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A Method to Measure $\cos(2\beta)$ Using Time-Dependent Dalitz Plot Analysis of $B^0 \rightarrow D_{CP}\pi^+\pi^-$

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We present a feasibility study of a previously outlined method to measure both the sine and the cosine of twice the CKM Unitarity Triangle angle β using a time-dependent Dalitz plot analysis of $B^0 \rightarrow D\pi^+\pi^-$ decays, where the neutral D meson is reconstructed in decays to CP eigenstates. We show that this method can be used at the B factories to make a measurement of $\cos(2\beta)$ that is competitive with, or more precise than, other techniques using different quark-level transitions, while $\sin(2\beta)$ can be measured to a better precision than any existing measurement using $b \rightarrow c\bar{u}d$ transitions. Furthermore, this technique has great potential to be employed at LHCb.

I. INTRODUCTION

Precise determinations of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) [1, 2] matrix are important to check the consistency of the Standard Model and to search for new physics. Measurements of the $B^0 - \bar{B}^0$ mixing phase determined from $B^0 \rightarrow J/\psi K^0$ (and similar) decays at the B factories give $\sin(2\beta) = 0.680 \pm 0.025$ [3, 4, 5], where β is one of the angles of the CKM Unitarity Triangle (for an introduction, see, for example [6, 7, 8]). These results confirm the Kobayashi-Maskawa mechanism as the origin of CP violation within the Standard Model. Nonetheless, the effects of “new physics” are expected to be seen as non-negligible corrections to the Standard Model, particularly if new particles are present at energies as low as the TeV scale. The purpose of flavour physics in the LHC era, as discussed in several recent reviews [9, 10, 11, 12, 13, 14, 15], is to constrain the new physics parameter space and – once observed – to measure the couplings of the new physics particles.

An important part of this programme is the precise measurement of the angles of the CKM Unitarity Triangle using different quark-level transitions. For example, comparisons of the value of $\sin(2\beta)$ measured in $b \rightarrow c\bar{c}s$ transitions (such as $B^0 \rightarrow J/\psi K^0$) with those obtained in penguin (loop) dominated $b \rightarrow s\bar{s}s$ transitions (such as $B^0 \rightarrow \phi K^0$) or $b \rightarrow c\bar{u}d$ transitions (*e.g.* $B^0 \rightarrow D\pi^0$) can probe for new physics effects [16, 17, 18, 19]. In order to achieve the best possible precision, as well as to remove ambiguities in the results, it is important to use channels that can measure $\cos(2\beta)$ as well as $\sin(2\beta)$. This can be achieved, in general, by using any final state that contains interfering amplitudes.

In this paper, we present the results of a study of the feasibility of measuring $\cos(2\beta)$ from $B^0 \rightarrow D\pi^+\pi^-$ decays, where the neutral D meson is reconstructed in decays to CP eigenstates. This method has previously been described in outline [20]. In this work, we significantly extend the earlier study, taking advantage of results from the B factories that provide information on the composition of the Dalitz plot [21, 22]. We estimate the sensitivity that can be achieved with the BaBar dataset, and comment on the potential of the LHCb experiment.

The paper is organised as follows. In Section II we briefly review alternative approaches to measure $\cos(2\beta)$. The main body of the paper is in Sections III and IV, in which we give a description of the method and discuss the results of our feasibility study. Finally, we present our conclusions.

II. REVIEW

The weak phase 2β can be probed through mixing-induced CP violation effects in B decays mediated by a number of different quark-level transitions. This enables a powerful test of the Standard Model, since models of physics beyond the Standard Model that introduce new particles at the TeV scale can produce effects that differ between different decay modes. In this section we briefly review the various techniques that have been suggested, in order to illustrate the need for methods that can be used to provide precise measurements at LHCb and future experiments.

$b \rightarrow c\bar{c}s$

Since the most experimentally precise measurement of $\sin(2\beta)$ is made using $B^0 \rightarrow J/\psi K^0$ decays, one might expect that a similar channel can be used to extract also $\cos(2\beta)$. Indeed, methods based on $B^0 \rightarrow J/\psi K^0$ decays using the subsequent evolution of the neutral kaon have been proposed [23, 24, 25], but appear experimentally challenging.

The first experimental measurement of $\cos(2\beta)$ used the decay mode $B^0 \rightarrow J/\psi K^*(892)$, with $K^*(892) \rightarrow K_S^0 \pi^0$ [26]. This method relies on the interference between CP -even and CP -odd helicity states [27, 28, 29]. A residual ambiguity due to the unknown sign of the strong phase difference can be resolved using input from $K\pi$ scattering [30] or from theory [31]. The most recent measurements [26, 32] prefer $\cos(2\beta) > 0$ but with large uncertainty. It will be difficult for LHCb to improve on these measurements since it is necessary to measure accurately the momentum of the neutral pion in the final state.

Other methods using doubly-charmed final states $D^{(*)+} D^{(*)-} K_S$ have been proposed. In principle, time-dependent amplitude analyses of these states would yield information on the weak phase (see the discussion in the next section). This is simplest for the decay $B^0 \rightarrow D^+ D^- K_S$ [20, 33], where all final state particles are pseudoscalars so that the Dalitz plot gives a complete description of the phase space. The B factory statistics have not yet enabled this analysis, and due to the high multiplicity of charged tracks in the final state it may be difficult to study at LHCb. If either or both charmed mesons are reconstructed as D^* there are additional degrees of freedom that further complicate the amplitude analysis. Fortunately, by integrating over regions of the $B^0 \rightarrow D^{*+} D^{*-} K_S$ phase space, some simplifications are possible, but at the price of considerable theoretical uncertainty [34]. With some input from theory, the current experimental measurements [35, 36] prefer $\cos(2\beta) > 0$.

$b \rightarrow c\bar{u}d$

The possibility to measure $\cos(2\beta)$ from $B^0 \rightarrow D_{CP}\pi^+\pi^-$ decays [20] is the subject of this paper. A similar analysis using $B^0 \rightarrow D_{CP}^*\pi^+\pi^-$, with $D_{\pm}^* \rightarrow D_{\pm}\pi^0$ or $D_{\pm}^* \rightarrow D_{\mp}\gamma$ [37] is possible in principle, but requires a more involved amplitude analysis including the D^* decay angles.

Another method to measure $\cos(2\beta)$ in $b \rightarrow c\bar{u}d$ transitions, using $B^0 \rightarrow Dh^0$ (with h^0 being a light neutral meson such as a π^0) with time-dependent Dalitz plot analysis of the subsequent neutral D meson decay to $K_S^0\pi^+\pi^-$ has been proposed [38] and implemented [39, 40]. To analyze the latter decay channel, the B decay vertex position must be determined from the pion tracks that originate from the D decay. The lack of primary particles from the B vertex, together with the necessity to reconstruct the neutral meson, will make this analysis difficult to carry out in the hadronic environment of LHCb.

Another interesting approach could be to carry out a simultaneous analysis of the B and D meson decay Dalitz plots in the $B^0 \rightarrow D\pi^+\pi^-$, $D \rightarrow K_S^0\pi^+\pi^-$ decay chain, thereby combining the method of Ref. [20] and this paper with that of Ref. [37]. Large statistics could be available for such an analysis, since the relevant branching fractions for both B and D decays are reasonably high. Although the complete four-dimensional amplitude analysis would be quite complicated, it may be possible to select regions of the phase space where simplifications are possible (for example, selecting the $D\rho^0$ dominated region of the B decay phase space). In order to reach high precision, however, a complete modelling of the amplitude is likely to be necessary.

$b \rightarrow c\bar{c}d$

Several possibilities to measure $\cos(2\beta)$ from $b \rightarrow c\bar{c}d$ transitions have been discussed in the literature, but all are experimentally challenging and none have yet been implemented. For example, the interference between $B^0 \rightarrow D^{*+}D^-$ and $B^0 \rightarrow D^+D^{*-}$ decays could be measured in a time-dependent Dalitz plot analysis of $B^0 \rightarrow D^+D^-\pi^0$ [20, 41], or the interference between CP -even and CP -odd helicity states could be probed in a time-dependent analysis of $B^0 \rightarrow D^{*+}D^{*-}$ [27, 29]. Both these techniques require the reconstruction of a high multiplicity final state as well as precise understanding of potential misreconstruction effects. Another interesting possibility is provided by the vector-vector decay $B^0 \rightarrow J/\psi\rho^0$ [29], though due to the large natural width of the ρ , a time-dependent analysis of $B^0 \rightarrow J/\psi\pi^+\pi^-$ may be necessary to incorporate correctly all interference effects [42]. This would be a highly

challenging analysis, though it could potentially be studied at LHCb.

Another interesting possibility that has been proposed involves a study of helicity amplitudes in the dibaryon decay $B^0 \rightarrow \Lambda_c \bar{\Lambda}_c$ [28]. However, this decay has not yet been observed [43], meaning that it will be difficult to accrue sufficient statistics for a precise analysis.

$b \rightarrow q\bar{q}s$

Since measurements of mixing-induced CP violation phenomena in $b \rightarrow q\bar{q}s$ transitions provide one of the most interesting approaches to search for effects of physics beyond the Standard Model [16, 17, 18, 19], it is clearly important to be able to probe $\cos(2\beta)$ in these transitions. Recently, this has been achieved using time-dependent Dalitz plot analyses of $B^0 \rightarrow K_S^0 K^+ K^-$ (containing contributions from ϕK_S^0 and $f_0 K_S^0$ among others) [44] and $B^0 \rightarrow K_S^0 \pi^+ \pi^-$ (containing contributions from $\rho^0 K_S^0$ and $f_0 K_S^0$ among others) [45, 46]. The approach that has been adopted is to obtain values of β , rather than $\cos(2\beta)$ and $\sin(2\beta)$ separately. The current results indicate that values of β closer to the Standard Model solution than those with $\cos(2\beta) < 0$ are preferred, but much more precise results are needed. Although LHCb is expected to make some improvement on the current measurements, these channels provide one of the motivations for a very high-luminosity electron-positron flavour factory [12, 13, 14, 15].

In Table I we summarise the current status of experimental measurements of $\cos(2\beta)$. We have not included results from time-dependent Dalitz plot analyses of $B^0 \rightarrow K_S^0 K^+ K^-$ [44] and $B^0 \rightarrow K_S^0 \pi^+ \pi^-$ [45, 46] where the results have been presented in a different format. Discounting the results in $B^0 \rightarrow D^{*+} D^{*-} K_S^0$, which suffer from a large theoretical uncertainty, we see that no measurement has a precision better than about 0.50, and therefore additional approaches are very welcome.

In Table II we summarise the existing measurements of $\sin(2\beta)$ in $b \rightarrow c\bar{u}d$ transitions. Additional approaches that can improve the precision beyond that achieved in $B^0 \rightarrow D_{CP}^{(*)} h^0$ would be very helpful to test the Standard Model prediction that the value of $\sin(2\beta)$ measured should be the same as that obtained from $B^0 \rightarrow J/\psi K^0$ decays.

TABLE I: Summary of measurements of $\cos(2\beta)$. For more details and world averages, see Ref. [5]. For all quoted results, the first uncertainty is statistical and the second systematic. Note that the parameter measured in analyses of $B^0 \rightarrow D^{*+}D^{*-}K_S^0$ marked by (*) is not $\cos(2\beta)$ but $(2J_{s2}/J_0)\cos(2\beta)$, where the combination of hadronic parameters $(2J_{s2}/J_0)$ is expected to be positive. In the BaBar results on $B^0 \rightarrow Dh^0$ with $D \rightarrow K_S^0\pi^+\pi^-$, the third uncertainty is due to the D decay model; Belle include these effects together with other systematic uncertainties. The symbol h^0 denotes a light neutral meson such as a π^0 .

Experiment	$\cos(2\beta)$
$B^0 \rightarrow J/\psi K^*(892)$ with $K^*(892) \rightarrow K_S^0\pi^0$	
BaBar [26]	$3.32^{+0.76}_{-0.96} \pm 0.27$
Belle [32]	$0.56 \pm 0.79 \pm 0.11$
$B^0 \rightarrow D^{*+}D^{*-}K_S^0$	
BaBar [35] (*)	$0.38 \pm 0.24 \pm 0.05$
Belle [36] (*)	$-0.23^{+0.43}_{-0.41} \pm 0.13$
$B^0 \rightarrow Dh^0$ with $D \rightarrow K_S^0\pi^+\pi^-$	
BaBar [40]	$0.42 \pm 0.49 \pm 0.09 \pm 0.13$
Belle [39]	$1.87^{+0.40}_{-0.53} \begin{smallmatrix} +0.22 \\ -0.32 \end{smallmatrix}$

TABLE II: Summary of measurements of $\sin(2\beta)$ in $b \rightarrow \bar{c}ud$ transitions. For more details and world averages, see Ref. [5]. For all quoted results, the first uncertainty is statistical and the second systematic. In the BaBar results on $B^0 \rightarrow Dh^0$ with $D \rightarrow K_S^0\pi^+\pi^-$, the third uncertainty is due to the D decay model; Belle include these effects together with other systematic uncertainties.

Experiment	$\sin(2\beta)$
$B^0 \rightarrow D_{CP}^{(*)}h^0$	
BaBar [47]	$0.56 \pm 0.23 \pm 0.05$
$B^0 \rightarrow Dh^0$ with $D \rightarrow K_S^0\pi^+\pi^-$	
BaBar [40]	$0.29 \pm 0.34 \pm 0.03 \pm 0.05$
Belle [39]	$0.78 \pm 0.44 \pm 0.22$

III. METHOD

Detailed descriptions of the method to extract CP violating phases from $B \rightarrow Dh^0$ (where h^0 is a light neutral meson such as π^0 , η , *etc.*) can be found elsewhere [38, 48, 49]. (Similar discussions where h^0 is a K_S^0 meson, relevant for the extraction of the angle γ , can also be found in the literature [50, 51].) Here we provide only an outline of the method. To simplify the discussion, we initially treat B decays to charm as being flavour-specific – *ie.* we neglect $b \rightarrow u\bar{c}d$ amplitudes, that are suppressed by a factor of approximately 0.02 compared to the favoured $b \rightarrow c\bar{u}d$ amplitudes [52]. We consider the effects of suppressed amplitudes at a later stage.

Consider the amplitude for a B^0 decay to a point in the $\bar{D}^0\pi^+\pi^-$ Dalitz plot, described by the coordinates $m_+^2 \equiv m^2(D\pi^+)$ and $m_-^2 \equiv m^2(D\pi^-)$. We define the amplitude as

$$A(B^0 \rightarrow \bar{D}^0\pi^+\pi^-) \equiv A(m_+^2, m_-^2) = \sum_i c_i F_i(m_+^2, m_-^2), \quad (1)$$

where we have used the isobar formalism to express the amplitude as a sum of contributions from interfering resonances. The complex coefficients c_i describe the magnitude and phase of the contribution from each resonance i , while the strong dynamics (lineshapes and angular distributions) are contained within the F_i functions. The sum over i will include excited D mesons (*eg.* D_2^{*-}) and $\pi^+\pi^-$ resonances (*eg.* ρ^0). Feynman diagrams representing these amplitudes are shown in Fig. 1.

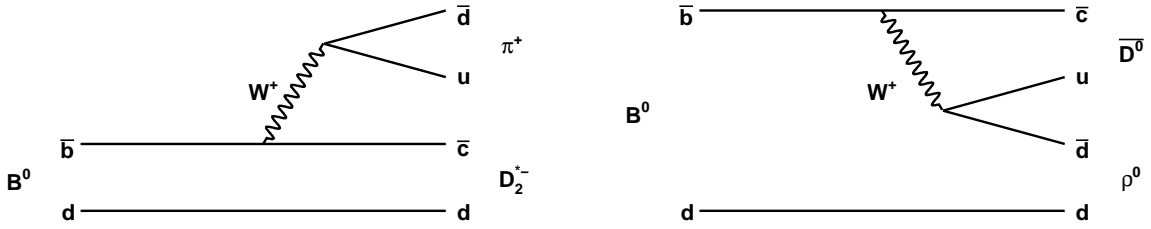


FIG. 1: Feynman diagrams for (left) a colour-favoured $\bar{b} \rightarrow \bar{c}u\bar{d}$ transition resulting in a $B^0 \rightarrow D_2^{*-}\pi^+$ decay, and (right) a colour-suppressed $\bar{b} \rightarrow \bar{c}u\bar{d}$ transition resulting in a $B^0 \rightarrow \bar{D}^0\rho^0$ decay.

The amplitude for the \bar{B}^0 decay to $D^0\pi^+\pi^-$ can be written similarly,

$$A(\bar{B}^0 \rightarrow D^0\pi^+\pi^-) \equiv \bar{A}(m_+^2, m_-^2) = \sum_i \bar{c}_i \bar{F}_i(m_+^2, m_-^2), \quad (2)$$

where the index i is understood to run over the same set of resonances, except that the charge of the excited D mesons will be opposite compared to the B^0 decay case. Since F contains strong dynamics only, we can assert $F_i(m_+^2, m_-^2) = \bar{F}_i(m_-^2, m_+^2)$ and, neglecting direct CP in B decay, we have $c_i = \bar{c}_i$ both for the excited D mesons and for the $\pi^+\pi^-$ resonances (which can only be CP -even).

Following the usual formalism (see, for example, [6, 7, 8]), we write the time-dependent decay rate of a B meson that is known to have specific flavour content (being either B^0 or \bar{B}^0 at time $\Delta t = 0$) to a particular final state f or its CP conjugate \bar{f} as

$$\Gamma(B_{\text{phys}}^0 \rightarrow f(\Delta t)) \propto e^{-|\Delta t|/\tau_{B^0}} (1 - S_f \sin(\Delta m \Delta t) + C_f \cos(\Delta m \Delta t)) , \quad (3)$$

$$\Gamma(\bar{B}_{\text{phys}}^0 \rightarrow f(\Delta t)) \propto e^{-|\Delta t|/\tau_{B^0}} (1 + S_f \sin(\Delta m \Delta t) - C_f \cos(\Delta m \Delta t)) , \quad (4)$$

where $S_f = 2 \text{Im}(\lambda_f)/(1 + |\lambda_f|^2)$, $C_f = (1 - |\lambda_f|^2)/(1 + |\lambda_f|^2)$ and $\lambda_f = \frac{q}{p} \frac{\bar{A}}{A}$. In the above, τ_{B^0} is the average lifetime of the neutral B meson, Δm is the mass difference between the two eigenstates of the B^0 - \bar{B}^0 system which are described in terms of the flavour specific states by the mixing parameters q and p as $|B_{L(H)}\rangle = p|B^0\rangle + (-)q|\bar{B}^0\rangle$. (We have neglected the lifetime difference and assumed CPT invariance.)

In the case at hand, f represents a point in the $D\pi^+\pi^-$ Dalitz plot. If we consider D decays to CP eigenstates (and neglect direct CP violation in the D system), then we have

$$\lambda(m_+^2, m_-^2) = \frac{q}{p} \eta_D \frac{\bar{A}(m_+^2, m_-^2)}{A(m_+^2, m_-^2)} , \quad (5)$$

where η_D is the CP eigenvalue of the D_{CP} state. With the further substitution $|q/p| = 1$, $\arg(q/p) = -2\beta$, both good approximations in the Standard Model, we can write

$$S(m_+^2, m_-^2) = \frac{2 \text{Im}(e^{-2i\beta} \eta_D A^* \bar{A})}{|A|^2 + |\bar{A}|^2} = \frac{2 \text{Im} \left(e^{-2i\beta} \eta_D \sum_i c_i^* F_i(m_+^2, m_-^2)^* \sum_j \bar{c}_j \bar{F}_j(m_+^2, m_-^2) \right)}{|\sum_i c_i F_i(m_+^2, m_-^2)|^2 + |\sum_i \bar{c}_i \bar{F}_i(m_+^2, m_-^2)|^2} , \quad (6)$$

$$C(m_+^2, m_-^2) = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{|\sum_i c_i F_i(m_+^2, m_-^2)|^2 - |\sum_i \bar{c}_i \bar{F}_i(m_+^2, m_-^2)|^2}{|\sum_i c_i F_i(m_+^2, m_-^2)|^2 + |\sum_i \bar{c}_i \bar{F}_i(m_+^2, m_-^2)|^2} . \quad (7)$$

The numerator of the expression for $S(m_+^2, m_-^2)$ can be written as $2\eta_D \left(\cos(2\beta) \text{Im}(A^* \bar{A}) - \sin(2\beta) \text{Re}(A^* \bar{A}) \right)$, making explicit the dependence of mixing-induced CP violation on both $\sin(2\beta)$ and $\cos(2\beta)$. We note that one can choose to fit for $\sin(2\beta)$ and $\cos(2\beta)$ independently, or alternatively one can fit directly for β . Although the latter appears attractive, the expressions above make clear that in the former case both parameters appear as coefficients of physically observable functions of the amplitudes,

and thus one might expect somewhat better statistical behaviour for these observables, in particular in regions close to physical boundaries.

The sensitivity to $\cos(2\beta)$ is proportional to $\text{Im}(A^*\bar{A})$, and therefore depends strongly on interference in the Dalitz plot between resonances with non-trivial phase differences. To illustrate this point, we show in Fig. 2 the values of $\text{Im}(A^*\bar{A})$ and $\text{Re}(A^*\bar{A})$ across the $B^0 \rightarrow D_{CP}\pi^+\pi^-$ Dalitz plot, calculated using our nominal model as described in the next section. The Dalitz plot is drawn as $m_-^2 \equiv m_{D\pi^-}^2$ vs. $m_+^2 \equiv m_{D\pi^+}^2$. We emphasise that the Dalitz plot model can be experimentally determined from the higher statistics $B^0 \rightarrow \bar{D}^0\pi^+\pi^-$ sample with flavour-specific \bar{D}^0 decays. Consequently, the model uncertainty on the obtained value of $\cos(2\beta)$ should be controllable.

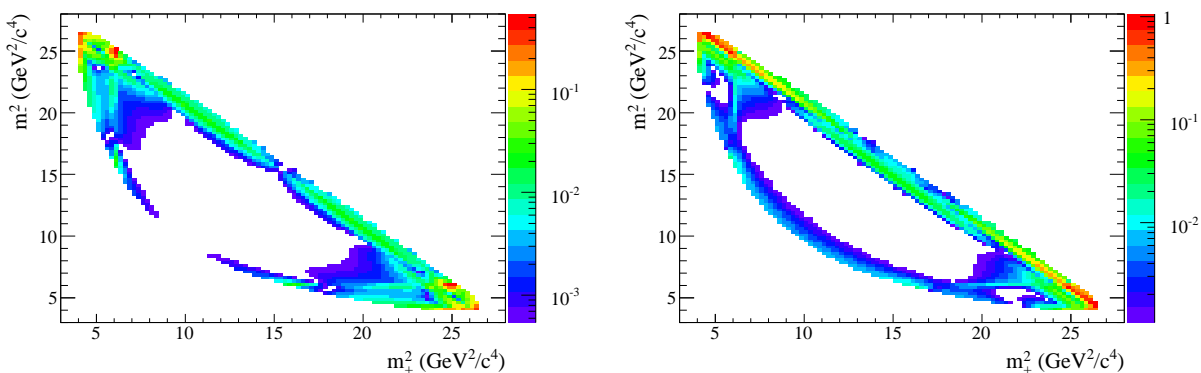


FIG. 2: Dalitz plot distributions of (left) $|\text{Im}(A^*\bar{A})|$ and (right) $|\text{Re}(A^*\bar{A})|$, which govern the sensitivity to $\cos(2\beta)$ and $\sin(2\beta)$, respectively, using our nominal $B^0 \rightarrow D_{CP}\pi^+\pi^-$ decay model. Note that the z -axis is shown on a log scale.

We conclude this section with a brief discussion of some of the approximations that have been made in the above formalism. We have assumed that the $B^0-\bar{B}^0$ system contains no CP violation in mixing ($|q/p| = 1$), negligible lifetime differences ($\Delta\Gamma_d = 0$) and no CPT violation. Furthermore, we have assumed no direct CP violation in D meson decays or in B decays to $D\pi^+\pi^-$. All of these are valid approximations in the Standard Model, and have been experimentally tested to good precision.

We have also until now neglected contributions from the suppressed $b \rightarrow u\bar{c}d$ amplitudes, illustrated in Fig. 3. The relative weak phase between $b \rightarrow c\bar{u}d$ and $b \rightarrow u\bar{c}d$ amplitudes is given by the CKM Unitarity Triangle angle γ . If the contribution of the suppressed decays is significant, it leads to some interesting phenomenology, including the potential to measure

$\sin(2\beta+\gamma)$ from resonant amplitudes such as $D_2^{*\pm}\pi^\mp$, and to measure γ from rates and direct CP violation effects in modes such as $D\rho^0$. Both measurements would, however, benefit from the larger statistics that are available by reconstructing the D meson in a flavour-specific decay mode and are anyway unlikely to be competitive with similar measurements using $D^{(*)\pm}\pi^\mp$ and DK final states, respectively. This serves to illustrate how flavour-specific D decay modes can be used to control model uncertainties, as well as other experimental systematic uncertainties, in the analysis. A further corollary is that, unless the suppressed amplitudes are accounted for, there can be small biases on the extracted values of 2β that are measured from $B^0 \rightarrow D_{CP}\pi^+\pi^-$, with the biases opposite in sign for D mesons reconstructed in CP -even and CP -odd final states [49]. The suppressed amplitudes therefore introduce some model dependence into the results, which can be tested by adding suppressed (“wrong-sign”) D_2^* resonances (for example) into the model, by relaxing the constraint $c_i = \bar{c}_i$, and by checking the consistency of results of independent fits to the samples with CP -even and CP -odd D decays.

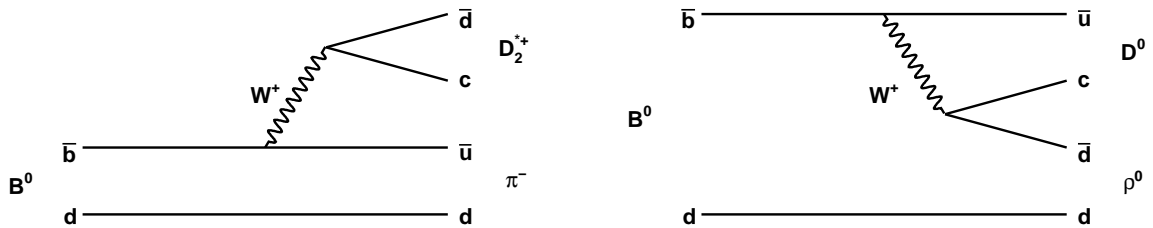


FIG. 3: Feynman diagrams for Cabibbo-suppressed transitions: (left) $\bar{b} \rightarrow \bar{u}c\bar{d}$ resulting in $B^0 \rightarrow D_2^{*+}\pi^-$, and (right) $\bar{b} \rightarrow \bar{u}c\bar{d}$ resulting in $B^0 \rightarrow D^0\rho^0$.

IV. FEASIBILITY STUDY

A. Nominal Model

We form a nominal $B^0 \rightarrow \bar{D}^0 \pi^+ \pi^-$ Dalitz plot model based on results from the Belle Collaboration [22]. The most prominent contributions to the Dalitz plot are found to be $B^0 \rightarrow D_2^{*-} \pi^+$, $D_2^{*-} \rightarrow \bar{D}^0 \pi^-$; $B^0 \rightarrow D_0^{*-} \pi^+$, $D_0^{*-} \rightarrow \bar{D}^0 \pi^-$; $B^0 \rightarrow D_v^{*-} \pi^+$, $D_v^{*-} \rightarrow \bar{D}^0 \pi^-$; $B^0 \rightarrow \bar{D}^0 \rho^0$, $\rho^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow \bar{D}^0 f_2$, $f_2 \rightarrow \pi^+ \pi^-$. The parameters of these resonances are taken from the Particle Data Group (PDG) [53] and are summarised in Table III. The particle denoted D_v^{*-} is a virtual D^{*-} meson. The D^{*-} itself is too long-lived to cause interference and, following Belle [22], we veto the region dominated by this contribution by excluding the invariant mass range $2.00 \text{ GeV}/c^2 < m_{D\pi} < 2.02 \text{ GeV}/c^2$ from the analysis.

TABLE III: Parameters of the resonances used in our nominal model. These values are taken from the PDG [53].

Resonance	Mass (MeV/ c^2)	Width (MeV)
D_2^{*-}	2461.1 ± 1.6	43 ± 4
D_0^{*-}	2352 ± 50	261 ± 50
D_v^{*-}	2010.27 ± 0.17	0.096 ± 0.022
ρ^0	775.49 ± 0.34	149.4 ± 1.0
f_2	1275.1 ± 1.2	$185.0^{+2.9}_{-2.4}$

The model described above is sufficient for the purposes of our feasibility study, although the $\bar{D}^0 \pi^+ \pi^-$ decay amplitude may include additional resonant or nonresonant terms. Belle found a possible contribution from $B^0 \rightarrow \bar{D}^0 f_0(600)$ (the $f_0(600)$ is sometimes known as the σ meson). Ultimately the decay model and its uncertainty should be determined from $B^0 \rightarrow \bar{D}^0 \pi^+ \pi^-$ with flavour-specific \bar{D}^0 decays as a part of the analysis.

B. Event Generation

We estimate the number of $B^0 \rightarrow D_{CP} \pi^+ \pi^-$ events that we expect in the final BaBar dataset as follows. Belle found $2909 \pm 115 B^0 \rightarrow \bar{D}^0 \pi^+ \pi^-$, $\bar{D}^0 \rightarrow K^+ \pi^-$ events in a data

sample of 388×10^6 $B\bar{B}$ pairs. We assume similar selection efficiency to that in the Belle analysis, and scale according to the size of the final BaBar dataset: 467×10^6 $B\bar{B}$ pairs. We calculate the expected numbers of events in CP -even (K^+K^- and $\pi^+\pi^-$) and CP -odd ($K_S^0\pi^0$ and $K_S^0\omega$) events using scaling factors calculated from event yields in a recent analysis of $B^+ \rightarrow DK^+$ from BaBar [54]. These factors are $(8.6 \pm 0.2)\%$ for K^+K^- and $(3.1 \pm 0.1)\%$ for $\pi^+\pi^-$, giving a total $(11.7 \pm 0.2)\%$ for CP -even; correspondingly $(8.9 \pm 0.2)\%$ for $K_S^0\pi^0$ and $(3.3 \pm 0.1)\%$ for $K_S^0\omega$ sum to a total $(12.2 \pm 0.2)\%$ for CP -odd. Taking all these factors into account, we expect approximately 410 CP -even events and 430 CP -odd events in the final BaBar data sample.

We use the `Laura++` package to generate and fit events. This package has been largely developed by the authors and used in several recent analyses published by the BaBar Collaboration [55, 56, 57, 58, 59]. We generate events using c_i parameters based on results of the Belle analysis of $B^0 \rightarrow \bar{D}^0\pi^+\pi^-$ [22] and $c_i = \bar{c}_i$, using a convention in which all F_i functions are normalised to unity when integrated over the Dalitz plot. We assume a reconstruction efficiency that does not vary across the Dalitz plot and neglect misreconstruction effects. Initially, we also neglect backgrounds from other B decays or other processes, but we include these events in the study at a later stage, discussed below. We simulate effects due to Δt resolution and misidentification of the flavour of the tagging B meson using standard resolution functions and parameters from BaBar [60]. In the fit we float the real and imaginary parts of all c_i except those for the D_2^{*-} which are fixed as reference parameters. We also float both $\cos(\phi_{\text{mix}})$ and $\sin(\phi_{\text{mix}})$, where $\phi_{\text{mix}} = 2\beta$ in the Standard Model. The generated distribution of events in the Dalitz plot for ten times the expected statistics can be seen in Fig. 4. Projections onto $m_{D_{CP}\pi^\pm}$ and $m_{\pi^+\pi^-}$ can be seen in Fig. 5.

C. Results

The distributions of the fitted results for $\cos(\phi_{\text{mix}})$ and $\sin(\phi_{\text{mix}})$ from 500 pseudo-experiments generated as described in the previous subsection are shown in Fig. 6. We are clearly able to determine both $\cos(\phi_{\text{mix}})$ and $\sin(\phi_{\text{mix}})$, with spreads of the distributions of about 0.50 and 0.18 respectively. (For comparison, the means of the distributions of the uncertainties on these parameters that result from the fits are found to be 0.43 and 0.17 respectively, confirming the approximately Gaussian nature of these parameters that is also

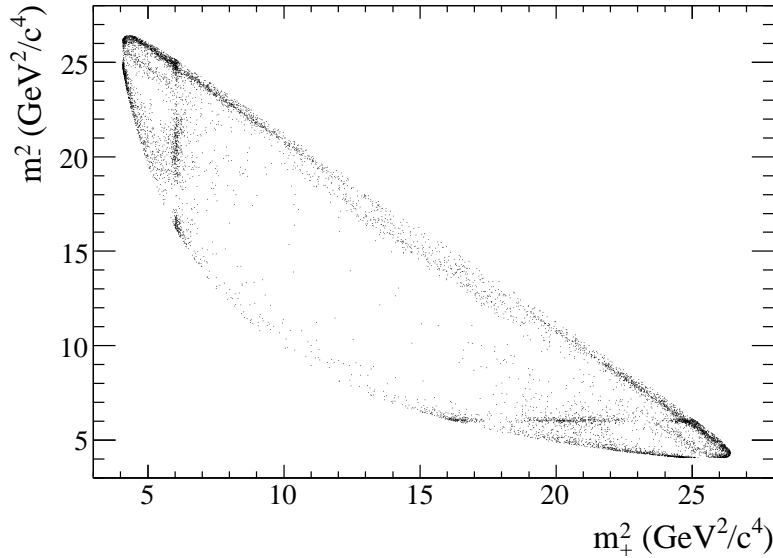


FIG. 4: Dalitz plot distribution of the generated events using our nominal model. The statistics shown here are ten times that expected in the full BaBar dataset.

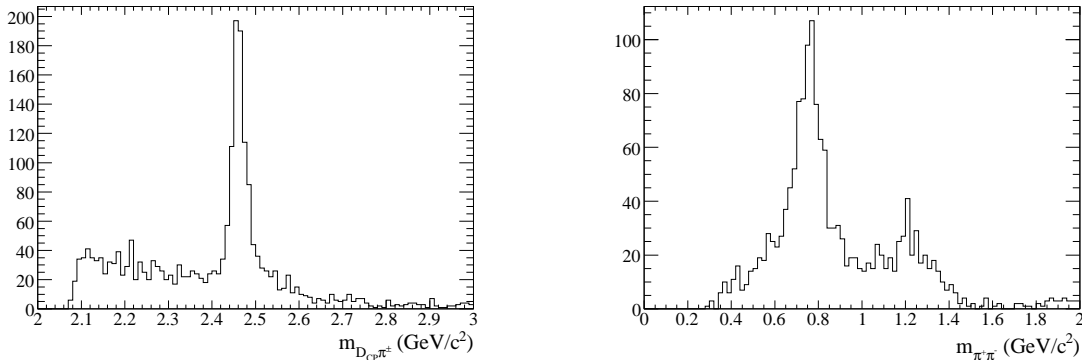


FIG. 5: Projections onto (left) $m_{D_{CP}\pi^\pm}$ and (right) $m_{\pi^+\pi^-}$ of the generated events using our nominal model. The statistics shown here are ten times that expected in the full BaBar dataset. To avoid artefacts due to reflections, in the $m_{D_{CP}\pi^\pm}$ projection we require $m_{\pi^+\pi^-} > 2.0 \text{ GeV}/c^2$; in the $m_{\pi^+\pi^-}$ projection we require $m_{D_{CP}\pi^\pm} > 2.75 \text{ GeV}/c^2$ (both combinations). Structures due to (left) D_2^{*-} and (right) ρ^0 and f_2 resonances are clearly apparent.

evident in Fig. 6.) Small biases in the fit results are found to disappear when the number of events per experiment is increased. All fitted c_i parameters are found to be similarly consistent with the input values.

Further to illustrate the method, we show in Fig. 7 the Δt asymmetry for events in

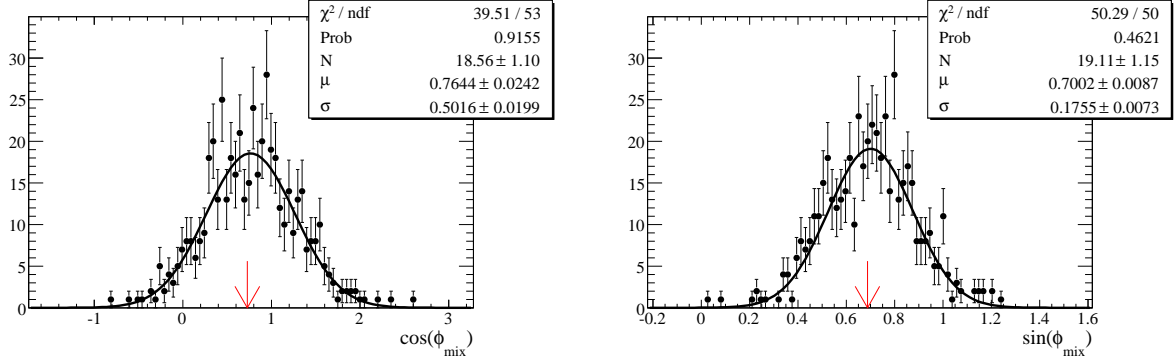


FIG. 6: Distributions of the fitted values of (left) $\cos(\phi_{\text{mix}})$ and (right) $\sin(\phi_{\text{mix}})$ from the pure signal study. The resolutions are 0.50 ± 0.02 and 0.18 ± 0.01 , respectively. The red arrows indicate the generated values, which are based on the value $2\beta = 21.7^\circ$.

the region of the ρ^0 resonance in the $m_{\pi^+\pi^-}$ distribution. The asymmetry is between events where the other B meson has been identified (“tagged”) as a B^0 or a \bar{B}^0 by the charge of the lepton produced through its semileptonic decay. If this region contained $B^0 \rightarrow D_{CP}\rho^0$ decays alone, the asymmetry would give $\sin(2\beta) \sin(\Delta m \Delta t)$, smeared by experimental effects (Δt resolution and misidentification of the flavour of the tagging B meson). The plot contains only events containing a D meson reconstructed in a CP -even decay mode and has 500 times the statistics expected from the final BaBar dataset.

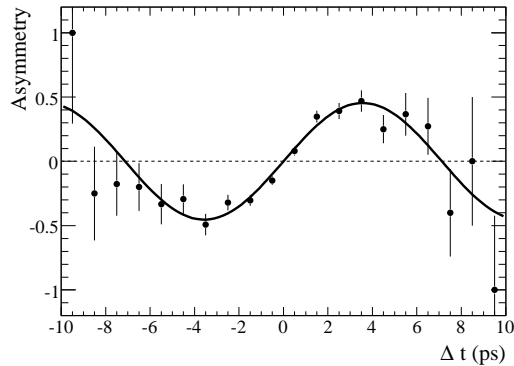


FIG. 7: Asymmetry between B^0 and \bar{B}^0 tags as a function of Δt in the region of the ρ^0 resonance ($0.70 \text{ GeV}/c^2 < m_{\pi^+\pi^-} < 0.85 \text{ GeV}/c^2$). Only events in the best tagging category are shown. The statistics shown here are 500 times that expected in the full BaBar dataset.

To ensure that our results do not depend on the true value of ϕ_{mix} we repeat the process with a number of different input values. As shown in Fig. 8, we determine the correct values

of $\cos(\phi_{\text{mix}})$ and $\sin(\phi_{\text{mix}})$ in all cases. We also find that uncertainties on these parameters do not depend strongly on the input values – a variation of 4% (18%) is found in the mean uncertainty on the fitted value of $\cos(\phi_{\text{mix}})$ ($\sin(\phi_{\text{mix}})$), with the uncertainty being largest when ϕ_{mix} is smallest.

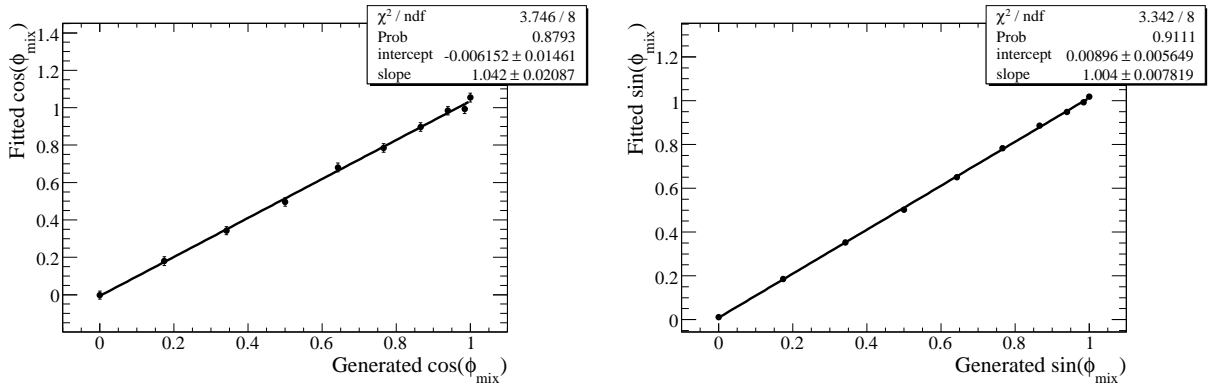


FIG. 8: Fitted values as a function of the generated values of (left) $\cos(\phi_{\text{mix}})$ and (right) $\sin(\phi_{\text{mix}})$. In the latter plot the error bars are too small to be seen.

D. Experimental Complications

In a true experimental environment there will be events from background processes that will have a specific structure in the Dalitz plot and also in Δt . The presence of these events will complicate the analysis and could degrade the sensitivity to the parameters of interest. In order to attempt to estimate the scale of these effects we repeat the study including background events. The Belle analysis [22] indicates that the level of background is approximately the same as that of signal once they apply a selection on the discriminating kinematic variables m_{ES} and ΔE (for definitions of these variables, see for example [58]). As such we include the same number of background events as signal in our samples. We take the distribution of the background events in the Dalitz plot from the Belle paper and use a delta function for the true Δt distribution. In order to provide further discrimination between signal and background we also include m_{ES} and ΔE in the fit. We use Gaussian shapes to describe the signal distribution of both of these variables, whilst for background we use the ARGUS shape [61] and a linear function, respectively.

The results of this extended study are shown in Fig. 9. The fitted values of $\cos(\phi_{\text{mix}})$ and

$\sin(\phi_{\text{mix}})$ are still unbiased and the uncertainties are largely unchanged from the pure signal case.

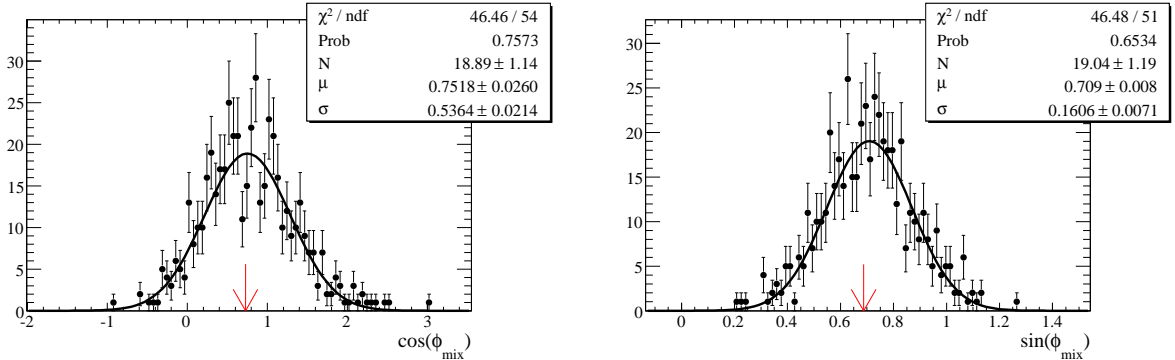


FIG. 9: Distributions of the fitted values of (left) $\cos(\phi_{\text{mix}})$ and (right) $\sin(\phi_{\text{mix}})$ from the signal and background study. The resolutions are 0.54 ± 0.02 and 0.16 ± 0.01 , respectively. The red arrows indicate the generated values, which are based on the value $2\beta = 21.7^\circ$.

There are a number of additional potential experimental complications that we have not simulated. Effects due to smearing of the reconstructed Dalitz plot position should be negligible since our Dalitz plot model does not contain any very narrow resonances. More significant misreconstruction effects can occur when one of the particles from the signal side is exchanged with a particle from the decay of the other B in the event. This effect tends to occur most frequently near the corners of the Dalitz plot. Since we veto the regions in the corners of the Dalitz plot that are dominated by decays of the D^* meson, we do not expect this to cause any serious difficulty. Finally, we have assumed that the reconstruction efficiency does not vary across the $D\pi^+\pi^-$ phase space, but in a real experiment it is likely that the corners of the Dalitz plot have a lower efficiency. This may lead to a small reduction in the precision of the $\cos(2\beta)$ measurement, since these are the regions that are most sensitive to this parameter, as shown in Fig. 2. However, in an experimental analysis the efficiency would be measured using detailed Monte Carlo simulation together with data control samples, and therefore we do not expect any bias on the results.

E. Prospects at LHCb

The potential to utilize this method at LHCb has not been explicitly studied, but some useful extrapolations can be made. We consider only the CP -even D meson decay to K^+K^- ,

which is well-suited for study at LHCb since it has a final state consisting of only charged tracks including two kaons. (The use of particle identification information from LHCb's ring imaging Cherenkov detectors being essential to reduce combinatoric background in the hadronic environment [62].)

An estimation of the likely yield can be made by comparison with the decay $B^0 \rightarrow D^- \pi^+$, $D^- \rightarrow K^+ \pi^- \pi^-$. In this channel, which has a product branching fraction of $(2.47 \pm 0.13) \times 10^{-4}$ [53], LHCb expects to trigger and reconstruct 1.34×10^6 events in 2 fb^{-1} of data (one nominal year of data taking) [63]. For $B^0 \rightarrow D \pi^+ \pi^-$, $D \rightarrow K^+ K^-$ the product branching fraction is $(3.3 \pm 0.4) \times 10^{-6}$ so that if the trigger and reconstruction efficiencies are the same, one expects approximately 18,000 events. Taking an effective tagging efficiency at LHCb of 5% [64], the equivalent number of perfectly tagged events in one year of LHCb data is approximately 900, which compares very well to the yields from the final BaBar dataset (where the effective tagging efficiency is about 30% [60]). Although it may be necessary to apply less efficient selection criteria to suppress background, this provides an indicative measure of the potential at LHCb. With 2 fb^{-1} of data, it will be possible to achieve a precision better than any previous measurements. With the complete LHCb data set, it should be possible to measure $\cos(2\beta)$ and $\sin(2\beta)$ with precisions of 0.12–0.17 and 0.03–0.05 respectively.

The prospects for LHCb to make precise measurements of $\cos(2\beta)$ and $\sin(2\beta)$ using $B^0 \rightarrow D \pi^+ \pi^-$, $D \rightarrow K^+ K^-$ therefore look rather good. However, any firm conclusion on this point requires a detailed study including proper simulation of detector effects as well as consideration of the Dalitz plot model including effects of the suppressed amplitudes. We leave such studies to further work.

V. CONCLUSION

We have presented the results of a feasibility study of a method to measure $\cos(2\beta)$ using a time-dependent Dalitz plot analysis of $B^0 \rightarrow D\pi^+\pi^-$ decays, where the neutral D meson is reconstructed in decays to CP eigenstates. We estimate that, with the final BaBar dataset, $\cos(2\beta)$ can be measured with a precision of ~ 0.50 , making this approach competitive with, or superior to, all other methods that have been attempted to date. Furthermore, $\sin(2\beta)$ can be measured to within ~ 0.16 , which is more precise than any existing measurement using $b \rightarrow c\bar{u}d$ transitions.

We have argued that uncertainties relating to the composition of the $B^0 \rightarrow D\pi^+\pi^-$ Dalitz plot can be tamed using the much larger data samples that are available when the D is reconstructed in a flavour-specific hadronic decay mode such as $D^0 \rightarrow K^-\pi^+$. Therefore, the sensitivity is limited by statistics only, and more precise measurements will be possible at electron-positron colliders with higher luminosity. Moreover, since final states containing only charged particles can be used, there is great potential for this analysis to be employed at LHCb, where use of the CP -even decay $D \rightarrow K^+K^-$ looks particularly promising.

Application of this method at future experiments will allow a determination of $\cos(2\beta)$ that would definitively establish its sign, hence resolving the ambiguity on 2β . Moreover, measurements of $\sin(2\beta)$ in $b \rightarrow c\bar{u}d$ transitions can be made with precision comparable to that obtained by the B factories in $b \rightarrow c\bar{c}s$ transitions ($B^0 \rightarrow J/\psi K^0$). Discrepancies between these values would be unambiguous signs of new physics.

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