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Joint Location and Transmit Power Optimization for NOMA-UAV Networks via Updating Decoding Order

Abstract—Unmanned aerial vehicle (UAV) can be combined with non-orthogonal multiple access (NOMA) to achieve better performance. However, jointly optimizing the location, transmit power and decoding order for NOMA-UAV networks remains difficult, due to the change of decoding order as a result of UAV mobility. In this letter, a low-complexity scheme is proposed to maximize the sum rate of NOMA-UAV networks via updating decoding order, which can be decomposed into two steps. First, the joint location and power optimization can be divided into two non-convex sub-problems, which are further approximated via successive convex optimization. Then, the decoding order is updated according to the optimized UAV location. An iterative algorithm is proposed to execute the two steps alternately. In addition, the asymptotic performance is analyzed. Simulation results demonstrate the effectiveness of the proposed scheme.

Index Terms—NOMA, power and location optimization, successive interference cancellation, UAV.

I. INTRODUCTION

Recently, unmanned aerial vehicles (UAVs) have been widely used as carrying platforms of base stations in wireless communications [1], [2]. Many recent studies have been dedicated to UAV communications [3]–[7]. In [3], the joint optimization of trajectory and transmit power was studied by Wu et al. to maximize the sum rate in multi-UAV networks. Yang et al. studied UAV energy tradeoff for the data collection in UAV networks via trajectory optimization in [4]. Zhao et al. [5] proposed a novel channel tracking scheme for UAV mmWave multi-antenna systems. In [6], Gong et al. considered a UAV-assisted cellular network, applying the superimposed training sequence with imperfect channel statistics. UAV-aided jamming for secure communication with unknown location of the eavesdropper was investigated in [7] by Nnamani et al.

On the other hand, to improve the spectrum efficiency, non-orthogonal multiple access (NOMA) is emerging as a crucial technique for future wireless networks [8]–[10]. In [8], Chen et al. have proved that NOMA has a better performance than orthogonal multiple access (OMA). A resource allocation algorithm for NOMA networks was proposed by Chang et al. to improve the secrecy energy efficiency [9]. In [10], Lei et al. proposed a max-min transmit antenna selection scheme for NOMA systems with secrecy outage performance analyzed.

Due to their advantages, it becomes natural to integrate UAV and NOMA for enhancing the performance, and some fundamental works have been done in [11], [12]. Liu et al. set up a general framework for NOMA-UAV networks in [11]. Mei and Zhang proposed a cooperative NOMA scheme for cellular-connected UAV networks in [12]. Recently, plenty of research on resource allocation of NOMA-UAV network has been conducted [13]–[17]. In [13], Tang et al. proposed a UAV placement scheme to maximize the number of users in a NOMA-UAV network. A joint placement and power optimization scheme was proposed by Liu et al. for NOMA-UAV networks in [14]. The joint optimization of altitude and beamwidth was considered in a NOMA-UAV network by Nasir et al. in [15]. Liu et al. proposed a distributed NOMA-UAV scheme to assist emergency communications [16]. Wang et al. [17] proposed a UAV-aided NOMA scheme with secure simultaneous wireless information and power transfer. Resource allocation for optimizing the energy efficiency in NOMA-UAV network was introduced in [18], [19].

Motivated by above works, we focus on the system design for NOMA-UAV networks. The location and transmit power of UAV are jointly optimized to maximize the sum rate. Different from above works with fixed decoding order, we propose an iterative algorithm to update the current decoding order in the ascending order of channel gains after each iteration. Numerical results show that the proposed scheme can effectively improve the performance of NOMA-UAV networks.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

Consider a NOMA-UAV network with one UAV and $K$ ground users. All are equipped with a single antenna. Define
$U_i$ as the $i$th user, $i \in K \triangleq \{1, 2, \ldots, K\}$. The superimposed information is transmitted from the UAV to the users via NOMA. The received signal at $U_i$ is given by

$$s_i = h_i \sum_{j=1}^{K} z_j + n_i, \quad i \in K,$$  \hspace{1cm} (1)

where $h_i$ represents the channel coefficient from the UAV to $U_i$, and $n_i$ denotes the additive white Gaussian noise (AWGN) at $U_i$. $z_j$ is the message for $U_j$ with $|z_j|^2 = a_j P_{\text{sum}} = P_j$, where $P_{\text{sum}}$ is the sum transmit power of UAV, $a_j$ is the power coefficient of $U_j$, and $P_j$ is the transmit power for $U_j$.

Assume that the horizontal coordinates of $U_i$ is denoted as $q_i = [x_i, y_i]^T \in \mathbb{R}^{2 \times 1}$. The UAV hovers at a fixed altitude $H$ and its horizontal coordinate is $L = [X, Y]^T \in \mathbb{R}^{2 \times 1}$. The distance between the UAV and $U_i$ can be expressed as

$$d_i = \sqrt{H^2 + \|q_i - L\|^2}.$$  \hspace{1cm} (2)

The probability of UAV-to-ground links dominated by line-of-sight (LoS) can be expressed as

$$P_i^{\text{LoS}} = \frac{1}{1 + a_0 \exp(-b_0 (\theta_i - a_0))},$$  \hspace{1cm} (3)

where $a_0$ and $b_0$ denote the environment constants, $\theta_i = \arcsin (\frac{H}{d_i})$ represents the elevation angle between the UAV and $U_i$. According to an extensive survey for UAV channel modeling [20], when the UAV is located high enough (e.g., 120m), the LoS probability is approximate to 1. Thus, the channel from the UAV to $U_i$ can be denoted as

$$h_i = \sqrt{\rho_0 d_i^{-2}} = \sqrt{\frac{\rho_0}{H^2 + \|q_i - L\|^2}}.$$  \hspace{1cm} (4)

where $\rho_0$ denotes the reference channel gain at the unit distance.

According to NOMA, the successive interference cancellation (SIC) is applied for users to receive their own messages, and users with low channel gains are compensated by high power allocation ratios. Therefore, according to the distance from the UAV to users, we assume $|h_1|^2 \geq \cdots \geq |h_i|^2 \geq \cdots \geq |h_K|^2 > 0$ with $0 < a_1 \leq \cdots \leq a_i \leq \cdots \leq a_K$. $U_i$ needs to decode the messages from $U_{i+1}$ to $U_K$ and removes them from the superposed signal. Then, we can express the signal-to-interference-plus-noise-ratio (SINR) for $U_i$ ($2 \leq i \leq K$) as

$$\text{SINR}_i = \frac{|h_i|^2 P_i}{\sum_{k=1}^{i-1} P_k + \sigma^2} = \frac{P_i}{\sum_{k=1}^{i-1} P_k + \frac{\sigma^2}{|h_i|^2}},$$  \hspace{1cm} (5)

where $\sigma^2$ is the power of AWGN at user receivers. When $i = 1$, the received SINR can be calculated by

$$\text{SINR}_1 = \frac{P_1 |h_1|^2}{\sigma^2}.$$  \hspace{1cm} (6)

Thus, the transmission rate for $U_i$ can be denoted as

$$R_i = \log_2 (1 + \text{SINR}_i), \quad i \in K.$$  \hspace{1cm} (7)

We assume $\gamma_i$ denotes the SINR threshold of $U_i$, which can be expressed as

$$\text{SINR}_i \geq \gamma_i, \quad i \in K.$$  \hspace{1cm} (8)

Accordingly, the rate threshold of $U_i$ is obtained from (8) as

$$\eta_i = \log_2 (1 + \gamma_i), \quad i \in K.$$  \hspace{1cm} (9)

### B. Problem Formulation

Define $P = \{P_i, i \in K\}$. To maximize the sum rate of ground users via jointly optimizing $L$ and $P$ based on (7) and (8), the optimization problem can be formulated as

$$\max_{L, P} \sum_{i \in K} \log_2 (1 + \text{SINR}_i)$$  \hspace{1cm} (10a)

subject to

$$\text{SINR}_i \geq \gamma_i, \quad i \in K,$$  \hspace{1cm} (10b)

$$0 < P_1 \leq \cdots \leq P_i \leq \cdots \leq P_K,$$  \hspace{1cm} (10c)

$$\sum_{i=1}^{K} P_i \leq P_{\text{sum}},$$  \hspace{1cm} (10d)

Combining (5), (6) and (7), the problem (10) can be rewritten as

$$\max_{L, P} \sum_{i \in K} \log_2 (1 + \text{SINR}_i)$$  \hspace{1cm} (11a)

subject to

$$\frac{P_i}{\sum_{k=1}^{i-1} P_k + \frac{\sigma^2}{|h_i|^2}} \geq \gamma_i, \quad i \in K \setminus \{1\},$$  \hspace{1cm} (11b)

$$P_i \frac{|h_i|^2}{\|h_i|^2} \geq \gamma_i,$$  \hspace{1cm} (11c)

$$0 < P_1 \leq \cdots \leq P_i \leq \cdots \leq P_K,$$  \hspace{1cm} (11d)

$$\sum_{i=1}^{K} P_i \leq P_{\text{sum}}.$$  \hspace{1cm} (11e)

In (11), the constraints (11b) and (11c) are non-convex with respect to $L$ and $P$. Thus, approximation will be used in the next section.

Different from the fixed SIC order [14], the decoding order is updated after each iteration according the ranking of channel gains in this letter, with the stronger user decoded later.

### III. Iterative Algorithm for the Optimization

The problem (11) is difficult to solve due to its non-convexity. Thus, we propose a scheme to optimize the location and power alternately via successive convex optimization.

#### A. Transmit Power Optimization

First, we fix the UAV location and (11) becomes

$$\max_P \sum_{i \in K} \log_2 (1 + \text{SINR}_i)$$  \hspace{1cm} (12a)

subject to

$$\frac{P_i}{\sum_{k=1}^{i-1} P_k + \frac{\sigma^2}{|h_i|^2}} \geq \gamma_i, \quad i \in K \setminus \{1\},$$  \hspace{1cm} (12b)

$$P_i \frac{|h_i|^2}{\|h_i|^2} \geq \gamma_i,$$  \hspace{1cm} (11c)

$$0 < P_1 \leq \cdots \leq P_i \leq \cdots \leq P_K,$$  \hspace{1cm} (11d)

$$\sum_{i=1}^{K} P_i \leq P_{\text{sum}}.$$  \hspace{1cm} (11e)

(12c) is convex. (12b) is non-convex and its left-hand-side can be replaced by $R_i$. Thus, it can be changed into two concave functions with respect to $P$ as

$$R_i = \log_2 (1 + \text{SINR}_i)$$

$$= \log_2 \left(1 + \frac{|h_i|^2 P_i}{\sum_{k=1}^{i-1} P_k + \frac{\sigma^2}{|h_i|^2}}\right)$$  \hspace{1cm} (13)

$$= \log_2 \left(|h_i|^2 \sum_{k=1}^{i-1} P_k + \frac{\sigma^2}{|h_i|^2}\right) - \log_2 \left(|h_i|^2 \sum_{k=1}^{i-1} P_k + \frac{\sigma^2}{|h_i|^2}\right).$$
We need to approximate the second concave function via the first-order Taylor expansion at a specific point to obtain its global upper-bound. Define the transmit power in the $r$th iteration and the second concave function as $P_r$ and $R_i$, respectively, and we have

$$\tilde{R}_i = \log_2 \left( |h_i|^2 \sum_{k=1}^{i-1} P_k + \sigma^2 \right)$$

$$\leq \sum_{k=1}^{i-1} A_i' (P_k - P_{i-1}) + B_i' \triangleq \tilde{R}_i^{[ab]},$$

where $A'_i$ and $B'_i$ can be calculated as

$$A'_i = \frac{|h_i|^2 \log_2(e)}{|h_i|^2 \sum_{j=1}^{i-1} P_j + \sigma^2},$$

$$B'_i = \log_2 \left( |h_i|^2 \sum_{k=1}^{i-1} P_k + \sigma^2 \right).$$

Therefore, (12) becomes convex as

$$\max_{\mathbf{P}} \sum_{i \in \mathcal{K}} \log_2(1 + \text{SINR}_i)$$

$$s.t. \quad \log_2 \left( |h_i|^2 \sum_{k=1}^{i} P_k + \sigma^2 \right) - \tilde{R}_i^{[ab]} \geq \eta_i, \forall i \in \mathcal{K}\{1\},$$

$$\log_2 \left( |h_i|^2 \sum_{k=1}^{i-1} P_k + \sigma^2 \right) = \tilde{R}_i - \tilde{R}_i^{[ab]},$$

which is convex and can be solved by CVX.

B. Location Optimization

Then, we fix the transmit power to transform (11) into

$$\max_{\mathbf{L}} \sum_{i \in \mathcal{K}} \log_2(1 + \text{SINR}_i)$$

$$s.t. \quad \frac{\rho_0}{H^2 + \|q_i - L_i\|^2} P_i \geq \gamma_i, \forall i \in \mathcal{K}\{1\},$$

$$\rho_0 \sum_{i=1}^{i-1} P_k + \sigma^2 \geq \gamma_1.$$

The constraints (18b) and (18c) are non-convex with respect to $\mathbf{L}$. For (18b), it can be split into two convex functions with respect to $\|q_i - L_i\|^2$

$$R_i = \log_2 \left( 1 + \text{SINR}_i \right)$$

$$= \log_2 \left( 1 + \frac{\rho_0}{H^2 + \|q_i - L_i\|^2} \sum_{k=1}^{i-1} P_k + \sigma^2 \right)$$

$$= \tilde{R}_i - \tilde{R}_i^{[ab]},$$

where

$$\tilde{R}_i = \log_2 \left( \frac{\rho_0}{H^2 + \|q_i - L_i\|^2} \sum_{k=1}^{i} P_k + \sigma^2 \right),$$

$$\tilde{R}_i^{[ab]} = \log_2 \left( \frac{\rho_0}{H^2 + \|q_i - L_i\|^2} \sum_{k=1}^{i-1} P_k + \sigma^2 \right).$$

Notice that $\tilde{R}_i$ is neither concave nor convex with respect to $\mathbf{L}$. Thus, we define the local point $\mathbf{L}'$ in the $r$th iteration and derive the lower-bounded expression of $\tilde{R}_i$ via the first-order Taylor expansion as

$$\tilde{R}_i = \log_2 \left( \frac{\rho_0}{H^2 + \|q_i - L_i\|^2} \sum_{k=1}^{i} P_k + \sigma^2 \right)$$

$$\geq \sum_{k=1}^{i} C_i' \left( \|q_i - L_i\|^2 - \|q_i - L_i'\|^2 \right) + D_i' \triangleq \tilde{R}_i^{[ab]},$$

where $C_i'$ and $D_i'$ can be calculated as

$$C_i' = \frac{\rho_0}{H^2 + \|q_i - L_i'\|^2} \sum_{l=1}^{i} P_l + \sigma^2$$

$$D_i' = \log_2 \left( \frac{\rho_0}{H^2 + \|q_i - L_i'\|^2} \sum_{l=1}^{i} P_l + \sigma^2 \right).$$

With (19) and (22), (18b) can be transformed into

$$\tilde{R}_i^{[ab]} - \tilde{R}_i \geq \eta_i.$$
C. Iterative Algorithm

Based on Section III-A and Section III-B, (11) can be solved iteratively using Algorithm 1. In Step 3, the decoding order is updated according to the optimized UAV location.

Algorithm 1 Iterative Algorithm for (11)
Initialization: Set the geometric center of users as the starting location \( L^0 = \sum_{i=1}^{K} q_i / K \). The initial decoding order and the power \( P^0 \) are set according to \( L^0 \) and the minimum transmit power. Set the initial index of iterations as \( r = 0 \).

while \((R(P^{r+1}, L^{r+1}) - R(P^r, L^r) \leq \epsilon_1)\) do

1. Solve (17) via \( L^r \), and obtain \( P^{r+1} \).
2. Solve (30) via \( P^r+1 \), and obtain \( L^{r+1} \).
3. Update the decoding order according to \( L^{r+1} \).
4. Update: \( r = r + 1 \).

end

Output: \( R_{\text{sum}}^* = R(P^*, L^*) \).

The convergence of Algorithm 1 is proved in Proposition 1.

**Proposition 1:** Algorithm 1 is convergent.

**Proof:** Define the objective value of \( r \)th iteration as \( R(P^r, L^r) \). In the \((r + 1)\)th iteration, we obtain the objective value \( R(P^{r+1}, L^{r+1}) \) by Step 1 of Algorithm 1, and it is the lower bound of the original problem (11). Thus we have

\[
R(P^r, L^r) \leq R(P^{r+1}, L^{r+1}),
\]

for Step 2 of Algorithm 1, and we can obtain the objective value \( R(P^{r+1}, L^{r+1}) \). Similarly, we have

\[
R(P^{r+1}, L^r) \leq R(P^{r+1}, L^{r+1}).
\]

Step 3 in Algorithm 1 can always adjust the current decoding order in each iteration, and the sum rate will not decrease. Thus, combining (31) with (32), we prove the objective value of (11) is non-decreasing after each iteration, and is upper bounded by a finite value. Algorithm 1 is convergent.

D. Analysis of the Last Decoding User

The last decoding user is the closest one to the UAV and last decoded via SIC. The last decoding user is determined when the UAV location is initialized, and will not change during iterations, which is proved in Proposition 2. To simplify the derivation, we introduce an auxiliary variable \( \alpha_i \) as

\[
\alpha_i = \frac{\sigma^2}{|h_i|^2}.
\]

**Proposition 2:** The last decoding user is not changed during iterations and the optimal UAV location is getting closer to this user with increasing transmit power.

**Proof:** Define \( U_1 \) as the initial last decoding user, and we can always find suitable power allocation at \( L^0 \) to satisfy

\[
R_i(\alpha_i^*) = R_i(\alpha^*_i), \forall i \in K \setminus \{1\}.
\]

The derivative of \( R_i(\alpha_i) \) and \( R_1(\alpha_1) \) can be expressed as

\[
R_i'(\alpha_i) = \frac{-P_i \log_2 e}{\left( \alpha_i + \sum_{k=1}^{i-1} P_k \right) \left( \alpha_i + \sum_{k=1}^{i} P_k \right)}, 2 \leq i \leq K,
\]

\[
R_1'(\alpha_1) = \frac{-P_1 \log_2 e}{\left( \alpha_1 + \sum_{k=1}^{i-1} P_k \right) \left( \alpha_1 + \sum_{k=1}^{i} P_k \right)}, 2 \leq i \leq K.
\]

According to the decoding order at \( L^0 \), we have \( h_i^* \geq h_i^*, i \in K \setminus \{1\} \) and \( \alpha_i^* \leq \alpha_i^*. \) Thus, \(|R_i'(\alpha_i^*)| < |R_1'(\alpha_1^*)|\) is met under the assumption in (34). In order to increase the sum rate, the UAV location will approach \( U_1 \). The user \( U_1 \) always has the best channel condition and the last decoding user is not changed. We can observe that the increase of power has a much greater influence on \( R_1 \) from (37). Therefore, with the rate thresholds are satisfied, the optimal UAV location will approach \( U_1 \) with larger \( P_{\text{sum}} \).

From Proposition 2, we can conclude that the UAV location should move to the last decoding user, if we need to improve the sum rate with higher transmit power.

IV. SIMULATION RESULTS AND DISCUSSION

In the simulation, assume that the UAV hovers at \( H = 150 \) m. We set \( \sigma^2 = -110 \) dBm and \( \rho_0 = -60 \) dB.

First, we set \( K = 3 \) and users are marked by red triangles, as shown in Fig. 1. The optimal location of UAV is presented for different transmit power, when \( \eta = (1, 1, 1) \) bit/s/Hz. From the result, we can observe that the optimal UAV location becomes closer to the last decoding order when \( P_{\text{sum}} \) increases, which is consistent with the conclusion from Proposition 2.

The sum rate of the proposed scheme is compared in Fig. 2 with different \( \eta \) according to the topology in Fig. 1 with the same threshold for each user. The result shows that the sum rate increases when the transmit power of the UAV is higher.
Furthermore, the sum rate decreases as the rate threshold increases. This is because that the lower threshold provides more degree of freedom for the power allocation and location selection, and thus the UAV can allocate more transmit power for the interference-free (last decoding) user, which leads to a higher throughput.

In Fig. 3, the average sum rate of the proposed scheme is compared with the scheme in [14], with and without power control (PC). We assume all the users are randomly deployed in a square area of $400 \times 400$ m$^2$, and we set $\eta = (1, 1, 1)$ bit/s/Hz for all the schemes. The result shows PC can effectively improve the rate performance for both schemes. In addition, we can observe that the average sum rate of the proposed scheme is much higher than the benchmark scheme in [14]. For example, the average sum rate can be improved by 5% when $P_{\text{sum}} = 30$ mW.

V. CONCLUSIONS

In this letter, we have jointly optimized the UAV location and transmit power in NOMA-UAV networks via updating decoding order, which can be divided into two sub-problems. The non-convex sub-problems are approximated into convex ones, and an iterative algorithm is proposed to optimize the location and power alternately via successive convex optimization. Its convergence is proved. In addition, the closer performance of the algorithm has been further analyzed. Finally, simulation results have been shown to verify the effectiveness of the proposed scheme over benchmarks.

REFERENCES