF
Sh^2	0.021	0.015	0.014
	[0.056]	[0.269]	[0.119]

Panel C: Squared Sharpe ratio comparisons

	HKM	HKMSF
CAPM	0.006	0.007
	[0.934]	[0.610]
HKM		0.001
		[0.277]

Panel D: GRS

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
CAPM	3.75	4.74	3.13	4.01	3.10	1.16	5.92
	[0.000]	[0.000]	[0.010]	[0.000]	[0.004]	[0.303]	[0.000]
HKM	3.84	4.70	2.86	4.05	3.24	1.15	5.33
	[0.000]	[0.000]	[0.017]	[0.000]	[0.003]	[0.314]	[0.000]
HKMSF	4.08	4.99	2.93	3.61	3.18	0.98	5.32
	[0.000]	[0.000]	[0.014]	[0.000]	[0.003]	[0.504]	[0.000]
N	25	20	6	18	20	23	12
T	172	148	65	103	47	105	135

Table 2

OLS cross-sectional asset-pricing tests by asset class.

The table presents the OLS estimates of the prices of beta and covariance risks for HKM's two-factor model. *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) t-ratio (t-stat$_c$), the KRS model misspecification-robust t-ratio (t-stat$_m$), and the bootstrap p-value (boot p-val). In addition, we present the sample OLS cross-sectional R^2 (R^2) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) p-values. Finally, RANK TEST and fs p-val are the (approximate) finite-sample rank test discussed in the Internet Appendix and its finite-sample p-value. N and T represent the number of assets and time series observations, respectively. p-values are in square brackets.

Panel A: Price of beta risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
CPTL	6.88	7.56	7.05	22.42	11.08	7.31	19.38	9.35
t-stat$_c$	(2.16)	(2.59)	(1.66)	(2.02)	(3.44)	(1.90)	(3.12)	(2.52)
t-stat$_m$	(2.10)	(1.73)	(1.73)	(2.12)	(1.81)	(0.82)	(2.06)	(1.12)
boot p-val	[0.032]	[0.112]	[0.119]	[0.112]	[0.118]	[0.634]	[0.088]	
MKT	1.19	1.43	1.24	2.81	1.11	−0.56	10.13	1.49
t-stat$_c$	(0.78)	(0.82)	(0.32)	(0.67)	(0.41)	(−0.25)	(2.17)	(0.80)
t-stat$_m$	(0.77)	(0.53)	(0.33)	(0.66)	(0.20)	(−0.15)	(1.32)	(0.86)
boot p-val	[0.423]	[0.624]	[0.723]	[0.480]	[0.866]	[0.867]	[0.174]	
INT	0.48	0.41	0.34	−1.11	−0.39	1.15	−0.94	−0.00
t-stat$_c$	(0.36)	(1.44)	(0.33)	(−0.31)	(−2.77)	(0.83)	(−0.83)	(−0.00)
t-stat$_m$	(0.34)	(1.45)	(0.35)	(−0.31)	(−4.31)	(0.88)	(−0.51)	(−0.00)
boot p-val	[0.731]	[0.131]	[0.713]	[0.754]	[0.010]	[0.426]	[0.564]	
R^2	0.53	0.84	0.81	0.99	0.67	0.25	0.53	0.71
SPEC TEST	69.26	60.41	9.32	15.33	47.13	20.22	15.19	
asy p-val	[0.000]	[0.000]	[0.025]	[0.428]	[0.000]	[0.444]	[0.086]	
boot p-val	[0.004]	[0.127]	[0.116]	[0.647]	[0.875]	[0.910]	[0.414]	
RANK TEST	1.33	2.20	1.39	0.66	2.20	0.98	0.74	
fs p-val	[0.156]	[0.006]	[0.248]	[0.821]	[0.031]	[0.500]	[0.684]	
N	25	20	6	18	20	23	12	124
T	172	148	65	103	47	105	135	172

Panel B: Price of covariance risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
CPTL	8.63	10.03	7.75	29.53	13.25	12.01	12.92	11.95
t-stat$_c$	(2.73)	(1.50)	(0.86)	(1.78)	(1.94)	(1.83)	(1.16)	(2.19)
t-stat$_m$	(2.54)	(0.89)	(0.91)	(1.91)	(0.95)	(1.22)	(0.53)	(1.06)
boot p-val	[0.012]	[0.407]	[0.354]	[0.130]	[0.371]	[0.529]	[0.581]	
MKT	−7.95	−9.43	−7.27	−29.44	−14.59	−14.37	−0.36	−11.20
t-stat$_c$	(−2.68)	(−1.05)	(−0.52)	(−1.54)	(−1.37)	(−1.63)	(−0.02)	(−2.12)
t-stat$_m$	(−2.58)	(−0.61)	(−0.55)	(−1.67)	(−0.64)	(−1.29)	(−0.01)	(−0.84)
boot p-val	[0.006]	[0.566]	[0.581]	[0.170]	[0.568]	[0.473]	[0.992]	

Table 3

GLS cross-sectional asset-pricing tests by asset class.

The table presents the GLS estimates of the prices of beta and covariance risks for HKM's two-factor model. *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) t-ratio (t-stat$_c$), the KRS model misspecification-robust t-ratio (t-stat$_m$), and the bootstrap p-value (boot p-val). In addition, we present the sample GLS cross-sectional R^2 (R^2) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) p-values. N and T represent the number of assets and time series observations, respectively. p-values are in square brackets.

Panel A: Price of beta risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
CPTL	0.35	3.58	4.28	7.98	5.48	4.29	8.67
t-stat$_c$	(0.17)	(2.28)	(1.47)	(2.66)	(2.36)	(1.72)	(2.02)
t-stat$_m$	(0.10)	(1.80)	(1.19)	(1.51)	(1.88)	(1.25)	(1.27)
boot p-val	[0.923]	[0.078]	[0.266]	[0.343]	[0.181]	[0.401]	[0.198]
MKT	−1.04	2.08	2.91	2.84	2.78	−0.10	10.00
t-stat$_c$	(−1.05)	(1.89)	(1.18)	(1.75)	(1.76)	(−0.08)	(3.08)
t-stat$_m$	(−0.81)	(1.50)	(0.84)	(1.33)	(1.48)	(−0.07)	(2.33)
boot p-val	[0.416]	[0.152]	[0.396]	[0.191]	[0.273]	[0.943]	[0.147]
INT	2.67	0.08	0.97	−1.61	−0.09	0.23	−2.12
t-stat$_c$	(3.67)	(6.12)	(1.65)	(−1.47)	(−3.68)	(0.58)	(−2.59)
t-stat$_m$	(2.38)	(5.72)	(1.33)	(−0.96)	(−2.39)	(0.51)	(−1.81)
boot p-val	[0.020]	[0.000]	[0.208]	[0.367]	[0.198]	[0.671]	[0.164]
R^2	0.04	0.08	0.22	0.15	0.07	0.21	0.37
SPEC TEST	86.75	70.33	10.00	60.35	73.98	26.06	21.81
asy p-val	[0.000]	[0.000]	[0.019]	[0.000]	[0.000]	[0.164]	[0.009]
boot p-val	[0.001]	[0.018]	[0.071]	[0.056]	[0.663]	[0.609]	[0.198]
N	25	20	6	18	20	23	12
T	172	148	65	103	47	105	135

Panel B: Price of covariance risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
CPTL	2.29	2.11	1.37	7.35	2.92	6.66	−4.46
t-stat$_c$	(1.05)	(0.97)	(0.33)	(1.40)	(1.36)	(2.32)	(−0.62)
t-stat$_m$	(0.57)	(0.58)	(0.19)	(0.77)	(1.06)	(1.70)	(−0.34)
boot p-val	[0.602]	[0.561]	[0.829]	[0.598]	[0.430]	[0.240]	[0.740]
MKT	−3.76	0.32	1.91	−4.57	−0.26	−7.70	19.24
t-stat$_c$	(−1.71)	(0.10)	(0.29)	(−0.69)	(−0.08)	(−2.09)	(1.67)
t-stat$_m$	(−1.00)	(0.06)	(0.17)	(−0.37)	(−0.06)	(−1.63)	(1.01)
boot p-val	[0.331]	[0.945]	[0.855]	[0.783]	[0.965]	[0.231]	[0.429]

Table 4

Constrained OLS cross-sectional asset-pricing tests by asset class.

The table presents the constrained OLS estimates of the prices of beta and covariance risks for HKM's two-factor model. *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. For each parameter estimate, we report the Jagannathan and Wang (1998) t-ratio (t-stat$_c$), the KRS model misspecification-robust t-ratio (t-stat$_m$), and the bootstrap p-value (boot p-val). In addition, we present the sample uncentered OLS cross-sectional R^2 (R^2_U) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) p-values. Finally, RANK TEST and fs p-val are the (approximate) finite-sample rank test discussed in the Internet Appendix and its finite-sample p-value. N and T represent the number of assets and time series observations, respectively. p-values are in square brackets.

Panel A: Price of beta risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
CPTL	7.82	7.00	8.13	24.69	6.57	5.55	23.75	9.24
t-stat$_c$	(3.81)	(3.21)	(3.49)	(1.39)	(2.58)	(1.74)	(2.57)	(2.83)
t-stat$_m$	(3.93)	(0.17)	(3.34)	(1.42)	(0.97)	(0.35)	(2.86)	(1.25)
boot p-val	[0.001]	[0.821]	[0.002]	[0.290]	[0.391]	[0.754]	[0.068]	
MKT	1.59	4.89	1.48	1.50	0.36	1.25	6.64	1.52
t-stat$_c$	(2.09)	(2.15)	(0.33)	(1.11)	(0.15)	(0.68)	(1.17)	(0.92)
t-stat$_m$	(2.08)	(0.23)	(0.31)	(1.11)	(0.05)	(0.31)	(1.02)	(0.94)
boot p-val	[0.048]	[0.777]	[0.750]	[0.199]	[0.961]	[0.790]	[0.326]	
R^2_U	0.96	0.83	0.95	0.99	0.62	0.09	0.81	0.68
SPEC TEST	67.87	90.32	10.49	11.57	76.05	23.83	12.52	
asy p-val	[0.000]	[0.000]	[0.033]	[0.773]	[0.000]	[0.301]	[0.252]	
boot p-val	[0.012]	[0.032]	[0.221]	[0.729]	[0.712]	[0.790]	[0.648]	
RANK TEST	2.12	2.12	1.29	0.80	2.09	1.10	1.42	
fs p-val	[0.003]	[0.007]	[0.282]	[0.683]	[0.039]	[0.363]	[0.171]	
N	25	20	6	18	20	23	12	124
T	172	148	65	103	47	105	135	172

Panel B: Price of covariance risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
CPTL	9.41	2.53	8.86	35.26	8.35	6.25	26.66	11.73
t-stat$_c$	(3.29)	(0.46)	(1.28)	(1.23)	(2.19)	(1.50)	(1.67)	(2.17)
t-stat$_m$	(3.29)	(0.02)	(1.15)	(1.26)	(0.45)	(0.35)	(1.79)	(1.03)
boot p-val	[0.002]	[0.976]	[0.255]	[0.292]	[0.652]	[0.746]	[0.200]	
MKT	−8.31	3.51	−8.23	−37.57	−9.59	−5.47	−20.95	−10.92
t-stat$_c$	(−2.62)	(0.42)	(−0.64)	(−1.21)	(−1.44)	(−0.92)	(−0.89)	(−1.86)
t-stat$_m$	(−2.63)	(0.02)	(−0.58)	(−1.24)	(−0.31)	(−0.32)	(−0.89)	(−0.76)
boot p-val	[0.015]	[0.978]	[0.567]	[0.295]	[0.744]	[0.736]	[0.337]	

Table 5

Constrained GLS cross-sectional asset-pricing tests by asset class.

The table presents the constrained GLS estimates of the prices of beta and covariance risks for HKM's two-factor model. *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. For each parameter estimate, we report the Jagannathan and Wang (1998) t-ratio (t-stat$_c$), the KRS model misspecification-robust t-ratio (t-stat$_m$), and the bootstrap p-value (boot p-val). In addition, we present the sample uncentered GLS cross-sectional R^2 (R_U^2) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) p-values. N and T represent the number of assets and time series observations, respectively. p-values are in square brackets.

Panel A: Price of beta risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
CPTL	5.98	1.77	6.54	7.75	4.41	3.87	11.86
t-stat$_c$	(3.94)	(1.18)	(2.84)	(2.39)	(1.91)	(1.61)	(3.40)
t-stat$_m$	(2.56)	(0.87)	(2.41)	(1.35)	(1.49)	(1.16)	(1.77)
boot p-val	[0.041]	[0.398]	[0.028]	[0.479]	[0.265]	[0.432]	[0.240]
MKT	1.56	1.51	2.64	0.96	2.05	−0.09	5.30
t-stat$_c$	(2.25)	(1.43)	(0.96)	(1.03)	(1.31)	(−0.07)	(2.62)
t-stat$_m$	(2.21)	(1.03)	(0.65)	(1.00)	(1.16)	(−0.06)	(1.59)
boot p-val	[0.031]	[0.325]	[0.549]	[0.325]	[0.381]	[0.959]	[0.196]
R_U^2	0.16	0.02	0.34	0.13	0.04	0.20	0.21
SPEC TEST	83.29	112.28	12.39	57.39	87.74	26.68	36.86
asy p-val	[0.000]	[0.000]	[0.015]	[0.000]	[0.000]	[0.182]	[0.000]
boot p-val	[0.003]	[0.000]	[0.128]	[0.078]	[0.625]	[0.640]	[0.054]
N	25	20	6	18	20	23	12
T	172	148	65	103	47	105	135

Panel B: Price of covariance risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
CPTL	6.61	0.13	4.89	10.22	2.65	6.02	9.59
t-stat$_c$	(3.16)	(0.06)	(1.44)	(2.11)	(1.27)	(2.31)	(1.91)
t-stat$_m$	(1.75)	(0.03)	(0.71)	(1.18)	(0.90)	(1.66)	(1.05)
boot p-val	[0.170]	[0.980]	[0.468]	[0.508]	[0.506]	[0.276]	[0.351]
MKT	−5.30	1.81	−2.39	−10.20	−0.79	−6.96	−3.40
t-stat$_c$	(−2.27)	(0.58)	(−0.37)	(−1.94)	(−0.24)	(−2.03)	(−0.56)
t-stat$_m$	(−1.27)	(0.33)	(−0.20)	(−1.07)	(−0.18)	(−1.58)	(−0.32)
boot p-val	[0.295]	[0.746]	[0.862]	[0.547]	[0.897]	[0.225]	[0.760]

Table 6

OLS cross-sectional asset-pricing tests by asset class (price of beta risk in single factor models).

The table presents the OLS estimates of the prices of beta risk for the CAPM and HKM's single-factor model (HKMSF). *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) t-ratio (t-stat$_c$), the KRS model misspecification-robust t-ratio (t-stat$_m$), and the bootstrap p-value (boot p-val). In addition, we present the sample OLS cross-sectional R^2 (R^2) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) p-values. Finally, RANK TEST and fs p-val are the (approximate) finite-sample rank test discussed in the Internet Appendix and its finite-sample p-value. N and T represent the number of assets and time series observations, respectively. p-values are in square brackets.

Panel A: CAPM

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
MKT	−0.99	3.41	4.21	8.60	5.85	−0.88	12.48	1.77
t-stat$_c$	(−0.80)	(2.97)	(1.74)	(2.91)	(2.85)	(−0.53)	(2.93)	(1.04)
t-stat$_m$	(−0.76)	(2.91)	(1.76)	(2.99)	(2.35)	(−0.28)	(2.22)	(1.06)
boot p-val	[0.437]	[0.005]	[0.103]	[0.009]	[0.020]	[0.784]	[0.097]	
INT	3.27	0.40	0.56	−5.98	−0.34	0.52	−1.74	0.09
t-stat$_c$	(3.12)	(1.86)	(0.88)	(−2.72)	(−3.54)	(0.84)	(−1.91)	(0.16)
t-stat$_m$	(2.95)	(1.85)	(0.97)	(−2.80)	(−4.25)	(0.61)	(−1.89)	(0.14)
boot p-val	[0.005]	[0.079]	[0.346]	[0.016]	[0.001]	[0.529]	[0.093]	
R^2	0.08	0.81	0.69	0.91	0.61	0.01	0.49	0.32
SPEC TEST	80.31	66.85	9.09	48.54	139.20	31.97	17.01	
asy p-val	[0.000]	[0.000]	[0.059]	[0.000]	[0.000]	[0.059]	[0.074]	
boot p-val	[0.000]	[0.060]	[0.199]	[0.181]	[0.740]	[0.373]	[0.465]	
RANK TEST	7.91	8.31	7.92	4.31	5.45	3.19	2.28	
fs p-val	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.001]	
N	25	20	6	18	20	23	12	124
T	172	148	65	103	47	105	135	172

Panel B: HKMSF

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
CPTL	0.07	5.05	5.92	13.34	8.49	0.95	19.31	3.45
t-stat$_c$	(0.02)	(3.05)	(1.83)	(3.12)	(3.25)	(0.32)	(3.28)	(1.08)
t-stat$_m$	(0.02)	(3.00)	(1.83)	(3.15)	(2.55)	(0.11)	(2.88)	(1.04)
boot p-val	[0.983]	[0.003]	[0.108]	[0.009]	[0.015]	[0.905]	[0.037]	
INT	2.14	0.40	0.41	−4.73	−0.37	0.31	−0.96	−0.01
t-stat$_c$	(1.63)	(1.78)	(0.51)	(−2.75)	(−3.71)	(0.49)	(−1.15)	(−0.02)
t-stat$_m$	(1.37)	(1.77)	(0.56)	(−2.78)	(−5.47)	(0.35)	(−1.14)	(−0.02)
boot p-val	[0.180]	[0.101]	[0.612]	[0.019]	[0.001]	[0.704]	[0.243]	
R^2	0.00	0.83	0.77	0.95	0.64	0.00	0.53	0.41
SPEC TEST	83.80	68.41	9.91	49.28	72.70	29.26	18.25	
asy p-val	[0.000]	[0.000]	[0.042]	[0.000]	[0.000]	[0.108]	[0.051]	
boot p-val	[0.001]	[0.061]	[0.189]	[0.157]	[0.791]	[0.460]	[0.476]	
RANK TEST	2.63	7.65	10.49	3.30	6.75	1.77	2.26	
fs p-val	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.035]	[0.015]	

Table 7

GLS cross-sectional asset-pricing tests by asset class (price of beta risk in single factor models).

The table presents the GLS estimates of the prices of beta risk for the CAPM and HKM's single-factor model (HKMSF). *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) t-ratio (t-stat$_c$), the KRS model misspecification-robust t-ratio (t-stat$_m$), and the bootstrap p-value (boot p-val). In addition, we present the sample GLS cross-sectional R^2 (R^2) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) p-values. N and T represent the number of assets and time series observations, respectively. p-values are in square brackets.

Panel A: CAPM

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
MKT	−1.42	2.31	3.35	3.64	3.05	−0.49	9.33
t-stat$_c$	(−1.51)	(2.15)	(1.54)	(2.59)	(1.99)	(−0.40)	(3.36)
t-stat$_m$	(−1.29)	(1.84)	(1.37)	(2.10)	(1.61)	(−0.33)	(2.53)
boot p-val	[0.194]	[0.091]	[0.208]	[0.078]	[0.253]	[0.739]	[0.125]
INT	3.07	0.08	1.04	−2.33	−0.09	−0.08	−1.81
t-stat$_c$	(4.93)	(6.30)	(2.14)	(−2.71)	(−4.05)	(−0.24)	(−2.50)
t-stat$_m$	(3.83)	(5.98)	(1.77)	(−1.96)	(−2.43)	(−0.21)	(−2.26)
boot p-val	[0.001]	[0.000]	[0.120]	[0.114]	[0.196]	[0.868]	[0.079]
R^2	0.03	0.07	0.22	0.10	0.05	0.01	0.36
SPEC TEST	79.32	70.25	9.83	64.41	105.52	31.25	24.46
asy p-val	[0.000]	[0.000]	[0.043]	[0.000]	[0.000]	[0.070]	[0.006]
boot p-val	[0.001]	[0.021]	[0.140]	[0.032]	[0.587]	[0.354]	[0.232]
N	25	20	6	18	20	23	12
T	172	148	65	103	47	105	135

Panel B: HKMSF

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
CPTL	−1.45	3.63	4.54	6.74	5.47	2.02	11.72
t-stat$_c$	(−0.80)	(2.35)	(1.58)	(2.93)	(2.33)	(0.90)	(3.55)
t-stat$_m$	(−0.60)	(2.04)	(1.44)	(2.08)	(1.89)	(0.63)	(2.08)
boot p-val	[0.555]	[0.055]	[0.230]	[0.137]	[0.180]	[0.570]	[0.144]
INT	2.89	0.08	0.93	−2.10	−0.09	−0.04	−1.12
t-stat$_c$	(3.90)	(6.13)	(1.65)	(−2.76)	(−3.71)	(−0.13)	(−2.27)
t-stat$_m$	(2.84)	(5.87)	(1.45)	(−1.95)	(−2.41)	(−0.11)	(−2.06)
boot p-val	[0.008]	[0.000]	[0.214]	[0.132]	[0.201]	[0.928]	[0.048]
R^2	0.01	0.08	0.22	0.14	0.07	0.03	0.26
SPEC TEST	79.28	70.81	10.54	64.05	75.43	28.61	33.11
asy p-val	[0.000]	[0.000]	[0.032]	[0.000]	[0.000]	[0.124]	[0.000]
boot p-val	[0.001]	[0.016]	[0.137]	[0.039]	[0.684]	[0.503]	[0.074]

Table 8

Constrained OLS cross-sectional asset-pricing tests by asset class (price of beta risk in single factor models).

The table presents the constrained OLS estimates of the prices of beta risk for the CAPM and HKM's single-factor model (HKMSF). *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. For each parameter estimate, we report the Jagannathan and Wang (1998) t-ratio (t-stat$_c$), the KRS model misspecification-robust t-ratio (t-stat$_m$), and the bootstrap p-value (boot p-val). In addition, we present the sample uncentered OLS cross-sectional R^2 (R^2_U) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) p-values. Finally, RANK TEST and fs p-val are the (approximate) finite-sample rank test discussed in the Internet Appendix and its finite-sample p-value. N and T represent the number of assets and time series observations, respectively. p-values are in square brackets.

Panel A: CAPM

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
MKT	1.89	5.37	5.44	1.61	3.58	0.32	−6.53	1.86
t-stat$_c$	(2.60)	(3.29)	(2.36)	(1.77)	(1.77)	(0.18)	(−1.24)	(1.12)
t-stat$_m$	(2.60)	(3.40)	(2.33)	(1.81)	(1.63)	(0.14)	(−1.08)	(1.15)
boot p-val	[0.014]	[0.001]	[0.049]	[0.073]	[0.122]	[0.896]	[0.272]	
R^2_U	0.86	0.83	0.92	0.56	0.60	0.00	0.14	0.49
SPEC TEST	108.24	93.58	12.98	76.57	125.95	29.90	54.92	
asy p-val	[0.000]	[0.000]	[0.024]	[0.000]	[0.000]	[0.121]	[0.000]	
boot p-val	[0.000]	[0.063]	[0.210]	[0.009]	[0.659]	[0.505]	[0.001]	
RANK TEST	830.72	8.17	7.63	58.74	5.57	3.07	2.68	
fs p-val	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	
N	25	20	6	18	20	23	12	124
T	172	148	65	103	47	105	135	172

Panel B: HKMSF

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
CPTL	3.38	7.96	7.08	3.12	5.02	2.02	22.29	3.44
t-stat$_c$	(2.55)	(3.15)	(2.66)	(1.77)	(1.86)	(0.58)	(3.01)	(1.11)
t-stat$_m$	(2.55)	(3.28)	(2.60)	(1.79)	(1.67)	(0.28)	(0.54)	(1.12)
boot p-val	[0.018]	[0.004]	[0.046]	[0.072]	[0.134]	[0.787]	[0.604]	
R^2_U	0.90	0.83	0.94	0.60	0.61	0.02	0.27	0.54
SPEC TEST	105.64	91.78	11.72	80.15	88.71	28.70	16.67	
asy p-val	[0.000]	[0.000]	[0.039]	[0.000]	[0.000]	[0.154]	[0.118]	
boot p-val	[0.000]	[0.049]	[0.286]	[0.007]	[0.673]	[0.590]	[0.767]	
RANK TEST	14.47	7.73	12.33	7.16	6.98	1.74	2.08	
fs p-val	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.037]	[0.023]	

Table 9

Constrained GLS cross-sectional asset-pricing tests by asset class (price of beta risk in single factor models).

The table presents the constrained GLS estimates of the prices of beta risk for the CAPM and HKM's single-factor model (HKMSF). *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. For each parameter estimate, we report the Jagannathan and Wang (1998) t-ratio (t-stat$_c$), the KRS model misspecification-robust t-ratio (t-stat$_m$), and the bootstrap p-value (boot p-val). In addition, we present the sample uncentered GLS cross-sectional R^2 (R^2_U) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) p-values. N and T represent the number of assets and time series observations, respectively. p-values are in square brackets.

Panel A: CAPM

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
MKT	1.62	1.52	4.47	0.91	2.31	−0.51	5.27
t-stat$_c$	(2.33)	(1.50)	(2.07)	(1.01)	(1.52)	(−0.41)	(2.93)
t-stat$_m$	(2.31)	(1.16)	(1.79)	(0.98)	(1.30)	(−0.35)	(1.63)
boot p-val	[0.026]	[0.274]	[0.111]	[0.339]	[0.341]	[0.733]	[0.219]
R^2_U	0.05	0.02	0.28	0.01	0.02	0.01	0.12
SPEC TEST	108.39	112.24	14.28	78.95	112.45	31.31	50.45
asy p-val	[0.000]	[0.000]	[0.014]	[0.000]	[0.000]	[0.090]	[0.000]
boot p-val	[0.000]	[0.001]	[0.133]	[0.004]	[0.581]	[0.427]	[0.002]
N	25	20	6	18	20	23	12
T	172	148	65	103	47	105	135

Panel B: HKMSF

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
CPTL	4.03	2.03	6.33	3.09	4.37	2.07	11.25
t-stat$_c$	(3.32)	(1.42)	(2.55)	(1.86)	(1.88)	(0.92)	(3.53)
t-stat$_m$	(3.11)	(1.13)	(2.32)	(1.32)	(1.50)	(0.65)	(1.92)
boot p-val	[0.003]	[0.275]	[0.044]	[0.262]	[0.263]	[0.590]	[0.215]
R^2_U	0.11	0.02	0.33	0.04	0.04	0.03	0.21
SPEC TEST	104.42	115.34	12.25	80.16	90.18	28.67	38.92
asy p-val	[0.000]	[0.000]	[0.032]	[0.000]	[0.000]	[0.155]	[0.000]
boot p-val	[0.001]	[0.000]	[0.243]	[0.004]	[0.647]	[0.585]	[0.059]

Table 10
Sharpe ratio analysis for the CAPM, HKM, and AEM models.

Panel A reports factor (mimicking portfolio) means (Fac. Mean), standard deviations (Fac. SD), and correlations (Fac. Corr). In Panel B, we present bias-adjusted squared Sharpe ratios (Sh^2) for the CAPM, the two-factor models of HKM (HKM) and AEM (AEM), and the single-factor models of HKM (HKMSF) and AEM (AEMSF). Finally, Panel C is for differences in bias-adjusted sample squared Sharpe ratios between models. *MKT*, *CPTLT*, and *LMP* denote the market factor, the traded capital factor of HKM, and the leverage mimicking portfolio of AEM, respectively. The number of time series observations is $T = 172$. p-values are in square brackets.

Panel A: Summary statistics

	MKT	*CPTLT*	*LMP*
Fac. Mean	0.015	0.019	0.032
	[0.030]	[0.062]	[0.000]
Fac. SD	0.091	0.133	0.072
	Fac. Corr		
MKT	1.000	0.838	0.312
CPTLT		1.000	0.332
LMP			1.000

Panel B: Squared Sharpe ratios

	CAPM	HKM	HKMSF	AEM	AEMSF
Sh^2	0.021	0.015	0.014	0.215	0.240
	[0.056]	[0.269]	[0.119]	[0.000]	[0.000]

Panel C: Squared Sharpe ratio comparisons

	HKM	HKMSF	AEM	AEMSF
CAPM	0.006	0.007	−0.194	−0.219
	[0.934]	[0.610]	[0.000]	[0.034]
HKM		0.001	−0.200	−0.225
		[0.277]	[0.054]	[0.029]
HKMSF			−0.201	−0.226
			[0.052]	[0.028]
AEM				−0.025
				[0.841]

Table 11
OLS cross-sectional asset-pricing tests by asset class for AEM's two-factor model.

The table presents the OLS estimates of the prices of beta and covariance risks for AEM's two-factor model. *MKT* and *LevFac* denote the market and nontraded leverage factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) t-ratio (t-stat$_c$), the KRS model misspecification-robust t-ratio (t-stat$_m$), and the bootstrap p-value (boot p-val). In addition, we present the sample OLS cross-sectional R^2 (R^2) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) p-values. Finally, RANK TEST and fs p-val are the (approximate) finite-sample rank test discussed in the Internet Appendix and its finite-sample p-value. N and T represent the number of assets and time series observations, respectively. p-values are in square brackets.

Panel A: Price of beta risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
LevFac	13.69	14.31	7.91	−58.39	−23.83	2.39	−13.77	11.77
t-stat$_c$	(2.54)	(2.05)	(0.75)	(−1.52)	(−2.25)	(0.54)	(−1.46)	(1.14)
t-stat$_m$	(2.62)	(0.89)	(0.62)	(−1.89)	(−2.81)	(0.29)	(−0.86)	(0.61)
boot p-val	[0.044]	[0.488]	[0.648]	[0.514]	[0.177]	[0.780]	[0.382]	
MKT	0.89	4.05	3.17	1.73	4.56	−0.48	8.86	1.73
t-stat$_c$	(0.57)	(1.77)	(1.00)	(0.22)	(2.24)	(−0.31)	(2.62)	(0.91)
t-stat$_m$	(0.58)	(1.49)	(0.79)	(0.17)	(2.30)	(−0.14)	(1.44)	(0.89)
boot p-val	[0.555]	[0.129]	[0.407]	[0.870]	[0.033]	[0.883]	[0.130]	
INT	0.79	0.26	1.03	−1.96	−0.12	0.43	−1.86	−0.07
t-stat$_c$	(0.56)	(0.64)	(1.65)	(−0.29)	(−1.15)	(0.66)	(−2.22)	(−0.05)
t-stat$_m$	(0.59)	(0.51)	(1.22)	(−0.24)	(−1.19)	(0.41)	(−2.14)	(−0.05)
boot p-val	[0.559]	[0.542]	[0.233]	[0.826]	[0.359]	[0.660]	[0.054]	
R^2	0.70	0.87	0.73	0.98	0.93	0.03	0.59	0.48
SPEC TEST	44.35	47.06	8.47	6.18	45.22	29.10	17.15	
asy p-val	[0.003]	[0.000]	[0.037]	[0.977]	[0.000]	[0.086]	[0.046]	
boot p-val	[0.235]	[0.496]	[0.171]	[0.996]	[0.950]	[0.429]	[0.149]	
RANK TEST	1.48	3.04	1.40	0.92	5.33	1.72	0.97	
fs p-val	[0.086]	[0.000]	[0.244]	[0.551]	[0.000]	[0.043]	[0.476]	
N	25	20	6	18	20	23	12	124
T	172	148	65	103	47	105	135	172

Panel B: Price of covariance risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
LevFac	5.94	5.38	2.30	−20.25	−5.82	0.87	−5.69	5.01
t-stat$_c$	(2.19)	(1.36)	(0.60)	(−1.59)	(−2.07)	(0.53)	(−1.61)	(0.49)
t-stat$_m$	(2.13)	(0.69)	(0.59)	(−1.52)	(−2.35)	(0.33)	(−1.04)	(0.54)
boot p-val	[0.088]	[0.520]	[0.579]	[0.425]	[0.121]	[0.754]	[0.246]	
MKT	−0.38	4.12	3.60	6.14	6.66	−0.79	13.42	0.87
t-stat$_c$	(−0.21)	(1.10)	(0.75)	(0.40)	(1.49)	(−0.39)	(2.45)	(0.29)
t-stat$_m$	(−0.20)	(1.14)	(0.61)	(0.36)	(1.50)	(−0.19)	(1.56)	(0.18)
boot p-val	[0.838]	[0.209]	[0.485]	[0.683]	[0.107]	[0.851]	[0.081]	

Table 12

GLS cross-sectional asset-pricing tests by asset class for AEM's two-factor model.

The table presents the GLS estimates of the prices of beta and covariance risks for AEM's two-factor model. *MKT* and *LevFac* denote the market and nontraded leverage factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) t-ratio (t-stat$_c$), the KRS model misspecification-robust t-ratio (t-stat$_m$), and the bootstrap p-value (boot p-val). In addition, we present the sample GLS cross-sectional R^2 (R^2) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) p-values. N and T represent the number of assets and time series observations, respectively. p-values are in square brackets.

Panel A: Price of beta risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
LevFac	3.19	−0.38	−9.96	−14.24	−11.78	−4.10	−11.42
t-stat$_c$	(1.15)	(−0.14)	(−1.20)	(−2.36)	(−3.67)	(−1.36)	(−1.36)
t-stat$_m$	(0.65)	(−0.11)	(−0.75)	(−0.91)	(−2.95)	(−1.27)	(−0.94)
boot p-val	[0.555]	[0.908]	[0.486]	[0.602]	[0.119]	[0.247]	[0.498]
MKT	−1.19	2.32	4.45	1.97	3.24	−0.67	10.14
t-stat$_c$	(−1.17)	(2.21)	(2.06)	(1.02)	(2.37)	(−0.50)	(3.44)
t-stat$_m$	(−0.96)	(1.87)	(1.52)	(0.62)	(1.73)	(−0.44)	(2.81)
boot p-val	[0.342]	[0.072]	[0.150]	[0.556]	[0.226]	[0.647]	[0.075]
INT	2.83	0.08	0.54	−1.19	−0.09	−0.11	−2.21
t-stat$_c$	(4.09)	(6.35)	(0.92)	(−0.90)	(−3.39)	(−0.31)	(−3.05)
t-stat$_m$	(3.03)	(6.17)	(0.56)	(−0.52)	(−2.52)	(−0.28)	(−2.64)
boot p-val	[0.006]	[0.000]	[0.568]	[0.651]	[0.182]	[0.784]	[0.050]
R^2	0.05	0.07	0.34	0.25	0.21	0.06	0.44
SPEC TEST	67.13	70.61	5.76	31.76	58.94	28.82	15.83
asy p-val	[0.000]	[0.000]	[0.124]	[0.007]	[0.000]	[0.091]	[0.071]
boot p-val	[0.006]	[0.014]	[0.253]	[0.435]	[0.809]	[0.428]	[0.411]
N	25	20	6	18	20	23	12
T	172	148	65	103	47	105	135

Panel B: Price of covariance risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
LevFac	1.54	−0.35	−3.07	−5.04	−2.93	−1.40	−4.87
t-stat$_c$	(1.17)	(−0.31)	(−1.20)	(−2.03)	(−2.27)	(−1.37)	(−1.61)
t-stat$_m$	(0.69)	(−0.23)	(−0.68)	(−0.89)	(−2.03)	(−1.01)	(−1.09)
boot p-val	[0.516]	[0.804]	[0.551]	[0.530]	[0.138]	[0.369]	[0.405]
MKT	−1.82	3.08	5.49	3.52	4.45	−0.62	15.10
t-stat$_c$	(−1.44)	(2.06)	(1.67)	(1.16)	(1.96)	(−0.36)	(3.13)
t-stat$_m$	(−1.28)	(1.76)	(1.42)	(0.82)	(1.56)	(−0.31)	(2.50)
boot p-val	[0.190]	[0.084]	[0.178]	[0.442]	[0.246]	[0.750]	[0.115]

Table 13

Cross-sectional asset-pricing tests by asset class for AEM's single-factor model.

The table presents the OLS (Panel A) and GLS (Panel B) estimates of the prices of beta risk for AEM's single-factor model (AEMSF). *LevFac* denotes the nontraded leverage factor. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) t-ratio (t-stat$_c$), the KRS model misspecification-robust t-ratio (t-stat$_m$), and the bootstrap p-value (boot p-val). In addition, we present the sample cross-sectional R^2 (R^2) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) p-values. Finally, RANK TEST and fs p-val are the (approximate) finite-sample rank test discussed in the Internet Appendix and its finite-sample p-value. N and T represent the number of assets and time series observations, respectively. p-values are in square brackets.

Panel A: OLS

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
LevFac	14.13	25.46	19.11	−112.61	−26.22	2.86	−26.27	14.34
t-stat$_c$	(2.60)	(1.25)	(0.67)	(−0.37)	(−2.84)	(0.60)	(−2.27)	(0.90)
t-stat$_m$	(2.73)	(0.32)	(1.03)	(−0.44)	(−0.62)	(0.39)	(−1.92)	(0.88)
boot p-val	[0.047]	[0.808]	[0.644]	[0.829]	[0.727]	[0.739]	[0.302]	
INT	0.41	0.43	2.14	2.73	0.49	0.32	−1.51	0.10
t-stat$_c$	(0.26)	(0.93)	(1.08)	(0.25)	(1.06)	(0.49)	(−1.62)	(0.04)
t-stat$_m$	(0.27)	(0.40)	(1.13)	(0.26)	(2.16)	(0.44)	(−1.53)	(0.03)
boot p-val	[0.781]	[0.701]	[0.531]	[0.827]	[0.422]	[0.634]	[0.160]	
R^2	0.69	0.21	0.57	0.91	0.35	0.03	0.37	0.47
SPEC TEST	44.07	36.02	6.00	2.15	57.39	28.52	28.43	
asy p-val	[0.005]	[0.007]	[0.199]	[1.000]	[0.000]	[0.126]	[0.002]	
boot p-val	[0.326]	[0.888]	[0.604]	[0.999]	[0.800]	[0.520]	[0.195]	
RANK TEST	1.42	2.94	1.56	0.97	7.48	1.67	0.99	
fs p-val	[0.107]	[0.000]	[0.186]	[0.494]	[0.000]	[0.050]	[0.464]	
N	25	20	6	18	20	23	12	124
T	172	148	65	103	47	105	135	172

Panel B: GLS

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
LevFac	3.26	0.12	−2.38	−16.58	−11.56	−4.03	−4.38
t-stat$_c$	(1.19)	(0.05)	(−0.39)	(−2.79)	(−3.83)	(−1.33)	(−0.92)
t-stat$_m$	(0.61)	(0.04)	(−0.14)	(−1.04)	(−3.08)	(−1.25)	(−0.32)
boot p-val	[0.571]	[0.958]	[0.902]	[0.638]	[0.097]	[0.252]	[0.749]
INT	2.20	0.08	1.20	0.07	−0.08	−0.11	−1.15
t-stat$_c$	(4.28)	(6.40)	(2.34)	(0.09)	(−3.25)	(−0.33)	(−3.47)
t-stat$_m$	(3.23)	(6.25)	(1.33)	(0.06)	(−2.48)	(−0.30)	(−2.09)
boot p-val	[0.006]	[0.000]	[0.245]	[0.971]	[0.182]	[0.784]	[0.032]
R^2	0.02	0.00	0.01	0.19	0.14	0.06	0.01
SPEC TEST	72.07	80.51	19.76	30.14	72.90	28.38	80.99
asy p-val	[0.000]	[0.000]	[0.001]	[0.017]	[0.000]	[0.130]	[0.000]
boot p-val	[0.001]	[0.001]	[0.004]	[0.594]	[0.646]	[0.506]	[0.000]

Table 14
LMW model (tests of equality of unconditional and downside risk betas).

The table reports individual and joint tests of equality of unconditional and downside betas. $\hat{B}_{1,i}$ (for $i = 1, \ldots, N$) denotes the sample beta difference for asset i. JT is a joint test of beta equality. The asymptotic p-values (asy p-val) of a t-test of $H_0: B_{1,i} = 0$ and of a χ^2-test of $H_0: B_1 = 0_N$ are based on Proposition 1 of the Internet Appendix. We also include bootstrap p-values (boot p-val) based on the methodology described in the Internet Appendix. Test assets are (i) Six currency portfolios (FX), monthly resampled based on the interest rate differential with the US (Cur_i for $i = \mathrm{low}, 2, 3, 4, 5, \mathrm{high}$). High inflation countries in the last currency portfolio are excluded; (ii) Five currency portfolios of developed countries (Developed FX), monthly resampled based on the interest rate differential with the US (Dev. Cur_i for $i = \mathrm{low}, 2, 3, 4, \mathrm{high}$); (iii) Six Fama and French portfolios (FF6) sorted on size, i, and book-to-market, j (FF_{ij} for $i = \mathrm{s}, \mathrm{b}$ and $j = \mathrm{g}, \mathrm{n}, \mathrm{v}$), where "s" and "b" are for small and big, respectively, while "g", "n", and "v" are for growth, neutral, and value, respectively; (iv) Five commodity futures portfolios (Commod.), monthly resampled based on basis (Comm_i for $i = \mathrm{low}, 2, 3, 4, \mathrm{high}$); (v) Six sovereign bond portfolios (Sov. bonds) sorted by the probability of default, i, and bond beta, j (Sov_{ij} for $i = \mathrm{l}, \mathrm{h}$ and $j = \mathrm{l}, \mathrm{m}, \mathrm{h}$), where "l", "m", and "h" are for low, medium, and high, respectively; and (vi) 18 portfolios of call (C) and put (P) options (Options) on the Standard & Poor's 500 with maturity, i, between 30 and 90 days and moneyness, j, between 90 and 110 ($\mathrm{C}_{i,j}$ and $\mathrm{P}_{i,j}$ for $i = 30, 60, 90$ and $j = 90, 100, 110$). The factor is the value-weighted CRSP US equity market portfolio. The data is monthly and the sample periods are reported in the table.

	FX 1974:01–2010:03 ($T = 435$)				Developed FX 1974:01–2010:03 ($T = 435$)		
	$\hat{B}_{1,i}$	asy p-val	boot p-val		$\hat{B}_{1,i}$	asy p-val	boot p-val
Port.				Port.			
$\mathrm{Cur}_{\mathrm{low}}$	−0.01	0.931	0.919	Dev. $\mathrm{Cur}_{\mathrm{low}}$	0.00	0.991	0.985
Cur_2	−0.01	0.930	0.928	Dev. Cur_2	−0.02	0.872	0.874
Cur_3	0.00	0.993	0.995	Dev. Cur_3	−0.00	0.990	0.989
Cur_4	0.03	0.833	0.799	Dev. Cur_4	0.09	0.580	0.465
Cur_5	0.07	0.618	0.674	Dev. $\mathrm{Cur}_{\mathrm{high}}$	0.11	0.529	0.590
$\mathrm{Cur}_{\mathrm{high}}$	0.19	0.315	0.462				
	JT	asy p-val	boot p-val		JT	asy p-val	boot p-val
All	1.87	0.931	0.702	All	1.39	0.925	0.626

Table 14 (cont'd)
LMW model (tests of equality of unconditional and downside risk betas).

	FX and FF6 1974:01–2010:03 ($T = 435$)		
Port.	$\hat{B}_{1,i}$	asy p-val	boot p-val
Cur_{low}	−0.01	0.931	0.934
Cur_2	−0.01	0.930	0.934
Cur_3	0.00	0.993	0.994
Cur_4	0.03	0.833	0.807
Cur_5	0.07	0.618	0.683
Cur_{high}	0.19	0.315	0.457
FF_{sg}	0.16	0.641	0.127
FF_{sn}	0.27	0.281	0.008
FF_{sv}	0.35	0.169	0.003
FF_{bg}	−0.11	0.754	0.121
FF_{bn}	0.03	0.900	0.599
FF_{bv}	0.09	0.764	0.467
	JT	asy p-val	boot p-val
All	6.22	0.904	0.598

	FX and Commod. 1974:01–2008:12 ($T = 420$)		
Port.	$\hat{B}_{1,i}$	asy p-val	boot p-val
Cur_{low}	−0.01	0.971	0.972
Cur_2	−0.01	0.960	0.955
Cur_3	0.01	0.942	0.931
Cur_4	0.04	0.712	0.660
Cur_5	0.08	0.583	0.621
Cur_{high}	0.20	0.317	0.440
Comm_{low}	0.50	0.302	0.137
Comm_2	0.24	0.500	0.432
Comm_3	0.24	0.542	0.377
Comm_4	−0.03	0.933	0.891
$\text{Comm}_{\text{high}}$	0.14	0.726	0.558
	JT	asy p-val	boot p-val
All	2.49	0.996	0.843

	FX, FF6, and Commod. 1974:01–2008:12 ($T = 420$)		
Port.	$\hat{B}_{1,i}$	asy p-val	boot p-val
Cur_{low}	−0.01	0.971	0.961
Cur_2	−0.01	0.960	0.957
Cur_3	0.01	0.942	0.936
Cur_4	0.04	0.712	0.660
Cur_5	0.08	0.583	0.627
Cur_{high}	0.20	0.317	0.431
FF_{sg}	0.16	0.651	0.142
FF_{sn}	0.28	0.262	0.005
FF_{sv}	0.36	0.129	0.001
FF_{bg}	−0.12	0.741	0.097
FF_{bn}	0.03	0.898	0.611
FF_{bv}	0.10	0.713	0.395
Comm_{low}	0.50	0.302	0.145
Comm_2	0.24	0.500	0.440
Comm_3	0.24	0.542	0.378
Comm_4	−0.03	0.933	0.890
$\text{Comm}_{\text{high}}$	0.14	0.726	0.553
	JT	asy p-val	boot p-val
All	8.39	0.957	0.727

	FX and Sov. bonds 1995:01–2010:03 ($T = 183$)		
Port.	$\hat{B}_{1,i}$	asy p-val	boot p-val
Cur_{low}	0.16	0.299	0.169
Cur_2	0.21	0.121	0.071
Cur_3	0.18	0.274	0.138
Cur_4	0.09	0.502	0.265
Cur_5	0.32	0.068	0.136
Cur_{high}	0.55	0.061	0.032
Sov_{ll}	0.69	0.063	0.193
Sov_{lm}	0.60	0.065	0.315
Sov_{lh}	1.27	0.032	0.225
Sov_{hl}	0.60	0.071	0.163
Sov_{hm}	0.68	0.057	0.139
Sov_{hh}	1.26	0.044	0.121
	JT	asy p-val	boot p-val
All	7.36	0.833	0.841

Table 14 (cont'd)
LMW model (tests of equality of unconditional and downside risk betas).

FX, FF6, and Sov. bonds
1995:01–2010:03 ($T = 183$)

Port.	$\hat{B}_{1,i}$	asy p-val	boot p-val
Cur_{low}	0.16	0.299	0.169
Cur_2	0.21	0.121	0.072
Cur_3	0.18	0.274	0.130
Cur_4	0.09	0.502	0.261
Cur_5	0.32	0.068	0.120
Cur_{high}	0.55	0.061	0.030
FF_{sg}	−0.03	0.963	0.869
FF_{sn}	0.23	0.606	0.176
FF_{sv}	0.28	0.571	0.140
FF_{bg}	−0.17	0.753	0.025
FF_{bn}	0.26	0.465	0.085
FF_{bv}	0.14	0.796	0.626
Sov_{ll}	0.69	0.063	0.190
Sov_{lm}	0.60	0.065	0.307
Sov_{lh}	1.27	0.032	0.219
Sov_{hl}	0.60	0.071	0.171
Sov_{hm}	0.68	0.057	0.145
Sov_{hh}	1.26	0.044	0.119
	JT	asy p-val	boot p-val
All	44.32	0.001	0.634

Options
1986:04–2010:03 ($T = 288$)

Port.	$\hat{B}_{1,i}$	asy p-val	boot p-val
$\text{C}_{30,90}$	−0.39	0.503	0.042
$\text{C}_{30,100}$	−0.46	0.456	0.023
$\text{C}_{30,110}$	−0.40	0.462	0.059
$\text{C}_{60,90}$	−0.39	0.497	0.043
$\text{C}_{60,100}$	−0.45	0.459	0.026
$\text{C}_{60,110}$	−0.40	0.473	0.042
$\text{C}_{90,90}$	−0.39	0.496	0.029
$\text{C}_{90,100}$	−0.46	0.455	0.018
$\text{C}_{90,110}$	−0.45	0.452	0.039
$\text{P}_{30,90}$	0.44	0.298	0.198
$\text{P}_{30,100}$	0.03	0.934	0.880
$\text{P}_{30,110}$	−0.14	0.741	0.348
$\text{P}_{60,90}$	0.41	0.198	0.097
$\text{P}_{60,100}$	0.03	0.945	0.885
$\text{P}_{60,110}$	−0.12	0.773	0.464
$\text{P}_{90,90}$	0.37	0.206	0.101
$\text{P}_{90,100}$	0.02	0.951	0.896
$\text{P}_{90,110}$	−0.10	0.808	0.569
	JT	asy p-val	boot p-val
All	8.20	0.976	0.924

FX, Commod., and Options
1986:04–2008:12 ($T = 273$)

Port.	$\hat{B}_{1,i}$	asy p-val	boot p-val
Cur_{low}	0.02	0.904	0.877
Cur_2	0.03	0.847	0.806
Cur_3	0.01	0.937	0.924
Cur_4	0.04	0.734	0.652
Cur_5	0.09	0.689	0.656
Cur_{high}	0.17	0.573	0.587
Comm_{low}	0.56	0.376	0.180
Comm_2	0.34	0.450	0.379
Comm_3	0.37	0.433	0.274
Comm_4	0.11	0.753	0.572
$\text{Comm}_{\text{high}}$	0.15	0.639	0.547
$\text{C}_{30,90}$	−0.37	0.522	0.032
$\text{C}_{30,100}$	−0.44	0.477	0.014
$\text{C}_{30,110}$	−0.38	0.475	0.037
$\text{C}_{60,90}$	−0.38	0.516	0.031
$\text{C}_{60,100}$	−0.44	0.479	0.021
$\text{C}_{60,110}$	−0.38	0.490	0.030
$\text{C}_{90,90}$	−0.37	0.514	0.016
$\text{C}_{90,100}$	−0.44	0.475	0.008
$\text{C}_{90,110}$	−0.44	0.471	0.027
$\text{P}_{30,90}$	0.44	0.350	0.233
$\text{P}_{30,100}$	0.04	0.918	0.846
$\text{P}_{30,110}$	−0.13	0.758	0.363
$\text{P}_{60,90}$	0.42	0.231	0.111
$\text{P}_{60,100}$	0.04	0.919	0.844
$\text{P}_{60,110}$	−0.11	0.793	0.476
$\text{P}_{90,90}$	0.38	0.228	0.108
$\text{P}_{90,100}$	0.03	0.929	0.859
$\text{P}_{90,110}$	−0.09	0.831	0.615
	JT	asy p-val	boot p-val
All	33.23	0.269	0.558

Table 15
LMW model (currencies and equities).

The table reports the constrained price of unconditional beta risk, *MKT*, and the price of downside market risk, *DR*, for the CAPM and DR-CAPM. Fama and MacBeth (1973) t-ratios ($t\text{-stat}_{fm}$) and t-ratios under correctly specified ($t\text{-stat}_c$) and potentially misspecified ($t\text{-stat}_m$) models based on Propositions 2 and 3 of the Internet Appendix are in parentheses. Panel A is for OLS, while Panel B is for GLS. R^2 (R^2_U) is the centered (uncentered) CSR R^2. RMSPE is the root mean squared pricing error. SPEC TEST is the χ^2-statistic testing for the joint significance of the pricing errors based on the Fama and MacBeth (1973) asymptotic covariance of the sample pricing errors. Asymptotic and bootstrap p-values (asy p-val and boot p-val) are in square brackets. Test assets are (i) Six currency portfolios (FX), monthly resampled based on the interest rate differential with the US. High inflation countries in the last currency portfolio are excluded; (ii) Five currency portfolios of developed countries (Developed FX), monthly resampled based on the interest rate differential with the US; and (iii) Six Fama and French portfolios (FF6) sorted on size and book-to-market. The factor (included as a test asset) is the value-weighted CRSP US equity market portfolio. The data is monthly. N and T represent the number of assets and time series observations, respectively.

Panel A: OLS

	FX		Developed FX		FX and FF6	
	CAPM	DR-CAPM	CAPM	DR-CAPM	CAPM	DR-CAPM
MKT	0.39	0.39	0.39	0.39	0.39	0.39
DR		2.18		2.34		1.41
$t\text{-stat}_{fm}$		(2.83)		(2.23)		(3.55)
$t\text{-stat}_c$		(1.03)		(1.11)		(0.85)
$t\text{-stat}_m$		(1.02)		(1.10)		(0.75)
boot p-val		[0.364]		[0.278]		[0.242]
R^2	0.09	0.79	0.35	0.85	0.24	0.71
R^2_U	0.46	0.87	0.58	0.90	0.68	0.88
RMSPE	0.19	0.09	0.15	0.07	0.26	0.16
SPEC TEST	42.28	24.61	22.36	9.81	114.54	63.40
asy p-val	[0.000]	[0.000]	[0.000]	[0.044]	[0.000]	[0.000]
N	7	7	6	6	13	13
T	435	435	435	435	435	435

Panel B: GLS

	FX		Developed FX		FX and FF6	
	CAPM	DR-CAPM	CAPM	DR-CAPM	CAPM	DR-CAPM
MKT	0.39	0.39	0.39	0.39	0.39	0.39
DR		2.32		1.95		2.29
$t\text{-stat}_{fm}$		(4.20)		(3.54)		(7.15)
$t\text{-stat}_c$		(1.74)		(2.09)		(3.49)
$t\text{-stat}_m$		(0.66)		(0.58)		(1.25)
boot p-val		[0.402]		[0.269]		[0.172]
R^2	0.04	0.44	0.14	0.62	−0.04	0.42
R^2_U	0.06	0.46	0.12	0.61	0.02	0.46
RMSPE	0.19	0.09	0.15	0.08	0.26	0.21

Table 16

LMW model (currencies, equities, commodities, and sovereigns).

The table reports the constrained price of unconditional beta risk, MKT, and the price of downside market risk, DR, for the CAPM and DR-CAPM. Fama and MacBeth (1973) t-ratios ($t\text{-stat}_{fm}$) and t-ratios under correctly specified ($t\text{-stat}_c$) and potentially misspecified ($t\text{-stat}_m$) models based on Propositions 2 and 3 of the Internet Appendix are in parentheses. Panel A is for OLS, while Panel B is for GLS. R^2 (R^2_U) is the centered (uncentered) CSR R^2. RMSPE is the root mean squared pricing error. SPEC TEST is the χ^2-statistic testing for the joint significance of the pricing errors based on the Fama and MacBeth (1973) asymptotic covariance of the sample pricing errors. Asymptotic and bootstrap p-values (asy p-val and boot p-val) are in square brackets. Test assets are (i) Six currency portfolios (FX), monthly resampled based on the interest rate differential with the US. High inflation countries in the last currency portfolio are excluded; (ii) Six Fama and French portfolios (FF6) sorted on size and book-to-market; (iii) Five commodity futures portfolios (Commod.), monthly resampled based on basis; and (iv) Six sovereign bond portfolios (Sov. bonds) sorted by the probability of default and bond beta. The factor (included as a test asset) is the value-weighted CRSP US equity market portfolio. The data is monthly. N and T represent the number of assets and time series observations, respectively.

Panel A: OLS

	FX and Commod.		FX, FF6, and Commod.		FX and Sov. bonds		FX, FF6, and Sov. bonds	
	CAPM	DR-CAPM	CAPM	DR-CAPM	CAPM	DR-CAPM	CAPM	DR-CAPM
MKT	0.32	0.32	0.32	0.32	0.41	0.41	0.41	0.41
DR		1.47		1.40		0.53		0.56
$t\text{-stat}_{fm}$		(2.79)		(3.66)		(2.52)		(2.69)
$t\text{-stat}_c$		(0.86)		(1.04)		(0.44)		(0.47)
$t\text{-stat}_m$		(0.91)		(1.02)		(0.45)		(0.47)
boot p-val		[0.142]		[0.116]		[0.153]		[0.122]
R^2	−0.43	0.81	−0.17	0.74	−0.21	0.66	−0.23	0.57
R^2_U	0.16	0.89	0.46	0.88	0.43	0.84	0.53	0.83
RMSPE	0.30	0.11	0.31	0.15	0.41	0.22	0.38	0.22
SPEC TEST	52.66	28.24	128.27	64.48	40.26	39.68	88.31	86.54
asy p-val	[0.000]	[0.002]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
N	12	12	18	18	13	13	19	19
T	420	420	420	420	183	183	183	183

Panel B: GLS

	FX and Commod.		FF6, FX, and Commod.		FX and Sov. bonds		FF6, FX, and Sov. bonds	
	CAPM	DR-CAPM	CAPM	DR-CAPM	CAPM	DR-CAPM	CAPM	DR-CAPM
MKT	0.32	0.32	0.32	0.32	0.41	0.41	0.41	0.41
DR		1.76		2.06		0.15		0.25
$t\text{-stat}_{fm}$		(4.94)		(7.99)		(0.77)		(1.33)
$t\text{-stat}_c$		(1.92)		(3.59)		(0.12)		(0.22)
$t\text{-stat}_m$		(0.69)		(1.40)		(0.06)		(0.11)
boot p-val		[0.278]		[0.054]		[0.634]		[0.503]
R^2	0.02	0.47	−0.09	0.45	0.18	0.19	0.02	0.04
R^2_U	0.04	0.48	0.02	0.51	0.03	0.05	0.01	0.03
RMSPE	0.30	0.12	0.31	0.20	0.41	0.33	0.38	0.28

Table 17

LMW model (currencies, commodities, and equity index options).

The table reports the constrained price of unconditional beta risk, *MKT*, and the price of downside market risk, *DR*, for the CAPM and DR-CAPM. Fama and MacBeth (1973) t-ratios ($t\text{-stat}_{fm}$) and t-ratios under correctly specified ($t\text{-stat}_c$) and potentially misspecified ($t\text{-stat}_m$) models based on Propositions 2 and 3 of the Internet Appendix are in parentheses. Panel A is for OLS, while Panel B is for GLS. R^2 (R^2_U) is the centered (uncentered) CSR R^2. RMSPE is the root mean squared pricing error. SPEC TEST is the χ^2-statistic testing for the joint significance of the pricing errors based on the Fama and MacBeth (1973) asymptotic covariance of the sample pricing errors. Asymptotic and bootstrap p-values (asy p-val and boot p-val) are in square brackets. Test assets are (i) Six currency portfolios (FX), monthly resampled based on the interest rate differential with the US. High inflation countries in the last currency portfolio are excluded; (ii) Five commodity futures portfolios (Commod.), monthly resampled based on basis; and (iii) 18 portfolios of call and put options (Options) on the Standard & Poor's 500. The factor (included as a test asset) is the value-weighted CRSP US equity market portfolio. The data is monthly. N and T represent the number of assets and time series observations, respectively.

Panel A: OLS

	Options		FX, Commod., and Options	
	CAPM	DR-CAPM	CAPM	DR-CAPM
MKT	0.40	0.40	0.29	0.29
DR		1.14		1.13
$t\text{-stat}_{fm}$		(4.19)		(4.11)
$t\text{-stat}_c$		(1.31)		(1.21)
$t\text{-stat}_m$		(1.12)		(1.02)
boot p-val		[0.008]		[0.008]
R^2	0.19	0.81	−0.02	0.74
R^2_U	0.36	0.85	0.25	0.81
RMSPE	0.44	0.21	0.39	0.20
SPEC TEST	162.71	162.62	211.94	211.89
asy p-val	[0.000]	[0.000]	[0.000]	[0.000]
N	19	19	30	30
T	288	288	273	273

Panel B: GLS

	Options		FX, Commod., and Options	
	CAPM	DR-CAPM	CAPM	DR-CAPM
MKT	0.40	0.40	0.29	0.29
DR		−0.05		0.04
$t\text{-stat}_{fm}$		(−0.30)		(0.23)
$t\text{-stat}_c$		(−0.04)		(0.03)
$t\text{-stat}_m$		(−0.02)		(0.02)
boot p-val		[0.876]		[0.902]
R^2	−0.00	−0.00	−0.00	−0.00
R^2_U	0.01	0.01	0.01	0.01
RMSPE	0.44	0.45	0.39	0.38

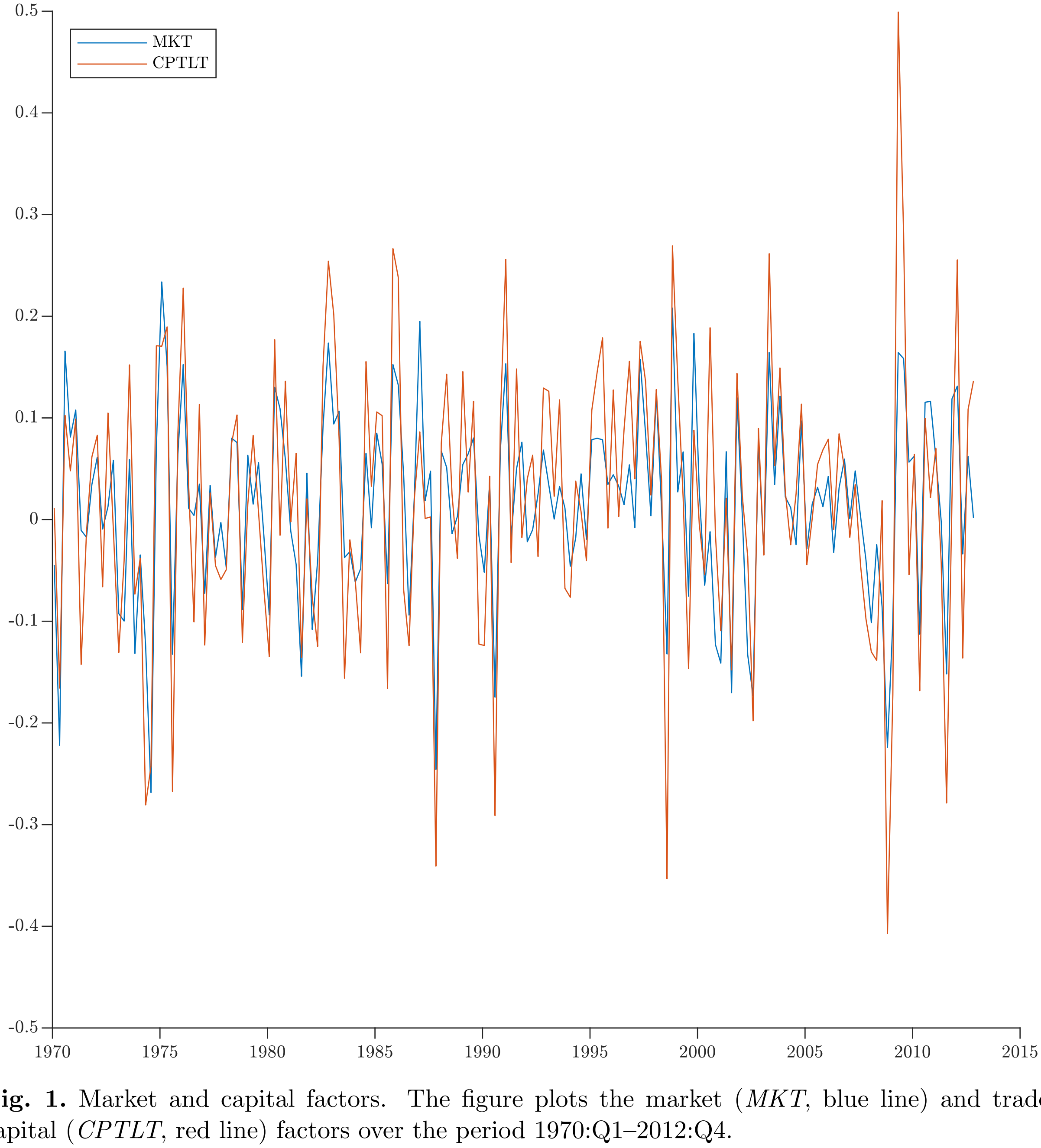

Fig. 1. Market and capital factors. The figure plots the market (MKT, blue line) and traded capital ($CPTLT$, red line) factors over the period 1970:Q1–2012:Q4.

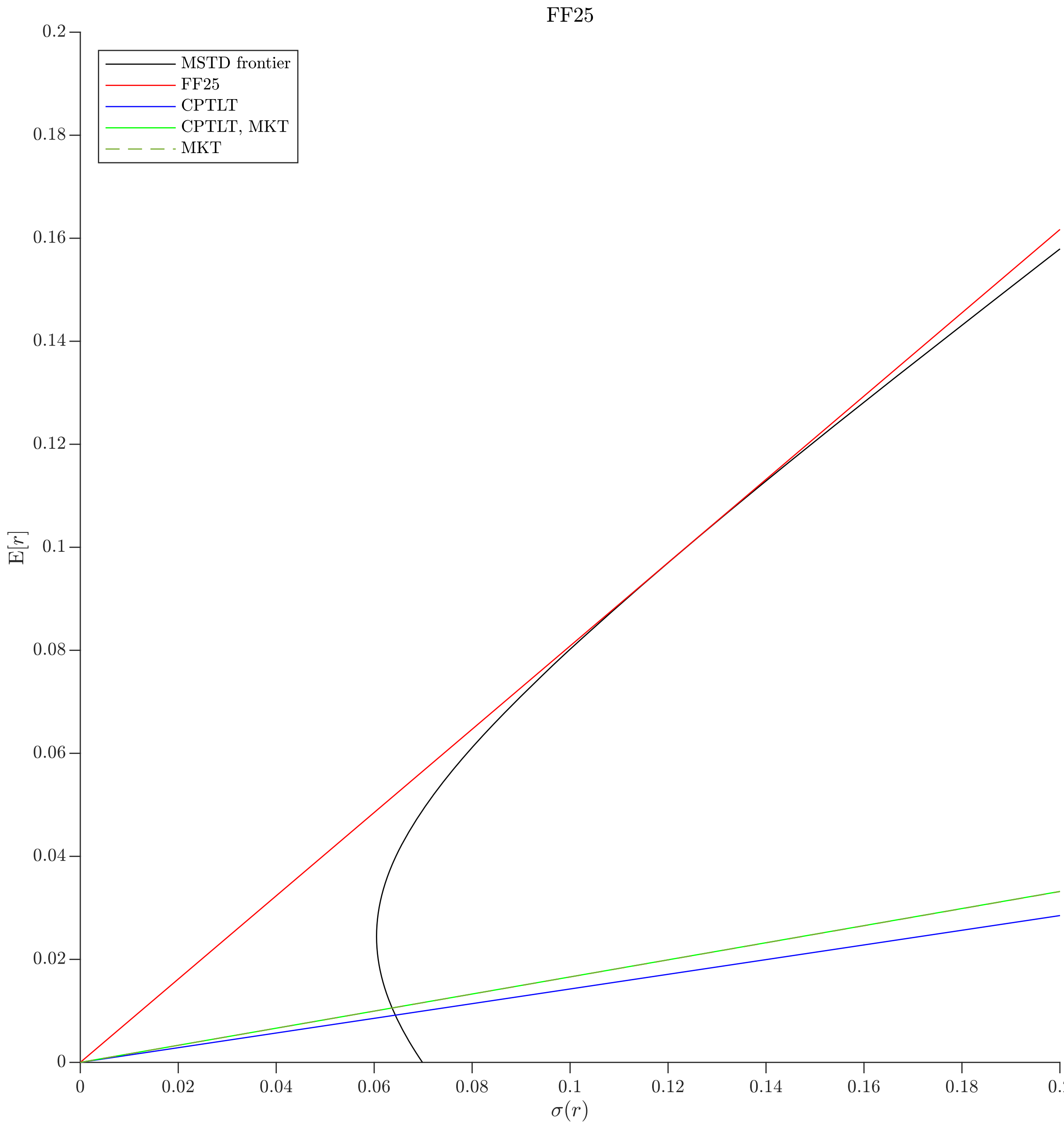

Fig. 2. Mean-standard deviation frontier for equity portfolios and maximum Sharpe ratios. The figure plots the mean-standard deviation frontier (MSTD frontier, black line) for the excess returns on the 25 Fama-French equity portfolios. The slope of the red line (FF25) represents the maximum Sharpe ratio that can be achieved by investing in these 25 portfolios. Similarly, the slope of the solid green line ($CPTLT$, MKT) represents the maximum Sharpe ratio from optimally combining the market and traded capital factors. Finally, the slopes of the dashed green (MKT) and blue lines ($CPTLT$) are the Sharpe ratios of the market and traded capital factors, respectively. The sample period is 1970:Q1–2012:Q4.

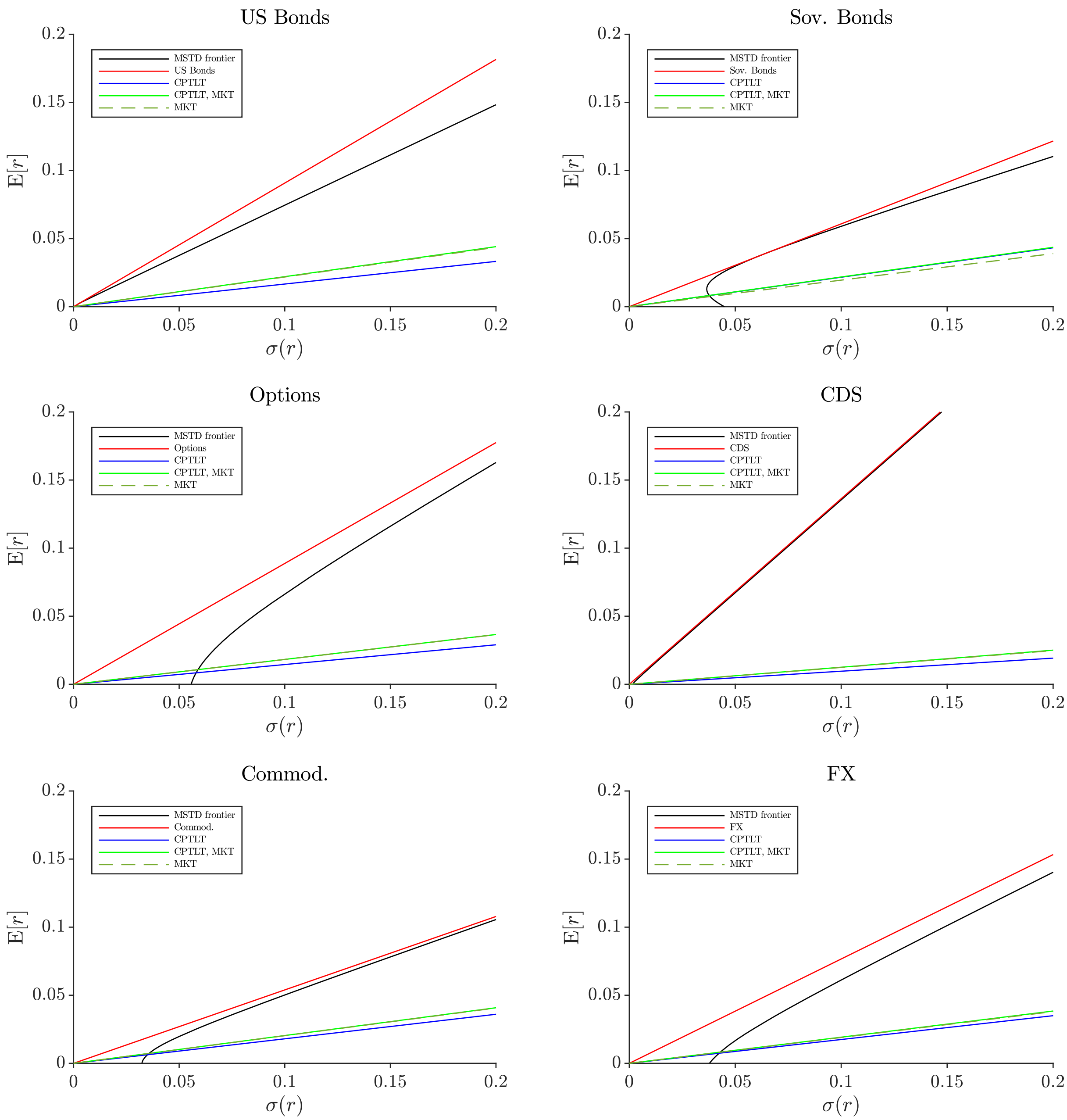

Fig. 3. Mean-standard deviation frontiers for various asset classes and maximum Sharpe ratios. The figure plots the mean-standard deviation frontiers (MSTD frontier, black line) for the excess returns on various portfolios (US bonds, Sov. bonds, Options, CDS, Commod., and FX). The slope of the red line represents the maximum Sharpe ratio that can be achieved by investing in the asset class-specific portfolios. Similarly, the slope of the solid green line (*CPTLT*, *MKT*) represents the maximum Sharpe ratio from optimally combining the market and traded capital factors. Finally, the slopes of the dashed green (*MKT*) and blue lines (*CPTLT*) are the Sharpe ratios of the market and traded capital factors, respectively. The sample periods for US bonds, sovereign bonds, options, CDS, commodities, and FX are 1975:Q1–2011:Q4, 1995:Q1–2011:Q1, 1986:Q2–2011:Q4, 2001:Q2–2012:Q4, 1986:Q4–2012:Q4, and 1976:Q2–2009:Q4, respectively.

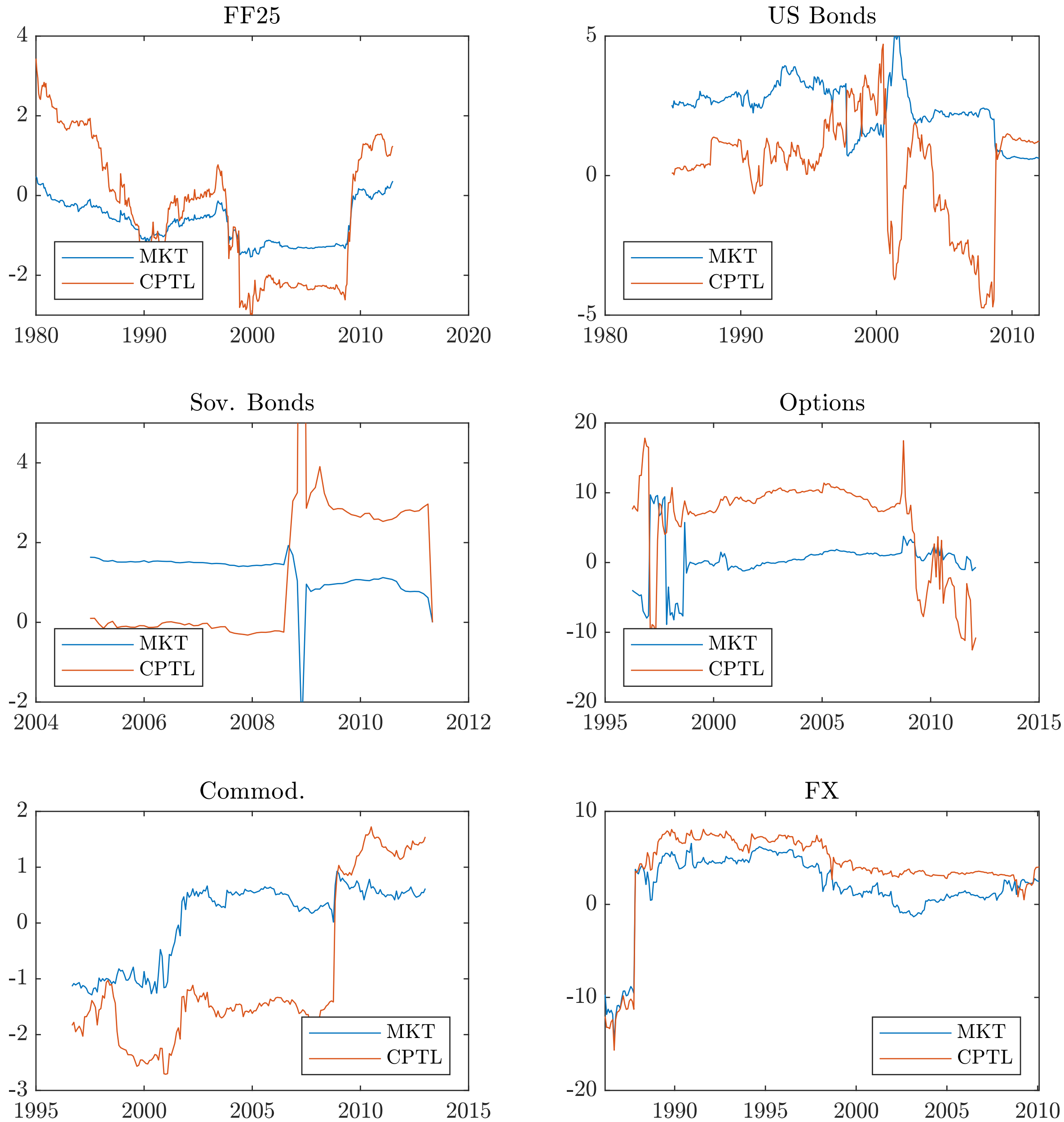

Fig. 4. Rolling window OLS estimates for market and capital. Based on rolling windows of 120 months, the figure displays the behavior of the OLS CSR prices of beta risk for market (*MKT*, blue line) and nontraded capital (*CPTL*, red line) in HKM's two-factor model for six asset classes (FF25, US bonds, Sov. bonds, Options, Commod., and FX). The monthly samples for equities, US bonds, sovereign bonds, options, commodities, and FX are 1970:01–2012:12, 1974:12–2011:12, 1995:01–2011:04, 1986:04–2012:01, 1986:09–2012:12, and 1976:03–2010:01, respectively.

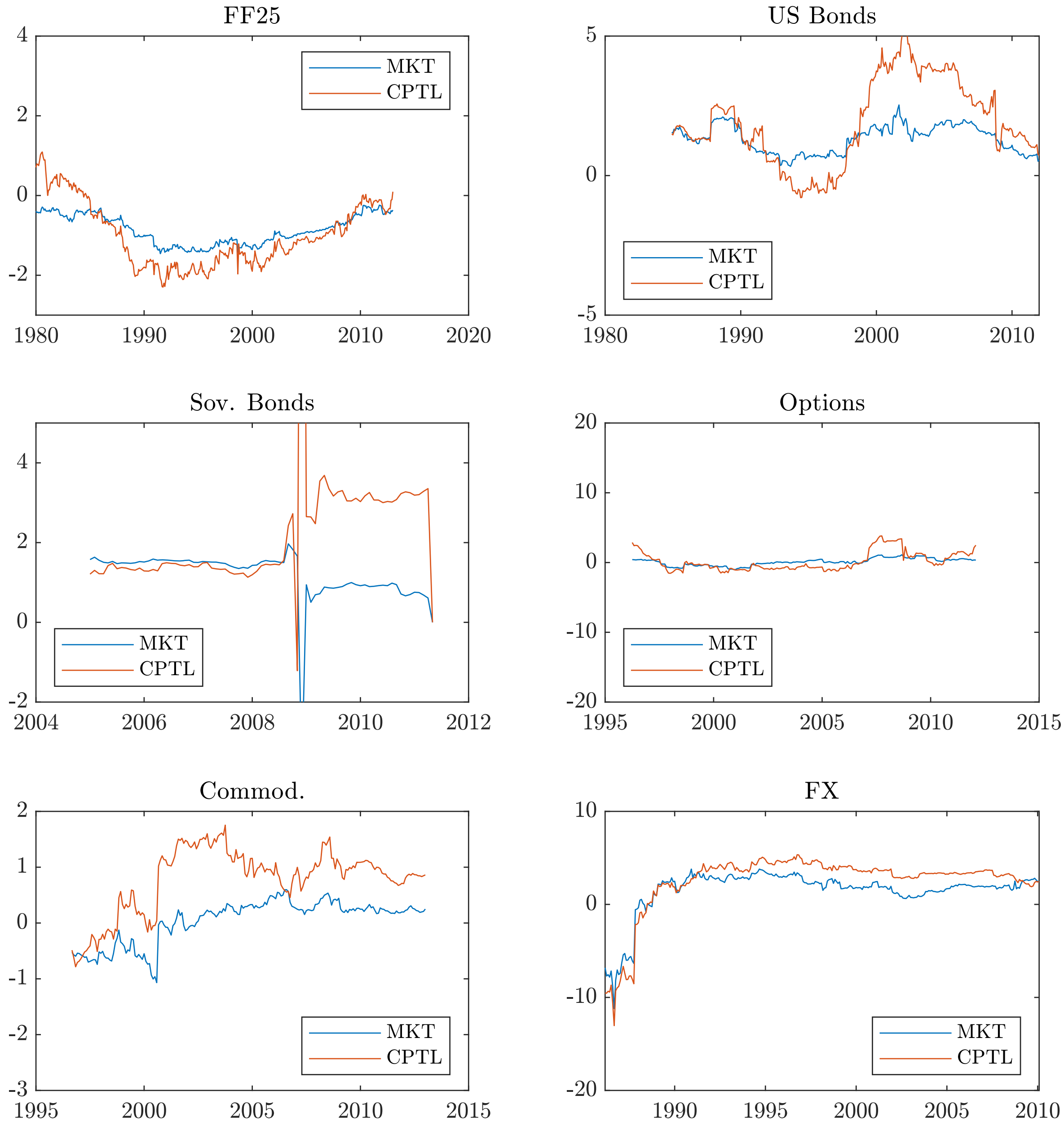

Fig. 5. Rolling window GLS estimates for market and capital. Based on rolling windows of 120 months, the figure displays the behavior of the GLS CSR prices of beta risk for market (*MKT*, blue line) and nontraded capital (*CPTL*, red line) in HKM's two-factor model for six asset classes (FF25, US bonds, Sov. bonds, Options, Commod., and FX). The monthly samples for equities, US bonds, sovereign bonds, options, commodities, and FX are 1970:01–2012:12, 1974:12–2011:12, 1995:01–2011:04, 1986:04–2012:01, 1986:09–2012:12, and 1976:03–2010:01, respectively.

Fig. 6. Rolling and recursive window OLS estimates for downside market risk. Based on rolling windows of 180 months (blue line) and recursive windows with an initial sample of 180 months (red line), the figure displays the behavior of the OLS CSR price of downside beta risk in the DR-CAPM for four sets of assets (currencies, currencies and equities, currencies and commodities, and currencies, commodities, and equities). The monthly samples for currencies and currencies and equities are 1974:01–2010:03, while the monthly samples for currencies and commodities, and currencies, commodities, and equities are 1974:01–2008:12.

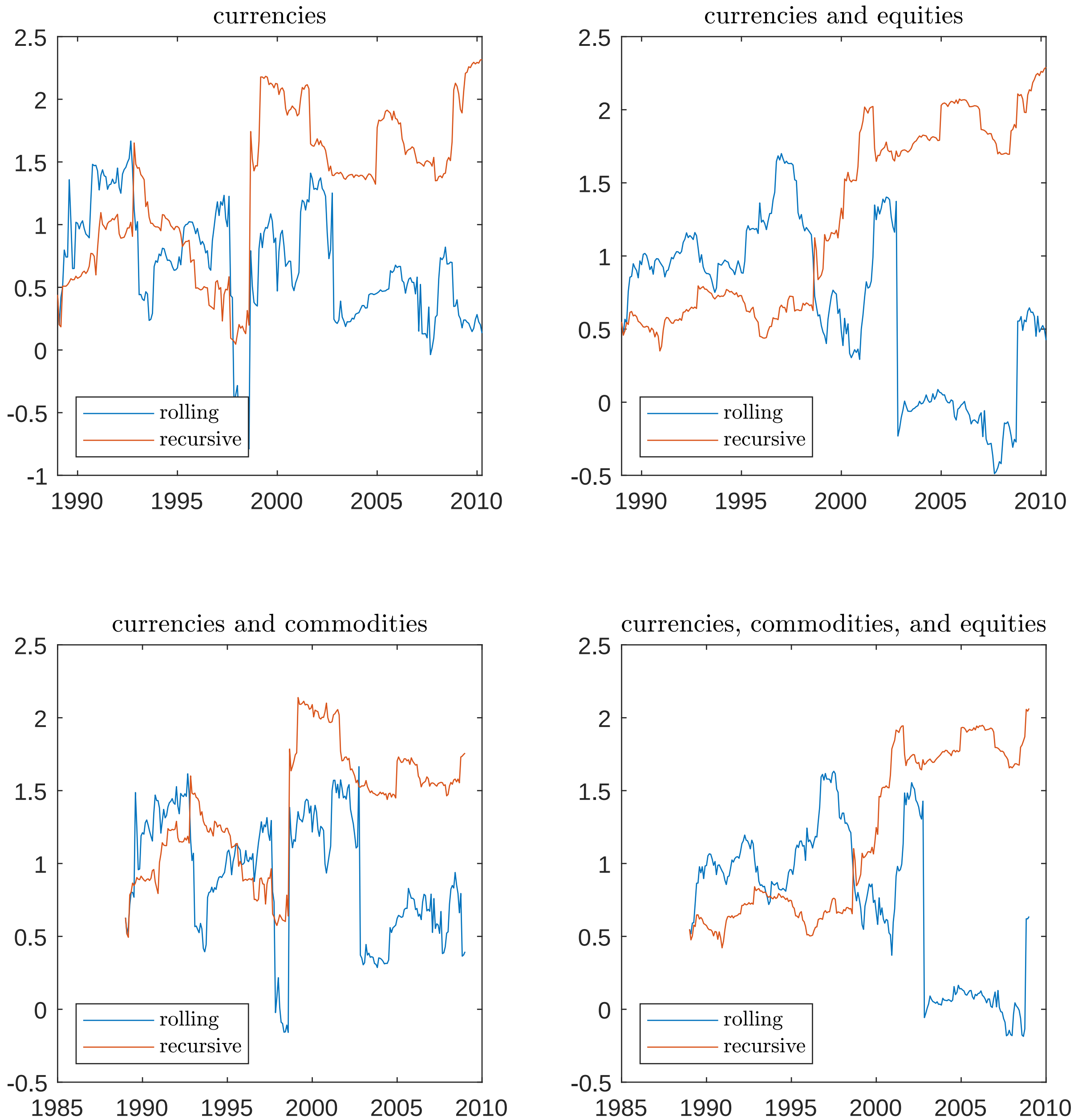

Fig. 7. Rolling and recursive window GLS estimates for downside market risk. Based on rolling windows of 180 months (blue line) and recursive windows with an initial sample of 180 months (red line), the figure displays the behavior of the GLS CSR price of downside beta risk in the DR-CAPM for four sets of assets (currencies, currencies and equities, currencies and commodities, and currencies, commodities, and equities). The monthly samples for currencies and currencies and equities are 1974:01–2010:03, while the monthly samples for currencies and commodities, and currencies, commodities, and equities are 1974:01–2008:12.