# If You Must Do Confirmation Theory

## Do It This Way

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CONTENTS

# 0 Uncertain inference

It has been a standard response to Hume's criticism of induction that, of course, inductive arguments are not deductively valid, but they are inductively valid. An inductive conclusion $c$ does not follow with certainty from the empirical assumptions or premises $a$. At best, $c$ is partially deducible from $a$.

The conclusion of a valid inductive argument is not proved from the assumptions of the argument, but it is said to be confirmed by them. In what philosophers call inductive logic or confirmation theory, it is usually taken for granted that the degree of deducibility or the uncertainty of an inductive argument from $a$ to $c$ is best measured by the probability $\mathfrak{p}(c \mid a)$.

## 0 Confirmation is not partial deducibility

**The hypothesis $\mathfrak{c}$ is confirmed by the evidence $\mathfrak{a}$, and has the degree of confirmation $\sigma(\mathfrak{c}, \mathfrak{a}) = \mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a}) - \mathfrak{p}(\mathfrak{c})$, if & only if $\mathfrak{a}$ is relevant to $\mathfrak{c}$, that is, if $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a}) > \mathfrak{p}(\mathfrak{c}\,|\,\top) = \mathfrak{p}(\mathfrak{c})$. There are other measures, for example $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a})/\mathfrak{p}(\mathfrak{c})$, but none of them is a measure of degree of deducibility. In particular, probabilistic independence or irrelevance, $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a}) = \mathfrak{p}(\mathfrak{c})$ (or $\sigma(\mathfrak{c}, \mathfrak{a}) = 0$), is not a generalization of logical independence.**

**The probability $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a})$, often identified with the degree of belief $\mathfrak{c}(\mathfrak{c}\,|\,\mathfrak{a})$ that $\mathfrak{c}$ enjoys on $\mathfrak{a}$, may be substantial even when $\mathfrak{a}$ intuitively counts against $\mathfrak{c}$. The function $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a})$ measures degree of deducibility, but it does not measure confirmation.**

## 0 The aim of this talk

Several serious mistakes have led to this divorce between logic and confirmation theory. My aim today is to expose and to correct the most interesting mistake — the uncriticized assumption that the probability $\mathfrak{p}(c \mid a)$ is the best generalization of the relation of deducibility; that is, that $\mathfrak{p}(c \mid a)$ adequately measures the degree to which $c$ is deducible from $a$.

There are at least five other functions $\varphi$ definable in terms of $\mathfrak{p}$ that generalize deducibility in the sense that $\varphi(c \mid a) = 1$ whenever $c$ is deducible from $a$. I shall suggest that one of them, the function $\mathfrak{q}(c \mid a)$, which I shall call the deductive dependence of $c$ on $a$, combines the best features $\mathfrak{p}$ and $\sigma$.

# 1 Elementary theory of probability

We begin with an uninterpreted abstract binary measure function $\mathfrak{p}$ on an algebra of sentences or propositions. It will be assumed that $\mathfrak{p}$ obeys the axiom system for probability given by Popper in The Logic of Scientific Discovery, appendix $*$v.

This system is more general than the system of Kolmogorov (and more general even than those of Hosiasson-Lindenbaum and of Rényi). In particular, $\mathfrak{p}(c\,|\,a)$ is defined for all $a$, $c$, even inconsistent $a$, and $\mathfrak{p}(c\,|\,a) = 1$ whenever $c$ is deducible from $a$. It follows that $\mathfrak{p}(c\,|\,\perp) = 1$ for all $c$, and hence that the complementation law $\mathfrak{p}(c\,|\,a) + \mathfrak{p}(c'\,|\,a) = 1$ fails when $a \equiv \perp$. The identity $\mathfrak{p}(b\,|\,b) = 1$ is universally valid.

# 1 Popper's axioms

$$
\begin{array}{lrcl}
\textbf{A0} & \exists c \exists d \mathfrak{p}(a \mid c) & \neq & \mathfrak{p}(b \mid d) \\
\textbf{A1} & \forall b(\mathfrak{p}(a \mid b) = \mathfrak{p}(c \mid b)) & \Rightarrow & \mathfrak{p}(b \mid a) = \mathfrak{p}(b \mid c) \\
\textbf{A2} & \mathfrak{p}(a \mid a) & = & \mathfrak{p}(c \mid c) \\
\textbf{B1} & \mathfrak{p}(ac \mid b) & \leq & \mathfrak{p}(a \mid b) \\
\textbf{B2} & \mathfrak{p}(ac \mid b) & = & \mathfrak{p}(a \mid cb)\mathfrak{p}(c \mid b) \\
\textbf{C} & \mathfrak{p}(a|a) \neq \mathfrak{p}(b|a) & \Rightarrow & \\
 & \mathfrak{p}(a \mid a) & = & \mathfrak{p}(c \mid a) + \mathfrak{p}(c' \mid a).
\end{array}
$$

**When factored by $a \sim c =_{\text{df}} \forall b(\mathfrak{p}(a, b) = \mathfrak{p}(c, b))$ (indistinguishability), the domain is reduced to a Boolean algebra.**

# 1 Probabilistic definition of deducibility

We shall assume here that logically distinguishable sentences are distinguishable in measure; that is, that if $\mathfrak{a}$ and $\mathfrak{c}$ are not interdeducible, then $\mathfrak{p}(\mathfrak{a} \mid \mathfrak{b}) \neq \mathfrak{p}(\mathfrak{c} \mid \mathfrak{b})$ for some sentence $\mathfrak{b}$. We do not make the stronger assumption that $\mathfrak{p}$ is a strictly positive or regular measure (sometimes called a Carnap function). That is, $\mathfrak{p}(\mathfrak{c}|\mathfrak{a}) = 1$ is a necessary condition, but not always a sufficient condition, for the deducibility of $\mathfrak{c}$ from $\mathfrak{a}$.

The universal identity $\forall\mathfrak{b}\, \mathfrak{p}(\mathfrak{c} \mid \mathfrak{ab}) = 1$ is the simplest definition of the deducibility of $\mathfrak{c}$ from $\mathfrak{a}$ in terms of the measure $\mathfrak{p}$. But since $\forall\mathfrak{b}\, \mathfrak{p}(\mathfrak{c} \mid \mathfrak{ab}) = \mathsf{r}$ implies that $\mathsf{r} = 1$, degrees of deducibility can be introduced only by generalizing $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{a}) = 1$.

# 1 A few words from W. E. Johnson's *Logic*

**The position assumed by probability in logical discussion has always been dubious. On the one side the topic has been assumed to be the exclusive property of the mathematician, or rather more precisely, the arithmetician. On this view the quantity called probability is a mere abstract fraction, and the rules of probability are merely those of arithmetic. The fraction is, in short, the ratio of two numbers, the number holding for a species to that holding for its proximate genus, this ratio being necessarily a proper fraction, the limits of which are zero and unity. If this view were correct, there would be no separate topic to be called probability. A precisely reverse account of probability is that it is a measure of a certain psychological attitude of thought to which the most obvious names that could be given are belief or doubt, taken as subject to different degrees. On either of these two extreme views probability would have no particular connection with logic.**

# 2 Degrees of deducibility

The credence function $\mathfrak{c}(\mathrm{c} \mid \mathrm{a})$, is here defined to have the same value everywhere as the measure $\mathfrak{p}(\mathrm{c} \mid \mathrm{a})$, taking the value $1$ when $\mathrm{c}$ is deducible from $\mathrm{a}$, and, provided that $\mathrm{a} \not\equiv \bot$, the value $0$ when the negation $\mathrm{c}'$ of $\mathrm{c}$ is deducible from $\mathrm{a}$. Values of $\mathfrak{c}$ between $0$ and $1$ represent degrees of deducibility.

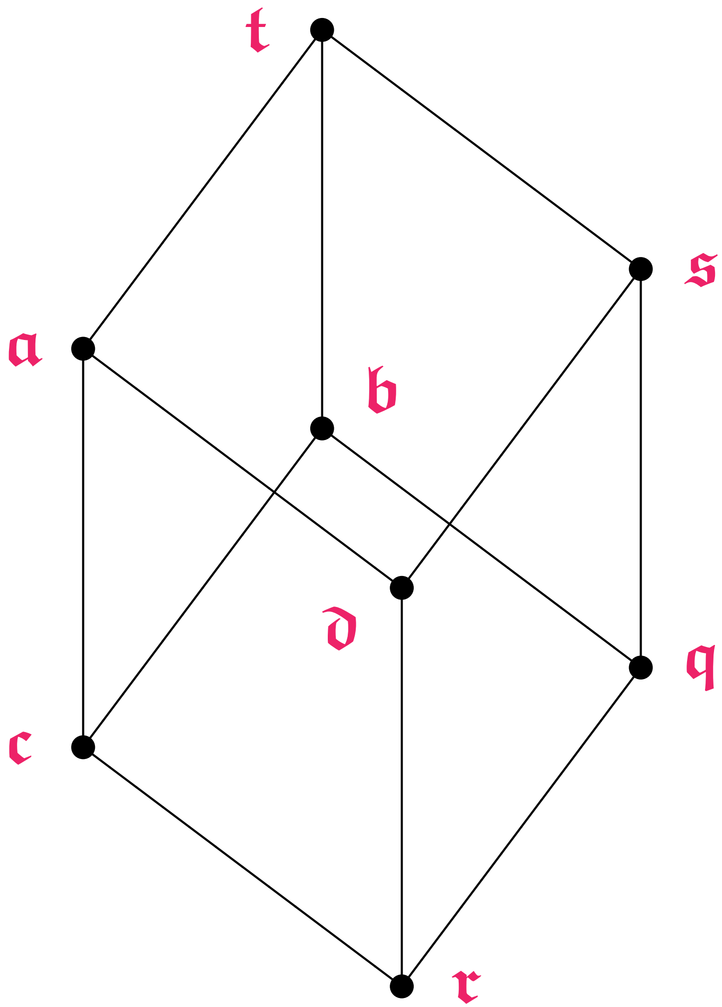
There are other ways to generalize the relation of deducibility. The function $\mathfrak{t}(\mathrm{c} \mid \mathrm{a}) = \mathfrak{p}(a \to c \mid \top) = 1$ takes the value $1$ when $\mathrm{c}$ is deducible from $\mathrm{a}$, and the value $0$ when $\mathrm{a} \equiv \top \equiv \mathrm{c}'$. The function $\mathfrak{a}(\mathrm{c} \mid \mathrm{a}) = \mathfrak{p}(\mathrm{c} \mid \mathrm{a} \vee \mathrm{c})$ takes the value $1$ when $\mathrm{c}$ is deducible from $\mathrm{a}$, and the value $0$ when $\mathrm{a} \not\equiv \top \equiv \mathrm{c}$. If $\mathfrak{p}$ is regular, these sufficient conditions for $0$ are also necessary.

# 2 Eight functions that grade deducibility

**There are essentially eight distinct pairs $\mathsf{X}$, $\mathsf{Z}$ of truth functions of $\mathfrak{a}$ and $\mathfrak{c}$ such that $\mathfrak{p}(\mathsf{Z}\,|\,\mathsf{X}) = 1$ whenever $\mathfrak{c}$ is deducible from $\mathfrak{a}$. $\uparrow$ is Sheffer stroke, $\triangle$ is symmetric difference.**

| function | in terms of $\mathfrak{p}$ | mnemonic |
|---|---|---|
| $\mathfrak{t}(\mathfrak{c}\,|\,\mathfrak{a})$ | $\mathfrak{p}(\mathfrak{a} \to \mathfrak{c}\,|\,\top)$ | tautology |
| $\mathfrak{s}(\mathfrak{c}\,|\,\mathfrak{a})$ | $\mathfrak{p}(\mathfrak{a}'\,|\,\mathfrak{a} \uparrow \mathfrak{c})$ | Sheffer stroke |
| $\mathfrak{a}(\mathfrak{c}\,|\,\mathfrak{a})$ | $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a} \vee \mathfrak{c})$ | alternation |
| $\mathfrak{b}(\mathfrak{c}\,|\,\mathfrak{a})$ | $\mathfrak{p}(\mathfrak{a} \to \mathfrak{c}\,|\,\mathfrak{c} \to \mathfrak{a})$ | biconditional |
| $\mathfrak{c}(\mathfrak{c}\,|\,\mathfrak{a})$ | $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a})$ | credence |
| $\mathfrak{q}(\mathfrak{c}\,|\,\mathfrak{a})$ | $\mathfrak{p}(\mathfrak{a}'\,|\,\mathfrak{c}')$ | deductive dependence |
| $\mathfrak{d}(\mathfrak{c}\,|\,\mathfrak{a})$ | $\mathfrak{p}(\mathfrak{a}'\mathfrak{c}\,|\,\mathfrak{a} \triangle \mathfrak{c})$ | directed distance |
| $\mathfrak{r}(\mathfrak{c}\,|\,\mathfrak{a})$ | $\mathfrak{p}(\perp\,|\,\mathfrak{a}\mathfrak{c}')$ | refutation |

# 2 The eight functions ordered by magnitude



## 2 The extremal functions $\mathfrak{r}(c \mid a)$ and $\mathfrak{t}(c \mid a)$

Note that $\mathfrak{r}(c \mid a)$ equals $0$ for all $a$, $c$ except those for which $ac' \equiv \bot$. In other words, $\mathfrak{r}(c \mid a)$ equals $1$ if & only if $c$ is deducible from $a$, and equals $0$ otherwise. It is a degenerate case of a numerical measure of deducibility. The function $\mathfrak{t}$ is also unsatisfactory as a measure of deducibility. Since relative measures are not definable in terms of absolute measures, full deducibility itself is not definable from $\mathfrak{t}$.

Closest to the refutation function $\mathfrak{r}$, but pairwise incomparable by magnitude, are the credence function $\mathfrak{c}$, the deductive dependence function $\mathfrak{q}$, and the directed distance function $\mathfrak{d}$. The names given to $\mathfrak{q}$ and $\mathfrak{d}$ will be explained as we proceed.

## 3 The function $\mathfrak{q}$

The focus of today's talk is the deductive dependence function $\mathfrak{q}$. This properly generalizes the deducibility relation since $\mathfrak{q}(c\,|\,a) = 1$ if & only if $\mathfrak{p}(a'\,|\,c') = 1$, which holds if $a'$ is deducible from $c'$, which holds if & only if $c$ is deducible from $a$.

It will be suggested that $\mathfrak{q}(c\,|\,a)$ is in many respects a better measure of the confirmation of $c$ by $a$ than is either the credence function $\mathfrak{q}(c\,|\,a)$ or the relevance function $\sigma(c\,|\,a) = \mathfrak{c}(c\,|\,a) - \mathfrak{c}(c)$. Indeed, $\mathfrak{q}(c\,|\,a)$ combines the virtue of $\mathfrak{c}(c\,|\,a)$, that it is a measure of deducibility, with the virtue of $\sigma(c\,|\,a)$ that it takes the value $0$ when $a \equiv \top$ (except when $c \equiv \top$); that is to say, if there is no evidence, there is no confirmation.

# 3 Content

The **content** of the proposition $\mathfrak{b}$ is commonly measured by $\mathfrak{p}(\mathfrak{b}') = 1 - \mathfrak{p}(\mathfrak{b})$. Since $\mathfrak{a} \vee \mathfrak{c}$ is the content that $\mathfrak{a}$ shares with $\mathfrak{c}$, its measure is $\mathfrak{p}((\mathfrak{a} \vee \mathfrak{c})')$. The 'proportion' of the content of $\mathfrak{c}$ that is included within the content of $\mathfrak{a}$ is thus $\mathfrak{p}((\mathfrak{a} \vee \mathfrak{c})')/\mathfrak{p}(\mathfrak{c}') = \mathfrak{p}(\mathfrak{a}'\mathfrak{c}')/\mathfrak{p}(\mathfrak{c}') = \mathfrak{p}(\mathfrak{a}' \mid \mathfrak{c}') = \mathfrak{q}(\mathfrak{c} \mid \mathfrak{a})$. It measures how much of $\mathfrak{c}$ is deducible from $\mathfrak{a}$. Miller & Popper (1986) called $\mathfrak{q}(\mathfrak{c} \mid \mathfrak{a})$ the **deductive dependence** of $\mathfrak{c}$ on $\mathfrak{a}$.

Hempel & Oppenheim (1948) interpreted $\mathfrak{Q}(\mathfrak{c} \mid \mathfrak{a}) = \mathfrak{q}(\mathfrak{a} \mid \mathfrak{c})$ as a measure of the **systematic power** of the hypothesis $\mathfrak{c}$ to organize the evidence $\mathfrak{a}$. Hilpinen (1970) interpreted $\mathfrak{q}(\mathfrak{a} \mid \mathfrak{c})$ as a measure of the **information transmitted** by $\mathfrak{a}$ about $\mathfrak{c}$.

# 3 Approximate validity

Deductive validity is defined by the principle of transmission of truth: it is necessary, given that the assumption $\alpha$ of a valid inference is true, that its conclusion $c$ is true. Falsificationists prefer an equivalent formulation, the principle of retransmission of falsity: it is necessary, given that the conclusion $c$ of a valid inference is false, that its assumption $\alpha$ is false.

There is no equivalence for approximate validity. It is probable, given that the assumption $\alpha$ of an approximately valid inference is true, that its conclusion $c$ is true is not equivalent to It is probable, given that the conclusion $c$ of an approximately valid inference is false, that its assumption $\alpha$ is false.

## 3 Numerical comparison of $\mathfrak{c}$ and $\mathfrak{q}$

The approximate identities $\mathfrak{p}(c\,|\,a) \approx 1$ and $\mathfrak{p}(a'\,|\,c') \approx 1$ are not equivalent. Salmon (2005) gives this (frequentist) example: '[f]rom the statement that the large majority of men are not physicists, it does not follow that the vast majority of physicists are not men'. $\mathfrak{c}$ and $\mathfrak{q}$ are not the same function.

More exactly, if the assumption $a$ is consistent, and $\mathfrak{p}(c) \neq 1$,

$$\mathfrak{q}(c \mid a) - \mathfrak{c}(c \mid a) = [1 - \mathfrak{p}(c \mid a)][1 - \mathfrak{p}(c) - \mathfrak{p}(a)]/[1 - \mathfrak{p}(c)],$$

and therefore $\mathfrak{c}(c \mid a) \lesseqgtr \mathfrak{q}(c \mid a)$ if & only if $\mathfrak{p}(c) + \mathfrak{p}(a) \lesseqgtr 1$.

# 3 Lower limits of $\mathfrak{c}$ and $\mathfrak{q}$

By construction, if $\varphi$ is any one of the eight functions $\mathfrak{a}$, $\mathfrak{b}$, $\mathfrak{c}$, $\mathfrak{d}$, $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$ listed above then $\varphi(c \mid a)$ takes the value $\mathbf{1}$ when $c$ is deducible from $a$ (and if $\mathfrak{p}$ is a regular measure, only then). But the conditions are different under which each function takes the value $\mathbf{0}$, as we have already noted for $\mathfrak{c}$, $\mathfrak{a}$, and $\mathfrak{t}$.

The functions $\mathfrak{c}$ and $\mathfrak{q}$ are dual. Indeed $\mathfrak{c}(c \,|\, a) = \mathfrak{p}(c \,|\, a) = \mathbf{0}$ when $a \not\equiv \bot$ and $ac \equiv \bot$; that is, when $a$ is consistent (non-contradictory) and $c'$ is deducible from $a$ (or $a$ and $c$ are contraries). Likewise $\mathfrak{q}(c \mid a) = \mathfrak{p}(a' \,|\, c') = \mathbf{0}$ when $c \not\equiv \top$ and $a \vee c \equiv \top$; that is, when $c$ is contentful (non-tautological), and $a$ is deducible from $c'$ (or $a$ and $c$ are subcontraries).

## 3 Some properties of $\mathfrak{q}$

It follows that $\mathfrak{q}(c\,|\,\top) = 0$ unless $c$ is a tautology $\top$ (in which case $\mathfrak{q}(c\,|\,\top) = 1$). Since $\top$ is deducible from every assumption $a$, we have $\mathfrak{q}(\top\,|\,a) = 1$. At the other extreme, $\mathfrak{q}(c\,|\,\bot) = 1$ for every $c$, while $\mathfrak{q}(\bot\,|\,a)$ may assume any value.

The monotony laws for the first argument of $\mathfrak{p}$ translate readily into monotony laws for the second argument of $\mathfrak{q}$; that is, $\mathfrak{q}(c\,|\,a \vee b) \leq \mathfrak{q}(c\,|\,a) \leq \mathfrak{q}(c\,|\,ab)$. The general addition law assumes the following form: $\mathfrak{q}(c\,|\,a) + \mathfrak{q}(c\,|\,b) = \mathfrak{q}(c\,|\,a \vee b) + \mathfrak{q}(c\,|\,ab)$. This implies that whenever $a$ and $b$ are mutual subcontraries, $\mathfrak{q}(c\,|\,a) + \mathfrak{q}(c\,|\,b) = \mathfrak{q}(c\,|\,ab)$ (unless $c \equiv \top$, in which case all three terms have the value $1$).

# 4 Hypothesis and evidence

**We see that if $\mathfrak{c}$ and $\mathfrak{a}$ in the expression $\mathfrak{q}(\mathfrak{c}\,|\,\mathfrak{a})$ represent a non-tautological hypothesis $\mathfrak{c}$ and a statement of evidence $\mathfrak{a}$, as they do in $\mathfrak{c}(\mathfrak{c}\,|\,\mathfrak{a})$, then $\mathfrak{q}(\mathfrak{c}\,|\,\mathfrak{a})$ takes the value $\mathbf{0}$ when there is no evidence, that is, $\mathfrak{a} \equiv \top$, and never decreases when more evidence is adduced [weak monotonic increase].**

**Sentences $\mathfrak{a}$ and $\mathfrak{b}$ are maximally independent if they are simply independent and mutually subcontrary; that is, $\mathfrak{a} \not\equiv \top \equiv \mathfrak{a} \vee \mathfrak{b} \equiv \top \not\equiv \mathfrak{b}$. Provided that $\mathfrak{c} \not\equiv \top$, the effects on $\mathfrak{c}$ of two subcontrary items $\mathfrak{a}$, $\mathfrak{b}$ of evidence are additive: $\mathfrak{q}(\mathfrak{c}\,|\,\mathfrak{a}) + \mathfrak{q}(\mathfrak{c}\,|\,\mathfrak{b}) = \mathfrak{q}(\mathfrak{c}\,|\,\mathfrak{ab})$. If $\mathfrak{a}$ and $\mathfrak{c}$ are maximally independent, and $\mathfrak{p}$ is regular, $\mathfrak{q}(\mathfrak{c}\,|\,\mathfrak{ab})$ exceeds both $\mathfrak{q}(\mathfrak{c}\,|\,\mathfrak{a})$ and $\mathfrak{q}(\mathfrak{c}\,|\,\mathfrak{b})$.**

## 4 $\mathfrak{q}$ is not a measure of credence

**Since $\mathfrak{q}$, is monotone increasing, $\mathfrak{q}(c\,|\,a)$ may exceed $0$ when the assumption $a$ and the conclusion $c$ contradict each other. Unless it can be rational to believe a hypothesis that contradicts the evidence, $\mathfrak{q}(c\,|\,a)$ cannot be interpreted as a measure of credence, as a (rational) degree of belief in $c$ given $a$.**

**It is easily shown that the two-valued function $\mathfrak{r}(c \mid a)$ is the only other one among the eight functions here enumerated that takes the value $1$ whenever $c$ is deducible from $a$ and the value $0$ whenever $c'$ is deducible from $a$ (and $c \not\equiv \perp$). Only $\mathfrak{c}$ is a genuine measure of (consistent) belief. But there are other measures that score this trick, such as the product $\mathfrak{c} \cdot \mathfrak{q}$.**

## 4 Using $\mathfrak{q}$ as a measure of confirmation

Because $\mathfrak{q}(c\,|\,\top) = 0$ (when $c \not\equiv \top$), the distinction between credence $\mathfrak{c}$ and relevance $\sigma$, in this context often called the initial confirmation and the incremental confirmation of $\mathfrak{c}$ by $\mathfrak{a}$ dissolves in the case of $\mathfrak{q}$. Confirmation is measured by $\mathfrak{q}(c\,|\,a)$. We note in passing, however, that relativizing $\mathfrak{q}$ to $a$ is not (as it is for $\mathfrak{c}$) the same operation as updating $\mathfrak{p}$ by $a$.

If $\mathfrak{q}$ is taken to measure degree of confirmation, it may appear to be harmless that $\mathfrak{q}(c\,|\,a)$ may exceed $\mathbf{0}$ when $ac \equiv \bot$. For in science it often happens that the evidence that is cited in favour of a hypothesis actually contradicts it. On this point, though not elsewhere, I agree with Lakatos and Feyerabend.

## 4 Degrees of truth

**Such a misfit between hypothesis and evidence is tolerable, however, only for hypotheses that are approximately, if not exactly, true. We cannot allow every false hypothesis to receive unlimited confirmation even when it has been refuted.**

**The worry can be relieved if the evidence $a$ is required (reasonably) to be true; that is, to belong to the set $T$ of all (empirical) truths. The value of $q(c \mid a)$ cannot then exceed $q(c \mid T)$, which is naturally equated with $\sup\{q(c \mid a) : a \in T\}$. In 1994 I suggested that $q(c \mid T)$ is a good measure of degree of approximation to truth. In this sense only approximately true hypotheses can be substantially confirmed by true evidence.**

# 4 Classical versus Bayesian statistics

**If we admit the non-classical assumption that probabilities of hypotheses exist, the function $\mathfrak{q}$ may be used to contrast classical and Bayesian statistics. Let $\mathrm{h}$ be the alternative to the null hypothesis $\mathrm{H}$. Typically $\mathrm{h}$ states that there is a chance effect manifested in a positive value of a statistic $\mathbf{t}$, and $\mathrm{H}$ denies it. The evidence $\mathrm{e}$ is that $\mathbf{t}$ has the value $\rho$. The p-value (of $\mathrm{h}$) is the probability, given $\mathrm{H}$, that $\mathbf{t} \geq \rho$. Since $\mathrm{H} \equiv \mathrm{h}'$, this may be written as: $\mathfrak{c}(\mathbf{t} \geq \rho \,|\, \mathrm{h}') = 1 - \mathfrak{c}(\mathbf{t} < \rho \,|\, \mathrm{h}') = 1 - \mathfrak{q}(\mathrm{h} \,|\, \mathbf{t} \geq \rho) \geq 1 - \mathfrak{q}(\mathrm{h} \,|\, \mathbf{t} = \rho)$. A low $\mathbf{p}$-value means a high value of $\mathfrak{q}(\mathrm{h} \,|\, \mathbf{t} = \rho)$; that is, $\mathfrak{q}(\mathrm{h} \,|\, \mathrm{e}) \approx 1$. Bayesian statistics, in contrast, values hypotheses $\mathrm{h}$ for which $\mathfrak{c}(\mathrm{h} \,|\, \mathrm{e}) \approx 1$. The two approaches differ only in how they measure deducibility.**

# 4 Anglo–American criminal law

**The distinction between credence $\mathfrak{c}$ and deductive dependence $\mathfrak{q}$ needs to be rigorously maintained also in criminal trials, in which the accused can be convicted only if his guilt $c$ is proved beyond reasonable doubt by the evidence $a$. This can mean only that $c$ is almost deducible from $a$ given the court's extensive (though indefinite) background knowledge.**

**The defence may proceed in two ways. One is to call witnesses so that the evidence $a$ before the court renders open to doubt the conclusion $c$ that the accused is guilty: $\mathfrak{c}(c\,|\,a) << 1$. A more fundamental defence is that there is no case to answer: that the evidence $a$ has little bearing on $c$: $\mathfrak{q}(c\,|\,a) << 1$.**

## 4 When high credence may be a bad thing

It can be proved that, provided $\mathfrak{q}(c \mid a) \neq 1$, the smaller $\mathfrak{c}(c)$ and $\mathfrak{c}(a)$ are, the smaller $\mathfrak{c}(c \mid a)$ is, so that a high value of $\mathfrak{c}(c \mid a)$ may not be achievable if both the charge $c$ and the evidence $a$ are a priori improbable. If it is the closeness of $\mathfrak{q}(c \mid a)$ to $1$ that is needed for a guilty verdict, this failure to prove the charge 'beyond reasonable doubt' should be of no concern.

There is much high-minded talk in courts of law of the probability (or rational indubitability) $\mathfrak{c}(c \mid a)$ of the accused's guilt $c$, given the evidence $a$, but what should be in the forefront of intelligent minds (judges, advocates, and, one hopes, jurors) is $\mathfrak{q}(c \mid a)$; that is, how much $c$ deductively depends on $a$.

## 5 Popper follows Waismann, and goes further

**We have seen that the function $\mathfrak{q}$ is in some ways similar to the credence function $\mathfrak{c}$, and in other ways similar to the relevance function $\sigma$. It has at times been confused with both.**

**In his Logic Popper repeated Waismann's metaphor that probability $\mathfrak{p}$ is a measure of logical proximity. Adding content to this idea in Part II of Realism, he assimilated $\mathfrak{p}$ (and $\mathfrak{c}$) to $\mathfrak{q}$:**

> **[in] the logical interpretation of probability, '$c$' and '$a$' are interpreted as names of statements (or propositions) and $\mathfrak{p}(c \mid a) = r$ is an assertion about the contents of $c$ and $a$ and their degree of logical proximity; or more precisely, about the degree to which the statement $c$ contains information which is contained in $a$.**

# 5 Misidentification of $\mathfrak{q}$ and $\mathfrak{c}$

It is of course correct that '*if $a$ is consistent and contradicted by $c$, then $\mathfrak{p}(c \mid a) = 0$*'. But a consistent proposition $a$ may contain much information that is also contained in a proposition $c$ that contradicts it (though it contains no information that is also contained in its contradictory $a'$). For an example, take any two competing scientific theories, which inevitably share many consequences. A more abstract example is provided by any two distinct maximal theories in a rich language.

But this shows that neither $\mathfrak{p}(c, a)$ nor $\mathfrak{c}(c, a)$ is a measure of '*the degree to which the statement $c$ contains information which is contained in $a$*'. It is $\mathfrak{q}(c, a)$ that measures this.

## 5 Misidentification of $\mathfrak{c}$ and $\mathfrak{d}$

**Waismann's idea (1930), adopted by Popper, that logical probability (that is, credence) '*could be called the degree of "logical proximity" of two statements*', displays another unintended entangling of different generalizations of deducibility.**

**Since $\mathfrak{c}(c\,|\,a) = \mathfrak{c}(a\,|\,c)$ usually fails, $1 - \mathfrak{c}$ is not a (pseudo)-metric operation. Yet there is a clear sense in which distance is not symmetric: the *directed distance* from Cuicuilco to the summit of Xitle exceeds the distance in the other direction. The triangle inequality also fails for $1 - \mathfrak{c}$ (and for $1 - \mathfrak{q}$), but it holds (in finite cases) for $1 - \mathfrak{d}$. The function $\mathfrak{d}(c\,|\,a) = \mathfrak{p}(a'c\,|\,a \vartriangle c)$ truly measures the *logical proximity* of $c$ to $a$.**

## 5 Misidentification of $\mathfrak{q}$ and $\sigma$

**Zwirn & Zwirn (1989), Rivadulla (1994), and Howson (2000) have been puzzled by the conclusion of the Popper–Miller theorem that the excess content $a \to c$ of the hypothesis $c$ — that part of its content that goes beyond the evidence $a$ — is in general countersupported by $a$: that $\sigma(a \to c, a) < 0$ (except in some unexciting cases in which equality obtains).**

**In Howson's view this infringes 'the sceptical tenet [which he attributes to us] that $a$ only informs us about $a$ and nothing beyond it'. But since $a \to c$ and $a$ are maximally independent, what sceptics demand is that $\mathfrak{q}(a \to c \mid a)$ take the value $0$. (It does.) The value of $\sigma(a \to c, a)$ is beside the point.**

## 6 A disclaimer

**Having said all this, I must end by emphasizing that my aim has not been to lay the foundations of a new theory of confirmation based on the function $\mathfrak{q}$, but to raze to the foundations all those old theories of confirmation that are based on the functions $\mathfrak{c}$ and $\sigma$. I submit that if $\mathfrak{c}$ or $\sigma$ (or some related measure) is the best measure of confirmation, then $\mathfrak{q}$ is better, and therefore neither $\mathfrak{c}$ or $\sigma$ is the best one.**

**I am not recommending $\mathfrak{q}$, or anything like it, as a substitute measure of confirmation, since I hold that in science we do not need any such measure. The outcome of hypothesis testing is not subsequent selection but immediate deselection.**

# 6 Falsificationism

**For the purpose of testing hypotheses, and their further examination, but not for their evaluation after testing, selection rules may be appropriate. I am saying nothing new if I say that falsificationism prefers for testing a hypothesis $c$ for which $\mathfrak{q}(c \mid a)$ is low, not one for which $\mathfrak{q}(c \mid a)$ is high, since little of the content of such a hypothesis $c$ is deducible from the evidence $a$, and it is therefore susceptible to genuine tests.**

**Only in legal trials (and similar activities, such as professional examinations) does the yearning for proof and for positive evidence make any real sense. Here, it seems, a high value of $\mathfrak{q}(c|a)$ may be a sine qua non of a positive conclusion.**