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The Ins and Outs of UK Unemployment

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Abstract

This paper shows that in the UK, increases in unemployment in a recession are driven by rises in the separation rate. A new decomposition of unemployment dynamics is devised that does not require unemployment to be in steady state at all times. This is important because low UK transition rates – one quarter the size of the US – imply substantial deviation of unemployment from steady state near cyclical turning points. In periods of moderation, the job finding rate is shown to have most influence on UK unemployment dynamics. Evidence comes from the first study of monthly data derived from individuals’ labour market spells recorded in the British Household Panel Survey from 1988 to 2008.

Keywords: Unemployment dynamics, Job finding rate, Separation rate

JEL classification: E24, E32

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1 Introduction

Do inflows or outflows drive unemployment dynamics? Hall’s (2005) and Shimer’s (2005, 2007) claims that job finding dominates and separations have no cyclical impact have inspired both development of labour market theories with an acyclical separation rate (Blanchard and Gali, 2008; Gertler and Trigari, 2009) and several further empirical investigations using US data. Fujita and Ramey (2009), using Current Population Survey gross flows data, and Elsby, Michaels and Solon (2009), using duration-based data, arrive at the contrary finding that the separation, or inflow, rate accounts for just under half of unemployment variance and leads cyclical changes in unemployment.¹ For the UK, Petrongolo and Pissarides (2008) followed a similar analysis using quarterly Labour Force Survey data over a period of ‘great moderation’ between 1993 and 2005, also finding that the inflow rate explained around half of unemployment dynamics. Previous UK research had attributed most unemployment changes to inflow shocks (Burgess and Turon, 2005).

This paper contributes the first empirical analysis of monthly gross flows data created from the British Household Panel Survey. These data cover the start of the 1990s recession, which is important to investigate unemployment over the cycle, although the latest usable data do not cover the recession of 2008-09. The data allow unemployment dynamics to be investigated in a three-state model involving flows to and from inactivity as well as between unemployment and employment.

Several recent papers calculate the relative importance of changes in inflow and outflow rates to equilibrium unemployment dynamics (Shimer, 2007; Fujita and Ramey, 2009; Petrongolo and Pissarides, 2008; Gomes, 2009). My first pieces of evidence for the UK show that this might be misleading: actual unemployment deviates from equilibrium, particularly around cyclical turning points. In the early 1990s recession, the equilibrium unemployment rate peaked five quarters before the actual unemployment...

¹Yashiv (2007) surveys and analyses data used by Shimer (2007) and a variety of previous studies.
ment rate (Figure 2 graphs these). As noted by Hall (2005), this indicates that past changes in transition rates are relevant to current unemployment developments. This occurs because inflow and outflow rates are relatively low in the UK: only 11% of the unemployed exited unemployment each month, on average between 1998 and 2008 – and only 0.6% of workers lost their jobs each month, according to BHPS data.\(^2\)

In this paper I present an alternative decomposition based on actual unemployment. This ‘dynamic’ decomposition allows past changes in inflow and outflow rates to influence current actual unemployment. Elsby, Hobijn and Sahin (2009) present a dynamic decomposition of (log) actual unemployment that is applicable to a ‘two-state’ world in which workers can either be employed or unemployed. The decomposition in section 3.2 covers a ‘three-state’ world in which workers can also be out of the labour force (inactive). A disadvantage of Elsby, Hobijn and Sahin’s use of log unemployment changes – and one of the major motivations for this paper’s development of a dynamic decomposition based on a non-log change – is the difficulty of further decomposing the influence of log outflow and inflow rate changes to disentangle the impact of changes in job finding and separation rates from the effect of unemployment exits to and accessions from inactivity. This is potentially important in the context of the effects of policy measures to aid job finding (such as the introduction in the UK of Jobcentre Plus in 2002), since this type of measure might move the outflow rates from unemployment to each of employment and inactivity in opposite directions.

Allowing for the relative importance of inflow and outflow rates to vary over time reveals that, since 1989, rising unemployment has generally been associated with separation rate changes (a higher rate of job loss). But there is a lengthy period between the mid-1990s and early 2000s when unemployment movements are

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\(^2\)These figures refer to overall inflow and outflow rates – the sum of direct flows between unemployment and employment and indirect flows involving inactivity – between 1988q4 and 2008q2. (These overall flow rates are defined formally in Section 3.) The comparable US figures over 1988q4 to 2007q2 are 50% for the monthly outflow rate and 3% for the inflow rate, according to my calculations using Shimer’s (2007) raw transition probabilities based on CPS gross flows data. I thank Robert Shimer for making his data available at http://sites.google.com/site/robertshimer/research/flows (see Shimer, 2007).
dominated by changes in the rate of job finding.

2 Data

The numbers of individuals moving between labour market states in the UK are measured using data from the British Household Panel Survey, which has interviewed several thousand individuals each year since 1990. I use weights to ensure that the sample remains representative of Britain (below the Caledonian Canal in Scotland). Because this paper is the first to use the BHPS to estimate the cyclical properties of gross flows, in this section I explain how the estimates are constructed, illustrate the estimated transition rates and describe their co-movement with unemployment. In section 2.1 I discuss important data-related issues and compare the BHPS with the other data source that can be used to create three-state transition rates for the UK, the Labour Force Survey (LFS).³

The BHPS is an annual survey, but in principle all labour market spells are captured through recalled job history. The main source of BHPS data used here is the 1990-91 to 2008-09 panel (Waves A to R, using the ‘Employment’ and ‘Employment history’ sections of the individual interview questionnaire). In addition, I use post-1988 information from the (largely) pre-panel labour market and job histories recorded in Waves B and C.⁴ Individuals are asked for the day, month and year of any status change.⁵ All individuals aged 16 and above are included. I use Paull’s

³A comparison of BHPS and LFS transition rates is included in Appendix A to this paper. For information about how UK inflow and outflow rates compare with other countries’, see Elsby, Hobijn and Sahin (2009), who calculate transition rates for fourteen OECD countries based on a two-state model using aggregate duration-based unemployment data, rather than the micro data used here. UK transition rates lie roughly mid-way between those of continental European countries (low) and other Anglo-Saxon and some Scandinavian countries (high).

⁴Previous studies (Elias, 1986; Paull, 2002) have indicated that recall bias over a two-year period is not generally significant.

⁵Prior to the introduction of ‘dependent interviewing’ in Wave P, each year the respondent was asked their current labour force status and start date. If the status started after the start of interviewing for the previous wave (1 September in the previous year) the individual was asked about all labour force statuses and start dates backwards from the current spell to 1 September the previous year. From Wave P onward, with dependent interviewing, the information obtained is essentially identical, but individuals are now reminded what they were doing at the last interview,
(2002) method of reconciling data inconsistencies between waves (this reconciliation
method is discussed further in section 2.1).

131,906 spells across 31,216 individuals are allocated to months between July 1912
and April 2009. Although these data could in principle generate almost a century
of transition rates, in the earlier years there are very few individuals observed, with
limited transitions and many gaps in the data. More recent years, before data based
on the usual one-year recall of the panel survey itself begin in September 1990, must
also be treated with care, for two reasons: recall error becomes increasingly likely the
further back one goes; and there are no weights provided to make pre-panel years
representative. Another restriction on the usable sample arises at the end: from the
start of the latest survey period (September 2008), transition data are unreliable,
since there is no information on any individual’s labour market status after their
interview. The sample must therefore be curtailed before the third quarter of 2008.

I aggregate labour force statuses as coded in the BHPS into the categories typically
used in three-state studies of unemployment dynamics. The advantage of using three
states to model unemployment dynamics lies in accurately capturing the influence
of flows between employment and unemployment, and in being able to discern the
separate influence of flows involving inactivity. These distinctions are missed by
many non-micro data sources, because they allow only two states to be examined
(employment and not in employment).6

Employment $E$ includes full- and part-time employment, self-employment, ma-
ternity leave and government training schemes (64% of the BHPS adult sample).7
Unemployment $U$ simply includes unemployment (4% of adults).8 Inactivity $I$ in-
and if it has changed they are asked to go forward from their last interview listing statuses and start
dates.

6In the UK, this applies to Claimant Count data and LFS data released by the ONS that is
disaggregated by duration.
7All statistics calculated from BHPS data are weighted unless stated otherwise. Figures relate
to all individuals aged 16 and over during September 1990 to August 2008.
8Unemployment is self-reported – “unemployment” is given as one of range of possible labour
market statuses. Therefore it does not formally match the recognised ILO definition ("not employed,
available and looking for work, or waiting to start a job"). It also does not imply that the individual
cludes retirement, family care, long-term sick or disabled, full-time study, national or war service, and anything else (32% of adults). Within the panel period (September 1990 to August 2008) there are 3.1 million (unweighted) $E, U$ or $I$ individual-month observations, 1.8 million with non-zero weight.

Two consecutive monthly status observations for an individual constitute a ‘match’. I obtain 1,527,559 matches within the main panel – 7,072 matches per month, on average. 21,440 of these matches are ‘transitions’ involving a change of status between $E, U$ or $I$ (so I observe on average about 100 transitions per month). Transitions between months are summed (using weights) to give the various (weighted) flows.

Monthly flow rates are calculated as the sum of the relevant weighted flows between period $t - 1$ and $t$ divided by the weighted sum of the number of observations on the relevant status at $t - 1$. For example, the flow rate between unemployment and employment is calculated as $UE_t/U_{t-1}$, where $UE_t$ is the number of $U \rightarrow E$ flows between $t - 1$ and $t$. Flow rates are individually seasonally adjusted using the Census Bureau’s X12 program. I treat the flow rates as measures of the monthly probability of making the relevant transition (the assumptions behind this are discussed below). Quarterly averages of monthly probabilities are used. Transition rates are then calculated. For example, the job finding rate $\lambda^UE_t = - \ln(1 - UE_t/U_{t-1})$.

Figure 1 plots monthly separation and job finding rates measured using BHPS data (annual averages are taken to smooth out noise resulting from the relatively small sample size). The data span the 1990s’ recession but, because they end in the is claiming unemployment-related benefit (which underlies the definition of ‘Claimant Count’ unemployment in the UK). The BHPS does include information about search activity and benefit receipt, but that is not used here.

The term ‘match’ has previously been used in relation to CPS data (notably by Shimer, 2007), where – as in the UK LFS – researchers must create their own person-level identifiers to match individuals from one period to the next, and individuals – by design – remain in the survey for few observations (two sets of four months in the case of the CPS, and five quarters for the LFS). Because the BHPS is constructed as a panel, individual identifiers are already provided, and individuals are observed in all months since they were first surveyed (and since they left school, using recall data). Nevertheless, the algorithm to match status observations on individuals across months (with potentially missing data) is the same.

This noise does not appear to be problematic in conducting steady state decompositions: as will be seen, the subject of this decomposition – the steady state unemployment rate – is itself
second quarter of 2008, they miss the upturn in actual unemployment that began in mid-2008.

Fig. 1. UK separation and job finding rates

Notes. Annual averages of monthly data. Job finding rate is the $U \rightarrow E$ transition rate $\lambda^{UE}$. Separation rate is the $E \rightarrow U$ transition rate $\lambda^{EU}$. Source: BHPS Waves 1 to 18, 1988q4-2008q2.

BHPS estimates suggest that the separation rate rose from a very low level of 0.2% before the 1990-91 recession to a peak of just over 0.8% in 1991, after which it declined to about 0.4% in 1999, the level about which it has subsequently fluctuated. Constructed from these transition rates. To counter the effects of noise on the (non-steady state) decomposition of the actual unemployment rate, I will focus at that point on annual, rather than quarterly, changes.
The countercyclicality of the separation rate appears to contradict the Hall-Shimer claim that it is comparatively acyclic.

Surprisingly, during the recession of 1990q3-1991q3, the job finding rate does not appear notably procyclical, according to BHPS data. It is already possible to anticipate results confirmed below: the rise in unemployment in the early 1990s recession is more than accounted for by increases in the inflow rate. Subsequently, the job finding rate exhibited the expected procyclicality: the gradual decline in the unemployment rate during the 1990s was mirrored by a rise in the job finding rate from approximately 7.5% after the recession to about 12% in the late 1990s. During the 2000s, the upticks in unemployment are generally matched by dips in the job finding rate, which also shows a gentle secular decline over that period.

Figure 2 plots estimates of UK transition rates involving inactivity, calculated from BHPS data. Panel (a) features a prolonged rise in the unemployment to inactivity transition rate during the 1990s, consistent with the large increase in numbers on inactivity related benefits during that period. In the 2000s there are notable dips and rises in the UI transition rate, in some cases consistent with countercyclicality in the unemployment exit rate to inactivity, but the cyclical match is not very close in the last few years of the sample. The transition rate from inactivity to unemployment (panel (c)) also appears somewhat countercyclical, although again turning points in the IU transition rate do not consistently coincide with unemployment movements.

BHPS estimates of the transition rate from inactivity to employment (panel (d)) display some similarity in movement with the job finding rate (see Figure 1), although there is an additional notable ‘hump’ covering 2000-07. The ‘hump’ somewhat mirrors unemployment movements in the period when unemployment declined to stabilise around 4-5% and then rose in 2007. During the period of low unemployment, a higher proportion of individuals joining the labour force were able to find a job, so the IE transition rate rose; it subsequently fell as the labour market began to tighten. A ‘hump’ during the 2000s is also a feature of the transition rate from employment to

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11 Before that recession (at the start of the sample), there is a rise in the job finding rate, a feature common to other UK data sources (transition rates calculated from recall data from the annual LFS by Elsby, Smith and Wadsworth, 2010, and Claimant Unemployment overall outflow rates from Petrongolo and Pissarides, 2008). However, both these sources indicate a fall in the job finding rate over the next two years, whereas it remains relatively static according to BHPS data.
inactivity (panel (b)). One possible rationalisation might be that workers were more confident to leave the labour force (temporarily) during a time of low unemployment. Another feature of the $EI$ transition rate is countercyclicality in the 1990s recession: more workers exited to inactivity as well as to unemployment during that downturn.

Fig. 2. Transition rates involving inactivity

Notes. Annual averages of monthly data. Source: BHPS Waves 1 to 18, 1988q4-2008q2.

Section 3 will formally calculate the relative importance of these transition rate changes for unemployment dynamics. Before then, I highlight a number of problems that need to be faced when estimating transition rates and briefly discuss advantages and disadvantages of using the BHPS in relation to other datasets.
2.1 Data issues

This paper uses the BHPS rather than the LFS. What are the pros and cons of each dataset? The LFS has a much larger sample size: whereas the BHPS sample that is representative of the UK population numbers between 7,500 and 10,000, LFS estimates could be drawn from a sample of between 75,000 and 115,000 individuals. The advantage of frequency of observation goes to the BHPS, however: despite being an annual survey, estimates can be constructed on a monthly basis, whereas the LFS is restricted to quarterly frequency.

Sample design also favours the BHPS, in at least four ways. First, the BHPS ‘following rules’ ensure that individuals are tracked as they move between households and move address. In contrast, the LFS samples by address and will therefore miss transitions associated with house or household moves. Second, the BHPS follows individuals over a much longer timespan. Whereas the LFS tracks individuals over five quarters, enabling four data points on labour force flows per individual, the BHPS has followed some individuals continuously from 1990 (others have been added over the 18 waves of the BHPS, since the ‘following rules’ entail interviewing all of a new household formed by original sample members, and also following offspring of original household members, keeping the sample reasonably representative). Third, the level of proxy responses in the LFS (32% in 2010q2) is much higher than in the BHPS (just over 1% of all interviews in Wave 18); the BHPS attempts to interview all adult members of households.\(^\text{12}\) Fourth, whereas in the BHPS all individuals are interviewed face-to-face and separately (where possible), in the LFS, after the first face-to-face interview, the four subsequent interviews are carried out by telephone. The BHPS has only a slight advantage over the LFS in terms of response: the LFS response rate was 55% over the five waves in 2010q2, compared to an average reinterview rate (for full interviews) in BHPS Wave 18 of around 57% of those that the BHPS attempted to interview in Wave 17. In both surveys, response rates have been declining over time.

I turn now to particular problems that can affect estimates derived from micro

\(^{12}\)Details of the LFS survey are taken from the LFS Performance and Quality Monitoring Report, published each quarter by the ONS. Details for the BHPS are taken from Taylor et. al. (2010).
data, beginning with time aggregation error. If individuals can both find and lose a job within a month, discrete data will yield biased measures of underlying instantaneous transition rates. Consider, for example, the flow of individuals observed in discrete data as moving from unemployment to employment. This will incorrectly measure the continuous-time transition rate $\lambda_{t}^{UE}$. The measured $UE$ flow over $t$ will include as $U \rightarrow E$ movers those who went from unemployment via inactivity to employment ($U \rightarrow I \rightarrow E$), and will exclude those with short employment spells who went from unemployment to employment but then (also in the time between discrete observations) left employment for another state ($U \rightarrow E \rightarrow U$ or $U \rightarrow E \rightarrow I$). $UE$ flows measured from discretely-observed data thus reflect the true continuous time transition rate and these two different sources of error.

US research indicates that although the level of turnover is affected by the time aggregation bias (Shimer, 2007; Elsby, Michaels and Solon, 2009; Fujita and Ramey, 2009), the cyclicality of turnover is hardly affected (Fujita and Ramey, 2006). According to Nekarda (2009), this arises because although time aggregation in both inflows to and outflows from unemployment co-moves positively with the business cycle, these effects roughly offset each other. The absence of cyclical bias from time aggregation is confirmed in previous work using UK LFS data: Petrongolo and Pissarides (2008) report little difference even with (two-state) quarterly data between steady-state unemployment decomposition results using transition rates corrected for time aggregation and uncorrected estimates. This suggests that time aggregation bias is not very important for the analysis of this paper. In fact, BHPS data suggest intra-month transitions might be rare in the UK: the weighted total is just 379 during the BHPS panel period September 1990 to August 2008. This is 1.8% of the 21,440 weighted total transitions during the period. Therefore I aggregate states observed in raw BHPS data to the monthly frequency and do not correct further for time aggregation.

There are several further issues to consider when using survey data to measure labour market dynamics. The most important are probably recall error and classification error (when a state is wrongly reported, leading to spurious transitions). These affect most sources of micro data on labour force status, but to differing ex-
tents. Because the BHPS is an annual survey, recall error might be more substantial than in quarterly LFS data (or in monthly US CPS data). But the same infrequency of observation and reliance on recalled data should mean that classification error is lower in the BHPS, since classification error will affect the BHPS estimates only once a year, at the time of interview.

I use a method of reconciling data across waves devised by Paull (2002) which tries to reduce remaining classification error. This method gives preference to information from the interview closest to the relevant period, but in cases where this information is inconsistent with the overall history, information from later interviews is used instead, in an attempt to minimise spurious transitions. In Appendix B I describe the reconciliation method in more detail and show that the pattern of inflow and outflow rate contributions to unemployment dynamics is not sensitive to the method of constructing labour force histories.\(^\text{13}\)

The fact that the design of the BHPS enables later information to be used to minimise spurious transitions is a further advantage over the LFS, where analysis is restricted to closest interview-derived data because the LFS lacks the recall data needed to assess and eliminate inconsistencies within labour force histories. The use of later recalled information to eliminate spurious transitions information could in principle induce further recall error, but this is probably minimised by requiring that the information is only used when the resulting labour force history appears consistent.

Although Poterba and Summers’ (1986) CPS study indicated an upward bias in labour turnover from spurious transitions, Paull (2002) and Elias (1996) both show that for the BHPS there is no bias in aggregate transition rates. In Appendix B, I also find no apparent cyclical bias from spurious transitions. Some additional evidence comes from Fujita and Ramey (2006), who applied Poterba and Summers’ (1986)\(^\text{13}\) In Appendix B I examine transition rates generated through four different methods of constructing labour force histories and find, consistent with Paull’s (2002) study, that the reconciliation method used in this paper produces quite similar findings to the closest-interview method. Both these methods produce somewhat higher levels of transition rates than alternative methods in which all problematic observations are discarded or preference given to recalled data. However, the cyclical pattern of transition rates is very similar no matter what method is used, as is the decomposition of unemployment dynamics.
estimates of classification error (using fixed correction factors based on Poterba and Summers’ analysis of reinterview surveys during January to June 1980) and found that the cyclicality of transition rates from CPS data was hardly changed.

How serious is recall error in the BHPS? First, some recall errors can be eliminated by ensuring that reports remain consistent if the same period is recalled in various interviews. Second, recall error in the BHPS has been studied by Elias (1996) and Paull (2002), who both reach the comforting conclusion that recall error is not bad over a period of around three years, but can have severe biasing effects over longer periods. For this reason, the sample used here is restricted to start two years prior to the first wave of interviewing.

Abowd and Zellner (1985), among others, noted a further problem with gross flows data: that they are not missing at random. Through its following rules, the BHPS should already feature lower selection bias than fixed-address surveys such as the LFS. The method I use to reconcile data across waves attempts to reduce selection bias further, using a series of rules to impute states and dates in certain cases, where this seems reasonable. This method increases sample size, particularly drawing in individuals with very dynamic labour market behaviour (typically younger workers), who are otherwise most likely to be ruled out due to inconsistencies and missing data.

Finally, ‘margin error’ arises when an individual surveyed in the past has no current observation. This paper follows the usual ‘missing at random’ correction procedure (employed by Poterba and Summers, 1986; Shimer, 2007; Petrongolo and Pissarides, 2008, among others), whereby measured transitions are simply reweighted to match the observed population distribution.

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14 The prime-age women subgroup is most prone to recall error, with unemployment spells being understated.

15 Some previous research has used an alternative procedure, estimating weights that minimise distance between the stocks implied by CPS micro data and the aggregate totals (also based on the CPS) published by the Bureau of labour Statistics. These weights can be constant (Abowd and Zellner, 1985) or time-varying (Fujita and Ramey, 2006).
3 Do ins or outs win?

The main aim of this paper is to describe the contributions of transition rates relating to unemployment inflows and outflows to UK unemployment dynamics: to measure whether the inflow rate or outflow rate is most important. The data used enable a further decomposition to be made, to measure the contributions of the theoretically-important job finding and separation rates.

Previous work by Elsby, Michaels and Solon (2009), Petrongolo and Pissarides (2008) and Fujita and Ramey (2009) has calculated the relative contributions of inflow and outflow rates using a neat decomposition based on the identity describing the dynamics of unemployment assuming no deviations from steady state. In this section, I will show how this decomposition relates to a simple description of unemployment dynamics in a three-state model and present estimates for the UK based on BHPS data.

Acknowledging that these estimates are only valid if unemployment is closely approximated by its steady state level, in the subsequent section I will elaborate on evidence that this approximation might not hold for the UK. The unemployment rate will be close to steady state only when transition rates are high – in countries such as the US, where the sum of monthly inflow and outflow rates averages over 50%.\textsuperscript{16} But for the UK, where monthly transition rates sum to only 12% on average,\textsuperscript{17} unemployment will deviate noticeably from steady state. Therefore, in section 3.2 I will propose a ‘dynamic’, or non-steady state, decomposition that allows past flow rates to affect current unemployment.\textsuperscript{18} But first, in this section, I investigate the relative importance of inflow and outflow rates to changes in steady state unemployment.

The dynamics of unemployment can be shown simply in the context of a two-state world where workers are either employed or unemployed.\textsuperscript{19} Let $s_t$ be the (instanta-

\textsuperscript{16}According to my calculations based on Shimer’s (2007) CPS gross flows data, inflow and outflow rates between 1967q2 and 2007q2 sum to 54% on average.

\textsuperscript{17}This figure is calculated from the BHPS, 1988q4 to 2008q2.

\textsuperscript{18}This dynamic decomposition of unemployment encompasses a three-state world where workers can be inactive as well as employed or unemployed. Elsby, Hobijn and Sahin (2009) present an alternative decomposition based on log unemployment and a two-state world.

\textsuperscript{19}What follows also assumes zero labour force growth so that the labour force can be normalised to unity.
neous) unemployment inflow rate and \( f_t \) be the outflow rate. In continuous time, the unemployment rate \( u_t \equiv U_t / (U_t + E_t) \) evolves according to

\[
\dot{u}_t = s_t e_t - f_t u_t \equiv s_t (1 - u_t) - f_t u_t. \tag{1}
\]

In steady state, with steady state unemployment rate \( \bar{u}_t \), inflows equal outflows (so \( \dot{u}_t = 0 \)):

\[ s_t (1 - \bar{u}_t) = f_t \bar{u}_t. \]

Rearranging, the steady state unemployment rate is given by

\[
\bar{u}_t = \frac{s_t}{s_t + f_t}. \tag{2}
\]

In a three-state world, where individuals can either be employed, unemployed, or inactive, the continuous time dynamics of unemployment, employment and inactivity are

\[
\begin{align*}
\dot{U}_t &= \lambda_t^{EU} E_t + \lambda_t^{IU} I_t - (\lambda_t^{UE} + \lambda_t^{UI}) U_t \tag{3} \\
\dot{E}_t &= \lambda_t^{UE} U_t + \lambda_t^{IE} I_t - (\lambda_t^{EU} + \lambda_t^{EI}) E_t \tag{4} \\
\dot{I}_t &= \lambda_t^{UI} U_t + \lambda_t^{EI} E_t - (\lambda_t^{IU} + \lambda_t^{IE}) I_t, \tag{5}
\end{align*}
\]

where \( \lambda_t \) denotes an instantaneous transition rate at time \( t \) and the superscripts describe the relevant transition.

In steady state where \( \dot{U}_t = \dot{E}_t = 0 \), (3) to (5) can be rearranged to express steady state unemployment rate \( \bar{u}_t \equiv U_t / (U_t + E_t) \) as a function of all six transition rates:

\[
\bar{u}_t = \frac{\lambda_t^{EU} + \lambda_t^{EI} \lambda_t^{IU}}{\lambda_t^{EU} + \lambda_t^{EI} \lambda_t^{IU} + \lambda_t^{UE} + \lambda_t^{UI} \lambda_t^{IE}} = \frac{s_t}{s_t + f_t}. \tag{6}
\]

The second term in the numerator multiplies \( \lambda_t^{EI} \), the outflow rate from employment to inactivity, by the proportion of all outflows from inactivity that go to unemploy-
ment \( \lambda_{t}^{IU} / (\lambda_{t}^{IU} + \lambda_{t}^{IE}) \). It can therefore be interpreted as the likelihood of transiting \( E \rightarrow I \rightarrow U \). The numerator \( s_t \) thus gives the overall inflow rate to unemployment from employment – the direct \( E \rightarrow U \) transition rate rate plus the transition rate working through inactivity. Similarly, \( f_t \) (the sum of the third and fourth terms in the denominator) is the outflow rate from unemployment to employment – directly and working through inactivity.

The expression for the steady state unemployment rate (6) illustrates the key advantage of working with a three-state model. It is possible to disentangle the theoretically-important effects of the separation rate \( \lambda_{t}^{EU} \) and job-finding rate \( \lambda_{t}^{UE} \) from the overall unemployment flow rates, and to separately identify flows involving inactivity.

I now demonstrate how changes in unemployment can be decomposed into two components resulting from changes in \( s_t \) and from changes in \( f_t \) if we assume that the unemployment rate is well approximated by its steady state – or if what we are interested in is the dynamics of steady-state unemployment. Following Petrongolo and Pissarides (2008) and differencing (2):

\[
\Delta \bar{u}_t = \frac{s_t}{s_t + f_t} - \frac{s_{t-1}}{s_{t-1} + f_{t-1}}
\]

\[
= (1 - \bar{u}_t) \frac{\Delta s_t}{s_{t-1}} - \bar{u}_t (1 - \bar{u}_{t-1}) \frac{\Delta f_t}{f_{t-1}}
\]

(7)

Percentage changes in unemployment are approximately described by

\[
\frac{\Delta \bar{u}_t}{\bar{u}_{t-1}} \approx \frac{\Delta s_t}{s_{t-1}} \frac{1}{\bar{u}_{t-1}} - \frac{\Delta f_t}{f_{t-1}} \frac{1}{\bar{u}_{t-1}}
\]

(8)

where the upper bar represents steady state values. \( \bar{C}_t^s \) measures the contribution of changes in inflow rate \( s_t \) to changes in steady state unemployment and \( \bar{C}_t^f \) is the contribution of outflow rate.\(^{20}\) In this decomposition, the primary drivers of

\[^{20}\text{See Appendix C for mathematical details. This decomposition of the change in the level of steady state unemployment is very similar to the decomposition of log steady state unemployment changes in Elsby, Michaels and Solon (2009) and Fujita and Ramey (2009), where the percentage changes}\]
unemployment dynamics are percentage changes in inflow and outflow rates ($\Delta s_t / s_{t-1}$ and $\Delta f_t / f_{t-1}$, respectively), since $(1 - \pi_t) \approx 1$ at all times. This was noted by Elsby, Michaels and Solon (2009), who proceed to measure the inflow and outflow contributions using simply the log changes in the transition rates.

The contribution of changes in the inflow rate each period can be further divided into contributions from the employment-unemployment transition rate (separation rate $\lambda_{EU}^t$) and from the transition rate for employment-to-unemployment inflows working through inactivity, since from (6)

$$\frac{\Delta s_t}{s_{t-1}} = \frac{1}{s_{t-1}} \left[ \Delta \lambda_{EU}^t + \Delta \left( \frac{\lambda_{IE}^t \lambda_{UI}^t}{\lambda_{IU}^t + \lambda_{IE}^t} \right) \right].$$

Then the contribution of the separation rate to the dynamics of steady state unemployment is

$$\overline{C}_{EU}^t = (1 - \pi_t) \frac{\Delta \lambda_{EU}^t}{s_{t-1}}.$$  \hspace{1cm} (10)

A sub-division of the overall unemployment-to-employment outflow contribution enables the contribution of the job finding rate, $\overline{C}_{UE}^t$, to be recovered. Contributions can similarly be defined for flow rates working through inactivity $\overline{C}_{EIU}^t$ and $\overline{C}_{UIE}^t$.

The contributions to unemployment dynamics that can be measured using expressions such as (10) are time-varying. A useful summary statistic of each component’s relative importance is its overall contribution to the variance of steady state unemployment. These variance contributions mirror finance ‘betas’. For the inflow rate, for example, the variance contribution to steady state unemployment dynamics is

$$\overline{\beta}^s = \frac{cov(\Delta \pi_t, \overline{C}_t^s)}{var(\Delta \pi_t)}.$$  \hspace{1cm} (11)

Similar betas can be defined for the other transition rates. By definition, $\overline{\beta}^s + \overline{\beta}^f = \overline{\beta}^{EU} + \overline{\beta}^{EIU} + \overline{\beta}^{UE} + \overline{\beta}^{UIE} \approx 1$, where the difference from unity is accounted for by approximation error.

In separation and job finding rates are measured as $\Delta \ln (s_t)$ and $\Delta \ln (f_t)$. Indeed, Elsby, Michaels and Solon (2009) point out that their log decomposition can be reformulated as a decomposition in levels in which the premultipliers of the transition terms are both $(1 - \pi_t) \pi_t$ (which shows that the log and level decompositions both involve very similar approximations).
Table 1 presents results from BHPS data, decomposing steady state unemployment using only current transition rate contributions.

<table>
<thead>
<tr>
<th>Contributions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\beta}^s$ inflow rate</td>
<td>0.57</td>
</tr>
<tr>
<td>$\bar{\beta}^f$ outflow rate</td>
<td>0.40</td>
</tr>
<tr>
<td>$\bar{\beta}^{EU}$ separation rate</td>
<td>0.41</td>
</tr>
<tr>
<td>$\bar{\beta}^{UE}$ job finding rate</td>
<td>0.31</td>
</tr>
<tr>
<td>$\bar{\beta}^{EIU}$ inflow rate via inactivity</td>
<td>0.16</td>
</tr>
<tr>
<td>$\bar{\beta}^{UIE}$ outflow rate via inactivity</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 1. Covariance contributions to steady state unemployment variance

Notes. $\bar{\beta}$ is the contribution of current changes in transition rates to the variance of steady state unemployment. Contributions do not sum to unity due to approximation error. Source: BHPS Waves 1 to 18, 1988q4-2008q2.

Overall, changes in the inflow rate explain nearly 60% of movements in steady state unemployment. Within this, changes in the separation rate have the dominant role: by themselves, they account for around two fifths of steady state unemployment dynamics. The role of the outflow rate is lower: its changes explain only 40% of steady state unemployment dynamics. Movements in the job finding rate account for most of this, explaining approximately one third of steady state unemployment changes.

The estimates suggest that the role of inflow transitions working via inactivity in explaining movements in steady state unemployment is quite high (with a beta of 16%), which is much larger than the effect of changes in outflow rate via inactivity (9%). The high contribution of the $E IU$ flow stems in part from the cyclical properties of separations from employment to inactivity. As noted in Section 2, the $EI$ transition rate moves in a similar way to the separation rate to unemployment, rising as the labour market tightens (see Figure 2(b)). The inflow rate via inactivity ($E IU$) multiplies this $EI$ transition rate by the proportion of flows out of inactivity that go to unemployment, $IU / (IU + IE)$. This also shows substantial countercyclicality (see Figure 3): as unemployment rises, those leaving inactivity are more likely to join the pool of the unemployed. It is not surprising that, given these two countercyclical com-
ponents, the $EIU$ inflow rate via inactivity accounts for quite a sizeable proportion of unemployment dynamics.

Fig. 3. Proportion of inactivity exits going to unemployment

Notes. Annual averages of monthly data. Source: BHPS Waves 1 to 18, 1988q4-2008q2.

The $UIE$ outflow rate via inactivity combines multiplicatively the procyclical proportion of flows out of inactivity that go to employment, $IE/(IU+IE)$ (which is simply 1 minus the proportion plotted in Figure 3) and the $UI$ transition rate. The $UI$ transition rate was only somewhat countercyclical since around 2000. Before then it was dominated by a secular increase (see Figure 2(a)). All these features combine to explain the relatively low contribution of unemployment outflows via inactivity to UK unemployment dynamics.

3.1 Discussion

The steady state unemployment rate for the UK, calculated using BHPS data on flows between employment, unemployment and inactivity as in (6), is shown as the solid line in Figure 4.\(^{21}\) The steady-state unemployment rate exhibits particularly large rapid rises at several points in the sample: most clearly during the recession of 1990q3-1991q3, and also before smaller upticks in the actual unemployment rate.

\(^{21}\) The ‘noise’ visible in the steady state unemployment rate stems from noise in quarterly averages of monthly transition rate estimates.
around 2000, 2003 and 2005-6. Over the rest of the period, the unemployment rate fell gradually – during which time the steady state unemployment rate typically lay below the actual rate – and then stabilised at a historically low level. The data end too early to pick up any rise associated with the 2008q3-2009q3 recession.

![Fraction of labour force](image)

**Fig. 4. Actual and steady state unemployment rates**

Notes. Actual unemployment rate is $U_t/(U_t + E_t)$: stock of unemployed divided by sum of stocks of unemployed and employed. Steady state unemployment rate is $s_t/(s_t + f_t)$: overall inflow rate divided by sum of inflow and outflow rates, based on a three-state model. Source: BHPS Waves 1 to 18, 1988q4-2008q2.

Comparison of the steady-state and actual unemployment rates in Figure 4 suggests that actual unemployment based on BHPS stocks moves quite closely with equilibrium unemployment – except when actual unemployment is changing fast.

As Hall (2005b, p. 398) notes, if turnover dynamics are relevant we would expect the equilibrium unemployment rate to lead the actual rate, and that is exactly what we see for the UK. In contrast, US actual and equilibrium rates are coincident: the approximation $u_t \approx s_t/(s_t + f_t)$ holds well (Hall, 2005b; Shimer, 2007; Fujita and Ramey, 2009). Hall (2005) notes that a key reason for the irrelevance of turnover dynamics in the US is the high exit rate from unemployment, which is around 30% to 60% per month. In contrast, as Figure 1 shows, since the mid-1990s the UK job finding rate has averaged under 11% per month (and the total exit rate 12% per month, thus the quarterly rate being under 40%). Intuitively, it just takes a great
deal longer for a worker who loses a job in the UK to find another one, so a rise in the rate of job loss will raise unemployment not just this period, but in several subsequent months as well.

The importance of the relative magnitudes of the speed at which unemployment is changing and the rates at which workers lose and find jobs is clear from rearranging (1):

\[ u_t = \frac{s_t}{s_t + f_t} - \frac{u_t}{s_t + f_t}. \]  

(12)

\( u \) will be less important the larger is \( s_t + f_t \), i.e. the faster is labour turnover.

Expression (12) is also key in explaining why and how the dominance of the inflow rate shown in Table 1 differs from estimates previously published for the UK. Whereas Table 1 claims a 57% contribution from the inflow rate, Petrongolo and Pissarides (2008) report a roughly 50-50 split in influence between inflow and outflow rates, using LFS gross flows data. The difference arises because Petrongolo and Pissarides excluded periods where the deviation between the change in steady state unemployment and the change in actual unemployment is large compared to the actual unemployment rate. If I likewise exclude from BHPS data periods of substantial deviation of actual from steady state unemployment,\(^{22}\) BHPS estimates are identical to Petrongolo and Pissarides’ (2008) estimates: the inflow rate explains only 51% of steady state unemployment dynamics in ‘normal’ times. This, of course, suggests time variation in the relative contributions. But it is possible to go further: equation (12) shows that periods when the actual and steady state unemployment rates deviate substantially will be those when unemployment is changing most rapidly.\(^{23}\) In recent history, these have been periods associated with rising unemployment; the decline in unemployment rates in recoveries tends to be gradual. The difference in

\(^{22}\) For the BHPS, the 17 (out of 78) periods when actual and steady state unemployment rates deviate substantially are concentrated in 1990-91, 1996-97, 2000-01 and 2005-06. Although only the first is normally defined as a recession, all are periods when annual real GDP growth dropped below trend (roughly 3%) for several quarters (markedly so, and falling below 2%, in the last two cases). Not surprisingly, as Figures 1 to 3 show, all are periods which saw a rise in the unemployment rate.

\(^{23}\) A recent paper by Barnichon (2009) finds that for the US it is also the case that job separation is responsible for most of unemployment movements at cyclical turning points, even though, in the US, job finding (which Barnichon measures by vacancy posting) is on average more important.
BHPS estimates – between a 57% inflow rate contribution using all data and 51% when unusually fast unemployment changes are excluded – suggests that the inflow rate has a relatively high influence on unemployment increases. The BHPS sample period incorporates the recession of the early 1990s, whereas LFS data start only in 1992, which might explain why – once all data are included – the BHPS estimate of the inflow rate contribution (57%) is larger than the LFS estimate (52%).

The discussion so far has two implications. First, for the UK it seems likely that an accurate assessment of the relative importance of inflow and outflow rates will require a decomposition of actual, rather than just steady state, unemployment. Second, it will be sensible to allow estimates to vary over time. Both of these are dealt with in the next section.

3.2 Non-steady state decomposition

I now use the identity describing unemployment dynamics (12) to devise a decomposition of changes in the actual unemployment rate that will allow the relative contributions of inflow and outflow rates to be measured. This decomposition is based on changes in the level of unemployment. The model here is comparable to that developed by Elsby, Hobijn and Sahin (2009) to describe the dynamics of log unemployment away from steady state. But whereas Elsby, Hobijn and Sahin are restricted by model and duration-based data to just two states, this paper’s development of a dynamic decomposition based on a non-log change (and use of gross flows data) enables the impact of changes in job finding and separation rates to be distinguished from the effect of outflows and inflows via inactivity.

(12) can be written

\[ u_t = \frac{s_t}{s_t + f_t} - \frac{du_t}{dt} \frac{1}{s_t + f_t}. \]  

(13)

Differentiating (13), unemployment dynamics out of steady state can be expressed in

24This LFS estimate using all data is my calculation based on publicly available unadjusted transition rates from Petrongolo and Pissarides (2008) that I seasonally adjusted using X12-ARIMA.

A recent paper by Barnichon (2009) finds that for the US it is also the case that job separation is responsible for most of unemployment movements at cyclical turning points, even though, in the US, job finding (which Barnichon measures by vacancy posting) is on average more important.
terms of the rate of change in the steady state unemployment rate and the rate of acceleration of the actual unemployment rate:

\[
\frac{du_t}{dt} = \frac{d}{dt} \left( \frac{s_t}{s_t + f_t} \right) - \frac{d}{dt} \left( \frac{du_t}{dt} \frac{1}{s_t + f_t} \right)
\]  

(14)

Rearranging gives a second-order differential equation:\(^{25}\)

\[
\frac{d^2 u_t}{dt^2} = \frac{d\pi_t}{dt} (s_{t-1} + f_{t-1}) - \frac{d}{dt} \left[ (s_t + f_t) - \frac{d (s_t + f_t)}{dt} \frac{1}{(s_t + f_t)} \right]
\]

(15)

Treating (15) as a first-order differential equation in \(du_t/dt\), discretizing, and using \(\omega_t\) to represent the sum of inflow and outflow rates \(s_t + f_t\) gives the following recursive expression for actual unemployment dynamics:

\[
\Delta u_t = \frac{\Delta \bar{u}_t \omega_t \omega_{t-1}}{\omega_t^2 + \omega_{t-1}^2} + \Delta u_{t-1} \frac{\omega_t}{\omega_t^2 + \omega_{t-1}^2}
\]

\[
= \frac{\Delta \bar{u}_t \omega_t s_{t-1}}{\bar{u}_{t-1} \omega_t^2 + \omega_{t-1}^2} + \Delta u_{t-1} \frac{\omega_t}{\omega_t^2 + \omega_{t-1}^2}
\]

(16)

(17)

where the change in unemployment \(\Delta u_t\) is the change over period \(t\). The coefficient on \(\Delta \bar{u}_t\) in (16) gives the rate of convergence to the steady state.

The higher are transition rates, the closer will movements in actual unemployment approximate changes in equilibrium unemployment. Conversely, the lower are transition rates, the larger the relative impact of past changes in transition rates and past changes in equilibrium unemployment. Low UK transition rates demonstrate why caution should be used before attempting decompositions of UK unemployment that depend on it being in steady state at all times.

Contributions to actual unemployment dynamics from changes in the outflow and inflow rates can be defined on the basis of equation (17) as:

\(^{25}\)Mathematical details can be found in Appendix C. That Appendix shows that a similar picture of the relative impact of inflow and outflow rates is obtained from the Elsby, Hobijn and Sahin’s (2009) non-steady state decomposition.

In earlier work, Fujita and Ramey (2009) and Burgess and Turon (2005) allow past changes in transition rates to affect current unemployment rates by estimating VARs.
\[ C_s^t = \frac{C_t s_{t-1}}{\omega_t^s + \omega_{t-1}} + C_{s-1}^t \frac{\omega_t}{\omega_t^s + \omega_{t-1}} \]  \hspace{1cm} (18) \\
\[ C_f^t = \frac{C_t f_{t-1}}{\omega_t^f + \omega_{t-1}} + C_{f-1}^t \frac{\omega_t}{\omega_t^f + \omega_{t-1}} \]  \hspace{1cm} (19)

where by definition \( C_0^s = C_0^f = 0 \), and the contributions to steady state unemployment dynamics \( C_s^t \) and \( C_f^t \) are as defined in (8) above. As with the steady state decomposition, contributions from both flow rates to actual unemployment dynamics can be subdivided into direct and indirect flows between unemployment and employment.

In contrast to the steady state decomposition described in Section 3, \( C_s^t \) and \( C_f^t \) do not by definition account for all unemployment rate changes. There is also a contribution from the initial condition at time \( t = 0 \). The path of this contribution over time can be expressed recursively:

\[ C_t^0 = C_{t-1}^0 \frac{\omega_t}{\omega_t^s + \omega_{t-1}} \]

where \( C_0^0 \) is defined as the initial deviation from steady state: \( C_0^0 = \Delta u_0 - \Delta \bar{u}_0 = u_0 - \bar{u}_0 \). Finally, there is a residual contribution that arises from violation of maintained assumptions (transition rates constant within months, no labour force growth, linearity).

As with the steady state decomposition, the relative importance of the various flow rates can be summarised in betas. For example, the beta for the inflow rate is

\[ \beta^s = \frac{\text{cov}(\Delta u_t; C_t^s)}{\text{var}(\Delta u_t)}. \]  \hspace{1cm} (20)

To counter substantial short-run variation (noise) in transition rates, in the empirical work below I decompose the annual change in the unemployment rate. Monthly transition rates are converted to annual rates through multiplication by 12.

To allow for time variation in betas, I calculate rolling 5-year betas. These are shown in Figure 5, where the horizontal axis states the relevant five-year roll period.
Figure 5 shows substantial variation over time in the relative contributions of inflow and outflow rates. In the early part of the sample, covering the recession of the early 1990s, the inflow rate is dominant. During this time, changes in the inflow rate more than account for unemployment rate dynamics – the outflow rate negatively covarying with the unemployment rate. From 1992, the contribution of the outflow rate rises. Initially this is due to falling job finding rates contributing to higher unemployment, while unemployment continues to rise despite fewer job losses. As unemployment declines during the middle of the sample, the contribution of the outflow rate dominates: better job finding prospects seem to have driven the improvement in the unemployment situation. Towards the end of the sample, the unemployment rate stabilises, reflecting relatively stable inflow and outflow rates. The gentle decline in the job finding rate and, in particular, the correlation of the notable uptick in unemployment from end-2001 to 2003 with movements in the separation rate and its negative co-movement with the job finding rate (see Figure 1) is marked by increasing relative influence of the inflow rate.

Disaggregating the contributions further, Figure 6 confirms that most action comes
through job finding and separation rates themselves, which display the characteristics already noted for outflow and inflow rates. The contribution to unemployment dynamics of the inflow rate working through inactivity shows similarity in movement to its counterpart inflow rate directly from employment, generally adding to the effect of the separation rate. The outflow rate via inactivity has a contribution that often mirrors in movement that of the inflow contribution via inactivity and does not generally comove with the job finding rate; its lower contribution to actual unemployment dynamics on average matches the steady state contribution shown in Table 1.

![Graph](https://example.com/graph.png)

**Fig. 6. Variance contributions of changes in job finding, separation and transition rates via inactivity to actual unemployment rate dynamics (rolling 5-year betas)**

Source: BHPS Waves 1 to 18, 1988q4-2008q2.

### 4 Conclusion

This paper presented the first use of information on labour market transitions calculated from the BHPS to examine the relative importance of job finding and separation rates in driving UK unemployment dynamics. UK transition rates are low –

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27An investigation into why the relative contributions to unemployment dynamics exhibit these marked changes over time is beyond the scope of this paper, as it would require a different, causal analysis. One possibility is that active labour market policies designed to aid job finding might have played a role, as might variations in policy regarding availability of inactivity-related benefits. However, recent work by Manning (2009) and Petrongolo (2009) found little beneficial impact from the introduction of Jobseekers’ Allowance (JSA) in the fourth quarter of 1996, although Karagiannaki (2007) identified positive effects from the merging of benefits and employment assistance agencies into Jobcentre Plus in 2002.
one quarter the size of US rates – so workers who lose their job will on average be out of employment for that much longer. This means that unemployment deviates noticeably from steady state, and past changes in transition rates will affect current unemployment.

Previous literature, focused on the US, has used a simple decomposition of unemployment changes – into components that can be attributed to changes in each of the inflow and outflow rates – that is valid only if unemployment is well approximated by its steady state value. Because this does not hold well for the UK, I proposed a decomposition of actual unemployment rate changes that incorporates the impact of past transition rates.

Results suggest that at different points in the cycle, job finding and separations both play an important role in changing unemployment rates. At times when unemployment is changing fast – which are typically associated with rising unemployment – the separation rate is most important. But at other times – and during the period of great moderation – changes in the job finding rate were found to have the primary role. Overall, then, the results challenge the Shimer-Hall (2005) contention that all unemployment changes are due to primarily to the outflow rate. Results are more consistent with subsequent work showing that both inflows and outflows matter (Petrongolo and Pissarides, 2008; Elsby, Michaels and Solon, 2009; Fujita and Ramey, 2009). However, unlike this recent work, this paper emphasises changes over time, and the consequent finding that an upsurge in the inflow rate drives rising unemployment during recessions has clear policy implications.

The method developed in this paper would be suitable for application to other countries where labour market transition rates are low – including, for example, Continental European countries. Further additional work would be useful to obtain information about the relative importance of job finding and separation rates for particular groups, such as youth workers, who have been hit hardest by the rise in UK unemployment during the recent recession.
5 References


Karagiannaki, Eleni (2007), “Exploring the effects of integrated benefit systems and active labour market policies: evidence from Jobcentre Plus in the UK”, *Journal
of Social Policy, 36 (2) 177-195.

Manning, Alan (2009), “You can’t always get what you want: the impact of the
UK Jobseeker’s Allowance”, Labour Economics, 16 (3) 239-250.

Nekarda, Christopher J (2009), “Understanding unemployment dynamics: the
role of time aggregation”, Working Paper, Federal Reserve Board of Governors.

Paull, Gillian (2002), “Biases in the reporting of labour market dynamics”, Institute

Petrongolo, Barbara (2009), “The long-term effects of job search requirements:
evidence from the UK JSA reform”, Journal of Public Economics, 93 (11-12) 1234-
1253.

Petrongolo, Barbara and Christopher A Pissarides (2008), “The ins and outs of
European unemployment”, American Economic Review, 98 (2) 256-262.

Poterba, James M and Lawrence H Summers (1986), “Reporting errors and labor
market dynamics”, Econometrica, 54 (6) 1319-1338.

Shimer, Robert (2005), “The cyclical behavior of equilibrium unemployment and
vacancies”, American Economic Review, 95 (1) 25-49.


Taylor, Marcia Freed (ed) with John Brice, Nick Buck and Elaine Prentice-Lane

of Economics, 109 (4) 779-806.
Appendix A: Comparison of BHPS and LFS data

It is useful to compare the two major available sources of gross flows data for the UK: the British Household Panel Survey (BHPS), as used in the main text, and the Labour Force Survey (LFS) – the source that has been used previously by Petrongolo and Pissarides (2008) and Gomes (2009) to investigate unemployment dynamics.

The main text includes information on how differences in sampling design tend to favour the BHPS, whereas the LFS has the advantage of larger sample size. In this Appendix I will focus on how estimates derived from the two datasets compare.

Figure A1 plots monthly separation and job finding rates measured using BHPS data and estimates from the quarterly UK LFS. Whereas BHPS data allow ‘true’ monthly transition rates to be calculated, Figure A1 shows LFS quarterly transition rates converted to monthly values by simply dividing by three. Because the LFS estimates are based on changes in status between one quarter and the next, they ignore any intervening transitions and might therefore be downward-biased through time aggregation – and, consistent with this, the LFS estimates do, in the main, lie below the BHPS estimates. However, the extent of the difference is not as large as might have been expected. Using the analytical method pioneered by Shimer (2007) and adapted by Elsby, Michaels and Solon (2009), Gomes (2009) calculated monthly estimates from LFS data adjusted for time aggregation bias, and they average 18-20% above the unadjusted monthly rates (for the UE and EU transition rates Gomes focused on), which is a bigger difference than is apparent on average in Figure A1. Factors that might be responsible are examined below, after I outline economic implications of the estimates.

LFS estimates can be calculated from the third quarter of 1992, and thus only capture the tail-end of the 1990s recession, but they are plotted up to the first quarter of 2010 and so cover the recession of 2008-09. The BHPS data span the earlier recession but, because they end in the second quarter of 2008, they miss the upturn in actual unemployment that began in mid-2008. (In the second quarter the LFS unemployment rate was still 5.3% – very close to its pre-recession low of 5.2% in the previous quarter – whereas over the next year to mid-2009 unemployment rapidly
increased, rising 2.5 percentage points.)

Fig. A1. UK separation and job finding rates: BHPS and LFS estimates

Notes. BHPS data are quarterly averages of monthly transition rates. LFS data are quarterly rates divided by 3 (representing monthly rates without adjustment for time aggregation). Job finding rate is the $U \rightarrow E$ transition rate $\lambda^{UE}$. Separation rate is the $E \rightarrow U$ transition rate $\lambda^{EU}$. Sources: BHPS Waves 1 to 18, 1989q1 to 2008q2, and QLFS 1992q2 to 2010q1.

BHPS and LFS transition rate estimates generally follow the same pattern (substantial noise is visible in the BHPS transition rates due to the relatively small sample size). Separation rate estimates are very close between the two datasets. The BHPS job finding rate rose faster during the 1990s, whereas it lies below LFS estimates in the first half of the 2000s.
Combining the data sources indicates that UK recessions are marked with a burst of job loss (contrary to the claims of Hall, 2005). BHPS estimates suggest that separation rates rose from a low level before the 1990-91 recession to peak at over 0.8% in 1991. LFS estimates indicate a sharp rise in the separation rate at the beginning of the 2008-09 recession. However, the peak rate of job loss was 0.55% per month, substantially lower than in the 1990s recession. Since the second half of 2009 there has been a rapid, although incomplete, decline in the separation rate.

Hall (2005) and Shimer (2007) suggested that the primary driving force of unemployment over the cycle is a procyclical job finding rate. Even later US studies that have found more influence from changes in the separation rate at the start of recessions have confirmed the procyclical nature of the job finding rate (Elsby, Michaels and Solon, 2009; Fujita and Ramey, 2009; Nekarda, 2009). According to LFS estimates, after a rapid rise during the second half of 2007, the job finding rate began to decline from the beginning of 2008 – roughly coincident with the start of the rise in the separation rate – and the job finding rate has continued to fall quite sharply since then. However, LFS estimates indicate that a decline in the job finding rate had afflicted the UK labour market for a much longer period – since around 2004-05 (BHPS estimates are not inconsistent with this picture). In the 1990s recession, BHPS estimates do not show a similar decline in the job finding rate: during 1990q3-1991q3 the job finding rate appears relatively acyclical, according to BHPS data.

Figure A2 plots UK transition rates involving inactivity based on BHPS data (taking annual averages to smooth out noise), and again compares them with LFS estimates. In contrast to the transition rates between employment and unemployment, there are substantial differences in the inactivity-related transition rates between the two datasets. In terms of levels of transition rates, the LFS estimates are almost everywhere higher than BHPS figures – and substantially higher in the case of transitions between unemployment and inactivity. On the basis of the time aggregation of the estimates, this difference should be reversed, as the LFS estimates should miss transitions occurring within quarters. How can the difference be explained?
Fig. A2. **UK transition rates involving inactivity: BHPS and LFS estimates**

Notes. BHPS data are annual averages of monthly transition rates. LFS data are quarterly rates divided by 3 (representing monthly rates without adjustment for time aggregation). Sources: BHPS Waves 1 to 18, 1989q1 to 2008q2, and QLFS 1992q2 to 2010q1.

One possibility is that the LFS estimates are subject to the spurious transition bias that the BHPS estimates have been designed to avoid (and inherently avoid within waves through reliance on recall data). Poterba and Summers (1986) comparison of CPS data with the CPS validation study showed that UE transitions are more subject to (classification) error than EU transitions (that were found to have the lowest error), which might explain why the LFS UE rate is at times higher than the BHPS estimate. UI transition errors are high, according to the CPS, consistent with the difference in
Figure A2 between LFS and BHPS transition rate estimates. But $EI$ error was also found to be surprisingly high by Poterba and Summers, whereas Figure A2 indicates that LFS and BHPS estimates are often quite close. Although it has previously been argued that some respondents’ difficulty in distinguishing between unemployment and inactivity may be particularly likely to generate spurious movements in and out of the labour force, Poterba and Summers’ estimates suggest that the difference between $IU$ transition rates is also unlikely to be totally due to spurious transitions inflating LFS estimates, as the CPS $IU$ error rate was found to be low.

Another possibility is that the BHPS estimates suffer from recall bias. Note that this implies that transitions involving inactivity are forgotten (or not reported) even over the relatively short period of the year between interviews, and that transitions between inactivity and unemployment are particularly prone to be forgotten. The closeness (in some cases) of the two non-employment states implies that it would not be difficult for respondents to justify reclassification between them, but there is surprisingly little published evidence that these transitions are most prone to be redefined.

A third possibility is that the self-definition of labour force states within the BHPS, differing from the ILO definitions used by the LFS, leads to the difference in transition rates. BHPS unemployment and inactivity rates both typically lie below the LFS rates, and the BHPS employment rate is higher (the unemployment data are shown in Figure A3; estimates for the other states are not shown here).

It seems likely that all three explanations for the difference in transition rates play a role. This is suggested by the examination of the impacts of errors from spurious transitions, recall and selection in Appendix B, based on methods of creating BHPS estimates that maximise or minimise these errors.

Turning to the comparative cyclicality of inactivity transitions, LFS estimates of inflow rates into inactivity show some similarities in cyclical profile to the BHPS data: the $UI$ transition rate is procyclical and the $EI$ rate relatively procyclical (though less so than BHPS estimates). As noted in the main text, the BHPS proportion of exits from inactivity to unemployment ($IU/(IU + IE)$) is countercyclical: in recessions, new labour force entrants are more likely to join the pool of the unemployed.
Figure A2 confirms countercyclicality also in the LFS $IU$ transition rate and signs of procyclicality in the $IE$ transition rate.

![Graph showing fraction of labour force over time]

**Fig. A3. UK transition rates involving inactivity: data for the BHPS and LFS**

**Notes.** BHPS data are annual averages of monthly transition rates. LFS data are quarterly rates divided by 3 (representing monthly rates without adjustment for time aggregation). Sources: BHPS Waves 1 to 18, 1989q1 to 2008q2, and QLFS 1992q2 to 2010q1.

As noted above, the LFS unemployment rate generally exceeds the BHPS unemployment rate (see Figure A3). The questions used to define unemployment differ between the two datasets. The BHPS question simply asks respondents to self-define themselves as unemployed; “Unemployed” is one of ten possible options a respondent can use to describe their current labour force state. In the LFS, labour force state is a variable derived from a number of questions. To be classified as unemployed, a non-working respondent must answer affirmatively that they have been looking for a job (or place on a government training scheme) in the past four weeks and that they are available to start a new job, or that they are waiting to start a job. The difference in the definitions could well be responsible for part of the contrast in the unemployment rates, as it might well be the case that someone in the BHPS reports themselves as inactive if they looked for work only once earlier in the month, for example, which would be consistent with under-reporting unemployment in the BHPS. There could also be over-reporting, of course, since individuals may consider them-
selves unemployed even if they do not satisfy the search criteria to be classified as such on the ILO definition.

Is the unemployment difference due to classification error? Poterba and Summers (1986) suggest not: according to them, errors in classification lead a large number of persons to be spuriously classified as unemployed, artificially inflating the stock of short term unemployed. However, they argue that many longer term unemployed are spuriously not measured as unemployed, a bias that offsets the former, so that the measured stock of unemployment is relatively unaffected.

Recall error might well be part of the explanation for the unemployment rate difference. As Elias (1997), Paull (2002) and others have argued, unemployment spells are particularly likely to be redefined, as they may not appear particularly important to the respondent in retrospect, or the respondent might (consciously or unconsciously) redefine their past status to accord with what they now believe (should have) happened.

The contrast between BHPS and LFS unemployment is very reminiscent of Akerlof and Yellen’s (1985) comparison between CPS and Work Experience Survey (WES) unemployment for the US. The reliance in BHPS data on retrospective reporting between annual surveys is identical to that in the WES. Akerlof and Yellen argue that the ratio between WES and CPS unemployment represents a measure of the seriousness or ‘salience’ of unemployment: unemployment is more likely to be recalled and reported if it is more important (salient) to an individual. Akerlof and Yellen confirm that WES unemployment is closer to CPS unemployment at times and for subgroups when other indicators suggest unemployment is more serious.

From Figure A3 it is clear that the BHPS-LFS unemployment ratio is greatest in periods of high unemployment. At those times, competition for jobs will be fierce and, often, vacancies are more scarce, so (expected and actual) unemployment duration will be longer. This is likely to increase the salience (and memorability) of an unemployment experience. It might also be the case that respondents feel more comfortable reporting themselves unemployed when they feel many others are in the same position. Relatedly, like WES unemployment compared with CPS, BHPS unemployment shows greater cyclicality than LFS unemployment.
Additional reference

Appendix B: Comparison of methods for constructing labour force histories from BHPS data

Various methods could potentially be used to join data across waves to form a labour force history for each individual in the BHPS. In the main text I use one of four methods devised and investigated by Paull (2002), the ‘Reconciliation’ method. The Paull methods are particularly useful, as they span the range from the ‘Closest Interview’ that (as its name suggests) takes information only from the interview closest to the relevant event and discards all later information, to the ‘Latest Interview’ method that lets data from later interviews overrule information from previous interviews (gaining consistency at the cost of using data involving longer recall). The ‘Reconciliation’ method lies between these, in that it does use later information, but only where this is able to resolve inconsistencies arising when only closest interview data are used. A final method that could be used is to simply to discard all cases where there are inconsistencies or missing information (‘Selected No Problems’). This method is useful to illustrate the consequences of selection bias.

The Reconciliation method was designed to counter the two major problems encountered when constructing gross flows from micro data: non-randomly missing data and classification error (Abowd and Zellner, 1985). Labour force histories constructed from panels such as the LFS in the UK and the CPS in the US typically (and in some cases of necessity) rely on a closest interview-type method, as respondents are asked about their labour force status in a particular week, and are not (generally) required to recall over a longer period of time. The work of Abowd and Zellner and others also emphasised the selection problem arising from the need to match individuals across months in the CPS: those who cannot be matched will include those who move location, for example, because the CPS samples addresses and does not follow individuals. Because such moves are often undertaken for work reasons, gross flows will understate related transition rates.
Fig. B1. Differences across methods for constructing labour force histories: transition rates

Notes. Annual averages of monthly data. The four methods of constructing labour force histories are based on an update of algorithms provided by Paull (2002).
Reconciliation method uses a series of rules to govern when inconsistencies arising from closest interview data should be overruled by later information, and to impute certain missing information. The Closest Interview method uses information only from the closest interview. The Latest Interview method overrides closest interview data with information from the subsequent wave(s). The Selected No Problems method discards all cases where there are missing data or any other problems. Source: BHPS Waves 1 to 18, 1988q4-2008q2.

How do estimates from the four methods compare? Figure B1 shows the six transition rates between employment, unemployment and inactivity calculated from the four labour force histories, while Figure B2 presents unemployment data based on the four methods. In all cases the base calculation involves monthly transition rates, which are then averaged over quarters. In these Figures, four-quarter moving averages are taken to remove noise and make differences between the methods easier to see, and the graphs are scaled so as to make the differences between the methods as clear as possible.

To minimise recall error, the Reconciliation method prefers to use data from the closest interview. It generally results in estimates that are quite similar to the Closest Interview method. Both these methods generate higher transition rate estimates than the method that prefers later interviews and the one that selects only observations with no problems. This bipartite distinction is most evident in the panels (c) and (d) of Figure A2, showing transition rates between unemployment and inactivity, and least evident in job finding rates (panel (a) of Figure B1).

The Reconciliation method attempts to reduce spurious transitions that might affect Closest Interview estimates. This is done by replacing inconsistent status or date observations with later information, as long as this is consistent with the whole history. Logically, then, the Reconciliation estimates should lie between Closest Interview estimates (with over-inflated transition rates) and Latest Interview estimates (with transition rates biased downwards by recall error).

Why, then, are Reconciliation estimates typically at least as high as, and often higher than, Closest Interview transition rates? The answer lies in the other part of the Reconciliation method, which tries to minimise selection bias by imputing dates and in some cases corrects statuses. These corrections are based on a complex se-
ries of rules devised by Paull (2002) “by examining case-by-case inconsistencies and missing information” and are “designed to generate consistency for a high number of cases without making imputations that dramatically changed the data” (p.21). That selection bias results in too few transitions can be seen by the universally low rates for the Selected No Problems method. As previous evidence suggests, those discarded (who make errors in reporting or recall) are not missing at random (Abowd and Zellner, 1985; Poterba and Summers, 1986): instead, they will disproportionately include those with the most complex labour force histories, who by definition make the most transitions. Because they include these highly-mobile individuals, the Reconciliation method estimates are also based on the largest sample (with the Selected No Problems sample being smallest, followed by the Closest Interview method sample).

There is some indication from Figure B1 that Closest Interview estimates might be subject to classification error. In the case of outflows from inactivity – $IU$ and $IE$ transition rates – the bottom row of Figure B1 indicates that Closest Interview rates exceed even the Reconciliation estimates that have been boosted by adjustment to counter sample selection. This is quite strongly suggestive that spurious transitions might be affecting Closest Interview estimates, although there is no clear test of this that can be applied.

A final point worth noting concerning transition rate estimates is that the introduction of Dependent Interviewing in Wave 16 (P) has, in general, substantially reduced differences across the four methods. With computer-assisted Dependent Interviewing, respondents are reminded what they were doing at the time of the last interview and asked if this is still the case. If a respondent’s status has changed, they are asked to list statuses and start dates going forwards from the last interview date (or 1 September last year if information on last year’s status is missing). The use of last interview’s status in this year’s interview should substantially reduce spurious transitions. Dependent Interviewing should therefore reduce the role of information from later interviews in the Reconciliation method, and bring those estimates nearer to those based only on the Closest Interview. Another effect is to reduce the difference between the Selected No Problems estimates and those based on Latest Interview. Recalled data from later interviews is now only available when there is no information
from interviews closer, and so by definition the Latest Interview method from around 2006 generates observations with no conflicts or missing data.

As Figure B2 shows, the different methods can generate quite different estimates of unemployment rates. The Reconciliation estimates tend to be highest as they include very mobile individuals with short unemployment spells, missed by all the other methods. The increasing tendency of people to forget (or redefine) unemployment spells as the period of recall lengthens is demonstrated by the Latest Interview unemployment rates typically being lowest.

Fig. B2. Differences across methods for constructing labour force histories: unemployment rates

Notes. Annual averages of monthly data. Source: BHPS Waves 1 to 18, 1988q4-2008q2.

To summarise so far, Figures B1 and B2 indicate that there are some differences in the levels of transition and unemployment rates across the various methods of creating labour force histories from BHPS data. However, standing back and looking at how the series move over time (and, in particular, how transition rate estimates vary with unemployment estimates), there seems quite substantial commonality across the methods. How much does the choice of method influence the results in the main text concerning the contributions of the various flows to unemployment dynamics? In what follows I replicate the steady-state and dynamic decompositions of movements in the unemployment rate for the three methods not used in the main text and compare results with the preferred Reconciliation method.
First, as a check, I show in Figure B3 that steady state unemployment rates calculated on the basis of all methods lead the actual unemployment rate (also calculated from each method) (this replicates Figure 4 in the main text).

![Fig. B3. Actual and steady state unemployment rates](image)

**Notes.** Actual unemployment rate is $U_t/(U_t + E_t)$: stock of unemployed divided by sum of stocks of unemployed and employed. Steady state unemployment rate is $s_t/(s_t + f_t)$: overall inflow rate divided by sum of inflow and outflow rates, based on a three-state model. Source: BHPS Waves 1 to 18, 1988q4-2008q2.

Table B1 presents estimates of the contribution of the various transition rates to *steady state* unemployment dynamics. In terms of general findings, the methods agree on the dominance of the inflow rate in determining UK unemployment dynamics over the sample period, with estimates ranging from 57% to 76% of unemployment changes explained by inflows into unemployment. Within inflows, there is also agreement that changes in the separation rate are mainly responsible (accounting for between 41% and 61% of unemployment movements).
Reconciliation method estimates place relatively high importance on the two outflow rate contributions compared to the other methods, and relatively little weight on separation rate changes. The key results in the main text concerning the dominance of the separation rate in the UK would therefore be reinforced if any of the other methods were used instead.

The high inflow rate contribution according to the Latest Interview method is the counterpart of its overly-low estimate of the outflow rate contribution. From Figure B1, panel (a) shows that the job finding rate for this method is less procyclical than other methods, particularly in the second half of the sample. The Latest Interview estimate of the outflow contribution related to inactivity is also extremely low, consistent with previous evidence suggesting that short spells (often involving inactivity transitions) are most likely to be forgotten (see Paull, 2002).

The Selected No Problems estimates place least weight on transitions between unemployment and inactivity, in accordance with this method’s low transition rate estimates in Figure B1 panels (c) and (e). The Selected No Problems method discards too many individuals with very short spells of unemployment and retains low numbers of individuals moving in and out of activity.
The Closest Interview method appears to inflate the importance of inflow rate changes from employment and via inactivity. Why might this be? Panel (b) of Figure B1 indicates that the Closest Interview separation rate estimates are relatively high when unemployment peaks (the distance between this method and the Reconciliation estimates is countercyclical). This suggests particularly countercyclical separation rate estimates from the Closest Interview method, which explains, proximately, their large contribution to unemployment dynamics shown in Table B1. Regarding the inactivity inflow contribution, Poterba and Summers (1986) showed that spurious transitions – perhaps surprisingly – particularly affect the transition rate from employment to inactivity. Panel (d) of Figure B1 also indicates that for the affected Closest Interview method this transition rate is not only raised in level, but is also increased in countercyclically: it has a temporal profile more similar to the separation rate than is the case for the other methods. This countercyclicality, possibly related to spurious transitions, helps explain the large contribution of the inflow rate via inactivity for the Closest Interview method.

Despite the recorded differences across methods in transition rate levels, Figures B4 and B5 show that all methods result in similar decompositions of actual unemployment dynamics. This is due to the commonality of cyclical movements evident in Figures B1 to B3, and suggests that results in the main text are robust to different ways of constructing labour force histories.
Fig. B4. Variance contributions of changes in outflow and inflow rates to actual unemployment rate dynamics (rolling 5-year betas)

Notes. Source: BHPS Waves 1 to 18, 1988q4-2008q2.
B.1 Details of the Reconciliation method

This section details the amendments made in the Reconciliation method for combining BHPS data across waves. It is based on Paull (2002, Appendix B), where examples are also given.

Some amendments correct for inconsistencies in spell dates (where spells overlap or there are gaps between spells):

a) Inconsistent months originally imputed from a reported season are adjusted (within the season) to match the reported month from another interview.

b) End dates for spells with a gap before the following spell which are in the same or consecutive waves are extended to the start date of that following spell.

c) The start date for the second spell in overlapping spells is set to the end date of the first spell. If the spells have the same start date or the first spell begins after the second, the second spell is dropped.
Other amendments impute missing dates:

a) Missing start dates for the first spell after leaving full-time education are replaced with the date left full-time education if the spell ends within 12 months of leaving full-time education.

b) Missing ‘dividing’ dates between two spells (that is, where the start date for the first spell is known but the end date unknown and where the end date for the second spell is known but the start date unknown) are replaced with the midpoint of the feasible period for the start date.

c) Missing start dates are replaced by the previous spell’s end date where the end dates of the two spells are within 12 months of each other and the spells are of different types.

d) Unemployment and inactivity spells with missing end dates are merged with subsequent spells of the same type.

e) Employment spells with missing end dates are merged with subsequent spells of employment if the subsequent spell starts at the same time or before the initial spell.

f) Spells with missing start dates are merged with previous spells of the same type if the end dates are within 12 months of each other.

Other amendments impute missing spell types:

a) Spells of unknown type are merged with overlapping spells of known type.

b) Spells of unknown type with either start or end date missing are merged with spells of known type if either the start or end dates are within 3 months of each other.
Appendix C: Mathematical details of the model of unemployment dynamics in the main text

C.1 The non-steady state decomposition

Start from the equation for actual unemployment dynamics ((12) in the main text):

\[ u_t = \frac{s_t}{s_t + f_t} - \frac{1}{dt} \frac{u_t}{s_t + f_t}. \] (21)

Differentiating, unemployment dynamics out of steady state can be expressed in terms of the rate of change in the steady state unemployment rate and the rate of acceleration of the actual unemployment rate:

\[ \frac{du_t}{dt} = \frac{d}{dt} \left( \frac{s_t}{s_t + f_t} \right) - \frac{d}{dt} \left( \frac{1}{dt} \frac{u_t}{s_t + f_t} \right). \] (22)

Using the quotient and product rules for differentiation and rearranging:

\[ \frac{du_t}{dt} = \frac{1}{(s_t + f_t)^2} \left( f_t \frac{ds_t}{dt} - s_t \frac{df_t}{dt} \right) - \frac{1}{(s_t + f_t)^2} \left[ \frac{d^2 u_t}{dt^2} \frac{(s_t + f_t)}{dt} - \frac{du_t}{dt} \frac{d(s_t + f_t)}{dt} \right]. \]

\[ \frac{d^2 u_t}{dt^2} = \left( f_t \frac{ds_t}{dt} - s_t \frac{df_t}{dt} \right) \frac{1}{(s_t + f_t)} - \frac{du_t}{dt} \left( s_t + f_t \right) - \frac{d(s_t + f_t)}{dt} \frac{1}{(s_t + f_t)}. \] (23)

Although this is a second-order differential equation, I will treat it as a first-order differential equation in \( y_t = du_t / dt \) (since it is changes in unemployment that are of interest), with time varying coefficient \( \beta_t = s_t + f_t - \frac{d(s_t+f_t)}{dt} \frac{1}{(s_t + f_t)} \) and variable term \( \alpha_t = \left( f_t \frac{ds_t}{dt} - s_t \frac{df_t}{dt} \right) \frac{1}{(s_t + f_t)}. \)\(^{28}\)

\(^{28}\)The general solution to this first-order differential equation (dropping time subscripts – which apply to \( y, u, s, f, \alpha \) and \( \beta \) – for brevity) is

\[ y = \frac{du}{dt} = \exp \left( - \int \beta dt \right) \left[ A + \int \alpha \exp \left( \int \beta dt \right) \right] \]

\[ = A \exp \left[ -0.5 \left( s^2 + f^2 \right) - \ln (s + f) \right] + \exp \left[ -0.5 \left( s^2 + f^2 \right) - \ln (s + f) \right] \]

\[ \times \int \left( f \frac{ds}{dt} - s \frac{df}{dt} \right) \frac{1}{(s + f)} \exp \left[ 0.5 \left( s^2 + f^2 \right) - \ln (s + f) \right]. \]
\[
\frac{dy_t}{dt} = \alpha_t - y_t \beta_t.
\]

Discretizing and rearranging gives a recursive expression – an inhomogeneous recurrence relation with variable term:\(^{29}\)

\[
y_t - y_{t-1} = \alpha_t - y_t \beta_t
\]

\[
y_t = \frac{\alpha_t}{1 + \beta_t} + y_{t-1} \frac{1}{1 + \beta_t}.
\]

The variable term \(\frac{\alpha_t}{1 + \beta_t}\) involves changes in steady state unemployment, since from (23), discretizing where relevant:\(^{30}\)

\[
\alpha_t = (f_t \Delta s_t - s_t \Delta f_t) \frac{1}{(s_t + f_t)}
\]

\[
= (1 - \bar{u}_t) \Delta s_t - \bar{u}_t \Delta f_t
\]

\[
= \Delta \bar{u}_t (s_{t-1} + f_{t-1}),
\]

where \(\Delta\) denotes a monthly discrete difference. The coefficients in the recursive expression are therefore:

\(^{29}\)The recursive expression has general solution

\[
y = \Delta u_t = C \prod_{i=1}^{t-1} \frac{1}{1 + \beta_{j+1}} + \left( \prod_{i=1}^{t-1} \frac{1}{1 + \beta_{j+1}} \right) \sum_{j=0}^{t-1} \frac{\alpha_{j+1}}{1 + \beta_{j+1}} \prod_{i=1}^{j} \frac{1}{1 + \beta_{i+1}}.
\]

where \(C\) is an unspecified constant (that would be defined by the initial conditions) and \(\alpha\) and \(\beta\) are as defined in the text.

\(^{30}\)See the Petrongolo and Pissarides (2008) decomposition in Appendix C.2 for related intervening steps.
\[
\frac{1}{1 + \beta_t} = \frac{1}{1 + [(s_t + f_t) - \frac{\Delta(s_t + f_t)}{(s_t + f_t)}]} = \frac{(s_t + f_t)}{(s_t + f_t)^2 + (s_{t-1} + f_{t-1})}
\]
\[
\frac{\alpha_t}{1 + \beta_t} = \frac{\Delta \pi_t}{\pi_{t-1}} \frac{(s_t + f_t)^2 + (s_{t-1} + f_{t-1})}{s_{t-1} (s_t + f_t)}
\]

Thus actual unemployment dynamics are given by

\[
\Delta u_t = \Delta \pi_t \left( \frac{(s_t + f_t) (s_{t-1} + f_{t-1})}{(s_t + f_t)^2 + (s_{t-1} + f_{t-1})} + \Delta u_{t-1} \frac{(s_t + f_t)}{(s_t + f_t)^2 + (s_{t-1} + f_{t-1})} \right)
\]
\[
= \frac{\Delta \pi_t}{\pi_{t-1}} \frac{s_{t-1} (s_t + f_t)}{(s_t + f_t)^2 + (s_{t-1} + f_{t-1})} + \Delta u_{t-1} \frac{(s_t + f_t)}{(s_t + f_t)^2 + (s_{t-1} + f_{t-1})}
\]

as stated in the main text (equation (17)).

If \(s_t + f_t\) is constant, (24) simplifies to\(^{31}\)

\[
\Delta u_t = \Delta \pi_t \frac{1}{1 - r} - \Delta u_{t-1} \frac{r}{1 - r},
\]

where \(r = -\frac{1}{s + f}\), since then

\[
\Delta u_t = \Delta \pi_t \frac{s + f}{s + f + 1} + \Delta u_{t-1} \frac{1}{s + f + 1}
\]
\[
= \frac{\Delta \pi_t}{(s + f)^2 + s + f} \frac{(s + f)^2}{(s + f)^2 + s + f} + \Delta u_{t-1} \frac{s + f}{(s + f)^2 + s + f}.
\]

\(^{31}\)In the UK, \(s_t + f_t\) averages 12%, and the average of \(\Delta (s_t + f_t)\) is less than one-hundredth the size of the transition rates themselves (according to BHPS data), so this is an empirically reasonable approximation.

Monthly transition rates summing to 12% imply that current equilibrium contributions will contribute to current movements in actual unemployment with a factor of approximately 0.6. So about 40% of the variance of UK annual unemployment rate changes is due to past transition rates. Monthly transition rates summing to 50% (similar to US rates) imply a greater contribution of current equilibrium contributions: the factor is approximately 0.9 – so only 10% of the variance of annual US unemployment rate changes remains to be explained by past transition rates.
One way to think of (26) is as resulting from a discretized version of (12) with constant transition rates (using \( r = -1/(s + f) \)):

\[
\Delta u_t = \Delta \bar{u}_t + \Delta (r \Delta u_t) = \sum_{i=0}^{\infty} r^i \Delta \bar{u}_t - \sum_{i=1}^{\infty} r^i \Delta u_{t-1} = \frac{\Delta \bar{u}_t}{1-r} - \frac{r \Delta u_{t-1}}{1-r}.
\]

C.1.1 The relation between actual and steady state unemployment: the role of unemployment acceleration

Table C1 brings out the implications of (a discretized version of) (21) for how the actual unemployment rate varies with changes in the steady state rate. Contrast (22) with (21): the ‘levels’ version states that the unemployment rate will be further away from steady state the larger are changes in the unemployment rate, while the interpretation of the ‘differences’ version is that actual unemployment rate dynamics will deviate more from steady state movements the quicker the unemployment rate changes.

\[\begin{array}{ccc}
\Delta \left( \frac{\Delta u_t}{\bar{u}_t + f_t} \right) > 0 & \Delta \bar{u}_t > 0 & \Delta \bar{u}_t < 0 \\
\Delta u_t < \Delta \bar{u}_t; \Delta u_t \geq 0 & \Delta u_t < \Delta \bar{u}_t; \Delta u_t < 0 \\
\Delta \left( \frac{\Delta u_t}{\bar{u}_t + f_t} \right) < 0 & \Delta u_t > \Delta \bar{u}_t; \Delta u_t > 0 & \Delta u_t > \Delta \bar{u}_t; \Delta u_t \geq 0
\end{array}\]

Table C1. Deviation of actual unemployment dynamics from steady state unemployment dynamics

C.2 The Petrongolo and Pissarides (2008) steady state decomposition

The model in the text makes use of a decomposition of steady state unemployment proposed by Petrongolo and Pissarides (2008). For completeness, the mathematical
details of this decomposition are included here. Initially consider the decomposition of absolute changes in the level of steady state unemployment. The following algebra demonstrates that these changes can be decomposed into two (time-varying) additive components, one involving percentage changes in the inflow rate, $s_t$, and the other involving percentage changes in the outflow rate, $f_t$.

$$\Delta \bar{u}_t = \frac{s_t}{s_t + f_t} - \frac{s_{t-1}}{s_{t-1} + f_{t-1}}$$

$$= \frac{s_t (s_{t-1} + f_{t-1}) - s_{t-1} (s_t + f_t)}{(s_t + f_t) (s_{t-1} + f_{t-1})}$$

$$= \frac{s_t f_{t-1} - s_{t-1} f_t}{(s_t + f_t) (s_{t-1} + f_{t-1})}$$

$$= \frac{f_t (s_t - s_{t-1}) - s_t (f_t - f_{t-1})}{(s_t + f_t) (s_{t-1} + f_{t-1})}$$

$$= \frac{f_t \Delta s_t - s_t \Delta f_t}{(s_t + f_t) (s_{t-1} + f_{t-1})}$$

$$= \frac{f_t \Delta s_t}{(s_t + f_t) (s_{t-1} + f_{t-1})} - \frac{s_t \Delta f_t}{(s_t + f_t) (s_{t-1} + f_{t-1})}$$

$$= \left(1 - \frac{s_t}{s_t + f_t}\right) \frac{s_{t-1}}{s_{t-1} + f_{t-1}} \frac{\Delta s_t}{s_{t-1}} - \frac{s_t}{s_t + f_t} \left(1 - \frac{s_{t-1}}{s_{t-1} + f_{t-1}}\right) \frac{\Delta f_t}{f_{t-1}}$$

$$= (1 - \bar{u}_t) \bar{u}_{t-1} - \bar{u}_t (1 - \bar{u}_{t-1}) \frac{\Delta f_t}{f_{t-1}}$$

(27)

Rearranging (27) gives an expression for the percentage change in steady state unemployment (involving the approximation $\bar{u}_t \approx \bar{u}_{t-1}$):

$$\frac{\Delta \bar{u}_t}{\bar{u}_{t-1}} \approx \left(1 - \bar{u}_t\right) \frac{\Delta s_t}{s_{t-1}} - \left(1 - \bar{u}_{t-1}\right) \frac{\Delta f_t}{f_{t-1}}$$

Because $\left(1 - \bar{u}_t\right) \approx \left(1 - \bar{u}_{t-1}\right) \approx 1$ (an approximation also used in Elsby, Michaels and Solon, 2009, and Fujita and Ramey, 2009, in their decomposition of changes in steady state log unemployment), unemployment dynamics will be proximately
determined by the percentage changes in inflow and outflow rates, $\Delta s_t / s_{t-1}$ and $\Delta f_t / f_{t-1}$.

C.3 Comparison with an alternative non-steady state decomposition due to Elsby, Hobijn and Sahin (2009)

An alternative non-steady state decomposition was devised by Elsby, Hobijn and Sahin (2009) (EHS). EHS also start from the expression for unemployment dynamics (1):

$$\frac{du_t}{dt} = s_t (1 - u_t) - f_t u_t$$

EHS work with annual data so solve this forward one year to give:

$$u_t = \rho_t \bar{u}_t + (1 - \rho_t) u_{t-1}$$

where $t$ now denotes years and $\rho_t$ is the annual rate of convergence of actual unemployment rate to the steady state, which EHS calculate as:

$$\rho_t = 1 - \exp^{-12(s_t + f_t)}.$$  \hspace{1cm} (28)

$\rho_t$ will be close to unity when the sum of separation and job finding rates is large. In the special case where $\rho_t = 1, \forall t$, there are no deviations from steady state.\textsuperscript{32}

EHS take a log linear approximation to (28) around $u_{t-1} = \bar{u}_{t-1}, s_t = s_{t-1}$ and $f_t = f_{t-1}$;\textsuperscript{33}

$$\ln u_t \approx \ln \bar{u}_{t-1} + \rho_{t-1} (\ln \bar{u}_t - \ln \bar{u}_{t-1}) + (1 - \rho_t) (\ln u_{t-1} - \ln \bar{u}_{t-1})$$

Then the following relation is used:

\textsuperscript{32}The relationship between flow probabilities and transition rates $s_t + f_t$ implies that $\rho_t$ is the sum of the annual probabilities of flowing into or out of unemployment, since $12(s_t + f_t) = -\ln (1 - \rho_t)$.

\textsuperscript{33}This can be viewed as taking deviations of unemployment variables from last period’s steady state $\bar{u}_{t-1}$ and dividing through by $\bar{u}_{t-1}$, then approximating the resulting percentage deviations from steady state by log differences and approximating transition rates with their lags where appropriate.
\[ \ln u_t - \ln u_{t-1} = -\rho_{t-1} (\ln u_t - \ln \bar{u}_t) + \rho_{t-1} (\ln u_t - \ln u_{t-1}) , \]

implying

\[ \ln u_t - \ln \bar{u}_t = -\frac{1 - \rho_{t-1}}{\rho_{t-1}} \Delta \ln u_t , \]

the dynamics of actual log unemployment when transition rates are low (so \( \rho_t < 1 \)) can be expressed recursively as:

\[ \Delta \ln u_t \approx \rho_{t-1} (1 - \bar{\mu}_{t-1}) \left[ \Delta \ln s_t - \Delta \ln f_t \right] \]

\[ + \frac{1 - \rho_{t-2}}{\rho_{t-2}} \Delta \ln u_{t-1} . \]

The overall contributions of current and past changes in transition rates to actual log unemployment changes are, from (29)

\[ C_s^t = \rho_{t-1} \left[ (1 - \bar{\mu}_{t-1}) \Delta \ln s_t + \frac{1 - \rho_{t-2}}{\rho_{t-2}} C_s^{t-1} \right] \]

\[ C_f^t = \rho_{t-1} \left[ (1 - \bar{\mu}_{t-1}) \Delta \ln f_t + \frac{1 - \rho_{t-2}}{\rho_{t-2}} C_f^{t-1} \right] . \]

In (30) and (31), the first terms in the square brackets are the contributions of the current change in the log separation rate and log job finding rate respectively. The last terms are the contributions of previous changes in log transition rates. As in the decomposition presented in the main text, there are also contributions from the initial deviation from steady state and a residual.

Calculating rolling betas based on annual changes in log unemployment using BHPS data shows a picture of the relative contributions of outflow and inflow rate changes that is in many ways similar to Figure 5 in the main text. In the early years of the sample, unemployment dynamics are (more than) completely explained by log
inflow rate changes. The outflow rate dominates in mid-sample. The main difference when log unemployment changes are decomposed using the EHS method is that the contribution of the exit rate remains dominant until the end of the sample (with a noticeable rise in the importance of log inflow rate changes in the final years).

Fig. C1. Variance contributions of changes in log outflow and inflow rates to actual log unemployment rate dynamics (rolling 5-year betas) (EHS method)

A disadvantage of working with log unemployment changes – and one of the major motivations for this paper’s development of a dynamic decomposition based on a non-log change – is the difficulty of further decomposing the influence of log outflow and inflow rate changes to disentangle the impact of changes in job finding and separation rates from the effect of outflows and inflows via inactivity.