SLEEPING PATENTS AND COMPULSORY LICENSING: AN OPTIONS ANALYSIS

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Abstract

Why should a firm wish to create a new technology that it will leave unexploited for some time? Sleeping patents have long been perceived as anticompetitive devices, used by dominant firms to block entry into their market. In a real options framework with both economic and technological uncertainty, we show that a sleeping patent may arise as the result of optimal forward-looking behavior, in the absence of any anticompetitive motive. We also consider the effect of possible measures to enforce the development of sleeping patents and find that these are likely to harm incentives for firms to engage in research.

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1 Introduction

Is it ever rational for a potentially profitable invention to be left unexploited? And why should a firm ever wish to create a patent that it does not use? In law, economics and popular mythology it is generally presumed that any situation in which a firm wishes to hold a sleeping patent is detrimental to society as a whole. In the *Xerox* case, for example, the company was alleged to have taken out patents over inferior technologies in order to block entry and protect its dominant position. Another possibility, perhaps more significant in popular perception than in reality, is an invention that is 'too drastic', threatening to wipe out an entire industry with its improved efficiency.[1] In consequence of this view, UK patent law (and similar provisions in a number of other countries) provides for compulsory licensing of a new technology in cases where the patent-holder itself refuses to exploit it, despite market potential.[2]

This issue is the concern of on-going policy debate. In a recent issue of the *Antitrust Law Journal*, a symposium of articles considers possible legal remedies to address the problem of sleeping patents, essentially taking it for granted that the refusal to license an unused patent is an abusive and anticompetitive practice (see, *inter alia*, Crew (1998), Cohen and Burke (1998) and Chin (1998)). In the United States neither intellectual property law nor the antitrust laws permit any measures to be taken against non-exploitation of a patent. Following a number of unsuccessful antitrust cases relating to this issue there is some pressure to introduce compulsory licensing provisions into US law.

In this paper we analyze a two-stage investment project, with research undertaken in the first stage and irreversible market entry in the second, and consider circumstances under which the project will be suspended at the intermediate point between the two stages. We show that, in the presence of economic and technological uncertainty, optimizing behavior may result in a firm suspending the project at the intermediate point, leaving a potentially profitable technology unexploited for some period of time. This outcome arises in the absence of rivalry, either for the patent or in the product market, removing the anticompetitive motive for sleeping patents. Although it would be incorrect to conclude from this analysis that a sleeping patent is *never* held for anticompetitive purposes, the fact that sleeping patents may arise in other circumstances means that the observation of an unused yet marketable technology cannot, in itself, be taken as proof of anticompetitive behavior. This conclusion should be borne in mind when considering any proposed reform of the legal treatment of sleeping patents.

We demonstrate that, if economic uncertainty is high or the expected rate of innovation is low, it may be rational for a firm to carry out research at times when market profitability is below the level that would induce exploitation of the new technology. Thus research is carried out *prospectively* with a strong possibility that, once obtained, the patent will be left dormant for some time. The intuition for this result lies in a trade-off between two potential costs facing the firm. By carrying out research while conditions are not sufficiently favorable to induce market entry, the firm incurs the costs of research to gain an investment option that may not be exercised for an extended period of time. During this period it bears an opportunity cost given by the risk-free return on this investment. If, on the other hand, the firm delays research until conditions improve, it is likely that the discovery will not be achieved until some time after market entry would have been desirable. During this period the firm forgoes the product market profits which it would have earned had it been in a position to enter the market immediately.

The ability to suspend the project after the research phase, holding an exclusive patent over the newly-acquired technology, is crucial to the decision to carry out prospective research. We consider the effects of policy measures to enforce the immediate exploitation of a patent, such as compulsory licensing, and analyze their impact on the firm's incentives to engage in research. We find that restrictions on the firm's ability to suspend the project by holding a sleeping patent are likely to suppress research activity itself, an outcome that is particularly harmful in the presence of positive spillovers from R&D.

A number of papers analyze two-stage investment projects with possible suspension. Dixit and Pindyck (1994; pages 327-328) consider a general two-stage investment project with uncertain returns, where the first stage may be interpreted as research. When investment takes no time to complete, the firm waits until market entry is optimal and then sinks both the first- and second-stage investments at the same time. Thus, research will never be undertaken without the entire project being completed and sleeping patents will never arise. In their discussion Dixit and Pindyck note that if the first stage investment takes time to complete, then suspension at the intermediate stage is possible. If the market price were to fall while the first stage is being completed and reaches a level below the trigger point for the second stage investment, the project would then be shelved until conditions improve.

A number of models explain initial investment in the early stages of a multi-stage project through learning-by-doing. When research activity itself contributes to the resolution of uncertainty over, for example, total investment costs or final productivity, exploratory research may be carried out even when the expected NPV of the project is negative; see, for example, Roberts and Weitzman (1981). Pindyck (1993) analyzes a model in which the technical difficulty (in terms of time, materials and effort) of completing the project is uncertain and this information is revealed only as the investment is undertaken. In this case investment has a shadow value above its contribution to completion of the project, lowering the trigger point at which the project will be commenced.

In a model closely related to our own, Bar-Ilan and Strange (1998) analyze a two-stage investment model with time-to-build. Using numerical solution techniques they find that if the first stage investment lag is sufficiently long and output price uncertainty is high, the first-stage trigger may fall below the second-stage trigger and exploratory investment takes place. Although the models and their results are similar, uncertainty over the timing of discovery, rather than a fixed time to completion, is arguably better suited to the context of research activity where the timing of any breakthrough is highly unpredictable. Furthermore, this formulation allows closed-form solutions to be derived, avoiding the need to rely on numerical results.

The paper is organized as follows. Section 2 describes the structure of the model and the sources of uncertainty faced by the firm. Optimal trigger points for the second (capital investment) and first (research) stages respectively are derived in sections 3 and 4. In section 5 it is proven that, under certain conditions, the second stage trigger point exceeds the first stage trigger and sleeping patents are possible. Section 6 considers the effects of measures to enforce the exploitation of a sleeping patent. Section 7 concludes.

2 The model

A single, risk-neutral firm faces the following sequence of investment opportunities. First, it may carry out a research project which, if successful, leads to the creation of a new product design. When discovery takes place the design is patented immediately so that protection is gained from any subsequent rival discoveries. Second, the firm may choose when to invest in productive capacity and enter the product market, with costless suspension at this stage. The second stage investment is completely irreversible. The sequence of the firm's investment decisions is illustrated in figure 1.

The firm faces both technological and economic uncertainty. While the firm invests in research, discovery is a Poisson arrival. The economic value of the patent obtained by the successful inventor is also uncertain, with the price in the relevant product market following a stochastic process. The two-stage problem facing the firm can be analyzed as a compound option: the payoff from exercising the option to engage in research is the option to make a second investment which, if exercised, will generate a direct monetary return.

The underlying state variable in the model is the product market price P,[3] which follows a geometric Brownian motion process described by

$$dP = \mu P dt + \sigma P dz \tag{1}$$

where μ is the drift parameter, measuring the expected growth rate of P;
$\sigma > 0$ is the instantaneous standard deviation or volatility parameter; and
dz is the increment of a standard Wiener process; $dz \sim \mathrm{N}(0, dt)$.

The drift parameter, μ, must be strictly less than the risk-free interest rate, r, or otherwise the option to invest will never be exercised. For ease of exposition it is assumed that $\mu = 0$; since our focus is on the effects of uncertainty (i.e. the volatility parameter, σ) rather than the expected growth rate this restriction is unimportant.

In the first stage the firm invests by establishing and operating a research unit. When the firm engages in research activity, it makes the discovery according to a Poisson distribution with parameter (or hazard rate) $h > 0$. Thus the conditional probability that the firm makes the breakthrough in a short time interval of length dt

given that it has not done so before this time is hdt and the density function for the duration of research is given by the exponential distribution he^{-ht}. With a constant hazard rate the research technology is memoryless; there is no stock of accumulated knowledge that is lost if research activity is ceased.

At any time before discovery occurs investment in research is fully reversible, with no sunk (or fixed) costs. A flow cost of $C > 0$ per unit time is incurred during any period of research activity, but this ceases as soon as the project is abandoned. There is no actual or potential rivalry in the relevant research area, thus the firm is able to determine the timing of its research activity free from strategic considerations. The trigger point at which the first stage investment is undertaken is denoted P_1.[4] Since the research process is frictionless, with a memoryless research technology and no sunk costs, research is both commenced (as P rises) and subsequently abandoned (as P falls) at this point.[5]

For ease of exposition the research program can be thought of as h independent lines of research, each with a hazard rate of unity and an individual flow cost of c.[6] Thus the cost parameter can be rewritten as $C = ch$. This formulation allows the cost and hazard rate parameters to be changed in numerical simulations without affecting the expected value of the project, which would otherwise obscure the option value effects.

When discovery takes place the firm obtains a patent of infinite duration giving it exclusive rights over its new design and blocking all rival products. Patenting is assumed to be costless and thus will never be delayed; a patent is taken out as soon as the breakthrough occurs. For simplicity patent length is assumed to be infinite; this assumption is irrelevant to the results and the model could be modified to incorporate a finite patent length.

In order to manufacture and market the new product, physical capital must be sunk. This investment, along with market entry itself, is assumed to be totally irreversible. The initial investment cost is denoted I and production incurs a flow cost, w. Demand is completely inelastic at one unit, thus flow profits per unit time are given by $(P–w)$. As in the McDonald and Siegel (1986) model of an irreversible investment opportunity, investment may be delayed indefinitely, thus the situation is equivalent to a perpetual call option over the underlying project. The trigger point for the second stage investment is denoted P_2.

If it is found that $P_1 < P_2$, this may be interpreted as a situation in which a sleeping patent is likely to arise. Since discovery occurs randomly, commencement of research at a price level below P_2 does not *guarantee* the creation of a sleeping patent, since it is possible for P to move above P_2 before discovery actually takes place. For a sleeping patent to arise discovery itself must take place while the market price is below P_2; it is not sufficient that the firm merely engages in research over this range. However, sleeping patents become a possibility whenever the firm carries out research at a price level below P_2.

Although the patent will not immediately be developed if innovation occurs while the market price is in the range (P_1, P_2), this does not mean that exploitation is unprofitable. Rather the firm prefers to hold its option over the second stage due to the irreversible nature of this investment combined with uncertainty in the product market. However, since the patent is potentially profitable, other firms would be willing to develop it. If a rival firm obtains a compulsory license on the grounds that the patent-holder has failed to exploit the technology, it cannot then itself leave the patent dormant, thus the licensee does not have the option to delay. The ability for other firms to gain access to the technology in this way effectively introduces competition for development of the patent, restricting the ability of the inventor to delay the second stage investment.

In section 6 the threat of compulsory licensing, or another similar measure, is assumed to compel the firm to sink the second stage investment as soon as discovery takes place. Suspension at the intermediate stage is prevented and the compound option problem is reduced to the simple choice of whether or not to engage in research at any particular level of P. This is something of a simplification. In practice, compulsory licensing will not take place unless there is another firm that wishes to exploit the patent at this time. However, as explained above, assuming that there are other firms with the technical ability to use the patent, it is profitable for them to obtain a license to do so. Moreover, the payment of royalties to the patent-holder does not overcome the problem. If the license fee is less than the profit the patent-holder would gain by exploiting the patent itself, then it will develop the technology rather than allow it to be licensed. If, on the other hand, the same level of profit can be extracted from the licensee by means of royalty payments, the situation is analytically equivalent to the case in which the patent-holder itself exploits the patent without delay. Thus, on the assumption that there is a potential licensee who would wish to develop the patent, the assumption is a reasonable one.

The decision rule for an optimizing firm in the two-stage model with suspension is derived as follows. Due to the compound option structure of the model, the optimization problem must be solved backwards. First, the value of the second stage investment option and the level of the trigger point P_2 must be found. Once these solutions have been obtained expressions for firm value during the first stage can be derived. A different set of optimality conditions is derived for each possible ordering of the trigger points P_1 and P_2, which must both be solved in order to determine optimal investment behavior.

3 The second stage: Irreversible capital investment

Viewed from the second stage alone, the firm's decision problem is similar to the McDonald and Siegel (1986) model of an irreversible investment project with a constant investment cost, I. The optimal investment time is found by solving the following optimal stopping problem

$$V = \max_T E\left(\int_T^\infty e^{-rt} (P_t - w) dt - I \right) \tag{2}$$

where E denotes the expectation, T is the unknown future stopping time at which the investment is made, P_t is the market price at t, which evolves according to the stochastic differential equation (1), and w and I are the flow and sunk costs respectively.

Prior to investment at T the firm holds the opportunity to invest. It receives no expected cashflows but may experience a capital gain or loss on the value of its option. Hence, in the continuation region (values of P for which it is not yet optimal to invest) the Bellman equation for the value of the investment opportunity $V_0(P)$ is given by

$$rV_0 dt = E(dV_0). \tag{3}$$

Expanding dV_0 using Itô's lemma we can write

$$dV_0 = V_0'(P)dP + \frac{1}{2}V_0''(P)(dP)^2 .$$

Substituting from (1), noting that μ = 0 and $E(dz)$ = 0, we derive

$$E(dV_0) = \frac{1}{2}\sigma^2 P^2 V_0''(P)dt .$$

Hence the Bellman equation (3) becomes the following second-order differential equation

$$\frac{1}{2}\sigma^2 P^2 V_0''(P)dt - rV_0 = 0 . \tag{4}$$

From (1) we can see that if P goes to zero it stays there forever. Thus when P = 0 the option to invest has no value and $V_0(P)$ must satisfy the following boundary condition

$$V_0(0) = 0 \tag{5}$$

Solving the differential equation (4) subject to the boundary condition (5) we obtain the following solution for the value of the option to invest in research

$$V_0(P) = BP^{\beta_0} \tag{6}$$

where $B \geq 0$ is a constant whose value is yet to be determined, and $\beta_0 = \frac{1}{2}\left\{1+\sqrt{1+\frac{8r}{\sigma^2}}\right\}$ > 1 is the positive root of the characteristic equation $\varepsilon^2 - \varepsilon - \frac{2r}{\sigma^2} = 0$.

Next we consider the value of the firm in the stopping region (values of P for which is it optimal to invest at once). Since investment is irreversible, the value of the firm in the stopping region $V_1(P)$ is given by the expected value of product market profits, with no option value terms. Thus, the expected value of investment when the current market price is P_t is given by

$$V_1(P_t) = E\left(\int_t^{\infty} e^{-r\tau}(P_t - w)d\tau\right).$$

Since P has no expected trend (μ = 0) the expected value of immediate investment at t is given by

$$V_1(P_t) = \frac{P_t - w}{r}. \tag{7}$$

The boundary between the continuation region and the stopping region is given by a critical value of the stochastic process, or "trigger point", P_2 such that continuation is optimal for values of P below P_2 and stopping (i.e. immediate investment) is optimal above P_2. The optimal stopping time T is then the first time that the stochastic process P hits the interval $[P_2, \infty)$.

Two conditions must hold at the optimal investment point. First, the value of the option held by the firm prior to investment, V_0, must equal the value of the completed project, V_1, minus the sunk investment cost I (the "value-matching condition")

$$BP_2^{\beta_0} = \frac{P_2 - w}{r} - I. \tag{8}$$

Optimality requires a second condition, known as "smooth-pasting," to hold. This requires the value functions V_0 and V_1 to meet smoothly at P_2 with equal first derivatives[7]

$$\beta_0 B P_2^{\beta_0 - 1} = \frac{1}{r}. \tag{9}$$

Using conditions (8) and (9), the following solutions for the second stage trigger point P_2 and constant of integration B can be derived

$$P_2 = \frac{\beta_0}{\beta_0 - 1}(w + rI); \tag{10}$$

and

$$B = \left(\frac{\beta_0}{\beta_0 - 1}(w + rI)\right)^{1-\beta_0} (r\beta_0)^{-1}. \tag{11}$$

4 The first stage: Research activity

To determine P_1, the value of P at which the firm will commence and abandon research, the value of the firm in the idle and active states must be analyzed as before. In this case, however, the value function in the active state may take one of two forms, depending upon whether or not P is sufficiently high that the second stage investment would be sunk immediately if the discovery were to be made. The three cases are considered in turn, deriving the appropriate value function in each case.

4.1 The inactive firm

At low values of P no research activity is undertaken and the firm simply holds the option to invest. The derivation of the value of the firm follows that for $V_0(P)$ and is given by

$$U_0(P) = ZP^{\beta_0} \tag{12}$$

where $Z > 0$ is an unknown constant and β_0 is as given above.

4.2 The active firm

If the firm undertakes research at a level of P for which it would choose to delay the second stage investment (at any $P < P_2$) the return to discovery is the value of the option to invest. Alternatively, if research is undertaken at a point where capital investment would be undertaken immediately (at any $P \geq P_2$) the return to invention is the expected NPV of the second stage investment project. Thus, the value of the active firm falls into the following two cases depending upon the current level of P.

(i) For $P < P_2$

If the firm undertakes research at a level of P for which, if successful, it would not wish to proceed with the irreversible capital investment at once (i.e. any $P < P_2$), the immediate payoff to discovery is the second stage option value $V_0(P) = BP^{\beta_0}$. The value of a firm which carries out research over this range, derived in Appendix 1, is given by

$$U_1(P) = BP^{\beta_0} - \frac{ch}{(r+h)} + XP^{-\alpha_1} + YP^{\beta_1} \tag{13}$$

where X and Y are unknown constants;

$\alpha_1 = \frac{1}{2}\left(\sqrt{1+\frac{8(r+h)}{\sigma^2}} - 1\right) > 0$; and

$\beta_1 = \frac{1}{2}\left(1+\sqrt{1+\frac{8(r+h)}{\sigma^2}}\right) > 1$.

(ii) For $P \geq P_2$

If the firm carries out research at a level of P for which, if successful, it will undertake the second stage investment and start production immediately (i.e. any $P \geq P_2$), the return to discovery is the expected NPV, $V_1(P)$. The value of a firm which undertakes research over this range, also derived in Appendix 1, is given by

$$U_2(P) = \frac{h}{r(r+h)}\left(P - w - r(I-c)\right) + WP^{-\alpha_1} \tag{14}$$

where W is an unknown positive constant.

4.3 The first stage optimization problem

Since there are two possible orderings of the trigger points P_1 and P_2, two separate sets of optimality conditions must be constructed and solved. The two cases, with the relevant value functions in each range, are illustrated in figure 2. When $P_1 \geq P_2$ any level of P at which the firm undertakes research is also one at which it would be willing to proceed with the second stage investment immediately. Thus, sleeping patents can never arise in this case. When $P_1 < P_2$, by contrast, the firm carries out research over the range (P_1, P_2) and sleeping patents become a possibility, arising if the discovery itself happens to occur within this range.

When the research technology is frictionless such that switching between research activity and inactivity is costless, the value-matching and smooth-pasting conditions will hold at *any* arbitrary switching point. Thus, these conditions alone are not sufficient to determine the optimal location of the trigger point P_1. To ensure that the chosen switching point maximizes firm value an additional, first-order condition is needed, while the second derivative must be negative.

(i) For $P_1 \geq P_2$

When $P_1 \geq P_2$ the set of optimality conditions consists of three equations with three unknowns (P_1, W and Z). At P_1 the following value-matching and smooth-pasting conditions must hold

$$U_0(P_1) = U_2(P_1);$$
$$U_0'(P_1) = U_2'(P_1).$$

The first-order condition determining the optimal location of P_1 is given by

$$U_0'(P_1) = Z'(P_1) = 0.$$

Solving this system the following expression for the first stage trigger point P_1 is obtained (details are given in Appendix 2); this expression for P_1 is valid when the resulting value is greater than or equal to P_2

$$P_1 = \frac{\alpha_1 \beta_0}{(\alpha_1 + 1)(\beta_0 - 1)}\left[w + r(I + c)\right]. \tag{15}$$

(ii) For $P_1 < P_2$

When $P_1 < P_2$ there are five equations with five unknowns (P_1, W, X, Y and Z). The following value-matching and smooth-pasting conditions hold at P_1

$$U_0(P_1) = U_1(P_1);$$
$$U_0'(P_1) = U_1'(P_1).$$

At P_2, value-matching and smooth-pasting conditions are given respectively by

$$U_1(P_2) = U_2(P_2);$$
$$U_1'(P_2) = U_2'(P_2).$$

As before, the first-order condition determining the optimal location of P_1 is given by

$$U_0'(P_1) = Z'(P_1) = 0.$$

The following expression for P_1, which is valid when the resulting value is below P_2, can be derived (as in Appendix 2)

$$P_1 = P_2\left(\frac{\alpha_1\beta_0(\beta_0 - 1)}{(\beta_1 - \beta_0)(\alpha_1 + \beta_0)}\frac{ch}{(w + rI)}\right)^{\frac{1}{\beta_1}} > 0. \tag{16}$$

Although optimality conditions and solutions for the two cases can be derived, this does not yet prove that it is possible for P_2 to exceed P_1 so that sleeping patents may arise. The next section derives conditions under which research is carried out prospectively and demonstrates that, for sufficiently high uncertainty or a slow speed of discovery, sleeping patents become a realistic possibility.

5 Prospective research and sleeping patents

In this section conditions are derived under which research is carried out prospectively and sleeping patents may arise. For sleeping patents to be a possibility the first stage trigger point P_1 must lie strictly below the second stage trigger P_2. Then if discovery occurs while the market price is in the range (P_1, P_2) the patent will be left dormant until P_2 is reached. The ratio of first- and second-stage trigger points is denoted by ρ as follows

$$\rho = \frac{P_1}{P_2}. \tag{17}$$

Using this notation, sleeping patents are a possibility if and only if ρ falls below unity. When $P_1 \geq P_2$, expression (15) for P_1 can be used along with (10) for P_2 to derive an expression for ρ. Thus we can derive

$$\rho = \frac{(w + r(I + c))}{(w + rI)} \frac{\alpha_1}{(\alpha_1 + 1)}. \tag{18}$$

Although this expression is valid only when $P_1 \geq P_2$ it can be used to demonstrate results over the full range of parameter values. If it can be shown that ρ as defined by expression (18) falls below unity at some point, then it must be true that the alternative case holds at this level and $P_1 < P_2$. Although the true value of ρ will no longer be given by expression (18), it must nonetheless take some value less than unity.

The two propositions given below set out conditions under which research will be undertaken prospectively and sleeping patents are likely to arise.

Proposition 1: *For any set of parameter values* $\{h, c, w, I, r\}$, *there exists a critical value of the volatility parameter,* σ^*, *above which the first stage trigger point* P_1 *lies strictly below the second stage trigger point* P_2 *and sleeping patents may arise.*

Proof: The proof can be sketched as follows (full details are given in appendix 3). The *existence* of at least one critical value σ^* at which ρ equals unity is demonstrated using the intermediate value theorem. Applying this theorem to the function $\rho(\sigma)$, the existence of some $\sigma^* \in (0, \infty)$ at which $\rho = 1$ can be demonstrated by showing that

(i) $\rho(\sigma)$ tends towards some value greater than unity as σ approaches zero,

(ii) $\rho(\sigma)$ tends to zero as σ approaches infinity, and

(iii) the function $\rho(\sigma)$ is continuous.

The *uniqueness* of the critical value σ^* is proven by demonstrating that $\rho(\sigma)$ is strictly monotonic; i.e. that $\rho'(\sigma)$ is strictly negative for all values of σ in the range $(0, \infty)$. Thus, once the critical value σ^* is exceeded, ρ is always less than unity (i.e. $P_1 < P_2$) and sleeping patents are a possibility.

Proposition 2: *For any set of parameter values* {σ, *w*, *I*, *r*} *and as long as the flow research cost c is sufficiently small, there exists a critical value of the hazard rate, h*, below which the first stage trigger point* P_1 *falls below the second stage trigger point* P_2 *and sleeping patents may arise.*

Proof: The structure of the proof is similar to that for proposition 1 above. Details are given in appendix 4.

An expression for the critical value σ^* can be derived by setting ρ = 1 and rearranging for σ to obtain

$$\sigma^* = rc\sqrt{\frac{2(r+h)}{(w+rI)(w+r(I+c))}}\,. \tag{19}$$

Similarly for h^* we can derive

$$h^* = \frac{(w+rI)(w+r(I+c))\sigma^2}{2(rc)^2} - r\,. \tag{20}$$

Note, however, that the resulting value of h^* must be positive, which will be the case for sufficiently small c or large σ.

The results are illustrated using numerical simulations. In the simulations described below the following central parameter values are used. The volatility parameter in the geometric Brownian motion governing P is given by σ = 0.2. The risk-free interest rate is 0.05. The parameters of the research technology are h = 0.5 and c = 4. The sunk and flow costs in the second stage are I = 10 and w = 0 respectively. With these parameter values the first stage trigger point is P_1 = 1.08 while the second stage trigger is P_2 = 0.93. Thus sleeping patents are impossible in this example: any value of P at which the firm undertakes research is also one at which it would exploit the patent immediately.

According to propositions 1 and 2 there are two cases in which research will be carried out prospectively: firstly, if the degree of uncertainty is sufficiently high, or secondly, if both the hazard rate and flow research costs are sufficiently low. These cases, and the combination of the two parameters which together may result in sleeping patents, are illustrated in the following three graphs.

Figure 3 shows the effect of increasing the volatility parameter, σ, while holding all other parameter values constant. Both P_1 and P_2 rise as volatility increases, but the effect on P_2 is more pronounced: although product market conditions clearly affect research decisions, the ability to suspend the project after discovery reduces their impact. As described in proposition 1, a critical value is reached at which P_1 crosses P_2; in this example σ^* is approximately 0.355.

Figure 4 shows the effect of changing the hazard rate, h. Unsurprisingly, there is no effect on the second-stage trigger point P_2. However, the first-stage trigger point P_1 rises with h, causing research activity to be more delayed. For values of h below a critical level, $h^* = 0.125$, P_1 lies below P_2 and sleeping patents are possible.

Figure 5 shows the locus of (σ, h) combinations at which the first and second stage trigger points coincide ($P_1 = P_2$). Below this locus, with a higher value of σ (greater economic uncertainty) or lower value of h (a low expected rate of discovery), P_2 exceeds P_1 and sleeping patents are possible.

6 The constrained firm

We now consider the case of a firm that is constrained (under threat of compulsory licensing or some other measure) to use its patent as soon as it is created. The compound option is reduced to a simple one in which the only decision taken by the firm is the timing of its research activity; in this case the timing of the second stage investment is given by the discovery date itself. The trigger point at which the constrained firm commences and abandons research is denoted P_C.

When the firm is constrained to use the patent immediately its optimization problem follows the analysis of section 4.3(i), with the solution denoted P_C (rather than P_1 as above). However, this result now holds over the complete range of P, regardless of whether the resulting value is greater or less than P_2. If parameter values are such that the (unconstrained) trigger point P_1 exceeds P_2, the constraint is non-binding and the "constrained" trigger point P_C coincides with P_1 (the two have identical derivations). In cases where P_1 lies below P_2, however, the constraint is binding and P_C will take some intermediate value between the two points. Since P_C is, in effect, the trigger point for the second as well as the first stage investment, the firm faces a trade-off between delaying research beyond the optimal trigger point P_1 and potentially having to sink the second stage investment before reaching the critical value P_2.

Figure 6 shows a numerical simulation comparing the constrained trigger point P_C with the unconstrained values P_1 and P_2. Levels of P_1, P_2 and P_C are shown for values of the volatility parameter σ over the range (0.2, 0.7); other parameters take the values given in section 5. For values of σ below $\sigma^* = 0.35$, the unconstrained first stage trigger P_1 exceeds P_2 and the constraint is non-binding, thus P_C coincides with P_1. For values above σ^*, however, the constraint binds and P_C takes an intermediate value between P_1 and P_2. Thus in the constrained case research activity is further delayed, until product market conditions are more favorable, compared with the behavior of the unconstrained firm.

This finding has the following implications. Policy measures to enforce the development of sleeping patents, such as compulsory licensing, may cause one or both of the following detriments. By forcing the firm to exploit its patent immediately, investment costs may be sunk too soon and society loses the value of the option to delay. When firms are forward-looking their research behavior is also affected. As a substitute for the option to hold a sleeping patent, the firm may instead choose to delay its research activity relative to the social optimum. Although new designs will be brought to market more rapidly, they will be produced more slowly.

The constraint imposed on the firm's option to delay is socially damaging in itself, even in the absence of externalities. However, if there are external benefits from research, such as spillovers of knowledge to other firms, the delay in research activity will be even more harmful. In cases where the right to invest is auctioned, as is often the case for oil leases, another consideration is the amount of revenue raised. Any constraint on the holder's ability to hold options by delaying investment will reduce the amount of revenue raised in auctioning these rights.

7 Conclusions

This paper has studied the effects of technological uncertainty combined with risky market conditions on the incentive for firms to engage in research. In particular, it has been shown that a sleeping patent may arise as the foreseeable outcome of socially optimal behavior by the firm. Thus, although the circumstances of individual cases must be examined in detail before this conclusion can be drawn, observation of a dormant but potentially profitable technology does not necessarily prove an anti-competitive motive or otherwise socially harmful behavior.

When discovery occurs randomly, a firm that delays research until market conditions are favorable cannot be certain that discovery will occur as soon as desired. When market uncertainty is high or the expected speed of discovery is low, it may be rational for the firm to engage in research at a time when the prevailing conditions would not justify immediate market entry were the technology already available. In other words, the firm may find it worthwhile to incur initial expenditures to gain the option to invest even at a time when this option would not itself be exercised.

In the context of the real options literature, this finding shows that the ability to gain options for the future may make it rational to exercise current ones sooner. Thus, the existence of more options does not necessarily increase delay. In such situations the sequence of options available to the firm must be considered in its entirety, as serious errors could be made if the early options were to be valued on their own.

The analysis has a number of implications for policy towards R&D. First, by demonstrating that sleeping patents may arise as the result of socially optimal investment behavior under uncertainty, it suggests that they are not necessarily anti-competitive or socially undesirable. There are a number of reasons why a patent may be left dormant and each case must be examined carefully before any such verdict is made. Secondly, the analysis suggests caution in adopting measures such as compulsory licensing to enforce the exploitation of sleeping patents. Forward-looking firms will foresee the implications of such measures and may instead choose to delay research activity itself, an outcome which could cause considerable social as well as private detriment.

References

Bar-Ilan, Avner, and William C. Strange. 1998. "A model of sequential investment." *Journal of Economic Dynamics and Control* 22: 437-463.

Chin, Yee Wah. 1998. Unilateral technology suppression: Appropriate antitrust and patent law remedies." *Antitrust Law Journal* 66: 441-453.

Cohen, Joel M., and Arthur J. Burke. 1998. "An overview of the antitrust analysis of suppression of technology." *Antitrust Law Journal* 66: 421-439.

Cornish, W. R. 1996. *Intellectual Property: Patents, Copyright, Trade Marks and Allied Rights* (third edition). Sweet and Maxwell, London.

Crew, Eugene. 1998. "Symposium: Antitrust and the suppression of technology in the United States and Europe: Is there a remedy?" *Antitrust Law Journal* 66: 415-419.

Dixit, Avinash K., and Robert S. Pindyck. 1994. *Investment Under Uncertainty*. Princeton University Press.

McDonald, Robert, and Daniel Siegel. 1986. "The value of waiting to invest." *Quarterly Journal of Economics* 101 (November): 707-728.

Pindyck, Robert S. 1993. "Investments of uncertain cost." *Journal of Financial Economics* 34: 53-76.

Roberts, Kevin, and Martin L. Weitzman. (1981). "Funding criteria for research, development, and exploration projects." *Econometrica* 49: 1261-1288.

Appendix 1: The compound (first stage) option value

While the firm is idle, the derivation of the (option) value of the firm follows that for the second stage value function $V_0(P)$. When the firm is active, however, there are now two separate value functions to be derived, according to whether the current value of P is below or above P_2.

(i) For $P < P_2$

If the firm undertakes research at a value of P for which, if successful, it would not develop the patent at once ($P < P_2$), the prize for discovery is the option value $V_0(P)$. Incorporating this payoff, the dynamic programming equation is given by

$$U_1(P) = \left(hBP^{\beta_0} - ch\right)dt + \left(1-(r+h)dt\right)\left(U_1(P) + \frac{1}{2}\sigma^2 P^2 U_1''(P)dt\right)$$

which rearranges to give

$$(r+h)U_1(P) = hBP^{\beta_0} - ch + \frac{1}{2}\sigma^2 P^2 U_1''(P). \qquad \text{(A1.1)}$$

Solving this differential equation, we obtain

$$U_1(P) = \frac{hBP^{\beta_0}}{r+h-\tau} - \frac{ch}{r+h} + XP^{-\alpha_1} + YP^{\beta_1} \qquad \text{(A1.2)}$$

where X, Y are unknown constants;

$\alpha_1 = \frac{1}{2}\left(\sqrt{1+\frac{8(r+h)}{\sigma^2}} - 1\right) > 0$;

$\beta_1 = \frac{1}{2}\left(1+\sqrt{1+\frac{8(r+h)}{\sigma^2}}\right) > 1$; and

$\tau = ½\sigma^2\beta_0(\beta_0 - 1) = r$.

The term τ is the expected growth rate of the second stage option $V_0(P)$. However, this simplifies to the interest rate *r*: since the option yields no cashflows, the return on holding it is the capital growth alone which, in the absence of arbitrage opportunities, must equal the risk-free interest rate *r*. Thus, (A1.2) becomes

$$U_1(P) = BP^{\beta_0} - \frac{ch}{r+h} + XP^{-\alpha_1} + YP^{\beta_1}. \qquad \text{(A1.3)}$$

Since this value function holds only over the range (P_1, P_2), there is no boundary condition at zero or infinity that can be used to eliminate one of the constant terms.

(ii) For $P \geq P_2$

If the firm undertakes research at a level of *P* for which, if successful, it will sink the capital investment and start production immediately (i.e. any $P \geq P_2$), the benefit from discovery is the expected NPV of the second stage investment, $V_1(P)$. Incorporating this payoff into the usual dynamic programming equation we obtain

$$U_2(P) = \left[h\left(\frac{P-w}{r} - I\right) - ch\right]dt + \left(1 - (r+h)dt\right)\left(U_2(P) + \frac{1}{2}\sigma^2 P^2 U_2''(P)dt\right)$$

which rearranges to give

$$(r+h)U_2(P) = h\left(\frac{P-w}{r} - I - c\right) + \frac{1}{2}\sigma^2 P^2 U_2''(P). \qquad \text{(A1.4)}$$

Solving this equation subject to the boundary condition $U_2(P) \to V_1(P)$ as $P \to \infty$ (i.e. the option value approaches the expected NPV of the project), the following solution is obtained (where *W* is an unknown constant)

$$U_2(P) = \frac{h}{r(r+h)}\left(P - w - r(I+c)\right) + WP^{-\alpha_1}. \qquad \text{(A1.5)}$$

Appendix 2: Finding the first stage trigger point P_1

Solutions must be found for two separate cases, $P_1 \geq P_2$ and $P_1 < P_2$, which have different optimality conditions. Since an assumption concerning the relative sizes of P_1 and P_2 is made, the solution obtained in each case is valid only if the relevant assumption is not violated by the outcome.

(i) For $P_1 \geq P_2$

Value-matching, smooth-pasting and first order conditions at P_1 are given by

$$ZP_1^{\beta_0} = \frac{h}{r(r+h)}\left(P_1 - w - r(I+c)\right) + WP_1^{-\alpha_1}; \tag{A2.1}$$

$$\beta_0 ZP_1^{\beta_0 - 1} = \frac{h}{r(r+h)} - \alpha_1 WP_1^{-\alpha_1 - 1}; \tag{A2.2}$$

$$\partial Z(P_1)/\partial P_1 = 0. \tag{A2.3}$$

Using (A2.1) and (A2.2) to eliminate W, we obtain

$$(\alpha_1 + \beta_0)Z(P_1) = \frac{hP_1^{-\beta_0}}{r(r+h)}\left((\alpha_1 + 1)P_1 - \alpha_1\left(w + r(I+c)\right)\right).$$

Differentiating with respect to P_1 (the first order condition) we obtain

$$\frac{\partial Z}{\partial P_1} = \frac{hP_1^{-\beta_0 - 1}}{r(r+h)}\left(\alpha_1\beta_0\left(w + r(I+c)\right) - (\beta_0 - 1)(\alpha_1 + 1)P_1\right) = 0.$$

Therefore

$$P_1 = \frac{\alpha_1\beta_0}{(\alpha_1 + 1)(\beta_0 - 1)}\left[w + r(I+c)\right]. \tag{A2.4}$$

The second derivative of $Z(P)$ is negative, ensuring that P_1 is a maximum.

(ii) For $P_1 < P_2$

At P_2, the value-matching and smooth-pasting conditions are given by

$$\frac{h\left(P_2 - w - rI\right)}{r(r+h)} + \left(W - X\right)P_2^{-\alpha_1} = BP_2^{\beta_0} + YP_2^{\beta_1}; \tag{A2.5}$$

$$\frac{h}{r(r+h)} - \alpha_1\left(W - X\right)P_2^{-\alpha_1 - 1} = \beta_0 BP_2^{\beta_0 - 1} + \beta_1 YP_2^{\beta_1 - 1}. \tag{A2.6}$$

Eliminating $(W - X)$ and substituting the solutions for B and P_2 we obtain

$$(\alpha_1 + \beta_1)YP_2^{\beta_1} = -\frac{(\alpha_1 + \beta_0)(w + rI)}{(\beta_0 - 1)(r+h)}. \tag{A2.7}$$

At P_1, value-matching, smooth-pasting and first order conditions are given by

$$ZP_1^{\beta_0} = BP_1^{\beta_0} - \frac{ch}{(r+h)} + XP_1^{-\alpha_1} + YP_1^{\beta_1}; \tag{A2.8}$$

$$\beta_0 ZP_1^{\beta_0 - 1} = \beta_0 BP_1^{\beta_0 - 1} - \alpha_1 XP_1^{-\alpha_1 - 1} + \beta_1 YP_1^{\beta_1 - 1}; \tag{A2.9}$$

$$\partial Z\left(P_1\right)/\partial P_1 = 0. \tag{A2.10}$$

Eliminating X and substituting for Y from (A2.7) we obtain

$$Z\left(P_1\right) = B - \frac{P_1^{-\beta_0}\alpha_1 ch}{(\alpha_1 + \beta_0)(r+h)} - \frac{P_1^{\beta_1 - \beta_0}P_2^{-\beta_1}(w + rI)}{(\beta_0 - 1)(r+h)}$$

Setting the first order condition $Z'\left(P_1\right)$ to zero, the following solution for P_1 can be derived (which is valid as long as the resulting value is strictly less than P_2)

$$P_1 = P_2\left(\frac{\alpha_1\beta_0\left(\beta_0 - 1\right)}{(\beta_1 - \beta_0)(\alpha_1 + \beta_0)}\frac{ch}{(w + rI)}\right)^{\frac{1}{\beta_1}}. \tag{A2.10}$$

Appendix 3: The possibility of sleeping patents when σ is large

The ratio of trigger points $\rho(\sigma)$ is given by

$$\rho = \frac{\left(w + r(I + c)\right)}{(w + rI)} \frac{\alpha_1(\sigma)}{\left(\alpha_1(\sigma) + 1\right)}. \tag{A3.1}$$

To prove the existence and uniqueness of the critical point σ^* at which $\rho = 1$, four properties of the function $\rho(\sigma)$ must be demonstrated. First, that $\rho(\sigma)$ approaches a value greater than unity as $\sigma \to 0$; second, that $\rho(\sigma) \to 0$ as $\sigma \to \infty$; third, that the function is continuous; and fourth, that it is strictly monotonic for all values of σ in the range $(0, \infty)$.

(i) Limiting result as $\sigma \to 0$

As $\sigma \to 0$, $\alpha_1(\sigma) \to \infty$ and $\dfrac{\alpha_1}{\alpha_1 + 1} \to 1$.

Thus

$$\rho \to \frac{\left(w + r(I + c)\right)}{(w + rI)} > 1 \text{ as } \sigma \to 0. \tag{A3.2}$$

Since $c > 0$, the limit of $\rho(\sigma)$ as $\sigma \to 0$ is a value greater than one.

(ii) Limiting result as $\sigma \to \infty$

As $\sigma \to \infty$, $\alpha_1(\sigma) \to 0$.

Thus

$$\rho \to 0 \text{ as } \sigma \to \infty. \tag{A3.3}$$

(iii) Continuity and monotonicity

In expression (A3.1) for $\rho(\sigma)$, α_1 is a continuous function of σ and all other elements are independent of σ, thus $\rho(\sigma)$ is continuous over the range (0, ∞). In combination with the limiting results presented in (i) and (ii), this proves the existence of at least one critical value σ^* at which $\rho = 1$.

To prove monotonicity of $\rho(\sigma)$ and hence the uniqueness of σ^*, the sign of the (partial) derivative $\rho'(\sigma)$ must be determined.

$$\frac{\partial \rho}{\partial \sigma} = \frac{\left(w + r(I + c)\right)}{(w + rI)} \frac{\partial}{\partial \sigma}\left(\frac{\alpha_1}{\alpha_1 + 1}\right). \tag{A3.4}$$

Thus,

$$\operatorname{sgn}\frac{\partial \rho}{\partial \sigma} = \operatorname{sgn}\frac{\partial}{\partial \sigma}\left(\frac{\alpha_1}{\alpha_1 + 1}\right) = \operatorname{sgn}\frac{\partial \alpha_1}{\partial \sigma}.$$

$$\frac{\partial \alpha_1}{\partial \sigma} = -\frac{2}{\sigma^3}\left\{2(r+h)\left[1 + \frac{8(r+h)}{\sigma^2}\right]^{-\frac{1}{2}}\right\} < 0. \tag{A3.5}$$

The function $\rho(\sigma)$ is strictly decreasing over the range (0, ∞). Thus, the critical value σ^* at which $\rho = 1$ is unique.

Appendix 4: The possibility of sleeping patents when *h* is small

The ratio of trigger points $\rho(h)$ is given by

$$\rho = \frac{\left(w + r(I + c)\right)}{(w + rI)} \frac{\alpha_1(h)}{\left(\alpha_1(h) + 1\right)}. \tag{A4.1}$$

(i) Limiting result as $h \to 0$

When $h = 0$, α_1 becomes

$$\alpha_1 = \alpha_0 = \frac{1}{2}\left\{\sqrt{1 + \frac{8r}{\sigma^2}} - 1\right\} > 0.$$

Since $\frac{\alpha_1}{\alpha_1 + 1} < 1$ and this ratio falls as σ rises, then, for sufficiently small c or large σ, ρ will be less than unity.

(ii) Limiting result as $h \to \infty$

As $h \to \infty$, $\alpha_1(h) \to \infty$ and $\frac{\alpha_1}{(\alpha_1 + 1)} \to 1$.

Thus

$$\rho \to \frac{\left(w + r(I + c)\right)}{(w + rI)} > 1 \text{ as } h \to \infty. \tag{A4.2}$$

(iii) Continuity and monotonicity

In expression (A4.1), α_1 is a continuous function of h while all other elements are independent of h, thus $\rho(h)$ is continuous over the range $[0, \infty)$. In combination with the limiting results presented in parts (i) and (ii), the existence of at least one critical value h^* at which $\rho(h) = 1$ can be demonstrated using the intermediate value theorem.

To prove strict monotonicity and hence the uniqueness of h^*, the sign of the (partial) derivative $\rho'(h)$ must be determined.

$$\frac{\partial \rho}{\partial h} = \frac{\left(w + r(I + c)\right)}{(w + rI)} \frac{\partial}{\partial h}\left(\frac{\alpha_1}{(\alpha_1 + 1)}\right). \tag{A4.3}$$

But

$$\frac{\partial}{\partial h}\left(\frac{\alpha_1}{(\alpha_1 + 1)}\right) = \frac{1}{(\alpha_1 + 1)^2} \frac{\partial \alpha_1}{\partial h}.$$

Thus,

$$\operatorname{sgn}\frac{\partial \rho}{\partial h} = \operatorname{sgn}\frac{\partial \alpha_1}{\partial h}.$$

Taking the partial derivative of α_1 with respect to h, we derive

$$\frac{\partial \alpha_1}{\partial h} = \frac{2}{\sigma^2}\left(1 + \frac{8(r + h)}{\sigma^2}\right)^{-\frac{1}{2}} > 0. \tag{A4.4}$$

The function $\rho(h)$ is strictly increasing over the range $(0, \infty)$. Thus, the critical value h^* at which $\rho = 1$ is unique.

Figure 1: The compound option model

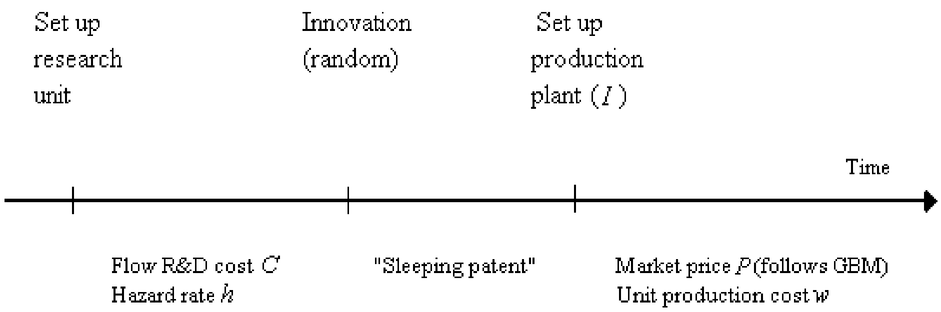


Figure 2: Trigger points in the no sunk costs case

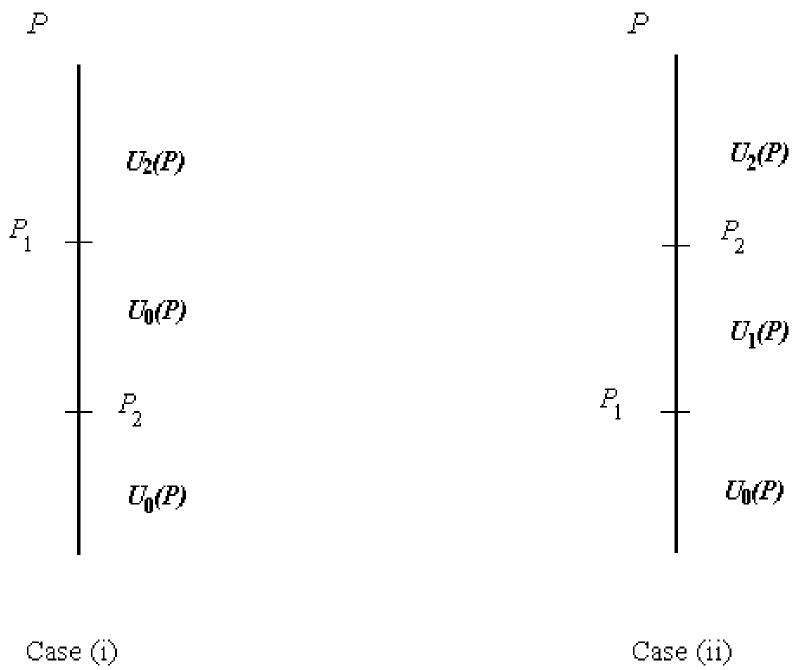


Figure 3: The possibility of sleeping patents when σ is large

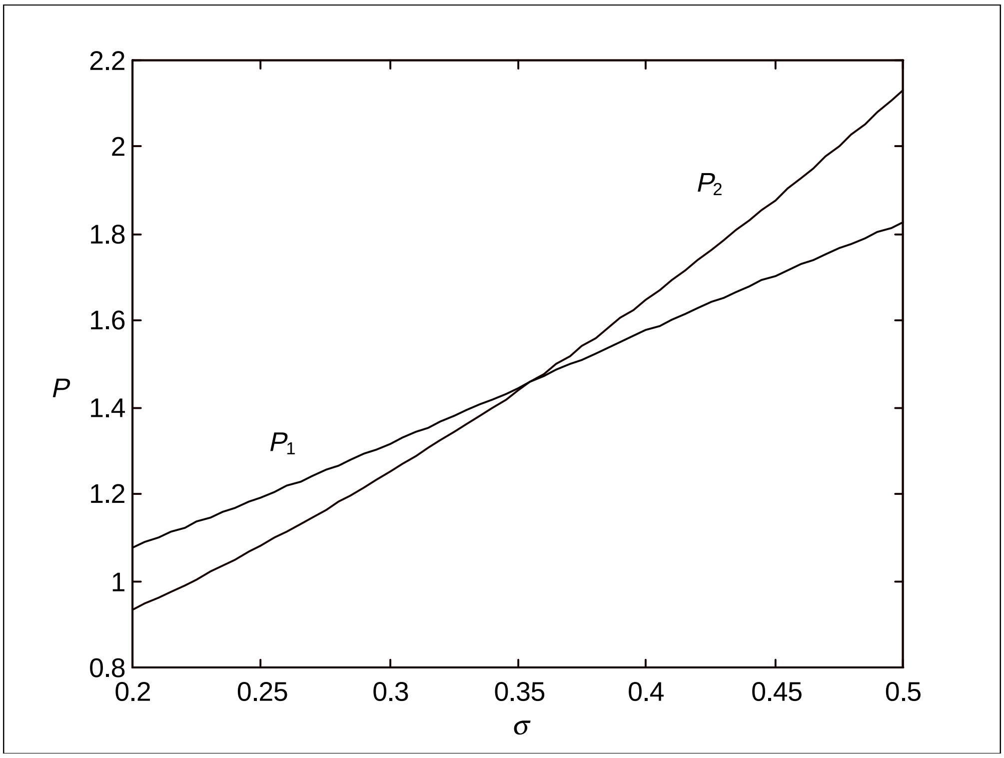


Figure 4: The possibility of sleeping patents when *h* is small

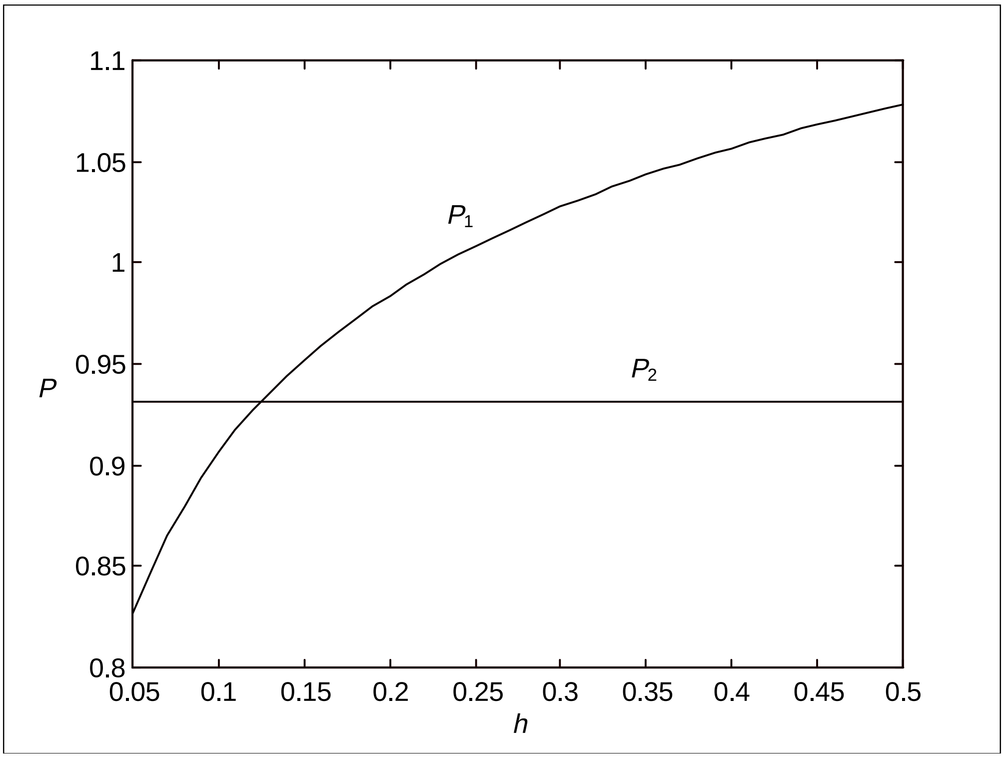


Figure 5: (σ, *h*) combinations giving rise to sleeping patents

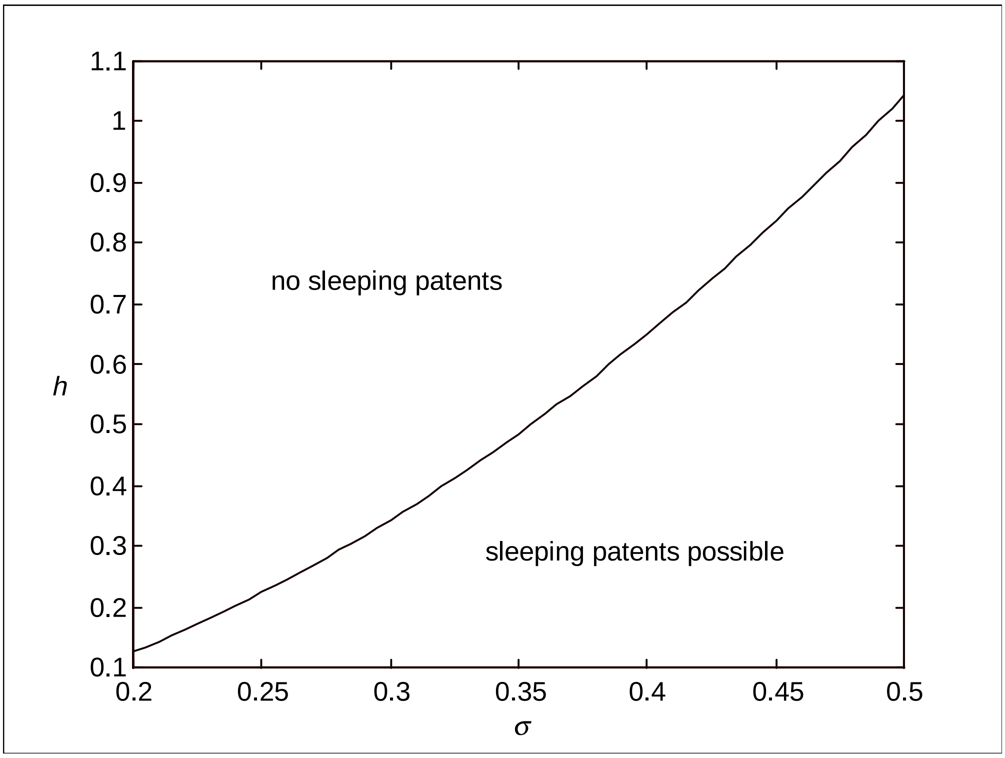
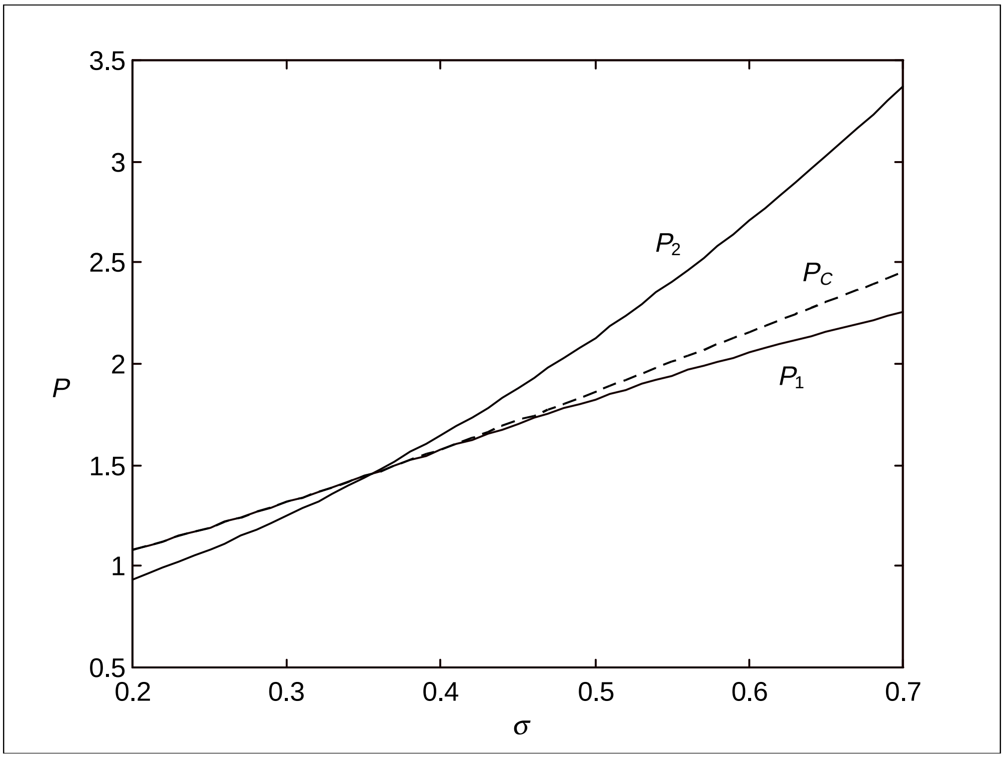


Figure 6: Trigger points with and without ability to delay



[1] This concept is illustrated by the 1955 Ealing comedy, "The Man in the White Suit," in which the creator of a dirt-repellent and (seemingly) indestructible fiber is offered £¼ million (then a considerable fortune) by the owners of the clothing industry to suppress the invention.

[2] The 1977 Patents Act allows for compulsory licensing in three cases: (i) so that another patented invention may be produced; (ii) so that the invention is "worked" (or the product manufactured) in the UK or EC; and (iii) where the patentee refuses to exploit the new technology, yet it has market potential and demand is not being met on reasonable terms. For further details on UK and EC patent law see Cornish (1996).

[3] Note that, if the patent is for a new product, this price may not be directly observable. However, the potential demand and market price may be inferred from customer surveys or the prices of existing products.

[4] To be precise, the statement that a firm invests at a trigger point P^* means that the firm invests at the time when the stochastic process P first hits P^*, approaching this level from below.

[5] Although the flow costs of research cannot be recovered, these should be thought of in the following way. In a period dt during which research is undertaken, the firm incurs a cost $chdt$ in return for which it receives a random return with an expected value given by hdt multiplied by the value of the patent at that time. Thus, the flow research cost is equivalent to production costs in a product market model (although the return is stochastic in this case) and should not be recovered if the project is later abandoned.

[6] Note that the number of research lines is not a choice variable for the firm. Given that individual lines are independent, the hazard rate of the entire project is given by $h.1 = h$.

[7] If smooth-pasting were violated and instead a kink arose at P_2, a deviation from the supposedly optimal policy would raise the firm's expected payoff. By delaying for a small interval of time after the stochastic process first reached P_2, the next step dP could be observed. If the kink were convex, the firm would obtain a higher expected payoff by entering if and only if P has moved (strictly) above P_2, since an average of points on either side of the kink give it a higher expected value than the kink itself. If the kink were concave, on the other hand, second order conditions would be violated. Continuation along the initial value function would yield a higher payoff than switching to the alternative function and switching at P_2 could not be optimal. More detailed explanation of this condition can be found in appendix C of chapter four in Dixit and Pindyck (1994).