MULTIPRODUCT FIRMS
AND PRODUCT DIFFERENTIATION:
A SURVEY

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Multiproduct Firms and Product Differentiation: A Survey[a]

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Abstract

We start the survey by reviewing the implications of horizontal and vertical product differentiation on market structure under the assumption of single-product firms. Then, we analyse the main results of the multi-product firm models, both when variants are assumed differentiated in vertical attributes only and when variants are assumed differentiated in two dimensions (vertical and horizontal). Finally, we review the empirical literature about discrete-choice models of product differentiation.

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1. Introduction

In recent years the study of models of multi-product competition with product differentiated variants has become an important field both for theoretical and empirical industrial organization. This is no surprise, since multi-product firms are ubiquitous, but what is noteworthy is how difficult it is to model them satisfactorily. This survey examines the modelling of firms' product range and pricing decisions in situations in which variants are differentiated in one or more dimensions. The literature that is of potential relevance to a survey of this field is extensive. We take the approach here of only analysing a relatively small numbers of papers in depth under different assumptions about product differentiation.

In order to develop the intuition necessary to understand more complex models, we start the survey with the "finiteness property" of models of vertical product differentiation. The aim is to stress the different effects on market structure of horizontal and vertical product differentiation. In section 2, we first present a model of single product competition in which goods are differentiated both horizontally and vertically (two-dimensional product differentiation) and, then present a reconsideration of the "finiteness property" with two-dimensional product differentiation.

We start the analysis of multi-product competition by reviewing models in which variants are differentiated only by horizontal (section 3) or by vertical characteristics (section 4). Then in section 5, we analyse multi-product competition when goods are differentiated in two dimensions (vertical and horizontal dimensions).

In recent years, the most interesting empirical approach to oligopolistic models of product differentiation is through discrete-choice models of product differentiation. Therefore, we devote the section 6 of this survey to reviewing these models with the aim of using their empirical results as modelling suggestions.

2. Price competition and product differentiation: single product firms

A standard result in the analysis of markets with horizontally differentiated products is that in free-entry equilibrium the number of firms increases without bound when the size of the fixed setup costs tends to zero or the size of the market becomes very large. Additionally, the limiting price approaches the marginal cost[1].

However, in markets in which products are vertically differentiated (only), so long as consumers' willingness to pay for quality grows faster than the unit variable costs of providing this quality, the number of firms in the market is independent both of the market size and fixed costs. In the free-entry equilibrium of these models, the number of firms in the market will never be greater than a maximum determined by the income distribution. Thus, price will be above marginal costs and the firm will get positive profits. In order to illustrate this phenomenon, usually referred to as a "finiteness property", we will follow Shaked and Sutton [1982, 1983]. First, we present the general framework of the model and then analyse the particular case of a single product duopoly selling vertically differentiated products.

Let us assume that n products of different quality produced by n single-product firms are available for sale. If we denote quality by q we can sort the products (indexed j) in increasing order of quality: $q_1 < q_2 < ... < q_n$ with prices for each quality variant: $p_1 < p_2 < ... < p_n$. As finiteness is a property of the price equilibrium, we consider qualities as given and focus on the Nash equilibrium in prices[2]. All consumers have the same tastes but differ in income. Consumers' incomes (q) are assumed to be uniformly distributed with density s over $[a, b]$ with $a > 0$. Consumers, indexed by i, either choose to consume one unit of one of the available qualities or instead consume none. Utility from consuming the variant of quality k is assumed to take the specific form[3].

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[1] For an analysis of the different effects of horizontal and vertical product differentiation on market structure see Sutton [1986]. For extensive reviews of product differentiation per se, see Eaton and Lipsey [1989], Tirole [1988, ch2], Ireland [1987], Waterson [1989] and Beath and Katsoulacos [1991].

[2] In general Shaked and Sutton consider a two-stage competitive game. In Stage 1, firms choose quality and in Stage 2, prices (Bertrand competition)

[3] Here we use the Mussa and Rosen [1978] specification of preferences instead of Shaked and Sutton [1982] specification. The reason is that the papers considering both horizontal and vertical product differentiation reviewed later use this function to specify the vertical product differentiation component. Other applications for markets in which products are vertically differentiated using this specification can be found for example in Tirole [1988; ch7] and Moorthy [1988]

$$U_i(q_i, k) = q_i q_k - p_k \tag{1}$$

with the utility from consuming nothing being:

$$U_i(q_i, 0) = q_i q_0 \tag{2}$$

A consumer with income q_k will be indifferent between buying variants of quality k and k-1 if

$$\begin{aligned} &U(q_k, k) = U(q_k, k-1) \\ &\text{i.e.}\; q_k q_k - p_k = q_k q_{k-1} - p_{k-1} \end{aligned} \tag{3}$$

We can rearrange (3) as

$$q_k = r_k (p_k - p_{k-1}) \tag{4}$$

where $r_k = 1/(q_k - q_{k-1})$. Thus, given prices, those consumers with an income greater than q_k will buy the variant k and those consumers with an income lower than q_k will buy the variant k-1. By repeating this operation for every pair of adjacent qualities, we can partition the distribution of consumers into income bands such that everyone within a specific income band buys a certain quality. The higher the income band, the higher the quality. We can use this income-splitting property to obtain the demands for each one of the quality variants, starting with the highest, n,

$$\begin{aligned} x_n &= M(b - q_n) \\ x_{n-1} &= M(b - q_{n-1}) \\ & \\ x_1 &= M(q_2 - a) \end{aligned} \tag{5}$$

where M may be understood as a measure of the size of the economy. By substituting (4) into each of the demands in (5) we find that the demand of each of the quality variants depends on its own price and quality but also on prices and qualities of its lower and upper quality neighbours.

With respect to costs, it is assumed (for simplicity) that fixed costs depend upon quality and that unit variable costs are independent of quality, i.e. the whole burden of quality improvements is placed upon fixed costs.

$$C_k = cx_k + F(q_k) \tag{6}$$

Furthermore, it is assumed that the range of the income distribution is such that $2a < b < 4a$. If, without further loss of generality, we set $c = 0$, the net revenues of the firms producing the variant of quality q_n and the variant of quality q_1 are, respectively

$$M.R_n = Mp_n(b - q_n) \tag{7}$$

and

$$M.R_1 = \begin{cases} M.p_1(q_2 - a), q_1 \leq a \\ M.p_1(q_2 - q_1), q_1 > a \end{cases} \tag{8}$$

The first order condition of profit maximisation for the firm producing the variant of quality q_k (given qualities) is given by $\partial R_k / \partial p_k$. Therefore, for the firms producing quality variants q_n and q_{n-1} these are respectively,

$$\frac{\partial R_n}{\partial p_n} = b - 2q_n - r_n p_{n-1} = 0 \tag{9}$$

$$\frac{\partial R_{n-1}}{\partial p_{n-1}} = q_n - 2q_{n-1} - r_n p_{n-1} - r_{n-1} p_{n-2} = 0 \text{ if } q_{n-1} \geq a \tag{10}$$

$$\frac{\partial R_{n-1}}{\partial p_{n-1}} = q_n. - q_{n-1} - r_n p_{n-1} = 0 \text{ if } q_{n-1} < a \tag{11}$$

Using (9) and (10), Shaked and Sutton [1982] establish that if $4a > b > 2a$ only two firms stay in the market. Since $p_k > 0$ and $r_k > 0$, (9) and (10) require respectively that

$b > 2q_n$ and $q_n > 2q_{n-1}$

which implies $b > 4q_{n-1}$. Since by assumption we consider $b < 4a$, this implies that $q_{n-1} < a$, i.e. at most two firms have positive market shares at the Nash equilibrium in prices. Thus, the number of firms in the market is independent of both the size of the market and the size of the fixed costs and depends only upon the breadth of the income distribution.

The Nash equilibrium in prices in this model is derived from the best-reply functions of the two duopolists. Given the direct relationship between income and price (through the marginal consumer conditions) we can study the best reply functions in (q_1, q_2) space [Shaked and Sutton, 1982]. Accordingly, there are up to three possible areas where a Nash equilibrium can arise depending on the values of $v = \frac{q_2 - q_0}{q_2 - q_1}$. If $v \geq (b+a)/3a$ the Nash equilibrium prices are $p_1 = \frac{b-2a}{3(v-1)r_1}, p_2 = \frac{2b-a}{3r_2}$. If $(b-a)/3a \leq v < (b+a)/3a$ then Nash equilibrium prices are $p_1 = \frac{a}{r_1}, p_2 = \frac{b+a(v-1)}{2r_2}$. In both cases the market is covered (i.e. all the consumers buy one or the other good). The third region ($v < (b-a)/3a$) is ruled out by our assumption $2 < b < 4a$.

Both equilibria involve two established firms producing different qualities (the quality decision has not been made explicit here; it is determined by the relationship between fixed costs and quality enhancement) at a price which in general implies supernormal profit but which does not attract entry. In a model of *pure* vertical product differentiation, the existence of (even very small) sunk costs implies that no firm will enter the market producing an existing quality variant, since in that case, Bertrand competition will drive prices down to unit variable costs, meaning sunk costs will not be covered.

2.1 Two-dimensional models of product differentiation

Actually, most of the products that we purchase and use embody both horizontal and vertical product characteristics. We can combine these two characteristics in the following utility function:

$$v_{ij} = r_i + q_i q_j - z(d_i - l_j) - p_j \qquad (12)$$

where r is the basic willingness to pay for the product. Each consumer type i is defined by its willingness to pay for quality (q) and its parameter for horizontal specification (d) in function z. A product variant j can be specified as (q_j, l_j). The indirect utility function (12) is additively separable in the horizontal and vertical characteristics. The underlying assumption behind this additivity is the independence of the two characteristics.

2.1.1 The Neven and Thisse model of two-dimensional product differentiation

Neven and Thisse [1990][4] analyse duopoly product selection using (12). They assume quadratic transportation costs, $z(d_i - l_j) = z.(d_i - l_j)^2$ Consumers are uniformly distributed over $[0, \theta_M] \times [0, L]$. Qualities q_1, q_2 are chosen from $[q_m, q_M]$ and locations from $[0, L]$. Without loss of generality, it is assumed that $q_2 > q_1$ and $l_2 > l_1$. Production costs are assumed zero.

The firm's decision process is modelled as two stage game: in the first stage firms simultaneously choose product characteristics; in the second stage firms choose prices. Depending on the two variants it is possible to distinguish two regimes: a horizontal dominance regime characterises those situations in which variants are closer vertically than horizontally $(2zL(l_2\text{-}l_1) > q_M(q_2\text{-}q_1))$; the vertical dominance regime is characterised by the opposite situation, variants are closer horizontally than vertically. Let us start by briefly noting the properties of the price equilibrium. Neven and Thisse show that, "when firm 1 improves upon its position (higher $q_1 < q_2$ or more central location $l_1 < l_2$) along the dominated characteristic, its equilibrium price generally increases despite the fact that variants are getting closer". In contrast to the one-dimensional models of horizontal or vertical product differentiation, prices do not necessarily fall when variants get closer[5]. The cause of this result is that variants are sufficiently separated by the dominant characteristic. However, when variants become closer in the dominant characteristic, the results of the one-dimensional model hold and prices fall.

[4] For other papers in which single product firms compete in more than one-dimensional space, see: Vandenbosch and Weinberg [1995] for a two-dimensional vertical product differentiation model and Degryse [1996] for the application of a two-dimensional model of horizontal/vertical product differentiation to study the interaction between remote access (vertical attribute) and location (horizontal attribute) as determinants of the market equilibrium in the banking sector.

Two configurations appear as equilibria depending on the preference intensity in the vertical dimension relative to the horizontal dimension. If preference intensity in the vertical dimension is large enough (relative to that in the horizontal dimension) then vertical differentiation is maximal and horizontal differentiation is minimal, $(q_1^*, l_1^*) = \left(q_m, L/2\right)$ and $(q_2^*, l_2^*) = \left(q_M, L/2\right)$. If the opposite is true, then the equilibrium configuration involves maximum horizontal product differentiation and minimum vertical product differentiation, $(q_1^*, l_1^*) = \left(q_M, 0\right)$ and $(q_2^*, l_2^*) = \left(q_M, L\right)$. Whereas, in a one-dimensional model of vertical/horizontal product differentiation, the equilibrium implies maximum product differentiation, in this model the maximum product differentiation configuration $((q_1^*, l_1^*) = \left(q_m, 0\right), (q_2^*, l_2^*) = \left(q_M, L\right)$ never arises. This result demonstrates that interplay between horizontal and vertical characteristics (even under the assumption of additive preferences) has an important impact upon the firms' product selection process.

2.1.2 Reconsidering the finiteness property in two-dimensional models of product differentiation

The finiteness property of the models of vertical product differentiation ceases to hold when the horizontal dimension is introduced. Assume that consumer preferences are given by (12). Independent of firm numbers in the market, an entrant can make positive profits by introducing a new variant with quality equal to the quality ranked first for all consumers and horizontal specification different from that of the established firms. Since nearby consumers will strictly prefer this new variant, the entrant will capture a positive market share selling at a price above unit costs of production. The limiting result is that all firms will choose the same quality specification but different locations.

Notwithstanding, Shaked and Sutton [1987][6] suggest the existence of some form of finiteness property in two-dimensional models (vertical-horizontal) of product differentiation when fixed costs depend on quality. They show that in a free entry

[5] See for example the prices resulting from the Nash equilibrium in prices of the vertical product differentiation model.

[6] In order to employ the same specification of indirect utility function that we will use when analysing multi-product competition, we follow chapter 8 of Anderson, de Palma and Thisse [1992] in reviewing Shaked and Sutton [1987].

equilibrium, at least one firm has a market share bounded away from zero, even when the market becomes arbitrarily large.

Firms costs are decomposed into quality-dependent fixed costs, *K(q)*, and quality-dependent unit costs of production, *c(q)*. Shaked and Sutton make three further assumptions about the costs:

$$c'(q) < q_M \text{ for all } q \in [0, \infty[\tag{13}$$

$$0 < \frac{\partial \log K(q)}{\partial q} \leq b \text{ for all } q \in [0, \infty[\tag{14}$$

$$c(q) < r \text{ for all } q \in [0, \infty[\tag{15}$$

where marginal costs and fixed costs are assumed to be continuously differentiable with respect to *q*, and *b* is a constant. The first two conditions ensure that the burden of quality improvement is placed on fixed cost rather than on marginal costs, but fixed cost does not grow too fast with quality. The third condition ensures that unit costs of production are lower than income for all qualities.

The objective is to show that in free entry equilibrium at least a firm will have a strictly positive market share ($m > 0$). The proof is by contradiction. Assume a free entry equilibrium at which all firms have positive market shares smaller than an arbitrary $e > 0$, that is independent of size of the market, *M*. Since price cannot be higher than income, the maximum possible revenue is given by *Mer*, therefore fixed cost must be less than *Mer*. What is shown in Appendix B is that there are strictly positive values for Δ, *m*, *m*, such that if a firm enters the market producing a variant of quality ($q^+ + \Delta$) (where q^+ is the highest quality available in the market) at an arbitrary location *l* in [0, *L*] and incurring a fixed cost $K(q^+ + D)$, it can obtain a market share *mM* selling at a price $c(q^+ + D) + m$. Thus, the entrant would make positive profits, contradicting the original assumption of free entry equilibrium. Therefore, we cannot find a free entry equilibrium under the condition that none of the firms has a market share $e > 0$. At least one of the firms must have a market share bounded away from zero independent of the size of the market. From this, we can conclude that even when horizontal product differentiation is allowed, the presence of vertical attributes implies a minimum degree of concentration in the market.

3. Horizontal Product Differentiation and Multi-Product Firms

The standard Hotelling (1929) line or Circular Road (e.g. Salop, 1979) model of horizontal product differentiation has consumers arrayed evenly along the space in terms of their tastes for an ideal product, suffering disutility from buying a variant which differs from their ideal. Each firm produces a single product, which sells to a range of consumers in the vicinity. However, there is no particular reason why firms should be constrained to producing only one variant; in practice they clearly are not so constrained. If they produce several, the manner in which locational and neighbour effects are assumed to enter the model plays a significant part in determining the outcome.

For example, consider products A, B, C and D in figure 1. The locations are fixed. Each consumer buys only one unit (or none). If we suppose that A and C are owned by the same player, there is no necessary difference in the pricing outcome compared with the situation in which each firm produces only one product. A's price is not a function of C's, or vice versa, because they do not compete directly (so long as B's price is not too high), so there is no strategic advantage in owning both products. Nevertheless in alternative situations, strategic effects appear.

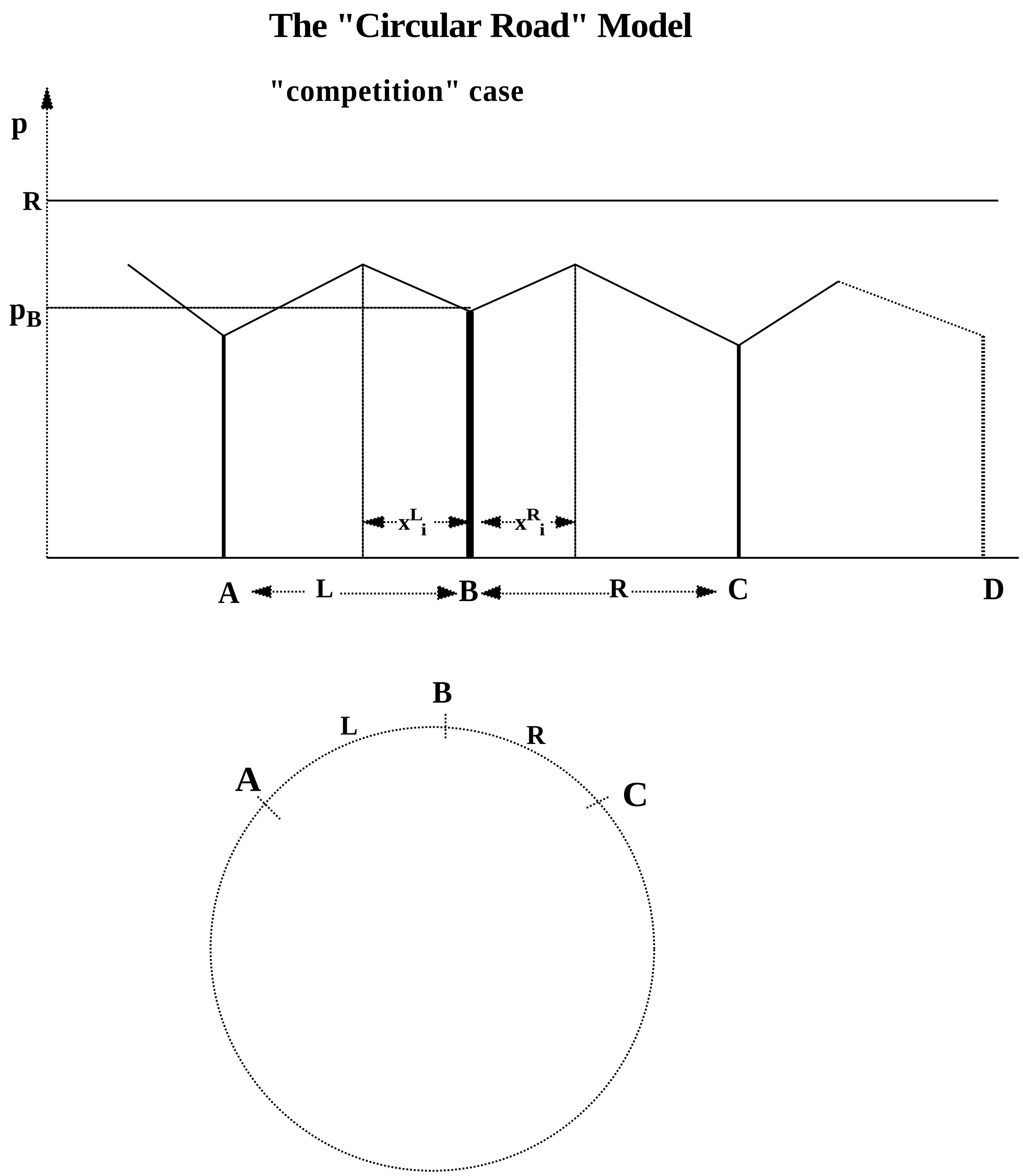

Figure 1: The Circular Road Model

For example, suppose that currently in the market only product A exists, but that the market is developing and that in time products at B, C and D will become profitable. The most likely player to enter products in these positions, it may be argued, is A. Indeed, A is likely to be able to do anything better than new players. To illustrate, it might raise prices on product B, which faces no competition from other firms. Or indeed, it might avoid the expense of producing B at all, by shifting C marginally to the left and thereby leaving just insufficient room for anyone to produce at B. In either case, it is difficult to see what an entrant can do against an intelligent and farsighted incumbent. Consequently, the incumbent can make supernormal profits without entry occurring, as Eaton and Lipsey [1979] and Schmalensee [1978] have pointed out. Notice the assumption here is that firms freely choose both price and location, whereas in the example of the previous paragraph, they only choose price. It is also important to the outcome that one firm moves first.

Both these structures described above assume that locations, once chosen, are sunk. If not, the incumbent is in a much weaker position, since its first mover advantage is removed. As Judd [1985] points out, an incumbent might be forced by an entrant to exit. To see the argument, consider only positions A and B in figure 1. The incumbent produces both products. But now suppose that locations are not fixed ex post, as a result of the absence of sunk costs in location. An entrant comes in adjacent to A. This leads to significant price competition, driving the price of B (an imperfect substitute) down also. The incumbent may well be better off dropping A from its line, to avoid direct competition with the entrant. We may observe the crucial importance of there being no sunk costs in determining the outcome here.

Really, the problem with using address models to examine multi-product firms is that results are so dependent upon the precise assumptions of the game, meaning that the modeller needs to be quite precise in making their choice of assumptions. It is significant that Judd constrains the location of the existing player's products to obtain his results. Most recent modelling in this area either constrains the range of decisions made within an address framework (normally by fixing locations), or employs a non-address model, which itself constrains relationships between the products without necessarily entailing strategic issues. Examples illustrating this last point include Klemperer [1992] on address models, Shaked and Sutton (1990) and Dobson and Waterson [1996] on non-address models, and Lal and Matutes [1989] who have different consumers. In order to gain a greater appreciation of the field, we discuss these papers in more detail below.

3.1 Horizontal product differentiation: multi-product competition with exogenous product-range

In the extension of the standard Hotelling model that Klemperer [1992] proposes, the number of products in the firm is considered as fixed (n) and a general choice of product location is not allowed. Therefore, let us consider two firms (A and B) producing a range of n products. Products are distributed around a circumference of unit length and product x can be understood as located at distance x from an arbitrary starting point. It is assumed that every consumer has identical tastes for variety and desires to buy the entire range of possible products. Consumer shopping cost can be divided into two components: as in the standard Hotelling model the consumer costs from buying a product are given by $p_j + g(x)$, where p_j is the price of the product and $g(x)$ ($g'(x) > 0$) the costs of substituting a product of less than ideal characteristics. In addition, consumers incur a fixed shopping cost of using suppliers: density $f(y, z)$ of consumers each incurs a fixed shopping cost of y if it buys any output from firm A and a fixed shopping cost of z if it buys any output from firm B. Furthermore, it is assumed that firms have constant and equal cost of production for the n products (set equal to zero without loss of generality) and that each firm simultaneously and independently chooses prices for its n products.

In this framework Klemperer compares the symmetric non-cooperative equilibrium prices that result from two alternative forms of competition: head-to-head competition and interlaced competition. Whilst in head-to-head competition each firm's product line comprises the products located at $x = i/n$ for $i = 1,...,n$, in interlaced-product-line competition, products of firm A are located at $x = i/n$ for $i = 1,...,n$ and products of firm B are located at $x = (i- 1/2)/n$.

Call the price resulting from interlaced-product-line competition p_I and the price resulting from head-to-head competition p_H. If shopping costs are assumed to be equal to zero, then Bertrand price competition between identical products drives price down to zero with head-to-head competition. It is product differentiation what allows firms to set a positive price (above marginal cost) with interlaced competition. Nevertheless, Klemperer shows that in presence of shopping costs both p_H and p_I are greater than zero and that in general there is no ranking between them.

In head-to-head competition, consumers do not find it worthwhile to buy from two shops. As the range of products offered by the two firms is identical, only consumers with equal shopping costs at the two firms are attracted by small price cuts from one of the firms. Thus, prices in head-to-head competition depend only on the density of consumer with equal shopping costs, $f(z, z)$. With interlaced-product-lines, competition also depends on other parts of the density function of shopping costs and on the form of the substitution cost function. In the interlaced competition equilibrium consumers that buy from A, consumers that buy from B and consumers that buy from both firms may be found. The effect of a small reduction in the price of one of the firms, say A, will provoke consumer switching from buying only from B to buying only from A, consumer switching from buying from both firms to buying only from A, and consumer switching only from B to buy from both firms. Therefore, with interlaced-product-lines competition, prices do not depend only on $f(z, z)$ but also depends on other parts of the density function of shopping costs and on the form of the substitution cost function.

In general the ranking between p_H and p_I depends on the nature of the distribution of the shopping costs across consumers and on the form of the substitution-costs function. With respect to the former, the greater the number of consumers with different shopping costs for each firm (i.e. the smaller $f(z, z)$) the less competitive head-to-head price is relative to interlaced price. With respect to the latter, decreasing the substitution costs can increase the sensitivity to price competition of the group of consumers that buy from both firms in interlaced competition and so make it more competitive (in head to head competition, no consumer finds it worthwhile to buy from two firms).

Therefore, if $f(z, z)$ is small and substitution costs are small, industry profits may be higher (less competition) when firms offer identical product ranges than when the product offerings are interspersed. The intuition behind this result is that offering identical products gives no reason for the consumers to buy from more than one supplier and so reinforces firm loyalty. If few consumers make the investment of buying from more than one firm, then the price-sensitive share of the market is small and equilibrium prices may be higher.

3.2 Horizontal product differentiation: multi-product competition with endogenous product-range decision

From amongst the papers that consider the product-range decision of the firm in this framework, we will focus our attention on Shaked and Sutton [1990], Dobson and Waterson [1996] and Lal and Matutes [1989]. In Shaked and Sutton [1990], goods are differentiated solely by their intrinsic characteristics and it is unimportant where the product is bought, whereas in Dobson and Waterson, goods provide utility to the consumer not only through their intrinsic characteristics but also through the firm from which they are bought. Lal and Matutes [1989] analyse price competition and product-line decisions in a duopoly model in which each firm can sell two products and there exist different consumers. These two products are neither complements nor substitutes and product differentiation comes through the firm from which the product is bought. We start this section by reviewing Shaked and Sutton [1990], then we present Dobson and Waterson [1996] stressing differences and similarities. We then introduce Lal and Matutes [1989]. Table I summarises the main assumptions and results of these three models.

3.2.1 Demand factors and market structure

Shaked and Sutton's [1990] main aim is to explore the role played by demand factors in explaining market structure in relation to concentrated equilibria versus fragmented equilibria. Therefore, in order to abstract from any cost-based influence it is assumed that: the cost of the production of any particular good is the same for the two firms and it is independent of the range of goods the firm produces (no economies of scope).

Most of the analysis is carried out in a two goods framework. Here, only two varieties of the good can be produced and each variety can be produced by at most one firm. The market is characterised by a two-stage game. In Stage 1, each firm simultaneously chooses which product(s) it will produce and incurs an exogenous sunk cost $e > 0$ per product entered; in Stage 2 firms simultaneously choose their respective prices (with a Bertrand-Nash equilibrium in prices). Since products enter symmetrically into firms' profit functions, the menu of configurations that form equilibria in this setting depends only on Stage 2 profits and is given by: p (2, 0), p (1, 1) or p (1, 0) where p (k, l)

denotes profits of a firm with k products when the rival is producing l products. Furthermore, it is assumed that p (1,0) > e, i.e. the single product monopolist is profitable; p (2,0) ≥ p (1,0) i.e. the introduction of a second product does not reduce monopolist profits; and p (2, 0) > 2p (1,1), i.e. monopolisation does not reduce total profits.

The two main features of market demand used to characterise market structure are based on comparing the incentives of the incumbent and a new entrant to introduce a new product. These incentives depend on two effects. The first, the **expansion effect** (E), is given by the fractional increase in profits enjoyed by a incumbent monopolist when introducing a second product, and measures the degree to which total demand increases when a new product is introduced:

$$E = \frac{p\,(2,0) - p\,(1,0)}{p\,(1,0)}$$

The second effect, the **competition effect** (C), is given by the fractional difference between the industry profits under monopoly and the industry profits under competition and measures the incentive to monopolise. The tougher the degree of competition in the industry the larger is C.

$$C = \frac{p\,(2,0) - 2p\,(1,1)}{2p\,(1,1)}$$

The simultaneous Nash equilibria that correspond to different values of E and C are determined through identifying the optimal reply functions of each of the firms. To illustrate, consider the following example: if firm 1 is producing only one product then (1→1) will be the optimal reply of firm 2 if the profits associated with this reply are greater than the profits associated with (1→0). These optimal reply functions can be used to subdivide the (E, C) space into areas that correspond to the different patterns of optimal replies. From these patterns the simultaneous entry Nash equilibria can be determined. Each equilibrium configuration is determined by a pair of strategies that are optimal one to the other. Shaked and Sutton show that the interplay of competition and substitution effects leads broadly to the following configurations: for weak (strong) competition and expansion effects the fragmented equilibrium (1,1) (monopoly equilibrium, (2,0)) prevails. The stronger is post-entry competition and the larger the increment to demand when a new product is introduced, the stronger the tendency for the monopoly solution to appear.

Next, Shaked and Sutton use the competition-substitution effect framework to re-examine a simple linear demand schedule model of horizontal product differentiation derived from a quadratic utility function, extended to allow for the number of consumers in the market to increase with the number of variants. Denote by j the ratio between the number of consumers when one and two variants are sold in the market. Now assume that $j = 1$ to analyse the effects of the degree of substitution between products (s) over the competition and expansion effects and so over the market structure. We can consider the two extreme cases: when the substitutability between two variants tends to zero ($s \rightarrow 0$) the competition effect tends to zero and the expansion effect is maximum (C$\rightarrow$ 0 and E$\rightarrow$ 1). However when the goods are perfect substitutrs here (this implies $\sigma \rightarrow 2$) the competition effect tends to infinity and the expansion effect to zero ($C \rightarrow \infty$ and $E \rightarrow 0$), and the incentive to monopolise is maximised. Hence, the market equilibrium that prevails depends upon the degree of substitutability between variants. When high $(C \rightarrow \infty)$ the prevailing equilibrium is a single product monopolist; when intermediate ($C > 0$ and $E > 0$) the fragmented equilibrium prevails; and when low either the fragmented equilibrium or multi-product monopolist prevail.

Finally, Shaked and Sutton analyse a three-good framework in which the assumption of simultaneous entry is replaced by sequential entry. The result they obtain in this model when one of the players has a first-mover advantage contrasts with most of the literature since although pre-emption always happens in the "Hotelling type" models, in this model it does not necessarily occur. Fragmented equilibrium occurs for intermediate values of the substitution parameter (s) for which both competition and expansion effects are positive. Shaked and Sutton point out that the reason behind these unusual results is that in "Hotelling type" models it is always assumed that the market expansion effect is equal to zero.

The possibility of fragmented equilibrium in the three goods model with sequential entry arises from the fact that for a certain underlying parameter combinations (s, j, e) the optimal reply structure takes following form: (2→1), but (1,1→0). If the first mover introduces two products it does not deter entry, however, if the first mover introduces one product and the second mover one additional product then no further entry occurs. Entry is avoided in the second case because prices are lower due to competition. For (1,1→0) to be an optimal reply, the expansion effect should be weak and the competition effect strong otherwise the optimal reply would be (1,1→1). For (2→1) to be an optimal reply the competition effect should be weak or the expansion effect strong otherwise the optimal reply would be (2→0). Therefore, for this to arise, both competition and expansion effect should be of intermediate size, requiring the substitution parameter to fall in an intermediate range[7].

3.2.2 Inter-product vs. intra-product competition as determinants of the market structure

Shaked and Sutton [1990] strictly follow a "goods-are-goods" [Eaton and Lipsey, 1989] approach in which consumer preferences depend only on the intrinsic nature of the good and not on the firm from which the product is acquired. By contrast, in Dobson and Waterson [1996] goods provide utility to the consumer not only through their intrinsic characteristics but also through their origin, for example, some consumers may prefer to buy margarine X at Sainsbury rather than at Tesco.

The model focuses on two decisions facing consumers, which product (e.g. which margarine) to buy and where (supermarket A or B) to buy it. Thus, in this model product differentiation is represented by two parameters: on the one hand, the parameter g reflects consumer preferences regarding the product characteristics and can be interpreted as a measure of inter-product competition (if $g > 0$, products are substitutes;

[7] This are the necessary conditions for (1,1) to appear as an equilibrium. With respect to the sufficient conditions:

$p(1,1)-e > p(2,1)-2e$

$p(1,1)-e > p(3,0)-3e$

if g < 0, products are complements). On the other hand, the parameter b reflects consumer preferences regarding seller characteristics. It is this second form of product differentiation which allows to the retailers to differentiate the products and set a price above marginal cost when selling variants with the same intrinsic characteristic[8]. Parameter b can be interpreted as a measure of intra-product competition (if b = 0, retailers services are considered as independent; $b \to 1$ retailer services are close substitutes; b < 0, not considered).

In this model with two firms and two products to sell, firms product range and price decisions are modelled as a two-stage game. In Stage 1, firms simultaneously choose the number of products to sell: none, one or both. Stage 2 is modelled as a Bertrand-Nash price setting competition. By assuming that consumers

$$\max U(q_{11}, q_{12}, q_{21}, q_{22})$$
$$s.t. \quad I = y + q^T p$$

and that $U(\cdot)$ is quadratic and strictly concave, Dobson and Waterson obtain a linear demand structure that takes the following form for product i sold by retailer h if both products are sold by both retailers:

$$p_{ih} = 1 - q_{ih} - bq_{ik} - gq_{jh} - dq_{jk}, 0 \leq b < 1, -1 < g < 1 \qquad (16)$$
$$d = bg, |d| < b, |g|$$

where $p^T = [p_{11}, p_{12}, p_{21}, p_{22}]$ is the vector of prices, $q^T = [q_{11}, q_{12}, q_{21}, q_{22}]$ is the vector of demands and y is the quantity of the numeraire commodity consumed. If both firms do not produce both products the relevant elements of the vector q in (16) are set equal to zero.

8 In Shaked and Sutton [1990] a variant with the same intrinsic characteristics cannot be produced by more than one firm. Furthermore, as firm differentiation of the product is not allowed in presence of arbitrary small ($e > 0$) sunk costs, firms will never choose to produce the same variant. This is because price competition between identical products will equate price to marginal costs.

Unlike Shaked and Sutton [1990], Dobson and Waterson [1996] allow for the possibility of economies of scope when producing (selling) more than one good and assume that fixed costs depend on the number of products sold.

$$F = \begin{cases} 0 \text{ if } n = 0 \\ f \text{ if } n = 1 \\ f + (1-e) f = 2f(1-e/2) \text{ if } n = 2 \end{cases}$$

where $e \in [0,1]$ is a measure of the economies of scope with $e = 0$ meaning no economies of scope.

The equilibrium outcomes for different values of the parameters (b, g) are determined through identifying the optimal response of each retailer to the other's product and price choice. It is assumed that the single-product monopoly is profitable. Again the procedure for generating an optimal reply is similar: if retailer 1 sells only one product then (1→1) will be the optimal reply of retailer 2 if the profits associated with this reply are greater than the profits associated with (1→0) and (1→2) replies. Once the necessary and sufficient condition for each of the possible optimal replies is identified, it is possible to use them to subdivide the (b, g) space into areas that correspond to the different patterns of optimal replies. From this pattern the simultaneous move Nash equilibrium outcomes can be determined. Each equilibrium configuration is represented by a pair of strategies that are optimal replies one to the other.

Dobson and Waterson start by assuming $e = 0$ (absence of economies of scope). Their results suggest that for intense intra-product (high values of b, i.e. tough competition between retailers) and inter-product competition ($g > 0$ and close to one, i.e. each product takes sales from the other) the market is able only to support a specialised monopoly (1, 0). When intra-product competition and inter-product competition are low, both firms diversify and (2, 2) arises as market structure. Finally, the specialised duopoly (1, 1) arises as a market configuration when the products are substitutes ($g > 0$) even with values of b close to one (suggesting that product differentiation rather than firm differentiation can be enough for the retailers to cover their costs). There is an area in which both (1, 1) and (2, 0) are both Nash equilibrium outcomes which suggests the possibility of a deterrence equilibrium in other game structures.

As expected, the introduction of economies of scope ($e > 0$) makes diversification more likely. The most noticeable effect of the consideration of economies of scope is the restriction of the values of $g > 0$ that are able to support the specialised duopoly for moderate values of b (higher values of $g > 0$ are needed). This restriction goes along with an extension of the range of values of $g > 0$ that support the diversified duopoly.

Finally, Dobson and Waterson analyse the effect over the set of equilibria of changing the assumption of simultaneous product choice for sequential product choice and explore the influence of introducing exit costs as in Judd [1985]. To consider sequential product choice they introduce a three-stage game. In the first stage, the first-mover (firm 1) selects its product range and has the possibility of pre-empting entry by entering as a diversified multi-product firm. In the second stage, the potential entrant (firm 2) makes its product choice. In the third stage both firms set prices simultaneously[9]. Whilst with simultaneous product choice when products are substitutes ($g > 0$) and intra-product competition is intense (high b), specialised duopoly was a possibility, with sequential entry pre-emption always occurs. Note however that, as in Shaked and Sutton [1990], even when products are substitutes, pre-emption is not a predominant outcome. The single-product duopoly remains an equilibrium configuration when firms are able to reduce intra-product differentiation (lower values of b). Dobson and Waterson also show that the higher are exit costs, the greater the range of b and g over which product proliferation can deter entry.

3.2.3 Multi-product competition with a dual consumer population

Lal and Matutes [1989] is largely devoted to the analysis of the implications of multi-product competition on equilibrium prices and profitability in a static duopoly in which each firm is assumed to produce two products. However, we include it in this section because they briefly consider duopolists' product range decisions.

In their model, firms A and B (located at the end points of a line of unit length) are price competitors in a market for two unrelated products in which the differentiation of each product is given by the firm selling it, not by the intrinsic nature of the product. They have constant variable cost of production for each product. Consumers are assumed to be uniformly located along the line. Each consumer buys one unit of *each* of the two products so long as the total cost of the product does not exceed his reservation price. The total costs of the product, as in the Hotelling-type models, is the sum of the price of the product and the opportunity cost of the shopping trip and consumers are assumed to buy the product in such a way as to minimise their total cost.

Furthermore, it is assumed that the population of consumers is divided into two segments. Rich consumers with a high reservation price (*H*) for each product represent a proportion a of the total population of consumers. Poor consumers with low reservation price ($L < H$) represent a proportion $(1 - a)$ of the total population. The opportunity cost of shopping is positively related to transportation costs and assumed to be different for each of the two segments of consumers. The transportation cost per unit distance of the poor consumers is set equal to zero and for the rich consumers is assumed to be $c > 0$. Therefore, while the poor consumers buy the product at the lowest available price if this is lower than its reservation price, rich consumers must take into account their transportation costs in order to decide how many stores to visit and, if only one, which to visit. This is equivalent to assuming that whereas rich consumers are concerned about firm differentiation, poor consumers are not.

Lal and Matutes show that whether a Nash equilibrium exists depends upon the level of product differentiation. The critical level of product differentiation above which a Nash equilibrium in which firms set prices above marginal costs exists depends on the ratio of rich consumer transportation costs to poor consumer willingness to pay for quality $\left(c/L\right)$ and on the ratio of poor to rich consumers $V = \frac{(1-c)}{a}$. With $c = 0$, the products of the two firms are considered perfect substitutes for consumers (no product differentiation) and therefore the resulting Nash equilibrium will be the standard Bertrand outcome of price competition with two goods, i.e. prices equal marginal costs.

[9] This game is referred as "distinct first mover advantage", Dobson and Waterson also describe a game with "mild first mover advantage".

For small values of c/L ($0 < c/L \leq 2$), the Nash equilibrium fails to exist. Independent of the value taken by V, the unit transportation cost for the rich consumers is not high enough to remove the incentive to undercut. However, once prices reach marginal costs (assumed equal to zero in this model) for positive unit transportation costs the firm has an incentive to increase prices and serve consumers located close to the firm. Thus, an Edgeworth cycle is generated leading to the non-existence of a pure strategy Nash equilibrium (product differentiation is not enough for a Nash equilibrium to exist).

For intermediate values of c/L ($2 < c/L \leq 4$), whether the critical level of product differentiation that allows a Nash equilibrium to exist is achieved or not depends on V. If V is higher than a critical level V^* (function of c and L) then the critical level is not achieved. For $V > V^*$ the proportion of poor consumers is so high that the firms find it profitable to serve the entire poor segment of consumers. As a result, an Edgeworth cycle like the one described above is generated. If $V < V^*$ then the critical level of product differentiation needed for the Nash equilibrium to exist is satisfied and two possible equilibria arise. The first of these entails both firms competing to supply the good demanded by the poor consumers. For this good it is always profitable for the firms to undercut their rival with the result that both firms' prices equal zero ($p_{1A} = p_{1B} = 0$). In this equilibrium the price of the other good is set equal to the unit transportation cost of the rich consumer ($p_{2A} = p_{2B} = c$).

In the second of the equilibria, the so-called *reverse* equilibrium, the firms set prices $p_{1A} = p_{2B} = L$ and $p_{2A} = p_{1B} = c-L$, and discriminate between poor and rich consumers. The rich consumer who buys from a single store pays a higher price for the bundle than the poor consumer who shops around. This reverse equilibrium can be interpreted as a two-part tariff in which consumers with the lowest willingness to pay are left with no surplus while firms obtain higher profits from those consumers with higher willingness to pay for the two goods[10]. In this reverse equilibrium firm profits are greater than in the first equilibrium described above. Furthermore, price discrimination in this equilibrium can

[10] The price dispersion arising in the reverse equilibrium offers an alternative explanation to the loss-leader phenomenon observed in multi-product firms. It links loss-leader pricing to the nature of competition between multi-product firms serving a heterogeneous population of consumers.

be so effective as to equal the profits obtained by the firm under collusive arrangements[11].

For high values of c/L (c/L >4), a Nash equilibrium exists if and only if $V > V^*$. This equilibrium involves both firms competing for the one good bought by poor consumers. The prices associated with this equilibrium are $p_{1A} = p_{1B} = 0$ and $p_{2A} = p_{2B} = c$. The reverse equilibrium ceases to exist for values of $c/L > 4$. For this range of c/L, if prices at one of the firms are L and $L-c$, reverse pricing by the other firm would create such a price differential that rich consumers would have an incentive to shop around. Equilibrium prices make the price of the bundle equal to that of the standard Hotelling model.

In the reverse equilibrium both firms would be serving different goods to poor consumers and firms would maximise their profits from serving both consumer segments. However, when such an outcome cannot be supported as an equilibrium of the non-cooperative game proposed, the firms compete for the poor segment demand with the same good. As a result, the price of this good goes to marginal cost. It is the existence of the rich segment of consumers in the market which allows firms to charge full market price for the second good. Since the poor consumers are left without this good, the price that firms charge for it is equivalent to that in the standard Hotelling model with one good.

For most of the paper it is assumed that both firms produce two products, but in the final section, Lal and Matutes study the product line decision of the firm by analysing whether it is more profitable for the firms to sell one or two products. This is carried out using a three stage sequential game. In this game it is assumed that firm one is in the market selling the two goods. In Stage 1, a potential entrant (firm 2) decides whether to enter and if so with which products; in Stage 2, both the incumbent and the entrant decide whether to drop the production of one or two goods. Finally, in stage 3, firms compete in prices. They find that if a Nash equilibrium in which both firms sell both products exists, the profits of the firm from selling both products are larger than its share of the profits in an equilibrium in which this firm sells only one product.

[11] The profits of the firms in the reverse equilibrium are equal to those obtained under collusion when $c \rightarrow 4L$ and $a \rightarrow 2/3$.

4. Multi-product competition in a model of vertical product differentiation with endogenous quality range decision

Champsaur and Rochet [1989] extend the single-product firm models of vertical product differentiation presented in section 2 to allow for multi-product competition. Restricting their attention to the demand side (in the costs side no economies of scope are allowed and production activities of the firm are fully additive) they analyse quality range and price decisions in the multi-product duopoly. They consider a two-stage non-cooperative game in which firms first take the quality decision then the price decision. They highlight the point that two opposing effects influence the quality range decision. On the one hand, in order to discriminate among consumers with different characteristics, firms would like to offer a broad range of qualities (as in the monopoly situation). On the other hand, price competition lowers the profits of a firm when it offers qualities close to those of its rivals and so firms have an incentive to differentiate their products from those of their rivals. They show that for intermediate quality ranges the second of the effects dominates the first, so that in the Nash equilibrium of the quality game in which firms make positive profits there is a quality gap between the product lines offered by the two firms. In this model, it is the assumption of price competition among homogeneous products that rules out head-to-head competition.

Table 1: Summary 1

	Shaked and Sutton (1990)	Dobson and Waterson (1996)	Lal and Matutes (1989)
Firms/Product Variants	2 Firms 2 Product Variants	2 Firms 2 Product Variants	2 Firms 2 Product Variables (no subst. nor compl.)
Costs	No economies of scope	Economies of Scope are allowed	No Economies of Scope
Differentiation	Product characteristics	Product characteristics Firm characteristics	Firm characteristics
Consumers			Rich Consumers(α)/Poor Consumers (1-α)
Relevant Parameters	s: degree of substitution	g: degree of inter-product competition b: degree of inter-firm competition	Rich Cons: c transp.costs and R res. Price Poor Cons: 0 transp.costs and L res. Price $R = (1-a)/a$
Product Line	A product variant cannot be produced by more than one firm	A product variant can be produced simultaneously by the two firms	Two firms produce two variants
Results	Simultaneous game	Simultaneous game	Existence of Nash Equilibrium
	High $s \rightarrow$ $(c \rightarrow \infty, E \rightarrow 0)$ Intermediate s $\rightarrow (C > 0, E > 0) \rightarrow (1,1)$ Low s Indeterminate area (1,1) or (2,0)	1. No Economies of Scope High b and γ (γ>0) $\rightarrow$ (1,0) Low b and $\gamma \rightarrow$(2,2) g > 0 even with high $b \rightarrow$(1,1) 2. Economies of scope More likely differentiation	If $0 < c/L \leq 2$ Edgeworth Cycle If $2 < c/L \leq 4$ then depends on R - if $R > R^*$ Edgewoth cycle - if $R < R^*$ Two Nash Equilibria 1. $p_{1A} = p_{1B} = 0$; $p_{2A} = p_{2B} = c$ 2. $p_{1A} = p_{1B} = L$; $p_{2A} = p_{2B} = c-L$ If c/L>4 then depends on R - if $R < R^*$ no Nash equilibrium - if $R > R^*$ no Nash equilibrium $p_{1A} = p_{1B} = 0$; $p_{2A} = p_{2B} = c$
	Sequential Game	Sequential game	Sequential game
	No pre-emption with intermediate-s ® (E>0, C>0)	Indeterminate area and part of the area where g > 0 even with high b $\rightarrow$pre-emption	If a Nash equilibrium in which the two firms sell both products exists, no firm is interested in withdrawing a product

5. Multi-product competition in two-dimensional models of product differentiation

The next step is to study models of multi-product competition in which products are differentiated in two dimensions: quality and horizontal attributes. We propose here a general framework to analyse Katz [1984], Gilbert and Matutes [1993] and Canoy and Peitz [1997]. In Katz [1984] the firm's product line decision is taken as given and firms simultaneously choose prices and qualities of the products in their product lines. However, in Gilbert and Matutes [1993] and Canoy and Peitz [1997] qualities are taken as given and firms choose product line (niche or proliferation) and prices. These last two papers analyse firm entry decisions using sequential games. Table 2 summarises the main assumptions and results of these papers.

We start by providing a common benchmark for the three papers with respect to the assumptions about firms and consumers.

Firms

Consider a market in which two firms can produce more than one variant of a good. In this market variants are differentiated in two dimensions: quality and horizontal characteristics. It is assumed that there are two goods: a low-quality (good 1) and a high-quality (good 2). Lets us denote the realised choice from amongst the space of possible production choices for firm f by R_f. Firm f sets prices p_{fj}, j Î R_f such that it maximises:

$$p_f = \sum_{j \in R_f} \left[p_{fj} - c(q_j) \right] s_{fj} - K_{R_f}$$

where s_{fj} is the market share of the variant of quality j produced by firm f and $c(q_j)$ the unit cost of production of this variant (with $c'(q_j) > 0$). As regards the fixed cost of production K, assumptions differ across the papers: Katz [1984] does not introduce any fixed costs, Canoy and Peitz [1997] assume weak economies of scale and Gilbert and Matutes [1993] strict economies of scale.

Customers

Each consumer, *i*, buys one unit of one of the variants and none of the other variants. His indirect utility function is given by the following variant of (14)

$$v_{ij} = r_i + q_j q_i - z\left|d_i - l_j\right| - p_{fj} \qquad (17)$$

Each consumer type is defined by its willingness to pay for quality (*q*) and its parameter for horizontal specification (*d*). All three papers assume that consumers' preference for horizontal specification is distributed in the interval [0 ,L] with density *d*. Gilbert and Matutes[12] and Canoy and Peitz [1997] assume that at each location *d*, consumers' willingness to pay for quality is uniformly distributed over [0 ,*q*], whereas Katz [1984] assumes a finite number of *q*-types in the market (here, we will assume two: high *q* and low *q*-types). The "transport cost" parameter *z* measures the intensity of the consumer's preference amongst firms.

A consumer located at δ with a willingness to pay for quality θ will buy the product *j* from firm *i* if:

$$\{f, j\} = \arg\max\left[r_i + q_i q_j - z\left|d_i - l_j\right| - p_{fj}\right]$$

All the three papers assume maximum differentiation. In Katz [1984] and Gilbert and Matutes [1993] the firms (in respect of the variant's horizontal characteristic) are located at the end points of the linear segment of length L, and in Canoy and Peitz [1997] variants are located at the corners of a symmetric triangle.

5.1 A model with homogeneous brand preferences

Gilbert and Matutes [1993] present two different models. In the first of them, two duopolists compete in a one-stage game in which they set the prices for all the possible variants. The second model is a three-stage sequential game in which the firms can make credible commitments not to produce one or more products.

They set the quality of the low-quality variant to zero (q_1 = 0) and that of the high-quality variant to 1 (q_2 = 1). In their model z does not vary with q and it is assumed to equal one. The density of consumers in each location d is assumed to be equal to N/L. Therefore, their indirect utility function takes the following form:

$$v_{ij} = r + qq_j - |d - l_i| - p_j$$

Then, using this utility function and by means of the condition defining the customer indifferent between any two particular variants they obtain the sales regions (market shares) for each of the variants.

They assume that each of the firms has a cost function with strict economies of scope

$$C_i(x_{i1}, x_{i2}) = c(q_1)x_{i1} + c(q_2)x_{i2} + F = c_1 x_{i1} + c_2 x_{i2} + F$$

where x_{ij} denotes the sales of quality variant j by firm i. The cost advantage of the low-quality over the high quality variant is defined as $D = c_2 - c_1$.

Consider first the one-stage game in which the two firms simultaneously set price for the low-quality and high-quality variants and firms have no ability to pre-commit to their product offerings. Gilbert and Matutes show that in a symmetric Nash equilibrium the profits of firm 1 are

$$p_1 = \frac{N}{L}\left[(p_{11} - c_1)t_1 q_1 + (p_{12} - c_2)t_2(1 - p_{12} + p_{11})\right]$$

and the profits of firm 2 are

[12] Actually, Gilbert and Matutes [1993] do not make any assumption about the shape of the distribution, we make the assumption here to facilitate comparisons with the other two models.

$$p_2 = \frac{N}{L}\left[(p_{21} - c_1)(L - t_1)q_1 + (p_{22} - c_2)(L - t_2)(1 - p_{12} + p_{11})\right]$$

where:

$$t_1 = \frac{p_{21} - p_{11} + L}{2}$$

$$t_2 = \frac{p_{22} - p_{12} + L}{2}$$

The equilibrium prices obtained by solving the system of first order conditions resulting from the maximisation problems of firms 1 and 2 are: $p_{11} = p_{21} = L+c_1$ and $p_{12} = p_{22} = L+c_2$. Therefore, in the symmetric equilibrium: both firms set the same prices, the mark-up is the same for each of the products ($m_{11} = m_{12} = L$ and $m_{21} = m_{22} = L$), this mark-up is the same as in the single product competition, and profits are independent of the number of variants produced[13]. In equilibrium, mark-ups are independent of consumer tastes for quality. Although firms are no better off by offering the product line than by offering a single product, this symmetric Nash equilibrium in pure strategies involves both firms producing the product line (q_1 and q_2). This is because if one firm is producing a single variant, for any product choice of the other firm, this firm could introduce a second variant with the same mark-up that would generate additional sales with no loss of profits from the consumers that switch from one variant to the other. The sales of the new variant come from consumers switching from the variant that the firm was already producing and from the variant of the same quality produced by the other firm. Consumer switches between variants produced by the firm are costless to the firm and the additional sales to consumers previously buying from the rival represent additional profits[14]. Therefore, in a one-stage game of product-choice and pricing and without possible commitments to product choices, firms introduce the maximum number of product variants but they do not benefit thereby.

[13] This result contrasts with the higher mark-ups for the high-quality variant obtained in the analysis of vertical product differentiated-only models by Mussa and Rosen [1978] and Moorthy [1988]. The equilibrium price when each firm only sells the same quality variant is $p=c+L$. Notice that since locations are fixed, competition between the players is softened compared with vertical product differentiation models. It is also assumed that $0 < D \leq \overline{?}$ and that therefore in an efficient equilibrium both variants would be produced.

[14] Given that in this symmetric Nash equilibrium both firms produce the product line if the introduction of any new product involves any overhead cost the firms are worse off as multi-product firms than as single product.

They model product commitment as a three-stage sequential game in which firms can commit to withhold one or more quality variants from their production possibility sets. The stages of this game are as follows: in Stage 1, firm 1 takes the necessary actions to produce 1, 2 or the product line; in Stage 2 firm 2 takes the same decision with full information about the actions taken for firm 1; in Stage 3 firms choose prices simultaneously and can decide whether or not to drop one or more quality variants from their actual production. Two specifications of this sequential game are analysed. In the first specification firms 1 and 2 incur the sunk cost of entry before Stage 1. In the second specification, in order to analyse the possibility of entry in an industry with an established incumbent, it is assumed that firm 1 incurs the sunk cost of entry at Stage 1 and firm 2 at Stage 2 before choosing products. In both specifications, the choice of niche or product line strategy depends on the degree of firm-specific differentiation; we will focus on the analysis of the second, which we consider more interesting.

For small firm-specific differentiation (L) (relative to vertical differentiation ($\bar{q}$) the incumbent's (firm 1) optimal strategy is to specialise in one of the quality variants and allow entry to occur in the other. In the limit when $L \to 0$ firms' profits are zero for the quality variant they both sell, so in Stage 1 firm 1 will commit to produce only one of the quality variants, and knowing that, in Stage 2 firm 2 will choose the other quality variant. For large firm-specific differentiation entry occurs and both firms produce both quality variants. For intermediate values of firm-specific differentiation the incumbent chooses the product line as a defensive strategy even if conditional on entry both firms would be better off if the incumbent chose a niche strategy. The incumbent chooses the product line strategy in order to avoid product proliferation by the entrant. If the incumbent chooses a niche strategy then the entrant will choose a product line strategy and the incumbent will be worse off than by having chosen a product line strategy. This strategy deters entry when the level of profits from both firms producing the product line is lower than the fixed costs of entry. Even though entry would be profitable under specialisation by both firms, for the incumbent, product line is a dominant strategy. Gilbert and Matutes [1993] find an exception to Judd's [1985][15] result on the inability of multi-product firms to deter entry by product proliferation. In this model, when firms

[15] Whilst, Schmalensee [1978] and Eaton and Lipsey [1979] suggested that product proliferation could be used as an strategy to deter entry, Judd [1985] shows that multiproduct firms are specially vulnerable to

are able to pre-commit to a given product choice and the level of firm-specific differentiation is sufficiently large, product proliferation is a credible strategy to deter entry.

5.2 A model of separate quality sub-markets

Katz [1984] assumes that the purchasing decision of the consumer can be broken into two stages. First, the consumer chooses her preferred quality variant from each firm. In the second stage, the consumer chooses the firm whose optimal quality variant yields the higher surplus. Therefore, in the first stage, the quality variants of a given firm compete against each other. In the second stage the products of a firm compete against the products of the other firms.

In this model firms choose price and qualities simultaneously with quantities determined by consumer demand. Before analysing multi-product competition Katz analyses the two polar cases of vertical product differentiation only and horizontal product differentiation only. By assuming that $r \geq 0$ and $z = 0$ in (17) for both high-θ and low-θ consumers, he analyses the quality and pricing decision of a multi-product monopolist. In Appendix A, we show the two main results of this analysis. First, discrimination between consumers that differ in their willingness to pay for quality leads to the monopolist providing (socially) sub-optimal quality to the low-q consumers[16]. Second, the monopolist's price-cost margin is greater for the high-quality than the low-quality variant. The multi-quality monopolist discriminates against the high-q consumers.

In order to analyse the horizontal product differentiation case in isolation, the existence of some exogenous mechanism that allows each of the firms to prevent a given q_j-type from consuming any other variant that the variant of quality q_j is assumed. This assumption allows the market to be divided into two independent sub-markets, one for each of the two q-types considered. The analysis is restricted to symmetric equilibria

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entry. Entry affects not only the profits of the variant facing direct competition by the entrant but also the profits obtained by the variants neighbouring it

[16]In general the multi-quality monopolist provides sub-optimal quality to all q-types but the highest q-type.

($p_{1j} = p_{2j}$ and $q_{1j} = q_{2j}$ for $j = 1,2$) and two further assumptions are made: all the q_i-type have the same intensity of firm preference ($q_2 > q_1 \Rightarrow z_2 > z_1$) and there are N_j q_j-consumers uniformly distributed in the interval $[0,L]$ with $L = 1$. Given the assumption of the exogenous allocation mechanism, firms need not set q_1 lower than the efficient quality for low-q consumers in order to discriminate between high and low q-type consumers. Thus, qualities will be set at the efficient level in the two sub-markets (q_1^e, q_2^e) [17]. The analysis of each one of these independent sub-markets is identical to the analysis of the standard linear-city model of horizontal product differentiation assuming transportation costs equal to z_j. This analysis reveals that the higher the intensity of consumer's firm preference (z_j) the greater the market power of each one of the firms, and consequently the higher the price-cost margin.

Thus, both the analysis of the price and quality setting for the multi-quality monopolist and for the horizontally differentiated only sub-markets suggest that firms' price-cost margins will be higher for the high-quality variant than for the low quality variant [18]. This means that firms will be especially concerned about possible trading down by high-q consumers from consumption of the high-quality variant to consumption of the low-quality variant. Katz shows that in any symmetric equilibrium this possibility results in firms supplying qualities (q_j) that are below the optimal levels of quality and it has two additional effects over the pattern of competition across variants. On the one hand, price competition for the low-quality variant may be blunted by strong firm differentiation in the high-quality variant. If (i) the resulting equilibrium prices (p_1, p_2) and efficient qualities (q_1^e, q_2^e) with an exogenous type-allocation mechanism violate the quality discrimination requirements needed in absence of this exogenous mechanism and (ii) the high-q prefer the low-quality good, then to prevent switching firms must either increase the surplus that the high-q consumers obtain from the high-quality variant or decrease the surplus that they obtain from the consumption of the low-quality variant. Since reducing the price of the high-quality variant would reduce the profits of this variant, firms will opt for increasing the price of the low-quality variant. Therefore,

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[17] For the θ_i-type consumer the efficient level of quality is the q_j^e that maximises $q_j q - c(q)$ and therefore $q_j = c'(q_j^e)$.

[18] This result contrasts with the equal price-cost margin result obtained in Gilbert and Matutes [1993] presented before.

it is likely that competition between product line firms engaged in quality discrimination will be softer than between single product firms. As a result, price-cost margins for the low-quality variant will be higher when firms engage in quality discrimination than when there exists an exogenous allocative mechanism that makes it unnecessary[19].

On the other hand if the low-quality segment of the market is sufficiently large, firms will compete for sales in this segment even though this competition may involve a reduction of the prices for the high quality variant. The presence of low-*q* consumers confers a positive externality to high-*q* consumers.

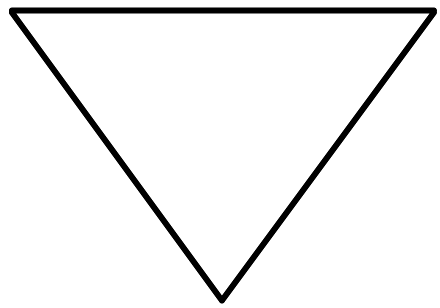
5.3 A model with heterogeneous brand-preferences and quality determined horizontal differentiation

Canoy and Peitz [1997] propose a “differentiation triangle” to analyse the trade-off between strategic (nature of resulting price competition and possibilities of entry deterrence) and cost factors to explain firms' product choice between niche and product line strategies. They assume that variants are positioned at the corners of a symmetric triangle and that firms i =1,2,3 can choose their product from the product set V_i={0, (L_i), (H_i), (L_i,H_i)}. In order to limit the first-mover advantage of incumbent 1 it is assumed that firms cannot produce both high-quality variants.

Like Gilbert and Matutes [1993], they set the quality of low-quality variants at 0 and the high-quality variant equal to 1. However, whilst Gilbert and Matutes [1993] make the horizontal characteristic independent of the quality level, they make the horizontal characteristic depend upon the quality level by assuming that the low-quality variant cannot be horizontally differentiated from any other variant of the same quality. Each variant is described by two numbers q_j, l_j in [0, 1] and so variant L is described as (0, l_j), H_1, as (1, 0) and H_2 as (1, 1)

[19] The presence of high-*q* consumers may be a negative externality for low-*q* consumers because the

Figure 2: The differentiation Triangle



They assume consumers' willingness to pay for quality (θ) is uniformly distributed in the interval [0, 1]. Like Katz [1984], they assume that intensity of consumer's firm preference (z) is increasing in θ and so they make $z=qq_j$ in (17).

They propose a sequential game similar to the three-stage sequential game of Gilbert and Matutes [1993] in which an additional intermediate stage is added to allow for the possibility of entry of a third firm. In Stage 1, incumbent 1 develops $\{L\}$, $\{H_1\}$, $\{L, H_1\}$, $\{0\}$ and incurs the associated fixed costs; in Stage 2, incumbent 2 develops $\{L\}$, $\{H_2\}$, $\{L, H_2\}$, $\{0\}$ and incurs the associated fixed costs; in Stage 3 the potential entrant (firm 3) develops $\{L\}$, $\{H_1\}$, $\{H_2\}$, $\{L, H_1\}$, $\{L, H_2\}$ or $\{0\}$ and incurs the associated fixed costs; finally at Stage 4, firms set prices simultaneously (at this stage fixed costs are sunk).

In the model, for a particular combination p_L, p_{H1}, p_{H2} such that all market shares are positive, only customers of one particular type are indifferent between the three variants offered. Therefore, by identifying the conditions characterising the customer type that is indifferent between any two variants it is possible to calculate the market shares that correspond to each of the variants.

Canoy and Peitz follow two steps to finding the perfect equilibrium of the game. In the first they show that only four scenarios such that all corner points are occupied are candidate subgame perfect equilibria and using numerical methods they calculate the prices, market shares and profits before deduction of fixed costs that correspond to each

surplus they obtain from the consumption of the low-quality variant can as a result be reduced.

of them. In the second step, assuming that fixed costs are sufficiently small for all the corners to be occupied in equilibrium, they show that there exists a unique subgame perfect equilibrium. Which scenario constitutes the equilibrium depends on the interaction between costs (economies of scope) and strategic factors (pricing and product choice as an entry deterrence instrument).

In order to understand this interaction they distinguish the motivations of each firm in developing a product line or niche strategy. On the one hand, incumbent 1 will choose to produce the vertical product line or a single variant based on the cost structure (the first mover advantage allows it to guarantee profits at least as high as the profits of incumbent 2). Therefore, if economies of scale are very strong, incumbent 1 chooses the product line. If production of the low-quality variant is very favourable then incumbent 1 chooses a niche strategy producing only the low-quality variant and the resulting market structure is a three-firm oligopoly. Otherwise, incumbent 1 follows a niche strategy with the high-quality variant. On the other hand, incumbent 2's choice is restricted by the choice of the incumbent 1. If incumbent 1 develops the product line, incumbent 2 selects the only option in the market, namely a high-quality niche. However, if incumbent 1 chooses a high-quality niche strategy, incumbent 2 will choose between niche and product line strategies based upon entry deterrence principles (whereas brand proliferation is an entry deterrence strategy for incumbent 2 it is not for incumbent 1). So far as prices are concerned, they show that the firm producing the vertical product line sets a higher price for its high-quality variant than the firm following a niche strategy. By these means, a firm may relax competition between the two quality variants produced.

6. Discrete choice models of product differentiation

Following Bresnahan [1987][20] a major theme in the empirical analysis of oligopoly models with product differentiation has been the joint analysis of a demand function and

[20] Whereas most of the analyses involving discrete-choice models of product differentiation use *non-address* models (all the products are in competition with all others), Bresnahan [1987] develops a discrete-choice model with vertically differentiated products that is an *address* model (each product competes just with its neighbours).

a supply relationship using discrete-choice models of product differentiation[21]. Significant papers include Berry [1994] which establishes the theoretical benchmark for the treatment of unobserved product characteristics and Berry, Levinsohn and Pakes [1995][22]which carries out an equilibrium analysis of the automobile industry in the US. In this paper, BLP empirically implement the theoretical framework proposed by Berry [1994] when carrying out the joint estimations of demand and supply relationships. Hence, we devote this section to the analysis of the framework proposed by Berry [1994] with some references to BLP.

6.1 A benchmark for the estimation of discrete choice models of product differentiation

First, we offer a brief description of the models used in the empirical analysis of discrete choice models of product differentiation. The objective of these models is the joint estimation of supply and demand equations in markets with product differentiation. The general demand framework is that consumers' utility depends both on product characteristics and on consumers' characteristics and demand for each product is obtained by aggregating over all the consumers that prefer this product to all the other products. On the supply side, firms are modelled as price setting oligopolists and the existence of a Nash equilibrium in prices is assumed. The characteristics of the products are taken as given.

[21] For an extensive survey of the discrete-choice models of product differentiation, see Anderson, de Palma and Thisse [1992].
[22] From now on this paper will be referred as BLP.

Table 2: Summary II

	Framework		
Firms/variants	Two VariantsLQ (q_1)/ HQ (q_2)		
Relevant Parameters	q: willingness to pay for quality z : strength of brand preference L: firm (horizontal) differentiation $\bar{q}$: maximum willingness to pay for quality		
	Katz [1984]	Gilbert and Matutes [1993]	Canoy and Peitz [1997]
Assumptions about distribution of θ	Two q-types $q_2 > q_1$	q uniformly distributed [0, $\bar{q}$]	[0,1]
Assumptions about brand preferences Horizontal characteristics	$q_2 > q_1 \rightarrow z_2 > z_1$ Independent of quality	z = 1 for all q Independent of quality	$Z = qq_i$ Depends upon quality
Results	Vertical Product Differentiation Only	One Stage Price Setting Game	Sequential Game
	- $m_1 > m_2$ (m: price cost margin) - $q_2 = q_2^e$ and $q_1 < q_1^e$ Horizontal Product Differentiation Only	Two firms produce the product line $m_1 = m_2$ Sequential game	• Incumbent 1 (I1): product line decision based on cost structure - Strong economies of scope $\rightarrow$ product line - Very favourable L_1 $\rightarrow$resulting market structure: three firms oligopoly - Otherwise H_1 • Incumbent 2 product line decision: importance of strategic motives - If I1 product line $\rightarrow$ H_2 - If I1 H_1 then niche or product line on the basis of entry deterrence principle • Price of the H is higher for the firm that develops the product line
	$m_1 > m_2$ ↓ ↓ Combining $m_1 > m_2$ $q_2 = q_2^e$ and $q_1 < q_1^e$	- small $L/\bar{q}$ $\rightarrow$incumbent niche and entrant niche - large $L/\bar{q}$ $\rightarrow$ incumbent product line and entrant product line - intermediate $L/\bar{q}$ $\rightarrow$ incumbent product line strategy to deter entry (pre-emption strategy)	

We start our analysis with the demand side of the problem. The utility of consumer i for product j depends both on a vector of individual characteristics (v_i) unobserved by the econometrician and on a vector of characteristics of the product, x_j

$$u_{ij} = U(x_j, x_j, p_j, v_i; q)$$

where p_j is the price of the product j, x_j is a vector of product characteristics unobserved by the econometrician, q is the vector of parameters determining the distribution of consumer characteristics.

Assume this utility function is additively separable in a term (d_j) depending exclusively on the characteristics of the products and on an individual specific term (v_{ij}) that for the moment, we consider as resulting from the interaction of consumer and product characteristics,

$$u_{ij} = d_j(x_j, p_j, x_j) + v_{ij} \qquad \text{for } j = 0, 1, \ldots, J \tag{18}$$

$j = 1,...,J$ represent the purchase of competing differentiated products, and $j = 0$ the outside option, i.e. where the consumer does not purchase any of the products available in the market. The term d_j can be interpreted as the mean utility that consumers obtain from the purchase of product j. Furthermore, if we assume a linear specification for d_j, we can express the mean utility level for product j as:

$$d_j = x_j b - a p_j + x_j \tag{19}$$

Each consumer purchases one unit of the good that gives him the highest utility, $\{u_{ij} \geq u_{ir}$ for $j = 0, 1,...r\}$. If we define the set of unobservable taste parameters, v_{ij}, that result in the purchase of product j as $A_j(\delta)=\{v_i | d_j+v_{ij} > d_r+v_{ir};$ "r ¹ j$\}$, the market share of firm j is given by the probability that v_i falls into the region A_j. Given the distribution of consumer preferences over the product characteristics, $F(\cdot)$, the discrete choice market share of product j is[23]:

$$s_j = (d(x, p, x), x; q) = \int_{A_j(d)} f(v, x) dv \qquad (20)$$

With a total of M consumers (a market size of M), the demand for product j is given by

$$q_j = Ms_j(x, x, p; q)$$

Let us consider the following demand equation relating observed market shares (S_j) to the market shares predicted by the model (s_j),

$$S_j = s_j(x, p, x; q) \qquad (21)$$

We expect unobserved product characteristics to be correlated with prices, which makes prices endogenous. The traditional solution to this problem of endogenous prices (when prices and unobserved characteristics enter in a linear fashion in the demand equation) is to use instrumental variables. However, from (20) we see that unobserved product characteristics enter (21) in a non-linear fashion which prevents the application of this approach. Berry [1994] solves this problem by using a transformation which makes market shares linear in unobserved product characteristics. For simplicity, let us start assuming that the distribution of consumer unobservables (v_{ij}) is known (by the econometrician) so market shares depend only on mean utility levels

$$S_j = s_j(d) \quad \text{for } j = 1, \ldots, n \qquad (22)$$

Using the fact that at the true values of d the above equation must hold exactly, if it is possible to obtain a closed form solution for the integral in (20) we can invert (22) to obtain the vector $d = s^{-1}(S)$. Thus, the vector of observed market shares uniquely

[23] In other words, the discrete-choice model market share for product j can be calculated as the probability of purchase of product j given the distribution of consumer preferences over the product characteristics.

determines the vector of mean utility levels. From (19), at the true values of the parameters (b, α), the demand equation is

$$d_j(S) = x_j b - ap_j + x_j \qquad (23)$$

If in (23) we consider x_j as an unobserved error term, we can obtain the unknown parameters (b, a) by instrumental variable regression of $d_j(S)$ on (x_j, p_j).

An alternative to assuming a known distribution of consumer unobservables is to assume that the density of v_{ij} is unknown but depends on a vector of parameters (s) to be estimated. With this assumption, the market share function and the implied mean utility levels depend also on s, i.e. $S = s(d, s)$. Again, inverting this last equation we may obtain the demand equation as

$$d(s, s) = x_j b - ap_j + x_j$$

We can still make use of instrumental variables to estimate the above equation. Now, we have an additional parameter (vector) to estimate, s

Different assumptions about consumer preferences lead to different specifications of the utility function and thus to different demand specifications and patterns of substitution. The simplest is the assumption of homogeneous preferences across consumers. Under this assumption the utility function (18) takes the form

$$u_{ij} = x_j b - ap_j + x_j + e_{ij}$$

where x_j can be understood as the mean of consumer's valuation of an unobserved product characteristic and e_{ij}, representing the distribution of consumer preferences about this mean, is assumed to be mean zero independently and identically distributed across consumers and products. Furthermore, if we assume that e_{ij} follows an extreme value distribution and normalise the utility of the outside good to zero, the probability of purchase of product j (market share of the product j) is

$$s_j(d) = \frac{e^{d_j}}{1+\sum_{j=1}^{n} e^{d_j}} \qquad \text{for all j= 0,1,…,}n \qquad (24)$$

from which we can obtain the following linear model in price and product characteristics[24]

$$\ln(S_j) - \ln(S_0) = d = x_j b - ap_j + x_j \qquad (25)$$

Therefore, if we consider x_j as an error term we can estimate the structural form demand parameters (b, a) by instrumental variables.

The main problem with this simple logit specification is that it does not allow the interaction of consumer and product characteristics. Products are just differentiated by mean utility levels (d_j), implying market shares and own and cross price elasticities are determined exclusively by them. The result is unreasonable patterns of substitution: in the logit model substitution effects are the same independently of the degree of similarity between product characteristics[25].

The obvious solution is to allow heterogeneous preferences. In discrete-choice models, we can generate heterogeneous preferences by interacting consumer and product characteristics. One possibility is the use of nested logit models that, albeit in a restricted way, allow consumer tastes to be correlated across products. In the nested logit models, prior to the estimation products are grouped in sets of products of similar characteristics and a higher correlation for the products within the same set than for product belonging to different sets is imposed[26]. Thus, the utility of consumer i from buying product j belonging to group g is:

$$u_{ij} = d_j + z_{ig} + (1-s)e_{ij}$$

[24] The transformation requires weights for the outside option and taking logs.

[25] In the logit model the general expression for the cross price elasticity is given by $h_{jr} = \frac{ds_j}{dp_r}\frac{p_r}{s_j} = ap_r s_r$, and so a change in the price of the Ford Fiesta will have the same effect over the market shares of the Ford Scorpio and Fiat Punto.

For consumer *i*, the variable ζ is common to all products in *g* and has a distribution that depends on s. The parameter s ($0 \pounds s < 1$) can be interpreted as a substitution parameter. The perturbations (e_{ij}) that (as in the logit model) follow a maximum value distribution are assumed correlated for products belonging to the same set but uncorrelated for products belonging to different sets. In this nested logit model the probability of purchase of product *j* (market share of good *j*) is given by:

$$s_j(d, s) = \frac{e^{\frac{d_j}{1-s}}}{D_g^s \left[\sum_g D_g^{(1-s)}\right]} \quad \text{where} \quad D_g = \sum_{j \in J_g} e^{\frac{d_j}{1-s}} \tag{26}$$

This market share of product *j* ($j \in J_g$) can be expressed as the product of the share of product *j* within group *g* (conditioned share) and the share of group (*g*) over the total of products (marginal share), i.e.

$$s_j = \bar{s}_{j/g} \cdot \bar{s}_g \tag{27}$$

where:

$$\bar{s}_{j/g}(d, s) = \frac{e^{\frac{d_j}{1-s}}}{D_g} \quad \text{for } j \in g \tag{28}$$

$$\bar{s}_g(d, s) = \frac{D_g^{(1-s)}}{\sum_g D_g^{(1-s)}} \qquad \text{for } g = 0,1,\ldots,G \tag{29}$$

Normalising the mean utility level of the outside good to zero, then $D_0 = 1$ and

$$s_0(d, s) = \frac{1}{\sum_g D_g^{(1-s)}} \tag{30}$$

Using (28), (29) and (30), and after rearranging (27), we obtain the linear model used for the estimation of the parameters of the model (b, a, s)

[26] *G+1* exhaustive and mutually exclusive sets, $g = 0,1,\ldots,G$. The outside good is assumed to be the only product in group 0.

$$\ln(S_j) - \ln(S_0) = x_j b - a p_j + s \ln(\bar{S}_{j/g}) + x_j \qquad (31)$$

The estimating equation of the nested logit includes an endogenous extra term on the market share of the model with respect to the group of products to which model j belongs. Estimation of the parameters (a, b, s) can by obtained by linear instrumental variables regression.

In contrast to the simple logit model, the nested logit model generates reasonable patterns of substitution. Cross-price elasticities between products belonging to the same set (with similar characteristics) are greater than between products belonging to different sets (with more heterogeneous characteristics).

If we think of quality as the criterion used to group the products, the parallelism between the nested logit model and the models of vertical-horizontal product differentiation presented in the previous sections are obvious. Those models could be thought as nested logit models with two groups: a low and a high-quality group.

The nested logit's main limitation is that correlation patterns between products and consumer characteristics depend on grouping of products carried out prior to the estimation and therefore imposing some a priori patterns of substitution. The random coefficients model proposed by BLP that allows for full interaction between consumer and product characteristics solves this problem and obtains sensible patterns of substitution at the cost of a substantial complication in the calculation of the market share equation (20). In their model, the vector of consumer taste parameters, b, for observed characteristic k is modelled as:

$$b_{ik} = b_k + s_k z_{ik} \qquad \text{for } k = 1,\dots, k$$

where b_k is the mean of the taste parameter for characteristic k and z_{ik} is assumed to have an identically and independently distributed standard normal distribution. Thus, the utility function can be expressed as:

$$u_{ij} = x_j b - a p_j + x_j + v_{ij} = d_j + v_{ij}$$

and $v_{ij} = \sum_{k=1}^{K} s_k x_{jk} z_{ik} + e_{ij}$ with e_{ij} independently and identically distributed across consumers and products. The main drawback of the random coefficient models is that the market share equation (20) is difficult to calculate and usually it is necessary to use simulation procedures.

So far we have only considered the analysis of the demand-side of the model, therefore the next step will be to consider the supply side. Consider a market with *N* firms, each of them producing just one product. In order to simplify, we can assume that fixed costs are equal to zero, and that marginal costs are independent of output levels and linear in a vector of cost characteristics. Furthermore it is assumed that cost characteristics can be decomposed into observed product characteristics (w_j) and product characteristics (w_j)[27] unobserved by the econometrician. Given these assumptions, the cost function takes the following form

$$c_j = w_j g + w_j$$

and profits of firm *j* are

$$p_j = (p_j - c_j) M s_j (x, p, x; q)$$

If following Caplin and Nalebuff [1991] it is assumed that a pure strategy Nash equilibrium exists for this pricing game, then the price set by firm *j* must satisfy the following first order condition

$$\left[p_j - c_j\right]\left[\frac{\partial s_j (x, p, x; q)}{\partial p_j}\right] + s_j (x, p, x; q) = 0$$

which, after rearrangement, can be expressed as

$$p_j = c_j + \frac{s_j}{\left|\partial s_j / \partial p_j\right|} = w_j g + \frac{s_j}{\left|\partial s_j / \partial p_j\right|} + w_j \qquad (32)$$

From (19) and (20), $\partial s_j / \partial p_j = -a \, \partial s_j / \partial d_j$ and so (33) can be rewritten as a function of $\partial s_j / \partial d_j$. Given the vector $\boldsymbol{d}$ obtained from the inverse market share function, $\boldsymbol{d} = s^{-1}(S)$, it is possible to obtain $\partial s_j / \partial d_j$ by analytical or numerical differentiation of the market share evaluated at the adequate value of $\boldsymbol{d}$. Therefore, $\boldsymbol{dj}$ and $\partial s_j / \partial d_j$ can be treated as known transformations of the data and (33) can be written

$$p_j = w_j g + \frac{1}{a}\left[S_j / \left(\partial s_j / \partial d_j\right)\right] + w_j \tag{34}$$

We now consider the pricing equation (34) for the three models considered above: logit, nested logit and random coefficients model. For the logit model, (24) yields $\partial s_j / \partial d_j = S_j(1 - S_j)$ and so, using (34), the pricing equation for the logit model is given by

$$p_j = w_j g + \frac{1}{a}\left(\frac{1}{(1 - S_j)}\right) + w_j \tag{35}$$

where the parameters to estimate are $(g,\ a)$. The logit joint estimation problem is given by (25) and (35).

For the nested logit model, from (27)

$$\frac{\partial s_j}{\partial d_j} = \frac{1}{(1 - s)} S_j \left[1 - s\bar{S}_{j/g} - (1 - s) S_j\right]$$

and so using the pricing equation, the estimation equation for the nested logit model is given by

$$p_j = w_j g + \left[\frac{(1 - s)}{a} \Big/ \left[1 - s\bar{S}_{j/g} - (1 - s) S_j\right]\right] + w_j \tag{36}$$

---

[27] In general it is expected that x_j will be a part of w_j and xj will be correlated with wj.

where the parameters to estimate are (a, g, s). Therefore, (31) and (36) give the estimating equations for the joint nested logit estimation.

In both the logit and the nested logit specification cases, with an analytical solution for d_j an analytical solution for $\partial s_j / \partial d_j$ exists. However, for the full random effects model numerical differentiation is required.

Of course, the firm often supplies multiple products. To sketch the implications of this for the estimation procedure, equation (32) must be replaced by

$$\sum_{1}^{R} [p_r - c_r] \left[\frac{\partial s_r(x, p, x; q)}{\partial p_j} \right] + s_j(x, p, x; q) = 0, \qquad (37)$$

which holds for each of the j, r $\in$ R products this firm sells. Then if (following Nevo, 1998) we let

$B_{jr} = - \partial s_r / \partial p_j$
q_{jr} = 1 if the firm sells r and j, 0 otherwise
$W_{jr} = q_{jr} . B_{jr}$

we may rewrite the set of equations (37) in matrix notation as

$\mathbf{Q(p)} - W(\mathbf{p} - \mathbf{c}) = \mathbf{0}$

whence the "supply" equation may be written:

$\mathbf{p} = \mathbf{c} + W^{-1}\mathbf{Q(p)}$

From this, special cases akin to (36) may be developed.

Table 3. Summary III

	Logit	Nested Logit	Full Random Coefficients
Consumer Preferences	Homogeneous	Heterogeneous	Heterogenous
Advantages	Simplicity of Estimation	Reasonable patterns of substitution	Reasonable patterns of substitution
Drawbacks	Unreasonable patterns of substitution	Patterns of Substitution determined by a priori grouping	No analytical solution for the market share function

7. Concluding Remarks: Learning from the empirical results

The main shortcoming of the discrete choice models described is that they take as their starting point an equilibrium situation without considering product range decisions explaining this equilibrium. With this in mind, the aim of this section is to use the empirical results of recent empirical work to suggest the direction that future theoretical modelling on competition in markets where products are differentiated both vertically and horizontally should follow. For their relevance in the recent empirical work, we will focus our attention on the results obtained when analysing the car market. The car industry is a good example of a market in which variants of a given quality compete with horizontally differentiated variants of the same quality and with variants of another quality.

The most important application of discrete choice models of product differentiation to the car market is BLP, which has heterogeneous consumer preferences within a full random coefficients framework to interact consumer and product characteristics. Goldberg [1995], also for the US automobile market, uses nested logit models to capture consumer sequential choice characterising the car purchase decision. Her analysis focuses on the study of the effects of a voluntary export restraint and exchange-rate pass-through. Verboven [1996] uses a two-level nested logit model to study the causes of international price discrimination in the European automobile market. Cars are first grouped by segments and then by country of origin with the aim of explicitly modelling the national segmentation of the European automobile market[28]. Common results of these three papers relevant to our enquiry are:

1. own price elasticities are decreasing with quality
2. cross price elasticities are decreasing with quality
3. mark-ups are increasing in quality

[28] Other applications of discrete choice models of product differentiation are: Berry, Levinsohn and Pakes [1995b] study of the effect of a voluntary export restraint placed on exports of automobiles from Japan to the US (May,1991) over the US automobile industry and welfare; Berry, Grilli and Lopez de Silanes [1992] analysis of the possible effect on the automobile industry of a free trade agreement between Mexico and the US; Berry, Spiller and Carnall [1996] analysis of airline competition

One possible explanation for the observed quality-dependence of price elasticities and mark-ups is that in general customers' preference for diversity is more intense for the high than for the low quality products. In other words, whereas the purchasing decision of a consumer buying the low-quality variant will be mainly determined by the price, the horizontal characteristics of the variant play a relevant role in determining the purchasing decision of a consumer buying the high-quality variant. Hence, the possibilities of horizontal product differentiation are directly related to quality. It would be desirable for the theoretical models described in section 5 (in which variants are both vertically and horizontally differentiated) to incorporate these features with the aim of generating results matching with the observed facts.

Let us consider a model with two firms that produce two variants of a good: a high and a low quality variant. The qualities of the high and the low quality variants are the same in the two firms. The horizontal characteristic of the variant is set by the firm selling it. Possibly, the easiest way of couching the relationship between horizontal product differentiation and quality is to assume that variants are located in the four corners of a trapezoid. The low-quality variants would be located in the corners of the short side and the high-quality variants in the corners of the long side. Among the three papers reviewed in section 5, only Canoy and Peitz [1997] considers this possibility, and then only in a limiting case. Although, this assumption probably simplifies the analysis, it prevents an examination of predictions about cross price elasticities within the low quality variant.

A possible way of incorporating the idea that consumers' preference for diversity is increasing in consumers' willingness to pay for quality (q) is to make the parameter representing consumers' intensity of firm preference (z) depend on consumers' willingness to pay for quality, as in Katz [1988] and Canoy and Peitz [1997]. Katz considers a discrete distribution for q to establish a one to one relationship between q and z; i.e. if $q_2 > q_1$ then $z_2 > z_1$. This assumption contributes in his model to generating lower price elasticities and higher mark-ups for the high-quality variant. However, the main problem in Katz [1988] is the assumption that there are as many products as consumer types when actually there are fewer. Canoy and Peitz [1997] solve this problem by assuming a continuous distribution of consumer types and making z an increasing function of q. Gilbert and Matutes [1994] locate the low-quality variants at the same horizontal distance as the high-quality variants and assume a constant z that does not vary with q. The result of these assumptions is that in the symmetric equilibrium, mark-ups are independent of quality.

Therefore, in some ways, the most realistic of the models considered in section 5 is Canoy and Peitz [1997]. However, the introduction of the elements described above complicates the model and closed form solutions cannot be obtained; the authors have to use numerical methods to find a solution. In conclusion, recent theoretical modelling and empirical studies go in the same direction. The main problem in introducing more realistic assumptions into the theoretical models is that they become cumbersome so they can only be solved by numerical simulation.

8. Appendix

A: The monopolist price-quality decision

In this Appendix we solve analytically the efficient and multi-product monopolist price-quality problems for the case of a vertical product differentiation model as described by Katz [1984]

Let us assume that each consumer buys one unit of the variant and none of the others. His indirect utility function is given by:

$$v_j = q_j q_j - p_j \qquad \text{for } j = 0, 1, \ldots, n$$

where $q_j = 0 = p_j$ denotes the outside option.

As in Katz [1984], assume a discrete distribution for consumer willingness to pay for quality: there are only two types of consumers with willingness to pay for quality, q_1 and q_2, with $q_2 > q_1$. Let N_j denote the number of consumers with willingness to pay for quality qj. It is also assumed that firms offer only two products q_1 and q_2, with $q_2 > q_1$.

In order to capture the idea of marginal cost increasing with quality in the simplest manner[29]., the marginal cost of producing a good of quality q_j, is $C(q_j) = aq_j^2$.

Efficient Solution

The efficient quality for the q_j-consumer is the quality that maximises the total surplus from serving her, i.e. the quality that maximises the difference between her willingness to pay for quality, $q_j q$, and the marginal cost of providing this quality, $c(q) = aq^2$. ($TS = CS+PS$; $CS = q_j q - p_j; PS = p_j - c(q); TS = q_j q - c(q)$). Therefore, it is the quality that maximises consumer q_j surplus under marginal-cost pricing.

The efficient qualities in an industry with two products are given by the solution to

$$\max_{q_1, q_2} \sum_{j=1}^{2} \left[N_j \left(q_j q_j - aq_j^2 \right) \right]$$

These efficient qualities are $q_j^e = \frac{q_j}{2a}$ and the associated prices $p_j^e = \frac{q_j^2}{4a}$. Another interesting characteristic of the efficient solution is that consumer surplus is greater for the q_2-consumers ($CS_2 = \frac{1}{4}\frac{q_2^2}{a}$) than for the q_1-consumers ($CS_1 = \frac{1}{4}\frac{q_1^2}{a}$), i.e. consumer surplus increases with willingness to pay for quality.

Monopoly Solution

Let us consider the price and quality decisions of a monopolist producing the two quality variants. The monopolist would like to charge any consumer its reservation price but it cannot observe the consumer's willingness to pay for quality directly. In order to induce to the q_1-type consumer to buy the quality variant q_1 and to the q_2-type consumers to buy the quality variant q_2, the monopolist must set prices p_1, and p_2 satisfying the following conditions:

$$q_1 q_1 - p_1 = 0$$

[29] This quadratic specification of marginal costs is also used in Moorthy [1988].

$$q_2 q_2 - p_2 = q_2 q_1 - p_1$$

Therefore, the optimum quality choices for the multi-product monopolist (q_1^m, q_2^m) are given by the solution to

$$\max_{q_1, q_2} \Pi_m = N_1(p_1 - aq_1^2) + N_2(p_2 - aq_2^2)$$

$$s.t.$$

$$p_1 = q_1 q_1$$

$$p_2 = q_2(q_2 - q_1) + q_1 q_1$$

These optimum quality choices are $q_1^m = \frac{q_1}{2a} - \frac{N_2(q_2 - q_1)}{2N_1 a}$, and $q_2^m = \frac{q_2}{2a}$. Whereas the monopolist supplies the q_2-type consumers with their efficient quality, the quality supplied to the q_1-consumers is lower than the efficient level $\left(q_1^e = \frac{q_1}{2a}\right)$. Equally, it is possible to prove that when the multi-product monopolist produces *n* quality variants to serve *n* consumer types, it provides all consumer types except the highest one with qualities lower than would be efficient.

The prices that maximises profits of the multi-product monopolist serving qualities q_1^m and q_2^m are

$$p_1^m = \frac{q_1(N_1 q_1 + N_2(q_2 - q_1))}{2aN_1}$$

$$p_2^m = \frac{1}{2aN_1}\left[(N_1 + N_2)(q_2 - q_1)^2 - N_1 N_2 q_1\right]$$

The price-cost margins for each of the quality variants are

$$PCM_1 = p_1 - aq_1^2 = \frac{1}{4aN_1^2}\left[N_1^2 q_1^2 - N_2^2(q_2 - q_1)\right]$$

$$PCM_2 = p_2 - aq_2^2 = \frac{1}{4aN_1}\left[(N_1 + 2N_2)(q_2 - q_1)^2 + N_1 q_1^2\right]$$

and $d = PCM_2 - PCM_1 = \frac{1}{4aN_1^2}(q_1 - q_2)^2(N_1 + N_2)^2 > 0$. Therefore, the multi-product monopolist obtains a higher price-cost margin for the high quality variant than for the low quality one. This is a consequence of the fact that higher quality variants serve higher quality types, and higher quality types (as we saw above) bring with them higher consumer surplus (see Moorthy, 1988).

B. Finite market share in a large market (from section 2.1.2)

The utility for a consumer of type (*q*, *d*) from consuming the variant of quality (q^{+}+*D*) (the variant offered by the new entrant) is

$$V_{(q^{+}+\Delta)} = r - \left[c(q^{+} + \Delta) + m\right] + q\,(q^{+} + \Delta) - z\left|d - l\right|$$

and the highest utility that a consumer can obtain from consuming any of the other variants available in the market is

$$V_{q^{+}} = r - c(q^{+}) + q\,(q^{+})$$

Subtraction of $V_{q^{+}}$ from $V_{(q^{+}+\Delta)}$ results in the left-hand side of condition (A1)[30]. This implies that there exists a positive fraction *m* of consumers for whom

$$\left[q - \max c'(q)\right]\Delta - m - z(L) > 0 \qquad \text{(A1)}$$

strictly holds. This fraction *m* of consumers strictly prefers the variant offered by the new entrant at price *c(*q^{+}*+D)+m* to any other variant offered at marginal cost. Thus, the entrant sells at least *sm* units at a markup of *m*. Consider a value of *e* such that

$$e < \frac{mm}{re^{b\Delta}} \qquad \text{(A2)}$$

Conditional on market shares smaller than *e* and prices less or equal to *r*, the maximum revenue that firms can obtain is *Mer*. In equilibrium,

$$Mer \geq K(q^{+}) \qquad \text{(A3)}$$

With $p = r$ the revenue for the entrant is *Mmr*, and integration of (14) implies

$$K(q^+ + \Delta) \leq e^{b\Delta} K(q^+)$$

and so given that

$$\begin{aligned} Mmr - K(q^+ + \Delta) &\geq Mmr - e^{b\Delta} K(q^+) \\ &\geq Mmr - e^{b\Delta} Mer \qquad \text{from (A3)} \\ &\geq Mmr - Mmm \qquad \text{from (A2)} \end{aligned}$$

$p_e > 0$ and entry will occur.

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[30] Where $\Delta > 0$ is large enough and m is small enough.

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