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CORRECTION

PERFECT SIMULATION FOR A CLASS OF POSITIVE RECURRENT MARKOV CHAINS

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In [1] we introduced a class of positive recurrent Markov chains, named tame chains. A perfect simulation algorithm, based on the method of dominated CFTP, was then shown to exist in principle for such chains. The construction of a suitable dominating process was flawed, in that it relied on an incorrectly stated lemma ([1], Lemma 6). This claimed that a geometrically ergodic chain, subsampled at a stopping time σ, satisfies a geometric Foster–Lyapunov drift condition with coefficients not depending on σ. This is true if σ is a stopping time independent of the chain, but *not* if this independence does not hold. Reference [1], Lemma 6 is therefore false as stated.

We now indicate a corrected construction of a dominating process. As described in [1], Section 3.1, the process D is defined by starting with a process Y and pausing it using a function S. In the following modified construction this is simplified by taking $S = F$, where F is the function taming X. We restate [1], Theorem 16, and give a shorter proof, which avoids the faulty Lemma 6 but pays a price in terms of consequences for the perfect simulation algorithm of Section 3.3. The discussion of tameness (Section 4) is unaffected.

THEOREM 16. *Suppose X satisfies the weak drift condition $PV \leq V + b\mathbf{1}_C$, and that X is tamed with respect to V by the function*

$$F(z) = \begin{cases} \lceil \lambda z^{\delta} \rceil, & z > d', \\ 1, & z \leq d', \end{cases}$$

with the resulting subsampled chain X' satisfying a drift condition $PV \leq \beta V + b'\mathbf{1}_{[V \leq d']}$, with $\log \beta < \delta^{-1} \log(1-\delta)$. Then there exists a stationary ergodic process D which dominates $V(X)$ at the times $\{\sigma_n\}$ when D moves.

PROOF. Suppose that $D_{\sigma_n} = z$, and that $V(X_{\sigma_n}) = V(x) \leq z$. We wish to show that $D_{\sigma_{n+1}}$ can dominate $V(X_{\sigma_{n+1}})$, where $\sigma_{n+1} = \sigma_n + F(z)$ is the time at which D next moves. Domination at successive times σ_j at which D moves then follows inductively. For simplicity in the calculations below we set $\sigma_n = 0$.

First choose $\beta^* > \beta$ such that

$$\log \beta < \log \beta^* < \delta^{-1} \log(1-\delta). \tag{1}$$

Our aim is to control $\mathbb{E}_x[V(X_{F(z)})]$, recalling that $F(z)$ is deterministic and that $F(V(x)) \leq F(z)$:

$$\begin{aligned}
\mathbb{E}_x\big[V\big(X_{F(z)}\big)\big] &= \mathbb{E}_x\big[V\big(X_{F(V(x))}\big)\big] + \mathbb{E}_x\big[V\big(X_{F(z)}\big) - V\big(X_{F(V(x))}\big)\big] \\
&= \mathbb{E}_x[V(X'_1)] + \mathbb{E}_x\big[V\big(X_{F(z)}\big) - V\big(X_{F(V(x))}\big)\big] \\
&\leq \beta V(x) + b'\mathbf{1}_{[V(x)\leq d']} + b[F(z) - F(V(x))] \\
&\leq \beta z + b' + b(\lambda+1)z^{\delta} \\
&\leq \beta^* z \qquad \text{for } z \geq h^*, \qquad (2)
\end{aligned}$$

where $h^* < \infty$ is a constant chosen sufficiently large for inequality (2) to hold. The first inequality in this sequence holds due to the drift conditions satisfied by X' and X. The second follows from the definition of F and the assumption that $V(x) \leq z$.

Now define the process $Y = h^* \exp(U)$, where U is the system workload of a $D/M/1$ queue with arrivals every $\log(1/\beta^*)$ time units and service times being independent and of unit Exponential distribution. As in the original proof of Theorem 16, Y may be paused using F to obtain the process D which is positive recurrent and has a proper equilibrium distribution by virtue of inequality (1).

Finally, observe that D takes values in $[h^*, \infty)$. As in the proof of Theorem 5 of [2], it follows from inequality (2) that $V(X_{F(z)})$ can be dominated by $D_{F(z)}$, as required. □

The majority of Section 3.3 remains valid when the dominating process is constructed as above. The only issue is that by taking $S = F$ in this new method we are no longer assured that $S(h^*) \geq m$, where the set $C^* = \{x : V(x) \leq h^*\}$ is m-small. Unfortunately, there no longer seems to be a simple way to ensure this since our attempts to increase S in the above always result in an increased value of h^*.

If it so happens that $F(h^*) \geq m$ for a given chain, then the original perfect simulation algorithm remains unchanged. If this is not the case, then the algorithm must be altered. It now becomes necessary, when $D_0 = h^*$, for D to dominate $V(X)$ not at time $\sigma_1 = F(h^*)$ but at time

$$\sigma^* = \inf_{j\geq 2}\{\sigma_j : \sigma_j \geq m\}.$$

This is an example of the composite nondeterministic sampling schemes we had originally hoped to avoid (cf. the comment before [1], Theorem 15]). Furthermore, we need to be able to couple target chains and dominating process at σ^* in such a way that the target chains may regenerate at this time (using the fact that C^* is σ^*-small). This unfortunately reduces the impact of the result, which is an issue that we are currently trying to resolve.

REFERENCES

[1] CONNOR, S. B. and KENDALL, W. S. (2007). Perfect simulation for a class of positive recurrent Markov chains. *Ann. Appl. Probab.* **17** 781–808.

[2] KENDALL, W. S. (2004). Geometric ergodicity and perfect simulation. *Electron. Comm. Probab.* **9** 140–151. MR2108860

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