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Evaluation of Transmission Reach and Information Rates in Nonlinear Optical Fiber Communication Systems

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ABSTRACT

Coherent optical fiber systems can achieve long-distance, large-capacity and high data-rate transmissions. The system performance of communication systems is generally evaluated with regard to the data capacity and the transmission reach. In this work, the performance of multi-channel (up to C-band) Nyquist-spaced coherent optical communication systems has been assessed in terms of achievable information rates, transmission distances and signal-to-noise ratios, considering different influencing factors, such as nonlinearity compensation, signal input power and modulation format. Numerical simulations and enhanced Gaussian noise (EGN) model have been carried out for different modulation formats including quadrature phase shift keying (QPSK), 16-ary quadrature amplitude modulation (16-QAM), 64-QAM and 256-QAM. It is found that in C-band (151-channel) Nyquist-spaced systems, the achievable information rates at the transmission distance of 6000 km are 19.3 Tbit/s for dual-polarization QPSK (DP-QPSK), 30.9 Tbit/s for DP-16QAM, 32.0 Tbit/s for DP-64QAM and 32.2 Tbit/s for DP-256QAM, respectively, when electronic dispersion compensation is applied only. Such achievable information rates can be increased up to 38.3 Tbit/s for DP-16QAM, 47.2 Tbit/s for DP-64QAM and 47.8 Tbit/s for DP-256QAM, respectively, when the nonlinearity compensation is employed.

Keywords: Coherent optical communication, Kerr fiber nonlinearities, Digital signal processing, Achievable information rate, Transmission distance

1. INTRODUCTION

Nowadays, more than 95% of data transmission is carried by optical fiber networks. The demand of the information rate is increasing dramatically, with the rapid development of various new technologies, such as big data, Internet of Things (IoT), artificial intelligence (AI), virtual reality (VR), and the fifth-generation mobile communications (5G) 1. The transmission distance and data rate of the coherent optical fiber communication systems have been promoted by utilizing the digital signal processing (DSP) technology 2,3. The achievable information rates (AIRs) will decrease due to the distortions from the chromatic dispersion (CD), the laser phase noise (LPN), the polarization mode dispersion (PMD), and the Kerr fiber nonlinearity interference (NLI) 4–10. These impairments can be separately suppressed using the DSP approaches. For example, the frequency domain equalizer can compensate for the CD, and digital back-propagation (DBP) is an effective technology to compensate CD and NLI simultaneously 11–13. The system performance of communication systems is generally evaluated with regard to the data capacity and the transmission reach. In this work, considering different scenarios such as modulation format, digital compensation algorithm and optical launch power per channel, theoretical values of the AIRs under different transmission distances of the multi-channel (up to 151) Nyquist-spaced coherent optical communication system have been evaluated based on the enhanced Gaussian noise (EGN) model 8,14–16. Numerical simulations have also been performed to verify the accuracy of the EGN model in 7-ch Nyquist-spaced 32Gbaud optical transmission systems using the electronic dispersion compensation (EDC) and multi-channel DBP (MC-DBP), when dual-polarization quadrature phase shift keying (DP-QPSK), DP 16-ary quadrature amplitude modulation (DP-16QAM), and DP-64QAM are applied.

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This paper is organized as follows: Section 2 introduces the analytical model for evaluating the signal-to-noise ratio (SNR) and the AIRs of the transmission system. Section 3 describes the transmission setup. The simulation results and AIR-distance relationship based on EGN model will be presented in Section 4. Finally, Section 5 provides the conclusions.

2. ANALYTICAL MODEL AND ACHIEVABLE INFORMATION RATE

In this work, the EGN model is applied to numerically estimate the SNR of the system. The Gaussian noise (GN) model uses the following assumption and treatment: (1) The NLI noise in the optical fiber link is relatively small. (2) The transmitted signal is represented as Gaussian noise. (3) The signal distortion generated by the nonlinearity is represented as additive Gaussian noise. Considering the linear and the NLI noise, the SNR in the Nyquist optical fiber communication system can be described as follows \(^{14,17}\):

\[
SNR \approx \frac{P_{ch}}{P_{ASE} + P_{NLI}}
\]

where \(P_{ch}\) is the average optical power per channel, \(P_{ASE}\) is the total power of amplified spontaneous emission (ASE) noise, \(P_{NLI}\) is the total power of NLI noise, which composed of the inter-signal distortion \(\sigma_{2-S}^2\) and the signal-noise interaction \(\sigma_{2-N}^2\). For the dual-polarization multi-span EDFA amplified system, the ASE noise of the system totally comes from the accumulation of the EDFA amplified noise, which can be described as follows \(^{18}\):

\[
P_{ASE} = N_S(G - 1)F_nh
\]

where \(N_S\) is the total number of spans in the optical fiber link, \(G\) is the EDFA gain, \(F_n\) is the EDFA noise figure, \(h\) is the frequency of the optical carrier, \(\nu_0\) denotes the average photon energy, and \(R_S\) is the symbol rate of the transmitted signal. Then the distortion of inter-signal noise and signal-noise interaction noise are respectively expressed as \(^{17,19}\):

\[
\sigma_{2-S}^2 = N_S^{3+1}\eta P_3
\]

\[
\sigma_{2-N}^2 \approx 3\xi\eta P_{ASE} \cdot P^2
\]

where \(\xi\) is the coherence factor of the interaction between signals that increased with the transmission distance, \(\eta\) is the non-linear distortion coefficient given by the GN model \(^{20,21}\):

\[
\eta_0(N_{ch}) \approx \left(\frac{2}{3}\right)^3 \frac{\alpha \gamma^2 L_{eff}^2}{\beta_2 |R_S^2|} \arcsinh \left(\frac{\pi^2}{2 |\beta_2| L_{eff}} \cdot N_{ch} R_S^2 \right)
\]

The EGN model takes into account the influence of the modulation format, and the revised \(\eta\) can be expressed as:

\[
\eta(N_{ch}) \approx \eta_0(N_{ch}) - \frac{80 \kappa \gamma^2 L_{eff}^2}{81 \pi |\beta_2| L_{eff} R_S^2} \left[\psi \left(\frac{N_{ch} + 1}{2}\right) + C + 1\right]
\]

where, \(L_s\) is the fiber span length, \(L_{eff}\) is the effective fiber span length which equals \((1 - e^{-2\alpha L_s})/(2\alpha)\). \(\alpha\) is the attenuation coefficient, \(\beta_2\) is the group velocity dispersion coefficient, and \(\kappa\) is the coefficient related to the modulation format. For QPSK, 16QAM, 64QAM and 256QAM, the values of \(\kappa\) are respectively 1.17/25, 13/21 and 121/200. \(\psi(x)\) is the digamma function, \(C=0.577\) is the Euler-Mascheroni constant. \(\xi\) denotes to the distortion caused by interaction between the signal and the ASE noise which accumulates with the transmission distance.

When MC-DBP is used at partial bandwidth to compensate the non-linearity, the total power of inter-signal noise \(\sigma_{2-S}^2 = N_S^{\xi+1}\left[\eta(N_{ch}) - \eta(N_{ch}^{DBP})\right]P_3\) where \(N_{ch}^{DBP}\) is the number of MC-DBP channels. The above formulas can be used to estimate the SNR of the system under different modulation formats, transmission distances and fiber input power, to predict the performance of the transmission system.

The AIR is the maximum data rate that can be reliably transmitted through a channel, for a given modulation format. For the dual-polarization Nyquist-spaced optical fiber communication system, the calculation formula of AIR is equals to \(N_{ch} \cdot R_S \cdot MI\), where MI represents the mutual information. The dual-polarization symbol-wise soft-decision MI between input variable \(X\) and output variable \(Y\) for a discrete QAM signal input distribution can be calculated as follows:
\[ M1 = \frac{2}{M} \sum_{x \in X} \int P_{Y|X}(y|x) \log_2 \left( \frac{P_{Y|X}(y|x)}{\frac{1}{M} \sum_{x' \in X} P_{Y|X}(y|x')} \right) dy \]  

where \( M = |X| = 2^m \) denotes the cardinality of the M-QAM constellation with the number of bits per symbol \( m \). \( P_{Y|X}(y|x) \) represents the probability of receiving the symbol \( y \) when sending the symbol \( x \). For a given channel input \( X \), the conditional probability density function of channel output \( Y \) is given by:

\[ P_{Y|X}(y|x) = \frac{1}{\pi \sigma_z^2} \exp \left( -\frac{|y - x|^2}{\sigma_z^2} \right) \]  

### 3. TRANSMISSION SYSTEM

The setup of 32-Gbaud multi-channel Nyquist-spaced coherent optical fiber transmission system is illustrated in Fig. 1. At the transmitter, a 32-GHz spaced laser comb is launched as the source of the optical carrier. The symbol sequences of transmitted signals in each channel are independent and random. A root-raised cosine (RRC) filter with a roll-off of 0.1% is used for the Nyquist pulse shaping (NPS) in each channel. The optical carriers are modulated by the I-Q modulators, and are then fed into the optical transmission link. The transmission link is composed of the standard single mode fiber (SSMF) with a span length of 80 km, and an EDFA with 4.5dB noise figure to compensate for the loss in the optical fiber. The signal propagation over the optical fiber is simulated based on the split-step Fourier solution of the Manakov equation. At the receiver, the signal is mixed with a local oscillator (LO) laser for coherent detection. The signals are detected by photodetectors (PDs) and sampled through analog-to-digital converters (ADCs).

![Diagram](https://via.placeholder.com/150)

**Figure 1.** Simulation setup of the Nyquist-spaced WDM optical fiber communication system using EDC and MC-DBP. PBS: polarization beam splitter; PBC: polarization beam combiner; Tx: transmitter; Rx: receiver.

In DSP modules after the ADC, a RRC filter is applied to filter out signals of a certain bandwidth. The EDC and MC-DBP are used to compensate for the transmission distortions to restore the original signals. The EDC is implemented by a frequency domain equalizer. The MC-DBP is a reverse operation of the signal propagation in optical fiber, calculated based on the inverse split-step Fourier solution of Manakov equation. The matched filter is used to select the center channel and eliminate the ASE noise. Finally, the performance of the center channel is estimated in terms of the SNR and MI.
4. RESULT AND DISCUSSION

In this section, two aspects of works are carried out. Firstly, for the 7-channel Nyquist-spaced WDM optical communication system with different modulation formats, transmission distances, and optical launch power per channel, the SNR prediction of the EGN model is compared and verified with the simulation results. Then the EGN model is used to estimate the AIRs of C-band (151-channel) Nyquist-spaced optical communication system versus transmission distance. The simulation parameters are listed in Table 1.

Table 1. Parameters in transmission system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol rate</td>
<td>32 Gbaud</td>
</tr>
<tr>
<td>Channel center wavelength</td>
<td>1550 nm</td>
</tr>
<tr>
<td>Channel spacing</td>
<td>32 GHz</td>
</tr>
<tr>
<td>Number of channels</td>
<td>7 or 151</td>
</tr>
<tr>
<td>Span length of SSMF</td>
<td>80 km</td>
</tr>
<tr>
<td>EDFA noise</td>
<td>4.5 dB</td>
</tr>
<tr>
<td>Dispersion coefficient (D)</td>
<td>17 ps/nm/km</td>
</tr>
<tr>
<td>Attenuation coefficient (α)</td>
<td>0.2 dB/km</td>
</tr>
<tr>
<td>Nonlinear coefficient (γ)</td>
<td>1.2 /W/km</td>
</tr>
</tbody>
</table>

Figure 2 shows the simulation results of SNR versus the launched optical power per channel for the situation of different modulation formats and transmission distances in the cases of EDC and partial-bandwidth or full-field DBP. For DP-QPSK, DP-16QAM, and DP-64QAM modulation formats, the transmission distances are set as 8000 km, 2400 km, and 800 km, respectively. The solid lines in Fig.2 represent the theoretical calculation results of the SNR based on EGN model, and the markers represent the numerical simulation results. It is found that an acceptable agreement can be achieved between the theoretical EGN model and numerical simulations at both linear regime and weak-nonlinearity regime.

Figure 2. SNR versus power per channel in Nyquist-spaced 32 Gbaud optical transmission system using the EDC and different bandwidth DBP. Solid line: EGN model. Symbol: Simulation results. (a) DP-QPSK; (b) DP-16QAM; (c) DP-64QAM.

Figure 3 shows the relationship between the SNR and the number of transmission spans (related to the transmission distance) using different modulation format. Here we evaluate the SNR versus the communication distance within 10,000 km (125 spans), which shows a practical application significance. The input fiber power per channel is set to 1 dBm, and
The numbers of spans are set to 10, 30, 50, 75, and 100 respectively (corresponding transmission distances are 800 km, 2400 km, 4000 km, 6000 km, 8000 km). As the transmission distance of the system increases, the accumulated noise of the system continues to increase, and the curve between SNR and the number of transmission spans (transmission distance) shows an obvious downward trend. In the case of 1 dBm input fiber power, the EGN model is basically consistent with the simulation results.

Figure 3. SNR versus number of spans in Nyquist-spaced 32 Gbaud optical transmission system using the EDC and different bandwidth DBP. Solid line: EGN model. Symbol: Simulation results. (a) DP-QPSK; (b) DP-16QAM; (c) DP-64QAM.

Figure 4. Theoretical results of AIRs versus transmission distance in 7-ch Nyquist-spaced 32Gbaud optical transmission system using EDC and MC-DBP for different modulation formats based on EGN model.

Figure 4 shows the relationship between AIRs and the communication distance of the 7-channel Nyquist-spaced WDM coherent communication system. For DP-QPSK system, within the transmission distance of 10,000 km, when the channel power is set to the optimal power, the AIRs can reach the saturation value of 0.9 Tbit/s, regardless of the use of EDC or MC-DBP. For DP-16QAM transmission system, the AIRs can be maintained up to 1.7 Tbit/s when full-field DBP (FF-DBP) is used, the AIRs will decrease with the transmission distances when over 2000 km for the use of EDC and partial-bandwidth DBP. For DP-64QAM system, the downward trend of AIRs is more pronounced, and the AIRs in DP-64QAM system will gradually approach the values in DP-16QAM system when the transmission distance is over 5000 km. Using
FF-DBP, the AIRs in DP-64QAM can achieve the saturation value of 2.7 Tbit/s within 2000 km, and will significantly reduce when the transmission distance exceeds 2000 km.

In the case of C-band transmission (151 channels), the AIRs of the systems versus the transmission distance are presented in Fig. 5 when DP-QPSK, DP-16QAM, DP-64QAM and DP-256QAM are applied, and the maximum AIRs can reach 19.33 Tbit/s, 38.66 Tbit/s, 58.0 Tbit/s and 77.32 Tbit/s respectively, in the case of FF-DBP. For DP-QPSK system using EDC or MC-DBP, the AIRs can maintain the value of 19.33 Tbit/s within 10,000 km. For DP-16QAM system, the use of FF-DBP can makes the AIRs maintain 38.66 Tbit/s for the distance within 4000 km and realizes an AIR figure above 35 Tbit/s within the whole 10,000 km transmission distance. While the EDC is used, the AIRs will decrease over 1000 km, and reduce to about 26 Tbit/s at 10,000 km. For DP-64QAM system, the AIRs will keep decreasing with the distance in case of EDC and maintain the saturated values within 2000 km in the use of FF-DBP. In the schemes that DP-256QAM is used to modulate the optical carrier, the AIRs cannot maintain the saturated value in the system using EDC and partial-bandwidth DBP. At the transmission distance of 6000 km the AIRs are 19.3 Tbit/s for DP-QPSK, 30.9 Tbit/s for DP-16QAM, 32.0 Tbit/s for DP-64QAM and 32.2 Tbit/s for DP-256QAM, respectively, when EDC is applied only, and can be increased up to 38.3 Tbit/s for DP-16QAM, 47.2 Tbit/s for DP-64QAM and 47.8 Tbit/s for DP-256QAM, respectively, when the nonlinearity compensation (NLC) is employed. Obviously over such a transmission distance, the AIRs are similar between DP-64QAM and DP-256QAM. The use of FF-DBP can greatly improve the AIRs for DP-16QAM, DP-64QAM and DP-256QAM systems.

![Figure 5. Theoretical results of AIRs versus transmission distance in 151-ch Nyquist-spaced 32Gbaud optical transmission system using EDC and MC-DBP for different modulation formats based on EGN model.](image)

5. **CONCLUSIONS**

In this work, the analytical model accounting for the relationship between AIR and transmission distance is developed to evaluate the performance of the large-capacity nonlinear Nyquist-spaced optical fiber communication system. The accuracy and effectiveness of the EGN model are verified via the split-step Fourier numerical simulations. It is found that in C-band (151-channel) Nyquist-spaced systems, the achievable information rates at the transmission distance of 6000 km are 19.3 Tbit/s for DP-QPSK, 30.9 Tbit/s for DP-16QAM, 32.0 Tbit/s for DP-64QAM and 32.2 Tbit/s for DP-256QAM, respectively, when EDC is applied only. Such achievable information rates can be increased up to 38.3 Tbit/s for DP-16QAM, 47.2 Tbit/s for DP-64QAM and 47.8 Tbit/s for DP-256QAM, respectively, when the FF-NLC is employed.
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REFERENCES