

**SURVIVAL OF THE SMALL FIRM
AND THE ENTREPRENEUR
UNDER DEMAND AND EFFICIENCY UNCERTAINTY**

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Survival of the Small Firm and the Entrepreneur under Demand and Efficiency Uncertainty^α

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Abstract

The objective of this paper is to offer an answer to the question: why do some entrepreneurs wish to own another firm in the future after having closed an unsuccessful one? We first show this question is relevant because making use of a sample of entrepreneurs in the UK who have experienced a business closure, we show that 45% of them have the desire to own another firm in the future, despite having an unsuccessful experience in small firm ownership. To tackle our question we develop a model where the profits of the small firm depend on two firm-specific parameters: the efficiency parameter, which represents the skills of the entrepreneur to manage and cope efficiently with the everyday tasks of the small firm, and the demand parameter, which denotes the success of the firm's product to attract demand or capture a market niche. It is found that our model answers our initial question by revealing the existence of a mechanism of entrepreneurial self-selection. Under such mechanism, skilful entrepreneurs are the only ones who wish to own another firm in the future, regardless of the degree of success in their previous venture, whereas unskilful entrepreneurs prefer to go to wage work. We show this mechanism accounts not only for the empirical evidence relevant to our initial question, but also for the rest of cases of entrepreneurs' attitudes after experiencing a business closure.

Keywords: Small Firm, Entrepreneur, Survival, Entrepreneurial Skills

JEL Classification: J23, L11, L25, M13

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1 Introduction

Why do some small firm owners wish to start a new firm after having closed one that was not successful? In this paper we try to answer this question. Why is this question relevant? Firstly, because straightforward intuition provides two conflicting hypotheses as answers. On one hand, we can argue that the small firm owner has learned from his mistakes and believes that in the following venture he can do better. This suggests the entrepreneur believes he has the potential talent to run successful firms, he was just unlucky in his previous venture. On the other hand, we can argue that the lack of success in his previous venture is a reflection of the fact that he does not have the skills to run a successful firm. If he does not have what it takes, why would he want to start a new firm? Secondly, because we show that the desire of opening a new firm after having closed an unsuccessful one is a rather common occurrence amongst entrepreneurs. Utilising a survey of entrepreneurs in the United Kingdom who have recently experienced a business closure, we show that 45% of these entrepreneurs wish to start a new firm, despite the lack of success in their previous venture. Do these entrepreneurs have the talent to run successful firms?, or are they being irrational by hoping they will do better the next time given that it has been proved they do not have the necessary entrepreneurial skills? Both questions cannot be answered affirmatively. In any case, we believe an answer in one way or another will constitute findings which should be considered by policy makers interested in the promotion and encouragement of start-ups and entrepreneurial activity. Hence, the importance of tackling our initial question.

Our question is also relevant in the context of research on small firms. In the Economics literature, the interest in research on small firms has been increasing in recent years. This can be regarded as a consequence of the recognition during the 80's and onwards that small firms account for a considerable share of economic activity (Acs and Audretsch, 1990; Braunerhjelm, 2000). In spite of the growing importance of small firms in the economy, it has been found that small firms consistently have lower likelihood of survival than larger firms do, a finding which is now regarded as a stylised result in small firm research (Geroski, 1995; Storey and Wynarczyk, 1996). In the light of these results, efforts have been directed to unravel the determinants of small firm survival (Bates, 1990; Audretsch, 1991; Kalleberg and Leicht, 1991; Mata and Portugal, 1994; Storey and Wynarczyk, 1996). Despite the fact that in some of these research the role the entrepreneur plays as owner and manager has been acknowledged to some degree, none of them look at the survival of the entrepreneur, in other words, at the outcome of the decision between staying in entrepreneurship by starting another firm after having closed the previous one, or going to wage work. Glad-

stone and Lee (1995, p. 27) on discussing the impact of the insolvency system in the UK on entrepreneurialism assert that ‘...there is still little or no concomitant recognition of, or accompanying policy, to rehabilitate entrepreneurs forced out of entrepreneurialism (...rm owners whose ...rms did not survive, my brackets), even though they are a vital part of the competitive cycle. The impact of failure on the individual entrepreneur has been of minimal interest to most theorists...’ Therefore, by trying to answer why some small ...rm owners wish to start a new ...rm after having closed an unsuccessful one, we also contribute to the efforts aimed to understand the processes which determine the survival of small ...rms, and we also attempt to reduce the gap in the study of the forces which influence the survival of the entrepreneur¹.

The paper has the following structure. In the next section, after specifying the definition of entrepreneur to be used throughout our analysis, we present empirical evidence on entrepreneurial survival which shows the relevance of our initial question. Such empirical evidence also reveals the whole spectrum of entrepreneurs’ attitudes, which derive from the combination of having or not the desire to own another ...rm in the future, after having the experience of running successful ...rms or unsuccessful ones. In Section 3 the model and the assumptions we use to try to answer our initial question are introduced. In contrast to current literature, in our model we specify that the profits of the small ...rms depend on two ...rm-specific determinants: the efficiency parameter, which represents the skills of the entrepreneur to manage and cope efficiently with the everyday tasks of the small ...rm, and the demand parameter, which denotes the success of the ...rm’s product to attract demand or capture a market niche. We assume such parameters are independent of each other. In Section 4 we introduce an alternative model in which we assume the entrepreneur’s ability are not restricted to skills to manage, but that they represent some type of “overall” skills. The purpose of this is to provide a benchmark case for the predictions obtained by our general model. In this section we also present the predictions offered by our general model and by the alternative one, and we discuss the answers both models offer to our initial question. It is found that our general model establishes that those entrepreneurs who start or have the desire to start a new ...rm after having closed an unsuccessful one, have skills to manage as large as those who are running successful ...rms. Contrary to this, the alternative model gives the explanation that such entrepreneurs should not exist, because the fact that the

¹ Interestingly, in Labour Economics, research on the survival of individuals as self-employed has attracted a lot of attention (for a survey article see Le, 1999). However, such studies concentrate only on the outcome of staying in or exiting self-employment by the individual, without considering the fact that these individuals may or may not have owned several ...rms in the process. That is, that they may have decided to close a ...rm and have started a new one, which is our main concern.

small firm did not survive is simply the result that they do not have the necessary skills to run a successful firm. We believe our general model provides a more sensible answer to our initial question. However, to determine which model offers more credible results, we state that such model should provide an explanation not only to our initial question, but also to the whole empirical evidence contained in Section 2.

In Section 5 we discuss which model is able to explain the empirical evidence. We found that our general model is successful in this task, whilst the alternative model fails. By means of revealing the existence of a mechanism of entrepreneurial self-selection, our general model is able to account for the empirical evidence on entrepreneurial survival. Under the mechanism of entrepreneurial self-selection, skilful entrepreneurs are the only ones who wish to own another firm in the future after having closed one, regardless of the degree of success in their previous venture, whereas unskilful entrepreneurs prefer to go to wage work. In this section, given that the alternative model, which uses the assumption that entrepreneurs have some type of “overall” skills, is unable to explain the empirical evidence, we also reach the following conclusion: when undertaking research on entrepreneurship if we consider data comprising small firm owners without any other discriminatory criterion, it is reasonable to suppose that the skills of such business owners are limited to skills to manage, and that they do not comprise skills to innovate or to spot profitable opportunities. In Section 6 we address the robustness of some of our assumptions and the implications of alternative ones. Finally, Section 7 summarises our findings and mentions paths for future research.

In summary, our paper answers our initial question by saying that the reason why some small firm owners wish to start a new firm after having closed an unsuccessful one is because they have the skills to run potentially successful firms. In fact, our findings state that such behaviour is part of a mechanism of entrepreneurial self-selection. Under such mechanism, skilful entrepreneurs are the only ones who wish to own another firm in the future, regardless of the degree of success in their previous venture, whereas unskilful entrepreneurs prefer to go to wage work. Although this mechanism accounts for the empirical evidence on entrepreneurial survival in Section 2, we recognise that further research is needed to reject or fail to reject the prediction that those entrepreneurs who have the desire to start another firm, have indeed higher skills than those who prefer to go to wage work. In any case, we believe the findings of this paper are a first step in a research agenda aimed at the study of the survival of the entrepreneur.

2 Preliminaries

2.1 The Definition of Entrepreneur

Despite being a fertile and growing area of research, there is no agreement amongst academics on the definition of entrepreneur. Across the literature, one article may use entrepreneur to refer to those who simply own and manage a firm, whereas others may include more characteristics, for instance, capacity of innovation or risk taking. Moreover, in some cases these definitions have been used interchangeably. Holtz-Eakin (2000) on addressing the question "Are small businesses entrepreneurial or just small?" points to the problem. He argues that 'The research literature to date, however, has not provided a clear resolution to this question. In part, this question has been avoided; the entrepreneurial virtues of new business are often assumed rather than examined' (p. 284).

Contrary to this, we restrict the definition of entrepreneur to be used in this article. To accomplish this we resort to one of the few articles which distinguishes and groups the different definitions of entrepreneurship. Tyson et al (1994) offers four categories to classify the different definitions. The first one is "Entrepreneurship as innovation". They include in this category those definitions built upon the schumpeterian view of entrepreneurs as agents of innovation, in terms of developing new products or methods of productions, identifying new markets or sources of supply, or developing new organisational forms. The second category is "Entrepreneurship as risk-taking". They group in this category those definitions which view entrepreneurs as having relatively low aversion risk, which enables them to undertake an innovation that others are not willing to attempt. The third category is "Entrepreneurship as stabilising force". The definitions included here are those which regard entrepreneurs as taking advantage of the state of disequilibrium of the economy, where the inefficient allocation of resources generates the opportunity for profitable activities. The final category they suggest is "Entrepreneurship as founding or owning and managing a small business". They assert that '...this view surfaces quite often as the empiricist's definition - understandably, given the practical difficulties of identifying the population of "Schumpeterian innovators" or "risk takers" in an economy' (Tyson et al, 1994, p. 167).

This paper adheres to the definition used more by the empiricists and regards entrepreneurs simply as those who start or own and manage a small firm. Several reasons account for this decision. Firstly, we make use of empirical data to show why our initial question "why do some small firm owners wish to start a new firm after having closed one that was not successful?" is relevant, in the sense that starting a business after having closed

an unsuccessful one is a rather common occurrence amongst small firm owners. Clearly, the database we use focuses on individuals who own and manage their firms. Accordingly, we are bound to adopt the definition of entrepreneurship as founding or owning and managing a business. Our decision gains strength when we acknowledge the growing interest in the measurement of entrepreneurship amongst researchers and institutions, which in general implicitly or explicitly define an entrepreneur as an individual who manages his own business². Secondly, in order to try to answer our initial question, this paper proposes that the entrepreneurial skills of the small firm owners are in general of a restricted nature. Specifically we will argue that small firms owners have an endowment of skills to manage the business, that is, to deal efficiently with the day to day activities of the business³. This endowment varies from individual to individual. We argue that these managerial skills are in general not related with the ability to generate revenue for the firm, in other words, with the ability to select a product or service for which there is enough demand or there is a market niche. In summary, this paper will suggest that the entrepreneurial skills of small firm owners are in general restricted to managerial skills. Note that by no means we state that it is impossible for some small firm owners to have more than one type of skills. As will be shown, our argument states that when we define entrepreneur as a small firm owner, then it is more sensible to assume that their skills are limited to managerial tasks in the sense defined later on in this paper, and that in general this managerial skills are not correlated with the ability to select and offer an attractive product. Finally, it must be added that as a consequence of our definition of entrepreneur, we use the terms "firm owner", "business owner", and "owner-manager" as synonyms of entrepreneur.

2.2 Empirical Evidence on Entrepreneurial Survival: Entrepreneurs who Re-enter and Obstinate Entrepreneurs

The study by Stokes and Blackburn (2001) "Opening up Business Closures: a Study of Businesses that Close and Owners' Exit Routes" is one in a kind. It is, perhaps, the only research which has addressed the question of what happens to the entrepreneur once his firm has not survived. The study was carried out in the United Kingdom. It comprised, as a first stage, the elaboration of the questionnaire to be used to interview the small firm

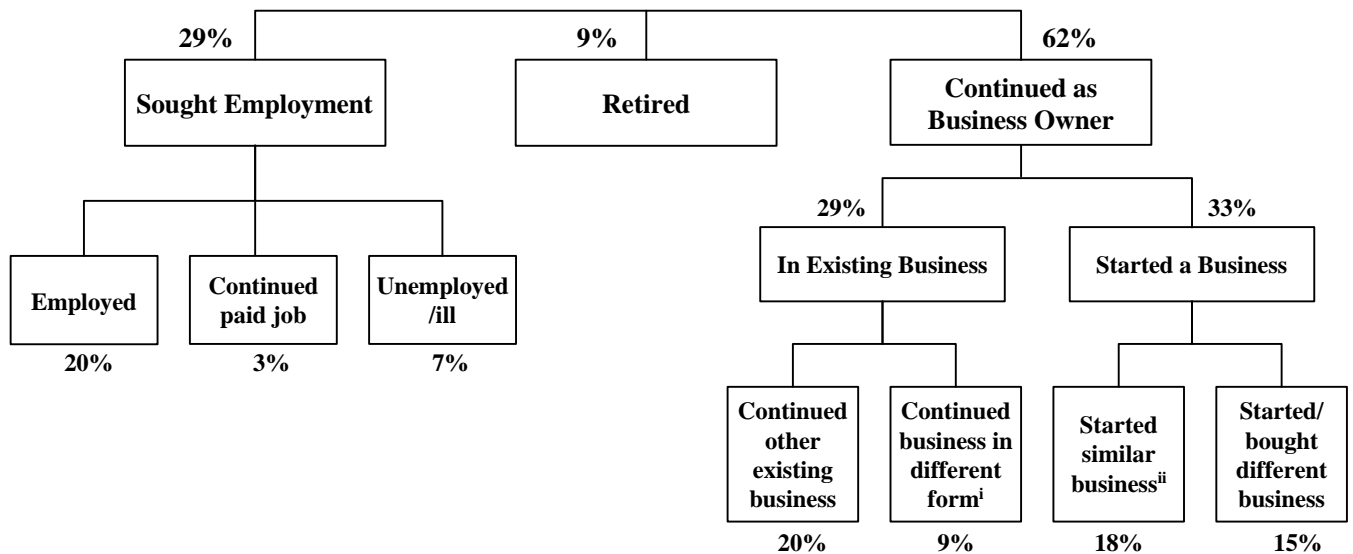
²For instance, in the project "Global Entrepreneurship Monitor" where one of the principal objectives is to measure the level of entrepreneurial activity across several countries, entrepreneurship is defined as 'Any attempt at new business or new venture creation, such as self-employment, a new business organization, or the expansion of an existing business, by an individual, a team of individuals, or an established business' (Reynolds et al, 1999, p. 3).

³A more extended exposition of what these skills might embody is contained in the next section.

owners. This questionnaire was built from interviews with 20 organisations of different type, including private organizations (e.g. banks, accountants), non-profit organisations, and government consultants, which had experience in advising small business. In a second stage, the questionnaire was sent to a sample of 2719 owner-managers who had recently closed their businesses, from which 388 usable observations were obtained. The study collects a rich set of variables, which includes information on the firm (e.g. size, age), the owner-manager (e.g. reasons to start firm), financial situation of the business at the time of closure, path followed by the business owner, attitudes to the experience of closing a business, learning experience, amongst others. The results of the study are impressive inasmuch as they provide the existing picture of the entrepreneurs' decision between sticking with entrepreneurialism or other alternatives, once the experience of business closure has occurred. As Stokes and Blackburn state, SB henceforth, 'The research has considerable implications for business educationalists, policy makers and financiers. Those who have closed a business are strong candidates to become involved in a new venture, and should therefore be a prime target for those seeking to advise or finance new business' (p. ii). In this sense, the findings by SB are subject to provide support on the relevance of our initial question. We will proceed as follows. As introduction, we first provide evidence that some entrepreneurs start new firms after having closed one, that is, evidence on entrepreneurial re-entry. However, our question concentrates on those entrepreneurs who wish to start new firms, after having closed unsuccessful ones, that is, on the obstinate entrepreneurs. The SB study is also helpful in this respect, as it provides evidence that these entrepreneurs do exist, and that they might represent a considerable share of the population of small firm owners. Thus, in a second stage, we reproduce this evidence.

In this paper we make use of two figures from SB. The first one, Figure 1, reproduces the figure "Business owner exits - what they did next" (SB, p. 17), except for the notes (i) and (ii) which were added for explanatory purposes. Figure 1 displays the answers by the owner-managers, and the percentage which answered in this way, to the question "What did you do after leaving the business?" Therefore, Figure 1 effectively provides evidence on entrepreneurial re-entry. The relevance of the findings from Figure 1 is that, to the best of my knowledge, a serious research on entrepreneurial re-entry had not been undertaken, until the SB study. Figure 1 speaks for itself, although some features are worth pointing out. Firstly, the figure shows that 62% of the entrepreneurs in the sample remained in entrepreneurialism after having experienced a business closure, as opposed to 29% who opted for paid employment. Thus, we can effectively say that 29% of entrepreneurs in the sample

Fate of the business owner



ⁱ It refers to a different organisational-legal form
ⁱⁱ It refers to start a similar business in a different location

Figure 1: “Business owner exits - what they did next” from Stokes and Blackburn (2001, p. 17)

did not “survive”. The percentage of surviving entrepreneurs is further classified into those who started a new business, 33%, and those who continued in an existing business, 29%. One point is worth noting. Looking at the sub-category “Continued business in different legal form”, it does not seem to fit in the definition of closing a business as understood for economic purposes. That is, this sub-category refers to those entrepreneurs who did a legal change to their businesses, but from an economic point of view it is still the same firm offering the same product. As a consequence, the observations in this sub-category should be ignored. This is not the case with the sub-category “Started a similar business in a different location”, because location itself is a source of product differentiation. Hence, it can be argued that these entrepreneurs indeed started a new firm. Finally, the sub-category “Continued other existing business” reveals that at least 20% of the sample were entrepreneurs that were running more than one firm at the same time. This is consistent with the term “Portfolio owners” suggested by Storey (1994), on referring to the findings of Storey et al (1987) that it is not uncommon for some entrepreneurs to have ownership interests in more than one firm. Overall, removing the observations in the sub-category “Continued business in

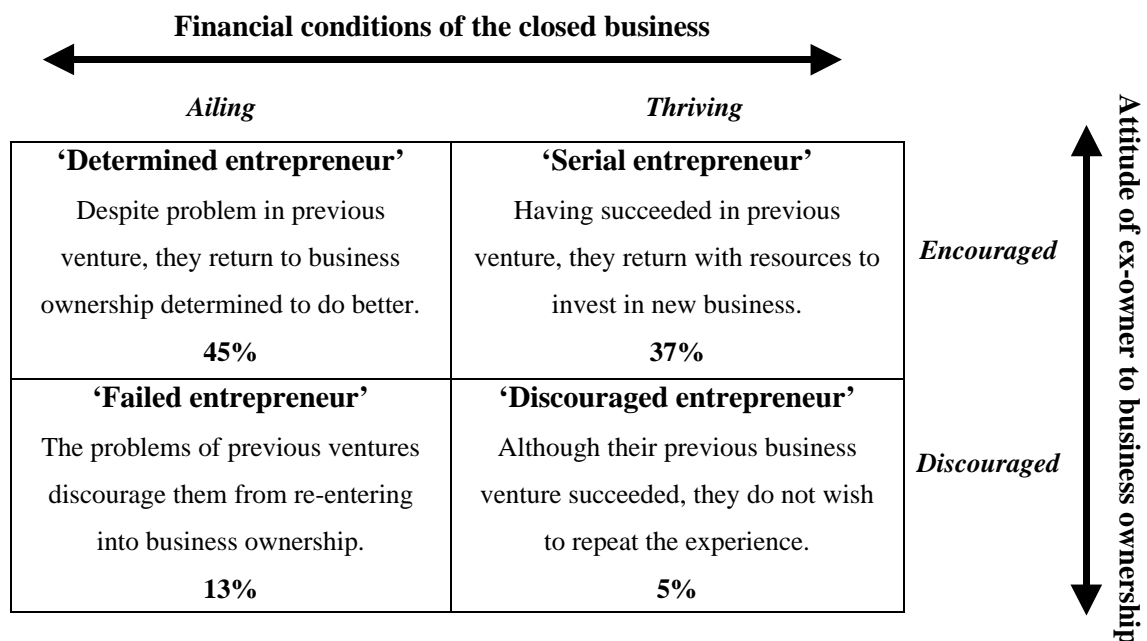


Figure 2: “A matrix of owners’ responses to business closure” from Stokes and Blackburn (2001, p.29)

different legal form”, Figure 1 reveals that 58% percent of the owner-managers in the SB study were entrepreneurs who re-entered with a new firm or continued as business owners, despite having closed a previous business.

These figures on entrepreneurial re-entry do not offer any information on whether the closed business was successful or not. Furthermore, they only show those entrepreneurs which in fact started a new firm, and not those who wished to do so but did not re-enter. To amend this situation Figure 2 reproduces the figure “A matrix of owners’ responses to business closure” from SB (p. 29). It classifies the entrepreneurs in four quadrants, depending on the owner’s view whether the business was financially ailing or thriving at the time of closure, and on whether the owner felt encouraged or discouraged to own a business in the future given the experience of closure. Each quadrant also shows the percentage of owners which fit into the category. It is important to add that when answering these questions the owners had the option of choosing from 1 to 5, where 5 meant thriving (encouraged) and 1 ailing (discouraged). Thus, Figure 2 should be seen as having in the axis the scale 1 to 5. Given the nature of the questions, we can state that the top row of Figure 2 includes

those entrepreneurs who wish to start a new firm, as opposed to those in the bottom row who want to leave entrepreneurialism. On the other hand, the left column comprises those entrepreneurs who saw their firms as financially ailing at the time of closure. Common sense would suggest that firm's success is related to its financial performance. We can argue then that the left column shows those entrepreneurs who were running unsuccessful firms, contrary to those in the right column who regarded their firms as successful. In this sense, the top-left quadrant represents the obstinate entrepreneurs, i.e., those entrepreneurs who wish to start a new firm, even though their previous ventures were unsuccessful. According to SB, these are the "determined entrepreneurs", who represent 45% of the sample, and in fact they are the single largest group of the four. Figure 2 also shows the other three possibilities: those who had successful firms and wish to start a new firm, the "serial entrepreneurs"; those who despite having successful firms do not want to remain in entrepreneurialism, the "discouraged entrepreneurs"; and those who were running unsuccessful firms and do not want to own a business in the future, the "failed entrepreneurs". Interestingly, the group of "discouraged entrepreneurs" represents the antagonistic case to the one we try to explain. Why do these individuals want to leave entrepreneurialism if they had the experience of running a successful business? Is not this proof enough that they are skilful entrepreneurs? As will be shown, in our attempt to explain the reasons which drive the "determined entrepreneurs" to start a new firm despite not being successful in the previous one, we also try to explain the other three possibilities included in Figure 2.

Regarding Figure 2, an issue must be addressed. The right column of Figure 2 may reveal an apparent inconsistency: why would some entrepreneurs close their firms despite being financially thriving? Although not the only ones, two possibilities can explain this situation. Firstly, it is hard to imagine a small firm which was financially thriving throughout its life span. Thus, it could be possible that by answering thriving or ailing the owners tended to describe the recent average performance of their firms. Consequently, it is not unlikely to suppose that some entrepreneurs in the right column of Figure 2 had firms who were meeting their expectations once, i.e., that were successful, but its performance started to decline. Secondly, for those firms in the right column of Figure 2 which were indeed financially thriving, it can be the case that such performance might have attracted the attention of other entrepreneurs. In this sense, it is not unlikely that if a price large enough was offered to the owners of these successful firms, some of them might have accepted to sell their firms. Under these views, the apparent inconsistency between success and closure reduces.

The most important point from Figure 2 is that it reveals that 45% of entrepreneurs in the

sample have the desire of keeping in entrepreneurialism, even though they did not succeed in their previous venture. We have no grounds to assert that the sample in the SB study is not fully representative of the populations of entrepreneurs who have experienced a business closure in the UK⁴. Nevertheless, we believe that this rather high percentage indicates the relevance of the question we try to tackle. We now introduce the model which might help us to explain this evidence.

3 The Model

The model assumes the objective of the individual is to maximise his lifetime utility. He will do so by choosing between working in a paid job or setting up and running a firm. Here we are invoking the so called "income choice model" which can be traced back to Knight (1921). The individual obtains utility $U = W$ when he is an employee and $U = \pi + \xi$ when he is running his own firm, where W is the wage from the paid job and π are the profits from running the firm. Following Blanchflower and Oswald (1998), $\xi \geq 0$ denotes the extra satisfaction to the individual for being his own boss, which we assume as non-constant across individuals. We also define ξ as the "entrepreneurial spirit" of the individual. The individual discounts the future with rate $0 < \delta < 1$ over his life span T . Thus, the problem for the individual is $\max \sum_{t=1}^T \delta^{t-1} U_t$ by deciding between a paid job or running his own firm.

The individual knows his own ξ , which is constant over time, and has perfect foresight on future values of W . For the sake of simplicity we assume W remains constant. If the individual decides to run a firm, he will enter an industry characterised by monopolistic competition where the technological conditions of such industry and the phase of the product life cycle suggest incumbent firms are not affected by economies of scale, and as a result they do not need to grow to survive. The purpose of these assumptions is to simplify the analysis by allowing the firm to ignore its competitors⁵ and by eliminating the requirement to grow to survive found as empirical regularity for small firms in the manufacturing sector⁶. We

⁴According to SB, the 388 usable responses were obtained from HSBC bank customers who closed a business account (36%), from Dun and Bradstreet lists of existing business (5%), and from closures identified during research on other projects by the Small Business Research Centre, Kingston University (59%).

⁵We use the definition of monopolistic competition proposed by Hart (1985). He points out, in the spirit of Chamberlin (1933), that monopolistic competition might be defined as one where there are many firms producing differentiated commodities, each firm is negligible so it can ignore its impact and reactions from other firms, and each firm faces a downward sloping demand curve.

⁶There is an abundant literature which provides empirical support to the fact that small firms' presence and survival in the manufacturing sector depend on the technological conditions of the industry, suggesting that economies of scale play a role. See, for instance, White (1982), Acs and Audretsch (1989), Audretsch

now discuss the nature of the entrepreneurial skills to be used in this paper.

On the small firm and entrepreneurship literature there is now a tradition to assume that the profits of the small firm depend on the talent or skills of its owner. Lucas (1978) was the first who put forward the notion that firm's profits depend on the talent of the managers. In his model he shows how variations in talent across managers lead to a size distribution of firms. Jovanovic (1982) proposed that efficiency was the firm-specific determinant of profits. In his model, firms start small and those which discover they are efficient grow and survive. Later on, Frank (1988) merged the two ideas by assuming that the skills or talent of the entrepreneur determines the efficiency of the small firm, and thereby its profits and survival. Further theoretical and empirical studies show that it became rather common to assume that small firm profits depend on the skills of the entrepreneur⁷. The points to highlight in these papers are that they assume the skills of the entrepreneur are the only firm-specific determinant of profits, and that they do not discuss what these skills entail. Contrary to the models such as the ones in Jovanovic (1982) and Frank (1988), we assume the profits of the small firm depend on two firm-specific determinants, which we call the efficiency and the demand parameters, and we restrict the nature of the entrepreneurial skills of the small firm owner. More specifically, we assume a profit function for the small firm of the following form

$$\pi^{ij} = pQ(p, \alpha^{ij}) - \frac{1}{\theta^j} cQ(p, \alpha^{ij}) \quad (1)$$

These are the profits for entrepreneur $j \in J$ running firm $i \in I$, where $I \setminus J = \emptyset$, and where p is the price of the good/service offered or to be offered by the firm, and $Q(\cdot)$ represents the demand function. The demand parameter is represented by α . It denotes how successful the firm's product is in attracting customers or in capturing a market niche. A high value in α^{ij} would mean that entrepreneur j is running the small firm i which is offering a product or service that is very appealing to customers, which in turn is generating relatively large volumes of revenue to the small firm. The efficiency parameter is represented by θ . It denotes the skills of the entrepreneur to run a business, which are directly translated as the efficiency of the firm. A skilful entrepreneur would be represented by a high value in θ^j ,

(1991), Mata and Portugal (1994), Geroski (1995). Interestingly, the very few studies that have looked at the service sector (e.g. Audretsch et al, 1999), or at industries in the mature phase of the product life cycle (Agarwal and Audretsch, 1999 and 2001; Agarwal and Gort, 1996) have found that small firms in these circumstances do not need to grow survive. This can be explained by accounting for the small size of the minimum efficient scale in these industries.

⁷See, for instance, Evans and Jovanovic (1989), Bates (1990), Holtz-Eakin et al (1994), Cressy (1996), Storey and Wynarczyk (1996).

which in turns means he is able to run his own firm more efficiently⁸. We further assume that these parameters are a source of uncertainty for the entrepreneur because they are unknown to him. He only knows that his “true” skills and “true” demand for his firm’s product are random draws from some distributions. The only way the entrepreneur can learn the parameters α^{ij} and θ^j is by running a firm. Specifically, we assume that

A.1 α is an i.i.d. stochastic variable with cumulative distribution function (CDF) $A(\alpha)$, with support $[0, 1)$. $A(\alpha)$ is continuous and twice differentiable. $A(\alpha)$ is known by the individual.

A.2 θ is an i.i.d. stochastic variable. It has a CDF $G(\theta)$ with support $(0, 1)$. $G(\theta)$ is continuous and twice differentiable. The individual knows the form of $G(\theta)$.

One consequence of these assumptions is that the parameters α and θ are uncorrelated with each other. Furthermore, note that by assuming the profits of the small firm depend on these two uncorrelated firm-specific determinants, and by assuming that the firm’s demand function is only affected by the parameter α^{ij} , we restrict the nature of the entrepreneurial skills of the small firm owner. In other words, our framework supposes that the entrepreneurial skills θ^j are limited to skills to manage the firm, and do not include the ability to select and offer an attractive product, as represented by α^{ij} , which might guarantee large revenue to the firm. Without attempting an exhaustive lists, our definition of the parameter θ^j , include the entrepreneur’s ability to organise and assume the responsibility of the activities of the small firm, to select the cheapest inputs given a desire level of quality, to deal with suppliers and, perhaps, employees, to ensure that the production of goods or services in the firm is done in the best possible way using the resources available, amongst others. In general, they represent the skills to undertake efficiently the everyday activities of the firm and to tackle the problems that might arise. As can be seen, our definition of entrepreneurial skills do not includes the notion of “Schumpeterian” innovators, but is closer to the definition of the entrepreneur’s “general ability” put forward by Marshall⁹.

⁸Two other important dimensions of entrepreneurship which might affect the profits of the small firm are the entrepreneur’s degree of risk aversion and the entrepreneur’s degree of disutility of effort. To keep our model simple, we omit them in the present analysis. In a later section we discuss the likely impact of their inclusion.

⁹He defines the entrepreneur’s “general ability” as the one which allows him ‘to be able to bear in mind many things at a time, to have everything ready when wanted, to act promptly and show resource when anything goes wrong, to accommodate oneself quickly to changes, to be steady and trustworthy, to have always a reserve of force...’ Marshall (1930, pp. 206 - 207).

Similarly, this paper follows those which have explicitly acknowledge the possibility that the entrepreneur may have skills which are restricted to specific activities¹⁰.

The following additional assumption are made about the profit function and about the industry the small firm would operate in:

A.3 $Q(p, \alpha^{ij}) = D(\alpha^{ij})$; b_p is the demand function the small firm faces, where $b > 0$ and p is the price of the good/service. Since the industry is subject to monopolistic competition, elasticities of substitution between any firm's good and the rest are assumed to be zero, and only the price of the firm's own good affects the firm's demand. The demand function has the following properties:

$$\frac{\partial D(\alpha^{ij})}{\partial \alpha} > 0, \quad \frac{\partial^2 D(\alpha^{ij})}{\partial \alpha^2} < 0. \quad (2)$$

A.4 All operating firms have the same cost structure which they acquire after paying the exogenous sunk cost $k > 0$, which is the same for every firm.

A.5 The cost function $\frac{1}{\theta^j} c Q(p, \alpha^{ij})$ presents constant marginal costs, where $c > 0$. We also assume that there exist a Q^{\max} such that for any $Q > Q^{\max}$, $c = 1$.

These assumptions serve different purposes. Conditions A.4 and A.5 together imply all firms can potentially produce the same amount of output. Note that, due to the assumption of monopolistic competition, all firms are by definition small, which means that should the size of firms be measured by production levels, Q^{\max} is small enough for all firms to fit in the small size category. Condition A.5 also tells us that marginal cost is constant over the feasible production range. Intuitively, this captures the assumption that firms are not affected by economies of scale, since the average cost is constant over the production range. Thus, growing to reach the minimum efficient scale is not a requisite for the firms to remain competitive.

Prior to owning a firm, the individual ignores his entrepreneurial skills and whether or not the product/service he is planning to offer will attract enough demand, i.e., he ignores his endowment of α^{ij} and θ^j . Once the individual runs a firm for the first time, nature

¹⁰Our model is similar to the ones in Holmes and Schmitz (1990) and Giord (1993) where it is assumed there are two types of entrepreneurial abilities: talent to manage and talent to develop new products. However, our model differs from theirs in we are interested in the study of small firm and entrepreneurial survival, whilst Holmes and Schmitz (1990) concentrate on the sale and buy of business, and Giord (1993) focuses on the distribution of firm sizes and the choice of specialisation as innovative entrepreneur or managerial entrepreneur.

determines these values from the CDFs $A(\cdot)$ and $G(\cdot)$, respectively. Additionally, when the individual is running a firm, he does not observe these values directly, but indirectly through realisations in every period. This process is represented by

$$\begin{aligned}\varphi_t^{ij} &= \alpha^{ij} + \omega_t, \\ \eta_t^j &= \theta^j + \omega_t,\end{aligned}\tag{3}$$

where ω is an i.i.d. normal random variable with mean zero and variance σ^2 . Once the firm is in operation, the entrepreneur learns about its own efficiency and demand by observing the realisations φ_t^{ij} and η_t^j in every period. We assume that the learning process, or in other words, the method the entrepreneur uses to estimate and update his beliefs about α^{ij} and θ^j in every period is a bayesian method.

Profit Maximisation

The objective of the individual when he is running a firm is profit maximisation. Since profits depend on the parameters α and θ , the maximised value of profits will depend on the specific parameters the individual is endowed with. The maximisation problem is represented as

$$\max_{p_t} \sum_{t=e}^{\infty} \delta^{t-e} \pi_t^{ij} \quad \text{subject to } \alpha_B^{ij} \text{ and } \theta_B^j,$$

where p_t represents the price the monopolistic firm chooses to apply, and α_B^{ij} and θ_B^j denotes the bayesian estimator of α^{ij} and θ^j , respectively. We also assume the individual starts a firm in period e . Since the expected value of future values of a bayesian estimator is equal to the current bayesian estimator, i.e., $E[\alpha_B]_{t+z} = [\alpha_B]_t$ for any $z > 0$, where $E[\cdot]$ denotes the expectation operator¹¹, the problem reduces to calculate the maximisation problem just for the period $t = e$. The first order conditions give

$$p \frac{\partial Q}{\partial p} + Q \left(\frac{1}{p} - \frac{1}{\theta_B^j} \right) - \frac{c}{\theta_B^j} \frac{\partial Q}{\partial p} = 0,\tag{4}$$

which rearranged provides the standard mark-up rule

$$\frac{p}{p} \left(1 - \frac{1}{\theta_B^j} \right) = \frac{1}{\varepsilon} \frac{1}{\alpha_B^{ij}},\tag{5}$$

¹¹See Winkler (1972, p. 204).

Equation (5) spells out the effect the demand and efficiency parameters have on the pricing strategy of the firm and, ultimately, on the profits of the firm. The more skilful the entrepreneur is, i.e., the higher θ^j , the higher the mark-up is. Likewise, the higher the demand parameter, the more inelastic the price elasticity $\varepsilon^i_{\alpha_B}$ is, and the higher the mark-up is.

Let the optimum price and quantity be expressed as

$$\begin{aligned} p^* &= p(\alpha_B^{ij}, \theta_B^j) \\ Q^* &= Q(\alpha_B^{ij}, \theta_B^j) \end{aligned}$$

and the solution to the maximisation problem as

$$\begin{aligned} \max_{p_t} & \mathbf{P} \sum_{t=e}^{T-1} \delta^t e^{-\rho t} \left[\alpha_B^{ij} + \xi^j \left(\frac{1}{\delta} - 1 \right) \right] p^* + \xi^j \left(\frac{1}{\delta} - 1 \right) \delta^t e^{-\rho t} \\ \text{s.t. } & \alpha_B^{ij} \text{ and } \theta_B^j \end{aligned} \quad (6)$$

This is the discounted value of the maximum expected utility from running a firm. It can be shown that the second order conditions for a maximum are satisfied. As can be seen, the maximum expected profits $\pi^* = p^* Q^*$, from now on simply profits, are a function of the parameters α^{ij} and θ^j which are updated in a bayesian fashion every period. The next lemmas are presented for mathematical rigorosity and establish the properties of the profits with respect to the demand and efficiency parameters. To save on notation we drop the superscripts i , and j , and the subscripts t and B .

Lemma 1. The profits are an increasing function of efficiency: $\frac{\partial \pi^*(\alpha, \theta)}{\partial \theta} > 0$.

Proof. The first order conditions (4) are expressed as $\alpha + bp + D(\alpha) - bp - \frac{cb}{\theta} = 0$, thus $p(\alpha, \theta) = \frac{D(\alpha)}{2b} + \frac{c}{2\theta}$. The demand function is $Q(p, \alpha) = D(\alpha) + bp$. Hence, $Q(\alpha, \theta) = \frac{D(\alpha)}{2} + \frac{bc}{2\theta}$, and $\pi^*(\alpha, \theta) = \frac{(bc + \theta D(\alpha))^2}{4b\theta^2}$. From here it follows that $\frac{\partial \pi^*(\alpha, \theta)}{\partial \theta} = \frac{c}{2\theta^3} (\theta D(\alpha) + bc)$, which rearranged yields $\frac{\partial \pi^*(\alpha, \theta)}{\partial \theta} = \frac{c}{\theta^2} \left[\frac{D(\alpha)}{2} + \frac{bc}{2\theta} \right]$. Now, the terms inside the brackets in the right-hand-side are equivalent to $Q(\alpha, \theta)$. In this way we can write $\frac{\partial \pi^*(\alpha, \theta)}{\partial \theta} = \frac{c}{\theta^2} Q(\alpha, \theta)$ and thus $\frac{\partial \pi^*(\alpha, \theta)}{\partial \theta} > 0$. \square

Lemma 2. The profits are a function of the demand parameter in the following way: whenever $p(\alpha, \theta) > MC$ we have $\frac{\partial \pi^*(\alpha, \theta)}{\partial \alpha} > 0$, and for $p(\alpha, \theta) > MC + \frac{1}{2b} \frac{\partial D(\alpha)}{\partial \alpha} + \frac{\partial^2 D(\alpha)}{\partial \alpha^2}$ we have $\frac{\partial^2 \pi^*(\alpha, \theta)}{\partial \alpha^2} < 0$, where $MC = \frac{c}{\theta}$ is the marginal cost.

Proof. From the proof of Lemma 1 we obtain $\frac{\partial \pi^*(\alpha, \theta)}{\partial \alpha} = \frac{\partial D(\alpha)}{\partial \alpha} \left[\frac{D(\alpha)}{2b} + \frac{c}{2\theta} \right]$. From the value of $p(\alpha, \theta)$ we have $\frac{D(\alpha)}{2b} = p(\alpha, \theta) - \frac{c}{2\theta}$, which inserted in the term in brackets renders

$\frac{\partial V^*(\alpha, \theta)}{\partial \alpha} = \frac{\partial D(\alpha)}{\partial \alpha} p(\alpha, \theta) - \frac{c}{\theta}$. The sign of this expression is positive whenever $p(\alpha, \theta) > \frac{c}{\theta}$. Regarding the second part, we calculate $\frac{\partial^2 V^*(\alpha, \theta)}{\partial \alpha^2} = \frac{1}{2b} \left(\frac{\partial D(\alpha)}{\partial \alpha} \right)^2 + \frac{\partial^2 D(\alpha)}{\partial \alpha^2} p(\alpha, \theta) - \frac{c}{\theta}$. We require $\frac{\partial^2 V^*(\alpha, \theta)}{\partial \alpha^2} < 0$, so we evaluate under which conditions $\frac{1}{2b} \left(\frac{\partial D(\alpha)}{\partial \alpha} \right)^2 + \frac{\partial^2 D(\alpha)}{\partial \alpha^2} p(\alpha, \theta) - \frac{c}{\theta} < 0$. Rearranging for $p(\alpha, \theta)$ we find that $\frac{\partial^2 V^*(\alpha, \theta)}{\partial \alpha^2} < 0$ takes place whenever $p(\alpha, \theta) > \frac{c}{\theta} + \frac{1}{2b} \frac{\partial D(\alpha)}{\partial \alpha} \left(\frac{\partial D(\alpha)}{\partial \alpha} \right)^{-1} \frac{\partial^2 D(\alpha)}{\partial \alpha^2}$ where the second term on the right-hand-side is positive. \square

3.1 The Decision to Start a Firm

The individual will choose to work in a paid job or to set up and run a firm depending on whether or not the expected utility in one state is larger than it is in the other. When the individual has not started a firm before, the only information he has available to calculate the expected profits from running a firm is the distribution of the parameters α and θ . At this stage the only two alternative states are to start a firm or to work in a paid job. Under this situation, the maximisation problem for the individual can be expressed as choosing

$$\arg \max_{j \in \{1, 2\}} \left[W^j, E \max_p \left[\pi^{ij} + \xi^j \right] \right], \quad (7)$$

where π^{ij} is specified by (1), and $E[\cdot]$ denotes the expectation operator. At this stage, the individual is constrained to rely on the expected value of the maximum profits as he has no other way to infer better estimators for his true parameters. This expression is rewritten as

$$\arg \max_{j \in \{1, 2\}} \left[W^j, E_{\alpha, \theta} \left[\pi^*(\alpha, \theta) \right] + \xi^j \right] \quad (8)$$

where we define $\pi^*(\alpha, \theta)$ as the value of maximised profits, and therefore

$$E_{\alpha, \theta} \left[\pi^*(\alpha, \theta) \right] = \int_0^{\infty} \int_0^{\infty} \pi^*(\alpha, \theta) dA(\alpha) dG(\theta) \quad (9)$$

as its expected value. Rearranging expression (8) we obtain

$$\arg \max_{j \in \{1, 2\}} \left[W^j - \xi^j, E_{\alpha, \theta} \left[\pi^*(\alpha, \theta) \right] \right]. \quad (10)$$

We are now in position to state that whenever the following expression is true, the individual has an incentive to turn down paid job and start a firm

$$E_{\alpha, \theta} \left[\pi^*(\alpha, \theta) \right] > W^j - \xi^j. \quad (11)$$

Note that the above inequality does not state that individual j will enter the industry in case it is fulfilled. To enter the industry the individual requires funds to set up the firm and cover the sunk costs. The process whereby individuals gather these funds and the difficulties they may face are issues not included in the scope of this paper^{12 13}.

Note that the assumptions of our model are consistent with the definition of entrepreneur we use in this paper as those who started or own and manage his firm. Firstly, by assuming that the individual seeks to maximise utility by choosing between running his own firm or being an employee, we eliminate any possibility that entrepreneurs would start a firm because they have generated any kind of innovation or because they have spotted a profitable opportunity. This is reinforced with the fact that we assume the demand parameter α to be a random draw and that the entrepreneur is not able to provide a better forecast apart from the mean of the parameter's distribution. Additionally, as has been already discussed, by assuming that α and θ are independent random variables not correlated with each other, we restrict the skills of the entrepreneurs to manage. Secondly, by making no specific assumptions about those individuals who start and run a firm, our model implies all entrepreneurs have the same degree of risk aversion. We now turn to the predictions this model offers to answer our initial question.

4 Results

Once the individual has decided to set up and run a firm at the end of a given period, it is assumed that the firm is formally in operation in the next one. As defined before, the problem for the individual is to maximise his lifetime utility. Therefore, when he is running a firm he has to determine which state provides the largest utility. What are these states? It is straightforward to see that one possibility is to keep his current firm running. A second possibility is the wage worker state. This would imply to close the current firm and go to paid job. Furthermore, a third possibility is the option of running a different firm. In other words, the possibility of closing the current firm and starting a new one. Why is this last

¹²For research on the topic see, for instance, Evans and Jovanovic (1989), Blanchflower and Oswald (1998), Taylor (2001).

¹³Regarding the risk nature of the individual, consider the expression $E_{\alpha, \theta} [U_1^m(\alpha, \theta)] + R^j > W^j + \xi^j$. From here, we can define when the individual is risk averse ($R^j > 0$), risk neutral ($R^j = 0$), or a risk lover ($R^j < 0$). Thus, in this paper we are assuming that the individual is risk neutral. A more extended discussion on the reasons why it is more sensible to include the entrepreneur's degree of risk aversion on the decision between employment and self-employment, and not as a parameter affecting the profits of the small firm, is contained in a subsequent section.

possibility relevant? In line with monopolistic competition markets, the situation we are looking at is, for example, one where the entrepreneur was running a Chinese take away and is deciding to move to Mexican food, or to open a sport clothes retail outlet. In this sense, by new firm we refer to one where the good or service to be offered by such firm is different from the one offered by the previous firm. Therefore, once the entrepreneur is running a firm, to maximise his utility he must decide whether keeping his current firm is the best choice, or if closing and starting a new firm provides the largest utility, or whether he is better off by closing and going to a paid job.

Let e represent the period in the individual's life span when he started a firm. At the end of every period $e, e + 1, \dots, T$ the individual evaluates which state provides the largest utility. Let $s \in \{1, 2, \dots, T\}$ represent the number of periods the individual has been running a firm. Thus, the utility from keeping the current firm running is calculated by

$$\max_{p_t} \sum_{t=s}^T \delta^{t-s} u_t^{ij} \text{ s.t. } \alpha_B^{ij} \text{ and } \theta_B^j \text{ are } \alpha_B^{ij}, \theta_B^j \frac{1 - \delta^{T-s}}{1 - \delta} \quad (12)$$

This is the same expression as in (6), in which the entrepreneur uses the bayesian method to calculate his expectations about the parameters α^{ij} and θ^j .

Given the nature of the parameters α^{ij} and θ^j , the procedure to calculate the profits the entrepreneur would obtain for starting a new firm is different from the one shown by the previous expression. Recall that θ^j represents the skills of the entrepreneur to manage the firm efficiently. These skills are time invariant, that is, talented entrepreneurs are always talented, whereas unskillful entrepreneur are not able to enhance such skills¹⁴. Therefore, once the entrepreneur has started a firm for the first time and has learned his "true" value of efficiency, he is constrained to this value in the future. This means every time he starts a new firm, his efficiency parameter remains the same. However, this is not true for the case of the demand parameter, for we allow different products to face different levels of demand. In other words, every time the entrepreneurs runs a new firm, he receives a different value of α . In the context of the above example, it is easy to think that the new Mexican food outlet or the new sport clothes retail outlet the entrepreneur is planning to set up might not

¹⁴This assumption may seem very restrictive at first glance, since it eliminates any possibility for the improvement of skills to manage as a result of learning. However, in our model this is not the case because we assume the parameter θ^j embodies the endowment of potential skills, which can be regarded as time invariant. In this sense, entrepreneurs with low efficiency may improve the way they manage their firms over time, but they have a very limited space to improve these skills because by definition they have low potential.

attract enough demand, regardless of the fact that the Chinese take away was a successor not. In terms of our model, this implies the expected profits of a new firm are calculated as follows :

$$E_{\alpha} \max_{p_t} \sum_{t=s}^{\infty} \delta^{t-s} \pi_t^j \text{ s.t. } \theta_B^j \leq \int_0^1 \pi(\alpha, \theta_B^j) dA(\zeta) \frac{1 - \delta^{T-s}}{1 - \delta} . \quad (13)$$

Note the difference between expression (9) and (13). Expression (9) represents the expected profits for a de novo entrepreneur, that is, for an entrepreneur which does not have any experience in running a firm. This means that a de novo entrepreneur ignores his endowment of both parameters α and θ , and as a consequence he would calculate expected profits by applying expectation over both parameters. In contrast, in our model an incumbent entrepreneur, i.e. one which has experience in running a firm, has already discovered his efficiency parameter. Therefore, if he is to start a new firm the unique uncertainty is the demand parameter. Hence, the fact that the expected profits of a new firm as shown by (13) are calculated by applying expectation only over the demand parameter. To save on notation we define

$$E_{\alpha} \int_0^1 \pi(\alpha, \theta_B^j) dA(\zeta) \equiv \int_0^1 \pi(\alpha, \theta_B^j) dA(\zeta) .$$

Finally, the third alternative for the entrepreneur is to close his current firm and go to employment. Given that expression (11) was the decision rule to enter the market, we set

$$\pi_R^j = W^j \xi^j \quad (14)$$

to indicate the reservation level of profits. Additionally, we assume that once the entrepreneur is running his firm, the value of the alternative of returning to wage work starts to depreciate. This can be regarded as the increase in satisfaction the individual receives for being his own boss. This is indicated by $\gamma^s \pi_R^j$, where $0 < \gamma < 1$ is the rate of depreciation and s is the number of periods the individual has been running a firm as defined before. The purpose of this is to adhere to the stylised result that the probability of dropping from self-employment decreases the more time the individual spends as self-employed (Evans and Leighton, 1989; Blanchflower and Meyer, 1994). The utility from the employment state are thus expressed as

$$\sum_{t=s}^{\infty} \gamma^t \delta^{t-s} \pi_R^j = \gamma^s \pi_R^j \frac{1 - (\gamma\delta)^{T-s}}{1 - \gamma\delta} .$$

In this sense the problem for the entrepreneur is written as

$$\arg \max_{\alpha, \theta_B^j} \pi(\alpha, \theta_B^j, d(\gamma, \delta, T, s)) \quad (15)$$

where $d(\gamma, \delta, T, s) = \frac{\gamma^s(1-\delta)(1-\gamma\delta)^{T-1-s}}{(1-\gamma\delta)(1-\delta)^{T-1-s}}$, and the first argument stands for the profits of the current firm, the second argument for the profits of a new firm, and the third one for the value of going to wage work. Before proceeding with the predictions offered by expression (15) we will look first at the predictions obtained from an alternative model, which will provide a benchmark for our general model, and will provide a case in support of the definition we have given to the parameter θ^j .

4.1 The Alternative Model

In Subsection 2.1 we mention that there is a lack of consistency in the definition of entrepreneur used by academics across the research literature. To surmount this problem we also state that the definition of entrepreneur to be used throughout this paper refers to those individuals who started or own and manage his own business. In agreement with this definition we specify that the efficiency parameter θ^j denotes the ability of the entrepreneur, which we restrict to skills to manage and deal efficiently with the everyday activities of the firm. In this context, the objective of this subsection is to build an alternative model where this assumption is relaxed, and to try to provide an answer to our initial question using such alternative model. In turn, the predictions of both the alternative model and the general one will be confronted. The justification to do this is simple: if in the specialised literature on business and economics there is no agreement on the definition of entrepreneur, in the non-specialised literature like newspapers and magazines and in the popular knowledge the feeling is that an entrepreneur is or should rather be an individual which possesses many qualities: good management skills, vision, discipline, innovative ideas, willingness to take risks, amongst others. Hence, the attractiveness of modifying the definition of our parameter θ^j to encompass one or more of these characteristics.

To this end consider the profit function

$$\pi^j = \pi(\alpha^j, \theta^j) \quad (16)$$

Let α^j represents some sort of "overall" entrepreneur skills for individual j . Accordingly, we assume that the profits are a strictly increasing function of α^j , i.e., $\partial \pi / \partial \alpha^j > 0$. This function generalises the profit function that can be found in Jovanovic (1982) and Frank

(1988), and in fact, in terms of the assumptions of our general model, it would imply that the parameters α^{ij} and θ^j are perfectly correlated. To appreciate this consider a profit function of the form revenue function minus cost function

$$\pi_i = R(\phi_i) - C(\phi_i),$$

where the technology of production is included in the latter and the demand function in the former. Recall that this is the form of the profit function we consider in our model as shown by expression (1), where the demand parameter α^{ij} affects the revenue function directly, and the efficiency parameter θ^j influences the cost function. Jovanovic (1982) assumes the small firm produces in an industry with homogeneous product and perfect competition. Hence, he eliminates the need for the existence of a parameter like α^{ij} and concentrates on the cost function. In his model the efficiency parameter affects the cost function and is the only firm-specific determinant. Frank (1988) assumes that the talent of the entrepreneur is the only firm-specific determinant and that it affects the technology of production as well. He does not make any assumption on the type of competition the small firm faces. His model is equivalent to Jovanovic's in the sense that it discards the existence of a parameter like α^{ij} by implicitly assuming homogeneous product and concentrates in the technology of production. We believe our general model offers a more realistic case by assuming monopolistic competition and hence the existence of differentiated products, than by simply assuming small firms compete in markets with homogeneous products. Under our general model, if we assume the profits of the small firm depend on a single parameter which embodies the overall entrepreneurial skills of the owner, then it must be the case that such skills affect both the revenue function and the cost function. Hence, our argument that under such scenario, it is as though the parameters α^{ij} and θ^j were perfectly correlated.

Note that the parameter α^j therefore represents more than one characteristic of entrepreneurial skills, because under the form of a profit function like (16), the implications is that the entrepreneur has talent to generate revenue for the small firm, as he does to manage the firm efficiently. In other words, it implies that if the entrepreneur is a skilful manager, then he also has the ability to identify or develop products which are very appealing to customers, in other words, he also has skills to innovate or to spot profitable opportunities. Under this assumptions, a skilful entrepreneur running a small restaurant would be as successful as he would in running a small video shop, because having a high value on the parameter α^j implies that he is a skilful managers and skilful in distinguishing profitable opportunities.

Following a similar procedure as in Subsection 3.1, assume a is an i.i.d. stochastic variable which follows a specific cumulative distribution function, and let

$$E[\pi^j(a)] = \int_R^j \quad (17)$$

represent the entry decision, where $\int_R^j = W^j | \xi^j$. If inequality (17) is satisfied, the entrepreneur has an incentive to turn down paid job and start a firm. Once the entrepreneur is running a firm, the entrepreneur evaluates which state provides the largest utility. Thus, he has the option of keeping his current firm, or closing down and going to wage work, which is represented by the following expression

$$\arg \max_{p_t} \left(\max_{t=s} \delta^{t-s} \pi^j(a)_t \text{ s.t. } a_B^j, \max_{t=s} \gamma^t \delta^{t-s} \int_R^j \right)$$

This expression is equivalent to

$$\arg \max_{\int_R^j} \left(\int_R^j, \int_R^j d(\gamma, \delta, T, s) \right) \quad (18)$$

where \int_R^j are the maximum profits given the bayesian estimation of a^j . The first argument represents the value for the entrepreneur of keeping his current firm, and the second denotes the value of the alternative of returning to wage work for an entrepreneur who has been running a firm for s periods.

Note that the difference between expression (18) and (15) is that the profits of a possible new firm are not included in (18), because these profits would be the same as the profits of the current firm. In other words, given that the small firm profits depend only on the "overall" skills of the entrepreneur as denoted by a^j , and since these skills do not change from running one firm to another, the entrepreneur would expect the performance in the new firm to be the same as the one he experiences in the current firm: the same success in the small restaurant as in the video shop¹⁵. This means it would not make sense for the entrepreneur to shut the current firm and try to start a new one because he will not do any better in the new venture.

From expression (18) whenever the first argument is at least as large as the second one the individual decides to keep running his firm, i.e., the firm survives. Therefore, for the firm

¹⁵As in the case of θ^j in our general model, we assume a^j entails the endowment of potential skills, so entrepreneurs with low values of a will always perform poorly.

to survive, the problem for the entrepreneur reduces to obtain profits satisfying the weak inequality

$$v^a i_B^j \geq v_R^j d(\gamma, \delta, T, s). \quad (19)$$

From here we obtain the next proposition.

Proposition 1 Let $v^j \geq v^a(a^j)$ where $j \in J$ and $\partial v^j / \partial a^a > 0$. The maximisation problem is the one shown by (18). Define a_F as the minimum level of expected entrepreneurial skills which would induce the entrepreneur to start a firm, and a_{\min} as the minimum level of entrepreneurial skills required to keep the firm running. Let $H \subseteq J$ be the subgroup for which expression (19) is not satisfied. Therefore, $\exists j \in H, a_B^j < a_{\min} < a_F$. As a consequence, the wage work state will always provide the largest rewards to subgroup H .

Proof. Let a_F solve $v_R^j = v^a(a_F)$. Since $\partial v^j / \partial a^a > 0$, any $a^j > a_F$ would satisfy (17). Let a_{\min} solve expression (19), $v^a(a_{\min}) = v_R^j d(\gamma, \delta, T, s)$. Thus, we rewrite this expression as $v^a(a_{\min}) = v^a(a_F) d(\gamma, \delta, T, s)$. Since the term in square brackets is smaller than one for $\gamma < 1$, this implies $a_{\min} < a_F$. If for the subgroup H expression (19) is not satisfied and since $\partial v^j / \partial a^a > 0$, then it must be the case that $\exists j \in H, a_B^j < a_{\min} < a_F$. Given that the value a^j does not vary overtime, the second argument of expression (18), which represent the reward in the wage work state, will always be the largest for subgroup H . ■

Proposition 1 states that when the only firm-specific determinant of the profits of the small firm are the overall entrepreneurial skills of the owner-manager, those entrepreneurs whose firms do not survive have skills lower than they expected and lower than the ones required to run a firm, therefore, it is optimal for them not to try to start another firm in the future. In other words, this framework unambiguously implies that for every failed firm there is a failed entrepreneur. As a consequence, not only the firm will not survive, but also the entrepreneur will not attempt to start another firm again.

Figure 3 presents graphically the arguments contained in Proposition 1. Recall that v_R^j represents the value of the wage worker state, and we refer to $v_R^j d(\gamma, \delta, T, s)$ as the updated reservation level, that is, once the individual has started a firm¹⁶. An individual would start a firm provided he believes he will receive profits larger than the reservation level. In Figure

¹⁶In Figure 3 we are further assuming $v_B(0) = 0$ and $\frac{\partial^2 v}{\partial \theta^2} < 0$. None of these assumptions are central to the results in Proposition 1.

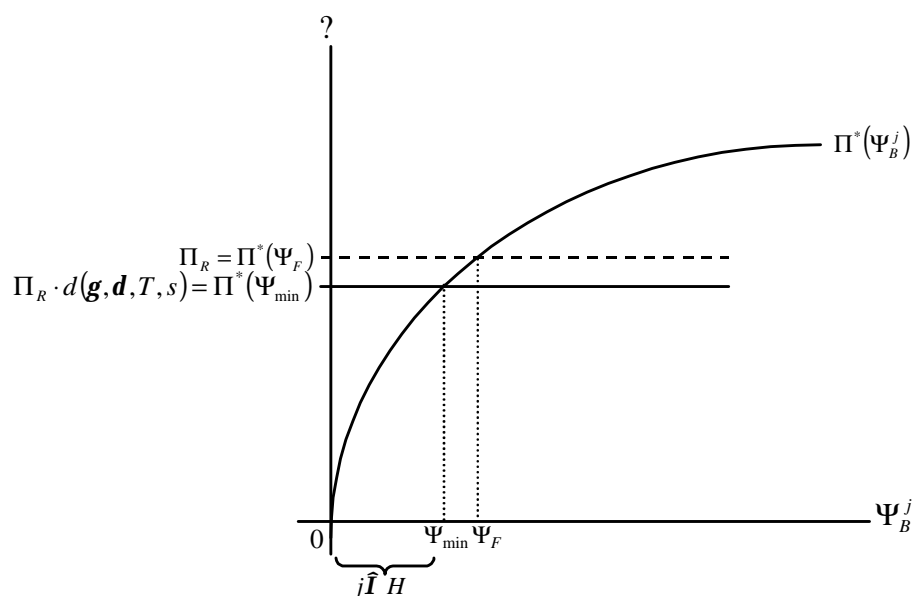


Figure 3: When the profits of the small firm depend only on the overall entrepreneurial skills of the owner-manager.

3 this is depicted as anticipating profits over or above the horizontal dotted line, which involves that the entrepreneur expects to have skills no smaller than a_F . Once the firm is running, the entrepreneur will keep the firm and refuse to become a wage worker whenever the expected profits are larger than the updated reservation level, i.e., whenever the expected profits are above the horizontal solid line. This suggests the entrepreneur must have skills larger than or equal to a_{\min} , where we interpret this last parameter as the minimum skills to run a firm. The first fact to note is that $a_{\min} < a_F$. In other words, the entrepreneur can afford to have skills smaller than the ones which would draw him into the market, and still opt for running a firm. This result is a direct consequence of including the depreciation rate γ . Focusing on Figure 3, it is easy to see that a firm that did not survive means the owner-manager estimated the expected profits to be lower than the rewards in the wage worker state. In turn, this unequivocally means the entrepreneur has skills to run a firm below a_{\min} . Given that the entrepreneur is confined to this level of skills, he will find optimal not to start another firm in the future.

More specifically, when the profits of the small firm depend only on the overall entrepreneurial skills of the owner, Proposition 1 indicates two points. Firstly, if the small firm does not survive, it means the entrepreneur has skills lower than he required. In other words,

he is not good in running firms. Secondly, under this scenario, this failed entrepreneur would find it optimal not to own another firm in the future, because having low entrepreneurial skills means he will receive higher rewards from the wage worker state. As it stands, this alternative model does not seem to provide an intuitive answer to our initial question as to why some small firm owners wish to start a new firm after having closed one that was not successful. According to Proposition 1 such entrepreneurs should not exist, because the fact that the small firm did not survive is simply the result that the entrepreneur does not have the necessary skills to run a successful firm, and therefore it is not rational for him to own another firm in the future since the wage work state provides higher rewards. However, the empirical evidence provided in Section 2 shows that a rather high share of entrepreneurs who close a firm wish to own another one in the future, despite the lack of success in their previous venture. We conclude that a model wherein the profits of the small firm depend only on the overall entrepreneurial skills of the small firm owner, which implicitly assumes the owner has the same level of skills to select an attractive product which generates high revenue for the small firm as he does to manage the firm efficiently, does not help us to answer our initial question and to explain the empirical evidence. In the following subsection we present and evaluate the predictions of our general model, which provide a better explanation. Finally, note that another implication of the alternative model is that the terms “survival of the firm” and “survival of the entrepreneur” are equivalent, even though they have different meanings. We can roughly define “survival of the firm” as the term which refers to the situation when the small firm continues trading, and “survival of the entrepreneur” as the one which denotes when the entrepreneur keeps running firms instead of going to wage work. According to the alternative model, failure by the entrepreneur to keep a small firm trading -the non survival of the small firm- indicates that he does not have the necessary entrepreneurial skills and therefore he is better off in a paid job -the non survival of the entrepreneur-.

4.2 The General Model

When the profits of the small firm depend on the demand parameter α^{jj} and the efficiency parameters θ^j as shown by (1), where the former denotes how successful the firm’s product is in attracting customers and the latter represents the skills of the owner to manage the firm efficiently, and under assumptions A.1 through A.5, the problem for the entrepreneur is

$$\arg \max_{\alpha_B^j, \theta_B^j} \left[\alpha_B^j, \theta_B^j \right], \quad E_{\alpha} \left[\alpha, \theta_B^j \right], \quad \int_R^j d(\gamma, \delta, T, s) \quad (20)$$

which is a repetition of expression (15), where the first argument stands for the profits of the current firm, the second one for the expected profits of a new firm, and the third one for the

value of going to wage work. The important feature to keep in mind is that the efficiency parameter is the same in the first and second arguments, whilst the parameter for demand changes. Depending on the value of the parameters α^{ij} and θ^j , the entrepreneur will decide whether keeping the current firm, opening a new one, or going to wage work, provides the highest rewards.

The entrepreneur's current firm will survive whenever the first argument is the largest. Therefore, sufficient conditions for the current firm to survive are

$$E[\alpha_B^{ij}, \theta_B^j] > E[\alpha, \theta_B^j] \quad (21)$$

and

$$E[\alpha, \theta_B^j] > E[d(\gamma, \delta, T, s)]. \quad (22)$$

Our interest is to provide an explanation as to why some entrepreneurs wish to start a new firm after having closed a previous one that was not successful. In terms of our model, it is as though the second term in (20) had the largest value. In other words, the situation is similar to one where inequality (21) is not satisfied, i.e., when the current firm does not survive, whilst inequality (22) is satisfied, or in other words, whilst the entrepreneur wishes to start another firm. This paves the way to our next proposition.

Proposition 2 Let the profit function be the one shown by (1). The maximisation problem is the one in (20). Define θ_{\min} as the minimum level of entrepreneurial skills which satisfies the sufficient conditions for the current firm to survive (21) and (22). Let $D \subseteq J$ be the subgroup for which the second term in (20) is the largest. Therefore, $\forall j \in D, \theta_B^j > \theta_{\min}$. As consequence, the wage worker state will always provide the smallest rewards to subgroup D .

Proof. Let $\bar{\alpha}$ solve expression (21) as an equality. Lemma 3 in Appendix A establish the conditions under which $\bar{\alpha}$ exist. Utilising Lemma 2 and assuming $p > MC$, expression (21) is satisfied whenever $\alpha_B^{ij} > \bar{\alpha}$, where $i \in I, j \in J$, and $I \setminus J = \emptyset$. From expression (11) let θ_F solve $E[d(\gamma, \delta, T, s)] = E[\alpha, \theta_F]$, and from expression (22) let θ_{\min} solve $E[\alpha, \theta_{\min}] = E[d(\gamma, \delta, T, s)]$. Lemma 4 in Appendix A establish under which conditions θ_F and θ_{\min} exist. Thus, $E[\alpha, \theta_{\min}] = E[\alpha, \theta_F] = E[d(\gamma, \delta, T, s)]$. Making use of Lemma 1, any $\theta_B^j > \theta_{\min}$ satisfies (22). If for subgroup D expression (21) is not satisfied, but expression (22) is, we must therefore have $\alpha_B^{ij} < \bar{\alpha}$ and $\theta_B^j > \theta_{\min}, \forall j \in D, i \in I$. Under this circumstances the second argument in (20) is the largest. Once the entrepreneurs runs the new firm he receives

a new draw for the demand parameter. Define the new draw as α_B^{lj} , where $i \in I, j \in D$. It can be the case that $\alpha_B^{lj} \geq \bar{\alpha}$. Under any of these situations, either the first argument or the second one in (20) are the largest. This implies the third argument in expression (20), which represents the wage working state, has always the smallest value for subgroup D . ■

Proposition 2 says that when the profits of the small firm depend on the efficiency and demand parameters, those entrepreneurs whose firms do not survive, but wish to start a new one, have skills to manage the firm at least as large as the ones required to run a potentially successful firm. This implies that every time their firms fail to survive, the optimal action for them is to start another one, rather than becoming a wage worker. Note that this is the opposite result to the one obtained from Proposition 1, namely, that under the alternative model failure by the entrepreneur to keep his firm afloat implies he does not have the necessary skills, whereas under our general model, the fact that the entrepreneur wishes to own another firm is a reflection that he is skilful, the only thing his firm lacked was a more appealing product. Additionally, Proposition 2 establishes that such a skilful entrepreneur will always find that the prospects of starting another firm render a higher value to him than the alternative of going to wage work does.

Regarding our initial question as to why some entrepreneurs wish to own another firm after having closed an unsuccessful one, the answer by Proposition 2 is: because these entrepreneurs have enough skills to run potentially successful firms, assuming such skills are restricted to managerial skills; the other condition to run a successful firm is to find a product or service which is appealing to customers or for which there is a market niche. Two important questions to ask are: how plausible these assumptions are and hence, how reliable the prediction offered by our general model by means of Proposition 2 is. We provide the following arguments to tackle such questions. Firstly, such assumptions are consistent with the view suggested by Audretsch (1995), who asserts that entrepreneurs are always in the process of discovering whether they possess the “the right stuff” in terms of the product they offer for which sufficient demand exists and whether they can produce it more efficiently than their rivals. More specifically he states that ‘when a new firm is launched, its prospects are shrouded in uncertainty. If the new firm is built around a new idea, it is uncertain whether there is sufficient demand or if some competitor will have the same or even superior idea. Even if the new firm is a clone, there is the question whether there is enough demand’ (p. 67) He further asserts that ‘an additional layer of uncertainty is how competent the new firm really is’ (p. 67), which in terms of our model this last statement can be easily translated as referring to how skilful the owner-manager is. Storey (1990)

acknowledges a possibility which is also in line with our assumptions. He declares that ‘the creation and subsequent closure of a ...rm may merely be part of the small entrepreneurs way of searching out profitable opportunities...’ (p. 47). Secondly, we believe the predictions of our model will gain strength if they account for all the possibilities as provided by the empirical evidence in Figure 2. That is, whilst Proposition 2 offers an explanation as to why the “determined entrepreneurs” in Figure 2 have the desire to own another ...rm in the future, such explanation does not involve the other three cases. The following proposition present the complete results of our general model, which we believe suggests an explanation for all the empirical evidence contained in Figure 2.

Proposition 3 Let the profit function be the one shown by (1). The maximisation problem is the one in (20). Define θ_{\min} as the minimum level of entrepreneurial skills which satisfies the sufficient conditions for the current ...rm to survive (21) and (22), and define the group R as the one which contain the different combinations of α^{ij} and θ^j such that $\exists (\alpha^r, \theta^r) = \exists_0 (\bar{\alpha}, \theta_F) [f(\gamma, \delta, T, s)]$, $\exists (\alpha^r, \theta^r) \in R$

(i) Let $M \subseteq J$ be the subgroup for which the first term in (20) is the largest. Therefore $\alpha_B^{ij} > \bar{\alpha} \ \forall j \in M$, $\theta_B^j > \theta_{\min} \ \forall j \in M_1$, $\theta_{\min} > \theta_B^j > \theta^r \ \forall j \in M_2$, $i \in I$, $r \in R$, where $M_1 \cup M_2 = M$ and $M_1 \cap M_2 = \emptyset$.

(ii) Let $D \subseteq J$ be the subgroup for which the second term in (20) is the largest. Therefore $\alpha_B^{ij} < \bar{\alpha}$, $\theta_B^j > \theta_{\min}$, $\forall j \in D$, $i \in I$.

(iii) Let $Z \subseteq J$ be the subgroup for which the third term in (20) is the largest. Therefore $\alpha_B^{ij} < \bar{\alpha}$ and $\theta_B^j < \theta_{\min} \ \forall j \in Z_1$, $\alpha_B^{ij} > \bar{\alpha}$ and $\theta_B^j < \theta^r < \theta_{\min} \ \forall j \in Z_2$, $i \in I$, $r \in R$ where $Z_1 \cup Z_2 = Z$, and $Z_1 \cap Z_2 = \emptyset$.

(iv) For subgroups M_1 and M_2 the sale of their ...rms or any shock which makes $\alpha_B^{ij} < \bar{\alpha}$, $j \in M$, $i \in I$, assuming α_B^{ij} is exposed to unforeseen shocks, has the effect of making the second argument in (20) the largest for subgroup M_1 , and the third argument in (20) the largest for subgroup M_2 .

Proof. See Appendix A. ■

Proposition 3 is represented graphically in Figure 4. It shows under which conditions, in terms of combinations of the parameters α and θ , it is optimal for the entrepreneur to keep his current ...rm, start a new ...rm, or go to wage work. More specifically, Figure 4 is an isoprofit map in the space (α, θ) , where every point in its surface entails specific coordinates which form part of an isoprofit curve. In turn, every isoprofit curve is associated with a single

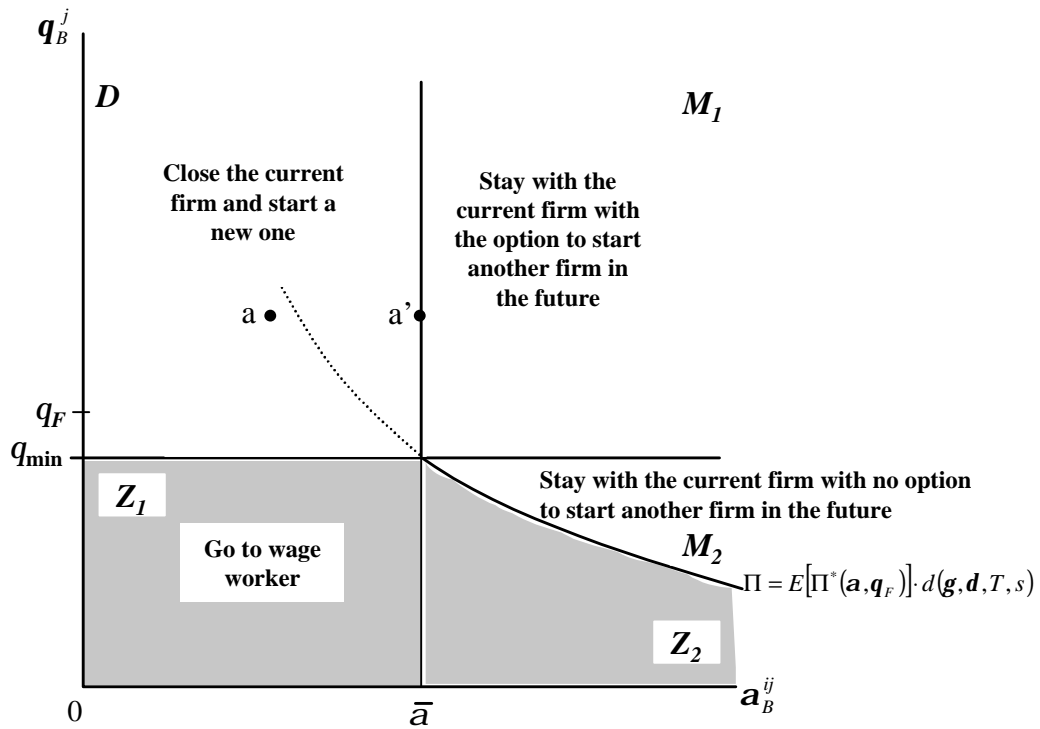


Figure 4: Optimal decision for the entrepreneur when the profits of the small firm depend on the efficiency and demand parameters

value of profits for different combinations of α and θ . Lemma 5 in Appendix A establishes that such isoprofit curves have negative slope and are convex with respect to the origin. The farther to the north-east the isoprofit is placed, the higher the associated value of profits.

Figure 4 is divided in five areas. Areas M_1 and M_2 contain the combinations of parameters α_B^j and θ_B^j such that the best option for the entrepreneur is to keep his current firm, because it is providing profits larger than the updated reservation level $E[\pi^*(\alpha, \theta_F)]d(\gamma, \delta, T, s)$, which is represented by the isoprofit curve that divides areas M_2 and Z_2 and once it crosses the coordinates (\bar{a}, θ_{\min}) continues upward as a dotted line. Recall that such updated reservation level represents the rewards in the wage working state. Therefore, any point to the right of its associated isoprofit curve represents profits that exceed such rewards. This explains why entrepreneurs placed in areas M_1 and M_2 are better off keeping his current firm, whilst those in areas Z_1 and Z_2 find optimal to close their current firm and go to wage work. The best option for entrepreneurs in area D is also to close their current firm, but instead of becoming wage workers, they are better off by starting a new firm. Note that area D comprises

some combinations of α_B^{ij} and θ_B^j which are placed to the right of the isoprofit curve of the updated reservation level, and some combinations located to the left of it. However, according to Proposition 3 the best course of action for entrepreneurs in area D is not to keep the current firm or close down and go to wage work, but to shut their current firm and start a new one. The reason is because by starting a new firm these entrepreneurs have the option of offering a different product in the hope of drawing a higher α : closing the Chinese take away and opening a sport clothes retail outlet. Take for example an entrepreneur who finds himself in a situation where his coordinates of parameters are represented by point a in Figure 4. His current firm's profits are smaller than the reservation level. However, by starting a new firm his expectations are to reach a point like a' which is placed in an isoprofit to the right of the reservation level. As a result, this entrepreneur does not find optimal to go to wage work, but to keep in entrepreneurialism by starting a new firm. The key point is that Proposition 3 states that all entrepreneurs in area D have skills above the cutoff point θ_{\min} , contrary to those in area Z_1 who do not. In this sense, what entrepreneurs in areas D and M_1 have in common is that all of them have skills above the cutoff point θ_{\min} . Notice from Figure 4 that we can say that entrepreneurs in area M_1 are the ones who are running successful firms because their firms' demand is above average. Therefore, it is fair to say the only thing an entrepreneur in area D lacked was to offer a more appealing product, but his skills to manage the firm efficiently are as good as the ones of those entrepreneurs who run successful firm. This is the case previously pointed out by Proposition 2.

Proposition 3 also establishes that the difference between entrepreneurs in areas M_1 and M_2 is that, in case their current firms do not survive or in case they sell them, the former have the option of keeping in entrepreneurialism by starting a new firm, whereas the latter do not and have to go to wage work. The intuition is as follows. Suppose an entrepreneur in area M_1 sells his firm. He has the option of starting a new firm or going to wage work. If he plans to start a new firm, his expectation about his new firm's demand is $\bar{\alpha}$. Given that he has skills above θ_{\min} , his expectation with a new firm in term of Figure 4 is to reach a point over the vertical line which starts from the coordinates $(\bar{\alpha}, \theta_{\min})$ and goes upward. As can be seen, any point in this line above the point $(\bar{\alpha}, \theta_{\min})$ renders profits higher than the updated reservation level. Therefore, it is optimal for an entrepreneur in M_1 to start a new firm. This is not the case for an entrepreneur in area M_2 , since this entrepreneur has skills lower than θ_{\min} . In other words, by starting a new firm he expects to be placed in area Z , which means he would receive profits lower than the reservation level. Therefore, entrepreneurs in area M_2 find optimal to go to wage work after having sold their firms. This intuition is equivalent to the one where there is a shock in demand and the firm does not

survive. For instance, the shock can represent a downturn in demand because another firm entered the market and is offering a product similar enough to impinge in the demand of the original firm. If the shock is big enough, the small firm's demand may fall below \bar{a} . Thus, it is as if entrepreneurs in area M_1 were "moved" to area D , where according to Proposition 3, these entrepreneurs are better off by closing their current firms and starting a new one. On the other hand, entrepreneurs in area M_2 would be placed in area Z after the shock, where they are better off by going to wage work.

Notice that Proposition 3 predicts that only entrepreneurs with skills above θ_{\min} have the option of starting new firms, that is, only those in areas M_1 and D . This is regardless of whether they were previously running firms with high demand or low demand. In this sense, Proposition 3 suggests the existence of a mechanism of entrepreneurial self-selection whereby skilful entrepreneurs self-select themselves by starting new firms after having closed one, regardless of the degree of success on the previous firm, whereas unskilful entrepreneurs go to paid employment. We are now in position to discuss whether or not this mechanism of entrepreneurial self-selection explains the empirical evidence in Section 2 and thereby, whether or not there is support for our general model. Before proceeding, note that unlike the alternative model our general model does differentiate between survival of the small firm and survival of the entrepreneur. According to Figure 4, survival of the small firm takes place under the conditions described by areas M_1 and M_2 , whereas survival of the entrepreneur occurs under the conditions described by areas D and M_1 .

5 Explaining the Evidence

Why do some entrepreneurs wish to start new firms after having closed one that was not successful? This is the question which the present paper tries to tackle. We showed the relevance of such question by means of the empirical evidence in Figure 2 in page 9. By doing so, we not only revealed there is potentially a large share of entrepreneurs who have closed a firm that was not successful and still want to own another firm in the future, but that there also exist, amongst other cases, some entrepreneurs that despite having successful firms do not have the desire to run their own firms again. In order to offer an explanation to our initial question and to the empirical evidence in general, we proposed and developed a model whose principal assumption is that the profits of the small firm depend on two firm-specific parameters: the first one captures the success of the firm's product in attracting demand, and the second the skills of the owner to manage the firm efficiently, that is, to deal

efficiently with the day-to-day activities. We assume such parameters are not correlated, i.e., we assume the entrepreneur's skills to manage are not correlated with the ability to select and offer an appealing product. On the contrary, we suppose the demand parameter varies every time the entrepreneur starts a new firm. From our model we obtained Propositions 2 and 3. To compare such results with the ones from an alternative model wherein the entrepreneur has some type of overall skills which includes skills to manage and skills to innovate or spot profitable opportunities, we also obtained Proposition 1.

As regards our initial question, Proposition 2 and 3 establish that those entrepreneurs who start or have the desire to start a new firm after having closed an unsuccessful one have skills to manage as great as those who are running successful firms. Our model goes further by suggesting the existence of a mechanism of entrepreneurial self-selection. Under such mechanism, entrepreneurs who are skilful are the only ones who start or have the desire to start another firm in the future after having closed a previous one. Such desire is independent of whether or not the previous experience of running a firm is regarded as a success or failure. This is not true of unskilful entrepreneurs, who prefer to go to employment. Does this mechanism offer an explanation to the empirical evidence provided in Figure 2? We believe it does. By means of Figure 4 our theory argues that the reason why the "determined entrepreneurs" and the "serial entrepreneurs" in Figure 2 have the intention to own another firm in the future is because they are skilful entrepreneurs. They would be placed in areas D and M_1 in Figure 2, respectively. Our theory interprets skilful entrepreneurs as those who have managerial skills at least as large as those needed to run potentially successful small firms, i.e., firms which would provide larger economic and non-economic rewards than the wage worker state would. In fact our model states that the "determined entrepreneurs" are as skilful as the "serial entrepreneurs". The latter were just "luckier" with their firms in the sense that they were running a firm with a very appealing product or service. Similarly, the mechanism of entrepreneurial self-selection also accounts for the cases of "failed entrepreneurs" and "discouraged entrepreneurs" shown by Figure 4. The former would be placed in area Z in Figure 4 and the latter in area M_2 . According to our general model, the fact that a "discouraged entrepreneur" wishes to leave entrepreneurialism despite having owned a firm that was successful, can be explained by saying that he was "lucky" in running a firm with a very attractive product or service, whose success indeed compensated for his low skill to manage the firm. However, this unskilful entrepreneur does not expect to "hit the jackpot" again, so he prefers to go to a paid job after having closed his current firm¹⁷.

¹⁷We believe the closest interpretation to the demand parameter α^{ij} in our model is luck. This is because

To offer such explanations our model relies on the assumption that the skills of the small firm owners are restricted to skills to manage the firm efficiently, and that such skills are not related with the ability to offer an attractive and appealing product. Let us assume the entrepreneur has both abilities. Thus, an entrepreneur who deals efficiently with the day-to-day activities of the small firm is also good at innovating or spotting profitable opportunities. This is the case of the model discussed in Subsection 4.1, from which Proposition 1 was obtained. Such proposition establishes that those entrepreneurs whose firms do not survive have skills lower than required, and that it is optimal for them to not attempt to own another firm in the future. This means that for those entrepreneurs who run successful firms, they are better off starting new firms. In other words, under such model only the “serial entrepreneurs” and the “failed entrepreneurs” in Figure 2 exist. As can be seen, the alternative model fails to explain the whole spectrum of empirical evidence, whereas our general model does not. Therefore, we can state that in general, the empirical evidence supports our assumption that the skills of the small firm owners are restricted to skills to manage the firm efficiently. The present discussion should be placed in its correct context. Firstly, we do not suggest that it is impossible for some entrepreneurs to have both high skills to manage and high skills to innovate or spot profitable opportunities. Secondly, regardless of the definition of entrepreneurship, our conclusion supports the view that when undertaking research on entrepreneurship if we consider data comprising small firm owners without any other discriminatory criterion, it is reasonable to suppose that the ability to manage and the ability to innovate or spot profitable opportunities are in general not correlated amongst small firm owners.

Therefore, given that the assumptions of our general model seem plausible, the mechanism of entrepreneurial self-selection gains strength. After all, the central argument of such mechanism is analogous to the idea of survival of the fittest: only the skilful entrepreneurs survive, whilst the unskilful do not. However, by skills and entrepreneurial survival such mechanism refers, in the case of the former, to skills to manage defined in this paper as skills to undertake efficiently the responsibilities of the everyday activities of the small firm, and in the case of the latter, to the entrepreneur’s desire to own another firm, which should not be mistaken for the survival of the small firm.

we specify it as a random variable which the entrepreneur is unable to predict every time he starts a new firm, even if he has experience in running previous businesses.

6 Extensions and Discussion

6.1 Robustness and Alternative Assumptions

In this subsection we discuss the robustness of some of the assumptions in our model, and we address some alternative hypotheses which might also explain the empirical evidence shown in Section 2.

Transferability of Skills

In our model we define the parameter θ^j as the one which embodies the entrepreneur's skills to manage his own firm efficiently. As we developed our model and defined the maximisation problem the entrepreneur faces once he is running a firm, which is shown by expression (??), one underlying assumption is that the entrepreneur can perfectly transfer his skills from his old firm to the new one. In our model a new firm is defined as one where the product or service is different from the one offered by the previous firm. Given that in our model we do not restrict the nature of product or services offered by small firms, one can argue the assumption of perfect transferability of skills seems not plausible when we contemplate the hypothetical example of an entrepreneur who was running a small restaurant and is planning to close and start a small manufacturing firm to produce computer microchips. Although we agree that such assumption is rather strong whenever we look at the possibility of an entrepreneur moving across small firms with very different activities, we point out two arguments which we think lessen the severity of such assumption. Firstly, we believe our definition of skills to manage included in Section 3 is general enough to make the assumption of transferability of skills across different firm activities more palatable. We defined such skills as the '...ability to organise and assume the responsibility of the activities of the small firm, to select the cheapest inputs given a desired level of quality, to deal with suppliers and, perhaps, employees, to ensure that the production of goods or services in the firm is done in the best possible way using the resources available, amongst others. In general, they represent the skills to undertake efficiently the everyday activities of the firm and to tackle the problems that might arise'. In this sense, it is not hard to think of a restaurateur or bed and breakfast owner opening a retail shop or vice versa. Secondly, we believe situations similar to the example mentioned above are rather unlikely. Figure 1 in page 8 provides some hints to support our argument. Amongst those who closed a firm but continued as business owners, Figure 1 shows that 34% started a similar business in a different location, compared to 28% who started or bought a different business¹⁸. As can be seen the first group

¹⁸In Figure 1, ignoring those who continued business in a different legal form, the results were obtained

would have no problems in transferring his skills to manage as the activities of the old and new firms are similar, whilst the fact that the new firm is placed in a different location fits into the definition of differentiated product. Ideally, information on the old and new firm's activities for the group of entrepreneurs who started or bought a different business in Figure 1 would provide us with stronger evidence to accept or refute our second argument. We lack such information. However, the fact that the empirical evidence shows that a larger share of entrepreneurs are owning new firms with activities similar to the old ones', compared to those who are running firms with different activities, suggests that cases where entrepreneurs move across small firms with very different activities might be minimal. After all, it makes sense for the entrepreneur to own firms where he can make use of the skills he has acquired in previous experiences.

Low Risk Aversion and Effort

In this part we ask ourselves to what extent the entrepreneur's attitudes towards risk and towards effort, in place of our original assumptions or additional to them, can explain the empirical evidence contained in Section 2 and, hence, answer our initial question. Consider the degree of risk aversion of entrepreneurs. Most of the classic economists who reflected on entrepreneurship converged on the notion that a specific characteristic of entrepreneurs is that they shoulder the risks of the economic activities they undertake. On over-viewing historical contributions to entrepreneurship research, van Praag (1999, pp. 327-328) declares that '[Richard] Cantillon's entrepreneur bears the risks as a consequence of selling (buying) at a certain price and buying (selling) at an uncertain price. [Jean-Baptiste] Say's entrepreneur bears the risk of losing capital and reputation due to experimenting and the chance of failure. The Marshallian entrepreneur is responsible for undertaking the business risk associated with the activities of his firm. ...[Frank] Knight...defines the entrepreneur as the decision maker whenever uncertainty is involved', where the square brackets are mine. In modern economic research this idea has been translated as the entrepreneur's decision between receiving a risk-free wage from employment, or engaging in self-employment, which includes owning and running a firm, with the inherent risks involved. In turn, the entrepreneurs' low risk aversion is a force which drives them to join self-employment¹⁹. Moreover, taking advantage of recent development on databases, some authors have been able to prove empirically that entrepreneurs indeed experience a lower degree of risk aversion²⁰.

from the calculations $0.18/(0.62-0.09)$ and $0.15/(0.62-0.09)$, respectively.

¹⁹For instance, see Kihlstrom and LaPorte (1979).

²⁰Uusitalo (2001), van Praag and Cramer (2001).

What the above deliberation suggests is that, in terms of our model, the entrepreneur's degree of risk aversion has to do more with the decision to start a firm in the first place, and less with the desire of starting a new firm after having closed one, which is our main concern. Therefore, it is sensible to say that it is unlikely that a model to investigate the survival of small firms and entrepreneurs based solely on the entrepreneur's degree of risk aversion, would explain the empirical evidence in Section 2.

With respect to the entrepreneur's attitude towards effort, popular knowledge and anecdotal evidence would assert that entrepreneurs, defined as owners and managers of their firms, exert more effort whilst working at their firms than an employee working for somebody else would. Moreover, it is often regarded that to run successful firms, the entrepreneur, compared to a common employee, must be willing to sacrifice more leisure in favour of hours of work²¹. This suggests that the entrepreneur's attitude towards effort is an important element to be considered in any research theorising on the behaviour of small firms and entrepreneurs. Surprisingly, very limited studies in research in Economics have incorporated such element²². van Praag (1999) compares the determinants of successful entrepreneurship as discussed by some classic economists. None of them seem to refer to the entrepreneur's effort as an important element to success, at least not directly²³. The standard consumption-leisure model which captures the notion that individuals value leisure, i.e., that they dislike work effort, and which has had a significant impact in particular areas of Economics²⁴, has been almost ignored when it comes to models of small firms or entrepreneurship. This is

²¹The empirical evidence contained in Blanchflower (2004) supports these beliefs. Using a survey which contains information for more than 35 countries he shows that self-employed work longer hours than employees. Similarly, using different surveys and through econometric estimation he arrives to the conclusion that self-employed place more weight on work than they do on leisure, which can be interpreted as self-employed experiencing less disutility of effort.

²²This is not the case in our sister disciplines Sociology, Psychology and Business Studies. Douglas and Shepherd (2002) and Gatewood et al (2002) are cases in context for Business Studies. The latter also surveys some relevant articles to our topic in the discipline of Sociology and Psychology.

²³Amongst other determinants of successful entrepreneurship, Jean Baptiste Say refers to "perseverance", Schumpeter to "willingness", Knight to "willingness and motivation" (van Praag, 1999, p. 329). It can be argued that in these and some other characteristics specified by these economists, it is implied, to various degrees, that the entrepreneur requires to apply considerably effort in his tasks to be successful.

²⁴Since in the consumption-leisure model work effort is a bad, it asserts that individuals are willing to "endure" work effort in exchange for a wage which would enable them to consume other goods, hence generating labour supply. This model is the basis of an entire branch of Economics: Labour Economics. It also spurred the research on the Principal-Agent problem, which in the beginning looked into the relationship employer-employee and how inefficiencies arise given the incentive by the employee to shirk in the job due to his disutility of effort (some early relevant articles are Alchian and Demsetz, 1972; Jensen and Meckling, 1976). In Macroeconomics, this idea of incentive to shirk also gave way to theories of unemployment, through the development of the efficiency-wage models (see Romer, 1996).

particularly relevant because, as discussed above, one would expect entrepreneurs to be more willing to forego leisure, thereby influencing the decision to join self-employment and, perhaps, the success in entrepreneurship. Three notable exceptions are Frank (1988), De Fraja (1996), and Douglas and Shepherd (2000). Although published in a business journal, Douglas and Shepherd (2000) use an economic model in essence. Amongst other results, they show that employees who generate less disutility of effort have a greater incentive to join self-employment, other things being equal (e.g. ability, attitude towards risk and independence). De Fraja (1996) specifies a model where the entrepreneur has the choice of carrying out a project by himself, or selling it to an outside investor and work as manager, where he explicitly acknowledges that the entrepreneur experiences disutility of effort. The product or service generated by the project can have either high or low demand, which is unknown before undertaking the project. De Fraja's model reveals that under conditions of high (low) demand, the entrepreneur exerts less (more) effort, in other words, that the entrepreneur has a more "laid back" attitude during good times. He declares this is inefficient because '...efficiency requires higher efforts in good times, when higher returns can be reaped' (De Fraja, 1996, p. 90). However, he shows that this result is exactly the opposite when the entrepreneur sells his project and acts as manager. Frank (1988) also takes into account the aversion to effort by the individual. His model focuses in the entrepreneur's decision to continue running a firm or exit self-employment. In his model he shows more explicitly that the optimal decision by the entrepreneur is one which is essentially based on the criterion marginal benefit of entrepreneurship must be equal to marginal cost of entrepreneurship. In his model, the former is equal to the entrepreneur's ability, and the former to the disutility of effort²⁵. Consequently, Frank reaches the conclusion that more able entrepreneurs exert more effort, whereas less able entrepreneurs act more lazily.

The models by Frank (1988) and De Fraja (1996), although with rather strong restrictions²⁶, suggest that in our model, if the entrepreneur's effort and his attitude towards it are considered, the structure of our model itself make it unclear what to expect in terms of the intensity of the entrepreneur's effort. In other words, De Fraja's model ignores the role of the entrepreneur's ability - in our model this is the efficiency parameter θ^j -, but he does consider the project's demand -the demand parameter α^{ij} in our model-. In turn, he finds that high values of α^{ij} mean low effort from the entrepreneurs. On the other hand,

²⁵In the model by De Fraja (1996), once the entrepreneur has decided to run the project by himself, the maximisation problem basically entails the same criterion.

²⁶One of De Fraja's assumptions is that the entrepreneur is the only one capable of carrying out the project, constraining any potential outside buyer of the project to hire him as manager. Regarding Frank's model, we have already mentioned that he implicitly assumes the entrepreneur enters a market of homogeneous goods.

Frank ignores the demand parameter α^{ij} and finds that more skilful entrepreneurs, i.e., those with higher values of θ^j , work harder. Assuming such results hold when both parameters are considered, like in our model, the final results are ambiguous or counter-intuitive. For instance, considering the four areas in Figure 4 in page 30, those entrepreneurs in area M_1 who have high skills and run firms with high demand would have, on one hand, the incentive to exert greater effort, but on the other, the incentive to not. The final result is ambiguous. This is not the case with entrepreneur in area D , who are also skilful but owning firms with low demand. If the results of the models of Frank (1988) and De Fraja (1996) hold, it means these entrepreneurs apply greater effort, even greater than those in area M_1 . If these results were to be used to explain the empirical evidence in Figure 2 in page 9, the implications would be that the determined entrepreneurs work harder than the serial entrepreneurs do, and yet the firms of the former are not successful. The intuition in this sense is hard to swallow. In any case, to disentangle the role of the entrepreneur's attitude towards effort in the survival of the small firm and the entrepreneur, and to conclude whether or not such new element changes our results and helps to explain the evidence in Section 2, a more formal model has to be developed. We believe this is material for a different paper.

Finally, notice that in the heart of Frank's and De Fraja's models and in ours, lies the assumption that the entrepreneur's effort does not alter the demand or the efficiency parameter. They are fixed parameters determined by nature, whose values influence the optimal decision by the entrepreneur to stay with his firm or as entrepreneur, and how hard to work. This makes a call for the development of models where the causality is assumed to be the other way around, e.g., models where the entrepreneur's efforts may improve his firm's demand. We conclude by recognising the importance of considering the value entrepreneurs allocate to effort whenever research in small firm or entrepreneurship is undertaken. We also recognise that including such element in our model may render more insights into the determinants of the survival of the small firm and the entrepreneur. We believe this task is relevant enough to be considered in an alternative paper.

7 Conclusions and Final Remarks

This paper offers an addition to the theories aimed at disentangling the forces and processes which determine the survival of the small firm and the entrepreneur. The principal objective of this paper is to provide an answer to the question: why do some small firm owners wish to start a new firm after having closed one that was not successful? By attempting to do

so, the paper also tries to explain some empirical evidence on small firm and entrepreneurial survival. The paper specifies and develops a model, based on specific assumptions, which provides an explanation to the empirical evidence and, hence, proposes an answer to our initial question. The paper also discusses alternative assumptions to the ones used in our model. It is found that such assumptions do not help to explain the empirical evidence. Our findings and the implications which derive from them can be summarised as follows.

Firstly, to answer our initial question our model proposes the following explanation. The reason why some small firm owners wish to start a new firm after having closed an unsuccessful one is because they have the skills to run potentially successful firms, that is, firms which would provide larger economic and non-economic rewards to the entrepreneur than those from being an employee. In fact, our theory states that such behaviour is part of a mechanism of entrepreneurial self-selection. Under such mechanism, skilful entrepreneurs, where entrepreneurs are defined as small firm owners, are the only ones who wish to own another firm in the future after having closed one, regardless of the degree of success in their previous venture, whereas unskilful entrepreneurs prefer to go to wage work. We showed this mechanism accounts for the empirical evidence on entrepreneurial survival provided at the outset of this paper. All in all, we believe this findings are a contribution to the efforts aimed to change the view that failure to keep a small firm afloat implies a failed entrepreneur. In other words, through the mechanism of entrepreneurial self-selection, this paper suggests that those entrepreneurs who wish to own another firm, even if they have had unsuccessful experiences, should not be regarded with suspicion. On the contrary, our findings argue these entrepreneurs have the talent to run successful firms. Clearly, one disadvantage of the present paper with relation to the mechanism of entrepreneurial self-selection is that it predicts only skilful entrepreneurs survive, without offering any supporting evidence. That is, even though such mechanism accounts for the empirical evidence on entrepreneurial survival included in this paper, it is still necessary to undertake further research to accept or reject the prediction that those entrepreneurs who started another firm or have the desire to start another firm, after having closed one, have indeed higher skills than those who prefer to go to wage work. We believe this is a task for a separate paper. In the same way, we believe that any possible policy implications should be addressed once such research has been carried out and its results obtained. In this sense, the findings of this paper are a first step in a research agenda aimed at the study of the survival of the entrepreneur.

To reach the above result, our model relies on the assumption that the skills of the small firm owner are restricted to skills to manage, defined as skills to undertake efficiently the

everyday tasks of the small firm. Specifically, we assume that such skills do not include the ability to develop and offer a successful product or for which there is a market niche, that is, that they do not include skills to innovate or to spot profitable opportunities. We assumed that the success of the small firm's product depends more on luck. This assumption is put to the test by specifying that entrepreneurs has some type of overall skills which incorporates skills to manage and skills to innovate or to spot profitable opportunities. We found that by making use of such alternative assumption we fail to explain the empirical evidence. This leads us to our second important result: regardless of the definition of entrepreneurship, our findings support the view that when undertaking research on entrepreneurship if we consider data comprising small firm owners without any other discriminatory criterion, it is reasonable to suppose that the skills of such business owners are limited to skills to manage, and that they do not comprise skills to innovate or to spot profitable opportunities.

Thirdly, the paper emphasises the need to differentiate between survival of the small firm and survival of the entrepreneur. Small firms are run by entrepreneurs. We say a small firm has not survived when it has stopped trading, though such an event does not necessarily mean the entrepreneur has not survived. The survival of the entrepreneur is the outcome of the decision between going to wage work or owning another firm, after having closed the previous one.

Finally, one may argue that the present paper does not offer any suggestion on how the survival of small firms could be improved. However, it does hint that perhaps we have been looking at the wrong problem in the sense that the survival of the entrepreneur might be of more significance. After all, as mentioned above, entrepreneurs are the ones who start and run the small firms.

Appendix

A

Lemma 3. Let $M^\alpha = \int_0^1 D(\alpha) dA(\alpha)$ and $N^\alpha = \int_0^1 D(\alpha)^2 dA(\alpha)$. Assume both M^α and N^α are positive constants. Thus, under the condition $M^\alpha = (N^\alpha)^{1/2}$ there exist a value of α^{ij} , call it $\bar{\alpha}$, which solves $E_\alpha^i(\bar{\alpha}, \theta_B^j) = E_\alpha^i(\alpha, \theta_B^j)$.

Proof. From Lemma 1 $\psi^i(\alpha, \theta) = \frac{(bc_i \theta D(\alpha))^2}{4b(\theta^j)^2}$, which we re-write as

$$\psi^i(\alpha, \theta) = \frac{bc^2}{4(\theta^j)^2} \left(\frac{cD(\alpha)}{2\theta} + \frac{D(\alpha)^2}{4b} \right).$$

Thus,

$$E_\alpha^i(\bar{\alpha}, \theta_B^j) = \frac{bc^2}{4(\theta_B^j)^2} \int_0^1 D(\alpha) dA(\alpha) + \frac{1}{4b} \int_0^1 D(\alpha)^2 dA(\alpha),$$

so

$$E_\alpha^i(\bar{\alpha}, \theta_B^j) = \frac{bc^2}{4(\theta_B^j)^2} \int_0^1 D(\alpha) dA(\alpha) + \frac{1}{4b} \int_0^1 D(\alpha)^2 dA(\alpha).$$

If $\bar{\alpha}$ solves $E_\alpha^i(\bar{\alpha}, \theta_B^j) = E_\alpha^i(\alpha, \theta_B^j)$, then it solves the equation

$$\frac{bc^2}{4(\theta_B^j)^2} \int_0^1 D(\bar{\alpha}) dA(\bar{\alpha}) + \frac{1}{4b} \int_0^1 D(\bar{\alpha})^2 dA(\bar{\alpha}) = \frac{bc^2}{4(\theta_B^j)^2} \int_0^1 D(\alpha) dA(\alpha) + \frac{1}{4b} \int_0^1 D(\alpha)^2 dA(\alpha),$$

or

$$\frac{1}{4b} \int_0^1 D(\bar{\alpha})^2 dA(\bar{\alpha}) - \int_0^1 N^\alpha dA(\bar{\alpha}) + \frac{c}{2\theta_B^j} (D(\bar{\alpha}) - M^\alpha) = 0.$$

Hence we require $D(\bar{\alpha}) = (N^\alpha)^{1/2}$ and $D(\bar{\alpha}) = M^\alpha$. By virtue of the inverse-function theorem²⁷ we get $\bar{\alpha} = D^{-1}((N^\alpha)^{1/2})$ and $\bar{\alpha} = D^{-1}(M^\alpha)$. Since $D(\bar{\alpha})$ is an increasing function we must therefore have $M^\alpha = (N^\alpha)^{1/2}$. \forall

Lemma 4. Let $M^\alpha = \int_0^1 D(\alpha) dA(\alpha)$ and $N^\alpha = \int_0^1 D(\alpha)^2 dA(\alpha)$. Assume both M^α and N^α are positive constants.. Suppose that $N^\alpha \int_0^1 \frac{1}{R} > 0$ and $(M^\alpha)^2 > N^\alpha \int_0^1 \frac{1}{R}$. Thus,

²⁷De la Fuente (2000, p. 181).

under these conditions there exist values of θ^j , which we name as θ_F and θ_{\min} , which solve $E_{\alpha}[\cdot^{\alpha}(\alpha, \theta_F)] = \cdot^j_R$ and $E_{\alpha}[\cdot^{\alpha}(\alpha, \theta_{\min})] = \cdot^j_R d(\gamma, \delta, T, s)$, respectively.

Proof. From Lemma 3 we know that

$$E_{\alpha}[\cdot^{\alpha}(\alpha, \theta_F)] = \frac{bc^2}{4(\theta_F)^2} \cdot \frac{c}{2\theta_F} M^{\alpha} + \frac{1}{4b} N^{\alpha} .$$

If θ_F solves $E_{\alpha}[\cdot^{\alpha}(\alpha, \theta_F)] = \cdot^j_R$, then it solves the equation

$$\frac{bc^2}{4(\theta_F)^2} \cdot \frac{c}{2\theta_F} M^{\alpha} + \frac{1}{4b} N^{\alpha} \cdot \cdot^j_R = 0 .$$

The positive root which we obtain from here is

$$\theta_F = \frac{1}{N^{\alpha} \cdot 4 \cdot^j_R} \sqrt[3]{bcM^{\alpha} + bc \cdot (M^{\alpha})^2 \cdot N^{\alpha} + 4b \cdot^j_R \sqrt[3]{1/2}} ,$$

where θ_F will be a positive constant as long as $N^{\alpha} \cdot 4 \cdot^j_R > 0$ and $(M^{\alpha})^2 > N^{\alpha} \cdot 4 \cdot^j_R$. Similarly, θ_{\min} solves the equation

$$\frac{bc^2}{4(\theta_{\min})^2} \cdot \frac{c}{2\theta_{\min}} M^{\alpha} + \frac{1}{4b} N^{\alpha} \cdot \cdot^j_R d(\gamma, \delta, T, s) = 0 .$$

The positive root which solves this equation is

$$\theta_{\min} = \frac{1}{N^{\alpha} \cdot 4b \cdot^j_R d(\gamma, \delta, T, s)} \sqrt[3]{bcM^{\alpha} + bc \cdot (M^{\alpha})^2 \cdot N^{\alpha} + 4b \cdot^j_R d(\gamma, \delta, T, s) \sqrt[3]{1/2}} .$$

It can be shown that $d(\gamma, \delta, T, s) \cdot 1$ for $\gamma \cdot 1$ and $s \cdot T$. Thus the conditions mentioned above are sufficient for θ_{\min} to be a positive constant. \forall

Proof of Proposition 3. Sufficient and necessary conditions which make the ...rst term in (20) to be the largest are

$$\cdot^{\alpha} \cdot \alpha_{B}^{ij}, \theta_{B}^j \cdot > E_{\alpha}[\cdot^{\alpha} \cdot \alpha, \theta_{B}^j \cdot] \quad (23)$$

and

$$\cdot^{\alpha} \cdot \alpha_{B}^{ij}, \theta_{B}^j \cdot > \cdot^j_R d(\gamma, \delta, T, s) . \quad (24)$$

Whenever the following inequality is true

$$E_{\alpha}[\cdot^{\alpha} \cdot \alpha, \theta_{B}^j \cdot] > \cdot^j_R d(\gamma, \delta, T, s) \quad (25)$$

such conditions are satisfied. However we can have

$$E_{\alpha}[\alpha, \theta_B^j] < \int_R d(\gamma, \delta, T, s) \quad (26)$$

and still satisfy them. From the two last expressions and using Lemma 4 let θ_F solve $E_{\alpha}[\alpha, \theta_F] = \int_R d(\gamma, \delta, T, s)$, and let θ_{\min} solve $E_{\alpha}[\alpha, \theta_{\min}] = \int_R d(\gamma, \delta, T, s)$. We obtain

$$E_{\alpha}[\alpha, \theta_{\min}] = E_{\alpha}[\alpha, \theta_F] \int_R d(\gamma, \delta, T, s) \quad (27)$$

If for subgroup M the first term in (20) is the largest, then from (23) and Lemma 1 and Lemma 3 we must have $\alpha_B^{ij} > \bar{\alpha} \quad \forall j \in M$. From (27) and Lemma 2, any $\theta_B^j > \theta_{\min}$ satisfies (25) and consequently (24). We define M_1 such that $\theta_B^j > \theta_{\min}, \quad \forall j \in M_1$. Note that the left hand side of equation (27) contains the rewards in the wage worker state, i.e., it is the updated reservation level. Therefore, there must be an isoprofit curve representing this updated reservation level. As defined, let R represent the subgroup which contains the different combinations of α^{ij} and θ^j , respectively, which account for such isoprofit curve. That is they satisfy

$$\int (\alpha^r, \theta^r) = E_{\alpha}[\alpha, \theta_F] \int_R d(\gamma, \delta, T, s), \quad \forall (\alpha^r, \theta^r) \in R \quad (28)$$

Thus, for some $(\alpha^r, \theta^r) \in R$, say (α_i, θ_i) we have

$$\int (\alpha_i, \theta_i) = E_{\alpha}[\alpha, \theta_F] \int_R d(\gamma, \delta, T, s) \quad ,$$

then from (27) we have that

$$\int (\alpha_i, \theta_i) = E_{\alpha}[\alpha, \theta_{\min}] \quad ,$$

so

$$\frac{bc^2}{4(\theta_i)^2} \int \frac{cD(\alpha_i)}{2\theta_i} + \frac{D(\alpha_i)^2}{4b} = \frac{bc^2}{4(\theta_{\min})^2} \int \frac{c}{2\theta_{\min}} M^{\alpha} + \frac{1}{4b} N^{\alpha} \quad ,$$

or

$$\frac{bc^2}{4} \int \frac{1}{(\theta_i)^2} \int \frac{1}{(\theta_{\min})^2} \int \frac{c}{2} \int \frac{D(\alpha_i)}{\theta_i} \int \frac{M^{\alpha}}{\theta_{\min}} + \frac{1}{4b} \int D(\alpha_i)^2 \int N^{\alpha} = 0 \quad ,$$

which will be satisfied if $\theta_i = \theta_{\min}$, and $D(\alpha_i) = M^{\alpha}$ and $D(\alpha_i)^2 = N^{\alpha}$. Using Lemma 3 we know the last two equalities hold for $\alpha_i = \bar{\alpha}$. This means that the isoprofit curve represented by (28) crosses through the point $(\bar{\alpha}, \theta_{\min})$. Lemma 3 below establishes that this isoprofit curve has negative slope and is convex to the origin. By means of (23), we

have the restriction $\alpha_B^{ij} > \bar{\alpha}$ and given that the isoprofit curve (28) has negative slope, is convex to the origin, and crosses through $(\bar{\alpha}, \theta_{\min})$, (26) and (24) are satisfied if and only if $\theta_{\min} > \theta_B^j > \theta^r$. We define M_2 such that $\theta_{\min} > \theta_B^j > \theta^r$, $\forall j \in M_2$. Note that $M_1 \setminus M_2 = \emptyset$ and $M_1 \cup M_2 = M$. This accounts for (i).

Given that for subgroup D the second argument in (20) is the largest, sufficient and necessary conditions are $E_{\alpha}^j(\alpha, \theta_B^j) > E_{\alpha}^j(\alpha_B^{ij}, \theta_B^j)$ and $E_{\alpha}^j(\alpha, \theta_B^j) > E_R^j(d(\gamma, \delta, T, s))$. By means of Lemma 1 and Lemma 3 the first inequality is satisfied whenever $\alpha_B^{ij} < \bar{\alpha}$, and making use of (27) and Lemma 2 the second inequality is satisfied whenever $\theta_B^j > \theta_{\min}$. Therefore, $\forall j \in D$, $\alpha_B^{ij} < \bar{\alpha}$ and $\theta_B^j > \theta_{\min}$. This accounts for (ii).

For subgroup Z the third argument is the largest. Thus, the following are sufficient and necessary conditions for such situation to take place

$$E_{\alpha}^j(\alpha_B^{ij}, \theta_B^j) < E_R^j(d(\gamma, \delta, T, s)) \quad (29)$$

and

$$E_{\alpha}^j(\alpha, \theta_B^j) < E_R^j(d(\gamma, \delta, T, s)) \quad (30)$$

where we can either have

$$E_{\alpha}^j(\alpha_B^{ij}, \theta_B^j) > E_{\alpha}^j(\alpha, \theta_B^j) \quad (31)$$

or

$$E_{\alpha}^j(\alpha_B^{ij}, \theta_B^j) < E_{\alpha}^j(\alpha, \theta_B^j) \quad (32)$$

Using Lemma 2 and equation (27), condition (30) is satisfied whenever $\theta_B^j < \theta_{\min}$. Additionally, if (32) is met, then condition (29) is also satisfied. This is accomplished by means of Lemma 1 and Lemma 3 whenever $\alpha_B^{ij} < \bar{\alpha}$. We define Z_1 such that $\forall j \in Z_1$, $\alpha_B^{ij} < \bar{\alpha}$ and $\theta_B^j < \theta_{\min}$. If (31) is met, then condition (30) is satisfied, provided (29) is also satisfied. Thus, (31) is true whenever $\alpha_B^{ij} > \bar{\alpha}$. Given this restriction and since the left-hand-side term of (29) represent a level of profits equivalent to the isoprofit curve depicted by (28), which in turn is characterised by Lemma 5, condition (29) is satisfied if and only if $\theta_B^j < \theta^r < \theta_{\min}$. We define Z_2 such that $\forall j \in Z_2$, $\alpha_B^{ij} > \bar{\alpha}$ and $\theta_B^j < \theta^r < \theta_{\min}$. Note that $Z_1 \setminus Z_2 = \emptyset$, and $Z_1 \cup Z_2 = Z$. This accounts for (iii).

An entrepreneur in subgroup M will be willing to sell his current firm if and only if the offered price is higher than the sum of expected discounted profits. That is, if the price is

larger than the first term in expression (20). Thus, the problem for entrepreneurs in subgroup M becomes

$$\arg \max_{\alpha, \theta_B^j} E_{\alpha}^j(\alpha, \theta_B^j) - d(\gamma, \delta, T, s) \quad (33)$$

where the first argument is the expected profits from starting a new firm, and the second the value of going to wage work. The entrepreneur will start another firm if $E_{\alpha}^j(\alpha, \theta_B^j) > d(\gamma, \delta, T, s)$, from which we can obtain equation (27). Given that $\theta_B^j > \theta_{\min}$ $\forall j \in M_1 \cup M_2$, $\theta_{\min} > \theta_B^j > \theta^r$ $\forall j \in M_2 \cup M$, $M_1 \setminus M_2 = \emptyset$ and $M_1 \cup M_2 = M$, the second argument in expression (20) is now the largest for subgroup M_1 , and the third argument the largest for subgroup M_2 . This accounts for the first part of (iv). If α_B^{ij} changes such that $\alpha_B^{ij} < \bar{\alpha}$, then condition (23) is no longer met. For subgroup M_1 this situation is equivalent to the one for subgroup D for which the second argument in (20) is the largest, because $\theta_B^j > \theta_{\min}$ $\forall j \in (M_1 \cup D)$, and $\alpha_B^{ij} < \bar{\alpha}$ $\forall j \in D$. However, for subgroup M_2 the situation is equivalent to the one for subgroup Z_1 for which the third argument in (20) is the largest, because $\theta_B^j < \theta_{\min}$ $\forall j \in (M_1 \cup Z_1)$ and $\alpha_B^{ij} < \bar{\alpha}$ $\forall j \in Z_1$. This completes the proof. \square

Lemma 5. The isoprofit curves of the (maximum expected) profits function in the space (α, θ) have negative slope and are convex to the origin.

Proof. For simplicity all superscripts and subscripts are dropped. The slope of the isoprofit is determined by

$$\frac{\partial \theta}{\partial \alpha} = - \frac{\partial D(\alpha, \theta) / \partial \alpha}{\partial D(\alpha, \theta) / \partial \theta}$$

From the proof of Lemma 1 and Lemma 2 we obtain,

$$\frac{\partial \theta}{\partial \alpha} = - \frac{\theta^2}{bc} \frac{\partial D(\alpha)}{\partial \alpha}$$

From assumption A.3 we have that $\frac{\partial D(\alpha)}{\partial \alpha} > 0$, therefore $\frac{\partial \theta}{\partial \alpha} < 0$. The convexity of the curves is established by

$$\frac{\partial^2 \theta}{\partial \alpha^2} = - \frac{\theta^2}{bc} \frac{\partial^2 D(\alpha)}{\partial \alpha^2}$$

From assumption A.3 we have that $\frac{\partial^2 D(\alpha)}{\partial \alpha^2} < 0$. Therefore, $\frac{\partial^2 \theta}{\partial \alpha^2} > 0$ and the isoprofit are convex to the origin. \square

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