Probabilistic Modelling of Walking Excitation for Building Floors

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Abstract

Slender floor structures are becoming increasingly prone to excessive vibration due to human-induced walking excitation. To prevent discomfort of floor occupants and/or malfunctioning of sensitive equipment, it is necessary to have a reliable means of estimating floor vibration in the design phase. For accurate estimation of the floor vibration, both reliable excitation and structural models are required. This paper concentrates on the former by evaluating the performance of the existing force models and suggesting their improvement. For this a force model adopted in the UK by the Concrete Society was applied to four nominally identical floors using their experimentally identified modal properties. After comparison with experimental data the drawbacks of the force model were identified after which an improved model of the walking-induced dynamic force, based on combination of two existing methodologies used separately for low- and high-frequency floors, is proposed. The improved model accounts for the inter-subject variability in the walking force with respect to the pacing frequency, step length and forcing magnitude. Moreover, it includes all relevant frequency components of the walking force into analysis, removing the need for classification of floors as low- or high-frequency. The proposed approach should help designers and building owners to make more informed decisions when evaluating vibration serviceability of floor structures.

Keywords: floors, vibration, walking force, uncertainty, probability, serviceability.

Subject headings:

- serviceability
- floors
- guidelines
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1 Introduction

The vibration serviceability assessment of floor structures under human-induced walking excitation has, for long time, been based on deterministic design procedures. Some well-known examples are the procedures featuring the design guidelines published by the UK Steel Construction Institute (Wyatt, 1989) and American Institute for Steel Construction (Murray et al., 1997). These two guidelines were important tools for civil engineers in addressing the vibration serviceability issue of floors over the last 15 years. Their implementation, however, revealed some drawbacks and stimulated new research. This, among other benefits, resulted in an updated version of the Steel Construction Institute guideline (Smith et al., 2007).

Apart from using simplified formulas for predicting vibration response, the two procedures from 1989 and 1997 were limited to specific type of construction, such as composite steel-concrete floors. Due to their deterministic nature (that yields a single response value which is then used in a binary pass-fail decision mode) these procedures could not account for variability in the dynamic force induced by different people. This variability is well known in biomechanics research (Giakas and Baltzopoulos, 1997; Masani et al., 2002), and only recently it has come into the focuss of researchers in the field of civil and structural engineering (Kerr, 1998; Brownjohn et al., 2004; Pachi and Ji, 2005; Sahnaci and Kasperski, 2005; Živanović, 2006; Pavić and Živanović, 2007). This recent research shows that the variability in the human-induced force moving across an area of interest (such as a corridor in a building) could be studied by looking at the variability of gait parameters, such as the forcing (i.e. walking) frequency, the amplitude of the dynamic force and the step length, to name just a few. The last parameter multiplied by the walking frequency defines the walking speed and therefore influences the duration of the walking excitation.

As recently as in 2005 Pavić and Willford wrote Appendix G (denoted as 'AppG' in further text) of Technical Report 43 published by the UK Concrete Society. This is probably the first vibration serviceability design guideline that takes into account some aspects of randomness in the walking force. Namely the random character of magnitude of the walking force is taken into account. Moreover, the guideline is based on basic dynamic principles and therefore is not limited to any specific type of floor construction. The AppG procedure has been originally developed and used for the last 10 years by ARUP consulting engineers (Willford et al., 2006).

The guideline proposes a checking procedure that depends on the natural frequency of the fundamental mode of vibration. If the fundamental mode is below 10 Hz the floor is called the low-frequency floor (LFF). In this case the floor is considered to be prone to resonant vibration under the walking-induced force. The force is modelled using its first four harmonics that are believed to have the potential to cause the resonance of one or more vibration modes. If the fundamental natural frequency is above 10 Hz then the floor is called the high-frequency floor (HFF). In this case the human walking typically causes a transient response to the heel impact in each step. This is the rationale behind modelling the footfall load in AppG as an impulse having sufficient energy to excite higher structural modes. This division

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into low- and high-frequency floors suggests that a floor has to behave in one of the two described ways when exposed to human-induced walking excitation. However, this scenario might not be appropriate for a floor that has strong responses in both the high- and low-frequency region.

The procedure defined in AppG was quickly recognised as an improvement on previous guidelines. Other comprehensive guidelines published by the, previously mentioned, UK Steel Construction Institute (Smith et al., 2007) and the UK Concrete Centre (Willford and Young, 2006) have adopted similar provisions in support of the approach.

In this paper AppG has been used for the vibration serviceability assessment of four nominally identical beam-and-block (B&B) floors which appear not to belong to either of the two floor types defined in AppG. The main aim is to check the applicability of the AppG procedure to this type of floors, identify its drawbacks and suggest an improved probabilistic force model. For this purpose five different procedures (called 'models' hereafter) for vibration response assessment are presented. The paper starts with a description of the structure to be used in the analysis. After this, measurements of the vertical floor vibration response to single person walking are described (Model 1). This is compared with the responses calculated when a set of continuous walking forces measured using an instrumented treadmill (Brownjohn et al., 2004) was applied to the structure (Model 2). Then the two forcing models (for LFFs and HFFs) defined in AppG are briefly explained and their ability to predict the measured responses is evaluated (Model 3 and Model 4). Finally, an improved probabilistic model (Model 5) is proposed, followed by conclusions. Response estimates according to the five models have been calculated using VSATs (Vibration Serviceability Assessment Tools) software, developed in-house for vibration serviceability assessment of slender structures.

The B&B structure chosen for this study was interesting not only because it had both low- and high-frequency components in its response but also because it consisted of four nominally identical structures, whose similarity in vibration behaviour was interesting to check. Furthermore, the vibration serviceability of this widely utilised type of floor system is under-researched.

2 Structural Description

The structure analysed is a part of a primary school consisting of four nominally identical classrooms (Figure 1). Approximate dimensions of these four open-plan floors are 9.0x7.0 m, as indicated in Figure 1. They are built as B&B system, consisting of 225 mm deep prestressed beams and 440x215x100 mm lightweight thermalite blocks (Figure 1). The beams span 7.0 m at 315 mm centers. The floor features a 75 mm screed layer with no reinforcement (Figure 1).

The modal properties (natural frequencies, modal damping ratios, modal masses and mode shapes) for the four floors were identified via FRF-based modal testing utilising multi-shaker excitation (Pavić et al., 2008). This was done for all modes with

natural frequencies below 30 Hz. In this frequency range eight modes of vibration were identified for Floor 1, 11 for Floors 2 and 4, and 10 for Floor 3. The modes mentioned are the main contributors to the measured response during all walking tests, suggesting that in the vibration analysis all modes above 30 Hz could be neglected. The modal properties of the modes below 30 Hz are shown in Table 1.

The table of modal properties shows that the four floors had different number of vibration modes in the frequency range considered, despite the fact they were constructed in nominally the same way and they were tested using the same testing procedures. This suggests that it is difficult to build identical civil engineering structures. Therefore some differences in vibration behaviour of the four floors could be expected, and will be quantified in this paper.

Table 1 also shows that the damping values identified for the lowest vibration modes were higher than those typical for other types of floor construction, such as concrete or steel-concrete composite floors. This was due to the non-monolithic construction and behaviour of the B&B floors investigated. It was also clear that the second mode was much more damped in Floor 1 than in the other three floors (Pavić et al., 2008). This is probably a consequence of difficulties in constructing identical structures. Other reasons for this difference could be traditional sensitivity of damping estimates in the process of FRF curve fitting and the fact that the fitting process assumes linear behaviour (of the typically mildly non-linear) structure. While the assumption of linearity could lead to some inconsistencies in the damping estimates, it is still the simplest and most reliable method for practicing engineers to use when modelling structures.

It is interesting to note that although the lowest modes were highly damped, the structure was still perceptibly responsive to walking excitation, as will be demonstrated later.

3 Model 1: Response Measurements due to Walking

The vertical acceleration response was measured due to walking at 1.4, 1.6, 1.8, 2.0 and 2.2 Hz, where the pacing rates were controlled by a metronome. Two well trained test subjects took part in the testing programme, one at a time. The measurement grid, as used for Floor 1, is shown in Figure 2a. The response was measured at 11 test points (TPs) that are circled in Figure 2a. The sampling frequency was 256 Hz. In all tests the test subjects walked along a line defined by test points 3-8-13-18-23 and back to TP3. Nominally identical measurements were repeated on the other three floors. The measurement grid was the same for all floors, in the sense that two-axis symmetry of the floors was taken into account. This means that, for example, the test point closest to the door in all setups was TP21, while the one furthest away from the door was TP5 (Figure 2a).

The acceleration records measured were weighted to take into account the variability in human perception of vibration with respect to the vibration frequency (Griffin,

1996). An example of the weighted acceleration record measured at TP18 for walking at step frequency of 1.8 Hz is shown in Figure 2b. Since occupants of these floors are normally either sitting, standing or walking, the weighting was performed according to weighting curve W_b (BSI, 1987). The weighting process attenuated the measured signal only slightly in this particular structure.

Figure 2b shows that the impulsive response to each heel strike is clearly visible, which means that the floor primarily responds to high-frequency components of the walking force. In comparison, the steady-state response due to low-frequency components of the walking force is difficult to see in Figure 2b due to the small contribution of these components to the total vibration response and movement of the load along the walking path. The same conclusion can be reached by observing that the Fourier amplitude spectrum of the vibration response is dominated by the frequency content above 10 Hz (Figure 2c).

After performing weighting, the running 1s RMS trend (Figure 2b) was calculated for each acceleration time history, with a 0.5 s window overlap. The maximum value of the 1s RMS trend was also extracted (Figure 2b). After this, a time varying 'running' response factor (or 'R factor') and its maximum value (maxR) were calculated by normalising the 1s RMS trend by the RMS perception threshold of 0.005 m/s^2 (ISO, 1989). The R factor is nowadays being increasingly used for vibration response assessment in buildings and footbridges (Pavić and Willford, 2005; Willford and Young, 2006; Smith et al., 2007), which was the reason to adopt it as the vibration measure in this study. An R factor of one suggests that the vibration level is just perceptible for an average person, while an R factor that is greater than one indicates that the vibration level is R times higher than the perceptible one.

The maximum measured vibration response occurred when the test subjects walked at the fastest adopted pacing rate of 2.2 Hz due to greater amount of energy released during faster, but still regular and comfortable, walking. The maximum measured R factors (from either test subject) across the measurement points are shown in Figure 3. For these nominally identical floors subjected to nominally identical walking loads it is interesting to note that the maximum response not only ranges between R=10.5 on Floor 2 and R=15.6 on Floor 3 but also occurs at different locations on the floors (Figure 3). Because of this, it is useful to investigate performance of the floors by extending the comparison to time varying R factor, instead of using only the absolute maximum value (as usually done in practice) which occurs only once and lasts for a very short period of time. This refinement of the R factor analysis was done by plotting the probability distribution for the running R factor and the corresponding cumulative probability function at all 11 monitored locations, for each floor individually. Results for TP18 only are presented here (Figure 4) because this was the point of maximum response for two of the four floors (Figure 3). Figure 4 shows that the probability distributions of running R factors across four floors (as well as the corresponding cumulative distribution functions) are quite similar. This indicates that relying on the maximum values only (e.g. R=10.5 for Floor 2 versus R=15.6 for Floor 3 shown in Figure 3) might not be the best way to compare results. Similar agreement in the results was obtained for the rest of the measurement points. Finally, results in Figure 4 for TP18 show that for more than 90% of the time, the

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R factor is less than eight on all four floors. This gives quite a different impression about the liveliness of, say, Floor 3 which had a maximum R factor, at the same point, of 15.6.

It is interesting to mention that one of the two test subjects who took part in the response measurements is believed to be a very efficient dynamic exciter. Namely, when taking part in measurement programmes on a number of different structures, he was often generating the strongest response of the floor tested. This observation also supports the view that a single value of maximum R factor can be quite misleading as it may be generated by an unrepresentable but highly efficient human exciter.

4 Model 2: Vibration Response to Measured Walking Forces

The fact that the experimental measurements were limited to two test subjects only motivated the response analysis of the four floors subject to a larger set of walking forces measured on an instrumented treadmill by Brownjohn et al. (2004). These 118 force time histories were generated by 10 test subjects, some being well trained and therefore quite effective dynamic exciters. The forces, one at a time, were applied to the measured modal model (Pavić et al., 2008) of the four floors (Table 1) to calculate the vibration response. This is a rare example of a publicly documented attempt to use the experimentally measured (as opposed to numerically calculated) modal properties in conjunction with the experimentally acquired continuous walking excitation force to estimate a floor vibration response. This use is very convenient since it does not require any mathematical modelling of the structure and walking force, which often introduces simplifications and errors that might not be warranted. It is also a good way of checking the reliability of the modal properties used in the response calculation via comparison of measured and simulated responses.

The measured force records were applied along the walking path shown in Figure 2a and calculated responses were weighted using W_b weighting function (BSI, 1987). The walking frequencies in the set ranged from 1.24 Hz to 2.52 Hz. Among these, 102 force records had pacing frequencies in the range 1.4-2.2 Hz corresponding to that employed in measurements described in the previous section. The contours of maxR factors calculated under these 102 forces are shown in Figure 5.

The comparison of Figure 3 with Figure 5 reveals that the absolute maximum R factors under measured forces (Model 2) are generally greater than those measured in-situ (Model 1), except for Floor 2. All maximum R factors in the measured responses occurred during the tests in which the test subject who was identified earlier as very efficient dynamic exciter took part. Therefore, this test subject managed to produce responses of the similar order as the most efficient test subject whose measured forces are used in Model 2. This kind of agreement is expected, having in mind the properties of the test subjects. This also gives confidence that the

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modal properties used in the calculation are reliable enough for response simulations. This is important as it means that the measured modal model could be used for verification of AppG approach. This is demonstrated in the next section.

5 Vibration Estimation based on AppG

Before using the AppG procedure for assessment of the vibration response, the basic theory behind the guideline will briefly be explained. More detailed explanation is available in papers by Willford et al. (2007a; 2007b).

5.1 Force Models in AppG

As previously explained, AppG divides floors into low- and high-frequency classes, depending on the natural frequency of the fundamental mode of vibration - the frequency separating the two types being 10 Hz. The force models used for vibration serviceability assessment of the two types of floors are fundamentally different: the model used for LFFs is composed of four force harmonics (Figure 6a) that should be applied to all vibration modes with natural frequency below 12 Hz. The model used for HFFs consists of an impulse in the time domain, i.e. it is a broad band excitation in the frequency domain (Figure 6b), that should be applied to all structural modes in the range from f_{n1} to $2f_{n1}$, where f_{n1} is the natural frequency of the fundamental vibration mode. Both force models are supposed to include 75% of the human population in terms of the forcing amplitude generated during walking at a predefined step frequency, meaning that the estimated vibration response at a given step frequency has a 25% chance of being exceeded. The two force models are to be applied to the floor as stationary forces at relevant points across the floor. These points are usually chosen to belong to a likely walking path. The amplitudes of the walking harmonics are functions of the pacing frequency of interest (chosen by an analyst for simulation purposes) while the magnitude of the step impulse is defined as a function of both the pacing frequency and the natural frequency of the relevant mode being excited.

For the LFFs the steady state response at point k in mode n to each of four harmonics ($h = 1, 2, 3, 4$) can be calculated as a complex function:

$$a_{k,n}(h) = \mu_{k,n}\mu_{j,n}\left(\frac{hf_p}{f_n}\right)^2 \frac{P_{j,h}(f_p)}{M_n} \frac{1}{\left[1-\left(\frac{hf_p}{f_n}\right)^2\right] + i\left[2\zeta_n\frac{hf_p}{f_n}\right]} \tag{1}$$

where $\mu_{k,n}$ and $\mu_{j,n}$ are amplitudes of the nth mode shape at point k at which the response is being calculated and at point j at which the harmonic excitation having amplitude $P_{j,h}(f_p)$ (related to harmonic h as defined in Table 2) is applied, respectively. f_p is the pacing frequency while f_n, M_n and ζ_n are the natural frequency, modal mass and damping ratio for the nth mode of vibration, respectively. The (complex) responses across all relevant vibration modes to a harmonic excitation

are summed up to get the response a_h $(h = 1, 2, 3, 4)$ to the hth harmonic. Then the magnitudes of these responses are summed in the square root of the sum of squares (SRSS) sense to get an estimated total peak acceleration response at point k:

$$a_k = \sqrt{|a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2}. \tag{2}$$

If a walking harmonic considered matches one of the natural frequencies of vibration modes, then the assumed resonant response could be reduced by a factor that takes into account the limited duration of the walking force and its movement along the walking path, which often do not allow the full resonant response to be developed.

For HFFs the total velocity response at point k to impulsive load at, say, point j across all relevant vibration modes should be calculated as:

$$v_k(t) = \sum_n \mu_{k,n}\mu_{j,n}\frac{I_j}{M_n}e^{-\zeta_n\omega_n t}\cos(\omega_{nd}t). \tag{3}$$

where I_j is the impulse applied at point j that depends on pacing f_p and natural frequency f_n and is equal to $54\frac{f_p^{1.43}}{f_n^{1.30}}$. Further, $\omega_{nd} = 2\pi f_n\sqrt{1-\zeta_n^2}$ while the rest of the variables are the same as in Equation 1. The final time-domain acceleration response could be obtained by differentiating the velocity signal.

AppG suggests using the modal properties (natural frequencies and mode shapes) of floors obtained from a detailed FEM while the damping ratios should be adopted based on experience with similar floors. In this study, however, the measured modal properties are used since they are generally considered to be more reliable than those obtained from an FEM. The accuracy of modal properties is crucial in this analysis since the aim is to check the appropriateness of the force models in AppG which could be done properly only without introducing additional uncertainties (of modal properties) into the analysis. As demonstrated in Section 4, the four experimentally estimated modal models featuring measured natural frequencies, modal damping ratios, mode shapes and modal masses, could be used for this purpose.

5.2 Categorisation of Four Beam-and-Block Floors

The four floors investigated all have the fundamental natural frequency below 10 Hz (Table 1). Therefore, strictly speaking, according to the AppG procedure, these floors are LFFs. However, it can be seen in Figure 2c that the high-frequency content above 10 Hz in the measured responses is far stronger than the low-frequency one. Based on this, each floor appears to be behaving more like a HFF. This is the reason to conduct the vibration serviceability assessment of the floors in both ways (despite AppG classification of the floors as low-frequency), as explained in the next two sections.

5.3 Model 3: AppG Model for LFFs

The force model representing the first four walking-induced harmonics was applied at five points belonging to the walking path (Figure 2a), one at a time. The pacing rates in simulations corresponded to those used in experiments: 1.4, 1.6, 1.8, 2.0 and 2.2 Hz. The response was then calculated at different locations across the floor. The maximum R factor at a response point was calculated for all five positions of the forcing function, and the maximum of these was chosen to represent the maximum response at this particular location. After summing contributions from the four forcing harmonics to get the peak acceleration response a_k (Equation 2), the maximum R factor at point k could be obtained as $\frac{0.707a_k}{0.005}$, where the multiplier 0.707 is used for conversion of the peak (assumed to be sinusoidal) to the RMS response and 0.005 [m/s^2] is the RMS perception threshold used in defining R factors. Note that this AppG procedure, for a chosen excitation frequency, results in an estimate of the maximum acceleration response only, i.e. it does not produce the response time history.

The absolute maximum responses calculated occurred when the walking frequency was set to 2.2 Hz, since the fourth harmonic of this walking force was closer to the natural frequency of the first mode of vibration (and it was of greater relative magnitude) than the same harmonic of forces at lower pacing rates. These maximum values are shown in 'Model 3' column of Table 3. It can be seen that when comparing the values obtained experimentally ('Model 1' in Table 3) and obtained from the measured force model ('Model 2' in Table 3), 'Model 3' response factors are significantly lower. The calculated maximum responses are approximately equal only to 22-56% of the maximums calculated from Model 2. Model 2 is used as a benchmark model since it is statistically more reliable than Model 1, and seems to incorporate better the ability of different test subjects to dynamically excite the floor by walking. Therefore, it is apparent that Model 3 is unable to represent 75% of responses induced by various test subjects on this particular structure. This means that treatment of the floors as low-frequency ones is problematic and could lead to a significant underestimation of the vibration response.

5.4 Model 4: AppG Model for HFFs

Similar to the force model for LFFs (Model 3), the model for HFFs was applied at five points belonging to the walking path (Figure 2a), one at a time. Again the five pacing rates used in experiments were taken into consideration. As specified by AppG, the modes considered were in the range $f_{n1} - 2f_{n1}$, where f_{n1} is the fundamental frequency of the floor. The time varying acceleration response at a response point was calculated for all five positions of the forcing impulse based on Equation 3. Then the maximum 1s RMS of the highest response was extracted and divided by 0.005 to get the maximum R factor for the response point considered. The absolute maximum responses occurred again when walking frequency was set to 2.2 Hz. These maximum values for each floor are shown in the column entitled 'Model 4' in Table 3. The maximum responses estimated are equal to 52%, 67%,

53% and 52% of the maximum response from Model 2 (Table 3) for each of the four floors. Therefore, Model 4 seems to be more representative than Model 3, but still only applies to approximately 50% of the population rather than the assumed 75% described in the the benchmark model.

5.5 Drawbacks of AppG

Based on results from Models 3 and 4, it could be concluded that both models underestimate the vibration response of the floors investigated. In this section the possible drawbacks of the models are summarised.

1. Division into low- and high-frequency floors seems to not always be warranted, as is the case for the four floors studied. This seems to be due to the presence of non-negligible frequency content in the measured response time history both below and above the 10 Hz division (Figure 2c). To illustrate the ability of the AppG to represent what was measured, a response of Floor 1 is calculated at TP18 to four harmonics used in Model 3 assuming walking at 1.8 Hz and random phases between the harmonics. The time domain response (using three modes below 12 Hz) and its spectrum are as in Figure 7a and 7b, while the response to impulse (taking into account first six modes) and its spectrum are as in Figure 7c and 7d. The qualitative comparison between the frequency content of the two responses with the one in Figure 2c reveals that the response to Model 4 is more similar to the experimental one. Also, Model 3 has little energy capable of exciting properly any of the three vibration modes present in the simulation. Instead, the four walking harmonics dominate the response (Figure 7b), which is quite opposite to what was measured (Figure 2c). On the other hand, the four harmonics are not present in the response to Model 4 (Figure 7d) despite the fact that they could be seen in Figure 2c, although overall the frequency content is closer to the measured one. However, this model would not be used in practice due to the fact that the fundamental natural frequency of the floor is less than 10 Hz. Therefore, the division between the two types of floors based on the fundamental frequency might be problematic and points out the need for developing a more universal forcing model that could be applied to any floor regardless of its fundamental frequency.

2. The two models defined in AppG cannot be applied at the same time and their responses summed up since Model 3 produces a single number only (i.e. the maximum acceleration response) while Model 4 results in the response time history. Therefore, the AppG model cannot be used on floors that respond strongly in modes both below and above 10 Hz.

3. Model 4 for HFFs uses a 1 s averaging time to produce an RMS estimate of the response to a single impulse (see Figure 7c). This neglects the additional steps made in 1s and therefore underestimates the calculated response. To rectify this a more appropriate averaging time interval of $\frac{1}{f_p}$ could be chosen, where f_p is the pacing frequency or alternatively, a 1s averaging time could be

used in conjunction with modelling a few successive steps. This would lead to an increase of the maximum R factors from aforementioned 52%, 67%, 53% and 52% to values closer to the expected 75%.

4. Model 4 for HFFs uses unweighted response time histories for calculation of R factors, neglecting differences in human response to vibrations of different frequencies (Griffin, 1996). Although this approach is conservative, the frequency weightings should be taken into account if the model is to be improved.

5. The distribution of pacing frequencies is not considered in AppG. This precludes the estimation of the probability of certain vibration response levels when all relevant walking frequencies (and not only one) characterising the pedestrian population are taken into account at the same time.

Based on this overview of possible issues with the AppG procedure, a more refined model that builds on the AppG current provision and aims to overcome some of its drawbacks is presented in the next section.

6 Model 5: Probabilistic Force Model

The force model suggested is probabilistic in nature and is a mixture of two models similar to those in AppG: one to be used for modes up to 12 Hz and the other to be used for modes above 10 Hz. However, the models are to be applied at the same time and their aggregate effect is to be taken into account for vibration serviceability assessment. For this purpose Model 3 from AppG is replaced by a forcing model that takes into account the continuous frequency content of the walking force up to the fifth harmonic. This model is described in detail elsewhere (Živanović et al., 2007) and only main features of it will be outlined in this section. Model 4 from AppG is improved to take into account inter-subject variability in the force induced and the effect of force travelling along the walking path.

The resulting model accounts for natural variability within the human population not only in the walking force magnitude, as is the case in AppG, but also in the step frequency and step length. This is done by modelling the three parameters via their probability density functions. Also, this model could be applied to any floor structure regardless of the frequency of its fundamental mode of vibration.

The output of the procedure is an estimate for the structural vibration response to a single person walking in a probabilistic sense. Once the model is verified against the possible acceleration responses induced by different people in experiments, the model can be extended to modelling multi-person traffic, under the condition that the arrival distribution of pedestrians and possible directions of walking are known, or can be assumed.

6.1 Modelling Low Frequency Content of the Force

Typical low-frequency content in the spectrum of a dynamic force induced by human walking consists of a number of harmonics and subharmonics (Figure 8a). The harmonics are the consequence of the fact that the walking force is approximately repeating itself with each step. Since the walker's two legs induce slightly different forces depending on one's style of walking, subharmonics will also appear in the spectrum (Sahnaci and Kasperski, 2005). The spectra for walking forces measured by Brownjohn et al. (2004) were overlaid for each harmonic and subharmonic separately. Then, the mean spectrum for each (sub)harmonic was fitted using a set of exponential functions (Živanović et al., 2007). The fit for the first harmonic only is presented in Figure 8b. Each fit is a function of the pacing frequency and DLF (i.e. the magnitude of the forcing harmonic divided by the pedestrian's weight). Assuming random phases between frequency lines in the forcing spectra (Živanović et al., 2007) the force could be converted into the time domain and applied along the walking path to get the modal force for each mode of interest. The modal force is then applied to the corresponding mode to get the vibration response in this mode. Using the mode superposition method the total vibration response could be calculated.

The model has been applied to the four floors investigated in this paper by simulating 500 pedestrians crossing the floor one at a time. A step frequency and a step length from appropriate normal distributions are associated with each pedestrian, as well as the magnitudes of each of the five harmonics (Table 4). The mean and the standard deviation for the walking frequency are taken as 1.87 Hz and 0.186 Hz, respectively, while the mean step length and the corresponding standard deviation are 0.71 m and 0.071 m (Živanović et al., 2007). This uniquely determines the force induced by each person, as well as its duration.

6.2 Modelling High Frequency Content of the Force

The high frequency content of the walking force is modelled using series of impulses associated with each step. The magnitude of the impulse is taken as normally distributed, with mean equal to the AppG value of $42\frac{f_p^{1.41}}{f_n^{1.30}}$ and a standard deviation equal to 40% of the mean value (Pavić and Willford, 2005), where f_p and f_n are the pacing and natural frequency, respectively. For the application of this model to all modes between 10 and 30 Hz the same 500 pedestrians (i.e. using previously generated walking frequencies and step lengths) are used, with added information about the magnitude of the impulse.

6.3 Verification of Model 5

A typical weighted vibration response at TP18 on Floor 1 under a pedestrian walking at a step frequency of 1.8 Hz is shown in Figure 9a with its spectrum shown in Figure 9b. It can be seen that both the time record and the frequency content resemble

those from measurements (Figure 2) quite well.

The maximum R factors across the four floors obtained when walking frequencies are in the range 1.4-2.2 Hz are shown in Table 3, in 'Model 5' column. These values are slightly higher than those from Model 2, due to better statistical representation of the pedestrians in Model 5. It is worth noting that the mean frequency of the walking forces in the two models was almost the same (1.87 Hz in Model 5 and 1.84 Hz in Model 2).

6.4 Vibration Estimation based on Model 5

Instead of looking only into results associated with a particular subset of people (i.e. those walking with step frequency between 1.4 and 2.2 Hz) a more reliable vibration serviceability check is done by taking into account the parameters of the whole human population using the floor, the most important parameter being the step frequency. If the previously assumed distributions for the pacing frequency and step length are considered as appropriate then the contour plots of the maximum R factor, with 5% exceedance probability, across the four floors are as shown in Figure 10. At the same time the cumulative distributions of the maxR factors as well as the running R factors, for say TP3, across the four floors are shown in Figure 11. It could be seen that the distributions of the running R factors across the four floors (Figure11b) are better correlated than the distributions for maxR factors per time history (Figure11a), suggesting again that running R factors might be a more informative way of response evaluation. Figure 11 also shows that the four floors of nominally the same construction exhibited similar, but not identical, vibration behaviour. This is probably due to slight differences in as-built boundary conditions, the degree of monolithic action between beams and blocks, and so on. These factors are likely causes of differences in modal properties and number of modes below 30 Hz reported in Table 1. These results are indicative of the order of magnitude of similarities/differences one could expect on nominally identical B&B floors.

Based on the information in Figure 11b it is possible to determine how long the R factor at TP3 spends above or below some predefined level. For example, it can be read from Figure 11b that the response factor is below 8 for at least 90% of the time for all of the floors. This contrasts the fact that considerably higher maximum R values of approximately 24, 13, 17 and 20 have been estimated on the floors.

Therefore, the cumulative distribution functions for R factors that are the output of the estimation procedure proposed could be used in conjunction with predefined acceptable vibration levels to estimate the proportion of time when the vibrations do not exceed these levels. However, it would be even more informative if the distribution of R factors (of the kind presented in Figure 4a) found for all relevant points (for example workstations) on the floor could be used in conjunction with a set of probabilistic human perception scales that would define the percentage of people unhappy with various vibration levels. This idea has been demonstrated in a paper related to footbridge vibration (Živanović and Pavić, 2007) but much more

research is needed before reliable probability-based scales can be defined.

It should be noted that the response estimation according to Model 5 was conducted for parameters characterising the population of adults, using the data published by Živanović et al. (2007). This was partly done in order to compare responses with those from Model 2, also generated by adults, and partly because of the lack of data characterising the population of children, who are the expected users of the floors investigated.

7 Conclusions

The paper demonstrates that a black-box usage of AppG guideline for classification of a floor depending on its fundamental frequency only, which in turn influences the choice of the procedure for vibration estimation, might be erroneous. This is particularly so for borderline floors that cannot easily be classified either as LFFs or HFFs. Also, some of the additional drawbacks of AppG, such as excessive averaging time for RMS calculation in the (single impulse) model for HFFs and the use of unweighted responses for vibration evaluation, were identified.

A probabilistic model that overcomes the problems identified in AppG was developed by extending and combining the AppG methodology and a low-frequency force model defined by the authors in the past (Živanović et al., 2007). The force model proposed removed the need to classify floors as low- and high-frequency ones and can be applied to any type of floor construction. The model takes into account the differences in the walking force induced by different people and, as a result, can estimate the probability distribution of vibration responses generated by a pedestrian population.

The paper further shows that taking into account only extreme values of vibration response might be misleading when assessing vibration serviceability of floors. Instead it seems more suitable to consider how much time the response spends above or below a predefined vibration limit. This is more appropriate way of judging the vibration serviceability state of a structure than using the binary pass-fail criterion, which better suits ultimate limit state checks.

Finally, the damping ratio of the floor structures studied was found to be larger than that typical for concrete and steel-concrete composite floors, probably due to the non-monolithic construction and behaviour of B&B floors. Interestingly, the four floors, although constructed in nominally identical way, exhibited some differences in the modal properties and vibration behaviour under walking-induced dynamic loading.

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8 Tables

Table 1: Natural frequency [Hz], modal damping [%] and modal mass [tonne] for measured modes of vibration (Pavić et al., 2008).

Mode	**Floor 1**			**Floor 2**			**Floor 3**			**Floor 4**		
#	f_n [Hz]	ζ_n [%]	m [t]	f_n [Hz]	ζ_n [%]	m [t]	f_n [Hz]	ζ_n [%]	m [t]	f_n [Hz]	ζ_n [%]	m [t]
1	8.2	11.6	7.9	8.3	12.3	24.4	8.7	17.5	8.2	8.5	7.6	5.5
2	10.2	10.3	3.8	10.2	3.9	13.6	10.3	4.7	11.6	10.9	2.6	14.0
3	11.2	3.7	7.4	10.8	3.4	17.7	11.1	7.1	4.0	11.6	2.7	42.6
4	12.4	2.6	34.0	11.7	7.9	9.1	11.2	1.8	56.6	12.5	4.2	18.6
5	14.0	2.9	20.5	12.3	3.4	9.6	12.4	3.7	8.3	13.2	2.4	20.8
6	16.9	4.4	5.9	14.3	4.7	14.7	13.7	4.6	15.8	14.3	3.8	18.4
7	21.8	3.6	13.2	15.4	5.3	7.2	14.6	2.5	3.9	15.4	2.9	28.5
8	25.7	3.0	46.2	18.4	4.6	10.0	16.4	2.8	12.4	16.2	3.0	10.0
9	above 30Hz			20.5	2.6	39.0	19.0	1.4	40.4	20.0	6.8	5.5
10	above 30Hz			23.5	8.5	5.5	23.6	7.1	4.5	26.0	4.5	8.9
11	above 30Hz			29.5	4.4	6.9	above 30Hz			28.0	4.9	2.1

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Table 2: Force amplitude for the first four harmonics of walking (Pavić and Willford, 2005).

Harmonic #	**Frequency range f_h [Hz]**	**Force amplitude**
1	1.0-2.8	$0.41W(f_h - 0.95)^*$
2	2.0-5.6	$0.0056W(f_h + 12.3)$
3	3.0-8.3	$0.0064W(f_h + 5.2)$
4	4.0-11.2	$0.0065W(f_h + 2.0)$

* capped at 392N.
$W = 700$N is the assumed mean weight of people.

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Table 3: MaxR factors according to different models (location of maxR is given in brackets).

Floor #	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**
1	12.4 (TP18)	16.4 (TP8)	7.0 (TP8)	8.5 (TP8)	19.3 (TP3)
2	10.5 (TP13)	10.4 (TP13)	2.4 (TP23)	7.0 (TP3)	12.6 (TP23)
3	15.6 (TP18)	19.2 (TP13)	4.2 (TP18)	10.2(TP23)	21.4 (TP13)
4	13.1 (TP3)	14.3 (TP3)	8.0 (TP8)	7.4 (TP3)	18.6 (TP3)

Table 4: Normal distribution of force amplitude for first five harmonics (Živanović et al., 2007).

Harmonic #	**Mean**	**Standard deviation**
1	$\mu = (-0.2649 f_p^3 + 1.3206 f_p^2 - +1.7597 f_p + 0.7613)W$	0.16μ
2	$0.07W$	$0.030W$
3	$0.05W$	$0.020W$
4	$0.05W$	$0.020W$
5	$0.03W$	$0.015W$

f_p is the step frequency.
$W = 750$N is the assumed mean weight of people.

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9 Figures

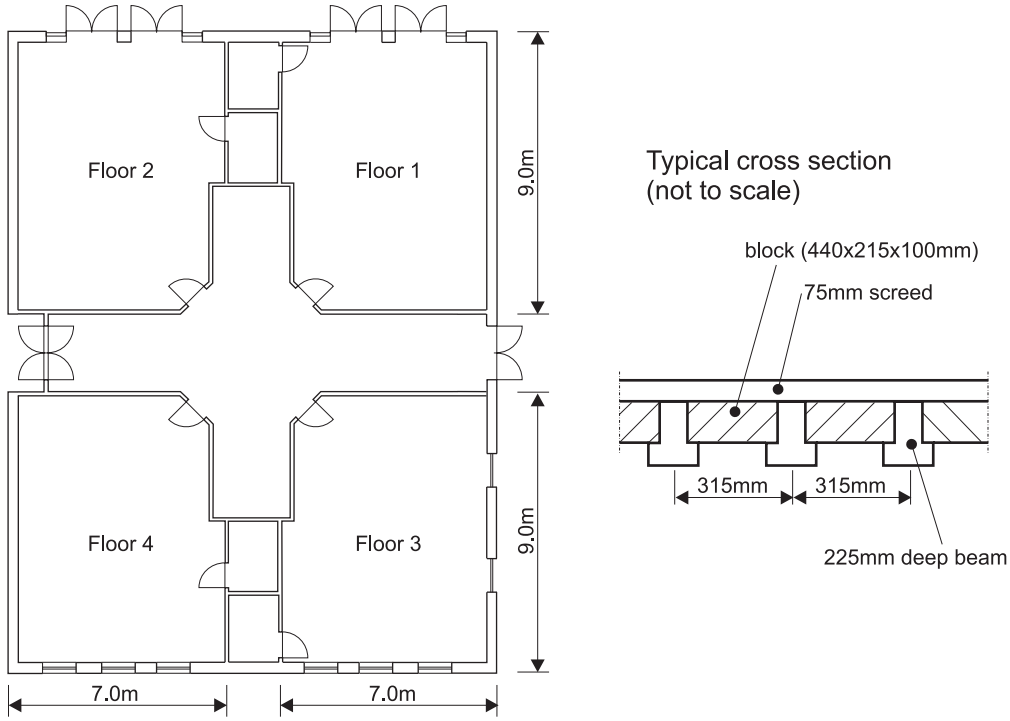

Figure 1: Plan of the structure (not to scale) with typical floor cross section.

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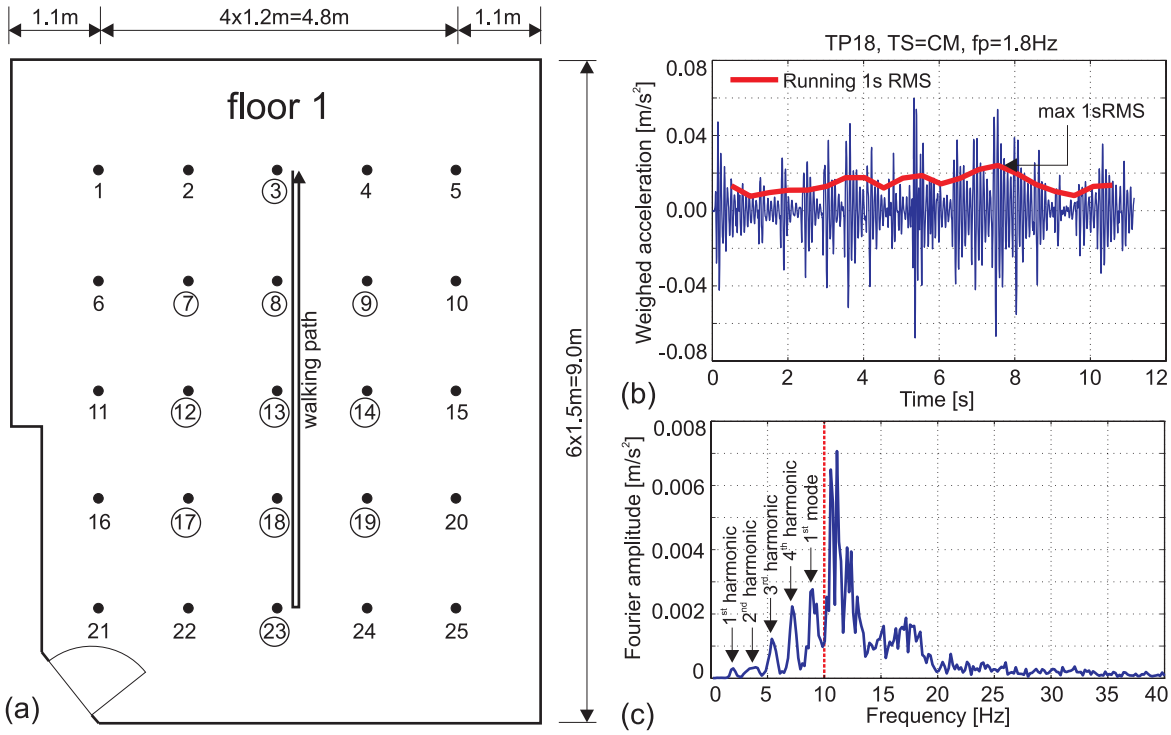

Figure 2: (a) Measurement grid with the walking path. (b) A weighted measured time history at TP18 of Floor 1 and its running 1s RMS trend. (c) The corresponding Fourier amplitude spectrum.

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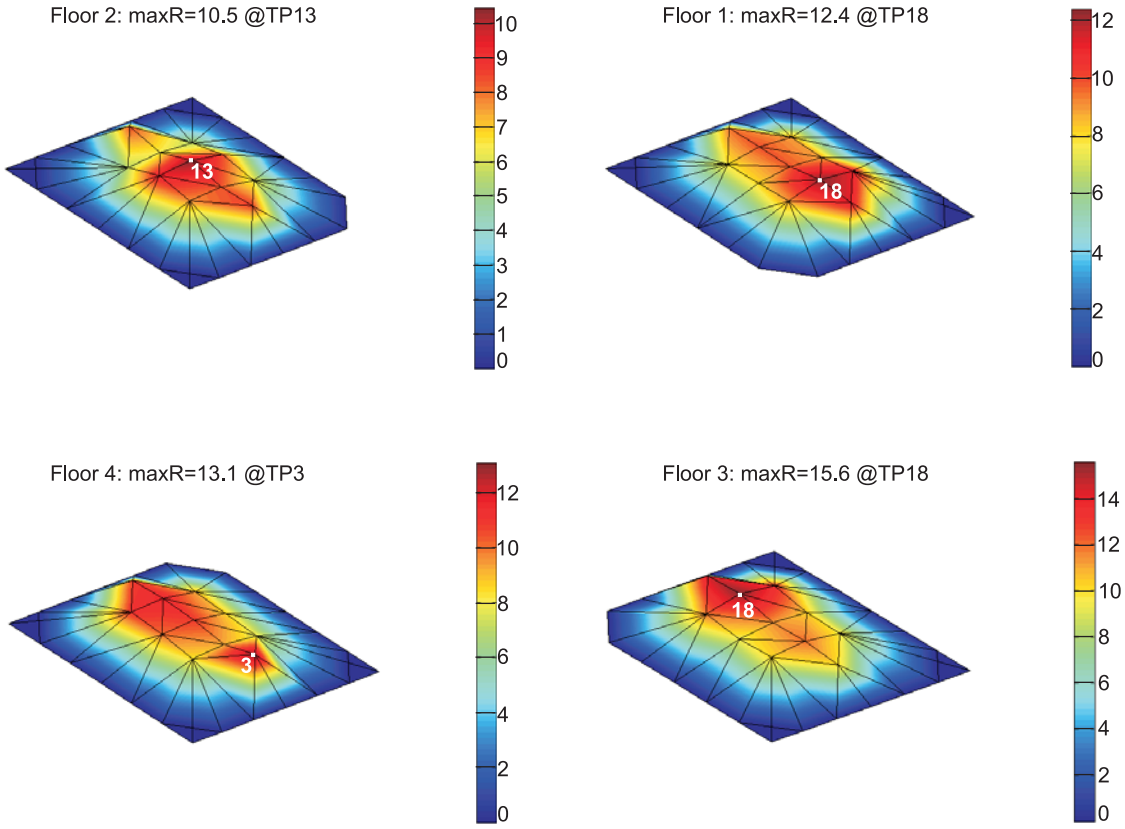

Figure 3: Maximum measured R factors across four floors.

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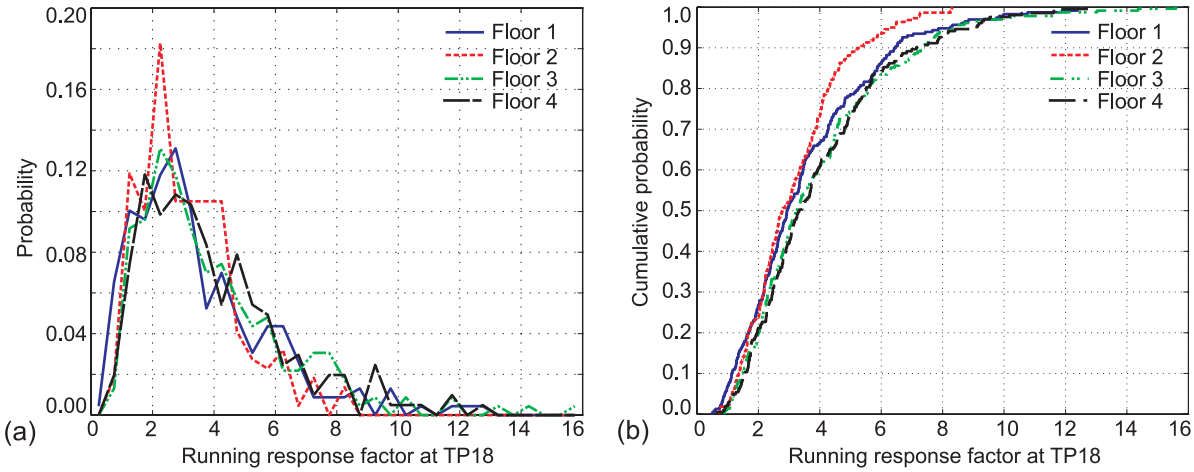

Figure 4: (a) The probability distribution of running R factors at TP18 taking into account all pacing rates and both test subjects. (b) The corresponding cumulative distribution function.

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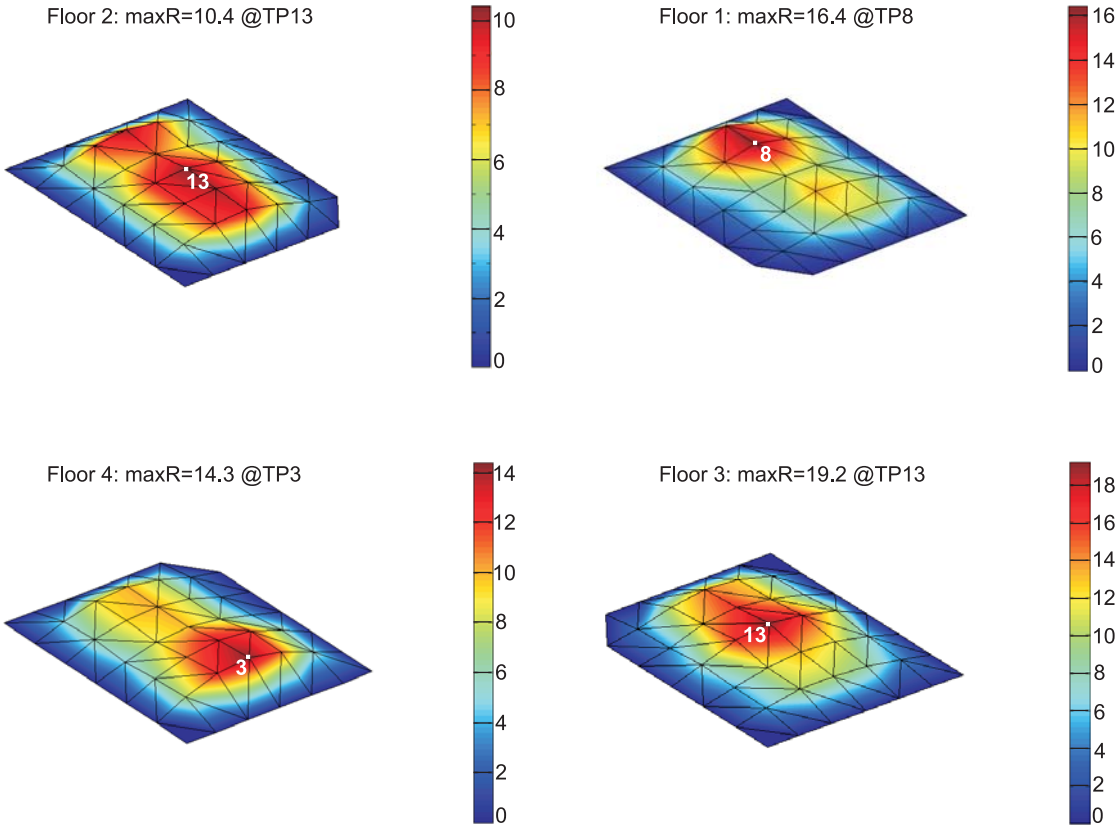

Figure 5: Maximum calculated R factors under measured walking forces with pacing frequencies in the range 1.4-2.2 Hz.

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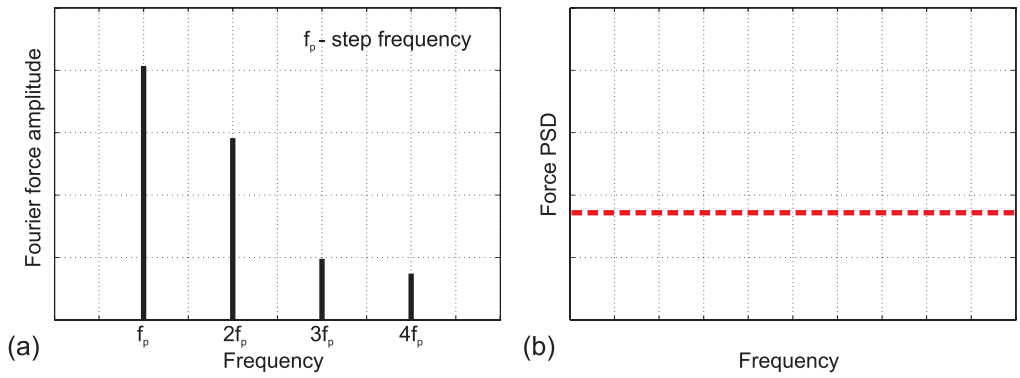

Figure 6: Force models for (a) low-frequency floor and (b) high-frequency floor.

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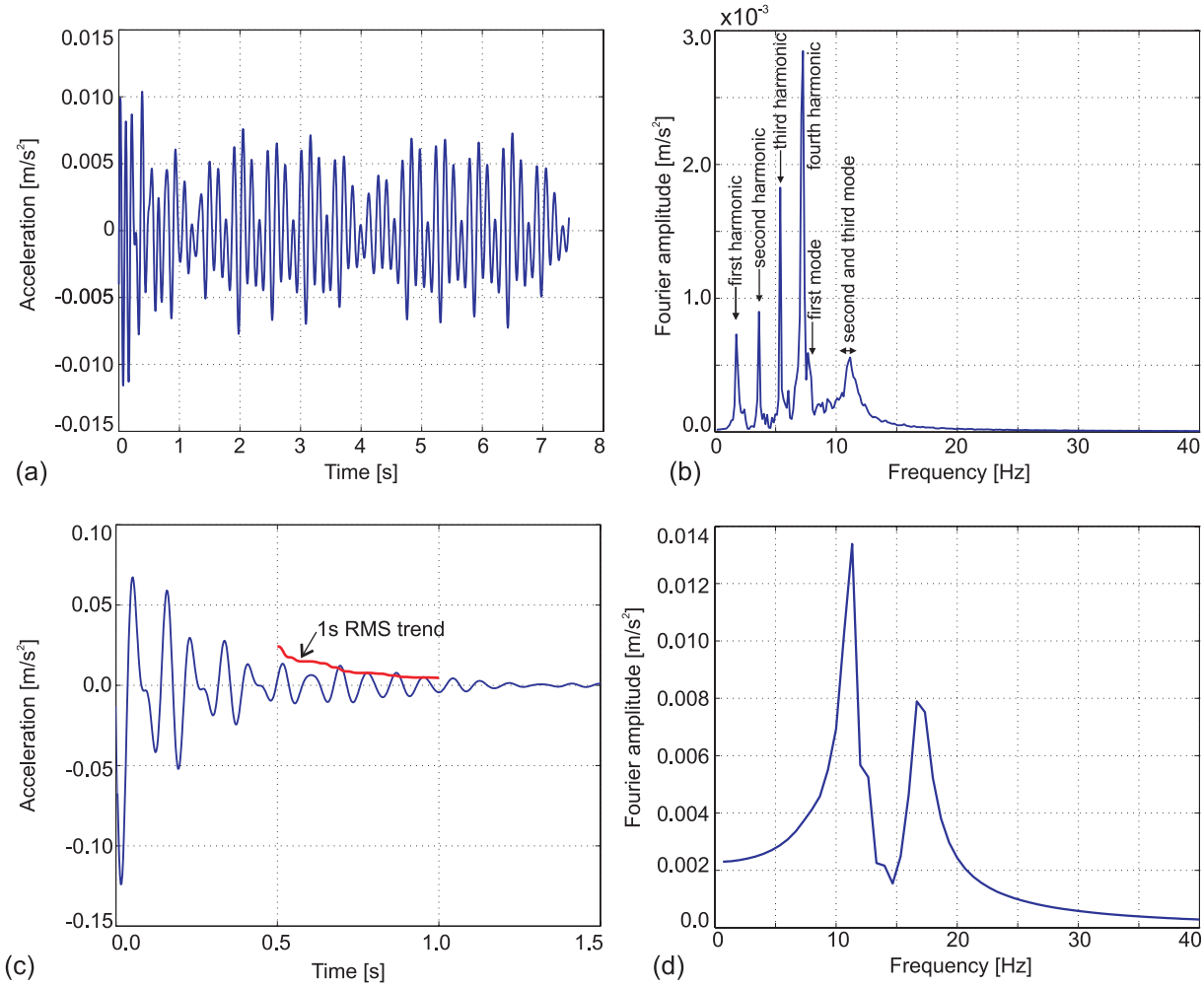

Figure 7: The response of Floor 1 at TP18 to Model 3 in (a) the time and (b) the frequency domain. The response to Model 4 in (c) the time and (d) the frequency domain. Assumed walking frequency is 1.8 Hz.

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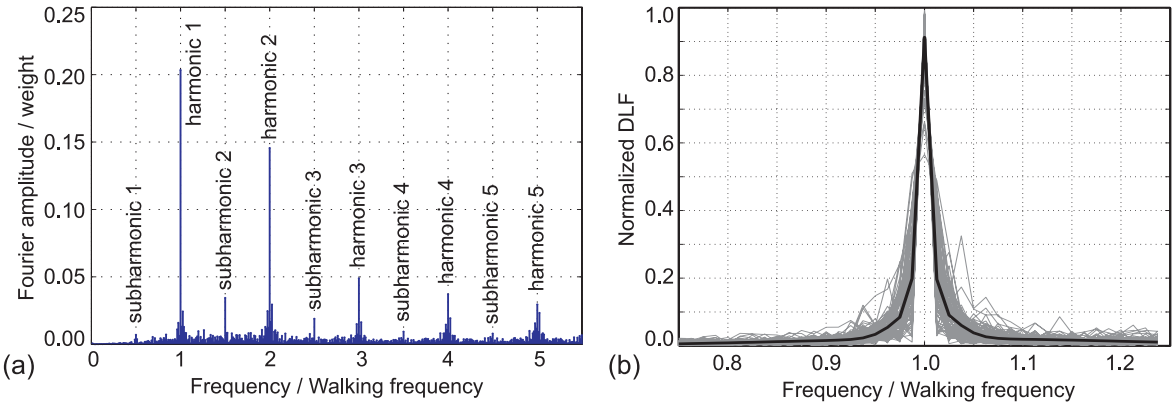

Figure 8: (a) Spectrum of a measured walking force. (b) Fitting function for the first harmonic.

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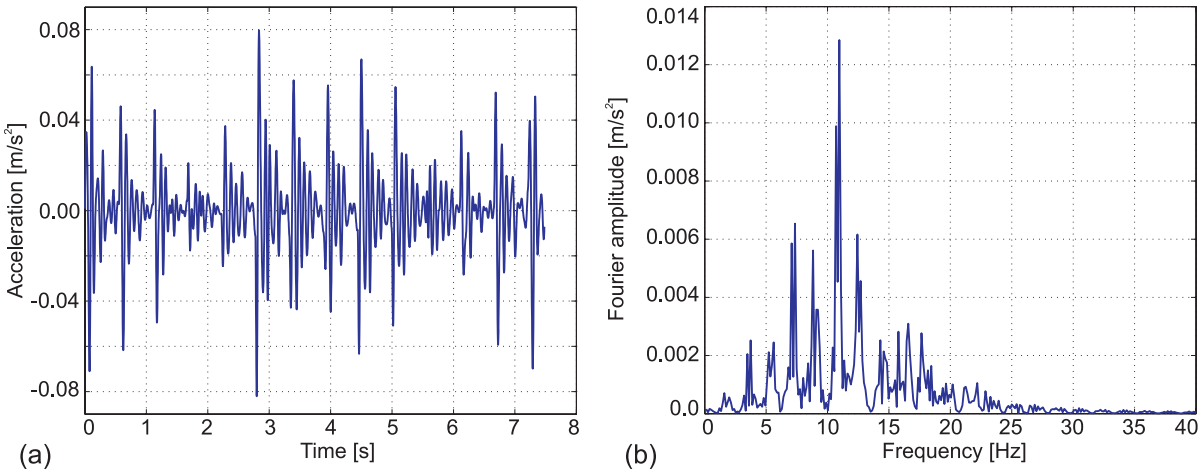

Figure 9: A response at TP18 of Floor 1 to walking at 1.8 Hz calculated using Model 5 in (a) the time domain and (b) the frequency domain.

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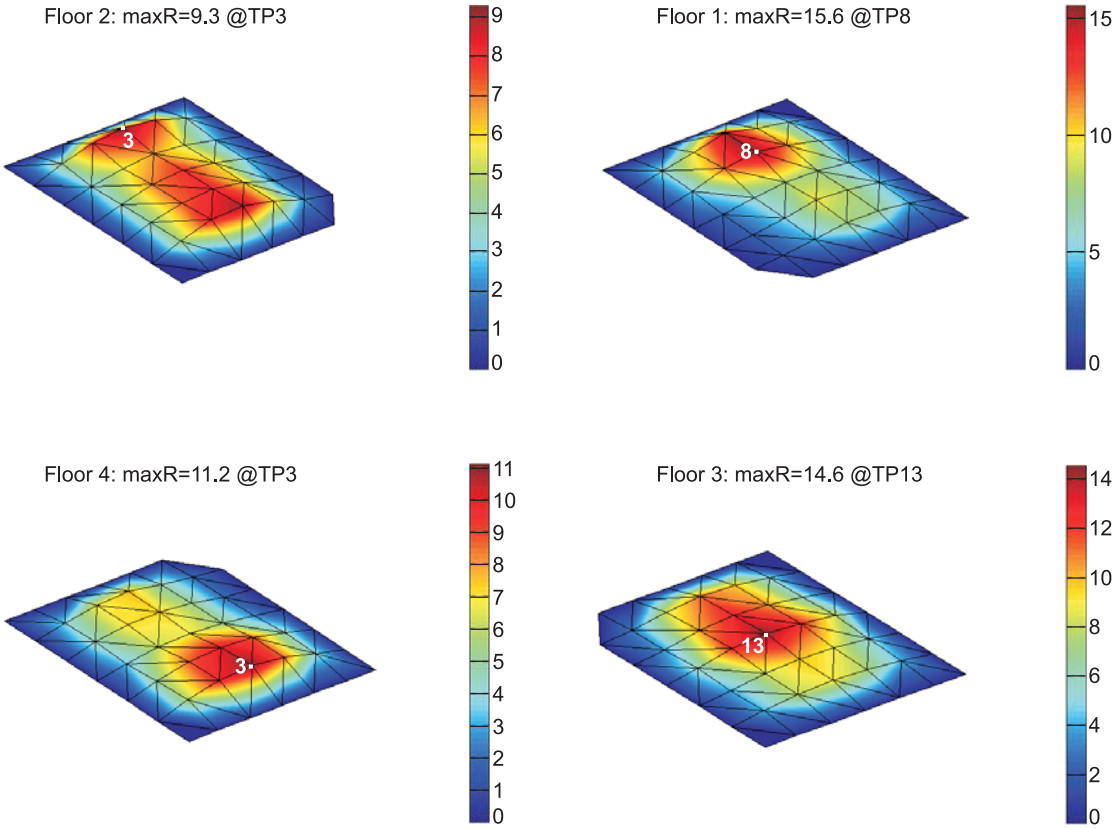

Figure 10: MaxR factor having 5% chance of being exceeded.

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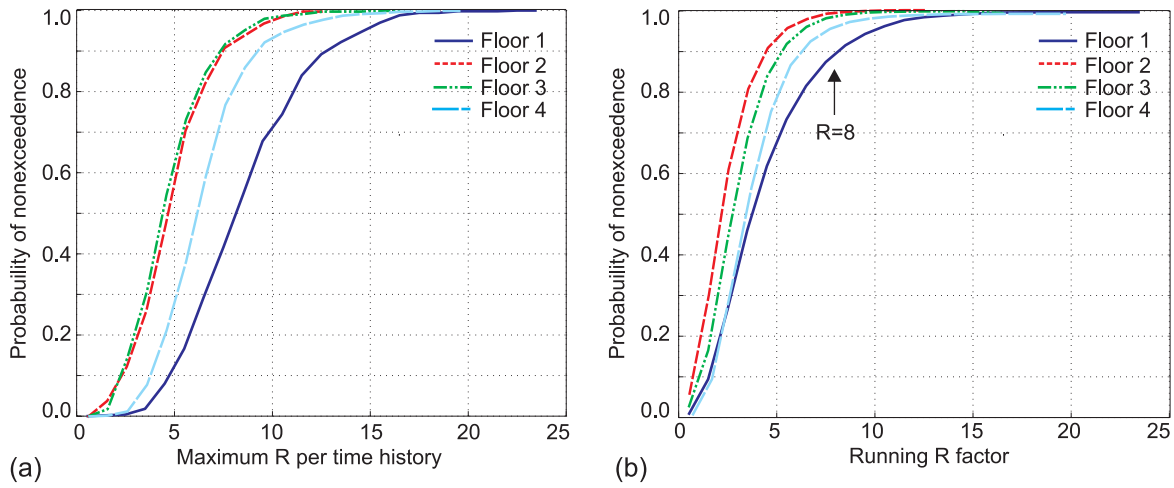

Figure 11: Probability of nonexceedance of (a) maxR factor and (b) running R factor at TP3 for four floors.