Quantum-limited absorption estimation with ring resonators

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Abstract: Coherent-state probes in appropriately designed all-pass ring resonators outperform any quantum probe single-pass absorption estimation strategy, including those using Fock or bright squeezed states.

Introduction

Numerous studies have leveraged non-classical probes such as Fock and squeezed states to enhance precision in parameter estimation over equally bright classical strategies [1]. Remarkably, we find coherent-state probes in all-pass ring resonators can estimate absorption more precisely than any quantum-probe single-pass strategy [2].

Results

The system we consider is an all-pass ring resonator comprised of a bus waveguide coupled to a ring resonator with length L. An analyte with an absorption coefficient α_A, which we seek to estimate, is evanescently coupled to the ring as shown in Fig. 1

According to the Beer-Lambert law, the attenuation coefficient of the ring resonator $a = e^{-\alpha_T L/2}$ where $\alpha_T = \alpha_I + \Gamma\alpha_A$ is the total linear absorption coefficient. Here Γ quantifies the fraction of guided light in the ring interacting with the analyte and α_I is the intrinsic absorption coefficient.

We quantify the performance of pure single-mode Gaussian probes in estimating α_A using the Quantum Fisher information (QFI) formalism. For an experimental strategy repeated ν times, the variance $\Delta^2\alpha_A$ is related to the QFI $Q(\alpha_A)$ through the Quantum Cramér-Rao bound $\Delta^2\alpha_A \geq [\nu\, Q(\alpha_A)]^{-1}$ [3]. We find that maximum information is obtained when the ring resonator is on resonance and critically coupled, yielding a QFI

$$Q_G = (|\beta|^2 + \sinh^2 s)\frac{L^2\Gamma^2 B}{1 - e^{-\alpha_T L}}, \tag{1}$$

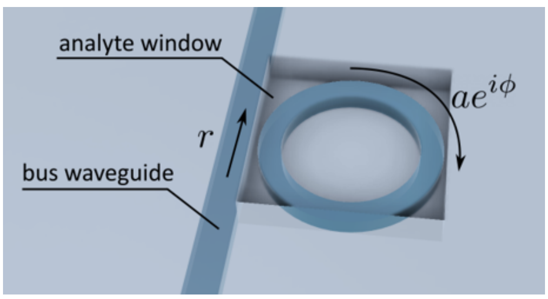

Fig. 1. All-pass ring resonator with self-coupling coefficient r, attenuation a, and round trip phase ϕ. We seek to estimate the absorption coefficient α_A of an analyte evanescently coupled to the light circulating in ring resonator.

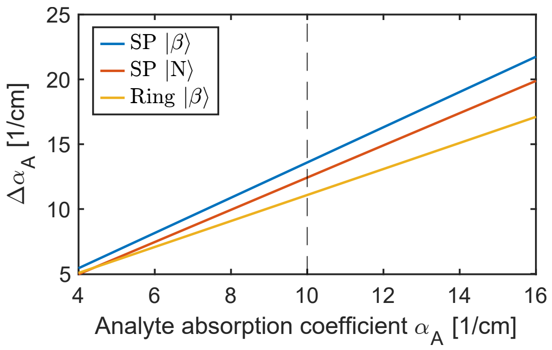

Fig. 2. Standard deviation $\Delta\alpha_A$ for an all-pass ring resonator probed by a coherent-state $|\beta\rangle$ (orange) and for single-pass (SP) strategies probed by coherent-state (blue) and Fock state $|\mathrm{N}\rangle$ (red) probes. The latter two correspond to the classical and quantum limits for the SP strategies, respectively. At a target analyte $\alpha_A = 10\,\mathrm{cm}^{-1}$, the ring resonator is critically coupled and probing it with a classical coherent-state outperforms any single-pass strategy.

where B is the resonator's buildup factor. The performance of a coherent-state probe with mean photon number $|\beta|^2$ is readily obtained by setting the squeezing factor $s = 0$, resulting in a QFI

$$Q_{|\beta\rangle} = |\beta|^2 \frac{L^2 \Gamma^2 B}{1 - e^{-\alpha_T L}}. \tag{2}$$

The term in parenthesis in Eq. (1) is equal to the mean number of input photons in a bright single-mode squeezed state. As such, arbitrarily bright squeezed states provide no advantage over coherent-state probes.

Fock states have been shown to be optimal in estimating absorption in single-pass strategies [4]. For a probe photon number N_0, they attain a QFI

$$Q_{\mathrm{SP},|N\rangle} = N_0 \frac{L^2}{e^{\alpha_A L} - 1}. \tag{3}$$

Coherent-state probes in single-pass strategies yield a QFI

$$Q_{\mathrm{SP},|\beta\rangle} = |\beta|^2 L^2 e^{-\alpha_A L}. \tag{4}$$

Comparing Eqs. (2) to (4), appropriately designed all-pass ring resonators with coherent-state probes are capable of estimating α_A with higher precision than any quantum-probe single-pass strategy, including those using optimal Fock state probes as depicted in Fig. 2.

Conclusion

By leveraging interference and resonant enhancement effects, photonic integrated ring resonator circuits with shot-noise limited classical coherent-state sources can surpass the precision attainable with any single-pass quantum probe absorption estimation strategy. Our findings are especially relevant for lab-on-chip sensors, which have important practical applications in environmental monitoring [5] and biochemical analysis [6].

References

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