Improved Inference and Estimation in Regression With Overlapping Observations

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Abstract

We present an improved method for inference in linear regressions with overlapping observations. By aggregating the matrix of explanatory variables in a simple way, our method transforms the original regression into an equivalent representation in which the dependent variables are non-overlapping. This transformation removes that part of the autocorrelation in the error terms which is induced by the overlapping scheme. Our method can easily be applied within standard software packages since conventional inference procedures (OLS-, White-, Newey-West- standard errors) are asymptotically valid when applied to the transformed regression. Through Monte Carlo analysis we show that they perform better in finite samples than the methods applied to the original regression that are in common usage. We illustrate the significance of our method with two empirical applications.

JEL classification: $C20, G12$

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1 Introduction

Researchers in empirical finance often regress long-horizon returns onto explanatory variables. Such regressions have been used to assess stock return predictability, to test the expectations theory of the term structure of interest rates, to test the cross-sectional pricing implications of the CAPM and consumption-CAPM, to investigate the forward premium puzzle, and to test the efficiency of foreign exchange markets. These regressions involve overlapping observations which raise econometric issues that are addressed in this paper.

Regressions with long horizon returns often show much higher R^2's than regressions with one-period returns. But work by Valkanov (2003), Hjalmarsson (2006), and Boudoukh, Richardson & Whitelaw (2008) suggests that long-horizon return regressions have no greater statistical power to reject the null of no predictability than their short-horizon counterparts. For testing predictability the use of long-horizon returns (as opposed to one-period returns) would appear to be of little value.

Nonetheless, the analysis of long-horizon returns can contribute significantly to understanding predictability (or dependence) and its economic significance. For example, one concern with the interpretation of short-horizon regressions is measurement error. Cochrane & Piazzesi (2005, p. 139) forecast annual bond returns using monthly data and claim that "to see the core results you must look directly at the one-year horizon" and further find that estimating a typical one-month return model "completely misses the single factor representation" due to measurement error. Another reason for analyzing longer horizons is lengthy and uncertain response times. A number of recent studies of the consumption-CAPM, including Daniel & Marshall (1997), Parker (2001), Parker & Julliard (2005), Jagannathan & Wang (2005, 2007), and Malloy, Moskowitz & Vissing-Jorgensen (2005) measure consumption risk using consumption growth and returns measured over several periods. In general this approach works better than the standard consumption-CAPM. If consumers face costs associated with changing consumption, or if information acquisition is constrained, then consumption may change more slowly than implied by the standard consumption-CAPM, and this justifies a focus on returns and consumption growth measured over longer horizons.

Long-horizon return regressions potentially suffer from two econometric problems. The first is bias in the usual OLS coefficient estimates. The bias is not caused by the presence of overlapping observations but arises when the predictor variable is persistent and its innovations are strongly correlated with returns (see Gregory Mankiw & Shapiro (1986) and Stambaugh (1999)). These conditions may also arise in short-horizon regressions.

Our paper does not address this problem of bias but focuses instead on the second problem, one which is specific to overlapping observations: the strong autocorrelation pattern induced by the overlapping scheme. It is now well known that commonly used methods to deal with the autocorrelation are inadequate and can lead to misleading estimates of the confidence intervals associated with coefficient estimates obtained from finite samples. Despite this, many studies still resort to standard inference techniques such as applying White or common Newey-West standard errors within an overlapping regression framework.[1]

This paper presents a simple procedure that can markedly improve inference in regressions with overlapping observations. The beauty of our approach, for practical purposes, is that it can be readily implemented in standard econometric software packages and no serious programming is needed to obtain our inference statistics.

We consider an overlapping regression in which a multi-period return is regressed onto a set

[1]See for instance Lamont (1998), Lettau & Ludvigson (2001), Baker, Greenwood & Wurgler (2003), Evans & Lyons (2005) and Bacchetta, Mertens & van Wincoop (2009).

of regressors, and for which observations are available each period. This regression is transformed into a non-overlapping regression in which one-period returns are regressed onto a set of transformed regressors. The OLS coefficient estimates from the original and transformed regressions are numerically identical, but inference based on the transformed regression is simplified because the autocorrelation induced by overlapping observations is no longer present. The procedure is equally applicable to time-series regressions and to panel regressions. It can be applied to both predictive (forecasting) and contemporaneous (explanatory) regressions.

We show that standard inference procedures, such as OLS, White (1980) and Newey & West (1987), are asymptotically valid when applied to the transformed regression. To assess the finite-sample performance of our procedure we run Monte Carlo simulations. These show that the standard inference procedures perform substantially better when based on the transformed regression rather than on the original specification. Indeed, simpler procedures, such as OLS and White, when applied to the transformed regression, perform better than more sophisticated techniques such as Hansen & Hodrick (1980) and Newey-West applied to the original regression. The superior performance of our procedure is most marked when the return horizon in the original specification is long in comparison to the sample length, and Hansen-Hodrick and Newey-West standard errors tend to be severely biased down. The standard errors obtained from our transformed regression have much less bias and lower standard deviation. The result is that confidence intervals using our method have coverage probabilities much closer to their nominal levels than confidence intervals constructed using standard techniques.

Other papers have documented problems with conventional inference applied to long-horizon regressions (for example Ang & Bekaert (2007), Nelson & Kim (1993), and Hodrick (1992)) and utilize or advocate simulation techniques for inference. Another strand of the literature develops covariance estimators for specific cases, imposing additional structure on the serial correlation of moment conditions. These structured estimators generally have excellent small-sample properties, but their applicability is limited. For example, the estimator of Richardson and Smith (1991) provides valid inference only under the null hypothesis that returns are serially uncorrelated, and only when the explanatory variables are past returns. Even then, valid inference requires the unpalatable assumption (for asset returns) of conditional homoscedasticity.

The methodology that is most similar to ours is Hodrick (1992). He presents a structured covariance estimator that generalizes Richardson & Smith (1991) in that regressors need not be past returns and returns need not be conditionally homoscedastic. A drawback of Hodrick's derivation is that it is complex, and as a result his estimator has not gained widespread acceptance. For example Campbell, Lo & MacKinlay (1997) do not mention it in their well-known textbook, despite having a detailed discussion on statistical inference in long-horizon regressions. It has also not been widely used in the empirical literature, with the exception of Ang & Bekaert (2007). They use Hodrick (1992) standard errors and argue that much of the empirical evidence for the time-series predictability of stock returns has been overstated in the literature due, in part, to the use of OLS or Hansen & Hodrick (1980) standard errors which they find 'lead to severe over-rejections of the null hypothesis'. Our method is not complex, easy to implement and therefore is more likely to be adopted by empirical researchers.

Our method naturally extends to cases where the error term in short-horizon regressions is autocorrelated. By transforming the regression equation, the autocorrelation in the error term induced by the use of overlapping data is stripped out. The researcher can then focus on addressing any remaining autocorrelation present in short-horizon returns by applying the standard econometrics toolbox.

Jegadeesh (1991) and Cochrane (1991) advocate an approach which is similar in appearance to ours. They bypass the problem of overlapping observations by regressing one-period returns onto the sum of lags of the explanatory variable. However, this is strictly a procedure for testing the null of no-predictability. It does not provide a coefficient estimate for a long-horizon regression, and it is restricted to regressions with a single explanatory variable, so it is of little use for understanding the sources of long-horizon predictability.

Our approach is particularly useful for panel data, where the complexity and size of the data precludes some of the more sophisticated methods for dealing with overlapping observations such as bootstrapping and the Hodrick (1992) procedure. The Fama-MacBeth methodology is a simple approach that neatly accounts for cross-sectional correlation in errors. When multi-period returns are involved, we can use our transformed regression approach in combination with the Fama-MacBeth methodology to remove the serial correlation induced by overlapping observations.

Section 2 develops the basic idea in the context of inference for a linear regression with overlapping observations. Section 3 presents results from Monte Carlo studies demonstrating the advantages of our approach. Section 4 illustrates our approach with two empirical examples. The first example analyses the predictability of long-horizon US stock market returns and the second example analyses reversal in relative country stock index returns. Section 5 concludes.

2 Linear Regression with Overlapping Observations

Let r denote the $T \times 1$ vector of one period log returns and A the $(T-k+1) \times T$ matrix, that has entries $a_{ij} = 1$ if $i \leq j \leq i+k-1$ and 0's otherwise, with $i = 1, \ldots, T-k+1$. Thus, A is the transformation matrix with 1's on the main diagonal and the first $k-1$ right off-diagonals and 0's otherwise. Hence, Ar is the $(T-k+1)\times 1$ vector of k period log returns[2]. X denotes the $(T-k+1)\times \ell$ matrix of explanatory variables (with the first column of X consisting of 1's). We consider the following (predictive) linear regression setup with overlapping returns

$$Ar = X\beta + u, \tag{1}$$

in which u denotes the $(T-k+1)\times 1$ error term vector. The OLS parameter estimate of β in equation (1) is given by

$$\hat{\beta} = (X'X)^{-1}X'Ar. \tag{2}$$

It can be rewritten as

$$\hat{\beta} = (X'X)^{-1}(A'X)'r, \tag{3}$$

which shows that the OLS estimate of β can be rewritten in terms of the original non-overlapping one period returns.

Moreover, $\hat{\beta}$ as given in equation (3) can be obtained from an associated transformed regression

$$r = \tilde{X}\beta + \tilde{u}, \tag{4}$$

in which $\tilde{X}$ is the $T\times \ell$ matrix of transformed explanatory variables given by

$$\tilde{X} \equiv A'X(X'AA'X)^{-1}X'X, \tag{5}$$

and $\tilde{u}$ is the $T\times 1$ error term vector of this transformed regression.[3] To see this, note that

[2] We assume that the single period returns as well as the overlapping long period returns are available to the researcher. This is generally the case, though the Hansen & Hodrick (1980) study of the foreign exchange market is an exception since the returns in that case are on three month forward contracts, and the prices of one week forward contracts are not available.

[3] We use the convention that the tilde marks quantities from the transformed regression throughout the paper.

the OLS estimate of β in equation (4) is given by

$$\begin{aligned}\hat{\beta} &= (\tilde{X}'\tilde{X})^{-1}\tilde{X}'r, \\ &= \left((X'X(X'AA'X)^{-1}X'AA'X(X'AA'X)^{-1}X'X\right)^{-1}X'X(X'AA'X)^{-1}X'Ar, \\ &= (X'X)^{-1}X'Ar, \qquad (6)\end{aligned}$$

which is indeed the same as the estimator in equation (2).

Substituting for Ar from (1) into (2), and for r from (4) into (6) gives the following pair of equations for the error in the estimate of β which conventional inference procedures are based on:

$$\begin{aligned}\hat{\beta} - \beta &= (X'X)^{-1}X'u, \\ \hat{\beta} - \beta &= (X'X)^{-1}X'A\tilde{u}. \qquad (7)\end{aligned}$$

The benefit of using the second formulation is that the error depends explicitly on the autocorrelation structure of $\tilde{u}$, the noise in the transformed, non-overlapping regression, rather than on u, the noise in the overlapping regression. The autocorrelation structure of $\tilde{u}$ is generally much simpler than the autocorrelation structure of u since that part of the autocorrelation in u induced by the deterministic aggregation scheme A is explicitly known and accounted for. The efficiency gain lies in accounting for a known dependence pattern explicitly, without the need to estimate it in a noisy way.

We do not claim that either model (1) or model (4) is the true data generating process. If model (1) is misspecified so will be model (4) and thus issues such as inconsistency, bias, endogeneity and omitted variable problems, are not addressed and cannot be mitigated by the use of our transformation. However, our transformation will be of particular help in improving the accuracy of the standard errors when the model is misspecified, because the misspecification is likely to induce autocorrelation in the error terms. It is much easier to deal with this autocorrelation by itself (as in the transformed regression) rather than to deal with both it and the autocorrelation induced by the use of overlapping observations at the same time.

2.1 Inference on β

We now show how the asymptotic covariance matrix of $\hat{\beta}$ derived from the overlapping regression (1) is related to the asymptotic covariance matrix of $\hat{\beta}$ derived from the non-overlapping regression (4). Let $\hat{\beta}_{(T-k+1)}$ denote the OLS estimate of β from a data sample of size $T-k+1$ obtained from regression equation (1). Rearranging equation (1) yields

$$\frac{T-k+1}{\sqrt{T-k+1}}\left(\hat{\beta}_{(T-k+1)}-\beta\right)=\left(\frac{1}{T-k+1}X'X\right)^{-1}\frac{1}{\sqrt{T-k+1}}X'u,$$

and under certain regularity conditions[4] we obtain a central limit theorem of the following form

$$D_{(T-k+1)}^{-1/2}\left(\hat{\beta}_{(T-k+1)}-\beta\right)\overset{asy}{\sim}N(0,I_\ell),$$

with

$$D_{(T-k+1)}=\frac{1}{T-k+1}Q_{(T-k+1)}^{-1}S_{(T-k+1)}Q_{(T-k+1)}^{-1},$$

where $Q_{(T-k+1)}$ is given by

$$Q_{(T-k+1)}\equiv\mathrm{E}\left(\frac{1}{T-k+1}X'X\right),$$

and $S_{(T-k+1)}$ is given by

$$\begin{aligned}S_{(T-k+1)} &\equiv \mathrm{V}\left(\frac{1}{\sqrt{T-k+1}}X'u\right)\\ &= \frac{T}{T-k+1}\mathrm{V}\left(\frac{1}{\sqrt{T}}(A'X)'\tilde{u}\right)\\ &= \frac{T}{T-k+1}\tilde{S}_{(T)}.\end{aligned}$$

[4]See for example White (2001) chapter 6.4 for the general case of dependent and heterogeneously distributed observations.

The second line follows by exploiting the relationship in (7), and it links the asymptotic covariance matrix $S_{(T-k+1)}$ in the overlapping regression with its twin asymptotic covariance matrix $\tilde{S}_{(T)}$ in the non-overlapping regression in which the deterministic transformation scheme A is explicitly visible. Under the assumption of consistent estimates $\hat{\tilde{S}}_{(T)}$ for $\tilde{S}_{(T)}$ and $\hat{Q}_{(T-k+1)}$ estimated by $\hat{Q}_{(T-k+1)} = \frac{1}{T-k+1}X'X$, a consistent estimate for the asymptotic covariance matrix of $\hat{\beta}$ can be obtained by

$$\hat{\tilde{D}}_{(T-k+1)} = \frac{T}{(T-k+1)^2}\hat{Q}^{-1}_{(T-k+1)}\hat{\tilde{S}}_{(T)}\hat{Q}^{-1}_{(T-k+1)}. \tag{8}$$

Alternatively it can be obtained under the assumption of consistent estimates $\hat{S}_{(T-k+1)}$ for $S_{(T-k+1)}$ as

$$\hat{D}_{(T-k+1)} = \frac{1}{T-k+1}\hat{Q}^{-1}_{(T-k+1)}\hat{S}_{(T-k+1)}\hat{Q}^{-1}_{(T-k+1)}.$$

In this latter case, we do however lose the advantage of accounting for the deterministic transformation scheme A a priori and the estimate $\hat{S}_{(T-k+1)}$ will not be as precise as the estimate $\hat{\tilde{S}}_{(T)}$.

The most common procedure to estimate the covariance matrix of $\hat{\beta}$ under the suspicion of an unknown autocorrelation pattern in the error terms of a linear regression is to resort to the Newey-West HAC covariance matrix. The Newey-West covariance estimate of $\hat{\tilde{S}}_{(T)}$ is given by

$$\hat{\tilde{S}}_{(T)} = \hat{\Gamma}_{A'X}(0) + \sum_{j=1}^{J} w(j,J)\left(\hat{\Gamma}_{A'X}(j) + \hat{\Gamma}_{A'X}(j)'\right), \tag{9}$$

with

$$\hat{\Gamma}_{A'X}(j) \equiv \sum_{t=1}^{T-j}(A'X)'_t\hat{\tilde{u}}_t\hat{\tilde{u}}_{t+j}(A'X)_{t+j},$$

and

$$w(j,J) \equiv 1 - \frac{j}{J+1},$$

where J denotes the lag length and a subscript t denotes the t^{th} row of a matrix. From the definition of $\tilde{X}$ in equation (5) it can be seen that

$$\tilde{X}_t(X'X)^{-1}(X'AA'X) = (A'X)_t,$$

so that

$$\hat{\Gamma}_{A'X}(j) = (X'AA'X)(X'X)^{-1}\hat{\Gamma}_{\tilde{X}}(j)(X'X)^{-1}(X'AA'X).$$

Substituting this expression into equation (9) and afterwards into equation (8) yields

$$\hat{\hat{D}}_{(T-k+1)} = T(\tilde{X}'\tilde{X})^{-1}\left(\hat{\Gamma}_{\tilde{X}}(0) + \sum_{j=1}^{J} w(j,J)\left(\hat{\Gamma}_{\tilde{X}}(j) + \hat{\Gamma}_{\tilde{X}}(j)'\right)\right)(\tilde{X}'\tilde{X})^{-1}, \tag{10}$$

which is the standard Newey-West HAC covariance matrix for the transformed non-overlapping regression. This estimator is simple, it is consistent and it is guaranteed to be positive definite. Without further knowledge of the autocorrelation structure in the error terms that remains after correcting for the autocorrelation induced by the transformation A, it is the most reliable estimator at hand.

The White heteroscedasticity consistent covariance matrix is obtained as a special case ($J = 0$) and can be used under the assumption of no further autocorrelation in the error terms $\tilde{u}$. In this case equation (10) simplifies to

$$\hat{\hat{D}}_{(T-k+1)} = T(\tilde{X}'\tilde{X})^{-1}\hat{\Gamma}_{\tilde{X}}(0)(\tilde{X}'\tilde{X})^{-1}. \tag{11}$$

Our result is of key interest for practical purposes, since it shows that reliable standard errors in the case of regressions with overlapping observations can be obtained simply by i) constructing the transformed regressor matrix $\tilde{X}$, ii) running regression (4) and iii) relying on conventional Newey-West HAC standard errors on the basis of this regression for parameter inference. The beauty of this is that it can be achieved almost effortlessly in any standard econometric software package.

3 Monte Carlo Analysis

We have outlined the relationship between the asymptotic covariance of β in the transformed and the overlapping regression. We argued that accounting for a known transformation scheme a priori as done by the transformed regression will yield more precise estimates of the covariance of β than applying conventional methods such as Hansen-Hodrick or Newey-West directly to the overlapping regression. In this section we show that inference based on the transformed regression has indeed better finite-sample properties than conventional approaches based on the overlapping regression.

We run Monte Carlo simulations using a variety of values for k and for the length of the data T. We compare the performance of procedures based on the transformed regression with the more conventional approaches based on Newey-West and Hansen-Hodrick estimators of the covariance matrix of β applied to the overlapping data regression.[5]

An alternative approach to improving the inference in the presence of auto-correlated errors is to use pre-whitening, as in Andrews & Monahan (1992) and Sul, Phillips & Choi (2005). In our simulations, pre-whitening at best performs comparably with the Newey-West and Hansen-Hodrick estimators applied also to the overlapping regressions. For the shorter datasets we consider, the estimation of the pre-whitening VAR comes so close to non-stationarity that the estimated standard error is unstable. In the interests of space, the simulations with the four alternative HAC estimators and with pre-whitening are not reported here but are available from the authors' website.

[5] Andrews (1991) examines and compares a variety of other HAC estimators, which differ from Newey-West in their weighting function $w(j, J)$. We have done the simulations using the four other HAC estimators he considers (see Andrews (1991 p. 829)). The results are substantially unaltered.

Our main finding is that the transformation we propose does indeed lead to substantial improvements in inference for small samples. Conventional OLS standard errors obtained from the transformed regression provide the most accurate small-sample inference for homoscedastic data generating processes. In the presence of heteroscedasticity, White (1980) heteroscedasticity-consistent standard errors from the transformed regression provide the most accurate inference in small samples. For the case of autocorrelated error terms in the non-overlapping data Newey & West (1987) from the transformed regression performs best. When the forecast return horizon is long in comparison to the sample period, and when the regressors are strongly positively autocorrelated, the Newey-West and Hansen-Hodrick procedures produce standard errors that are severely biased downwards.

The underlying data generating process for our simulations of the one period return process takes the following form

$$r_{t+1} = \alpha + \gamma_1 X_{1t} + \nu_{t+1},$$

where X_{1t} is a stationary AR(1) processes with unit variance and AR parameter 0.8. We consider four particular cases:

1) In the base case (Table 1) there is no predictability at any horizon ($\gamma_1 = 0$) and the error term $\nu_{t+1} \sim N(0,1)$.

2) In the second case (Table 2) there is no predictability as in 1), but the error term ν_{t+1} is heteroscedastic with $\nu_{t+1} = X_{1t}\varepsilon_{t+1}$, where $\varepsilon_{t+1} \sim N(0,1)$.

3) In the third case (Table 3) returns are predictable ($\gamma_1 = 0.5$) and the error term $\nu_{t+1} \sim N(0,1)$; the coefficient γ_1 is chosen so that the R^2 for one period returns is 20%.

4) The fourth case (Table 4) is included to show the implications of autocorrelation in the non-overlapping observations. We induce the autocorrelation by using a misspecified model. Returns are predictable as in 3), but instead of X_{1t} a false regressor X_{2t} is used in the regressions. X_{2t} is mutually uncorrelated with X_{1t} and also follows a stationary AR(1) processes with unit variance and AR parameter 0.8.

We consider two scenarios for the choice of the sample length T and overlapping periods k. The first scenario is $T = 250$ and $k = 3$; and the second scenario is $T = 100$ and $k = 12$. The overlapping regression in equation (1) is estimated. The standard error on $\hat{\beta}_1$ is reported. For each data generating process, and for each sample length and return horizon we present results for four conventional covariance estimators applied to the overlapping regression: 'OLS' is the standard OLS covariance estimator, 'White' is the White (1980) heteroscedasticity-consistent covariance estimator, 'NW' is the Newey & West (1987) heteroscedasticity and autocorrelation consistent (HAC) estimator, and 'HH' is the heteroscedasticity-consistent version of Hansen & Hodrick (1980). The choice of bandwidth for HAC estimators depends on the assumed correlation structure. With an overlapping regression it is conventional to use a bandwidth equal to the overlap or twice the overlap (see for example Cochrane & Piazzesi (2005)). Since there is no qualitative difference between the results for a lag length equal to $J = k$ or $J = 2k$ we only report the first case 'NW(k)'. In addition, we also report Newey-West standard errors 'NW' with the common lag length as suggested by Newey & West (1987), which is $J = \left\lfloor 4\left(\frac{T}{100}\right)^{2/9} \right\rfloor$.

We then present results for covariance estimators based on the transformed regression. We consider the three estimators presented in the previous section: OLS, White, and Newey-West. For each covariance estimator and each scenario we report the bias, standard deviation, and RMSE (root mean squared error), as well as the true confidence levels of the nominal 99%, 95%, and 90% regression coefficient confidence intervals. 50000 simulations are used for each scenario.

Table 1 shows that the OLS and White estimators from the overlapping regression are severely biased down since they fail to account for serial correlation induced by the overlapping scheme. However, the Newey-West and Hansen-Hodrick estimators also exhibit a downward bias, which is particularly strong when the return horizon is long and the sample length short. For example with a forecast return horizon of 12 and 100 observations the bias in the NW and HH estimators is sufficiently large to result in the 99% confidence intervals from these estimators having coverage frequencies below 88%.

In contrast, the estimators based on the transformed regression have much better properties. In particular, the standard OLS estimator of covariance obtained from the transformed regression performs very well in this situation, exhibiting low bias and coverage frequencies that are close to their nominal levels. Note however that the Newey-West estimator applied to the transformed regression is also biased down, though not by as much as the Newey-West estimator applied to the overlapping regression. The bias is induced by the fact that the estimated error has zero mean, and this gives rise to a spurious negative autocorrelation. The Newey-West estimator is also noisier than the White and standard OLS estimator because it estimates cross product terms that the other estimators set to zero by construction.

Obs.	k	Variance Est.	Bias	Std.	RMSE	99%	95 %	90 %
			Overlapping Regression					
250	3	OLS	-0.019	0.003	0.020	89.4%	78.2%	70.0%
		White	-0.020	0.004	0.020	88.8%	77.5%	69.3%
		NW(k)	-0.008	0.008	0.011	96.8%	90.2%	83.5%
		NW	-0.005	0.009	0.011	97.4%	91.5%	85.4%
		HH	-0.007	0.009	0.011	97.0%	90.8%	84.4%
			Transformed Regression					
		OLS	0.000	0.006	0.006	98.9%	94.9%	89.6%
		White	-0.001	0.007	0.007	98.8%	94.5%	89.3%
		NW	-0.002	0.009	0.010	98.1%	93.1%	87.6%
			Overlapping Regression					
100	12	OLS	-0.629	0.077	0.634	71.0%	57.9%	49.9%
		White	-0.643	0.079	0.648	67.7%	54.8%	47.2%
		NW(k)	-0.386	0.293	0.485	88.2%	78.1%	70.5%
		NW	-0.443	0.220	0.494	86.9%	75.9%	68.1%
		HH	-0.442	0.222	0.495	86.8%	75.9%	68.1%
			Transformed Regression					
		OLS	-0.020	0.223	0.224	98.8%	94.8%	89.6%
		White	-0.033	0.259	0.261	98.6%	94.2%	88.7%
		NW	-0.129	0.309	0.335	96.9%	91.0%	84.7%

Table 1: Monte Carlo simulations: No return predictability. Homoscedastic error terms. For each estimator, the bias in the estimate, its standard error, its root mean square error and true confidence levels of 99%, 95% and 90% are shown.

Obs.	k	Variance Est.	Bias	Std.	RMSE	99%	95 %	90 %
			Overlapping Regression					
250	3	OLS	-0.063	0.002	0.063	69.7%	56.5%	48.7%
		White	-0.049	0.010	0.050	85.9%	73.8%	65.2%
		NW(k)	-0.023	0.024	0.033	96.1%	88.5%	81.3%
		NW	-0.017	0.028	0.033	96.8%	90.1%	83.4%
		HH	-0.020	0.027	0.034	96.3%	89.2%	82.2%
			Transformed Regression					
		OLS	-0.045	0.005	0.045	89.6%	78.4%	70.2%
		White	-0.003	0.023	0.024	98.6%	94.2%	88.6%
		NW	-0.010	0.030	0.032	97.7%	91.8%	85.8%
			Overlapping Regression					
100	12	OLS	-0.718	0.059	0.721	66.2%	53.0%	45.4%
		White	-0.711	0.073	0.714	66.9%	54.1%	46.4%
		NW(k)	-0.485	0.285	0.562	87.0%	75.6%	67.5%
		NW	-0.521	0.221	0.566	86.1%	74.4%	66.2%
		HH	-0.522	0.224	0.568	85.9%	74.3%	66.0%
			Transformed Regression					
		OLS	-0.182	0.242	0.303	97.7%	92.0%	85.9%
		White	-0.037	0.465	0.466	98.9%	94.2%	88.5%
		NW	-0.153	0.483	0.506	97.6%	91.4%	84.9%

Table 2: Monte Carlo simulations: No return predictability. Heteroscedastic error terms. For each estimator, the bias in the estimate, its standard error, its root mean square error and true confidence levels of 99%, 95% and 90% are shown.

Obs.	k	Variance Est.	Bias	Std.	RMSE	99%	95 %	90 %
			Overlapping Regression					
250	3	OLS	-0.021	0.003	0.021	90.4%	79.2%	70.7%
		White	-0.021	0.004	0.021	89.8%	78.4%	70.0%
		NW(k)	-0.009	0.008	0.012	96.9%	90.5%	83.9%
		NW	-0.006	0.010	0.011	97.5%	91.8%	85.8%
		HH	-0.007	0.010	0.012	97.2%	91.1%	84.8%
			Transformed Regression					
		OLS	-0.002	0.006	0.006	98.9%	94.7%	89.4%
		White	-0.002	0.007	0.007	98.7%	94.4%	89.1%
		NW	-0.004	0.010	0.010	98.0%	92.8%	87.1%
			Overlapping Regression					
100	12	OLS	-1.319	0.112	1.323	68.2%	55.4%	47.6%
		White	-1.352	0.116	1.357	64.6%	51.9%	44.3%
		NW(k)	-0.923	0.439	1.022	83.2%	72.7%	65.2%
		NW	-0.973	0.335	1.029	84.4%	73.0%	65.1%
		HH	-0.973	0.340	1.031	84.1%	72.8%	65.0%
			Transformed Regression					
		OLS	-0.712	0.240	0.751	93.3%	83.5%	75.5%
		White	-0.734	0.281	0.786	92.5%	82.4%	74.3%
		NW	-0.638	0.431	0.770	92.4%	83.0%	75.6%

Table 3: Monte Carlo simulations: Return predictability. Homoscedastic error terms. For each estimator, the bias in the estimate, its standard error, its root mean square error and true confidence levels of 99%, 95% and 90% are shown.

Obs.	k	Variance Est.	Bias	Std.	RMSE	99%	95 %	90 %
			Overlapping Regression					
250	3	OLS	-0.051	0.005	0.052	82.9%	70.1%	61.7%
		White	-0.052	0.006	0.052	82.0%	69.2%	60.8%
		NW(k)	-0.028	0.015	0.032	94.7%	86.3%	78.9%
		NW	-0.022	0.018	0.028	96.0%	88.7%	81.9%
		HH	-0.023	0.018	0.029	95.7%	88.1%	81.2%
			Transformed Regression					
		OLS	-0.033	0.007	0.033	94.3%	85.3%	77.5%
		White	-0.033	0.009	0.034	94.0%	84.9%	77.0%
		NW	-0.019	0.018	0.026	96.4%	89.6%	83.1%
			Overlapping Regression					
100	12	OLS	-1.656	0.200	1.668	68.0%	55.0%	46.9%
		White	-1.690	0.202	1.702	64.7%	51.7%	44.3%
		NW(k)	-1.030	0.753	1.276	87.9%	77.2%	69.2%
		NW	-1.188	0.567	1.316	85.7%	74.4%	66.3%
		HH	-1.184	0.574	1.315	85.8%	74.4%	66.4%
			Transformed Regression					
		OLS	-1.072	0.272	1.106	92.3%	81.8%	73.7%
		White	-1.103	0.307	1.145	91.2%	80.6%	72.6%
		NW	-0.919	0.546	1.069	92.3%	82.8%	75.3%

Table 4: Monte Carlo simulations: Misspecification. For each estimator, the bias in the estimate, its standard error, its root mean square error and true confidence levels of 99%, 95% and 90% are shown.

Table 2 reports results from simulations where the errors are conditionally heteroscedastic. The presence of heteroscedasticity has a clear effect, significantly worsening the performance of the OLS covariance estimates obtained from the transformed regression. Here, the White (heteroscedasticity consistent) covariance estimate obtained from the transformed regression performs very well and clearly better than the Hansen-Hodrick estimator in the overlapping regression. The Newey-West estimator applied to the transformed regression is again inferior to the White estimator, since there is no autocorrelation structure left that may be captured by the Newey-West estimator.

Table 3 reports results from simulations where the regressors and errors follow the same processes as in Table 1, but the actual returns are predictable with a one-period ahead R^2- of 20% percent. The results are broadly similar to those in Table 1. Again the procedures based on the transformed regression perform best, and the best performing estimator is again OLS applied to the transformed regression. Note however that when returns are predictable, the OLS covariance estimator applied to the transformed regression is biased down as it ignores the serial correlation in one-period returns due to the predictability of returns. As in Table 1, the Newey-West estimator applied to the transformed regression is inferior to the OLS estimator in respect of both noise and bias.

The data generating processes simulated so far have a noise term that is serially uncorrelated. Where there is some correlation structure in the noise, our approach should be helpful in stripping out the autocorrelation induced by the overlapping scheme, making it easier to account for any remaining underlying autocorrelation in estimating the standard error of the parameter estimates. A very common cause of autocorrelated noise process is the use of a misspecified regression model. To examine this case, we rely on the same data generating model as in 3), but use a false regressor in the regression model.

The results, in Table 4, confirm our intuition. As in the previous tables, the covariance estimators from the overlapping regression are severely biased downwards, with the use of Newey-West and Hansen-Hodrick reducing but not eliminating the bias. The bias and coverage ratios are all much worse than in Table 1, the base case, because of the autocorrelation in the error term. Again, the covariance estimators from the transformed regressions perform better than ones from the original regressions, but the autocorrelation in the error term induces significant bias in the OLS and White estimators. The biases are reduced by the use of Newey-West, and the coverage ratios are closer to their nominal levels.

The broad conclusions drawn from these tables seem to be robust to the choice of parameters. In particular, if the regressors are less persistent (AR parameter of 0.1 rather than 0.8) simulations (not reported here) also show that the procedures performed on the transformed regressions work best, with the Newey-West estimator being the best in presence of remaining autocorrelation, the White estimator being the best in the presence of heteroscedasticity and the OLS estimate being best otherwise.

4 Review of Two Financial Studies

Overlapping regressions have been central to the debate over the predictability of stock market returns (Fama & French (1988); Campbell & Shiller (1988)). To illustrate the relevance of our approach we conduct two analyses using real rather than simulated data. In the first we re-examine the issue of the predictability of long-horizon US stock market returns using Robert Shiller's data on stock market returns and earnings. In the second we illustrate our approach to Fama-MacBeth regressions by looking at the predictability of relative country stock returns. Although we introduced our methodology only in the context of plain linear regression it is straightforward to extend it to the Fama-MacBeth panel regression framework.

It should be emphasised that the purpose of this analysis is to illustrate the approach to overlapping regressions we have developed in this paper rather than to cast new light on the debate over the predictability of stock prices. In particular, we have not attempted to allow for other econometric issues raised by the use of a highly persistent regressor, or the joint endogeneity of the dependent and independent variables. Here, we only show the effect our methodology in terms of efficiency gains for standard errors under the same assumptions under which they were originally derived in the literature.

4.1 US Stock Market Predictability

Taking data from Robert Shiller's website (http://www.econ.yale.edu/ shiller/ data.htm) for annual US stock returns (S&P 500) and price-earnings ratios from 1871 to 2008, we estimate the regression

$$r_{t,t+k} = \beta r_{t-k,t} + u_{t+k}$$

where $r_{t,t+k}$ is the k period log real return from t to $t+k$. The results are set out in Table 5. Where the dependent variable is the ten-year return ($k = 10$), the standard approach, with either Newey-West or Hansen-Hodrick estimates of the covariance matrix, leads to severe underestimates of the standard error, with corresponding over-estimates of the t-statistics, in comparison to the analysis based on the transformed regression. This bias is observable also in each of the sub-periods. The coefficient on the lagged return, which appears to be significantly negative over the whole period and in the first half period, is indistinguishable from zero when using the transformed regression. For the five year return, the position is broadly similar except that the coefficient is not significantly different from zero in either the standard or the transformed regression except when looking at the first half of the period.

We now examine the predictability of long-period returns from the price earnings ratio. We consider the regression

$$r_{t,t+k} = \beta_1 X_t + \beta_2 r_{t-k,t} + u_{t+k}$$

where $r_{t,t+k}$ is the k period log real return from t to $t+k$ and X_t the year t ratio of price to smoothed earnings. The results are set out in Table 6. The results are similar to the previous regression in that the standard approach, with either Newey-West or Hansen-Hodrick estimates of the covariance matrix, leads to severe underestimates of the standard error, with corresponding over-estimates of the t-statistics, in comparison to the analysis based on the transformed regression. This bias is observable also in each of the sub-periods. 'n/a' indicates cases in which the estimated Hansen-Hodrick covariance matrix is not positive definite. The coefficient on the price earnings ratio is significantly negative at conventional significance levels both over the period as a whole, and in the second half, under both the standard approach and the transformed regression, but the standard errors are roughly doubled. This holds true both for five and ten year rolling returns. The coefficient on lagged returns, which appears to be significantly positive in the second half of the period for both 5 and 10 year returns, and to be significantly negative in the first half for 10 year returns, turns out to be insignificantly different from zero when using the transformed variables. In almost every transformed regression we observe the tendency that the Newey-West t-statistics are slightly higher than the White ones, indicating that they are picking up some remaining autocorrelation pattern left in the non-overlapping error terms.

Dependent variable	Variance Estimator	1881-2008 $\hat{\beta}$	1881-2008 t-stat	1881-1944 $\hat{\beta}$	1881-1944 t-stat	1945-2008 $\hat{\beta}$	1945-2008 t-stat
10-year log			Overlapping Regression				
real return	NW(k)	-.299	-2.70	-.489	-3.40	-.223	-1.60
	NW		-2.36		-3.10		-1.36
	HH		n/a		-5.07		n/a
			Transformed Regression				
	White	-.299	-1.37	-.489	-1.24	-.223	-0.88
	NW		-1.41		-1.23		-1.02
5-year log			Overlapping Regression				
real return	NW(k)	-.123	-0.89	-.368	-2.68	.138	0.66
	NW		-0.89		-2.68		0.66
	HH		-0.85		-3.49		0.64
			Transformed Regression				
	White	-.123	-0.65	-.368	-1.33	.138	0.54
	NW		-0.67		-1.61		0.55
1-year log			Standard Regression				
real return	White	.034	0.35	0.001	0.01	.077	0.49
	NW		0.41		0.01		0.73

Table 5: Estimation results for regression $r_{t,t+k} = r_{t-k,t}\beta + u_{t+k}$, where $r_{t,t+k}$ is the k-year log real return on the S&P 500 index. The estimate of beta and its t-statistic are shown for 10, 5 and 1 year returns, for a variety of different time periods. The estimates based on overlapping regressions apply OLS to the data as is, and use Newey-West and Hansen-Hodrick to estimate the covariance matrix. The transformed regression uses the methodology described in this paper and uses White and Newey-West estimates for the covariance matrix. The last row of each panel shows the non-overlapping case where annual returns are regressed annually. The data are from http://www.econ.yale.edu/ shiller/data.htm.

Dependent variable	Variance Estimator	1881-2008		1881-1944		1945-2008	
Panel A: Coefficient on the price earnings ratio (β_1)							
		$\hat{\beta}_1$	t-stat	$\hat{\beta}_1$	t-stat	$\hat{\beta}_1$	t-stat
10-year log		Overlapping Regression					
real return	NW(k)	-.057	-3.09	-.012	-0.82	-.079	-3.16
	NW		-3.28		-0.89		-3.28
	HH		-2.84		-0.81		-3.34
		Transformed Regression					
	White	-.057	-2.13	-.012	-0.20	-.079	-2.52
	NW		-3.14		-0.31		-3.50
5-year log		Overlapping Regression					
real return	NW(k)	-.028	-3.58	-.036	-1.51	-.032	-5.95
	NW		-3.47		-1.51		-5.95
	HH		-3.07		-1.34		-6.53
		Transformed Regression					
	White	-.028	-2.27	-.036	-1.26	-.032	-2.27
	NW		-2.67		-1.12		-3.38
1-year log		Standard Regression					
real return	White	-.006	-2.73	-.013	-2.51	-.005	-2.02
	NW		-2.29		-2.12		-2.08
Panel B: Coefficient on lagged returns (β_2)							
		$\hat{\beta}_2$	t-stat	$\hat{\beta}_2$	t-stat	$\hat{\beta}_2$	t-stat
10-year log		Overlapping Regression					
real return	NW(k)	.156	0.99	-.394	-2.42	.414	1.99
	NW		0.96		-2.43		1.96
	HH		1.15		-3.06		2.58
		Transformed Regression					
	White	.156	0.52	-.394	-0.57	.414	1.24
	NW		0.84		-0.83		1.76
5-year log		Overlapping Regression					
real return	NW(k)	.150	1.05	-.056	-0.26	.472	2.87
	NW		1.03		-0.26		2.87
	HH		0.93		-0.28		2.37
		Transformed Regression					
	White	.150	0.65	-.056	-0.14	.472	1.69
	NW		0.72		-0.16		2.17
1-year log		Standard Regression					
real return	White	.090	0.92	.120	0.91	.115	0.74
	NW		1.10		0.90		1.07

Table 6: Estimation results for regression $r_{t,t+k} = X_t\beta_1 + r_{t-k,t}\beta_2 + u_{t+k}$, where $r_{t,t+k}$ is the k-year log real return on the S&P 500 index and X_t is the ten year rolling price earnings ratio. The estimate of beta and its t-statistic are shown for 10, 5 and 1 year returns, for a variety of different time periods. The estimates based on overlapping regressions apply OLS to the data as is, and use Newey-West and Hansen-Hodrick to estimate the covariance matrix. The transformed regression uses the methodology described in this paper and uses White and Newey-West estimates for the covariance matrix. The last row of each panel shows the non-overlapping case where annual returns are regressed annually. The data are from http://www.econ.yale.edu/ shiller/data.htm.

4.2 Country Stock Returns

Since our approach can easily be generalized to Fama-MacBeth panel regressions we illustrate its implications by analyzing return predictability in international equity indices. Richards (1997) documents reversal in the relative returns of international equity indices. Countries that have done relatively well in the past period tend to under-perform their peers in the future. The reversal is strongest at the three year horizon. The finding is confirmed by Balvers, Wu & Gilliland (2000).

A natural way of exploring the predictability of relative country returns at different horizons is to follow the Fama-MacBeth procedure using country stock indices as the assets. Specifically we run the following cross-sectional regressions for every month t

$$r_{t,t+k} = r_{t-k,t}\beta_t + u_{t+k}, \tag{12}$$

where $r_{t,t+k}$ is the vector of k-month returns across different countries from month t to $t+k$. We then test whether the estimated slope coefficient differs from 0. Like Richards (1997), the country returns are the MSCI equity index returns less the return on the US market, taken from Datastream. The period is January 1982 to May 2007, and the countries are Austria, Australia, Belgium, Canada, Denmark, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, Norway, Singapore, Sweden, Switzerland, and the UK. The results are shown in Table 7.

The point estimate of beta is positive at the one year horizon. This is consistent with the findings of Bhojraj (2006), and suggests some momentum in returns at shorter horizons. The value of beta goes negative at longer horizons, taking its largest negative values at around the six year horizon. According to the untransformed regression, the beta is significantly less than zero for horizons of four years or more (the reason that no Hansen-Hodrick t-statistics are available beyond 6 years is that the covariance matrix is not positive definite). According to the transformed regression with simple OLS standard errors, however, the positive beta at the one year horizon is just significant, but at all other horizons the beta does not significantly differ from zero. Using Newey-West standard errors in the transformed regression suggests that no beta is significantly different from zero.

Horizon in years	Mean $\hat{\beta}$	t-statistic obtained from Overlapping Regression			Transformed Regression	
		NW(k)	NW	HH	OLS	NW
1	0.123	1.72	2.06	1.44	1.98	1.69
2	0.041	0.39	0.64	0.32	0.52	0.43
3	-0.083	-1.01	-1.59	-1.10	-0.84	-0.85
4	-0.179	-3.12	-4.23	-4.69	-1.59	-1.79
5	-0.172	-3.20	-5.19	-3.41	-1.25	-1.49
6	-0.240	-5.36	-9.81	-6.03	-1.61	-1.97
7	-0.212	-5.54	-5.92	n/a	-1.47	-1.89
8	-0.174	-6.85	-7.14	n/a	-1.22	-1.34
9	-0.139	-2.78	-4.04	n/a	-0.84	-0.83
10	-0.096	-7.32	-3.69	n/a	-0.50	-0.48

Table 7: The table shows the estimate of the regression coefficient of long horizon country index returns on lagged returns. The basic regression is $r_{t,t+k} = r_{t-k,t}\beta_t + u_{t+k}$ where $r_{t,t+k}$ is the vector of k-month log returns (in excess of the US log return) across 22 different countries from month t to $t+k$, and the slope parameter estimates are then pooled and tested for whether the mean ('mean beta') differs from zero. Since the regressions are done each month, the data are overlapping, so the t-statistics are adjusted for autocorrelation using a Newey-West (NW) or Hansen-Hodrick (HH) procedure. As an alternative the regression is transformed as described in the text and the standard error is calculated from the transformed regression ('transformed regression'). The data are from Datastream.

5 Conclusion

The main contribution of this paper is to introduce a simple transformation of the regressor matrix which turns a long horizon regression with overlapping observations into a short horizon regression with non-overlapping observations. This transformation greatly simplifies parameter inference. We show that standard inference techniques such as OLS, White and Newey-West parameter standard errors can be applied to the transformed regression and perform better than more sophisticated methods utilized directly with the overlapping regression. Our transformation can readily be implemented in standard software packages.

We show, using Monte Carlo studies, that our method dominates conventional techniques in all three cases of homoscedastic, heteroscedastic and autocorrelated error terms in small samples. In the cases of homoscedastic and heteroscedastic error terms our method is asymptotically equivalent to Hansen-Hodrick type estimators, but when there is residual autocorrelation (not induced by the overlapping scheme) our method dominates asymptotically as well as in small samples.

The intuition behind the efficiency gain in our method is that we explicitly account for a known deterministic aggregation pattern a priori and do not try to estimate it in a noisy way, as for example through pre-whitening or through Hansen Hodrick or Newey-West type approaches. The efficiency gain of our method in comparison to methods designed to account only for the autocorrelation induced by the overlapping scheme (Hansen-Hodrick) is higher the more autocorrelation structure is present in the error terms of the non-overlapping regression.

In this paper we introduce our methodology in the context of overlapping regressions, but the methodology is applicable to a wider range of data aggregation schemes, as long as they are deterministic and known to the researcher.

The importance of using more reliable standard errors have been shown by reviewing two important empirical studies. The transformation of a long-horizon regression into a short-horizon regression shows the limits to the statistical power of long-horizon regressions.

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