Organising Competition for the Market*

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Abstract

We study competition for the market in a setting where incumbents (and, to a lesser extent, neighbouring incumbents) benefit from a cost or information advantage. We first compare the outcome of staggered and synchronous tenders, before drawing the implications for market design. We find the timing of tenders interrelates with the likelihood of monopolisation. For high incumbency advantages and/or discount factors monopolisation is expected, in which case synchronous tendering is preferable as it strengthens the pressure that entrants exercise on the monopolist. For low incumbency advantages and/or discount factors other firms remain active, in which case staggered tendering is preferable as it maximises competitive pressure coming from the other firms. We use bus tendering in London to illustrate our insights and draw policy implications.

Keywords: Dynamic procurement, incumbency advantage, local monopoly, competition, asymmetric auctions, synchronous contracts, staggered contracts.

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1 Introduction

In 2018, the citizens of Rome requested a referendum to open up the provision of local transport services to competitive tendering. This is, in microcosm, part of a movement which has seen numerous countries and municipalities worldwide progressively introducing competitive tendering for public services, such as rail and bus transport, refuse collection, waste management, and school meals.\footnote{Competitive tendering in some sectors has been introduced by law. See e.g. the European Regulation 1370/2007 on public passenger transport services.} This trend is also reflected in the increasing adoption amongst countries, currently 48 but with 12 seeking to join, of the WTO Agreement on Government Procurement, dating from 2014, which emphasises the need for competitive processes. However, simply opening the market by no means guarantees that it becomes contestable. Incumbents’ positions often remain dominant, with a few large players winning contracts repeatedly within and across sectors.\footnote{In Europe, between 2006 and 2016, the number of public tenders with only one bid has grown from 17% to 30% and the average number of offers per tender has fallen from five to three in the same period - See European Commission (2017). Evidence of lack of competition in public procurement tenders is also reported at country level. For example, Amaral et al. (2009) document that 60% of local bus tenders in France between 2002 and 2005 received only one bid and the sector as a whole is dominated by three companies. Weiergraebner and Wolf (2020) report that in the German rail sector the former state monopoly still operates the majority of traffic routes - 67.1% in 2016 - despite the liberalisation process started in the 1990s.}

With about 13% of GDP spent by OECD countries on public procurement (OECD, 2019), and recurrent tendering characterising the award of many public contracts, designing the market so as to maintain effective competition is therefore a key issue.

We address two questions in relation to organising the market for competition. The first concerns the liberalisation phase. As historical operators typically benefit from sunk cost and information advantages vis-à-vis potential competitors, measures should be taken to ensure a level playing field. Breaking up the historical operator may contribute to achieving this.

The second and little-examined question concerns the evolution of competition over time, given that the service will be required for the foreseeable future. Having sunk the entry cost, or learnt relevant market information, the initial winners may be advantaged in subsequent tenders. The timing of these tenders therefore needs to be carefully planned. We focus on one key aspect, namely, the synchronicity of repeated tenders.

We find that these two questions should not be treated in isolation. Instead, industry structure and tendering timing interplay significantly to ensure a competitive environment. Whether tenders should be synchronous or not both affects and depends on whether a competitive market structure can result from an appropriate market design.

We consider an infinitely repeated setting with two-period contracts for two adjacent
markets; the tenders can either be synchronous, in which case they take place in the same period in both markets, or be staggered, in which case they alternate across the two markets. All firms face the same variable cost of service provision but, in order to operate in a market, must sink a fixed cost, which is lower if the firm is already operating in the other market and zero if the firm is the incumbent. This is the source of incumbency advantages, which we show can be reinterpreted as stemming from superior information on relevant market conditions. Any firm that loses a market must again sink the fixed cost in order to re-enter. There are many potential entrants and cost information is public knowledge. We focus on Markov equilibria and compare per-period prices.

We show that one of two market structures may arise in equilibrium: either one firm serves both markets (monopoly), or in each market a distinct firm operates (duopoly). Which market structure arises depends on the initial market structure, the tendering regime, the incumbency advantages, as summarised by the relative levels of sunk costs, and the discount factor. A duopoly always yields lower prices than a monopoly, but such a more competitive structure can be sustained only when the incumbency advantages and/or the discount factor are low; monopolisation prevails otherwise.

We derive the implications for the organisation of competitive tendering by considering an extended setting in which a public authority, liberalising a number of markets, must choose whether to break up the incumbent, as well as whether to tender the markets in a synchronous or staggered pattern. Abstracting from transaction costs, we find that total discounted prices are minimised when the historical operator is broken up, so as to start the tendering process with a competitive industry structure, and then either staggered or synchronous tendering is used, depending on whether competition can be sustained or not: staggered tendering is preferable when competition can be sustained over time, whilst synchronous tendering is preferable when monopolisation is inevitable. The key insight is that synchronous contracts enhance the competitive pressure exercised by potential entrants, and therefore should be used in case monopolisation is expected, whilst staggered contracts enhance the competitive pressure exercised by the firms already active in the markets, and therefore should be used where competition is sustainable. Our analysis also sheds light on the choice of contract duration, the impact of tendering timing on entry, and the implications of alternative objectives for the public authority.

These issues are very much alive. When privatising British Rail, there was extensive (and acrimonious) discussion on how to organise the horizontal split in passenger franchises in order to maximise competition. There was also significant discussion on the length of franchises, with the Treasury arguing for three-year franchise awards in each case, all issued nearly simultaneously; eventually the franchises offered were for longer periods, varying between 5 and 15 years, with the most common being approximately 7
years, so there was a natural staggering (Gourvish, 2002; Shaw, 2000). Private operators face similar issues of tender design. For example, National Express coaches (NX), Britain’s largest inter-city and regional express coach operator with around 150 timetabled routes and hundreds of daily destinations, contracts out almost all its routes to a large number of coach companies (around 20). Flixbus manages a similar network of bus routes in continental Europe. More generally, firms face similar problems when they outsource services such as maintenance, logistics or IT, and tender contracts recurrently. Incumbency advantages can arise there from the acquisition of knowledge on the firm’s products and resource management system.

To stress the practical relevance of our insights, in the final part of the paper we show how they help in identifying key features of tendering process used in the London bus market, considered an example of good practice in maintaining competition (OECD, 2009), and in understanding the observed patterns. We also discuss several policy implications.

This paper relates to a vast body of research on how to increase competition in procurement with asymmetries among bidders. Asymmetries may arise from technology choices, locations of firms, capacity constraints, switching costs, better information and familiarity with local rules and regulations, or from ownership of important assets. To maintain competition in such a context, a number of papers have pointed out potential benefits from using discriminatory procurement rules (in a static setting, see Myerson, 1981 and Maskin and Riley, 2000; in a dynamic setting see Laffont and Tirole, 1988, Lewis and Yildirim, 2002 and 2005; and Barbosa and Boyer, 2017), discounting switching costs in the evaluation of bids (Cabral and Greenstein, 1990), splitting supply (Anton and Yao, 1987 and 1992), using shorter and more frequent contracts (Saini 2012), or enhancing the information on the common value components of potential entrants (De Silva et al. 2009).3 We look more specifically at the choice of the initial market structure and the timing of tenders.4

In the context of repeated interaction, the relative merits of staggered and synchronous contracts have been left almost unexplored. One important exception is Cabral (2017), who compares staggered and synchronous contracts in an industry with an infinite sequence of short-lived buyers.5 The main point of departure is that Cabral focuses on

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3Empirical procurement studies present evidence in support of these predictions. See e.g. De Silva et al. (2003, 2009) for evidence on the bidding behaviour of entrants; Athey et al. (2013) on the impact of discriminatory procurement rules; De Silva (2005) and De Silva et al. (2005) on the role of synergies across projects auctioned; Jofre-Bonet and Pesendorfer (2000) on the effect of capacity constraints; and Weiergraebner and Wolf (2020) on incumbency advantage due to lower cost or better information in a common value setting.

4This links our paper also to the literature on the endogenous determination of market structure when property rights are auctioned (see, e.g., Dana and Spier 1994).

5The impact of the timing of contracts on competition has been studied also by Dana and Fong (2011)
economies of scale or scope, whereas we focus on incumbency advantages arising from sunk costs or superior information. The implication of economies of scale is that, with homogeneous products, monopolisation always arises; as a result, synchronous tenders always yield lower prices. By contrast, in our setting competition is sustainable when the incumbency advantages and/or the discount factor are not too large, and staggered tenders are desirable in that case. Another point of departure is that our focus on procurement leads us to study initial market design as well.

The rest of the paper proceeds as follows. In Section 2 we describe the set-up. In Sections 3 and 4 we characterise the equilibria under staggered and synchronous contracts. In Section 5 we derive the implications for market design. In Section 6, we analyse some extensions, specifically, welfare maximisation and contract duration. In Section 7, we use our framework to discuss empirical evidence on tendering for London bus services and draw some policy implications. In Section 8 we provide some concluding remarks.

2 Setup

We consider two markets $A$ and $B$ that are repeatedly up for tender over an infinite horizon, discrete time setting indexed by $t = 0, 1, \ldots$. Specifically, we suppose that each market is tendered every other period, so that all contracts cover two consecutive periods of operation, and compare two scenarios:

- synchronous tenders: both markets are tendered in even periods;
- staggered tenders: market $A$ is tendered in even periods, whereas market $B$ is tendered in odd periods.

The winner of a tender receives a lump-sum price $p$ for servicing the market. All firms face the same (total discounted) cost of providing the service for two periods, which we normalise to zero.

In addition, to start servicing a market a firm must sink a fixed cost, which is however lower if the firm has already been servicing the other market in the past period. In each tender, there are therefore potentially three types of firms:

- synchronous tenders: both markets are tendered in even periods;
- staggered tenders: market $A$ is tendered in even periods, whereas market $B$ is tendered in odd periods.

In what follows, firms’ prices can be interpreted as firms’ margins, net of operating costs.
• the firm currently servicing the market that is up for tender has already sunk this cost;

• the firm currently servicing the other market (if it is not the same as the first one) needs to sink a fixed cost $s > 0$;

• potential entrants need to sink a higher fixed cost $S > s$.

The firms currently servicing the markets thus benefit from incumbency advantages with $S$ representing the incumbency advantage vis-a-vis outside entrants and $s$ the advantage relative to the firm in the adjacent market.\footnote{Our model assumes away any economies of scale or scope in operating costs or entry. The absence of economies of scale or scope in operating costs plays a key role in our results (see the discussion following Proposition 1). By contrast, our key insights continue to hold in the presence of economies of scope in entry; see the discussion at the end of Section 3.} This can stem from existing infrastructure, better knowledge of market conditions or necessary technologies, or from consolidated management practices (see Remark 1 below). We further assume that there is a large number of potential entrants, and that any firm that loses a market will have to sink again the fixed cost ($s$ or $S$, depending on whether the firm is servicing the other market).

In each tendering date, there are thus two states:

• State $I$ (One-Incumbent): one firm, which we denote by $M$, currently services both markets, and faces competition only from potential entrants, which we denote generically by $E$; we will sometimes refer to this as a “monopoly”.

• State $II$ (Two-Incumbents): two distinct firms currently service one market each, and compete against each other as well as against potential entrants; we will sometimes refer to this as a “duopoly”. In the case of staggered tenders, we will denote the “defender” currently servicing the market that is up for tender by $D$, and the “challenger” currently servicing the other market by $C$. In the case of synchronous tenders, we will denote both incumbents by $D$.

For the sake of exposition, we assume that the tender takes the form of a combinatorial first-price auction, which is consistent with current practice.\footnote{See the discussion on the management of London bus services in section 7.1. In our setting, where bidders’ values are common knowledge, all classic auction formats (e.g., first-price or second-price sealed bid auctions, ascending or descending auctions) yield similar outcomes. Allowing for combinatorial bids matters for synchronous tenders, but does not affect the relevant equilibria – see Remark 2.} With staggered tenders, each firm simply submits a price for the market that is up for tender; the lowest bidder then wins and services the market for the offered price $p$. The resulting profit for the
winner is \( p \) if it already services the market (firm \( M \) in state \( I \), or firm \( D \) in state \( II \)), \( p - s \) if it does not but currently services the other market (firm \( C \) in state \( II \)), and \( p - S \) if it is not currently operating in any market for any firm \( E \)).

With synchronous tenders, each firm instead submits a price for every combination of markets, that is, a price \( P \) for both markets and prices \( p_A \) and \( p_B \) for markets \( A \) and \( B \). The winning allocation minimises the total price of servicing the two markets, and each winner receives the offered price for each market it obtains.\textsuperscript{11} The profits are then computed as above.

Firms maximise the sum of their discounted profits, using the same discount factor \( \delta \in (0, 1) \). We study the subgame-perfect equilibria and, to eliminate any scope for tacit collusion, focus on Markov equilibria, in which firms’ equilibrium strategies in a given period can only depend on the current state, \( I \) or \( II \). Finally, to discard dominated equilibria, we restrict attention to Coalition-Proof Nash equilibria.\textsuperscript{12} In our setting, this amounts to focusing on Pareto-efficient continuation equilibria. In case of staggered tenders, this rules out losing bids that are lower than firms’ values; in case of synchronous tenders, this selects the most profitable equilibrium. In what follows, “equilibrium” thus stands for “coalition-proof Markov subgame-perfect equilibrium”.

\textbf{Remark 1 (incumbency advantages).} We focus on sunk costs, as they are a natural source of incumbency advantages. An incumbency advantage may instead stem from learning through experience about market specific conditions. For example, the market may provide information that helps in reducing the costs of servicing the market in the future. However, as conditions may change over time, the information acquired in a recent period is more useful than the information gathered in previous periods, which again confers an advantage to the incumbent currently operating a given market. We provide in Appendix A a micro-foundation, based on such information decay, for the sunk costs \( S \) and \( s \) used to model incumbency advantages.

We first characterise below the equilibria for both staggered and synchronous tenders. We then derive the implications for market design.\textsuperscript{13}

\textsuperscript{11}With observable costs, the procurer could simply set prices directly. Our simple setup is therefore not suitable to discuss optimal price regulation or the possibility of discriminatory procurement, which we rule out.

\textsuperscript{12}See Bernheim, Peleg and Whinston (1987).

\textsuperscript{13}For the sake of exposition, we assume that the two markets are already serviced in period 0, so that the game already starts in state \( I \) or \( II \). This is consistent with our market design analysis, which considers the transition from in-house to competitive tendering.
3 Staggered tenders

We first consider the case of staggered tenders. Competition among entrants implies that they obtain zero equilibrium profit. By contrast, incumbent firms may obtain positive profits. Furthermore, intuitively, an incumbent firm is better off when it monopolises both markets than when a different firm is operating in the other market. Hence, letting $V_i$ denote the equilibrium continuation value of firm $i = M, D, C$ at the beginning of each period, we expect:\footnote{We confirm in Appendix B that this indeed holds in equilibrium.}

$$V_M > V_D + V_C \geq 0.$$ 

It follows that, in state $I$, $M$ has more to gain from winning: it then avoids facing a challenger in the next tender and thus gains $\delta (V_M - V_D)$, which exceeds $E$’s gain from becoming a challenger, given by $\delta V_C$. Moreover, as $E$ faces the entry cost $S$, in equilibrium $M$ wins by matching the best price that entrants are willing to offer, namely:

$$p_E = S - \delta V_C,$$

which corresponds to their cost of entering the market, minus the discounted value of becoming a challenger. We thus have:

$$V_M = p_E + \delta V_M = S + \delta (V_M - V_C). \tag{1}$$

Likewise, in state $II$, $D$ prevails over $E$, as the gain from winning is equal to $\delta V_C$ for both firms, but $D$ does not face the entry cost $S$. By contrast, the comparison between $D$ and $C$ is less clear-cut: $C$ must sink cost $s$ but gains more from winning because this enables it to monopolise both markets. Specifically, driving the defender out again gives $C$ an expected gain $\delta (V_M - V_D)$, which exceeds $D$’s gain from being a challenger in the next tender, $\delta V_C$. It follows that which firm prevails depends on the comparison between the challenger’s sunk cost, $s$, and the “value of monopolisation”:

$$\Delta_{Stag} \equiv \delta (V_M - V_D - V_C),$$

reflecting the impact of eliminating the challenger on total discounted industry profit.

Two types of equilibrium can therefore arise:

- **Single-state equilibrium**: if $s < \Delta_{Stag}$, $C$ wins and monopolises both markets; the equilibrium then remains in state $I$ forever.
• **Dual-state equilibrium:** if instead \( s > \Delta_{\text{Stag}} \), \( D \) wins and thus both firms remain active. The equilibrium path then remains forever in the initial state: starting from state \( I \), \( M \) keeps servicing both markets; starting instead from state \( II \), each incumbent keeps servicing its own market.

If \( \Delta_{\text{Stag}} > s \), then in state \( II \) \( D \) exits and obtains \( V_D = 0 \), whereas \( C \) wins by matching the best price that \( D \) is willing to offer, namely:

\[
p_D = -\delta V_C,
\]
reflecting the discounted value of being a challenger in the next tender. \( C \) thus obtains:

\[
V_C = p_D - s + \delta V_M = -s + \delta (V_M - V_C).
\]

Combining (1), \( V_D = 0 \) and (2) yields:

\[
\Delta_{\text{Stag}} = \delta (S + s).
\]

The value of monopolisation increases with the discount factor, \( \delta \), since the monopoly rent is only enjoyed from the next tender on, and with the entrants’ cost handicap, \( S \), which reduces the competitive pressure they impose on \( M \) in state \( I \), as illustrated by (1). It also increases with the challenger’s cost handicap, \( s \), which reduces its profit from winning in state \( II \), as shown by (2).

If instead \( \Delta_{\text{Sync}} < s \), then in state \( II \) \( D \) wins against \( C \) and their roles are swapped in the next tender; hence, \( C \) obtains \( V_C = \delta V_D \), whereas \( D \) pays the best price that \( C \) is willing to offer, namely:

\[
p_C = s - \delta (V_M - V_D),
\]
reflecting the cost of entering the neighbouring market and the discounted value of monopolising both markets. \( D \) thus obtains:

\[
V_D = p_C + \delta V_C = s - \delta (V_M - V_D - V_C).
\]

Combining (1) with \( V_C = \delta V_D \) and (3) yields:

\[
\Delta_{\text{Stag}} = \frac{\delta (S - s)}{1 - 2\delta}.
\]

As before, the value of monopolisation increases with \( \delta \) and \( S \). By contrast, it is now decreasing in \( s \): because here the defender prevails, an increase in the challenger’s cost
handicap now increases the winner’s profit in state II, as can be seen from (2).

The equilibrium price then depends on the initial state: starting from state I, the incumbent monopolises both markets forever and the price always equals \( p_E = S - \delta V_C \); starting instead from state II, in every tender the defender prevails and the price is equal to \( p_C = s - \delta (V_M - V_D) \).

Elaborating on this leads to:

Proposition 1 (staggered tenders). Under staggered tendering, generically there exists a unique coalition-proof Markov equilibrium outcome,\(^{15}\) which can be of two types:

- **Single-state:** if
  \[
  \sigma \equiv \frac{S}{s} > \sigma^{Stag} (\delta) \equiv \frac{1 - \delta}{\delta},
  \]
  then monopolisation arises: regardless of the state in the initial period, from the next period on the same firm services both markets forever and the equilibrium price is
  \[
  p^{Stag}_I \equiv (1 - \delta) [(1 + \delta) S + \delta s].
  \]

- **Dual-state:** if instead \( \sigma < \sigma^{Stag} (\delta) \), then the equilibrium path depends on the initial state:

  - **persistent monopoly:** if in the initial state the same firm services both markets, this firm continues to do so forever and the equilibrium price is
    \[
    p^{Stag}_I \equiv \frac{1 - \delta}{1 - 2 \delta} [(1 - \delta - \delta^2) S - \delta^2 s];
    \]

  - **sustainable competition:** if in the initial state different firms serve the two markets, these firms will continue to do so forever and the equilibrium price is
    \[
    p^{Stag}_{II} \equiv \frac{1 - \delta^2}{1 - 2 \delta} [(1 - \delta) s - \delta S].
    \]

In the limit case where \( \sigma = \sigma^{Stag} (\delta) \), there are infinitely many equilibrium outcomes, which yield the same prices, \( p^{Stag}_I = S \) and \( p^{Stag}_{II} = 0 \), and differ only in the probability of switching from state II to state I, which can take any arbitrary value.\(^ {16}\)

**Proof.** See Appendix B.
Whether competition is sustainable or not thus critically depends on the incumbents’ cost advantages over the entrants— not only for the firm servicing the market up for tender, but also for the firm servicing the neighbouring market. The impact of these incumbency advantages is captured by the parameter \( \sigma = S/s \), which is increasing in \( S \) and decreasing in \( s \). Raising \( S \) reduces the pressure exerted by the entrants on the firm currently servicing the market, which increases the value of monopolisation. Reducing \( s \) increases the advantage of a neighbouring incumbent vis-à-vis the entrants, given by \( S - s \), which makes it easier for this neighbouring incumbent to prevail and become a monopolist.\(^{17}\) Increasing either incumbency advantage tilts the balance in favour of monopolisation, which always arises when \( \sigma \) exceeds \( \sigma^{Stag}(\delta) \). Therefore it is useful to think of \( \sigma \) as the magnitude of the incumbency advantages vis-à-vis the entrants. Proposition 1 also highlights the role of the weight put on future profits: the larger this weight, the greater the value of monopolisation, which therefore arises for a broader range of \( \sigma \): the threshold \( \sigma^{Stag}(\delta) \) is decreasing in \( \delta \).

Our model assumes away economies of scale or scope in both operation\(^{18}\) and entry.\(^{19}\) The crucial assumption, however, is the absence of economies of scale or scope in operation, which is the reason why competition can be sustained in state \( II \). Otherwise, as shown by Cabral (2017), one firm would inevitably outbid the other and retain both markets for ever. By contrast, our qualitative results are robust to introducing economies of scope in entry, as we show in Online appendix A.

4 Synchronous tenders

We now turn to the case of synchronous tenders, in which firms compete both in stand-alone and bundle prices. We show in Appendix C that coalition-proof equilibria are symmetric, and thus focus on symmetric strategies here.

As before, competition among entrants implies that they obtain zero profit, whereas incumbent firms may obtain positive profits, all the more so when the same firm operates in both markets. Hence, letting \( V_M \) and \( V_D \) denote the equilibrium continuation values of an incumbent firm in states \( I \) and \( II \), we have:

\[
V_M > 2V_D \geq 0.
\]

\(^{17}\) Reducing \( s \) also affects the value of monopolisation, and actually decreases it in case of monopolisation; however, this indirect effect is discounted and never offsets the direct impact on the neighbouring incumbent’s capacity to outbid the firm currently servicing the market.

\(^{18}\) Namely, the cost of operating a market – normalised here to zero – does not depend on how many markets the firm operates.

\(^{19}\) Namely, an entrant wishing to enter both markets simultaneously face total costs of \( 2S \), rather than, say, \( S + s \).
It follows from the above that in state $I$, $M$ prevails over the entrants, giving rise to a persistent monopoly. Furthermore, the total price that two entrants are willing to offer for becoming duopolists, $2p_E = 2S - \delta^2 (2V_D)$, exceeds the bundle price $P_E = 2S - \delta^2 (V_M)$ that an entrant is willing to offer for monopolisation.\(^{20}\) Hence, in order to win, $M$ must match the latter price, and thus obtains:

$$V_M = P_E + \delta^2 V_M = 2S. \quad (4)$$

In state $II$, the two incumbents again constitute the relevant source of competition. Hence, in equilibrium, either one incumbent wins both markets, or each incumbent wins a market – which must then be the one it already services.\(^{21}\) We consider in turn these two types of equilibrium.

We first note that, by offering an aggressive bundle price and high enough stand-alone prices, each incumbent can force the other incumbent to compete for the bundle as well;\(^{22}\) as a result, there always exists a monopolisation equilibrium in which one incumbent wins both markets. Competition for survival then drives the incumbents’ profits down to zero ($V_D = 0$) and leads them to offer their best prices namely:

$$P_D = s - \delta^2 V_M = s - 2\delta^2 S.$$ 

By contrast, whether competition is sustainable or not involves the same trade-off as for staggered tenders. Winning the adjacent market costs $s$, but enables a firm to monopolise both markets, generating an expected gain, $\delta^2 (V_M - V_D)$, higher than the gain from remaining a duopolist, $\delta^2 V_D$. It follows that competition is sustainable only if:

$$s \geq \Delta^{sync}, \quad (5)$$

where the value of monopolisation is equal here to:

$$\Delta^{sync} \equiv \delta^2 (V_M - 2V_D).$$

Each incumbent then matches the best price that its rival is willing to offer to take over

\(^{20}\)The bundled price expression accords with our assumption that such an entrant must incur $S$ for each market, which assumes away any economies of scope in entry. This is a natural assumption if incumbency advantages take time to develop; it is also in line with the micro-foundation based on information decay (see Appendix A).

\(^{21}\)Starting from a candidate equilibrium in which incumbents win each other’s markets, either one could profitably deviate by instead targeting its own market so as to save the entry cost $s$.

\(^{22}\)We show in Appendix C that stand-alone prices matching entrants’ best offers suffice to achieve this.
its market, namely:

$$p = s - \delta^2(V_M - V_D),$$  \hspace{1cm} (6)

which reflects the cost of entering the neighbouring market and the discounted gain from monopolising both markets. Using $V_D = p/(1 - \delta^2)$ and (4) then yields:

$$\Delta^{Sync} = \frac{2\delta^2(S - s)}{1 - 2\delta^2}$$ and $$V_D = s - \Delta^{Sync} = \frac{s - 2\delta^2 S}{1 - 2\delta^2}. \hspace{1cm} (7)$$

As for staggered tenders, the value of monopolisation: (i) increases with the weight $\delta$ of future rents and with the entrants’ cost $S$, which here entirely determines the size of the monopoly rent; and (ii) it is instead decreasing in the neighbouring incumbent’s cost of entering the market, $s$, which limits the competitive pressure exerted by this firm and thus raises the price in state $II$, as illustrated by equation (6). Condition (5) (which, from (7), ensures $V_D \geq 0$) amounts to:

$$s \geq 2\delta^2 S.$$

It follows that competition is again sustainable if being a monopolist is not too profitable ($S$ low), there is a large cost of serving the additional market ($s$ high) and firms discount the future substantially ($\delta$ low). Furthermore, when such an equilibrium exists, it Pareto-dominates the monopolisation equilibrium, in which competition for survival drives profits to zero.

This leads to:

**Proposition 2 (synchronous tenders).** Under synchronous tendering, generically there exists an essentially unique\textsuperscript{24} coalition-proof Markov equilibrium outcome, which can be of two types:

- **Single-state.** If

  $$\sigma = \frac{S}{s} > \sigma^{Sync}(\delta) \equiv \frac{1}{2\delta^2},$$

  then monopolisation arises and the equilibrium price is:

  $$p^{Sync}_I \equiv (1 - \delta^2) S.$$

- **Dual-state:** if instead $\sigma < \sigma^{Sync}(\delta)$, then the equilibrium path depends on the initial state:

\textsuperscript{23}We have $(1 - \delta^2)V_D = p = s - \delta^2(V_M - V_D)$, which implies both $V_D = s - \delta^2(V_M - 2V_D) = s - \Delta^{Sync}$ and $(1 - 2\delta^2)V_D = s - \delta^2V_M = s - \delta^2 S$.

\textsuperscript{24}In case of monopolisation, either incumbent may prevail in state $II$. 

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- **persistent monopoly**: if initially the same firm services both markets, the equilibrium price is again \( p_I^{\text{Sync}} = (1 - \delta^2) S \);

- **sustainable competition**: if instead different firms initially service the two markets, the equilibrium price is:

\[
p_{II}^{\text{Sync}} \equiv (1 - \delta^2) \frac{s - 2\delta^2 S}{1 - 2\delta^2}.
\]

In the limit case where \( \sigma = \sigma^{\text{Sync}}(\delta) \), there are infinitely many coalition-proof equilibrium outcomes, which yield the same per-market prices, \( p_I^{\text{Sync}} = (1 - \delta^2) S \) and \( p_{II}^{\text{Sync}} = 0 \), and differ only in the probability of switching from state II to state I, which can take any arbitrary value.

**Proof.** See Appendix C.

**Remark 2 (role of bundled bids).** Preventing firms from submitting bundled bids does not generate additional equilibria,\(^{25}\) but limits the scope for monopolisation in state II.\(^{26}\) However, the discarded equilibria deliver the lowest possible payoffs, as competition for survival eliminates all profits; hence, ruling out bundled bids has no impact on coalition-proof equilibria, and Proposition 2 remains valid.

## 5 Market design

We first compare equilibrium prices across tendering regimes and states of competition. We then note that competition is easier to sustain under synchronous tendering, and draw the implications for market liberalisation.

### 5.1 Price comparisons

The following proposition shows that equilibrium prices can be ranked:

**Proposition 3 (price comparisons).** The equilibrium prices satisfy:

\[
p_I^{\text{Stag}} > p_I^{\text{Sync}} > p_{II}^{\text{Sync}} > p_{II}^{\text{Stag}}.
\]

\(^{25}\)To see this, note that any equilibrium arising when bundled bids are ruled out still exist when they are allowed: expanding the firms’ offers by adding a bundled price equal to the sum of the stand-alone prices does not affect the equilibrium payoffs nor the feasible deviations.

\(^{26}\)Specifically, we show in Online appendix B that, when bundled bids are ruled out, monopolisation occurs only when the value of monopolisation \( \Delta^{\text{Sync}} \) exceeds the sunk cost \( s \). It follows that monopolisation no longer occurs when competition is sustainable and yields positive profits (i.e., \( \sigma < \sigma^{\text{Sync}}(\delta) \)).
The intuition relies on three observations:

- **Monopoly prices are lower under synchronous tendering:** $p_{I}^{\text{Stag}} > p_{I}^{\text{Sync}}$.

In state $I$, synchronous tendering allows the entrants to exert greater competitive pressure on the incumbent. Winning both tenders would enable them to replace the incumbent immediately, whereas under staggered tendering, an entrant would need to win two subsequent tenders before becoming a monopolist.

- **Duopoly prices are lower under staggered tendering:** $p_{II}^{\text{Sync}} > p_{II}^{\text{Stag}}$.

In case of sustained competition, the price is determined by the bid of the neighbouring incumbent, which in turn is driven by the prospect of becoming a monopolist. As monopoly prices are higher under staggered tendering, the neighbouring incumbent bids more aggressively under staggered tendering than under synchronous tendering.

- **Prices are lower under duopoly:** $p_{I}^{\text{Stag}} > p_{II}^{\text{Stag}}$ and $p_{I}^{\text{Sync}} > p_{II}^{\text{Sync}}$.

In state $I$, entrants need to incur a large fixed cost, $S$, in order to challenge the incumbent. By contrast, in state $II$, the other incumbent only needs to incur $s < S$, and thus exerts greater competitive pressure.

### 5.2 On the sustainability of competition

The above analysis shows that, in both tendering regimes, the competitive price is lower than any monopoly price – from either regime. The following proposition shows further that competition is sustainable for a broader range of parameters under synchronous tendering:

**Proposition 4** (sustainability of competition). There is more scope for sustainable competition under synchronous tendering: $\sigma_{\text{Sync}}(\delta) > \sigma_{\text{Stag}}(\delta)$.

**Proof.** We have:

\[ \sigma_{\text{Sync}}(\sigma) - \sigma_{\text{Stag}}(\sigma) = \frac{1}{2\delta^2} - \frac{1 - \delta}{\delta} = \frac{\delta^2 + (1 - \delta)^2}{2\delta^2} > 0. \]
This is illustrated by Figure 1. Under both tendering regimes, competition is sustainable only if the incumbency advantages (captured by $\sigma$) and/or the weight on future profits (measured by $\delta$) are low enough. However, there exists a middle range (namely, when $\sigma$ lies between $\sigma^{\text{Stag}}(\delta)$ and $\sigma^{\text{Sync}}(\delta)$) where competition is sustainable only under synchronous tendering. The intuition follows from the first two observations above: together, they imply that, in a dual-state equilibrium, the price increase from switching to monopoly is lower under synchronous tendering (that is, $p^{\text{Stag}}_I - p^{\text{Stag}}_{II} > p^{\text{Sync}}_I - p^{\text{Sync}}_{II}$), which reduces the value of monopolisation.\(^{27}\) As long as competition is sustainable, the values of monopolisation under staggered and synchronous tendering can respectively be expressed as:

$$\Delta^{\text{Stag}} = \frac{\delta}{1 - \delta} \left( p^{\text{Stag}}_I - p^{\text{Stag}}_{II} \right) \quad \text{and} \quad \Delta^{\text{Sync}} = \frac{2\delta}{1 + \delta} \frac{\delta}{1 - \delta} \left( p^{\text{Sync}}_I - p^{\text{Sync}}_{II} \right).$$

It follows that $p^{\text{Stag}}_I - p^{\text{Stag}}_{II} > p^{\text{Sync}}_I - p^{\text{Sync}}_{II}$ implies $\Delta^{\text{Stag}} > \Delta^{\text{Sync}}$.

Propositions 3 and 4 compare stationary equilibrium prices and do not study the convergence towards these equilibrium paths. In the next subsection, we study how market design can affect this convergence and determine the equilibrium outcome.

\(^{27}\)That tenders only take place every other period further reduces the value of monopolisation.
5.3 Market liberalisation

In practice, many services with natural monopoly features, such as urban transportation or the local distribution of water, electricity and gas, have traditionally been provided by regulated monopolies, commonly state or municipally-owned, before being opened to competition – in the form of “competition for the market”, rather than “competition in the market”. Our analysis can shed some light on the design of the liberalisation process.

For the sake of exposition, we assume that competition prevails in the limit cases where monopolisation could also occur – i.e., when \( \sigma = \sigma^\tau(\delta) \), for \( \tau \in \{ \text{Sync}, \text{Stag} \} \). This not only yields lower long-term prices, but is also the efficient outcome (as monopolisation would require the challenger to incur the sunk cost \( s \)) and thus minimises the total bill paid over time (as firms are indifferent between the two outcomes).

**Long-term prices.** For strong incumbency advantages and/or high discount factors (namely \( \sigma > \sigma^\text{Sync}(\delta) \)), monopolisation always occurs, as in Cabral (2017); long-term prices are then lower under synchronous tendering, which strengthens entrants’ competitive pressure on the monopolist: \( p^{\text{Sync}}_I < p^{\text{Stag}}_I \). When instead the the incumbency advantages are small or the discount factor is low (namely, \( \sigma \leq \sigma^\text{Stag}(\delta) \)), then – in contrast to Cabral (2017) – competition is sustainable; opting for staggered tendering is then optimal as this enhances the competitive pressure that the incumbents exert on each other: \( p^{\text{Stag}}_{II} < p^{\text{Sync}}_{II} \). Finally, for medium levels of the incumbency advantages or the discount factor (i.e., when \( \sigma^\text{Stag}(\delta) < \sigma \leq \sigma^\text{Sync}(\delta) \)), synchronous tenders are again optimal, in order to avoid monopolisation: \( p^{\text{Sync}}_{II} < p^{\text{Stag}}_{II} \). Summing-up, we have:

**Corollary 1** (market design: long-term prices). To minimise long-term equilibrium prices:

- if \( \sigma \leq \sigma^\text{Stag}(\delta) \), then staggered tenders are optimal;
- if instead \( \sigma > \sigma^\text{Stag}(\delta) \), then synchronous tenders are optimal.

**Breaking up the incumbent.** We first note that it is always optimal to break up the historical incumbent into two firms – assigning each of them the necessary human and physical capital needed to service a market – so as to start in state \( II \) rather than in state \( I \). If competition is sustainable, this leads to lower stationary prices (from the first period onward). In case of monopolisation, competition for survival generates lower prices in the initial tender:

**Corollary 2** (market design: breaking up the incumbent (or break-up decision)). To minimise equilibrium prices in both the short-term and in the long-term, breaking up the historical operators is always strictly optimal.
Proof. See Appendix E.1. \qed

In what follows, we thus suppose that the initial state is in a duopoly.

**Total discounted prices.** Corollary 1 is limited to the comparison of long-term prices, which arise from \( t = 1 \) onward. The same prices emerge at \( t = 0 \) when competition is sustainable. It follows that staggered tenders remain optimal for \( \sigma \leq \sigma_{\text{Stag}}(\delta) \), as competition is then sustainable in both regimes. By contrast, in case of monopolisation firms initially compete aggressively for survival. We show in Appendix E that the resulting prices are actually below cost (normalised here to zero), and even more so under staggered tenders:

\[
p_{\text{Stag}}(SSE) < p_{\text{Sync}}(SSE) < 0,
\]

where \( SSE \) stands for single-state equilibrium. We now consider the implications of this initial intense competition for market design.

To ensure that the same number of markets are opened to competition in each period under both tendering regimes, we consider a setting in which: (i) two “pairs” of neighbouring markets, \( \{A_i - B_i\}_{i=1,2} \), with each pair being serviced by a regulated local monopoly, are to be liberalised – for example, a “pair” of markets can be interpreted as a city, and a market as a set of bus routes;\(^{28}\) and (ii) liberalisation must take place progressively – this can be justified, for example, by limited capacity within the regulator.\(^{29}\) The regulator must select which markets to open in period 0, namely:

- opening the markets to competition in one city in period 0, and in the other city in period 1; tenders are then synchronous in both cities; or
- in each city, opening one market to competition in period 0 and the other market in period 1; tenders are then staggered in both cities.

When \( \sigma > \sigma_{\text{Sync}}(\delta) \), firms initially compete for survival under both regimes, and this competition is tougher under synchronous tenders, where firms are in a more symmetric position: both firms obtain \( V_D = 0 \), whereas under staggered tenders the winner obtains \( V_C > 0 \); as total costs are the same under both regimes, it follows that synchronous tenders are optimal. In the intermediate range \( \sigma_{\text{Stag}}(\delta) < \sigma \leq \sigma_{\text{Sync}}(\delta) \), synchronous tenders yield sustainable competition and thus lower long-term prices, but staggered tenders still trigger a competition for survival that delivers lower initial prices; as a result, the desirability of

\(^{28}\) The reasoning readily extends to any number of cities and any number of market pairs for each city, provided that there exists overall an even number of market pairs.

\(^{29}\) An alternative approach would consist in initially tendering some of the markets for a single period. See Remark 4 at the end of this section.
staggered tenders increases, relative to the previous comparison based solely on long-term prices. Building on these insights leads to:

**Proposition 5** (market design: total discounted prices). There exists a threshold \( \hat{\sigma}(\delta) \in [\sigma^{\text{Stag}}(\delta), \sigma^{\text{Sync}}(\delta)] \), which lies strictly above \( \sigma^{\text{Stag}}(\delta) \) for \( \delta \) large enough and otherwise coincides with \( \sigma^{\text{Stag}}(\delta) \), such that in period 0, to minimise total discounted prices:

- if \( \sigma < \hat{\sigma}(\delta) \) (which, by construction, includes the entire region where \( \sigma < \sigma^{\text{Stag}}(\delta) \)), then it is strictly optimal to open one market to competition in each city (staggered tenders);

- if \( \sigma > \hat{\sigma}(\delta) \) (which, by construction, includes the entire region where \( \sigma > \sigma^{\text{Sync}}(\delta) \)), then it is strictly optimal to open both markets to competition in one city (synchronous tenders);

- if \( \sigma = \hat{\sigma}(\delta) \), then both tendering regimes are optimal.

**Proof.** See Appendix E.2. \( \square \)

These insights are illustrated in Figure 2, where the solid line represents the threshold \( \hat{\sigma}(\delta) \). The future rents that accrue to the monopolist under staggered tenders induce more aggressive bidding in the first period, which raises the benefit of staggered tendering. The
effect is greater when future profits are not too discounted, which explains why \( \hat{\sigma}(\delta) \) lies above \( \sigma^{Stag}(\delta) \) when \( \delta \) is large.\(^{30}\)

**Remark 3 (On coalition-proofness).** As noted in Remark 2, under synchronous tenders, there always exists (even if \( \sigma < \sigma^{Sync}(\delta) \)) a single-state Markov perfect equilibrium in which, in state II, the two incumbents bid aggressively to become an entrenched monopolist, which is Pareto-dominated from the firms’ standpoint – and, thus, not coalition-proof – but may make the overall bill smaller than under the (single- or dual-state) equilibrium that arises under staggered tenders. Removing the coalition-proofness requirement may therefore further tilt the balance in favour of synchronous tenders.

**Remark 4 (On alternative market liberalisation processes).** To identify the sole impact of break-up and tendering pattern decisions, we have considered a setting in which the number of tenders and the contract length are both invariant across scenarios and over time. Alternatively, staggered tendering can be achieved by initially tendering a one-period contract in one of the markets, and a two-period contract in the other market. We show in Online appendix C that our insights continue to hold in this scenario: it remains optimal to break up the incumbent as the competition for the market then generates lower prices, particularly so when the equilibrium is single-state and tenders are staggered. The only noticeable difference is that tendering both markets in the initial period fosters competition for monopolisation, even when subsequent tenders are staggered; this tends to tilt the balance in favour of staggered tendering—see Online appendix C.2.

## 6 Extensions

### 6.1 Welfare

Our focus so far has been on the price paid by the auctioneer, that is, on the buyer’s surplus. We now extend our analysis by accounting for efficiency considerations as well; specifically, we consider here the problem of a planner maximising:

\[
W \equiv \omega - \mathcal{P} + \alpha(\mathcal{P} - \mathcal{C}),
\]

where \( \alpha \in [0, 1] \) is the weight assigned to the firms’ profits, \( \omega \) is the total discounted gross surplus from the service, \( \mathcal{P} \) is the average total bill per city and \( \mathcal{C} \) is the average total

\(^{30}\)In principle, under staggered tendering the public authority could also choose to auction off which market should be tendered first. Whilst the public authority is indifferent between starting with market \( A \) or market \( B \), the firms are not indifferent: in the duopoly state, if the equilibrium is a single-state one, then each incumbent firm would rather see the other market being auctioned first, so as to get an opportunity to monopolise both markets. Hence, by auctioning-off the choice of the initial market, the public authority could extract all the rents, as the two broken-up incumbents are symmetric at that stage.
discounted cost incurred to service a city. Increasing \( \alpha \) amounts to downplaying the role of prices and focusing more on efficiency.\(^{31}\) The case \( \alpha = 0 \) corresponds to our baseline model, while \( \alpha = 1 \) corresponds to total welfare maximisation, which boils down here to pure cost minimisation.

The Proposition below describes the implications on the break up decision and the optimal tendering regime of accounting for this social cost.

**Proposition 6 (welfare).** When the buyer attaches a weight \( \alpha \) to the firms’ profits, in each tendering regime \( \tau \in \{ \text{Sync, Stag} \} \):

- There exists a threshold \( \bar{\sigma}^\tau(\alpha) \) such that breaking up the incumbent remains optimal unless both \( \alpha > 1/2 \) and \( \sigma^\tau(\delta) < \sigma \leq \bar{\sigma}^\tau(\alpha) \); the threshold \( \bar{\sigma}^\tau(\alpha) \) increases from 1 to \( \infty \) as \( \alpha \) goes from 1/2 to 1, and is higher for staggered tenders.

- There exists a threshold \( \check{\sigma}^\tau(\delta;\alpha) \in [\sigma^\tau(\delta), \hat{\sigma}(\delta)] \) such that staggered tenders are optimal if and only if \( \sigma \leq \check{\sigma}^\tau(\delta) \); the threshold \( \check{\sigma}^\tau(\delta;\alpha) \) is decreasing in \( \alpha \) and coincides with \( \hat{\sigma}(\delta) \) for \( \alpha = 0 \), and with \( \sigma^\tau(\delta) \) for \( \alpha \geq 1/3 \).

**Proof.** See Online Appendix D. \( \Box \)

In a dual-state equilibrium, breaking up the incumbent remains optimal under either tendering regime, since the equilibrium remains in the initial state forever and thus no sunk cost is ever incurred. By contrast, in a single-state equilibrium, breaking up the incumbent now generates a social cost \( \alpha s \), because an incumbent will incur \( s \) to monopolise the market. Yet, the Proposition shows that breaking up the incumbent stays optimal even in that case if \( \alpha \) is low and/or incumbency advantages are so high that competition for monopolisation generates low enough prices in the initial period.

Furthermore, when sunk costs are socially costly, the welfare obtainable in state II is unaffected if the equilibrium is dual-state, but is reduced if the equilibrium is single-state, due to the sunk costs incurred in monopolising the market. As a result, when the equilibrium is dual-state only under synchronous tenders (i.e., \( \sigma^{\text{Stag}}(\delta) < \sigma \leq \sigma^{\text{Sync}}(\delta) \)), the scope for synchronous tenders increases. In the other regions, the comparison between the two regimes is unaffected when the equilibrium is dual-state under both regimes (i.e., \( \sigma \leq \sigma^{\text{Stag}}(\delta) \)), since sunk costs are not incurred under either regime; when instead the equilibrium is single-state under both regimes (i.e., \( \sigma > \sigma^{\text{Sync}}(\delta) \)), the break-up decision has no bearing on the ranking of equilibrium prices.\(^{32}\)

---

\(^{31}\)As in Section 5.3, we favour the efficient outcome in case of indifference.

\(^{32}\)In addition, when breaking up the incumbent, synchronous tenders postpone the sunk cost incurred in one of the cities.
6.2 Contract duration

We explore the effect of contract length. Following Cabral (2017), we suppose that the underlying model is defined in continuous time. Denoting the period length by $2L$, the discount factor is then:

$$\delta(L) \equiv \exp(-L),$$

where $r$ is the continuous time interest rate. In other words, a lower $\delta$ corresponds to a longer contract length.

The following proposition shows that, if it is possible to choose the contract length optimally, full efficiency can be achieved under both tendering regimes; hence, the tendering regime itself no longer matters.

**Proposition 7** (contract duration). For each tendering regime $\tau \in \{\text{Sync}, \text{Stag}\}$, it is optimal to break up the incumbent and choose the contract length $L$ such that $\delta(L) = \delta^\tau(\sigma)$.

This ensures allocative efficiency and leads the incumbents to price at cost:

$$p_{II}^{\text{Stag}}\big|_{\sigma = \sigma^{\text{Stag}}(\delta)} = p_{II}^{\text{Sync}}\big|_{\sigma = \sigma^{\text{Sync}}(\delta)} = 0.$$

**Proof.** See Appendix F. □

Starting from a duopoly, the optimal contract length makes the incumbents indifferent between monopolising the markets or not. They are therefore willing to keep to their respective markets. This ensures sustainable competition, as well as cost efficiency, by avoiding the cost $s$ needed to enter the neighbouring market. Yet, they obtain the same zero payoff as when competing for monopoly, which implies that they price at cost.\(^{33}\)

Finally, recall that monopolisation is more likely to arise under staggered tendering; as a result, the optimal contract length is also longer under staggered tendering (that is, $[\sigma^{\text{Stag}}]^{-1}(\sigma) < [\sigma^{\text{Sync}}]^{-1}(\sigma)$).

7 Applications

7.1 London buses

London has a very large bus market with over 600 routes, and over two billion passenger journeys annually, amounting to over half of English bus travel. The UK Government broke up then privatised the London historical operator, creating 12 divisions in 1994, each related to particular garaging facilities (see Cantillon and Pesendorfer, 2006, Ameral et al., 2013 and Iossa and Waterson, 2019). Since 2001, there has been route-by-route

\(^{33}\)Recall that the equilibrium continuation values are continuous functions of the discount factor $\delta$. 

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tender competition on a gross cost basis, fares going directly to the organiser, Transport for London (TfL), with tenders being on the basis of supplying buses, drivers, maintenance and garaging to match the timetabling requirements of TfL for a period of five (commonly extendable to seven) years.\(^{34}\) Tranches of route tenders are issued approximately every nine days on a rotating basis throughout the year, each including on average fewer than four significant routes. Bids for the whole tranche are extremely rare, although bundled bids are common (sometimes across tranches).\(^{35}\) We collected data on all tender outcomes from 2003 to end 2014, totalling 884 contracts; in most cases the same route has been tendered at least twice.\(^{36}\)

The features of London bus market are in line with our key modelling assumptions. For one thing, there appear to be no significant economies of scale or scope in the market. If anything, Cantillon and Pesendorfer (2007) report some negative cost synergies. Although economies of scale and scope can arise when bus operators face revenue risk (Berechman and Giuliano, 1985; see also Cowie, 2002), they do not do so in London, where contracts are on a gross cost basis.

The absence of significant economies of scale and scope is also supported by the fact that the market as a whole has remained competitive, which is consistent with the equilibrium remaining in “state II”, whereas such economies would lead to monopolisation (see Cabral, 2017). Despite a number of acquisitions, entry, and exit of operators, there have been for some time around 10 companies active in the market, with five having a share greater than 10%. Over the years, on average around three firms have bid for each route (see Iossa and Waterson, 2019). Further revealed evidence comes from Waterson and Xie (2019), who find no monopolisation of any particular area of London.

By contrast, there appear to be significant incumbency advantages. A first important factor is garage ownership. Garages are a sunk cost and the firm which has the garage most convenient for the bus route in question is likely to face lower variable costs of operation than other firms. The data on tender outcomes from 2003 to end 2014 confirms the relevance of garage proximity: 47% of contracts are won by the company whose garage

\(^{34}\)See Transport for London (2017); there is also a quality monitoring element, with payments to or from the operator.

\(^{35}\)Cantillon and Pesendorfer (2007) examine cost synergies and within-tranche bundled bids (called combination bids) in an earlier period (December 1995 to May 2001), using a structural econometric model based on a sample of 118 tranche auctions. Cross-tranche bids are also frequently observed: in the sample used for Table 2 below, we find that 30% of bundled bids were cross-tranche.

\(^{36}\)We excluded routes that are predominantly school transport routes, as well as the few stand-alone night services (most of which are otherwise operated by the day operator). The main data-set comprises all the information available for 884 route outcomes between 2003 and end 2014 from the TfL website, including winning bid and bidder identity, equivalent price per mile, number of bids, constitution of winning package where applicable and package price (in addition to individual route bid). Separately, we examined all the numbered routes’ start and finish points and their relation to existing garages, as discussed in relation to Table 1 (and Table 3 in Appendix G).
is nearest to the route in question and, as can be seen in table 1, the occupant of the route (the “winner”) is less than 15 minutes driving distance away from the route over three quarters of the time.

However, Table 1 also indicates that there is significant potential competition: three firms’ garages are within 15 minutes almost half the time, and within 20 minutes three quarters of the time. To investigate this further, we computed rough estimates of the time (and so cost) penalty applying to a rival. We find that cost differences are relatively slight among the nearest firms. The median time penalty for the second nearest firm is a mere 3 minutes per trip, whereas for the third nearest it is 7 minutes per trip. Using the median bid-per-mile and assuming an average of four trips per day, this translates into a cost handicap for the second nearest operator of approximately 1.7% of the median bid per route, which appears too small to be material. Therefore, while it clearly matters, garage proximity alone may not suffice to determine the winner.

<table>
<thead>
<tr>
<th>Winner</th>
<th>Lowest</th>
<th>Second lowest</th>
<th>Third lowest</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;15 minutes</td>
<td>77.6</td>
<td>99.3</td>
<td>83.3</td>
</tr>
<tr>
<td>&lt;20 minutes</td>
<td>89</td>
<td>99.8</td>
<td>96.8</td>
</tr>
</tbody>
</table>

Table 1: Distance from garage

Note: Percent of routes for which firms’ garages are within a given time distance (less than 15 or 20 minutes) from the route.
Source: All routes in sample.

Another important source of incumbency advantages stems from information decay. Indeed, a key factor in determining the bidder’s quote is their decision on the number of buses deployed at peak times to meet the punctuality and frequency targets set by TfL. Failure to estimate this correctly is in effect penalised whichever direction the error takes. If the winning bidder commits too few buses to the service, it will pay a penalty to TfL for failure to meet the targets. If it commits too many, its costs are higher than they need be. Compared with other operators, the current incumbent has the advantage of knowing far more intimately where on the route congestion black-spots occur, enabling it to bid accurately. Amongst other operators, the best-placed will be those currently running similar routes, in line with our model.  

37We assumed that each bus needs to leave and return to garage/depot twice per day (i.e. four trips). Taking a mapping program to plot the distance between either end of a route to all garages in London (based on their detailed postcodes), and calculating the driving distance for each company’s closest bus garage, we then obtained the time penalty, which we converted into a cost penalty by using estimates of miles per hour and the price per mile bid. Garage capacity is a potential issue, occasionally given as a reason by TfL as to why a contract is not awarded to the lowest bidder.
38Calculation available from the authors on request.
39Uncertainty has been reduced since 2014 as a result of improved location and congestion technology.
To test whether garage proximity leads to a given company operating a route, or whether merely being an incumbent per se is more important, we utilise the 402 instances in which a route has been re-tendered within our period of observation. If the incumbent wins again and is the nearest firm, this does not distinguish the hypotheses. If instead the route moves to a nearer firm, this favours the garage explanation. On the other hand, if the incumbent remains but it is not the nearest, this favours the incumbency explanation. In the sample, the latter strongly dominates the former, 143 cases to 49; see Appendix G.

The patterns observed in the London bus market are also in line with the predictions of our analysis. In particular, for the 402 routes for which we have repeated observations, the incumbent wins almost 60% of the time, a challenger 40% and on only three occasions is the winner an entrant.

Another prediction is that, in the absence of monopolisation – which, as discussed above, appears to be the relevant situation in the London bus market –, prices are higher under synchronous contracts (see Proposition 3). Testing this prediction is a challenging task, given the lack of information on actual costs and margins. However, an implication of this prediction is that firms will prefer to win contracts that are offered for tender at approximately the same time, possibly across close tranches. Interestingly, TfL does enable firms to alter the timing of tenders, by allowing them to extend their initial five-year contract to seven years, which they commonly do so, and to combine bids across tranches, which is aided by the frequency of tranche announcements. We can therefore test the prediction by studying whether firms use this possibility to make tenders more synchronous.

As TfL provides almost no details concerning unsuccessful bids, it is difficult to determine the number of neighbouring routes on which a given firm bids simultaneously. Fortunately, however, TfL invites bundled bids, which we can use as indicative of simultaneous bids. To investigate this, we collected data on the annual tendering programmes planned by TfL for 2014/15 to 2019/20.\(^\text{40}\) We use the first two years (2014/16) only to determine which routes were carried over to the last four years (2016/20). We find that, to a significant extent, successful bidders have used the above-mentioned possibilities to create their own bundles and render staggered routes subsequently synchronous. Table 2 summarises our examination. The top row enumerates all the extended routes that appear in the last four plans. The middle row shows that around 2/3 of these routes were won as part of a bundled bid. The final row shows that about a third of these bundled bids combined extended and non-extended routes, which shows the clearest evidence of a tendency by winning firms to synchronise routes, providing a powerful confirmation of

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\(^{40}\)These were all the plans available on the TfL website for which the outcomes were known.
Proposition 3. A more detailed Table and year-by-year examination of these points are provided in Appendix G.

<table>
<thead>
<tr>
<th>Annual plans</th>
<th>16/17 to 19/20 (4 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extended routes</td>
<td>223</td>
</tr>
<tr>
<td>Of which: winning bid is bundled</td>
<td>145 (65%)</td>
</tr>
<tr>
<td>Of which: combined with new routes</td>
<td>46 (32%)</td>
</tr>
</tbody>
</table>

Table 2: Firms aim at synchronicity
Note: 249 routes in the 2014/15 to 2017/18 plans remained unlet. 20 could not be traced to a later plan and 6 were incomplete at the time of collecting data, leaving 223 as our sample.
Source: TfL plan schedules and outcomes.

7.2 Policy implications

We now briefly discuss the policy implications of our analysis. The first thing to emphasise is that, whilst in our model entry costs are given, in practice it need not be so. Sector, market and technology characteristics may constitute entry barriers, but policies can be put in place to reduce their impact. For example, governments can retain ownership of essential assets, or ensure dispersed private ownership, as was done for the garages in the London bus tendering. They can also collect information and impose transparency requirements on current operators, so as to ensure that potential competitors have access to relevant information on market conditions.\(^{41}\)

Taking as given the entry costs, our findings highlight that breaking up the incumbent brings two types of benefits: it reduces the initial prices and fosters sustainable competition. However, the windfall gain in the first period generated in case of monopolisation may create an illusion about the gain from liberalisation, as well as distribution concerns across generations of users, when it is not invested or partly saved to be redistributed over time. Furthermore, in practice, breaking up the incumbent can involve substantial political and administrative costs. Breaking up the incumbent may then be more desirable when it generates long-term benefits, that is when the incumbency advantages and/or the discount factor are small. When instead breaking up the incumbent is infeasible, it follows from the above analysis that monopoly remains the only possible equilibrium state; synchronous tenders then always lead to lower prices.

\(^{41}\)Ensuring that potential entrants have access to demand information is particularly relevant in the case of net cost contracts, as then the operators bears demand risk. Interestingly, the UK Competition Commission (2011) inquiry into local bus services outside London discovered that competition in tendering was more easily achieved using gross cost rather than net cost contracts.
Following a breakup, our analysis shows that when incumbency advantages are small (formally, when $\sigma \leq \sigma^{Stag}$), competition can be sustained and staggered contracts are preferable. When they are large (formally, when $\sigma > \sigma^{Sync}$), monopolisation is expected and synchronous contracts are preferable. For intermediate values (formally, when $\sigma^{Stag}(\delta) < \sigma \leq \sigma^{Sync}(\delta)$), the choice of tendering regime depends on an assessment of market conditions vis-à-vis the threshold $\hat{\sigma}(\delta)$. In practice, governments may lack sufficiently detailed information to determine where exactly they are in this range. This may tilt the balance in favour of synchronous tenders, which increase the scope for sustainable competition and, in any event, deliver lower long-term prices (specifically, duopoly prices rather than the monopoly prices that arise under staggered tenders in this range).

The same considerations apply to contract duration: whilst the optimal choice lies on the boundary (i.e., set $\delta$ so as to equate $\sigma^{Stag}(\delta)$ or $\sigma^{Sync}(\delta)$ to $\sigma$), in practice identifying these thresholds may be complicated. Moreover, the cost of errors is highly asymmetric, as total bills vary abruptly across the boundary; it might therefore be advisable to play it safe by increasing the length of the contract, so as to ensure sustainable competition.

Our analysis also suggests that it may be desirable to use a bid preference scheme, like those favouring the participation of small and medium size firms (SMEs), to offset the incumbency advantages. This could be implemented either through bid credits to the entrants (or the challengers) or by handicapping the incumbent’s bid. In Online Appendix E we show that, in both tendering regimes, introducing a large enough bias in favour of the entrants expands the scope for sustainable competition and reduces equilibrium prices. Furthermore, as the bias tends to offset perfectly the incumbent’s advantage, the range in which monopolisation arises tends to vanish, and all equilibrium prices tend to cost.\footnote{Of course, the capacity of estimating incumbency advantages may be constrained in practice if only limited information on market and supply conditions is available.}

By contrast, a set aside scheme, like those used in the U.S. to favour minority groups or SMEs, would not be useful to enhance competition, and it is therefore suboptimal in our context.\footnote{If the historical operator cannot be broken up, a set aside scheme may however constitute a useful instrument to ensure that the equilibrium moves to state II when competition is sustainable.}

The London bus tendering case strongly suggests that these recommendations are not limited to situations where there are two markets and two active firms. Furthermore, given that there is continuing competition, our analysis suggests that staggered tendering is superior to synchronous tendering. What this implies is for TfL to schedule tenders for neighbouring large routes in a staggered manner, that is approximately 2.5 years apart. In addition, as shown by previous research (Grimm \textit{et al.} 2006), the choice of how many routes to put into each tender can also contribute to fostering effective competition.
8 Concluding remarks

We have studied the design of competition for the market in a setting where incumbents (and, to a lesser extent, neighbouring incumbents) benefit from a cost or information advantage. We have focused on two main instruments: market structure and the timing of tenders, and shown that these are inherently interlinked. Our findings suggest that breaking up the historical operator helps to ensure lower prices and to sustain competition. When the incumbency advantages are small and/or the discount factor is low, a competitive market structure can be maintained over time. Staggered tendering is then preferable, as it maximises the competitive pressure that incumbents exert on one another. Instead, when either incumbency advantages are strong and/or the discount factor is high, monopolisation is inevitable. Synchronous tendering is then preferable, as it strengthens the pressure that entrants exert on the monopolist. We have also shown that efficient entry always occurs in the case of sustainable competition, whilst it can be blocked in the case of monopolisation. A careful choice of contract duration also helps to sustain competition and reduce prices.

In our stylised model, firms face no uncertainty about cost conditions or reliability. Recent episodes in Europe, with large service operators going into liquidation whilst holding hundreds of public sector contracts, suggest that having existing operators ready to replace a failing contractor could yield significant benefits to consumers, making it even more desirable to maintain competition over time. Future research should consider introducing cost and reliability uncertainty to explore how to best manage this type of termination risk.


Appendix

A  Information decay

Suppose that there is uncertainty about how to service a market efficiently. Specifically:

- in each period $t$:
  
  - each market is characterised by a state of nature $\theta_t \in \Theta \equiv \{0, 1\}$, which is uniformly distributed over $\Theta$;
  
  - the operator servicing the market must choose an action $a_t \in \Theta$ and the cost of providing the service is then:
    
    $$c_t = \begin{cases} 
    \hat{c} & \text{if } a_t = \theta_t, \\
    \hat{c} + \hat{S} & \text{if } a_t \neq \theta_t; 
    \end{cases}$$

  - the operator servicing the market obtains a signal $\sigma_t \in \Theta \equiv \{0, 1\}$ and the operator servicing the neighbouring market obtains a signal $\hat{\sigma}_t \in \Theta \equiv \{0, 1\}$, whereas the other operators observe no signal;

- the signals convey useful information about the state in the next period (and only that one), namely:
  
  - for the next period,
    
    $$\Pr [\theta_{t+1} = \sigma_t] = 1 \text{ and } \Pr [\theta_{t+1} = \hat{\sigma}_t] = \rho > \frac{1}{2};$$

  - for the following periods, that is, for $\tau > 1$:
    
    $$\Pr [\theta_{t+\tau} = \sigma_t] = \Pr [\theta_{t+\tau} = \hat{\sigma}_t] = \frac{1}{2}.$$ 

We now characterise the cost, for the various types of firms, of a contract for servicing the market in periods $t + 1$ and $t + 2$.

The incumbent $D$ already operating the market in the previous period, $t$, observes $\sigma_t$ in that period and will also observe $\sigma_{t+1}$ if it wins the contract. Hence, it will optimally choose $a_{t+1} = \sigma_t$ and $a_{t+2} = \sigma_{t+1}$; its total cost is thus given by:

$$c_D = \hat{c} + \delta \times \hat{c} = (1 + \delta) \hat{c}.$$
Consider now the case of an entrant. Having observed no signal in period $t$, the entrant expects the state of nature to be uniformly distributed over $\Theta$ in the first period of the contract, and then observes $\sigma_{t+1}$. Therefore, regardless of the action it chooses in period $t+1$, it will pay the extra cost $\hat{S}$ with probability $1/2$ in that period, and then choose $a_{t+2} = \sigma_{t+1}$ in period $t+2$, and its total expected cost is given by:

$$c_E = \hat{c} + \hat{S} + \delta \times \hat{c} = (1 + \delta) \hat{c} + \hat{S}.$$  

Consider finally the case of a challenger $C$ operating the neighbouring market in period $t$. Having observed the signal $\hat{\sigma}_t$ in that period, if it wins the contract it will optimally choose $a_{t+1} = \hat{\sigma}_t$ and, having observed $\sigma_{t+1}$, will then choose $a_{t+2} = \sigma_{t+1}$. Hence, its its total expected cost is given by:

$$c_C = \hat{c} + (1 - \rho) \hat{S} + \delta \times \hat{c} = (1 + \delta) \hat{c} + (1 - \rho) \hat{S}.$$  

These expected costs correspond to $c$, $c + S$ and $c + s$ for

$$\hat{c} \equiv \frac{c}{1 + \delta}, \: \hat{S} \equiv 2S \: \text{and} \: \rho \equiv 1 - \frac{s}{2S}.$$  

B Proof of Proposition 1

We study here the equilibria of the game in which, in every period, one market is up for tender. We first note that equilibrium continuation values cannot be negative, as any firm can secure a non-negative payoff by offering above-cost prices. Furthermore, entrants necessarily obtain $V_E = 0$, as $V_E > 0$ would require winning a market with a positive margin, in which case any losing entrant could profitably undercut any winning one.

B.1 Coalition-proof Nash equilibria

We first characterise the coalition-proof Nash equilibria for given continuation values satisfying $V_E = 0$ and $V_i \geq 0$ for $i = M, D, C$. We start by characterising firms’ best offers, before studying the equilibrium outcomes in each state.

44The associated expected extra cost is given $(1 - \rho) \hat{S}$ if $a_{t+1} = \hat{\sigma}_t$ and $\rho \hat{S}$ if $a_{t+1} \neq \hat{\sigma}_t$; as $\rho > 1/2$, it is optimal to choose the former option.
**Best offers**

In both states, a potential entrant obtains \( p - S + \delta V_C \) if it wins at price \( p \), and 0 if it loses; hence, potential entrants are willing to lower their prices down to

\[
p_E \equiv S - \delta V_C. \tag{8}\]

In state \( I \), \( M \) obtains instead \( p + \delta V_M \) if it wins at price \( p \) and \( \delta V_D \) if it loses; hence, it is willing to lower its price down to

\[
p_M \equiv -\delta (V_M - V_D). \tag{9}\]

In state \( II \), \( D \) obtains \( p + \delta V_C \) if it wins at price \( p \), and 0 if it loses; hence, it is willing to lower its price down to

\[
p_D \equiv -\delta V_C. \tag{10}\]

\( C \) obtains instead \( p - s + \delta V_M \) if it wins at price \( p \), and \( \delta V_D \) if it loses; hence, it is willing to lower its price down to

\[
p_C \equiv s - \delta (V_M - V_D). \tag{11}\]

It is useful to note that

\[
p_D < p_E; \tag{12}\]

and

\[
p_M < p_C. \tag{13}\]

**State I**

We first show that \( M \) is willing to outbid the entrants:

**Lemma 1** (\( M \) makes a better offer). *The best offers satisfy:*

\[
p_E > p_M. \tag{12}
\]

*Proof.* Suppose that \( p_E \leq p_M \). Using (12) and (13), this yields:

\[
p_D < p_E \leq p_M < p_C.
\]

Therefore, in state \( II \), \( D \) wins (as its best offer is the lowest, and thus it can profitably undercut any rival) at a price not exceeding \( p_E \) (otherwise, any entrant could profitably undercut \( D \)); \( C \) thus obtains \( V_C = \delta V_D \), where

\[
V_D \leq p_E + \delta V_C = S.
\]
Furthermore, in state $I$, either $M$ loses for sure (if $p_E < p_M$) or competition drives prices down to $p_M$ (if $p_E = p_M$), in which case $M$ is indifferent between winning or not; in both cases, we have: $V_M = \delta V_D = V_C$. Using (8) and (9), $p_E \leq p_M$ then amounts to $S \leq \delta V_D \leq \delta S$, a contradiction.

Lemma 1 implies that $M$ prevails in state $I$, which leads to:

**Lemma 2** (state $I$ for staggered tenders). In state $I$, there exists a unique coalition-proof Nash equilibrium outcome, in which $M$ wins at price $p_E$ and obtains $\bar{V}_M = S - \delta V_C + \delta V_M$.

**Proof.** As already noted, in equilibrium the entrants obtain $V_E = 0$. Furthermore, $M$ cannot charge more than $p_E$, otherwise any entrant could profitably undercut it. Conversely, all firms offering $p_E$ and the auctioneer assigning the market to $M$ constitutes a Nash equilibrium, as no firm could profitably increase its price, which would lead to exit, and no firm can profitably decrease its price either: an entrant would make a loss, and $M$ would make a lower profit. The same holds as long as at least one entrant offers $p_E$, and no entrant undercuts that price.

It follows that there is a unique coalition-proof Nash equilibrium outcome, in which $M$ wins at price $p_E$. Using (8), $M$’s payoff is then equal to:

$$\bar{V}_M = p_E + \delta V_M = S - \delta V_C + \delta V_M.$$  

Lemma 2 provides a partial characterisation of $M$’s equilibrium payoff which confirms that, in any coalition-proof equilibrium, the incumbents obtain (weakly) lower payoffs in state $II$:

**Corollary 3** (the value of monopolisation for staggered tenders). The equilibrium continuation values are such that:

$$V_M = \frac{S - \delta V_C}{1 - \delta},$$

and

$$\Delta^{Stag} \equiv \delta (V_M - V_D - V_C) \geq 0.$$  

**Proof.** In any coalition-proof equilibrium, we must have $V_M = \bar{V}_M = S - \delta V_C + \delta V_M$, which yields (14). Furthermore, no equilibrium price can exceed $p_E$, otherwise any entrant could profitably undercut it. Hence, in state $II$, we have:

$$V_D + V_C \leq \sum_{t=0}^{+\infty} \delta^t p_E = V_M,$$
where the equality follows from (8) and (14).

\[ \square \]

**State II**

We first show that the incumbents are willing to outbid the entrants:

**Lemma 3** *(D and C make better offers).* The best offers satisfy:

\[ p_E > \max\{p_D, p_C\}. \]

**Proof.** Corollary 3 and \( S > s \) together imply \( p_E - p_C = S - s + \Delta_{stag} > 0 \). The conclusion follows from (12).

**Lemma 3** implies that the incumbents prevail in state II, which leads to:

**Lemma 4** *(state II for staggered tenders).* In state II, the coalition-proof Nash equilibrium outcomes are as follows:

- If \( \Delta_{stag} < s \), there exists a unique coalition-proof Nash equilibrium outcome, in which \( D \) wins at price \( p_C = s - \delta(V_M - V_D) \); \( D \) then obtains \( \tilde{V}_D = s - \Delta_{stag} \) and \( C \) obtains \( \tilde{V}_C = \delta V_D \).

- If instead \( \Delta_{stag} > s \), there exists a unique coalition-proof Nash equilibrium outcome, in which \( C \) wins at price \( p_D = -\delta V_C \); \( D \) then obtains \( \tilde{V}_D = 0 \) and \( C \) obtains \( \tilde{V}_C = \delta (V_M - V_C) - s \).

- Finally, in the boundary case where \( \Delta_{stag} = s \), there are infinitely many coalition-proof Nash equilibrium outcomes, in which either incumbent wins at price \( p_C = p_D \), and both obtain \( \tilde{V}_C = \tilde{V}_D = 0 \); these outcomes only differ in the probabilities that either incumbent wins, which can take any arbitrary values.

**Proof.** Lemma 3 implies that competition takes places between \( D \) and \( C \). If \( \Delta_{stag} < s \), then \( p_D < p_C \) and the unique coalition-proof equilibrium outcome is such that \( D \) wins by matching \( p_C \); the associated payoffs are:

\[ \tilde{V}_C = \delta V_D \text{ and } \tilde{V}_D = p_C + \delta V_C = s - \Delta_{stag}. \]

Conversely, if \( \Delta_{stag} > s \), then \( p_D > p_C \) and \( C \) thus wins at a price not exceeding \( p_D \); the unique coalition-proof equilibrium outcome is such that \( C \) wins by matching \( p_D \); the associated payoffs are:

\[ \tilde{V}_D = 0 \text{ and } \tilde{V}_C = p_D - s + \delta V_M = \Delta_{stag} - s. \]
Finally, if $\Delta_{\text{Stag}} = s$, then $p_D = p_C$ and the Nash equilibria are such that either firm wins at that price, with arbitrary probability; as firms always obtain the same zero payoff, all the equilibria are coalition-proof. □

B.2 Equilibrium characterisation

In equilibrium, in state II the payoffs $\tilde{V}_D$ and $\tilde{V}_C$ must coincide with the continuation values $V_D$ and $V_C$. Furthermore, if $D$ wins, then the equilibrium path remains forever in the initial state (dual-state equilibrium). If instead $C$ wins in state II, then the equilibrium switches to state I forever (single-state equilibrium). We consider in turn these two types of equilibrium.

Dual-state equilibrium

From Lemma 4, a dual-state equilibrium exists if and only if $\Delta_{\text{Stag}} \leq s$. We then have:

$$V_D = \tilde{V}_D = s - \delta (V_M - V_D - V_C) \quad \text{and} \quad V_C = \tilde{V}_C = \delta V_D.$$  

Together with (14), this yields:

$$V_M = \frac{(1 - \delta - \delta^2) S - \delta^2 s}{1 - 2\delta}, \quad V_D = \frac{(1 - \delta) s - \delta S}{1 - 2\delta}, \quad V_C = \frac{\delta (1 - \delta) s - \delta S}{1 - 2\delta},$$  

and

$$\Delta_{\text{Stag}} = \frac{\delta (S - s)}{1 - 2\delta}. \quad (15)$$

Corollary 3 then implies $\delta < 1/2$. Hence, $\Delta_{\text{Stag}} \leq s$ if and only if $\delta (S - s) \leq (1 - 2\delta)s$, which amounts to:

$$\sigma \equiv \frac{S}{s} > \sigma_{\text{Stag}}(\delta) \equiv \frac{1 - \delta}{\delta}. \quad (16)$$

Conversely, whenever $\sigma \leq \sigma_{\text{Stag}}(\delta)$ (which, together with $\sigma > 1$, implies $\delta < 1/2$), the above continuation values are non-negative – furthermore, $V_M$ is positive and the other two continuation values are also positive if $\sigma < \sigma_{\text{Stag}}(\delta)$. Hence, there exists a dual-state equilibrium, in which the per market prices in the two states are respectively given by:

$$p^{\text{Stag}}_I = p_E = p^{\text{Stag}}_I (DSE) \equiv \frac{1 - \delta}{1 - 2\delta} \left[ (1 - \delta - \delta^2) S - \delta^2 s \right], \quad (17)$$

$$p^{\text{Stag}}_{II} = p_C = p^{\text{Stag}}_{II} (DSE) \equiv \frac{1 - \delta^2}{1 - 2\delta} \left[ (1 - \delta) s - \delta S \right]. \quad (18)$$
Single-state equilibrium

From Lemma 4, a single-state equilibrium exists if and only if $\Delta^{Stag} \geq s$. We then have:

$$V_D = \bar{V}_D = 0 \text{ and } V_C = \bar{V}_C = \delta (V_M - V_C) - s.$$

Together with (14), this yields:

$$V_M = (1 + \delta) S + \delta s, V_D = 0, V_C = \delta S - (1 - \delta) s,$$

and

$$\Delta^{Stag} = \delta (S + s).$$

Hence, $\Delta^{Stag} \geq s$ if and only if $\sigma \geq \sigma^{Stag}(\delta)$, which in turn ensures that the continuation values are non-negative. Conversely, if this condition holds, then there exists a single-state equilibrium, in which from period 1 onward the equilibrium path remains forever in state $I$. The per market prices in the two states are respectively given by:

$$p^{Stag}_{I} = p_E = p^{Stag}_{I} (SSE) \equiv (1 - \delta) [(1 + \delta) S + \delta s],$$

$$p^{Stag}_{II} = p_D = p^{Stag}_{II} (SSE) \equiv (1 - \delta) \delta s - \delta^2 S.$$

Recap

Summing-up, we have:

- If $\sigma < \sigma^{Stag}(\delta)$, there exists a unique coalition-proof Markov perfect equilibrium outcome, which is dual-state: the equilibrium path remains in the initial state forever; the prices in the two states, $p^{Stag}_{I}$ and $p^{Stag}_{II}$, are respectively given by (17) and (18).

- If instead $\sigma > \sigma^{Stag}(\delta)$, there exists a unique coalition-proof Markov perfect equilibrium outcome, which is single-state: regardless of the initial state, from period 1 onward the equilibrium path stays in the monopoly state, and the price, $p^{Stag}_{I}$, is given by (21).

- Finally, in the limit case $\sigma = \sigma^{Stag}(\delta)$, there exist infinitely many equilibria, which all yield $V_M = S / (1 - \delta)$ and $V_D = V_C = 0$, and are thus coalition-proof; these equilibria only differ in the probability that the equilibrium switches from state $II$ to state $I$ (any probability can be sustained in equilibrium, and the equilibrium path remains forever in state $I$ when it reaches it).
It can moreover be checked that, in both states, the equilibrium prices vary continuously with $\delta$:

$$\lim_{\delta \uparrow \delta^{stag} (\sigma)} p_{I}^{stag} (DSE) = \lim_{\delta \downarrow \delta^{stag} (\sigma)} p_{I}^{stag} (SSE) = S,$$

$$\lim_{\delta \uparrow \delta^{stag} (\sigma)} p_{II}^{stag} (DSE) = \lim_{\delta \downarrow \delta^{stag} (\sigma)} p_{II}^{stag} (SSE) = 0.$$

C Proof of Proposition 2

We now study the equilibria of the game in which, in every even period, both markets are up for tender. As before, in equilibrium (i) every firm obtains a non-negative continuation value, which it can secure by offering above-cost prices; and (ii) the continuation value of entrants is $V_E = 0$, as $V_E > 0$ would require winning a market with a positive margin, in which case any losing entrant could profitably undercut the winning one.

We denote again by $V_M$ the continuation value of the incumbent in state $I$; in state $II$, in which two incumbent firms compete with each other as well as with potential entrants, we now denote the incumbent servicing market $i = A, B$ by $D_i$ and its continuation value by $V_i$.

C.1 Coalition-proof Nash equilibria

We first characterise the coalition-proof Nash equilibria for given continuation values satisfying $V_E = 0$ and $V_i \geq 0$ for $i = M, A, B$. We start by characterising potential entrants’ best offers, which are the same in both states, before studying the equilibrium outcomes in each state.

Entrants’ best offers

A potential entrant obtains $P - 2S + \delta^2 V_M$ if it wins both markets at total price $P$, $p_i - S + \delta^2 V_i$ if it wins market $i$ at price $p_i$, and 0 if it loses both markets. Hence, potential entrants are willing to service market $i$ for a stand-alone price

$$p_{Ei} \equiv S - \delta^2 V_i,$$

or both markets for a bundled price

$$P_E \equiv 2S - \delta^2 V_M.$$
State I

We now show that, in state \( I \), there exists a unique coalition-proof Nash equilibrium outcome:

**Lemma 5** (state \( I \) for synchronous tenders). *In state \( I \), there exists a unique coalition-proof Nash equilibrium outcome, in which \( M \) wins both markets at total price \( P_E \) and obtains \( \hat{V}_M = 2S \).*

**Proof.** As already noted, in equilibrium the entrants obtain \( V_E = 0 \). Furthermore, \( M \) cannot obtain more than \( \hat{V}_M = 2S \), otherwise any entrant could profitably undercut it. To establish existence, consider a candidate equilibrium in which all firms offer the bundle price \( P_E \) and, for each market \( i = A, B \), the stand-alone price 

\[
\hat{p}_i = \max \{ p_{Ei}, P_E - p_{Ej} \}.
\]

As \( \hat{p}_A + \hat{p}_B \geq P_E \), the auctioneer is willing to assign both markets to \( M \), which gives \( M \) a payoff equal to \( P_E + \delta^2 V_M = 2S = \hat{V}_M \). No firm can then benefit from increasing any of its prices, as this can only induce exit, or from decreasing its bundle price: the entrants would make a loss, and \( M \) would lower its profit. Furthermore, in order to win market \( i \) on a stand-alone basis, a firm must charge a price \( p_i \) such that \( p_i + \hat{p}_j \leq P_E \) (so as to undercut the bundle price \( P_E \) by “teaming up” with the offered stand-alone price \( \hat{p}_j \) for market \( j \)), that is:

\[
p_i \leq P_E - \hat{p}_j \leq P_E - (P_E - p_{Ei}) = p_{Ei},
\]

where the second inequality follows from the definition of \( \hat{p}_j \). It follows that entrants cannot profitably do so, and that \( M \) cannot profitably deviate either, as this would yield at most \( p_{Ei} + \delta^2 V_i = S < 2S = \hat{V}_M \). \( \square \)

It directly follows from Lemma 5 that, in any coalition-proof equilibrium, the incumbents obtain again (weakly) lower payoffs in state \( II \):

**Corollary 4** (the value of monopolisation for synchronous tenders). *The equilibrium continuation values are such that:

\[
V_M = 2S,
\]

and

\[
\Delta_{Sync} = \delta^2 (V_M - V_A - V_B) \geq 0.
\]

**Proof.** In any coalition-proof equilibrium, we must have \( V_M = \hat{V}_M = 2S \). Furthermore, the total equilibrium price for the two markets can never exceed \( P_E \), otherwise any entrant

\footnote{By construction, \( \hat{p}_A \geq p_{EA} \) and \( \hat{p}_B \geq P_E - p_{EA} \).}
could profitably undercut it. Hence, in state II, we have:

\[ V_A + V_B \leq \sum_{t=0}^{+\infty} \delta^t P_E = V_M, \]

where the equality follows from (24) and \( V_M = 2S \).

**State II**

Building on this, the following lemma shows that, generically, there exists an essentially unique\(^{46}\) coalition-proof Nash equilibrium outcome in state II:

**Lemma 6** (state II for synchronous tenders). In state II, the coalition-proof Nash equilibrium outcomes are as follows:

- If \( \Delta_{sync} < s \), there exists a unique coalition-proof Nash equilibrium outcome, in which each \( D_i \) wins market \( i \) at price \( \tilde{p}_i \equiv s - \delta^2 (V_M - V_j) \) (for \( i \neq j \in \{A,B\} \)) and obtains \( \tilde{V}_i = \tilde{V}_D \equiv s - \Delta_{sync} > 0 \).

- If instead \( \Delta_{sync} > s \), there are infinitely many coalition-proof Nash equilibrium outcomes, in which both incumbents offer the bundle price \( P_D = s - \delta^2 V_M \), and obtain \( \tilde{V}_A = \tilde{V}_B = 0 \); these outcomes only differ in the probabilities that either incumbent wins, which can take any arbitrary values.

- Finally, in the boundary case where \( \Delta_{sync} = s \), there are again infinitely many coalition-proof Nash equilibrium outcomes, in which both incumbents offer a total price equal to \( P_D = s - \delta^2 V_M \) and obtain \( \tilde{V}_A = \tilde{V}_B = 0 \); these outcomes only differ in the probabilities that either of the incumbent wins both markets and/or that each incumbent wins its market, which can take any arbitrary values.

**Proof.** From Corollary 4, \( \Delta_{sync} \geq 0 \). Furthermore, in equilibrium, potential entrants cannot win any market. Indeed, if an entrant \( E \) were to win one or both markets, then at least one incumbent \( D_i \) would exit and obtain zero payoff. But then, \( D_i \) could slightly undercut \( E \)'s winning bids (and underbid the losing bids, so as to obtain only the markets won by \( E \) in the candidate equilibrium) and, facing lower fixed costs (namely, \( s \) or \( 0 \) instead of \( S \)), obtain in this way almost \( V_E + S - s = S - s > 0 \), a contradiction. Likewise, the incumbents cannot win each other’s markets; otherwise, the incumbent with the lowest payoff (and both incumbents, in case of a tie) could profitably deviate by targeting its

\(^{46}\)As indicated in footnote 24, “essentially unique” refers here to the fact that, when \( \Delta_{sync} \geq s \), either incumbent may win both markets.
own market, so as to enjoy the (weakly) greater payoff and save the entry cost \( s \). It follows that, in equilibrium, either one incumbent wins both markets, or each incumbent wins its own market. We consider both types of equilibrium in sequence, and denote \( D_i \)'s equilibrium payoff by \( \tilde{V}_i \).

We first show that there always exists an equilibrium in which either incumbent wins both markets. Any such equilibrium necessarily yields \( \tilde{V}_A = \tilde{V}_B = 0 \): the losing incumbent exits and thus obtains zero payoff; and the winning incumbent cannot obtain a positive payoff, otherwise the losing one would profitably undercut it. Conversely, suppose that entrants offer their best prices, \( \{P_E, p_{EA}, p_{EB}\} \), whereas the two incumbents offer a bundle price equal to \( P_D \equiv s - \delta^2 V_M \), together with the stand-alone prices \( p_{EA} \) and \( p_{EB} \). The conditions \( \Delta^{Sync} \geq 0 \) and \( S > s \) imply:

\[
P_D < P_E \leq p_{EA} + p_{EB}.
\]

Hence, the auctioneer is willing to assign both markets to either incumbent with any arbitrary probability, and the entrants cannot profitably undercut them. Furthermore, each \( D_i \) obtains \( \tilde{V}_i = 0 \) and thus cannot profitably deviate by exiting. Therefore, to establish existence it suffices to check that no incumbent \( D_i \) can benefit from targeting its own market (which ensures that targeting the rival’s market is not profitable either). Indeed, \( D_i \)'s best price for market \( i \), given by

\[
p_{Di} = -\delta^2 V_i,
\]

does not allow \( D_i \) to undercut \( P_D \): combined with the equilibrium price for market \( j \), \( p_{Ej} \), it yields a total price equal to:

\[
p_{Di} + p_{Ej} = -\delta^2 V_i + S - \delta^2 V_j = P_D + S - s + \Delta^{Sync} > P_D,
\]

where the inequality follows from \( S > s \) and \( \Delta^{Sync} \geq 0 \).

We now consider a candidate equilibrium in which each incumbent keeps its market. In any such equilibrium, \( D_j \) would be willing to service market \( i \) as well at any price exceeding, for \( i \neq j \in \{A, B\} \):

\[
\tilde{p}_i \equiv s - \delta^2 (V_M - V_j).
\]

Hence, such an equilibrium can exist only if \( \tilde{p}_i \geq p_{Di} \), given by (25), which amounts to \( s \geq \Delta^{Sync} \). Conversely, if \( s \geq \Delta^{Sync} \), then there exists an equilibrium in which each \( D_i \) wins market \( i \) by matching its rival’s best price, \( \tilde{p}_i \), and thus obtains \( \tilde{p}_i + \delta^2 V_i = \tilde{V}_D \). To see this, suppose that both incumbents offer the stand-alone prices \( \tilde{p}_A \) and \( \tilde{p}_B \), together
with a bundle price \( \tilde{P} \equiv \tilde{p}_A + \tilde{p}_B \). The conditions \( \Delta^{Sync} \geq 0 \) and \( S > s > 0 \) imply:

\[
p_{Ei} - \tilde{p}_i = S - s + \Delta^{Sync} > 0 \quad \text{and} \quad P_E - \tilde{P} = 2(S - s) + \Delta^{Sync} > 0.
\]

Hence, the auctioneer is willing to allocate each market to its current operator, and the entrants cannot profitably undercut them. Furthermore, by definition of \( \tilde{p}_A \) and \( \tilde{p}_B \), no incumbent can benefit from winning both markets. Therefore, to establish existence, it suffices to note that, as the incumbents obtain \( \tilde{V}_D = s - \Delta^{Sync} \geq 0 \), no incumbent can benefit from raising its price, which would induce exit, or from targeting the rival’s market, which would yield at most \( \tilde{V}_D - s < \tilde{V}_D \).

To recap, there always exist Nash equilibria in which both incumbents obtain zero payoff and, with arbitrary probability, either of them wins both markets. If \( \Delta^{Sync} > s \), these are the only equilibria; as they are payoff equivalent, they are also coalition-proof. If instead \( \Delta^{Sync} < s \), these equilibria are Pareto-dominated by other ones, in which each incumbent keeps its market; among all equilibria, there is a unique Pareto-dominant outcome, in which both incumbents obtain \( \tilde{V}_D = s - \Delta^{Sync} > 0 \). Finally, in the boundary case where \( \Delta^{Sync} = s \), implying \( \tilde{p}_i = p_{Di} \) and \( p_{DA} + p_{DB} = P_D \), it is straightforward to check that the Nash equilibria are such that both incumbents obtain zero payoff, the total price is \( \tilde{p}_{DA} + \tilde{p}_{DB} = P_D \) and, with arbitrary probability, either incumbent wins both markets and/or each incumbent keeps its market. \( \square \)

## C.2 Equilibrium characterisation

In equilibrium, \( V_M = 2S \) (from Corollary 4) and, in state II, the Nash equilibrium payoffs must coincide with the continuation values \( V_A \) and \( V_B \). Hence, from Lemma 6 we have:

\[
V_A = V_B = V_D \equiv \max \left\{ s - \Delta^{Sync}, 0 \right\}.
\]  

(26)

Furthermore, once in state I the equilibrium path remains in that state forever; using Corollary 4, the per market price is then given by

\[
p_i^{Sync} \equiv \frac{P_E}{2} = (1 - \delta^2) S.
\]  

(27)

By contrast, in state II, the equilibrium path may either remain in that state, or switch to state I. We consider in turn these two types of equilibrium.
Dual-state equilibrium

From Lemma 6, a dual-state equilibrium exists if and only if $\Delta^{Sync} \leq s$. Using Corollary 4 and equation (26), we then have:

$$\Delta^{Sync} = \delta^2 (V_M - 2V_D) = 2\delta^2 (S - s + \Delta^{Sync}).$$

Together with $\delta > 0$, $S > s$ and $\Delta^{Sync} \geq 0$ (from Corollary 4), this implies $\delta^2 < 1/2$. Solving for $\Delta^{Sync}$ and using again (26) then yields:

$$\Delta^{Sync} = \frac{2\delta^2 (S - s)}{1 - 2\delta^2} (> 0) \text{ and } V_D = \frac{s - 2\delta^2 S}{1 - 2\delta^2}. \quad (28)$$

It follows that the working condition $\Delta^{Sync} \leq s$ holds if and only if:

$$\sigma > \sigma^{Sync}(\delta) \equiv 2\delta^2,$$

Conversely, whenever $\sigma < \sigma^{Sync}(\delta)$ (which implies $\delta^2 < 1/2$, $\Delta^{Sync} < s$ and $V_D > 0$), there exists a dual-state equilibrium, in which the equilibrium path remains forever in the initial state. Furthermore, in state II the per market price is given by:

$$p^{Sync}_{II} = p^{Sync}_{II}(DSE) \equiv (1 - \delta^2) V_D = \frac{1 - \delta^2}{1 - 2\delta^2} (s - 2\delta^2 S). \quad (29)$$

Single-state equilibrium

From Lemma 6, a single-state equilibrium exists if and only if $\Delta^{Sync} \geq s$. From (26), we then have $V_A = V_B = 0$, implying that the working condition $\Delta^{Sync} \geq s$ boils down to $\delta > \delta^{Sync}(\sigma)$. Conversely, if $\delta > \delta^{Sync}(\sigma)$, there exists a single-state equilibrium, in which from period 1 onward, the equilibrium path remains in state I and the per market price is then $p^{Sync}_{I} = (1 - \delta^2) S$. If the game starts in state II, then the price in the initial periods is:

$$p^{Sync}_{II} = p^{Sync}_{II}(SSE) \equiv \frac{P_D}{2} = \frac{s}{2} - \delta^2 S. \quad (30)$$

Recap

Summing-up, we have:

---

$^47$ As $\delta^2 < 1/2$, $\Delta^{Sync} \leq s$ amounts to:

$$2\delta^2 (S - s) \leq (1 - 2\delta^2) s \iff 2\delta^2 S \leq s.$$
• If \( \sigma < \sigma^{Sync}(\delta) \), there exists a unique coalition-proof Markov perfect equilibrium outcome, which is dual-state: the equilibrium path remains in the initial state forever; the prices in the two states, \( p^{Sync}_{I} \) and \( p^{Sync}_{II} \), are respectively given by (27) and (30).

• If instead \( \sigma > \sigma^{Sync}(\delta) \), there exist an essentially unique coalition-proof Markov perfect equilibrium outcome, which is single-state: regardless of the initial state, from period 1 onward the equilibrium path stays in the monopoly state, and the price, \( p^{Sync}_{I} \), is again given by (27).

• Finally, in the limit case \( \sigma = \sigma^{Sync}(\delta) \), there exist infinitely many equilibria, which all yield \( V_{M} = \delta S \) and \( V_{A} = V_{B} = 0 \), and are thus coalition-proof these equilibria only differ in the probability that the equilibrium transition from state II to state I (any probability can be sustained in equilibrium, and the equilibrium path remains forever in state I when it reaches it).

In state I, the equilibrium price is the same in both types of equilibrium: \( p^{Sync}_{I} = (1 - \delta^{2}) S \); it can moreover be checked that, in state II, the equilibrium price varies continuously with \( \delta \):

\[
\lim_{\delta \uparrow \delta^{Sync}(\sigma)} p^{Sync}_{II}(DSE) = \lim_{\delta \downarrow \delta^{Sync}(\sigma)} p^{Sync}_{II}(SSE) = 0.
\]

D Proof of Proposition 3

As already noted, \( p^{Sync}_{I} > p^{Sync}_{II} \) whenever a dual-state equilibrium exists under synchronous tenders. We now check that the monopoly price is higher under staggered tendering; noting that monopoly prices are continuous functions of the parameters across the entire range, we have:

• If \( \delta \geq \delta^{Stag}(\sigma) \), then:

\[
p^{Stag}_{I} - p^{Sync}_{I} = (1 - \delta) [(1 + \delta) S + \delta s] - (1 - \delta^{2}) S = (1 - \delta) \delta s > 0.
\]

• If instead \( \delta \leq \delta^{Stag}(\sigma) \), then:

\[
p^{Stag}_{I} - p^{Sync}_{I} = \frac{1 - \delta}{1 - 2\delta} \left[ (1 - \delta - \delta^{2}) S - \delta^{2} s \right] - (1 - \delta^{2}) S = (1 - \delta) \delta^{2} \frac{S - s}{1 - 2\delta} > 0.
\]

• By contrast, duopoly prices are higher under synchronous tendering whenever a
dual-state equilibrium exists under both tendering regimes:

\[ p_{II}^{\text{Sync}} - p_{II}^{\text{Stag}} = (1 - \delta^2) \frac{s - 2S\delta^2}{1 - 2\delta^2} - \frac{1 - \delta^2}{1 - 2\delta} [(1 - \delta)s - \delta S] \]

\[ = \frac{1 + (1 - 2\delta)^2 (1 - \delta^2) \delta (S - s)}{2 (1 - 2\delta) (1 - 2\delta^2)} > 0, \]

where the inequality follows from the fact that, under staggered tenders, a dual-state equilibrium exists only when \((\delta^2 < \delta \leq \delta^{\text{Stag}}(\sigma) < 1/2)\).

\[ \frac{s}{2} - \delta^2 S = \delta^2 s [\sigma^{\text{Sync}}(\delta) - \sigma] \leq 0, \]

\[ \frac{s}{2} - \delta^2 S = \delta^2 s [\sigma^{\text{Sync}}(\delta) - \sigma] \leq 0, \]

E Market liberalisation

We compare here the various liberalisation scenarios:

- starting with a duopoly (break-up) versus a monopoly (no break-up)
- staggered versus synchronous tendering

As indicated in the text, we assume that competition prevails in the limit cases where monopolisation could also occur.

E.1 Break-up decision: Proof of Corollary 2

In either tendering regime (staggered or synchronous), it is always optimal to break up the incumbent, so as to start in state II. If the equilibrium is dual-state, doing so lowers the prices forever as, from Proposition (3), duopoly prices are lower than monopoly ones: \( p_{II}^\tau < p_I^\tau \) for \( \tau \in \{\text{Sync, Stag}\} \). If instead the equilibrium is single-state, then breaking up the incumbent has no impact on subsequent tenders, but does lower prices in the initial tenders. Indeed, whereas monopoly prices are by construction positive (i.e., strictly above cost), duopoly prices are instead non-positive (i.e., below cost); from (21) and (30), we have:

\[ p_{II}^{\text{Stag}}(SSE) = (1 - \delta) \delta s - \delta^2 S = \delta^2 s [\sigma^{\text{Stag}}(\delta) - \sigma] \leq 0, \]

\[ p_{II}^{\text{Sync}}(SSE) = \frac{s}{2} - \delta^2 S = \delta^2 s [\sigma^{\text{Sync}}(\delta) - \sigma] \leq 0, \]
where the inequalities follow from $\sigma \geq \sigma^\tau(\delta)$, for $\tau \in \text{Sync, Stag}$.\textsuperscript{48} It follows that breaking up the incumbent is always optimal.

### E.2 Tendering regime: Proof of Proposition 5

In what follows, we focus on the case where the incumbent has been broken up. We first derive the average total bill per city, $P^\tau(\theta)$, for each tendering regime $\tau \in \{\text{Sync, Stag}\}$ and each type of equilibrium $\theta \in \{\text{DSE, SSE}\}$.

#### Total bill for staggered tenders

Under staggered tenders, each city runs one tender in every period. The total bill per city is therefore of the form $\sum_{t=0}^{\infty} \delta^{t-1} p_t^{\text{Stag}}$.

If the equilibrium is dual-state, competition is sustainable and the price remains equal to $p_{II}^{\text{Stag}} (DSE)$, given by (18). Hence, the total bill per city is:

$$P^{\text{Stag}} (DSE) = \frac{1}{1 - \delta} p_{II}^{\text{Stag}} (DSE) = \frac{1 + \delta}{1 - 2\delta} [s - \delta (S + s)].$$

If instead the equilibrium is single-state, monopolisation occurs; the price is thus given by (22) in the initial period, and by (21) afterwards. Hence, the total bill per city is:

$$P^{\text{Stag}} (SSE) = p_{II}^{\text{Stag}} (SSE) + \frac{\delta}{1 - \delta} p_{I}^{\text{Stag}} (SSE) = \delta (S + s).$$

As indicated in the text, in each city the total bill jumps by $s$ at the boundary between the two types of equilibrium: it is equal to $P^{\text{Stag}} (DSE) \big|_{\sigma = \sigma^{\text{Stag}}(\delta)} = 0$ for the dual-state equilibrium and to

$$P^{\text{Stag}} (SSE) \big|_{\sigma = \sigma^{\text{Stag}}(\delta)} = \delta s (1 + \sigma) \big|_{\sigma = \frac{1 + \delta}{\delta}} = s$$

for a single-state equilibrium. The reason is that for $\sigma = \sigma^{\text{Stag}}(\delta)$, firms are by construction indifferent between staying in state II (dual-state equilibrium) or switching to state I (single-state equilibrium); hence, any difference in incurred costs must be offset by a compensating difference in total bills. The conclusion then follows from the fact the former option is costless whereas the latter one involves a sunk cost $s$. This observation

\textsuperscript{48}The resulting initial prices are lower under staggered tenders:

$$p_{II}^{\text{Sync}} (SSE) - p_{II}^{\text{Stag}} (SSE) = \left(\frac{s}{2} - \delta^2 S\right) - \left[(1 - \delta) \delta s - \delta^2 S\right] = \left[\delta^2 + (1 - \delta)^2\right] \frac{s}{2} > 0.$$
implies that, in the limit case $\sigma = \sigma^{\text{Stag}}(\delta)$, the dual-state equilibrium Pareto-dominates (when including the auctioneer among the interested parties) the single-state one.

**Total bill for synchronous tenders**

Under synchronous tenders, each city runs two tenders every other period, and one city starts in the first period whereas the other starts one period later. The average total bill per city is therefore of the form $\sum_{t=0}^{\infty} \delta^{t-1} (p_t^{\text{Sync}})$.

If the equilibrium is dual-state, the price is forever equal to $p_{II}^{\text{Sync}} (\text{DSE})$, given by (29). Hence, the average total bill per city is:

$$\mathcal{P}^{\text{Sync}} (\text{DSE}) = \frac{1}{1 - \delta} p_{II}^{\text{Sync}} (\text{DSE}) = \frac{1 + \delta}{1 - 2\delta^2} (s - 2\delta^2 S).$$

If instead the equilibrium is single-state, monopolisation occurs successively in each city. The price is thus equal to $p_{II}^{\text{Sync}} (\text{SSE})$, given by (30), in periods 0 and 1, and to $p_{I}^{\text{Sync}}$, given by (27), afterwards. Hence, the average total bill per city is:

$$\mathcal{P}^{\text{Sync}} (\text{SSE}) = (1 + \delta) p_{II}^{\text{Sync}} (\text{SSE}) + \frac{\delta^2}{1 - \delta} p_{I}^{\text{Sync}} = \frac{1 + \delta}{2} s.$$

As for staggered tenders, at the boundary between the two types of equilibrium, in each city the total bill is equal to 0 for the dual-state equilibrium and to $s$ for a single-state equilibrium; hence, the average total bill per city jumps by $(1 + \delta) s/2$ – namely, from $\mathcal{P}^{\text{Sync}} (\text{DSE})|_{\sigma = \sigma^{\text{Sync}}(\delta)} = 0$ to $\mathcal{P}^{\text{Sync}} (\text{SSE})$.

**Optimal tendering regime**

Three cases can be distinguished, depending on whether competition is sustainable in either of the two regimes.

**Case 1:** $\sigma \leq \sigma^{\text{Stag}}(\delta)$. In this region, competition is sustainable under both tendering regimes. As duopoly prices are lower under staggered tenders, it follows that staggered tenders are preferable.

**Case 2:** $\sigma > \sigma^{\text{Sync}}(\delta)$. In this region, monopolisation occurs under both tendering regimes. The comparison of the total bills shows that synchronous tenders are preferable:

$$\mathcal{P}^{\text{Stag}} (\text{SSE}) - \mathcal{P}^{\text{Sync}} (\text{SSE}) = \frac{s}{2} [2\delta\sigma - (1 - \delta)] > 0,$$
where the inequality follows from the condition $\sigma > \sigma^{\text{Sync}}(\delta) = 1/2\delta^2$, which implies:

$$2\delta\sigma > \frac{1}{\delta} > (1 \cdot 1) - \delta.$$  

**Case 3:** $\sigma^{\text{Stag}}(\delta) < \sigma \leq \sigma^{\text{Sync}}(\delta)$. In this region, competition is sustainable only under synchronous tenders. For $\sigma = \sigma^{\text{Sync}}(\delta)$, synchronous tenders are optimal as the total bill is then zero; that is, $P^{\text{Sync}}(DSE) = 0 < P^{\text{Stag}}(SSE)$. For $\sigma < \sigma^{\text{Sync}}(\delta)$, the comparison of the total bills yields:

$$P^{\text{Sync}}(DSE) - P^{\text{Stag}}(SSE) = \frac{(1 + 2\delta^3) s - (1 + 2\delta) \delta S}{1 - 2\delta^2}.$$  

The denominator of the right-hand side is positive, as $1 < \sigma \leq \sigma^{\text{Sync}}(\delta) = 1/2\delta^2$ implies $2\delta^2 < 1$. The sign of this expression is therefore the same as that of its numerator, which is negative if and only if $\sigma$ is large enough, namely:

$$\sigma > \tilde{\sigma}(\delta) \equiv \frac{1 + 2\delta^3}{(1 + 2\delta) \delta}, \quad (31)$$

The threshold $\tilde{\sigma}(\delta)$ satisfies:

$$\frac{d\tilde{\sigma}}{d\delta}(\delta) = -\frac{1 + 4\delta - 4\delta^3 - 4\delta^4}{(1 + 2\delta)^2 \delta^2} < 0,$$

where the inequality follows from $\delta < 1$ and $2\delta^2 < 1$ (see above), which respectively imply $4\delta^3 < 4\delta$ and $4\delta^4 < 1$. Furthermore:

$$\sigma^{\text{Sync}}(\delta) - \tilde{\sigma}(\delta) = \frac{1}{2\delta^2} - \frac{1 + 2\delta^3}{(1 + 2\delta) \delta} = \frac{1 - 4\delta^4}{2\delta^2 (1 + 2\delta)} > 0,$$

where the inequality follows again from $2\delta^2 > 1$ in the range $1 < \sigma \leq \sigma^{\text{Sync}}(\delta)$,\footnote{However, like $2\delta^2$, $\sigma^{\text{Sync}}(\delta)$ and $\tilde{\sigma}(\delta)$ both tend to 1 as $\delta$ tends to $\sqrt{2}/2$, as illustrated by Figure 2.} and

$$\tilde{\sigma}(\delta) - \sigma^{\text{Stag}}(\delta) = \frac{1 + 2\delta^3}{(1 + 2\delta) \delta} - \frac{1 - \delta}{\delta} = \frac{2\delta^2 + 2\delta - 1}{1 + 2\delta},$$

which is positive for delta large enough, namely:

$$\delta > \tilde{\delta} \equiv \frac{\sqrt{3} - 1}{2} \simeq 0.37.$$
Recap

Staggered contracts are therefore strictly preferred if

\[ \sigma < \tilde{\sigma}(\delta) \equiv \max\{\tilde{\sigma}(\delta), \sigma^{\text{Stag}}(\delta)\} \]

(which, by construction, includes the entire region where \( \sigma < \sigma^{\text{Stag}}(\delta) \)), whereas synchronous tenders are strictly preferred if \( \sigma > \tilde{\sigma}(\delta) \) (which includes the entire region where \( \sigma > \sigma^{\text{Sync}}(\delta) \), as \( \delta^{\text{Sync}}(\sigma) \) lies above \( \tilde{\delta}(\sigma) \) and \( \delta^{\text{Stag}}(\sigma) \)), as illustrated in Figure 2. Finally, in the boundary case where \( \sigma = \tilde{\sigma}(\delta) \), both tendering regimes deliver the same total bill.

F  Contract Duration: Proof of Proposition 7

From (29), we have

\[ p^{\text{Sync}}_{II}(DSE) = \frac{1 - \delta^2}{1 - 2\delta^2} (s - 2S\delta^2) = \frac{1 - \delta^2}{1 - 2\delta^2} 2\delta^2 s \left[ \sigma^{\text{Sync}}(\delta) - \sigma \right]. \]

Likewise, from (18), we have

\[ p^{\text{Stag}}_{II}(DSE) = \frac{1 - \delta^2}{1 - 2\delta} [s - \delta (s + S)] = \frac{1 - \delta^2}{1 - 2\delta} \delta s \left[ \sigma^{\text{Stag}}(\delta) - \sigma \right]. \]

It follows that:

\[ p^{\text{Sync}}_{II}(DSE)_{\sigma = \sigma^{\text{Sync}}(\delta)} = p^{\text{Stag}}_{II}(DSE)_{\sigma = \sigma^{\text{Stag}}(\delta)} = 0. \]

G  Empirical evidence

Here we examine in more detail two points made in Section 7.1 using Transport for London as an example. The first is that incumbency is at least as important as distance to nearest garage when assessing which company runs which route. The second examines the synchronisation of routes by operators in their bidding.

G.1 Garage proximity or incumbency?

To test the proposition that garage proximity itself causes a given company to operate a route, rather than the fact of being an incumbent per se, we turn to the set of 402 cases where we observe the same route being tendered twice (or more). For each case, we record
the proximity of the first and second occupant in terms of ranked nearness. Based on the assumption that the cause of initial incumbency is not modelled, there are four relevant cases to consider: (i) The firm whose garage is nearest is the successful bidder on both occasions; (ii) on the second occasion, a firm ranked X loses to a firm ranked x < X; (iii) on the second occasion, a firm ranked x loses to a firm ranked X; (iv) on the first and second occasions, a firm ranked z > 1 wins the tender. Of these cases, we argue that (i) is incapable of distinguishing between the two available hypotheses (garages being the key factor or incumbency per se being the key). Case (ii) is a case in favour of the garage explanation. Case (iv) suggests incumbency, not garage proximity, is the important factor determining outcomes. The test results are shown in table 3. It is difficult to think of an appropriate statistical test, but the numbers are clear-cut. Whereas case (iv) covers at least 120 of our routes, case (ii) covers only 49. Based on the outcome of this test, we assert that our model as interpreted above more accurately reflects the situation than does the leading alternative.

<table>
<thead>
<tr>
<th>Repeated observations in paired sample</th>
<th>Case</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanation of rank</td>
<td>(1,1)</td>
<td>(X,x)</td>
<td>(x,X)</td>
<td>(z,z)</td>
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</tr>
<tr>
<td>Number</td>
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<td>49</td>
<td>44</td>
<td>143</td>
<td></td>
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<tr>
<td>Number excluding takeovers</td>
<td>144</td>
<td>49</td>
<td>44</td>
<td>120</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Own sample of 402 routes let at least twice.

G.2 Synchronisation and Proposition 3

TfL bus routes are tendered according to schedules, published around two years in advance of the route being run under the new schedule. Based on these schedules, bus companies decide their bidding strategy. The schedules consist of a series of tranches, being bundles of routes. Each bidder needs to submit a compliant bid for a route but may also propose a bundled bid for a set of routes.

Proposition 3 shows that, conditional on continuing competition in the market as a whole, prices relative to costs are higher under synchronous contracts. To a significant extent then, the synchronicity or otherwise of bids is a property of the tranches that TfL sets out. TfL bundles bids into tranches in part because there might be savings in the use of vehicles (for example, bundling four relatively infrequent routes in a particular area of London into a single tranche). However, what we examine is the role of companies in synchronising routes themselves, outside the TfL tranche structure. Proposition 3 predicts that companies will choose to do this, where possible.
We identify two mechanisms that companies can use to synchronise bids. The first relies on the frequency of tranche competitions throughout the year. On average, tranches are only around nine days apart. Given that the gap between the issue of a tender document and the date for response is around two months, there is scope for a bidder to create a bundle ranging across two or three tranches and yet be compliant.

The second and more important mechanism relies on an institutional feature of the market. Successful bidders are initially awarded the contract for five years, but almost universally may extend this to seven years on application (and acceptable performance). Therefore, it is possible for a company to create a synchronous bundle by putting together routes that will come into their seventh year with routes that are only coming into their fifth year, i.e. previously staggered routes. In more detail, once having won tenders for two or more routes which were originally tendered at different times, roughly two years apart (a five year and a seven year contract), they will become synchronous for five years and potentially beyond, since a company can choose to curtail both after five years, so rendering them synchronous the next time. At the time TfL first creates the schedule, at least two years in advance, the companies’ decisions on whether to extend will not be apparent, so that each year a number of contracts are held over and added to future schedules, normally two years later, once extensions are determined.

Table 4 sets out our detailed examination of these possibilities within the TfL context. Six plan years are included, from 2014/15, the earliest currently on the TfL website, to 2019/20, the latest in which a reasonably complete picture has emerged. Of course, we are unable to examine how many routes were held over to the 2014/15 and 2015/16 plans. Apart from that, the table sets out the plans for these years including the number of tranches and routes initially in the plan.

The table shows that the average tranche consists of 3.5 routes, although this number varies between 1 and 10; it is also notable that the average tranche size decreases markedly in the 2019/20 plan, which affects the tendering pattern. Just under 2/3 of the routes are let as planned overall, the remainder being carried over. Of those carried over, more than 90% appear in the schedule two years later, so that, for example, of the 77 routes in the 2014/15 schedule carried over, 74 of these reappear in the 2016/17 schedule.

Of those in the plans that are let at the time, 25% more are let as part of a bundle rather than as a route by itself (or 48% more if we consider only the last four years, hence avoiding overlaps). However, our main interest in this segment of the table is in the routes which are let as cross-tranche bundles. On average, these constitute 30% of the routes.

---

50 We have excluded route numbers in the 600s in constructing this table because these are almost all “school bus” routes at each end of the day, which are an order of magnitude smaller than the typical route in intensity. For the same reason we have excluded the small number of night-only routes not paired with a daytime route.
that are bundled, showing that despite the TfL tranche structure, companies compose their own bundles to a significant extent, as predicted by Proposition 3.\textsuperscript{51}

\textsuperscript{51}We are talking only of successful bundles, of course. TfL does not reveal information on unsuccessful bundle bids.
<table>
<thead>
<tr>
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<td>167</td>
<td>203</td>
<td>146</td>
<td>117</td>
<td>65</td>
<td>3.52 Route/Tranche</td>
</tr>
<tr>
<td>Let as planned</td>
<td>44</td>
<td>82</td>
<td>150</td>
<td>112</td>
<td>93</td>
<td>45</td>
<td>0.64 Let/Planned</td>
</tr>
<tr>
<td>Of which</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sole</td>
<td>29</td>
<td>44</td>
<td>64</td>
<td>38</td>
<td>33</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>Bundled</td>
<td>15</td>
<td>38</td>
<td>86</td>
<td>74</td>
<td>60</td>
<td>18</td>
<td>1.24 Bundled/Sole*</td>
</tr>
<tr>
<td>Carried over</td>
<td>77</td>
<td>85</td>
<td>53</td>
<td>34</td>
<td>24</td>
<td>20</td>
<td>0.36 Carried/Planned</td>
</tr>
<tr>
<td>Bundle cross-tranche</td>
<td>4</td>
<td>5</td>
<td>24</td>
<td>24</td>
<td>23</td>
<td>8</td>
<td>0.3 Cross- v in- Tranche</td>
</tr>
<tr>
<td>Let 2 years later</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Of those</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sole</td>
<td>24</td>
<td>22</td>
<td>18</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bundled</td>
<td>50</td>
<td>57</td>
<td>28</td>
<td>10</td>
<td>18</td>
<td>18</td>
<td>1.86 Bundled/Sole</td>
</tr>
<tr>
<td>Undetermined</td>
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</tr>
</tbody>
</table>

* Last 4 years, 1.47

Table 4: Examining the synchronicity of routes
We now turn to the routes that are carried over and emerge in the plan for two years later, in the lower part of the table. Of the 92% that can be traced to that plan, a sharper tendency emerges: Now 86% more are bundled than are let as sole contracts. Moreover, examining the second mechanism described above, almost 1/3 of the routes that are in the plan this second time are bundled with routes that would not have been available but now are as a result of the five-year/seven-year mechanics of the system. This is strongly confirmatory of Proposition 3, because it shows how companies are using the system to create synchronous contractual positions. The final row of the table shows that previously unavailable packages involve half of all the carried-over routes that are bundled, so illustrating the impact of synchronisation.

We engaged in a further, more forensic investigation of the cases moving from plan 2014/15 to plan 2016/17, in terms of the companies obtaining the routes compared with the companies that previously ran the routes. However, little of interest emerged from this investigation- clearly one thing that a company will aim for is to take routes from other companies, perhaps by proposing a new bundle. In some cases, this happens, in others not.

H References


\(^{52}\)On occasion, TfL will rethink the decision to have the route at all.


A Synchronous tenders under economies of scope for entry

We revisit here the case of synchronous tenders under the assumption that entrants benefit from economies of scope: entering both markets requires sinking $S + s$ rather than $2S$.

A.1 Robustness of the insights

We have:

**Proposition 8** (synchronous tenders under economies of scope for entry). Under synchronous tendering, generically there exists an essentially unique coalition-proof Markov equilibrium outcome, which can be of two types:

- **Single-state**: if $\sigma > \bar{\sigma}_{\text{Sync}}(\delta)$, where

  $$ \bar{\sigma}_{\text{Sync}}(\delta) \equiv \frac{1 - \delta^2}{1 + \delta^2}, $$

  then monopolisation arises and the equilibrium price is:

  $$ p^I_{\text{Sync}} \equiv \left(1 - \delta^2\right) \frac{S + s}{2}. $$

- **Dual-state**: if instead $\sigma < \bar{\sigma}_{\text{Sync}}(\delta)$, then the equilibrium path depends on the initial state:

  - **persistent monopoly**: if initially the same firm services both markets, the equilibrium price is again $p^I_{\text{Sync}} = \left(1 - \delta^2\right) \frac{S + s}{2}$;
  
  - **sustainable competition**: if instead different firms initially service the two markets, the equilibrium price is:

  $$ p^I_{\text{Sync}} \equiv \frac{1 - \delta^2}{1 - 2\delta^2} \left( s - \delta^2 (S + s) \right). $$

In the limit case where $\sigma = \bar{\sigma}_{\text{Sync}}(\delta)$, there are infinitely many coalition-proof equilibrium outcomes, which yield the same per-market prices, $p^I_{\text{Sync}} = \left(1 - \delta^2\right) S$ and $p^I_{\text{Sync}} = 0$, respectively.
and differ only in the probability of switching from state II to state I, which can take any arbitrary value.

Proof. See Section A.2 below.

Comparing these findings with that of Proposition 2 shows that economies of scope for entry make a dual-state equilibrium more likely to arise, as:

\[ \hat{\sigma}^{sync}(\delta) = \frac{1 - \delta^2}{\delta^2} > \frac{1 - \delta}{\delta} = \sigma^{sync}(\delta), \]

where the inequality follows from \( \delta < 1 \). This comparison also shows that, under synchronous tenders, entry economies of scope tend to lower equilibrium prices in the monopoly state, and increase them instead in the duopoly state:

\[ p^{sync}_I - \hat{p}^{sync}_I = (1 - \delta^2) \frac{S - s}{2} > 0 \]
\[ \hat{p}^{sync}_I - p^{sync}_I = (1 - \delta^2) \frac{\delta^2 (S - s)}{1 - 2\delta^2} > 0. \]

With economies of scope for entry (i.e., \( S + s \) rather than \( 2S \)), the entrants exert more pressure on the incumbent in state I, which lowers the monopoly price; this, in turn, reduces the intensity of competition among the two incumbents in state II, which tends to increase duopoly prices. Together, these two observations imply that the value of monopolisation is reduced, making dual state more likely.

Still, monopoly prices remain higher than duopoly ones:

\[ \hat{p}^{sync}_I - \hat{p}^{sync}_II = \frac{1 - \delta^2}{1 - 2\delta^2} \frac{S - s}{2} > 0. \]

As a result, the equilibrium prices remain however ranked in the same order:

**Proposition 9** (price comparison under economies of scope for entry). We have:

\[ p^{stag}_I > \hat{p}^{sync}_I > \hat{p}^{sync}_II > p^{stag}_II. \]

Proof. Proposition 3 and the above conditions together imply \( p^{stag}_I > \hat{p}^{sync}_I > \hat{p}^{sync}_II > p^{stag}_II \).

A.2 Proof of Proposition 8

As before, in equilibrium (i) every firm obtains a non-negative continuation value, which it can secure by offering above-cost prices; and (ii) the continuation value of entrants is \( V_E = 0 \), as \( V_E > 0 \) would require winning a market with a positive margin, in which case
coalition-proof Nash equilibria

We first characterise the coalition-proof Nash equilibria for given continuation values satisfying $V_E = 0$ and $V_i \geq 0$ for $i = M, A, B$. We start by characterising potential entrants’ best offers, which are the same in both states, before studying the equilibrium outcomes in each state.

In state $I$, potential entrants are willing to service market $i$ for a stand-alone price $p_{Ei} = S - \delta^2 V_i$, as before. By contrast, their best price becomes

$$\hat{P}_E \equiv S + s - \delta^2 V_M.$$ (32)

Going through the same steps as in the proof of Proposition 2 shows Lemma 5 and Corollary 4 still hold, with the caveat that $M$’s continuation value becomes $V_M = \tilde{V}_M = S + s$ and the equilibrium bundled price thus becomes:

$$\hat{P}_I = \frac{\hat{P}_E}{2} = (1 - \delta^2) \frac{S + s}{2}.$$ (33)

In particular, it is still the case that the monopolisation value is non-negative; that is (dropping the superscript Sync for ease of exposition), $\Delta \geq 0$, as the total equilibrium price for the two markets can never exceed $P_E$, implying $V_A + V_B \leq \sum_{t=0}^{+\infty} \delta^t P_E = V_M$.

In state $II$, the two incumbents are the relevant competitors. It follows that the analysis developed in Appendix C.1 remains valid “as is:” if $\Delta < s$, then there exists a unique coalition-proof Nash equilibrium outcome, in which each $D_i$ wins market $i$ at price $\hat{p}_i \equiv s - \delta^2 (V_M - V_j)$ (for $i \neq j \in \{A, B\}$) and obtains $\tilde{V}_D = s\Delta > 0$; if instead $\Delta > s$, there are infinitely many coalition-proof Nash equilibrium outcomes, in which both incumbents offer the bundle price $P_D = s - \delta^2 V_M$, and obtain $\tilde{V}_D = 0$, and these outcomes only differ in the probabilities that either incumbent wins the two markets (and, in the limit case $\Delta^{Sync} \Delta = s$, that each incumbent keeps its market).

Equilibrium characterisation

From the above, in equilibrium $V_M = S + s$ and, for $i = A, B$:

$$V_i = \tilde{V}_D = \max \{s - \Delta, 0\}.$$ (34)
Furthermore, once in state I the equilibrium path remains in that state forever, and the per market price is then \( p_{I}^{\text{sync}} \), given by (33). By contrast, in state II, the equilibrium path may either remain in that state, or switch to state I. We consider in turn these two types of equilibrium.

- **Dual-state equilibrium.** A dual-state equilibrium exists if and only if \( \Delta \leq s \). We then have:
  \[
  \Delta = \delta^2 (V_M - 2V_D) = \delta^2 (S - s + 2\Delta).
  \]
  Together with \( \delta > 0, S > s \) and \( \Delta \geq 0 \), this implies \( \delta^2 < 1/2 \). Solving for \( \Delta \) and using again (34) then yields:
  \[
  \Delta = \frac{\delta^2 (S - s)}{1 - 2\delta^2} (> 0) \quad \text{and} \quad V_D = \frac{s - \delta^2 (S + s)}{1 - 2\delta^2}.
  \]
  As \( \delta^2 < 1/2 \), the working condition \( \Delta \leq s \) holds if and only if:
  \[
  \delta^2 (S - s) \leq (1 - 2\delta^2) s \iff \sigma \leq \sigma^{\text{sync}} (\delta) \equiv \frac{1 - \delta^2}{\delta^2}.
  \]
  Conversely, whenever \( \sigma < \sigma^{\text{sync}} (\delta) \) (which implies \( \delta^2 < 1/2, \Delta < s \) and \( V_D > 0 \)), there exists a dual-state equilibrium, in which the equilibrium path remains forever in the initial state. Furthermore, in state II the per market price is given by:
  \[
  p_{II}^{\text{sync}} = p_{II}^{\text{sync}} (DSE) \equiv (1 - \delta^2) V_D = \frac{1 - \delta^2}{1 - 2\delta^2} (s - \delta^2 (S + s)).
  \]

- **Single-state equilibrium.** A single-state equilibrium exists if and only if \( s \leq \Delta \). We then have \( V_A = V_B = 0 \), implying that the working condition \( s \leq \Delta = \delta^2 (S + s) \) boils down to \( \sigma \geq \sigma^{\text{sync}} (\delta) \). Conversely, if \( \sigma \geq \sigma^{\text{sync}} (\delta) \), there exists a single-state equilibrium, in which from period 1 onward, the equilibrium path remains in state I and the per market price is then \( p_{I}^{\text{sync}} = (1 - \delta^2) (S + s) / 2 \). If the game starts in state II, then the price in the initial periods is:
  \[
  p_{II}^{\text{sync}} = p_{II}^{\text{sync}} (SSE) \equiv \frac{P_D}{2} = \frac{s}{2} - \delta^2 S.
  \]

**B  Synchronous tenders without bundled bids**

We consider here the case of synchronous tenders when firms cannot submit bundled bids. That is, we study the equilibria of the game in which (i) both markets are up for tender in every even period, and (ii) firms can only submit stand-alone prices for each market. As before, in equilibrium the continuation value of entrants is \( V_E = 0 \) and every other
firm obtains a non-negative continuation value, which it can secure by offering above-cost prices. We denote again by $V_M$ the continuation value of $M$ in state $I$, and by $V_i$ the continuation value of each $D_i$ in state $II$. Without loss of generality, we can restrict attention to $V_M \in [0, 2S]$ (if $M$ were obtaining more than $2S$, then an entrant could profitably undercut it) and $V_i \in [0, S]$ (likewise, if $D_i$ were obtaining more than $2S$, then an entrant could profitably undercut it); it follows from $V_M \geq 0$, $V_i \leq S$ and $\delta < 1$ that the value of monopolisation satisfies (dropping again the superscript $Sync$ for ease of exposition):

$$\Delta = \delta^2 (V_M - V_A - V_B) > -2S.$$ 

**B.1 Coalition-proof Nash equilibria**

As before, the entrants are willing to service $D_i$’s market at any price above $p_{Ei} \equiv S - \delta^2 V_i$, or both markets for a total price above $P_E \equiv 2S - \delta^2 V_M$. We first characterise the coalition-proof Nash equilibria in state $I$, for any given continuation values $V_M$, $V_A$ and $V_B$:

**Lemma 7** (state $I$ for synchronous tenders – no bundling). In state $I$, the coalition-proof Nash equilibrium outcomes are as follows:

(i) If $\Delta \geq 0$, there exists a unique coalition-proof Nash equilibrium outcome, in which $M$ wins both markets at total price $P_E$ and obtains a payoff equal to $\tilde{V}_M = 2S$.

(ii) If instead $\Delta < 0$:

- if $\Delta < -S$, there exist an essentially unique equilibrium outcome, in which $M$ wins one market $i \in \{A, B\}$ at price $p_{Ei}$ and obtains $\tilde{V}_M = S$, whereas an entrant wins the other market, $j$, at price $p_{Ej}$;

- if instead $\Delta > -S$, there exists a unique coalition-proof Nash equilibrium outcome, in which $M$ wins both markets at price $p_{EA}$ and $p_{EB}$, and obtains $\tilde{V}_M = 2S - \Delta (< S)$;

- finally, if $\Delta = -S$, the outcomes described above constitute the only coalition-proof Nash equilibrium outcomes and give $M \tilde{V}_M = S$.

**Proof.** As before, $M$ must win at least one market in equilibrium, as it can profitably undercut any viable offer from the entrants. Hence, either $M$ wins both markets, or it wins one market and an entrant wins the other market. In the former case, $M$ cannot obtain more than $\tilde{V}_M = 2S$, otherwise an entrant could profitably undercut $M$. In
the latter case, $M$ cannot obtain more than $\tilde{V}_M = S$, otherwise an entrant could again profitably undercut it.

Suppose first that $\Delta \geq 0$, which implies $P_E \leq p_{EA} + p_{EB}$, and consider a candidate equilibrium in which all firms offer each market at price $p_i$ satisfying $p_A + p_B = P_E$ and $p_i \leq p_{Ei}$, and the auctioneer assigns both markets to $M$. By construction, the entrants cannot profitably undercut these prices and $M$ obtains $\tilde{V}_M = P_E + \delta^2 V_M = 2S$, which is positive and exceeds the payoff from a deviation that would target a single market, as such a deviation cannot yield more than $p_{Ei} + \delta^2 V_i = S$. Hence, there exists a Nash equilibrium giving the maximal payoff $\tilde{V}_M = 2S$; conversely, any other Nash equilibrium (in which either $M$ would win both markets at lower prices, or win a single market) would generate a lower payoff for $M$. As the entrants’ equilibrium payoffs are always zero, this Nash equilibrium is the unique coalition-proof Nash equilibrium.

Suppose next that $\Delta < 0$, and first consider a candidate equilibrium in which $M$ wins market $i$ at price $p_i$, and thus obtains $\tilde{V}_M = p_i + \delta^2 V_i$, whereas an entrant wins market $j \neq i$ at price $p_j$. Competition among entrants then imply that $p_j = p_{Ej}$ and $p_i \leq p_{Ei}$. Furthermore, by targeting market $j$, $M$ could obtain $p_{Ej} + \delta^2 V_j = S = p_{Ei} + \delta^2 V_i$. To ensure that this deviation is not profitable, we must have $p_i \geq p_{Ei}$, which, combined with the previous equilibrium condition $p_i \leq p_{Ei}$, yields $p_i = p_{Ei}$ and $\tilde{V}_M = S$. Finally, to ensure that $M$ cannot profitably deviate by winning both markets, it must be the case that:

$$\tilde{V}_M = S \geq p_A + p_B + \delta^2 V_M = p_{EA} + p_{EB} + \delta^2 V_M = 2S - \Delta,$$

or:

$$\Delta \geq S.$$

Conversely, when this condition holds, all firms offering $p_{Ei}$ for each market $i = A, B$ constitutes a Nash equilibrium in which $M$ obtains $\tilde{V}_M = S$.

Consider now a candidate equilibrium in which $M$ wins both markets at prices $p_A$ and $p_B$. The price $p_i$ cannot exceed $p_{Ei}$ (otherwise, an entrant could profitably undercut $M$) and $M$ should not find it profitable to target market $j$ only:

$$p_A + p_B + \delta^2 V_M \geq p_j + \delta^2 V_j,$$

which implies

$$p_i \geq \delta^2 (V_j - V_M).$$

Therefore, we must have:

$$S - \delta^2 V_i = p_{Ei} \geq p_i \geq \delta^2 (V_j - V_M),$$

6
implying:

\[ \Delta \geq -S. \]

The most profitable candidate Nash equilibrium of this type is such that \( p_i = p_{Ei} \), which gives \( M \) a payoff of:

\[ p_{EA} + p_{EB} + \delta^2 V_M = 2S + \Delta > 0, \]

where the inequality stems from \( \Delta > -2S \). Conversely, as long as \( \Delta \geq -S \), these prices do constitute a Nash equilibrium:

- as \( \Delta < 0 \), \( P_E > p_{EA} + p_{EB} \); hence, the entrants cannot profitably undercut the equilibrium prices;
- by construction, the condition \( \Delta \geq -S \) ensures that \( M \) cannot profitably deviate by targeting a single market (either one);
- as \( M \) makes a non-negative payoff, it has no incentive to raise its price and lose the two markets.

To conclude the proof, it suffices to note that, in the limit case where \( \Delta = -S \), both of the above equilibria coexist, and they yield the same payoff. \( \square \)

In equilibrium, the equilibrium payoffs must coincide with the equilibrium continuation values, which leads to:

**Corollary 5** (the value of monopoly for synchronous tenders – no bundling). *The equilibrium continuation values satisfy*

\[ V_M = 2S \quad \text{and} \quad \Delta \geq 0. \]

*Proof.* From Lemma 7, if \( \Delta < -S \), then in equilibrium we must have: \( V_M = \bar{V}_M = S \); using \( \delta < 1 \) and \( V_i \leq S \) for \( i = A, B \), this would imply \( \Delta = \delta^2 (V_M - V_A - V_B) > -S \), a contradiction. Likewise, if instead \( \Delta \in (-S, 0) \), then in equilibrium we must have:

\[ V_M = \bar{V}_M = 2S - \Delta = 2S - \delta^2 (V_M - V_A - V_B) = \frac{2S + \delta^2 (V_A + V_B)}{1 - \delta^2} \geq \frac{2S}{1 - \delta^2}. \]

Using the last inequality and \( V_i \leq S \) for \( i = A, B \), this would imply:

\[ (1 - \delta^2) (V_A + V_B - V_M) \leq (1 - \delta^2) (V_A + V_B) - 2S \leq -2\delta S < 0, \]
a contradiction. It follows that the equilibrium continuation values must satisfy $\Delta \geq 0$; Lemma 7 then yields $V_M = \tilde{V}_M = 2S$, which indeed exceeds $V_A + V_B$, as no $V_i$ can exceed $S$.

Building on the previous analysis, the following lemma shows that there exists an essentially unique coalition-proof Nash equilibrium outcome in state $II$:

**Lemma 8** (state $II$ for synchronous tenders – no bundling). In state $II$, the coalition-proof Nash equilibrium outcomes are as follows:

- If $\Delta < s$, there exists a unique coalition-proof Nash equilibrium outcome, in which each incumbent $D_i$ wins market $i$ at price $\tilde{p}_i \equiv s - \delta^2 (V_M - V_j)$ (for $i \neq j \in \{A, B\}$) and obtains $\tilde{V}_i = \tilde{V}_D \equiv s - \Delta > 0$.

- If instead $\Delta > s$, there are infinitely many coalition-proof Nash equilibrium outcomes, in which both incumbents offer the same total price, $P_D \equiv s - \delta^2 V_M$, and obtain $\tilde{V}_A = \tilde{V}_B = 0$; these outcomes only differ in the probabilities that either incumbent wins the markets, which can take any arbitrary values.

- Finally, in the boundary case where $\Delta = s$, there are again infinitely many coalition-proof Nash equilibrium outcomes, in which both incumbents offer the same total price, $P_D = s - \delta^2 V_M$, and obtain $\tilde{V}_A = \tilde{V}_B = 0$; these outcomes only differ in the probabilities that either of the incumbents wins both markets and/or that each incumbent wins its market, which can take any arbitrary values.

**Proof.** The same argument as before can be used to show that the relevant competition takes place between the two incumbents; hence, in equilibrium, one incumbent $I_i$ wins its market, and either it also wins the other market, or the other incumbent keeps its own market. Which outcome prevails depends on the comparison between the two incumbents’ best offers for the second market: *conditional on winning market $i$, $I_i$ is willing to service market $j$ as well for any price up to*\(^{53}\)

$$\tilde{p}_i = s - \delta^2 (V_M - V_j),$$

whereas $I_j$ is willing to keep its market – rather than exiting – at any price up to:

$$p_{Ij} = -\delta^2 V_j.$$  

It follows that monopolisation occurs if $\tilde{p}_i > p_{Ij}$, which amounts to $\Delta > s$, whereas competition is sustainable if $\Delta < s$. In the boundary case, both outcomes can arise. \(\square\)

\(^{53}\)Note that $p_{Ei} - \tilde{p}_i = S - s + \Delta > 0$, which confirms that competition takes place between the two incumbents.
B.2 Equilibrium characterisation

The rest of the proof is the same as for the proof of Proposition 2 (see Appendix C.2) and can be summarised as follows:

- In equilibrium, $V_D = \tilde{V}_D$. Using Corollary 5 and Lemma 8, the condition $\Delta < s$ then amounts to $\sigma < \sigma^{Sync}(\delta)$ coalition-proof Markov perfect equilibrium outcome, which is dual-state: the equilibrium path remains in the initial state forever and the per market equilibrium prices are $p^{Sync}_I = (1 - \delta^2) S$ for state $I$ and $p^{Sync}_{II} = \frac{1-\delta^2}{1-2\delta^2} (s - 2\delta^2 S)$ for state $II$.

- The condition $\Delta > s$ amounts instead to $\sigma > \sigma^{Sync}(\delta)$, in which case there are infinitely many coalition-proof Markov perfect equilibrium outcomes, which are single-state: regardless of the initial state, from period 1 onwards the equilibrium path stays in the monopoly state, and the per market price is again equal to $p^{Sync}_I = (1 - \delta^2) S$; these equilibria only differ in the probability that one or the other incumbent prevails in state $II$.

- In the limit case $\sigma = \sigma^{Sync}(\delta)$, the coalition-proof equilibrium outcomes described above coexist and give the same payoffs to all firms.

C Market liberalisation: alternative setting

We consider here a variant of the market liberalisation analysis in which the choice between staggered and synchronous tendering is achieved by adjusting the duration of an initial tender. That is, we consider a single pair of markets, as in the baseline model, and compare the following scenarios:

- Initial state: one or two incumbents (i.e., breaking up or not the historical incumbent).

- Tendering regime:
  - synchronous tenders: from period 0 onwards, two period-contracts are tendered every other period for each market;
  - staggered tenders: in period 0, a one-period contract is tendered for market $A$ and a two-period contract is tendered for market $B$; at the end of these contracts, a two-period contract is tendered in every period (for market $A$ in even periods, and for market $B$ in odd periods).
The analysis of synchronous tenders is the same as in baseline model. Hence, from period 2 onward the prices are stationary and characterised by Proposition 2. In period 0, the same prices arise in state I, as well as in state II if the equilibrium is dual-state; if instead the first tenders take place in state II and the equilibrium is single-state, then competition for survival yields lower prices than the stationary ones.

We now study the case of staggered tenders, before deriving the optimal policy.

C.1 Staggered tenders

Suppose that the auctioneer opted for staggered tendering and consider period 0, in which a one-period contract for market A and a two-period contract for market B are tendered. From period 1 onward, tenders are staggered as in the baseline model; the incumbents’ continuation values, \( V_M \) in state I and \( \{V_D,V_C\} \) in state II, are therefore those characterised by Proposition 1, and satisfy \( \Delta^{Stag} \equiv \delta (V_M - V_D - V_C) > 0 \).

State I

In the absence of break-up, in period 0 a single incumbent, \( M \), faces only the entrants. The entrants’ best offers are given by:

\[
p_{Ei} \equiv S - \delta V_i \quad \text{for} \quad i = A, B \quad \text{and} \quad P_E \equiv 2S - \delta V_M,
\]

with the convention:

\[
V_A = V_D \quad \text{and} \quad V_B = V_C, \tag{36}
\]

and they satisfy:

\[
p_{EA} + p_{EB} - P_E = \Delta^{Stag} > 0. \tag{37}
\]

\( M \) faces the same continuation payoffs but need not sink \( S \); hence, entrants can never win both markets in equilibrium, as \( M \) would then obtain zero payoff and could profitably undercut any winning entrant. Consider now a candidate equilibrium in which \( M \) wins market \( i \) at some price \( p_i \) and an entrant wins market \( j \) at some price \( p_j \), for some \( j \neq i \in \{A,B\} \). Competition among entrants yields \( p_j = p_{Ej} \); but then, \( M \) could profitably undercut the entrant so as to win both markets, as this would increase its total discounted payoff:

\[
(p_i + p_{Ej} + \delta V_M) - (p_i + \delta V_i) = S + \Delta^{Stag} > 0,
\]

Thus, the only candidate equilibrium is one in which the incumbent wins both markets. In any such equilibrium, \( M \) cannot obtain more than \( \tilde{V}_M = 2S \), otherwise any entrant could
profitably undercut it. To establish existence, suppose that all firms offer the entrants’ best prices, \( \{p_{EA}, p_{EB}, P_E\} \). From (37), the auctioneer is then willing to assign both markets to \( M \), which gives \( M \) a total payoff equal to

\[
P_E + \delta V_M = 2S = \tilde{V}_M.
\]

To see that this constitutes indeed an equilibrium, it suffices to note that:

- by construction, \( E \) cannot profitably undercut \( M \);
- conversely, \( M \) cannot benefit from exiting, and it cannot profitably deviate either by targeting market \( i \) on a stand-alone basis: this would require charging no more than \( p_{Ei} \) and thus yield at most \( p_{Ei} + \delta V_i = S < 2S = \tilde{V}_M \).

State II

If instead the historical incumbent is broken-up, in period 0 there are different incumbents in the two markets; we denote by \( D_i \) the incumbent in market \( i \in \{A, B\} \). \( D_A \) obtains zero payoff if it does not win any market, and otherwise obtains:

\[
\begin{align*}
p_A + \delta V_D & \quad \text{if it wins only market } A \text{ (one-period contract) at price } p_A, \\
p_B - s + \delta V_C & \quad \text{if it wins only market } B \text{ (two-period contract) at price } p_B, \\
P - s + \delta V_M & \quad \text{if it wins both markets at total price } P.
\end{align*}
\]

Likewise, \( D_B \) obtains zero payoff if it does not win any market, and otherwise obtains:

\[
\begin{align*}
p_A - s + \delta V_D & \quad \text{if it wins only market } A \text{ (one-period contract) at price } p_A, \\
p_B + \delta V_C & \quad \text{if it wins only market } B \text{ (two-period contract) at price } p_B, \\
P - s + \delta V_M & \quad \text{if it wins both markets at total price } P.
\end{align*}
\]

The same arguments as in the proof of Proposition 2 can be used to show that, in equilibrium, entrants cannot win any market and, if both incumbents win a market, then they must win “their” own market. Thus, consider first a candidate equilibrium in which each \( D_i \) wins market \( i \) at some price \( p_i \), for \( i = A, B \). Denoting \( D_i \)'s equilibrium continuation value by \( \tilde{V}_i \) and using as before the convention given by (36), the following conditions must hold:

- \( D_j \) should not find it profitable to deviate by outbidding its rival \( D_i \) so as to win both markets:

\[
\tilde{V}_j = p_j + \delta V_j \geq p_A + p_B - s + \delta V_M,
\]
which implies:
\[ p_i \leq \tilde{p}_i \equiv s - \delta (V_M - V_j), \]
and thus:
\[ \tilde{V}_i = p_i + \delta V_i \leq \tilde{V}_D \equiv s - \Delta^{Stag}. \]

• The payoffs cannot be negative:
\[ \tilde{V}_i \geq 0. \]

Together, the above conditions yield \( \tilde{V}_D \geq \tilde{V}_i \geq 0 \), which implies \( \Delta^{Stag} \leq s \), or \( \sigma \leq \sigma^{Stag}(\delta) \). Conversely, whenever \( \sigma \leq \sigma^{Stag}(\delta) \), in period 0 there exists an equilibrium in which each \( D_i \) wins market \( i \) and obtains
\[ \tilde{V}_D = s - \Delta^{Stag} = s - \frac{\delta (S - s)}{1 - 2\delta} = s - \frac{\delta (S + s)}{1 - 2\delta}. \quad (38) \]

To see this, suppose that entrants offer their best prices, \( \{P_E, p_{EA}, p_{EB}\} \), and that both incumbents offer to service each market \( i = A, B \) at price \( \tilde{p}_i \), and the bundle at price \( \tilde{P} \equiv \tilde{p}_A + \tilde{p}_B \). The auctioneer is then willing to allocate each market \( i \) to \( D_i \), and each \( D_i \) obtains \( \tilde{V}_D \); hence, it cannot benefit from exiting (as \( \tilde{V}_D \geq 0 \)), from targeting the other market \( j \) (which would yield at most \( \tilde{V}_D - s < \tilde{V}_D \)), or from winning both markets (from the definition of \( \tilde{p}_j \)). To establish existence, it therefore suffices to note that the entrants cannot profitably undercut the incumbents’ prices, as \( \tilde{p}_i - \tilde{p}_{Ei} = -(S - s) - \Delta^{Stag} < 0 \)
and \( \tilde{P} - P_E = -2(S - s) - \Delta^{Stag} < 0 \).

We now show that there always exists an equilibrium in which one incumbent wins both markets. Such candidate equilibrium necessarily yields \( \tilde{V}_A = \tilde{V}_2 = 0 \): the losing incumbent exits and thus obtains zero payoff; and the winning incumbent cannot obtain a positive payoff, otherwise the losing one would profitably undercut it. To establish existence, suppose that both incumbents offer a bundle price equal to
\[ P_D \equiv s - \delta V_M, \]
whereas entrants offer the bundle price \( P_E \), and that all firms (entrants and incumbents) offer the stand-alone price \( p_{Ei} \) for each market \( i = A, B \). As \( P_D - P_E = s - 2S < 0 \) and \( P_D - p_{EA} - p_{EB} = -(2S - s) - \Delta^{Stag} < 0 \), the auctioneer is then willing to assign both markets to either incumbent, and the entrants cannot profitably undercut the incumbents. Furthermore, each \( D_i \) obtains zero payoff and thus cannot profit deviate from exiting. To establish existence, it thus suffices to check that no \( D_i \) can benefit from targeting its own market \( i \) only (this ensures that \( D_j \) would not benefit from doing this either, as continuation payoffs are the same but \( D_j \) would need to sink \( s \)). To win market \( i \), \( D_i \)
should offer a price \( p_i \) such that \( p_i + p_{Ej} \leq P_D \), yielding a deviation payoff at most equal to:

\[
\tilde{V}_i^D = (P_D - p_{Ej}) + \delta V_i = - (S - s) - \Delta^{Stag} < 0.
\]

The deviation is therefore not profitable.

Summing-up, (i) there always exists two equilibria, in which one incumbent (either one) wins both markets at price \( P_D \) and all firms obtains zero payoff, and (ii) if \( \sigma \leq \sigma^{Stag}(\delta) \), there also exists an equilibrium in which each \( D_i \) wins market \( i \) at price \( \tilde{p}_i \) and obtains \( \tilde{V}_D \). Furthermore: (i) if \( \sigma < \sigma^{Stag}(\delta) \), then \( \tilde{V}_D > 0 \) and the last equilibrium is therefore the unique coalition-proof Nash equilibria; if instead \( \sigma > \sigma^{Stag}(\delta) \), the two equilibria first described are both coalition-proof; finally, in the limit case where \( \sigma = \sigma^{Stag}(\delta) \), all three equilibria yield zero payoff and are coalition-proof.

C.2 Optimal policy

Breaking-up the incumbent

Conditional on opting for synchronous tenders, as before it is optimal to break up the historical incumbent: if the continuation equilibrium is dual-state, this yields lower stationary prices from period 0 onwards; if instead it is single-state, breaking up the historical incumbent does not affect the stationary prices from period 2 onwards, but yields lower prices in period 0.

It is straightforward to check that the same holds when opting instead for staggered tenders. In the absence of break-up, the competition game starts and thus remains forever in state \( I \). As \( p_i^{Stag} > p_{II}^{Stag} \), the long-term prices are then the highest possible ones. Furthermore, in period 0, the total price for the two markets is also higher in state \( I \): from the above analysis, in state \( I \) it is equal to \( P_E \), whereas in state \( II \) it is given by \( P_D < P_E \) if the continuation equilibrium is single-state, and is otherwise equal to \( \tilde{P} = \tilde{p}_A + \tilde{p}_A < P_E \).

Tendering regime

We now focus on the case where the incumbent has been broken-up, so that the first tenders take place in state \( II \), and compare the total bill for the two markets generated by the two tendering regimes, \( \mathcal{P}_V^{Sync} \) and \( \mathcal{P}_V^{Stag} \) (with the subscript \( V \) referring to the variant considered here). As in the baseline setting we assume that competition prevails whenever it is sustainable. Depending on whether the continuation equilibria are single- or dual-state, three cases can be distinguished.
Dual-state equilibrium in both regimes  If $\sigma < \sigma^{\text{Stag}}(\delta) < \sigma^{\text{Sync}}(\delta)$, a dual-state equilibrium arises under both tendering regimes. The total bill paid over time for the two markets is then equal to the sum of the incumbents’ total discounted payoffs in period 0. Under staggered tenders, using (38) the total bill is is equal to:

$$\mathcal{P}^{\text{Stag}}_V (DSE) = 2\tilde{V}_D = \frac{2}{1-2\delta} \left[ s - \delta (S + s) \right].$$

Under synchronous tenders, we have instead:

$$\mathcal{P}^{\text{Sync}}_V (DSE) = 2V_D = \frac{2}{1-2\delta^2} \left( s - 2\delta^2 S \right).$$

Staggered tendering thus yields a lower total bill:

$$\mathcal{P}^{\text{Sync}}_V (DSE) - \mathcal{P}^{\text{Stag}}_V (DSE) = 2\delta \frac{(1-\delta)^2 + \delta^2}{(1-2\delta)(1-2\delta^2)} (S - s) > 0.$$

Single-state equilibrium in both regimes  If instead $\sigma \geq \sigma^{\text{Sync}}(\delta) > \sigma^{\text{Stag}}(\delta)$, a single-state equilibrium arises under both tendering regimes. Competition among the incumbents induces them to bid up to the entire continuation value from becoming a monopolist forever; as the winning incumbent must incur a sunk cost $s$, it follows that, under both tendering regimes, the total bill is equal to $\mathcal{P}^{\text{Sync}}_V (SSE) = \mathcal{P}^{\text{Stag}}_V (SSE) = s$.

Single-state equilibrium under staggered tenders only  Finally, if $\sigma^{\text{Stag}}(\delta) \leq \sigma < \sigma^{\text{Sync}}(\delta)$, a single-state equilibrium arises under staggered tenders, implying again that the total bill is then equal to $s$, whereas a dual-state equilibrium arises under synchronous tenders. From the above, it follows that:

$$\mathcal{P}^{\text{Sync}}_V (DSE) - \mathcal{P}^{\text{Stag}}_V (SSE) = \frac{2}{1-2\delta} \left( s - 2\delta^2 S \right) - s = \frac{s - 2\delta^2 (2S - s)}{1-2\delta^2}.$$

It follows that synchronous tenders generate a lower total bill if and only if $\delta$ and/or $\sigma$ are large enough, namely:

$$\sigma \geq \hat{\sigma}_V (\delta) \equiv \frac{1 + 2\delta^2}{4\delta^2},$$

where $\hat{\sigma}_V (\delta)$ is decreasing in $\delta$ and lies between $\sigma^{\text{Stag}}(\delta)$ and $\sigma^{\text{Sync}}(\delta)$:

$$\hat{\sigma}_V (\delta) - \sigma^{\text{Stag}}(\delta) = \frac{1 + 2\delta^2}{4\delta^2} - \frac{1-\delta}{\delta} = \frac{1-4\delta + 6\delta^2}{4\delta^2} = \frac{(1-2\delta)^2 + 2\delta^2}{4\delta^2} > 0.$$
and:
\[
\sigma^{\text{sync}}(\delta) - \hat{\sigma}_V(\delta) = \frac{1}{2\delta^2} - \frac{1 + 2\delta^2}{4\delta^2} = \frac{1 - 2\delta^2}{4\delta^2} > 0,
\]
where the last inequality follows from the fact that, as already noted, \(1 < \sigma^{\text{Stag}}(\delta)\) implies \(2\delta^2 < 1\).

**Comparison with the baseline setting**

In our baseline setting, in order to maintain the same number of tenders in every periods, two cities (each with two markets) were liberalised and two of the four markets were opened with a one-period lag. We now show that the insights from the above analysis are similar to those obtained in the baseline setting. Furthermore, adjusting for the fact that half of the markets are opened with a one-period delay, in most instances the two variants actually deliver the same average bill per city:

- If \(\sigma < \sigma^{\text{Stag}}(\delta) < \sigma^{\text{sync}}(\delta)\) (i.e., both tendering regimes yield a dual-state equilibrium), both variants favour staggered tenders and deliver the bill:
  \[
  \frac{1 + \delta}{2} P^{\text{sync}}_V(DSE) > \frac{1 + \delta}{2} P^{\text{Stag}}_V(DSE) = \frac{1 + \delta}{1 - 2\delta} [s - \delta (S + s)],
  \]
  \[
  P^{\text{sync}}(DSE) > P^{\text{Stag}}(DSE) = \frac{1 + \delta}{1 - 2\delta} [s - \delta (S + s)].
  \]

- If \(\delta \geq \delta^{\text{sync}}(\sigma) \geq \delta^{\text{Stag}}(\sigma)\) (i.e., both tendering regimes yield a single-state equilibrium), it is optimal to opt for synchronous tenders under both variants, and both variants then deliver the same bill:
  \[
  \frac{1 + \delta}{2} P^{\text{Stag}}_V(SSE) = \frac{1 + \delta}{2} P^{\text{sync}}_V(SSE) = \frac{1 + \delta}{2} s,
  \]
  \[
  P^{\text{Stag}}(SSE) \geq P^{\text{sync}}(SSE) = \frac{1 + \delta}{2} s.
  \]
However, in the new variant, staggered tenders would also deliver the same bill.

- If \(\sigma^{\text{Stag}}(\delta) \leq \sigma < \sigma^{\text{sync}}(\delta)\), staggered tenders yield a single-state equilibrium, whereas synchronous tenders yield a dual-state equilibrium. Synchronous tenders are optimal when \(\delta\) and/or \(\sigma\) are large enough, and the new variant tilts the balance in favour or staggered tenders: \(\hat{\sigma}_V(\delta) > \hat{\sigma}(\delta)\).

Three sub-cases can thus be distinguished:

\[^{54}\text{This amounts to } (1 - 2\delta^2) (1 - 2\delta + 4\delta^2) > 0.\]
If \( \sigma < \hat{\sigma}(\delta) < \sigma_V(\delta) \), both variants favour staggered tenders and deliver the same bill:

\[
\frac{1 + \delta}{2} \mathcal{P}_{S synced}^{sync}(DSE) > \frac{1 + \delta}{2} \mathcal{P}_{V}^{stag}(SSE) = (1 + \delta) \frac{s}{2},
\]

\[
\frac{\mathcal{P}_{sync}(DSE)}{2} \geq \frac{\mathcal{P}_{stag}(SSE)}{2} = (1 + \delta) \frac{s}{2}.
\]

If instead \( \sigma > \sigma_V(\delta) \), both variants favour synchronous tenders—and only those, contrary to the region where \( \delta \geq \delta^{sync}(\sigma) \)—and deliver the same bill:

\[
\frac{1 + \delta}{2} \mathcal{P}_{sync}(DSE) > \frac{1 + \delta}{2} \mathcal{P}_{stag}(SSE) = \frac{1 + \delta}{1 - 2\delta^2} (s - 2\delta^2 S),
\]

\[
\mathcal{P}_{stag}(DSE) > \mathcal{P}_{syn}^{sync}(DSE) = \frac{1 + \delta}{1 - 2\delta^2} (s - 2\delta^2 S).
\]

Finally, if \( \hat{\sigma}(\delta) < \sigma < \sigma_V(\delta) \), synchronous tenders are favoured in the original variant, whereas staggered tenders are favoured in the new variant:

\[
\frac{1 + \delta}{2} \mathcal{P}_{V}^{stag}(SSE) > \frac{1 + \delta}{2} \mathcal{P}_{V}^{stag}(SSE) = (1 + \delta) \frac{s}{2},
\]

\[
\mathcal{P}_{stag}(SSE) > \mathcal{P}_{syn}^{sync}(DSE) = \frac{1 + \delta}{1 - 2\delta^2} (s - 2\delta^2 S).
\]

The new variant delivers a lower total bill in that case:

\[
\frac{1 + \delta}{2} \mathcal{P}_{V}^{stag}(SSE) - \mathcal{P}_{syn}^{sync}(DSE) = \frac{(1 + \delta) \delta^2 s}{2 (1 - 2\delta^2)} [\sigma - \hat{\sigma}_V(\delta)] < 0.
\]

D Welfare: Proof of Proposition 6

Total welfare can be expressed as

\[
W \equiv \omega - (1 - \alpha) \mathcal{P} - \alpha C.
\]

In each tendering regime \( \tau \in \{Sync, Stag\} \), it is clearly optimal to break up the incumbent whenever the equilibrium is dual-state (i.e., \( \sigma \leq \sigma^*(\delta) \)): this does not generate any sunk cost, as each market then remains forever serviced by the same firm (hence, \( C = 0 \) no matter what), and reduces the price, which is stationary and lower under competition than under monopoly \( (p_{II} < p_I) \).

When instead the equilibrium is single-state i.e., \( \sigma > \sigma^*(\delta) \), breaking up the incumbent has no impact on long-term prices, which remain at the monopoly level, but affects the
outcome of the initial tender, where it creates a fight for monopolisation: this yields lower price(s) but the winning incumbent must incur a cost $s$. It follows that, in each tendering regime $\tau \in \{\text{Sync}, \text{Stag}\}$, breaking up the incumbent is optimal if the weight $\alpha$ is not too large, namely, if:

$$\alpha s \leq (1 - \alpha) \Delta_p^\tau, \tag{39}$$

where $\Delta_p^\tau$ denotes the average price increase in the initial tendering period, induced by a switch from competition to monopoly; from (21) and (22), we have:

$$\Delta_p^{\text{Stag}} = p_1^{\text{Stag}}(SSE) - p_{II}^{\text{Stag}}(SSE) = S,$$

and, from (27) and (30):\(^{55}\)

$$\Delta_p^{\text{Sync}} = 2 \left[ p_1^{\text{Sync}}(SSE) - p_{II}^{\text{Sync}}(SSE) \right] = 2S - s.$$

Condition (39) thus amounts to $\sigma \geq \tilde{\sigma}^\tau (\alpha)$, where:

$$\tilde{\sigma}^{\text{Stag}} (\alpha) \equiv \frac{\alpha}{1 - \alpha} \text{ and } \tilde{\sigma}^{\text{Sync}} (\alpha) \equiv \frac{1}{2(1 - \alpha)}.$$

The condition $\sigma \geq \tilde{\sigma}^\tau (\alpha)$ is trivially satisfied for $\alpha \leq 1/2$,\(^ {56}\) in which case breaking up the incumbent remains optimal. When instead $\alpha > 1/2$, breaking up the incumbent is optimal unless $(\sigma, \delta)$ lies in a range defined by $\sigma^\tau(\delta) < \sigma \leq \tilde{\sigma}^\tau(\alpha)$ which increases with $\alpha$ but is smaller under synchronous tendering, as $\sigma^{\text{Sync}}(\delta) > \sigma^{\text{Stag}}(\delta)$ and $\tilde{\sigma}^{\text{Sync}}(\alpha) < \tilde{\sigma}^{\text{Stag}}(\alpha)$ for $\alpha > 1/2$.

We now turn to the second part of Proposition 6. When $\sigma \leq \sigma^{\text{Stag}}(\sigma) \left( < \sigma^{\text{Sync}}(\delta) \right)$, the equilibrium is dual-state under both tendering regimes; hence, it is always optimal to break up the incumbent, and no sunk costs are ever incurred. From Proposition 5, staggered tendering is therefore optimal, as it delivers a lower total bill: $\mathcal{P}^{\text{Stag}}(DSE) < \mathcal{P}^{\text{Sync}}(DSE)$.

When instead $\sigma > \sigma^{\text{Sync}}(\delta) \left( > \sigma^{\text{Stag}}(\sigma) \right)$, the equilibrium is single-state under both tendering regimes; hence, breaking up the incumbent in regime $\tau \in \{\text{Sync}, \text{Stag}\}$ is optimal only if $\sigma \geq \tilde{\sigma}^\tau(\alpha)$. However, as in the baseline setting, synchronous tendering is always optimal, as: (i) when breaking up the incumbent, synchronous tendering yields a lower average cost, by postponing the sunk cost $s$ in one city; and (ii) regardless of the break-up decision, synchronous tendering delivers a lower bill: in case of break-up, the

\(^{55}\)Recall that implementing synchronous tenders requires one city to start tendering its markets (both of them) in period 0, and the other to start doing the same in period 1.

\(^{56}\)Specifically, $\tilde{\sigma}^\tau(\alpha)$ lies below 1 for $\alpha \leq 1/2$. 

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total bill involves competitive prices in the initial period followed forever by monopoly prices and, from Proposition 5, \( \mathcal{P}_{\text{Sync}}(\text{SSE}) < \mathcal{P}_{\text{Stag}}(\text{SSE}) \); in the absence of break-up, monopoly prices prevail from the start and, from Proposition 3, \( p_{\text{Sync}}^{S} < p_{\text{Stag}}^{S} \). It follows that synchronous tenders dominate staggered ones even when adopting the optimal break-up decision for staggered tender, and therefore a fortiori dominate them when optimising over the break-up decision for synchronous tenders.

Finally, when \( \sigma_{\text{Stag}}^{\delta} < \sigma \leq \sigma_{\text{Sync}}(\delta) \), the equilibrium is dual-state under synchronous tenders (hence, in this regime the incumbent is broken up and no sunk cost is ever incurred) but single-state under staggered tenders. Therefore:

- As long as \( \sigma \leq \tilde{\sigma}_{\text{Stag}}(\alpha) \), not breaking up the incumbent is optimal under staggered tenders; hence, in both regimes: (i) no sunk cost is ever incurred, and (ii) stationary prices prevail from the start. From Proposition 3, synchronous tendering is therefore optimal, as duopoly prices are lower than monopoly ones \( (p_{II}^{\text{Sync}} < p_{II}^{\text{Stag}}) \).

- If instead \( \sigma > \tilde{\sigma}_{\text{Stag}}(\alpha) \), the incumbent is broken up under staggered tenders as well, which generates a social cost \( \alpha s \). For \( \alpha = 0 \), we know from Proposition 5 that synchronous tendering is optimal (i.e., \( \mathcal{P}_{\text{Sync}}(\text{DSE}) \leq \mathcal{P}_{\text{Stag}}(\text{SSE}) \)) for \( \sigma \geq \hat{\sigma}(\delta) \). As \( \alpha \) increases, welfare decreases under staggered tendering, due to the social cost \( \alpha s \); hence, synchronous tendering becomes optimal for a wider range of \( \sigma \) and/or \( \delta \). Specifically, synchronous tendering is optimal if and only if:

\[
0 \leq \alpha s + (1 - \alpha) \left[ \mathcal{P}_{\text{Stag}}(\text{SSE}) - \mathcal{P}_{\text{Sync}}(\text{DSE}) \right] = \frac{(1 + 2\delta) \delta (1 - \alpha) S - \left[ 1 + 2\delta^3 - 2(1 - \delta^2 + \delta^3) \alpha \right] s}{1 - 2\delta^2},
\]

where the denominator of the last expression is positive, as \( 1 < \sigma < \sigma_{\text{Sync}}(\delta) \) implies \( \delta < \delta \equiv \sqrt{2}/2 \). Hence, synchronous tendering is optimal when \( \sigma \) is large enough, namely, when \( \sigma > \tilde{\sigma}(\delta; \alpha) \), where:

\[
\tilde{\sigma}(\delta; \alpha) \equiv \frac{1 + 2\delta^3 - 2\alpha (1 - \delta^2 + \delta^3)}{(1 - \alpha) \delta (1 + 2\delta)}.
\]
This function satisfies $\hat{\sigma}(\cdot; 0) = \hat{\sigma}(\cdot, \cdot) = 1$ and, for $\delta < \hat{\delta}$:

$$\frac{\partial \hat{\sigma}}{\partial \alpha} (\delta, \alpha) = -\frac{1 - 2\delta^2}{(1 - \alpha)^2 \delta (1 + 2\delta)} < 0.$$ 

In addition, for $\alpha = 1/3$ we have $\hat{\sigma}(1/2; 1/3) = 1$ and:

$$\sigma^{Stag}(\delta) - \hat{\sigma}(\delta, 1/3) = \frac{(1 - 2\delta)(1 + 4\delta + 2\delta^2)}{2\delta (1 + 2\delta)}, \quad 1 - \hat{\sigma}(\delta, 1/3) = \frac{(2\delta - 1)(1 - 2\delta^2)}{2\delta (1 + 2\delta)},$$

implying that $\hat{\sigma}(\delta; 1/3)$ lies below $\sigma^{Stag}(\delta)$ for $\delta \leq 1/2$ and below 1 for $1/2 \leq \delta \leq \tilde{\delta}$. Summing-up, $\hat{\sigma}(\cdot; \alpha)$ coincides with $\hat{\sigma}(\cdot)$ for $\alpha = 0$, is decreasing in $\alpha$, and lies below $\sigma^{Stag}(\cdot)$ for $\alpha \geq 1/3$.

It follows that, in the range $\sigma^{Stag}(\delta) < \sigma \leq \sigma^{Sync}(\delta)$, synchronous tendering is always optimal if $\alpha \geq 1/3$; if instead $\alpha < 1/3$, synchronous tendering is optimal for $\sigma \geq \hat{\sigma}(\delta; \alpha) \equiv \min\{\hat{\sigma}(\delta; \alpha), \sigma^{Stag}(\delta)\}$, where the threshold $\hat{\sigma}(\delta; \alpha)$ coincides with $\hat{\sigma}(\delta) = \min\{\hat{\sigma}(\delta), \sigma^{Stag}(\delta)\}$ for $\alpha = 0$, is decreasing in $\alpha$ as long as it remains above $\sigma^{Stag}(\delta)$, and coincides with $\sigma^{Sync}(\delta)$ for $\alpha = 1/3$.

### E Biasing the tenders

We consider here an extension in which entrants benefits from a favourable bias $b$ that enables them to exert a stronger competitive pressure on the incumbents (i.e., $b > 0$), but not so large that they could overtake the incumbent currently operating the market (i.e., $b < S$). The bias $b$ can for instance be obtained by granting a bidding credit $b$ to every entrant and/or by handicapping the current operator by $b$; either way, it means that the winner obtains the actual price it bid, but the current operator must underbid every bidding entrant by at least $b$ in order to win. In the case of synchronous tenders, a bias of $2b$ applies to bundled bids.

The following two propositions show that, in both tendering regimes, introducing a large enough bias expands the scope for sustainable competition and reduces equilibrium prices. Furthermore, when the bias tends to offset perfectly the incumbent’s advantage (i.e., $b$ tends to $S$), the range in which monopolisation arises tends to vanish, and all equilibrium prices tend to cost.

$^{57}$By definition, $2\delta^2 = 1$ and:

$$\hat{\sigma}(\delta; \alpha) \big|_{2\delta^2 = 1} = \frac{1 + 2\delta^3 - 2\alpha(1 - \delta^2 + \delta^3)}{(1 - \alpha)\delta(1 + 2\delta)} \big|_{2\delta^2 = 1} = \frac{1 + \delta - \alpha(1 + \delta)}{(1 - \alpha)(1 + \delta)} = 1.$$
Under staggered tenders, we focus on the case where the bias is large enough to enable entrants to exert a stronger competitive pressure than a neighbouring incumbent; that is, $b > S - s$. We have:

**Proposition 10** (biased staggered tenders). *Under staggered tendering, for $b \in (S - s, S)$ there exists a unique coalition-proof Markov equilibrium outcome, which can be of two types:

- If the first tender takes place in state I and $\sigma \geq \sigma_{Stag}(\delta; b/S)$, where
  \[
  \sigma_{Stag}(\delta; \frac{b}{S}) \equiv \frac{1 - \delta}{\delta \left(1 - \frac{b}{S}\right)},
  \]
  then the equilibrium is single-state; the same firm then services both markets forever and the equilibrium price is
  \[
  p_{Stag}^{I}(SSE; b) \equiv (1 - \delta) \left[(1 + \delta) (S - b) + \delta s\right].
  \]
- If instead the first tender takes place in state II or $\sigma < \sigma_{Stag}(\delta; b/S)$, the equilibrium is dual-state; the same firm then services both markets forever and the equilibrium price is
  \[
  p_{Stag}^{II}(DSE; b) \equiv (1 - \delta^2) (S - b).
  \]

Furthermore, as $b$ tends to $S$, the range in which the first equilibrium can arise tends to vanish, and all equilibrium prices tend to zero.

*Proof.* See Section F below.

In the case of synchronous tenders, to ensure again that entrants exert a stronger competitive pressure than a neighbouring incumbent, we focus on the case where $2(S - b) < s$, which amounts to $b > S - s/2$. We have:

**Proposition 11** (biased synchronous tenders). *Under synchronous tendering, for $b \in (S - s/2, S)$ there exists a unique coalition-proof Markov equilibrium outcome, which is dual-state: the same firms service the markets forever and the equilibrium per market price is

\[
 p_{Sync}^I(b) \equiv (1 - \delta^2) (S - b),
 \]

which tends to zero as $b$ tends to $S$.

*Proof.* See Section G below.
F Proof of Proposition 10

We consider here the case of staggered tenders in which entrants benefit from a favourable bias \( b \in (S - s, S) \). We first note that, as before, equilibrium continuation values cannot be negative, as any firm can secure a non-negative payoff by offering above-cost prices. Furthermore, entrants necessarily obtain \( V_E = 0 \), as \( V_E > 0 \) would require winning a market with a positive margin, in which case any losing entrant could profitably undercut any winning one.

F.1 Coalition-proof Nash equilibria

We first characterise the coalition-proof Nash equilibria for given continuation values satisfying \( V_E = 0 \) and \( V_i \geq 0 \) for \( i = M, D, C \). We start by characterising firms’ best offers, before studying the equilibrium outcomes in each state.

Best offers

In both states, a potential entrant obtains \( p - S + \delta V_C \) if it wins at price \( p \), and 0 if it loses; hence, potential entrants are willing to lower their prices down to

\[
p_E \equiv S - \delta V_C.
\]  

(40)

In state I, \( M \) obtains instead \( p + \delta V_M \) if it wins at price \( p \) and \( \delta V_D \) if it loses; hence, it is willing to lower its price down to

\[
p_M \equiv -\delta (V_M - V_D).
\]  

(41)

In state II, \( D \) obtains \( p + \delta V_C \) if it wins at price \( p \), and 0 if it loses; hence, it is willing to lower its price down to

\[
p_D \equiv -\delta V_C.
\]  

(42)

\( C \) obtains instead \( p - s + \delta V_M \) if it wins at price \( p \), and \( \delta V_D \) if it loses; hence, it is willing to lower its price down to

\[
p_C \equiv s - \delta (V_M - V_D).
\]  

(43)

It is useful to note that:

\[
p_M < p_C.
\]  

(44)

and, as \( b < S \):

\[
p_D < p_E - b.
\]  

(45)
State I

We first show that \( M \) is willing to outbid the entrants:

**Lemma 9** (\( M \) makes a better offer - biased tenders). The best offers satisfy:

\[ p_E - b > p_M. \]

*Proof.* Suppose that \( p_E - b \leq p_M \). Using (45) and (44), this yields:

\[ p_D < p_E - b \leq p_M < p_C. \]

Therefore, in state II, \( D \) wins (as its best offer is both lower than \( C \)’s, and lower than \( E \)’s by more than \( b \)), at a price not exceeding \( p_E - b \) (otherwise, any entrant could profitably undercut \( D \)); \( C \) thus obtains \( V_C = \delta V_D \), where

\[ V_D \leq p_E - b + \delta V_C = S - b. \]  \hspace{1cm} (46)

Furthermore, in state I, either \( M \) loses for sure (if \( p_E < p_M + b \)) or competition drives prices down to \( p_M \) (if \( p_E = p_M + b \)), in which case \( M \) is indifferent between winning or not; in both cases, we have: \( V_M = \delta V_D = V_C \). Using (40) and (41), \( p_E - b \leq p_M \) then amounts to \( S - b \leq \delta V_D \), which, together with (46), implies \( S - b \leq \delta (S - b) \), a contradiction. \( \square \)

Lemma 9 implies that \( M \) prevails in state I, which leads to:

**Lemma 10** (state I for staggered tenders - biased tenders). In state I, there exists a unique coalition-proof Nash equilibrium outcome, in which \( M \) wins at price \( p_E - b \) and obtains \( \tilde{V}_M = S - b - \delta V_C + \delta V_M \).

*Proof.* As already noted, in equilibrium the entrants obtain \( V_E = 0 \). Furthermore, \( M \) cannot charge more than \( p_E - b \), otherwise any entrant could profitably undercut it. Conversely, \( M \) offering \( p_E - b \) and all entrants offering \( p_E \), together with the auctioneer assigning the market to \( M \), constitutes a Nash equilibrium, as no firm could profitably increase its price, which would lead to exit, and no firm can profitably decrease its price either: an entrant would make a loss, and \( M \) would make a lower profit. The same holds as long as at least one entrant offers \( p_E \), and no entrant undercuts that price.

It follows that there is a unique coalition-proof Nash equilibrium outcome, in which \( M \) wins at price \( p_E - b \). Using (40), \( M \)'s payoff is then equal to:

\[ \tilde{V}_M = p_E - b + \delta V_M = S - b - \delta V_C + \delta V_M. \]

\( \square \)
Lemma 10 provides a partial characterisation of $M$’s equilibrium payoff which confirms that, in any coalition-proof equilibrium, the incumbents obtain (weakly) lower payoffs in state $II$:

**Corollary 6** (state $I$ for staggered tenders - biased tenders). *The equilibrium continuation values are such that:*

$$V_M = \frac{S - b - \delta V_C}{1 - \delta},$$

(47)

and

$$\Delta \equiv \delta (V_M - V_D - V_C) \geq 0.$$

**Proof.** In any coalition-proof equilibrium, we must have $V_M = \tilde{V}_M = S - b - \delta V_C + \delta V_M$, which yields (47). Furthermore, no equilibrium price can exceed $p_E - b$, otherwise any entrant could profitably undercut it. Hence, in state $II$, we have:

$$V_D + V_C \leq \sum_{t=0}^{+\infty} \delta^t (p_E - b) = V_M,$$

where the equality follows from (40) and (47).

**State II**

We first show that the entrants are willing to outbid the current operator:

**Lemma 11** (relevant competition - biased tenders). *If $D$ wins, the main competitive pressure comes from the entrants, that is:*

$$p_D < p_E - b \leq p_C.$$

**Proof.** Suppose to the contrary that $D$ wins and $C$ constitutes an effective source of competition, i.e.:

$$p_D \leq p_C < p_E - b.$$

It follows that there is a unique coalition-proof Nash equilibrium, in which $D$ wins at price $p_C$; hence $C$ and $D$ respectively obtain

$$\tilde{V}_C = \delta V_D \text{ and } \tilde{V}_D = p_C + \delta V_C = s - \Delta.$$

In equilibrium, we then have:

$$V_C = \tilde{V}_C = \delta V_D \text{ and } V_D = \tilde{V}_D = p_C + \delta V_C = s - \Delta.$$
Together with (47), this yields:

\[
V_M = \frac{(1 - \delta - \delta^2) (S - b) - \delta^2 s}{1 - 2\delta},
\]

\[
V_D = \frac{(1 - \delta) s - \delta (S - b)}{1 - 2\delta},
\]

\[
V_C = \frac{\delta (1 - \delta) s - \delta (S - b)}{1 - 2\delta},
\]

and

\[
p_C - (p_E - b) = s - \delta (V_M - V_D) - (S - \delta V_C - b)
\]

\[
= s - \delta \left( \frac{(1 - \delta - \delta^2) (S - b) - \delta^2 s}{1 - 2\delta} - \frac{(1 - \delta) s - \delta (S - b)}{1 - 2\delta} \right)
\]

\[
- \left( S - \delta \delta \frac{(1 - \delta) s - \delta (S - b)}{1 - 2\delta} - b \right)
\]

\[
= \frac{1 - \delta}{1 - 2\delta} [b - (S - s)] > 0,
\]

contradicting the working assumption \( p_C < p_E - b \).

Lemma 11 implies that, in state \( II \), either \( C \) wins at price \( p_D \) or \( D \) wins at price \( p_E - b \). Building on this leads to:

**Lemma 12 (state \( II \) for staggered tenders).** In state \( II \), the coalition-proof Nash equilibrium outcomes are as follows:

- If \( \Delta < s \), there exists a unique coalition-proof Nash equilibrium outcome, in which \( D \) wins at price \( p_E - b = S - b - \delta V_C \) and obtains \( \tilde{V}_D = S - b \), whereas \( C \) obtains \( \tilde{V}_C = \delta V_D \).

- If instead \( \Delta > s \), there exists a unique coalition-proof Nash equilibrium outcome, in which \( C \) wins at price \( p_D = -\delta V_C \) and obtains \( \tilde{V}_C = \delta (V_M - V_C) - s \), whereas \( D \) obtains \( \tilde{V}_D = 0 \).

- Finally, in the boundary case where \( \Delta = s \), there are infinitely many coalition-proof Nash equilibrium outcomes, in which either incumbent wins at price \( p_C = p_D \), and both obtain \( \tilde{V}_C = \tilde{V}_D = 0 \); these outcomes only differ in the probabilities that either incumbent wins, which can take any arbitrary values.

**Proof.** If \( \Delta < s \), then \( p_D < p_C \), which, together with (45), implies \( D \) wins; from Lemma 11, we must therefore have:

\[
p_D < p_E - b \leq p_C.
\]
Hence, the unique coalition-proof equilibrium outcome is such that $D$ wins by matching $p_E - b = S - b - \delta V_C$; the associated payoffs are:

$$\tilde{V}_C = \delta V_D \text{ and } \tilde{V}_D = p_E - b + \delta V_C = S - B.$$ 

If instead $\Delta > s$, then $p_C < p_D < p_E - b$ and the unique coalition-proof equilibrium outcome is such that $C$ wins by matching $p_D$; the associated payoffs are:

$$\tilde{V}_D = 0 \text{ and } \tilde{V}_C = p_D - s + \delta V_M = \Delta - s.$$ 

Finally, if $\Delta = s$, then $p_D = p_C$ and the Nash equilibria are such that either incumbent wins at that price, with arbitrary probability; as firms always obtain the same zero payoff, all the equilibria are coalition-proof. If instead $\Delta = s - (S - b)$, \[\square\]

**F.2 Equilibrium characterisation**

In equilibrium, in state $II$ the payoffs $\tilde{V}_D$ and $\tilde{V}_C$ must coincide with the continuation values $V_D$ and $V_C$. Furthermore, if $D$ wins, then the equilibrium path remains forever in the initial state (dual-state equilibrium). If instead $C$ wins in state $II$, then the equilibrium switches to state $I$ forever (single-state equilibrium). We consider in turn these two types of equilibrium.

**Dual-state equilibrium**

From Lemma 12, a dual-state equilibrium exists if and only if $\Delta \leq s$, in which case we must have:

$$V_D = \tilde{V}_D = V_D^{Stag}(SSE; b) \equiv S - b \text{ and } V_C = \tilde{V}_C = V_C^{Stag}(SSE; b) \equiv \delta (S - b),$$

and, using (47):

$$V_M^{Stag}(SSE; b) = (1 + \delta) (S - b) = V_D^{Stag}(SSE; b) + V_C^{Stag}(SSE; b).$$

Conversely, these continuation values yield

$$p_D = p_M = -\delta^2 (S - b) < p_E - b = (1 - \delta^2) (S - b) < p_C = s - \delta^2 (S - b).$$
implying that there always exists a dual-state equilibrium, in which in both states the current incumbent wins the market at a price equal to:

\[ p_E - b = p_{\text{Stag}}^{\text{DSE}; b} \equiv (1 - \delta^2) (S - b). \]  

(48)

**Single-state equilibrium**

From Lemma 12, a single-state equilibrium exists if and only if \( \Delta \geq s \), in which case we must have:

\[ V_D = \tilde{V}_D = 0 \quad \text{and} \quad V_C = \tilde{V}_C = \delta (V_M - V_C) - s. \]

Together with (47), the continuation values are respectively given by:

\[ V_{\text{Stag}}^M(\text{SSE}; b) \equiv (1 + \delta) (S - b) + \delta s, \]

\[ V_{\text{Stag}}^D(\text{SSE}; b) \equiv 0, \]

\[ V_{\text{Stag}}^C(\text{SSE}; b) \equiv \delta (S - b) - (1 - \delta) s, \]

The value of monopolization is therefore equal to

\[ \Delta_{\text{Stag}}(\text{SSE}; b) \equiv \delta (S - b + s), \]

and satisfies \( \Delta \geq s \) if and only if:

\[ \sigma \geq \sigma_{\text{Stag}}^{\text{SSE}}(\delta; b/S) \equiv \frac{1 - \delta}{\delta (1 - b/S)}, \]

where \( \sigma_{\text{Stag}}(\delta; b/S) \), increases with \( b \) and tends to \(+\infty\) as \( b \) tends to \( S \). Conversely, whenever \( \sigma \geq \sigma_{\text{Stag}}(\delta; b/S) \), these continuation values satisfy the condition \( \Delta \geq s \), which in turn ensures that the continuation values are all non-negative. Hence, whenever \( \sigma \geq \sigma_{\text{Stag}}(\delta; b/S) \), there exists a single-state equilibrium, in which from period 1 onward the equilibrium path remains forever in state \( I \). The prices in the two states are respectively given by:

\[ p_{\text{I}}^{\text{Stag}}(\text{SSE}; b) = p_D = p_{\text{I}}^{\text{Stag}}(\text{SSE}; b) \equiv (1 - \delta) [\delta s - \delta^2 S]. \]  

(49)

\[ p_{\text{II}}^{\text{Stag}} = p_E - b = p_{\text{II}}^{\text{Stag}}(\text{SSE}; b) \equiv (1 - \delta) [(1 + \delta) (S - b) + \delta s], \]  

(50)

**Recap**

Note that \( V_{\text{D}}^{\text{Stag}}(\text{DSE}; b) > 0 = V_{\text{C}}^{\text{Stag}}(\text{SSE}; b) \) and

\[ V_{\text{C}}^{\text{Stag}}(\text{SSE}; b) - V_{\text{C}}^{\text{Stag}}(\text{SSE}; b) = (1 - \delta) s > 0. \]
By contrast:

\[ V_{M}^{\text{Stag}}(SSE; b) - V_{M}^{\text{Stag}}(SSE; b) = -\delta s < 0. \]

Hence, the dual-state equilibrium yields higher profits in state II, but lower ones in state I. Summing-up, we have:

- If the first tender takes place in state II, there exists a unique coalition-proof Markov perfect equilibrium outcome, which is dual-state: the equilibrium path thus remains forever in state II and the price is given by (48), which tends to zero as \( b \) tends to \( S \).

- If instead the first tender takes place in state I, then:
  - if \( \sigma < \sigma_{\text{Stag}}^{\text{Stag}}(\delta, s; b) \), the above equilibrium still constitutes the unique coalition-proof Markov perfect equilibrium;
  - if instead \( \sigma \geq \sigma_{\text{Stag}}^{\text{Stag}}(\delta, s; b) \), there exists a unique coalition-proof Markov perfect equilibrium outcome, which is single-state: the equilibrium path remains forever in state I and the price, \( p_{I}^{\text{Stag}}(SSE, b) \), is given by (49), which, using \( \sigma \leq \sigma_{\text{Stag}}^{\text{Stag}}(\delta, s; b) \), satisfies:

\[
p_{I}^{\text{Stag}}(SSE, b) \leq \frac{(1 + \delta)(S - b) + \delta s}{S - b + s}.
\]

Hence, as \( b \) tends to \( S \), the range in which this equilibrium exists tend to disappear, as \( \sigma_{\text{Stag}}^{\text{Stag}}(\delta; b/S) \) tends to infinity, and the equilibrium price tends to zero.

**G Proof of Proposition 11**

We now consider the case of synchronous tenders in which entrants benefit from a favourable bias \( b \) to enter any market – in case of bundle bids, the incumbents thus need to undercut the entrants by at least \( 2b \); we focus on the case where \( b \) is close enough to \( S \), namely,

\[ b \in (S - \frac{s}{2}; S), \tag{51} \]

to ensure that entrants exert a competitive pressure on incumbents in state II: \( 2(S - b) < s \).

As before, equilibrium continuation values cannot be negative, as any firm can secure a non-negative payoff by offering above-cost prices, and competition among entrants implies that they necessarily obtain \( V_{E} = 0 \). We denote again by \( V_{M} \) the continuation value of
the incumbent in state \( I \); in state \( II \), in which two incumbent firms compete with each other as well as with potential entrants, we now denote the incumbent servicing market \( i = A, B \) by \( D_i \) and its continuation value by \( V_i \).

G.1 Coalition-proof Nash equilibria

We first characterise the coalition-proof Nash equilibria for given continuation values satisfying \( V_E = 0 \) and \( V_i \geq 0 \) for \( i = M, A, B \). We start by characterising potential entrants’ best offers, which are the same in both states, before studying the equilibrium outcomes in each state.

Entrants’ best offers

A potential entrant obtains \( P - 2S + \delta^2V_M \) if it wins both markets at total price \( P \), \( p_i - S + \delta^2V_i \) if it wins market \( i \) at price \( p_i \), and 0 if it loses both markets. Hence, potential entrants are willing to service market \( i \) for a stand-alone price

\[
p_{Ei} \equiv S - \delta^2V_i, \tag{52}
\]

or both markets for a bundled price

\[
P_E \equiv 2S - \delta^2V_M. \tag{53}
\]

State \( I \)

We now show that, in state \( I \), there exists a unique coalition-proof Nash equilibrium outcome:

**Lemma 13** (state \( I \) for synchronous tenders). In state \( I \), there exists a unique coalition-proof Nash equilibrium outcome, in which \( M \) wins both markets at total price \( P_E - 2b \) and obtains \( \tilde{V}_M = 2(S - b) \).

**Proof.** As already noted, in equilibrium the entrants obtain \( V_E = 0 \). Furthermore, \( M \) cannot obtain more than \( \tilde{V}_M = 2(S - b) \), otherwise any entrant could profitably undercut it. Specifically, if \( M \) loses both markets, it would obtain \( \tilde{V}_M = 0 < 2(S - b) \). If instead \( M \) wins market \( i \) (and only that one), it could not do so at a price exceeding \( p_{Ei} - b \), and thus cannot obtain more than \( \tilde{V}_M = p_{Ei} - b + \delta^2V_i = S - b < 2(S - b) \). Finally, if \( M \) wins both markets, it cannot do so at a total price exceeding \( P_E - 2b \), and thus cannot obtain more than \( \tilde{V}_M = P_E - 2b + \delta^2V_M = 2(S - b) \).
To establish existence, consider a candidate equilibrium in which all entrants offer the bundle price $P_E$ and, for each market $i = A, B$, the stand-alone price

$$\hat{p}_i = \max\{p_{Ei}, P_E - p_{Ej}\},$$

whereas $M$ offers the bundle price $P_E - 2b$ and stand-alone prices $\hat{p}_i - b$ for $i = A, B$. As $\hat{p}_A + \hat{p}_B \geq P_E$, the auctioneer is willing to assign both markets to $M$, which gives $M$ a payoff equal to $P_E - 2b + \delta^2 V_M = 2(S - b) = \tilde{V}_M$. No firm can benefit from increasing any of its prices, as this can only induce exit, or from decreasing its bundle price: the entrants would make a loss, and $M$ would lower its profit. Furthermore, in order to win market $i$ on a stand-alone basis, an entrant must charge a price $p_i$ such that $p_i + \hat{p}_j \leq P_E$ (so as to undercut the bundle price $P_E$ by “teaming up” with the offered stand-alone price $\hat{p}_j$ for market $j$), that is:

$$p_i \leq P_E - \hat{p}_j \leq P_E - (P_E - p_{Ei}) = p_{Ei},$$

where the second inequality follows from the definition of $\hat{p}_j$, implying that it cannot profitably do so; likewise, to win market $i$ $M$ must charge a price $p_i$ such that $p_i + \hat{p}_j - b \leq P_E - 2b$, which amounts to $p_i \leq p_{Ei} - b$, and thus cannot benefit from such a deviation, as this would yield at most $p_{Ei} - b + \delta^2 V_i = S - b < 2(S - b) = \tilde{V}_M$.

It directly follows from Lemma 13 that, in any coalition-proof equilibrium, the incumbents obtain again (weakly) lower payoffs in state $II$:

**Corollary 7** (the value of monopolisation for synchronous tenders). The equilibrium continuation values are such that:

$$V_M = V_M^{\text{Sync}}(b) \equiv 2(S - b) > 0,$$

implying

$$P_E - 2b = P_E^{\text{Sync}} - 2b \equiv (1 - \delta^2) V_M^{\text{Sync}}(b) > 0,$$

and

$$\Delta^{\text{Sync}}(b) \equiv \delta^2 (V_M^{\text{Sync}}(b) - V_A - V_B) \in [0, 2(S - b)].$$

Proof. In any coalition-proof equilibrium, we must have $V_M = \tilde{V}_M = 2(S - b)$. Furthermore, the total equilibrium price for the two markets can never exceed $P_E - 2b$, otherwise any entrant could profitably undercut it. Hence, in state $II$, we have:

$$V_A + V_B \leq \sum_{t=0}^{+\infty} \delta^{2t} (P_E - 2b) = V_M^{\text{Sync}}(b),$$

29
where the equality follows from (53) and $V_M^{Sync}(b) = 2(S - b)$; hence, $\Delta^{Sync}(b) \geq 0$. Finally, we have:

$$\Delta^{Sync}(b) \leq \delta^2 V_M^{Sync}(b) < 2(S - b),$$

where the first inequality stems from $V_A + V_B \geq 0$ and $\delta > 0$, and the second one from $V_M^{Sync}(b) = 2(S - b)$ and $\delta < 1$.

State II

Building on this, the following lemma shows that, generically, there exists an essentially unique coalition-proof Nash equilibrium outcome in state II:

**Lemma 14** (state II for synchronous tenders). *In state II, the coalition-proof Nash equilibrium outcomes are such that each incumbent keeps its market and their payoffs satisfy $\tilde{V}_i \geq 0$ and $\tilde{V}_A + \tilde{V}_B = V_M^{Sync}(b) - \Delta^{Sync}(b) > 0$.*

*Proof.* Consider a candidate equilibrium in which different firms win the two markets. In such an equilibrium, to operate market $i = A, b$ on a stand-alone basis, $D_i$’s best offer is

$$p_{Di} \equiv -\delta^2 V_i,$$

whereas the other incumbent, $D_j$, is not willing to offer less than $p_{Di} + s$. Assumption (51), which implies that $0 < S - b < s$, then ensures that

$$p_{Di} < p_{Ei} - b < p_{Di} + s.$$

It follows that, in any such equilibrium, each $D_i$ incumbent keeps its market at a price $p_i^*$ not exceeding $p_{Ei} - b$. In addition, $p_j^*$ cannot exceed the price at which $D_j$ would be willing to service market $i$ as well as its own, which is equal to:

$$\tilde{p}_i \equiv s - \delta^2 (V_M - V_j).$$

Hence, the equilibrium price for market $i$, $p_i^*$, should not exceed

$$\hat{p}_i \equiv \min\{p_{Ei} - b, \tilde{p}_i\}.$$

Finally, the total price $p_A^* + p_B^*$ should not exceed:

$$P_E^{Sync} - 2b = (1 - \delta^2) V_M^{Sync}(b),$$

otherwise an entrant could profitably win both markets, and each price must be at least 30.
equal to \( p_{Di} \), otherwise an incumbent would make a loss. Hence, we must have:

\[
p^*_i \in [p_{Di}, \hat{p}_i] \text{ for } i = A, B, \text{ and } p^*_A + p^*_B \leq \left(1 - \delta^2\right) V_{M}^{Sync}(b).
\]  

(55)

This range is not empty (i.e., \( \hat{p}_i > p_{Di} \)), as: (i) Corollary 7 ensures that \( (1 - \delta^2) V_{M}^{Sync}(b) > 0 \), (ii) (51) implies:

\[
p_{Ei} - b - p_{Di} = S - b > 0,
\]

and (iii):

\[
\hat{p}_i - p_{Di} = s - \Delta^{Sync}(b) > s - 2(S - b) > 0
\]

where the first inequality stems from Corollary 7 and the second one from (51) (namely, \( b > S - s/2 \)).

Conversely, for any prices \((p^*_A, p^*_B)\) satisfying (55), there exists an equilibrium in which each \(D_i\) wins market \(i\) at price \(p^*_i\). To see this, suppose that both incumbents offer the stand-alone prices \(p^*_A\) and \(p^*_B\), together with a bundle price \(P^* \equiv p^*_A + p^*_B\). The auctioneer is then willing to allocate each market to its current operator. Furthermore, the conditions \(p^*_i \leq \hat{p}_i\) ensure both that the entrants cannot profitably undercut them, and that no incumbent can benefit from winning both markets. Finally, the conditions \(p_{Di} \leq p^*_i\) ensures that each \(D_i\) obtains a non-negative profit, given by \(\tilde{V}_i = p^*_i + \delta^2 V_i = p^*_i - p_{Di} \geq 0\), which in turn implies that no \(D_i\) can benefit from raising its price, and the same conditions also ensures that no \(D_i\) can benefit either from targeting its rival \(D_j\)'s market, as this would yield at most:

\[
p^*_j + \delta^2 V_j - s \leq \hat{p}_j + \delta^2 V_j - s = -\Delta^{Sync}(b) \leq 0.
\]

Hence, there always exists equilibria in which the two incumbents keep their respective markets. Among these equilibria, the Pareto-efficient ones yield payoffs \(\tilde{V}_A\) and \(\tilde{V}_B\) satisfying \(\tilde{V}_i \geq 0\) for \(i = A, B\) and

\[
\tilde{V}_A + \tilde{V}_B = P_E^{Sync} - 2b + \delta^2 (V_A + V_B) = (1 - \delta^2) V_{M}^{Sync}(b) + \delta^2 (V_A + V_B) = V_{M}^{Sync}(b) - \Delta^{Sync}(b) > 0.
\]

To show that these equilibria are the only coalition-proof ones, it suffices to note that any alternative equilibrium in which one firm would win both markets is such that both incumbents exit and thus obtain zero payoff: indeed, in such an equilibrium the incumbents’ best offer for the bundle is \(P_D \equiv s - \delta^2 V_M\), and Assumption (51) (namely, \(b > S - s/2\)) therefore ensures that an entrant wins: \(P_D - (P_E - 2b) = s - 2(S - b) > 0\).
G.2 Equilibrium characterisation

In equilibrium, \( V_M = V^{\text{Sync}}_M(b) = 2 (S - b) \) (from Corollary 7) and, from Lemma 14, in state II each incumbent keeps its market and their payoffs satisfy:

\[
V_A + V_B = \tilde{V}_A + \tilde{V}_B = V^{\text{Sync}}_M(b) - \Delta^{\text{Sync}}(b) > 0. \tag{56}
\]

The equilibrium is therefore dual-state. Once in state I, the equilibrium path remains in that state forever; using Corollary 7, the per market price is then given by

\[
p_I = p^{\text{Sync}}(b) \equiv \frac{P_E}{2} = (1 - \delta^2) (S - b). \tag{57}
\]

Likewise, once in state II, the equilibrium path remains forever in that state. Furthermore, Corollary 7 and equation (56) together yield:

\[
\Delta^{\text{Sync}}(b) = \delta^2 \left( V^{\text{Sync}}_M(b) - V_A - V_B \right) = \delta^2 \Delta^{\text{Sync}}(b),
\]

implying \( \Delta^{\text{Sync}}(b) = 0 \). It follows that \( P_E = p_{EA} + p_{EB} \) and \( p_{Ei} - b < \tilde{p}_i \), as:

\[
\tilde{p}_i - (p_{Ei} - b) = s - \delta^2 \left( V^{\text{Sync}}_M(b) - V_j \right) - (S - b - \delta^2 V_i) = (S - s - \delta^2 V_i) > \frac{s}{2} > 0,
\]

where the last equality stems from \( V_A + V_B = V^{\text{Sync}}_M(b) \) and the first inequality stems from (51) (namely, \( b > S - s/2 \)). Hence, \( \tilde{p}_i = p_{Ei} - b \) and the payoffs thus satisfy:

\[
V_i \in \left[ p_{Di} + \delta^2 V_i, p_{Ei} - b + \delta^2 V_i \right] = [0, S - b]
\]

for \( i = A, B \) and:

\[
V_A + V_B = V^{\text{Sync}}_M(b) = 2 (S - b).
\]

It follows that

\[
V_A = V_B = \frac{V^{\text{Sync}}_M(b)}{2} = S - b,
\]

and the per market price is given by:

\[
p_{II} = (1 - \delta^2) \frac{V^{\text{Sync}}_M(b)}{2} = (1 - \delta^2) (S - b) = p^{\text{Sync}}(b).
\]