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A High-speed Test of the Equivalence Principle

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1. Introduction

It is well-known that Newton’s work on mechanics depended in a crucial way on the previous observations of Galileo. The key insight of Galileo was that one can analyse the motion of bodies using experiments and mathematical equations. One experimental observation which roughly emerges from this work in modern terms is that two objects of different mass which are simultaneously released from rest and allowed to fall under the influence of gravity through a vacuum should hit the ground at the same time (this is essentially what is called the equivalence principle in general relativity) [1]. In popular legend, it is said that Galileo tested this by dropping two balls of different masses from the Leaning Tower of Pisa and showing that they hit the ground at the same time, but the historical evidence suggests that this is unlikely as he seems to have experimented mostly with balls rolling down inclined slopes [2].

One way to state this observation is as a corollary of Newton’s law of universal gravitation. It might be a good exercise to ask students to consider why this is. The equation for universal gravitation is

\[ F = G \frac{m_1 m_2}{r^2}, \]  

where \( F \) is the gravitational force between two objects of mass \( m_1 \) and \( m_2 \), \( G \) is the gravitational constant, and \( r \) is the distance between the centres of mass of the two objects. However, in the case we are considering, \( m_1 = M \) where \( M \) is the mass of the Earth. Since this mass is a constant, it can be absorbed into a quantity \( g \) which we define to be \( g = G M / r^2 \), so the equation becomes

\[ F = g m, \]  

where \( m \) is the mass of the falling object. Students should recognise the similarity with Newton’s second law

\[ F = m a, \]  

where \( a \) is the acceleration, which states that the acceleration due to a force is proportional to that force. Comparison of (2) and (3) shows that two objects of different mass undergo the same acceleration due to gravity \( g \). The only way to oppose this falling motion is with the drag force due to the presence of air. For example, an object with a greater surface area will feel more of an effect from air resistance. It follows that if this drag force is very small two objects of different mass which are dropped from a height should hit the ground at almost the same time.

There is a long history of experiments to demonstrate this prediction, including experiments of Robert Boyle [3]. A more recent version of the demonstration (known as the Apollo 15 hammer-feather drop) is maybe the most famous demonstration experiment of all time [4]. Kagan carried out a series of experiments where two pumpkins were dropped from a height of around 20 m. It was found that it was possible to have the two pumpkins hit the ground at almost the same time, with the difference being attributed to the difficulty of releasing both of the pumpkins simultaneously and not due to air resistance. The neglect of air resistance is not due to the small
size of the drag force (which can be large), but because the increase in mass between a large and a small pumpkin is roughly proportional to the increase in surface area [5]. In a study similar to [5], Pendrill et al. recorded drops of two different objects at a time from a large height and discussed the pedagogical use of such demonstrations for middle-school students, finding that students typically needed some convincing of the equivalence principle and that they were amazed to see the Apollo 15 demonstration [6, 7]. Various other free-fall experiments have been carried out in a high-school environment, including a version of the Einstein elevator, but there do not seem to be any demonstrations of objects falling through a vacuum at this level [8–10].

2. Experimental Set-up

We will now describe our experimental set-up which is shown schematically in Figure 1. A stainless steel sphere of diameter 1.5 cm was held above the ground by a 12 V tubular solenoid electromagnet connected to a power supply. A hole was drilled in a Perspex block and the electromagnet was fixed into the block. A cylindrical Perspex tube was placed on top of a rubber ring around the target landing zone which sealed it from the bottom. A second rubber ring was placed on top of the tube which was then covered by the Perspex block: this sealed the tube and placed the ball inside. In effect, the tube is the vacuum chamber so the size of our chamber is the length of the tube which we have available, which was 65 cm (bespoke tubes can be ordered from an acrylics and Perspex supplier). A vacuum pump (Leybold) connected to the bottom of the chamber was switched on to reduce the ambient pressure inside the tube to a level which was read by a pressure sensor connected to the chamber. A piece of tissue paper was trapped between the sphere and the magnet. Once the tube was sealed, a high vacuum was pulled (pressure $P = 1 \times 10^2$ Pa). The sphere and tissue paper were then released simultaneously by switching off the electromagnet and allowed to fall under the influence of gravity. The impact was captured with a Photron FASTCAM SA-X2 high-speed video camera at frame rate of 12 500 fps. Backlighting was provided by a single light source (Phlox) of luminance 178 256 cd m$^{-2}$ and uniformity 99.44 %.

Fig 1: Schematic and photo of experiment.

3. Results and Discussion

At atmospheric pressure $P_A = 1.01 \times 10^5$ Pa, the density of air is $\rho_A = 1.2$ kg m$^{-3}$. A computation shows that the density for our high vacuum is

$$\rho = \frac{\rho_A P}{P_A} = 1.2 \times \frac{1 \times 10^2}{1.01 \times 10^5} \approx 1 \times 10^{-3} \text{ kg m}^{-3}. \quad (4)$$

The influence of air resistance on the falling body is given by the following standard equation for the drag force [11]:

$$F_D = \frac{1}{2} CA\rho v^2, \quad (5)$$

where $C$ is the drag coefficient, $A$ is the cross-sectional area, $\rho$ is the gas density, and $v$ is the speed of the object. Given that the drop height is around 65 cm, the impact speed is approximately
\[ v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times \frac{65}{100}} \approx 3.57 \, \text{m} \, \text{s}^{-1}, \]

where we have used the standard value for the acceleration due to gravity \( g \). \( C \) is usually taken to be 0.45 for a sphere, so plugging into equation (5) we have

\[ F_D = \frac{1}{2} \times 0.45 \times \left( \pi \times \left( \frac{0.75}{100} \right) \right)^2 \times 0.001 \, \text{kg m}^{-3} \times (3.57 \, \text{m} \, \text{s}^{-1})^2 \approx 5 \times 10^{-7} \, \text{N}. \]  

Similarly, for a piece of tissue paper with surface area \( A = 4 \, \text{cm} \times 4 \, \text{cm} \), the drag coefficient is taken to be 1.05, so plugging into equation (5) we obtain

\[ F_D = \frac{1}{2} \times 1.05 \times (0.04 \, \text{m})^2 \times 0.001 \, \text{kg m}^{-3} \times (3.57 \, \text{m} \, \text{s}^{-1})^2 \approx 1 \times 10^{-5} \, \text{N}. \]

We weighed the masses of the steel ball and the paper to be \( 1.74 \times 10^{-3} \, \text{kg} \) and \( 5 \times 10^{-5} \, \text{kg} \) respectively, hence the ratio of \( F_D \) to the force due to gravity \( F_G = mg \) is \( 2 \times 10^{-5} \) and \( 2 \times 10^{-2} \) for the ball and paper, respectively. The expectation is that due to the relative smallness of the drag force in both cases the sphere and the paper should hit the ground at almost the same time, but not exactly simultaneously.

This is what we see in Figure 1 but the nice thing about using the high-speed camera is that we can make this statement more quantitative. The video shows that the difference in frames between both objects touching the surface is around 50 frames. Since the camera is 12 500 frames per second, the time difference between the two impacts is around 4 milliseconds (or four thousandths of a second). The two objects were released with exact simultaneity so this must be due to differences of accelerations

\[ a = \frac{F_G - F_D}{m} = g - \frac{CA \rho v^2}{2m} \]

for both objects. Kagan avoided this problem by using two objects with the same ratio \( A/m \), since all objects with the same ratio \( A/m \) fall at the same rate, regardless of drag force effects [10, 5]. If the drag force were to vanish completely, equation (9) would become \( a = g \). However, plugging our values into equation (9) we see that \( a \) takes slightly different values for the objects, hence they fall with slightly different rates (\( g = 9.8 \, \text{ms}^{-1} \) and \( a = 9.7 \, \text{ms}^{-1} \) for the ball and paper, respectively, to two significant figures). We also point out that in our experiment the bottom edge of the paper was slightly below the ball where it was pinned to the magnet and so fell a slightly shorter distance.

The next step in our investigation which students could look into is the relation between terminal speed and pressure (or density). The terminal speed of an object is the maximum possible speed which it can reach as it falls through the gas, being attained when the drag force \( F_D \) on the object is equal to \( F_G \). By adding the two forces to obtain the net force and re-arranging, the equation for the terminal speed \( V_T \) is

\[ V_T = \sqrt{\frac{2mg}{\rho CA}}. \]  

One can see that it should be possible to alter the terminal speed by varying the gas pressure, which can be checked experimentally.
The main question to answer is what use the demonstration is for students, and whether it could be carried out by high-school instructors with sufficient experimental skills. Our high-precision experimental set-up is very expensive, but we emphasise that if one wishes simply to repeat the phenomenon in a classroom demonstration, it should be possible to repeat it with cheaper equipment. A vacuum can be pulled with a cheap vacuum pump (cost around $140) and a modern smartphone can do 400 frames per second, which is sufficient to check that both objects are falling almost simultaneously. Similarly, the solenoid electromagnet and Perspex block can be obtained very cheaply online. The main objective of the experiment is to introduce students to the concepts of drag force and acceleration due to gravity. An issue with existing demonstrations is that they rely on being able to judge whether two objects have hit the ground with approximate simultaneity without much precision or even with the naked eye, so that it is a bit hard to communicate to students that the prediction that the two objects hit the ground simultaneously is typically only correct up to a very close approximation and that there are some subtle corrections when one brings in all of the physics [5]. This notion is much easier to communicate with a high-precision experiment.

Furthermore, as noted earlier in Section 1, it leads directly to the principle of equivalence and Einstein elevator experiments which essentially form the physical basis of general relativity [12]. To start obtaining physical intuition for these types of thought experiments at high-school level would be highly desirable. We have not carried out the activity described in this article in a classroom environment, but we hope that instructors will attempt to incorporate it into their classes and give feedback on how effective it is or what problems they encounter. To develop further insight, students with access to a similar experimental set-up might like to study the dependence of terminal speed on the gas density.

Fig 2: A steel sphere and a piece of tissue paper falling in a vacuum.
Acknowledgments

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References


