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Cooperative Satellite-Aerial-Terrestrial Systems: A Stochastic Geometry Model

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Abstract—Nowadays, satellite and aerial platforms are playing an important role in realizing global seamless wireless coverage. In this paper, a cooperative satellite-aerial-terrestrial network (SATN) is considered, in which two kinds of relaying links, satellite and aerial relaying links, are used to assist a group of aerial terminals to forward their information to a remote terrestrial destination (D). Specifically, we model these aerial platforms sharing the same frequency band as a Matérn hard-core point process type-II. Also, a group of aerial jammers at D's side is modeled as a Poisson point process. To demonstrate the end-to-end (e2e) performance of the two relaying links, the statistical characteristics of the received signal-to-interference are characterized and then a closed-form expression for the outage probability (OP) over the uplink from the aerial source to the satellite/the aerial relay, the downlink from the satellite/the aerial relay to D, and the inter-aerial relay link are derived. Numerical results are presented to verify the proposed analysis models and compare the outage performance of the considered cooperative SATN with the two relay links under numerous scenarios.

Index Terms—Matérn hard-core point process, outage probability, satellite-aerial-terrestrial communication, stochastic geometry

I. INTRODUCTION

Owing to the recent advancement of materials and manufacturing technologies in the electronics industry, satellites and aerial platforms have been deployed to serve as space and aerial relays or base stations (BS) to provide global seamless coverage. For example, thousands of low orbit satellites have already been launched into space by SpaceX to provide satellite Internet access to most of the Earth. On the other hand, high-altitude platforms, usually unmanned airships or airplanes positioned above 20 km, and low-altitude platforms, like unmanned aerial vehicles (UAVs) operating 100s to 1000s meters, have already been deployed for commercial, emergency, and military applications. However, compared with traditional ground BSs with uninterruptible power supply, satellites and aerial platforms suffer from limited power budget and hardware resources [1]–[3].

Due to the unique characteristics of satellite/aerial-terrestrial and satellite-aerial-terrestrial communication systems, e.g., system complexity, rapid variability, and large-scale operation space, it is crucial but challenging to optimally allocate system resources for optimal system performance, e.g., energy efficiency/power consumption, coverage, capacity/throughput, etc. For instance, recent works aim to maximize the energy efficiency/the power consumption for satellite-aerial-terrestrial networks (SATNs)/satellite-terrestrial systems [4]–[9]. Other researchers paid their attention to improving the transmission rate/the throughput for SATNs/satellite-terrestrial systems [10]–[12] or to achieving other optimal performance indices for SATN/satellite-terrestrial systems [13]–[17].

In practice, except from optimizing the performance of SATNs/satellite-aerial/terrestrial systems, it is also vital to thoroughly understand how such systems operate, what kinds of system factors can affect their performance, and the principles that these system factors influence the performance. Therefore, some other researchers engaged in modeling and analyzing the performance of SATNs/satellite-aerial/terrestrial systems to uncover their operation principles in numerous working scenarios.

As the most popular system architecture for the applications of communication satellites, satellite-aerial/terrestrial communications have gained considerable attention from researchers [18]–[27]. However, sometimes, the terrestrial terminals may not be able to catch the signal transmitted by the satellite over such a long-distance space-to-ground link, due to the unavailability of line-of-sight (LoS) transmissions between the satellite and the terrestrial receivers coming from the deep fading or shadowing. Moreover, increasing the transmit power at the satellite/ground-user will not be able to get out of this trouble. Intuitively, to effectively work out such problems, introducing relays to bridge an alternative way for the information transmissions between the satellite and terrestrial terminals will be the first but best choice. Generally, there are two kinds of relays that have been brought to satellite communication systems: terrestrial and aerial relays. By integrating terrestrial relay, conventional satellite communication systems evolve into hybrid satellite-terrestrial relay networks (HSTRN), which have been extensively investigated. Authors of [28] derived a closed-form an-
alytical expression of the \( b \)-th moments of the complementary cumulative distribution functions of signal-to-interference-to-noise ratio (SINR) for the satellite-to-relay link and relay-to-user link in an integrated low earth orbit (LEO) decode-and-forward (DF) millimeter-wave HSTRN. The average symbol error rate performance in a multiuser HSTRN with opportunistic scheduling was studied in [29]. In [30], the authors investigated the outage performance of a multi-relay multiuser HSTRN operating at millimeter-wave bands, while considering that the dominant fading factor of the mmWave band is rain attenuation. In [31], the bit error rate and spectral efficiency were analyzed for a multi-relay broadcast HSTRN by considering the joint impact of carrier frequency offset and phase noise. Authors of [32] derived novel and exact outage probability (OP), symbol error probability, and achievable rate expressions for a two-way HSTRN over generalized fading channels. In [33], the outage performance of non-orthogonal multiple access (NOMA)-based HSTRN was analyzed.

On the other hand, due to the flexibility of deployments, aerial relays, e.g., high and low altitude platforms including drones, air-crafts, and airships, have also been introduced into satellite-terrestrial communication scenarios to increase the probability of LoS propagation and further improve the transmission performance of satellite-terrestrial communication systems. Then, SATNs emerged as the most promising network configuration of satellite communications and are becoming a hot topic in the satellite communication area. However, the system complexity of SATNs is more complicated than that of traditional satellite-terrestrial systems, resulting in the fact that depth and extensive research is urgently required. So far, some related works have already been proposed to study the outage, capacity, symbol error, and secrecy performance of SATNs [34]–[39].

Observing these aforementioned studies, one can find that only the transmission over downlink or uplink has been investigated and the end-to-end (e2e) transmission performance has not been uncovered yet. As everyone knows, satellites normally serve as space relays for two remote ground communication partners. So, one sees that the study of the e2e transmission performance for the whole satellite communication system still keeps blank and remains to be understood.

Moreover, it is also easy to find that, till now, only very few works [4], [18]–[20], [25], [26], [34], [36] have studied the impacts of the randomness of the distribution of space/aerial/terrestrial terminals on the overall performance of these targeted systems. However, the positions of space/aerial/terrestrial terminals in SATNs exhibit stronger randomness in such large-scale operation space, causing the strong randomness of the transmission distance among these terminals. Therefore, the path-loss experienced by the transmitted signal in SATNs also shows strong randomness.

Furthermore, none of these aforementioned works have studied the performance of satellite/aerial-terrestrial or satellite-aerial communication systems in presence of spatially multi-user interference (MUI)/co-channel interference, which can readily reflect the adverse characteristics of the practical large-scale three-dimensional operation space, though inter-user interference under NOMA scheme has been considered at the satellite in [23].

Inspired by such observations, this work considers a cooperative satellite-aerial-terrestrial communication (SATN) system, in which there are two relaying links to assist a group of aerial terminals to transmit their information to a remote terrestrial destination (\( D \)). Specifically, these aerial information sources are modeled by adopting the Matérn hard-core point process type-II to mimic and address the random properties of the deployment of practical aerial platforms. It is also assumed that these aerial sources access the satellite or the aerial relay while sharing the same frequency band. For example, code-division multiple access (CDMA) scheme is employed at the satellite/aerial relay due to the ability of frequency reuse, the flexibility in traffic management, and the orbit/spectrum resources. Then, the impacts of MUI have been considered over the uplink from aerial sources to the satellite/aerial relay. Moreover, to address the harshness of wireless communication, a group of aerial jammers, which is modeled as a Poisson point process (PPP), are taken into consideration at \( D \)’s side. The main contributions of this work are summarized as follows.

1. The statistical characteristics of the received signal-to-interference (SIR) over the aerial source-satellite/aerial relay uplink have been characterized by deriving the closed-form analytical expressions for the probability density function (PDF) and the cumulative distribution function (CDF) of SIR, while considering aerial sources accessing the satellite/the aerial relay via the same frequency channel, simultaneously.

2. The statistical characteristics of the received SIR over the satellite/aerial relay-\( D \) downlink have been characterized and the closed-form analytical expressions for the PDF and CDF of the SIR are presented, while considering hostile interfering scenarios.

3. The closed-form analytical expressions for the OP over the uplink from the aerial source to the satellite/aerial relay, the downlink from the satellite/aerial relay to \( D \), and the inter-aerial relay link among the aerial relays are respectively derived. Then, the e2e OP for the two considered relay links is also achieved.

4. The outage performance of the considered cooperative SATN system with the two relay link under numerous scenarios are compared via numerical results, and further meaningful insights are concluded accordingly.

The remainder of this work is structured as follows. In Section II, the considered system model is presented. Outage analysis of the uplink and the downlink is presented in Sections III and IV, respectively. In Section V, the OP of the inter-aerial relaying link and the e2e OP are evaluated. The proposed analytical models are verified by Monte-Carlo simulations in Section VI. Finally, the paper is concluded with some meaningful insights in Section VII.

\[ \text{Actually, it is very common that MUI exists in practical wireless communication systems, e.g., MUI always exists in practical CDMA systems which arises from the imperfect orthogonality existing among CDMA users or comes from the imperfect synchronization existing in time-division multiple access systems in which the users share the same frequency channel.} \]
In this work, we consider a cooperative SATN, which includes a satellite \( S \), a number of aerial relays \( R_n, n \in [1, \ldots, N], N \geq 1 \), a group of aerial transmitters \( A_m, m \in [1, \ldots, M], \) e.g., UAVs, and \( M \geq 1 \), and a destination \( D \), as shown in Fig. 1. The aerial transmitters, e.g., UAVs, aim to transmit their information to a remote terrestrial destination \( D \). However, as the distances between \( A_m \) and \( D \) are quite large, the direct links from the transmitters to the terrestrial receiver are unavailable. Therefore, satellite communications are exploited as a solution to aid the targeted remote wireless transmission.

Additionally, there is an alternative relaying link via a series of aerial relays \( R_n \) to link the aerial transmitters and \( D \). The \( N \) aerial relays are ordered from one to \( N \) to perform cooperative transmission with \( N + 1 \) hops\(^2\). Therefore, the received signals at \( D \) can be enhanced to achieve enlarged performance. We split our system into a downlink, an uplink, and inter-aerial relaying links\(^3\), shown in Fig. 2.

A. The Uplink

For the uplink, we consider two types of channels, i.e., the channels between the aerial transmitters and aerial relays and the channels between aerial transmitters and the satellite.

II. System Model

1) Multi-Access Mechanism at \( S/R_1 \): As there are a group of aerial transmitters trying to deliver their information to \( D \) via the help of \( S/R_1 \), the multi-access mechanism adopted at \( S/R_1 \) is a key factor affecting the transmission performance of the uplink.

In this work, we assume that these aerial transmitters share the same frequency by using CDMA. In this case, the received signal at \( S/R_1 \) will suffer from the MUI arising from other aerial transmitters, which will be discussed in the following analysis.

2) Deployment of Aerial Transmitters: In this work, we exploit the Matérn hard-core point process (MHCPP) type-II to mimic and reflect the random deployment of practical aerial scenarios for the aerial transmitters\(^4\). As considered, the aerial transmitters are randomly deployed in the ball space, denoted as \( V \). Each candidate point has a minimum distance \( D_{\text{min}} \) to maintain aviation safety. Before presenting the detailed distributions for the candidate points, we introduce the concept of the MHCPP type-II first as follows.

We define that the locations of the candidate points have a density \( \lambda_A \) and each candidate point has the minimum distance \( D_{\text{min}} \). Then, we present the processes to build up a MHCPP, denoted as \( \Phi_A \), within three steps.

- In the first step, we follows the homogeneous Poisson point processes (HPPP) to initially generate the candidate points. Hence, the candidate points are randomly chosen from the space \( V \) with the density \( \lambda_P \). If we denote the number of the candidate points as \( N_P \), the probability mass function of the Poisson distribution with a mean \( \lambda_P V \), when the number of the candidate points equals to \( s \), can be expressed as \( \Pr \{ N_P = s \} = (\lambda_P V)^s \exp(-\lambda_P V), \) where \( V \) is the volume of space \( V \).
- In the second step, we generate an independent mark for each candidate point. The value of the marks is chosen based on a uniform distribution with a range from \([0,1]\).

\(^1\) The MHCPP is adopted here is to accurately reflect and model the practical deployment of aerial terminals, as the aerial terminals cannot be too close to each other to promise their safety or to hold their own serving space.
In the final step, we only choose the point with the smallest mark within a small space with the distance $D_{\text{min}}$, while the others are eliminated. More specifically for a certain point $K$, we choose $K$ as the center of a small spherical repulsion space with the radius $D_{\text{min}}$. If there are other candidate points in the spherical repulsion space, we compare the mark value from one to another. We only keep the point with the smallest mark in each spherical repulsion space. Then, we move to another candidate point to do the same processes until each spherical repulsion space has only one point. Based on this process, we have the relationship of densities, i.e., $\lambda_A$ and $\lambda_P$, which is expressed as $\lambda_A = \frac{1 - \exp(-\frac{\pi}{4}D_{\text{min}}\lambda_P)}{\frac{3}{4}\pi D_{\text{min}}}$.

3) Channel Fading: Though LoS propagation plays a leading role over air-to-air/space links, the path impairments in the aeronautical mobile-satellite communications include surface reflection (multi-path) effects [45]. Thus, without loss of generality and facilitating the following analysis, we assume that all uplink channels suffer independent and identically distributed (i.i.d.) Nakagami-$m$ fading, which covers the statistical characteristics of LoS propagation. Hence, the expressions of the PDF and CDF are presented as

\[
    f_{|h_{A_m,x}|^2}(x) = \left(\frac{m_n}{\Omega}\right)^{m_n} x^{m_n-1} \exp\left(-\frac{m_n}{\Omega} x\right)
\]

and

\[
    F_{|h_{A_m,x}|^2}(x) = 1 - \sum_{i=0}^{m_n-1} \left(\frac{m_n}{\Omega}\right)^{m_n-i-1} \frac{x^{m_n-i-1}}{(m_n-i-1)!} \exp\left(-\frac{m_n}{\Omega} x\right),
\]

respectively, where $X \in \{S, R_1\}$ to present the satellite and the first aerial relay, respectively, $|h_{A_m,x}|^2$ is the channel gain of the channel from the aerial transmitter $A_m$ to the relay or the satellite, $X$, and $m_n$ and $\Omega$ is the parameters of Nakagami-$m$ fading channels.

4) Signal Model: We consider a practical satellite-aerial-terrestrial scenario where the aerial transmitters suffer the interference from each other. Hence, the SINR for an aerial transmitter $A_m$ to $X \in \{S, R_1\}$ can be expressed as

\[
    \gamma_{A_m,X} = \frac{P_{A_m} |h_{A_m,X}|^2 d_{A_m,X}^{-\alpha_X}}{\sigma^2 + \sum_{k=1}^{M} P_{A_k} |h_{A_k,X}|^2 d_{A_k,X}^{-\alpha_X}}
\]

\[
    \approx \frac{P_{A_m} |h_{A_m,X}|^2 d_{A_m,X}^{-\alpha_X}}{\sum_{k=1,k\neq m}^{M} P_{A_k} |h_{A_k,X}|^2 d_{A_k,X}^{-\alpha_X}},
\]

where $P_{A_m}$ is the transmit power at $A_m$, $d_{A_m,X}$ is the distance from $A_m$ to the relay or the satellite $X$, $\alpha_X$ is the path loss exponent for the channel to the relay or the satellite, and $\sigma^2$ is the strength of additive white Gaussian noise (AWGNN).

B. The Downlink

In this work, we assume that we exploit the decode-and-forward (DF) relay scheme to process and transmit the signals. Over the downlink, the signals from the aerial transmitters have been decoded and re-transmitted by the satellite or the aerial relays. At the terrestrial destination, we consider that there are a group of near-ground aerial jammers, denoted as $I_j$ with $j \in \mathbb{Z}^+$ and $j \in \{1, \cdots, J\}$ ($J \geq 1$), interfering the signals transmitted from the satellite or the aerial relay.

1) Deployment of Aerial Jammers: We consider that these aerial jammers follow a PPP\(^6\), which are uniformly distrib-

\(^6\) Here, for the downlink, the interference may not only come from the hostile neighboring terminals distributed in the considered space but also be the ones which are non-hostile and do not locate in the considered interfering space. For the later ones, the interferer may be far away from the receiver, but we can cast it into the considered hemisphere region by varying the transmission power. In this way, the interferer may be coincide in the hemisphere. Thus, the aerial jammers are modeled with normal PPP without restriction on the inter-node distance to reflect the worst case possibly existing in the practice.
ed in a hemisphere, $V_h$, whose bottom is on the ground with the radius $R_I$ and the original, $D$. The number of the aerial jammers is Poisson distributed with density $\lambda_J$, i.e., $\Pr \{ N = k \} = (\mu \nu h / k !) \exp (-\mu \nu h)$, where $\mu \nu h = \frac{2\pi \nu R_I^2}{3} \lambda_J$ is the mean measure.

Define the distance from a jammer $I_j$ to the center of the hemisphere, $D$, as $d_{I_j}$. The PDF of $d_{I_j}$ is expressed as

$$f_{d_{I_j}}(x) = \begin{cases} 0, & x < 0; \\ \frac{3x^2}{R_I^3}, & 0 \leq x \leq R_I; \\ 0, & x > R_I. \end{cases} \quad (4)$$

2) Channel Fading: For the $S - D$ and $R_N - D$ links, we consider the Shadowed-Rician model, which has been widely utilized by the researches in satellite/aerial-terrestrial communication area. We denote the channel gains between $S$ and $D$, and $R_N$ and $D$ as $|h_{iD}|^2$ ($i \in \{S, R_N\}$), thus the PDF of $|h_{iD}|^2$ is expressed as [41]

$$f_{|h_{iD}|^2}(x) = \chi \frac{\exp (-\beta x)}{(2b) \beta} F_1(m_n; 1; \beta x), \quad (5)$$

where $\chi = \left( \frac{2m_n (m_n + \Theta)}{m_n (m_n + \Theta) + \delta} \right)^{m_n} / (2b) \beta$, $\delta = \frac{\Theta}{2(m_n + \Theta)}$, $\Theta$ and $2b$ are the average power for the line of sight component and the multi-path component, respectively, $m_n$ is the fading parameters based on the Nakagami-$m$ fading channels, and $F_1(\cdot; \cdot; \cdot)$ is the confluent hypergeometric function of the first kind.

Moreover, for the jamming links between the aerial jammers and $D$, without loss of generality, it is also assumed that all jamming channels suffer i.i.d. Nakagami-$m$ fading to facilitate the following analysis since LoS propagation always plays a key role in aerial-to-ground transmission scenarios.

3) Signal Model: For the $S - D$ and $R_N - D$ links, the SINR of $|h_{iD}|^2$ ($i \in \{S, R_N\}$) is

$$\gamma_{iD} = \frac{P_S |h_{iD}|^2 d_{iD}^{-\alpha_S}}{\sum_{j=1}^N P_j |h_{jD}|^2 d_{jD}^{-\alpha_S} + \sigma^2} \approx \frac{P_S |h_{iD}|^2 d_{iD}^{-\alpha_S}}{\sum_{j=1}^N P_j |h_{jD}|^2 d_{jD}^{-\alpha_S}}, \quad (6)$$

where $P_j$ is the transmit power at $j$, $P_S$ is the transmit power at the $j$th jammer, $d_{iD}$ is the distance between $i$ and $D$, and $\alpha_S$ is the path-loss exponent.

C. Inter-Aerial Relaying Links

1) Channel Fading: In this work, it is assumed that all inter-aerial relaying links suffer Nakagami-$m$ fading channels, which covers typical LoS propagation scenarios. Hence, the expressions of PDF and the CDF of the channel power gain between the $n$th ($n = 1, \cdots, N - 1$) and the $(n + 1)$th aerial relays, $|h_{R_n R_{n+1}}|^2$, are presented as

$$f_{|h_{R_n R_{n+1}}|^2}(x) = \left( \frac{\Omega}{m_n} \right)^{m_n} \frac{x^{m_n-1}}{(m_n - 1)!} \exp \left( -\frac{x \Omega}{m_n} \right). \quad (7)$$

2) Signal Model: Next, we present the SNR over the link between the $n$th ($n = 1, \cdots, N - 1$) and the $(n + 1)$th aerial relays as $\gamma_{R_n R_{n+1}} = \frac{P_{R_n} |h_{R_n R_{n+1}}|^2}{\sum_{N=n+1}^{R_n R_{n+1}} |h_{R_n R_{n+1}}|^2 + \sigma^2}$, where $P_{R_n}$ is the transmit power at the $n$th aerial relay, $|h_{R_n R_{n+1}}|^2$ is the distance between the $n$th and the $(n + 1)$th aerial relays, and $\alpha_{R_n}$ is the path-loss exponent.

III. OUTAGE ANALYSIS OF THE UPLINK

For simplification, assume that all the aerial transmitters except the target transmitter share the same transmit power with $P_{Ak} = P_{A_n}$ and denote $m_i = m_n - i - 1$.

Lemma 1. The OP for the received signal at $X$ ($X \in \{S, R_1\}$), which is transmitted by the $n$th aerial transmitter, can be derived as

$$P_{A_n X}^{out} = 1 - \sum_{i=0}^{m_n-1} (\frac{(-1)^m}{m_i}! \mathbb{E}_{d_{A_n X}} \left\{ s_i \frac{d^{m_i} \exp \{ s \}}{d s^{m_i}} \right\}) + \frac{m_n d^{m_n} \exp \{ s \}}{P_{A_n}} \mathbb{E}_{d_{A_n X}} \left\{ \left( \frac{\Omega m_n d^{m_n} X_{\gamma_{out}}}{\Omega m_n d^{m_n} X_{\gamma_{out}} - 1} \right) \right\}. \quad (8)$$

Proof: Please refer to Appendix A.

To obtain a closed-form expression for (8), the $\mathbb{E}_{d_{A_n X}} \{ \exp \{ s \} \}$ should be calculated. Thus, we need to first derive $\mathbb{E}_{d_{A_n X}} \{ \exp \{ s \} \}$.

Lemma 2. $\mathbb{E}_{d_{A_n X}} \{ \exp \{ s \} \}$ can be expressed as

$$\mathbb{E}_{d_{A_n X}} \{ \exp \{ s \} \} = \exp \{ \lambda_A V_1 (D_2 - 1) \}, \quad (9)$$

where $V_1 = \frac{4x}{m_n} \left( R_A^3 - D_1^{3 \min} \right)$ and $D_2 = \mathbb{E}_{d_{A_k X}} \left\{ \left( \frac{m_n d^{m_n} X_{\gamma_{out}}}{\Omega m_n d^{m_n} X_{\gamma_{out}} - 1} \right) \right\} - 1, \quad (10)$

Proof: Please refer to Appendix B.

Lemma 3. The OP is

$$P_{A_n X}^{out} = 1 - \sum_{i=0}^{m_n-1} (\frac{(-1)^m}{m_i}! \mathbb{E}_{d_{A_k X}} \left\{ \left( \frac{\Omega m_n d^{m_n} X_{\gamma_{out}}}{\Omega m_n d^{m_n} X_{\gamma_{out}} - 1} \right) \right\}) \times \exp \{ \lambda_A V_1 (D_2 - 1) \} \times \left( \lambda_A V_1 D_2^{(1)} \cdots \lambda_A V_1 D_2^{(m_n - k + 1)} \right), \quad (10)$$

where $\mathbb{I} \{ \cdot \}$ is the indicator function and $D_2^{(k)} (k = 0, 1, \cdots, m_n)$ is

$$D_2^{(k)} = \frac{(m_n + k - 1)!}{(m_n - 1)!} \mathbb{E}_{d_{A_k X}} \left\{ \left( \frac{\Omega}{m_n d^{m_n} X_{\gamma_{out}}} \right)^{m_n + k} \right\}. \quad (11)$$

Proof: Please refer to Appendix C.
A. Uplink With the Satellite

When the aerial transmitters communicate with the satellite, we assume that all the aerial transmitters have the same transmission distance $d_{OS}$⁸. By substituting $d_{A,nX} = d_{OS}$ and $d_{A,R} = d_{OS}$ into (10), we can get the OP of the uplink from the aerial transmitter cluster to the satellite as

$$P_{out}^{OS} = 1 - \sum_{i=0}^{m_n-1} \frac{1}{(m_i)!} \left( \frac{m_n d_{OS}^{P_R X} \gamma_{out} P_{A,n}}{P_{A,m} \Omega} \right)^{m_i} \times \exp \left[ \lambda A V_1 (D_2 - 1) \right] \left[ 1 + \mathbb{I} \{ m_i > 0 \} \sum_{k=1}^{m_i} B_{m,k} \right] \times \left( \lambda A V_1 D_2^{(m_i-k+1)} \right).$$

(16)

Interestingly, comparing (12) and (16), we can find that the closed-form analytical expression for the OP over the uplink from the aerial transmitter to the satellite is similar to that for the OP over the uplink from the aerial transmitter to the aerial relay. Thus, in Section VI, we will study the outage performance of the uplink by merging these two cases into one for simplicity.

B. Uplink with the Aerial Relay

In this case, we suppose the transmitter is located in the center of the aerial transmitter cluster and the distance from the cluster center to the aerial relay is a constant, namely, $d_{A,R} = d_{OR}$.

To get $D_2^{(k)}$, we should derive the PDF of $d_{A,R}$.  

**Lemma 4.** The PDF of $d_{A,R}$ is

$$f_{d_{A,R}} (x) = \frac{\pi [R^2_A - \tau(x)^2]}{2V_1 d_{OR}},$$

(13)

where $\tau(x) = \max \{ D_{min}, \sqrt{x - d_{OR}} \}$ and $d_{max} = (d_{OR} - R_A)^2 \leq x \leq (d_{OR} + R_A)^2 = d_{max}$.

Proof: Please refer to Appendix D.

Then, by denoting $\Lambda = \frac{d_{OR}^{P_R X} \gamma_{out} P_{A,n}}{P_{A,m} \Omega}$, $D_2^{(k)}$ in (45) can be achieved as (14) shown on the top of next page.

When $\tau(x) = D_{min}$ which means $D_{min} > \sqrt{x - d_{OR}}$, we can get $\frac{d_{max}}{d_{min}} = (d_{OR} - R_A)^2 < x < (d_{OR} + R_A)^2$, $d_{max} = d_{min}$, and $R_A^2 - \tau(x)^2 = R_A^2 - D_{min}^2$.

When $\tau(x) = \sqrt{x - d_{OR}}$, which indicates $\sqrt{x - d_{OR}} > D_{min}$, thus $(d_{OR} - R_A)^2 < x < (d_{OR} + R_A)^2$, $(d_{OR} + R_A^2)^2 < x < (d_{OR} + 2R_A)^2$, and $R_A^2 - \tau(x)^2 = R_A^2 - d_{OR}^2 - x + 2d_{OR}\sqrt{x - \tau(x)}$.

By using [42, Eq. (3.194.1)], $D_2^{(k)}$ can be obtained as (15) shown on the top of next page, where $F(a, b, c, k) = \sum_{i=0}^{k} \frac{\Gamma(a+i)}{\Gamma(a+1)} F_{21} \left[ \begin{array}{c} \frac{(m+n+\alpha)x}{a} \\ \frac{(m+n+\alpha)x}{a} \end{array} \right] \left( m + k, m + a + 1; \frac{b}{a} \right)$.  

(15)

and the detailed derivation of (15) is given in Appendix E.

Substituting (15) and $d_{A,nX} = d_{OR}$ into (10), we can get the OP over the uplink from the aerial transmitter to the first aerial relay as

$$P_{out}^{OR} = 1 - \sum_{i=0}^{m_n-1} \frac{1}{(m_i)!} \left( \frac{m_n d_{OR}^{P_R X} \gamma_{out} P_{A,n}}{P_{A,m} \Omega} \right)^{m_i} \times \exp \left[ \lambda A V_1 (D_2 - 1) \right] \left[ 1 + \mathbb{I} \{ m_i > 0 \} \sum_{k=1}^{m_i} B_{m,k} \right] \times \left( \lambda A V_1 D_2^{(m_i-k+1)} \right).$$

(16)

C. Asymptotic Outage Performance of the Uplink

In this part, we will present the asymptotic analysis of the outage performance of the uplink of the considered system. In the following, we derive the approximate expressions assuming $\lambda = \frac{P_{A,nX}}{N_0} \rightarrow \infty$.

Adopting the series representations of the exponential function, $\exp \left( \frac{-m x}{\Omega} \right) = \sum_{n=0}^{\infty} \left( \frac{-m x}{\Omega} \right)^n$, and keeping the first two terms while ignoring the higher order term, the asymptotic CDF of $\lambda h_{A,nX} x^2$ can be obtained as

$$F_{\lambda h_{A,nX} x^2} (x) = 1 - \sum_{i=0}^{m_n-1} \frac{1}{(m_i)!} \left( \frac{m_n}{\Omega} \right)^{m_n-i-1} \frac{x^{m_n-i-1}}{(m_n - i - 1)!} \exp \left( \frac{-m x}{\Omega} \right).$$

(17)

**Lemma 5.** The asymptotic outage probability can be presented as

$$P_{out}^{\infty} = 1 - \sum_{i=0}^{m_n-1} \left( \frac{1}{m_i} \right) \frac{\mathbb{E}_{d_{A,nX}} \{ s_{m_i} \} \mathbb{E}_I \{ I^{m_i} \}}{\mathbb{E}_{d_{A,nX}} \{ s_{m_i+1} \} \mathbb{E}_I \{ I^{m_i+1} \}},$$

(18)

where

$$\mathbb{E}_I \{ I \} = \sum_{M=0}^{\infty} \frac{(\Lambda A V_1)^M}{(M - 1)!} \exp (-\Lambda A V_1) \times \left( 1 - \sum_{i=0}^{m_n-1} \left( \frac{m_n}{\Omega} \right)^{m_n-i-1} \frac{x^{m_n-i-1}}{(m_n - i - 1)!} \right) \exp \left( \frac{-m x}{\Omega} \right) \mathbb{E}_{d_{A,nX}} \{ s_{A,nX} \} \}$$

and

$$\mathbb{E}_{d_{A,nX}} \{ s \} = \mathbb{E}_{d_{A,nX}} \left\{ \frac{m_n d_{A,nX}^{P_R X} \gamma_{out} P_{A,n}}{P_{A,m} \Omega} \right\}.$$  

(19)

Proof: Please refer to Appendix F.

When the aerial transmitters communicate with the satellite,
\[ D_2^{(k)} = \frac{(m_n + k - 1)!}{(m_n - 1)!} \mathbb{E}_{d_{A_k}X} \left\{ \left( \frac{\Omega}{m_n d_{A_k}^T X} \right)^{\frac{m}{2}} \left( \frac{d_{A_k} R_{2}^\alpha}{d_{A_k}^T X + \frac{d_{A_k} R_{1}^\alpha}{\gamma_{out} P_{A_n}}} \right)^{m_n + k} \right\} \]

\[ = \frac{(m_n + k - 1)!}{(m_n - 1)!} \left( \frac{\Omega}{m_n} \right)^{\frac{m}{2}} \int_{d_{\text{max}}^T}^{d_{\text{max}}^k} \left( \frac{x}{x R_1^\alpha + \Lambda} \right)^{m_n + k} f_{d_{A_k}X}(x) dx \]

\[ = \frac{\pi (m_n + k - 1)!}{2 V_1 d_{OR_i} (m_n - 1)!} \left( \frac{\Omega}{m_n} \right)^{\frac{m}{2}} \int_{d_{\text{max}}^T}^{d_{\text{max}}^k} \left( \frac{x}{x R_1^\alpha + \Lambda} \right)^{m_n + k} dx \]

\[ \approx \frac{\pi (m_n + k - 1)!}{2 V_1 d_{OR_i} (m_n - 1)!} \left( \frac{\Omega}{m_n} \right)^{\frac{m}{2}} \Lambda^{-m_n-k} \left\{ (R_A^2 - D_{\text{min}}^2) \mathcal{F} \left( \frac{2}{\alpha R_1}, \frac{d_{\text{min}}^d, d_{\text{max}}^d, k}{\alpha R_1}, \frac{d_{\text{min}}^u, d_{\text{max}}^u, k}{\alpha R_1} \right) \right\} \]

\[ \approx \frac{\pi (m_n + k - 1)!}{2 V_1 d_{OR_i} (m_n - 1)!} \left( \frac{\Omega}{m_n} \right)^{\frac{m}{2}} \Lambda^{-m_n-k} \left\{ (R_A^2 - D_{\text{min}}^2) \mathcal{F} \left( \frac{2}{\alpha R_1}, \frac{d_{\text{min}}^d, d_{\text{max}}^d, k}{\alpha R_1}, \frac{d_{\text{min}}^u, d_{\text{max}}^u, k}{\alpha R_1} \right) \right\} \]

Thus, \( P_{iD}^\infty \) can be rewritten as

\[ P_{iD}^\infty = 1 - \sum_{i=0}^{m-1} \frac{(-1)^{m-i}}{(m-i)!} s^{m_i} \{ E_I \{ I \} \}^{m_i} \]

\[ - s^{m_i+1} \{ E_I \{ I \} \}^{m_i+1} \]

When the aerial transmitters communicate with the aerial relay, we can obtain \( E_{d_{A_k}X} \left\{ d_{A_k}^\alpha X \right\} \) as (23) shown on the next page.

Thus, we can obtain the asymptotic OP over the uplink from the aerial transmitter to the satellite by substituting (23) into (18).

IV. OUTAGE ANALYSIS OF THE DOWNLINK

Similar to Section III, the OP of the downlink depicted in Fig. 2(b) can be given as

\[ P_{out}^{iD} = \Pr \left\{ \gamma_{out} \leq \gamma_{iD} \right\} \]

\[ \approx \Pr \left\{ \frac{P_i | h_{iD} |^2 d_{iD}^{-\alpha_s}}{\sum_{j=1}^{J} P_j | h_{jD} |^2 d_{jD}^{-\alpha_s}} \leq \gamma_{out} \right\}, \quad i \in \{S, R_N\}. \]

Moreover, as the focus of this section is to investigate the impacts of these spatial distribution aerial jammers on the outage performance of the downlink, here we also assume that the distance between the satellite/the last aerial relay and \( D \) is fixed for the feasibility of the analysis. Namely, \( d_{iD} \) (\( i \in \{S, R_N\} \)) is with a given value. Thus, we can rewrite the previous equation as

\[ P_{out}^{iD} = \Pr \left\{ \frac{Z}{Y} \leq \frac{d_{iD}^{-\alpha_s}}{P_i \gamma_{out}} \right\}, \]

where \( i \in \{S, R_N\}, Z = |h_{iD}|^2, \) and \( Y = \sum_{j=1}^{J} P_j | h_{jD} |^2 d_{jD}^{-\alpha_s}. \)

Recalling (5) and using the Kummer’s transform of the hypergeometric function, we easily rewrite the PDF of \( |h_{iD}|^2 \) as

\[ f_{|h_{iD}|^2}(x) = \sum_{k=0}^{m_n-1} \Psi(k) x^k \exp \left( - (\beta - \delta) x \right), \]

where \( \Psi(k) = \frac{(z)}{(k)} x(1-m_n)_k \) and \( (\cdot)_n \) is the Pochhammer
symbol. Then, the CDF of \(|h_{iD}|^2\) can be presented as

$$F_{|h_{iD}|^2} (x) = \sum_{k=0}^{m_n-1} \Psi (k) \int_0^x t^k e^{-(\beta-\delta)t} dt = \sum_{k=0}^{m_n-1} \frac{\Psi (k)}{\beta-\delta} x^{k+1} \gamma (k + 1, (\beta - \delta) x).$$  \hfill (26)

Further, using (25) and (26) and denote

$$\kappa = \frac{\gamma_{out}}{P_i} (\beta - \delta),$$

we can obtain

$$P_{out} = \Pr \{ |h_{iD}|^2 \leq \gamma_{out} \} = \sum_{k=0}^{m_n-1} \frac{\Psi (k)}{\beta-\delta} x^{k+1} \gamma (k + 1, (\beta - \delta) Y).$$  \hfill (27)

As shown in [43], \(\gamma (k + 1, x) < \Gamma (k + 1) (1 - \exp (-\zeta x))^{k+1}\), \(\zeta = (\Gamma (k + 2))^{-\frac{1}{k+1}}\).

Then, \(P_{out}^{\delta}\) can be approximated as

$$P_{out}^{\delta} \approx \sum_{k=0}^{m_n-1} \frac{\Psi (k) \Gamma (k + 1)}{\beta-\delta} (1 - \exp (-\zeta x))^{k+1}\Gamma (k + 1) \left(1 - \exp (-\zeta x)\right)^{k+1}$$

$$= \sum_{k=0}^{m_n-1} \frac{\Psi (k) \Gamma (k + 1)}{\beta-\delta} \sum_{t=0}^{k+1} \left(1 + t\right)^t \Gamma (k + 1) \left(1 - \exp (-\zeta x)\right)^{k+1}$$

$$= \sum_{k=0}^{m_n-1} \frac{\Psi (k) \Gamma (k + 1)}{\beta-\delta} \sum_{t=0}^{k+1} \left(1 + t\right)^t \Gamma (k + 1) \left(1 - \exp (-\zeta x)\right)^{k+1}$$

\hfill (28)

Furthermore, considering the randomness of \(Y\), we can obtain

\[ P_{out} = \sum_{k=0}^{m_n-1} \frac{\Psi (k) \Gamma (k + 1)}{\beta-\delta} \sum_{t=0}^{k+1} \left(1 + t\right)^t \Gamma (k + 1) \left(1 - \exp (-\zeta x)\right)^{k+1} \]

\hfill (29)

where \(\mathcal{L}_Y (s)\) is the Laplace transform of random variable \(Y\).

**Lemma 6.** Considering that all jamming channels between the aerial jammers and the terrestrial receiver, \(D\), suffer i.i.d. Nakagami-m fading, namely, \(f_{|h_{iD}|^2} (x) = m_n \frac{x^{m_n-1}}{\Gamma (m_n)} \exp (-\frac{x}{\Omega} x)\), \(\mathcal{L}_Y (\zeta x)\) can be calculated as

\[ \mathcal{L}_Y (\zeta x) = \exp \left(-\pi R_l \lambda \sum_{i=1}^{V} \sqrt{1 - \zeta^2 \omega_i v_i} \right) \times \left(1 - \frac{m_n}{\Omega} \right)^{m_n} \left(1 + \zeta x + \zeta x P_i \right)^{\frac{1}{\Omega}}\]

\hfill (30)

where \(v_i = \frac{R_l (t_i + 1)}{2}, t_i = \cos \left(\frac{2\pi - \pi}{4}\right), \omega_i = \pi\).

**Proof:** Please refer to Appendix G.

Therefore, the OP over the downlink can be achieved via inserting (30) into (29).

Adopting the series representations of the exponential function, \(\exp \left(-\beta x\right) = \sum_{n=0}^{\infty} \left(-\beta x\right)^n \frac{n^m}{n!}\), and keeping the first two terms while ignoring the higher order term, the asymptotic
CDF and PDF of $\lambda_{dn} = P_1|h_{1D}|^2$ can be expressed as
\[
 f_{\lambda_{dn}}(x) = \sum_{k=0}^{m_n-1} \Psi(k) x^k \left(1 - \frac{\beta - \delta}{\lambda_{dn}} x\right) \tag{31}
\]
and
\[
 F_{\lambda_{dn}}(x) = \sum_{k=0}^{m_n-1} \frac{\Psi(k)}{k+1} x^{k+1} - \frac{\Psi(k) (\beta - \delta)}{(k+2) \lambda_{dn}} x^{k+2}. \tag{32}
\]

Then, we can obtain $P_{\text{out}}$ as
\[
 P_{\text{out}} = \sum_{k=0}^{m_n-1} \frac{\Psi(k)}{k+1} (\kappa \mathbb{E}_Y \{Y\})^{k+1} - \frac{\Psi(k) (\beta - \delta)}{(k+2) \lambda_{dn}} (\kappa \mathbb{E}_Y \{Y\})^{k+2}, \tag{33}
\]
where
\[
 \mathbb{E}_Y \{Y\} = \mathbb{E}_{|h_{1D}|^2, d_{ij}} \left[ \sum_{j=1}^{J} \mathbb{E}_{d_{ij}} \left[ \int_0^\infty \frac{P_j |h_{1D}|^2}{d_{ij}^2} (|h_{1D}|^2(x)) dx \right] \right] = \sum_{j=1}^{J} \int_\mathbb{R}^3 I_2 (d_{ij}) dd_{ij}, \tag{34}
\]
where
\[
 I_2 = \left( \frac{m_n}{\Omega} \right) ^{m_n} \frac{P_j}{d_{ij}^2 (m_n - 1)!} \int_0^\infty x^{m_n-1} \exp \left( -\frac{m_n}{\Omega} x \right) dx = \left( \frac{m_n}{\Omega} \right) ^{-1} \frac{P_j}{d_{ij}^2 (m_n - 1)!} \Gamma (m_n + 1), \tag{35}
\]
and $\mathbb{E}_Y \{Y\}$ can be finally derived as
\[
 \mathbb{E}_Y \{Y\} = \sum_{j=1}^{J} \left( \lambda_1 \int_\mathbb{R}^3 I_2 (d_{ij}) dd_{ij} \right) = \sum_{j=1}^{J} \left( -\pi R_t \lambda_t \sqrt{V} \int_{i=1}^{V} \sqrt{1 - t_i^2 \omega_i v_i^2} \left( \frac{m_n}{\Omega} \right) ^{-1} \times \frac{P_j \Gamma (m_n + 1)}{v_i^{\alpha_S} (m_n - 1)!} \right). \tag{36}
\]

Finally, the asymptotic OP over the downlink can be achieved by substituting (36) into (33).

V. OUTAGE OF INTER-AERIAL RELAYING AND E2E LINKS

A. Outage Analysis of Inter-Aerial Relaying Links

Considering Part C of Section II and denoting $\Delta_{n,n+1} = \frac{d_{R_n R_{n+1}}}{P_{\text{out}}} \sigma^2 \gamma_{\text{out}}$, one easily has the OP over the link between the $n$th ($n = 1, \ldots, N - 1$) and the $(n+1)$th aerial relays as
\[
 P_{\text{out}}^{R_n R_{n+1}} = \Pr \{ \gamma_{R_n R_{n+1}} \leq \gamma_{\text{out}} \} = \Pr \left\{ \frac{P_{R_n} |h_{R_n R_{n+1}}|^2}{d_{R_n R_{n+1}}^2 \sigma^2} \leq \gamma_{\text{out}} \right\} = 1 - \sum_{i=0}^{m_n-1} \left( \frac{m_n}{\Omega} \right) ^{m_n-i-1} \Delta_{n,n+1}^{m_n-i-1} \frac{1}{(m_n - i - 1)!} \times \exp \left( -\frac{m_n}{\Omega} \Lambda_{n,n+1} \right). \tag{37}
\]

Moreover, here we also assume that the transmission distance over each aerial relay hop is fixed to facilitate the analysis, as the main concern of this work is the impacts of the distribution of the aerial sources and jammers.

B. E2E OP Analysis

Using the analysis offered in previous sections, the e2e OP of the target system will be presented by considering the adopted relay types.

1) Satellite Relay Link: As observed from Fig. 1 and 2 and considering DF scheme is adopted at the satellite, the e2e OP over satellite relay link can be easily written as
\[
 P_{\text{out}}^{SR} = 1 - \left( 1 - P_{\text{out}}^{OS} \right) \left( 1 - P_{\text{out}}^{SD} \right). \tag{38}
\]

2) Aerial Relay Link: Similar to the previous subsection, we can present the e2e OP over the aerial relay link as
\[
 P_{\text{out}}^{AR} = 1 - \left( 1 - P_{\text{out}}^{OR_1} \right) \left( 1 - P_{\text{out}}^{OR_2} \right) \prod_{n=1}^{N-1} \left( 1 - P_{\text{out}}^{R_n R_{n+1}} \right). \tag{39}
\]

VI. NUMERICAL RESULTS

In this section, numerical results will be presented to investigate the performance of the considered cooperative satellite/aerial-terrestrial communication system. We run $10^6$ times of the realizations of the considered system and $10^6$ trials of Monte-Carlo simulations to model the randomness of the positions of the aerial transmitters and jammers and channel gains over each link.

Following the transmit power and the orbit altitude shown in [44], [45], in this subsection, unless otherwise explicitly specified, the parameters are set as follows: 1) Uplink: $d_{OS} = 300 \text{ km}, R_A = 10 \text{ km}, d_{min} = 1 \text{ km}, d_{OR_1} = 20 \text{ km}, \alpha_S = 2, \alpha_X = 2, \gamma_{\text{out}} = -1 \text{ dB}, \Lambda_A = 1 \times 10^{-4}, m_n = 2, \text{ and } \Omega = 1$; 2) Downlink: $d_{SD} = 300 \text{ km}, R_t = 10 \text{ km}, d_{R_N D} = 20 \text{ km}, \alpha_S = 2, \gamma_{\text{out}} = -1 \text{ dB}, \lambda_t = 1 \times 10^{-11}, m_n = 2, \text{ and } \Omega = 1, b = 1 \text{ dB}, P_S = 30 \text{ dBW}, \text{ and } P_{A_n} = 30 \text{ dBW}.$

A. The Outage Performance over the Uplink

Fig. 3 presents the influence of $R_A$ on the outage performance of the uplink. One can see that the OP over the uplink gets worse as the size of the distribution space of the aerial transmitters increases. This observation can be easily understood, since a large distribution space of the aerial transmitters indicates that there will be more aerial transmitters.
accessing the satellite/the aerial relay simultaneously, resulting in increased MUI accordingly.

Next, in Fig. 4, the impacts of the distribution density of the aerial transmitters on the OP over the uplink are investigated. It is noted from this figure that a large distribution density of the aerial transmitter incurs a reduced outage performance. Because more aerial transmitters will be brought by a large \( \lambda_A \) in the given distribution space, causing large MUI at the satellite/the aerial relay.

Furthermore, the relationship between the OP of the uplink and the transmission distances of the uplink is investigated in Fig. 5. As expected, the transmission distances of the uplink do not have a large impact on the outage performance over the uplink. Because the target transmission signal and corresponding self-interfering signal emitted by other aerial transmitters suffer from the same path loss when traveling over the same uplink.

Finally, the effects of the minimum safety distance among the aerial transmitters, \( D_{\text{min}} \), on the outage performance over the uplink are shown in Fig. 6. We can observe that \( D_{\text{min}} \) exhibits a positive impact on the OP over the uplink. In other words, a large \( D_{\text{min}} \) stands for a small OP over the uplink, which can be interpreted by the fact that for the given distribution space, a large \( D_{\text{min}} \) implies a low distribution density of the aerial transmitters and further low MUI at the satellite/the aerial relay. However, one can also see that such a positive effect is very weak as the OP curves plotted in this figure overlap each other from a macroscopic view. Because the range of \( D_{\text{min}} \) to promise the safety of the aerial transmitters is relatively quite smaller than the size of their distribution space and the length of the uplinks.

### B. The Outage Performance over the Downlink

In Figs. 7-9, the OP over the downlink between \( S/R_N \) and \( D \) is depicted. \( P_J \) exhibits an obvious negative impact on the outage performance over the downlink. Because a large \( P_J \) represents a large interfering power at \( D \).

In Fig. 7, the OP with different \( d_{iD} \) (\( i \in \{S, R_N\} \)) is presented. The outage performance over the downlink degrades...
while the transmission distance over the downlink enlarges. More specifically, one can see that the OP over the downlink between the satellite and the terrestrial receiver gets worse when the link length increases from 300 km to 800 km. Similar observations can be achieved for the downlink between \( R_N \) and \( D \) when \( d_{R_ND} \) reaches 30 km from 20 km.

Fig. 8 studied the influence of the size of the distribution space of the aerial jammers, \( R_I \), on the outage performance over the downlink. The OP over the downlink enlarges while \( R_I \) increases. For example, when \( P_J = 10 \) dBm, the OP of the downlink is on the order of \( 10^{-3} \) for \( R_I = 5 \) km, and that is on the order of \( 10^{-2} \) for \( R_I = 15 \) km. This observation can be easily explained by the fact that a large \( R_I \) means a large distribution space and further denotes more aerial jammers, which finally incurs the degraded outage performance.

In Fig. 9, we studied the relationship between the outage performance and the distribution density of the aerial jammers, \( \lambda_I \). For a given distribution space for the aerial jammers, a large \( \lambda_I \) leads to worse outage performance since a large \( \lambda_I \) implies more aerial jammers operate in the given 3D distribution space. This observation is similar to the one achieved from Fig. 3, which uncovers the impact of the distribution density of the aerial transmitters on the OP over the uplink in the previous subsection.

Moreover, it can be seen from Figs. 3-9 that simulation and analysis results agree with each other very well, which verify the correctness of our proposed analytical models.

C. The E2e Outage Performance

As illustrated in Figs. 10 and 11, for a given transmission distance between the aerial transmitters and the terrestrial destination, \( N \) offers a positive impact on the e2e outage performance of the considered system via the aerial relay path, but such a positive influence gets weak while \( N \) enlarges as flat bottoms can be seen from the OP lines plotted in both figures. This observation can be explained as follows: The transmission distance between every two neighboring aerial relays gets smaller and smaller when more aerial relays are introduced into the aerial relay link, leading to the improved OP over each aerial relay hop; then, when the outage performance of each aerial relay hop is improved to a certain degree, the e2e outage performance of the considered system will be only decided by the transmission quality over the uplink and the downlink shown in Fig. 2, which agrees with the expression of the e2e OP with aerial relay addressed by (39). Therefore, we can conclude: Though the e2e outage performance can be enhanced somewhat, bringing more aerial relays into the aerial relay link is not always a good choice for the considered system and optimal system design should be carried out to achieve a trade-off between the e2e OP and the increased system resource overhead arisen from the introduced aerial relays.

Moreover, another interesting finding here is that the e2e OP via the satellite relay link does not always outperform that via the aerial relay links because the e2e outage performance under these two cases varies with numerous system factors, e.g., the transmit power at the aerial transmitter/the satellite/the aerial relays, the number of aerial relays, the link distance among all terminals included in the considered system, etc. Thus, to realize an optimal considered system for a practical
Fig. 10. The e2e OP for various $P_S$ and $P_R$.

Fig. 11. The e2e OP for various $d_{OS}/d_{OR}$ and $d_{SD}/d_{RD}$.

scenario setting, the selection of the relay link depends on the feasibility of achieving a balance between the e2e OP and other performance indices, for example, the time delay incurred by the multi-hop forwarding over the aerial relay link.

VII. CONCLUSIONS

In this work, we have investigated the e2e outage performance of a cooperative SATN, in which there are two relaying choices for a group of aerial sources to forward their information to a remote terrestrial destination. The e2e outage performance of the considered system via two different relay links has been studied and compared. Some remarks are obtained:

1. In presence of co-channel interference among the aerial sources, the transmission distances over the uplinks have little impact on the outage performance of the uplinks;

2. The distribution density and the size of the distribution space of the aerial sources and the aerial jammers play a negative role on the OP over the uplink and the downlink, respectively;

3. The safety distance among the aerial sources exhibits a positive impact on the OP over the uplink;

4. For the aerial relay link, the number of aerial relays can be optimized to realize an optimal trade-off between the e2e OP and system resource overhead.

5. The choice of the relay link for the considered system depends on the practical scenario settings. In other words, both two kinds of relay links can provide optimal trade-offs between the e2e outage performance and the other performance indices/constraints, and then people should choose the relay type to satisfy the given system settings.

APPENDIX A: PROOF OF LEMMA 1

According to the definition of OP, the OP over the link from the $m$th aerial transmitter to $X$ ($X \in \{S, R_1\}$) can be written as

$$P_{out}^{A_mX} = Pr\{\gamma_{A_mX} \leq \gamma_{out}\}$$

$$= Pr\left\{\frac{P_{A_m} |h_{A_mX}|^2 d^{-\alpha_X}_{A_mX}}{\sum_{k=1,k\neq m}^M P_{A_k} |h_{A_kX}|^2 d^{-\alpha_X}_{A_kX}} \leq \gamma_{out}\right\}$$

$$= Pr\left\{|h_{A_mX}|^2 \leq \frac{d^{\alpha_X}_{A_mX} \gamma_{out} P_{A_m}}{P_{A_m}} \sum_{k=1,k\neq m}^M |h_{A_kX}|^2 d^{-\alpha_X}_{A_kX}\right\}$$

$$= 1 - \sum_{i=0}^{m-1} \frac{(-1)^{m_i}}{(m_i)!} \mathbb{E}_{d_{A_mX}} \{I^{m_i} \exp (sI)\}$$

$$= 1 - \sum_{i=0}^{m-1} \frac{(-1)^{m_i}}{(m_i)!} \mathbb{E}_{d_{A_mX}} \{s^{m_i} \mathbb{E}_{I} \{I^{m_i} \exp (sI)\}\},$$

(40)

where $I = \sum_{k=1,k\neq m}^M |h_{A_kX}|^2 d^{-\alpha_X}_{A_kX}$ and $s = -\frac{m_i d^{\alpha_X}_{A_mX} \gamma_{out} P_{A_m}}{P_{A_m}}$.

It is easy to get

$$D_1 = \frac{d^{m_i} \mathbb{E}_{I} \{\exp (sI)\}}{d^{m_i} s}.$$  

(41)

Thus, we can obtain (8) via substituting (41) into (40).

APPENDIX B: PROOF OF LEMMA 2

As $|h_{A_kX}|^2$ and $d_{A_kX}$ are i.i.d random variables, we can get

$$\mathbb{E}_I \{\exp (sI)\}$$

$$= \mathbb{E}_M \left\{\mathbb{E}_{|h_{A_kX}|^2,d_{A_kX}} \left\{\exp \left(s \sum_{k=1,k\neq m}^M \frac{|h_{A_kX}|^2}{d^{\alpha_X}_{A_kX}}\right)\right\}\right\}$$
Fig. 12. Aerial transmitter-aerial relay link model

\[
= \mathbb{E}_M \left\{ \prod_{k=1}^{M} \mathbb{E}_{|h_{A_k,X}|^2,A_{k,X}} \left( \exp \left( \frac{s|h_{A_k,X}|^2}{d_{A_k,X}} \right) \right) \right\}
\]

\[
= \sum_{M=0}^{\infty} \frac{(\lambda_A V_1)^M}{M!} \exp(-\lambda_A V_1)
\]

\[
\times \mathbb{E}_{|h_{A_k,X}|^2,A_{k,X}} \left\{ \exp \left( \frac{s|h_{A_k,X}|^2}{d_{A_k,X}} \right) \right\}^{M}
\]

\[
= \sum_{M=0}^{\infty} \frac{(\lambda_A V_1 D_2)^M}{M!} \exp(-\lambda_A V_1)
\]

\[
= \exp[\lambda_A V_1(D_2 - 1)],
\]

where \( V_1 = \frac{4\pi}{3}(R_A^3 - D_{\text{min}}^3) \).

Using the moment generating function (MGF) of \(|h_{A_k,X}|^2|\), it deduces

\[
D_2 = \mathbb{E}_{d_{A_k,X}} \left\{ \left( \frac{m_d d_{A_k,X}}{m_n d_{A_k,X} - 1} \right)^m_n \right\}.
\]

Then, (9) can be reached by combining (42) and (43).

**APPENDIX C: PROOF OF LEMMA 3**

After achieving \( \mathbb{E}_I \{\exp(sI)\} \), we will derive its \((m_i)\)th derivative. We know that the 0th derivative of any function is itself. When \( m_i > 0 \), according to the Fa di Bruno’s formula, we can get

\[
\frac{d^{m_i}}{da^{m_i}} \mathbb{E}_I \{\exp(sI)\} = \exp[\lambda_A V_1(D_2 - 1)] \sum_{k=1}^{m_i} B_{m_i,k} \lambda_A V_1 D_2^{(1)} \cdots \lambda_A V_1 D_2^{(m_i-k+1)},
\]

where \( B_{m,0}(\cdot) \) is the Bell polynomials and \( D_2^{(k)} \) is the \( k \)th derivative of \( D_2 \) w.r.t. \( s \).

It is easy to get \( D_2^{(k)} \) after substituting \( s = -\frac{m_n d_{A_k,X}^X \gamma_{out} P_{A_k}}{P_{A_k} m \Omega} \) as

\[
D_2^{(k)} = \frac{(m_n + k - 1)!}{(m_n - 1)!} \mathbb{E}_{d_{A_k,X}} \left\{ \left( \frac{\Omega}{m_n d_{A_k,X}^X} \right)^k \right\}
\]

\[
\times \left( \frac{m_n d_{A_k,X}^X}{m_n d_{A_k,X} - \Omega s} \right)^m_n + k \}
\]

\[
= \frac{(m_n + k - 1)!}{(m_n - 1)!} \mathbb{E}_{d_{A_k,X}} \left\{ \left( \frac{\Omega}{m_n d_{A_k,X}^X} \right)^k \right\}
\]

\[
\times \left( \frac{d_{A_k,X}^X}{d_{A_k,X} + \frac{d_{A_k,X}^X \gamma_{out} P_{A_k}}{P_{A_k} m}} \right)^m_n + k \}
\]

Substituting (44), (45), and \( s = -\frac{m_n d_{A_k,X}^X \gamma_{out} P_{A_k}}{P_{A_k} m \Omega} \) into (8), the OP can be presented as (10).

**APPENDIX D: PROOF OF LEMMA 4**

As aerial transmitters are assumed to be independently and uniformly distributed in \( \mathcal{V} \), the joint CDF of \( r \) and \( \theta \) can be expressed as

\[
F_{r,\theta}(x,y) = \frac{2\pi x^2 \sin \theta}{V_1} \int_0^\pi d\theta \int_0^x d\rho
\]

\[
= \frac{2\pi(1 - \cos^2 \theta)(x^3 - D_{\text{min}}^3)}{3V_1}.
\]

Then, the joint PDF of \( r \) and \( \theta \) can be written as

\[
f_{r,\theta}(x,y) = \frac{\partial^2 F_{r,\theta}(x,y)}{\partial x \partial y} = \frac{2\pi x^2 \sin \theta}{V_1}.
\]

From Fig. 12, the relationships between \( r, \theta \), and \( d_{A_k,R_1}^2 \) can be represented as \( d_{A_k,R_1}^2 = r^2 + d_{\text{OR}}^2 - 2r d_{\text{OR}} \cos \theta \). It can be easily seen that \( d_{\text{OR}} < R_A \leq d_{A_k,R_1} \leq d_{\text{OR}} + R_A \). To obtain the PDF of \( d_{A_k,R_1}^2 \), we first derive the joint PDF of \( d_{A_k,R_1}^2 \) and \( r \).

According to the multivariate change of variables formula, the Jacobian determinant of matrix \( \frac{\partial(d_{A_k,R_1},r)}{\partial(r,\theta)} \) is

\[
\left| \frac{\partial(d_{A_k,R_1},r)}{\partial(r,\theta)} \right| = \left| \begin{array}{cc} 2r - 2d_{\text{OR}} \cos \theta & 2r d_{\text{OR}} \sin \theta \\ -1 & 0 \end{array} \right| = 2r d_{\text{OR}} \sin \theta.
\]

Then, the joint PDF of \( d_{A_k,R_1}^2 \) and \( r \) can be achieved as

\[
f_{d_{A_k,R_1},r}(x,y) = f_{r,\theta}(x,y) \left| \frac{\partial(d_{A_k,R_1},r)}{\partial(r,\theta)} \right| = \frac{\pi y}{V_1 d_{\text{OR}}}.
\]
The PDF of $d_{Ak}^2 R_k$ can be acquired through the integration of $f_{d_{Ak}^2 R_k}(x,y)$ in (49) according to $r$ and finally shown in (13).

**APPENDIX E: PROOF OF (15)**

By using [42, Eq. (3.194.1)], $D_2^{(k)}$ can be obtained as (50) on the top of next page, where $\mathcal{F}(a, b, c, k) = \left[ b^{(m+a+n)\frac{1}{\alpha}} \right] _2 F_1 \left( m_n + k; m_n + a + 1; \frac{4 v_i}{\Omega} \right) - e^{\left( m_n + k; m_n + a + 1; \frac{4 v_i}{\Omega} \right) \left[ 2 F_1 \left( m_n + k; m_n + a + 1; \frac{4 v_i}{\Omega} \right) \right]} \times 2^{(m_n+a)\frac{1}{\alpha}}$.

**APPENDIX F: PROOF OF LEMMA 5**

By substituting (17) into (40), we can obtain that

$$P_{\text{out}}^\infty = 1 - \sum_{i=0}^{m_n-1} \frac{(-1)^{m_i}}{(m_i)!} E_{d_{A_{Ik}},x} \left\{ (sI)^{m_i} (1 - sI) \right\}$$

$$= 1 - \sum_{i=0}^{m_n-1} \frac{(-1)^{m_i}}{(m_i)!} E_{d_{A_{Ik}},x} \left\{ s^{m_i} E_I \left( I^{m_i} (1 - sI) \right) \right\}$$

$$= 1 - \sum_{i=0}^{m_n-1} \frac{(-1)^{m_i}}{(m_i)!} \left\{ E_{d_{A_{Ik}},x} \left\{ s^{m_i} E_I \left( I^{m_i} \right) \right\} - E_{d_{A_{Ik}},x} \left\{ s^{m_i+1} E_I \left( I^{m_i+1} \right) \right\} \right\}.$$  (51)

As $|h_{Ak}|^2$ and $d_{A_{Ik},x}$ are i.i.d random variables, $E_I \left\{ I \right\}$ can be derived as

$$E_I \left\{ I \right\} = EM \left\{ E_{|h_{Ak}|^2, d_{A_{Ik},x}} \left\{ \sum_{k=1}^{M} \frac{|h_{Ak}|^2}{d_{A_{Ik},x}} \right\} \right\}$$

$$= \sum_{M=0}^{\infty} \frac{(\lambda_A V_1)^M}{(M-1)!} \exp \left(-\lambda_A V_1 \right)$$

$$\times E_{|h_{Ak}|^2} \left\{ |h_{Ak}|^2 \right\} E_{d_{A_{Ik},x}} \left\{ d^{-\alpha}_{A_{Ik},x} \right\}$$

$$= \sum_{M=0}^{\infty} \frac{(\lambda_A V_1)^M}{(M-1)!} \exp \left(-\lambda_A V_1 \right)$$

$$\times \left\{ 1 - \sum_{i=0}^{m_n-1} \frac{m_n^{m_n-i-1}}{(m_n - i - 1)!} Z^{m_m-i} \right\} E_{d_{A_{Ik}},x} \left\{ d^{-\alpha}_{A_{Ik},x} \right\}. \quad (52)$$

Then, (18) can be reached by combining (51) and (52).

**APPENDIX G: PROOF OF LEMMA 6**

As it has been assumed that the channels between the aerial jammers and the terrestrial receiver, $D$, suffer i.i.d. Nakagami-$m$ fading, we have $f_{|h_J|D^2}(x) = (m_n)^{m_n} x^{m_n-1} \exp \left( -\frac{m_n}{\Omega} x \right)$. Thus, we derive $\mathcal{L} \left[ Y \right]$ as

$$\mathcal{L}_Y(s) = E \left[ \exp \left( -sY \right) \right] = E \left[ \exp \left( -s \sum_{j=1}^{J} \frac{P_J |h_J|D^2}{d_{I_j}^2} \right) \right]$$

$$= E \left[ \prod_{j=1}^{J} \exp \left( -s \frac{P_J |h_J|D^2}{d_{I_j}^2} \right) \right]$$

$$= E \left[ \prod_{j=1}^{J} \int_0^{\infty} \exp \left( -s \frac{P_J |h_J|D^2}{d_{I_j}^2} x \right) \left( \frac{1}{(|1 - I_1 (d_I)) d_I} \right) \right], \quad (53)$$

where $\mathbb{R}^3$ is the 3D distribution space for these aerial jammers presented in Fig. 2(b),

$$I_1 = \left( \frac{m_n}{\Omega} \right)^{m_n} \frac{1}{(m_n - 1)!} \int_0^{\infty} x^{m_n-1}$$

$$\times \exp \left( -\frac{m_n}{\Omega} + \frac{s P_J}{d_{I_j}^2} x \right) dx$$

$$= \left( \frac{m_n}{\Omega} \right)^{m_n} \frac{1}{(m_n - 1)!} \left( 1 - f(x) \right) \int_0^{\infty} x^{m_n-1} \exp \left( -\frac{m_n}{\Omega} x \right) \int_0^{\infty} \exp \left( -s \frac{P_J |h_J|D^2}{d_{I_j}^2} x \right) \left( \frac{1}{(|1 - I_1 (d_I)) d_I} \right) \right], \quad (54)$$

and the last step follows the probability generating functional of the PPP, which means for function $f(x)$ that $E \left[ \prod_{\mathbb{R}^3} f(x) \right] = \exp \left( -\lambda \int_0^{\infty} \left( 1 - f(x) \right) dx \right)$.

Then, thinking that $\mathbb{R}^3$ is a hemisphere space depicted in Fig. 2(b), one can finally derive $\mathcal{L}_Y(s)$ as (55) presented on the top of the page after next page, where $v_i = \frac{R_i (t_i + 1)}{2}$, $t_i = \cos \left( \frac{2\pi i}{2m_n} \right)$, and $\omega_i = \frac{2\pi i}{m_n}$.

Finally, replacing $s$ in (55) by $\zeta \chi t$ accomplishes the proof of Lemma 5.

**REFERENCES**


\[
D_2^{(k)} = \frac{\pi (m_n + k - 1)!}{2V_1d_{OR_1}(m_n - 1)!} \left( \frac{\Omega}{m_n} \right)^k \left\{ (R_A^2 - D_{2_{\min}}) \int_{d_{\min}^{u_{\min}}}^{d_{\max}^{u_{\min}}} \frac{x}{(x - \frac{\alpha_{R_1}}{2})^2 + \Lambda} \frac{dx}{m_n + k} \right. \\
+ \int_{d_{\max}^{u_{\min}}}^{d_{\max}^{u_{\max}}} (R_A^2 - d_{OR_1}^2) x^\frac{\alpha_{R_1}}{2} \left[ \int_{x - \frac{\alpha_{R_1}}{2} + 1}^{2d_{OR_1} x^\frac{\alpha_{R_1}}{2} + 1} \frac{dx}{(x - \frac{\alpha_{R_1}}{2} + \Lambda)^{m_n + k}} \right] \right. \\
+ \int_{d_{\min}^{u_{\min}}}^{d_{\min}^{u_{\max}}} (R_A^2 - d_{OR_1}^2) x^\frac{\alpha_{R_1}}{2} \left[ \int_{x - \frac{\alpha_{R_1}}{2} + 1}^{2d_{OR_1} x^\frac{\alpha_{R_1}}{2} + 1} \frac{dx}{(x - \frac{\alpha_{R_1}}{2} + \Lambda)^{m_n + k}} \right] \right. \\
\left. \left. + \frac{\pi (m_n + k - 1)!}{2V_1d_{OR_1}(m_n - 1)!} \left( \frac{\Omega}{m_n} \right)^k \Lambda^{m_n - k} \frac{2}{\alpha_{R_1}} \left( R_A^2 - D_{2_{\min}}^2 \right) \right. \\
\left. \frac{\int_{d_{\min}^{u_{\min}}}^{d_{\max}^{u_{\min}}} x^\frac{\alpha_{R_1}}{2} \left[ \int_{x - \frac{\alpha_{R_1}}{2} + 1}^{2d_{OR_1} x^\frac{\alpha_{R_1}}{2} + 1} \frac{dx}{(x - \frac{\alpha_{R_1}}{2} + \Lambda)^{m_n + k}} \right] \right. \\
\left. + \int_{d_{\max}^{u_{\min}}}^{d_{\max}^{u_{\max}}} (R_A^2 - d_{OR_1}^2) x^\frac{\alpha_{R_1}}{2} \left[ \int_{x - \frac{\alpha_{R_1}}{2} + 1}^{2d_{OR_1} x^\frac{\alpha_{R_1}}{2} + 1} \frac{dx}{(x - \frac{\alpha_{R_1}}{2} + \Lambda)^{m_n + k}} \right] \right. \\
\left. + \int_{d_{\min}^{u_{\min}}}^{d_{\min}^{u_{\max}}} (R_A^2 - d_{OR_1}^2) x^\frac{\alpha_{R_1}}{2} \left[ \int_{x - \frac{\alpha_{R_1}}{2} + 1}^{2d_{OR_1} x^\frac{\alpha_{R_1}}{2} + 1} \frac{dx}{(x - \frac{\alpha_{R_1}}{2} + \Lambda)^{m_n + k}} \right] \right) \} \right) \right) (50)
\]


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