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Multiple Priors and No-Transaction Region

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Abstract

We study single period asset allocation problems of the investor who maximizes the expected utility with respect to non-additive beliefs. The non-additive beliefs of the investor model the presence of an uncertainty and they are assumed to be consistent with the Maxmin expected utility theory of Gilboa and Schmeidler (1989). The proportional transaction costs are incorporated into the model. We provide the explicit form solutions for the bounds of no-transaction regions which completely determine the optimal policy of the investor.

Key words: uncertainty modelling; utility theory; maxmin portfolio selection; transaction costs.

JEL Classification: G11, C44, C61.

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Introduction

The interest in an investigation of investors' behavior dominates in finance during the last decades and the main topic of this research is the asset allocation problem. The dominant theory in this field is subjective expected utility theory (SEU), which was developed by Savage (1954). Since Knight (1921) has made a distinction between risk and uncertainty these notions form the basis of modern theories of decision making. According to Knight, the notion of risk relates to the situations where a probability measure can represent the likelihoods of events while uncertainty refers to cases when an investor has an incomplete information to assign probabilities to events. SEU was the first theory which tried to model such a distinction. Elsberg's paradox (Ellsberg (1961)) demonstrates, however, that it has many disadvantages and does not take into account the fact that the beliefs of the investor might not be additive. This argument shows that SEU is not an appropriate model of decision making under uncertainty. Alternative models and possible extensions of SEU in the direction of modelling uncertainty have been proposed by Schmeidler (1989) and Gilboa and Schmeidler (1989), who model investor's beliefs as non-additive subjective probabilities (capacities) and sets of additive probabilities, respectively. In his Choquet expected utility (CEU) Schmeidler provides an axiomatic foundation and a mathematical representations of investor's preferences, using a notions of expectation due to Choquet (1953). Gilboa and Schmeidler (1989) develop the Maxmin expected utility theory (MMEU) characterizing preference relation over acts which have a numerical representation by the functional of the form $V(X) = \min_{Q \in \mathcal{P}} E_Q(U(X))$, where X is an act, $U \colon \mathbb{R}^+ \to \mathbb{R}^+$ is a von Neumann-Morgenstern utility (von Neumann and Morgenstern (1944)) and $\mathcal{P}$ is a set of probability measures. In fact, MMEU theory of Gilboa and Schmeidler is a partial case of more general Choquet expected utility framework. On the other hand it can be regarded as robust to the model misspecification. The investor consider, in some sense, the neighborhood of possible distributions, defined by the set of priors, and makes a decision based on the worst case of possible distributions of the risky asset.

However, there are no so many application of the Choquet utility theory for the portfolio selection models in the literature. Dow and Werlang (1992) were the first who applied the Choquet expected utility model of Schmeidler (1989) for the asset allocation problem and found out an important implications of Schmeidler's model. They showed that, in the model with one risky and one riskless asset, there is a non-degenerate price interval at which the investor will strictly prefer to take zero position in the risky asset. In contrast to this, in the traditional expected utility theory the non-degenerate price interval is reduced to the point. Carlier and Dana (2003) investigate behavior of the investor within CEU framework. An example of capacity which they use in the investigation is a distorted probability — a composition of a continuous increasing function $h\colon [0,1] \to [0,1]$ and a probability measure P_0, i.e. $\nu(A) = h(P_0(A))$ for every event A. They obtain the result that under some conditions on the stock price the optimal policy for the investor is to set a weight of stocks in his/her portfolio equal to zero. Similar non-degenerate price region has been derived by Dow and Werlang (1992).

In this paper we solve the decision making problem within the MMEU approach in the economy with one riskless asset and one risky asset, which pays no dividends. Returns of the risky asset are assumed to be normally distributed. Although the normal distribution can not describe the behavior of high-frequency data, monthly stock returns could be modeled by normally distributed random variables. An appropriate model for the high-frequency data is GARCH process therefore these results are also useful for models conditionally normal distributions for the returns of risky asset.

It is shown that analogical to Dow and Werlang (1992) and Carlier and Dana (2003) results also have place and the explicit form of price no-trade condition is given.

In order to provide a model which is more relevant to real markets we incorporate proportional transaction costs under consideration. As it turned out that the no-transaction region for the investor who is MMEU maximizer has different forms depending on distributions of assets prices. The main contribution of the paper is that we derive explicit formulae of optimal policies and the bounds no-transaction region and the dynamic of their changes with respect to parameters of assets prices distributions.

The investor's attitude to the risk is represented by the exponential utility function of the form $U(x) = 1 - e^{-\gamma x}$, $0 < \gamma < 1$. A special structure of the exponential utility function allows us to derive explicit solutions of the investor's problem. The ambiguity is incorporated into the model by the set of priors. We consider all probability measures in this set to absolutely continuous with respect to a predefined measure P_0 and their

Radon-Nikodym derivatives are assumed to be log-normally distributed under P_0. It is shown how incorporating the uncertainty into the model impacts optimal policies of the investor.

The paper is organized as follows. Section 1 gives necessary definitions and preliminary results which we use in the sequel. In particular, we provide a short description of MMEU model. In the Section 2 we consider the single period asset allocation model and derive its solution for the investor whose preferences are consistent with MMEU framework. The proportional transaction costs are incorporated into the model in Section 3. The main result, presented in this Section, is a derivation of different forms of the no-transaction region depending on the parameters of the model. Section 5 briefly summarizes the contributions of the paper.

1 Definitions and setup

Let us consider a state space $(\Omega, \mathcal{F})$, where $\mathcal{F}$ is an algebra on Ω. Denote by $\mathcal{X}$ the set of acts, i.e. the set of all measurable function on $(\Omega, \mathcal{F})$. The object of study is choice behavior relative to $\mathcal{X}$. We postulate that there exists a preference relation $\succeq$ on $\mathcal{X}$ consistent with the axioms of MMEU (see Gilboa and Schmeidler (1989)), that is, there exist utility function $U\colon \mathbb{R}^+ \to \mathbb{R}^+$ and a set of probability measures $\mathcal{P}$ on $(\Omega, \mathcal{F})$ such that for every $X, Y \in \mathcal{X}$

$$X \succeq Y \Rightarrow V(X) \geq V(Y),$$

where the preference functional V can be represented as

$$V(X) = \min_{Q \in \mathcal{P}} E_Q(U(X)) \tag{1.1}$$

for each $X \in \mathcal{X}$. Here E_Q denotes the expectation with respect to probability measure Q.

Let us fix a measure P_0 on $(\Omega, \mathcal{F})$. In order to simplify the research we assume that all measures in $\mathcal{P}$ are absolutely continuous with respect to the measure P_0. By the Radon-Nikodym theorem for every measure $Q \in \mathcal{P}$ there exists a non-negative random variable η_Q with $E_{P_0}(\eta_Q) = 1$, such that $dQ = \eta_Q dP_0$. Therefore, we can identify the set $\mathcal{P}$ with the set of their Radon-Nikodym derivatives with respect to the probability measure P_0. Let us assume that these derivatives are log-normally distributed. In the sequel we need the following lemma.

Lemma 1.1. *Every normally distributed under measure P_0 random variable $Z \sim N(\mu, \sigma^2)$ is normally distributed under measure $Q \in \mathcal{P}$.*

Proof. We can express the Radon-Nikodym derivative of measure Q in the form $\eta_Q = e^{\alpha+\beta Z+u}$, where u is zero-mean normally distributed random variable which is independent from Z.

Let us consider the moment generating function $M_Q(s) = E_Q(e^{sZ}) = E_{P_0}(e^{sZ+\ln(\eta_Q)})$. Since random variables u and Z are independent we have that

$$M_Q(s) = e^{\alpha} E_{P_0}(e^u) E_{P_0}(e^{(s+\beta)Z}) = e^{\alpha+E_{P_0}(u^2)+\beta\mu+\frac{\beta^2\sigma^2}{2}+(\mu+\beta\sigma^2)s+\frac{s^2\sigma^2}{2}}.$$

Let $s = 0$ and use the fact that $E_Q(1) = 1$ to yield

$$M_Q(s) = e^{(\mu+\beta\sigma^2)s+\frac{s^2\sigma^2}{2}}.$$

This implies that $Z \sim N(\mu + \beta\sigma^2, \sigma^2)$ under measure Q which completes the proof.

2 Portfolio optimization

Let us consider a model, where the investor makes his/her investment decisions in the economy with one riskless asset (*bond*) and one risky asset (*stock*), which pays no dividends. This model we use further in all sections. The rate of return of the riskless asset here is denoted by r, the return of risky one is $Z = \mu + \sigma\varepsilon$, and the random variables ε are independent and normally distributed under the measure P_0 with zero mean and unit variance.

We restrict ourselves on the case of normally distributed stock return, first of all, because this simplifies the research. Although, an empirical study gives us evidence that stock returns are not normally distributed, one of possible explanations why we use normal distribution in our model, is that there are a lot of results postulating that the stock returns could be modelled by the GARCH process which has the conditional normal distribution.

According to the Maxmin Expected Utility model the aim of the investor is to maximize the preference functional (1.1) of his/her wealth at the end of the period.

The set of priors $\mathcal{P}$ consists of the absolute continuous with respect to P_0 probability measures, whose Radon-Nikodym derivatives are P_0-log-normally distributed.

In the sequel we assume that the investor's attitude to the risk is represented by the exponential utility function

$$U(x) = 1 - e^{-\gamma x},\ 0 < \gamma < 1.$$

Let us denote $\mu(\beta) = \mu + \beta\sigma^2$. Due to Lemma 1.1 the preference functional (1.1) in the context of our assumptions can be rewritten in the form

$$V(W) = \min_{\beta \in [\beta_{\min}, \beta_{\max}]} E_{P_0}(U(W)),$$

where

$$W = W_0((1+r)(1-w) + w(\mu(\beta) + \sigma\varepsilon)).$$

Here w denotes the proportion of the wealth invested in the risky asset. Since the function $\mu(\beta)$ is linear we can change the argument of the minimization problem and provide it with respect to the parameter μ, which belongs to the interval $[\mu_{\min}, \mu_{\max}]$, where $\mu_{\min} = \mu(\beta_{\min})$ and $\mu_{\max} = \mu(\beta_{\max})$.

Under such assumptions and notations we claim that the optimization problem for the investor is

$$\min_{\mu \in [\mu_{\min}, \mu_{\max}]} E_{P_0}(U(W(w))) \underset{w}{\rightarrow} \max \tag{2.1}$$

subject to the budget constraint

$$W = W_0((1+r)(1-w) + w(\mu + \sigma\varepsilon)).$$

The following theorem gives an optimal strategy of (2.1).

Let $C_{\min} = \mu_{\min} - (1+r)$ and $C_{\max} = \mu_{\max} - (1+r)$.

Theorem 2.1. *The optimal strategy of the investor is*

$$w^{opt} = \begin{cases} \frac{C_{\min}}{\gamma W_0 \sigma^2}, & \textit{if} \quad C_{\min} > 0, \\ \frac{C_{\max}}{\gamma W_0 \sigma^2}, & \textit{if} \quad C_{\max} < 0, \\ 0, & \textit{if} \quad C_{\min} \le 0 \le C_{\max}. \end{cases}$$

Proof. Given w let us find a form of the preference functional V.

$$\begin{aligned}
V(W(w)) &= \min_{\mu\in[\mu_{\min},\mu_{\max}]} E_{P_0}(U(W(w))) \\
&= 1+\min_{\mu\in[\mu_{\min},\mu_{\max}]} E_{P_0}(-e^{-\gamma W_0((1+r)(1-w)+w(\mu+\sigma\varepsilon))}) \\
&= 1+e^{-\gamma W_0(1+r)}e^{\frac{\gamma^2 W_0^2\sigma^2 w^2}{2}}\min_{\mu\in[\mu_{\min},\mu_{\max}]}(-e^{-\gamma W_0 w(\mu-(1+r))}).
\end{aligned}$$

In order to find the min-value of the above equation we notice that the expression

$$\frac{\partial(-e^{-\gamma W_0 w(\mu-(1+r))})}{\partial\mu} = \gamma W_0 w e^{-\gamma W_0 w(\mu-(1+r))}$$

is greater than or equal to 0 if $w\geq 0$ and less than 0 if $w<0$. Since possible values of μ are bounded by $\mu_{\min}$ and $\mu_{\max}$ we conclude that the function

$$\mu^*(w)=\begin{cases}\mu_{\min}, & w\geq 0\\ \mu_{\max}, & w<0\end{cases} = \operatorname*{argmin}_{\mu\in[\mu_{\min},\mu_{\max}]}(-e^{-\gamma W_0 w(\mu-(1+r))}).$$

In fact, the explicit form of the preference functional is

$$V(W(w)) = 1-e^{-\gamma W_0(1+r)}e^{-\gamma W_0(\mu^*(w)-(1+r))w+\frac{\gamma^2 W_0^2\sigma^2 w^2}{2}}. \tag{2.2}$$

Our aim, according to (2.1), is to maximize (2.2) with respect to w. It is worth to notice that

$$\lim_{w\to+0} V(W(w)) = \lim_{w\to-0} V(W(w)) = 1-e^{-\gamma W_0(1+r)}, \tag{2.3}$$

therefore $V(W(w))$ is a continuous function.

The function $V(W(w))$ is differentiable on $w\in(-\infty,0)\cup(0,+\infty)$ and its possible extremal points are $w'=\frac{C_{\min}}{\gamma W_0\sigma^2}$ on $(0,+\infty)$, $w''=\frac{C_{\max}}{\gamma W_0\sigma^2}$ on $(-\infty,0)$ and $w=0$.

Let us consider three cases. If $C_{\min}>0$ then $w'\in(0,+\infty)$ but $w''\notin(-\infty,0)$. This fact and (2.3) imply that the global maximum of the preference functional occurs at point w' which is the optimal portfolio weight for the investor.

If $C_{\max}<0$ then $w'\notin(0,+\infty)$ and $w''\in(-\infty,0)$. Therefore the optimal strategy in this case is $w^{opt}=w''$.

In the case if $C_{\max}>0$ and $C_{\min}<0$ there is the only maximum point $w=0$ and this means that is is optimal for the investor no invest all available wealth into the bond. The theorem is proved.

3 Model with Transaction costs

In this section transaction costs are incorporated in the model. As above, the portfolio of the investor consists of a risky asset and a riskless asset. Before the investment decision the investor owns x_0^* dollars in the risky asset and y_0^* dollars in the bond. After making an additional investment of Δ dollars in the stock, the agent incurs proportional transaction costs $\theta|\Delta|$ for $0 \leq \theta < 1$. The costs of transactions are assumed to be charged to the riskless asset. Let us define

$$\tau = \begin{cases} 1, & \text{the investor buys stocks,} \\ -1, & \text{the investor sells stocks,} \\ 0, & \text{the investor makes no transactions.} \end{cases}$$

After the transaction the stock and bond holdings, which form the portfolio at the end of period, become

$$x = x_0^* + \tau\Delta,$$

$$y = y_0^* - \tau\Delta - \theta\Delta,$$

where $\Delta \geq 0$ is the traded dollar amount of the risky asset.

The final portfolio holdings are then given by

$$x^* = xZ = (x_0^* + \tau\Delta)Z, \tag{3.1}$$

$$y^* = y(1+r) = (y_0^* - \tau\Delta(1+\tau\theta))(1+r). \tag{3.2}$$

Given the initial portfolio (x_0^*, y_0^*) the variables Δ and τ represent a trading strategy of the investor, whose objective is to maximize the preference functional

$$V(x^*(\Delta,\tau) + y^*(\Delta,\tau)) = \min_{\mu \in [\mu_{\min}, \mu_{\max}]} E_{P_0}(x^*(\Delta,\tau) + y^*(\Delta,\tau)) \underset{\Delta \geq 0, \tau}{\longrightarrow} \max, \tag{3.3}$$

subject to the bond and stock wealth dynamic (3.1) and (3.2). Here we want to emphasize functional dependence of x^* and y^* on Δ and τ.

Let us make the following notations:

$$A_{\min} = \mu_{\min} - (1+\theta)(1+r),\ A_{\max} = \mu_{\min} - (1+\theta)(1+r),$$
$$B_{\min} = \mu_{\min} - (1-\theta)(1+r),\ B_{\max} = \mu_{\max} - (1-\theta)(1+r).$$

The following theorems show how optimal policies for the investor depend on the relations between these parameters.

Theorem 3.1. *Let $A_{\min} \geq 0$. Then the optimal strategy of the investor for the investment problem (3.3), (3.1) and (3.2) is given by*

$$\begin{array}{lll} \Delta^{opt} = \frac{A_{\min}}{\gamma\sigma^2} - x_0^* \textit{ and } \tau^{opt} = 1 & \textit{if} & x_0^* < \frac{A_{\min}}{\gamma\sigma^2}, \\ \Delta^{opt} = -\frac{B_{\min}}{\gamma\sigma^2} + x_0^* \textit{ and } \tau^{opt} = -1 & \textit{if} & x_0^* > \frac{B_{\min}}{\gamma\sigma^2}, \\ \Delta^{opt} = 0 \textit{ and } \tau^{opt} = 0 & \textit{if} & \frac{A_{\min}}{\gamma\sigma^2} \leq x_0^* \leq \frac{B_{\min}}{\gamma\sigma^2}. \end{array}$$

Proof. Given μ one can rewrite the expected utility of the terminal wealth as

$$\begin{aligned} I(\mu) = E_{P_0}(U(x^* + y^*)) &= 1 - e^{-\gamma(x_0^*\mu + y_0^*(1+r))} \cdot E_{P_0}(e^{-\gamma\tau\Delta D - \gamma\sigma(x_0^* + \tau\Delta)\varepsilon}) \\ &= 1 - e^{-\gamma(x_0^*\mu + y_0^*(1+r))} \cdot e^{-\gamma\tau\Delta D} \cdot e^{\gamma^2\sigma^2(x_0^* + \tau\Delta)^2}, \end{aligned}$$

where $D = \mu - (1 + \tau\theta)(1 + r)$.

In order to find the form of the preference functional $V(x^* + y^*)$ we should solve the minimization problem

$$I(\mu) \underset{\mu \in [\mu_{\min}, \mu_{\max}]}{\longrightarrow} \min$$

with respect to μ. The derivative

$$\frac{\partial I(\mu)}{\partial \mu} = \gamma(x_0^* + \tau\Delta) \cdot e^{-\gamma(x_0^*\mu + y_0^*(1+r))} \cdot e^{-\gamma\tau\Delta D + \frac{\gamma^2\sigma^2(x_0^* + \tau\Delta)^2}{2}}.$$

Thus, $\frac{\partial I(\mu)}{\partial \mu} \geq 0$ if $x_0^* + \tau\Delta \geq 0$ and $\frac{\partial I(\mu)}{\partial \mu} < 0$ if $x_0^* + \tau\Delta < 0$. Denote

$$\mu(\Delta, \tau) = \begin{cases} \mu_{\min} & \text{if} \quad x_0^* + \tau\Delta \geq 0, \\ \mu_{\max} & \text{if} \quad x_0^* + \tau\Delta < 0 \end{cases} = \underset{\mu \in [\mu_{\min}, \mu_{\max}]}{\operatorname{argmin}} I(\mu). \tag{3.4}$$

Hence, we conclude that

$$V(\Delta, \tau) = 1 - e^{-\gamma(x_0^*\mu(\Delta,\tau) + y_0^*(1+r))} \cdot e^{-\gamma\tau\Delta(\mu(\Delta,\tau) - (1+\tau\theta)(1+r)) + \frac{\gamma^2\sigma^2(x_0^* + \tau\Delta)^2}{2}}. \tag{3.5}$$

Let us note that the case $\Delta = 0$ is equivalent to $\tau = 0$. Therefore for the case $\tau \neq 0$ we take under consideration only positive values of Δ. Given x_0^* the boundary conditions

$$V(0, 1) = V(0, -1) = V(\Delta, 0) \tag{3.6}$$

are satisfied. At points with $x_0^* + \tau\Delta = 0$ the preference functional

$$V(\Delta, \tau) = 1 - e^{-\gamma(1+r)(y_0^* - (1+\tau\theta))}$$

does not depends on $\mu(\Delta, \tau)$ and, hence, is continuous function on $[0, +\infty) \times \{-1, 0, 1\}$.

The derivatives of the preference functional are given by the expression

$$\frac{\partial V}{\partial \Delta} = \begin{cases} \tau\gamma L e^{-\gamma(x_0^*\mu_{\min}+y_0^*(1+r))} e^{-\gamma\tau\Delta D_{\min}}(D_{\min} - \gamma\sigma^2(x_0^* + \tau\Delta))), & x_0^* + \tau\Delta > 0, \\ \tau\gamma L e^{-\gamma(x_0^*\mu_{\max}+y_0^*(1+r))} e^{-\gamma\tau\Delta D_{\max}}(D_{\max} - \gamma\sigma^2(x_0^* + \tau\Delta))), & x_0^* + \tau\Delta < 0, \end{cases} \tag{3.7}$$

where $L = e^{\frac{\gamma^2\sigma^2 x^2}{2}}$, $D_{\min} = \mu_{\min} - (1+r)(1+\tau\theta)$ and $D_{\max} = \mu_{\max} - (1+r)(1+\tau\theta)$.

Let us consider the following four cases.

1). $x_0^* \in (\frac{B_{\min}}{\gamma\sigma^2}, +\infty)$.

If $\tau = 1$ we have that $x_0^* > -\Delta$. In this case the

$$\frac{\partial V(\Delta,1)}{\partial\Delta} = \gamma e^{\frac{\gamma^2\sigma^2 x^2}{2}} \cdot e^{-\gamma(x_0^*\mu_{\max}+y_0^*(1+r))} e^{-\gamma\Delta A_{\min}} \cdot (A_{\min} - \gamma\sigma^2(x_0^* + \Delta))) < 0$$

because of the fact that

$$-\gamma\sigma^2(x_0^* + \Delta) \leq -\gamma\sigma^2 x_0^* \leq -B_{\min} < -A_{\min}.$$

If $\tau = -1$ the derivative on the interval $0 < \Delta < x_0^*$ is

$$\frac{\partial V(\Delta,-1)}{\partial\Delta} = -\gamma e^{\frac{\gamma^2\sigma^2 x^2}{2}} \cdot e^{-\gamma(x_0^*\mu_{\min}+y_0^*(1+r))} e^{-\gamma\Delta B_{\min}} \cdot (B_{\min} - \gamma\sigma^2(x_0^* - \Delta))).$$

On this interval there exists the maximum which is the solution of the equation $\frac{\partial V(\Delta,-1)}{\partial\Delta} = 0$ and this point is

$$\Delta = -\frac{B_{\min}}{\gamma\sigma^2} + x_0^*.$$

On the semi-interval $(x_0^*, +\infty)$ the function $V(\Delta,-1)$ is decreasing. Indeed, $x_0^* - \Delta < 0 < \frac{B_{\max}}{\gamma\sigma^2}$ implies that $\frac{\partial V(\Delta,-1)}{\partial\Delta} < 0$.

Continuity of the preference functional leads to the fact that the global maximum is unique and the optimal strategy under the condition 1) is to sell $\Delta^{opt} = -\frac{B_{\min}}{\gamma\sigma^2} + x_0^*$ dollars of stocks.

2). $x_0^* \in [\frac{A_{\min}}{\gamma\sigma^2}, \frac{B_{\min}}{\gamma\sigma^2}]$.

If $\tau = 1$ we have that $\Delta + x_0^* > 0$ therefore $\mu(\Delta,1) = \mu_{\min}$. It is easy to see that the inequality $x_0^* \geq \frac{A_{\min}}{\gamma\sigma^2}$ leads to the inequality $\frac{\partial V(\Delta,1)}{\partial\Delta} < 0$. Thus, the function $V(\Delta,1)$ is decreasing.

If $\tau = -1$ the inequality $x_0^* \leq \frac{B_{\min}}{\gamma\sigma^2}$ also implies $\frac{\partial V(\Delta,-1)}{\partial\Delta} < 0$ and the function $V(\Delta,-1)$ decreases. Therefore, the only point of the maximum exists at $\Delta = 0$ which determines the optimal policy of the investor, i.e. $\Delta^{opt} = 0$.

3). $x_0^* \in [0, \frac{A_{\min}}{\gamma\sigma^2})$.

We have that under this condition $\mu(\Delta, 1) = \mu_{\min}$ and the equation $\frac{\partial V(\Delta,1)}{\partial\Delta} = 0$ has a unique solution at point $\Delta = \frac{A_{\min}}{\gamma\sigma^2} - x_0^*$.

If $\tau = -1$ the inequality $x_0^* \leq \frac{B_{\min}}{\gamma\sigma^2} < \frac{B_{\max}}{\gamma\sigma^2}$ implies $\frac{\partial V(\Delta,-1)}{\partial\Delta} < 0$. Therefore, the optimal policy of the investor is to buy $\Delta^{opt} = \frac{A_{\min}}{\gamma\sigma^2} - x_0^*$ dollars of stocks.

4). $x_0^* \in (-\infty, 0)$.

In this case $\mu(\Delta, -1) = \mu_{\max}$ for all $\Delta \geq 0$ and as in the previous case $x_0^* < \frac{B_{\min}}{\gamma\sigma^2} < \frac{B_{\max}}{\gamma\sigma^2}$ implies $\frac{\partial V(\Delta,-1)}{\partial\Delta} < 0$.

In the another case $\mu(\Delta, 1) = \mu_{\max}$ if $\Delta < -x_0^*$ and $\mu(\Delta, 1) = \mu_{\min}$ if $\Delta \geq x_0^*$. Since $\frac{A_{\max}}{\gamma\sigma^2} > 0 > x_0^* + \Delta$ we have that $\frac{\partial V(\Delta,1)}{\partial\Delta} > 0$ and the function $V(\Delta, 1)$ is increasing on the interval $(0, |x_0^*|)$. On the interval $(|x_0^*|, +\infty)$ this function has the unique maximum at the point $\Delta^{opt} = \frac{A_{\min}}{\gamma\sigma^2} - x_0^*$.

The interval $[\frac{A_{\min}}{\gamma\sigma^2}, \frac{B_{\min}}{\gamma\sigma^2}]$, where $\Delta^{opt} = 0$ is called no-transaction region. The optimal policy of the investor is completely determined by this interval. As long as the amount of wealth invested in the stock is within the no-transaction region, the portfolio is not adjusted. If this amount of wealth strays outside the bounds the transaction is made to restore the amounts of stocks to the closest boundary of the no-transaction region.

Theorem 3.2. *Let $A_{\min} < 0$, $A_{\max} \geq 0$ and $B_{\min} \geq 0$. Then the optimal strategy of the investor for the investment problem (3.3), (3.1) and (3.2) is given by*

$$\begin{array}{lll} \Delta^{opt} = |x_0^*| \text{ and } \tau^{opt} = 1 & \text{if} & x_0^* < 0, \\ \Delta^{opt} = -\frac{B_{\min}}{\gamma\sigma^2} + x_0^* \text{ and } \tau^{opt} = -1 & \text{if} & x_0^* > \frac{B_{\min}}{\gamma\sigma^2}, \\ \Delta^{opt} = 0 \text{ and } \tau^{opt} = 0 & \text{if} & 0 \leq x_0^* \leq \frac{B_{\min}}{\gamma\sigma^2}. \end{array}$$

Proof. We prove this theorem similarly to the proof of Theorem 3.1. As a matter of fact, some of cases are proved in the same manner.

1). $x_0^* \in (\frac{B_{\min}}{\gamma\sigma^2}, +\infty)$.

If $\tau = 1$ the derivative of the preference functional (3.7) is $\frac{\partial V(\Delta,1)}{\partial\Delta} < 0$ because

$$-\gamma\sigma^2(x_0^* + \Delta) \leq -\gamma\sigma^2 x_0^* \leq -B_{\min} < -A_{\min}.$$

If $\tau = -1$ the preference functional has a local maximum on the interval $0 < \Delta < x_0^*$

at point

$$\Delta = -\frac{B_{\min}}{\gamma\sigma^2} + x_0^*.$$

which is the solution of the equation $\frac{\partial V(\Delta,-1)}{\partial \Delta} = 0$.

On the interval $(x_0^*, +\infty)$ the function $V(\Delta, -1)$ is decreasing due to the fact that $x_0^* - \Delta < 0 < \frac{B_{\max}}{\gamma\sigma^2}$.

It turns out that the point $\Delta^{opt} = -\frac{B_{\min}}{\gamma\sigma^2} + x_0^*$, $\tau = -1$ is the point of global maximum of the preference functional and defines the optimal policy of the investor.

2). $x_0^* \in [0, \frac{B_{\min}}{\gamma\sigma^2}]$.

Similar to case 2) in Theorem 3.1 the inequalities $x_0^* \geq \frac{A_{\min}}{\gamma\sigma^2}$ and $x_0^* \leq \frac{B_{\min}}{\gamma\sigma^2}$ imply that both of the functions $V(\Delta, 1)$ and $V(\Delta, -1)$ are decreasing and, hence, the maximum of the preference functional occurs at the point $\Delta^{opt} = 0$.

3). $x_0^* \in (-\infty, 0)$.

If $\tau = -1$ we have that $x_0^* < \frac{B_{\min}}{\gamma\sigma^2} < \frac{B_{\max}}{\gamma\sigma^2}$ and this implies that $\frac{\partial V(\Delta,-1)}{\partial \Delta} < 0$.

If $\tau = 1$ we get that $\mu(\Delta, 1) = \mu_{\max}$ for $\Delta < |x_0^*|$ and $\mu(\Delta, 1) = \mu_{\min}$ for $\Delta > |x_0^*|$. On the interval $(0, |x_0^*|)$ the inequality $\frac{A_{\max}}{\gamma\sigma^2} > 0 > x_0^* + \Delta$ leads to the fact $\frac{\partial V(\Delta,1)}{\partial \Delta} > 0$. On the semi-interval $(|x_0^*|, +\infty)$ the inequality $\frac{A_{\min}}{\gamma\sigma^2} < 0 < x_0^* + \Delta$ implies $\frac{\partial V(\Delta,1)}{\partial \Delta} < 0$. Therefore the only maximum occurs at the point $\Delta^{opt} = |x_0^*|$.

Under the assumptions of Theorem 3.2 the no-transaction region is $[0, \frac{B_{\min}}{\gamma\sigma^2}]$. Its asymmetry can be explain by the non-additivity of the preferences of the investor. Similar to the case without the transaction costs, where the no-trade condition is $C_{\min} < 0 < C_{\max}$, the non-additivity in preferences makes an impact if $A_{\min} < 0 < A_{\max}$.

Theorem 3.3. *Let $A_{\max} < 0$ and $B_{\min} \geq 0$. Then the optimal strategy of the investor for the investment problem (3.3), (3.1) and (3.2) is given by*

$$\begin{array}{lll} \Delta^{opt} = \frac{A_{\max}}{\gamma\sigma^2} - x_0^* \text{ and } \tau^{opt} = 1 & \text{if} & x_0^* < \frac{A_{\max}}{\gamma\sigma^2}, \\ \Delta^{opt} = -\frac{B_{\min}}{\gamma\sigma^2} + x_0^* \text{ and } \tau^{opt} = -1 & \text{if} & x_0^* > \frac{B_{\min}}{\gamma\sigma^2}, \\ \Delta^{opt} = 0 \text{ and } \tau^{opt} = 0 & \text{if} & \frac{A_{\max}}{\gamma\sigma^2} \leq x_0^* \leq \frac{B_{\min}}{\gamma\sigma^2}. \end{array}$$

Proof. Similar to the previous theorems we consider three cases.

1). $x_0^* \in (\frac{B_{\min}}{\gamma\sigma}, +\infty)$.

It can be proved analogically to case 1) in Theorem 3.1 that the optimal for the investor is to sell $\Delta^{opt} = -\frac{B_{\min}}{\gamma\sigma^2} + x_0^*$ amount of stocks.

2). $x_0^* \in [\frac{A_{\max}}{\gamma\sigma^2}, \frac{B_{\min}}{\gamma\sigma^2}]$.

The inequalities $x_0^* \leq \frac{B_{\min}}{\gamma\sigma^2} < \frac{B_{\max}}{\gamma\sigma^2}$ in the case $\tau = -1$ and $x_0^* \geq \frac{A_{\max}}{\gamma\sigma^2} > \frac{A_{\min}}{\gamma\sigma^2}$ in the case $\tau = 1$ imply the functions $V(\Delta, 1)$ and $V(\Delta, -1)$ are decreasing and therefore the interval $[\frac{A_{\max}}{\gamma\sigma^2}, \frac{B_{\min}}{\gamma\sigma^2}]$ is a subset of the no-transaction region. This means that the optimal strategy is $\Delta^{opt} = 0$.

3). $x_0^* \in (-\infty, \frac{A_{\max}}{\gamma\sigma^2})$.

In the case of $\tau = -1$ the inequality $x_0^* \leq \frac{B_{\min}}{\gamma\sigma^2} < \frac{B_{\max}}{\gamma\sigma^2}$ leads to $\frac{\partial V(\Delta,-1)}{\partial\Delta} < 0$.

If $\tau = 1$ the derivative on the interval $0 < \Delta < |x_0^*|$ is

$$\frac{\partial V(\Delta, 1)}{\partial\Delta} = \gamma e^{\frac{\gamma^2\sigma^2 x^2}{2}} \cdot e^{-\gamma(x_0^*\mu_{\max} + y_0^*(1+r))} e^{-\gamma\Delta A_{\max}} \cdot (A_{\max} - \gamma\sigma^2(x_0^* + \Delta))).$$

On this interval there exists the maximum which is the solution of the equation $\frac{\partial V(\Delta,1)}{\partial\Delta} = 0$ and this point is

$$\Delta = \frac{A_{\max}}{\gamma\sigma^2} - x_0^* > 0.$$

On the interval $(-\infty, |x_0^*|)$ the function $V(\Delta, 1)$ is increasing. Indeed, $\frac{A_{\max}}{\gamma\sigma^2} < 0 < x_0^* + \Delta$ which implies $\frac{\partial V(\Delta,1)}{\partial\Delta} < 0$. The optimal strategy in this case is to invest additional $\Delta^{opt} = \frac{A_{\max}}{\gamma\sigma^2} - x_0^*$ dollars in the stock. The theorem is proved.

As we can observe, the no-transaction region $[\frac{A_{\max}}{\gamma\sigma^2}, \frac{B_{\min}}{\gamma\sigma^2}]$ in this case is much more narrower that in previous cases.

Theorem 3.4. *Let $A_{\max} < 0$, $B_{\min} < 0$ and $B_{\max} \geq 0$. Then the optimal strategy of the investor for the investment problem (3.3), (3.1) and (3.2) is given by*

$$\begin{array}{lll} \Delta^{opt} = \frac{A_{\max}}{\gamma\sigma^2} - x_0^* \text{ and } \tau^{opt} = 1 & \text{if} & x_0^* < \frac{A_{\max}}{\gamma\sigma^2}, \\ \Delta^{opt} = |x_0^*| \text{ and } \tau^{opt} = -1 & \text{if} & x_0^* > 0, \\ \Delta^{opt} = 0 \text{ and } \tau^{opt} = 0 & \text{if} & \frac{A_{\max}}{\gamma\sigma^2} \leq x_0^* \leq 0. \end{array}$$

Proof. The idea of the proof remains the same as in the previous theorem. In fact, the results of this theorem is symmetric to those of Theorem 3.2. Let us consider the following possibilities.

1). $x_0^* \in (0, +\infty)$.

If $\tau = 1$ we have the relationship $x_0^* > \frac{A_{\max}}{\gamma\sigma^2} > \frac{A_{\min}}{\gamma\sigma^2}$ which implies that, according to (3.7) and inequality $x_0^* + \Delta > 0$, $\frac{\partial V(\Delta,1)}{\partial\Delta} < 0$.

If $\tau = -1$ we get that $\mu(\Delta, -1) = \mu_{\min}$ for $\Delta < x_0^*$ and $\mu(\Delta, -1) = \mu_{\max}$ for $\Delta > x_0^*$. On the interval $(0, x_0^*)$ the inequality $\frac{B_{\min}}{\gamma\sigma^2} < 0 < x_0^* - \Delta$ leads to the condition

$\frac{\partial V(\Delta,-1)}{\partial \Delta} > 0$. On the semi-interval $(x_0^*, +\infty)$ the inequality $\frac{B_{\max}}{\gamma\sigma^2} > 0 > x_0^* + \Delta$ implies $\frac{\partial V(\Delta,-1)}{\partial \Delta} < 0$. Therefore, according to the continuity of the preference functional $V(\Delta, \tau)$, the only maximum occurs at the point $\Delta^{opt} = x_0^*$. For all this $\tau^{opt} = -1$.

2). $x_0^* \in [\frac{A_{\max}}{\gamma\sigma^2}, 0]$.

The inequalities $x_0^* \geq \frac{A_{\max}}{\gamma\sigma^2} > \frac{A_{\min}}{\gamma\sigma^2}$ in the case $\tau = 1$ and $x_0^* \leq \frac{B_{\max}}{\gamma\sigma^2}$ in the case $\tau = -1$ imply that both of the functions $V(\Delta, 1)$ and $V(\Delta, -1)$ are decreasing on $(0, +\infty)$ and, hence, the maximum of the preference functional occurs at the point $\Delta^{opt} = 0$.

3). $x_0^* \in (-\infty, \frac{A_{\max}}{\gamma\sigma^2})$.

If $\tau = -1$ the derivative of the preference functional (3.7) is $\frac{\partial V(\Delta,-1)}{\partial \Delta} < 0$ because

$$\gamma\sigma^2(x_0^* - \Delta) \leq \gamma\sigma^2 x_0^* \leq A_{\max} < B_{\max}.$$

If $\tau = 1$ the preference functional has a local maximum on the interval $\Delta \in (0, -x_0^*)$ at the point

$$\Delta = \frac{A_{\max}}{\gamma\sigma^2} - x_0^*.$$

which is the solution of the equation $\frac{\partial V(\Delta,1)}{\partial \Delta} = 0$.

On the interval $(-x_0^*, +\infty)$ the function $V(\Delta, 1)$ decreases because $x_0^* + \Delta > 0 > \frac{A_{\min}}{\gamma\sigma^2}$.

It turns out that the point $\Delta^{opt} = \frac{A_{\max}}{\gamma\sigma^2} - x_0^*$, $\tau = 1$ is the point of global maximum of the preference functional and defines the optimal policy of the investor. The theorem is proved.

Theorem 3.5. *Let $A_{\min} < B_{\min} < 0 < A_{\max}$. Then the optimal strategy of the investor the investment problem (3.3), (3.1) and (3.2) is given by*

$$\Delta^{opt} = -x_0^* \text{ and } \tau^{opt} = 1 \quad \text{if} \quad x_0^* \leq 0,$$
$$\Delta^{opt} = x_0^* \text{ and } \tau^{opt} = -1 \quad \text{if} \quad x_0^* > 0.$$

Proof. The form of the preference functional and its derivatives is given in by (3.5) and (3.7). As in the previous cases the optimal strategy depends on the value of initial stock holdings of the investor.

1). $x_0^* \in (0, +\infty)$.

If $\tau = 1$ the derivative of the preference functional (3.7) is $\frac{\partial V(\Delta,1)}{\partial \Delta} < 0$ because $x_0^* + \Delta > 0 > \frac{A_{\min}}{\gamma\sigma^2}$.

If $\tau = -1$ the preference functional is increasing on the interval $0 < \Delta < x_0^*$ because the inequality $x_0^* - \Delta > 0 > \frac{B_{\min}}{\gamma\sigma^2}$ leads to the condition $\frac{\partial V(\Delta,-1)}{\partial \Delta} > 0$.

On the interval $(x_0^*, +\infty)$ the function $V(\Delta, -1)$ is decreasing due to the fact that $x_0^* - \Delta < 0 < \frac{B_{\max}}{\gamma\sigma^2}$.

This means that the global maximum of the preference functional occurs at the point $\Delta^{opt} = x_0^*$ with $\tau = -1$ which defines the optimal policy of the investor.

2). $x_0^* \in (-\infty, 0]$.

If $\tau = -1$ the inequality $x_0^* - \Delta < 0 < \frac{B_{\max}}{\gamma\sigma^2}$ and equation (3.7) imply the inequality $\frac{\partial V(\Delta,-1)}{\partial} < 0$.

In the case $\tau = 1$ the function $V(\Delta, 1)$ increases on the interval $(0, -x_0^*)$ because of the condition $\frac{A_{\max}}{\gamma\sigma^2} > 0 > x_0^* + \Delta$. On the interval $(-x_0^*, +\infty)$ this function is decreasing due to the inequalities $\frac{A_{\min}}{\gamma\sigma^2} < 0 < x_0^* + \Delta$. Therefore the only point of maximum of the preference functional is $\Delta^{opt} = -x_0^*$ with $\tau = 1$.

Under the condition of the last theorem the no-transaction region is reduced to the point 0. This means than the investor sells all stocks available in the initial portfolio. This is optimal for him/her even paying transaction costs for this operation. The condition $B_{\min} < 0 \geq A_{\max}$ is analogous to the non-degenerate price condition of Dow and Werlang (1992) and Carlier and Dana (2003).

4 Empirical example

In order to provide an example of described above models we consider a multiperiod myopic decision-making procedure under proportional transaction costs. It assumes that the investor has a criterion defined over the one-period rate of returns on the assets. In other words, he/she follows optimal policies of a series of single-period problems connected in such way that the final portfolio of every problem is the initial one of the decision-making problem in the next period of time.

In the capacity of risky asset we consider daily prices of the Dow Jones index in the period from July 1996 till May 1999. We assume that the daily returns of the Dow Jones index follow the GARCH(1,1) process. This model is an appropriate one because under the myopic strategy at the time t the investor takes into account only the past information at the period of time $t-1$. Hence all results which have been obtained in this chapter are valid under conditional normality of stock returns.

In order to estimate the GARCH process we use past 100 days as an estimation window every period of time. Therefore, mean and conditional variance are changing over time. The monthly riskless rate is considered to be $r = 0.002$. We adopt the transaction costs rate to be equal to 0.1%. Since we incorporate in the analysis only investor's beliefs about uncertainty we assume that the absolute risk aversion coefficient is constant and equals 0.05 and the coefficients $\beta_{\min} = -5$ and $\beta_{\max} = 10$. The investor starts with the initial stock holding $x_0^* = 50\$$ and the bond holding $y_0^* = 50\$$.

Figure 1 shows the dynamic of changes in investor's portfolio during the horizon and the bounds of no-transaction region. As we can make sure from this figure the no-transaction region, analogically to the Expected Utility model (Gennotte and Jung (1994), Boyle and Lin (1997), Kozhan and Schmid (2005)), completely determines the optimal strategy of the investor. During March 1998 the effect of Dow and Werlang (1992) is observed. In this period the no-transaction region is reduced to the point and the optimal policy for the decision-maker is to invest all his/her actives into the bond (i.e. $x_t^* = 0$).

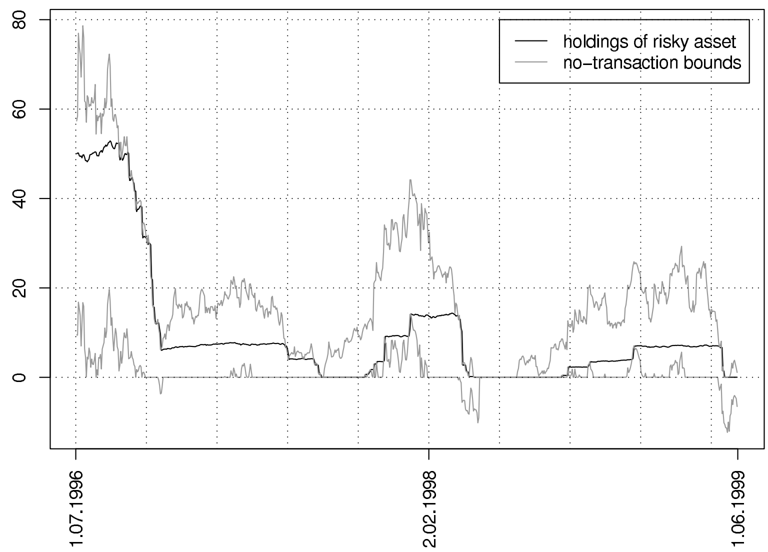

Figure 1: No-transaction bounds and dollar amounts of stock traded by MMEU maximizer, $\beta_{\min} = -5$ and $\beta_{\max} = 10$.

Figure 2 compares no-transaction regions of investors whose beliefs satisfy the ax-

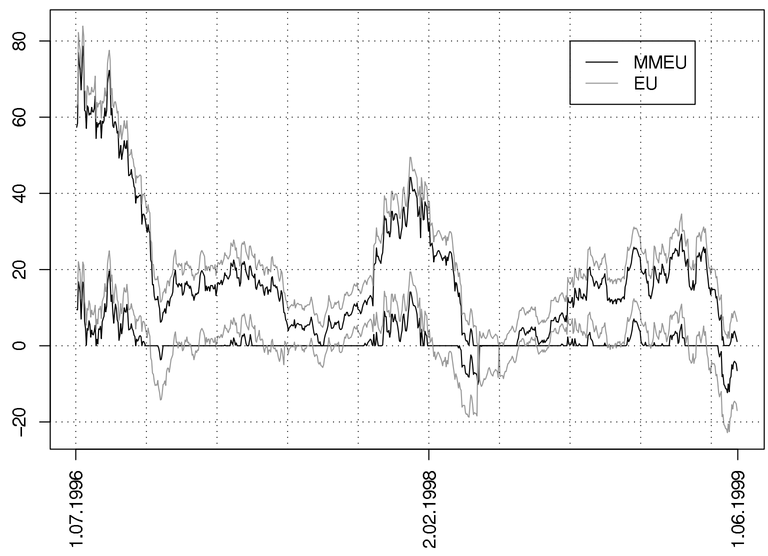

Figure 2: No-transaction bounds of EU and MMEU maximizers, $\beta_{\min} = -5$ and $\beta_{\max} = 10$.

ioms of two models: standard Expected Utility theory of von Neumann and Morgenstern (1944) and the Maxmin Expected Utility model. The first approach can be obtained as a special case of the second if we set $\beta_{\min} = \beta_{\max} = 0$. If any of EU bounds are situated above the axis $y = 0$ than the appropriate the MMEU bound is shifted down on the value $-\frac{\beta_{\min}}{\gamma}$ and cut from below by the line $y = 0$. If any of the EU no-transaction bounds are situated below the axis $y = 0$ than the appropriate MMEU no-transaction bound is shifted up on the value $\frac{\beta_{\max}}{\gamma}$ and cut from above by the axis. This implies that in general the MMEU no-transaction region is narrower as the EU one which makes the investor to be more active on the market and trade more frequently if uncertainty is presented in the model.

5 Conclusions

In the paper we have considered different types of asset allocation models within MMEU framework. Investor's attitudes to the risk correspond to the exponential utility function while his/her uncertainty aversion is represented by the set of priors $\mathcal{P}$. The main contribution is that explicit expressions for the bounds of the no-transaction region are derived.

In the model without transaction costs we have showed the existence of the non-degenerate price conditions, similar to those that were obtained by Dow and Werlang (1992) and Carlier and Dana (2003) within the CEU theory under the distorted probability.

Having incorporated proportional transaction costs we have seen that the non-additivity of the investor's preferences has an impact on an optimal policy of the investor. As in the case of standard Expected Utility framework of von Neumann and Morgernstern (see Boyle and Lin (1997), Gennotte and Jung (1994), Kozhan and Schmid (2005)) the optimal strategy is determined by the bounds of the no-transaction region. This bounds also divide real line on tree parts: the sell, the buy and the no-transaction regions. However, these bounds have different expressions depending on parameters of the model. It is clear that the model is reduced to the classical utility theory if we set $\mathcal{P} = \{P_0\}$, i.e. $\mu_{\min} = \mu_{\max} = \mu$. This leads to the fact that the no-transaction region is the interval of the form $[\frac{A}{\gamma\sigma^2}, \frac{B}{\gamma\sigma^2}]$, where $A = \mu - (1+\theta)(1+r)$ and $B = \mu - (1-\theta)(1+r)$ (see Kozhan and Schmid (2005)). The no-transaction regions under MMEU theory depends on the relationships between parameters $A_{\min}$, $A_{\max}$, $B_{\min}$ and $B_{\max}$ and are narrower as in the case of unique prior. It leads to the result that the investor is more restrictive in his/her decisions due the uncertainty faced in the model. From another hand, the investor becomes more active on the market because the probability that his/her holdings of stock are within the no-transaction region increases.

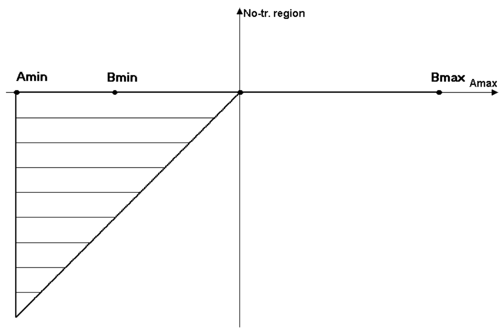

Figure 3: No-transaction region as a function of $A_{\max}$, $B_{\min} < 0$.

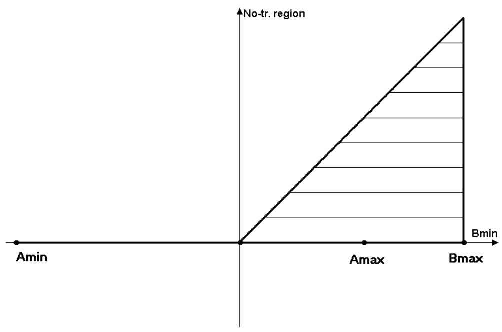

Figure 4: No-transaction region as a function of $B_{\min}$, $A_{\max} \geq 0$.

The dynamics how the bounds of the no-transaction region depends on parameters of the distribution of assets returns are shown on Figures 3 and 4. As it turns out, the non-degenerate price condition has place also in the model with the proportional transaction costs. Moreover, it is optimal for the investor under this condition to take a zero position in the risky asset even paying transaction costs for such portfolio reallocation.

In general, the paper provides a constructive analytical procedure for determining the no-transaction region, which completely solves the decision making problem of the investor with non-additive preferences.

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