A Short Note on the Problematic Concept of Excess Demand in Asset Pricing Models with Mean-Variance Optimization

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Abstract  
Referring to asset pricing models where demand is proportional to excess returns and said to be derived from a mean-variance optimization problem, the note formulates what probably is common knowledge but hardly ever made an explicit subject of discussion. This is an insufficient distinction between the desired holding of the risky asset on the part of the speculative agents, which is the solution to the optimization problem and usually directly presented as excess demand, and the desired change in this holding, which is what should reasonably constitute the excess demand on the market. The note arrives at the conclusion that in models with a market maker the story of the maximization of expected wealth should be dropped.

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This note makes an elementary observation in asset pricing models that derive the speculative agents’ formulation of excess demand from myopic mean-variance optimization of expected wealth. These models are often not sufficiently clear about their precise notion of demand and the corresponding changes in the assets the agents are holding.

The presentation of the mean-variance optimization approach usually begins with the equation for the wealth dynamics. If for concreteness the risky asset is a large stock or market index that pays a dividend \( y_t \) per share at the beginning of the market period \( t \), and the risk-free asset pays a fixed rate of return \( r \), the evolution of the wealth of an agent of type \( h \) from period \( t \) to period \( t+1 \) is described as

\[
W_{h,t+1} = (1+r)W_{h,t} + [p_{t+1} + y_{t+1} - (1+r)p_t] z_{h,t}
\]

where in addition \( p_t \) is the (uniform) price at which the asset is traded in period \( t \), and \( z_{h,t} \) is said to be “the number of shares of the risky asset purchased” at time \( t \) (emphasis added). The latter quotation can be found in Hommes et al. (2005, p. 1046) or He and Li (2007, p. 3400), and there are many other papers speaking of “purchasing” or “buying”.

More precisely, \( z_{h,t} \) is a desired quantity which, however, can always be realized in these models. The optimization problem itself uses (1) as its wealth constraint, subject to which the expected value of a CARA utility function \( U = U(W_{h,t+1}) = -\exp(-\mu_h W_{h,t+1}) \) is to be maximized, or in an equivalent formulation the term \( E_{h,t}(W_{h,t+1}) - (\mu_h/2) V_{h,t}(W_{h,t+1}) \) (where \( \mu_h > 0 \) is the agent’s risk aversion coefficient and \( E_{h,t}, V_{h,t} \) are his conditional expectations and conditional variance).\(^1\) The explicit solution to this problem is

\[
z_{h,t} = \frac{E_{h,t}[p_{t+1} + y_{t+1} - (1+r)p_t]}{\mu_h V_{h,t}[p_{t+1} + y_{t+1} - (1+r)p_t]}
\]

If one goes back to the derivation of eq. (1), the formulation that \( z_{h,t} \) is the number of shares purchased at time \( t \) is found to be somewhat careless. To see this let \( a_{h,t} \) and \( b_{h,t} \) be the number of shares and the risk-free asset (bonds) which are in the portfolio of agent \( h \) at the beginning of period \( t \). At that time he has also received the dividends and interest payments, so his wealth is (dropping the index \( h \))

\[
W_t = p_t a_t + b_t + a_t y_t + r b_t
\]

\(^1\)CARA stands for constant absolute risk aversion.
The agent invests his income on shares and bonds, while for speculative reasons he may also exchange shares for bonds or *vice versa*. Denote the shares and bonds he wishes to hold at the beginning of the next period as \( a_{t+1}^d \) and \( b_{t+1}^d \), respectively. The shares are bought at the current price \( p_t \) and, of course, subject to the constraint

\[
W_t = p_t a_{t+1}^d + b_{t+1}^d
\]  

With the change in the market price and the new dividends and interest receipts, the wealth at the beginning of \( t+1 \) amounts to (3) dated one period forward (and \( a^d, b^d \) in place of \( a \) and \( b \)). Using (4), rearrangement of this equation leads to

\[
W_{t+1} = (1+r)(p_t a_{t+1}^d + b_{t+1}^d) + [p_{t+1} + y_{t+1} - (1+r)p_t] a_{t+1}^d 
\]

It follows that \( z_{h,t} \) in (1) must be the desired holding of the risky asset. In contrast to the remark on (1), what agent \( h \) “purchases” on the market is not the entire stock \( a_{h,t+1}^d \) of the asset he wishes to hold but just the difference from his actual holding \( a_{h,t} \) at the beginning of period \( t \).

The problem is more serious than a careless use of words. Turning to the notion of demand it is reasonably also the aggregate differences between the agents’ desired and actual holdings that constitute the period \( t \) excess demand \( d_t \) on the market, and not the sum of the desired holdings themselves. Hence with \( H \) groups of speculative agents on the market,

\[
d_t = \sum_{h=1}^{H} a_{h,t+1}^d - \sum_{h=1}^{H} a_{h,t} = \sum_{h=1}^{H} z_{h,t} - \sum_{h=1}^{H} a_{h,t}
\]

It can now be argued that \( d_t \) is identical to \( \sum_h z_{h,t} \) if the total number of shares remains constant and is conveniently set equal to zero. This is indeed a consistent situation in models that employ a Walrasian auctioneer for continuous market clearing, \( d_t \equiv 0 \). One has then only to be aware that in a deterministic equilibrium the income from dividends and interest is exclusively invested in bonds, which implies that the proportion of wealth held in shares is steadily decreasing. This is the price one has to pay for a CARA utility function in whose maximization the level of current wealth is eliminated.
The idea that $\sum h z_{h,t}$ can be identified with $\sum h a_{h,t+1}$ does not work if market disequilibrium is admitted. The standard story abstains from rationing but instead introduces a market maker who, besides quoting the price, absorbs any excess of supply from the market and serves any excess of demand from his inventory. As in this context the assumption that the total number of shares is fixed has to include the inventory $a_{m,t}$ of the market maker, it now reads,

$$\sum_{h=1}^{H} a_{h,t} + a_{m,t} = \text{const}$$

In this identity $a_{m,t}$ is obviously fluctuating over time. Therefore, in a market maker setting, the sum $\sum_{h=1}^{H} a_{h,t}$ in (6) cannot be treated as a constant. In other words, a claim that $\sum_{h=1}^{H} z_{h,t}$ represents market excess demand would be unwarrantable.\footnote{Incidentally, the models in the two papers from which the quotation on the term $z_{h,t}$ in eq. (1) was taken do include a market maker.}

Two alternative consequences could be drawn from that. On the one hand, one may resort to the interpretation that the relevant notion of demand formulated on the market is that of desired holdings. The model would then be consistent but, we may say, too academic.

On the other hand, it may be postulated that $z_{h,t}$ as it is given by (2) is not the agent’s desired holding but his desired change in the asset. In this way the equation loses its optimization flavour; at least (2) cannot be sold as the solution to the maximization of expected wealth. Nevertheless, the right-hand side of (2) makes perfect economic sense, saying that excess demand is proportional to the expected excess return on the asset and discounted by some measure of risk or volatility.

Another price that has to be paid in the latter option should, however, not go unnoticed. As eq. (2) determines the desired as well as the realized change in the risky asset, the agent’s position is a mere appendix to the model that does not feed back on demand. Especially in a stochastic framework his position may thus easily wander around and become unduly large, with a positive or negative sign. This feature would certainly not be compatible with the risk constraints of real traders (that is, constraints in addition to their concerns about volatility).
If the latter observation is taken seriously, one is back to the problem that consistency requires the model to keep track of the individual holdings of the asset. As an outlook, a straightforward specification to guard against excessive positions may be mentioned. It maintains the original demand $z_{h,t} = a_{h,t+1} - a_{h,t}$ at its core but revises it downward (upward) in proportion to the current positive (negative) deviations of $a_{h,t}$ from some target level.\(^3\)

In conclusion, for an appropriate interpretation of models with a market maker and some mean-variance optimization formula it would be helpful for the reader if they were more explicit about their notion of demand. To us it seems most reasonable if they completely dropped their story of the maximization of expected wealth.

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\(^3\)And a similar rule for the market maker’s price quotes with which he seeks to limit his inventory. The numerical effort for introducing these concepts into the original models would be minimal. If they are sufficiently simple, even an analytical treatment can still be possible; see Franke and Asada (2008) for such an analysis in the framework of the seminal Beja–Goldman model (whose formulation of demand is sometimes said to be compatible with mean-variance optimization).
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