Manuscript version: Author’s Accepted Manuscript
The version presented in WRAP is the author’s accepted manuscript and may differ from the published version or Version of Record.

Persistent WRAP URL:
http://wrap.warwick.ac.uk/166620

How to cite:
Please refer to published version for the most recent bibliographic citation information. If a published version is known of, the repository item page linked to above, will contain details on accessing it.

Copyright and reuse:
The Warwick Research Archive Portal (WRAP) makes this work by researchers of the University of Warwick available open access under the following conditions.

Copyright © and all moral rights to the version of the paper presented here belong to the individual author(s) and/or other copyright owners. To the extent reasonable and practicable the material made available in WRAP has been checked for eligibility before being made available.

Copies of full items can be used for personal research or study, educational, or not-for-profit purposes without prior permission or charge. Provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way.

Publisher’s statement:
Please refer to the repository item page, publisher’s statement section, for further information.

For more information, please contact the WRAP Team at: wrap@warwick.ac.uk.
First observation and interpretation of spontaneous collective radiation from fusion-born ions in a stellarator plasma

To cite this article before publication: Bernard C G Reman et al 2022 Plasma Phys. Control. Fusion in press https://doi.org/10.1088/1361-6587/ac7892

Manuscript version: Accepted Manuscript

Accepted Manuscript is “the version of the article accepted for publication including all changes made as a result of the peer review process, and which may also include the addition to the article by IOP Publishing of a header, an article ID, a cover sheet and/or an 'Accepted Manuscript' watermark, but excluding any other editing, typesetting or other changes made by IOP Publishing and/or its licensors”

This Accepted Manuscript is © 2022 The Author(s). Published by IOP Publishing Ltd.

As the Version of Record of this article is going to be / has been published on a gold open access basis under a CC BY 3.0 licence, this Accepted Manuscript is available for reuse under a CC BY 3.0 licence immediately.

Everyone is permitted to use all or part of the original content in this article, provided that they adhere to all the terms of the licence https://creativecommons.org/licences/by/3.0

Although reasonable endeavours have been taken to obtain all necessary permissions from third parties to include their copyrighted content within this article, their full citation and copyright line may not be present in this Accepted Manuscript version. Before using any content from this article, please refer to the Version of Record on IOPscience once published for full citation and copyright details, as permissions may be required. All third party content is fully copyright protected and is not published on a gold open access basis under a CC BY licence, unless that is specifically stated in the figure caption in the Version of Record.

View the article online for updates and enhancements.
First observation and interpretation of spontaneous collective radiation from fusion-born ions in a stellarator plasma


$^1$Centre for Fusion, Space and Astrophysics, Department of Physics, Warwick University, Coventry CV4 7AL, UK

$^2$CCFE, Culham Science Centre, Abingdon, Oxfordshire OX14 3DB, UK

$^3$National Institute for Fusion Science, Toki, Gifu 509-5292, Japan

$^4$Department of Physics, Technical University of Denmark, Fysikvej, Building 309, 2800 Kongens Lyngby, Denmark

$^5$Research Institute for Applied Mechanics, Kyushu University, Kasuga,
Fukuoka, 816-8580, Japan

Department of Physics, Pohang University of Science and Technology,
Pohang, Gyeongbuk 37673, Korea

National Fusion Research Institute, Daejeon 34133, Republic of Korea

Abstract

During bursty MHD events, transient ion cyclotron emission (ICE) is observed from deuterium plasmas in the Large Helical Device (LHD) heliotron-stellarator. Unusually, the frequencies of the successive ICE spectral peaks are not close to integer multiples of the local cyclotron frequency of an energetic ion population in the likely emitting region. We show that this ICE is probably driven by a subset of the fusion-born protons near their birth energy $E_H = 3.02$ MeV. This subset has a kinetic energy component parallel to the magnetic field, $m_H v_{\parallel}^2/2$, significantly greater than its perpendicular energy $m_H v_{\perp}^2/2$, for which $v_{\perp} \sim V_A$, the Alfvén speed. First principles computations of the collective relaxation of this proton population, within a majority thermal deuterium plasma, are carried out using a particle-in-cell approach. This captures the full gyro-orbit kinetics of all ions which, together with an electron fluid, evolve self-consistently with the electric and magnetic fields under the Maxwell-Lorentz equations. The simulated ICE

*Now at Laboratoire Plasma et Conversion d’Energie, Université Paul-Sabatier Toulouse III, France
†Now at General Atomics, San Diego, CA, United States of America
spectra are derived from the Fourier transform of the fields which are excited. We find substantial frequency shifts in the peaks of the simulated ICE spectra, which correspond closely to the measured ICE spectra following the resonance condition \( \omega = k_{\parallel} v_{\parallel} + n\Omega_H \) for \( n \)th proton harmonic. This suggests that the transient ICE in LHD is generated by the identified subset of the fusion-born protons, relaxing under the magnetoacoustic cyclotron instability. So far as is known, this is the first report of a collective radiation signal from fusion-born ions in a non-tokamak magnetically confined plasma. Disambiguation between two or more energetic ion species that could potentially generate complex observed ICE spectra is an increasing challenge, and the results and methodology developed here will assist this. Our approach is also expected to be relevant to ICE driven by ion beams with lower parallel velocities, for example in cylindrical plasma experiments.

1 Introduction

The initial deuterium plasma campaign [1, 2, 3, 4, 5, 6, 7, 8] on the Large Helical Device (LHD) heliotron-stellarator has provided interesting new opportunities to study the fundamental physics of ion cyclotron emission (ICE) [9, 10]. ICE was detected both during perpendicular deuterium neutral beam injection (NBI) [11, 12, 13, 14, 15] and during transient events. In this paper, we focus on the ICE that arises during transient events [16, 17, 18, 19, 20, 21, 22] which may be caused by the helically trapped energetic-ion-driven resistive interchange MHD mode (EIC), characterised by the mode
1 INTRODUCTION

numbers $m = 1$ and $n = 1$ (poloidal and toroidal respectively) [16, 17, 18, 19]. The abrupt onset of an associated tongue-shaped magnetic surface deformation has been reported in LHD [21, 22, 23]. Contemporaneously, brief intense RF signals are detected in the hundreds of megahertz range. These signals exhibit successive ICE spectral peaks which have a typical frequency spacing of 20MHz to 25MHz, comparable to the proton cyclotron frequency. Importantly, these peaks are not located close to integer cyclotron harmonics. This is in contrast to ICE spectra from LHD where the spectral peaks are very close to successive ion cyclotron harmonics, driven by hydrogen (deuterium) NBI in hydrogen (deuterium) plasmas, see Refs. [11, 12, 13, 14, 15].

In this paper we argue that the ultimate origin of the transient ICE with spectral peaks highly shifted with respect to cyclotron harmonics lies in the fusion reactions within deuterium plasmas in LHD, which generate protons with birth energy $E_H = 3.02$ MeV. We propose here that a subset of these protons, with super-Alfvénic parallel velocities, are responsible for the transient ICE signal (shown, for example, in Figs. 1 and 2) that is observed during the bursting events outlined above. We arrive at this conclusion notwithstanding that the relative concentration of energetic fusion-born ions $n_{\text{energ}}/n_e$ is very low, less than $10^{-7}$. As we now briefly review, there are strong experimental precedents for the generation and detection of ICE from fusion-born ions at very low concentrations, comparable to those in LHD deuterium plasmas.

The first measurements of ICE from fusion-born protons in a deuterium plasma were from: JET Ohmically heated tokamak plasmas with currents as low as 1 MA, see the
1 INTRODUCTION

Figure 1: ICE phenomenology measured during the bursty MHD event in LHD deuterium plasma 133979. (Top) Time series of the electric field fluctuations (V). (Bottom) Corresponding windowed Fourier transform showing the time evolution at higher resolution of the frequency content in the hundreds of MHz.
Figure 2: ICE power spectra from LHD plasma 133979 before (top panel) and during (bottom) the bursty MHD event. The spectrum at the time of the bursty event exhibits three intense peaks b, c, and d, together with peak a, and there is additional activity above 300MHz. The peaks b, c, d are located between 250.04MHz and 255.36MHz, between 276.64MHz and 281.96MHz, and between 303.24MHz and 308.56MHz. The shifted harmonics of 26.6 \((n - 0.6)\) are indicated with blue dashed lines for \(n = 10, 11, 12\) and shifted harmonics of 26.6 \((n + 0.6)\) are indicated with red dashed lines for \(n = 9, 10, 11\) respectively; peak a is at 32.73MHz.
1 INTRODUCTION

Figure 3: Measured power spectra in LHD plasma 133979 taken at several successive times between \( t = 4.4430 \) s and \( t = 4.4447 \) in the magnetic configuration \((R_{ax}, B_{toroidal}) = (3.6m,2.75T)\). The panel at \( t = 4.4430 \) corresponds to the spectrum before the burst occurrence. The blue dashed vertical lines correspond to \( n = 26.6(n + 0.6) \) MHz, \( n = 8,9,10,11 \). The red dashed vertical lines correspond to \( n = 26.6(n - 0.6) \) MHz, \( n = 9,10,11,12 \).
1 **INTRODUCTION**

ICE spectra in Fig. 2 of Ref. [9]; and a 3 MA JET plasma heated with 4 MW of 55 keV proton NBI inclined at 58 degrees to the magnetic axis, see Fig. 1 of Ref. [9]. In these plasmas, the overall fusion reaction rate inferred from 2.5 MeV neutron fluxes ranged from $10^{12} \text{s}^{-1}$ at 1 MA to $10^{14} \text{s}^{-1}$ at 5 MA [9]. The core plasma electron density was a few times $10^{19} \text{m}^{-3}$ and the confinement time of energetic ions was a few times 0.1s, implying a relative concentration of energetic ions $\lesssim 10^{-7}$. In the first measurements of ICE from fusion-born alpha-particles in deuterium-tritium plasmas in JET, the ratio of local alpha-particle number density to local electron density was inferred from measured neutron fluxes and TRANSP computations. Its value ranged between $10^{-3}$ at the core and $10^{-5}$ at the edge, see Fig. 7 of Ref. [10] and the discussion in Sec. 3.3 thereof. The ICE phenomenology broadly resembled that from the prior deuterium plasma experiments at lower energetic ion concentration, see especially Figs. 2 and 5 of Ref. [10]. We note that Fig. 5 records ICE from Ohmic deuterium plasmas with measured neutron fluxes (and, by extension, approximate energetic ion concentrations) a million times lower than in the deuterium-tritium plasmas. Recent measurements of ICE from a subset of the fusion-born protons in KSTAR deuterium plasmas [24, 25, 26] and subsequent interpretation [27, 28] came as a surprise, because it was previously supposed that the fusion-born proton population was zero for all practical purposes, on the basis of plasma parameters and especially of particle orbits in relation to system size. In deuterium tokamak plasma discharges in JT-60U, ICE owing to fusion-born protons was detected [29, 30] during perpendicular and tangential deuterium NBI.
1 INTRODUCTION

There is thus a strong experimental record of ICE detection from fusion-born ions in plasmas where the fusion-born ion concentration is very low \((\leq 10^{-7})\), or indeed had previously been considered negligible. In this paper we therefore investigate how the ICE signals in Figs. 1, 2 and 3 may be excited by a subset of the diffuse population of fusion-born protons, and identify the likely physical origin of its distinctive frequency shift \([31, 30]\). We carry out direct numerical simulations using a hybrid particle-in-cell (PIC) approach \([32, 33, 34, 35]\). Our computations follow the full gyro-orbit kinetics of hundreds of millions of ions, including both minority energetic ions (protons at 3.02 MeV) and majority thermal deuterons, together with an electron fluid, evolving self-consistently with the electric and magnetic fields under the Maxwell-Lorentz system of equations. The simulation domain spans one spatial axis and all three velocity coordinates (1D3V).

This PIC-hybrid approach was recently applied successfully to the interpretation of ICE from NBI protons in hydrogen and deuterium plasmas in LHD \([14, 15]\). In related work, the applicability of PIC computations for interpreting ICE from a subset of fusion-born protons in deuterium plasmas was recently demonstrated for KSTAR observations \([27, 28]\), and also for ICE driven by NBI deuterons in KSTAR deuterium plasmas \([36]\), and more recently to ICE emitted from the core of ASDEX-U \([37, 38, 39, 40]\). In most contemporary ICE measurements, as in the early ICE observations in JET \([10]\) and TFTR \([41]\), the experimentally observed ICE spectral peaks are close to the cyclotron frequency of the energetic ions in the emitting region in the outer midplane plasma.
1 INTRODUCTION

In the corresponding PIC and PIC-hybrid computations [42, 34, 43, 35, 27, 28, 44, 36, 44, 40, 15], the minority energetic ion population is initialised with a physically motivated non-Maxwellian distribution in velocity space. This population then relaxes collectively under the magnetoacoustic cyclotron instability (MCI) [45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 42, 34, 35], which in these simulations manifests at the level of the Maxwell-Lorentz dynamics of the individual particles and the self-consistent fields. The spatiotemporal Fourier transforms of the excited electric and magnetic fields that arise in the PIC-based computations then yield simulated ICE spectra, which compare well with observations.

Identification of the initial distribution in velocity space of the candidate ICE-generating energetic ion population, prior to relaxation under the MCI, is central to this PIC-based approach. It typically rests on particle orbit studies, for example Fig. 14 of Ref. [10], Fig. 2 of Ref. [52], Fig. 2 of Ref. [27], Fig. 3 of Ref. [36] and Refs. [55, 56].

Here we examine the hypothesis that the bursting ICE signal is caused by a single energetic ion population (freshly fusion-born protons) at a single location (inferred from magnetic field strength assuming ion cyclotron phenomenology), operating under a single collective plasma physics process (the MCI). The work which follows shows that this hypothesis may well be valid, and this tends to weigh against more complex hypotheses involving a greater number of entities, for example more than one driving ion species at more than one location. For the present application, we develop, as follows, our hypothesis for the velocity space distribution of the emitting sub-population of
1 INTRODUCTION

the protons recently born in deuterium fusion reactions with energy \( E_H = 3.02 \text{ MeV} \). We first assume that, as usual, this ICE is dominated by waves propagating close to perpendicular to the local background magnetic field, and which are excited by the MCI of a highly non-Maxwellian energetic ion population. Specifically, these waves are on the fast Alfvén-cyclotron harmonic wave branch; this follows from the analytical theory of the MCI [45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57], and is confirmed by analysis of the oscillations self-consistently excited in first principles PIC-based computations [42, 34, 43, 35, 27, 28, 58, 14, 36, 40, 44, 15]. We assume that the ICE is primarily driven by a sub-population of energetic ions, which we can model approximately in terms of a drifting-ring velocity distribution [59]

\[
f(v_\parallel, v_\perp) = \frac{n_{\text{energ.}}}{2\pi v_\perp} \delta (v_\perp - u_\perp) \delta (v_\parallel - u_\parallel)
\]  

(1)

whose free parameters are its density \( n_{\text{energ.}} \), \( u_\perp \) and \( u_\parallel \). On the basis of many previous studies of ICE and its driving instability, the MCI [50, 52, 41, 57, 60, 34, 35, 14], we fix \( u_\perp \) to be close to the local Alfvén speed \( V_A \), because this is a preferred, but not a necessary, condition for ICE to be strongly driven. The centrality of the condition \( u_\perp \sim V_A \) has been re-emphasised by recent analytical and numerical studies [44, 61]. We assume that the driving ions are fusion-born protons at their birth energy \( E_H = 3.02 \text{MeV} \), and note that ICE unfolds on \( \mu \text{s} \) timescales during which no significant collisional energy
1 INTRODUCTION

loss has had time to occur. Their parallel component of kinetic energy follows from

\[ u_\parallel^2 = \frac{2}{m_H} E_H - u_\perp^2 \]  

(2)

This defines a value for \( u_\parallel > u_\perp \approx V_A \) which, when used in the PIC computations, gives rise to substantial spectral shifts in the simulated spectra. We shall show that these shifts correspond closely to the measured differences between ICE spectral peak frequencies and the integer harmonics of the local proton cyclotron frequency, that are observed during the transient events in LHD [16, 17, 21, 22, 23]. This tends to confirm our identification of this particular subset of the fusion-born protons, near their birth energy, as a candidate energetic ion population responsible for the bursting ICE.

An alternative scenario reported in Ref. [62] suggests that 178keV NBI protons, injected tangentially with low pitch angle, could also excite ICE with a spectrum displaying a substantial frequency shift from integer cyclotron harmonics. This approach differs from the one explored in this paper, in that the free energy available to destabilise electromagnetic waves is much lower than for the 3.02 MeV fusion-born protons we consider here. The lower velocities \( v_\parallel \) together with higher \( k_\parallel [62] \) result in \( k_\parallel v_\parallel \) being a small fraction of the proton cyclotron frequency \( \Omega_H \) when considering the resonant condition of the \( n \)th proton cyclotron harmonic including Doppler shift

\[ \omega = k_\parallel v_\parallel + n\Omega_H \]  

(3)
1 INTRODUCTION

Conversely the concentration of energetic ions $n_{\text{energ.}}/n_e$ is much higher for the NBI case; it is of order 5% for NBI protons in Ref. [62], whereas $n_{\text{energ.}}/n_e$ is estimated to be of the order of $10^{-7}$ for fusion-born protons in LHD. However, as we have reviewed earlier in this Section, ICE driven by fusion-born ions at very low concentrations $\leq 10^{-7}$ is a well established experimental phenomenon in tokamak plasmas.

The present study is topical, in that recent observations from ASDEX-Upgrade [37] and DIII-D [63, 64, 65] exhibit increasing diversity and complexity in the distribution of ICE spectral peaks. There may be multiple candidate energetic ion species, both fusion-born and NBI, either sub- or super-Alfvénic, furthermore ICE is detected from the core plasma [38, 39], as well as from its traditional locus near the outer midplane edge [13, 64]. This disambiguation is addressed in relation to contemporary ICE observations in the ASDEX-Upgrade tokamak [37, 38, 39, 40, 61]. Distinction between NBI drive and fusion-ion drive of ICE in early JET deuterium plasmas was achieved by observing ICE spectra for otherwise similar plasmas with widely different core density (and hence fusion reactivity) and the same NBI power, see Fig. 2 of Ref. [9]. This comparison might be more difficult to achieve in deuterium plasmas in LHD, which was not designed on a scale to achieve comprehensive energetic ion confinement. Frequency shifts of the kind examined here introduce a further degree of freedom to the frequency at which ICE spectral peaks are observed [50, 53, 52, 29, 31]. There is thus a new challenge, which one might term ICE plasma chemistry, in identifying the most likely ICE-generating ion species, together with their locations, from contemporary observed spectra where
these are not immediately obvious. The localisation of the ICE-generating subset in velocity space, which follows from the present analysis of the LHD data presented in Figs. 1 and 2, provides an example of the reconstruction of the zeroth-order features of the velocity space distribution function of an energetic ion population, based solely on ICE measurements [59, 66, 67, 68, 69]. In a recent development, this is being further examined using neural networks [70]. This in turn illustrates the diagnostic potential of ICE [71].

2 Observations of ICE during transient events in hydrogen and deuterium plasmas in LHD

2.1 ICE measurement system

The ICE acquisition system on LHD uses a dipole antenna developed in partnership with KSTAR [72, 73, 74]. The measurement system comprises a dipole antenna located in the 10-O port of LHD, inside the vacuum vessel whose centre is very close to the equatorial plane, about one degree below it. A fast digitizer performs direct sampling of the radiofrequency measurements at 1.25 GSamples/s. The time evolution of the RF signal intensity is collected by a 14-channel filter bank spectrometer in the range of 70 MHz to 2800 MHz, with intermediate spectral resolution and with μs time resolution for a duration spanning the whole plasma discharge [74]. A "discone" antenna, which is
2 OBS. OF ICE DURING LHD TRANSIENT EVENTS H & D PLASMAS

A type of dipole antenna designed to detect waves in the radiofrequency range 100 MHz to 2500 MHz, is also installed in the 9.5-L port of LHD inside the vacuum vessel, and the detected signal is acquired with a fast digitizer at 12.5 GSamples/s for 80 ms. The locations of tangential and perpendicular NBIs and RF antennas are shown in Fig. 4.

2.2 Previous observations from Hydrogen plasmas in LHD

In order to isolate the potential role of fusion-born protons in the ICE from LHD deuterium plasmas which is our main focus in this paper, it is first necessary to consider the potential role of NBI ions. Observations from earlier hydrogen plasmas in LHD can shed light on this, as follows. In some LHD hydrogen plasmas, transient events with bursty fluctuations were observed both as magnetic probe signals and as radiofrequency signals during heating by perpendicular NBI #4 and #5, as shown in Fig. 5 (a), (b) and (c).
Figure 4: Locations of NBIs and RF (dipole and discone) antennas. These are shown together with the typical values of the beam energy of each NBI.
Figure 5: Evolution of plasma parameters in LHD hydrogen plasma 129564 during which NBI #4 is turned off between $t = 4.38s$ and $t = 4.40s$. Time traces of (a): electric field fluctuations detected by the discone antenna, (b): emission intensities detected by the RF spectrometer, (c): magnetic field fluctuations detected by magnetic probe (d): NBI port through power, and radial profiles of (e): electron density, (f): electron and ion temperatures.

On the other hand, without NBI #4, the bursty fluctuations appear only in the radiofrequency signal detected by the discone antenna as shown in Fig. 6 (a), (b) and (c).
Figure 6: Evolution of plasma parameters in LHD hydrogen plasma 129574 during which NBI #4 is not operated throughout the whole duration of experiment. The time traces and radial profiles are similar to those of Fig. 5.

These plasmas were heated by tangential neutral proton beams at 186 keV (NBI #1), 170 keV (NBI #2), 170 keV (NBI #3), and by perpendicular proton beams at 43 keV (NBI #4), 40 keV (NBI #5). The magnetic configuration of these plasmas was \((R_{\text{ax}}, B_t) = (3,600 \, \text{m}, 2,850 \, \text{T})\) where \(R_{\text{ax}}\) is the distance between the centre of the torus and the magnetic axis, and \(B_t\) is the magnetic field strength at the magnetic axis. The corresponding spectrograms displaying the radiation power of the radio frequency waves detected by the discone antenna are shown in Fig. 7 for discharges with and without NBI #4. Unfortunately, the fast digitizer did not acquire the signal detected by the
Figure 7: Frequency spectrograms (Upper panels) obtained by FFT analysis of the electric field fluctuations (Lower panels) surrounding two bursty events in LHD hydrogen plasmas. (Left panels) Plasma 129564 heated by both NBI #4 and #5. (Right panels) Plasma 129574 heated only by NBI #5. The red and dark vertical lines show the times at which the ICE spectra shown in Fig. 8 are taken.

dipole antenna at this time. Figure 8 shows the frequency spectra before and during the burst with and without NBI #4. As shown in the upper left panel of Fig. 7 and in the upper panel of Fig. 8 corresponding to LHD hydrogen plasma 129564, radiation in the frequency range of $100 \text{MHz} < f < 350 \text{MHz}$ is strongly enhanced during the burst when NBI #4 is applied. The enhanced signal in the frequency range between 200 MHz and 300 MHz spans the 7th to 12th harmonics of a fundamental frequency considered to lie in the interval 28.22 MHz to 29.55 MHz, as shown in the upper panel of Fig. 8. As indicated in Fig. 9, the fundamental proton cyclotron resonances of 28.22 MHz to 29.55 MHz are located near the last closed flux surface (LCFS), which is defined as
Figure 8: Power spectra offset by the noise level taken before (brown solid line) and during (black solid line) the bursty events observed with NBI #4 (upper panel), and without NBI #4 (lower panel) in LHD hydrogen plasmas 129564 and 129574 respectively. The dashed vertical lines represent the locations of the 7th to 12th harmonics of 28.22 MHz (violet) to 29.55 MHz (magenta).
Figure 9: Contours of the proton cyclotron frequency with 5MHz interval (grey lines), contours of the closed magnetic flux surfaces (blue lines), locations of the fundamental proton cyclotron resonances of 28.22 MHz (purple line) and 29.55 MHz (magenta lines) at the vertically long cross section where the discone antenna is installed in the lower port.
2 OBS. OF ICE DURING LHD TRANSIENT EVENTS H & D PLASMAS

$r_{\text{eff}}/a_{99} = 1$, where $r_{\text{eff}}$ is the effective minor radius, such that 99\% of the electron kinetic energy in the plasma lies within $r_{\text{eff}} = a_{99}$. Conversely, the radiation in the frequency range $325\text{MHz} < f < 350\text{MHz}$ only is strongly enhanced when NBI #4 is not operated, as indicated on the right panel of Fig. 7 and in the lower panel of Fig. 8 which corresponds to LHD hydrogen plasma 129574. The profiles of electron density and electron and ion temperatures plotted versus the normalised minor radius defined by $r_{\text{eff}}/a_{99}$ are similar to each other as shown in panels (g), (h) of Figs. 5 and 6. These results from hydrogen plasmas suggest that emissions in the range of 200MHz - 300 MHz emitted near the LCFS following the burst originates from high energy protons supplied by perpendicular NBI and not by tangential NBI.

2.3 Observations of bursting ICE from LHD deuterium plasma

In LHD deuterium plasmas that are otherwise similar to the hydrogen plasmas discussed above, transient events with bursty fluctuations were observed in both magnetic fluctuations and radiofrequency signals during heating with perpendicular NBI #4 and #5, as shown in the panels (b) and (c) of Fig. 10. The magnetic configuration of this discharge was $(R_{\text{ax}}, B_t) = (3.600\text{m}, 2.750\text{T})$ and the plasma was heated by tangential NB proton beams at 185 keV (NBI #1), 162 keV (NBI #2), 175 keV (NBI #3), and perpendicular NB deuterium beams of 59 keV (NBI #4), 70 keV (NBI #5). A distinctive ICE signal, in the frequency range between 200MHz and 300MHz, was detected during a transient event with bursty fluctuations in LHD deuterium plasma.
Figure 10: Time evolution of plasma parameters in LHD deuterium plasma 133979. Time traces of (a): neutron flux, (b): emission intensities from the filter bank system (RF spectrometer) detected by 10-O dipole antenna, (c): magnetic fluctuation detected by magnetic probe, (d): NBI port through power, and radial profiles of (e): electron density, (f): electron and ion temperature.
133979. The measured perturbed electric field time series shown in Fig. 1 has spectral peaks (see also Fig. 2) whose separation could be related to the cyclotron frequency of an energetic ion species: in particular, the 3.02MeV fusion-born protons, born in deuteron-deuteron fusion reactions. As we describe below, these spectra are different from ICEs observed in hydrogen discharge and ICEs previously investigated. First, the peaks appear to have undergone substantial Doppler shifts with respect to local integer cyclotron harmonics; and second, the frequency interval between successive peaks is not strictly uniform. As an example of this ICE phenomenology, the top panel of Fig. 1 shows the time series of the electric field intensity of the bursting ICE at \( t \approx 4.443 \) s in LHD plasma 133979, and the bottom panel shows the corresponding spectrogram displaying intense radio frequency activity in the hundreds of megahertz range with increasingly high time resolution. We shall show that these effects can arise naturally from the substantial super-Alfvénic parallel velocity of the energetic protons that, we argue, drive this ICE. Figure 2 presents two ICE power spectra, taken (top) just before the bursty event at \( t = 4.440 \) s, and (bottom) during the event at \( t = 4.444 \) s. The major difference consists in the appearance during the burst of the three peaks labelled \( b, c, d \) in the lower panel, together with peak \( a \). The spectral peaks \( b, c, \) and \( d \), are located between 250.04 and 255.36MHz (b), between 276.64 and 281.96MHz (c), and between 303.24 and 308.56MHz. Here, the lower limits 250.04, 276.64, and 303.24 MHz can be written \( 26.6(n - 0.6) \) MHz for \( n = 10, 11, 12 \). The upper limits at 255.36 MHz, 281.96 MHz and 308.56 MHz, can be written \( 26.6(m + 0.6) \) MHz for \( m = 9, 10, 11 \). The shift of
±0.6 × 26.6 MHz corresponds closely to the Doppler shift due to large \(v_\parallel\) of fusion-born protons. Fusion-born ions are distributed randomly and uniformly with respect to solid angle in velocity-space at birth, hence \(v_\parallel\) can have both positive and negative values. Therefore the lead hypothesis, whose physical consistency we explore in this paper, is that this ICE is generated locally at the Doppler-shifted proton cyclotron resonance whose fundamental frequency is 26.6 MHz. Figure 10-(a) displays the time evolution of the measured neutron flux between \(t = 4.41\)s and \(t = 4.49\)s during deuterium discharge of LHD plasma 133979. Over this discharge, the neutron flux reaches its maximum value in this time range as shown in Fig. 10. The neutron flux increases until the occurrence of the bursty event, and decreases following it [6]. This must result in a transient surge in the number of fusion-born protons at 3.02 MeV, implying a transient inversion (local positive slope) of the velocity-space distribution of the protons, which is necessary for them to be able to drive ICE. Figure 10-(d) shows that the NBI power is steady around the time of the burst. Fig. 10-(b) plots the time evolution of the power radiated by the plasma in six different radio frequency channels, between 100MHz and 400MHz. Intense transient activity is visible. Moreover, similar transient activity is observed in frequency channels of 600MHz and 800MHz, however the origin of the activity in these highest observed frequencies are not clear. In the present paper, we focus on the energetic ion physics underlying the radio frequency bursts in the range 200MHz to 300MHz. The generation mechanism of these bursts could lie in the interplay between a quasi stable stationary (non-rotating) 1/1 MHD mode, a tongue-shaped deformation
OBS. OF ICE DURING LHD TRANSIENT EVENTS H & D PLASMAS

of the plasma at a non rational surface and a rotating 1/1 mode with MHD bursts [23]. As the tongue-shaped deformation arises, it hinders the stationary 1/1 mode and draws on the free energy of the instability which lies in the strong gradient of energetic ions. The tongue deformation results most likely from the pressure gradient of the trapped ions supplied by the NBI. A strong RF signal at a frequency $\approx 800$MHz accompanies the collapse along with RF signals which are observed in the frequency range between 200MHz and 300MHz during both MHD activity and tongue-shaped deformation (see bottom of Figure 2 in Ref. [23]).

There are some notable apparent parallels between recent ICE measurements from NSTX-U and the observed link between the LHD bursting ICE locale and the "Tongue" that we report here. Submillisecond bursts of ICE driven by marginally super-Alfvénic NBI deuterons in NSTX-U were observed [75] to be spatially collocated with internal transport barriers that are well within (by tens of centimetres) the plasma. Figures 4 and 5 of [75] display striking measurements of the correlation of the gently time-varying radial location of the ICE emission with the location where contours of electron density are concentrated, together with the steep ion temperature gradient. The experimental results from the present paper thus share several intriguing physical features with the results reported in Ref.[75]. Both relate to bursting ICE driven by a super-Alfvénic ion species - fusion-born protons in LHD and NBI deuterons in NSTX-U; and the location of the ICE in both cases correlates with a strongly nonlinear feature of the plasma - the "Tongue" in LHD [23] and the internal transport barrier in NSTX-U.
3 CANDIDATE SUB-POPULATION OF FUSION-BORN PROTONS

3 Identification of the candidate sub-population of fusion-born protons

The redistribution of the initially helically-trapped ions that are implicated in burst kinetic MHD [16, 17, 19] or abrupt tongue deformation [21, 22] could generate a distinct, transient, highly non-Maxwellian distribution in velocity space, perhaps involving the expelled ions and freshly trapped ions. Let us therefore make the assumption (for this, or some as-yet-unknown, reason) that a transient, spatially localised, highly non-Maxwellian population of 3.02MeV fusion-born protons could be responsible for the ICE spectral peaks b, c and d in Fig. 2. Our initial goal is then to perform multiple PIC-hybrid computations of the collective relaxation of these highly energetic protons, using the plasma parameters at an appropriately inferred ICE emission location; and to investigate whether the resulting simulated ICE power spectra are compatible with the measurements in Fig. 2. In this section, we identify the fundamental cyclotron frequency with the mean frequency spacing between the peaks b, c and d. This allows us to determine the emission location and thus the plasma parameters necessary to initialise our simulations in section 4. This approach assumes that these peaks have a common emission location. If the radiation is generated by protons, identifying the 26.6MHz mean frequency spacing between these peaks as an indicative local proton cyclotron frequency implies a local magnetic field strength of 1.75T. As shown in Fig. 11, the corresponding proton cyclotron resonance on the midplane is located at $R =$
3 CANDIDATE SUB-POPULATION OF FUSION-BORN PROTONS

4.521m that corresponds to the normalised minor radius \( r_{\text{eff}}/a_{99} = 0.956 \) and is close to the LCFS at the horizontally long poloidal cross section where the dipole antenna is installed. As indicated in Fig. 10-(g) and (h), the electron and ion temperatures at \( r_{\text{eff}}/a_{99} = 0.956 \) are 846eV and 907eV respectively, while the electron number density is \( 8.8 \times 10^{18} \text{m}^{-3} \) such that \( V_A = 0.9105 \times 10^7 \text{m/s} \); these are used as input parameters in our PIC-hybrid simulations. The rotation flow velocity of the LHD plasma is around \( 10^4 \text{m/s} \) and is negligibly small compared to the order of the Alfvén velocity.

Table 1 gives the computed time after which 3.02MeV protons are lost, as a function of their radial birth location and of their initial pitch angle, evaluated in the equatorial plane of LHD. The red boxes are the initial locations corresponding to unconfined fusion-born protons. Conversely, there is the possibility for fusion-born protons to remain on confined trajectories [76]. The ICE-relevant density of the fusion-born protons \( n_H \) can be evaluated by integration over the restricted range of pitch angle values, \( \alpha \), that contribute to exciting the MCI. We can estimate \( n_H \) from the expression

\[
n_H = f_{\text{ac}} (FC \times t_{sl}) / V = f_{\text{ac}} \times 10^{12} \text{m}^{-3}
\]

where \( t_{sl} \) is the slowing time indicated inside blue and red boxes in Table 1; \( FC \) is the neutron flux in neutron/s; and \( V \) is the plasma volume of LHD. Approximate values for these parameters are \( FC = 3.0 \times 10^{14}, V = 30 \text{m}^{-3}, t_{sl} = 50 \times 10^{-3} \text{s} \). Hence we may infer the fusion-born ion concentration \( n_H/n_e \) is less than \( 10^{-7} \) since \( n_e \) is of the order of \( 10^{19} \) and the factor value \( f_{\text{ac}} \) is less than 1. Further studies have shown that other
Figure 11: Contours of the proton cyclotron frequency with 5 MHz interval spacing (grey lines), locations of the fundamental proton cyclotron resonance of 26.6 MHz (red lines) and contours of closed magnetic flux surfaces (blue lines) sliced at vertically long poloidal cross section where the dipole antenna is installed.
3  CANDIDATE SUB-POPULATION OF FUSION-BORN PROTONS

| Radius (m) | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 | 2.7 | 2.8 | 2.9 | 3.0 | 3.1 | 3.2 | 3.3 | 3.4 | 3.5 | 3.6 | 3.7 | 3.8 | 3.9 | 4.0 |
|------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Pitch angle (degrees) | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 | 290 | 300 | 310 | 320 | 330 | 340 | 350 | 360 |

Table 1: Table showing the time after which a 3.02 MeV proton born in the horizontally elongated poloidal part of the equatorial plane in LHD is lost as a function of major radial location and pitch angle (defined by Eq. 8) at birth. Red boxes denote promptly lost protons and the number inside each indicates the time before loss, in milliseconds. The remaining protons, corresponding to the blue boxes, constitute the local confined population and define its velocity distribution. The underlying calculation [56] tracks the guiding centre and stops at 50 ms which corresponds to the slowing down time. Collisions are neglected because their timescale are long compared to the timescales on which ICE unfolds.

particles in the MeV energy range can be confined in LHD [76]. For example, some 15MeV protons resulting from the D + ^3 He → ^4 He (3.67 MeV) + p (15 MeV) reaction are confined over the chaotic field line region [77], but these are not obvious candidates to drive the ICE signal from deuterium plasmas considered here.

In order to make use of the information in Table 1 to establish whether the candidate ICE-generating protons at the location of the ICE bursts are indeed confined, we need to establish a mapping between particle velocity vectors there and at the birth location.

We first calculate the magnetic moment of the potentially confined sub-population of
3 CANDIDATE SUB-POPULATION OF FUSION-BORN PROTONS

3.02MeV fusion-born protons at the burst location, which we denote by

$$\mu_{ICE} = \frac{m_H v_{\perp,burst}^2}{2 |B_{burst}|}$$

(5)

Here the value of \(v_{\perp,burst}\), which denotes the perpendicular velocity of the ICE-emitting ions at the burst location, is not exactly known but is strongly constrained, as we describe below; and we assume \(|B_{burst}| = |B_{ICE}| = 1.75\,T\), as already established. We now invoke the conservation of magnetic moment \(\mu_{ICE} = \mu_R\), with \(\mu_R = m_H v_{\perp,R}^2/2 |B(R)|\) the magnetic moment at major radius \(R\), where the local perpendicular velocities of the protons is denoted by \(v_{\perp,R}\), and the corresponding local magnetic field strength is denoted \(|B(R)|\). It follows that

$$v_{\perp,R} = \sqrt{\frac{2 |B(R)| \mu_{ICE}}{m_H}}$$

(6)

Equating the total kinetic energy, which is the sum of perpendicular and parallel components, to the birth energy, we have

$$\frac{1}{2} m_H v_R^2 = \frac{1}{2} m_H v_{\perp,R}^2 + \frac{1}{2} m_H v_{\parallel,R}^2 = 3.02\,MeV$$

(7)

We can then compute the pitch angles \(\alpha\) at the inferred birth locations,

$$\alpha = \arcsin \left( \frac{v_{\perp,R}}{v_R} \right)$$

(8)
3  CANDIDATE SUB-POPULATION OF FUSION-BORN PROTONS

for $v_{⊥,\text{burst}} = [0.8, 0.9, 1.0, 1.1, 1.2]V_A$ at $R = 4.521m$. This range of values for $v_{⊥,\text{burst}}/V_A$ is chosen because it is known to give strong drive for the MCI and hence ICE [42]. Finally, we compare these values with those tabulated in Table 1, to find out whether these protons are born on confined trajectories. The values of $\alpha$ are displayed in Table 2, and suggest that the protons that intersect the burst location in the range of velocities considered were originally born on confined trajectories, as identified for LHD in Table 1.

| $R$ (m) | 3.10 | 3.20 | 3.30 | 3.40 | 3.50 | 3.60 | 3.70 | 3.80 | 3.90 | 4.00 | 4.10 | 4.20 | 4.30 | 4.40 | 4.50 | 4.60 |
|---------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| $|B|(T)$| 2.28 | 2.42 | 2.56 | 2.64 | 2.68 | 2.69 | 2.66 | 2.61 | 2.53 | 2.43 | 2.33 | 2.20 | 2.16 | 1.92 | 1.78 | 1.65 |
| 0.9$V_A$| 22.96| 23.71| 24.42| 24.82| 25.01| 25.07| 24.91| 24.68| 24.26| 23.76| 23.22| 22.53| 21.75| 20.97| 20.15| 19.36|
| 1.0$V_A$| 25.69| 26.54| 27.35| 27.80| 28.02| 28.09| 27.90| 27.64| 27.17| 26.59| 25.98| 25.19| 24.31| 23.44| 22.51| 21.61|
| 1.1$V_A$| 28.48| 29.44| 30.35| 30.87| 31.12| 31.19| 30.98| 30.68| 30.15| 29.50| 28.81| 27.92| 26.93| 25.94| 24.90| 23.90|
| 1.2$V_A$| 31.34| 32.42| 33.45| 34.04| 34.32| 34.40| 34.16| 33.83| 33.22| 32.49| 31.72| 30.72| 29.60| 28.51| 27.35| 26.23|

Table 2: Birth pitch angles $\alpha$, defined by Eq. 8, expressed in degrees at different major radial locations $R$, which for a 3.02 MeV proton lead to perpendicular velocities of $v_{⊥,\text{burst}} = (0.8, 0.9, 1.0, 1.1, 1.2) V_A$ at $R = 4.521m$. It follows that these protons lie in the confined region of $(\alpha, R)$ parameter space delineated by the lower blue region in Table 1 since the pitch angles calculated here do not exceed $35^\circ$. These pitch angle values are therefore smaller than those which lead to promptly lost ions indicated in Table 1 for the range $R = 3.10m$ to $R = 4.60m$. The box color meaning is identical to that of Table 1.

The foregoing suggests that 3.02 MeV protons with perpendicular velocities in the range $v_{⊥,\text{burst}} = [0.8 − 1.2] V_A$ at the location of interest are on confined trajectories. In the
next section, we will show that they could efficiently drive the observed transient ICE signal. In contrast, the velocities of the energetic deuterons from perpendicular NBI are very sub-Alfvénic at the ICE location. Typically, these have $v_{\perp}/V_A < 0.3$, which has led to ICE in previous LHD experiments and modeling [14, 28], although in a different frequency range. Their ability to drive the MCI in the frequency range 250MHz to 310MHz is weak as we show in Appendix C. We need not consider them further here and focus on the larger values of $v_{\perp}/V_A$ which are found to drive ICE more strongly in that regime.

4 Simulations of bursting ICE from LHD plasma 133979

4.1 Frequency shifts and energy partitioning

The three intense peaks $b$, $c$ and $d$ in Fig. 2 are approximately centred at frequencies 255.1MHz, 281.7MHz and 308.2MHz. In the preceding section, we used the average spacing between these frequencies, 26.6 MHz, as an interim value for the proton cyclotron frequency $\Omega_H$ at the ICE location. If we normalise the three spectral peak frequencies to 26.6MHz, they would correspond to proton cyclotron harmonics 9.6, 10.6 and 11.6, which are evidently not integer multiples of the fundamental. Figure 3 shows the measured power spectra in LHD plasma 133979 before the burst at $t = 4.4430$s (top left panel) and at successive times during the burst, from $t = 4.4437$s to $t = 4.4447$s, in the following panels. We find that the locations of the intense spectral peaks between
250MHz and 300MHz in Figs. 2 and 3 are very close to the dashed blue lines, which correspond to \( f = 26.6 \times (n + 0.6) \text{MHz} \), \( n = 8, 9, 10, 11 \); they are also close to the red lines, which correspond to \( f = 26.6 \times (n - 0.6) \text{MHz} \), \( n = 9, 10, 11, 12 \). In a different LHD plasma, as outlined in Appendix D, the peaks correspond to either \( 28 \times (n + 0.5) \text{MHz} \) or \( 28 \times (n - 0.5) \text{MHz} \). In either case, then, the measured frequency shifts relative to integer cyclotron harmonics have magnitude \( \simeq \Omega_H/2 \).

Let us first examine whether, in principle, in the present context, it is plausible that there could arise Doppler shifts of 15.7MHz (harmonics 9,10,11) or \(-10.9\text{MHz}\) (harmonics 10,11,12). It is well known [51, 42, 34] that to excite the MCI requires the energetic ions to have perpendicular velocity \( v_\perp \approx V_A \). The resonant condition of the \( n \)th proton cyclotron harmonic including Doppler shift is given as \( \omega = k_\| v_\| + n \Omega_H \). As discussed in Appendix A, both negative and positive values for \( k_\| v_\| \) are possible and are captured by the simulations, and they are also observed experimentally with the subpeaks \( n \pm 0.6 \) in Fig. 3. If the total kinetic energy of the ions is sufficiently large that this value of \( v_\perp \) is compatible with \( v_\| \simeq V_A \), also, this would provide scope for Doppler shifts satisfying \( k_\| V_A \sim \Omega_H \) if \( k_\| V_A \sim \Omega_H \). This is equivalent to \( (k_\|/k_\perp) k_\perp V_A \sim \Omega_H \). Therefore, in the MCI, a quasi-perpendicular fast Alfvén wave is resonant with the \( n \)th proton cyclotron harmonic: \( \omega_{fast} \simeq k_\perp V_A \simeq (n + \alpha) \Omega_H \), where \( \alpha \) is of order 1. The MCI is typically [10] most strongly driven around the tenth proton cyclotron harmonic in deuterium plasmas, i.e. \( n = 10 \), see Fig. 1 of Refs. [42, 34] and Fig. 4 of Ref. [28]. For sufficiently energetic ions undergoing the MCI, the resonance condition with the inclusion of a non
zero parallel velocity becomes \( k_A \sim \omega = n\Omega_H \pm |k|v| which suggests that it is therefore possible to satisfy \(|k|v| \sim \Omega_H at \omega \sim n\Omega_H if |k|/|k| \sim 1/n. It follows - but only at back-of-envelope level - that it might be possible for a strongly non-Maxwellian population of 3.02MeV protons to excite, through the MCI, waves at cyclotron harmonics that are Doppler shifted by the large amount \( \sim \Omega_H/2 \) that is observed. As an illustration, let us suppose 
\[
|k|v| \sim \Omega_H/2 \tag{9}
\]
for waves excited on the fast Alfvén branch at the tenth harmonic of \( \Omega_H \), so that 
\[
\omega \sim k_A \sim 10\Omega_H \tag{10}
\]
Then upon taking the ratio of each side of Eqs. 9 and 10, we find 
\[
\frac{k}{k_A} \sim \frac{V_A}{20v} \sim \frac{1}{48.4} \tag{11}
\]
for the case \( v = 2.42V_A \) which is inferred from Eq. 2 of Section 1 when \( v = 1.05V_A \). It follows that fast Alfvén waves propagating only \( 1^\circ or 2^\circ \) from perpendicular to \( B_0 \) could in principle undergo wave-particle cyclotron harmonic resonance at the required, highly shifted, frequency of approximately \( 10\Omega_H \pm \Omega_H/2 \). The question then is: are such waves actually excited? This motivates our direct numerical simulations reported below and in appendix B.

Table 3 displays the consequences of different partitions of the 3.02MeV proton birth
energy into perpendicular and parallel components, in terms of the corresponding perpendicular and parallel velocities normalised to the local Alfvén speed at the emission location in LHD. Guiding centre drifts enter into the MCI wave-particle resonance condition through an additional term $k \cdot v_{\text{drift}}$ [78]. A priori this will be extremely small, for reasons of vector orientation, as follows. The ICE is detected on LHD using a dipole antenna whose centre is very close to the equatorial plane, about one degree below it, and we know that MCI-excited waves have $k$-vectors that are quasi-perpendicular to the magnetic field. In combination, these conditions imply that the detected $k$-vector is close to horizontal. In contrast, the curvature and $\nabla B \times B$ guiding centre drifts are vertical for a particle in the equatorial plane. Thus $k$ and $v_{\text{drift}}$ are perpendicular to each other, or nearly so, for the particles and waves of interest. To check what "nearly so" means, we have calculated realistic orbits for the particles of interest using the standard code for LHD. The resulting magnitude of $v_{\text{drift}}$ is an order of magnitude lower than $v_\parallel$. Hence $k \cdot v_{\text{drift}}$ is down by a large factor, on two counts, compared to the effects that are retained in our 1D3V slab geometry PIC computations. We shall use PIC-hybrid simulations to explore the range of $k_\parallel$ which, together with $v_\parallel$, could result in frequency shifts consistent with the measured ICE power spectrum. Hitherto, no wavenumber measurements of ICE have been reported from LHD. Each PIC-hybrid computation is run at a given angle between the magnetic field $B_0$ and the 1D spatial simulation domain, which we identify as the outward radial direction in LHD, and which defines the orientation of possible $k$ vectors.
### Table 3: The consequences of different partitions of 3.02 MeV proton energy into perpendicular and parallel components, expressed in terms of velocities normalised to the local Alfvén speed $V_A = 0.9105 \times 10^7$ m/s in the ICE emitting region of LHD plasma 133979 during the bursty event at $t = 4.44$ sec. The last column shows the corresponding circulation frequency due to the combined curvature and grad $B$ guiding centre drifts, and demonstrates that this is small compared to $\Omega_H$.

<table>
<thead>
<tr>
<th>Species</th>
<th>$v_\perp/V_A$</th>
<th>Energy $\perp$ (keV)</th>
<th>$v_{\parallel}/V_A$</th>
<th>Energy $\parallel$ (keV)</th>
<th>$\omega_{\text{Drift}}/\Omega_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.50</td>
<td>108.16</td>
<td>2.59</td>
<td>2891.84</td>
<td>0.0144</td>
</tr>
<tr>
<td>H</td>
<td>0.60</td>
<td>155.76</td>
<td>2.56</td>
<td>2814.24</td>
<td>0.0143</td>
</tr>
<tr>
<td>H</td>
<td>0.63</td>
<td>170.00</td>
<td>2.63</td>
<td>2830.00</td>
<td>0.0142</td>
</tr>
<tr>
<td>H</td>
<td>0.70</td>
<td>212.00</td>
<td>2.54</td>
<td>2788.00</td>
<td>0.0141</td>
</tr>
<tr>
<td>H</td>
<td>0.80</td>
<td>276.90</td>
<td>2.51</td>
<td>2723.10</td>
<td>0.0140</td>
</tr>
<tr>
<td>H</td>
<td>0.90</td>
<td>350.45</td>
<td>2.47</td>
<td>2649.55</td>
<td>0.0138</td>
</tr>
<tr>
<td>H</td>
<td>1.00</td>
<td>432.66</td>
<td>2.44</td>
<td>2567.34</td>
<td>0.0136</td>
</tr>
<tr>
<td>H</td>
<td>1.10</td>
<td>523.52</td>
<td>2.39</td>
<td>2476.48</td>
<td>0.0134</td>
</tr>
<tr>
<td>H</td>
<td>1.20</td>
<td>623.03</td>
<td>2.34</td>
<td>2376.97</td>
<td>0.0131</td>
</tr>
</tbody>
</table>
4.2 Simulations with zero parallel velocity

As a first step in isolating the role of $v_\parallel$, we run simulations for which $u_\parallel = 0$, so that the initial velocity distribution function of protons is given by a ring beam

$$f_H = \left[ n_{proton} / (2\pi v_\perp) \right] \delta (v_\parallel) \delta (v_\perp - u_\perp).$$

The initial distribution in gyroangles is sampled randomly uniformly, and ion gyromotion is fully resolved, such that the physics of cyclotron resonant effects unfolds at the level of interactions between ions moving on their gyro-orbits and the local self-consistent electric and magnetic fields. This enables us to focus on the role of $u_\perp$. We know that $u_\perp \approx V_A$ is typical for the MCI to occur. With $u_\perp$ in this range, and $u_\parallel = 0$ for now, the total kinetic energy of such protons is less than 3.02MeV. We explore a range of perpendicular velocities from $u_\perp = 0.8V_A$ to $1.2V_A$. These computations use 500 particles per cell for the thermal deuterons and for the fast protons, and represent the electrons as a massless fluid. The grid has 1024 cells, and the cell size is chosen such that the cold plasma dispersion relation is recovered from the spatiotemporal Fourier transform of the electric and magnetic field fluctuations, in the appropriate limit. These provide good energy conservation, within 2% over the duration of a simulation. The time evolution of the fields and of the ion species energy density from our PIC-hybrid simulations show that the collective relaxation of the proton population with energy in the MeV range is governed by the MCI, as seen in Fig. 12. It is known that the MCI drive increases with the relative number density $\xi = n_{proton} / n_e$ in the relevant regime [46, 35]. In the simulations described below, the value of $\xi$ is chosen to saturate the MCI within $20\tau_D$: $\xi = 0.0025$ for orientation of the
Figure 12: Time evolution of the change in energy density of the fields and of the thermal deuterons and fusion-born proton population. The latter collectively relaxes under the MCI, giving up energy to excite the electric and magnetic fields, causing the thermal deuterons to oscillate self-consistently. The magenta and green traces correspond to the $y$ and $z$-components of the magnetic field. The blue and red curves show the energy density change of the $x$-component of the electric field and of the thermal deuterons respectively. The propagation angle between the simulation domain and the background magnetic field $B_0$ is $91^\circ$, $u_\perp = 1.0V_A$. The time is normalised to the proton gyroperiod $\tau_H$. 
spatial domain with respect to $B_0$ of 90.5°, 91.0° and 91.5°; and $\xi = 0.0050$ for 92.0°. This approach optimises the use of computational resources (each simulation requires two hours on 56 cores) without compromising the physics.

The resulting power spectra shown in Fig. 13 are for a range of values of the perpendicular component of velocity $0.8 \leq u_\perp/V_A \leq 1.2$. The spectral peaks are at successive proton cyclotron harmonics, and in this respect they differ, as expected, from the LHD ICE observations in Fig. 2 that we seek to explain. These preliminary simulation results are encouraging in relation to the essential feature of the ICE spectrum in Fig. 2, measured during the LHD transient, in that the simulated spectra are dominated by a few cyclotron harmonic peaks in the frequency range between $\omega = 8\Omega_H$ and $12\Omega_H$.

The magnitude of the most strongly driven spectral peaks in Fig. 13 tends to decrease monotonically as $u_\perp$ increases, and this feature is most notable when the propagation angle gets closer to 90°. Figure 14 shows the sensitivity of the spectral peak maxima to the propagation angle, as well as to the perpendicular beam velocity in the range $0.925 \leq u_\perp/V_A \leq 1.050$, using higher resolution computational parameters for these simulations, as anticipated from section 4.3, namely 2000 particles per cell and 8192 cells. It confirms that, in these initial computations for the restricted case $u_\parallel = 0$, the MCI of a ring-beam population of energetic protons with $u_\perp \sim V_A$ generates simulated ICE spectra whose dominant peaks are in the observationally significant range between $9\Omega_H$ and $12\Omega_H$. We have found that this spectral range is also dominant for an initial fast proton distribution which incorporates perpendicular thermal spread [59].
Figure 13: Power spectra of the excited $\delta B_z$ energy density in multiple computations of the relaxation of a ring-beam ($u_\parallel = 0$) distribution of protons for LHD plasma 133979 parameters at the time and location of the bursting ICE event. For the five spectra plotted in each panel, the protons have purely perpendicular velocity $u_\perp$: from top, $u_\perp = 1.2V_A$, $1.1V_A$, $1.0V_A$, $0.9V_A$ and $0.8V_A$. The propagation angle between $\mathbf{k}$ and $\mathbf{B}_0$ is $90.5^\circ$ (top left), $91.0^\circ$ (top right), $91.5^\circ$ (bottom left), $92.0^\circ$ (bottom right).
Figure 14: Power spectra for oscillations excited by a ring-beam ($v_\parallel = 0$ proton population) at a propagation angle of 91.0° (left), and 90.8° (right), for different perpendicular beam velocities (inset), in the range $0.925 \leq u_\perp / V_A \leq 1.050$. In both cases, the four dominant spectral peaks that result from the simulations are the ninth to twelfth harmonics of $\Omega_H$. These correspond exactly to the four cyclotron harmonics of greatest relevance to the interpretation of the measured ICE spectral peaks $b$, $c$ and $d$ in Fig. 2.
4 Simulations of Bursting Ice from LHD Plasma

\[ f_H \propto \exp \left[ -\frac{(v_\perp - u_\perp)^2}{v_{\perp,r}^2} \right] \] with \( v_{\perp,r} = 0.15u_\perp \). Proton cyclotron harmonics are therefore strongly driven in the range between \( 9\Omega_H \) and \( 12\Omega_H \) comparatively to other cyclotron harmonics. Cyclotron harmonics below \( 9\Omega_H \) tend to be more strongly excited as the ratio \( v_\perp/V_A \) further increases however our focus on the frequency range \( 9\Omega_H \) and \( 12\Omega_H \) is based on the experimental observations.

4.3 Simulations with realistic parallel velocity

The simulation results obtained in the preceding sub-section 4.2, for the case where the driving proton population has no velocity component parallel to the magnetic field, indicate that the range of angles and \( u_\perp/V_A \) values considered give rise to robust, and potentially experimentally relevant, MCI-driven power spectra. Let us now focus in particular on \( u_\perp/V_A = 1.05 \), and introduce parallel velocities into our approach. For fusion-born protons, the initial kinetic energy \( E = \left( u_\perp^2 + u_\parallel^2 \right) / 2m_H = 3.02\text{MeV} \). It follows from Eq. 2 that if \( u_\perp = 1.05V_A \), then \( u_\parallel = 2.42V_A \) for a 3.02MeV proton.

In the series of PIC-hybrid computations described in this sub-section, we use these values for \( u_\perp \) and \( u_\parallel \) (together with other pairings derived in the same way; see Table 3) as parameters in the simple model distribution function of the fusion-born protons. Following Eq. 1, this is

\[ f_H(v_\perp, v_\parallel) = \frac{n_H}{2\pi v_\perp}\delta(v_\parallel - u_\parallel)\delta(v_\perp - u_\perp) \] (12)
As previously, the initial distribution of proton is uniformly distributed in gyro-angles. We have run PIC-hybrid simulations for minority 3.02MeV proton populations, initialised using Eq. 12, in majority thermal deuterium plasmas. We use 2000 particles per cell for each ion species, with 8192 cells, and a duration of $15\tau_H$ with a relative proton density $\xi = 0.001$ in all of the following simulations. Relative energy does not change by more than 0.5%. Power spectra are constructed from the spatiotemporal Fourier transform of $\delta B_z$, taken over the full spatial domain and averaged over $15\tau_H$. This is shown on the right panel of Fig. 15, where the fast Alfvén branch and multiple cyclotron harmonic wave branches are clearly visible. Summing the Fourier transformed power between $k = 0$ and $k = 24\Omega_H/V_A$ yields the power spectrum shown on the left panel in Fig. 15. This is identical to the green trace shown in Fig. 16 (left). Figure 16 shows the power spectra of waves propagating in the $+\hat{x}$ direction (corresponding to the direction of the 1D simulation domain) at an angle of 91.0° and 90.8° with respect to the background magnetic field $B_0$. The green power spectra result from 3.02MeV protons whose parallel and perpendicular velocities are $u_\parallel = 2.415V_A = 2.199 \times 10^7\text{ms}^{-1}$ and $u_\perp = 1.050V_A = 0.956 \times 10^7\text{ms}^{-1}$; while the blue traces have the same value of $u_\perp$ but with $u_\parallel = 0$, for comparison (as in sub-section 4.2). The frequency resolution in the computed spectra is $\pm 0.07\Omega_H$. In the blue cases, for zero $u_\parallel$, three intense spectral peaks appear at $9\Omega_H$, $10\Omega_H$ and $11\Omega_H$. For the green traces in Fig. 16, with $u_\parallel = 2.42V_A$, the dominant spectral peaks are at: $9.50\Omega_H$, $10.44\Omega_H$ and $11.30\Omega_H$ for 91.0° propagation angle; and at $9.57\Omega_H$, $10.50\Omega_H$ and $11.30\Omega_H$ for 90.8°. We note im-
Figure 15: The power spectrum of $\delta B_z$ (left) is constructed from the spatiotemporal Fourier transform of $\delta B_z$ (right). These are plotted on a dB and $\log_{10}$ scale respectively, for the fields excited by the relaxation of a 3.02MeV proton population initialised with $[n_H/(2\pi v_\perp)] \delta(v_{\parallel} - u_{\parallel}) \delta(v_{\perp} - u_{\perp})$, $u_{\perp} = 1.05 V_A$ and $u_{\parallel} = 2.42 V_A$. The propagation angle between $\mathbf{k}$ and $\mathbf{B}_0$ is 91.0°, and the majority thermal ions are deuterons.
Figure 16: Power spectra of $\delta B_z$ from PIC-hybrid computations with the orientation of the spatial domain $\hat{x}$ at an angle of 91.0° (left) and 90.8° (right) with respect to the background magnetic field. Only excited waves propagating in the $+\hat{x}$ direction are included. The initial energetic proton distribution functions are $[n_H/(2\pi v_{\perp})]\delta(v_{\parallel} - u_{\perp})$ (blue trace) and $[n_H/(2\pi u_{\perp})]\delta(v_{\perp} - u_{\parallel})\delta(v_{\perp} - u_{\perp})$ (green trace). The velocities are $u_{\parallel} = 2.199 \times 10^7$ms$^{-1}$ and $u_{\perp} = 0.956 \times 10^7$ms$^{-1}$, corresponding to $u_{\perp} = 1.05V_A$ and $u_{\parallel} = 2.42V_A$ and are such that $m_H(u_{\parallel}^2 + u_{\perp}^2)/2 = 3.02$MeV. The dominant spectral peaks for the green traces are at: (left) 9.50$\Omega_H$, 10.44$\Omega_H$ and 11.30$\Omega_H$; (right) 9.57$\Omega_H$, 10.50$\Omega_H$ and 11.30$\Omega_H$.

Figure 16 immediately that the spectral peak frequencies are shifted by approximately $\Omega_H/2$, similar to the observational shifts noted at the start of sub-section 4.1.

For a second set of simulations using protons initialised with $u_{\perp} = 0.950V_A = 0.865 \times 10^7$ms$^{-1}$ and $u_{\parallel} = 2.470V_A = 2.244 \times 10^7$ms$^{-1}$, at a propagation angle of 89.0°, the MCI-excited spectrum is shown in Fig. 17. This has major peaks at 9.49$\Omega_H$, 10.57$\Omega_H$ and 11.57$\Omega_H$.

As noted at the start of sub-section 4.1, the observed frequencies of the ICE spectral
Table 4: Location of the four major peaks in the simulated ICE spectrum, in units of $\Omega_H$, for drifting ring-beam populations of minority 3.02 MeV protons initialised with five different combinations of $(u_{\perp}, u_{\parallel})$ as shown. In all cases the waves are forward propagating, and the propagation angle between the simulation domain and the background magnetic field $B_0$ is 91.0°. The frequency resolution in the computed spectra is ±0.07$\Omega_H$.

peaks $b$, $c$ and $d$ in Fig. 2 could be provisionally identified with 9.6$\Omega_H$, 10.6$\Omega_H$ and 11.6$\Omega_H$. The extent of agreement between these experimental values and the results of first principles simulation embodied in the green spectra of Figs. 16 and 17 appears encouraging. The four dominant spectral peaks as a function of $(u_{\perp}, u_{\parallel})$ are given in Tables 4 and 5 for waves propagating in the $+\hat{x}$ direction of the simulation domain, at angles of 91.0° and 89.0° with respect to the background magnetic field $B_0$.

There is a further interesting aspect to the simulated power spectra shown by green traces in Figs. 16 and 17, which correspond to simulations with super-Alfvénic $u_{\parallel}$: the peaks at lower frequencies lie very close to low integer cyclotron harmonics. This is in contrast to the substantial frequency shifts that are visible at higher harmonics, and are tabulated in Tables 4 and 5.
Table 5: The same as Table 4 for a propagation angle of 89°.

<table>
<thead>
<tr>
<th>$u_\perp$, $u_\parallel$ ($V_A$)</th>
<th>$\ell$</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_\perp = 0.90$, $u_\parallel = 2.48$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>10.57</td>
<td>11.64</td>
<td>12.64</td>
<td>13.7</td>
<td></td>
</tr>
<tr>
<td>$u_\perp = 0.95$, $u_\parallel = 2.47$</td>
<td>-</td>
<td>-</td>
<td>9.50</td>
<td>10.57</td>
<td>11.57</td>
<td>12.57</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$u_\perp = 1.00$, $u_\parallel = 2.44$</td>
<td>-</td>
<td>8.44</td>
<td>9.50</td>
<td>10.50</td>
<td>11.50</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$u_\perp = 1.05$, $u_\parallel = 2.42$</td>
<td>7.30</td>
<td>8.37</td>
<td>9.44</td>
<td>10.37</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$u_\perp = 1.10$, $u_\parallel = 2.39$</td>
<td>7.30</td>
<td>8.37</td>
<td>9.44</td>
<td>10.37</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Figure 17: Power spectrum of $\delta B_z$ from a PIC-hybrid computation for waves excited propagating in the +x direction, which is oriented at an angle of 89.0° with respect to the background magnetic field. The initial energetic proton distribution functions have the form defined in the caption to Fig. 16, with $u_\perp = 0.95V_A$ and $u_\parallel = 2.47V_A$. The dominant spectral peaks for the green traces are at $9.49\Omega_H$, $10.57\Omega_H$ and $11.57\Omega_H$. 
5 CONCLUSIONS

We have performed additional simulations spanning a range of parallel velocities in $v_{\parallel}/V_A = [0.0 : 0.1 : 2.6]$ at fixed $v_{\perp}$ showing that the resonance position varies smoothly with $v_{\parallel}$ and achieves significant spectral shifts for $v_{\parallel} \sim V_A$ and observed the same behaviour when repeating the process with a different value of $v_{\perp}/V_A$.

5 Conclusions

The transient ICE spectrum from LHD plasma 133979, and in particular peaks $b$, $c$ and $d$ of Fig. 2, present an interesting and potentially important challenge to the understanding of energetic ion physics in this LHD heliotron-stellarator plasma, and more widely. While the frequency separation of successive peaks can be identified with the proton cyclotron frequency $\Omega_H$ at a specific radial location in the outer edge plasma, the peak frequencies cannot be identified with integer multiples of $\Omega_H$: instead they are closer to integer-plus-one-half values. There are two energetic proton populations that are present in this deuterium plasma, with velocity-space inversions which could potentially enable them to drive the magnetoacoustic cyclotron instability and hence ICE. These are NBI-injected ions, and fusion-born protons which are created at 3.02 MeV. Careful analysis, see Table 1, shows that a distinct velocity-space subset of the fusion-born protons is confined, as distinct from being promptly lost, depending on pitch angle at birth in the spatial location of interest. In this paper, we have shown that this proton population is capable of driving the ICE signal, and is a more likely source than the NBI ions.
5 CONCLUSIONS

We have carried out multiple PIC-hybrid computations of the collisionless relaxation of a freshly fusion-born proton population at 3.02 MeV, with a restricted range of pitch angles reflecting Table 1, within a majority thermal deuterium plasma with parameters appropriate to LHD plasma 133979. The computations follow first-principles full ion-gyro-orbit self-consistent Maxwell-Lorentz physics for very large numbers of particles. Taken together, the phenomenology of particles and fields is consistent with the analytical theory of the MCI. The power spectrum constructed from the temporal Fourier transform of the excited fields, in the saturated regime of the instability, constitutes our simulated ICE spectrum.

All our simulated ICE spectra show dominant spectral peaks separated approximately by the proton cyclotron frequency $\Omega_H$ evaluated at $R = 4.234m$, in the frequency range between $8\Omega_H$ and $12\Omega_H$, see section 4. We find good mappings from the simulated ICE spectra to the three intense peaks from the measured LHD spectrum in Fig. 2, provided that the protons are initialised with $v_\parallel = 2.5V_A$, consistent with $v_\perp \sim V_A$ and $E_H = 3.02$ MeV. The excited waves propagate almost perpendicular to the background magnetic field, that is, radially out of the plasma. The simulated results have been obtained using a simple drifting ring distribution for the protons in velocity-space, and additional simulations show that the inclusion of thermal spread in the ring only slightly affects the relative strength of the most intense harmonics. The propagation angle also affects the calculated spectra. Having noted these relatively minor sensitivities, the underlying conclusion appears robust: the measured ICE spectrum from LHD
5 CONCLUSIONS

dideuteron plasma 133979 shown in Fig. 2 is probably excited by the fast relaxation of a transient local population of fusion-born protons at 3.02 MeV whose perpendicular velocity is close to the Alfvén speed, and whose parallel velocity is therefore $\sim 2.5$ times higher. We believe this may be the first observation of collective radiation from a confined population of fusion born-ions to be reported from a heliotron-stellarator plasma. Disambiguation between two or more energetic ion species that could potentially generate complex observed ICE spectra is an increasing challenge, and the results and methodology developed here will assist this. Our approach is also expected to be relevant to ICE driven by ion beams with lower parallel velocities, for example in cylindrical plasma experiments.

This first probable detection of collective electromagnetic radiation from fusion-born ions in a stellarator-type plasma is an encouraging development for MCF plasma physics, and shows the flexibility of the LHD heliotron-stellarator and its diagnostic systems. This result also demonstrates the role of ion cyclotron emission as a particularly sensitive diagnostic for fusion-born ion populations. Here on LHD, as previously in JET, TFTR and KSTAR [9, 10, 41, 24], ICE has been detected from a quasi-trace subset of the fusion-born ion population with distinctive properties in velocity-space, whose presence was unsuspected until their ICE was detected and then interpreted. Arguably this sensitivity reinforces the case for the adoption of ICE as a fast ion diagnostic in ITER.
6 ACKNOWLEDGMENTS

6 Acknowledgments

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 and 2019-2020 under grant agreement No 633053. The work received support from the RCUK Energy Programme [grant number EP/T012250/1], NIFS budget NIFS15KLPF045 and from NRF Korea grant no. 2014M1A7A1A03029881. The views and opinions expressed herein do not necessarily reflect those of the European Commission. ROD acknowledges the hospitality of Kyushu University. BCGR acknowledges helpful discussions with Prof. Mark Koepke, Dr. Kenneth G. McClements and with Dr. Leopoldo Carbajal-Gomez. The authors are grateful to both referees for their painstaking and constructive questions and commentary on the initially submitted version of this manuscript.

A Positive and negative frequency shifts

In evaluating the frequency shifts, we are interested in the values taken by $\omega - k_{\parallel}v_{\parallel}$ and particularly in the sign of the term $k_{\parallel}v_{\parallel}$. This sign depends on the respective orientations of $v$ and of $k$. In the simulations, $v_{\parallel}$ is an input which is always set positive, but could also be negative, while positive and negative values of $k_{\parallel}$ both co-exist in the simulations; this enables coverage of both possible signs of the frequency shift. In other words, for a fixed sign of $v_{\parallel}$, co- and counter-propagating waves will
A  POSITIVE AND NEGATIVE FREQUENCY SHIFTS

result in opposite signs of $k_{||}$. If instead, the direction of $k_{||}$ is set, fusion-born ions with either $v_{||} > 0$ or $v_{||} < 0$ could account for the sign. Typically, PIC simulations for ICE have $B_0 = B_0\hat{z}$, that is $B_0$ is perpendicular to the one-dimensional spatial simulation domain, whose direction is denoted by $\hat{x}$. In our current work, $B_0$ is almost perpendicular to the $x$-axis. If the angle between $B_0$ and $\hat{x}$ is let us say 89.0 degrees, the projection $B_{0,x}$ of $B_0$ on the $x$-axis is positive, whereas if this angle is 91 degrees, then $B_{0,x}$ is negative. The foregoing are the combinations of the parameters $(B_0, \pm ||k||, \pm v_{||})$ which eventually determine the sign of $k_{||}v_{||}$. There are symmetries, in that changing the angle between the background magnetic field and the $x$-axis from $(90^\circ - \alpha)$ degrees to $(90^\circ + \alpha)$ degrees, and changing the sign of $k$ at the same time yield the same power spectra. As a corollary, the power spectra summed over both positive and negative values of the wavevector along the simulation direction $\hat{x}$ ($k \cdot \hat{x} \leq 0$) are identical in these two cases. The Fourier transform decomposes our simulated signals in $\exp (+i (\omega t + k \cdot x)) = \exp (+i (\omega t + kx))$, $k$ the component along $\hat{x}$. The phase $\omega t + kx$ corresponds to waves propagating forward, in the $+\hat{x}$ direction when $k < 0$ and backward in the $-\hat{x}$ direction when $k > 0$. By convention, the power spectra are calculated by considering $\omega > 0$ in Fig. 18 and summing over the relevant range of $k$-values as shown in Figures 20 and 21.

We provide two possible explanations for the unequal frequency spacing in these power spectra. The first relates to the frequency resolution which depends on the length of the time series. The Nyquist criterion implies that the maximum frequency
Figure 18: Spatiotemporal Fourier transform of the fluctuating part of $\delta B_z$. The angle between $\mathbf{x}$ and $B_0$ is $91^\circ$. The physical quantities are real numbers and this implies that the top right and bottom left quadrants are identical, as are the top left and bottom right quadrants. This is because real signals $\cos(\omega \pm kx) = 1/2 (\exp(i(\omega t \pm kx)) + \exp(-i(\omega t \pm kx)))$ contain both positive and negative frequencies of equal amplitudes. When $\omega$ and $k$ have the same sign, the waves propagate backward, these correspond to waves that lie in the top right and bottom left quadrants. Conversely, if $\omega$ and $k$ have opposite signs, the waves propagate forward, these correspond to waves that lie in the top left and bottom right quadrants.
A POSITIVE AND NEGATIVE FREQUENCY SHIFTS

Figure 19: The proton cyclotron harmonics zoomed from Figure 18 are shifted differently depending on the sign of wavenumber $k$. The right (left) zone in the top panel is symmetric to the left (right) area in the bottom panel.
A  POSITIVE AND NEGATIVE FREQUENCY SHIFTS

Figure 20: Geometry setting (left panels) and power spectra of $\delta B_z$ (right panels) from PIC-hybrid computations with the orientation of the background magnetic field at an angle of $89.0^\circ$ (top panels) and $91.0^\circ$ (bottom panels) with respect to the spatial domain $+\hat{x}$. The initial value of $u_\parallel$ is strictly positive. Excited waves propagating in both the $+\hat{x}$ direction (spectrum summed over $k < 0$, see Fig. 18; top panels) and in the $-\hat{x}$ direction (spectrum summed over $k > 0$, see Fig. 18; bottom panels) result in the same power spectra when changing the angle from $-\alpha$ to $+\alpha$ between the vectors $\hat{z}$ and $B_0$. Proton cyclotron harmonics are shifted to the left since $k_\parallel u_\parallel > 0$. The initial energetic proton distribution functions are $\frac{[n_H/(2\pi v_{\perp})] \delta(v_\parallel) \delta(v_{\perp} - u_{\perp})}{(2\pi v_{\perp})} \delta(v_\parallel - u_\parallel)$ (blue trace) and $\frac{[n_H/(2\pi u_{\perp})] \delta(v_{\parallel} - u_{\parallel}) \delta(v_{\perp} - u_{\perp})}{(2\pi u_{\perp})}$ (green trace). The magnitude of the background magnetic field is 1.75T. The magnitude of the background magnetic field is 1.75T. The velocities are $u_\parallel = 2.199 \times 10^7 \text{ms}^{-1}$ and $u_{\perp} = 0.956 \times 10^7 \text{ms}^{-1}$, corresponding to $u_\parallel = 2.42V_A$ and $u_{\perp} = 1.05V_A$ and are such that $m_H(u_\parallel^2 + u_{\perp}^2)/2 = 3.02\text{MeV}$. 

\[ \cos^{-1}(\hat{k} \cdot \hat{b}_0) = 89^\circ \]
\[ \cos^{-1}(\hat{k} \cdot \hat{b}_0) = 91^\circ \]
A POSITIVE AND NEGATIVE FREQUENCYhiftS

Figure 21: Geometry setting (left panels) and power spectra of $\delta B_z$ (right panels) from PIC-hybrid computations with the orientation of the background magnetic field at an angle of $89.0^\circ$ (top panels) and $91.0^\circ$ (bottom panels) with respect to the spatial domain $\hat{x}$. The initial value of $u_\parallel$ is strictly positive. Excited waves propagating in both the $-\hat{x}$ direction (spectrum summed over $k < 0$; top panels) and in the $+\hat{x}$ direction (spectrum summed over $k > 0$; bottom panels) result in the same power spectra when changing the angle from $-\alpha$ to $+\alpha$ between the vectors $\hat{z}$ and $B_0$. Proton cyclotron harmonics are shifted to the right since $k_\parallel u_\parallel < 0$. The initial energetic proton distribution functions are $[n_H/(2\pi u_\perp)] \delta(v_\parallel - u_\parallel)$ (blue trace) and $[n_H/(2\pi u_\perp)] \delta(v_\parallel - u_\parallel) \delta(v_\perp - u_\perp)$ (green trace). The magnitude of the background magnetic field is 1.75T. The velocities are $u_\parallel = 2.199 \times 10^7 \text{ms}^{-1}$ and $u_\perp = 0.956 \times 10^7 \text{ms}^{-1}$, corresponding to $u_\parallel = 2.42V_A$ and $u_\perp = 1.05V_A$ and are such that $m_H(u_\parallel^2 + u_\perp^2)/2 = 3.02\text{MeV}$. 
A POSITIVE AND NEGATIVE FREQUENCY SHIFTS captured in our calculations corresponds to \( \nu_{\text{max}} = \frac{1}{2\Delta t} \) with \( \Delta t \) the time step of the simulation chosen to be \( 5 \times 10^{-4} \tau_H \), with \( \tau_H \) the proton gyroperiod. The time step for saving the numerical outputs used for the Fourier transforms is \( \Delta t = 5 \times 10^{-3} \tau_H \). Thus \( \nu_{\text{max}} = \frac{1}{(2 \times 5 \times 10^{-3} \times \tau_H)} = 100/\tau_H \) and therefore \( \omega_{\text{max}}/\Omega_H = 100 \).

Conversely, the frequency spacing \( \Delta \omega \) is given by \( 2\pi/T \), with \( T \) the duration of the simulation and taken to be \( T = 15 \times \tau_H \), which leads to \( \Delta \omega = 2\pi/(15 \times \tau_H) = \Omega_H/15 = 0.07\Omega_H \). This value is introduced in the caption of table 4 and can partially explain the different frequency spacings of \( 0.87 - 0.94\Omega_H \) and \( 0.92 - 1.07\Omega_H \). We have also performed simulations over \( 60\tau_H \) to increase the resolution in frequency space to \( \Delta \omega = 0.017\Omega_H \) and observe the same phenomenology: the spectral peaks spacing is not exactly constant. Nonlinear beating between cyclotron harmonics could give rise to additional excitations. This mechanism has been successfully tested in previous works, see for example Fig. 6 of Ref. [34] and additional nonlinear wave analysis in Fig. 4 of Ref. [28]. This phenomenon could give us a hint as to why the frequency spacing appears uneven: low cyclotron harmonics are linearly stable and grow due to the beating between high-harmonics-spectrally-shifted cyclotron waves. The spectral shift of the linearly stable cyclotron harmonics \( \omega_1 \), driven non-linearly, would result from the sum (or difference) of the high frequency, linearly unstable, waves \( \omega_2 \) and \( \omega_3 \), namely \( \omega_3 = \omega_1 + \omega_2 \). Therefore the spectral shift of \( \omega_3 \) would be the sum of the spectral shifts of \( \omega_1 \) and \( \omega_2 \) and explain the uneven frequency spacing. In particular, as shown in the power spectra of Fig. 22, which correspond to the bottom right panels of Figs 20 and 21, the
fundamental cyclotron frequencies are slightly offset. On the left panel, the first peak of
the green trace locates between 0.86 and 0.90Ω_H and the most intense peaks are located
at 9.503, 10.440 and 11.300Ω_H. The successive differences give 10.440 − 9.503 = 0.940
and 11.300 − 10.440 = 0.860Ω_H. On the right panel, the first peak of the green trace
is located at 1.034Ω_H and the strongest peaks are at 8.369Ω_H, 9.436Ω_H and 10.370Ω_H.
Therefore the differences between the three successive intense peaks correspond roughly
to the down-shifted and up-shifted fundamental proton cyclotron frequencies which
coexist in the simulations. These suggest the presence of nonlinear wave interactions
and are related to the faint spots away from the linear fast-Alfvén wave branch in the
dispersion relation graph of Fig. 18.
A POSITIVE AND NEGATIVE FREQUENCY SHIFTS

Figure 22: Power spectra of $\delta B_z$ from PIC-hybrid computations with the orientation of the spatial domain $\hat{x}$ at an angle of 91.0° with respect to the background magnetic field. Excited waves propagating in both the $+\hat{x}$ direction (left panel) and in the $-\hat{x}$ direction (right panel) are included. The initial energetic proton distribution functions are $\left[n_H/(2\pi v)\right] \delta(v|| - u||)$ (blue trace) and $\left[n_H/(2\pi u)\right] \delta(v|| - u||)$ (green trace). The velocities are $u|| = 2.199 \times 10^7$ ms$^{-1}$ and $u\perp = 0.956 \times 10^7$ ms$^{-1}$, corresponding to $u\perp = 1.05V_A$ and $u|| = 2.42V_A$ and are such that $m_H(u||^2 + u\perp^2)/2 = 3.02$ MeV. The dominant spectral peaks for the green traces are at: (left) 9.50$\Omega_H$, 10.44$\Omega_H$ and 11.30$\Omega_H$; (right) 8.37$\Omega_H$, 9.44$\Omega_H$ and 10.37$\Omega_H$. 
B Simulation at 85°, higher $k_\parallel$

We have shown in Section 4.1 that strong shifts can be achieved for angles as low as 1° or 2° from perpendicular to $B_0$. Strong shifts can in principle also be achieved at small $v_\parallel$ with high $k_\parallel$. We have run a hybrid-PIC simulation at 85 degrees which leads to higher values of $k_\parallel$ compared to those run at 89°. The thermal electron and deuteron temperatures are 846eV and 907eV respectively, while the electron number density is $8.8 \times 10^{18} \text{m}^{-3}$ and $B_0 = 1.75 \text{T}$ such that $V_A = 0.9105 \times 10^7 \text{m/s}$ as per section 3. The power spectrum corresponding to a propagation angle of 85° (89°) in Fig. 23 is obtained using 4000 (2000) macroparticles per cell, with 4096 (8192) grid cells and a relative fusion born-proton density $\xi = 0.001 (0.005)$ and calculated over $15\tau_H (35\tau_H)$. The instability unfolds more slowly at 85° and is the reason for using a higher $\xi$-value [14]. These power spectra further stress the asymmetry between spectral peaks obtained either with $k > 0$ or $k < 0$, the asymmetry being both in peak location and peak amplitude. We also note that the most intense peaks at 85° are obtained at lower cyclotron harmonics (1-6) while at 89°, they appear at higher cyclotron harmonics (8-12). This follows the trend of Figure 13 suggesting that ICE is less strongly driven between the 8th - 12th proton cyclotron harmonics when the propagation angle departs from perpendicular. Although other pairs of $(k_\parallel, k_\perp)$ and $(v_\parallel, v_\perp)$ are possible for significant spectral shift, we have focused on quasi-perpendicular propagating waves as these simultaneously show excitations in the range 8th to 12th proton cyclotron harmonics and generate substantial frequency shifts when they are driven by fusion-born protons.
Figure 23: Power spectra obtained at a propagation angle of 89° (left) and 85° (right). The simulation parameters are given in the text.

69 keV ⊥ D-NBI and 170 keV || H-NBI

The plasma studied in the present work was heated by 69 keV perpendicular NBI deuterons and 170 keV parallel NBI protons. We have therefore tested the hypothesis that 69 keV NBI deuterons drive ICE by carrying a simulation using the plasma parameters reported in the manuscript (thermal electron and deuteron temperatures $T_e=846\,\text{eV}$ and $T_D=907\,\text{eV}$ respectively, $n_e=8.8 \times 10^{18}\,\text{m}^{-3}$ and $B_0=1.75\,\text{T}$). The simulation grid consisted of 8192 cells, each initially loaded with 2000 thermal deuterons per cell and 2000 NBI deuterons per cell. These 69 keV NBI deuterons are initialised in velocity space with a ring-beam distribution $\propto \delta (v_\parallel) \delta (v_\perp - u_{NBI})$ and the relative density is chosen to be $n_{DNBI}/n_e = 0.020$. The angle between the background magnetic field and the simulation domain is set to 91°. The power spectrum appearing
Figure 24: Time evolution of the change in energy density of the electric and magnetic fields and of the ion species following the relaxation of a 69 keV deuteron NBI population (left). Power spectrum of the fluctuating $\delta B_z$ averaged over the simulation duration together with the thermal noise of the thermal deuteron plasma.

on the right panel of Fig. 24, which is normalised to the proton cyclotron frequency, suggests that ICE is excited in a different (higher) range of frequencies. It also shows that both even and odd deuteron cyclotron harmonics are excited and preferentially at higher frequencies compared to those of the present work. The fusion-born protons are able to drive strongly and simultaneously ICE at cyclotron harmonics 8th - 12th while the calculation with 69 keV NBI deuterons suggests strong excitations above $11 \Omega_H$. The inclusion of a parallel velocity component would further decrease the energy in the perpendicular component and thus lower the drive of ICE. Simulations of NBI deuterons in LHD deuterium plasma have also been presented in Ref. [62]. We have also focused on the disentanglement of ICE driven by fusion-born protons and 170keV
parallel-NBI protons. We have performed fully kinetic simulations that compare the drive of energetic protons initialised in velocity space with a ring-beam distribution \( \propto \delta(v_\perp - u_\perp) \) where the value \( u_\perp \) is chosen to correspond either to a 170keV NBI proton population or a 476keV Alfvénic subpopulation of fusion-born protons in LHD. We have initialised our full-PIC calculations using 1400 macroparticles per cell and per species (electrons, thermal deuterons and energetic protons) on a one-dimensional simulation domain consisting of 50000 cells of size \( 0.9 \lambda_{De} \), with the electron Debye length \( \lambda_{De} = 7.29 \times 10^{-5} \text{m} \). The relative energetic proton population is \( \xi = 0.002 \) and the angle between the background magnetic field and the simulation domain is 90°. The thermal electron temperature and density together with the thermal deuteron temperature are identical to those of the relaxation calculation of the 69keV NBI deuterons. The simulation results in Figs. 25 and 26 suggest that the frequencies that are most strongly driven by 170keV NBI protons locate around the 12th proton cyclotron harmonic while the LHD measured power spectra of Fig. 2 indicate excitations in the range 8th to 11th cyclotron harmonics.
Figure 25: Energy density time evolution of fields, thermal electrons and deuterons together with energetic minority proton population initialised as a ring-beam $n_H/(2\pi u_\perp)\delta (v_\parallel) \delta (v_\perp - u_\perp)$ in a full-PIC simulation. The electron density is $n_e = 8.8 \times 10^{18}\text{m}^{-3}$ and the background magnetic field is $B_0 = 1.75\text{T}$, perpendicular to the simulation domain. The electron and deuterium temperatures are respectively 846eV and 907eV. These correspond to the parameters during the transient ICE event in LHD plasma 133979. Left and right panels correspond respectively to 170keV beam protons and 476keV fusion-born protons characterised by $u_\perp/V_A = 0.627$ and $u_\perp/V_A = 1.050$. Since the propagation angle is exactly 90°, no spectral shifts would be observed if a non-zero parallel velocity component was included. This motivates the relaxation of two different energetic proton population of which energy is solely in the perpendicular velocity component. In addition, the lower hybrid frequency is minimal at 90° and leads to the lowest harmonics that these proton population can excite when the electron density is $8.8 \times 10^{18}\text{m}^{-3}$. The top panels share the same energy scales and show that the fusion-born proton population with $E_\perp = 476\text{keV}$ releases 5× more energy than the 170keV proton beam at equal relative density $\xi = n_H/n_e = 0.002$. The second qualitative difference comes from the nature of the fields excited: predominantly electrostatic with 170keV protons and electromagnetic with 476keV protons. The top left energy time evolution panel is zoomed on the bottom left panel and the panel on the top right is reproduced at the bottom right.
C 69 KEV ⊥ D-NBI AND 170 KEV || H-NBI

170 keV

δBz

Wavenumber |ΩH/Vₐ|

Frequency |ΩH|

Intensity (dB scale)

Eₓ

Wavenumber k |ΩH/Vₐ|

Frequency |ΩH|

Intensity (dB scale)

476 keV
\[ C \ 69 \text{KEV} \perp D-NBI \text{ AND } 170 \text{KEV} \parallel H-NBI \]

Figure 26: Spectral power on a log scale resulting from the spatiotemporal fast Fourier transform of the \( z \)-component of the perturbed magnetic field (top) taken over the simulation duration (10 proton gyropariods) and over the entire simulation domain \( (50000\lambda_{De}) \). The left panels relate to the relaxation of a 170keV proton beam and the right panels to a proton population of 476keV. In both cases, these energetic proton populations are initialised as ring-beams \( n_H/(2\pi u_\perp)\delta(v_\parallel)\delta(v_\perp - u_\perp) \) such that the energy is concentrated in the perpendicular velocity components and the parallel energies are initialised to zero. Averaging the dispersion relation on the top panels between \( k = 0 \) and \( k = 40\Omega_H/V_A \) gives rise to the power spectra on the middle panels represented on a dB scale as shown by the blue curves. The green curves represent the noise level obtained by running a simulation with thermal electrons and deuterons only. A similar procedure is conducted for the \( x \)-component of the electric-field and leads to the bottom panels. The spectral character differs in that excitations evolve from predominantly electrostatic excitation with the relaxation of 170keV protons to mainly electromagnetic excitations with 476keV protons. These populations have no parallel velocity component. The 170keV sub-Alfvénic proton population \( (u_\perp/V_A = 0.672) \) gives rise to one major peak, whose maximum is not necessarily at an integer cyclotron harmonic, and locates between \( 12\Omega_H \) and \( 13\Omega_H \). Increasing the energy leads to the Alfvénic regime of the 476keV energetic protons \( (u_\perp/V_A = 1.050) \) displaying excitations of the MCI between \( 8\Omega_H \) and \( 12\Omega_H \) which we shall refer to as the MCI sector.
D FREQUENCY SHIFTS NOT UNIQUE TO LHD PLASMA

D Frequency shifts not unique to LHD plasma

In LHD plasma 164962, for which $R_{ax} = 3.6m$ and $B_t = 2.85T$, bursting ICE was observed with spectral peaks corresponding to large frequency shifts with respect to cyclotron harmonics identified with the fusion-born protons. Figure 27 shows that these peaks correspond to either $28 \times (n + 0.5)MHz$ or $28 \times (n - 0.5)MHz$. 
Figure 27: Measured power spectrum detected by the dipole antenna in LHD plasma 164962 at $t = 4.255s$. The most intense spectral peaks have frequencies which correspond to $n = 28 \times (n + 0.5)\text{MHz}$ and $n = 28 \times (n - 0.5)\text{MHz}$, where $n$ is an integer, and are therefore identified as highly shifted cyclotron harmonics of the energetic protons.
Noise in the simulation.

We have quantified the noise level in our hybrid PIC calculations by running one simulation which consists of a deuterium thermal plasma only. We do not compare it with the noise arising from the LHD detection system. Our purpose is only to show that a good signal-to-noise ratio is achieved in our calculations. The thermal plasma shows preferential excitations at normal modes, which are the cyclotron harmonics as per the green trace in Fig. 28 in agreement with the fluctuation dissipation theorem [79].

![Figure 28: Simulated power spectra obtained for the simulation parameters given in section 4.3, namely 2000 macroparticles per cell and per ion species and 8192 cells, \( \xi = 0.001 \) and 91° propagation angle. The green trace corresponds to the spectrum obtained without energetic protons.](image-url)


REFERENCES

References


REFERENCES


REFERENCES


REFERENCES


REFERENCES


REFERENCES


REFERENCES


REFERENCES


[40] B Chapman, RO Dendy, SC Chapman, KG McClements, and R Ochoukov. Origin of ion cyclotron emission at the proton cyclotron frequency from the core of deu-
REFERENCES


[46] R.O. Dendy, Chris N. Lashmore Davies, and K.F. Kam. A possible excita-
REFERENCES


REFERENCES


[56] Ryonsuke Seki, Yutaka Matsumoto, Yasuhiro Suzuki, Kiyomasa Watanabe, and Masafumi Itagaki. Particle Orbit Analysis in the Finite Beta Plasma of the Large
REFERENCES


REFERENCES


[65] GH DeGrandchamp, KE Thome, WW Heidbrink, I Holmes, and RI Pinsker. Up-
REFERENCES


[70] BS Schmidt, M Salewski, B Reman, RO Dendy, D Moseev, R Ochoukov, A Fasoli, M Baquero-Ruiz, and H Järleblad. Determining 1d fast-ion velocity distribution


REFERENCES


