To pool or not to pool?
Security design in OTC markets

Vincent Glode\textsuperscript{a}, Christian C. Opp\textsuperscript{b,c,}\textsuperscript{*}, Ruslan Sverchkov\textsuperscript{d}

\textbf{ABSTRACT}

We study security issuers’ decisions on whether to pool assets when facing counterparties endowed with market power, as is common in over-the-counter markets. Our analysis reveals how buyers’ market power may render the pooling of assets suboptimal — both privately and socially — in particular, when the potential gains from trade are large. Pooling assets then reduces the elasticity of trade volume in the relevant part of the payoff distribution, exacerbating the inefficient rationing associated with the exercise of buyers’ market power. Our analysis provides insight on the determinants of asset-backed securities issuance, including regulatory reforms affecting financial institutions’ liquidity.

\textit{Keywords:} Pooling, Adverse selection, Imperfect competition, Decentralized markets

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\textsuperscript{a}The Wharton School, University of Pennsylvania, Steinberg-Dietrich Hall, 3620 Locust Walk, Philadelphia, PA 19104, USA
\textsuperscript{b}Simon Business School, University of Rochester, 3-110N Carol Simon Hall, Rochester, NY 14627, USA
\textsuperscript{c}National Bureau of Economic Research, 1050 Massachusetts Ave, Cambridge, MA 02138, USA
\textsuperscript{d}Warwick Business School, University of Warwick, Scarman Rd, Coventry, CV4 7AL, UK

\textit{Email address:} opp@rochester.edu (Christian C. Opp)

\textsuperscript{*}Corresponding author.
1. Introduction

Structured products are typically issued in over-the-counter (OTC) markets, where asymmetric information and market power have been shown to be prevalent frictions. For example, issuers may face prices that are not fully competitive when regulatory constraints raise the holding costs for many market participants, leaving only a few institutions willing to provide liquidity. In this paper, we study the security design problem of a privately informed issuer who possesses multiple assets and faces liquidity suppliers, or buyers, that are potentially endowed with market power.

We show how the allocation of market power has relevant and robust implications for security design that contrast with the takeaways from models that assume competitive environments. To isolate the impact of market power, we vary the number of prospective buyers, as well as the market structure in which they interact with the issuer. When several buyers act competitively, pooling all assets is optimal for the issuer, echoing arguments found in previous studies. As diversification reduces an issuer’s informational advantage, pooling assets helps alleviate adverse selection problems. Doing so is in the interest of the issuer who fully internalizes the benefits of improving the efficiency of trade when prices are set competitively.

However, we show that pooling assets has an important downside for the issuer once the demand side has market power and acts non-competitively, namely, a potential reduction in the issuer’s information rents. A privately informed issuer may prefer not to pool assets, especially when the potential gains from trade are large relative to the information asymmetry between the issuer and prospective buyers. In fact, any pooling decision that implies perfect diversification is never optimal for an issuer when facing buyer(s) with sufficiently strong market power. We provide explicit, sufficient conditions under which the issuer’s best option is to simply sell all assets separately. Under these conditions, separate sales are not only privately optimal but also achieve the first-best level of total trade surplus. In contrast, when assets are pooled, both the issuer’s private surplus and the total surplus from trade are strictly lower, as diversification invites strategic buyers with market power to choose pricing strategies that lead to inefficient rationing. Pooling affects the shape of the distributions characterizing information asymmetries between issuers and buyers, causing them to have thinner tails. As a result, the elasticity of trade volume — a key determinant of price-setting behavior in the presence of market power — decreases in the right tails of these distributions. Correspondingly, pooling typically worsens inefficient rationing when selling assets separately would lead to high trade volume.

Our analysis provides novel empirical predictions and policy implications regarding security issuances in OTC markets. Specifically, our results reveal why liquidity shortages among major institutions participating

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1 For evidence that OTC trading often involves heterogeneously informed traders, see Green et al. (2007), Jiang and Sun (2015), and Hollifield et al. (2017). For evidence that OTC trading tends to be concentrated among a small set of players, see Cetorelli et al. (2007), Atkeson et al. (2014), Begennau et al. (2015), Di Maggio et al. (2017), Li and Schürhoff (2019), Siriwardane (2019), and Hendershott et al. (2020).
in these markets can contribute to declines in asset-backed security (ABS) issuances despite concurrent increases in the volume of assets that are sold separately, as witnessed following the 2007-2008 financial crisis.² When market conditions or regulatory actions reduce the liquidity available to many potential buyers, thereby creating market concentration on the demand side, pooling assets does not only carry the benefit of reducing adverse selection but also can worsen inefficiencies associated with the exercise of market power. As a result, our analysis predicts that, ceteris paribus, issuers' propensity to pool assets (relative to selling assets separately) is negatively related to buyers' market power. Moreover, this effect should apply specifically when the gains from trade are large, which tends to be the case when sellers have high liquidity needs. That is, when trade is particularly valuable but potentially impeded by the presence of market power, our results become most relevant. These predictions apply both in the cross-section of assets and in the time series. In addition, they provide insight on the prevalence of partial pooling, whereby issuers allocate assets with distinct factor exposures to separate pools. This latter prediction is consistent with a common practice by originators to design securities that pool mortgages only from certain types of borrowers (e.g., geographic regions, residential vs. commercial), which creates pools with distinct factor exposures.

We also highlight how recent banking regulations might affect the issuance and trading of securities in OTC markets. In particular, we uncover the potential spillover effects of policies affecting the liquidity of many financial institutions participating in the structured securities market. Regulatory actions that reduce the number of prospective buyers willing to acquire newly issued securities (e.g., due to stricter liquidity requirements or other balance sheet constraints) can result in increased market power on the demand side and associated hesitancy by issuers to pool assets. Yet our results suggest that the reduction in pooling may be both a privately and socially optimal response to buyers' emergent market power. As such, policies reducing the concentration of liquidity among a few market participants may revive the issuance of securities backed by well-diversified pools, even when those policies are not specifically targeting the holdings of these types of securities.

We focus on the impact of market power on the decision to pool assets, thereby providing insight on security issuances in decentralized markets. Early contributions by Subrahmanyam (1991), Boot and Thakor (1993), and Gorton and Pennacchi (1993) emphasize the diversification benefits of pooling assets when securities are sold in competitive/centralized markets that are subject to asymmetric information problems. Building on the signaling-through-retention framework with price-taking buyers of DeMarzo and Duffie (1999), DeMarzo (2005) shows that the pooling of assets dampens an issuer’s ability to signal individual assets’ quality through retention. However, when the number of assets is large and the issuer can sell debt on the pool of assets, this “information destruction effect” is dominated by the above-mentioned

²In 2009, the issuance volume of ABS in the US was 73% lower than it was in 2006, while the issuance volume of CDO was 97% lower. In contrast, the total issuance volume in US fixed income markets was 13% higher in 2009 than in 2006. For more data, see the Securities Industry and Financial Markets Association’s website: http://www.sifma.org/research/statistics.aspx.
benefits of diversifying the risks associated with the issuer’s private information about each asset’s value. Issuing debt on a large pool of assets reduces residual risks and the information sensitivity of the security being issued. In contrast to DeMarzo (2005), whose setup can be thought of as a centralized market where (price-taking) buyers simultaneously compete for assets, we isolate the effects of buyers’ market power to capture a realistic feature of many OTC markets.

Our focus on the role of market power in an issuer’s security design decision relates our analysis to Biais and Mariotti (2005) who analyze a model where the security design stage is followed by a stage where either the issuer or the prospective buyer chooses an optimal trading mechanism (i.e., a price-quantity menu) for selling the designed security. When the buyer can choose the trading mechanism, he effectively screens the issuer, accounting for a generic volume-price trade-off. In contrast, when the issuer can choose the mechanism, the setup becomes equivalent to one with multiple competitive buyers. Biais and Mariotti (2005) show that issuing debt on a risky asset is optimal in both cases, since the debt contract’s low information sensitivity helps avoid market exclusion. In our paper, we are concerned with the security design problem of an issuer wishing to sell multiple assets, which is a key feature of the structured securities market. Our analysis reveals how in that setting, the optimal security design, in particular the optimal pooling decision, changes with the allocation of market power.

Axelson (2007) studies an uninformed issuer’s decision to design securities that are (centrally) traded in a uniform-price auction with privately informed buyers. The author finds that pooling assets and issuing debt on these assets is always optimal when the number of assets is large, otherwise selling assets separately might be optimal if the signal distribution is discrete and competition is high enough. Since the issuer is uninformed and buyers compete for assets through an auction, the author’s analysis is silent about how security design can be used to prevent being monopolistically screened by liquidity providers, which is a key result of our analysis.

Palfrey (1983) analyzes a firm’s decision to bundle products (or assets) sold in a second-price auction. In his model, customers have private information about their heterogenous valuations for the products. Selling the products separately is optimal when the sum of the expected second-highest valuation for each product is higher than the expected second-highest valuation for the bundle of all products. This comparison depends

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3 See also Hartman-Glaser et al. (2012), who model a moral hazard problem between a principal and a mortgage issuer and show that the optimal contract features pooling of mortgages with independent defaults, as it facilitates effort monitoring.

4 Gorton and Pennacchi (1990), Dang et al. (2015), Farhi and Tirole (2015), and Yang (2020) also study the optimal information sensitivity of securities issued in markets with asymmetric information. These papers highlight the benefits of designing securities that split cash flows into an information-sensitive part and a risk-less part. Yet, these contributions do not speak to how pooling imperfectly correlated assets affects the issuer’s ability to extract surplus when facing buyers with market power, which is the focus of our paper.

5 See also DeMarzo et al. (2005) and Inderst and Mueller (2006), who study optimal security design problems with informed buyers and only one asset.
on the number of prospective customers and the distribution of their product-specific valuations. Unlike Palfrey (1983), we examine how the degree of competition among buyers with identical valuations affect pooling decisions. The cross-buyer heterogeneity in valuations that is central for Palfrey’s (1983) results does not play a role for our findings.

In Section 2, we describe our model and provide an illustrative example in which the issuer sells a pool containing a continuum of assets. This example highlights how the presence of market power on the demand side greatly affects the issuer’s benefits from pooling assets. Section 3 presents our main analysis, contrasting the optimal security design in competitive and non-competitive environments. In Section 4, we discuss the robustness of our results to various alternative specifications of the model. We conclude in Section 5.

2. The environment

Suppose an issuer has a finite number $N \geq 2$ of fundamental assets to sell. These assets are indexed by $i$ and the set of all assets is denoted by $\Omega \equiv \{1, \ldots, N\}$. Each asset $i$ produces a random payoff $X_i$ at the end of the period. For tractability, the asset payoffs $X_i$ have identical marginal distributions as specified by the cumulative distribution function (CDF) $G(\cdot)$ with associated probability density function (PDF) $g(\cdot)$ that is positive and finite everywhere on its domain $\chi \equiv [0, \bar{x}]$. We consider a symmetric dependence structure between the individual payoffs $X_i$ for all $i \in \Omega$, excluding the degenerate case of perfect correlation. Identical marginal distributions and symmetric dependence together imply that the joint distribution of payoffs is invariant to permutations of assets. Going forward, we follow the convention of using capitalized letters for random variables and lower-case letters for their realizations.

2.1. Market participants and their liquidity needs

As is common in the security design literature, agents are risk neutral but can differ in their liquidity (or hedging) needs, which are captured by their discount factors. There is a number $B$ of deep-pocketed traders who are better equipped to hold claims to future cash flows than the issuer is (who needs liquidity today). Specifically, whereas the issuer applies a discount factor $\delta \in (0, 1)$ to future cash flows, these $B$ prospective buyers apply a discount factor of 1. Thus, the ex ante value of each fundamental asset is $\delta E(X_i)$ for the issuer and $E(X_i)$ for any of these buyers. As a result, gains from trade are realized when the issuer sells his assets to such a buyer in exchange for cash. Throughout the analysis, we occasionally refer to the $B$ prospective buyers with a discount factor of 1 as “liquidity suppliers” in line with previous studies (e.g., Biais and Mariotti, 2005).

To highlight the impact of market power on the issuer’s security design problem, we initially consider a benchmark scenario in which multiple buyers ($B \geq 2$) compete for the issuer’s assets in a centralized market. In this scenario, the buyers simultaneously make competitive bids that are equal to the expected value of
any security conditional on trade occurring, consistent with Bertrand competition. Then, we analyze the implications of the presence of market power on the demand side. To do so, we first consider a scenario in which only one buyer has a discount factor of 1, that is, $B = 1$. Capturing a common practice in OTC markets, the seller sends a request for quote for each security he wishes to sell to this buyer, and the buyer responds to each request with a take-it-or-leave-it offer. In Section 4, we further show that this monopolistic setting shares its key implications with an OTC market structure where the issuer contacts multiple buyers ($B \geq 1$) sequentially, selling assets to several distinct buyers in equilibrium. These results obtain due to the sequential and exclusive nature of bilateral interactions in the OTC market, which effectively sustains market power despite the presence of multiple buyers. Moreover, we discuss in Section 4 how our main insights extend to settings with uncertainty about the degree of buyer competition. In sum, we show that our key insights apply in settings where market power arises due to either the market structure (e.g., sequential OTC trade) or fundamentals, such as the concentration of liquidity among few agents (e.g., when most potential counterparties face similar regulatory constraints or liquidity needs as the issuer).

2.2. Timing and information structure

Our specification of the timeline follows previous studies (e.g., DeMarzo and Duffie, 1999; Biais and Mariotti, 2005). First, the issuer designs the securities he plans to sell. Second, the issuer becomes informed about the realizations of each asset payoff $X_i$. Third, the buyer(s) make(s) simultaneous take-it-or-leave-it offer(s) for each security to the issuer. Fourth, the issuer decides whether or not to accept any of these offer(s) in exchange for the securities; if multiple buyers offer an identical price that is accepted by the issuer, the counterparty for the trade is randomly selected among the highest bidders. Finally, all payoffs are realized.

Assuming that the issuer does not have private information at the initial security design stage increases the tractability of the analysis and shares similarities with the shelf registration process commonly used in practice [as also argued by DeMarzo and Duffie (1999) and Biais and Mariotti (2005)]. In that process, an issuer first specifies and registers the securities with a regulatory agency, such as the Securities and Exchange Commission in the US. Then, potentially after several months, the issuer brings these securities to the market. In the meantime, the issuer has typically collected additional private information about the securities’ future cash flows. In Section 4, we discuss the robustness of our main insights to various changes in this timeline, including some that would introduce signaling concerns at the security design stage.

Going forward, we refer to this scenario as monopolistic demand or monopolistic liquidity supply. In this context, the buyer can also be referred to as a monopsonist.
2.3. An illustrative example

Before proceeding with our main analysis, we present a simple, yet generic example that illustrates how the issuer’s benefits from pooling assets crucially depend on the presence of market power on the demand side. For this example, suppose the issuer owns a continuum of assets of measure 1 with i.i.d. payoffs \( X_i \) with finite mean and variance. The issuer considers selling the pool of these assets to the prospective buyer(s).

First, we analyze a market scenario in which \( B \geq 2 \) prospective buyers have abundant liquidity (that is, they have a discount factor equal to 1). In this case, they effectively compete in quotes à la Bertrand and offer the highest price that yields weakly positive expected profits conditional on the issuer accepting the offer. This scenario captures a competitive environment as commonly analyzed in the literature. When the issuer offers the assets as one pool, the law of large numbers applies, that is, perfect diversification implies that the pool’s payoff is \( \int_0^1 x_i \, di = E[X_i] \) almost surely. As a result, adverse selection concerns are completely eliminated, and buyers compete by offering a price \( p^c = E[X_i] \) for this pool. The maximum total surplus from trade, \( E[X_i] \cdot (1 - \delta) \), is attained and the issuer fully internalizes this surplus. That is, the issuer achieves the optimal expected payoff. The fact that pooling the continuum of assets eliminates information asymmetries is unambiguously beneficial for the issuer when facing competitive buyers, as he then fully internalizes the resultant improvements in trade efficiency.

In contrast, consider the market scenario in which \( B = 1 \) prospective buyer has liquidity to purchase the issuer’s assets (i.e., only one buyer has a discount factor of 1). Acting as a de-facto monopolist, this buyer can choose the price that maximizes his expected payoff. In this case, this optimally chosen price is the issuer’s reservation price for the pool of assets, that is, \( p^m = E[X_i] \delta \). As in the scenario with multiple prospective buyers, pooling the continuum of assets yields perfect diversification and eliminates adverse selection concerns. Yet, now that the demand side has market power, fully eliminating these information asymmetries has no upside for the issuer. Facing no informational disadvantage, the monopolistic liquidity supplier then offers a price that leaves the issuer indifferent between trading the security or not.

This generic result for asset pools that achieve perfect diversification strikingly highlights the relevance of market power for the optimality of pooling assets from the perspective of the issuer. In the presence of such market power, the issuer can extract rents only when retaining some private information. Thus, any pooling that leads to perfect diversification (as was the case in this example) is never optimal for an issuer when facing a prospective buyer with market power. On the other hand, when the issuer retains private information, a buyer with market power strategically chooses a price that potentially jeopardizes the realization of gains from trade. When deciding whether to pool assets, the issuer therefore faces an intuitive trade-off: he can only extract rents when retaining some private information, but he still partially internalizes the inefficiencies emerging from adverse selection and the exercise of market power under asymmetric information. As a result, he may only choose to pool a subset of assets in order to achieve partial diversification (but not perfect
diversification). Understanding these channels and how they affect the design of optimal securities is the focus of our main analysis below.

3. Main analysis

We now formalize our main insights. We follow previous studies in assuming that the issuer first decides on the pooling of the underlying assets and then chooses the security that is written on each of the pools (e.g., DeMarzo, 2005; Axelson, 2007). Formally, the issuer chooses a partition of the set Ω, that is, he groups the N assets into M ≤ N disjoint subsets denoted by Ω_j with j ∈ {1,...,M}. The corresponding M pools of assets then have the payoffs:

\[ Y_j = \sum_{i \in \Omega_j} X_i. \tag{1} \]

The CDF \( G_j \) of \( Y_j \) and the associated density \( g_j \) are then defined on the compact interval \( \chi_j = [0, \bar{y}_j] \), where \( \bar{y}_j = \sum_{i \in \Omega_j} \bar{x}_i \). In line with previous studies (e.g., Myerson, 1981), we assume that these distributions satisfy a regularity condition that ensures that first-order conditions in the trading game with a monopolistic buyer who is offered equity securities are sufficient conditions for the optimal pricing decisions.

**Assumption 1.** For any subset \( \Omega_j \) of \( \Omega \), the elasticity function:

\[ e_j(y) \equiv \frac{g_j(y)}{G_j(y)} \cdot y \tag{2} \]

is weakly decreasing on its respective support \( \chi_j \).

Throughout our main analysis below, we discuss examples with distributions satisfying Assumption 1 (see also the Appendix for additional illustrations). When interpreting elasticity functions, it is helpful to note that they represent the ratio of the local density \( g_j(y_j) \) to the average density \( G_j(y_j)/y_j \). These quantities play an important role in determining a monopolistic buyer’s optimal pricing strategy. We also denote by \( e(x_i) \equiv \frac{g(x_i)}{G(x_i)} \cdot x_i \) the elasticity function of each fundamental asset \( i \).

The issuer chooses for each pooled payoff \( Y_j \) a security that is backed by that payoff. Specifically, the security payoff \( F_j \) is contingent on the realized cash flow \( Y_j \) according to the function \( \varphi_j : \chi_j \to \mathbb{R}_+ \), such that \( F_j = \varphi_j(Y_j) \). We impose the standard limited liability condition:

\[ (LL) \ 0 \leq \varphi_j \leq Id_{\chi_j}, \]

where \( Id_{\chi_j} \) is the identity function on \( \chi_j \). In addition, as is standard in the security design literature (e.g., Harris and Raviv, 1989; Innes, 1990; Nachman and Noe, 1994), we restrict the set of admissible securities by
requiring that both the payoffs to the liquidity supplier and to the issuer be non-decreasing in the underlying cash flow:\(^7\)

(M1) \(\varphi_j\) is non-decreasing on \(\chi_j\),
and
(M2) \(Id_{\chi_j} - \varphi_j\) is non-decreasing on \(\chi_j\).

The sets of admissible payoff functions for the securities is therefore given by \(\{\varphi_j : \chi_j \rightarrow \mathbb{R}_+\mid (\text{LL}), (\text{M1}), \text{and} (\text{M2}) \text{ hold}\}\).

3.1. Competitive demand

In this subsection, we analyze the (benchmark) scenario in which the issuer faces \(B \geq 2\) liquidity suppliers who have a discount factor of 1. In this case, the issuer receives competitive ultimatum price quotes, a feature that is common in the literature (e.g., Boot and Thakor, 1993; Nachman and Noe, 1994; Friewald et al., 2015).\(^8\)

3.1.1. Optimality of pooling assets

Echoing previous studies, our analysis of this scenario predicts that issuing debt on the pool of all assets is optimal for the issuer, as summarized in Proposition 1.

**Proposition 1.** If \(E[X_i] \geq \delta \bar{x}\), the issuer is indifferent between selling assets separately and selling them as a pool. If \(E[X_i] < \delta \bar{x}\), the issuer optimally pools all \(N\) assets and issues a debt security on this pool.

To provide the intuition for this result, we present the proof of Proposition 1 in the main text. At the trading stage, the issuer has perfect knowledge of the realizations \(x_i\) of future cash flows \(X_i\). Since the payoff of any security \(F_j\) is only contingent on \(Y_j = \sum_{j} \Omega_j X_i\), the issuer also perfectly knows the realization \(f_j = \varphi_j(y_j)\) of \(F_j\). Suppose the issuer uses a simple equity security [what DeMarzo and Duffie (1999) refer to as a “passthrough” security]. If \(E[X_i] \geq \delta \bar{x}\), he can sell the assets separately (as equity), each at price \(p = E[X_i]\), since at this price even the highest issuer type \(\bar{x}\) finds it optimal to trade. The issuer obtains

\(^7\)As discussed by Innes (1990), these assumptions guarantee that the issuer does not have incentives to tamper with the underlying cash flows. (M1) implies that the issuer cannot reduce the security’s payoff to investors by secretly adding cash to the underlying pool (perhaps with the help of outside borrowing). (M2) implies that the issuer cannot increase the payoff he retains by secretly destroying some of the underlying cash flows.

\(^8\)DeMarzo (2005) also studies pooling decisions in a competitive environment. Yet, whereas DeMarzo (2005) studies a specific trading mechanism, namely a signaling game similar to Kyle (1985), we employ the trading mechanism used in Biais and Mariotti (2005), which the authors derive as the optimal mechanism in their setting. See Section 4 for a related discussion.
the same total payoff when pooling the assets and selling an equity security on the pool. Since the potential gains from trade are large enough ($\delta$ is sufficiently low), adverse selection does not impede the efficiency of trade even when assets are sold separately. The first-best level of total trade surplus is achieved, and the issuer fully internalizes this surplus.

In contrast, if $\mathbb{E}[X_i] < \delta\bar{x}$, the sale of an equity security on a single asset leads to adverse selection, since the highest issuer type $\bar{x}$ would not accept a price equal to $\mathbb{E}[X_i]$. Similarly, the sale of an equity security on a pool of $\tilde{N}$ assets leads to the exclusion of some issuer types, since the highest issuer type $\bar{y}_j = \tilde{N}\bar{x}$ would not accept a price equal to $\mathbb{E}[Y_j] = \tilde{N}\mathbb{E}[X_i]$. In this case, it is useful to recall the following result from Biais and Mariotti’s (2005) analysis of a setting with one underlying asset:

**Lemma 1.** Given an underlying asset with random payoff $Y$ and $\mathbb{E}[Y] < \delta\bar{y}$, the issuer optimally designs a debt security with the highest face value $d$ for which a buyer just breaks even when purchasing this debt security at a price $p = \delta d$.


Independent of his pooling choice that determines the underlying assets with payoffs $Y_j$, the issuer optimally uses a debt security when $\mathbb{E}[X_i] < \delta\bar{x}$ and equivalently, $\mathbb{E}[Y_j] < \delta\bar{y}_j$. Debt emerges as the optimal security, because it minimizes adverse selection costs. As a debt security’s payoff is less sensitive to the high cash flow realizations, it mitigates the lemons problem and facilitates the efficient transfer of cash flows from the issuer to the liquidity suppliers.

To determine the issuer’s optimal pooling decision, it is useful to first consider buyers’ expected net profits. A buyer purchasing debt with face value $d$ at a price $p = \delta d$ obtains the following expected net profit:

$$\int_0^d yg_j(y)dy + [1 - G_j(d)]d - \delta d = (1 - \delta)d - \left(G_j(d)d - \int_0^d yg_j(y)dy\right)$$ (3)

$$= (1 - \delta)d - \int_0^d G_j(y)dy,$$ (4)

where the last step follows from integration by parts. Next, we compare buyers’ expected net payoff from the sales of separate debt securities to that from the sale of a debt security on an underlying pool of assets. Consider first that the issuer sells $\tilde{N}$ individual debt securities with face value $d$. Further, suppose that each debt security is written on a separate underlying asset and the price in each transaction is $\delta d$. Then buyers’
total expected net profit (which may be negative)\(^9\) is:

\[
\tilde{N} \left( (1 - \delta)d - \int_0^d G(x)dx \right) = (1 - \delta)\tilde{N}d - \int_0^{\tilde{N}d} G \left( \frac{y}{\tilde{N}} \right) dy,
\]

(5)

where we used a change in variables, with \(y = \tilde{N}x\). In contrast, consider now that the issuer pools the \(\tilde{N}\) assets and issues one debt security with face value \(d_j = \tilde{N}d\) and buyers purchase this debt at price \(\delta d_j\). In this case, buyers’ total expected net profit (which again may be negative) is:

\[
(1 - \delta)\tilde{N}d - \int_0^{\tilde{N}d} G_j(y)dy.
\]

(6)

Lemma 2 provides insight on the relative magnitude of the profits in (5) and (6).

**Lemma 2.** The distribution of the pooled payoff \(Y_j = \sum_{i=1}^{\tilde{N}} X_i\) second-order stochastically dominates the distribution of the payoff \(\tilde{N}X_i\), that is,

\[
\int_s^\infty \left[ G \left( \frac{y}{\tilde{N}} \right) - G_j(y) \right] dy \geq 0
\]

(7)

for any \(s \in [0, \bar{y}_j]\).

Proof. See the Appendix.

Lemma 2 implies that buyers’ total expected net profit is higher in the scenario with pooling (i.e., (6) is greater than (5)). Next, recall that, according to Lemma (1), the optimal face value in each scenario would be set such that buyers break even, that is, the optimal face values would ensure that (5) and (6) are each equal to zero. The above result implies that if buyers break even at a given face value \(d^*\) on separate sales (first scenario), then they make positive profits on the pooled sale if the face value is set equal to \(\tilde{N}d^*\) (second scenario). It follows that the issuer can choose a face value \(d_j^* \geq \tilde{N}d^*\) on the pool while still ensuring that the buyers can break even (as buyers’ expected net profit is a continuous function of \(d_j\)). Finally, observe that when issuing debt with break-even face values under each of the two scenarios, the issuer’s total profits are \((1 - \delta)\delta \tilde{N}d^*\) and \((1 - \delta)\delta d_j^*,\) respectively, and the issuer extracts the full gains from trade in the competitive market. Since \(d_j^* \geq \tilde{N}d^*\), the issuer obtains a higher expected net profit when pooling the \(\tilde{N}\) assets and issuing debt with face value \(d_j^*\).

In sum, the argument for the optimality of pooling in this setting is intuitive. With competitive liquidity suppliers, the issuer extracts all of the realized gains from trade and, thus, fully internalizes any improvements in trade efficiency. As a result, when adverse selection impedes trade, the issuer seeks to minimize the

\[^9\text{At this point in the proof, the considered supposition does not impose that the buyers’ participation constraint is satisfied. That is, the expected net profit can be negative.}\]
information asymmetry between him and his prospective buyers by pooling assets. As pooling leads to
diversification, it reduces the information asymmetry and its associated inefficiencies. In other words, the
issuer does not face a trade-off when facing competitive buyers — reducing information asymmetry is always
weakly beneficial. However, we show below that this unambiguous optimality of pooling ceases to hold when
liquidity suppliers have market power.

3.2. Monopolistic demand

In this subsection, we derive our main results by considering a scenario in which the issuer faces a
monopolistic liquidity supplier, that is, there is only one buyer with a discount factor of 1. While our
baseline analysis considers this monopolistic setting, similar outcomes arise in the presence of multiple
prospective buyers that the issuer contacts sequentially (see Subsection 4.1 for details). In either case,
security demand is imperfectly competitive, which is a key feature of OTC markets in practice.\(^\text{10}\)

We start by examining a monopolistic buyer’s optimal pricing decision. Biais and Mariotti (2005) show
that for a given security offered, the optimal mechanism for the liquidity supplier with market power can be
implemented via a take-it-or-leave-it offer (see also Riley and Zeckhauser, 1983). Specifically, the prospective
buyer makes an ultimatum price offer \(p_j\) to maximize his ex ante profit from purchasing a security with
payoff \(F_j\):

\[
\Pr(\delta f_j \leq p_j) (\mathbb{E}[f_j | \delta f_j \leq p_j] - p_j) = \int \sup\{y: \varphi_j(y) \leq \frac{p_j}{\delta}\} (\varphi_j(y) - p_j) g_j(y) dy.
\]

The optimal price \(p^*_j\) set by this buyer identifies a marginal issuer type that is just willing to accept this
price: \(f^*_j \equiv \frac{p^*_j}{\delta}\). Issuer types with security payoffs below the threshold value \(f^*_j\) participate in the trade,
whereas issuer types with payoffs above \(f^*_j\) are excluded (i.e., they reject the offer).

3.2.1. Optimality of separate equity sales

We now establish our first key result, which identifies a sufficient condition for the strict optimality of
selling assets separately. This result also provides the necessary and sufficient condition under which selling
assets separately yields the first-best level of trade surplus.

**Proposition 2.** Suppose that the following condition holds:

\[
e(\bar{x}) \geq \frac{\delta}{1 - \delta}, \quad \text{or equivalently} \quad \delta \leq \bar{\delta},
\]

where \(\bar{\delta} \equiv \frac{e(\bar{x})}{1 + e(\bar{x})}\). Then the following results obtain:

---

\(^{10}\)The central feature of our analysis is the presence of some degree of market power, that is, a buyer can strategically affect
the prices of the securities being offered. Biais et al. (2000) show that this type of strategic pricing behavior also arises when
multiple risk-averse liquidity suppliers compete in mechanisms (see also Vives, 2011).
(i) The issuer optimally sells each asset separately to a monopolistic buyer, that is, 
\[ \Omega_j = \{j\} \quad \text{and} \quad \varphi_j(X_j) = X_j \quad \text{for} \quad j = 1, \ldots, N. \] 

The first-best level of total surplus from trade, \( N(1 - \delta)\mathbb{E}[X_i] \), is achieved and the issuer collects \( N\delta \bar{x} \), obtaining a surplus of \( N\delta(\bar{x} - \mathbb{E}[X_i]) \).

(ii) If the issuer pools any of the assets, the total surplus from trade is strictly below the first-best level \( N(1 - \delta)\mathbb{E}[X_i] \), and the issuer’s surplus is strictly below \( N\delta(\bar{x} - \mathbb{E}[X_i]) \).

To provide intuition for the central results of Proposition 2, we present the proof in the main text. First, consider part (i) of the proposition. Suppose that the issuer sells an equity claim on a pool \( j \), such that \( \varphi_j(Y_j) = Y_j \). When designing the optimal security, the issuer anticipates the buyer’s optimal pricing response. Using Eq. (8), we can write the buyer’s marginal benefit of increasing the threshold type \( f_{jm} = y_{jm} \) for \( f_{jm} \in [0, \bar{y}_j) \) as:

\[
(1 - \delta) f_{jm} g_j(f_{jm}) - \delta G_j(f_{jm}).
\] 

This last equation highlights the generic trade-off that a buyer with market power faces when choosing his price offer. When marginally increasing the threshold type by increasing the price, the buyer benefits from extracting the full gains from trade \( (1 - \delta)f_{jm} \) from this type, which has the local density \( g_j(f_{jm}) \). Yet, the associated price increase of magnitude \( \delta \) also comes at the cost of paying more when trading with all infra-marginal types, which have measure \( G_j(f_{jm}) \). In net, the buyer benefits from increasing the marginal issuer type if and only if expression (11) takes a strictly positive value (for any \( f_{jm} < \bar{y}_j \)). This condition can be equivalently expressed as a condition applying to the above-defined elasticity function:

\[
e_j(f_{jm}) > \frac{\delta}{1 - \delta}.
\] 

Now suppose the issuer simply sells all assets separately. Then the condition \( e(\bar{x}) \geq \frac{\bar{x}}{1 - \delta} \) together with Assumption 1 ensures that the buyer’s optimal price quote for each asset is \( p_i = \delta \bar{x} \), allowing the issuer to collect \( N\delta \bar{x} \). In this case, the marginal issuer type is the highest type on the support \([0, \bar{x}]\) and trade occurs with probability 1, ensuring that the first-best level of surplus from trade is achieved. Moreover, the issuer cannot collect a total payment greater than \( N\delta \bar{x} \) from the monopolistic buyer under any alternative security design, since the best possible payoff that all assets can deliver jointly is \( N\bar{x} \), and a buyer with market power would never offer a price above \( \delta N\bar{x} \) even if he believed that this maximum payoff on all assets was attained.

That is, even when going beyond the space of securities that are each backed by a distinct pool of assets and considering general security design functions \( \varphi_h(X_1, \ldots, X_N) \) subject to the joint limited liability constraint \( \sum_h \varphi_h(X_1, \ldots, X_N) \leq \sum_{i=1}^N X_i \), the issuer could not do better than under the security design described in Proposition 2.
To address part (ii) of the proposition, we show that the issuer’s surplus and the total surplus are strictly lower when assets are pooled. First, we introduce Lemma 3.

**Lemma 3.** For any subset $\Omega_j \subset \Omega$ that contains more than one element (i.e., if there is pooling), the following condition is satisfied:

$$e_j(\bar{y}_j) = 0 < \frac{\delta}{1-\delta}.$$  \hspace{1cm} (13)

**Proof.** See the Appendix.

This lemma states that if the issuer pools assets and issues an equity security on the pool, the elasticity for this security at the upper bound of the support $\bar{y}_j$ is zero, implying the exclusion of a positive measure of types. The elasticity is zero at the upper bound $\bar{y}_j$, since the density for the outcome that two assets simultaneously achieve their highest possible value $\bar{x}$ is zero. The intuitive reason for this result is diversification: the more diversified pool of assets is less likely to generate an extreme outcome than each idiosyncratic asset separately. Fig. 1 illustrates this result for the case where each underlying asset follows a uniform distribution and payoffs are independent.\(^{11}\) In the figure, we compare the shapes of the PDFs of a single asset, a pool of two assets, and a pool of four assets (see caption for details). The graph illustrates the familiar notion that diversification leads to a more peaked distribution with thinner tails.

These changes in the shapes of the PDFs map into corresponding changes in the elasticity functions $e_j(y_j)$, which govern the pricing behavior in the trading game (see Eq. (12)). Fig. 2 confirms that as soon as two assets are pooled, the elasticity at the upper bound of the support, $\bar{y}_j$, shrinks to zero. A thinner right tail of the PDF implies a lower elasticity in the right tail of the distribution (recall that the elasticity is the ratio of the local density $g_j(y_j)$ to the average density $G_j(y_j)/y_j$). Facing a less elastic response from the issuer in that part of the domain, a monopolistic buyer has stronger incentives to offer lower prices, which leads to the exclusion of high issuer types. If $\bar{N} \geq 2$ assets are pooled in a set $\Omega_j$, then the buyer optimally chooses a marginal issuer type strictly below $\bar{y}_j = \bar{N}\bar{x}$, since $e_j(\bar{y}_j) = 0 < \frac{\delta}{1-\delta}$. Correspondingly, the price offered by the buyer is strictly below $\delta\bar{N}\bar{x}$ for a pool of $\bar{N}$ assets, and the issuer obtains an expected payoff from pooling that is strictly below $\delta\bar{N}\bar{x}$.

To conclude the proof of part (ii) of Proposition 2, we address whether the issuer, after pooling assets, could still obtain an equally beneficial payoff as in the case of separate sales by designing an optimal security $F_j = \varphi_j(Y_j)$ on the pooled payoff $Y_j$. In Lemma 4, we characterize the optimal security on a given underlying asset $Y_j$ when an equity security leads to rationing.

\(^{11}\)In the Appendix, we also provide examples where asset payoffs are positively correlated.
Fig. 1. Effect of pooling on the shape of the probability density function. In the graph, we consider a setting with four independent assets \( N = 4 \), each of which has a payoff \( X_i \sim U[0,1] \). We plot the payoff distributions of a separate asset, a pool of two assets, and a pool of four assets. To compare the PDFs’ shapes relative to their respective domains \((0,1), (0,2), \text{ and } (0,4)\), we rescale the horizontal axis to represent the interval \( \chi_j = [0, \bar{y}_j] \) for each PDF \( g_j \).

Lemma 4. When the trading of an equity security on a payoff \( Y_j \) leads to the exclusion of issuer types (i.e., when \( e_j(\bar{y}_j) < \delta/(1-\delta) \)) but sustains trade with positive probability (i.e., when \( e_j(0) > \delta/(1-\delta) \)), the optimal security from the perspective of the issuer is a debt security with face value \( d^m_j \), i.e., \( \varphi = \min\{Id_{\chi_j}, d^m_j\} \), where \( d^m_j \) is the largest \( d \) such that:

\[
\int_0^d f_j g_j(f_j) df_j + [1 - G_j(d)]d - \delta d \geq \int_0^{f^m_j} (f_j - \delta f^m_j) g_j(f_j) df_j
\]

and where \( f^m_j \) solves:

\[
e_j(f^m_j) = \frac{\delta}{1-\delta}.
\]

That is, the optimal debt contract specifies the highest face value for which the buyer weakly prefers offering a price \( \delta d \) that is always accepted by the issuer over offering a lower price that is only accepted by issuer types below the threshold type \( f^m_j \).

Proof. As each of the pooled payoffs \( Y_j \) satisfies the regularity condition stated in Assumption 1, these results follow from Propositions 3, 4, and 5 in Biais and Mariotti’s (2005) analysis of a setting with one underlying asset.

As in the case of competitive demand, a debt contract mitigates adverse selection by making the security payoff less sensitive to high cash flow realizations. Yet, in the presence of market power on the demand
Fig. 2. Effect of pooling on the shape of the elasticity function. In the graph, we consider a setting with four independent assets \((N = 4)\), each of which has a payoff \(X_i \sim U[0, 1]\). We plot the elasticity functions of a separate asset, a pool of two assets, and a pool of four assets. To compare the elasticity functions’ shapes relative to their respective domains \(([0, 1], [0, 2], \text{and } [0, 4])\), we rescale the horizontal axis to represent the interval \(\chi_j = [0, \bar{y}_j]\) for each elasticity function \(e_j\).

On the supply side, the issuer is also concerned with the adverse consequences of buyers’ exercise of market power for the realization of gains from trade. A debt contract incentivizes a buyer with market power to offer a price that is just high enough to lead to the inclusion of all issuer types, as even marginally deviating to a lower price would discretely decrease the probability of trade (a positive-measure set of the highest issuer types would reject any lower offer).

Using this result, we can now evaluate whether the optimal security issued on a pool can deliver the same payoff as the securities outlined in Proposition 2. Since any pooling of \(\tilde{N} \geq 2\) assets in a set \(\Omega_j\) leads to exclusion when an equity security is offered and \(e_j(\bar{y}_j) = \delta/(1 - \delta)\), Lemma 4 implies that the best possible security written on that pool is a debt security with face value \(d^m_j\). Yet, since \(d^m_j < \bar{y}_j = \tilde{N}\bar{x}\), selling this debt security delivers a payoff to the issuer that is strictly below the one he obtains from selling the \(\tilde{N}\) assets separately. Thus, the effects of diversification cannot be undone by designing a security the payoff of which is a function of the pooled (diversified) cash flow \(Y_j\). This concludes our proof of Proposition 2.

In sum, when separate sales of assets are efficient, pooling assets leads to strictly worse outcomes, both in terms of the issuer’s surplus and the total trade surplus. This result emerges as pooling generically leads to a payoff distribution with thinner tails, and equivalently, a less elastic response to price quotes in the right tail of the payoff distribution (see Fig. 2). A less elastic response causes a liquidity supplier with market power to optimally set prices that lead to inefficient rationing, harming both the issuer and total trade efficiency. Thus, in contrast to the previously analyzed scenario with competitive liquidity suppliers (see Proposition 1), pooling assets hurts the issuer when the demand side has market power and the gains
from trade are sufficiently large.

3.2.2. Optimality of separate debt sales

Proposition 2 provided the condition under which selling assets separately as equity is optimal for the issuer and attains the first-best level of trade surplus. In the following analysis, we consider cases when this condition is violated and the elasticity function crosses \( \frac{\delta}{1-\delta} \) from above, such that \( e(\bar{x}) < \frac{\delta}{1-\delta} \), but it is nonetheless still optimal for the issuer to avoid the pooling of the underlying assets. However, in those cases, the issuer chooses to issue separate debt securities rather than equity securities, as we show in Proposition 3.

Proposition 3. Suppose that, for any subset \( \Omega_j \) of \( \Omega \), the elasticity function \( e_j \) is strictly decreasing on its respective support \( \chi_j \) (recall that Assumption 1 only required weak monotonicity) and that \( e(\bar{x}) < \frac{\delta}{1-\delta} \). There exists a \( \delta^* \in (\delta, 1] \) such that for all \( \delta \in (\delta, \delta^*) \), it is strictly optimal to issue a separate debt security on each asset payoff \( X_i \).

To prove this result, it is useful to introduce additional notation. Let \( \Pi(\delta) \) denote the issuer’s profit, as a function of the parameter \( \delta \), from selling one underlying asset separately, and issuing an optimal security on that underlying asset. Further, let \( \Pi_{\bar{\Omega}}(\delta) \) denote the issuer’s profit, also as a function of \( \delta \), from pooling \( \bar{\Omega} \) assets and issuing an optimal security on that underlying pool. The basic idea of the proof is to establish that these profits are continuous functions of \( \delta \), and to use the fact established in Proposition 2, which is that selling assets separately yields the issuer a strictly higher expected profit than pooling assets does when \( \delta = \bar{\delta} \):

\[
\bar{\Pi}(\delta) > \Pi_{\bar{\Omega}}(\delta).
\]  

First, suppose the issuer issues equity securities either on the pool of several assets or on each individual asset. For generality, suppose the underlying asset has the payoff \( y_j \), which could be a pool or a single asset. Then, for any \( \delta \in \left[ \frac{e_j(y_j)}{1+e_j(y_j)} , \frac{e_j(0)}{1+e_j(0)} \right] \), the monopolistic buyer would target an interior marginal issuer type \( f^m_j \) satisfying:

\[
e_j(f^m_j) = \frac{\delta}{1-\delta} \iff f^m_j(\delta) = e^{-1}_j \left( \frac{\delta}{1-\delta} \right),
\]

where \( e_j \) is an invertible function, since it is assumed to be strictly decreasing on its support.\(^{12}\) Thus, for all \( \delta \in \left[ \frac{e_j(y_j)}{1+e_j(y_j)} , \frac{e_j(0)}{1+e_j(0)} \right] \), this marginal issuer type \( f^m_j \) is a continuous function of the discount factor \( \delta \). This

\(^{12}\) In the special case of a pool of assets that each follow a uniform distribution, the elasticity function of the pool is only weakly decreasing everywhere on the domain, however it is strictly decreasing in the upper part of the domain, above a threshold value (see Fig. 2 for an illustration). As the logic of the proof only requires a strictly decreasing elasticity function in the right tail of the support, similar arguments can be used to extend the qualitative insights of Proposition 3 to this case of uniformly distributed asset payoffs, albeit at the cost of introducing additional notational complexity.
result is useful, since as shown in Lemma 4, the optimal debt security, which will be issued for \( \delta > \frac{e_j(\bar{y}_j)}{1+e_j(\bar{y}_j)} \), is implicitly characterized as a function of this marginal issuer type obtained when issuing an equity security. Specifically, the optimal security from the perspective of the issuer is a debt security with face value \( d^m_j \), where \( d^m_j \) is the largest \( d \) for which:

\[
\int_0^d f_j g_j(f_j) df_j + [1 - G_j(d)]d - \int_0^{f^m_j} (f_j - \delta f^m_j) g_j(f_j) df_j \geq 0,
\]

and where \( f^m_j = e_j^{-1}(\frac{\delta}{1+\delta}) \). Note that this optimal face value \( d^m_j \) is then also a continuous function of \( \delta \). This continuity result holds for any set \( \Omega_j \), including the case where \( \Omega_j \) includes only one asset.

Finally, note that if all the optimal face values \( d^m_j \) are continuous functions of \( \delta \), then the issuer’s profit functions \( \Pi(\delta) \) and \( \Pi_N(\delta) \) are also continuous functions of \( \delta \) since:

\[
\Pi(\delta) = \delta d^m(\delta) - \delta \int_0^{d^m(\delta)} f g(f) df - \delta [1 - G(d^m(\delta))] d^m(\delta) = \delta \int_0^{d^m(\delta)} G(f) df,
\]

and

\[
\Pi_N(\delta) = \delta d^m_N(\delta) - \delta \int_0^{d^m_N(\delta)} f g(f) df - \delta [1 - G(d^m_N(\delta))] d^m_N(\delta) = \delta \int_0^{d^m_N(\delta)} G_N(f) df,
\]

where we use integration by parts to simplify the expressions.

Given Eq. (16) and the continuity of the functions \( \Pi(\delta) \) and \( \Pi_N(\delta) \), we know that there is also a non-empty region \( (\delta, \delta^*) \), such that when \( \delta \) lies in that region, the following relation holds:

\[
\bar{N}(\delta) > \Pi_N(\delta).
\]

That is, selling \( \bar{N} \geq 2 \) assets separately (with debt) is strictly better for the issuer than selling debt on a pool of \( \bar{N} \) assets. The upper bound of the region, \( \delta^* \), is implicitly defined by the lowest \( \delta \) such that \( \bar{N}(\delta) = \Pi_N(\delta) \).

The main insight from Proposition 3 is that even when the potential gains from trade are smaller than required by the condition stated in Proposition 2, pooling assets may still be suboptimal for the issuer. The main difference relative to the result of Proposition 2 is that once separate equity securities do not trade fully efficiently, switching to separate debt securities is optimal. Yet, as the design of these debt securities is still intimately linked to the monopolistic liquidity supplier’s incentives to inefficiently screen the issuer (the marginal issuer type from equity sales enters expression (18)), the elasticity of trade volume is still an important determinant of the issuer’s net profit. As pooling assets reduces this elasticity in the right tail of the payoff distribution (see Fig. 2), it is undesirable to do so when the marginal issuer type from separate equity sales is sufficiently high, or equivalently, when the liquidity differences between the issuer and the buyer are sufficiently large (i.e., \( \delta \) is sufficiently low).

3.2.3. Optimality of pooling assets when adverse selection is severe

Unlike in the case of competitive demand where it is always optimal for the issuer to pool assets, the predictions for the scenario with monopolistic demand are more nuanced and feature a trade-off between
the benefits of diversification and the preservation of information rents. Propositions 2 and 3 highlight that the optimality of separate sales emerges when trade is particularly valuable, that is, when the prospective buyer and the issuer differ more in terms of their liquidity needs. In contrast, when potential gains from trade are smaller, adverse selection concerns and the exercise of market power lead to larger inefficiencies when assets are sold separately. Lower gains from trade (i.e., higher values of $\delta$) cause the liquidity supplier to choose a more aggressive pricing strategy, which leads to the exclusion of a larger range of issuer types when equity securities are issued. In this subsection, we focus on these cases. Specifically, when $e(0) < \frac{\delta}{1-\delta}$, the trading of separate securities — whether it is equity or debt — fails completely, as the elasticity function $e(x)$ then lies below $\frac{1}{1-\delta}$ everywhere on the support; all issuer types are excluded. However, as suggested by Fig. 2, pooling assets increases the elasticity in the left tail of the distribution, and thus can allow sustaining trade in these cases where separate sales would lead to trade breakdowns. That is, when adverse selection concerns are particularly severe, the trade-off faced by the issuer is tilted toward favoring the pooling of assets, even when the demand side is monopolistic.

Whereas our analysis up to this point did not require specifying a particular dependence structure for asset payoffs (beyond symmetry), we now make an additional assumption to maintain tractability. In particular, we consider a factor structure for asset payoffs as is commonly considered in the finance and economics literature. Formally, the asset payoffs are given by:

$$X_i = \sum_{k=1}^L C_k + Z_i, \quad \forall i \in \Omega,$$

where $C_k$ is a common factor and $Z_i$ is an asset-specific shock. Each common factor $C_k$ is i.i.d. and has a strictly positive and finite density function $g_{C}(x)$ on its support $[0, \bar{x}_C]$. The asset-specific component $Z_i$ is also i.i.d. and has a strictly positive and finite density function $g_Z(x)$ on its support $[0, \bar{x}_Z]$. As assumed throughout, the issuer is privately informed about the asset payoffs $X_i$.

As the elasticities at the lower bound of the support are key in determining whether trade breaks down in equilibrium, it is useful to establish the value of these elasticities for pools of assets. Let $e_q(\cdot)$ denote the elasticity function associated with a pool of $\tilde{N}$ assets. Lemma 5 characterizes the values of this function at the lower bound of its support.

**Lemma 5.** A pool of $\tilde{N}$ assets has the elasticity $e_q(0) = \tilde{N} + L$ at the lower bound of the support.

---

13. In the Appendix, we illustrate how pooling assets that exhibit this familiar factor structure affects distribution and elasticity functions.

14. Formally, we require that the first $(\tilde{N} + L - 1)$ derivatives of the density function of any pool of $\tilde{N} \leq N$ underlying assets with $L$ common factors exist at 0, which ensures that the elasticity functions of these pools exist at 0. See the proof of Lemma 5 for details.
Proof. See the Appendix.

The lemma confirms that, as suggested by Fig. 2, the elasticity at the lower bound increases when more assets are pooled. In fact, the elasticity increases by one for each asset that is added to a pool. Thus, when selling assets separately would lead to a complete trade breakdown (that is, when \( e(0) < \frac{\delta}{1-\gamma} \)), the issuer can do strictly better by pooling assets, provided that he has sufficiently many of them. We formalize this result in Proposition 4.

**Proposition 4.** Suppose that the issuer has \( N \geq \left( \frac{\delta}{1-\gamma} - L \right) \) assets. Then at least one of the subsets \( \Omega_j \) will optimally consist of \( N^* \) assets, where \( N^* \geq \frac{\delta}{1-\gamma} - L \).

As a complete trade breakdown yields zero surplus, it is optimal for the issuer to pool assets whenever a single asset has a lower-bound elasticity \( e(0) \) below \( \frac{\delta}{1-\gamma} \). Moreover, to ensure a lower-bound elasticity that is high enough to avert a complete trade breakdown, the issuer may have to pool more than two assets, specifically a number \( N^* \geq \left( \frac{\delta}{1-\gamma} - L \right) \).

While our analysis generally predicts that, ceteris paribus, issuers’ propensity to pool assets should be negatively related to buyers’ market power, Propositions 2, 3, and 4 highlight that the strength of this relationship depends on the magnitude of the potential gains from trade. When the gains from trade are sufficiently large (i.e., \( \delta \) is sufficiently low) and the demand side has market power, it is optimal to sell assets separately since adverse selection is less of a concern. Moreover, we have shown that when the issuer sells assets separately, the elasticity with which he responds to price changes is larger in the right tail of the distribution than when he is pooling assets. This elasticity in the right tail is relevant when the potential gains from trade are sufficiently large, causing the marginal issuer type that a monopolistic buyer targets to reside in that part of the distribution. Yet, when the potential gains from trade are sufficiently small, adverse selection concerns and the exercise of market power lead to complete market breakdowns if assets are sold separately. In this case, the issuer has to reduce information asymmetries to ensure that trade can occur at all. He thus pools assets. In particular, Lemma 5 shows that the elasticity in the left tail of the support rises with the number of assets that are pooled, allowing trade to occur once sufficiently many assets are included in a pool.

Our results also reveal that pooling assets on the one hand and using debt securities on the other play distinct roles in an issuer’s security design problem. An issuer is always concerned with buyers’ strategic exercise of market power and attempts to influence the type distribution in a way that maximizes his own rents. Yet, a debt contract cannot introduce the effects of diversification associated with pooling, which emerges only in the presence of multiple stochastic shocks. Conversely, pooling does not create the precisely defined point masses in the type distribution that a debt contract can achieve. Whereas using debt securities
is always weakly optimal in our environment (as in Biais and Mariotti, 2005), we show that the pooling of assets has more ambiguous effects. In particular, it tends to be suboptimal exactly when the potential gains from trade are large.

4. Robustness

In this section, we discuss the robustness of our main insights to various changes in the environment.

4.1. Multiple buyers participating in an OTC market

We now show that our main result regarding the optimality of separate sales with one buyer ($B = 1$) stated in Proposition 2 also applies when the issuer contacts multiple buyers sequentially. Suppose there are now $B \geq N$ prospective buyers with a discount factor of 1, so that each asset can in principle be sold to a distinct buyer. The issuer can contact these buyers in sequential bilateral interactions, requesting a quote for each security separately. Corresponding to the $B$ buyers, there are possibly up to $B$ rounds of requests for quote for a given security. The round in which a request for quote is made is common knowledge (more on this later). In response to a bilateral request for quote, a buyer makes a take-it-or-leave-it offer, a common practice in OTC markets [see, e.g., characterizations of OTC trading by Viswanathan and Wang (2004), Bessembinder and Maxwell (2008), and Duffie (2012)]. Consistent with these ultimatum offers, the issuer requests from a given buyer at most one quote for any given security he designed. In each round, a given security (or fraction of a security) that has not been sold yet can be offered only to one buyer, ensuring that delivery is always feasible. We also follow a simple tie-breaking rule that an issuer chooses to accept an offer in an earlier round when indifferent between an early and an anticipated later-round offer.

We can now conjecture and verify a perfect Bayesian equilibrium in this extension that mimics the main insight of our baseline model. Suppose that the condition stated in Proposition 2 is satisfied and that in the security design stage the issuer designed separate equity securities in a way that if the issuer were facing just one monopolistic buyer, each security would obtain an offer that is accepted by the issuer with probability 1. Given this security design decision, we consider the following outcome for the trading subgame of this OTC market with multiple buyers: for each security, the first buyer that obtains a request for quote for that security offers a price that is equal to the offer that would be obtained in a monopolistic setting, which we denote by $p^m = \delta x$. This offer also gets accepted with probability 1 in the considered OTC setting where multiple buyers are contacted sequentially. This is because a high-type issuer cannot expect a better offer on the security in any future round. Indeed, suppose that by rejecting, an issuer can perfectly signal that he is the highest possible type, which is the best-case scenario for any issuer. By backward induction, if a buyer gets a request for quote for this security in the $B$-th round, that buyer is effectively a true monopolist, as the issuer cannot request additional quotes from other buyers at that stage. In round $B$, a buyer thus
quotes the monopolistic offer $p^m$. Anticipating this behavior, a buyer who gets a request for quote in the $(B - 1)$-th round knows that the issuer’s outside option from rejecting a quote is $p^m$. This buyer thus has to offer a price weakly above $p^m$ to obtain an acceptance with probability 1 (given the tie-breaking rule). Thus, this buyer quotes $p^m$ as well. Iterating this argument, a buyer receiving a request for quote in the second round at best also quotes the monopolistic price $p^m$. Since this price is already offered in the first round, the issuer accepts the first-round offer. As a result, in equilibrium, the issuer sells each of his $N$ securities to a distinct buyer in the first round of requests for quote.

The central reason why equilibrium prices remain at their monopolistic levels in this extension, despite the presence of multiple unconstrained buyers, is the sequential and exclusive nature of bilateral interactions in OTC markets. In contrast, if quotes were collected simultaneously in a centralized market where the asset is allocated to the highest bidder, competitive prices would obtain (see Subsection 3.1). The OTC market structure is thus essential for maintaining effectively monopolistic offers, even when multiple buyers are endowed with liquidity ($\delta = 1$). Moreover, this result does not per se require that buyers in this market know in which round they receive a request for quote. Rather, we chose this assumption to capture the predictability of counterparties in OTC transactions (see Glode and Opp, 2020). However, the same prices and allocations would obtain in an equilibrium of the model if we assumed that buyers have no information about the round in which they are being contacted with a request for quote.\footnote{Specifically, suppose the round in which a request for quote is made is not known to a buyer and cannot be signaled by the seller. Then an equilibrium with the same prices and allocations as above exists where a buyer contacted in the first request for quote makes a monopolistic offer $p^m$ that is accepted by the seller with probability 1. Each buyer receiving a request for quote (including off-equilibrium requests) believes that he is the first to be contacted, which is consistent with the equilibrium. Given that every buyer would quote the price $p^m$, the seller optimally accepts the first quote, following the tie-breaking rule.}

Finally, we verify that in the security design stage, the issuer indeed designs separate equity securities such that, if he were to face a monopolistic buyer, each security would obtain an offer that is accepted by the issuer with probability 1. Given that the condition detailed in Proposition 2 holds, designing these securities implies that the trading game yields the issuer a payoff equal to $N\delta \bar{x}$. Suppose the issuer considers instead designing securities that invite screening in the first round (that is, offers that are not accepted with probability 1), and that this will allow high-type issuers to signal their quality. At best, such signaling could generate the belief that an issuer is of the highest possible type. But as shown above, by backward induction, the issuer can then also only expect a price equal to $\delta$ times this highest type. Yet, under the security design described in Proposition 2, the issuer already obtains a total payoff corresponding to $\delta$ times the highest type of her whole portfolio: $N\delta \bar{x}$. Thus, the issuer cannot do better than choosing the security design outlined in Proposition 2.
4.2. Uncertainty about the degree of buyer competition

A common feature of the baseline model and the analysis of a sequential OTC market was that the issuer could fully anticipate the degree of market power faced at the trading stage, which was either fully competitive or effectively monopolistic. One may, however, wonder what would happen if the issuer could not perfectly anticipate the market structure such that the expected degree of market power might be at an intermediate level. To consider this possibility, we now allow for uncertainty in the degree of buyer competition that the issuer faces when trying to sell securities. For this extension, we go back to our baseline setting where in the presence of multiple buyers \((B \geq 2)\), the demand side makes simultaneous offers and is fully competitive. With probability \(\phi\), the issuer faces one buyer \((B = 1)\), whereas with probability \((1 - \phi)\), he faces \(B \geq 2\) buyers. In this setting, one may gradually adjust the expected degree of market power at the trading stage by changing the parameter \(\phi\).

If the issuer now can choose the securities that will be offered for sale after having learned about how many buyers will compete but before having acquired private information (perhaps by shelf-registering a cleverly designed portfolio of securities), then the same results obtain as in our baseline setting: in this “interim” security design timeline, the issuer offers a debt security on a pool of assets when he learns that \(B \geq 2\) and offers separate debt securities for each asset when he learns that \(B = 1\).

If instead the issuer must choose whether to pool assets and set the face value for each debt security issued before knowing the degree of buyer competition, then his decision problem is different. In this “ex ante” security design timeline, the issuer considers the benefits and costs of issuing specific debt securities anticipating that ex post he might face competitive buyers or a monopolistic buyer.

It turns out that for each debt security the issuer considers issuing, he effectively determines whether it would be best to use the face value that is optimal in the competitive case or the face value that is optimal in the monopolistic case. The rationale for why this behavior is optimal is as follows: for a given pool \(\Omega_j\), let \(d^c_j\) and \(d^m_j\) denote the optimal face values of debt in the competitive and monopolistic cases, respectively. We know from our baseline analysis that \(d^c_j \geq d^m_j\). In a first step, we can rule out that the optimal face value for the case with an uncertain level of buyer competition, say \(d^\phi_j\), is such that \(d^\phi_j > d^c_j\). With such a high face value \(d^\phi_j\), competitive buyers would refuse to buy the issuer’s security (thus, delivering no profits for the issuer) and a monopolistic buyer would offer the screening equity price \(p^m_j\). The issuer would then be strictly better off designing a debt security with a face value \(d^\phi_j = d^c_j\) to extract a positive surplus from trade when buyers are competitive, while not affecting the profit collected when the buyer is a monopolist. Second, we can rule out the optimality of setting \(d^\phi_j < d^m_j\) since the profits in both bargaining scenarios would be lower than if the issuer picked a face value of \(d^\phi_j = d^m_j\). Finally, if the issuer picked \(d^\phi_j \in (d^m_j, d^c_j)\), competitive buyers would agree to purchase the debt security but the monopolistic buyer would offer the screening equity price \(p^m_j\). Since the monopolistic buyer would offer \(p^m_j\) for any face value above \(d^m_j\), the issuer could extract more surplus from trade in the competitive case by choosing a face value \(d^\phi_j = d^c_j\).
Overall, these arguments imply that the optimal $d^\phi_j$ for a given pool $j$ is either equal to $d^m_j$ or $d^c_j$.

Fig. 3. Effect of uncertainty about the number of prospective buyers on issuer’s expected profit. In the graph, we consider a setting with two independent assets ($N = 2$), each of which has a payoff $X_i \sim U[0, 1]$, and an issuer’s discount factor $\delta = 0.6$. We compare the issuer’s expected profit from selling separate debt securities for each asset and from selling one debt security on a pool of assets for all possible levels of $\phi$ (i.e., the probability that only one prospective buyer has a discount factor of 1), assuming an ex ante security design timeline.

In Fig. 3, we compare the issuer’s expected profit from selling separate debt securities for each asset and from selling one debt security on a pool of assets assuming an ex ante security design timeline, in a setting with two independent assets ($N = 2$), each of which has a payoff $X_i \sim U[0, 1]$, and with an issuer’s discount factor $\delta = 0.6$. When issuing separate debt securities for each asset, the issuer optimally sets the face value of debt at 0.80, regardless of the value of $\phi$. When issuing one debt security on a pool of two assets, the issuer optimally sets the face value of debt at 1.655 if $\phi < 0.012$ and at 1.576 if $\phi \geq 0.012$ (which causes the kink in the profit function for the pooling strategy). Consistent with the main intuition from our baseline model, pooling assets is optimal when the issuer is sufficiently likely to face several buyers, whereas selling separate securities is optimal when the issuer is sufficiently likely to face a monopolistic buyer.

4.3. Heterogenous exposures to common factors

In Subsection 3.2.3, we considered a setting where assets were symmetrically exposed to common factors. We now discuss the robustness of our results to the case where different assets are exposed to distinct factors. As an example, suppose that the issuer has two sets of assets that are exposed to distinct factors $C_1$ and $C_2$. That is, the asset payoffs in a set $k \in \{1, 2\}$ are given by $X^k_i = C_k + Z_i$, where $Z_i$ are i.i.d. asset-specific shocks, and each set $k$ contains a continuum of assets of total measure 1.

By the law of large numbers, it follows that the pool of assets contained in a given set $k$ will generate a payoff $\int X^k_i di = C_k + E[Z_i]$ almost surely. Since $E[Z_i]$ is constant, such a pool containing all assets exposed
to the same common factor $k$ can be efficiently sold to a monopolistic buyer, provided that the distribution of $C_k$ satisfies the condition stated in Proposition 2. Moreover, Proposition 2 implies that it would be strictly suboptimal for the issuer to pool all assets and partially diversify the common shocks.\footnote{Pooling all assets would result in a payoff $C_1 + C_2 + 2 \cdot E[Z_i]$, which has an elasticity of zero at the upper bound of the support, and would thus invite screening by a monopolistic buyer.} This result provides insight on the common practice by originators to design securities that pool mortgages only from certain types of borrowers (e.g., geographic regions, residential vs. commercial), thereby creating pools with distinct factor exposures.

4.4. Signaling through retention

As originally shown by Biais and Mariotti (2005), for a given security offered by an issuer, a monopolistic buyer’s optimal mechanism is a take-it-or-leave-it offer for the total supply of the security, rather than a menu of price-quantity offers allowing for signaling through retention. Since the buyer makes a take-it-or-leave-it offer for the securities, he could extract all the surplus from trade if he were able to infer the issuer’s type from a signal. In this case, the issuer’s profit from implementing fully revealing retention policies is therefore weakly lower than his profit without any signaling through retention [see also Glode et al. (2018) for related arguments]. On the other hand, in the scenario with competing liquidity suppliers, restricting our analysis to the specific trading mechanism considered in DeMarzo (2005) would yield results that are consistent with his results — issuers with assets of higher quality would retain a higher fraction of the issue.\footnote{See also Williams (2021), who studies the optimality and efficiency of security retention in the presence of search frictions.} Signaling would then allow the high issuer types to separate themselves from the low types and would resolve the lemons problem for high values of $\delta$.

4.5. Risk aversion

In line with previous studies, we have assumed that agents are risk neutral. It is worth noting that, even if we allowed for risk aversion, pooling assets would not by itself lead to better risk sharing among traders. This is because the issuer offers to sell all assets to the buyer(s) independent of whether he pools the assets or not. With risk-averse agents, the main impediment to risk sharing would be the fact that the issuer’s private information may result in socially inefficient trade breakdowns, which is already a force at play in our baseline model.

5. Conclusion

We study the optimality of pooling assets when security issuers face a market in which liquidity is scarce and buyers endowed with such liquidity have market power, as is common in over-the-counter markets. Our
analysis reveals how buyers’ market power may render the pooling of assets suboptimal — both privately and socially — in particular, when the potential gains from trade are large. While our results suggest that the dramatic decline of the ABS market post crisis may have been an efficient response by originators to significant changes in liquidity and market power in OTC markets, they also highlight the potential welfare implications of liquidity constraints imposed on financial institutions in the new market environment. Regulatory policies mitigating the concentration of liquidity among few market participants may be well suited to reduce market power and associated declines in the issuance of securities that are backed by well-diversified pools, even when those policies are not specifically targeting structured security holdings (such as general leverage and liquidity requirements).

More broadly, the principles we uncover could also be incorporated into analyses of firms’ capital structure decisions, specifically regarding their debt maturity structure. To illustrate the mapping between this problem and our setup, suppose a firm generates cash flows in different time periods and is privately informed about these future cash flows. Each cash flow can be viewed as one of the fundamental assets from our baseline setup. The firm then decides whether to pool all cash flows across time (e.g., by issuing an equity claim or a perpetual debt claim) or not (e.g., by issuing multiple zero coupon bonds of different maturities). Our findings suggest that when firms face investors with market power, it is relatively more beneficial for them to issue multiple debt securities with different maturities, a practice that is indeed quite common.
Appendix A: Proofs omitted from the text

Proof of Lemma 2: Without loss of generality, suppose \( i = 1 \). We can express \( \tilde{N}X_1 \) as follows:

\[
\tilde{N}X_1 = \sum_{i=1}^{\tilde{N}} X_i + \left( (\tilde{N} - 1)X_1 - \sum_{i=2}^{\tilde{N}} X_i \right),
\]

where \( (\tilde{N} - 1)X_1 - \sum_{i=2}^{\tilde{N}} X_i \) has a conditional expected value of zero:

\[
\mathbb{E} \left[ (\tilde{N} - 1)X_1 - \sum_{i=2}^{\tilde{N}} X_i \right] = (\tilde{N} - 1) \mathbb{E} [X_1] - \sum_{i=2}^{\tilde{N}} \mathbb{E} [X_i] = 0.
\]

The last equality follows from the symmetry of the joint distribution of asset payoffs. It directly follows that \( \tilde{N}X_1 \) is a mean-preserving spread of \( Y_j \), and the distribution of \( Y_j \) thus second-order stochastically dominates the distribution of \( \tilde{N}X_1 \).

Proof of Lemma 3: Consider the sum of \( Y_{\tilde{N}} = \sum_{i=1}^{\tilde{N}} X_i \) and \( X_k \), where \( k > \tilde{N} \), that is, \( Y_{\tilde{N} + 1} = Y_{\tilde{N}} + X_k \). For the density of \( Y_{\tilde{N} + 1} \), we can write:

\[
g_{\tilde{N}+1}(y_{\tilde{N}+1}) = \int_0^x g_{\tilde{N},1}(y_{\tilde{N}}+x-x)dx,
\]

where \( g_{\tilde{N},1}(y_{\tilde{N}+1} - x, x) \) is the density of the joint distribution of \( Y_{\tilde{N}} \) and \( X_k \). In the special case when assets' payoffs are independent, we have \( g_{\tilde{N},1}(y_{\tilde{N}+1} - x, x) = g_{\tilde{N}}(y_{\tilde{N}}+x-x)g(x) \).

Now evaluate \( g_{\tilde{N}+1} \) at the upper bound of the support \( \tilde{y}_{\tilde{N}+1} = (\tilde{N} + 1)x \):

\[
g_{\tilde{N}+1}(\tilde{N} + 1)x = \int_0^x g_{\tilde{N},1}(\tilde{N} + 1)x - x, x)dx = 0,
\]

since the joint density \( g_{\tilde{N},1}(y_{\tilde{N}}, x) \) is equal to zero for any outcome \( y_{\tilde{N}} \) of \( Y_{\tilde{N}} \) above \( \tilde{N}x \). As a result, the elasticity \( e_{\tilde{N}+1}(\tilde{y}_{\tilde{N}+1}) = g_{\tilde{N}+1}(\tilde{y}_{\tilde{N}+1})\tilde{y}_{\tilde{N}+1}/G(\tilde{y}_{\tilde{N}+1}) \) is also zero for all \( \tilde{N} \geq 1 \), that is, as soon as at least two assets are pooled, such that \( \tilde{N} + 1 \geq 2 \), the elasticity of the pool will be zero at the upper bound \( \tilde{y}_{\tilde{N}+1} \).

Proof of Lemma 5: First, suppose that \( g(0) > 0 \) and \( g'(0) \) is finite. By L'Hôpital's rule, the elasticity is:

\[
\lim_{y \to 0} \frac{g(y)y}{G(y)} = \lim_{y \to 0} \frac{g'(y)y + g(y)}{g(y)} = \frac{g'(0)y + g(0)}{g(0)} = 1.
\]

Next, suppose that \( g(0) = 0 \), \( g'(0) > 0 \), and \( g''(0) \) is finite. Then, applying the rule twice, the elasticity is:

\[
\lim_{y \to 0} \frac{g(y)y}{G(y)} = \lim_{y \to 0} \frac{g'(y)y + g(y)}{g(y)} = \lim_{y \to 0} \frac{g''(y)y + 2g'(y)}{g'(y)} = 2.
\]
More generally, if the \((k-1)\)-th derivative of the density function \(g\) is the first derivative to be positive, i.e., if \(g^{(k-1)}(0) > 0\) and \(g^{(l)}(0) = 0\) for all \(l < k - 1\), then the elasticity is equal to \(k\), since the recursive application of L'Hôpital's rule yields:

\[
\lim_{y \to 0} \frac{g(y)y}{G(y)} = \lim_{y \to 0} \frac{g'(y)y + g(y)}{g(y)} = \cdots = \lim_{y \to 0} \frac{g^{(k)}(y)y + kg^{(k-1)}(y)}{g^{(k-1)}(y)} = k,
\]

where the superscripts denote orders of derivative.

To prove the lemma, it remains to be shown that, if we construct a pool of \(N\) assets correlated through the common factors \(\sum_{k=1}^{L} C_k\), the first derivative of the density function of this pool that is positive (and non-zero) is the \((N + L - 1)\)-th derivative. We next proceed to prove the lemma in two steps: first for \(L = 0\), when assets' payoffs \(X_i\) are independent, and then for \(L > 0\), when asset payoffs \(X_i\) are correlated.

**Independent payoffs.** Absent common factors we obtain \(g(x) = g_2(x)\). Consider the convolution of \(Y_N = \sum_{i=1}^{N} X_i\) and \(X_k\), where \(k > N\), that is, \(Y_{N+1} = Y_N + X_k\). Since these \(Y_N\) and \(X_k\) are independent, we can write:

\[
g_{N+1}(y_{N+1}) = \int_0^y g_N(y_{N+1} - x)g(x)dx,
\]

which, for \(0 \leq y_{N+1} \leq \bar{x}\), simplifies to:

\[
g_{N+1}(y_{N+1}) = \int_{0}^{y_{N+1}} g_N(y_{N+1} - x)g(x)dx.
\]

Thus, the first two derivatives become:

\[
g'_{N+1}(y_{N+1}) = g_N(0)g(y_{N+1}) + \int_{0}^{y_{N+1}} g'_N(y_{N+1} - x)g(x)dx,
\]

\[
g''_{N+1}(y_{N+1}) = g_N(0)g'(y_{N+1}) + g'_N(0)g(y_{N+1}) + \int_{0}^{y_{N+1}} g''_N(y_{N+1} - x)g(x)dx.
\]

More generally, for \(k \geq 1\), the \((k)\)-th derivative is:

\[
g^{(k)}_{N+1}(y_{N+1}) = \sum_{l=0}^{k-1} g^{(l)}_N(0)g^{(k-1-l)}(y_{N+1}) + \int_{0}^{y_{N+1}} g^{(k)}_N(y_{N+1} - x)g(x)dx.
\]

Hence, when evaluated at \(y_{N+1} = 0\), we obtain the following derivative:

\[
g^{(k)}_{N+1}(0) = \sum_{l=0}^{k-1} g^{(l)}_N(0)g^{(k-1-l)}(0).
\]

Next we can prove the claim for all \(N \geq 1\) by induction. First, suppose we have \(N = 1\). Then \(g_1(0) = g(0) = g_2(0) > 0\), and adding an asset yields \(g_2(0) = 0\) (see above integral) while \(g'_2(0) = g_1(0)g(0) = g(0)^2 > 0\). Next, for the induction step, suppose the claim holds for \(N\), i.e., \(g^{(N-1)}_{N}(x) > 0\) and
\( g_N^{(k)}(x) = 0 \) for all \( k < \tilde{N} - 1 \). Then adding an asset yields
\[
g_{\tilde{N}+1}^{(k)}(0) = \sum_{l=0}^{k-1} g_{\tilde{N}}^{(l)}(0) g_{\tilde{N}+1}^{(k-1-l)}(0) = 0 \quad \text{for all} \quad k < \tilde{N} \quad \text{while:}
\]
\[
g_{\tilde{N}+1}^{(\tilde{N}+1)}(0) = \sum_{l=0}^{\tilde{N}-1} g_{\tilde{N}}^{(l)}(0) g_{\tilde{N}+1}^{(\tilde{N}-1-l)}(0) = g_{\tilde{N}}^{(\tilde{N}+1)}(0) g(0) > 0.
\] (A.14)

Therefore, the claim also holds for \( \tilde{N} + 1 \).

Thus, every time we add an asset to the pool, the next-higher derivative of the density function turns to zero, while leaving the derivatives thereafter positive. Ultimately, by induction for all \( \tilde{N} \geq 1 \), we obtain that \( g_{\tilde{N}}^{(\tilde{N}+1)}(x) > 0 \) and \( g_{\tilde{N}}^{(k)}(x) = 0 \) for all \( k < \tilde{N} - 1 \).

**Correlated payoffs.** For \( L \geq 1 \) and a pool of \( \tilde{N} \) assets, we obtain:
\[
Y_{\tilde{N}} = \sum_{i=1}^{\tilde{N}} X_i = \tilde{N} \sum_{k=1}^{L} C_k + \sum_{i=1}^{\tilde{N}} Z_i.
\] (A.15)

Denote the density function of \( \tilde{N} \sum_{k=1}^{L} C_k \) as \( c_{\tilde{N}}(x) \) and the density function of \( \sum_{i=1}^{\tilde{N}} Z_i \) as \( s_{\tilde{N}}(x) \). Since \( \tilde{N} \sum_{k=1}^{L} C_k \) and \( \sum_{i=1}^{\tilde{N}} Z_i \) are independent, the density \( g_{\tilde{N}}(y_{\tilde{N}}) \) of the pool \( Y_{\tilde{N}} \) is a convolution and we can write for \( 0 \leq y_{\tilde{N}} \leq \tilde{N} L \bar{x}_C \):
\[
g_{\tilde{N}}(y_{\tilde{N}}) = \int_{y_{\tilde{N}} - \tilde{N} L \bar{x}_C}^{y_{\tilde{N}}} s_{\tilde{N}}(y_{\tilde{N}} - x) c_{\tilde{N}}(x) dx.
\] (A.16)

Thus, proceeding in the same way as in the first part of the proof by taking the derivatives and evaluating them at \( y_{\tilde{N}} = 0 \), for \( k \geq 1 \), we obtain the \( (k) \)-th derivative:
\[
g_{\tilde{N}}^{(k)}(0) = \sum_{l=0}^{k-1} s_{\tilde{N}}^{(l)}(0) c_{\tilde{N}}^{(k-1-l)}(0).
\] (A.17)

Applying the above results for independent assets to the sum of \( \tilde{N} \) independent asset-specific components \( \sum_{i=1}^{\tilde{N}} Z_i \), we obtain that \( s_{\tilde{N}}^{(\tilde{N}+1)}(x) > 0 \) and \( s_{\tilde{N}}^{(l)}(x) = 0 \) for all \( l < \tilde{N} - 1 \). Similarly, we can apply the same results to the sum of \( L \) common factors \( \sum_{k=1}^{L} C_k \), since the factors \( C_k \) are assumed to be i.i.d., to obtain \( c_{\tilde{N}}^{(L-1)}(0) > 0 \) and \( c_{\tilde{N}}^{(l)}(0) = 0 \) for \( l \leq L - 1 \). Additionally, since \( c_{\tilde{N}}(x) = \frac{1}{\bar{x}_{\tilde{N}}} c_1(\tilde{x}/\bar{x}_{\tilde{N}}) \), we have \( c_{\tilde{N}}^{(L-1)}(0) > 0 \) and \( c_{\tilde{N}}^{(l)}(0) = 0 \) for \( l < L - 1 \) and all \( \tilde{N} \geq 1 \).

Combining the above, we obtain the desired result for all \( \tilde{N} \geq 1 \) and \( L \geq 1 \) as:
\[
g_{\tilde{N}}^{(k)}(0) = \sum_{l=0}^{k-1} s_{\tilde{N}}^{(l)}(0) c_{\tilde{N}}^{(k-1-l)}(0) = 0 \quad \text{for all} \quad k < \tilde{N} + L - 1 \quad \text{while:}
\]
\[
g_{\tilde{N}}^{(\tilde{N}+L-1)}(0) = \sum_{l=0}^{\tilde{N}+L-2} s_{\tilde{N}}^{(l)}(0) c_{\tilde{N}}^{(\tilde{N}+L-2-l)}(0) = s_{\tilde{N}}^{(\tilde{N}+L-1)}(0) c_{\tilde{N}}^{(L-1)}(0) > 0.
\] (A.18)

As with independent assets, every time we add an asset to the pool, the next-higher derivative of the density function turns to zero, while leaving the derivatives thereafter positive. Analogously, every time we add a factor to the common component \( C \), the next-higher derivative of the density function turns to zero, while leaving the derivatives thereafter positive. □
Appendix B: Additional examples of distributions

In this Appendix, we provide additional examples of distributions satisfying the requirements of our baseline setting (including Assumption 1).

B.1 Independent asset payoffs

The figures in this subsection illustrate the effects of pooling on the shapes of the PDF and the elasticity function for independent asset payoffs.

Fig. B.1. Effect of pooling on the shape of the probability density function. In the graph, we consider a setting with four independent assets ($N = 4$), each of which has a payoff $X_i$ that follows a beta distribution, with shape parameters $\alpha = 4$ and $\beta = 4$, that is truncated on the interval [0.001, 0.999]. We plot the payoff distributions of a separate asset, a pool of two assets, and a pool of four assets. To compare the PDFs' shapes relative to their respective domains, we rescale the horizontal axis to represent the interval $\chi_j = [0, \bar{y}_j]$ for each PDF $g_j$. 
Fig. B.2. Effect of pooling on the shape of the elasticity function. In the graph, we consider a setting with four independent assets ($N = 4$), each of which has a payoff $X_i$ that follows a beta distribution, with shape parameters $\alpha = 4$ and $\beta = 4$, that is truncated on the interval $[0.001, 0.999]$. We plot the elasticity functions of a separate asset, a pool of two assets, and a pool of four assets. To compare the elasticity functions’ shapes relative to their respective domains, we rescale the horizontal axis to represent the interval $\chi_j = [0, \bar{y}_j]$ for each elasticity function $e_j$.

Fig. B.3. Effect of pooling on the shape of the probability density function. In the graph, we consider a setting with four independent assets ($N = 4$), each of which has a payoff $X_i$ that follows a beta distribution, with shape parameters $\alpha = 2$ and $\beta = 3$, that is truncated on the interval $[0.001, 0.999]$. We plot the payoff distributions of a separate asset, a pool of two assets, and a pool of four assets. To compare the PDFs’ shapes relative to their respective domains, we rescale the horizontal axis to represent the interval $\chi_j = [0, \bar{y}_j]$ for each PDF $g_j$. 
Fig. B.4. Effect of pooling on the shape of the elasticity function. In the graph, we consider a setting with four independent assets \((N = 4)\), each of which has a payoff \(X_i\) that follows a beta distribution, with shape parameters \(\alpha = 2\) and \(\beta = 3\), that is truncated on the interval \([0.001, 0.999]\). We plot the elasticity functions of a separate asset, a pool of two assets, and a pool of four assets. To compare the elasticity functions’ shapes relative to their respective domains, we rescale the horizontal axis to represent the interval \(\chi_j = [0, \bar{y}_j]\) for each elasticity function \(e_j\).

B.2 Correlated asset payoffs

We now consider assets with correlated payoffs. Fig. B.5 and Fig. B.6 show the effects of pooling on the shapes of the PDF and the elasticity function when asset payoffs follow a uniform distribution. In particular, the figures highlight that the effects of diversification are present even if assets are highly correlated and become more pronounced for lower correlation coefficients.
Fig. B.5. Effect of pooling on the shape of the probability density function for two uniform assets with correlated payoffs. In the graph, we consider a setting with two correlated assets ($N = 2$), each of which has a payoff $X_i \sim U[0, 1]$, and where the joint dependence is characterized by a generalized diagonal band copula with a generator function given by the density function of a beta distribution $Beta \left( \frac{1}{2} \left[ \sqrt{\frac{49 + \rho}{1 + \rho}} - 5 \right], 1 \right)$. We plot the PDF of a pool of two assets for three scenarios with distinct correlation coefficients $\rho \in \{0, 0.5, -0.5\}$. 

![Graph showing the probability density function for different correlation coefficients.](image)

$g(y_j)$

$y_j$

$\rho = 0$

$\rho = 0.5$

$\rho = -0.5$

Fig. B.6. Effect of pooling on the shape of the elasticity function for two uniform assets with correlated payoffs. In the graph, we consider a setting with two correlated assets ($N = 2$), each of which has a payoff $X_i \sim U[0, 1]$, and where the joint dependence is characterized by a generalized diagonal band copula with a generator function given by the density function of a beta distribution $Beta \left( \frac{1}{2} \left[ \sqrt{\frac{49 + \rho}{1 + \rho}} - 5 \right], 1 \right)$. We plot the elasticity function of a pool of two assets for three scenarios with distinct correlation coefficients $\rho \in \{0, 0.5, -0.5\}$.

![Graph showing the elasticity function for different correlation coefficients.](image)

$e(y_j)$

$y_j$

$\rho = 0$

$\rho = 0.5$

$\rho = -0.5$

Fig. B.7 and B.8 illustrate the effects of pooling for assets with payoffs that follow the factor structure introduced in Subsection 3.2.3.
Fig. B.7. Effect of pooling on the shape of the probability density function for four assets with payoffs that follow the factor structure. In the graph, we consider a setting with four assets (N = 4), each of which has a payoff $X_i = C + Z_i$ with $C, Z_i \sim U[0, 1]$. We plot the payoff distributions of a separate asset, a pool of two assets, and a pool of four assets. To compare the PDFs’ shapes relative to their respective domains ([0, 2], [0, 4], and [0, 8]), we rescale the horizontal axis to represent the interval $\chi_j = [0, \bar{y}_j]$ for each PDF $g_j$.

Fig. B.8. Effect of pooling on the shape of the elasticity function for four assets with payoffs that follow the factor structure. In the graph, we consider a setting with four assets (N = 4), each of which has a payoff $X_i = C + Z_i$ with $C, Z_i \sim U[0, 1]$. We plot the elasticity functions of a separate asset, a pool of two assets, and a pool of four assets. To compare the elasticity functions’ shapes relative to their respective domains ([0, 2], [0, 4], and [0, 8]), we rescale the horizontal axis to represent the interval $\chi_j = [0, \bar{y}_j]$ for each elasticity function $e_j$. 


References


