ANONYMOUS SINGLE-PROFILE WELFARISM

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Abstract. This note reexamines the single-profile approach to social-choice theory. If an alternative is interpreted as a social state of affairs or a history of the world, it can be argued that a multi-profile approach is inappropriate because the information profile is determined by the set of alternatives. However, single-profile approaches are criticized because of the limitations they impose on the possibility of formulating properties such as anonymity. We suggest an alternative definition of anonymity that applies in a single-profile setting and characterize anonymous single-profile welfarism under a richness assumption. *Journal of Economic Literature* Classification Number: D63.

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1. Introduction

Welfarist principles for social evaluation rank social alternatives using information about individual well-being alone, ignoring non-welfare information. The most commonly used approach to social-choice theory employs multiple profiles of welfare (utility) information and uses a social-evaluation functional to assign a social ranking of the alternatives to each utility profile in the domain of the functional. Non-welfare information is implicitly fixed. Welfarism is a consequence of the axioms unlimited domain, Pareto indifference and binary independence of irrelevant alternatives; see, for example, Blackorby, Bossert and Donaldson [2002], Bossert and Weymark [2004], d'Aspremont and Gevers [1977], Guha [1972], Hammond [1979], Sen [1977, 1979] and Weymark [1998]. For any social-evaluation functional satisfying the three axioms, there exists a single ordering on the set of possible utility vectors that can be used, in conjunction with the information in a profile, to rank the alternatives.

In the traditional social-choice framework, non-welfare information is assumed to be fixed and does not appear explicitly as a possible input of social evaluation. This is unsatisfactory because, without having the option of varying the non-welfare information, it is not possible to identify its influence. For example, suppose that whether or not an agent is hardworking is to be taken into account in social evaluation and the individual levels of welfare are not the only determinants of a social ranking. If there is a single non-welfare-information profile, it is impossible to say whether the preferential treatment of an agent is due to the observation that he or she is hardworking or to some other feature of the alternatives to be ranked. To isolate the influence of the agent's attitude towards work, it is necessary to examine the counterfactual: a profile where the agent is not hardworking. Thus, it is desirable to exclude non-welfare information explicitly in a model! of soci al evaluation. One way of doing so is to introduce an expanded version of an information profile. In addition to a welfare component $(U_1, \ldots, U_n)$ that consists of one utility function defined on the set of alternatives for each member of society, the information profile contains a non-welfare component $(K_0, K_1, \ldots, K_n)$ where K_0 is a function that assigns social non-welfare information to the alternatives and, for i between 1 and n, K_i does the same for non-welfare information that is specific to individual i. Thus, a profile can be written as a pair (U, K) and K may matter in addition to U when establishing a social ranking.

Blackorby, Bossert and Donaldson [2004] allow for multiple profiles of non-welfare information. In that setting, the independence axiom is formulated in terms of both welfare and non-welfare information. If, in any two profiles, welfare and non-welfare information coincide on a pair of alternatives, binary independence requires the social orderings to rank them in the same way. This weak version of binary independence, together with unlimited domain and Pareto indifference, is shown to imply welfarism. In addition, this approach permits a compelling justification of anonymous welfarism. The standard anonymity axiom requires the social ordering to be unaffected by a permutation of utility functions across individuals with non-welfare information (implicitly) unchanged. However, some individual non-welfare information, such as being hardworking, may be thought to justify special consideration and this lessens the ethical attractiveness of the axiom. The anonymity axiom employed in Blackorby, Bossert and Donaldson [2004] requires the social ordering to be unaffected if *both* utility functions and individual non-welfare-information functions are permuted. Together with a restriction on the ranges of the individual non-welfare-information functions (which is needed to e! nsure th at permuted profiles are well-defined) and the welfarism axioms, it implies that the ordering of utility vectors must be anonymous: it ranks all permutations of a utility vector as equally good.

Although the multi-profile approach has many attractive features, it can be criticized on the grounds that it may not be suitable if the alternatives are assumed to be social states of affairs or histories of the world. In that case, it can be argued that both welfare information and non-welfare information are part of the descriptions of the alternatives. Consequently, only a single information profile, determined by the set of alternatives, is available. This view is strengthened by the requirement that utility information be consistent with a comprehensive account of well-being such as that of Griffin [1986].

In such a setting, a multi-profile approach requires some aspects of the descriptions of alternatives to change when profiles change. Consequently, alternatives must be regarded as labels for states of affairs or histories. If, for example, the set of alternatives consists of three elements, the multi-profile approach requires the social-evaluation functional to produce more than one ranking of the alternatives—one for each profile in its domain. The independence axiom ensures that these rankings are consistent.

This interpretation of the multi-profile approach may suggest its rejection in favour of a single-profile approach. Although the single-profile environment accommodates welfarism without any problems, there is a significant difficulty with the standard anonymity axiom because it requires multiple profiles (see, for example, Mongin [1994] for a discussion).

A social ordering R of alternatives is single-profile welfarist if and only if there exists a social-evaluation ordering $\overset{*}{R}$ defined on the set of attainable utility vectors such that any two alternatives are ranked according to R in the same way that their associated utility vectors are ranked by $\overset{*}{R}$. The purpose of this paper is to formulate a sensible anonymity axiom that can be used in the single-profile setting. As in Blackorby, Bossert and Donaldson [2004], non-welfare information is explicitly taken into consideration. In this model, it is straightforward to verify that single-profile welfarism is a consequence of Pareto indifference only: no further assumptions are required and no assumptions about the number of alternatives or information diversity are needed. The proof of the result is the same as in the case where non-welfare information is suppressed. This is no surprise because, in the single-profile case, whether the fixed profile of non-welfare information appears explicitly is of no consequence for the single social ordering. However, once further axioms that involve non-welfare information in a non-trivial way are imposed, this no longer is the case. We illustrate this observation by analyzing the consequences of a single-profile version of the anonymity axiom.

In its standard multi-profile formulation, anonymity requires that if a utility profile and the individual portion of the corresponding non-welfare-information profile are replaced by a common permutation with social non-welfare information unchanged, the resulting social ranking is unchanged as well. Clearly, this axiom makes no sense in a single-profile setting. Mongin [1994] proposes to extend the single-profile domain by adding (at least) all permutations of the profile.

In contrast, we retain the single-profile model and define a single-profile version of anonymity. It applies to any pair of alternatives x and y and requires the two to be ranked as equally good whenever there exists a permutation ρ of the set of individuals such that utilities and individual non-welfare information in y are obtained by applying ρ to the corresponding values in x, provided that social non-welfare information is the same in both alternatives. This axiom is easily defended because it is silent unless the permutation is applied to both welfare and non-welfare information. The social-evaluation ordering $\overset{*}{R}$ is anonymous if and only if it declares any two utility vectors in the set of attainable utility vectors to be equally good whenever one is a permutation of the other.

The anonymity axiom by itself, in conjunction with Pareto indifference, does not imply that $\overset{*}{R}$ is anonymous. This is the case because the axiom has no power unless alternatives exist in which both welfare and individual non-welfare information are permuted. If a richness condition is employed, however, anonymous single-profile welfarism can be characterized. This characterization is the main result of the paper and it provides an important step towards developing a satisfactory single-profile theory of welfarist social choice.

The next section introduces our notation and the axioms employed in the remainder of the paper. Section 3 contains the results and proofs, and Section 4 concludes.

2. Basic definitions

The set of all positive integers is denoted by $\mathcal{Z}_{++}$ and the set of real numbers by $\mathcal{R}$. For $n \in \mathcal{Z}_{++}$, let $\mathcal{R}^n$ be the n-fold Cartesian product of $\mathcal{R}$. We consider a society $N = \{1, \ldots, n\}$ of $n \in \mathcal{Z}_{++} \setminus \{1\}$ individuals. The set of alternatives is X and we assume that it is non-empty. For a vector $u \in \mathcal{R}^n$ and a bijection $\rho: N \to N$, we define $u^\rho = (u_{\rho(1)}, \ldots, u_{\rho(n)})$.

There is a single (fixed) utility profile $U = (U_1, \ldots, U_n)$, where $U_i: X \to \mathcal{R}$ is the utility function of individual $i \in N$. Utility is an index of individual well-being. We write $U(x) = (U_1(x), \ldots, U_n(x))$ for all $x \in X$.

Non-welfare information is described by a single profile $K = (K_0, K_1, \ldots, K_n)$, where $K_0: X \to \mathcal{K}_0$ is a function that associates social non-welfare information with each alternative in X and, for all $i \in N$, $K_i: X \to \mathcal{K}_i$ associates individual non-welfare information for individual i with each alternative in X. The set $\mathcal{K}_0 \neq \emptyset$ is the set of possible values of social non-welfare information and, for all $i \in N$, $\mathcal{K}_i \neq \emptyset$ is the set of possible values for individual i's non-welfare information. We write $K_{-0} = (K_1, \ldots, K_n)$ and, for all $x \in X$, we define $K(x) = (K_0(x), K_1(x), \ldots, K_n(x))$ and $K_{-0}(x) = (K_1(x), \ldots, K_n(x))$.

In single-profile social choice, a single social ordering R on X is to be established. I and P are the symmetric and asymmetric factors of R. The ordering R is welfarist if and only if there exists a social-evaluation ordering $\overset{*}{R}$ on $U(X) \subseteq \mathcal{R}^n$ such that the ranking of any two alternatives x and y according to R is obtained by the ranking of $U(x)$ and $U(y)$ according to $\overset{*}{R}$. The symmetric and asymmetric factors of $\overset{*}{R}$ are denoted by $\overset{*}{I}$ and $\overset{*}{P}$. As is shown in the following section, the Pareto-indifference axiom is necessary and sufficient for single-profile welfarism (see, for example, Blackorby, Donaldson and Weymark [1990] for a formulation without a non-welfare-information profile). Pareto indifference requires any two alternatives to be ranked as equally good if each individual is equally well off in both.

Pareto Indifference: For all $x, y \in X$, if $U(x) = U(y)$, then xIy.

As shown in Blackorby, Bossert and Donaldson [2002], Pareto indifference is a consequence of an axiom proposed by Goodin [1991] which we call minimal individual goodness. It requires that if one alternative is socially better than another, it must be the case that the former is better for at least one individual. Without this requirement, we run the risk of recommending social changes that are empty gestures, benefiting no one and, perhaps, harming some or all. To see that Pareto indifference is implied by minimal individual goodness, suppose that everyone is equally well off in two alternatives x and y. Minimal individual goodness implies that x is not better than y and that y is not better than x. Because R is assumed to be complete, it follows that x and y are equally good.

In a multi-profile setting, anonymity requires that if one profile is obtained from another by permuting the individual utility functions and non-welfare-information functions and, moreover, social non-welfare information is unchanged, the same social ranking results for the two profiles. Clearly, this definition cannot be employed in a single-profile setting. We propose the following single-profile anonymity condition instead.

Single-Profile Anonymity: For all $x, y \in X$ and for all bijections $\rho\colon N \to N$, if $K_0(y) = K_0(x)$ and $(U(y), K_{-0}(y)) = (U(x)^\rho, K_{-0}(x)^\rho)$, then xIy.

Single-profile anonymity requires the social ranking to be insensitive with respect to permutations of all individual information attained in a given alternative. Note that the axiom only applies if the permuted utility vector and the permuted non-welfare-information vector are in the image of U and of K_{-0}. The axiom is silent if this is not the case. Single-profile anonymity is easily defended because it allows non-welfare information to matter. All that is ruled out is the claim that an individual's identity justifies special treatment, no matter what non-welfare information obtains.

3. Welfarism and anonymity

We begin our discussion by stating the single-profile welfarism theorem (see Blackorby, Donaldson and Weymark [1990]) in our model where non-welfare information is explicitly taken into consideration. Without further requirements (such as anonymity), the proof is identical to that of the version without non-welfare information. We provide the proof for completeness.

Theorem 1: *R satisfies Pareto indifference if and only if there exists an ordering $\overset{*}{R}$ on $U(X)$ such that, for all $x, y \in X$,*

$$xRy \Leftrightarrow U(x)\overset{*}{R}U(y). \tag{1}$$

Proof. 'If.' Suppose $\overset{*}{R}$ is an ordering such that (1) is satisfied. Pareto indifference follows immediately from the reflexivity of $\overset{*}{R}$.

'Only if.' Suppose R satisfies Pareto indifference. Define the relation $\overset{*}{R}$ on $U(X)$ by

$$u\overset{*}{R}v \Leftrightarrow \text{ there exist } x, y \in X \text{ such that } U(x) = u,\ U(y) = v \text{ and } xRy \tag{2}$$

for all $u, v \in U(X)$. That $\overset{*}{R}$ is well-defined follows from Pareto indifference. (1) follows from the definition of $\overset{*}{R}$. It remains to show that $\overset{*}{R}$ is an ordering.

For all $u \in U(X)$, there exists $x \in X$ such that $U(x) = u$. Because R is reflexive, we have xRx and thus $u\overset{*}{R}u$. Hence $\overset{*}{R}$ is reflexive.

To show that $\overset{*}{R}$ is complete, let $u, v \in U(X)$ be such that $u \neq v$. By definition, there exist $x, y \in X$ such that $U(x) = u$ and $U(y) = v$. Because $u \neq v$, it follows that $x \neq y$ which, by the completeness of R, implies xRy or yRx. Consequently, $u\overset{*}{R}v$ or $v\overset{*}{R}u$.

Finally, to prove that $\overset{*}{R}$ is transitive, let $u, v, q \in U(X)$ be such that $u\overset{*}{R}v$ and $v\overset{*}{R}q$. By definition, there exist $x, y, z \in X$ such that $U(x) = u$, $U(y) = v$, $U(z) = q$, xRy and yRz. By the transitivity of R, xRz and, consequently, $u\overset{*}{R}q$. ■

An ordering $\overset{*}{R}$ on $U(X)$ is anonymous if and only if, for all $u \in U(X)$ and for all bijections $\rho: N \to N$ such that $u^\rho \in U(X)$, $u\overset{*}{I}u^\rho$. Adding single-profile anonymity to Pareto indifference does not guarantee that the ordering $\overset{*}{R}$ is anonymous. For example, suppose that $X = \{x, y\}$ and $N = \{1, 2\}$. Suppose further that the individual utility functions are such that $U_1(x) = U_2(y) = 1$ and $U_1(y) = U_2(x) = 0$. Finally, suppose that $\mathcal{K}_0 = \{$ 'freedom of speech' $\}$, $\mathcal{K}_1 = \mathcal{K}_2 = \{$ 'hardworking,' 'not hardworking' $\}$ and non-welfare information is given by $K_0(x) = K_0(y) =$ 'freedom of speech,' $K_1(x) = K_1(y) =$ 'hardworking' and $K_2(x) = K_2(y) =$ 'not hardworking.' The social ordering R such that xPy trivially satisfies Pareto indifference and single-profile anonymity but the associated social-evaluation ordering $\overset{*}{R}$ on $U(X) = \{(1, 0), (0, 1)\}$ is not anonymous because, by definition, $(1, 0)\overset{*}{P}(0, 1)$.

In the presence of a richness condition, however, Pareto indifference and single-profile anonymity together are necessary and sufficient for the existence of an anonymous social-evaluation ordering $\overset{*}{R}$. The richness property is needed to ensure that single-profile anonymity has any power at all (note that the axiom is vacuously satisfied in the above example). It requires that if a utility vector u and a permutation u^ρ of u are in the image of U, then there must exist alternatives x and y such that social non-welfare information is the same in x and in y, and welfare and non-welfare information in y is given by applying ρ to welfare and non-welfare information in x.

Richness: For all $u \in U(X)$ and for all bijections $\rho: N \to N$ such that $u^\rho \in U(X)$, there exist $x, y \in X$ such that $U(x) = u$, $U(y) = u^\rho$, $K_0(y) = K_0(x)$ and $K_{-0}(y) = K_{-0}(x)^\rho$.

Our main result characterizes anonymous welfarism in the single-profile setting.

Theorem 2: *Suppose richness is satisfied. R satisfies Pareto indifference and single-profile anonymity if and only if there exists an anonymous ordering $\overset{*}{R}$ on $U(X)$ such that, for all $x, y \in X$, (1) is satisfied.*

Proof. Suppose richness is satisfied (note that the property is required for the only-if part of the proof only).

'If.' Suppose $\overset{*}{R}$ is an anonymous ordering such that (1) is satisfied. Pareto indifference follows from Theorem 1. To prove that single-profile anonymity is satisfied, suppose that two alternatives $x, y \in X$ and a bijection $\rho: N \to N$ are such that $K_0(y) = K_0(x)$ and

$(U(y), K_{-0}(y)) = (U(x)^\rho, K_{-0}(x)^\rho)$. Because $U(y) = U(x)^\rho$ and $\overset{*}{R}$ is anonymous, it follows that $U(x)\overset{*}{I}U(y)$ and, by (1), xIy.

'Only if.' Suppose R satisfies Pareto indifference and single-profile anonymity. Theorem 1 establishes that the relation $\overset{*}{R}$ on $U(X)$ as defined in (2) is an ordering such that (1) is satisfied. It remains to show that $\overset{*}{R}$ is anonymous. Suppose that $u, u^\rho \in U(X)$ for a bijection $\rho: N \to N$. By the richness assumption, there exist $x, y \in X$ such that $U(x) = u$, $U(y) = u^\rho$, $K_0(y) = K_0(x)$ and $K_{-0}(y) = K_{-0}(x)^\rho$. By single-profile anonymity, xIy which, by (1), implies $U(x)\overset{*}{I}U(y)$ and thus $u\overset{*}{I}u^\rho$. ■

A slight modification of the example discussed above illustrates the result. Information for x and y is the same as in the example, and the richness axiom ensures that there is an alternative $z \in X$ with $U_1(z) = 0$, $U_2(z) = 1$, $K_0(z) =$ 'freedom of speech,' $K_1(z) =$ 'not hardworking' and $K_2(z) =$ 'hardworking.' The example is illustrated in Table 1. In it, H means 'hardworking' and N means 'not hardworking.' Because both x and z are in X with permuted welfare and non-welfare information, single-profile anonymity applies and requires xIz. Because utility information is the same in z and y, Pareto indifference requires zIy and transitivity of R implies xIy. Welfarism implies $(1,0)\overset{*}{I}(0,1)$.

Table 1

	Person 1		**Person 2**	
	Utility	**Effort**	**Utility**	**Effort**
Alternative x	1	H	0	N
Alternative y	0	H	1	N
Alternative z	0	N	1	H

4. Conclusion

Non-welfare information is explicitly modeled in Kelsey [1987], who provides a formulation of Arrow's [1951, 1963] theorem in this generalized framework, and in Blackorby, Bossert and Donaldson [2004], where a generalization of multi-profile welfarism is developed. The present paper complements the analysis of Blackorby, Bossert and Donaldson [2004] by examining the single-profile approach in the same general framework.

We motivated the necessity for an analysis of the single-profile approach by appealing to some conceptual difficulties with multi-profile social-choice theory that arise if alternatives are assumed to be social states of affairs or histories. The single-profile approach is also employed in models where alternatives have a more narrow interpretation, however. Clearly, our analysis remains valid for these alternative interpretations as well.

We view this note as a first step towards establishing a sound framework for welfarist social evaluation on the basis of a single information profile. Although we restrict attention to the anonymity axiom, it would be useful to examine other properties of multi-profile social-evaluation functionals and to attempt to formulate their analogues in the single-profile setting.

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