Essays on the Effects of Debt on Real Activity

by

Ghasan Saeed Asbool

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Declarations

All the chapters in this thesis contain original research work. They have not been submitted by me for any other assessments or previous degree courses.
Abstract

This thesis consists of two chapters and a dataset made available for future research. Chapter 1 aims to explain two economic observations. First, firms of different sizes face different credit treatments by banks. There is a heterogeneous credit treatment to firms with different sizes but share the same production technology; where size is defined by firms’ net worth (precisely, the amount of capital they own). I present a micro founded macroeconomic model, where lenders in their attempt to maximise their profits, will decide whether to pay high audit costs or not when extending credit to firms. Using audit reveals firms’ idiosyncratic shocks that are otherwise unobservable before engaging in production. The lenders’ optimisation will generate an endogenous audit threshold for the level of capital, \( k \), that implies different credit treatment for firms that fall below the threshold level, compared to the ones that fall above it. Second, during the financial crisis, firms’ investments have differed non-uniformly; where medium-sized firms exhibited the largest percentage decline in investment. Using the said model, I discuss how differing credit treatments and shifts in \( k \) can lead to differing average investment responses to economic shocks by firms based on their size. The model predictions are qualitatively consistent with the UK data. Although there are other factors that may have caused such empirical regularities to arise, what I am presenting is one of the channels, namely financial frictions, through which the given phenomena can be explained.

The second chapter investigates the issue of the economy wide stagnating labour productivity in the UK since 2008. Most of the theoretical literature that ties financial frictions with productivity, address the issue from the angle of resources misallocation whether it is labour or capital. However, most of the decline in UK’s labour productivity occurred within firm, rather than through depressed external restructuring. Accordingly, my theoretical explanation seeks to tie shocks to the financial sector with production changes within firms. The results are achieved by a version of the real business cycle model, which incorporates wealth shocks and redistributive wealth taxes that affect wealth inequality. The shocks in the model cause changes in relative factors’ prices during the recession and the following recovery, leading to capital shallowing within firms. The model’s theoretical predictions are qualitatively consistent with the movements in real variables and prices that occurred in the UK since the financial crisis.

Finally, the dataset facilitates future research on how public debt decomposition between domestic and external creditors, affects the size of fiscal multipliers. Following the decisive bail out measures, many governments have embarked on fiscal consolidation plans, to bring their finances back into control. However, economic growth has fallen short from what has been expected, when the fiscal consolidation plans were introduced. Accordingly, it became clear that fiscal multipliers were substantially higher than previously assumed. In the years leading to the financial crisis, some of the small European countries have witnessed a steady and substantial increases in the percentage of non-resident holders of their public debt. This may have reduced the crowding out effect of government spending. This chapter presents an update of the dataset of Ilzetzki, et al. (2013). It consists of 50 countries and expands on the previous dataset by including decomposition of debt data between domestic and foreign holders using multiple sources.
Chapter 1

Bank Lending Practices and their Effect on Firms’ Investment

1.1 Introduction

Most of the literature discussing the variation in performance between firms based on their size, have focused on dividing firms into two groups, Small and Medium Enterprises (SMEs) as a single block and large firms. Little work, at least within economics, have tried to theoretically discover the variation within SMEs. It is important to investigate this variation, since more than 95 per cent of firms around the world belong to this category. More than half of the workers in emerging economies are employed by firms with fewer than 100 employees (Ayyagari et al. (2011)). SMEs are equally important in developed economies as well. For instance, according to the UK’s Department for Business, Energy and Industrial Strategy, DBEIS (2017), at the start of 2017 SMEs accounted for 99.9 per cent of the businesses population, 60 per cent of private sector employment and 52 per cent of private sector turnover. The importance of SMEs extends beyond the absolute numbers and proportions they account for. Evidence on a negative relationship between firm size and net growth rates was found using US firms’ employment dynamics datasets in Haltiwanger et al. (2010) and Neumark et al. (2011).

Many other studies reflect a similar pattern of difference in performance between small and large firms, especially during the recent recession and the following recovery. Small firms suffered proportionately more and exhibited a slower recovery

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1 There is no universally agreed standard on classifying firms based on their size or other criteria such as their revenue or net worth. Definitions differ across countries and sometimes even between institutions within the same country. Among the standards used for classifying firms are workforce size, assets and turnover (Beck (2013)). For instance, most UK government departments including the Office for National Statistics (ONS) consider small firms as having 49 or less employees and medium firms to be the ones with 50 to 249 employees. However, the UK Companies Act consider a medium firm to be one with significant net worth of £4 millions and annual sales of £15 millions (ICAEW (2014)).
(see Aliprantis and Burgen (2012); Tasci and Burgen (2012); Sahin et al. (2011); Krueger and Charnes (2011); CBO (2012); Fort et al. (2013)). The main explanation provided for the aforementioned pattern, is the severe reduction in credit extended by banks to small firms. Those firms are typically more reliant on bank lending, and hence are more affected by banking distress. Those greater financial limitations faced by SMEs as opposed to large firms have translated to different real activity. Using US data, Duygan-Bump et al. (2015) have found that small firms’ financial limitations played an important role in contributing to the high unemployment during the recent recession, where small firms’ workers were more likely to become unemployed. Benito et al. (2010) have used UK data and showed more generally that the contraction in credit supply during the financial crisis, has caused a sharp fall in business investment.

However, all those empirical studies have either reported aggregate phenomena with no breakdown of firms’ behaviour based on pre-recession characteristics such as size, or made no distinction between firms within the subgroup of SMEs. In this chapter I focus my attention on the empirical regularities reported in Crawford et al. (2013), where a large sample of data between 1997 and 2009 on UK firms and individuals has been used. Their study examines the extent to which firms’ labour productivity, investment and profits have diverged during the Great Recession from their pre-recession linear trends. They report their results distinguishing between firms’ responses based on their size as being small, medium or large.\(^2\) I further restrict my attention to their results on firms’ investment based on their size and try to provide a theoretical framework that can explain how such empirical regularities may have occurred. Figure 1.1 below summaries the results of Crawford et al. (2013) with regards to firms’ investment responses during the recession relative to their pre-crisis trend. Their findings are consistent with Benito et al. (2010), where firms’ total real investment has fallen sharply during the recession. However, unlike Benito et al. (2010) their investigation into those investment responses highlights a great dispersion based on firms’ size, with medium-sized firms exhibiting the sharpest decline.

\(^2\) They follow the ONS classification of firms’ sizes, which depends on the size of a firm’s workforce.
Their findings contradict with the intuition provided in the financial frictions’ literature, where based on access to credit, investment declines are expected to be the sharpest in small firms followed by medium firms and then large firms. It is worth noting that such non-linear behaviour based on size is not unique to the empirical findings of this study. Audretsch and Elston (2002) have used German data from The Bonn Database and found results that are in line with Crawford et al. (2013) findings. Moreover, although such empirical investigations that look more into SMEs sub-classification are not widespread, recent reports by market research firms suggest that medium-sized firms have outperformed both small and large firms post the recent recession.3

The model in this chapter tries to explain the empirical regularities presented in Figure 1.1, based on bank lending practices. In the vast majority of commercial banks, lending activities to firms are split into two main divisions, each dealing with firms of a certain size. Usually firms’ turnover or net worth is used as a proxy by banks to determine in which banking division firms will be dealt with. In order to motivate the assumptions of the theoretical framework, further empirical evidences on such banking practices are presented in the literature review. The model endogenously generates this cutting threshold of firms’ net worth, \( \hat{k} \), where the threshold can shift

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due to changes in the economy’s exogenous parameters. Out of those economic parameters, I am most interested in the effects of neutral productivity shocks, $z$, and investment specific technology shock, $q$ (see Fisher (2006)).

Given the theoretical framework presented, each of the two shocks will generate different responses between small, medium and large firms. Each shock causes those different responses based on a certain channel. In terms of the empirical regularities given in the UK, the model predicts that two channels must have been acting at the same time. However, just having the two channels working at the same time (two shocks), will not be sufficient in explaining how the empirical regularity came into place. What also matters is the relative size of the shocks to each other. The model predicts that the effect of the neutral productivity shock, $z$, on $k$ should have been stronger than the effect of the investment specific technology shock, $q$, for medium firms to respond by more than other firms on average. In the same time, when comparing small and large firms’ investment responses, the effect of the channel of $q$ must have been stronger than the channel of $z$, such that small firms’ investment would respond by more than the large firms’ investment.

The chapter proceeds in the following order: Section 1.2 presents the literature that is most relevant to the theoretical framework and provides the necessary background to motivate the model’s environment. Section 1.3 presents the theoretical model. Section 1.4 solves for the equilibrium and discusses its properties. In Section 1.5, I amend the model by adding Investment. Section 1.6 presents the model results and discusses them. Finally, Section 1.7 concludes and suggests some policy recommendations and possible paths for future research.

1.2 Literature Review and Background

The literature review section of this chapter presents a background on a number of different areas. Those areas are not directly connected, however, combined they provide the necessary literature background that is most relevant to the two questions being addressed and the motivation behind them. The topics covered in the literature review are: medium sized firms, bank lending practices, credit discrimination and financial frictions in macroeconomics, and the role of investment specific technology shocks.
1.2.1 Medium Sized Firms

Little work, at least within economics, have tried to discover the variation within SMEs. Medium-sized firms’ performance and characteristics are different from small firms, which makes general statements about SMEs very broad brush. Nevertheless, the huge scale of the very large firms and their capabilities, as well as the view that the smallest firms are more deserving and adaptive to changes, mean that mid-sized firms are largely ignored in research and policy. In addition to the results of Crawford et al. (2013) and Audretsch and Elston (2002), and due to the little academic research on mid-sized firms, I also present in this subsection information that was obtained from market research firms and government agencies reports and surveys.

According to the UK’s Department for Business, Energy & Industrial Strategy (DBEIS (2017)), there were around 5.7 million private firms at the start of 2017, with SMEs representing 99.9 per cent of the total population. This is not industry sector dependent, as SMEs represent at least 99.5 per cent of businesses in each of the main sectors. Figure 1.2 presents the contribution of different sized firms to the total population, employment and turnover at the start of 2017.

![Figure 1.2: The contribution of different sized businesses to total population, employment and turnover, at the start of 2017 (Source: DBEIS (2017)).](image)

As can be seen in the figure above, although medium sized firms represent less than 1 per cent of the total UK business population, they punch way above their weight in terms of contribution to total private sector employment and turnover. BDO (2017)
found medium sized firms to be the driving force behind UK exports in the five years following the Great Recession. Medium sized firms have grown at a faster rate and generated more jobs and growth in profits in 2016 than small and large firms. According to the classification that BDO uses, those firms represented only 1.5 per cent of all UK businesses in 2016, yet they have contributed with a one third of the UK’s turnover. Moreover, Livesey and Thompson (2013), Grant Thornton (2015) and OUBS (2011), report similar qualitative results where medium sized firms have outperformed small and large firms. Other surveys stress that the distinction between medium sized businesses and other SMEs, also extends to their entrepreneurial behaviour and attitudes towards growth. OUBS (2009) has constructed an entrepreneurial index and found that larger SMEs were more entrepreneurial. Additionally, in a SME survey conducted by the Institute for Chartered Accountants in England and Wales (ICAEW (2014)), it was found that growth appetite was strongly correlated with firms’ size. While around half of small businesses in the survey were looking to grow, nearly 90 per cent of medium-sized businesses did. Furthermore, in a survey conducted by Driver (2007) that reports on the relation between business optimism and investment, medium sized manufacturing firms’ investment where found to be more sensitive to changes in the real interest rate.

Crawford et al. (2013), used a large sample of data between 1997 and 2009 on UK firms and individuals. As shown in Figure 1.1, they found that firms’ investment based on size have exhibited a non-linear behaviour, where medium sized firms had the sharpest contraction in their investments relative to pre-crisis trend. They suggest that the sharper investment decline exhibited by SMEs in general is consistent with those firms facing tougher credit condition during the financial crisis. However, they do not provide an explanation to the greatest investment decline exhibited by medium firms. A similar result was also found by Audretsch and Elston (2002) who used data on German firms between 1961 and 1989 from The Bonn Database. They examined the relationship between firms’ investments and their liquidity constraints. They found that mid-sized firms faced greater liquidity constraints during economic downturns,

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4 BDO classifies small businesses as those with a turnover of less than £10 million. Medium-sized businesses are those with a revenue of £10 million to £300 million. While large businesses are those on the FTSE 350.

5 They follow a different size criterion than Crawford, et al. (2013), with two mid-sized groups (small firms have 500 employees or less, small-mid 501 to 1300 employees, large-mid 1301 to 5500 employees and large 5501 employees and more).
thus, affecting their investments to a greater extent than small firms followed by large firms. Using survey data on firms in 90 countries, Aterido et al. (2009) have found that during adverse economic conditions, micro firms with less than 10 employees are less affected compared to bigger SMEs. They conclude that their results might be due to finance transmission channels affecting firms differently based on their size within the large SMEs population.

Financial constraints are faced by firms of all sizes. However, it is widely understood that the severity of those constraints is primarily size dependent, implying that the smaller a firm the tougher are the credit conditions it faces. Consequently, the more severe credit conditions faced by medium firms compared to small and large firms during economic downturns, presents a puzzle. I explain this through the way banks perform their lending activities to firms, which is explained in the following section.

1.2.2 Bank Lending Practices

In the literature that studies lender-borrower relationships, banks are mostly treated as a single entity without a thorough investigation of their internal structures and characteristics. Conversely, the majority of commercial banks follow a segment approach, where they split their lending activities to firms between two main divisions. Each of those divisions deals with firms of a certain size category. Usually firms’ turnover or net worth is used as a proxy by banks to determine in which division a firm will be dealt with. Here and after, I call those two bank divisions, SME banking and corporate banking. SME banking refers to financial services that are primarily catered for firms that fall below a certain threshold of firm size, while corporate banking provides services to firms that are considered large. This division of lending activities extends to different resource allocation, product differentiation and even delivery and client evaluation techniques carried out by banks (Beck (2013)).

In what follows I present the characteristics of those two main bank divisions, where I depend on various literature sources. However, I particularly make extensive use of information obtained from Albareto et al. (2011) who conducted a survey on behalf of the Bank of Italy on banks conducting lending activities to firms. Their sample covers 82.2 per cent of outstanding loans made by Italian banks to non-financial firms in 2007. The survey reports the differences in banking practices
between SME and corporate banking. In assessing their business customers, banks use a mixture of quantitative techniques and soft information on their clients to determine customers’ credit worthiness. However, the balance between quantitative methods and soft information gathering will differ between banks departments depending on the firm segment they deal with.

In corporate banking divisions, banks tend to employ more qualified staff and dedicate more time and effort to study the quality of large firms’ projects and financials. Moreover, rigid credit scoring techniques and statistical models are given less relative importance. On the other hand, qualitative information especially on the borrower’s organisational structure, management credentials, quality of investment projects and the extensiveness of the borrower’s banking relations with other competing banks, are given more importance. This allows for a judgmental decision making in extending loans (Albareto et al. (2011)). Additionally, when credit facilities are approved by banks they are usually granted with little collateral requirements, when compared to credit facilities extended to small firms. This is due to a number of reasons; the bargaining power of large firms that have larger network of banking relationships, allows them to substitute between lenders. Also, unlike small firms, large firms can get access to the commercial paper market, which remains largely a restricted domain for them. Furthermore, the in-depth analysis that banks conduct on large firms, allows them to identify good projects, and thus reducing the risk of defaults (Cressy and Olofsson (1997)).

The aforementioned is consistent with some of the relationship lending theories predictions, although those theories have not specifically approached the topic from the angle of bank lending practices and their division. The premises on which those theories depend is the existence of asymmetric information between borrowers and lenders, which results in heterogenous treatments that produce two main predictions. The first is that the cost to borrowers of switching lenders, declines as the borrowers’ transparency improves. Transparency should improve with size, as large firms generally have better prepared financial data and better reputational record. Second, the cost of asymmetric information will depend negatively on the borrower’s size, if the bigger is the loan size, the lower is the per unit cost of monitoring (Chodorow-Reich (2014)). Furthermore, Bester (1985) and Villas-Boas and Schmidt-Mohr (1999) build theoretical models where banks can screen borrowers. In their models, under asymmetric information, credit screening becomes a substitute for
credit rationing, where rationing occurs if some borrowers cannot obtain loans although they accept higher collaterals and borrowing costs.

In SME banking, less effort and resources are dedicated to study the quality of firms’ projects. This is due to a number of reasons, which include the large number of potential clients. Although, the total number of potential SME customers will be more than corporate customers, banks make less profit per SME banking relationship. Moreover, technological progress which significantly helped in reducing the cost of processing data, has led banks to depend more on credit scoring models and statistical techniques when selecting borrowers within the large segment of SME clients. This in return has allowed banks to direct more resources to auditing and screening large business borrowers (Albareto et al. (2011)). The decreasing dependence on soft information in SME banking is also due to the amount of available information on those firms. The availability and quality of information is strongly linked with firm size. Pertersen and Rajan (1994) have used data from the American National Survey of Small Business Finances and found that small firms are more likely to face credit rationing. The lack of sufficient information means that lenders are less able to screen out potential defaulting SME borrowers. Furthermore, in a survey on US banks, Cowan and Cowan (2006) found that the less dependence on soft information when considering SME customers, meant that banks depended more on traditional creditworthiness criterions such as the availability of collaterals. Credit rationing due to lack of information has been addressed in the theoretical literature. For example, Jaffee and Russell (1976) built a theoretical model in which lenders ration extended credit when they cannot distinguish the quality of borrowers. A similar outcome will arise in the model presented in this chapter, where some borrowers may not always be able to obtain their optimal borrowing demands due to uncertainty about the quality of their projects.

Binks and Ennew (1997), used data on six thousand UK small businesses to study the practices followed by banks in dealing with their SME customers. Banks use a combination of income and capital gearing approaches in evaluating loan applications, though the balance between both approaches vary with firm size. The income approach is preferable to the capital gearing approach, since it evaluates loan applications from a cash flow to loan size view instead of collateral to loan size. The earlier view depends on evaluating firms’ current and expected future performance, and hence their ability to repay debt. However, this approach requires banks to have
an in-depth understanding of a firm’s business and the market it operates in, as well as the provision of such information and financial data by the borrowing firm. This is less likely in the case of SMEs. Additionally, it is also the case that such evaluations are too costly for banks to carry, and in the case of SMEs will render unprofitable given the loans sizes. Unlike equity finance where equity investors share both the up and downside, banks only share the risk of debt default, which in turn further reduces banks’ incentive to follow the income evaluation approach when dealing with SME customers. Furthermore, banks also have less incentive for providing more flexible treatments, since SMEs are less likely to substitute between lenders. In a research conducted by the UK Competition and Market Authority (CMA (2015)), it was found that for most small business, once a business banking account have been established with a provider, there was little evidence of those businesses switching their bank accounts elsewhere. In another report (CMA (2016)), it was found that only 4 per cent of SME customers substitute to other banking providers in any year. This is usually due to SMEs being restricted by their geographical location as well as the search costs associated with finding alternative banking services. Therefore, banks will depend on the traditional capital gearing method, asking for high collaterals against credit provided. Both Fort et al. (2013) and Sahin et al. (2011) have found evidence that US states which suffered more severe fall in house prices were also the ones with greater decline in small businesses borrowing. This is due to the small business owners who used their homes as a collateral for obtaining credit to finance their businesses’ operations.

The phenomena of dividing banking activities for firms between two departments having the aforementioned characteristics, are more pronounced in larger banks. In the survey carried by Albareto et al. (2011), 70 per cent of large banks where internally organised in the said structure. They also applied more rigid quantitative methods and capital gearing approaches, in evaluating SME loan applications. It is worth noting that their survey did not cover subsidiaries of foreign banks with presence in Italy. In other words, such phenomenon is expected to be stronger in those banks, as they are international and typically larger. Indeed, Beck et al. (2011) confirm this hypothesis, using data on 91 banks across 45 countries. Furthermore, Small banks are less centralized and more efficient in collecting and processing soft information when dealing with small businesses. This is due to the geographical proximity between firms and the decision maker in a bank. This is rarely the case in large banks, where
final decisions to grant loans are not made in branch offices (Berger and Udell (1995); Stein (2002)). Both Sapienza (2002) and Berger et al. (2005) have used US data and obtained results that are consistent with this prediction. Canales and Nanda (2012) used Mexican data and found that less centralised lenders have provided relatively greater loan amounts to small firms. Furthermore, Presbitero et al. (2014) have used a sample of Italian SMEs and found that the effect of the financial crisis was stronger in regions that had a larger share of non-local and larger banks, where this organisation phenomena is more pronounced.

Bank concentration and centralisation is stronger in the UK and US. The deregulations of the 1970s and 1980s in the US, have resulted in the merger of many small banks into larger nationwide banks (Beck (2013)). In the UK the largest and oldest banks, which all have strong international presence, hold the majority of business accounts in the retail market. This high concentration by few very large banks, means that they do not have to compete hard to retain their SME customers (CMA (2016)). This had a profound effect on the ability of SMEs to obtain loans with more flexible terms and less rigid credit requirements. Indeed, as David Young, who was the UK Prime Minister’s adviser on enterprise and small business in 2014, puts it: “The real part of the problem is that banks’ expenses are too high for small loans. If you are running a bank and you have got a choice between doing a £500,000 loan and £25,000 loan, the cost of doing both is the same so they tend to make the larger loan. I do not think it is deliberate. It is a fact of life”.6 Although bank lending practices play a central role in the model developed in this chapter, I follow a ‘reduced form’ approach, which abstracts from modelling the financial intermediation process. Thus the lenders and the borrowers interact directly, with the lenders following the aforementioned credit-screening practices in their lending decisions.

1.2.3 Credit Discrimination and Financial Frictions in Macroeconomics

The large macroeconomic literature on financial frictions deals with informational frictions between lenders and borrowers. Such informational frictions will result in

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problems of adverse selection or moral hazard, which in turn will lead to feedback and propagation channels that amplify initial real or monetary shocks. Bernanke et al. (1996 and 1999) have named the changes in credit market conditions, caused by informational frictions which propagate initial shocks, as the financial accelerator. The amplification of initial shocks will cause more severe business cycles by affecting investment and production on several rounds. The numerous models in the literature differ greatly in their predictions and the ingredients that they use. This is due to the huge disagreement about which informational frictions are of paramount relevance and soundness. However, the models in the literature are divided between two main feedback and propagation channels; the net worth (balance sheet) channel and the bank-equity (bank lending) channel. In what follows, I present some models that employ those two feedback and propagation channels, then I discuss credit discrimination in theoretical literature more generally.

Models that employ the net worth channel are more established and they address the demand side of credit markets. Essentially, those models depend on high state verification costs, to be incurred by the lenders in order to verify the state of the borrowers. Accordingly, lenders are discouraged from performing the required audits, and instead will either ration credit or require firms to pledge collaterals against credit extended to them. However, the limited wealth of firms will create contracting problems, which propagate adverse shocks to the economy (Bernanke and Gertler (1989); Bernanke et al. (1999); Carlstrom and Fuerst (1997)).

Kiyotaki and Moore (1997) built a dynamic model in which borrowers’ net worth, represents a binding constraint on their ability to obtain credit. Moreover, the model incorporates endogenous procyclical changes in borrowers’ assets values, thus affecting the amount of collateral they can pledge with the lenders, and consequently affecting their ability to borrow and invest. Hart and Moore (1994) develop a model in which borrowers use their assets as both factors of production and collaterals in order to obtain credit. In their model, adverse economic shocks lower the value of those assets, which in turn

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7 Those models are also called the internal equity models of credit frictions.
8 To illustrate, starting from an adverse aggregate shock (for example, productivity, demand, etc), this shock will result in a decrease in firms’ net worth. Accordingly, lenders will be less willing to lend, and thus aggregate investment and production drops. Consequently, firms’ net worth will decrease even further, resulting in a further round of the same chain and so on. By symmetry, similar reasoning will apply when starting from a good aggregate shock. The key aspect of the informational frictions is that good investment projects are not funded, purely because of low net worth. In other words what determines if projects will be financed, is not whether projects have positive net present value (NPV) or not, but rather if there is sufficient net worth to pledge as a collateral against obtaining credit.
reduce credit extended by lenders. Consequently, the reduction in spending by borrowers will deteriorate the economy further resulting in a further lower value of their assets. Other papers have developed static models with adverse selection that focus on the properties of contracts between the lender and the borrower. The degree of access to credit and its effect on contracting efficiency and factor utilisation are presented in Farmer (1988a and 1988b). Also, Diamond (1984) and Holmstrom and Tirole (1997) have developed microeconomic models that link the borrower’s wealth level to the intermediation process and properties of the contracts they can obtain.

Models that employ the Bank equity channel are less established and they address the supply side of credit markets. However, they also present mechanisms in which macroeconomic shocks are amplified. Den Haan et al. (2003), develop a dynamic equilibrium model in which households’ loanable funds are channelled to borrowers through intermediaries, via long term banking relationships. However, intermediaries will need to have a minimum amount of funds in order to induce firms to produce and not choose their outside option. Adverse shocks which make households invest less in the credit market, will result in some of the relationships to severe since some of the intermediaries will end up with funds below the required threshold that is necessary to induce borrowers to produce. With less existing relationships in the credit market, returns on household investment will drop. This in turn induces households to invest even less, resulting in more relationships breaking up. If the initial shock is sufficiently large, then it can lead to a complete collapse of the credit market by destroying enough number of credit relationships.

A number of empirical studies have investigated the variation between large and small firms’ responses to adverse economic shocks, through the financial frictions channels. Gertler and Gilchrist (1994) have used US data on manufacturing firms to study the response to monetary contractions. They found that small firms have exhibited significant disproportionate response compared to large firms. This has

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9 Those models are also called the liquidity-flows models.

10 To illustrate, according to the Basel accords which regulate banks’ lending activities, bank equity constraints bank ability to issue or renew loans. Therefore, starting from an adverse shock, such as some firms defaulting on their loans, will result in a decrease in banks’ equity. Accordingly, banks will supply less loans, preventing firms from obtaining sufficient financing. Consequently, firms scale back their production plans, which will result in the economy deteriorating even further. This in turn will result in more firms defaulting on their debt obligations, triggering a further round of the same chain and so on. By symmetry, similar reasoning will apply when starting from a good aggregate shock that increases banks’ equity. Again, in a completely frictionless world, all that matters is that firms’ projects have positive NPV for banks to be able to extend credit to them.
mainly occurred through a decline in small firms’ demand for inventories due to the financial limitations they face. Crouzet and Mehrotra (2017) have also used data on manufacturing firms from the US Census Bureau, and they found that small firms’ turnover, inventories and investment were more procyclical than large firms. Moreover, they also found that independent of firm size, indebted firms were more sensitive to adverse shocks. This led them to conclude that the excess procyclicality of small firms is due to the tougher credit conditions they face. On the contrary, Kudlyak and Scanchez (2017) findings challenge the predictions of the financial frictions theoretical literature. They found that large firms turnover and short-term debt have contracted proportionately more than small firms during the recent recession.\footnote{Their dataset covered the period between 1958Q4 and 2014Q2. The data they used is aggregated into eight asset classes, with those classes remaining constant through the entire sample. However, they define small firms to be the ones below the twenty-fifth percentile of firms’ turnover distribution in each period. Thus, as the distribution of firms’ sales continues shifting to the right with time, the asset classes in which data is aggregated into, remains constant. For example, in the early years of their sample, firms with total assets of $5 million and under were small firms, while in the end of their sample (precisely by the start of the recession and onward) firms with total assets of $1000 million and under were considered to be small firms. I find this very implausible, nevertheless, I later show that my model can generate such results if I follow their definition of small firms. The implication of their definition of small firms on my model, is that medium firms will be grouped with large firms, where medium firms exhibit the largest proportionate contraction in debt and output, following adverse shocks.}

Some of the theoretical models in the literature, especially within the net worth channel, have tried to explain this variation in the response between small and large firms, through firms’ credit having varying vulnerability to the adverse shocks affecting the economy. Small firms in general are not just a scaled down version of large firms. Their financial characteristics differ from large firms. For example, they have lower ratios of fixed to total assets, higher liabilities to total assets ratios, and they are less productive than large firms due to economies of scale (Cressy and Olofsson (1997)). To the best of my knowledge, all the theoretical models generate discriminating credit treatments and contracts, by imposing exogenous heterogeneity between small and large firms in one or more dimensions, other than their initial condition, which is their size. Firms can be of multiple types in those models. Types can take the form of differing abilities in repaying loans with high and low risk borrowers, such as in Bester (1985), Besanko and Thakor (1987) and Broecker (1990). Additionally, types may reflect different production technologies, where one type of firms is more productive than the other type, and enjoys higher success rates, such as...
Moreover, some models impose that firms have access to different investment projects, that are either of different rates of success or returns for different firms, such as in Caminal and Matutes (1997) and Villas-Boas and Schmidt-Mohr (1999). Other models depend on lenders having different market power to generate heterogenous credit treatments, such as in Petersen and Rajan (1995). Finally, some models abstract from imposing different characteristics between firms that lead to credit discrimination, and simply assume the heterogenous credit treatment exogenously, such as in Bernanke et al. (1999).

1.2.4 The Role of Investment Specific Shocks

Investment specific technology (IST) shocks play an important role in the model presented later in this chapter. In this section of the literature review, I provide a brief overview of the theoretical and empirical literature on this type of shocks. Unlike neutral technology (total factor productivity) shocks, which affect the production of all goods homogeneously, investment shocks impact only the production of investment and capital goods. Technological advancements such as improved transportation, telecommunication and equipment, have made the production of other investment goods relatively cheaper over time, which in turn caused higher accumulation of capital. Therefore, the trend has been a reduction in the relative price at which those goods are sold at (Fisher (2006)). The literature on investment shocks considers two types of those shocks; investment specific technology (IST) shocks and marginal efficiency of investment (MEI) shocks. IST shocks affect the rate at which investment goods are produced from consumption goods, while MEI shocks affect the rate at which investment goods can be transformed to productive capital (Justiniano et al. (2011)). In my model, I consider IST shocks and implicitly assume that there are no shocks to the marginal efficiency of investment.

The theoretical literature on IST shocks argues that they represent an important alternative to neutral productivity shocks as a source of business cycles in a general equilibrium setup. They have a key role in explaining short run macroeconomic fluctuations, as well as the long-term movements of some key indicators such as labour productivity in growth models (check Greenwood et al. (1988); Greenwood et al. (1997); Fisher (1997); Christiano and Fisher (1998); Campbell (1998)). More recent studies include Greenwood et al. (2000) who develop a variation of the model in
Greenwood et al. (1997). They have used US data on the relative price of new equipment to calibrate their model and found that IST shocks can account for 30 per cent of the fluctuations in output. This is very significant when considering that investment in new equipment represent only 7 per cent of the US GNP. Moreover, both Smets and Wouters (2007) and Justiniano et al. (2010) have developed DSGE models, which are extended versions of the New Neoclassical Synthesis model and use Bayesian estimation. In both studies, in the short run, IST shocks explain a significant proportion of the variation in labour hours and output, and their estimates are comparable to the structural vector autoregression (SVAR) estimations in Fisher (2006).

Fisher (2006) adopts the same competitive growth model as in Greenwood et al. (1997), however, he has expressed it with a social planner problem and simplified by incorporating one instead of two capital goods. The simplification did not affect the analysis. When compared to the neutral productivity shock, he found that most of the variation in short run labour hours and output are explained by IST shocks. In Fisher (2006) the accumulation of capital in the model is given by:

\[
\begin{align*}
    k_{t+1} &\leq (1 - \delta) k_t + v_t i_t \\
    v_t &= \exp(\theta + \varepsilon_{vt}) v_{t+1}, \theta \geq 0
\end{align*}
\]

where \(k\) is the level of capital from a period to another as indicated by the time subscript, \(i_t\) is the amount of investment in capital, \(v_t\) is the level of the investment specific shock, and \(\varepsilon_{vt}\) denotes the shock to the investment-specific technology at time \(t\). Therefore, in a competitive economy, IST is equivalent to the inverse of the price of investment relative to the consumption good. I follow the specification given in equation (1.1) to introduce IST shocks later in my model. The models in the literature use different channels, through which shocks transmit and propagate to create large variations in the macroeconomic variables.

Fisher (2006) extends on his work by conducting a SVAR estimation and use the theoretical model to derive the identifying assumptions. He uses US data and splits the sample at 1982, due to the significant changes in the conduct of monetary policy,

\[\text{12 The welfare theorem holds in the competitive model, hence why Fisher (2006) has expressed it with a social planner problem.}\]
regulations and the rate of decline in real equipment price. Moreover, three econometrical specifications were considered and in the most preferred specification, he found that neutral productivity shocks and IST shocks account for 73 per cent and 44 per cent of the variation in hours and output, respectively before 1982. Furthermore, they have accounted for 38 per cent and 80 per cent of the variation in the same variables, post 1982. He has also found that IST shocks were behind most of the variation in labour hours and output. All the previously mentioned studies are based on the US. To the best of my knowledge, Bakhshi and Larsen (2001) is the only paper that is based on the UK. They have used a DSGE model for the UK, where the investment-specific technological change is limited to the advancements in the information and communications technology (ICT) sector. They arrived at similar qualitative conclusions as the previously mentioned studies.

In the model presented in the next section, I have used both investment specific and neutral productivity shocks in a financial frictions framework. I study the effects of both shocks within the context of bank lending practices, and how this map to variations in firms’ investment responses depending on their sizes and access to credit.

1.3 The Model

The model seeks to answer the following questions: (i) why banks offer different credit treatments to firms of different sizes but share the same production technology, (ii) why medium sized firms have exhibited the largest percentage decline in investment in the 2008 recession in the UK, followed by small firms and then large firms as presented in Crawford et al. (2013), and (iii) why large firms have exhibited larger decline in their sales (output) and debt relative to small firms in the 2008 recession in the US as presented by Kudlyak and Scanchez (2017).

1.3.1 The Model’s Environment

I consider a one period economy in which there is one good that may be used for consumption, investment and contracting, and its price is normalized to one. The consumption good can be transferred to capital at a rate $q$, where $q$ is a technology
parameter for transferring the final good to capital. Thus, $1/q$ is the number of consumption good units that must be exchanged to get a unit of the capital good. The agents in this economy are: (i) a unit mass of identical households, indexed by $n$, that I also call the lenders; and (ii) a unit mass of entrepreneurs, indexed by $j$, whom I also call the borrowers. In other words, since there is a unit mass of each agent, a one to one matching is assumed at the start of the economy, which eliminates the search problem and hence I can ignore the agents’ indices. At the start of the economy, each household (lender) is endowed with the production factor $E$. The household’s endowment is sufficiently large, such that the resource constraint in lending to the entrepreneur will not bind. Let us suppose that he receives an endowment of $E$ equals to $X$, where $X$ is a constant.

Each household maximizes $u(c)$, where $u(c) = c$, thus being risk neutral. The household owns a production technology $g(E)$, that depends only on the production factor $E$, and yields one to one in terms of the final consumption good. Each household can also lend part of his endowment of $E$ to the entrepreneur he is matched with, where the entrepreneur uses it as an input in one of her two production technologies. When the household lends, he is payed back in terms of the final consumption good. Lending takes the form of a contract with a Nash Bargaining solution. Finally, I assume that the production factor $E$ fully depreciates when used in production, regardless of the production technology it is used in.

Each entrepreneur also maximizes $u(c)$, where $u(c) = c$, thus being risk neutral. The assumption that both agents are risk neutral simplifies the analysis and ensures that the equilibrium is tractable. Entrepreneurs are endowed with physical capital $k$ which is fully observable, however, their endowments of capital differ at the start of the economy. Accordingly, the aggregate capital stock in the economy is given by:

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13 I follow Fisher (2006) in the way he has included the investment specific shocks in the RBC model that he used to derive the identifying assumptions used in the empirical estimation. However, unlike Fisher (2006) where those shocks are persistent, I assume those shocks to be a white noise since the model presented here is static.

14 Therefore, as in Fisher (2006), in a competitive economy $1/q$ will be the real price of the capital good.

15 I abstract from introducing a banking sector and a financial intermediation process in the model. Instead, I follow a reduced form approach, where I assume that the lenders and the borrowers are matched directly, with the lenders performing the credit screening that is typically performed by the banking sector.

16 It can be thought of as a raw material or a working capital that enters into the production of the final good.
Each entrepreneur owns a firm with two production technologies, both producing the final consumption good, albeit at different productivities. The first production technology is \( h(k) \), where \( h(k) = (1/q)k \). This production technology depends only on capital and converts the capital good to the final consumption good. Capital also fully depreciates when used in this production technology. The second production technology is \( f(k, x) = zk^\alpha x^{1-\alpha} \), where \( \alpha \in (0,1) \) and \( z \) is the standard neutral productivity shock.\(^{17}\) The latter is the production technology used when lending occurs. Production using the second technology requires two inputs; the entrepreneur’s capital \( k \), and \( x \) which is the borrowed amount from the household endowment of the production factor \( E \). In other words, both agents can stay in autarky, or engage in lending and borrowing, where the entrepreneur uses her second production technology \( f(k, x) \). I follow Quadrini (2011) in assuming multiple sectors, where the final good can be produced \( g(E) \), \( h(k) \) and \( f(k, x) \), although at differing productivities. Finally, I assume that a fraction \( \delta \in (0,1) \) of capital \( k \), depreciates when used in the second production technology \( f(k, x) \).

My assumption that borrowing is used for working capital needs rather than investment in physical capital is supported by a number of empirical studies. Kashyap et al. (1994) have studied firms’ inventories behaviours using micro US data and found that bank-leveraged firms have significantly cut their inventories relative to non-bank-dependent firms in response to adverse macroeconomic conditions. Moreover, Paravisini et al. (2015) have used Peruvian data to study the effect of credit shocks on firms’ export behaviour during the recent financial crisis. They found that bank-dependent firms have reduced their exports by more relative to non-bank-dependent firms, where this occurred mostly through raising firms’ variable production costs rather than affecting their stock of fixed capital. Although firms use credit for a number of reasons, the primary use of short-term debt is facilitating working capital needs rather than replacing exiting physical capital or facilitating expansions (Cressy and Olofsson (1997)).

\(^{17}\) As is Fisher (2006), \( z \) can be thought of as a number of factors that affect the production possibilities.
1.3.2 The Uncertainty Structure and Contractual Friction

The uncertainty structure in the economy is as follows: (i) an aggregate neutral productivity shock, $z$, that is symmetric across all firms; (ii) an investment specific technology shock, where $q$ can possibly vary and it is also symmetric across all firms; and (iii) an idiosyncratic shock for each entrepreneur that is specific to her technology $f(k,x)$. More specifically, when using the technology $f(k,x)$, the entrepreneur’s production can either be successful, generating positive output with a prior probability $P \in [0,1]$, or unsuccessful with a prior probability $1 - P$, which generates zero output and thus wastes the borrowed amount $x$.

Accordingly, the lender-borrower relationship functions under uncertainty due to the idiosyncratic shocks. At the start of the economy, all shocks are realized. The realisations of the aggregate neutral productivity shock and the investment specific shock are observable. However, an entrepreneur will not learn the realisation of her idiosyncratic shock until she borrows and starts the production using the technology $f(k,x)$, or when being audited by the household (lender) before producing. The household has an audit technology, but he incurs a high fixed cost in using it, $A > 0$. Using audit reveals the realisation of the entrepreneur’s idiosyncratic shock and it becomes a common knowledge. When audit is used by the lender and the entrepreneur’s production project is found to be successful, the household lends without imposing collateral requirements. This is because the entrepreneur is able to pay back her debt using output from the successful production. However, if audit reveals that the entrepreneur’s project is unsuccessful, then no loan will be extended by the household. Accordingly, both of the matched household and entrepreneur, will stay in autarky and use their production technologies $g(E)$ and $h(k)$, respectively.

On the other hand, if the lender chooses not to audit, then he still lends to the entrepreneur, yet he fully secures the loan to ensure the repayment of $x$, by using the entrepreneur’s undepreciated capital stock as a collateral. In other words, an entrepreneur who is not audited can borrow a maximum amount $x \leq (1/q)(1 - \delta)k$. Consequently, a borrower with a small capital stock $k$, may not be able to borrow her optimal demand, especially when $q$ is very high. This is in line with the discussion presented earlier in the literature review. Since banks do not incur the high audit costs when dealing with small firms, uncertainty will be present on whether those firms will
be able to satisfy their debt obligations or not. Accordingly, banks require firms to pledge high collaterals as a precaution against defaulting debt.

As will be shown later in the model’s equilibrium, a lender will only choose to pay the high fixed audit cost $A$, if the entrepreneur’s capital stock $k \geq \hat{k}$, where $\hat{k}$ is a capital threshold for auditing that arise through the lender’s optimisation problem.

### 1.3.3 The Lender-Borrower Relationship

When lending occurs, the matched household and entrepreneur engage in production and share output. According to the assumed structure of the credit contracts, the borrowed amount $x$, must be paid back regardless of the production process outcome. Moreover, the division of output between the lender and borrower is agreed through a Nash Bargaining solution. I let $\gamma \in [0,1]$ be the lender’s bargaining weight, while $1 - \gamma$ is the entrepreneur’s bargaining weight. Nash Bargaining is widely used in macro-labour economics literature. It is also used in macroeconomic financial frictions literature in setting credit contracts (check Den Haan et al. (2003)).

### 1.3.4 Timing within the Period

Although a period is expressed as a single point in time, I add a timing description within the period to facilitate understanding the model. The timing within a period is in the following order: (i) aggregate and idiosyncratic shocks are realized, however, the realisation of the idiosyncratic shock is unobservable; (ii) the household forms the decision whether to lend the entrepreneur he is matched with or not; (iii) if the household decides to lend, then he also decides whether to audit the entrepreneur or not; (iv) agents agree the division of output through a Nash Bargaining solution; (v) agents form their consumption plans, more specifically, the entrepreneur declares her demand for $x$ depending on whether she is audited or not; (vi) the outcome of the auditing process is declared for the audited entrepreneur, more precisely, agents find out whether the realisation of their idiosyncratic shock is 0 or 1; (vii) credit is extended to the unaudited entrepreneur and to the successful audited entrepreneur; (viii) production starts. This is when the unaudited entrepreneur will learn the realisation

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18 I have attempted to address the research questions in a general equilibrium set up, however, an equilibrium does not always exist. A detailed discussion of those attempts is presented in the appendix, section 1.8.1.
of her idiosyncratic shock; (ix) output from production is realized and the division of output takes place as agreed;\(^{19}\) (x) agents consume.

### 1.4 The Equilibrium

I now solve for the equilibrium of the model. This consists of the Nash Bargaining, the entrepreneur’s problem, the household’s problem and obtaining the capital threshold for auditing, \(\bar{k}\). I also discuss the properties of this equilibrium by conducting comparative statics, and I discuss other conditions that are necessary for the equilibrium. In all what follows below, the budget constraints are expressed in terms of the consumption good.

#### 1.4.1 Nash Bargaining

The lender and borrower negotiate and agree the credit contract. Credit contracts are assumed to be written for one period only. The division of the match surplus is agreed through a Nash Bargaining solution, where \(zk^\alpha x^{1-\alpha}\) is the output of production that the lender and borrower bargain over. As mentioned earlier, \(\gamma\) and \(1 - \gamma\) are the lender’s and borrower’s bargaining weights, respectively. I let \(w\) be the payoff to the lender, and therefore \(zk^\alpha x^{1-\alpha} - w\) is the entrepreneur’s payoff. Moreover, \(x\) and \(k/q\) are the lender’s and borrower’s outside option, respectively. It is the amount of the consumption good that they get when they choose to stay in autarky, using the production technologies \(g(E)\) and \(h(k)\). Accordingly, the surplus of the match is \(zk^\alpha x^{1-\alpha} - x - (k/q)\). The Nash Bargaining maximises the weighted combination of the lender’s and borrower’s shares:

\[
\max_w (w - x)^\gamma (zk^\alpha x^{1-\alpha} - w - (k/q))^{1-\gamma} \quad (1.3)
\]

I take logs and then the first-order condition. Thus, the lender’s share from the production surplus is given by:

\[
w = \gamma(zk^\alpha x^{1-\alpha} - x - (k/q)) + x \quad (1.4)
\]

\(^{19}\) i.e. contracts are enforced
while the entrepreneur’s share is given by:

\[ zk^\alpha x^{1-\alpha} - \gamma (zk^\alpha x^{1-\alpha} - x - (k/q)) - x \]  

\[ \text{(1.5)} \]

### 1.4.2 The Entrepreneur’s Problem

The entrepreneur maximizes \( u(c) = c \). Thus, the entrepreneur’s problem is to maximise her profits, which are given in terms of the final consumption good \( c \). Moreover, since an entrepreneur can be subjected to two different credit treatments, depending on whether she is being audited or not, there will be two versions of the optimisation problem.

#### 1.4.2.1 The Audit Case

The entrepreneur knows her share from the production surplus. I use the Nash bargaining solution as given in equation (1.5) to substitute into the optimisation problem. The entrepreneur maximises her share from the production surplus, and thus her profits, by choosing her optimal demand for \( x \):

\[
\max_x \pi = P \left( zk^\alpha x^{1-\alpha} - \gamma \left( zk^\alpha x^{1-\alpha} - x - \frac{k}{q} \right) - x + \frac{(1 - \delta)k}{q} \right) + (1 - P) \frac{k}{q} 
\]

\[ \text{(1.6)} \]

To illustrate more, the audited entrepreneur faces two prospects in the audit process, either her project is found to be successful or not. Hence, when her project is found to be unsuccessful with probability \( 1 - P \), she will use her entire capital endowment to produce the consumption good using the technology \( h(k) \). Furthermore, since only a fraction \( \delta \) of capital depreciates when using the technology \( f(k, x) \), she also use her undepreciated capital \( (1 - \delta)k \) to produce the consumption good using the technology \( h(k) \), when her project is found to be successful under audit. Taking the first order condition and solving for the optimal demand for \( x \) under audit:\(^{20}\)

---

\(^{20}\) The function is concave; thus, the second order condition is satisfied.
\[ x^*_N = \left( (1 - \alpha)z \right)^\frac{1}{\bar{\alpha}} k \]  

\[ (1.7) \]

1.4.2.2 The No Audit Case

The unaudited entrepreneur does not know the realisation of her idiosyncratic shock. Nevertheless, the lender still lends but uses the entrepreneur’s undepreciated capital as a collateral, to ensure the repayment of \( x \) in case the realisation of the idiosyncratic shock is equal to 0. The entrepreneur takes this uncertainty into account in her optimisation problem:

\[
\max_x \pi = P \left( zk^\alpha x^{1-\alpha} - \gamma \left( zk^\alpha x^{1-\alpha} - x - \frac{k}{q} \right) - x + \frac{(1 - \delta)k}{q} \right) \\
+ (1 - P) \left( -x + \frac{(1 - \delta)k}{q} \right) 
\]

\[ (1.8) \]

To illustrate, if the entrepreneur finds her production under the technology \( f(k, x) \) to be unsuccessful, she will use her undepreciated capital \( (1 - \delta)k \) to produce the consumption good using the technology \( h(k) \). However, she also needs to pay back the borrowed amount \( x \). Taking the first order condition and solving for the optimal demand for \( x \):

\[
x^*_N = \left( \frac{(1 - \alpha)Pz(1 - \gamma)}{1 - P\gamma} \right)^\frac{1}{\bar{\alpha}} k 
\]

\[ (1.9) \]

where the subscript \( NA \) indicates the optimal demand under no audit.

1.4.2.3 Non-Binding vs Binding Borrowing Constraint for The Unaudited Entrepreneur

Since the unaudited entrepreneur can borrow a maximum amount \( x \leq \frac{1}{q}(1 - \delta)k \), an entrepreneur with a small capital stock \( k \), may not be able to borrow her optimal demand. This is specifically when \( q \) is very high. To find this threshold level of \( q \), I compare the optimal demand \( x^*_N \) and the maximum borrowing amount \( (1/q)(1 - \delta)k \):

---

21 The function is concave; thus, the second order condition is satisfied.
I solve the above equation for $q$ and obtain:

$$
\bar{q} = (1 - \delta) \left( \frac{(1 - \alpha)Pz(1 - \gamma)}{1 - Py} \right)^{\frac{1}{\alpha}}
$$  (1.10)

where $\bar{q}$ is the value of $q$ at which the unaudited entrepreneur’s borrowing constraint binds.

### 1.4.3 The Household’s Problem

The household’s utility maximisation problem consists of two problems. The first problem is whether to lend or not. To solve this, I compare the household’s utility under autarky versus the case of lending. The second problem is when lending is found to be optimal compared to autarky, then should the household audit the borrowing entrepreneur or not. Accordingly, I obtain the household’s indirect utility for all the said cases and compare between them.

#### 1.4.3.1 Autarky

When the household chooses not to lend, then he uses all of his endowment of the production factor $E$, in producing the consumption good using the production technology $g(E)$. Therefore, the household utility is given by:

$$
u(c) = X
$$  (1.11)

#### 1.4.3.2 Lending with Audit

This is the case where the household chooses to incur the audit cost $A$ to reveal the realisation of the borrower’s idiosyncratic shock. When audit is used, then with prior probability $P$ the entrepreneur’s production project is found to be successful, and
hence lending occurs. When lending occurs, the household lends $x$, which is part of his endowment of the production factor $E$, and uses the remaining of it after deducting the audit cost $X - A - x$, in the other production technology that he owns, $g(E)$. Otherwise, with prior probability $1 - P$ the entrepreneur’s project is found to be unsuccessful and both agents stay in autarky. The household uses his endowment $X - A$ to produce the consumption good using the technology $g(E)$. Accordingly, the household’s expected utility when using audit is given by:

$$EU_H^U = P(X - A - x + w) + (1 - P)(X - A)$$

(1.12)

where the superscript $H$ indicates the household and the subscript $A$ indicates the case where audit is used. I substitute for $w$ and $x$ using the equations (1.4) and (1.7), respectively and obtain the indirect utility under audit:

$$EU_H^U = X - A + kPy \left( (z)\frac{1}{\alpha} \left( (1 - \alpha)^{\frac{1-\alpha}{\alpha}} - (1 - \alpha)\frac{1}{\alpha} \right) - (1/q) \right)$$

(1.13)

1.4.3.3 Lending with No Audit

When the lender does not use the audit technology, he will still lend to the entrepreneur. However, he will secure the loan to ensure the repayment of $x$ by using the entrepreneur’s undepreciated capital stock as collateral. The lender will be willing to lend the entrepreneur a maximum amount $x \leq (1/q)(1 - \delta)k$. Since $q$ can either be greater or less than $\bar{q}$, two cases will emerge under lending with no audit. Additionally, the entrepreneur uses the remainder of her endowment of $E$, $X - x$, to produce the consumption good using the technology $g(E)$. Accordingly, the household’s expected utility when lending under no audit is given by:

$$EU_{NA}^H = P(X - x + w) + (1 - P)(X - x + x)$$

(1.14)

When $q \leq \bar{q}$, the entrepreneur can obtain her optimal demand, $x_{NA}^\star$. I substitute for $w$ and $x$ using the equations (1.4) and (1.9), respectively. Therefore, the household’s expected utility when lending under no audit, and the entrepreneur’s borrowing constraint is non-binding, is given by:
\[ EU_{NA1}^H = X + k P \gamma \left( (z) \frac{1}{(1 - \alpha)P(1 - \gamma)} \right) \left( \frac{1 - \alpha}{1 - P \gamma} \right)^{\frac{1}{1 - P \gamma}} - \left( \frac{1 - \alpha}{1 - P \gamma} \right)^{\frac{1}{1 - P \gamma}} \left( \frac{1}{q} \right) \] (1.15)

where the subscript NA1 indicates the case where audit is not used and \( q \leq \bar{q} \). On the other hand, when \( q > \bar{q} \), the entrepreneur can borrow a maximum amount \( x = (1/q)(1 - \delta)k \), which is strictly less than \( x_{NA}^* \). I use the later expression to substitute for \( x \) and equation (1.4) to substitute for \( w \), in equation (1.14). The household’s expected utility when lending under no audit, and the entrepreneur borrowing constraint is binding, is therefore given by:

\[ EU_{NA2}^H = X + k P \gamma \left( z \left( \frac{1 - \delta}{q} \right)^{\frac{1 - \alpha}{1 - \delta}} - \frac{1 - \delta}{q} - \frac{1}{q} \right) \] (1.16)

where the subscript NA2 indicates the case where audit is not used and \( q > \bar{q} \).

### 1.4.4 Obtaining the Audit Threshold, \( \hat{k} \)

When lending without incurring the audit cost, \( A \), the household obtains \( x \) at least from the entrepreneur, by seizing the collateral. To clarify, the household’s utility when lending under no audit and the realisation of the idiosyncratic shock is 0 will be \( X \). If the realisation is equal to 1, then it will be strictly greater than \( X \), such that in expectation it is strictly greater than his utility under Autarky. However, this may not always be the case, and I need additional conditions on the model’s parameters to be satisfied, to ensure that I am not in the autarky equilibrium.\(^{22}\) Those conditions are discussed later in section (1.4.6). For now, I assume that those conditions are satisfied. Accordingly, household will never choose autarky over lending under no audit. As will be shown below, the household’s expected utility when using audit will only be greater than the case of no audit, when the entrepreneur’s capital is sufficiently large. Also, since \( q \) can either be greater or less than \( \bar{q} \), two cases will emerge.

\(^{22}\) The production technology \( f(k,x) \) must be sufficiently productive compared to the technologies \( g(E) \) and \( h(k) \), for agents to choose not to be in the autarky equilibrium.
1.4.4.1 The Non-Binding Borrowing Constraint Case

When $q \leq \bar{q}$, the unaudited entrepreneur can obtain her optimal demand, $x_{NA}^*$. The household will be indifferent between auditing and not, when:

$$EU^H_A = EU^H_{NA1}$$  \hspace{1cm} (1.17)

I use equations (1.13) and (1.15) to substitute in equation (1.17), and I solve for $k$:

$$\hat{k} = \frac{A}{\frac{1}{1 - \gamma}(\frac{1 - \alpha}{\alpha} - (1 - \alpha)\frac{1}{\gamma}) - \left(\frac{P(1 - \alpha)(1 - \gamma)}{1 - P\gamma} - \frac{(1 - \alpha)}{1 - P\gamma}\right)}$$  \hspace{1cm} (1.18)

where $\hat{k}$ is the endogenous audit threshold for the level of capital that arise through the lender’s optimisation. It will imply different credit treatments to firms depending on their level of capital stock. Accordingly, firms with a capital stock $k \geq \hat{k}$ are considered large firms by the lenders and thus will be audited, while firms with a capital stock $k < \hat{k}$ are considered small and will not be audited.

1.4.4.2 The Binding Borrowing Constraint Case

On the other hand, when $q > \bar{q}$ the unaudited entrepreneur can borrow a maximum amount $x = (1/q)(1 - \delta)k$ which is strictly less than $x_{NA}^*$. The household will be indifferent between auditing and not, when:

$$EU^H_A = EU^H_{NA2}$$  \hspace{1cm} (1.19)

I use equations (1.13) and (1.16) to substitute in equation (1.19), and I solve for $k$:

$$\hat{k} = \frac{A}{\frac{1}{1 - \gamma}(\frac{1 - \alpha}{\alpha} - (1 - \alpha)\frac{1}{\gamma}) - \left(\frac{1 - \delta}{q} - \frac{1 - \delta}{q}\right)}$$  \hspace{1cm} (1.20)

The intuition behind why audit occurs above a certain threshold of capital is as follows: For a given level of capital, the entrepreneur’s borrowing is greater under
audit compared to the case of no audit. This means that the division of output, \( w \), which is the lender’s revenue is bigger in absolute under auditing. However, the lender also incurs a fixed audit cost under auditing, while lending without audit is costless. Consequently, the wedge between the division of output that the lender gets back under audit and the no audit case (i.e. the difference in revenue between the two cases), will not be sufficient to offset the auditing cost at low levels of capital.

Nevertheless, this wedge in the lender’s revenues between the two cases is not constant in capital, unlike the auditing cost, \( A \). The entrepreneur’s borrowing demand depends positively on her capital stock, and the difference between those demands grows larger with capital, since \( (\partial x^*_A / \partial k) > (\partial x^*_N / \partial k) \). This implies that the difference in the lender’s revenue between the two cases depends positively on the entrepreneur’s capital. Accordingly, there will exist a certain level of capital, \( k^* \), where above it the wedge between the lender’s revenue under the two cases will surpass the fixed audit cost, and hence making it worthwhile to audit the entrepreneur.

### 1.4.5 Comparative Statics

In this section I look at the equilibrium properties, by conducting comparative statics on the borrowing demands given in equations (1.7) and (1.9). This is also done for the auditing thresholds that are given in equations (1.18) and (1.20) where I discuss how changes in the model’s exogenous parameters cause \( k \) to shift either to the left or right within the distribution of firms’ capital that was given in equation (1.2). When \( k \) shifts in either direction, some firms will experience changes in the credit treatment they are receiving as shown in Figure 1.3.
1.4.5.1 Optimal Borrowing Demands

When comparing $x_A^*$ and $x_{NA}^*$, I find that the entrepreneur’s borrowing demand is lower in the case where she is unaudited. This is due to the uncertainty she faces, where there is a possibility that part or all of her stock of undepreciated capital gets seized by the lender, due to a bad realisation of her idiosyncratic shock. In addition, the unaudited entrepreneur’s demand, $x_{NA}^*$, is positively related to the prior probability of the firm being successful, $P$. This is intuitive since the possibility of the unaudited entrepreneur being under a situation where all or part of her capital stock gets confiscated is less likely, she would demand more of $x$. On the contrary, $x_{NA}^*$ depends negatively on $\gamma$. The higher is $\gamma$, the lower is the unaudited entrepreneur’s share from the expected production surplus, and thus the lower is her incentive to borrow. This is due to the unaudited entrepreneur facing the uncertainty where the possibility of her undepreciated capital being seized exists. This is not the case with the audited entrepreneur, and hence why $x_A^*$ does not depend on $\gamma$.

1.4.5.2 $\hat{k}$ with Non-Binding Borrowing Constraint

As can be seen in equations (1.18) and (1.20), $\hat{k}$ is a function of exogenous economic parameters. Thus, as economic conditions change, so does $\hat{k}$. The direction in which $\hat{k}$ shifts will determine whether some firms will start getting better credit treatment or worse. The sign of some of the evaluated derivatives cannot be determined more
generally without assigning values to the economic parameters. Therefore, for those derivatives I used the same values that are used later to generate the numerical results.

When considering the effect of changes in the fixed audit cost, I find that 
\[
(\partial \hat{k}/\partial A) > 0. 
\]
This will also apply in the binding borrowing constraint case. As the audit cost increases, the difference in the lender’s revenue between the case of audit and no audit that is required to offset the fixed audit cost, increases. Since this difference in revenues between the two cases depends positively on the entrepreneur’s capital stock, the auditing threshold shifts to the right as shown in Figure 1.3. Accordingly, some of the firms which used to be audited will fall below the audit threshold, receiving a less favourable credit treatment than what they used to enjoy before the increase in A.

When \( z \) increases, both \( x_A^* \) and \( x_{NA}^* \) also increase. However, the increase in the audited entrepreneur demand is bigger, since \( (\partial x_A^*/\partial z) > (\partial x_{NA}^*/\partial z) \). This implies that an increase in \( z \) will cause the gap between the lender’s revenue between the case of audit and no audit to increase. Consequently, the difference in revenue between the two cases, that is required to offset the fixed audit cost will occur at a lower level of \( k \) than before, and hence \( \hat{k} \) will shift to the left. The difference in the lender’s revenue between the two cases will increase even further when the unaudited entrepreneur’s borrowing constraint is binding.

An increase in the entrepreneur’s prior probability of success \( P \), means that the possibility the lender’s audit will reveal a favourable realisation of the idiosyncratic shock, increases. This in turn will increase the difference between the lender’s expected revenue under audit, versus no audit, for a given level of the entrepreneur’s capital, \( k \). In other words, as \( P \) increases the difference in the expected revenue between the two cases, that is required to offset the fixed audit cost will occur at a lower level of \( k \) than before, and hence \( \hat{k} \) will shift to the left.

Increases in the lender’s bargaining power, \( \gamma \), will increase his share from the production surplus. This occurs both under the case of audit and no audit. However, his revenue from lending under audit, in absolute, will increase further than the case of no audit. To clarify, it was shown earlier that \( x_{NA}^* \) depends negatively on \( \gamma \), while \( x_A^* \) is independent of it. Therefore, as the lender’s bargaining power increases, a drop in \( x_{NA}^* \) will partly offset the increase in the share paid to the lender due to \( \gamma \) increasing. This in turn implies that the difference in lender’s revenue between the two cases, that
is required to offset the fixed audit cost will occur at a lower level of \( k \) than before, and hence \( \hat{k} \) will shift to the left.

**1.4.5.3 \( \hat{k} \) with Binding Borrowing Constraint**

In this case, the effects of the parameters \( A, z, P \) and \( \gamma \) are the same on \( \hat{k} \) as in the case where the entrepreneur’s borrowing constraint is non-binding. The parameters \( \delta \) and \( q \) do not affect \( \hat{k} \) when the entrepreneur’s borrowing constraint is non-binding, as her borrowing is not constrained by the maximum amount \( x = (1/q)(1 - \delta)k \). However, they do affect \( \hat{k} \) in this case, as shown in equation (1.20).

An increase in the depreciation rate, \( \delta \), decreases the maximum amount that the unaudited entrepreneur can borrow. This in turn increases the difference in the lender’s expected revenue between the case of audit and no audit, given the same level of \( k \). This in turn implies that the difference in revenue between the two cases, that is required to offset the fixed audit cost will occur at a lower level of \( k \) than before, and hence \( \hat{k} \) will shift to the left. This reasoning also applies when considering the effect of an increase in \( q \) on \( \hat{k} \). Table 1.1 below, summarises the effect of changes in the model’s parameters on \( \hat{k} \).

<table>
<thead>
<tr>
<th>Derivative</th>
<th>Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \partial \hat{k}/\partial A )</td>
<td>+</td>
</tr>
<tr>
<td>( \partial \hat{k}/\partial z )</td>
<td>-</td>
</tr>
<tr>
<td>( \partial \hat{k}/\partial P )</td>
<td>-</td>
</tr>
<tr>
<td>( \partial \hat{k}/\partial \gamma )</td>
<td>-</td>
</tr>
<tr>
<td>( \partial \hat{k}/\partial \delta )</td>
<td>-</td>
</tr>
<tr>
<td>( \partial \hat{k}/\partial q )</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1.1: The effects of changes in the model’s parameters on the auditing threshold, \( \hat{k} \).
1.4.6 Conditions on the Model’s Parameters

In order for the household and the entrepreneur to engage in lending and borrowing instead of choosing to stay in autarky, the production technology \( f(k, x) \) must be sufficiently productive compared to the technologies \( g(E) \) and \( h(k) \). In what follows, I discuss those required conditions for the household and the entrepreneur. When generating the numerical results, I calibrate the model to ensure that those conditions are satisfied.

Since \( q \) can either be greater or less than \( \bar{q} \), there will be a set of conditions required for each of the two cases. When \( q \leq \bar{q} \), the unaudited entrepreneur prefers borrowing and producing using the technology \( f(k, x) \), when her expected utility without being audited is greater than her utility under autarky. This will occur iff:

\[
P \left( zk^{\alpha} x^{1-\alpha} - y \left( zk^{\alpha} x^{1-\alpha} - x \frac{k}{q} \right) - x + \frac{(1 - \delta)k}{q} \right) + (1 - P) \left( -x + \frac{(1 - \delta)k}{q} \right) > \frac{k}{q}
\]

I substitute for \( x \) using equation (1.9). Thus, the condition on the parameters is:

\[
(1 - \gamma)Pz \left( \frac{(1 - \alpha)Pz(1 - \gamma)}{1 - P\gamma} \right)^{\frac{1-\alpha}{\alpha}} \quad > \quad 0 \quad (1.21)
\]

\[-(1 - P\gamma) \left( \frac{(1 - \alpha)Pz(1 - \gamma)}{1 - P\gamma} \right)^{\frac{1}{\alpha}} + P\gamma - \frac{\delta}{q} \quad > \quad 0 \quad (1.22)
\]

When \( q > \bar{q} \), I use \((1/q)(1 - \delta)k\) to substitute for \( x \) and the condition becomes:

\[
(1 - \gamma)Pz \left( \frac{1 - \delta}{q} \right)^{\frac{1-\alpha}{\alpha}} + \frac{2 P\gamma - P\gamma \delta - 1}{q} \quad > \quad 0
\]

When \( q \leq \bar{q} \), the household will prefer lending over staying in autarky when his expected utility as given in equation (1.15) is strictly greater than \( X \). I simplify, and thus the condition on the parameters is:
\[(z) \frac{1}{\alpha} \left( \frac{(1 - \alpha)P(1 - \gamma)}{1 - P\gamma} \right)^{\frac{1 - \alpha}{\alpha}} - \left( \frac{(1 - \alpha)P(1 - \gamma)}{1 - P\gamma} \right)^{\frac{1}{\alpha}} \frac{1}{q} > 0 \quad (1.23)\]

On the other hand, when \( q > \bar{q} \) then the household’s utility as given in equation (1.16) must be strictly greater than \( X \). Accordingly, the condition on the parameters is given by:
\[ z \left( \frac{1 - \delta}{q} \right)^{1-\alpha} - \frac{2 - \delta}{q} > 0 \quad (1.24)\]

### 1.5 Adding Investment to the Model

The model in its current setup, can address the first question of this research. However, it cannot provide an answer to why medium sized firms have exhibited the largest percentage decrease in investment in the 2008 recession in the UK, followed by small firms and then large firms as presented in Crawford et al. (2013). To address this question I amend the model, however, in a way that keeps it static. I have attempted to answer this question in an infinite horizon setting; however, it became clear that the question I want to address cannot be answered.23

I now amend the model, such that each entrepreneur \( j \) maximizes \( u(c) + v(i) \), where \( u(c) = \ln c \) and \( v(i) = \ln i \). The entrepreneur gets utility from consuming \( u(c) \), and I interpret \( v(i) \) as the utility she gets from bequeathing to a next generation. In other words, \( v(i) \) is a valuation of a bequest. Instead of bequeathing capital to the next generation, the entrepreneur bequests the consumption good. The next generation of entrepreneurs converts \( i \) to capital and then utilise it in production. Accordingly, I further interpret \( i \) as investment.24 I follow Galor and Zeira (1993) in the way I

---

23 This is not possible in an infinite horizon version, specially that I am assuming that firms only differ in the initial level of capital they start with. Dynamics in an infinite horizon model are studied by shocking the model in a steady state. This will not allow studying the behaviour of middle-sized firms. All firms will converge to a single steady state level of capital, regardless of the initial condition they start from. In other words, as firms converge to their steady state, a single representative firm will arise that will either be above or below \( \bar{k} \) in the steady state. The capital distribution at the start of the economy as given by equation (1.2), will collapse to a spike in the steady state. Therefore, the comparisons between firms’ responses (small, medium and large) which is the question I am seeking to answer, would not be possible.

24 Unlike Fisher (2006) where the investment specific shocks are persistent, I assume that those shocks take the form of white noise. An alternative set up where capital is bequeathed, clouds my analysis. When capital is bequeathed, I cannot isolate the adverse effect a positive shock to \( q \) would have on firms’ investment. A positive shock to \( q \), means that the amount in terms of the final consumption good that entrepreneurs can dedicate for investment declines. This is because of a decrease in the
amended the model, to allow for bequests to a next generation.

### 1.5.1 Nash Bargaining

Similar to before, the lender and borrower negotiate and agree the credit contract. Credit contracts are assumed to be written for one period only. The division of the match surplus is agreed through a Nash Bargaining solution. The solution will be as given in equations (1.4) and (1.5) previously.

### 1.5.2 The Entrepreneur’s Problem

Now the entrepreneur maximises $u(c) + v(i)$. She will maximise her profits, which are given in terms of the consumption good. Given the structure of her utility, she will divide those profits equally between consumption and investment.\(^{25}\)

#### 1.5.2.1 The Audit Case

The entrepreneur maximises her utility by choosing $c$ and $i$. Since her resources are divided equally between consumption and bequeathing, she maximises her share from the production surplus. This is done by choosing her optimal demand for $x$:

\[
\max_{c,i,x} U = \ln(c) + \ln(i) \quad (1.25)
\]

subject to

\[
c + i \leq P \left( zk^{\alpha} x^{1-\alpha} - \gamma \left( zk^{\alpha} x^{1-\alpha} - x + \frac{k}{q} \right) - x + \frac{(1-P)k}{q} \right) + \frac{k}{q} \quad (1.26)
\]

entrepreneur’s total resources in terms of the final consumption good. However, bequeathing capital means that the capital which will be passed to a future generation, is purchased in the same period of the shock occurrence. Therefore, since capital became cheaper in the period of the shock, the amount of capital bequeathed can be more than what the entrepreneur would have in the absence of the shock. The positive effect on the bequeathed capital coming from the positive shock to $q$ can partly, fully or more than fully offset the adverse effect on capital accumulation coming from the reduction in firms’ resources. I want to evaluate the change in investment (the amount spent on capital accumulation) in terms of the final consumption good, and not in terms of the capital good itself.

\(^{25}\) The entrepreneur’s utility function is a monotonic transformation of a Cobb-Douglas utility function with equal weights for both of its arguments.
I substitute for $c$ in equation (1.25), using the budget constraint (1.26). I then take the first-order conditions with respect to $x$ and $i$:26

$$\frac{dU}{dx} = 0 \Rightarrow x^*_i = \left((1 - \alpha)z\right)^{\frac{1}{\alpha}} k$$

(1.27)

$$\frac{dU}{di} = 0 \Rightarrow i = c$$

(1.28)

Using the budget constraint (1.26) to substitute for $c$ in the optimality condition obtained in (1.28):

$$i = \frac{1}{2} \left(P \left(zk^\alpha x^{1-\alpha} - \gamma \left(zk^\alpha x^{1-\alpha} - x - \frac{k}{q}\right) - x + \frac{(1 - \delta)k}{q}\right)ight.$$  
$$+ \left(1 - P\right) \frac{k}{q}\right)$$

(1.29)

Finally, using (1.27) to substitute for $x$ in (1.29), gives the optimal investment decision under audit, $i^*_A$:

$$i^*_A = \frac{1}{2} \left((1 - \gamma)P(z)^{\frac{1}{\alpha}} \left((1 - \alpha)^{\frac{1-\alpha}{\alpha}} - (1 - \alpha)^{\frac{1}{\alpha}}\right) + \frac{1 + P\gamma - P\delta}{q}\right) k$$

(1.30)

### 1.5.2.2 The No Audit Case

The unaudited entrepreneur does not know the realisation of her idiosyncratic shock. Nevertheless, the lender still lends but uses the entrepreneur’s undepreciated capital as a collateral to ensure the repayment of $x$, in case the realisation of the idiosyncratic shock is equal to 0. The entrepreneur takes this uncertainty into account in her optimisation problem:

$$\max_{c, x, i} U = u(c) + v(i)$$

subject to

26 The function is concave; thus, the second order condition is satisfied.
I substitute for $c$ in equation (1.31), using the budget constraint (1.32). I then take the first-order conditions with respect to $x$ and $i$:

\[
\frac{dU}{dx} = 0 \implies x^*_a = \left(\frac{(1 - \alpha)Pz(1 - \gamma)}{1 - Py}\right)^{1/\alpha} k
\]

\[
\frac{dU}{di} = 0 \implies i = c
\]

Using the budget constraint (1.32) to substitute for $c$ in the optimality condition obtained in (1.34):

\[
i = \frac{1}{2} \left( P \left( zk^\alpha x^{1-\alpha} - \gamma \left( zk^\alpha x^{1-\alpha} - x - \frac{k}{q} \right) - x + \frac{(1 - \delta)k}{q} \right) 
+ (1 - P) \left( -x + \frac{(1 - \delta)k}{q} \right) \right)
\]

Finally, using (1.33) to substitute for $x$ in (1.35), gives the optimal investment decision under no audit and non-binding borrowing constraint, $i^*_{NA1}$:

\[
i^*_{NA1} = \frac{1}{2} \left( (1 - \gamma)Pz \left( \frac{(1 - \alpha)Pz(1 - \gamma)}{1 - Py} \right)^{1-\alpha/\alpha} 
- (1 - Py) \left( \frac{(1 - \alpha)Pz(1 - \gamma)}{1 - Py} \right)^{1/\alpha} + \frac{1 + Py - \delta}{q} \right)
\]

\[27\text{ The function is concave; thus, the second order condition is satisfied.}\]
When $q > \bar{q}$, the entrepreneur’s borrowing constraint in binding, and I use the maximum borrowing amount $x = (1/q)(1 - \delta)k$ to substitute for $x$ in equation (1.35):

$$i_{NA2} = \frac{1}{2} \left( (1 - \gamma)Pz \left( \frac{1 - \delta}{q} \right)^{1-\alpha} + \frac{P\gamma(2 - \delta)}{q} \right) k$$

which the optimal investment decision under no audit and binding borrowing constraint.

1.5.3 The Household’s Problem

The household’s preference remained unchanged. Additionally, the entrepreneur’s optimal demand for $x$ is as given before in equations (1.7) and (1.9). Accordingly, the solution to the household’s problem is similar to what has been presented in section (1.4.3), and the audit threshold $\bar{k}$ is the same as given in section (1.4.4).

1.5.4 Conditions on the Model’s Parameters

In order for the household and the entrepreneur to engage in lending and borrowing instead of choosing to stay in autarky, the production technology $f(k, x)$ must be sufficiently productive compared to the technologies $g(E)$ and $h(k)$. The household’s and the entrepreneur’s resource constraints remained the same, and therefore the conditions given in equations (1.21) to (1.24) still apply.

1.6 Results and Discussion

As explained earlier, the lenders’ optimisation will generate an endogenous audit threshold for the level of capital, $\hat{k}$. It implies discriminating credit treatment for the firms that fall below the threshold level, compared to the ones that fall above it. Since $\hat{k}$ is a function of exogenous economic parameters, when economic conditions change, so does $\hat{k}$. The direction in which $\hat{k}$ shifts will determine whether some firms will start getting better credit treatment or worse.

In the results presented in this section, the distribution of the entrepreneurs’ net worth, which is their endowments of capital is divided into three intervals that are equal in length. The three intervals represent the three groups of firms; small, medium
and large. Moreover, I let $\hat{k}$ be at the centre of the capital distribution before the occurrence of any shock. Accordingly, with $\hat{k}$ being at the centre of the distribution, half of the medium firms will fall on the left and the other half on the right of the auditing threshold $\hat{k}$. In other words, in the absence of shocks, half of the medium firms are treated as large by lenders and thus being audited, while the other half are treated as small firms and forced to provide collaterals to obtain credit. This proposition is supported by a market research. The UK Competition and Market Authority (CMA (2015)) has carried out an investigation into SMEs’ banking relationships, where one-to-one interviews were conducted. A significant number of mid-sized firms within the sample reported being able to renegotiate banking fees, especially when a dedicated relationship manager was assigned to them. Despite firms across the sample attempting to negotiate banking terms, it was medium sized businesses in general and high turnover mid-sized firms in specific, that were able to develop a more peer-to-peer relationships.

Figure 1.4: Adverse economic shocks shifting $\hat{k}$ to the right, imply worse credit treatment to some previously audited firms.

I hypothesize that during adverse economic conditions which cause a reduction in credit extended to all firms, this threshold level of firm size shifts to the right as shown in Figure 1.4. Consequently, the firms which get affected the most (the ones that suffer the largest percentage reduction in their ability to obtain credit) following a shock, are the firms which were earlier on the right of this threshold but are now on the left of it. In other words, it is the firms that were earlier treated as large firms; being
audited and borrowing without the need for a collateral, but now are treated as small firms; not being audited and required to place a collateral to borrow. Therefore, although all firms face a reduced credit in a recession, some of the medium sized firms suffer a double hit. They face the general credit reduction like all firms, but they also suffer from a change in credit treatment due to the shifting audit threshold level of the banks.

Accordingly, the idea is to make a comparison in the average responses for entrepreneurs’ investment between: (i) audited entrepreneurs, who remain audited (i.e. large firms); (ii) unaudited entrepreneurs, who remain unaudited (i.e. small firms); and (iii) medium-sized firms, which some of them were audited but become unaudited following the shocks. Since the model is static, the results are generated by comparing the model outcomes with shocks, relative to the outcomes in the absence of shocks.28

1.6.1 The UK in the Great Recession

The 2008 recession in both the UK and US started with a large drop in asset prices, that later spread to a wide recession affecting the whole of their economies. Rather than attempting to quantitatively match the reported results in Figure 1.1, I try to qualitatively explain those variations in firms’ investment responses. The 2008 large drop in asset prices has led to a decline in the value of capital. This in turn has resulted in tougher credit conditions to small firms in particular, as they rely more on property collaterals to obtain loans (Chadha et al. (2017); Haldane (2017)). Moreover, both Bahaj et al. (2016) and Pinter (2019) have found empirical evidence that the decline in house prices has caused a reduction in SMEs’ loans and investment in the UK. Hence, I proxy for it by a positive shock to \( q \).

Additionally, I proxy for the economy-wide recession that followed by a negative shock to \( z \). Given the theoretical framework presented, each of the two shocks will generate different responses between small, medium and large firms. Each shock causes those different responses based on a certain channel. In terms of the empirical regularities given in the UK, the model predicts that two channels must have been acting at the same time. However, just having the two channels working at the

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28 The downside of this approach is that I am unable to track the evolution of the distribution of aggregate capital stock in the economy as given in equation (1.2). In future research, the model should be extended to multiple periods, in order to address this matter. I briefly discuss the way in which this extension can be performed in my concluding remarks in section 1.7.
same time (two shocks), will not be sufficient in explaining how the empirical regularity came into place. What also matters is the relative size of the shocks to each other.

Instead of considering the combined effect of the two shocks only, I first examine each shock in isolation. This is to allow understanding the underlying channel that is associated with each shock, $z$ and $q$. Moreover, in the numerical exercises presented, I consider a situation in which the economy moves from non-binding borrowing constraints to binding borrowing constraints. This is motivated by the credit crunch that characterised the 2008 recession. Net bank lending to the non-financial private sector, stood at 12 per cent of the UK’s GDP in 2007. However, it dropped to less than 1 per cent in 2009 (Martin and Rowthorn (2012)).

**1.6.1.1 Neutral Productivity Shock, $z$**

I examine the response of firms’ investment to a negative neutral productivity shock, while holding all other parameters unchanged. More precisely, I consider a 2 per cent reduction in the value of $z$. Table 1.2 provides the economy’s parameters values, both in the absence of a shock and when the shock occurs. It also provides the values of $\bar{q}$ and $\hat{k}$ that are implied by the given parameters’ values.

<table>
<thead>
<tr>
<th></th>
<th>$P$</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$A$</th>
<th>$z$</th>
<th>$q$</th>
<th>$\bar{q}$</th>
<th>$\hat{k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>2</td>
<td>1.124</td>
<td>1.13271</td>
<td>184.041</td>
</tr>
<tr>
<td>Post-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>1.96</td>
<td>1.124</td>
<td>1.21162</td>
<td>196.861</td>
</tr>
</tbody>
</table>

Table 1.2: The economy’s parameters values; in the absence of a shock and when a 2 per cent negative shock to $z$ occurs.

---

29 For the purpose of checking the robustness of my model’s predications, in the appendix, section 1.8.3, I replicate the exercises for other possible scenarios and the results remain intact. All other economic parameters were kept at the same values through all the exercises. To study the different situations, where the economy either stays with binding or non-binding borrowing constraints following the shocks, I only scaled the economy up or down by varying the productivity of the technology $f(k,x)$, such that the initial ratio $z/q$ remains the same for all the studied scenarios.

30 I followed the standard values assumed for $\alpha$ and $\delta$ in the literature, while the value of $A$ was chosen arbitrarily. Additionally, please check the appendix, section 1.8.2, where I justify my choice for the values of $P$ and $\gamma$. The results are robust to varying the value of $\gamma$ between 0.01 to 0.99 (please check the appendix, section 1.8.3.2).
Figure 1.5: The percentage impact on firms’ investment due to a 2 per cent negative shock to $z$.

As can be seen in Figure 1.5, when there is only a negative shock to $z$, ceteris paribus, an audited firm’s investment responds by more compared to the investment of an unaudited firm. This is formally given by equations (1.38) and (1.39). The derivatives in those two equations are positive, which imply that in the event of a negative shock to $z$, audited firms’ investment will drop by a higher percentage relative to unaudited firms' investment. In other words, the inequality signs will change direction, with the left-hand side being more negative than the right-hand side.

$$\left( \frac{\partial i_A^*}{\partial z} \right)_{i_A^*} > \left( \frac{\partial i_{NA1}^*}{\partial z} \right)_{i_{NA1}}$$ (1.38)$^{31}$

$$\left( \frac{\partial i_A^*}{\partial z} \right)_{i_A^*} > \left( \frac{\partial i_{NA2}^*}{\partial z} \right)_{i_{NA2}}$$ (1.39)$^{32}$

To understand the reason behind this result,$^{33}$ I first consider the percentage effect of a change in $z$ on the optimal borrowing demands, $x_A^*$ and $x_{NA1}^*$, and find them to be equal:

---

$^{31}$ The derivatives are evaluated using the pre-shock parameters’ values (i.e. when the unaudited firm borrowing constraint is non-binding).

$^{32}$ The derivatives are evaluated using the post-shock parameters’ values (i.e. when the unaudited firm borrowing constraint is binding). Even when the derivatives in equations (1.38) and (1.39) are evaluated at both pre-shock and post shock parameters’ values, the inequality sign remains in the same direction.

$^{33}$ Note that in this numerical example, the borrowing constraint remains non-binding following the negative shock to $z$. 

\[
\frac{\partial x_A^* / \partial z}{x_A^*} = \frac{\partial x_{NA1}^* / \partial z}{x_{NA1}^*} \tag{1.40}^{34}
\]

I let \( y \) be the firm’s output (i.e. \( y = z k^\alpha x^{1-\alpha} \)), and hence equation (1.40) will also imply the following:

\[
\frac{\partial y_A^* / \partial z}{y_A^*} = \frac{\partial y_{NA1}^* / \partial z}{y_{NA1}^*} \tag{1.41}
\]

This would lead us to think that the percentage investment responses should be the same between audited and unaudited entrepreneurs, in contrast to equation (1.38). However, this conclusion does not take into account the uncertainty that the unaudited entrepreneur faces when deciding her optimal demand for \( x \). With probability \( 1 - P \), the unaudited entrepreneur production will be unsuccessful, and she will be forced to pay back the amount she borrowed using her capital resources which have already depreciated at a rate \( \delta \).

Considering the impact that a shock to \( z \) would have on an entrepreneur’s expected resources would lead to an accurate conclusion. I let \( R \) be the total expected resources of the entrepreneur as given by the right-hand side of the budget constraints in equations (1.26) and (1.32). I substitute for \( x \) in \( R \) using \( x_A^* \) and \( x_{NA1}^* \), thus obtaining both the audited and unaudited entrepreneur’s total expected resources in equilibrium, \( R_A^* \) and \( R_{NA1}^* \) respectively.

\[
\frac{\partial R_A^* / \partial z}{R_A^*} > \frac{\partial R_{NA1}^* / \partial z}{R_{NA1}^*} \tag{1.42}
\]

When considering the case between an audited entrepreneur and an unaudited entrepreneur with a binding borrowing constraint, then the channel of transmission is clearer, since:

\[
\frac{\partial x_{NA2}^* / \partial z}{x_{NA2}^*} = 0 \tag{1.43}
\]

\(^{34}\) The derivatives are evaluated using the pre-shock parameters’ values (i.e. when the unaudited firm borrowing constraint is non-binding). Even if those derivatives are evaluated using the post-shock parameters’ values, the two expressions would remain equal.
The impact of the shock on the audited entrepreneur will come from both a direct and an indirect channel; a reduction in debt (indirect) and a reduction in output (direct). For the unaudited entrepreneur with a binding borrowing constraint, the impact of a shock to $z$ will only come through the direct channel, which is a reduction in output.

The above discussion explains the differences in investment response between small and large firms. However, to explain the largest decline in investment that is exhibited by medium firms, I need to look at the effect of a shock to $z$ on the capital threshold for auditing. Medium firms fall on both sides of $\tilde{k}$, and a negative shock to $z$ will cause $\tilde{k}$ to shift to the right. Therefore, some of the medium firms which were audited before the shock, will now fall under the threshold level $\tilde{k}$. This will result in those firms facing a discriminating credit treatment relative to before. It follows that for those firms, a negative shock to $z$ will cause a double hit. The first is in the form of a contraction in debt that both audited and unaudited firms suffer alike, as explained before. Additionally, the change in the credit conditions that those firms face will present a further contraction. Since $x_A^* > x_{MA1}$, it follows that those medium firms (the ones which post-shock fell below $\tilde{k}$) will have a much larger drop in their investment. Consequently, the average investment of all medium firms declines by a higher percentage relative to small and large firms.

### 1.6.1.2 Investment Specific Technology Shock, $q$

I now examine the response of firms’ investment to a positive IST shock, while holding all other parameters unchanged. More precisely, I consider a 10 per cent increase in the value of $q$. Table 1.3 provides the economy’s parameters values, both in the absence of a shock and when the shock occurs.

<table>
<thead>
<tr>
<th></th>
<th>$P$</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$A$</th>
<th>$z$</th>
<th>$q$</th>
<th>$\bar{q}$</th>
<th>$\tilde{k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>2</td>
<td>1.124</td>
<td>1.13271</td>
<td>184.041</td>
</tr>
<tr>
<td>Post-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>2</td>
<td>1.2364</td>
<td>1.13271</td>
<td>169.63</td>
</tr>
</tbody>
</table>

Table 1.3: The economy’s parameters values; in the absence of a shock and when a 10 per cent positive shock to $q$ occurs.
Figure 1.6: The percentage impact on firms’ investment due to a 10 per cent positive shock to $q$.

As can be seen in Figure 1.6, when there is only a positive shock to $q$, ceteris paribus, an unaudited firm’s investment responds by more than the investment of an audited firm. This is formally given by equations (1.44) and (1.45). The derivatives in those two equations are negative, which imply that in the event of a positive shock to $q$, audited firms’ investment will drop by a smaller percentage relative to unaudited firms’ investment.

$$\frac{(\partial i_A^*/\partial q)}{i_A^*} > \frac{(\partial i_{NA1}^*/\partial q)}{i_{NA1}^*} \quad (1.44)^{35}$$

$$\frac{(\partial i_A^*/\partial q)}{i_A^*} > \frac{(\partial i_{NA2}^*/\partial q)}{i_{NA2}^*} \quad (1.45)^{36}$$

A positive shock to $q$, means a decrease in the value of an entrepreneur’s net worth in terms of the final consumption good. Although this decline is equally shared between audited and unaudited entrepreneurs, unaudited entrepreneurs face the uncertainty where with probability $1 - P$ their production will be unsuccessful. In such event, they are forced to pay back the amount they borrowed using their capital resources which have already depreciated at a rate $\delta$. Therefore, an increase in $q$ would

---

35 The derivatives are evaluated using the pre-shock parameters’ values (i.e. when the unaudited firm borrowing constraint is non-binding).

36 The derivatives are evaluated using the post-shock parameters’ values (i.e. when the unaudited firm borrowing constraint is binding). Moreover, even when the derivatives in equations (1.44) and (1.45) are evaluated at both pre-shock and post shock parameters’ values, the inequality sign remains in the same direction.
have a relatively greater negative impact on the unaudited entrepreneurs expected total resources in equilibrium. This in turn will translate to a greater expected percentage reduction in the resources allocated for investment. This is formally given in equation (1.46), where the derivatives are negative.

\[
\frac{(\partial R^*_A/\partial q)}{R^*_A} > \frac{(\partial R^*_{NA1}/\partial q)}{R^*_{NA1}}
\]  

(1.46)

The effect of a positive shock to \( q \) on investment will be even stronger in the case where the unaudited entrepreneur borrowing constraint is binding. This is because an increase in \( q \) will also lower the maximum amount that the unaudited entrepreneur can borrow, \( x = (1/q)(1 - \delta)k \). This in turn will result in larger percentage decline her expected resources.

The above discussion explains the differences in investment response between small and large firms. However, as can be seen in Figure 1.6, medium firms exhibit the smallest percentage drop in investment. To explain this, I need to look at the effect of a shock to \( q \) on the capital threshold for auditing, since medium firms fall on both sides of \( \hat{k} \). A positive shock to \( q \) will cause \( \hat{k} \) to shift to the left. Consequently, some medium firms which were unaudited before the shock, will now fall above the threshold level \( \hat{k} \), resulting in them facing a better credit treatment relative to before. It follows that for those firms, a positive shock to \( q \) will form an improvement. The positive effect of the improvement in credit treatment on those firms is larger than the negative effect of an increase in \( q \) on their expected resources, \( R \). Accordingly, those firms (the ones which post-shock fell above \( \hat{k} \)) will experience an increase in investment following the shock. This increase for those firms will in turn make the average investment of all medium firms decline by a lower percentage relative to small and large firms.

1.6.1.3 Shocks to both \( z \) and \( q \)

As described earlier, each shock, \( z \) and \( q \), will give different responses between small, medium and large firms through two different channels of transmission. In terms of the empirical regularity given in the UK, the theoretical framework predicts that it must have been the case that there were two channels acting at the same time.
However, having the two channels working at the same time (two shocks), will not be sufficient on its own in explaining how the empirical regularity came into place. What also matters is the relative size of the shocks to each other. In light of the of the earlier discussion, the model predicts that the effect of the neutral productivity shock $z$ on $\hat{k}$, should have been stronger than the effect of the investment specific technology shock $q$. In this case, medium firms will respond by more compared to other firms on average. In the same time, when comparing small and large firms’ investment responses, the effect of the channel of $q$ must have been stronger than the channel of $z$. This is for small firms’ investment to respond by more compared to large firms’ investment. I now consider a 10 per cent increase in the value of $q$ combined with a 2 per cent reduction in the value of $z$, as shown in Table 1.4:

<table>
<thead>
<tr>
<th></th>
<th>$P$</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$A$</th>
<th>$z$</th>
<th>$\bar{q}$</th>
<th>$\hat{k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>2</td>
<td>1.124</td>
<td>1.13271</td>
</tr>
<tr>
<td>Post-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>1.96</td>
<td>1.2364</td>
<td>1.21162</td>
</tr>
</tbody>
</table>

Table 1.4: The economy’s parameters values; in the absence of a shock and when a 10 per cent positive shock to $q$ combined with a 2 per cent negative shock to $z$ occur.

In the results shown in the Figure 1.7, the economy has moved from $z/q = 1.77936$ to $z/q = 1.58525$ following the shocks to $z$ and $q$. The choice of relative size of the two shocks to each other, makes the two underlying channels interact to generate larger percentage decline in investment for small firms, relative to large firms. Furthermore, the relative size of the shocks is also crucial for determining the effect on the average investment of medium firms. A negative shock to $z$ would cause $\hat{k}$ to shift to the right, while a positive shock to $q$ would cause $\hat{k}$ to shift in the opposite direction. The model predicts that the effect of the shock to $z$ must have dominated the effect of shock to $q$ on $\hat{k}$, such that $\hat{k}$ has shifted to the right. This will result in a double hit to some medium firms, and therefore a greater percentage decline in medium firms’ investment on average.
To further clarify the above argument and how the two different channels work, I now consider two other alternatives. I run the two alternatives of: (i) a 2 per cent negative shock to \( z \) combined with a 5 per cent positive shock to \( q \), and (ii) a 0.5 per cent negative shock to \( z \) combined with a 10 per cent positive shock to \( q \). Those two exercises confirm that the relativity of the two shocks to each other matters. The two shocks shift \( \hat{k} \) in opposite directions, and the generated results will be determined by which of the two effects dominates the other.

<table>
<thead>
<tr>
<th></th>
<th>( P )</th>
<th>( \alpha )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>( A )</th>
<th>( z )</th>
<th>( q )</th>
<th>( \bar{q} )</th>
<th>( \hat{k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>2</td>
<td>1.124</td>
<td>1.13271</td>
<td>184.041</td>
</tr>
<tr>
<td>Post-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>1.96</td>
<td>1.1802</td>
<td>1.21162</td>
<td>196.861</td>
</tr>
</tbody>
</table>

Table 1.5: The economy’s parameters values; in the absence of a shock and when a 5 per cent positive shock to \( q \) combined with a 2 per cent negative shock to \( z \) occur.

As shown in Table 1.5, following the two shocks the economy remains in a situation of non-binding borrowing constraints for unaudited entrepreneurs. The combined effect of the two shocks have caused \( \hat{k} \) to shift to the right, resulting in a double hit to some medium firms as shown in Figure 1.8. Nevertheless, the negative
shock to $z$ was substantially large relative to the positive shock to $q$, such that large firms’ investment dropped by a larger percentage compared to small firms. In other words, when comparing large firms and small firms’ investment, the effect of the channel caused by a negative shock to $z$, has dominated the effect of the channel caused by a positive shock to $q$.

![Bar Chart]

Figure 1.8: The percentage impact on firms’ investment due to a 2 per cent negative shock to $z$, combined with a 5 per cent positive shock to $q$.

On the other hand, when I consider the other alternative of a 0.5 per cent negative shock to $z$ combined with a 10 per cent positive shock to $q$, then the economy moves to a situation where the borrowing constraints of unaudited entrepreneurs become binding:

<table>
<thead>
<tr>
<th></th>
<th>$P$</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$A$</th>
<th>$z$</th>
<th>$q$</th>
<th>$\bar{q}$</th>
<th>$\hat{k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>2</td>
<td>1.124</td>
<td>1.13271</td>
<td>184.041</td>
</tr>
<tr>
<td>Post-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>1.99</td>
<td>1.2364</td>
<td>1.1518</td>
<td>175.091</td>
</tr>
</tbody>
</table>

Table 1.6: The economy’s parameters values; in the absence of a shock and when a 5 per cent positive shock to $q$ combined with a 2 per cent negative shock to $z$ occur.
In this example, the combined impact of the shocks will make small firms investment decline by a greater percentage relative to large firms. This is consistent with the empirical facts. However, when considering the effect on $\hat{k}$, the negative shock to $z$ was substantially small relative to the positive shock to $q$, such that the auditing threshold has shifted to the left. This has improved the credit treatment for some medium firms, which caused their investment to increase. Accordingly, the total effect on the average investment of all medium firms, was to decline by a smaller percentage relative to small and large firms.

![Figure 1.9: The percentage impact on firms’ investment due to a 0.5 per cent negative shock to $z$, combined with a 10 per cent positive shock to $q$.](image)

1.6.2 The US in the Great Recession

Kudlyak and Scanchez (2017) have used US data covering the period between 1958Q4 and 2014Q2 to study the behaviour of small and large firms. Their results challenge the predictions of the financial frictions theoretical literature, where adverse shocks are propagated through the most financially vulnerable firms. They found that large firms turnover and short-term debt have contracted proportionately more than small firms during the recent recession.

When I examined their results further, I conclude that their results are due to the way in which they have defined small and larger firms. The data they used is aggregated into eight asset classes, with those classes remaining constant through the entire sample. However, they define small firms to be the ones below the twenty-fifth
percentile of firms’ turnover distribution in each period. Therefore, as the distribution of firms’ sales continues shifting to the right with time, the asset classes in which data is aggregated into remains constant. For example, in the early years of their sample, firms with total assets of $5 million and under were small firms, while in the end of their sample (precisely by the start of the 2008 recession and onward) firms with total assets of $1000 million and under were considered to be small firms. I find this very implausible; nevertheless, I show that my model can generate such results if I follow their definition of small firms. The implication of their definition of small firms on my model, is that medium firms will be grouped with large firms, where medium firms exhibit the largest average percentage contraction in debt and output, following adverse shocks.

I now repeat the same exercises that I conducted before; however, I perform two changes before obtaining the numerical results. The first is reducing the bargaining power of the lenders. The banking sector in the US is less concentrated than in the UK, hence the change. The second change is re-dividing the entrepreneurs’ net worth distribution, such that now small firms are the ones at the bottom twenty-fifth percentile, while large firms are the ones occupying the top twenty-fifth percentile. I now consider a 10 per cent increase in the value of $q$ combined with a 2 per cent reduction in the value of $z$, as shown in Table 1.7:

<table>
<thead>
<tr>
<th></th>
<th>$P$</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$A$</th>
<th>$z$</th>
<th>$q$</th>
<th>$\bar{q}$</th>
<th>$\hat{k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.35</td>
<td>0.1</td>
<td>30</td>
<td>2</td>
<td>1.124</td>
<td>0.867454</td>
<td>264.91</td>
</tr>
<tr>
<td>Post-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.35</td>
<td>0.1</td>
<td>30</td>
<td>1.99</td>
<td>1.2364</td>
<td>0.927882</td>
<td>275.793</td>
</tr>
</tbody>
</table>

Table 1.7: The economy’s parameters values; in the absence of a shock and when a 10 per cent positive shock to $q$ combined with a 2 per cent negative shock to $z$ occur.

The economy starts from a situation where the unaudited entrepreneurs’ borrowing constraints are binding and remain binding following the shocks. Figures

37 Even if I keep the bargaining power the same as before, the qualitative results would not change. Please refer to the appendix, section 1.8.2.2, for a brief discussion on the banking concentration in the United States.
1.10 and 1.11, present the results for small versus large firms, where large firms exhibit larger percentage decreases in their debt and output.

Figure 1.10: The percentage impact on firms’ debt due to a 2 per cent negative shock to $z$, combined with a 10 per cent positive shock to $q$.

Figure 1.11: The percentage impact on firms’ output due to a 2 per cent negative shock to $z$, combined with a 10 per cent positive shock to $q$.

However, in Figures 1.12 and 1.13 I breakdown the results to small, medium and large firms. As can be seen the larger decreases exhibited by large firms were due to medium firms being grouped with them. Following the previous definition of small firms would not change the qualitative results. However, it is worth noting that following the definition of Kudlyak and Sanchez (2017) would make the response of the “large” firms even more severe, since I am adding with them more small unaudited firms than I would have under the previous definition.
1.7 Conclusion

The model developed in this chapter has addressed the two main research questions. The first is the reason behind the discriminating credit treatments offered by banks to firms based on their size. The mechanism that explains this credit discrimination, can also explain the differences in firms’ investment responses to economic shocks based on their size, as was reported by Crawford et al. (2013). In addition, contrary to the proposition of Kudlyak and Sanchez (2017) that their results challenge the
predictions of the financial frictions theoretical literature, my model shows that such empirical results can be generated with a mechanism that propagates adverse shocks through the most financially vulnerable firms.

To the best of my knowledge, all the theoretical models generate discriminating credit treatments and contracts, by imposing exogenous heterogeneity between small and large firms. This exogenous heterogeneity can be in one or more dimensions where it is expressed as types, instead of the firms’ initial condition which is their size. The premises on which those models are built are different from the premises that I start from. Therefore, those models will not generate the empirical regularity which I am trying to explain. Some models abstract from imposing different characteristics between firms that lead to credit discrimination, and simply assume the heterogeneous credit treatment exogenously. For instance, in Bernanke et al. (1999) the credit treatments to firms are assumed to be different, and on the bases of this assumption they generate their results. On the other hand, my model implies that because firms are of different sizes, the differing credit treatments are arising endogenously due to the fixed audit cost that must be incurred by banks.

In the other financial frictions’ models, ceteris paribus, the amount of credit that firms need depend on their net worth. In other words, the difference between what they own and what they need to have. When considering models that apply the net worth channel more specifically, the amount of credit that firms can get depends on the value of the firms’ assets. Accordingly, when there is an adverse shock to the value of firms’ assets, shocks will propagate differently for firms that differ in their net worth, and hence their access to credit. The equivalent to this result in my model is when there is a positive shock to $q$, where the numerical results generated are compared to the results of the other models. However, in my model, when there is a negative shock to $z$ and in the absence of any shock to $q$, my results would differ. Large firms which enjoy a better access to credit, will have a greater response than small firms. As stated earlier, I am not taking many other heterogeneities into account. For instance, large firms enjoy higher success rates and greater bargaining power. Another criticism is that negative shocks to $z$, cause reduction in credit through a reduction in the demand for credit. A reduction in credit that is due to a reduction in supply, only occurs when there is a positive investment specific technology shock and the unaudited entrepreneurs’ borrowing constraints are binding.
Due to the importance of medium sized firms as discussed early in the chapter, and since they are the most vulnerable to adverse shocks as it was established by my model; policies that aim at reducing the informational asymmetries between lenders and borrowers should be beneficial. The establishment of systematic approaches to the assessment of businesses by independent bodies, and which can be accepted by banks should reduce the financing difficulties faced by mid-sized firms. This would involve outsourcing part of the banks auditing processes to independent third parties, where firms can contribute towards their auditing costs. A similar suggestion to reduce informational asymmetries was proposed by Binks and Ennew (1997). Finally, medium sized firms can be a candidate for obtaining finance outside of the traditional banking system, if equity financing can be improved. Mason and Harrison (2015) have studied equity financing for British SMEs during the financial crisis. They found that the matching between firms and investors followed a less systematic approach and was mostly a coincidence-based process.

Future research, that builds on the theoretical framework presented in this chapter, should look at the effect of including a full banking sector in the model. With this inclusion, financial intermediation and credit screening will be the role played by the banks, which receive deposits from the households and grant loans to the entrepreneurs. The inclusion of a banking sector would not in itself be sufficient to analyse financial fragility, however. Other essential elements to be added include: (i) the explicit introduction of a monetary sector in the model in order for liquidity to have a role; and (ii) agents’ intertemporal decision-making when the model is extended to multiple periods (Peiris et al. (2018)).

One possible way for adopting the aforementioned changes is to follow the framework developed by Baliga and Polak (2004) in order to compare German and Anglo-Saxon financial systems, which differ in the degree that firms’ credit is monitored. The Anglo-Saxon system is more geared towards traded bonds – pre-screening borrowers at the start, with little monitoring thereafter. The German system follows a more hands-on approach, with a very close relationship banking through non-traded bank loans. In their model either the lender or the entrepreneur makes the

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38 They have used data on six thousand UK small businesses, which responded to a survey conducted by the Forum of Private Businesses in 1994.

39 Other key elements for analysing financial fragility are already present in the model. Those are: (i) endogenous default, (ii) heterogeneity of economic agents, (iii) uncertainty, and (iv) missing financial markets, such that not all risks in the economy can be hedged with existing assets.
choice between monitored and non-monitored loans, depending on which agent being allocated the full bargaining power. The choice between the two types of loans is determined through a set of conditions that arise endogenously in the model. It is worth mentioning that their model follows a two periods setup. This would be especially beneficial since extending the model presented in this chapter to an infinite horizon will cause the distribution of aggregate capital stock given in equation (1.2) to collapse to a spike in the steady state – preventing the comparisons between firms’ responses (small, medium and large), which is the question I am addressing in this chapter.
1.8 Appendix

1.8.1 The General Equilibrium Model

I have attempted to address the research questions in a general equilibrium setup; however, an equilibrium does not always exist over the entire capital space. I start by a description of the model’s environment, and then follow it by a discussion of the equilibrium.

I consider a two-period economy, $t = 1, 2$. There are two agents in this economy; a household and an entrepreneur. Both agents get utility from consumption, but consumption only occurs in $t = 2$. Each agent has access to a storage technology that gives one-to-one between periods. The household receives an endowment $E$ of the consumption good in $t = 1$. The endowment is assumed to be sufficiently large, such that the resource constraint in lending to the entrepreneur will not bind. The entrepreneur owns a firm with a production technology $f(k, l) = z k^a l^{1-a}$. The output of this technology is the consumption good, and it is transferable one-to-one to the capital good. The entrepreneur starts in $t = 1$ with an endowment of capital $k$, and hires labour which is supplied by the household. Moreover, capital depreciates only when engaging in production. Precisely, a fraction $\delta \in (0,1)$ of the entrepreneur’s capital depreciates when she engages in production. The production starts in $t = 1$ with output being realised in $t = 2$. I assumed that labour costs are paid upfront in $t = 1$. To pay the upfront labour cost, the entrepreneur borrows from the household in $t = 1$. I let $b$ be the amount that the entrepreneur borrows to pay the upfront labour cost.

The output realisation in $t = 2$ is subject to an idiosyncratic shock. With prior probability $P$, output will be strictly positive (the ex-post realisation of the idiosyncratic shock is 1), and with probability $1 - P$ output will be zero (the ex-post realisation of the idiosyncratic shock is 0). In $t = 1$, the entrepreneur is either audited or not, by the household which she is borrowing from. Audit is a choice made by the household and reveals the realisation of the idiosyncratic shock, that is otherwise unobservable without engaging in production. If the entrepreneur is audited and found to be good, then she borrows, hires labour and produces. If she is found to be bad, then she would not borrow or hire labour, and simply stores her capital endowment and consumes it in $t = 2$. On the other hand, if the entrepreneur is not audited, then she
still borrows, hires labour and engages in production. The entrepreneur’s capital is assumed to be perfectly observable by the household.

In \( t = 1 \), the household has a choice between two options. He can either store his whole endowment and consumes it in \( t = 2 \), or engage in lending and supplying labour. If the household chooses to lend, then he has a further two options. He can either use part of the endowment to pay the fixed audit cost, \( A > 0 \), and based on the audit outcome he lends or not, or he can choose to lend without auditing. The household stores the remainder of his endowment for consumption in \( t = 2 \). Since consumption occurs only in \( t = 2 \), then the opportunity cost from lending is zero, thus \( r = 0 \). If audit is conducted, then the realisation of the idiosyncratic shock becomes a common knowledge. Therefore, if the realisation is found to be bad, the entrepreneur will not hire labour or borrow, although workers would like to work. The realisation of output from production in \( t = 2 \) is assumed to be perfectly observable to the household.

Finally, the entrepreneur who is unaudited and gets a bad realisation of her idiosyncratic shock, chooses to default. Specifically, if the output realisation of the unaudited entrepreneur is zero in \( t = 2 \), she defaults on her debt, and cannot be forced to pay back. Accordingly, the household’s incentive from auditing is to avoid the possibility of the entrepreneur defaulting on the debt if her production is unsuccessful. At low levels of capital, the loss in the household’s expected utility from paying the audit cost, will be greater than the loss in his expected utility from the entrepreneur defaulting on the debt, if the entrepreneur’s production is unsuccessful. As the entrepreneur’s capital grows larger, the loss in the household’s expected utility from paying the audit cost, will be smaller than the loss in his expected utility from the entrepreneur defaulting on the debt, if the entrepreneur’s firm project turns to be bad.

1.8.1.1 The Entrepreneur’s Problem

The entrepreneur maximizes her utility by choosing consumption in the second period \( c' \), the amount of the consumption good that she wants to borrow \( b \), and the amount of labour she wants to hire \( l \). Moreover, since an entrepreneur can be subjected to two different credit treatments, depending on whether she is being audited or not, there will be two versions of the optimisation problem.
1.8.1.1 The Audit Case

In $t = 1$, with a prior probability $1 - P$, the entrepreneur’s production will be found unsuccessful through audit, and hence she will not borrow to hire labour. Accordingly, her problem is given by:

$$\max_{c',b,l} E[\beta c']$$  \hspace{1cm} (1.47)

subject to

$$c' = zk^{\alpha} l^{1-\alpha} - b + (1 - \delta)k \quad \text{with probability } P$$

$$c' = k \quad \text{with probability } 1 - P$$  \hspace{1cm} (1.48)

$$b = wl$$

where $c'$ indicates that consumption takes place in the second period, $w$ is the wage rate, and $\beta \in (0,1)$ is the implied discount factor. I substitute for $c'$ in (1.47) using the entrepreneur’s lifetime budget constraint, which is given in (1.48):

$$\max_{t} P\beta [zk^{\alpha} l^{1-\alpha} - wl + (1 - \delta)k] + (1 - P)\beta k$$  \hspace{1cm} (1.49)

Taking the first order condition and solving for the optimal labour demand under audit, gives:\footnote{The function is concave; thus, the second order condition is satisfied.}

$$l = \left(\frac{z(1 - \alpha)}{w}\right)^{\frac{1}{\alpha}} k$$  \hspace{1cm} (1.50)

1.8.1.2 The No Audit Case

Under the no audit case, the entrepreneur will not be audited; however, she still borrows and hires labour. If the entrepreneur’s realisation of the idiosyncratic shock is unfavourable, then she defaults and cannot be forced to pay back. Accordingly, her problem is given by:

$$\max_{c',b,l} E[\beta c']$$  \hspace{1cm} (1.51)

subject to
\[ c' = zk^{\alpha} l^{1-\alpha} - b + (1 - \delta)k \quad \text{with probability } P \]
\[ c' = (1 - \delta)k \quad \text{with probability } 1 - P \] (1.52)
\[ b = wl \]

Substituting for \( c' \) in (1.51) using the entrepreneur’s lifetime budget constraint, which is given in (1.52):

\[ \max_l P \beta [zk^{\alpha} l^{1-\alpha} - wl + (1 - \delta)k] + (1 - P)\beta(1 - \delta)k \] (1.53)

Taking the first order condition and solving for the optimal labour demand under audit, gives:

\[ l = \left( \frac{z(1 - \alpha)}{w} \right)^{\frac{1}{\alpha}} k \] (1.54)

Since the entrepreneur can default on her debt if her production is unsuccessful, then her labour demand will be the same under audit and no audit.

1.8.1.2 The Household’s Problem

The household maximizes his utility by choosing consumption in the second period \( c' \) and the amount of labour he wants to supply \( l \). Moreover, since the household can either audit the entrepreneur or not, there will be two versions of the optimisation problem.

1.8.1.2.1 The Audit Case

\[ \max_{c',l} E \left[ -\frac{l^2}{2} + \beta c' \right] \] (1.55)

subject to

\[ c' = zc' = E - A + wl - b + b \quad \text{with probability } P \] (1.56)
\[ c' = E - A \quad \text{with probability } 1 - P \]

---

41 The function is concave; thus, the second order condition is satisfied.
42 In the good state of the world, with probability \( P \), the amount of debt \( b \) is subtracted indicating lending, then added back to indicate debt repayment.
I substitute for $c'$ in (1.55) using the household’s lifetime budget constraint, which is given in (1.56):

$$\max_{l} \quad P \left[ -\frac{l^2}{2} + \beta (E - A + wl) \right] + (1 - P)\beta (E - A)$$  \hspace{1cm} (1.57)

Taking the first order condition and solving for the optimal labour supply under audit, gives:\[43\]

$$l = \beta w$$ \hspace{1cm} (1.58)

1.8.1.2.2 The No Audit Case

$$\max_{c',l} \quad E \left[ -\frac{l^2}{2} + \beta c' \right]$$  \hspace{1cm} (1.59)

subject to

$$c' = E + wl - b + b \quad \text{with probability } P$$

$$c' = E + wl - b \quad \text{with probability } 1 - P$$  \hspace{1cm} (1.60)

Debt repayment only occurs in the good state of the world, since the entrepreneur cannot be forced to pay back. I substitute for $c'$ in (1.59) using the household’s lifetime budget constraint, which is given in (1.60):

$$\max_{l} \quad P \left[ -\frac{l^2}{2} + \beta (E + wl) \right] + (1 - P) \left[ -\frac{l^2}{2} + \beta E \right]$$  \hspace{1cm} (1.61)

Taking the first order condition and solving for the optimal labour supply under audit, gives:\[44\]

$$l = P \beta w$$ \hspace{1cm} (1.62)

Since the entrepreneur can default on her debt if her production is unsuccessful, the household’s labour supply is less than what it would be under the audit case. In other words, in the bad state of the world, the household will be supplying labour with no return.

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\[43\] The function is concave; thus, the second order condition is satisfied.

\[44\] The function is concave; thus, the second order condition is satisfied.
1.8.1.3 The Equilibrium

The equilibrium is characterised by the labour market clearing. Since I have two cases depending on whether audit takes place or not, two equilibrium levels of labour and wages will arise.

1.8.1.3.1 The Audit Case

I equate the labour supply and demand as given in equations (1.50) and (1.58), and solve for the equilibrium wage under audit, \( w^*_A \):

\[
 w^*_A = (z(1 - \alpha))^{1/(1+\alpha)} \left( \frac{k}{\beta} \right)^{\alpha/(1+\alpha)}
\]  

(1.63)

I use (1.63) to substitute for \( w \) in the labour supply given in equation (1.58) and obtain:

\[
l^*_A = [z\beta(1 - \alpha)]^{1/(1+\alpha)} \left( k \right)^{\alpha/(1+\alpha)}
\]  

(1.64)

which is the equilibrium level of labour under audit.

1.8.1.3.2 The No Audit Case

I equate the labour supply and demand as given in equations (1.54) and (1.62), and solve for the equilibrium wage under no audit, \( w^*_{NA} \):

\[
 w^*_{NA} = (z(1 - \alpha))^{1/(1+\alpha)} \left( \frac{k}{P\beta} \right)^{\alpha/(1+\alpha)}
\]  

(1.65)

I use (1.65) to substitute for \( w \) in the labour supply given in equation (1.62) and obtain:

\[
l^*_{NA} = [zP\beta(1 - \alpha)]^{1/(1+\alpha)} \left( k \right)^{\alpha/(1+\alpha)}
\]  

(1.66)

which is the equilibrium level of labour under no audit.
1.8.1.4 Obtaining the Audit Threshold, \( \hat{k} \)

This is a general equilibrium set up, where agents form their decisions based on the prices they receive in equilibrium. Accordingly, I first obtain the threshold for auditing in terms of the wage rate. I obtain the household’s utility under the audit and no audit cases, by substituting for \( l \) in equations (1.57) and (1.61) using equations (1.64) and (1.66), respectively.

\[
EU^H_A = \frac{1}{2} P \beta^2 w^2 + \beta E - \beta A \tag{1.67}
\]

\[
EU^H_{NA} = \frac{1}{2} P^2 \beta^2 w^2 + \beta E \tag{1.68}
\]

1.8.1.4.1 The Audit Case

The household prefers the audit equilibrium iff \( EU^H_A \geq EU^H_{NA} \). I solve this inequality for \( w \) and obtain:

\[
w \geq \left( \frac{2A}{\beta(P - P^2)} \right)^{\frac{1}{2}} = \hat{w} \tag{1.69}
\]

where \( \hat{w} \) is the wage threshold for the household’s auditing decision. To obtain the auditing threshold in terms of the entrepreneur’s observable capital stock \( \hat{k}_A \), I compare between the equilibrium wage under audit (1.63) and the wage threshold for auditing (1.69) and solve for \( k \):

\[
k \geq \left( \frac{2A}{P - P^2} \right)^{\frac{1+\alpha}{2\alpha}} (\beta)^{-\frac{1+\alpha}{2\alpha}} (z(1 - \alpha))^{-\frac{1}{\alpha}} = \hat{k}_A \tag{1.70}
\]

1.8.1.4.2 The No Audit Case

Similarly, the household prefers the no audit equilibrium iff \( EU^H_{NA} \geq EU^H_A \). I solve this inequality for \( w \) and obtain:

\[
w < \left( \frac{2A}{\beta(P - P^2)} \right)^{\frac{1}{2}} = \hat{w} \tag{1.71}
\]
To obtain the no auditing threshold in terms of entrepreneur’s observable capital stock $\hat{k}_{NA}$, I compare between the equilibrium wage under audit (1.65) and the wage threshold for auditing (1.71) and solve for $k$:

$$k \leq \left( \frac{2A}{P - P^2} \right)^{\frac{1+a}{2a}} (\beta)^{-\frac{1+a}{2a}} P (z(1-\alpha))^{-\frac{1}{a}} = \hat{k}_{NA} \quad (1.72)$$

### 1.8.1.5 Numerical Exercise

The auditing and no auditing capital thresholds, $\hat{k}_A$ and $\hat{k}_{NA}$, as given in equations (1.70) and (1.72) are not the same. They differ by a factor $P$. For instance, if I assume that $A = 0.5$, $P = 0.6$, $\alpha = 0.3$, $\beta = 0.95$ and $z = 1$, then $\hat{k}_A = 76.77$ and $\hat{k}_{NA} = 46.06$.

![Figure 1.14: The auditing and no auditing capital thresholds.](image)

Figure 1.14 illustrates the issue, where in the interval ($\hat{k}_{NA} = 46.06$, $\hat{k}_A = 76.77$) no equilibrium exists, and the incentive compatibility breaks down. To further illustrate the issue, I conduct a numerical exercise where for different values of the capital stock $k$, I calculate the equilibrium wage rate in the audit and no audit cases. I assume the parameter values to be: $A = 0.5$, $P = 0.6$, $\alpha = 0.3$, $\beta = 0.95$, $\delta = 0.1$, $E = 800$ and $z = 1$, which give the wage threshold for auditing $\hat{w} = 2.09427$. 
Table 1.8: The equilibrium wage rate in the audit and no audit cases, that correspond to different values of the entrepreneur’s capital $k$.

Recall from equations (1.69) and (1.71) that auditing is preferred when $w \geq \hat{w}$ and no auditing is preferred when $w < \hat{w}$. As can be seen in Table 1.8, an equilibrium exists, and incentive compatibility is preserved in the intervals $[0, \hat{k}_{NA} = 46.06]$ and $[\hat{k}_A = 76.77, \infty)$. However, this does not apply in the interval $(\hat{k}_{NA} = 46.06, \hat{k}_A = 76.77)$. This is due to the functions $w_A^*$ and $w_{NA}^*$ not intersecting in the space of $k$ and $w$, as shown in Figure 1.15.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$w_A^*$</th>
<th>$w_{NA}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>1.80174</td>
<td>2.02716</td>
</tr>
<tr>
<td>$\hat{k}_{NA} = 46.06$</td>
<td>1.86139</td>
<td>2.09427</td>
</tr>
<tr>
<td>46.5</td>
<td>1.86544</td>
<td>2.09883</td>
</tr>
<tr>
<td>55</td>
<td>1.93913</td>
<td>2.18174</td>
</tr>
<tr>
<td>60</td>
<td>1.97846</td>
<td>2.22599</td>
</tr>
<tr>
<td>76.2</td>
<td>2.09066</td>
<td>2.35222</td>
</tr>
<tr>
<td>$\hat{k}_A = 76.77$</td>
<td>2.09427</td>
<td>2.35629</td>
</tr>
<tr>
<td>80</td>
<td>2.11427</td>
<td>2.37879</td>
</tr>
</tbody>
</table>

Figure 1.15: A plot of the functions $w_A^*$ and $w_{NA}^*$, in the space of $k$ and $w$. 
Note that $w_{NA}^*$ is always greater than $w_A^*$, since $P < 1$. Counterfactually, if the functions are hypothetically given as in Figure 1.16, then there will be a single capital threshold for the auditing decision and an equilibrium will exist over the entire capital space.

![Figure 1.16: A hypothetical plot of the functions $w_A^*$ and $w_{NA}^*$ in the space of $k$ and $w$, when there is a unique capital threshold for auditing.](image)

I have amended the assumption on capital depreciation, such that capital always depreciates between $t = 1$ and $t = 2$. However, I arrive at the same conclusion, where an equilibrium does not always exist over the entire capital space. In addition, I have also tried to change the assumption on the enforceability of debt repayment, such that an unaudited entrepreneur is collaterally constrained in her borrowing. In this case, the wage threshold for the household’s auditing decision is given by:

$$ w \geq \left( \frac{2A}{\beta(P-1)} \right)^{\frac{1}{2}} = \bar{w} $$

(1.73)

The above expression implies that $\bar{w} < 0$, since $P \in (0,1)$. The same outcome is obtained when solving for the threshold level of no auditing (i.e. when $EU_{NA}^H \geq EU_A^H$). Since prices are positive, this suggests that for any wage rate that arise in equilibrium, the household will always choose to audit. Accordingly, the model fails in answering the first research question.
1.8.2 Choice of Parameters’ Values

1.8.2.1 The Prior Probability of Entrepreneurs’ Success, $P$

According to ONS (2012), the two years average survival rate for UK businesses just before the recession was around 80 per cent. I have chosen the two years average survival rate, since Crawford et al. (2013) results cover the period of 2008 and 2009. Moreover, according to the US Small Business Administration, SBA (2016), the one-year average survival rate for US businesses just before the recession started was around 80 per cent. I have chosen the one-year average survival rate, since Kudlyak and Sanchez (2017) results cover one year of recession only.

1.8.2.2 The Lenders’ Bargaining Power, $\gamma$

Instead of justifying the precise choice for the value of $\gamma$ that was reported in the results, I explain the reason behind choosing different values for $\gamma$ between the UK and the US. In an investigation by the UK Competition and Markets authority (CMA (2016)), it was found that the biggest four banks in the UK hold between them around 85 per cent of the banking sector that deals with firms. SNL Financial which is a subsidiary of S&P, has complied data on the US banking concentration.\textsuperscript{45} It found that the share of the industry’s assets held by the biggest five American banks, have grown from around 10 per cent in 1990 to more than 44 per cent by 2014. Moreover, the percentage of total liabilities and assets held by the biggest five banks as a share of the industry’s total is larger in the UK compared to the US (Bank of International Settlements (2001)). Accordingly, although the banking concentration in the US is high as well, it is not as severe as in the UK.

1.8.3 Robustness Checks

In this section I perform robustness checks. I replicate the numerical exercises presented in the results section of the chapter, under other possible scenarios. Precisely, I check the robustness of the results to the economy remaining in the same

borrowing constraints regime, binding or non-binding, following the shocks. The results robustness is also examined to a change in the lenders’ bargaining power. In all cases, the results remain intact.

1.8.3.1 Borrowing Constraints’ Regimes

In what is shown below, results remain qualitatively the same. The initial values of \( z \) and \( q \) and their values after the shocks \( z' \) and \( q' \), were chosen such that if the economy has started in a situation of binding or non-binding borrowing constraints, it would remain in the same situation following the shocks. To make those economies comparable to the economy presented in the results earlier, the ratios \( z/q \) and \( z'/q' \) for the economies considered were maintained the same as in the economy presented in the results earlier.

1.8.3.1.1 Binding Borrowing Constraints

As can be seen in Table 1.9, the economy starts from a situation of binding borrowing constraints and remains in that regime following the shocks. Additionally, Figure 1.17 demonstrates that the model’s qualitative results stay intact.

<table>
<thead>
<tr>
<th>Pre-shock</th>
<th>Post-shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>0.8</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.3</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.5</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.1</td>
</tr>
<tr>
<td>( A )</td>
<td>30</td>
</tr>
<tr>
<td>( z )</td>
<td>3</td>
</tr>
<tr>
<td>( q )</td>
<td>1.6856</td>
</tr>
<tr>
<td>( \bar{q} )</td>
<td>0.29319</td>
</tr>
<tr>
<td>( \hat{k} )</td>
<td>20.364</td>
</tr>
</tbody>
</table>

Table 1.9: The economy stays in a binding borrowing constraints regime; in the absence of a shock, and when a 10 per cent positive shock to \( q \) combined with a 2 per cent negative shock to \( z \) occur.
Figure 1.17: The results for a 2 per cent negative shock to $z$ combined with a 10 per cent positive shock to $q$, while the economy remains in a binding borrowing constraints regime.

1.8.3.1.2 Non-Binding Borrowing Constraints

As can be seen in Table 1.10, the economy starts from a situation of non-binding borrowing constraints and remains in that regime following the shocks. Additionally, Figure 1.18 demonstrates that the model’s qualitative results continue to hold.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$A$</th>
<th>$z$</th>
<th>$q$</th>
<th>$\bar{q}$</th>
<th>$\hat{k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>1</td>
<td>0.5612</td>
<td>11.417</td>
</tr>
<tr>
<td>Post-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>0.98</td>
<td>0.61732</td>
<td>12.2124</td>
</tr>
</tbody>
</table>

Table 1.10: The economy stays in a non-binding borrowing constraints regime; in the absence of a shock, and when a 10 per cent positive shock to $q$ combined with a 2 per cent negative shock to $z$ occur.
1.8.3.2 Lenders’ Bargaining Power

I examine the robustness of my results to a change in the lenders’ bargaining power. I replicate the numerical exercises for two alternative values of $\gamma$, 0.01 and 0.99.

1.8.3.2.1 Setting $\gamma = 0.01$

As can be seen in Table 1.11, the economy starts from a situation of binding borrowing constraints and remains in that regime following the shocks. Additionally, Figure 1.19 demonstrates that the model’s qualitative results remain unchanged.

<table>
<thead>
<tr>
<th></th>
<th>$P$</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$A$</th>
<th>$z$</th>
<th>$q$</th>
<th>$\bar{q}$</th>
<th>$\hat{k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.01</td>
<td>0.1</td>
<td>30</td>
<td>2</td>
<td>1.124</td>
<td>0.621018</td>
<td>9271.87</td>
</tr>
<tr>
<td>Post-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.01</td>
<td>0.1</td>
<td>30</td>
<td>1.96</td>
<td>1.2364</td>
<td>0.664279</td>
<td>9652.75</td>
</tr>
</tbody>
</table>

Table 1.11: A 10 per cent positive shock to $q$ combined with a 2 per cent negative shock to $z$, while setting $\gamma = 0.01$. 

Figure 1.18: The results for a 2 per cent negative shock to $z$ combined with a 10 per cent positive shock to $q$, while the economy remains in a non-binding borrowing constraints regime.
Figure 1.19: Investments’ responses to a 10 per cent positive shock to $q$ combined with a 2 per cent negative shock to $z$, while setting $\gamma = 0.01$.

1.8.3.2.2 Setting $\gamma = 0.99$

As can be seen in Table 1.12, the economy starts from a situation of non-binding borrowing constraints and remains in that regime following the shocks. Additionally, Figure 1.20 demonstrates that the model’s qualitative results remain unchanged.

<table>
<thead>
<tr>
<th></th>
<th>$P$</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$A$</th>
<th>$z$</th>
<th>$q$</th>
<th>$\bar{q}$</th>
<th>$\bar{k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>2</td>
<td>1.124</td>
<td>15266.1</td>
<td>28.8392</td>
</tr>
<tr>
<td>Post-shock</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>30</td>
<td>1.96</td>
<td>1.2364</td>
<td>16329.5</td>
<td>30.8482</td>
</tr>
</tbody>
</table>

Table 1.12: A 10 per cent positive shock to $q$ combined with a 2 per cent negative shock to $z$, while setting $\gamma = 0.99$. 
Figure 1.20: Investments’ responses to a 10 per cent positive shock to $q$ combined with a 2 per cent negative shock to $z$, while setting $\gamma = 0.99$. 
Chapter 2

Wealth Shocks, Inequality and Labour Productivity

2.1 Introduction

Prior to the Great Recession, the UK enjoyed a strong growth in labour productivity. It grew at a rate of 2.2 per cent per annum between 2000 and 2007 (Weale (2014)). Among the G6 industrialised countries, the UK’s labour productivity was only surpassed by the US. Nevertheless, the Great Recession which started as a global financial crisis originating in the US banking industry, has brought an end to this period of sustained growth. The recession in the UK has caused the biggest decline in output since the Second World War, where GDP contracted by 6.3 per cent between 2008Q1 and 2009Q2. In the same time, employment remained relatively robust, dropping by 2.1 per cent during the same period. Consequently, labour productivity has dropped across the whole economy; it was not concentrated in any particular sector (Crawford et al. (2013); Figure 2.1).

In the subsequent recovery, the UK’s real GDP growth was stronger than most G7 countries. However, it was characterised by a particularly poor labour productivity performance. In the decade before the recession, 77 per cent of growth in GDP was due to growth in labour productivity, with the remainder attributed to increase in labour hours. The picture contrasted in the recovery, with only 9 per cent coming from improvements in labour productivity. The economic recovery depended heavily on increasing employed labour instead of improvements in labour productivity, whether measured as output per worker or per hour (Cunliffe (2015)). This gave rise to the UK’s ‘productivity puzzle’ (Figure 2.2).

The decline in labour productivity growth is of a major concern to policy makers. Productivity is the main determinant of living standards and it is important to

46 Here and after, I refer to labour productivity by productivity, unless otherwise stated.
47 Due to the large number of figures in the second chapter, they have been included in a special section in the appendix. Please check section 2.6.1.
understand the reasons behind its growth decline to draw the correct economic policies. Rising labour productivity contributes to lower inflation as firms can produce at lower costs, hence it affects the supply side of the economy. Moreover, it is the key determinant of increases in real income, and therefore affects the demand side of the economy as well (Cunliffe (2015)). Inflation in the medium term is determined by the balance between what is demanded and the economy’s supply capacity. Accordingly, it is fundamental to the task of the monetary authority. It is also important for the government finances, as it tries to bring the deficit back in control through its austerity measures (Barnett et al. (2014)).

Decline in labour productivity growth is a repeated theme in all advanced economies following the financial crisis. Labour productivity is lower today in all G7 countries than it would have been, if the pre-recession trends have continued uninterrupted. Nevertheless, the decline in UK’s labour productivity growth has been the largest among all G7 countries. Productivity has nearly flat-lined since 2011 at its pre-crisis level, and by 2014 it was around 18 per cent below its pre-recession path. This compares to a gap of 7 per cent for the rest of the G7 countries (ONS (2015); Figure 2.3). Moreover, some other developed countries hit by the financial crisis have experienced different scenarios. For instance, although Germany had a weak labour productivity growth following the recession, it was concentrated in the export sectors. On the other hand, labour productivity in the US has increased during the crisis, as the decline in labour hours was greater than the GDP contraction (Bryson and Forth (2015); Figure 2.4). Furthermore, the recent decline in labour productivity growth is very different from previous economic downturns in the UK; it was more severe with the recovery being more sluggish (Riley et al. (2015); Figure 2.5). Although the crisis started as a shock to the global financial markets, the previously mentioned empirical facts lead us to believe that the marked decline in UK’s labour productivity growth, is more due to domestic rather than global factors.48

Many empirical and theoretical studies have tried providing explanations for the UK’s productivity problem. Some attribute it to a deficient demand since 2008 such as Martin and Rowthorn (2012), while others like Goodridge et al. (2013) blame

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48 It is worth noting that output statistics are not very accurate and measuring public sector labour productivity has been particularly challenging (Emmerson et al. (2013)). Additionally, Barnett et al. (2014) have estimated that around 25 per cent of the estimated productivity shortfall is attributed to measurement issues. Although this is quite sizeable, it cannot explain the significant productivity gap since those measurement issues have been persistent for years.
it on structural supply side reasons. It is probably the case that many factors have contributed to a poor UK labour productivity growth, as the Bank of England researchers Barnett et al. (2014) have suggested. In this chapter, I present a simple theoretical framework with a closed form solution, that qualitatively generates such outcomes. I consider the empirical regularities that took place during the recession and the following recovery. In considering the empirical regularities, I focus on the most significant phenomena. In other words, I depend on those factors that were more important and unique in the UK case compared to other advanced economies, through surveying the available literature. I later use those to motivate the assumptions used in building the theoretical model, which shows how such factors interact in a way that manifests itself in a prolonged low labour productivity.

The model presents a supply side explanation, where more persistent factors related to the financial crisis have led to changes in the supply side of the economy. This view is also shared by the Bank of England economists. The poor output growth and labour productivity performance has occurred at a time when employment has been expanding (Barnett et al. (2014); Broadbent (2012)). Most of the theoretical literature that ties financial frictions with productivity, addresses the issue from the angle of resources misallocation, whether it was labour or capital. However, most of the decline in UK labour productivity occurred within firm rather than through depressed external restructuring (BIS (2011); Crawford et al. (2013); Riley et al. (2015); Riley and Rosazza-Bondibene (2016); Schneider (2018)). Accordingly, my theoretical explanation will seek to tie financial frictions with production changes within firms.\textsuperscript{49} The results are achieved by a version of the real business cycle model that incorporates wealth shocks and wealth redistribution taxes which affect wealth inequality.

The chapter proceeds in the following order: Section 2.2 presents the empirical regularities and the literature that provides the motivation for the assumptions of the theoretical framework. Section 2.3 presents a theoretical model incorporating wealth shocks only, solves for the equilibrium and discusses its properties. In section 2.4, I amend the model by adding redistributive wealth taxes and discuss the results. Finally, section 1.7 concludes and suggests venues for future research.

\textsuperscript{49} More precisely, providing a theoretical explanation to why wealth shocks affect aggregate labour productivity.
2.2 Literature Review

Labour productivity is composed of three main components: (i) the degree to which factors of production are utilised within firms, (ii) the amount of capital inputs made available per unit of labour input,\(^{50}\) and (iii) the efficiency of combining the factors of production together in the production process, which is also known as total factor productivity (TFP). A cyclical reasoning for the decline in labour productivity growth, essentially depends on a deficient demand. As illustrated in Figure 2.6, in this scenario there will be a temporary opening up in firms’ spare capacity, as firms scale down their production in response to weak demand conditions. If this is true, then we would expect labour productivity to recover once the crisis subsides. Alternatively, more persistent factors related to the financial crisis could have either affected the degree of capital deepening, TFP or both, resulting in a prolonged impact on labour productivity by affecting the supply side of the economy (Barnett et al. (2014)). Oulton and Sebastia-Barriel (2013) have tested the later hypothesis using a panel of 61 countries, between 1955 and 2010. They found evidence that financial crises result in a greater decline in labour productivity in the short run compared to other types of recessions. Their results also established that productivity suffers from a permanent damage in the long run through capital shallowing. In what follows, I present explanations discussed in the literature that fall under both hypothesis, and examine those in greater depth using empirical studies and data.

The literature review is divided into three sections, where the issue of declining labour productivity in the UK is surveyed. The first section presents the empirical regularities that characterised the UK’s economy since the outbreak of the financial crisis. Those are the movements in variables and prices, which the theoretical framework tries to qualitatively replicate. The second section presents a number of theories that were proposed to explain the productivity puzzle. However, I also provide evidence from the data and empirical studies that refute those explanations. Finally, the third section presents a number of empirical phenomena which help in explaining parts of the puzzle. Nevertheless, each of those phenomena, on their own, cannot provide a full explanation of the facts that accompanied the stagnation in productivity. The model incorporates those explanations in a single theory that provides a plausible

\(^{50}\) Also called capital deepening and capital intensity.
mechanism, which explains how the movements in variables and prices came into place since 2008.

### 2.2.1 The UK’s Economy Since 2008: Output, Factors of Production, Labour Productivity and Prices

During the recession, real GDP fell by more than 6 per cent relative to its pre-recession peak. Although it has grown since from its lowest point in the middle of 2009, the loss in real GDP was not recovered until the middle of 2013. Real GDP has been recovering steadily since the middle of 2009 (Figure 2.7). Moreover, when compared to previous recessions in the UK, the contraction in GDP was greater, and the recovery was much slower (Figure 2.8).

Despite the fall in employment following the recession, the decline was small compared to the two recessions that preceded the financial crisis (Figure 2.9). The reduction in employment was substantially smaller than the decline in output, and it recovered faster. Labour input, whether measured at labour hours or quality adjusted labour input, has grown strongly in the recovery, surpassing its pre-crisis level by the beginning of 2012 (Figure 2.10; Figure 2.11). By mid 2017, UK unemployment rate has fallen to a four decades low, reaching to 4.3 per cent (Figure 2.12). The muted response of employment during the recession and its strong growth in the recovery, has been attributed to the fall in real wages, which were unusually flexible. This has resulted in a lost decade in real pay rise (Haldane (2018); Figure 2.13; Figure 2.14).

Since demand falls in recessions, firms adjust their expectations and hence it is normal for investment to fall. However, when comparing it with previous recessionary episodes, investment has fallen more sharply and remained weak for a long time (Emmerson et al. (2013); Figure 2.15). Relative to a continuation of its pre-recession trend, investment has fallen by around 10 per cent during the crisis, and it has only returned to its 2007 levels by the ends of 2014 (Crawford et al. (2013); Figure 2.16).

The 2008 recession has brought about a large decline in the cost of capital (Chadha et al. (2017)). Moreover, the Bank of England has drastically cut down the official interest rate, which reached close to the zero-lower bound in order to support the economy. Despite those two facts, there was a sharp increase in the cost of financing capital purchases during the crisis (Benito et al. (2010)). This is because the
lower official interest rate did not pass through to lower retail interest rates that firms faced, since the additional risk premiums charged by banks have went up (Emmerson et al. (2013)). By 2012, the cost of capital faced by large firms have increased to 10 per cent, compared to 8 per cent before the financial crisis. The expected increase in the financing costs for SMEs was even greater, since they are perceived to be riskier (Broadbent (2012)). A good indicator of changes in the financing costs faced by firms, is the credit default swaps (CDS) premiums, which banks need to pay to insure their unsecured bonds. Those premiums are highly correlated with banks’ funding costs and credit conditions. The CDS premiums for all major UK banks were substantially low before the crisis and close to zero. However, they have went up sharply during the crisis and never came down until 2013 (Riley et al. (2014); Figure 2.17). Even when those premiums came down eventually, they remained higher than the substantially low levels they were at in the previous decade. This was already expected since the crisis, due to the systematic under-pricing of risk pre-2008 (BIS (2011)).

Given the movements in output and employment, labour productivity dropped during the crisis. It started growing again following the crisis, and it regained its losses by 2011. Following this initial phase, labour productivity has nearly frozen despite the growth in GDP. Instead of firms achieving more output from their existing employees through productivity improvements, nearly all of the expansion in GDP was facilitated by increasing the workforce. This meant that labour productivity remained mostly flat since 2011 (Cunliffe (2015); Figure 2.2). The crisis was also characterised by a deterioration in the capital-labour ratio within firms, which led to capital shallowing. This was due to the moderate decline in employment during the crisis and its strong growth in the following years, combined with the sharp reduction in investment during the crisis and its sluggish growth during the recovery (Patterson (2012); Franklin et al. (2015); Roland and Valero (2015)). It was estimated that the amount of capital per worker has dropped by 9 per cent during the recession. This has slightly improved during the recovery; however, by mid 2012 the amount of capital per worker was still 5 per cent lower than its level in 2008Q2. This was attributed to the change in relative factor prices (Pessoa and Van Reenen (2013)).
2.2.2 Dismissed Theories

In this section I present a number of theories that were discussed in the literature as an explanation to the productivity problem, and the motivation behind them. Those are labour hoarding, inefficient allocation of resource due to the financial nature of the crisis and the declining R&D intensity of the UK’s economy. However, I also provide evidence from the data and empirical studies that refute those explanations.

2.2.2.1 Labour Hoarding

Labour hoarding has been suggested as a reason behind the persistent productivity issue. This argument essentially depends on a deficient demand reasoning. It refers to a situation when firms continue employing more labour input than is required by their current production plans, which have been scaled back in response to weak demand conditions. Accordingly, firms’ labour input will be underutilised and hence labour productivity drops. The rationale behind this firms’ behaviour is to reduce the costs associated with varying their labour input over the business cycle, especially when the reduction in demand is not expected to last for a long time. Firms can face substantial costs when firing their workers, due to workers protection laws and severance packages. Moreover, when the economy recovers and demand picks up, firms incur further search costs in filling their vacancies. Those costs can increase further, if the vacant jobs require specific skills. Firms will have to retrain new workers to replace the workers who were already trained but lost during the economic downturn (Crawford et al. (2013)). In a survey conducted by the Chartered Institute of Personnel and Development, CIPD (2012), the main reason cited by employers for retaining more workers than needed during the crisis, was the loss of skills that are specific to their firms.

In most previous recessions in the UK, labour productivity has dropped due to labour hoarding. It is argued that during the recent crisis, firms have hoarded labour by a greater extent than before, due to the flexibility exhibited by real wages. Therefore, most of the adjustment in firms’ labour costs came through a reduction in real wages, rather than a reduction in labour hours. This means that the low labour productivity was a natural symptom of an economy characterised by cheap labour and depressed demand, and not necessarily caused by a persistent damage to the supply
side (Martin and Rowthorn (2012)). Indeed, when the crisis subsided, output per hour rose by 3 per cent between 2009 and 2011 (Grice (2012)).

Although labour hoarding might explain the decline in labour productivity during the crisis, this argument carries less weight in the years following the recession. As described earlier, there is a rationale for firms to hold into underutilised workers when demand is depressed due to the costs of firing, rehiring and retraining. However, with labour productivity nearly frozen at the 2008 level for years, such rational erodes. This is because the financial burden from holding those workers, surpasses the costs of varying the labour input. In addition, it is important to note that the UK productivity puzzle is actually a twin puzzle; the stagnating labour productivity and the strong growth in employment. In the recovery firms went on a hiring spree, instead of achieving more output from their existing employees through productivity improvements. In other words, labour hoarding is inconsistent with the strong gains in employment at a time of moderate GDP growth (Patterson (2012); Goodbridge et al. (2013); Figure 2.10; Figure 2.11). It is also inconsistent with the fact that the strong growth in employment was more due to increase in the flows into employment rather than a decrease in the flows out of it (Figure 2.18).

Labour hoarding depends on a deficient demand reasoning, which leads to low capacity utilisation within firms. However, evidence from the Bank of England surveys suggest that capacity utilisation has been improving since mid 2009, and firms are now operating close to their full capacity (BoE (2012) and (2015a)). Similarly, the Confederation of British Industry has reported in its 2015 industrial trends survey, that the proportion of British firms operating below their full capacity, is at its lowest since records began in 1998 (CBI (2015)). Finally, a strong indicator of the recovery in demand following the crisis is the household’s saving ratio, which has dropped from its 11.9 per cent peak in 2010 to 4.4 per cent in 2015 (Figure 2.19). Consumer confidence has been improving since 2010. An important driver of GDP growth in the recovery has been the growth in household consumption, which exceeded the growth in household income (ONS (2016b)). Therefore, demand explanations have become less compelling over time.
2.2.2.2 Inefficient Allocation of Resources

A wide economic literature suggests that recessions triggered by financial crises are more severe and persistent (for example check Minsky (1992); Kindleberger and Aliber (2005); Reinhart and Rogoff (2009)). A damaged financial system where banks are focused on re-building their balance sheets, can impede the cleansing effect of recessions, by preventing the efficient reallocation of factors of production to sectors and firms that are more productive. In this section, I present the empirical literature that investigates the extent to which this phenomenon has affected aggregate labour productivity. The section is divided into three subsections that look at: (i) the divergence in productivity between different economic sectors following the crisis, (ii) the reduction in the cleansing effect of the recession, which improves resource allocation across firms, and (iii) the extent of loan forbearance carried out by UK banks, which may have inhibited capital being reallocated to the most productive firms.

2.2.2.2.1 The Divergence in Sectors’ Productivities

The burst of the bubble in the finance and insurance sector, has been put forward as one of the explanations for the decline in labour productivity growth (Figure 2.20). The argument is that the UK has become too dependent on the financial sector to boost its strong performance in the years before the financial crisis, and hence there should be a greater focus on manufacturing. However, this argument ignores the fact that only 0.4 per cent of the total 2.8 per cent annual growth in the UK market economy between 1997 and 2007, was attributed to the financial sector. A similar argument is proposed with regard to the falling productivity in the mining sector; specifically the declining productivity in the North Sea oil sector. Nevertheless, the steady decline of those remaining oil reserves has been taking place since 1999, and therefore cannot explain the sudden stagnation of labour productivity following the recession. The bulk of the 1997 to 2010 improvements in the UK economy were real and not due to the bubble sectors of finance, property and oil (Corry et al. (2011)).

Different economic sectors do differ between them in their productivity levels, nevertheless, labour productivity growth has dropped across the whole economy during the recent recession (Crawford et al. (2013)). Productivity returned to growth in some sectors following the crisis; most notably the administrative and support
service activities. This sector includes firms that operate in outsourcing and recruitment, such as Capita. Accordingly, the rising labour productivity in this sector can be misleading, due to the expansion in employment that occurred across the whole economy (Dolphin and Hatfield (2015)).\textsuperscript{51} As can be seen in Table 2.1 below, nearly all economic sectors have been contributing to the growth in labour productivity before the crisis, though with varying degrees. This has drastically changed since, with nearly all sectors making zero or close to zero contributions. Therefore, what caused UK labour productivity to nearly flat-line, must have occurred at an economy wide level, rather than through variations between economic sectors. The phenomenon of output expansion through employing more labour, was the dominant theme across most sectors (Haldane (2017)). A similar conclusion has been reached by Riley and Bondibene (2016), who used firm-level data between 1998 and 2013 to study the origins of productivity growth in the UK. They confirm that firm-level productivity has frozen within most sectors. Therefore, any explanation to the UK productivity puzzle cannot stem from divergence in different sectors’ productivities.

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|}
\hline
 & 1998-2008 & 2009-2016 & Difference \\
\hline
Agriculture, forestry and fishing & 0.0 & 0.0 & 0.0 \\
Mining and quarrying & -0.1 & -0.1 & 0.0 \\
Other manufacturing & -0.1 & -0.2 & -0.1 \\
Transport equipment & 0.0 & 0.1 & 0.0 \\
Utilities & 0.0 & 0.0 & -0.1 \\
Construction & 0.1 & 0.0 & -0.1 \\
Wholesale and retail & 0.1 & 0.1 & 0.0 \\
Financial and insurance activities & 0.3 & -0.2 & -0.5 \\
Real estate activities & 0.2 & 0.1 & 0.1 \\
Information and communication & 0.4 & 0.1 & -0.3 \\
Administrative and support service activities & 0.2 & 0.2 & 0.0 \\
Professional, scientific and technical activities & 0.3 & 0.2 & -0.2 \\
Other services & 0.4 & 0.0 & -0.4 \\
Total & 1.9 & 0.2 & -1.7 \\
\hline
\end{tabular}
\caption{Economic Sectors’ contributions to annual labour productivity growth in the UK (Source: Haldane (2017)).}
\end{table}

\textsuperscript{51} In other words, due to the high labour supply, outsourcing and recruitment officers can possibly achieve more job matches than they would have, if the labour market was under different conditions.
2.2.2.2  External Restructuring

Firms vary in their labour productivity levels. Firms can improve their productivity by employing more physical capital, or by investing more in intangibles such as software and R&D. However, an impaired financial sector can prevent or slow the efficient reallocation of capital from less to more productive firms. This happens when capital is locked under unproductive surviving firms, while more productive existing firms cannot expand, or potential productive entrants cannot enter the market without the necessary capital outlay. In other words, the cleansing effect of recessions – also called external restructuring – which is a key source of productivity improvements, is being impeded (Riley et al. (2015)).

Nevertheless, contrary to what is expected in a financial crisis, most of the decline in UK labour productivity growth cannot be explained by a depressed external restructuring. BIS (2011), Crawford et al. (2013) and Barnett et al. (2014), have confirmed that most of the decline in labour productivity growth was due to a stagnating within firm productivity.

Riley et al. (2015) have conducted an extensive empirical investigation, reporting on the effect of the banking crisis on external restructuring in the UK. Their research concludes that the decline in labour productivity has occurred within firms. They found that external restructuring has actually contributed positively to labour productivity during the crisis and following recovery. However, the sizeable decline in the within firm productivity, has more than offset the positive effect of resource allocation, such that growth of aggregate labour productivity has dropped (Figure 2.21). Their results hold across key groups of firms’ characteristics, whether depending on firm size or type of economic activities, even in those sectors that are capital intensive such as manufacturing. In all those cases, the within firm component was the key driver of the movements in aggregate labour productivity, rather than external restructuring. Furthermore, they have compared their results with the productivity dynamics of the 1990s recession, which was not triggered by a banking crisis.

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52 External restructuring can be decomposed into three components. The first is the between component, which is the productivity improvement that is due to reallocating capital between existing firms, such that the surviving firms with viable productive projects can expand. The second is the entry component, which is the productivity improvements that come from new firms that enter the market with productive projects. However, those firms may find it difficult to enter, if their projects require an upfront investment in capital. The last is the exit component, which is the productivity improvement that comes from unproductive firms exiting the market. A banking crisis can negatively affect those three components, if banks forbear debt to unproductive surviving firms due to the impact on their balance sheets (Riley et al. (2015)).
crisis. They found that the size of the positive contribution made by resource reallocation to aggregate labour productivity, was even greater during the recent recession. Therefore, they conclude that in terms of recessions’ cleansing effects, the recent recession was not different from other “normal” recessions.

If external restructuring contribution to productivity growth was less than expected during the recent crisis, then it was due to a significantly lower productivity of new entrants. This is what Riley and Bondibene (2016) have confirmed in their empirical study. They found that new entrants between 2010 and 2013, had lower productivity compared to incumbent firms. This has differed from previous recessionary episodes, when those new firms had similar productivity to the average firm in their industry. Accordingly, the financial crisis did not prevent productive new firms from entering, but rather those entrants were less productive than expected based on historical experiences. In addition, they confirmed the findings of previous empirical studies, which found that most of the decline in aggregate labour productivity relative to a continuation of its pre-recession trend, is due to stagnant productivity growth within incumbent firms.

When looking at the distribution of firms’ productivity, the evidence is inconsistent. Haldane (2017) argues that despite the large dispersion in productivity between firms in the UK, this dispersion did not grow larger after the recession. Moreover, he also suggested that the proportion of low-productivity firms has become smaller after the crisis. Haldane (2018), contradicts the earlier conclusion and argues that the gap between the highest and lowest firms in terms of productivity, has grown much faster since the recession in the UK compared to its other developed competitors (Figure 2.22). However, those top performing companies which according to Haldane (2018) have increased the dispersion of firms’ productivity since 2008, only represent the top 1 per cent of the distribution. Consequently, it cannot explain the decline in labour productivity growth across the whole economy. Furthermore, the empirical investigation conducted by Schneider (2018), contradicts with the conclusion of Haldane (2018). He has used firm level data to decompose productivity distributions, and found that that the dispersion in firm level productivity has actually slowed down since the crisis. He concluded that the decline in aggregate labour productivity growth, can be entirely blamed on the slowdown of productivity growth in the most productive firms in the economy since 2008. This conforms with Benito et al. (2010) findings, who concluded that firms with riskier activities, which are typically more productive
and innovative, have faced severe difficulties in obtaining working capital during the crisis, and thus has negatively affected their productivity.

2.2.2.2.3   Loan Forbearance

A banking crisis can inhibit capital being reallocated to the most productive firms, when banks forbeard debt to unproductive surviving firms in an attempt to protect their balance sheets.\(^{53}\) Arrowsmith et al. (2013) of the Bank of England, have investigated the extent and impact of loan forbearance to SMEs by major UK banks, on aggregate labour productivity.\(^ {54}\) It was estimated that around 14 per cent of total lending exposure to SMEs, was falling under forbearance by 2013. Nevertheless, two third of this forbearance exposure was judged by banks’ relationship managers as being extended to productive firms, that were facing temporary difficulties. Moreover, in very few cases did loan forbearance by banks take the form of direct debt relief to failing businesses. The vast majority of loan forbearance was granted through wavering loan covenants such as debt to income ratios, or through extending loan repayment periods for firms.

This contrasts with the “evergreening” policy during the 1990s Japanese recession, which was blamed for the survival of many unhealthy firms. In the 1990s, Japanese banks extended additional loans to struggling firms, to allow them to make their interest payments or cover their expenses to prevent them defaulting (Peek and Rosengren (2005); Caballero et al. (2008)). Arrowsmith et al. (2013) have found that UK firms which received assistance through forbearance were significantly less productive. However, they estimated that the impact of this phenomenon was to reduce aggregate labour productivity by only 1 per cent. This was at a time when the gap between labour productivity and a continuation of its pre-recession trend, was measured at 18 per cent. Accordingly, this cannot present a leading explanation to the productivity puzzle.

\(^ {53}\) Loan forbearance is a form of assistance provided by banks to their struggling business debtors, in order to allow them to meet their debt obligations. It is not part of banks’ normal activities, but rather an exceptional measure that is taken to protect banks’ interests. Banks may choose to employ forbearance rather than classify loans as non-performing, to avoid writing off those loans and thus damaging their own balance sheets (Arrowsmith et al. (2013)).

\(^ {54}\) The investigation used a definition of SMEs as being companies with turnover below £50 million.
2.2.2.3 Declining R&D Intensity leading to a decline in TFP

During the recession, physical and intangible capital investment declined. Nevertheless, investment in R&D was the least affected when compared to other types of investment (Figure 2.23). Moreover, in the recovery investment in R&D grew strongly at a rate that exceeded its pre-crisis growth rate (Figure 2.24). Despite this, the UK suffers from a chronic underinvestment when compared to its main developed competitors. It is particularly the case when looking at R&D investment (Roland and Valero (2015); Figure 2.25).

This is important, because some studies such as Goodridge et al. (2013 and 2015) and Jones (2016), attribute the decline in labour productivity growth to the historical decline in R&D expenditure. The view of those economists, who advocate a supply side explanation, is that the primary reason behind the stagnant labour productivity, is a drop in TFP that occurred due to the declining British economy R&D intensity over the past 30 years. For instance, Goodridge et al. (2013) have estimated that a 10 per cent reduction in public R&D would negatively impact private sector TFP by 1.46 to 1.49 per cent per annum. Additionally, Goodridge et al. (2015) have estimated that between 2003 and 2013, public sector real expenditure on R&D has dropped by 5 per cent, which lead to a further negative effect by crowding out private expenditure on R&D. However, this reasoning stops short at explaining why such long declining R&D intensity had a sudden dramatic effect on labour productivity in the past crisis. This explanation is puzzling, especially when considering that the benefits from R&D are slow to materialise, require substantial changes in business processes, and have gradual effects on productivity (Carson et al. (1994) and David (1990)).

On the other hand, it is worth noting that more than half of the average annual growth in output between 1998 and 2010, was due to growth in the capital input. This compares to a contribution of only 0.1 per cent from growth in TFP, to the total 2.1 per cent annual growth in output over the same period (Figure 2.26). Pessoa and Van Reenen (2013) have argued in their empirical work that TFP has been quite resilient during the recent crisis, with its trend being very similar to the 1970s and 1980s. Their estimates of decomposing sources of changes in productivity, suggest that most of the decline in its growth was due to capital shallowing within firms. They attribute that to the change in relative factors’ prices during the recession and the following recovery, which was due to the credit crunch and the unusual flexibility of real wages.
2.2.3 Other Theories

In this section I present a number of empirical phenomena which help in explaining parts of the productivity puzzle. Nevertheless, each of those phenomena, on their own, cannot provide a full explanation of the facts that accompanied the stagnation in productivity. The model incorporates those explanations in a single theory that provides a plausible mechanism, which explains how the movements in variables and prices came into place since 2008.

2.2.3.1 Wealth Shocks and Labour Supply

One of the puzzling characteristics of the recent recession was the ability of firms to reduce real wages significantly, which was historically unparalleled (Emmerson et al. (2013)). One of the theories suggests that the reduction in trade unions’ influence and employees’ wage bargaining power, have reduced the resistance to such adjustments. However, Bryson and Forth (2015) refute this hypothesis, as they found little change during the previous decade in unions’ powers, membership proportions and premiums enjoyed by unions’ members. It is usual for labour force participation rates to drop during recessions, since the reduction in employment opportunities discourages job seekers from putting the additional search effort, during unfavourable economic times. Nevertheless, when compared to previous recessions, there was little change in labour participation rate during the recent crisis. Older workers participation has specifically remained mostly flat during the crisis. Additionally, the reduction in real wages and increased job insecurity, have encouraged second earners within households to seek employment (Andre et al. (2013); Figure 2.27).

The Great Recession has started with a large drop in asset prices, that later spread to a wide recession affecting the whole economy. A negative shock to households’ wealth can force many people to re-join the labour market, or to increase their supplied labour hours when they are already employed, in order to mitigate the loss in their wealth. Therefore, aggregate labour supply can increase (Cheng and French (2000); Banks et al. (2012)). There is a number of theoretical models that explain how households respond to financial developments, by using their labour supply. Those models combine self-insurance with flexible labour supply, to generate the aforementioned households’ behaviour in response to wealth shocks. This wealth effect on labour supply can be larger for older workers, especially when a wealth shock
renders their retirement savings insufficient. This is reflected in the theoretical models by a greater labour supply elasticity for this group of workers, as well as workers who are constrained in their borrowing (Low (2005); Attanasio et al. (2005)).

This theoretical prediction has been supported by empirical evidence, which attribute the increase in labour supply that persisted well into the recovery, to the decline in households’ wealth (BoE (2015b)). During the recession, UK’s household net real wealth has dropped by around 8 per cent. This was mostly due to a 10 per cent decline in net real property wealth, which constitutes most of households’ net real wealth (Figure 2.28). Although households’ net real wealth has recovered some of its loses, in mid 2014 it was still more than 2 per cent lower than its level in 2008. This recovery was due to the improvement in net real financial wealth, as net real property wealth remained 10 per cent lower than its level when Lehman Brothers collapsed (Figure 2.28 and Figure 2.29). The improvement in net real financial wealth was mostly due to the Quantitative Easing (QE) measures taken by the Bank of England (Bryson and Forth (2015)). However, those capital gains were not shared across homogeneously, since around 50 per cent of aggregate net real financial wealth is owned by the nation’s richest 5 per cent of households (Figure 2.31). The QE measures were vital in preventing a greater loss of jobs and contraction in GDP. Despite asset purchases through QE improving the wealth position of all households, it was the richer households who enjoyed a better improvement in their net real total wealth. Accordingly, the richer the household, the better was the recovery in its net real total wealth (Haldane (2016)). Furthermore, private pension wealth represents a sizeable proportion of the UK households’ net real wealth, and it has improved since the financial crisis (Figure 2.30). Pension funds are a major institutional holder of financial assets. Unfortunately, there is no data available that aid in mapping the holdings of financial assets by pensions funds to the different households’ holdings in those pension funds. Therefore, I abstract from including it in the analysis. However, the distribution of net private pension wealth is more unequal when compared to net property wealth (Figure 2.31).

The house prices bubble was more pronounced in the UK compared to France and Germany, with an arguably weaker welfare state. This might help explain why employment behaved differently in those countries following 2008 (Figure 2.31). Furthermore, by the beginning of the current century, early retirement offers became less attractive than before, with disability benefits becoming harder to access.
Additionally, pension schemes contributions rose, and low interest rates since the crisis have reduced pensions annuity incomes (Facci and Hackworth (2010); Banks et al. (2011)). It is worth noting that the biggest contributors to the positive labour supply shock, which continued well into the recovery, were: (i) existing employees seeking longer hours, (ii) people who did not retire, contrary to what was expected, and (iii) people who were not previously participating in the labour market (Cunliffe (2015)).

Benito and Saleheen (2011) have used data from the British Household Panel Survey, to investigate individuals’ response to financial shocks. They found that employees with a deteriorating financial status relative to what they have previously expected, have responded by increasing their labour hours. Disney and Gathergood (2014) have investigated UK households’ behaviour to changes in housing wealth. Their results confirmed the earlier predictions, with the most significant response exhibited by young married female house owners and older male owner. Moreover, they found that the wealth effect of change in house prices, had a stronger impact on labour supply and participation than changes within the labour market. Empirical evidence on wealth effects on labour supply are not limited to the UK. For example, Bottazzi et al. (2017) have used Italian data and confirmed the same wealth effects on labour hours, with older workers results being the most significant.

2.2.3.2 Credit Supply Shocks and Capital Shallowing

The credit channel is the transmission mechanism, through which shocks to the financial system can affect real activity. The destruction of banks’ intermediation capital can affect the real economy, by limiting credit supply and increasing the real financing costs for firms (Bernanke (1983); Paravisini et al. (2015)). The impact of adverse shocks to banks’ funding can be further amplified through an accelerator effect, due to the capital adequacy requirements imposed on banks by regulators. As banks try to rebuild their balance sheets, it is typically their small, young and high-risk activity customers that suffer the most from the credit crunch and rising financing premiums (Bernanke and Gertler (1989); Beck (2013)).

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55 The credit channel is also called the bank lending channel.
56 According to Basel accords which regulate banks’ lending activities, bank equity constrains bank ability to issue or renew loans. For instance, in its latest version, Basel 3, the risk weight assigned to large firms with the highest creditworthiness rating is 20 per cent. On the other hand, unrated small
Empirically, there is strong evidence on how increase in credit tightening and financial constraints, negatively affects real economic activity. In the early 1990s Japanese crisis, banks’ capital ratios have dramatically deteriorated due to large declines in property and financial stocks values. Peek and Rosengren (2000) have used this event as a natural experiment to evaluate the impact of credit crunches. They found that the crisis has caused significant negative impact on real output in some US states, as Japanese banks sought to decrease their foreign lending instead of contracting their domestic loan supply. The contraction in loan supply affects the real economy mainly through the credit channel, by limiting the private sector’s ability to finance its consumption and investments (Rauh (2006); Campello et al. (2010)).

As mentioned earlier, it is small, younger and high-risk activity businesses which suffer the most. Apart from their creditworthiness, those type of customers are usually limited in the number of banking relationships they have, and find it difficult to substitute away from their lenders during times of economic distress. Iyer et al. (2014) have used firm-level data on loans in Portugal, during the European interbank market crisis. They found that banks with higher interbank borrowing ratios have cut their credit lines to their business customers more severely, compared to other less affected banks. They also found that the decrease in credit supply was worse for smaller and younger firms. This is consistent with the empirical findings of Crawford et al. (2013), which were presented in the first chapter. The decline in the investments of UK SMEs was larger than big firms (Figure 1.1). Contraction in credit supply can negatively affect productivity too. Levine and Warusawitharana (2014) have used European firm-level data and found that reduction in credit supply can slow productivity growth within firms. Crawford et al. (2013) also found that labour productivity fell by a greater extent in UK SMEs compared to large firms.

In 2007, before the financial crisis hit, the flow of net bank lending in the UK to private non-financial corporations (PNFCs) stood at 12 per cent of GDP. However, over the course of the recession, it has contracted to less than 1 per cent by 2009 (Martin and Rowthorn (2012)). This was the largest and sharpest contraction that occurred in the real stock of PNFCs’ outstanding bank debt in all UK recessions post 1970. Its decline exceeded 20 per cent, relative to its pre-recession level (Riley et al. and medium businesses are assigned a risk weight which is 4 to 5 times higher. This implies higher capital requirements on banks, when lending to SMEs or risky businesses (DFID (2013)).
(2015); Figure 2.33). Although decline in bank lending can be due to both demand and supply factors, the empirical investigation of Bell and Young (2010) has concluded that most of the decline in outstanding bank credit since 2007 was due to supply tightening. Moreover, Barnett and Thomas (2013) have used a SVAR estimation to investigate the reasons behind the weak bank lending in the UK since the start of the financial crisis. They found that most of the weakness can be attributed to credit supply shocks. They also found that those credit supply shocks are responsible for nearly half of the gap between the current level of GDP and a continuation of its pre-crisis trend.

Despite the Great Recession starting as a crisis originating in the US banking industry, the credit crunch in the UK was relatively more severe (Figure 2.34). Part of this is explained by the higher concentration in the UK banking sector, which became even more concentrated due to the banking mergers following the crisis (Roland and Valero (2015)). Moreover, Kahle and Stulz (2013) have empirically investigated the effect of the credit crunch on US firms’ investment. Contrary to what is expected, they found that the investments of unleveraged firms and bank dependent firms, have dropped by similar proportions. They even found that bank-dependent firms have hoarded their cash holdings more than unleveraged firms during the recession. Accordingly, they concluded that the shock to the demand for goods and services was more important in the US, such that firms’ investment would react similarly, despite the differences in their financial situation. This can partially explain why there is no equivalent productivity puzzle in the US.

In the wake of the crisis, banks’ CDS premiums have increased substantially (Figure 2.17). Since those premiums are highly correlated with the costs of banks’ funding, banks have passed those additional costs to their business customers, despite the drastic cuts in the official interest rate by the Bank of England (Riley et al. (2014)). Consequently, much of the private sector’s planned investments through bank financing have likely rendered unprofitable, as the return on those capital expenditures might not be able to compensate for the higher finance charges that firms faced. This in turn might have contributed to the sharp decline in gross fixed capital formation.

57 An investigation by the UK Competition and Markets authority (CMA (2016)), has found that the biggest four banks in the UK hold between them around 85 per cent of the banking sector that deals with firms. Moreover, the percentage of total liabilities and assets held by the biggest five banks as a share of the industry’s total, is larger in the UK compared to the US (Bank of International Settlements (2001)). Accordingly, although the banking concentration in the US is high as well, it is not as severe as in the UK.
which negatively affected firms’ productive capacity (Grice (2012); Figure 2.16). The empirical study of Bond et al. (2003) lends support to this prediction, as they have found that the investment of British businesses is more sensitive to changes in external finance, compared to their counterparts in other European countries. Another support for the prediction that financial difficulties have hampered UK businesses’ productive capacity, can be found in the Confederation of British Industry (CBI) surveys. The proportion of firms reporting that their output has been constrained by their ability to source credit, has increased from around 3 per cent before the outbreak of the recession, to around 25 per cent by the beginning of 2009 (Figure 2.35).

There is a number of empirical studies that tried to investigate the link between the credit crunch and the decline in UK’s labour productivity. In their study of the effect of the banking crisis on external restructuring in the UK, Riley et al. (2015) have found that the decline in within firm labour productivity was more severe in SMEs, compared to larger firms. They also found that the decline was larger in manufacturing firms, compared to firms operating in the service sector. Both SMEs and manufacturing firms are typically more reliant on bank credit. Crawford et al. (2013) have also found that investment per worker, has declined by a greater extent in SMEs compared to large firms. Moreover, Pessoa and Van Reenen (2013) have estimated that around 60 per cent of the decline in labour productivity relative to its pre-crisis trend, was due to the change in relative factor prices, leading to a decline in capital intensity. They have also estimated that capital per worker would have been 8 per cent higher, if investment continued growing at its pre-crisis trajectory. Finally, Franklin et al. (2015) have used a large dataset on British firms banking relationships, while controlling for demand for the goods and services of those businesses. They estimated that up to half of the decline in aggregate labour productivity relative to its pre-crisis trend, is due to a tightening credit supply. Shocks to the financial sector in general and to the banking sector in particular, play a major role in motivating the theoretical framework developed in this chapter. However, I follow a ‘reduced form’ approach, which abstracts from including a banking sector in the model. Instead for simplicity, I assume that adverse shocks to the financial sector affect the supply of bonds in the credit market.
2.3 The Model

To the best of my knowledge, despite the extensive empirical economic literature on the UK’s stagnating labour productivity, no theoretical study has tried to explain the persistent phenomenon, in light of the empirical facts that took place during the recession and the following recovery. In this section, I present a simple theoretical model with a closed form solution, that aims to explain the movements in real variables and prices which occurred in the UK since the financial crisis. As discussed in sections 2.2.3.1 and 2.2.3.2, both wealth shocks and credit supply shocks can partially explain the UK’s productivity puzzle. Nevertheless, each on their own, cannot provide a full explanation of the facts presented in section 2.2.1, which accompanied the extremely slow recovery in productivity. The model incorporates those explanations in a single theory that provides a plausible mechanism.

The results are achieved using a version of the real business cycle model, which incorporates wealth shocks and redistributive wealth taxes that affect wealth inequality. However, I first present a version of the model that incorporates wealth shocks alone. This version of the model is not successful in replicating all of the empirical facts presented in section 2.2.1. In particular, employment does not respond to changes in wealth. This is due to the use of the Cobb-Douglas functional form. Nevertheless, it facilitates understanding the final version of the model presented in section 2.4, which also includes wealth redistribution taxes that affect wealth inequality.

In addition to the previously presented empirical motivation for the exogenous inclusion of wealth and credit supply shocks, Chowla et al. (2014) have found that most of the weakness in UK GDP since 2007, was due to global rather than domestic shocks. They have used a SVAR estimation to measure which exogenous global shocks had more relative importance in affecting output between 2007 and 2013. Out of three different types of shocks – demand shocks, financial shocks, and prices and supply shocks –, financial shocks were found to be the most important. Moreover, Born and Enders (2019) have studied the transmission of the global financial crisis from the US to the economies of the UK and Germany. Their aim is to determine whether the trade channel – due to the collapse in the demand for exports – or the

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58 More precisely, despite the recent increase in empirical studies that study the link between wealth shocks and labour productivity, no theoretical study has provided a plausible explanation.
financial channel – through the decline in cross-border asset holdings, leading to a reduction in the banks’ capital – was responsible for most of the observed decline in GDP. They have, for the purpose, constructed an international RBC model with a small open economy that is linked with the rest of the world by trade in goods and holdings of banking sector assets. When calibrating the model using the UK data, they find that the financial channel dominates the effect of the trade channel. Furthermore, in order to confirm their results empirically, they have estimated a SVAR model. In the UK case, they estimate that the financial channel has nearly twice the impact of the trade channel, and leads to a longer-lasting impact on economic activity.

2.3.1 The Model’s Environment

I consider a one period economy, in which there is one final good that may be used for consumption, lending and investment. The price of the good is normalized to one. The agents in this economy are: (i) a representative household, that I also call the lender; and (ii) a representative entrepreneur, whom I also call the borrower.

The household consumes the final good $c$, and supplies labour $l$. He maximizes $u^H(c, 1-l)$, where $u^H(c, 1-l) = \phi \ln c + (1 - \phi) \ln(1 - l)$ and $\phi \in (0, 1)$, thus being risk averse. He starts in the period with a wealth level $\bar{I}$ of the final good. However, his wealth $\bar{I}$ is subject to shocks $v \sim \log N(\mu_v, \sigma_v^2)$, such that his total wealth at the start of the period is $I = v\bar{I}$. The household can either stay in autarky by only consuming his beginning of period total wealth $I$, or investing it through lending in the bond market and supplying labour. If the household chooses the later, then his total end of period resources made available for consumption are $wl + (1 + r)b$, where $b$ is the amount of bonds being lent, $l$ is the number of labour units worked, $w$ is the real wage rate and $r$ is the real interest rate.\(^{59}\)

The entrepreneur consumes the final good $c$, and maximizes $u^E(c)$, where $u^E(c) = c$, thus being risk neutral. She owns a firm with a production technology $f(k, l) = zk^\alpha l^{1-\alpha}$, where $\alpha \in (0, 1)$, $k$ is the amount of capital used in production, $l$ is the number of employed labour units, and the standard productivity level $z \sim \log N(\mu_z, \sigma_z^2)$. The output of this technology is $y = f(l, k)$, which is the final good. The entrepreneur starts in the period without an endowment of capital, but she can

\(^{59}\) It is important to note that $r$ is interpreted in the model as the real cost of credit facing firms in the economy, rather than the policy interest rate set by a monetary authority.
borrow $b$ in the bond market at a gross rate $(1 + r)$. The entrepreneur’s firm has another technology, $k(b)$, where $b$ is the amount of bonds borrowed at the start of the period from the household in the bond market, and the output is units of physical capital $k$. The consumption good is assumed to be transferable to the capital good and vice versa at a rate of one-to-one, such that $(dk(b)/db) = 1$. I assume that a fraction $\delta \in [0,1]$ of capital $k$ depreciates, when used in the production technology $f(l,k)$.

Therefore, the amount of undepreciated capital after production is $(1 - \delta)k$. Additionally, since the entrepreneur starts in the period without an endowment of capital, then investment and capital stock are equivalent in this model.

Since it is a one period model, the household will not have an investment decision, and therefore supplies funds perfectly inelastically. Accordingly, $b^H = I = \nu \bar{I}$, where $b^H$ is the amount of bonds supplied by the household. However, the entrepreneur’s demand for bonds will be downward slopping (Figure 2.36).

### 2.3.2 Timing within the Period

Although a period is expressed as a single point in time, I add a timing description within the period to facilitate understanding the model. The timing within a period is in the following order: (i) The productivity and wealth shocks $z$ and $\nu$, are realized; (ii) the agents form their optimal decisions; (iii) the entrepreneur borrows $b$ in the bond market and employs $l$; (iv) the entrepreneur transfers $b$ to units of capital $k$, using the technology $k(b)$; (v) the entrepreneur starts the production process, using the technology $f(l,k)$; (vi) the final consumption good is produced by the entrepreneur’s firm; (vii) the entrepreneur pays the wage bill $wl$, and repays her debt $(1 + r)b$ to the household; (viii) agents consume.

---

60 When I solve for the equilibrium, I consider a special case where capital is assumed to fully depreciate in production $\delta = 1$, in order to be able to obtain a closed form solution.

61 This will be the case when the entrepreneur’s production technology $f(k, l) = zk^{\alpha}l^{1-\alpha}$ is sufficiently productive, such that $r \geq 0$ in equilibrium. In this case, the household’s utility from lending and supplying labour, will be higher than his utility in the autarky equilibrium.
2.3.3 The Household’s Problem

Since it is a one period model, the household will not have a consumption-investment decision and consume all his resources at the end of the period.\textsuperscript{62} To maximize his utility, the household chooses how much labour to supply \( l^H \), while supplying bonds inelastically. Therefore, the household’s optimisation problem is:

\[
\max_{c, l} \quad u^H(c, 1 - l) = \phi \ln c + (1 - \phi) \ln(1 - l) \quad \quad (2.1)
\]

subject to

\[
c = wl + (1 + r)b \quad \quad (2.2)
\]

\[
b = I = vI \quad \quad (2.3)
\]

I substitute the constraints (2.2) and (2.3) into the objective function (2.1), thus obtaining the household’s unconstrained optimisation problem:

\[
\max_l \quad \phi \ln(wl + (1 + r)vI) + (1 - \phi)\ln(1 - l) \quad \quad (2.4)
\]

I take the first-order condition with respect to \( l \) and solve for the household optimal labour supply \( l^H \).\textsuperscript{63}

\[
l^H = \phi - \frac{(1 - \phi)(1 + r)vI}{w} \quad \quad (2.5)
\]

The household’s optimal labour supply \( l^H \), depends positively on the real wage rate \( w \), and negatively on the real interest rate \( r \). Moreover, a negative shock to \( v \) which reduces the household’s wealth \( I = vI \), causes an increase in the household’s labour supply.

\textsuperscript{62} There will be a consumption-investment decision if there is more than one period, where the household chooses the amount of wealth \( I \) at the start of each period.

\textsuperscript{63} The function is concave; thus, the second order condition is satisfied.
2.3.4 The Entrepreneur’s Problem

Since it is a one period model, the entrepreneur will not have a consumption-investment decision, and consume all her resources at the end of the period. To maximize her utility, the entrepreneur chooses how much labour to hire \( l^E \), and the amount of the final good to borrow \( b^E \) that will be used in producing the capital good \( k \). Therefore, the entrepreneur’s optimisation problem is:

\[
\max_{c,b,l,k} u^E(c) = c \tag{2.6}
\]

subject to

\[
c = zk^{\alpha}l^{1-\alpha} - wl - (1 + r)b + (1 - \delta)k \tag{2.7}
\]

\[
k(b) = b \tag{2.8}
\]

I substitute the constraints (2.7) and (2.8) into the objective function (2.6), thus obtaining the entrepreneur’s unconstrained optimisation problem:

\[
\max_{b,l} zb^{\alpha}l^{1-\alpha} - wl - (r + \delta)b \tag{2.9}
\]

I take the first-order condition with respect to \( l \) and \( b \), respectively:

\[
(1 - \alpha)zb^{\alpha}l^{-\alpha} = w \tag{2.10}
\]

\[
\alpha zb^{\alpha - 1}l^{1-\alpha} = r + \delta \tag{2.11}
\]

I divide the first-order conditions (2.10) and (2.11), and obtain the entrepreneur’s optimality condition:

\[
\frac{(1 - \alpha) b^E}{\alpha l^E} = \frac{w}{r + \delta} \tag{2.12}
\]

---

64 There will be a consumption-investment decision if there is more than one period, since there is a continuation value from holding capital to the next period(s).

65 The function is concave; thus, the second order condition is satisfied.
2.3.5 The Equilibrium

There are two equilibriums in this model; autarky and another equilibrium where agents engage in lending and borrowing, supplying labour and producing. The autarky equilibrium is not the interesting case. To be in the second equilibrium, the entrepreneur’s production technology \( f(k, l) = zk^{a}l^{1-a} \) must be sufficiently productive.\(^66\)

I assume that capital fully depreciates when used in production, \( \delta = 1 \). This is to obtain an explicit functional form for the ratio of factors’ prices in equilibrium, leading to a closed form solution.\(^67\) Equilibrium in the labour and bond markets implies that \( l^E = l^H \) and \( b^E = b^H = I = vI \), respectively. I use equations (2.5) and (2.12), together with the two equilibrium conditions, and obtain:

\[
\frac{1 + r}{w} = \frac{\alpha}{1 - \alpha} \left( \frac{\phi}{vl} - \frac{(1 + r)vI(1 - \phi)}{wvl} \right) \tag{2.13}
\]

I rearrange the above expression to solve explicitly for \((1 + r)/w\) and obtain:

\[
\left( \frac{1 + r}{w} \right)^* = \frac{\alpha \phi}{(1 - \alpha \phi)vl} \tag{2.14}
\]

which is the ratio of factors’ prices in equilibrium.\(^68\) Therefore, a negative wealth shock leads to an increase in the ratio of factors’ prices \((1 + r)/w\). I obtain an expression for \( b/l \), by rearranging the optimality condition (2.12):

\[
\frac{b^E}{I^E} = \frac{\alpha}{1 - \alpha} \frac{w}{1 + r} \tag{2.15}
\]

I now substitute for \( w/(1 + r) \) in (2.15), using the ratio of equilibrium factors’ prices in (2.14) and obtain:

\(^66\) When presenting the numerical results in section 2.4.9, I scale up the standard productivity level such that \( \mathbb{E}[z] = 8 \). This is to ensure that \( r \geq 0 \) in equilibrium. If \( r < 0 \) in equilibrium, then both agents will stay in autarky, instead of lending and borrowing, supplying labour and producing.\(^67\) Without assuming \( \delta = 1 \), an explicit functional form for the ratio of factors’ prices cannot be obtained. I have also considered a model where the household’s utility is given by a CES utility function; however, a closed form solution could not be obtained either.\(^68\) Please note that variables with asterisk superscript represent equilibrium values.
which is the equilibrium amount of capital per unit of labour, since \( k(b) = b \). Therefore, a negative wealth shock leads to a deterioration in equilibrium capital intensity. Moreover, since \( y = zk^\alpha l^{1-\alpha} \) and \( k(b) = b \), then:

\[
\frac{y}{l} = z \left( \frac{b}{l} \right)^\alpha
\]  

(2.17)

I now substitute for \( b/l \) in (2.17), using the equilibrium amount of capital per unit of labour in (2.16) and obtain:

\[
\left( \frac{y}{l} \right)^* = z \left( \frac{(1-\alpha\phi)vI}{(1-\alpha)\phi} \right)^\alpha
\]  

(2.18)

which is labour productivity in equilibrium. Therefore, a negative wealth shock leads to a deterioration in equilibrium output per unit of labour. However, I cannot determine from the derivates \( \frac{\partial (1+r)/w}{\partial v}, \frac{\partial (y/l)}{\partial v} \) and \( \frac{\partial (b/l)}{\partial v} \) alone, how movements in equilibrium prices, output and labour have led to the change in relative factor prices, and the decline in labour productivity and capital intensity. Therefore, I continue the solution by solving for equilibrium output, labour and prices. I use the optimality condition in (2.12) to substitute for \( w/(1+r) \) in the labour supply given in (2.5), and I get:

\[
l = \phi - \frac{\alpha l \, vI \, (1-\phi)}{(1-\alpha) \, b}
\]  

(2.19)

I substitute for \( b \) in (2.19) using the equilibrium condition in the bond market, then solve for \( l \) to obtain:

\[
l^* = \frac{\phi(1-\alpha)}{1-\alpha\phi}
\]  

(2.20)
which is the equilibrium level of labour. Accordingly, changes in wealth do not affect the level of labour in equilibrium. This is problematic as it cannot explain the movements in employment that were described earlier in the literature review. I discuss the reason behind the model’s muted employment response in section 2.3.6, where comparative statics are discussed. To calculate equilibrium output, I use (2.20), the equilibrium condition \( b^E = b^H = l = vI \) and \( k(b) = b \), to substitute into the production function \( y = zk^a l^{1-a} \):

\[
y^* = z(vI)^a \left( \frac{\phi(1-\alpha)}{1-\alpha\phi} \right)^{1-\alpha} \tag{2.21}
\]

Equation (2.21) implies that equilibrium output declines as a result of a negative shock to \( v \). To solve for the equilibrium wage rate \( w^* \), I use the equilibrium level of labour in (2.20) and the equilibrium condition in the bond market, to substitute into the first-order condition obtained in (2.10):

\[
w^* = z(vI)^a \left( \frac{\phi(1-\alpha)}{1-\alpha\phi} \right)^{1-\alpha} \tag{2.22}
\]

Therefore, a negative wealth shock leads to a decline in the equilibrium real wage rate. To solve for the equilibrium interest rate \( r^* \), I use the equilibrium level of labour in (2.20) and the equilibrium condition \( b^E = b^H = l = vI \), to substitute into the first-order condition obtained in (2.11):

\[
r^* = az \left( \frac{\phi(1-\alpha)}{(1-\alpha\phi)vI} \right)^{1-\alpha} - 1 \tag{2.23}
\]

Accordingly, a negative wealth shock leads to an increase in the equilibrium net interest rate.\(^69\)

---

\(^69\) As can be seen from the functional form given in (2.23), the equilibrium real net interest rate can possibly be negative. Therefore, when presenting the numerical results in section 2.4.9, the economy is scaled up by scaling up the standard productivity level, such that \( z \sim N(8, \sigma_z^2) \). Alternatively, the parameter \( \hat{I} \) can be given a very low value. In the numerical results section, I ensure that agents prefer to lend and borrow, supply labour and produce, instead of staying in autarky.
2.3.6 Comparative Statics

The effects of productivity and wealth shocks on equilibrium quantities and prices are summarised in Table 2.2. In this model, a negative shock to productivity \( z \), represents some of the features that characterize a demand driven recession. A negative productivity shock lowers the marginal productivity of capital in the entrepreneur’s production, \( az(l/k)^{1-\alpha} \). As can be seen in Figure 2.37, this causes the demand curve for bonds to shift to the left in the bond market. Since the supply of bonds is perfectly inelastic, the combined effect would result in a lower cost of borrowing \( r \), without a change in the equilibrium amount of bonds.

<table>
<thead>
<tr>
<th>Productivity Shock, ( z )</th>
<th>Wealth Shock, ( v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derivative</td>
<td>Sign</td>
</tr>
<tr>
<td>( \partial b / \partial z )</td>
<td>0 (no effect)</td>
</tr>
<tr>
<td>( (b = k) )</td>
<td></td>
</tr>
<tr>
<td>( \partial l / \partial z )</td>
<td>0 (no effect)</td>
</tr>
<tr>
<td>( \partial(k/l) / \partial z )</td>
<td>0 (no effect)</td>
</tr>
<tr>
<td>( \partial y / \partial z )</td>
<td>+</td>
</tr>
<tr>
<td>( \partial(y/l) / \partial z )</td>
<td>+</td>
</tr>
<tr>
<td>( \partial w / \partial z )</td>
<td>+</td>
</tr>
<tr>
<td>( \partial r / \partial z )</td>
<td>+</td>
</tr>
<tr>
<td>( \partial((1 + r)/w) / \partial z )</td>
<td>0 (no effect)</td>
</tr>
</tbody>
</table>

Table 2.2: The effects of productivity and wealth shocks on equilibrium quantities and prices, in a model with wealth shocks only.

A negative productivity shock also lowers the marginal productivity of labour in the entrepreneur’s production, \((1 - \alpha)z(k/l)^\alpha\). As shown in Figure 2.38, this causes the demand curve for labour to shift to the left in the labour market. On the other hand, the decline in the interest rate in the bond market would have an effect on the household’s total resources. When the real net return on lending goes down from \( r^*_1 \) to \( r^*_2 \) (Figure 2.37), there will be a negative wealth effect on the household. He gets less return on his financial assets. Therefore, he responds by increasing his labour supply. Accordingly, the labour supply curve shifts outward (Figure 2.38), since
The combined effect in the labour market would cause the equilibrium wage rate to drop from $w_1^*$ to $w_2^*$. However, there is no effect on equilibrium labour. The use of the Cobb-Douglas functional form in the household’s preference and the entrepreneur’s production, implies that the shifts in the labour demand and supply curves in opposite directions, are exactly equal. This offsets any change in the quantity of employed labour in equilibrium.

Moreover, a negative productivity shock causes a reduction in equilibrium output, where $y = zk^a l^{1-a}$. Labour productivity $y/l$ will also decrease in equilibrium. However, this reduction in labour productivity is only caused by changes in equilibrium output, since equilibrium employed labour is unaffected by shocks to $z$. Furthermore, we can also infer that equilibrium capital deepening $k/l$ and relative factor prices $(1 + r)/w$, will not change due to negative shock to $z$. Accordingly, a negative shock to productivity $z$, represents some of the features that characterize a demand driven recession. Capital-labour ratio within firms and relative factor prices may not necessarily change during a demand driven recession; however, labour productivity can drop. This is due to the spare capacity within firms, where labour productivity will improve when demand recovers.

Although a negative wealth shock causes some of the equilibrium quantities and prices to move in the same direction as a negative productivity shock, the economic reasonings behind those similar moves are different. A negative shock to $v$ reduces the household’s wealth, that is available for lending in the bond market. This causes the bonds supply curve to shift to the left, and therefore increasing the equilibrium interest rate (Figure 2.39). The rise in the real cost of borrowing and reduction in credit, would have further effects in the labour market. In the entrepreneur’s production technology $f(l,k)$, there is a complementarity between capital and labour. The reduction in the amount of capital used in production, reduces the marginal product of labour, $(1 - \alpha)z(k/l)^{\alpha}$. This causes the entrepreneur to demand less labour, and hence the labour demand schedule shifts to the left (Figure 2.40). Moreover, the negative shock to $v$ leads to wealth effects on the household’s labour supply, since he now has fewer financial resources. The household responds by increasing his labour supply, since $(\partial l^H/\partial v) < 0$. Therefore, the labour supply curve shifts outward. The combined effect would cause the equilibrium wage rate to drop.

\footnote{Despite the increase in $r$, the quantity effect due to loss of wealth is larger.}
drop from $w_1^*$ to $w_2^*$. However, there is no effect on equilibrium labour, since the Cobb-Douglas functional form in the household’s preference and the entrepreneur’s production, implies that the shifts in the labour demand and supply curves in opposite directions, are exactly equal. This offsets any change in the quantity of labour employed in equilibrium.

In addition, since $k(b) = b$, $b^* = vI$ and $(\partial l^*/\partial v) = 0$, we can infer that equilibrium capital intensity $k/l$ will deteriorate, due to a negative shock to $v$. This is accompanied by a rise in relative factor prices $(1 + r)/w$. The reduction in the household’s wealth, will have further effects on equilibrium output and labour productivity. A reduction in $v$ will cause a reduction in the amount of capital used in production due to the credit contraction. This in turn will result in a reduction in equilibrium output since $y = zk^{\alpha}l^{1-\alpha}$. Furthermore, labour productivity $y/l$ will also decrease in equilibrium. However, this reduction in labour productivity is only caused by changes in equilibrium output, since equilibrium employed labour is unaffected by shocks to $v$.

### 2.4 Adding Redistributive Wealth Taxes

The previously presented version of the model is not successful in replicating all of the empirical facts presented in section 2.2.1. In particular, employment does not respond to changes in wealth. This is due to the use of the Cobb-Douglas functional form in both the household’s preferences and the entrepreneur’s production. In the second version of the model, I include wealth redistribution taxes that affect wealth inequality. This is important for two reasons. First, the austerity measures taken by the government since 2010, have affected the way in which the British economy evolved in the recovery. In particular it may have affected employment outcomes and labour productivity since. I discuss this in greater details in the following subsection. Second, the redistributive tax which is included in the model will be distortionary. This will break the equality of the shifts in the labour demand and supply curves, and hence equilibrium employment will respond to wealth shocks.
2.4.1 Motivation for The Redistributive Wealth Taxes

It has been argued that the phenomena of slowing labour productivity growth and rising inequality are connected in most developed countries (Guest and Swift (2008); DiPietro (2014)). This is especially the case in Europe, where Gartner (2014) have used a panel dataset and found that a trade-off exists between lower unemployment and lower inequality. This became more evident since the financial crisis. According to Andre et al. (2013) and OECD (2016), one of the main reasons behind this, is the declining redistributive power of taxes and transfers. Welfare and labour market reforms can affect labour market outcomes, by influencing the behaviour of both employed and unemployed people, and shifting the bargaining power in the wage setting process. Besides people who are already employed; the welfare reforms implemented by the government since 2010, have increased the pressure on benefit claimants and individuals who were not previously participating in the labour force. This in turn have likely contributed to the increase in labour supply and forced many individuals to accept jobs with lower pay, which they would have not otherwise accepted. The flexibility of real wages which coincided with the rise in the cost of financing capital purchases, have increased the substitution of labour for capital, contributing to the deteriorating capital intensity (Pessoa and Van Reenen (2013)).

During the financial crisis, the government has intervened to save the banking sector from a complete collapse, by underwriting the sector’s liabilities and nationalising major banking institutions such as RBS. This was followed by the Quantitative Easing measures, which saw billions of pounds injected into the British economy (Bryson and Forth (2015)). As a result, the country’s debt to GDP ratio has dramatically increased from 36.4 per cent to 60 per cent, over the course of the recession. The 2010 and 2015 General Elections has brought Conservative led governments, with a strong focus on reducing the government’s budget deficit and the country’s indebtedness, through a widespread austerity program. The austerity measures have led to a decline in spending across the government hierarchy. However, the most severe cuts have targeted real spending on welfare, protection programs and public services, which dropped by 16 per cent (Fetzer (2018); Figure 2.41).

Some of the cuts in social transfers have been intertwined with labour market changes, by imposing more conditionality on access to them. The policies have either encouraged inactive individuals to participate in the labour force, or incentivised part
of the employed population to continue working instead of retiring (Andre et al. (2013)). For instance, older women started to have an incentive to stay longer in employment instead of retiring, due to the increase in the State Pension Age (SPA). Moreover, lone mothers have been forced to seek employment by reducing the age threshold of their children, at which they are entitled to receive income support. Accordingly, the affected groups have exhibited marked increases in their employment rates (Emmerson et al. (2013); Figure 2.42).

Public services and direct transfers have an important role in reducing inequality between households, as it decreases the Gini coefficient by at least 0.1 in the UK. Unlike the universal system that is implemented in most European countries, direct transfers in the UK are more directed at the bottom of households’ income distribution, with pensions constituting a lower proportion. Moreover, although in kind transfers in the form of public services are made available for the entire population, it is the low-income households that are most dependent on them (Andre et al. (2013); Figure 2.43). Accordingly, the suffering from the impact of austerity measures were not evenly shared across all households. Furthermore, I have already discussed earlier in section 2.2.3.1; that there was a partial recovery in UK household’s net real wealth since the crisis (Figure 2.28). This recovery was due to the improvement in net real financial wealth (Figure 2.29), which occurred mostly due to the Quantitative Easing measures taken by the Bank of England. Nevertheless, those capital gains were not shared across homogeneously, since around 50 per cent of net real financial wealth is owned by the nation’s richest 5 per cent of households (Figure 2.31).

The discussion about the effect of austerity on the UK economy, is mostly dominated by its impact on stimulating demand through the Keynesian effect. This is not the goal from including a government in the model. Rather, I am interested in the economic effects of the increase in wealth inequality, which occurred as a by-product of the policies endorsed by the government.

2.4.2 Amending the Model’s Environment

I consider the same model as before; However, now I add another two agents: (i) a representative agent who I call the capitalist, and (ii) a government that collects taxes $T > 0$ from the capitalist and redistributes to the household. In other words,
government taxes are purely used for wealth redistribution purposes. The capitalist consumes the final good $c$ and does not supply labour. She maximizes $u^c(c)$, where $u^c(c) = c$, thus being risk neutral. She starts in the period with a wealth level $W$ of the final good. However, her wealth $\bar{W}$ is subject to shocks $\nu \sim \log N(\mu_\nu, \sigma_\nu^2)$. Moreover, the government collects the wealth tax $T$ at the start of the period, such that the capitalist’s total wealth at the start of the period is $W - T$, where $W = v\bar{W}$. The capitalist can either stay in autarky by only consuming her beginning of period total wealth $W - T$, or investing it through lending in the bond market. If she chooses the later, then her total end of period resources made available for consumption are $(1 + r)(W - T)$.

Moreover, with the addition of the wealth tax, now the household’s total wealth at the start of the period is $I + T$, where $I = v\bar{I}$. Accordingly, if he chooses to invest his wealth through lending in the bond market and supplying labour instead of staying in autarky, then his total end of period resources made available for consumption will be $wl + (1 + r)(I + T)$. Since it is a one period model, the household and the capitalist will not have an investment decision, and therefore supply funds perfectly inelastically when $r \geq 0$. Accordingly, $b^H + b^C = v\bar{I} + T + v\bar{W} - T = v(\bar{I} + \bar{W})$, where $b^H$ and $b^C$ are the amount of bonds supplied by the household and the capitalist, respectively. Therefore, changes in the government’s taxes $T$, do not affect the supply of bonds. The entrepreneur’s demand for bonds will be downward sloping, similar to before (Figure 2.44).

### 2.4.3 Timing within the Period

With the addition of the redistributive wealth taxes, the timing within a period is amended to be in the following order: (i) the productivity and wealth shocks $z$ and $\nu$, are realized; (ii) the government chooses the amount of tax $T$, that it wants to collect from the capitalist and redistributes it to the household; (iii) the agents form their optimal decisions; (iv) the entrepreneur borrows $b$ in the bond market and employs $l$; (v) the entrepreneur transfers $b$ to units of capital $k$, using the technology $k(b)$; (vi) the entrepreneur starts the production process, using the technology $f(l, k)$; (vii) the

---

71 I have initially assumed that the government collects taxes on wealth at the end of the period. However, when solving for the equilibrium, I was not able to obtain an explicit functional form for the ratio of factors’ prices in equilibrium, which is necessary for a closed form solution.
final consumption good is produced by the entrepreneur’s firm; (viii) the entrepreneur pays the wage bill \(wl\), and repays her debt \((1 + r)b\) to the lenders; (ix) agents consume.

### 2.4.4 The Household’s Problem

Since it is a one period model, the household will not have a consumption-investment decision and consume all his resources at the end of the period.\(^{72}\) To maximize his utility, the household chooses how much labour to supply \(l^H\), while supplying bonds inelastically. Therefore, the household’s optimisation problem is:

\[
\max_{c,l} u^H(c,l) = \phi \ln c + (1 - \phi) \ln(1 - l) \tag{2.24}
\]

subject to

\[
c = wl + (1 + r)(l + T) \tag{2.25}
\]

\[
l = v\bar{I} \tag{2.26}
\]

I substitute the constraints (2.25) and (2.26) into the objective function (2.24), thus obtaining the household’s unconstrained optimisation problem:

\[
\max_l \phi \ln(wl + (1 + r)(v\bar{I} + T)) + (1 - \phi) \ln(1 - l) \tag{2.27}
\]

I take the first-order condition with respect to \(l\) and solve for the household optimal labour supply \(l^H\).\(^{74}\)

\[
l^H = \phi - \frac{(1 + r)(v\bar{I} + T)(1 - \phi)}{w} \tag{2.28}
\]

---

\(^{72}\) There will be a consumption-investment decision if there is more than one period, where the household chooses the amount of wealth \(\bar{I}\) at the start of each period.

\(^{73}\) I have initially assumed that the government collects taxes on wealth at the end of the period, such that the taxes are lump sum and the household’s constraint given in (2.25) will be \(c = wl + (1 + r)l + T\). However, when solving for the equilibrium, I was not able to obtain an explicit functional form for the ratio of factors’ prices in equilibrium, which is necessary for a closed form solution.

\(^{74}\) The function is concave; thus, the second order condition is satisfied.
The household’s optimal labour supply \( l_H \), depends positively on the real wage rate \( w \), and negatively on the real interest rate \( r \) and the amount of the redistributive taxes \( T \). Moreover, a negative shock to \( v \) which reduces household’s wealth \( l = vI \), causes an increase in the household’s labour supply.

### 2.4.5 The Entrepreneur’s Problem

Since it is a one period model, the entrepreneur will not have a consumption-investment decision and consume all her resources at the end of the period.\(^{75}\) To maximize her utility, the entrepreneur chooses how much labour to hire \( l^E \), and the amount of the final good to borrow \( b^E \) that will be used in producing the capital good \( k \). Therefore, the entrepreneur’s optimisation problem is:

\[
\max_{c,b,l,k} u^E(c) = c
\]

subject to

\[
c = zk^a l^{1-a} - wl - (1+r)b + (1-\delta)k
\]

\[
k(b) = b
\]

I substitute the constraints (2.30) and (2.31) into the objective function (2.29), thus obtaining the entrepreneur’s unconstrained optimisation problem:

\[
\max_{b,l} z b^a l^{1-a} - wl - (r+\delta)b
\]

I take the first-order condition with respect to \( l \) and \( b \), respectively:\(^{76}\)

\[
(1-\alpha)zb^a l^{-\alpha} = w
\]

\[
\alpha zb^a l^{-\alpha} = r + \delta
\]

I divide the first-order conditions (2.33) and (2.34), and obtain the entrepreneur’s optimality condition:

---

\(^{75}\) There will be a consumption-investment decision if there is more than one period, since there is a continuation value from holding capital to the next period(s).

\(^{76}\) The function is concave; thus, the second order condition is satisfied.
\[
\frac{(1 - \alpha) b^E}{\alpha} \frac{b^E}{l^E} = \frac{w}{r + \delta} \] (2.35)

### 2.4.6 The Capitalist’s Problem

Since it is a one period model, the capitalist will not have a consumption-investment decision and consume all her resources at the end of the period.\(^\text{77}\) She faces a binary choice in maximizing her utility. The capitalist can either stay in autarky; consuming her beginning of period total wealth \(W - T\), or lending and consuming \((1 + r)(W - T)\). As long as \(r \geq 0\), then the capitalist will participate in the bond market.

### 2.4.7 The Equilibrium

Like before, I ignore the autarky equilibrium and assume that the technology \(f(k, l)\) is sufficiently productive. I also assume that capital fully depreciates when used in production, \(\delta = 1\). This is to obtain an explicit functional form for the ratio of factors’ prices in equilibrium, leading to a closed form solution.\(^\text{78}\) Equilibrium in the labour and bond markets implies that \(l^E = l^H\) and \(b^E = b^H + b^C = v(i + \bar{W})\), respectively. I use equations (2.28) and (2.35), together with the two equilibrium conditions, and obtain:

\[
\frac{1 + r}{w} = \frac{\alpha}{1 - \alpha} \left( \frac{\phi}{v(i + \bar{W})} - \frac{(1 + r)(vI + T)(1 - \phi)}{wv(I + \bar{W})} \right) \] (2.36)

I rearrange the above expression to solve explicitly for \((1 + r)/w\) and obtain:

\[
\left( \frac{1 + r}{w} \right)^* = \frac{\alpha \phi}{(1 - \alpha \phi)vI + (1 - \alpha)v\bar{W} + \alpha(1 - \phi)T} \] (2.37)

which is the ratio of factors’ prices in equilibrium. Therefore, a negative wealth shock leads to an increase in the ratio of factors’ prices \((1 + r)/w\), so does a decrease in the

\(^\text{77}\) There will be a consumption-investment decision if there is more than one period, where the capitalist chooses the amount of wealth \(\bar{W}\), at the start of each period.

\(^\text{78}\) Without assuming \(\delta = 1\), an explicit functional form for the ratio of factors’ prices cannot be obtained. I have also considered a model where the household’s utility is given by a CES utility function, however, a closed form solution could not be obtained either.
amount of redistributive taxes $T$. I obtain an expression for $b/l$, by rearranging the optimality condition (2.35):

$$
\frac{b^e}{l^e} = \frac{\alpha w}{1 - \alpha} \frac{1}{1 + r}
$$  \hspace{1cm} (2.38)

I now substitute for $w/(1 + r)$ in (2.38), using the ratio of equilibrium factors’ prices in (2.37) and obtain:

$$
\left(\frac{b}{l}\right)^* = \frac{(1 - \alpha \phi) v I + (1 - \alpha) v \bar{W} + \alpha (1 - \phi) T}{(1 - \alpha) \phi}
$$  \hspace{1cm} (2.39)

which is the equilibrium amount of capital per unit of labour, since $k(b) = b$. Therefore, a negative wealth shock leads to a deterioration in equilibrium capital intensity. A decrease in the amount of redistributive taxes $T$ has a similar effect as well. Moreover, since $y = zk^{\alpha}l^{1-\alpha}$ and $k(b) = b$, then:

$$
\frac{y}{l} = z \left(\frac{b}{l}\right)^{\alpha}
$$  \hspace{1cm} (2.40)

I now substitute for $b/l$ in (2.40), using the equilibrium amount of capital per unit of labour in (2.39) and obtain:

$$
\left(\frac{y}{l}\right)^* = z \left(\frac{(1 - \alpha \phi) v I + (1 - \alpha) v \bar{W} + \alpha (1 - \phi) T}{(1 - \alpha) \phi}\right)^{\alpha}
$$  \hspace{1cm} (2.41)

which is labour productivity in equilibrium. Therefore, a negative wealth shock leads to a deterioration in equilibrium output per unit of labour. A decrease in the amount of redistributive taxes $T$ has a similar effect as well. However, I cannot determine from the derivate $\partial((1 + r)/w)/\partial v$, $\partial(y/l)/\partial v$, $\partial(b/l)/\partial v$, $\partial((1 + r)/w)/\partial T$, $\partial(y/l)/\partial T$ and $\partial(b/l)/\partial T$ alone, how movements in equilibrium prices, output and labour have led to the change in relative factor prices, and the decline in labour productivity and capital intensity. Therefore, I continue the solution by solving for equilibrium output, labour and prices. I use the optimality condition in (2.35) to substitute for $w/(1 + r)$ in the labour supply given in (2.28), and I get:
\[ l = \phi - \frac{\alpha \ell (v\bar{I} + T) (1 - \phi)}{(1 - \alpha) b} \]  

(2.42)

I substitute for \( b \) in (2.42) using the equilibrium condition in the bond market, then solve for \( l \) and obtain:

\[ l^* = \frac{\phi v(1 - \alpha)(\bar{I} + \bar{W})}{v((1 - \alpha\phi)\bar{I} + (1 - \alpha)\bar{W}) + \alpha(1 - \phi)T} \]  

(2.43)

which is the equilibrium level of labour. Accordingly, a negative wealth shock leads to a decrease in the level of employed labour in equilibrium, despite the increase in labour supply as indicated by (2.28). I explain the reason behind this in section 2.4.8, where comparative statics are discussed. On the other hand, a decrease in the amount of redistributive taxes \( T \) increases the amount of employed labour in equilibrium. To calculate equilibrium output, I use (2.43), the equilibrium condition \( b^E = b^H + b^C = v(\bar{I} + \bar{W}) \) and \( k(b) = b \), to substitute into the production function \( y = zk^\alpha l^{1-\alpha} \):

\[ y^* = z(v(\bar{I} + \bar{W}))^\alpha \left( \frac{\phi v(1 - \alpha)(\bar{I} + \bar{W})}{v((1 - \alpha\phi)\bar{I} + (1 - \alpha)\bar{W}) + \alpha(1 - \phi)T} \right)^{1-\alpha} \]  

(2.44)

A negative wealth shock leads to a decrease in equilibrium output, while a reduction in redistributive taxes increases it. To solve for the equilibrium wage rate \( w^* \), I use the equilibrium level of labour in (2.43) and the equilibrium condition in the bond market, to substitute into the optimality condition obtained in (2.33):

\[ w^* = z(1 - \alpha) \left( \frac{v((1 - \alpha\phi)\bar{I} + (1 - \alpha)\bar{W}) + \alpha(1 - \phi)T}{\phi(1 - \alpha)} \right)^\alpha \]  

(2.45)

Therefore, a negative wealth shock and a decrease in redistributive taxes, both lead to a decline in the equilibrium real wage rate. To solve for the equilibrium interest rate \( r^* \), I use the equilibrium level of labour in (2.43) and the equilibrium condition \( b^E = b^H + b^C = v(\bar{I} + \bar{W}) \), to substitute into the optimality condition obtained in (2.34):
Accordingly, a negative wealth shock and a decrease in redistributive taxes, both lead to an increase in the equilibrium net interest rate.\textsuperscript{79}

\[ r^* = \alpha z \left( \frac{\phi(1 - \alpha)}{\nu((1 - \alpha\phi)I + (1 - \alpha)\hat{W}) + \alpha(1 - \phi)T} \right)^{1-\alpha} - 1 \quad (2.46) \]

\textbf{2.4.8 Comparative Statics}

The effects of wealth shocks and change in redistributive wealth taxes on equilibrium quantities and prices are summarised in Table 2.3. The addition of a third agent and redistributive wealth taxes in this version of the model, does not change the effect of shocks to $z$ or the economic reasoning behind them. Accordingly, the previous discussion on productivity shocks in section 2.3.6 continues to apply. However, unlike the previous version of the model, wealth shocks will now affect the amount of equilibrium labour.

<table>
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<tr>
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<th>Decrease in wealth inequality, $T$ increasing</th>
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<tr>
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<td>$\partial (k/l)/\partial v$</td>
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<tr>
<td>$\partial y/\partial v$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\partial (y/l)/\partial v$</td>
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</tr>
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<td>$\partial w/\partial v$</td>
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</tr>
<tr>
<td>$\partial ((1 + r)/w)/\partial v$</td>
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</tbody>
</table>

Table 2.3: The effects of a change in $v$ and $T$ on equilibrium quantities and prices, in a model with wealth shocks and redistributive wealth taxes.

\textsuperscript{79} As can be seen from the functional form given in (2.46), the equilibrium real net interest rate can possibly be negative. Therefore, when presenting the numerical results in section 2.4.9, the economy is scaled up, by scaling up the standard productivity level such that $z \sim \log N(8, \sigma_z^2)$. Alternatively, the parameters $I$ and $\hat{W}$ can be given a very low value. In the numerical results section, I ensure that agents prefer to lend and borrow, supply labour and produce, instead of staying in autarky.
A negative shock to \( v \) reduces the household’s and the capitalist’s wealth, that is available for lending in the bond market. The contraction in credit causes the bonds supply curve to shift from \( S_1 \) to \( S_2 \), and thus increasing the real cost of borrowing (Figure 2.45). This would have further effects in the labour market.

In the entrepreneur’s production technology \( f(l, k) \), there is a complementarity between capital and labour. The reduction in the amount of capital used in production, reduces the marginal product of labour, \( (1 - \alpha)z(k/l)^\alpha \). Consequently, the entrepreneur’s demand for labour declines, and hence the labour demand schedule shifts to the left (Figure 2.46). Moreover, the negative shock to \( v \) leads to wealth effects on the household’s labour supply, since he now has fewer financial resources. The household responds by increasing his labour supply, since \( (\partial l^* / \partial v) < 0 \). Therefore, the labour supply curve shifts outward. The combined effect would cause the equilibrium wage rate to drop. However, unlike the previous version of the model, the household is now subsidised by the redistributive taxes \( T \). The subsidy partly offsets the effect of the negative wealth shock on the household’s labour supply. This causes the labour supply curve to shift outwards by less than before. This is reflected in Figure 2.46 by a shift from \( S_1 \) to \( S_3 \), instead of a shift to \( S_2 \). Accordingly, the shifts in the labour demand and supply curves in opposite directions are not equal. The effect of this is a decrease in the quantity of employed labour in equilibrium.

The decrease in the amount of employed labour in equilibrium also has effects on the bond market, unlike the previous version of the model. In the entrepreneur’s production technology \( f(l, k) \), there is a complementarity between capital and labour. The reduction in the amount of labour used in production, reduces the marginal product of capital, \( az(l/k)^{1-\alpha} \). Consequently, the entrepreneur’s demand for bonds declines, and hence the demand schedule shifts from \( D_1 \) to \( D_2 \) (Figure 2.45). Since \( \partial r / \partial v < 0 \), then the shift in the supply curve is greater than the shift in the demand curve, leading to an increase in equilibrium interest rate.

In addition, \( k(b) = b, \ b^* = v(\bar{I} + \bar{W}) \) and \( (\partial l^* / \partial v) > 0 \), imply that both investment and labour decline due to a negative shock to \( v \). Since capital intensity deteriorates due to negative wealth shock, \( (\partial (k/l)^* / \partial v) > 0 \), then it must be the case

---

80 Despite the increase in \( r \), the quantity effect due to loss of wealth is larger.
that investment falls proportionately more than employment. The reduction in the household’s and the capitalist’s wealth, will have further effects on equilibrium output and labour productivity. A reduction in $v$ will cause a reduction in the amount of capital used in production due to the credit contraction, and a reduction in employed labour. This in turn will result in a reduction in equilibrium output since $y = zk^{\alpha}l^{1-\alpha}$. Furthermore, labour productivity $y/l$ will also decrease in equilibrium, as indicated by $(\partial (y/l)'/\partial v) > 0$. Since both equilibrium output and labour decline in response to a negative wealth shock, then it must be the case that output falls proportionately more than employment.

A reduction in redistributive taxes leads to a negative wealth effect on the household’s labour supply, causing it to increase since $(\partial l'/\partial T) < 0$. Therefore, the labour demand curve shifts outward (Figure 2.47). Since labour demand is unaffected by changes in $T$, the result would be a decrease in the equilibrium wage rate, while the equilibrium amount of employed labour increases. These changes in the labour market would have further effects in the bond market. As mentioned before, changes in the government’s taxes $T$ do not affect the supply of bonds, since $b^{H} + b^{C} = vl + T + vW - T = v(l + W)$. However, in the entrepreneur’s production technology $f(l, k)$, there is a complementarity between capital and labour. The rise in the quantity of employed labour, increases the marginal product of capital in the entrepreneur’s production, $az(l/k)^{1-\alpha}$. This makes the entrepreneur demand more capital. Since all the bonds borrowed by her are used in producing capital, $k(b) = b$, the bonds demand schedule shifts outward (Figure 2.48). However, the supply of bonds is perfectly inelastic, and hence the combined effect would result in a higher equilibrium interest rate $r$, without a change in the equilibrium amount of bonds.

In addition, $k(b) = b$, $(\partial b^*/\partial T) = 0$ and $(\partial l^*/\partial T) < 0$, imply that capital intensity deteriorates due to a decrease in redistributive taxes, which occurs only through changes in equilibrium labour. The reduction in redistributive wealth taxes, will have further effects on equilibrium output and labour productivity. A reduction in $T$ will cause an increase in equilibrium labour. This in turn will result in an increase in equilibrium output since $y = zk^{\alpha}l^{1-\alpha}$. Nevertheless, labour productivity $y/l$ will decrease in equilibrium, as indicated by $(\partial (y/l)'/\partial T) > 0$. Since both equilibrium

81 Since the entrepreneur starts in the period without an endowment of capital, then investment and capital stock are equivalent in this model.
output and labour increase in response to a decrease in \( T \), then it must be the case that employment grows proportionately more than output.

### 2.4.9 Numerical Results

In this section of the chapter, I conduct a numerical exercise to show that the model can explain how the movements in variables and prices came into place since 2008. Rather than attempting to quantitatively match the empirical facts, I try to explain those movements qualitatively. Since the presented theoretical framework is a one period static model, I add additional simplifying assumptions that allow extending the model to multiple periods without the need for a dynamic solution. I assume that the final consumption good is perishable, without the possibility of storage between periods. Additionally, in each period the household and the capitalist are endowed with \( I \) and \( W \), respectively. Accordingly, in every period, all agents will fully consume their resources. This keeps the model static within each period.\(^{82}\) As shown in Figure 2.49 below, I consider five periods to obtain the numerical results. Each period represents two consecutive years, with the recession occurring in the third period \( t = 3 \). To allow for comparisons in the numerical exercise, I obtain a benchmark by assuming that a recession does not occur. Specifically, I assume that the economy grows through a 2 per cent exogenous growth in aggregate wealth, without being interrupted. In other words, I set the value of \( v = 1 \) in \( t = 1 \), then it grows by 2 per cent in each consecutive period.\(^{83}\) The results of these calculations are presented in Table 2.4.

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\(^{82}\) The downside of following this approach is that it abstracts from agents’ intertemporal decisions. Future research should seek to address this matter, by extending the model to an infinite horizon version. I briefly discuss the way in which this extension can be implemented in my concluding remarks in section 2.5.

\(^{83}\) The 2 per cent exogenous growth in the value of \( v \) was arbitrarily chosen, since no data on households’ aggregate wealth is available prior to the second half of the last decade. ONS data on wealth has only become available since 2006, while OECD data on wealth has only become available from 2009. The assumed 2 per cent growth in aggregate wealth every two years is close to 1.85 per cent, which is the average rate at which real business investment grew every two years between 1998 and 2008 (ONS (2019d)).
The 2008 recession has started with a banking crisis that led to a large decline in households’ aggregate net real wealth. As mentioned earlier in section 2.2.3.1, UK households’ net real wealth has dropped by around 8 per cent during the recession. Some of those losses were recovered in the post-recession period, however, by mid 2014 it was still more than 2 per cent lower than its level in 2008. Accordingly, in the numerical exercise, I proxy for the decline in real wealth by an 8 per cent reduction in the value of \( v \) in \( t = 3 \) relative to its value in \( t = 2 \). I also proxy for the partial recovery in aggregate wealth by a 6.52 per cent increase in the value of \( v \) over the periods \( t = 4 \) and \( t = 5 \), such that the value of \( v \) remains 2 per cent lower in \( t = 5 \) relative to its value in \( t = 2 \). The results of these calculations are presented in Table 2.5.

Moreover, as mentioned earlier, the partial recovery in aggregate real wealth was not shared equally across all households, since it came through the improvement in net real financial wealth due to the Quantitative Easing measures. Those capital gains were mostly enjoyed by the richest households, since around 50 per cent of aggregate net real financial wealth is owned by the nation’s richest 5 per cent (Figure 2.31). Furthermore, the austerity measures adopted by the government since 2010, have affected low-income households the most, since they are more dependent on public services and direct transfers (Figure 2.43). I proxy for this increase in wealth inequality, through a reduction in \( T \) in the periods \( t = 4 \) and \( t = 5 \). There is no precise data on the extent of the increase in wealth inequality, which occurred as a by-product of the policies endorsed by the government. Therefore, I run two different alternatives; a 25 and a 50 per cent reduction in \( T \) in the periods \( t = 4 \) and \( t = 5 \). The results of these calculations are presented in Table 2.6 and Table 2.7.
The parameter $\phi$ is the weight of consumption in the consumption-leisure multiplicative non-separable Cobb-Douglas utility function. Eichenbaum et al. (1998) have obtained an estimate of the parameter between 0.12 and 0.18, while others have used a calibration value of $\phi$ that is around twice as large (for example check: Domeij and Floden (2006); Heathcote et al. (2008); Collard and Dellas (2012)). On the other hand, Brissimis and Bechlioulis (2017) have obtained an estimate of the parameter equal to 0.74. Accordingly, since there is a wide range of estimated values, the value of $\phi$ was chosen to be 0.5. I followed the standard values assumed for $\alpha$ in the literature, while the depreciation rate $\delta$ is given a value of 1, due to the reasons mentioned earlier in the model section. Also, please note that in all the numerical calculations, I ensured that agents will have higher utility from interacting compared to staying in autarky.

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Table 2.4: The economy’s parameters and variables values; when there is no recession, and the economy continue growing without interruptions.\(^{84}\)

<table>
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\(^{84}\) The parameter $\phi$ is the weight of consumption in the consumption-leisure multiplicative non-separable Cobb-Douglas utility function. Eichenbaum et al. (1998) have obtained an estimate of the parameter between 0.12 and 0.18, while others have used a calibration value of $\phi$ that is around twice as large (for example check: Domeij and Floden (2006); Heathcote et al. (2008); Collard and Dellas (2012)). On the other hand, Brissimis and Bechlioulis (2017) have obtained an estimate of the parameter equal to 0.74. Accordingly, since there is a wide range of estimated values, the value of $\phi$ was chosen to be 0.5. I followed the standard values assumed for $\alpha$ in the literature, while the depreciation rate $\delta$ is given a value of 1, due to the reasons mentioned earlier in the model section. Also, please note that in all the numerical calculations, I ensured that agents will have higher utility from interacting compared to staying in autarky.
Table 2.5: The economy’s parameters and variables values; when a recession occurs, and no increase in wealth inequality occurs in the recovery through a change in \( T \).
Table 2.6: The economy’s parameters and variables values; when a recession occurs, and wealth inequality increases in the recovery through a 25 per cent reduction in $T$. 

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</tbody>
</table>

Table 2.6: The economy’s parameters and variables values; when a recession occurs, and wealth inequality increases in the recovery through a 25 per cent reduction in $T$. 

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The numerical results are plotted in Figures 2.50 to 2.61. As can be seen in Figure 2.50, the model can qualitatively replicate the relative movements in output and factors of production, that occurred during the recession and the following recovery. A reduction in aggregate wealth, leads to a decline in output with a slow recovery in economic activity (Figure 2.51). However, this recovery in economic activity improves with the increase in wealth inequality. This comes through increasing the amount of employed labour, which makes the production more labour intensive. To clarify, in the absence of a change in $T$, the model predicts employment to be lower than the level implied by a continuation of the pre-recession trend (Figure 2.52). Given the model parameters, a reduction of around 8.7 per cent or more in redistributive taxes in the recovery, will make the amount of employed labour surpasses the level implied by a continuation of the pre-recession trend. The more severe is the reduction in $T$, the higher is the increase in employed labour and the lower is the wage rate (Figure 2.53). The reason behind this has been explained earlier
in Figure 2.47, where the outward shift in the labour supply curve will be greater, the larger is the decline in redistributive taxes.

By construction in the model, when aggregate wealth decreases, so does investment. This is because a negative shock to \( v \) represents a credit contraction for the entrepreneur – in the form of a reduction in the supply of bonds – who uses all the funds she borrows to invest in capital. Changes in redistributive wealth taxes do not affect the path of investment (Figure 2.54), since \( b^H + b^C = v\bar{I} + T + v\bar{W} - T = v(\bar{I} + \bar{W}) \). On the other hand, the negative wealth shock leads to a sharp increase in the cost of credit, which gradually declines as aggregate wealth recovers in the post-recession periods (Figure 2.55). Nevertheless, when wealth inequality increases due to a decrease in \( T \), the decline in the cost of credit becomes slower in the economic recovery. This is because the increase in employed labour due to a reduction in redistributive taxes, increases the marginal product of capital in the entrepreneur’s production. Since all the bonds borrowed by her are used in producing capital, \( k(b) = b \), the bonds demand schedule shifts to the right (Figure 2.48). The outward shift becomes bigger, the larger is the decline in redistributive taxes.

The movements in production factors’ prices, implies that their relative price shoots up during the recession (Figure 2.56), and gradually improves in the recovery. However, despite the improvement, it never goes back to its pre-recession level, with the improvement being less pronounced the larger is the decrease in \( T \). This change in relative factor prices leads to the substitution of labour for capital, contributing to the deteriorating capital intensity within firms (Figure 2.57). In other words, production in the economy has become more labour intensive, with the economic recovery depending heavily on increasing employed labour, instead of improvements in labour productivity. The larger is the increase in wealth inequality through decreasing redistributive taxes, the larger is the gap between post-recession labour productivity and the continuation of its pre-crisis trend (Figure 2.58). Although a decrease in \( T \) increases output, it increases employed labour by a greater proportion. For instance, decreasing \( T \) by 50 per cent instead of 25 per cent in \( t = 4 \), makes output grows by an additional 0.74 per cent in \( t = 4 \) relative to \( t = 3 \). However, it makes employed labour grows by an additional 1.05 per cent over the same time.

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85 Since the entrepreneur starts in the period without an endowment of capital, then investment and capital stock are equivalent in this model.

86 The reason behind this has been explained earlier in the comparative statics section and Figure 2.44.
On the other hand, a decrease in redistributive taxes can improve total labour income \( w_l \) in the recovery (Figure 2.59). However, in light of the previously discussed movements in prices and quantities, this improvement comes from increasing labour hours rather than wage rate improvement. The overall effect of a decrease in \( T \) on the agents’ total resources are presented in Figures 2.60 and Figure 2.61.

2.5 Conclusion

The UK labour productivity has been recovering at an extremely low rate following the recession, which gives less weight to demand side explanations. The theoretical framework developed in this chapter has addressed the issue by adopting a supply side explanation, where more persistent factors related to the financial crisis have led to changes in the economy’s supply capacity. To the best of my knowledge, despite the extensive empirical economic literature on the UK’s labour productivity performance, no theoretical study has tried to explain the persistent phenomenon in light of the empirical facts that took place during the recession and the following recovery. More precisely, despite the recent increase in empirical studies that study the link between wealth shocks and labour productivity (for example check: Bernstein, et al. (2017)), no theoretical study has provided a plausible mechanism. Some of the theories proposed in the literature are able to explain different parts of the puzzle, without providing a full conclusive explanation. The model incorporates those explanations in a single theory, which explains how the empirical facts came into place since 2008.

Many factors may have contributed to a poor UK labour productivity growth. However, most of the decline in UK’s labour productivity occurred within firm rather than through depressed external restructuring. Accordingly, the model sought to explain the economy wide phenomenon, by linking shocks to the banking sector as well as changes in wealth distribution, with production changes within firms. The results are achieved by a version of the real business cycle model, which incorporates wealth shocks and redistributive taxes that affect wealth inequality. The shocks in the model cause changes in relative factors’ prices during the recession and the following recovery, leading to capital shallowing within firms. The model’s theoretical predictions are qualitatively consistent with the movements in real variables and prices that occurred in the UK since the financial crisis.
The government measures might have been necessary to save the economy from a deeper recession. Nevertheless, those measures have indirectly led to an increase in wealth inequality, with serious implications for how the British economy has evolved in the recovery. According to the model’s predictions, those policies have helped in accelerating GDP growth and lowering unemployment. On the other hand, it contributed to a deteriorating labour productivity growth, increased capital shallowing and lower real wages. Therefore, it becomes hard to celebrate the record low unemployment rate, as it is overshadowed by a dismal labour productivity performance and stagnating living standards for the majority of the working population. This view is supported by the fact that most of the growth in employment has occurred in low-skilled and low-paid jobs (Dolphin and Hatfield (2015)). Therefore, instead of economic outcomes constraining the government’s ability in boosting spending and ending its austerity program, it is likely that the policies endorsed in the recovery have contributed to those limitations on the government.

Shocks to the financial sector in general, and to the banking sector in particular, play a major role in motivating the theoretical framework developed in this chapter. However, I follow a ‘reduced form’ approach, which abstracts from including a banking sector in the model. Instead for simplicity, it is assumed that adverse shocks to the financial sector affect the supply of bonds. Accordingly, a future research that builds on the theoretical framework presented in this chapter, should look at the effect of including a full banking sector in the model. However, the inclusion of a banking sector in a general equilibrium setup, with unrestricted access to financial markets, would not in itself change the fact that the model satisfies the Modigliani-Miller Theorem on the irrelevance of the debt/equity balance in corporate financing. In other words, the bank will be a redundant institution, since the size and composition of its balance sheet would have no effect on agents’ decisions (Freixas and Rochet (2008)). Other essential elements must be included as well. These are: (i) the explicit introduction of a monetary sector in the model in order for liquidity to have a role; and (ii) agents’ intertemporal decision-making when the model is extended to multiple periods (Peiris et al. (2018)).

87 Other key elements for analysing financial fragility are already present in the model. Those are: (i) endogenous default, (ii) heterogeneity of economic agents, (iii) uncertainty, and (iv) missing financial markets, such that not all risks in the economy can be hedged with existing assets. This is represented by the fact that the household cannot insure against wealth shocks.

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One possible way for adopting the aforementioned changes is to follow the theoretical framework developed by Born and Enders (2019). As discussed earlier, they have constructed a RBC model with a small open economy, linked with the rest of the world by trade in goods and holdings of banking sector assets. They have followed a similar setup of a household who act as lenders but also provide labour, and an entrepreneur who borrows, hires labour and owns the production technology and capital. However, the financial intermediation process is done through a banking sector. Additionally, they have trade in domestic and foreign goods, since they want to investigate the trading channel. Accordingly, a future extension of my model could incorporate the relevant elements from Born and Enders (2019), and abstract from other features that do not assist in addressing the research question of this chapter.
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The further rises in CDS premiums in 2011 and 2012, were due to the European debt crisis.

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The further rises in CDS premiums in 2011 and 2012, were due to the European debt crisis.
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91 Social housing figures reflect the income increasing effect for the population of households that benefit from reduced rent, through the Local Housing Allowance. On the other hand, health care and education figures reflect the income increasing effect for the entire population of UK households.

92 The model’s parameters will be chosen to ensure the demand and supply curves intersect in the first quadrant. Otherwise, it is possible to end up with a negative real interest rate in equilibrium.
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Dataset

The Effects of Public Debt Decomposition on Fiscal Multipliers

3.1 Introduction and Background

At the wake of the financial crisis, governments in developed countries have stepped in to save their economies from a cliff fall. As the official interest rates were aggressively cut down reaching close to the zero-lower bound in many cases, interest in the macroeconomic effects of fiscal policy has been revived. Following the decisive bail out measures, many governments have embarked on fiscal consolidation plans, to bring their finances and indebtedness back into control. However, in many of these cases, economic growth has fallen short from what has been expected, when the fiscal consolidation plans were introduced. Accordingly, it became clear that fiscal multipliers were substantially higher than previously assumed by policy makers (Blanchard and Leigh, 2013). This in turn has intensified the revived interest in the topic, and since then there has been a proliferation in the number of studies investigating the factors that might affect the size of fiscal multipliers.94

Estimates of the size of fiscal multipliers before the crisis were primarily based on linear Vector Autoregression (VAR) models. The estimated multipliers in those models do not change based on the state of the economy (Arin et al. 2015). Those include the structural VAR (SVAR) models, which depend on economic theory or institutional information on government’s tax and transfer systems to identify fiscal shocks, such as Blanchard and Perotti (2002). The alternative was estimation techniques that depend on a narrative approach to identify fiscal innovations, such as Ramey and Shapiro (1998). Since the multipliers’ estimates in the aforementioned techniques did not vary with the business cycle, the effect was to lend support to the

94 Ramey (2011) provides an extensive review of the theoretical and empirical literature on the macroeconomic effects of fiscal policy. Nevertheless, since it reviews the literature until 2011, it does not cover the empirical techniques that became more widely used since, and which investigate the dependency of multipliers on countries’ characteristics or the state of their economy.
expansionary austerity hypothesis.\textsuperscript{95} Alesina and Ardagna (2010) is among the most recent empirical studies which supported the hypothesis, using a large sample of countries and periods.

The Smooth Transition VAR (STVAR) model which uses a regime switching framework, was first introduced by Anderson and Vahid (1998) and Weise (1999). However, it only became widely used in fiscal policy literature when the fiscal consolidation adopted by some European countries, have resulted in deeper economic downturns. One of the first attempts was by Auerbach and Gorodnichenko (2012), who used US data and found that government spending multipliers during economic booms can be very close to zero, while being as high as 2.5 in downturns. Many studies have followed confirming similar results, most notably Baum et al. (2012), Callegari et al. (2012), IMF (2012) and Blanchard and Leigh (2013). Other empirical investigations have continued to use SVAR estimation, however, they combined it with controlling for other country-specific characteristics, that were never investigated before. Ilzetzki et al. (2013), have constructed a very large panel dataset of 44 countries and used the SVAR method of Blanchard and Perotti (2002). In their estimations, countries were split in groups depending on certain characteristics to determine their effect on fiscal multipliers. Those country-specific characteristics included countries’ indebtedness, openness to trade, economic development and exchange rate flexibility. Other papers have followed in the footsteps of Ilzetzki et al. (2013), such as Hory et al. (2018) who looked at how foreign currency denominated debt affects fiscal multipliers.

In this chapter, I am interested in the effect of public debt decomposition between domestic and external creditors, on the size of fiscal multipliers. There has been a wealth of research on the effect of public debt decomposition based on the location of holders, on economic growth. Dimanod (1965) has predicted that external borrowing has different macroeconomic effects compared to domestic borrowing, where the later causes further reduction in the aggregate capital stock. On the other hand, Efthimiadis and Tsintzos (2012) have developed a model that predicts a decline

\textsuperscript{95} The expansionary austerity hypothesis (also called expansionary fiscal contraction hypothesis) suggests that small increases in taxes or reduction in government spending in the short run, can actually stimulate private consumption and investment in the short run as well. This occurs because such fiscal changes in the short run will create expectations of future increases in net earnings, as the government will be able to undertake bigger future cuts in taxes or higher increases in government spending (Blanchard (1990)).
in economic growth, when the external-internal public debt ratio increases. There is extensive empirical literature which confirms that high level of external debt leads to a deteriorating GDP growth, such as Afxentiou (1993), Cunningham (1993), Sawada (1994), Rockerbie (1994), Deshpande (1997) and Pattillo et al. (2011). Nevertheless, there has not been any empirical research looking at the effect of external borrowing on fiscal multipliers, except for the very recent work of Priftis and Zimic (2018) and Broner et al. (2019).

When global liquidity is high, sourcing public debt from abroad can be beneficial. It provides a cheap source of financing, when compared to domestic credit that requires higher rate of return. Andritzky (2012) have built a dataset that decomposes investors in government bonds for a number of developed countries. He found that yields drop by 32 to 43 basis points, when the ratio of non-resident creditors increases by 10 per cent. Moreover, the decrease in yields was found to be greater for the Eurozone government bonds. However, there are substantial risks associated with external debt dependence, that become more evident in economic downturns. Those risks include, but are not limited to: (i) possible external sovereign default which can bring about high inflation, financial autarky and the halt of capital flows, (ii) negative balance sheet effects due to currency mismatch between debt servicing and revenues raised domestically through taxes, and (iii) political cost of external borrowing which weakens state’s sovereignty (Reinhart and Trebesch (2015)).

Sovereign external borrowing has a possible further effect on fiscal policy, which has not received sufficient attention before. In general, fiscal expansions through debt issuance, can have a crowding out effect on domestic private investment and consumption. The crowding out effect of fiscal policy is expected to be greater, when it is financed by debt issued for domestic holders as opposed to foreign holders. This is because the government competes with the domestic private sector, for the domestically available loanable funds. The crowding out occurs through both; an interest rate channel as the cost of borrowing rises due to the higher demand, as well as a quantity channel where less loanable funds are made available for private investment and consumption (Broner et al. (2014)). Accordingly, it is possible that fiscal multipliers can be larger when the ratio of foreign public debt holders is large.

96 Those results however, do not hold for every country. Some countries such as Japan continue to enjoy low yields despite the large base of domestic investors, due to high pensions savings and heavy home bias (Fidora et al. (2006); Tokuoka (2010)).
The crowding out effect of fiscal expansion through debt issuance has been empirically investigated by Huang et al. (2018), who have used data for more than half a million firms, across 69 countries between 1998 and 2014. Although they did not explore the effects of debt issuance between domestic and foreign holders, they have found that higher public debt has led to a reduction in private investment. The effect they found was more severe for firms in industries which require higher external financing.

![Figure 3.1: The percentage of public debt held by non-resident creditors in a number of European countries.](image)

The dataset facilitates future research on this question, which is motivated by the fact that in the years leading to the financial crisis, some of the small European countries – namely Greece, Ireland and Portugal – have witnessed a steady and substantial increases in the percentage of non-resident holders of their public debt (Figure 3.1). This was mostly due to the introduction of the Euro, which encouraged more foreign investors to acquire the increasing supply of sovereign bonds (Merler and Pisani-Ferry (2012)). Other European countries like Spain, Italy and the UK

97 Those countries have specifically experienced a worse economic performance than previously forecasted, when their fiscal consolidation plans were introduced.
98 Chatzouz (2015) have estimated the size of Greek fiscal multipliers using the SVAR approach of Blanchard and Perotti (2002) and quarterly data between 1991 Q1 and 2012 Q1. He found the government spending multipliers to be large, which suggests a strong Keynesian effect. He concludes that this might be due to the large proportion of sovereign debt held by foreign holders.
have also faced concerns about their sovereign solvency. However, their economies are bigger, and their financial markets are more developed, which helped in keeping the holdings of different investors relatively stable. The financial crisis has led to a change in the distribution of investors in government bonds, with central banks playing a more prominent role through the quantitative easing measures (Andritzky (2012)).

My dataset presents an update of Ilzetzki et al. (2013) dataset. The updated dataset has an additional 6 countries, which brings the total number of countries to 50. It expands on the previous dataset by including the decomposition of debt data between domestic and foreign holders using multiple sources, in order to facilitate future empirical investigation of the topic. When the dataset was completed, two working papers have emerged, which discuss the same research question; Priftis and Zimic (2018) and Broner et al. (2019).

Broner et al. (2019) have used annual data between 1978 and 2014, which cover 18 developed countries. They follow Ramey and Zubairy (2018) IV approach, to instrument for the fiscal variables of interest. On the other hand, Priftis and Zimic (2018) have used quarterly data between 1995 Q1 and 2016 Q2 covering 33 countries, of which 6 are developing countries. They follow the SVAR method of Blanchard and Perotti (2002), which was also used in Ilzetzki et al. (2013). They combine this with a sign restriction on the direction of the domestic-external debt ratio, to identify fiscal shocks. Both studies find evidence that increase in the ratio of external debt holders, does indeed make fiscal multipliers bigger. Accordingly, my dataset can contribute to the literature by allowing for a further study that confirms Priftis and Zimic (2018) results, using a larger dataset with a larger number of developing countries. A further investigation can also look at how this effect can vary between economic booms and recessions.

3.2 Data

To follow the SVAR approach of Blanchard and Perotti (2002), quarterly data is essential for the identification of fiscal shocks, as annual data can lead to spurious results. Although there is an increasing number of countries reporting data with a quarterly frequency, in many of those cases it is annual data that has been interpolated. I ensured that the data I collected was actually recorded on quarterly bases, through
checking the sources metadata, or by directly contacting local authorities when data is obtained from local central banks and statistical agencies. The updated unbalanced panel dataset of Ilzetzki et al. (2013), consists of quarterly data from 1952 Q1 to 2018 Q2 for 50 advanced and emerging economies, for which the decomposition of public debt data between domestic (resident) and external (non-resident) holders are available. The dataset includes the following variables: government expenditure, decomposition of government debt into domestic and external creditors, real effective exchange rate, central bank discount rate, current account position, private consumption and total investment. In an alternative dataset where private investment is used instead of total gross fixed capital formation, the number of countries drops to 35 countries, with most of this drop affecting the number of emerging countries that are included. Following Ilzetzki et al. (2013), I do not collect data on government taxes, although this can bias regression results in principle. As presented in their study, the bias from the omission of taxes is not significant when they compared their results with Ilzetzki (2011), using countries that are shared between both studies.

The absence of a single source for all the data included in the dataset, implies inconsistency in the level of coverage between a country and another. For instance, data can be inconsistent in the following dimensions: general government versus central government, and varying debt maturities and instruments. Despite the inconsistency between countries in the level of government coverage, I follow Ilzetzki et al. (2013) in including all the countries for which data are available for all variables. Wherever possible, I include the broader coverage of government. To reduce other possible mismatches, all data obtained were unadjusted and in current prices, expect for the United Kingdom and the United States where only seasonally adjusted data were available. Moreover, data for each country was obtained in the local currency of the country, except for El Salvador as the only available data was in US Dollars. All nominal values were converted to real values at constant 2010 prices using the GDP deflator, except for Honduras, Moldova, Nicaragua and Nigeria, where only the CPI deflator is available. The deflator was obtained from the same source as the government expenditure data, GDP and other subcomponents of GDP. The real values were later converted to US Dollars, using exchange rate data obtained from the IMF-IFS series 64 for all countries. Finally, all series were seasonally adjusted using Eviews 10 routine TRAMO / SEATS, except for debt data. The fluctuations in the debt data are not related to seasonality, but are rather due to different chunks of debt
being issued or repaid at different times with different maturities. Information on data sources are included in the appendix.
3.3 Appendix

3.3.1 Appendix Data

**List of countries in the sample:** Albania, Argentina, Armenia, Australia, Austria, Bosnia and Herzegovina, Brazil, Bulgaria, Canada, Chile, Czech Republic, Dominican Republic, El Salvador, Finland, France, Georgia, Germany, Greece, Guatemala, Honduras, Hungary, Iceland, Indonesia, Ireland, Italy, Japan, South Korea, Latvia, Lithuania, Luxembourg, Malta, Mauritius, Mexico, Moldova, Netherlands, Nicaragua, Nigeria, Paraguay, Peru, Portugal, Romania, Russia, Slovakia, Slovenia, Spain, Sri Lanka, Sweden, Turkey, United Kingdom, United States.

**Gross Domestic Product:** GDP data were obtained from the same source as the government expenditure data for all countries, except Georgia, where National Bank of Georgia data were used.

**Private Consumption:** Household and NPISH final consumption expenditure data were obtained from the same source as the government expenditure data for all countries, except Georgia, where National Bank of Georgia data were used.

**Investment:** Gross fixed capital formation data were obtained from the same source as the government expenditure data for all countries, except Georgia, where National Bank of Georgia data were used.

**Private Investment:** Private gross fixed capital formation data were obtained by deducting government investment expenditure from total gross fixed capital formation.

**Real Effective Exchange Rate:** Following Ilzetzki et al. (2013), wherever possible the CPI based narrow index of the Bank for International Settlements was used. Otherwise the broad index was used. For Armenia, Dominican Republic, Georgia, Moldova, Nicaragua, Nigeria and Paraguay, no data was available in both indices and
therefore data from IMF-IFS series RECZF was used. For Albania, Bosnia and Herzegovina, El Salvador, Mauritius and Sri Lanka, no data was available in any of the aforementioned sources. Therefore, data from Darvas (2012) were used for those countries.

**Current Account:** Wherever possible, data on the current account were obtained from OECD Statistics, otherwise data from the IMF Balance of Payments and International Investment Position Statistics dataset were used.

**Central Bank Discount Rate:** To measure the impact of fiscal policy in the long run, Ilzetzki et al. (2013) calculate the present discounted value of the cumulative multiplier. In calculating the present value, they use the median central bank discount rate instead of the average over the periods in question, to prevent extreme episodes from affecting the results. Table 3.1 below summaries the data sources used to obtain the discount rate. In the case of Eurozone countries, their series are obtained by splicing their interbank rates up to the date of joining the Euro, with the ECB’s deposit facility rate in the period post joining the Eurozone.

Table 3.1: Data sources and coverage for the central bank discount rate.

<table>
<thead>
<tr>
<th>Country</th>
<th>Start</th>
<th>End</th>
<th>Source</th>
<th>Series and Comments</th>
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<td>2001q1</td>
<td>2018q2</td>
<td>IMF-IFS</td>
<td>Central bank policy rate</td>
</tr>
<tr>
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<td>1993q2</td>
<td>2018q2</td>
<td>Bank for International Settlements</td>
<td>Central bank policy rate</td>
</tr>
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<td>1999q4</td>
<td>2018q2</td>
<td>IMF-IFS</td>
<td>Central bank policy rate</td>
</tr>
<tr>
<td>Australia</td>
<td>1969q3</td>
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### Government Consumption:

Table 3.2: Data sources and coverage for government final consumption expenditure.

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**Government Investment:** Data on government investment is not available for all countries.

Table 3.3: Data sources and coverage for government gross fixed capital formation.

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<td>Ireland</td>
<td>1999q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>Italy</td>
<td>1999q1</td>
<td>2018q2</td>
<td>Eurostat</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>Japan</td>
<td>1994q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>Public sector gross fixed capital formation</td>
</tr>
<tr>
<td>South Korea</td>
<td>2000q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>Latvia</td>
<td>1999q1</td>
<td>2018q2</td>
<td>Eurostat</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>Lithuania</td>
<td>1999q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>1999q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
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<td>2000q1</td>
<td>2018q2</td>
<td>Eurostat</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>Mexico</td>
<td>1993q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>Public sector gross fixed capital formation</td>
</tr>
<tr>
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<td>1999q1</td>
<td>2018q2</td>
<td>Eurostat</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>Nicaragua</td>
<td>2006q1</td>
<td>2018q1</td>
<td>Central Bank of Nicaragua</td>
<td>Gross Public Fixed Capital Formation</td>
</tr>
<tr>
<td>Peru</td>
<td>2006q1</td>
<td>2018q2</td>
<td>Central Reserve Bank of Peru</td>
<td>General Government Gross Formation of Capital</td>
</tr>
<tr>
<td>Portugal</td>
<td>1999q1</td>
<td>2018q2</td>
<td>Eurostat</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>Romania</td>
<td>1999q1</td>
<td>2018q2</td>
<td>Eurostat</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>Slovakia</td>
<td>1999q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>Slovenia</td>
<td>1999q1</td>
<td>2018q2</td>
<td>Eurostat</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>Country</td>
<td>Start Year</td>
<td>End Year</td>
<td>Source</td>
<td>Data Description</td>
</tr>
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<td>----------</td>
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</tr>
<tr>
<td>Spain</td>
<td>1995q1</td>
<td>2018q2</td>
<td>Eurostat</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>Sweden</td>
<td>1995q1</td>
<td>2018q2</td>
<td>Eurostat</td>
<td>General government gross fixed capital formation</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1987q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government gross fixed capital formation, seasonally adjusted</td>
</tr>
<tr>
<td>United States</td>
<td>1952q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government gross fixed capital formation, seasonally adjusted</td>
</tr>
</tbody>
</table>
**Government Debt:** Government debt decomposed into domestic and external creditors. Wherever possible, debt data were obtained from OECD and the Quarterly Public Sector Debt (QPSD) database of the IMF – World Bank. Alternatively, when no data were not available from the aforementioned, data from Andritzky (2012) and Merler and Pisani-Ferry (2012) were used.

Table 3.4: Data sources and coverage for government debt and its decomposition into domestic and external creditors.

<table>
<thead>
<tr>
<th>Country</th>
<th>Start</th>
<th>End</th>
<th>Source</th>
<th>Series and Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albania</td>
<td>2009q3</td>
<td>2018q2</td>
<td>QPSD</td>
<td>Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Argentina</td>
<td>2011q1</td>
<td>2017q4</td>
<td>QPSD</td>
<td>Gross central government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Armenia</td>
<td>2014q2</td>
<td>2018q2</td>
<td>QPSD</td>
<td>Gross central government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Australia</td>
<td>1995q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government total gross debt at current prices – decomposed into domestic and external creditors.</td>
</tr>
</tbody>
</table>
| Austria  | 2010q1  | 2018q2  | QPSD   | Gross budgetary central government debt, all maturities, all instruments, nominal value, domestic currency – decomposed
<table>
<thead>
<tr>
<th>Country</th>
<th>Start Quarter</th>
<th>End Quarter</th>
<th>Database</th>
<th>Description</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bosnia and Herzegovina</td>
<td>2007q1</td>
<td>2017q4</td>
<td>QPSD</td>
<td></td>
<td>Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Brazil</td>
<td>2009q3</td>
<td>2018q2</td>
<td>QPSD</td>
<td></td>
<td>Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>2008q1</td>
<td>2018q1</td>
<td>QPSD</td>
<td></td>
<td>Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Canada</td>
<td>1990q1</td>
<td>2018q2</td>
<td>OECD</td>
<td></td>
<td>General government total gross debt at current prices – decomposed into domestic and external creditors. Gross budgetary central government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Chile</td>
<td>2009q1</td>
<td>2018q1</td>
<td>QPSD</td>
<td></td>
<td>Gross budgetary central government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors. Gross budgetary central government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>2009q1</td>
<td>2018q1</td>
<td>QPSD</td>
<td></td>
<td>Gross budgetary central government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors. Gross budgetary central government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Country</td>
<td>Start Period</td>
<td>End Period</td>
<td>Dataset</td>
<td>Description</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>Dominican Republic</td>
<td>2011q1</td>
<td>2018q2</td>
<td>QPSD</td>
<td>Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
<td></td>
</tr>
<tr>
<td>El Salvador</td>
<td>2009q4</td>
<td>2018q2</td>
<td>QPSD</td>
<td>Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td>2000q1</td>
<td>2018q1</td>
<td>OECD</td>
<td>General government total gross debt at current prices – decomposed into domestic and external creditors.</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>1998q4</td>
<td>2017q3</td>
<td>Merler and Pisani-Ferry dataset⁹⁹</td>
<td>Medium and long-term treasury bonds with a maturity of 2-50 years – decomposed into resident and non-resident holders.</td>
<td></td>
</tr>
<tr>
<td>Georgia</td>
<td>2009q3</td>
<td>2018q2</td>
<td>QPSD</td>
<td>Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>1991q1</td>
<td>2015q2</td>
<td>Merler and Pisani-Ferry dataset</td>
<td>General government debt – decomposed into resident and non-resident holders.</td>
<td></td>
</tr>
<tr>
<td>Greece</td>
<td>1997q4</td>
<td>2017q3</td>
<td>Merler and</td>
<td>General government securities (short and long term) –</td>
<td></td>
</tr>
</tbody>
</table>

⁹⁹ Merler and Pisani-Ferry (2012) database of sovereign bond holdings is publicly available and can be accessed online.
decomposed into resident and non-resident holders.

Gross central government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.\(^\text{100}\)

Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.

General government total gross debt at current prices – decomposed into domestic and external creditors.

Central government total gross debt at current prices – decomposed into domestic and external creditors.

General government total gross debt at current prices – decomposed into domestic and external creditors.

Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.

General government total gross debt at current prices – decomposed into domestic and external creditors.

\(^{100}\) General government data is available from the same source, but the series is very short. It starts from 2012q3.
<table>
<thead>
<tr>
<th>Country</th>
<th>Start Date</th>
<th>End Date</th>
<th>Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>1995q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government total gross debt at current prices – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Japan</td>
<td>2000q1</td>
<td>2012q1</td>
<td>Andritzky dataset</td>
<td>Central government bonds at nominal value – decomposed into resident and non-resident holders.</td>
</tr>
<tr>
<td>South Korea</td>
<td>2002q4</td>
<td>2011q3</td>
<td>Andritzky dataset</td>
<td>Central government bonds at nominal value – decomposed into resident and non-resident holders.</td>
</tr>
<tr>
<td>Latvia</td>
<td>2000q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government total gross debt at current prices – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Lithuania</td>
<td>2005q1</td>
<td>2018q1</td>
<td>QPSD</td>
<td>Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>2000q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government total gross debt at current prices – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Malta</td>
<td>2011q2</td>
<td>2018q2</td>
<td>QPSD</td>
<td>Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Mauritius</td>
<td>2009q3</td>
<td>2018q2</td>
<td>QPSD</td>
<td>Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.</td>
</tr>
</tbody>
</table>

The dataset is not publicly available. The author of Andritzky (2012) was contacted and he kindly provided access to the dataset.
<table>
<thead>
<tr>
<th>Country</th>
<th>Start Year</th>
<th>End Year</th>
<th>Database</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2018q2</td>
<td>OECD</td>
</tr>
<tr>
<td>Moldova</td>
<td>2009q3</td>
<td>2017q4</td>
<td>QPSD</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1999q4</td>
<td>2018q1</td>
<td>QPSD</td>
</tr>
<tr>
<td>Nicaragua</td>
<td>2010q1</td>
<td>2018q2</td>
<td>QPSD</td>
</tr>
<tr>
<td>Nigeria</td>
<td>2009q4</td>
<td>2017q4</td>
<td>QPSD</td>
</tr>
<tr>
<td>Paraguay</td>
<td>2011q2</td>
<td>2018q2</td>
<td>QPSD</td>
</tr>
<tr>
<td>Peru</td>
<td>2010q1</td>
<td>2017q4</td>
<td>QPSD</td>
</tr>
</tbody>
</table>

General government total gross debt at current prices – decomposed into domestic and external creditors.

Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.

Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.

Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.

Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.
<table>
<thead>
<tr>
<th>Country</th>
<th>Start Date</th>
<th>End Date</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1999q4</td>
<td>2018q2</td>
<td>OECD</td>
</tr>
<tr>
<td>Romania</td>
<td>2012q1</td>
<td>2017q4</td>
<td>QPSD</td>
</tr>
<tr>
<td>Russia</td>
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<td>2018q2</td>
<td>OECD</td>
</tr>
<tr>
<td>Slovakia</td>
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<td>2018q2</td>
<td>OECD</td>
</tr>
<tr>
<td>Slovenia</td>
<td>2010q1</td>
<td>2018q2</td>
<td>OECD</td>
</tr>
<tr>
<td>Spain</td>
<td>1995q1</td>
<td>2018q2</td>
<td>OECD</td>
</tr>
<tr>
<td>Sri Lanka</td>
<td>2010q1</td>
<td>2018q2</td>
<td>QPSD</td>
</tr>
</tbody>
</table>

value, domestic currency – decomposed into domestic and external creditors.

General government total gross debt at current prices – decomposed into domestic and external creditors.

Gross general government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.

Central government total gross debt at current prices – decomposed into domestic and external creditors.

General government total gross debt at current prices – decomposed into domestic and external creditors.

Central government total gross debt at current prices – decomposed into domestic and external creditors.

General government total gross debt at current prices – decomposed into domestic and external creditors.

Gross central government debt, all maturities, all instruments, nominal value, domestic currency – decomposed into domestic and external creditors.
<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>End Period</th>
<th>Agency</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>1995q4</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government total gross debt at current prices – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>Turkey</td>
<td>2005q4</td>
<td>2018q2</td>
<td>OECD</td>
<td>Central government total gross debt at current prices – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1995q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government total gross debt at current prices – decomposed into domestic and external creditors.</td>
</tr>
<tr>
<td>United States</td>
<td>1952q1</td>
<td>2018q2</td>
<td>OECD</td>
<td>General government total gross debt at current prices – decomposed into domestic and external creditors.</td>
</tr>
</tbody>
</table>
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