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Profit-raising entry under oligopolistic trade with endogenous input prices

Robin Naylor | Christian Soegaard

Department of Economics, University of Warwick, Coventry, UK

Correspondence
Christian Soegaard, Department of Economics, University of Warwick, Coventry CV4 7AL, UK.
Email: C.Soegaard@warwick.ac.uk

Abstract
Firms which face the threat of import competition from foreign rivals are conventionally seen as favouring import protection. We show that this is not necessarily the case when domestic firms’ input prices are determined endogenously. In a framework where the input price is determined through contracting with an upstream agent, which could be an input supplier or a labour union, the relationship between a domestic downstream firm’s profits and the number of foreign competitors depends on trade costs and the curvature of the demand function. If trade costs are sufficiently high, an increase in the number of foreign entrants can raise the profits of a downstream firm in a home market characterised by Cournot competition. For any concave and linear demand function and for any demand function which is not ‘too convex’, this occurs due to the upstream agent moderating its input price allowing downstream firms to increase profits. For sufficiently convex demand functions, on the other hand, profit-raising entry still occurs, albeit through a substantively different mechanism, and depending upon the extent to which an increase in the input price is passed on to the final consumer.

Key words
entry, international trade, oligopoly, profits, vertical markets
1 | INTRODUCTION

In the standard Cournot model of oligopoly, each firm's profits decrease as the number of firms competing in the product market increases (Seade, 1980a). In a model of trade, this implies that a domestic import-competing firm would have an unambiguous incentive to deter foreign competition, for example through aggressive ‘deep-pocket’ entry-deterrence behaviour or through investing in the effort or expense of lobbying government for import protection. In this paper, we show that when a firm's costs are determined endogenously through contracting with an upstream agent (either an input supplier or a labour union) in the domestic market, then the relationship between profits-per-firm and the number of foreign entrants depends on trade costs and demand curvature. The intuition for this result is that the adverse effect of increased product market competition through foreign entry is offset by profit-enhancing input price moderation in the domestic vertical market through the endogenous price-setting decisions of the upstream agent.

We show that with sufficiently high trade costs, entry by foreign firms may raise the profits of domestic downstream firms. Under Cournot competition in the downstream market, with domestic and foreign firms’ outputs as strategic substitutes, we show that the domestic upstream agent moderates input prices in response to foreign entry in order to mitigate the loss in output. We demonstrate that when trade costs are sufficiently high, this scenario of profit-raising entry arises for any concave and linear demand function, and for any demand function which is not ‘too convex’. If, on the other hand, outputs are strategic complements, there also exist situations in which downstream profits increase in the number of foreign competitors. In this case, the entry of foreign competitors, all else equal, leads domestic firms to raise output. This in turn leads the upstream agent to raise its input price. The increase in the input price (i.e. the marginal cost) incurred by the downstream firms would normally lead to a decrease in profits. We show, however, that if the inverse demand function is sufficiently convex and the output share of the domestic firms relative to foreign firms is sufficiently high, the increase in marginal costs is passed on to the consumer in the form of a higher final output price. The profit-enhancing increase in the final output price dominates the profit-reducing increase in marginal costs under these conditions.

The conditions for profit-raising entry in our model can thus be summarised as follows: when trade costs are sufficiently high and when outputs are strategic substitutes, profit-raising entry occurs provided the inverse demand function is not ‘too convex’, whereas when outputs are strategic complements, profit-raising entry occurs provided the inverse demand function is ‘sufficiently convex’. While our model does not rule out the occurrence of profit-raising entry for low levels of trade costs, it is more likely that the standard profit-reducing effect of entry identified in Seade (1980a) dominates the profit-enhancing effects we identify when trade costs are low.

One implication of our result is that downstream firms in vertical markets characterised by Cournot competition do not necessarily have incentives to deter entry from abroad: for example, the profits of a downstream firm can be greater in the presence of a foreign entrant than when import protection gives it a domestic monopoly. The trade literature has identified many channels through which domestic firms can benefit from trade. These channels, however, rely on a firm’s ability to experience a more-than proportionate increase in export-generated profits to compensate for the losses incurred in the domestic market. For example, in situations of intra-industry trade (see Brander & Krugman, 1983), oligopolistic firms gain through reciprocal dumping and so are more likely to favour trade liberalisation. We show that in a vertical market setting, the incentive on downstream firms to restrict market access to foreign rivals, such as through
lobbying for entry-restricting tariff and non-tariff barriers, is weakened or reversed—even in the absence of potential export market considerations. To the best of the authors’ knowledge, the finding of the present paper, which implies that increased trade can raise the profits of a domestic firm irrespective of the level of any exports, is novel. Our results are likely to be most relevant in situations where a small number of domestic downstream firms enjoy large market shares and where foreign competition faces some intrinsic competitive disadvantage due to transport costs or a domestic home bias.

Our model identifies a mechanism to counter that analysed in the classic model of Protection for Sale (Grossman & Helpman, 1994). In that model, import-competing firms pay campaign contributions to the incumbent government to obtain trade protection, which limits the degree of foreign competition in the domestic market. In contrast, in our framework, there is the potential for a domestic downstream firm to benefit from foreign competition. Thus, a pro-trade position of firms can stem not only from a desire to exploit export markets but also to gain strategic advantages in the domestic market. In a version of our model with linear demand, we examine how import tariffs affect the profits of import-competing domestic firms. If the number of foreign firms is fixed, downstream domestic firms will continue to have the Grossman–Helpman incentive to lobby for higher tariffs. This is because tariffs have a direct profit-shifting effect in favour of domestic firms. If the number of foreign firms is not fixed, however, it is possible, in our analysis, that the reduction or removal of import tariffs may increase the profits of a domestic downstream firm. This result emerges when the profit-enhancing effect of entry dominates the profit-reducing effect of the reduction or removal of import tariffs. Moreover, it follows readily from our analysis that any other non-tariff barrier (bureaucratic ‘red tape’ or regulatory barriers, for example) which might have an impact on the entry decision of foreign firms may not be in domestic firms’ interests.

Our findings also offer new insights into a long-standing puzzle regarding the relationship between market concentration and the extent to which firms lobby government for import protection. Hillman et al. (2001) examine whether firms in highly concentrated industries will be more or less likely to engage in lobbying. They note that a long-held argument based on impediments to collective action (Olson, 1965) suggests that domestic firms in more highly concentrated industries should be better able to coordinate and collude in lobbying for import protection. However, they find that the evidence is not consistent with this hypothesis. Similarly, Winters (2003), citing also Lavergne (1983), reports that concentration ratios do not explain protection. Our results provide an insight into this puzzle. If horizontally concentrated markets are also characterised by vertical market relationships, then the incentive to lobby for protection is mediated by the domestic vertical relationship.

We derive conditions under which the payoff of the upstream agent decreases with entry of foreign downstream rivals. This occurs for every concave or linear demand function or any demand function which is not ‘too convex’, or alternatively, when the competition in the downstream market is characterised by strategic substitutability. This implies potentially conflicting interests between firms (and/or labour unions) at different levels in the vertical hierarchy. Evidence supports the view that businesses tend to favour free trade, whereas labour unions tend to oppose it. In the United States, for example, labour unions lobbied

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1 See Maggi and Rodriguez-Clare (2000), for example, whose model generates the prediction that protection can increase with import penetration. See also Greenaway et al. (2008).

2 See also Bombardini and Trebbi (2012).
unsuccessfully against the North American Free Trade Agreement (NAFTA) whereas businesses were largely in favour. While labour unions mostly object to trade agreements with low-wage economies, they were also seen to oppose the Australia-US Free Trade Agreement (AUSFTA) more than two decades ago. Since the 2016 presidential election in the United States, many businesses have joined a lobby to preserve NAFTA. If the demand function is sufficiently convex, or alternatively that the competition in the downstream market is characterised by strategic complementarity, the profits of the upstream agent increase in response to entry. As such, it is possible in our model that all domestic agents (upstream and downstream) can benefit from foreign entry.

In a version of our model with linear demand, we also consider a demand function which allows for the possibility that consumer preferences are biased towards the consumption of domestic goods. We find that our apparently counter-intuitive results regarding the impact of import competition on profits, although not dependent on home bias, are nonetheless strengthened the more pronounced is the extent of any home bias. Hence, a greater home bias may be associated with a greater willingness on the part of domestic firms to accommodate foreign competition.

While we have applied our modelling to one of international trade, our results are not restricted to such settings. We could alternatively have assumed the foreign firms in the model are domestic firms with higher costs, and as such, our framework implies that relatively efficient (and possibly unionised) incumbent firms may have an incentive to accommodate less efficient entrants. Our main findings on profit-raising entry are not the result of very special assumptions on the inverse demand function and trade costs. Our central results may arise for a very large range of inverse demand functions and as our numerical simulations demonstrate, also a relatively wide range of trade costs.

The rest of this paper is organised as follows. In Section 2, we briefly locate our analysis within the context of the relevant literature on profit-raising entry. In Section 3, we set out the basic model and in Section 4, we examine how firms’ profits vary with the number of firms. Section 5 closes the paper with conclusions and further remarks.

2 | LITERATURE REVIEW

Our finding that a firm’s profits might increase with the overall number of firms can also arise in different environments. There now exists a growing literature exploring different channels through which entry might be associated with profit-raising effects. We identify at least three strands of this literature: (a) Vertical industrial relationships; (b) Stackelberg competition with cost asymmetry; and (c) Product differentiation.

Within the first strand of the literature, an early contribution (Tyagi, 1999) constructs a model of vertical industrial relationships with downstream firms competing in a Cournot setting. Entry into the market leads the upstream agent to increase the input price, which in turn leads downstream firms to compete less aggressively post entry. Under certain conditions on the demand function, this can lead to an increase in the profits of each firm. Subsequent papers have further explored the possibility of profit-raising entry under vertical relationships (Matsushima & Mizuno, 2012; Mukherjee, 2019; Mukherjee et al., 2008, 2009; Naylor & Soegaard, 2014, 2018). Within the second strand, Mukherjee and Zhao (2017) use a model of Stackelberg competition where firms differ in marginal costs. They find that if the incumbent Stackelberg leader is efficient, the entry of an inefficient Stackelberg follower increases its
profits. A similar result can be found in Pal and Sarkar (2001). These results arise without any vertical relationships in the market. The third strand involves the presence of some form of product differentiation. Ishibashi and Matsushima (2009) use a model with firms that produce high-end (or branded) products and firms that produce low-end (or non-branded) products. They show that the entry of low-end firms may increase the profits of high-end producers. This is because in the absence of low-end firms, high-end producers would compete for price-sensitive consumers, and thereby overproduce. In Mukherjee’s (2019) model of oligopoly, an increase in the number of firms in the final goods market generates entry in the upstream market. If the final goods are sufficiently differentiated, the input price moderation resulting from the free entry in the upstream market dominates the profit-reducing effect on the final output price, thus producing profit-raising entry. Papers which generate profit-raising entry due to some form of product differentiation also include Coughlan and Soberman (2005) and Chen and Riordan (2007).

Profit-raising entry arises under two distinct mechanisms in our model. The first is through input price moderation, which occurs when trade costs are sufficiently high and when inverse demand is linear, concave or not ‘too convex’. The fact that the profit-enhancing effect of input price moderation can dominate the profit-reducing effect from more intense product market competition has been shown previously in models of linear demand. Indeed, Arya et al. (2007), Mukherjee et al. (2009) and Naylor and Soegaard (2018) can be seen as variants of our model with linear demand. In Arya et al. (2007), the upstream agent competes directly with the downstream firm in the final goods market, and they show that if the cost advantage of the downstream firm relative to the upstream agent is sufficiently high, the former may profit from this direct competition. A similar result is produced in Mukherjee et al. (2009), where the entry of sufficiently inefficient entrants can lead to incumbent firms increasing their profits. Our modelling of inverse demand as a general function rather than a linear one adds further nuance to this literature. We demonstrate conditions under which input price moderation will occur (the case of strategic substitutability) in the final goods market. We show that a sufficiently high-cost asymmetry between incumbents and entrants (modelled as a trade cost in our framework) is not enough by itself to generate profit-raising entry, as the demand function must also not be ‘too convex’. For highly convex demand, the benefit of moderation in the input market is passed on to the consumer. The second mechanism arises for sufficiently convex demand functions. Under this scenario, final outputs are strategic complements and the upstream agent raises its price in response to entry. Under sufficiently convex demand, the cost of the increase in the input price is passed on to the consumer. This mechanism is similar to that analysed in Tyagi (1999), but our result arises in regions of high trade cost, and this cost asymmetry allows for the characterisation of sufficient conditions for profit-raising entry through this mechanism. Our contribution relative to existing studies of profit-raising entry is the identification of novel channels through which profit-raising entry occurs—contracting over the input price and international trade—and demonstrating how this arises under different assumptions on the curvature of the demand function.

In a closely related literature, Mukherjee et al. (2008) show that a monopolist may find it profitable to create competition by licencing its technology to competitors if the input market is unionised. Similarly, Mukherjee and Zhao (2012) show that firms have incentives to encourage parallel trade, whereby competition is increased with a foreign trader, to increase profits, also in the presence of input price determination. Ishida et al. (2011) show that the entry of high-cost competitors can benefit low-cost firms by stimulating R&D. The profit-enhancing effect of improved technology can dominate the profit-reducing effect from more intense competition.
Fanti and Buccella (2017) show the possibility of profit-raising entry in a model of networks and corporate social responsibility.

We also note that within the union-bargaining literature, Naylor (2002) shows conditions under which industry profits are increasing with the number of firms in the market, but does not address the issue of the individual firm’s profit level. It is less surprising that industry profits can increase with the number of firms as such a result is anyway consistent with falling profits-per-firm. Similarly, in a vertically integrated oligopoly setting, Dowrick (1989) develops a framework in which unions act as the upstream agent and shows how the bargained wage varies with market size, but does not focus on the relationship between profits and the number of firms. Horn and Wolinsky (1988) examine a differentiated oligopoly with upstream agents (unions) and downstream firms, but assume a duopolistic market in which the number of firms is fixed.

3 | THE MODEL

We develop a model in which $m$ downstream firms and an upstream agent (which could be regarded as either a firm supplying an input or a labour union) contract over the price of an input (i.e. the wage rate in the case of labour). All are located in country $h$. If we assumed autarky, the domestic market would be characterised as a situation of vertical oligopoly. However, we assume instead that the downstream firms compete against $n$ entrants from the RoW. We assume for simplicity that foreign firms are all identical and pay a competitive RoW input price. Foreign firms face a per-unit trade cost associated with exporting to country $h$. This, together with the assumption of quantity-setting competition among a finite number of downstream firms, generates the possibility that the domestic upstream agent can set an input price greater than the RoW level. This set-up seems particularly relevant for relatively small countries with domestic markets which do not support intense product market competition or for relatively affluent countries with high wages or other input costs. Our assumption that the input is not traded makes our framework particularly relevant to the case where the upstream agent is a labour union, or a productive input which faces intrinsic transport costs that make the good prohibitively expensive to ship. As we note below, however, qualitatively similar results can be derived without this assumption. In the first stage (the contracting stage), the upstream agent provides a take-it-or-leave-it offer to the downstream firms. In the second stage, the downstream home firms and the $n$ foreign entrants set their output choices—given the pre-determined input price from Stage 1—to maximise profits. We proceed by backward induction.

3.1 | Stage 2: the product market game

We model inverse demand as the general function $p = p(Q)$, where $Q = Q_h + Q_f$ is aggregate output, $Q_h = \sum_{i=1}^{m} q_{hi}$ is the output of the downstream home firms, and $Q_f = \sum_{j=1}^{n} q_{fj}$ is the aggregate output of downstream foreign firms. We assume that $p(Q)$ is thrice continuously differentiable and that $\frac{dp}{dQ} < 0$. We do not model demand in the RoW as the focus is on output decisions in country $h$. In Subsection 4.3, we allow for an alternative specification of inverse

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5This situation can also be described as a bargaining game in which the upstream agent has all the bargaining power.
demand, which enables us to capture either home or foreign bias. In this setting, we assume that the inverse demand facing domestic firms is the function, \( p_h \equiv p_h(Q) = B + p(Q) \), such that any value of \( B > 0 \) captures a bias towards domestically produced goods. In the general model presented in this section, however, we impose \( B = 0 \) such that goods are homogeneous.

The profits of home firm \( i \), and those of foreign firm \( j \), respectively, can be written as follows:

\[
\pi_{hi} = [p - c_{hi}] q_{hi}; \quad (1)
\]

\[
\pi_{fj} = [p - c_{fj} - \tau] q_{fj}. \quad (2)
\]

\( c_{hi} \) is the domestic input price, which results from the contracting between the upstream agent and the downstream firms in country \( h \). For simplicity and clarity of analysis, we assume a single monopoly upstream agent in the home country. As we show in the analysis of the stage 1 sub-game below, the upstream agent’s maximisation problem leads to a value of the input price identical for all domestic firms. Hence, we drop the subscript such that \( c_{hi} = c_h \forall i \). The implication is that domestic input prices or wages are determined through contracting with a centralised agent/labour union. Alternative assumptions regarding the nature of the vertical relationships produce qualitatively similar results. For example, in Appendix A we demonstrate that our results regarding profit-raising entry hold for the case of a centralised upstream agent supplying both home and foreign firms under linear demand. We also note that results apply: (i) when contracting over the input price is decentralised across multiple upstream–downstream pairs, and (ii) when the input price paid by foreign firms is also the result of contracting with an upstream agent located in RoW.

In Equation (2), \( c_{fj} \) is the input price incurred by foreign firm \( j \), and \( \tau \) is a per-unit trade cost faced by foreign firms.\(^4\) Under the assumption that they do not engage in contracting with an upstream agent, foreign firms pay a given input price which we denote as \( \bar{c} \).\(^5\) Hence, we have \( c_{fj} = \bar{c} \forall j \). We could have assumed an upstream agent for foreign firms, though again this does not impact on the qualitative nature of our results regarding the domestic market. Under the Cournot-Nash assumption, differentiation of (1) and (2), respectively, yields the first-order conditions for profit maximisation for the incumbent firms and the foreign entrants. The corresponding best-reply functions must satisfy:

\[
p' q_{hi} + p - c_h = 0, \quad i = 1 \ldots m; \quad (3)
\]

\[
p' q_{fj} + p - c_f - \tau = 0, \quad j = 1 \ldots n, \quad (4)
\]

where \( p' = \frac{dp}{dQ} \). Since home firms are identical, and similarly for foreign firms, we drop subscripts \( i \) and \( j \), respectively. Using this, the solutions to the system of \( n + m \) equations in (3) and (4) will give us the outputs in the final-stage Cournot equilibrium and we denote these solutions \( q_{hi}^C \) and \( q_{fj}^C \), respectively. The corresponding profits for a home and foreign firm, respectively, are \( \pi_{hi}^C \) and \( \pi_{fj}^C \).

\(^4\)Trade costs may include such things as transport technology, storage, inventory and preparation technology, communications networks, language barriers, import tariffs and so on.

\(^5\)Naylor (1999) considers a situation in which both domestic and foreign firms bargain over input prices with respective upstream agents, but without considering the effects of entry on the downstream firm’s profits.
second-order conditions for the home and foreign optimisation problems, respectively, are as follows:\(^6\)

\[
p'' q_{hi} + 2p' < 0, \quad i = 1 \ldots m; \tag{5}
\]

\[
p'' q_{fj} + 2p' < 0, \quad j = 1 \ldots n, \tag{6}
\]

where \(p'' = \frac{d^2 p}{dQ^2}\). We use the stability condition derived in Seade (1980b) to ensure the equilibrium is stable:\(^7\)

\[
p''Q + p' (m + n + 1) < 0. \tag{7}
\]

In what follows, it will be convenient to define the \textit{elasticity of slope of inverse demand} as follows:

\[
\varphi \equiv \frac{p''Q}{p'}. \tag{8}
\]

The construct of the elasticity of slope of inverse demand captures the degree of concavity/convexity of the inverse demand function and is commonly used in the analysis of oligopolistic competition (see, e.g., Dixit, 1986; Perry, 1982; Seade, 1980a; Tyagi, 1999). Since \(p' < 0\), the sign of \(\varphi\) is determined by the sign of \(p''\). Hence, any linear demand function is characterised by \(\varphi = 0\), any concave function by \(\varphi > 0\), and any convex function by \(\varphi < 0\). An alternative interpretation of \(\varphi\) which has been used in the literature (see, e.g., Kimmel, 1992), and which will be useful for our purposes, is the fraction of cost changes that are passed on to prices, with a lower value of \(\varphi\) indicating a higher pass-through. Using \(\varphi\), we can rewrite the stability condition as follows:

\[
\varphi > - (m + n + 1). \tag{9}
\]

\(^6\)We assume the second-order conditions must be satisfied with strict inequality. These conditions are necessary and sufficient for the existence of a unique Cournot-Nash equilibrium.

\(^7\)In our model with home and foreign firms with different marginal costs, the stability condition can be derived as follows. The marginal profit of home firm \(hi\) is \(\frac{\partial \pi_{hi}}{\partial q_{hi}} = p' q_{hi} + p - c_h\). Differentiating this expression with respect to home firm \(k\) yields \(\frac{\partial^2 \pi_{hi}}{\partial q_{hi} \partial q_{hk}} = p'' q_{hi} + p' \frac{\partial q_{hi}}{\partial q_{hk}} + p'\), and with respect to foreign firm \(j\), we have \(\frac{\partial^2 \pi_{hi}}{\partial q_{hi} \partial q_{fj}} = p'' q_{hi} + p' \frac{\partial q_{hi}}{\partial q_{fj}} + p'\).

Summing these two expressions for \(k = 1 \ldots m\) and \(j = 1 \ldots n\) yields

\[
\sum_{k=1}^{m} \frac{\partial^2 \pi_{hi}}{\partial q_{hi} \partial q_{hk}} + \sum_{j=1}^{n} \frac{\partial^2 \pi_{hi}}{\partial q_{hi} \partial q_{fj}} = p'' (nq_h + Q_h') + p' (m + n + 1) < 0.
\]

Violation of this inequality implies that the marginal profitability of home firm \(hi\) increases with an increase in total output. Summing this expression over all \(i = 1 \ldots m\) gives

\[
p'' (m + n) Q_h + mp' (m + n + 1) < 0.
\]

Using similar steps, we can find the equivalent expression for foreign firms as \(p'' (m + n) Q_f + np' (m + n + 1)\), and hence, summing over all domestic and foreign firms yields:

\[
(m + n) [p''Q + p' (m + n + 1)].
\]
For convenience, we aggregate the $m$ best-reply functions of the home firms to obtain an equation which determines the aggregate output of home firms:

$$p'Q_h + m\left(p - c_h\right) = 0,$$

and similarly, we aggregate the best-reply functions of all firms—home and foreign—to obtain an equation determining aggregate output in the home market:

$$p'Q + m\left(p - c_h\right) + n\left(p - \bar{c} - \tau\right) = 0.$$  

(11)

We denote the solutions to these last two equations as respectively, $Q^C_h$ and $Q^C$, and we denote the sum of the equilibrium profits of all $m$ domestic firms as $\Pi^C_h = \sum_{i=1}^{m} \pi^C_{hi} = m\pi^C_h$.

### 3.2 Stage 1: the contracting stage

The profits of the single domestic upstream agent can be written as follows:

$$\Pi'^I_h = (c_h - \bar{c}) Q^C_h,$$

where $\bar{c}$ denotes the marginal cost of the input, which is assumed to be a fundamental economic cost common across countries. We assume the upstream agent gives the downstream firms a ‘take-it-or-leave-it’ offer: in a situation of union-firm bargaining, this is the case described by the monopoly union model. Maximising (12) with respect to $c_h$ implies the first-order condition:

$$\frac{d\Pi'^I_h}{dc_h} = (c_h - \bar{c}) \frac{\partial Q^C_h}{\partial c_h} + Q^C_h = 0,$$

with second-order condition:

$$\left(c_h - \bar{c}\right) \frac{\partial^2 Q^C_h}{\partial c_h^2} + 2 \frac{\partial Q^C_h}{\partial c_h} < 0.$$  

(14)

The magnitude of trade costs may prohibit trade. We denote the critical level of trade costs above which foreign firms are unable to make non-negative profits as $\tilde{\tau}$. We have:

$\tilde{\tau}$This marginal cost captures workers’ reservation wages if the input is labour, or an underlying intrinsic cost of producing the intermediate input.

$\tilde{\tau}$We could alternatively have assumed a more generalised Nash bargain over the domestic input price. The maximand for such a problem would take the following form:

$$\Omega_h = (\Pi'^I_h)^{\beta} (\Pi^C_h)^{1-\beta},$$

where $\beta$ is the upstream agent’s Nash bargaining power. However, from analysis of this more general case, we conclude that such an extension significantly complicates the model without adding any insight which could not be derived using the assumed structure where the upstream agent has all the bargaining power.
\[ \lim_{\tau \to \tau} Q_f^C = 0, \]  
where \( Q_f^C \) is the total equilibrium output of the \( n \) foreign firms, \( Q_f^C = \sum_{j=1}^{n} q_f^C = n q_f^C \).

## 4 | The Effects of Entry on Profits

We now investigate how the profits of the domestic firms in subgame perfect Nash equilibrium vary with the number of foreign rivals in the home market. Our motivation is to examine whether there are conditions under which the domestic downstream firms might have an incentive to encourage or accommodate foreign entry. Differentiating the profits of home firm \( i \) in (1) with respect to \( n \) yields,

\[ \frac{d\pi_h^C}{dn} = \frac{\partial \pi_h^C}{\partial n} + \frac{\partial \pi_h^C}{\partial c_h} \frac{dc_h}{dn} . \]  

We have decomposed the effects of entry into the direct effects on profits (i.e. the effect on output and the final output price), and the indirect effect through input prices. In Appendix B, we derive explicit expressions for \( \frac{\partial \pi_h^C}{\partial n} \) and \( \frac{\partial \pi_h^C}{\partial c_h} \), respectively, using (3) and (11), to rewrite (16) as,

\[ \frac{d\pi_h^C}{dn} = - \frac{q_h (p - \bar{c} - \tau) \left[ p'' q_h + 2p' \right]}{p'' Q + p' (m + n + 1)} - \frac{q_h \left[ (2Q - Q_h) (p'') + 2p' (n + 1) \right]}{p'' Q + p' (m + n + 1)} \left( \frac{dc_h}{dn} \right). \]  

In the absence of endogenous setting of the input price by an upstream agent, the effect of entry by foreign rivals is well established by existing literature: entry of an additional firm reduces the profits of incumbents. In our model, this result is straightforward to show: for \( \frac{dc_h}{dn} = 0 \), we have,

\[ \frac{d\pi_h^C}{dn} = - \frac{q_h (p - \bar{c} - \tau) \left[ p'' q_h + 2p' \right]}{p'' Q + p' (m + n + 1)} < 0, \]  

where we have used the second-order condition for profit maximisation in (5), and the stability condition in (7) to determine the sign. However, if entry by a foreign rival induces the upstream agent to change the input price, the effect is no longer unambiguous. We first analyse the effect of a change in the input price on Cournot profits:

\[ \frac{\partial \pi_h^C}{\partial c_h} = - \frac{q_h \left[ (2Q - Q_h) (p'') + 2p' (n + 1) \right]}{p'' Q + p' (m + n + 1)} = - \frac{q_h \left[ \varphi (2 - s_h) + 2 (n + 1) \right]}{[\varphi + m + n + 1]}, \]
where \( s_h = \frac{Q_h}{Q} \) is the share of total output produced by domestic firms. The second equality follows by dividing, respectively, the numerator and the denominator by \( p' \), and using the definition of the elasticity of slope of inverse demand in (8). Since the denominator of (19) is strictly positive due to the stability condition in (9), (19) takes its sign from its numerator. It is therefore evident that \( \frac{\partial p_h}{\partial c_h} \) is unambiguously negative for any concave \((\varphi > 0)\) or linear \((\varphi = 0)\) inverse demand function, and for any convex function which is not ‘too convex’. A positive value of \( \frac{\partial p_c}{\partial c_h} \) occurs when \( \varphi \left( 2 - s_h \right) < -2(n + 1) \), which is not inconsistent with the stability condition (9). In order to understand the intuition for the counter-intuitive result that the profits of the domestic downstream firms are increasing in their own marginal cost, recall the alternative interpretation of \( \frac{\partial \pi_c}{\partial c_h} \) described above: the fraction of costs passed on to prices. A very negative value of \( \frac{\partial \pi_c}{\partial c_h} \) implies that a large fraction of the change in cost is passed through to prices. A rise in the marginal cost of a domestic firm leads to a profit-reducing decrease in its output and to a profit-enhancing rise in prices. The latter effect dominates if the pass-through of the change in cost to prices is sufficiently high, and the share of domestic output in the total output is also sufficiently high.

The sign of \( \frac{\partial \pi_c}{\partial c_h} \) is critical to the impact on domestic downstream firms’ profits of an increase in the number of foreign entrants. Suppose, for example, that \( \frac{\partial \pi_c}{\partial c_h} > 0 \), then any attempt by the upstream agent to moderate input prices in response to entry will, all else equal, have a negative impact on domestic downstream profits. Similarly, if \( \frac{\partial \pi_c}{\partial c_h} < 0 \), an attempt by the upstream firm to raise its price for the input will, all else equal, decrease the payoff to downstream firms. A necessary condition for downstream firms to benefit from foreign entry is therefore that the signs of, respectively, \( \frac{\partial \pi_c}{\partial c_h} \) and \( \frac{\partial c}{\partial n} \), are the same.

The input price is determined by the first-order condition for the upstream agent’s maximisation problem in (13). Rearranging this expression yields \( c_h - \bar{c} = -\frac{Q_h}{\partial \pi_c} \). In Appendix B, we derive an explicit expression for \( m \frac{\partial q}{\partial c_h} = \frac{\partial Q_h}{\partial c_h} \) using (3) and (11), to rewrite the equilibrium value of the input price, \( c^*_h \), as follows:

\[
\frac{\partial c^*_h}{\partial n} = -\frac{p' q_h \left[p'' Q + p' (m + n + 1)\right]}{p'' (Q - Q_h) + p' (n + 1)} > 0.
\]

Thus, the input price depends on the number of foreign entrants indirectly through total output \( Q \) and total domestic output \( mq_h = Q_h \), and directly through \( n \). Using this we have:

\[
\frac{dc^*_h}{dn} = \frac{\partial c^*_h}{\partial Q} \frac{\partial Q}{\partial n} + \frac{\partial c^*_h}{\partial Q_h} \frac{\partial Q_h}{\partial n} + \frac{\partial c^*_h}{\partial n}
\]

In Appendix B, we derive the explicit expressions for \( \frac{\partial Q}{\partial n} \) and \( \frac{\partial Q_h}{\partial n} \) using (3) and (11). These expressions take the form:

\[
\frac{\partial Q}{\partial n} = \frac{p' q_f}{\left[p'' Q + p' (m + n + 1)\right]} > 0;
\]
\[
m \frac{dq_h}{dn} = \frac{\partial Q_h}{\partial n} = - \frac{q_f [p''Q_h + mp']}{[p''Q + p'(m+n+1)]}.
\] (23)

By differentiating (20) with respect to \(n\), it is straightforward to determine the expression for the direct effect on the input price of a change in \(n\):

\[
\frac{dc^*_h}{dn} = \frac{(p')^2 q_h [p''Q_h + p'm]}{[p''(Q - Q_h) + p'(n+1)]^2} = \frac{p'q_h [\varphi s_h + m]}{[\varphi (1 - s_h) + n+1]^2}.
\] (24)

where the second equality is obtained by dividing, respectively, the numerator and the denominator by \(p'\) and using the definition of \(\varphi\) in (8). We are unable to sign the value of \(dc^*_h\) in (21) for every value of the trade cost \(\tau\). However, by using the expressions for \(\frac{\partial Q}{\partial n}\) and \(\frac{\partial Q_h}{\partial n}\) in, respectively, (22) and (23), as well as (15), we make the following observations:

\[
\lim_{\tau \to \tilde{\tau}} \frac{\partial Q}{\partial n} = 0;
\] (25)

\[
\lim_{\tau \to \tilde{\tau}} \frac{\partial Q_h}{\partial n} = 0.
\] (26)

That is, in the limit as the trade cost \(\tau\) approaches its prohibitive level, the indirect effects on the input price of a change in \(n\) through respectively, \(Q\) and \(Q_h\), converge to zero. Using this observation, we have,

\[
\lim_{\tau \to \tilde{\tau}} \frac{dc^*_h}{dn} = \lim_{\tau \to \tilde{\tau}} \frac{\partial c^*_h}{\partial n} = \frac{p'q_h [\varphi + m]}{(n+1)^2},
\] (27)

where we use (15) such that \(\lim_{\tau \to \tilde{\tau}} s_h = \lim_{\tau \to \tilde{\tau}} \frac{Q - Q_f}{Q} = 1\). We establish Proposition 1:

**Proposition 1**  The upstream agent moderates input prices in response to entry, that is \(\frac{dc_h}{dn} < 0\), if the trade cost, \(\tau\), is sufficiently high and,

\[
\varphi s_h + m > 0.
\] (28)

**Proof**  See Appendix C.

Hence, a sufficient condition for the upstream agent to moderate input prices in response to foreign entry is that the inverse demand function is not ‘too convex’, or that \(\varphi s_h > - m\). Condition (28) is found in several papers on oligopoly (see, e.g., Frank, 1965; Okuguchi, 1973; Ruffin, 1971; Seade, 1980a). As shown in Seade (1980a), (28) can be interpreted as the requirement that marginal revenue is steeper than market demand (in the Cournot case), a requirement which cannot be ruled out a priori. Seade (1980a) also demonstrates that if condition (28) holds, output per firm
decreases with entry, or alternatively, outputs of domestic downstream firms are strategic substitutes.  

Proposition 1 therefore implies that the upstream agent moderates input prices in response to entry if the downstream firms’ outputs are strategic substitutes in the limit as \( \tau \) approaches the prohibitive level. The intuition for this result is that when each downstream domestic firm decreases their outputs as a response to an increase in the output of foreign rivals, the upstream agent lowers its input price so as to mitigate the loss in output. On the other hand, for strategic complementarity, in which each domestic firm increases output in response to entry, the upstream agent is better off raising its price.

In the next proposition, we will be more specific about the effects of entry by foreign rivals on the profits of domestic downstream firms.

**Proposition 2** Profits of a domestic downstream firm increase in the number of foreign entrants if the trade cost, \( \tau \), is sufficiently high and,

\[
\left[ \varphi \left( 2 - s_h \right) + 2 \left( n + 1 \right) \right] \left[ \varphi s_h + m \right] > 0. \tag{30}
\]

**Proof** See Appendix C.

It is straightforward to use (30) to determine that for any concave (\( \varphi > 0 \)) or linear (\( \varphi = 0 \)) inverse demand function, domestic downstream firms will benefit from entry when trade costs are sufficiently high. Moreover, when domestic outputs are strategic substitutes, there is the possibility of profit-raising entry if the inverse demand function is not ‘too convex’. In particular, if:

\[
\varphi > \max \left[ -\frac{m}{s_h}, -\frac{2 (n + 1)}{2 - s_h} \right]. \tag{31}
\]

If, on the other hand, we have \( \varphi \) in the range,

\[
-\frac{m}{s_h} < \varphi < -\frac{2 (n + 1)}{2 - s_h}, \tag{32}
\]

then the impact of input price moderation by the upstream agent, attempting to mitigate the adverse effects of foreign entry on its profits, causes the Cournot-competing downstream firms to lower their final product price to such an extent that their profits fall, to the benefit of consumers. When domestic outputs are strategic complements (\( \varphi < -m \)), then for sufficiently high trade costs, there is the possibility of profit-raising foreign entry provided the inverse demand function is sufficiently convex. In particular, if:

\[^{10}\text{To see this more explicitly, multiply (28) by } p', \text{ such that,}
\]

\[
p'' Q_h + p' m = \frac{1}{m} \left( p'' q_{hi} + p' \right) < 0. \tag{29}
\]

The first-order condition for profit maximisation of a specific domestic downstream firm \( i \) is \( p'' q_{hi} + p - c_{hi} \), and differentiating this with respect to the output of all other firms \( Q_{-hi} \) yields \( p'' q_h + p' \). Thus, a negative value indicates that a domestic downstream firm decreases its output in response to an increase in the output of all other firms in the market: that is, that its best-reply function is downward sloping and hence that outputs are strategic substitutes.
In this case, entry will result in the upstream agent raising the price of the input. While this leads to lower outputs, domestic downstream firms will pass the higher marginal cost on to consumers, thus raising the profits of downstream firms. On the other hand, if $\varphi$ is in the range $\varphi > \max \left[-\frac{m}{s_h}, -\frac{2(n+1)}{2-s_h}\right]$, then the upstream agent’s higher price does not result in higher downstream profits: the increase in the price the downstream firms charge will not be sufficient to cover lost output. We summarise all of these results in Table 1.

| $\varphi$ | Input price effect $\left(\frac{du_i^*}{dn}\right|_{r=\bar{r}}$) | Profit effect $\left(\frac{dx_i^*}{dn}\right|_{r=\bar{r}}$) |
|-----------|-------------------------------------------------|-------------------------------------------------|
| $>0$      | $<0$                                            | $>0$                                            |
| $=0$      | $<0$                                            | $>0$                                            |
| $\varphi > \max \left[-\frac{m}{s_h}, -\frac{2(n+1)}{2-s_h}\right]$ | $<0$                                            | $>0$                                            |
| $-\frac{m}{s_h} < \varphi < -\frac{2(n+1)}{2-s_h}$ | $<0$                                            | $<0$                                            |
| $\varphi < \min \left[-\frac{m}{s_h}, -\frac{2(n+1)}{2-s_h}\right]$ | $>0$                                            | $>0$                                            |
| $-\frac{2(n+1)}{2-s_h} < \varphi < -\frac{m}{s_h}$ | $>0$                                            | $<0$                                            |

$$\varphi < \min \left[-\frac{m}{s_h}, -\frac{2(n+1)}{2-s_h}\right]. \quad (33)$$

In this case, entry will result in the upstream agent raising the price of the input. While this leads to lower outputs, domestic downstream firms will pass the higher marginal cost on to consumers, thus raising the profits of downstream firms. On the other hand, if $\varphi$ is in the range

$$-\frac{2(n+1)}{2-s_h} < \varphi < -\frac{m}{s_h}, \quad (34)$$

then the upstream agent’s higher price does not result in higher downstream profits: the increase in the price the downstream firms charge will not be sufficient to cover lost output. We summarise all of these results in Table 1.

What is the reason for this apparent counter-intuitive result that the profits-per-firm can increase in the number of foreign competitors? In the standard model of oligopoly, an increase in the number of entrants unambiguously reduces the profits of incumbents through an increase in product market competition. We saw this above in (18) and this result is proved more generally in Seade (1980a). This mechanism is still at work in the present model; however, the presence of the upstream agent creates the possibility that foreign entry leads to a change in input prices.

Analysing first the case of strategic substitutability, we note that the upstream agent balances its input price offer against the impact on its sales to the downstream firms. Since the upstream agent does not sell any input to foreign firms, increasing its price will make domestic downstream firms less competitive and reduce output. Hence, the upstream agent has an incentive to moderate input prices in the face of more intense downstream competition. If the effect of profit-enhancing moderation of input prices dominates the profit-reducing effect of increased product market competition in the downstream market, profits-per-incumbent downstream firm will increase in the number of foreign entrants. While we cannot, in the general model, rule out the possibility of profit-raising entry for low or even zero trade costs, the first effect is more likely to dominate the higher the trade costs faced by foreign entrants. This is because in the presence of higher barriers to trade, the market share of domestic

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11Seade (1980a) demonstrates that the finding that the profits per firm unambiguously decrease in the number of entrants follows from the stability condition (7).
agents (upstream and downstream) is higher. In this case, the markup of the upstream agent is higher which in turn gives it more leeway to offer discounts to downstream firms. Thus, if trade costs are sufficiently high, international trade will act to mitigate the double marginalisation problem in this vertical market. Our result holds for any linear and any concave demand function, and for some convex functions unless they are ‘too convex’. The profit-raising effect of entry from input price moderation can dominate the profit-reducing effect on the final output price has been shown in models using linear demand (see, e.g., Arya et al., 2007; Mukherjee et al., 2009; Naylor & Soegaard, 2018). In our model, which uses a general demand function, we show conditions under which input price moderation occurs in the range of sufficiently high trade costs (i.e. strategic substitutability). Moreover, we show that input price moderation and sufficiently high trade costs alone do not produce profit-raising entry as the inverse demand function also must not be ‘too convex’.

Moving on to the case of strategic complementarity. In this case, the behaviour of firms is such that entry by foreign rivals, all else equal, leads to an increase in the output of domestic firms. The upstream firm responds by increasing input prices. Surprisingly, if the inverse demand function is sufficiently convex and the market share of the domestic downstream firms is sufficiently high, the rise in the input price is passed on to consumer prices, with downstream firms raising their profits. A similar result arises in Tyagi (1999) in a framework of symmetric firms. For specific conditions on the demand function, entry leads to less aggressive competition in the downstream market, causing profit-raising entry.

4.1 | Examples

As we have observed, we are unable to draw any conclusions about the relationship between incumbent per-firm profits and foreign entry for low trade costs in the general model. This would require making assumptions about the sign of the third derivative of the inverse demand function (or the derivative of the elasticity of slope of inverse demand) which is unusual. However, analysing particular inverse demand functions allows us to be more specific about the range of trade costs for which profit-raising entry occurs.

4.1.1 | Linear demand function

For the case of the linear demand function,

\[ p(Q) = a - Q. \]  \hspace{1cm} (35)

we can use the first-order conditions in (3) and (4) to determine the Cournot outputs as follows:

\[ mq_h = Q_h = \frac{m [a - (n + 1) c_h + n (\bar{c} + \tau)]}{m + n + 1}; \]  \hspace{1cm} (36)

\[ nq_f = Q_f = \frac{n [a + m c_h - (m + 1) (\bar{c} + \tau)]}{m + n + 1}. \]  \hspace{1cm} (37)
The input price is obtained using (36) and (13):

\[ c_h = \frac{a + (2n + 1)\bar{c} + n\tau}{2(n + 1)}. \tag{38} \]

Substituting (38) into (36) and (37), respectively, equilibrium Cournot outputs can be determined as follows:

\[ mq_h^* = Q_h^* = \frac{m[a - \bar{c} + n\tau]}{2(m + n + 1)}; \tag{39} \]

\[ nq_f^* = Q_f^* = \frac{n[2(n + 1)[a - \bar{c} - \tau] + m(a - \bar{c}) - m\tau(n + 2)]}{2(n + 1)(m + n + 1)}. \tag{40} \]

The level of trade cost which prohibits trade in the linear case can be solved from (40). Trade costs are prohibitive in the linear model if and only if:

\[ \tau \geq \tilde{\tau} \equiv \frac{(a - \bar{c})(m + 2n + 2)}{m(n + 2) + 2(n + 1)}. \tag{41} \]

The profits of a domestic downstream firm can be found by substituting (38) into (36), and using the resulting expression in (1). This yields:

\[ \pi_h^* = \left(\frac{a - \bar{c} + n\tau}{2(m + n + 1)}\right)^2, \tag{42} \]

where we use an asterisk to indicate equilibrium values. Differentiating with respect to \( n \) yields:

\[ \frac{d\pi_h^*}{dn} = \frac{[(a - \bar{c}) + n\tau][\tau(m + 1) - (a - \bar{c})]}{2(m + n + 1)^3}, \tag{43} \]

which is monotonically increasing in \( \tau \). This expression allows us to determine a critical level of trade costs above which the profits of the downstream firms increase in the number of foreign entrants. In the case of a linear inverse demand curve, thus, profits of downstream firms increase in the number of entrants if and only if:

\[ \tau > \hat{\tau} \equiv \frac{a - \bar{c}}{m + 1}. \tag{44} \]

For any value of trade costs below this threshold, the profit-reducing effect on product market competition dominates the profit-enhancing effect of input price moderation whereas the reverse is true for any value above it. For this to be consistent with values of \( \tau \) for which trade costs are not prohibitively high, we require that \( \tilde{\tau} < \tau < \hat{\tau} \). From comparison of (41) and (44), the condition that \( \tilde{\tau} > \hat{\tau} \) is given by:
As the positivity condition is satisfied, it follows that there are always ranges of non-prohibitive trade costs for which profits-per-firm are increasing in the number of firms, a finding which is consistent with Proposition 2. This proposition says that downstream firms’ profits are rising in the entry of foreign rivals when trade costs are high and (30) is satisfied. Since in the case of a linear demand function, $\varphi = 0$, this is satisfied.

We can illustrate a specific situation of profit-raising entry by comparing the profits obtained by one home firm when it has no foreign competition in the downstream market with that

$$\hat{\tau} - \bar{\tau} > 0 \Rightarrow (a - \bar{c}) m [m + n + 1] > 0. \quad (45)$$

**Figure 1** Illustration of profits for different scenarios of competitors and inverse demand functions.
obtained with one foreign rival. Evaluating the home firm’s profits in (42) for \( m = 5 \) at \( n = 0 \) and \( n = 1 \), respectively, we obtain:

\[
\begin{align*}
\pi_h|_{n=0} & = \left( \frac{a-c}{12} \right)^2; \\
\pi_h|_{n=1} & = \left( \frac{a-c+c}{14} \right)^2.
\end{align*}
\]

In Panel (i) of Figure 1, we plot these functions for specific parameter values, and the range of trade costs for which the domestic downstream firms benefit from a foreign competitor is illustrated as the range \( \tau \in [0.5; 1.42) \).

### 4.1.2 | Constant elasticity of demand function

Following steps similar to those for the linear demand function, we can analyse the profits of domestic downstream firms for different numbers of foreign competitors using other types of demand function. The constant-elasticity demand function takes the form,

\[
p = \frac{1}{Q^\sigma},
\]

where \( \sigma > 0 \) is the price elasticity of demand. This function is particularly convenient because, similar to its linear counterpart, it allows for the derivation of explicit closed-form expressions for profits as functions of trade costs, thus enabling us to characterise the full range of trade costs for which profit-raising entry occurs. With a constant price elasticity, the elasticity of slope of inverse demand, defined in equation (8), is also a constant, \( \varphi = - (\sigma + 1) \). In Appendix D, we derive the full expressions for outputs (Equations A27 and A28), the input price (Equation A29) and the profits of domestic downstream firms (A31), where for convenience, we assume \( \sigma = 1 \). We also derive the level of trade costs which prohibit trade, \( \hat{\tau} \), and the level of trade cost, \( \bar{\tau} \), above which the profits of downstream firms increase in foreign entry (Equations A30 and A33), which we reproduce here,

\[
\hat{\tau} \equiv \frac{(m+2n)c}{n(m-2)}; \quad \text{and} \quad \bar{\tau} \equiv \frac{c}{m-1},
\]

noting that these thresholds refer to the case where the numbers of domestic and foreign firms, respectively, \( m \) and \( n \), are treated as continuous variables. In Panel (ii) of Figure 1, we illustrate the profits of the domestic downstream firms for the case of \( n = 1 \) and \( n = 0 \), respectively, clearly indicating the range of trade costs for which the downstream domestic firms benefit from having one more
foreign firm in the market. We note that for the assumed parameter values, that is \( m = 5 \), outputs are strategic substitutes, \( \phi |_{\tau = \bar{\tau}} = -2 < m = 5 \) at the prohibitive level of trade cost. As such, the upstream agent moderates its price in response to entry leading the downstream firm to earn higher profits when trade costs exceed \( \bar{\tau} = \frac{\bar{c}}{m-1} \).

4.1.3 | Other demand functions

We conduct a similar analysis for another two demand functions, respectively, \( p = \frac{1}{\sqrt{Q-b}} \) and \( p = \frac{1}{Q^2-b} \), in Panel (iii) and (iv) in Figure 1. Neither of these demand functions permits analytical closed-form solutions and so we use numerical methods to plot the diagrams. Noting first for the function in Panel (iii), we analyse the difference in the profits of a domestic downstream firm for the case where it faces, respectively, one and two competitors. Using numerical simulations, we find that the level of trade cost, \( \bar{\tau} \), above which it is more profitable for an incumbent firm to compete against two competitors \( \left( \tau_{n=2} \right) \) rather than one \( \left( \tau_{n=1} \right) \) is 3.16, and trade is prohibitive whenever trade costs exceed \( \bar{\tau}_{n=2} = 4.4 \) (To be precise, this is the level above which the second foreign firm would exit). For the demand function in Panel (iv), we compute the specific thresholds for trade costs as \( \bar{\tau} = 0.29 \) and \( \bar{\tau} = 0.81 \), respectively. We note in both cases that evaluating the elasticity of slope of inverse demand at the respective prohibitive levels of trade costs for the two demand functions that the condition in (28) is negative. Hence, at high trade costs, outputs are strategic complements, and the upstream agent therefore raises its price in response to foreign entry. The downstream firms benefit from foreign entry, however, since the higher marginal cost is passed on to the consumer.

4.2 | The effects of entry on the profits of the upstream agent

We can similarly analyse the effects on the profits of the upstream agent in the home country. The profits of the upstream agent are given by (12). Differentiating this with respect to \( n \) yields:

\[
\frac{d\Pi'_h}{dn} = \left( Q_h + (c_h - \bar{c}) \frac{\partial Q_h}{\partial c_h} \right) \frac{\partial c_h}{dn} + (c_h - \bar{c}) \frac{\partial Q_h}{dn} = (c_h - \bar{c}) \frac{\partial Q_h}{dn}
\]

(50)

+ \left[ \frac{\left( c_h - \bar{c} \right) q_f \left[ p''Q_h + mp' \right]}{\left[ p''Q + p' \left( m + n + 1 \right) \right]} - \frac{\left( c_h - \bar{c} \right) q_f \left[ \phi s_h + m \right]}{\left[ \phi + \left( m + n + 1 \right) \right]} \right],
\]

(51)

where the second equality uses the first-order condition for the agent’s optimisation problem, (13), and the third equality uses the expression for \( \frac{\partial Q_h}{dn} \) in (A12), and the fourth equality divides the numerator and the denominator, respectively, by \( p' \) and then uses the definition of the elasticity of the slope of the inverse demand function in (8). Since the denominator of this expression is positive due to the stability condition in (9), the sign of (51) is determined by the term in square brackets in the numerator. We can thus establish the following proposition.

**Proposition 3** The profits of the upstream agent increase in the number of foreign downstream firms if and only if,
This implies that if the downstream market is characterised by strategic complementarity, the upstream agent also benefits from entry of downstream rivals. It follows that there exist parameter values of trade costs and values of $\phi$ for which there are potentially conflicting interests within the vertical market with downstream firms favouring entry by foreign rivals and upstream firms having protectionist motives. This would be the case for any linear or concave demand function or any demand function which is not ‘too convex’. For more convex demand functions, the reverse may be true.

4.3 The impact of home bias

In this subsection, we consider the impact of a bias towards domestically produced goods, but in order to obtain analytically tractable results, we restrict attention to the linear model. These results do not generalise to every possible type of demand curvature. We thus augment the linear model by allowing for the possibility that consumers are biased towards domestically produced products. In particular, we assume that home firms in the downstream market face the following inverse demand function $p_h = B + p(Q)$, whereas that for foreign firms continues to be $p(Q)$. Following steps similar to (35) through (42), we can write the equilibrium profits of a domestic downstream firm as follows:

$$\pi^*_h = \frac{(B + \tau)n + B + a - \bar{c})^2}{4(m + n + 1)^2}. \quad (53)$$

The prohibitive level of trade cost is now:

$$\tau \geq \bar{\tau} \equiv \frac{m(a - \bar{c}) - mB(n + 1) + 2(n + 1)(a - \bar{c})}{m(n + 2) + 2(n + 1)}. \quad (54)$$

And the level of trade costs above which there is profit-raising entry is:

$$\tau > \bar{\tau} \equiv \frac{a - \bar{c} - mB}{m + 1}. \quad (55)$$

In Panel (i) of Figure 2, we illustrate how the profits of each domestic firm in (53) vary with the level of home bias. As is already clear from (53), equilibrium profit is a monotonically increasing function of the level of home bias $B$. In Figure 2, we note that the profits of the home firm when $B = 1.1$ is strictly greater than its profits when $B = 0$ for $n = 1$ and for $n = 0$, respectively. Moreover, we note that when $B = 1.1$, the profits of each domestic downstream firm when there is one foreign competitor in the market $\pi_h|_{n=1,B=1.1}$, exceed the profits obtained without foreign competition $\pi_h|_{n=1,B=1.1}$ for every level of trade cost, $\tau$.

4.4 The impact of the number of firms in the domestic industry

Similar to the previous subsection, we favour analytically tractable results and therefore restrict attention to the linear model. Taking the derivative of $\bar{\tau}$ in equation (44) with respect to $m$, it
follows that an increase in the number of downstream domestic firms lowers the cut-off level of trade costs for which there is profit-raising entry:

\[
\frac{d\tilde{\tau}}{m} = -\frac{(a - \bar{c})}{(m+1)^2} < 0.
\]

The implication of this is that our finding is not limited to industries with a small number of firms. While this is true, the value of \(m\) must be small enough to ensure strategic interaction since our result would not hold in an environment of price-taking behaviour where price is equal to marginal cost. In a similar vein, our result would not hold under price competition à la Bertrand where price is equal to marginal cost. As such, we believe our story of profit-raising entry is suitable to a setting in which production is concentrated with a small group of firms, which interact strategically in a quantity-setting fashion.

In Panel (ii) of Figure 2, we have plotted the profits of each downstream firm in (42) for different values of \(m\) and \(n\). We note that the threshold of trade cost above which there is profit-raising entry is lower when \(m = 10\) compared to when \(m = 5\).

### 4.5 The effects of trade liberalisation

In the classic model of Protection for Sale (Grossman & Helpman, 1994), import-competing firms pay campaign contributions to their government in exchange for a protectionist import tariff. Our finding that domestic downstream firms may have incentives to accommodate foreign entry is likely

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14 Notice from (41) that the prohibitive level of trade costs is also decreasing in \(m\). As such, the range of non-prohibitive trade costs for which profits may increase in foreign entry may not be larger when there is a greater number of domestic firms. Simple algebra, however, shows that for smaller values of both \(m\) and \(n\), this range increases in \(m\).

15 Our results can be extended to the Bertrand case by introducing product differentiation. It is possible to show that profit-raising entry may also occur in such a setting.
to have implications for their preferences regarding trade policy. In this subsection, we investigate whether domestic firms have incentives to lobby the government for less restrictive tariff barriers. In the interest of obtaining tangible results which allow for transparent economic interpretation, we restrict our analysis to the linear model. Previously, we noted that \( \tau \) can be interpreted as representing any element of the costs associated with trade, including any import tariff. Accordingly, in this section of the paper, we model changes in tariffs as operating through \( \tau \). We first show that for a given number of foreign firms, domestic firms continue to have the Grossman–Helpman incentive to lobby for tariffs to shield them from foreign competition. To see this, we differentiate the equilibrium profits of a domestic firm for the linear case in (42) with respect to the trade cost \( \tau \):

\[
\frac{d\pi^*_h}{d\tau} = \frac{n [a - \bar{c} + n\tau]}{2(m + n + 1)^2} > 0, \tag{56}
\]

which is monotonically increasing in \( \tau \). Hence, with a fixed number of firms, any increase in the import tariff will benefit domestic firms. An increase in the import tariff, however, has a similar moderating effect on input prices/wages as does entry. To see this, we differentiate the input price in (38) with respect to \( \tau \). We obtain:

\[
\frac{dc^*_h}{d\tau} = \frac{n}{2(n + 1)} > 0. \tag{57}
\]

This implies that as \( \tau \) decreases, the downstream firms’ input prices are moderated which by itself is a profit-enhancing effect. With import tariffs, however, the downward pressure on the final import price from the lowering of the tariff outweighs any benefits from a lower import price.

However, we next show that allowing the reduction in import tariffs to have an effect on the extensive margin of trade leads to a business incentive to favour a reduction in the import tariff. This is because if the lowering of tariffs induces entry, the benefits described in this section regarding the potential for entry to raise domestic firms’ profits can be realised. Consider the case where there is one domestic incumbent and one potential entrant, respectively, producing outputs \( q_h \) and \( q_f \). The entrant is unable to enter if the tariff exceeds the prohibitive level defined in (41). We choose parameter values, \( a = 3 \) and \( \bar{c} = 1 \), and compute the relevant profits of the incumbent under, respectively, domestic monopoly and international duopoly, using (42) as:

\[
\pi_h = \begin{cases} 
\pi_h|_{n=0} = \frac{1}{4} \frac{(\tau + 2)^2}{36} & \text{if } \tau \leq \bar{\tau} \\
\pi_h|_{n=1} = \frac{1}{4} \frac{(\tau + 2)^2}{36} & \text{if } \tau > \bar{\tau}
\end{cases} \tag{58}
\]

We depict the incumbent’s profit as a function of \( \tau \) in Figure 3. For high values of \( \tau \), the incumbent operates as a monopoly in the domestic market. As \( \tau \) falls below \( \bar{\tau} \), the foreign entrant enters the market resulting in a discontinuous jump in profits which comes about due to input price moderation. As such, the profit-enhancing effect we have identified in this section resulting from entry dominates the profit-reducing effect caused by the decrease in the tariff. The incumbent would favour a decrease in tariff barriers for particular values of \( \tau \).

It follows from our analysis that the reduction or removal of several types of non-tariff barriers, such as bureaucratic ‘red tape’, regulatory barriers or import licences, may similarly raise the profits of domestic firms provided they affect trade primarily at the extensive margin. That is, if
the removal of non-tariff barriers facilitates entry into the home market, then depending on the parameters of the model, it is possible that home firms would see it in their own self-interest to lobby for their removal.

5 | CONCLUSIONS

In this paper, we consider the potential impacts on profits of domestic downstream firms in vertical markets when confronted by foreign entry into the home market. We assume that foreign firms incur low and exogenous production costs but face additional—and exogenous—import costs, in the form of transportation or tariff costs, for example.

In our general model which accommodates any demand curvature, we have established that the profits of incumbent domestic downstream firms can be increasing with the entry of foreign rivals, a result which arises if trade costs are sufficiently high. For any concave and linear demand function, and for any demand function which is not ‘too convex’, the profit-raising result arises due to the upstream agent moderating the input price. We have also demonstrated that there may also be profit-raising entry for highly convex demand functions but this occurs through a different mechanism: when trade costs are sufficiently high, and the inverse demand function is sufficiently convex, the entry of foreign rivals leads the upstream agent to raise the input price. The higher final output price which domestic firms can charge for the output following entry outweighs the profit-reducing effect stemming from the higher marginal cost. We also show conditions under which the upstream agent benefits from foreign entry in the downstream market. The range of conditions on trade costs and on demand curvature consistent with profit-raising
entry in the downstream market does not always coincide with that in the upstream market, implying potentially conflicting interests: upstream agents will have the potential to benefit from downstream entry only for highly convex inverse demand functions.

The general findings of our paper on the relationship between the profits of domestic firms and foreign entry are not the result of special assumptions or a narrow range of parameter values. Rather, as we have shown, profit-raising entry occurs for a wide range of assumptions on the curvature of demand and trade costs. As our numerical examples demonstrated, profit-raising entry occurs for all linear and concave demand functions and for a large range of trade costs. Moreover, profit-raising entry occurs for a wide range of convex demand functions as well.

Overall, our results counter the standard notion in the literature on Protection for Sale (Grossman & Helpman, 1994), in which domestic import-competing firms have an unambiguous incentive to deter rather than accommodate foreign trade. In the version of our model with linear demand, we have examined how the profits of domestic downstream firms vary with the level of import tariff. We find that for a fixed number of firms, a decrease in the import tariff unambiguously harms the domestic firms in the linear model. However, if the reduction in the import tariff induces profit-enhancing entry, then domestic firms can benefit. We have demonstrated that for a range of tariffs, it is possible that the profit-enhancing extensive margin benefit of a reduction in the tariff (associated with entry) dominates the profit-reducing loss due to greater intensive-margin trade. As such, if the reduction or the removal of a trade policy instrument liberalises trade at the extensive margin, it is possible it will not be resisted by downstream firms when trade costs are above a threshold.

The existence of the profit-enhancing effect can help to explain why downstream firms in highly concentrated (oligopolistic) markets might be less likely to lobby for import protection (for tariff and non-tariff barriers) than might otherwise be expected. This suggests that empirical attempts to identify a relationship between market concentration and lobbying for protection should take into account the nature of the vertical relationships in oligopolistic markets.

ORCID
Christian Soegaard https://orcid.org/0000-0002-8250-6050

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APPENDIX A
In this appendix, we analyse the effects of entry on downstream home firms using the assumption of a single centralised upstream agent serving both home and foreign firms. We restrict attention to the linear model in the interest of achieving tangible results. The Cournot outputs of the domestic and foreign downstream firms, respectively, continue to be determined by Equations (36) and (37) with the assumption that the marginal cost incurred by foreign firms is now \( c = \tilde{c}_f = c_h = c \), the input price determined by contracting with the upstream agent. Thus, Equations (36) and (37) now become:

\[
\begin{align*}
mq_h &= Q_h = \frac{m[a - c + n\tau]}{m + n + 1}, \\
nq_f &= Q_f = \frac{n[a - c - (m + 1)\tau]}{m + n + 1}.
\end{align*}
\]

The profits of the upstream input supplier is now:

\[
\Pi^I = (c - \tilde{c}) Q^C.
\]

Using \( Q = Q_h + Q_f \) from (A1) and (A2) in (A3), then maximising with respect to the choice of \( c \) yields the following solution for the single input price:

\[
c^* = \frac{(a + \tilde{c})(m + n) - n\tau}{2(m + n)}.
\]

We can now solve for equilibrium output and profit of a domestic downstream firm by plugging (A4) into (A1) and using the expression for profits in (1):

\[
\begin{align*}
\frac{Q_h^*}{m} &= q_h^* = \frac{(a - \tilde{c})(m + n) + n\tau(2(m + n) + 1)}{2(m + n + 1)(m + n)}; \\
\pi_h^* &= \frac{[(a - \tilde{c})(m + n) + n\tau(2(m + n) + 1)]^2}{4(m + n + 1)^2(m + n)^2}.
\end{align*}
\]

Differentiating (A6) with respect to \( n \) and solving for \( \tau \) gives us the threshold of trade cost above which there is profit-raising entry as:

\[
\tilde{\tau}^S \equiv \frac{(a - \tilde{c})(m + n)^2}{2m^3 + (4n + 3)m^2 + (2n^2 + 4n + 1) m + n^2}.
\]

Foreign equilibrium output can be solved by plugging (A4) into (A2):

\[
\begin{align*}
\frac{Q_f^*}{n} &= q_f^* = \frac{(a - \tilde{c})(m + n) - \tau(2(m + 1)(m + n) - n)}{2(m + n + 1)(m + n)}.
\end{align*}
\]
This allows us to solve for the prohibitive level of the tariff as:

$$
\tau_S^* = \frac{(a - \bar{c})(m + n)}{2m(m + n) + 2m + n}.
$$

(A9)

It is easy to show that $\tau_S^* > \bar{\tau}_S$ since:

$$
\tau_S^* - \bar{\tau}_S \equiv \frac{(a - \bar{c}) m (m + n) (m + n + 1)}{(2m(m + n) + 2m + n) \left(2m^3 + (4n + 3)m^2 + (2n^2 + 4n + 1)m + n^2\right)} > 0.
$$

(A10)

**APPENDIX B**

The effect of the number of firms $n$ on the total output in the home market, $Q$, can be obtained by implicit differentiation of (11). This yields:

$$
\frac{dQ}{dn} = -\frac{(p - \bar{c} - \tau)}{[p''Q + p'(m + n + 1)]} = \frac{p'q_f}{[p''Q + p'(m + n + 1)]} > 0,
$$

where the second equality follows from the first-order condition in (4). Equation (A11) obtains its sign from the stability condition (7). We can write up an expression for the effect of $n$ on the output of a home firm $q_{hi} = q_h$ using (3) and (A11), we have,

$$
\frac{dq_{h}}{dn} = \frac{dQ_{h}}{dn} = \frac{(p - \bar{c} - \tau) [p''Q_{h} + mp']}{p' [p''Q + p'(m + n + 1)]} = \frac{-q_f [p''Q_{h} + mp']}{[p''Q + p'(m + n + 1)]},
$$

(A12)

noting that, $m\frac{dq_{h}}{dn} = \frac{dQ_{h}}{dn}$. The second equality follows from the first-order condition (4). The effect of $n$ on the profits of an incumbent home firm can be found by evaluating (1) at the Cournot-Nash solution and differentiating with respect to $n$. Using (3), (A11) and (A12), we get:

$$
\frac{d\pi_{h}}{dn} = -\frac{q_h (p - \bar{c} - \tau) [p''q_h + 2p']}{[p''Q + p'(m + n + 1)]} < 0,
$$

(A13)

which is negative due to the second-order condition in (5) and the stability condition (7). Using (11), we derive an expression for the effect on total output of a change in input price $c_h$ as:

$$
\frac{dQ}{dc_h} = \frac{m}{[p''Q + p'(m + n + 1)]} < 0,
$$

(A14)

which is negative due to the stability condition in (7). Using (3) and (A14), we can solve for the effect of the input price on the output of an incumbent home firm:

$$
\frac{dq_{h}}{dc_h} = \frac{dQ_{h}}{dc_h} = \frac{mp'' (Q - Q_{h}) + p' (n + 1)}{p' [p''Q + p'(m + n + 1)]}.
$$

(A15)

We can now write an expression for the effects on the profits of the incumbent home firms of a change in the input price, $c_h$, using (3), (A14) and (A15):
\[ \frac{d\pi_h}{dc_h} = -\frac{q_h \left( (2Q - Q_h) p'' + 2p' (n + 1) \right)}{p''Q + p' (m + n + 1)}. \]  

(A16)

**APPENDIX C**

Proof of Proposition 1. The effect of the number of foreign firms on the input price is determined through equation (21), which is reproduced here as:

\[ \frac{dc^*_h}{dn} = \frac{\partial c^*_h}{\partial Q} \frac{\partial Q}{\partial n} + \frac{\partial c^*_h}{\partial Q_h} \frac{\partial Q_h}{\partial n} + \frac{\partial c^*_h}{\partial n} = X (n, \tau) + \frac{\partial c^*_h}{\partial n}, \]  

(A17)

where,

\[ X (n, \tau) \equiv \frac{\partial c^*_h}{\partial Q} \frac{\partial Q}{\partial n} + \frac{\partial c^*_h}{\partial Q_h} \frac{\partial Q_h}{\partial n}. \]  

(A18)

Assume without loss of generality that \( X (n, \tau) \) is continuous in, respectively, \( n \) and \( \tau \), and thrice continuously differentiable in these arguments. We know from (25) and (26) that \( X (n, \tau) \) converges to zero in the limit as \( \tau \to \hat{\tau} \), that is,

\[ \lim_{\tau \to \hat{\tau}} X (n, \tau) = 0. \]  

(A19)

Suppose \( \tau' \) is a particular value of \( \tau \) strictly less than the prohibitive level, such that \( \tau' \in [0; \hat{\tau}) \). We have,

\[ \frac{dc^*_h}{dn} \bigg|_{\tau = \tau'} = X (n, \tau') + \frac{\partial c^*_h}{\partial n} \bigg|_{\tau = \tau'} = X (n, \tau') - \left( \frac{p' q_h \left[ \phi s_h + m \right]}{\phi (1 - s_h) + n + 1} \right)^2, \]  

(A20)

where the last equality uses (24). If \( \tau' = \hat{\tau} \), we have,

\[ \lim_{\tau \to \hat{\tau}} \frac{dc^*_h}{dn} = -\left( \frac{p' q_h \left[ \phi s_h + m \right]}{\phi (1 - s_h) + n + 1} \right)^2. \]  

(A21)

As such, a continuity argument establishes that \( \frac{dc^*_h}{dn} \bigg|_{\tau = \tau'} \) inherits the sign of \( \frac{dc^*_h}{dn} \bigg|_{\tau = \hat{\tau}} \) provided \( X (n, \tau) \) is sufficiently small, which is the case when the difference between \( \tau' \) and \( \hat{\tau} \) is sufficiently small.

Proof of Proposition 2. The effect of the number of foreign firms on the input price is determined through equation (16), which is reproduced here as:

\[ \frac{d\pi_c}{dn} = \frac{\partial \pi_c}{\partial n} + \frac{\partial \pi_c}{\partial c_h} \frac{dc_h}{dn}. \]  

(A22)

Using (21), we have:

\[ \frac{d\pi_c}{dn} = \frac{\partial \pi_c}{\partial n} + \frac{\partial \pi_c}{\partial c_h} \left( \frac{\partial c^*_h}{\partial Q} \frac{\partial Q}{\partial n} + \frac{\partial c^*_h}{\partial Q_h} \frac{\partial Q_h}{\partial n} + \frac{\partial c^*_h}{\partial n} \right) = Y (n, \tau) + \frac{\partial \pi_c}{\partial c_h} \frac{dc_h}{dn}, \]  

(A23)
where,

\[ Y(n, \tau) = \frac{\partial \pi^C_h}{\partial n} + \frac{\partial \pi^C_h}{\partial c_h} X(n, \tau), \]

where \( X(n, \tau) \) is defined in (A18). It is assumed that \( Y(n, \tau) \) is a continuous function of each of its arguments and thrice continuously differentiable. It is easy to see from (A13) that,

\[ \lim_{\tau \to \hat{\tau}} \frac{d\pi^C_h}{dn} = 0. \quad (A24) \]

Using this and (A19), we have:

\[ \lim_{\tau \to \hat{\tau}} \frac{d\pi^C_h}{dn} = \frac{\partial \pi^C_h}{\partial c_h} \frac{\partial c^*_h}{\partial n} = - \left( \frac{q_h [\varphi (2 - s_h) + (n + 1)]}{[\varphi + m + n + 1]} \right) \left( \frac{p' q_h [\varphi s_h + m]}{[\varphi (1 - s_h) + n + 1]^2} \right), \quad (A25) \]

where the second equality uses (19) and (24). The sign of this expression is determined by the sign of (30). Suppose \( \tau' \) is a particular value of \( \tau \) strictly less than the prohibitive level, such that \( \tau' \in [0; \hat{\tau}) \). We have,

\[ \frac{d\pi^C_h}{dn} \bigg|_{\tau = \tau'} = Y(n, \tau') + \left( \frac{\partial \pi^C_h}{\partial c_h} \frac{\partial c^*_h}{\partial n} \right) \bigg|_{\tau = \tau'}. \quad (A26) \]

It follows from a continuity argument that \( \frac{d\pi^C_h}{dn} \bigg|_{\tau = \tau'} \) inherits the sign of \( \left( \frac{\partial \pi^C_h}{\partial c_h} \frac{\partial c^*_h}{\partial n} \right) \bigg|_{\tau = \tau'} \) provided that \( Y(n, \tau') \) is sufficiently small, which is the case when the difference between \( \tau' \) and \( \hat{\tau} \) is sufficiently small.

**APPENDIX D**

We use the constant-elasticity inverse demand function (48) in the first-order conditions in (3) and (4), respectively, to determine the Cournot outputs as follows:

\[ mq^C_h = Q^C_h = \frac{m [m + n - 1] [n (\bar{c} + \tau) - (n - 1) c_h]}{[n (\bar{c} + \tau) + mc_h]^2}; \quad (A27) \]

\[ nq^C_f = Q^C_f = \frac{n [m + n - 1] [mc_h - (m - 1)(\bar{c} + \tau)]}{[n (\bar{c} + \tau) + mc_h]^2}. \quad (A28) \]

The input price is obtained using (A27) and (13):

\[ c^*_h = \frac{(\bar{c} + \tau) n [(2 (m + n) - 1) \bar{c} - \tau n]}{2n^2 (\bar{c} + \tau) + n [2 (m - 1) \bar{c} + (m - 2) \tau] - mc}. \quad (A29) \]
Substituting (A29) into (A28), the level of trade cost which prohibits trade in the linear case can be solved. Trade costs are prohibitive if and only if:

\[ \tau \geq \hat{\tau} \equiv \frac{(m + 2n)\bar{c}}{n(m - 2)}. \tag{A30} \]

The profits of a domestic downstream firm can be found by substituting (A29) into (A27), and using the resulting expression in (1). This yields:

\[ \pi_h^* = \frac{(\bar{c} + n\tau)^2}{4[(m + n)\bar{c} + n\tau]^2}, \tag{A31} \]

where we use an asterisk to indicate equilibrium values. Differentiating with respect to \( n \) yields:

\[ \frac{d\pi_h^*}{dn} = \frac{(n\tau + \bar{c})[(m - 1)\tau - \bar{c}]\bar{c}}{2[(m + n)\bar{c} + n\tau]^3}. \tag{A32} \]

We can now determine the critical level of trade costs above which the profits of the downstream firms increase in the number of foreign entrants. In the case of a constant-elasticity inverse demand curve, profits of downstream firms increase in the number of entrants if and only if:

\[ \tau > \hat{\tau} \equiv \frac{\bar{c}}{m - 1}. \tag{A33} \]