Imperfect Competition and Efficiency in Lemons Markets

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No 939

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

THE UNIVERSITY OF WARWICK

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ABSTRACT. This paper studies the impact of competition on the degree of inefficiency in lemons markets. More precisely, we characterize the second-best mechanism (i.e., the optimal mechanism with private information) in a stylized lemons market with finite numbers of buyers and sellers. We then study the relationship between the degree of efficiency of the second-best mechanism and market competitiveness. The relationship between the first-best and second-best mechanisms is also explored.

JEL Classification Numbers: C7, D4, D61, D82.

"... most cars traded will be the "lemons," and good cars may not be traded at all. The "bad" cars tend to drive out the good (in much the same way that bad money drives out the good)." GEORGE A. AKERLOF, *The Market for "Lemons": Quality Uncertainty and the Market Mechanism, 1970.*

1. INTRODUCTION

1.1. **Background.** It is conventional wisdom that competition is a good thing. The more the better. By fostering appropriate individual incentives, competition can help promote aggregate (or social) welfare. Economics textbooks are replete with models in which aggregate welfare increases with the degree of competition. One classic example of this key insight is provided by Cournot's model of imperfect competition: in this model, the difference between the Nash equilibrium market price and the constant marginal cost of production is strictly decreasing (and aggregate welfare is strictly increasing) in the number of competing firms.

Does competition have a similar beneficial impact in markets with asymmetric information? While it is well established that such markets tend in general to be inefficient (except perhaps in the restrictive, limiting scenario when they contain an arbitrarily large number of traders), much less is known about how the *degree* of inefficiency varies with the *degree* of competition. An overall aim of this paper

Date: 26th March 2010.

Key words and phrases. Lemons Markets, Second-Best Mechanism, Competition and Efficiency.

Acknowledgements. We thank V. Bhaskar, Tilman Börgers, Bhaskar Dutta, Matt Jackson, Claudio Mezzetti, Motty Perry, and participants at numerous seminars for their helpful comments and suggestions.

is to answer this question for markets with quality uncertainty, which, following Akerlof (1970), are termed lemons markets.

It goes without saying that a better understanding of the relationship between competition and efficiency in lemons markets is useful not only from a theoretical perspective but also from a practical (market-design and policy) perspective. Such understanding should provide insight into the role played by competition on how well lemons markets function and perform.

A distinguishing characteristic of a lemons market is that when contemplating the possibility of bilateral trade, one of the traders has relatively more information about something (eg., quality of the object for sale) that affects both traders' payoffs from trade. Such markets are ubiquitous, and have been intensely studied over the past three decades, both by economic theorists and by applied economists in the context of specific markets such as credit and labour markets. In his seminal paper, George Akerlof was the first to argue that lemons markets will typically be inefficient: sellers owning high quality objects may fail to trade, although there are buyers who would wish to trade with them. The basic intuition for this fundamental observation stems from the incentives of the sellers owning low quality objects: each such seller has an incentive to pretend to own a high quality object in order to command a high price for her, actually, low quality object. The consequence of such incentives is that buyers may not be prepared to buy at a price that is high enough for trade to be profitable for sellers owning high quality objects.

1.2. **Our Contribution.** We consider a stylized lemons market with finite numbers of buyers and sellers, in which each seller has private information about the quality of the object that she owns. A main objective is to characterize the maximal possible degree of efficiency that such a market can attain in any equilibrium. As such we do not study this market with any given, specific set of trading rules. This is because doing so would leave open the possibility that with a different set of trading rules it might be the case that a higher degree of efficiency is attained in equilibrium. Instead, we characterize the *second-best mechanism*, which defines the maximal achievable level of efficiency. Our approach thus involves considering all outcomes that can be achieved as a Bayesian Nash equilibrium of some game (induced by some set of trading rules). In order to conduct such a normative exercise, we use the mechanism design methodology, in which details of the trading rules are irrelevant. Of course, this exercise is made possible by appealing to the *Revelation Principle*, which allows us to confine attention to direct mechanisms in which agents truthfully report their private information.

A central contribution of this paper, therefore, is the characterization of the *second-best mechanism* (i.e., the optimal mechanism with private information) in a

stylized lemons market with finite numbers of buyers and sellers. That is, we characterize the mechanism which maximizes expected social surplus subject to satisfying appropriate incentive compatibility and individual rationality constraints, and being budget balanced. This analysis extends Samuelson (1984) who characterized the second-best mechanism for *bilateral* lemons markets. Samuelson, like us, uses the mechanism design methods introduced by Myerson and Satterthwaite (1983) who characterized the second-best mechanism for bilateral markets with *private* values. Gresik and Satterthwaite (1989) extend Myerson and Satterthwaite by characterizing the second-best mechanism for finite markets with private values.

Having characterized the second-best mechanism, we then study its properties. In particular, we show that the degree of efficiency of the second-best mechanism (and its relationship to the first-best) depends on the numbers of buyers and sellers, the likelihood of any seller owning a lemon rather than a peach, and whether the gains from trade are higher from trading a lemon or from trading a peach.[1] In order to highlight some of our results, we now briefly describe a few of the properties of the relationship between relative efficiency and market competitiveness, where the former is the "distance" (eg., ratio) between the degree of efficiency of the second-best mechanism and the first-best.[2]

Consider the scenario in which the market contains a single buyer and $N \geq 1$ sellers, in which the likelihood of any seller owning a lemon is sufficiently high, and the gains from trade are higher from trading a peach than from trading a lemon. In this case, we show that for relatively small lemons markets, the relationship between relative efficiency and competition is either monotonic (strictly increasing or strictly decreasing) or non-monotonic, depending on exact parameter values. For relatively large lemons markets, relative efficiency is strictly decreasing in the degree of competition (i.e., the optimal mechanism becomes less efficient relative to the first-best when the number of sellers is increased beyond a certain critical point). One (policy) implication is that the "optimal" degree of competition is uniquely defined and is bounded away from being "too large".

[1] A "lemon" denotes a low quality object, while a "peach" denotes a high quality object.

[2] We study this relationship in two alternative ways. First, we keep the number of buyers fixed, and study how maximal expected social surplus changes as the number of sellers is increased. This is the typical approach adopted by the literature that studies the same issue (of the relationship between efficiency and competition) for markets with symmetric information; for an example of recent work along these lines, see Janssen and Moraga-Gonzalez (2004). Second, we study how maximal expected social surplus changes as the numbers of buyers and sellers increases while keeping the buyer-seller ratio constant. This is the traditional replication scenario adopted by the literature on markets with private values; see, for example, Rustichini, Satterthwaite and Williams, 1994).

While the properties of the optimal mechanism when the number of buyers is arbitrary but strictly less than the number of sellers are similar to those when there is a single buyer, the optimal mechanism is distinctively different when the number of competing sellers is less than or equal to the number of buyers. In this latter case, we show that if the likelihood of any seller owning a lemon is sufficiently small, then the optimal mechanism implements the first-best. If, on the other hand, the likelihood of any seller owning a lemon is sufficiently large, then the ratio of the maximal achievable level of expected surplus to the first-best expected surplus is a constant, strictly less than one. Thus, when the number of buyers in the market is greater than or equal to the number of sellers, increasing the number of sellers (while maintaining the number of buyers at least equal to the number of sellers) does not affect maximal efficiency.

Before proceeding further, we should mention that our results establish that lemons markets will in general not attain the first-best outcome even in the limit as the number of competing traders becomes arbitrarily large. This specific result is perhaps not that unexpected, although it should be contrasted with the positive (limiting) result that has been established for markets with other kinds of asymmetric information such as with private values (see, for example, Rustichini, Satterthwaite and Williams, 1994).

1.3. **Organization of the Paper.** The remainder of the paper is organized as follows. Section 2 lays down our baseline model, as follows. Subsection 2.1 describes our basic market environment: in this benchmark set-up, there is a single buyer and a finite number of sellers, where each seller is one of two "types" and the sellers' types are independently and identically distributed. Furthermore, it is assumed that trading a peach generates a higher surplus than trading a lemon. Subsections 2.2 and 2.3 formulate the mechanism design problem. Section 3 solves this problem, characterizes the second-best mechanism, and derives the relationship between the efficiency of the second-best mechanism and market competitiveness (and compares all this to the first-best).

Section 4 considers several extensions to our baseline model, as follows. Subsection 4.1 studies the alternative scenario in which trading a lemon generates a higher surplus than trading a peach, and subsection 4.2 studies the general case with arbitrary numbers of buyers and sellers. Subsections 4.3–4.6 briefly consider several other extensions including ones that consider environments with a continuum of types, correlation in the sellers' types, and private information on the buyer's side.

We conclude in section 5 with a discussion of some aspects of our model and main results, and relate our contribution to the literature. In particular, in subsection 5.2, we study how the efficiency of the second-best mechanism responds to an increase in both the number of sellers and the number of buyers while keeping the ratio of these numbers constant. This exercise offers an alternative perspective on the relationship between efficiency and competitiveness. The results and insights obtained are similar to those obtained when we keep the number of buyers constant while changing the number of the sellers.

2. The Baseline Model

2.1. **Market Environment.** The market in the baseline model consists of a single buyer and a finite number N of sellers, where $N \geq 1$. Each seller owns one unit of an indivisible object whose quality q is her private information. It is either low quality (a "lemon"), $q = L$, or high quality (a "peach"), $q = H$. The probability that it is a lemon is α and the probability that it is a peach is $1 - \alpha$, where $0 < \alpha < 1$.

The buyer is interested in acquiring one and only one unit of the object. He values a lemon at v_L and a peach at v_H, where $v_H > v_L$. A seller's reservation values for a lemon and a peach are respectively c_L and c_H, where $c_H > c_L$. If the buyer acquires an object of quality q at price p, then his net payoff is $v_q - p$ (where $q = L, H$ and $p \geq 0$); and if a seller owning an object of quality q sells it at price p, then her net payoff equals $p - c_q$.[3] If an agent does not trade, then his or her net payoff is zero. All agents are risk-neutral and maximize expected utility.

The surpluses from trading a lemon and a peach are respectively $s_L \equiv v_L - c_L$ and $s_H \equiv v_H - c_H$, where $s_L > 0$ and $s_H > 0$. In the baseline model, we assume that $s_H > s_L$ — i.e., trading a peach generates a higher surplus than trading a lemon — as this is the relatively more interesting and more plausible case.

In subsection 4.1 we study the alternative scenario in which $s_L > s_H$. While the logic and much of the analysis in this case is similar to that of the baseline model in which $s_H > s_L$, the main results (concerning in particular the properties of the second-best mechanism) are rather different. Hence, for the sake of expositional clarity, it is instructive to deal with these two scenarios separately. Before proceeding further, we should note that for similar reasons the baseline model considers a market with a single buyer (and N sellers), and we extend the baseline model to the general case of M buyers and N sellers in subsection 4.2. The analysis and main insights when $M < N$ are similar to the single buyer ($M = 1$) case. But some of the analysis and the main results when $M \geq N$ are different from the $M < N$

[3] While the buyer may ascertain the quality of an object after acquiring it, the terms of trade cannot be made contingent on quality since that is non-verifiable by "third" parties (such as the courts).

case. This arises because in the former case it is feasible for all sellers to trade while that it is not so in the latter case.

2.2. **Incentive Constraints, Participation Constraints and Budget Balance.** By a standard appeal to the *Revelation Principle*, it suffices to focus on direct mechanisms where each seller announces her type and the mechanism selects an outcome conditional on the announcements of all sellers.[4] Furthermore, without loss of generality, we restrict attention to symmetric mechanisms where the outcome does not depend on the names of the sellers.[5] Following the Revelation Principle, the mechanism needs to satisfy the incentive compatibility constraints. We also require it to satisfy the participation (or individual rationality) constraints and be budget balanced.[6]

Given a direct, symmetric mechanism, let $\hat{p}^q$ be the probability that a seller sells her object if she reports that she is a q-type seller ($q = L, H$). Correspondingly, let $\hat{t}^q$ be the expected revenue that she obtains by doing so.[7] A mechanism is *incentive-compatible* (IC) if and only if no type of a seller benefits strictly by misrepresenting

[4]The institutional interaction that underlies a direct mechanism allows a seller to signal her privately held information through two instruments, namely, price and probability of trade. This is restrictive in the sense that other (costly) instruments such as warranties or "money burning" actions are not part of the structure of a direct mechanism. However, the key point of the *Revelation Principle* is that any outcome that can be generated by any indirect mechanism — including mechanisms that allow for a richer institution interaction, with various other instruments through which information could possibly be signalled and/or screened — can be attained through some direct mechanism. Hence, for the purposes of deriving and characterizing the *second-best* (which is the focus of the current paper), one can, without loss of generality, restrict attention to direct mechanisms.

[5]The argument for why the restriction to symmetric mechanisms is without loss of generality is based on the following two observations: (i) an asymmetric mechanism is equivalent to a random mechanism (one that picks a symmetric mechanism randomly), and (ii) a random mechanism can do no better than a symmetric mechanism (by definition).

[6]The former requirement is motivated by the notion that trade is voluntary, while the latter by the notion that there are no "third" parties (such as governments) who subsidize market trade.

[7]This characterization of a direct mechanism follows from the *Revelation Principle*, which says that any outcome of any indirect mechanism can be captured in this way, with type-contingent expected transfers and trade probabilities. Wilson (1979, 1980) are early contributions to the literature on lemons markets in which trade probabilities also feature, but his analysis is restricted to three specific classes of indirect mechanisms. He does not adopt the mechanism design methodology that underlies the analysis in the current paper which allows us to characterize the *second-best* mechanism, and maximal efficiency attainable across *all* possible trading mechanisms.

her type.[8]

$$\hat{t}^H - \hat{p}^H c_H \geq \hat{t}^L - \hat{p}^L c_H, \quad \text{and} \tag{1}$$

$$\hat{t}^L - \hat{p}^L c_L \geq \hat{t}^H - \hat{p}^H c_L. \tag{2}$$

Individual Rationality (IR) for an arbitrary seller of type q $(q = H, L)$ requires that the expected payoff from the mechanism be at least the payoff from the outside option, which is normalized to zero.

$$\hat{t}^H - \hat{p}^H c_H \geq 0, \quad \text{and} \tag{3}$$

$$\hat{t}^L - \hat{p}^L c_L \geq 0. \tag{4}$$

Individual Rationality for the buyer follows the same principle. Since the buyer does not know the type of object owned by any seller, individual rationality requires that his expected payoff from transacting with any individual seller must be non-negative. This yields (using the requirement that the mechanism be budget balanced, BB)

$$(1-\alpha)\left[\hat{p}^H v_H - \hat{t}^H\right] + \alpha\left[\hat{p}^L v_L - \hat{t}^L\right] \geq 0. \tag{5}$$

The symmetry of the mechanism entails restrictions on $\hat{p}^H$ and $\hat{p}^L$. From the point of view of an individual seller of type H, the probability that there are k other sellers of type H is given by $\binom{N-1}{k}(1-\alpha)^k\alpha^{N-1-k}$; and in this case, symmetry implies that she sells her product with a probability at most $1/(k+1)$. Taking expectations across all possible realizations of k, it follows that

$$\hat{p}^H \leq \frac{1-\alpha^N}{N(1-\alpha)}. \tag{6}$$

Using a similar argument, it also follows that

$$\hat{p}^L \leq \frac{1-(1-\alpha)^N}{N\alpha}. \tag{7}$$

The probability of sale between an arbitrarily chosen seller and the buyer is $(1-\alpha)\hat{p}^H + \alpha\hat{p}^L$. Since the total probability with which trade occurs must be less than or equal to one, the mechanism must also satisfy

$$N[(1-\alpha)\hat{p}^H + \alpha\hat{p}^L] \leq 1. \tag{8}$$

[8]The first inequality is an arbitrary high-type seller's IC condition while the second is an arbitrary low-type seller's IC condition.

2.3. **Mechanism Design Problem.** The expected surplus realized from a symmetric, direct mechanism is $N[(1-\alpha)\hat{p}^H s_H + \alpha\hat{p}^L s_L]$. Observe that the expression in the square brackets is the expected surplus from the transaction between an arbitrarily chosen seller and the buyer. The mechanism design problem is to choose a symmetric, direct mechanism amongst those that satisfy the two IC constraints, three IR constraints (within which has been factored the requirement of BB) and three admissibility constraints that generates the maximal expected surplus. Formally, this problem is:

$$E \equiv \max_{\hat{p}^H,\hat{p}^L,\hat{t}^H,\hat{t}^L} \quad N\left[(1-\alpha)\hat{p}^H s_H + \alpha\hat{p}^L s_L\right] \tag{9}$$

subject to (1)–(8).

The solution to this maximization problem defines the second-best mechanism (i.e., the optimal mechanism with private information), to which we now turn.

3. Second-Best Mechanism

3.1. **A Reduced-Form Problem.** We solve (9) by defining and solving a reduced-form problem that involves the following change of variables:

$$\tilde{p}^H = N(1-\alpha)\hat{p}^H \quad \text{and} \quad \tilde{p}^L = N\alpha\hat{p}^L,$$

where $\tilde{p}^H$ and $\tilde{p}^L$ are respectively the total probabilities with which trade occurs between the buyer and sellers owning peaches and lemons. Given this change of variables, the constraints (6)–(8) become

$$\tilde{p}^H \leq 1-\alpha^N, \tag{10}$$

$$\tilde{p}^L \leq 1-(1-\alpha)^N \quad \text{and} \tag{11}$$

$$\tilde{p}^L + \tilde{p}^H \leq 1, \tag{12}$$

and the maximand in (9) becomes $\tilde{p}^H s_H + \tilde{p}^L s_L$. Note that $1-\alpha^N$ (resp., $1-(1-\alpha)^N$) is the probability that there exists at least one seller amongst the N sellers who owns a peach (resp., a lemon). The two IC conditions are satisfied only if the following inequality holds:[9]

$$\hat{p}^H \leq \hat{p}^L,$$

which, using the change of variables defined above, becomes:

$$\tilde{p}^H \leq \left[\frac{1-\alpha}{\alpha}\right]\tilde{p}^L. \tag{13}$$

[9]This follows by rewriting (1) and (2) as $(\hat{p}^H - \hat{p}^L)c_H \leq \hat{t}^H - \hat{t}^L \leq (\hat{p}^H - \hat{p}^L)c_L$, and then applying the assumption that $c_H > c_L$.

This implication of the sellers' IC conditions — namely, that the probability $\hat{p}^H$ with which trade occurs with a high-type seller is no greater than the probability $\hat{p}^L$ with which trade occurs with a low-type seller — turns out to be the binding constraint for some scenarios. In the other scenarios, it is a buyer's induced IR constraint (defined below) which will be the binding constraint. The following lemma contains several other preliminary results:

Lemma 1. *At a solution to the mechanism design problem (9), the low-type seller's IC constraint, (2), binds as does the high-type seller's IR constraint, (3). That is:*

$$\hat{t}^H = \hat{p}^H c_H \quad \textit{and} \quad \hat{t}^L = \hat{p}^H c_H + (\hat{p}^L - \hat{p}^H) c_L.$$

Furthermore, the low-type seller's IR constraint is automatically satisfied with these transfers.

Proof. See Appendix A. □

Using Lemma 1, substitute for $\hat{t}^H$ and $\hat{t}^L$ in the individual rationality constraint of the buyer, (5), and it becomes (after using the change of variables defined above, and some simplification):

$$\tilde{p}^H \left[s_H - \alpha(v_H - c_L) \right] + (1-\alpha)\tilde{p}^L s_L \geq 0. \tag{14}$$

It should perhaps be emphasized that inequality (14) is not the buyer's IR constraint, but the buyer's "induced" IR constraint as it is derived after the transfers implied by two of the constraints are plugged into the buyer's IR constraint.

Now define the following *reduced-form* problem:

$$E^* \equiv \max_{\tilde{p}^H, \tilde{p}^L} \quad \tilde{p}^H s_H + \tilde{p}^L s_L \tag{15}$$

$$\text{subject to (10)–(14).}$$

The following lemma establishes the connection between the two maximization problems:

Lemma 2. *Using the change of variables defined above and the expected transfer payments stated in Lemma 1, any solution of (15) defines a solution of (9) and vice-versa. Moreover,* $E = E^*$.

Proof. See Appendix A. □

Given Lemma 2, we are now ready to solve for the second-best mechanism by solving the reduced-form maximization problem (15).

3.2. **Second-Best Expected Surplus.** Define $\alpha^* = s_H/(v_H - v_L)$. Note that $\alpha^* > 0$ since (by assumption) $v_H > v_L$ and $v_H > c_H$. Furthermore, $\alpha^* < 1$ if and only if $v_L < c_H$.

Propositions 1 and 2 (below) respectively state the expected surplus E associated with the second-best mechanism when $\alpha \leq \alpha^*$ and $\alpha > \alpha^*$. We sketch the main elements of the argument in the text below, but relegate the detailed calculations to Appendix A (where the solution that underpins the second-best expected surplus is also derived). As will become clear, some of the analysis and most of the results differ according to whether $\alpha \leq \alpha^*$ or $\alpha > \alpha^*$. This arises because in the former ("soft" buyer) case the buyer's induced IR constraint does not bind (it is the sellers' IC constraints that do) while in the latter ("tough" buyer) case the buyer's induced IR constraint is the binding constraint.

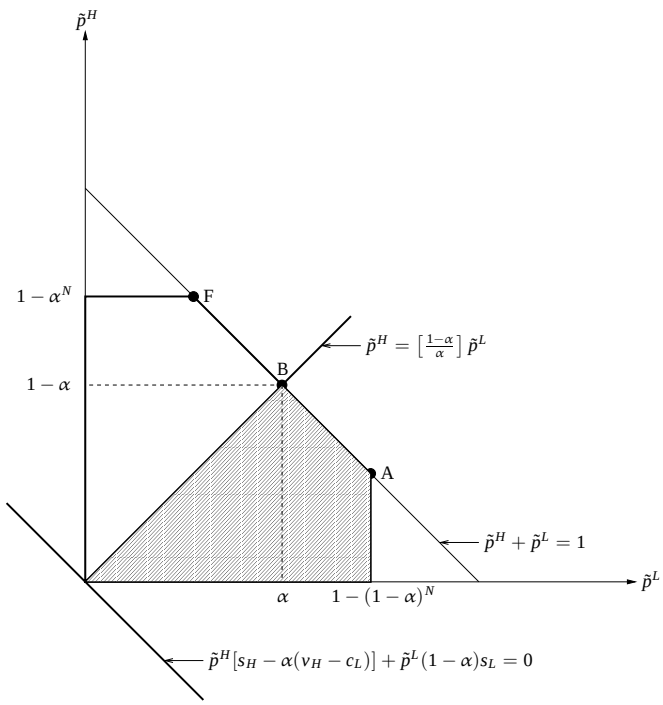

FIGURE 1. *Soft Buyer*: The feasible set when $\alpha \leq \alpha^*$

We begin by noting that the set of all pairs $(\tilde{p}^L, \tilde{p}^H)$ that satisfy (10)-(13) comprises the shaded region in Figure 1. In the absence of (14), therefore, the solution of (15) lies at point B. If $\alpha \leq \alpha^*$, then all points in the shaded region shown in Figure 1 satisfy (14). Hence:

Proposition 1 ("Soft" Buyer)**.** *If $\alpha \leq \alpha^*$, then the second-best expected surplus $E = (1-\alpha)s_H + \alpha s_L$.*

Proof. See Appendix A. □

We continue with our argument (which turns to the characterization of the second-best expected surplus when $\alpha > \alpha^*$) after the following remark that provides some intuition behind Proposition 1 (a fuller discussion of this proposition is deferred to subsections 3.2 and 3.3).

Remark 1. Point B in Figure 1 depicts the second-best solution, which entails setting $\tilde{p}^L = \alpha$ and $\tilde{p}^H = 1 - \alpha$ or equivalently, $\hat{p}^H = \hat{p}^L = 1/N$. The intuition for this solution runs as follows. Since $s_H > s_L$, maximizing expected surplus entails making $\hat{p}^H$ as large as possible. This means (given admissibility) getting it to be as close to $1 - \alpha^N$ as possible. But the IC constraint $\hat{p}^H \leq \hat{p}^L$ bites, and hence $\hat{p}^H = \hat{p}^L = \hat{p}$. The desired conclusion then follows immediately because of the admissibility requirement that the total probability with which trade occurs cannot exceed one (i.e., $N\hat{p} \leq 1$), and optimality then entails setting $\hat{p} = 1/N$. The following observations shed further light on the second-best mechanism in the case under consideration. Using Lemma 1, we obtain that in this case $\hat{t}^H = \hat{t}^L = c_H/N$. An indirect mechanism that implements the second-best is the following fixed-price mechanism: The buyer makes a "take-it-or-leave-it" fixed-price offer, and then he chooses amongst those sellers who accept to trade at the announced price. It is easy to verify that there exists a perfect Bayesian equilibrium of this indirect mechanism in which the buyer announces that he is willing to trade at price equal to c_H, each seller of either type accepts to trade at this price, and the buyer then selects to trade with each seller with equal probability (which is $1/N$). The expected surplus generated in this equilibrium is

$$N\left(\frac{1}{N}\left[(1-\alpha)s_H + \alpha s_L\right]\right),$$

which equals $(1-\alpha)s_H + \alpha s_L$, the second-best expected surplus. The buyer's expected payoff equals $N[(v^e - c_H)/N] = v^e - c_H$ (where $v^e \equiv (1-\alpha)v_H + \alpha v_L$), which is greater than or equal to zero if and only if $\alpha \leq \alpha^*$ (which is, of course, the hypothesis of Proposition 1). The expected payoffs to a high-type seller and a low-type seller are respectively zero and $(c_H - c_L)/N$.

Now let us return to the main argument and assume that $\alpha > \alpha^*$. In this case not all points in the shaded region shown in Figure 1 satisfy (14). Figures 2 and 3 show how (14) affects the feasible set depending on whether or not point A remains a feasible point. Point A remains a feasible point if and only if the following inequality holds:

$$s_L \geq \frac{(1-\alpha)^{N-1}}{1-(1-\alpha)^N}\left[\alpha(v_H - c_L) - s_H\right]. \tag{16}$$

Since the RHS of (16) is strictly decreasing in N, converges to zero in the limit as $N \to \infty$ and is strictly greater than s_L when $N = 1$, there exists an N^*, where $N^* \geq 2$, such that (16) holds if and only if $N \geq N^*$.

If point A does remain a feasible point (i.e., $N \geq N^*$), then the shaded region shown in Figure 2 comprises the feasible set; in this case the solution of (15) lies at point C. If, on the other hand, point A does not remain a feasible point (i.e., $1 \leq N < N^*$), then the shaded region shown in Figure 3 comprises the feasible set; in this case the solution of (15) lies at point D. Hence:

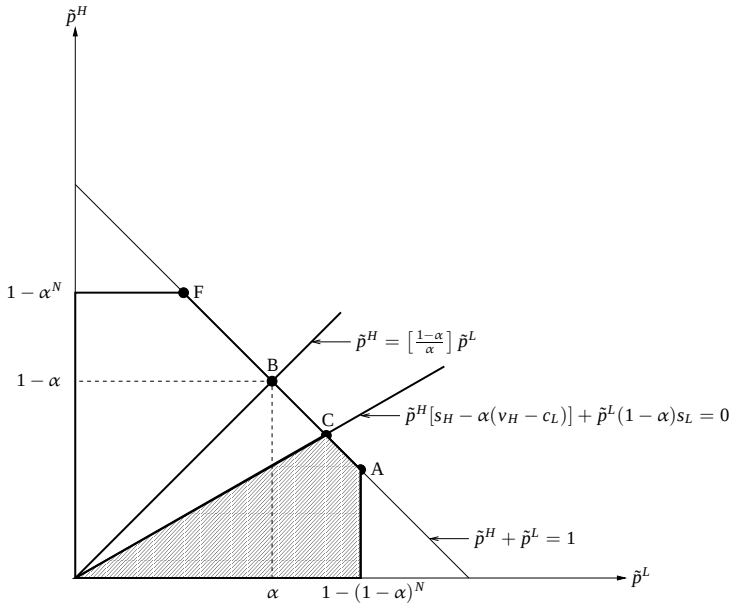

FIGURE 2. *Tough Buyer and Large Market*: The feasible set when $\alpha > \alpha^*$, and inequality (16) holds (i.e., $N \geq N^*$).

Proposition 2 ("Tough" Buyer)**.** *Assume that $v_L < c_H$. If $\alpha > \alpha^*$ and:*
(i) [Large Markets]. If $N \geq N^*$ (i.e., (16) holds), then the second-best expected surplus

$$E = \frac{\alpha(c_H - c_L)s_L}{\alpha(c_H - c_L) - (1-\alpha)(s_H - s_L)}.$$

(ii) [Small Markets]. If $1 \leq N < N^*$ (i.e., (16) does not hold), then the second-best expected surplus

$$E = \frac{\alpha[1-(1-\alpha)^N](c_H - c_L)s_L}{\alpha(c_H - c_L) - (1-\alpha)s_H}.$$

Proof. See Appendix A. □

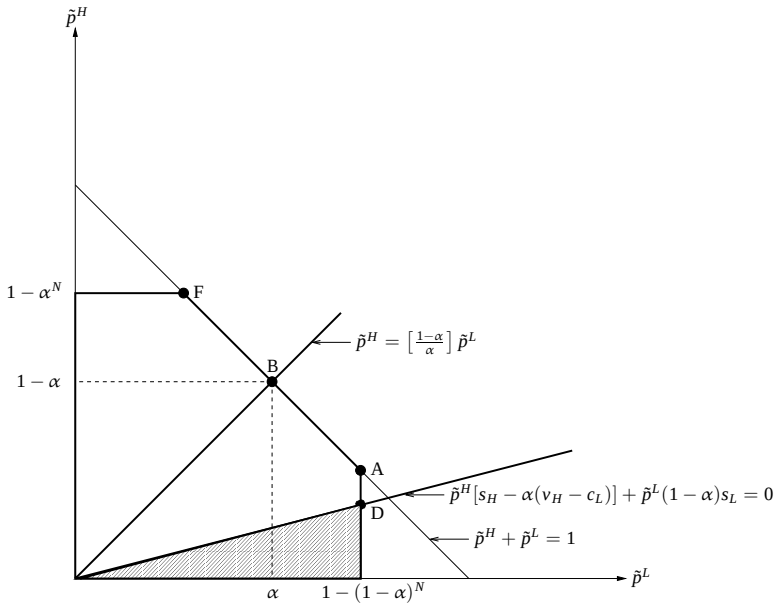

FIGURE 3. *Tough Buyer and Small Market*: The feasible set when $\alpha > \alpha^*$, and inequality (16) does not hold (i.e., $1 \leq N < N^*$).

Remark 2. Notice that the fixed-price mechanism of Remark 1 does not implement the second-best mechanism here since it would give the buyer a negative expected payoff.[10] This is consistent with the observation — obtainable from Figures 2 and 3 — that the buyer's induced IR constraint plays a decisive role in pinning down the second best mechanism. This constraint, it may be recalled, is the buyer's IR constraint after the transfers implied by the low-type's IC constraint and the high-type's IR constraint are factored into it. If N is sufficiently small — how small depends on the "tightness" of the buyer's induced IR constraint (cf. Figures 2 and 3) — then the total probability with which trade occurs is strictly less than one. Only when the market has enough sellers does trade occur for sure, although with positive probability the "wrong" object is traded (i.e., a lemon is traded instead of a peach).

3.3. **Comparison with the First-Best.** The first-best lies at point F; this is because (since $s_H > s_L$), in the first-best trade occurs with a high-type seller unless all sellers are of low type. Hence, the first-best has $\tilde{p}^H = 1 - \alpha^N$ and $\tilde{p}^L = \alpha^N$. It thus follows that the first-best cannot be attained by the second-best mechanism except in the special case when there is a single seller *and* the buyer is soft (in this special case, points F and B in Figure 1 coincide). We summarize this in the following corollary.

[10]Recall that his expected payoff in this indirect mechanism is non-negative if and only if $\alpha \leq \alpha^*$.

Corollary 1 (Comparison with the First-Best)**.** *The second-best mechanism cannot attain the first-best (except in the special case when the market contains a single seller who owns a lemon with a sufficiently small probability).*

So, the second-best mechanism does not in general implement the first-best outcome. This result shows that Akerlof's (1970) central message about the inefficiency of lemons markets — which he developed in the context of perfectly competitive markets — is robust to imperfect competition. As can be seen from Figures 1 and 2, when the market contains a sufficient number of sellers, trade occurs with probability one. The second-best mechanism is inefficient, however, because of *allocative inefficiency*: trade occurs with positive probability between the buyer and a low-type seller even when the market contains a high-type seller. To understand this better, note that when N is large, it is commonly known that a fraction α are low-type sellers and $1-\alpha$ high-type sellers. However, this information cannot be used to identify whether a given seller is a high-type or a low-type. The uncertainty regarding the type of a seller remains and this explains why the allocative inefficiency persists even in the limit.

3.4. **The Role of Competition.** Using the results established above, we can now answer the following two questions of interest: (i) What impact does an increase in the number of competing sellers have on the second-best expected surplus? and (ii) What impact does an increase in the number of competing sellers have on relative efficiency (i.e., the "distance" between the second-best and first-best expected surpluses)?

Using Figures 1–3, it is easy to see that (i) if $\alpha \leq \alpha^*$, then the level of second-best expected surplus E is independent of N; and (ii) if $\alpha > \alpha^*$, then there is an integer N^* (where $N^* \geq 2$) such that E is strictly increasing in N over the interval $[1, N^*)$, attains the same or a higher value at N^* as it does at N^*-1, and is a constant for all $N \geq N^*$. Figure 4 illustrates the various possibilities.

Thus, second-best expected surplus does not change with an increase in the number of competing sellers once the market contains a certain critical number of them. This is a surprising result. It implies, moreover, that if there is an infinitesimal cost of getting a seller into the market, then second-best welfare (i.e., second-best expected surplus minus the total cost of having N sellers in the market) is strictly decreasing in the degree of market competition N, once it is sufficiently intense. Contrary to conventional wisdom, only a limited degree of competition is good; too much is bad. In particular, if the buyer is "soft" then the optimal number of sellers is one, and if the buyer is "tough" then the optimal number of sellers is $N^* \geq 2$. To put it differently, *if the likelihood of sellers owning lemons is small then a*

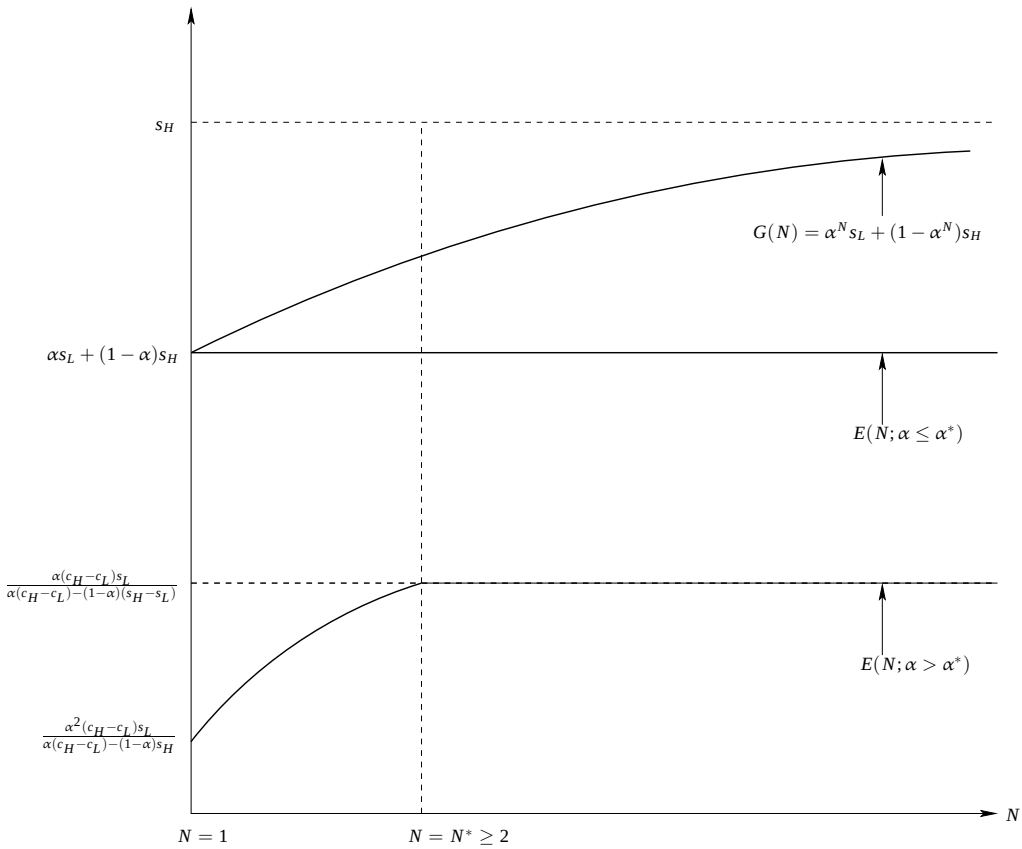

FIGURE 4. The relationship between efficiency and the number of sellers, where $G(.)$ and $E(.)$ respectively denote the first-best and second-best expected surpluses.

bilateral monopoly is the optimal market structure, but if the likelihood of sellers owning lemons is high then an oligopoly is the optimal market structure.

The first-best expected surplus G is strictly increasing in N.[11] Hence, if the buyer is soft, then relative efficiency is strictly decreasing in the degree of market competition. But if the buyer is tough, then the relationship between relative efficiency and the degree of competition is a bit more complex: For sufficiently large markets relative efficiency is strictly decreasing in the degree of market competition, but for sufficiently small markets it can be increasing, decreasing or non-monotonic since both E and G are strictly increasing in N over the range $[1, N^*)$. Finally, notice that in the limit as N tends to infinity, the second-best mechanism does not attain the

[11]First-best expected surplus (for any s_L and s_H) is $G = \alpha^N s_L + (1-\alpha)^N s_H + [1 - \alpha^N - (1-\alpha)^N] \max\{s_L, s_H\}$. Thus, when $s_H > s_L$, $G = \alpha^N s_L + (1-\alpha^N)s_H$.

first-best.[12] The following corollary summarizes two of our main insights regarding the role of market competition:

Corollary 2 (Role of Competition)**.** *The second-best expected surplus does not change with any increase in the number of competing sellers once the market contains a certain critical number of them. Moreover, the second-best mechanism becomes less efficient relative to the first-best with any further increase in the degree of competition beyond a certain critical point.*

4. Extensions

4.1. **Lemons Generate Higher Surplus.** We now consider our baseline model with the alternative (but relatively less plausible) scenario in which the surplus from trading a lemon is higher than the surplus from trading a peach (i.e., with the alternative assumption that $s_L > s_H$). It is easy to verify that all the arguments and preliminary results contained in subsections 2.2, 2.3 and 3.1 carry over to this scenario. The arguments leading to the characterization of the second-best mechanism are based on those presented in section 3.2, and the proofs of Propositions 1 and 2 in Appendix A deal with both scenarios at the same time. It is shown that if $s_L > s_H$, then the second-best solution is depicted by point A in Figures 1 and 2 for the soft buyer and tough buyer, large market cases, respectively. And by point D in Figure 3 for the tough buyer, small market case. Thus, when $s_L > s_H$, we have:

(a) If $\alpha \leq \alpha^*$, then $E = (1-\alpha)^N s_H + [1-(1-\alpha)^N]s_L$.
(b) If $\alpha > \alpha^*$ and $N \geq N^*$, then $E = (1-\alpha)^N s_H + [1-(1-\alpha)^N]s_L$
(c) If $\alpha > \alpha^*$ and $1 \leq N < N^*$, then $E = \dfrac{\alpha[1-(1-\alpha)^N](c_H - c_L)s_L}{\alpha(c_H - c_L) - (1-\alpha)s_H}$.

The first-best lies at point A; this is because (since $s_H < s_L$), in the first-best, trade occurs with a low-type seller unless all sellers are of high type; and hence the first-best has $\tilde{p}^H = (1-\alpha)^N$ and $\tilde{p}^L = 1-(1-\alpha)^N$. This means that the second-best mechanism attains the first best outcome provided that it is *not* the case that the buyer is tough *and* $1 \leq N < N^*$ (in this case Figure 3 applies, and points A and D are different). Consequently, if first-best requires trading a lemon rather than a peach, then the second-best mechanism attains the first-best except in the special case when the market contains a sufficiently small number of sellers each of whom owns a lemon with a sufficiently large probability. In this latter case,

[12] It may be noted that, in contrast, in markets with other kinds of asymmetric information the first-best is typically attainable in the limit as the number of traders increases without bound (see, eg., Rustichini, Satterthwaite and Williams (1994), who establish such a limiting result in markets with private values.)

second-best expected surplus is strictly increasing in the degree of market competition. Furthermore, relative efficiency, E/G, is strictly increasing in the degree of market competition, N, over the range $[1, N^*)$, and then $E/G = 1$ for all $N \geq N^*$. These results are consistent with conventional wisdom that more competition is better, but of course this is valid only in those lemons markets in which trading a lemon generates a higher social surplus than trading a peach.

4.2. **Many Buyers.** In Appendix B we extend our baseline model to a set-up in which the market contains an arbitrary number M of buyers. It turns out that the analysis and the results for the case when $M < N$ are essentially the same as for the single buyer case. In particular, the second-best mechanism becomes *less* efficient relative to the first-best when the number of competing sellers is increased beyond a certain critical number. The analysis and results for the case when $M \geq N$ are however different. For example, it is shown that increasing the number of sellers (while maintaining the number of buyers at least equal to the number of sellers) does not affect relative efficiency (the ratio of the second-best expected surplus to the first-best).

4.3. **Buyer Heterogeneity.** The negative results established above raises the question as to whether there is any way of overcoming them. In Muthoo and Mutuswami (2005) we have studied an extension to our baseline model that allows for private information on the buyer's side. One might think that introducing private information on the buyers side will add to the inefficiency; this, though, is not necessarily correct because the private information on the buyers side can be used to relax the low-type seller's incentive constraint.

In the extended set-up there are *two* types of the buyer, one for whom the surplus from trading a peach is more and the other for whom the surplus from trading a lemon is higher. Then, the socially optimal decision — viz., which type of good to transfer — depends on the type of the buyer which is unknown to the seller. In this extended scenario, it is no longer clear that a low-type seller wants to pretend to be a high-type: indeed, if the probability that a buyer is a "low" type is sufficiently high, then she would *not* want to do so. This raises the possibility that the inefficiency resulting from the seller's private information can be corrected by allowing for private information on the buyer's side.

Of course, introducing two types of the buyer makes the mechanism design problem more complex because we have to deal with additional individual rationality and incentive compatibility constraints. We have not been able to characterize the solution to the resulting mechanism design problem completely. However, we have been able to determine restrictions on parameters which ensure that when the number of sellers is large enough, then we can find an asymptotically efficient

mechanism. Our results in this regard suggest that while asymptotic efficiency in a market for lemons settings is not a generic phenomenon, there are still significant cases where it is possible.

4.4. **Correlation.** In our baseline model, the sellers' types are independently distributed. One might ask, if by allowing for correlation, one can obtain positive results as in Crémer and McLean (1988). They showed that if agents' types are correlated, then one can construct a two-stage 'augmented mechanism' which implements the same outcome as the original mechanism, but where all agents' informational rents are driven down to zero. This fact can be used to implement efficient (first-best) outcomes in some circumstances provided the efficient outcome is implementable without requiring budget balance: essentially, one can construct an 'augmented mechanism' which recovers from the agents the implicit subsidy needed to implement the efficient outcome. Thus, the key to seeing whether correlation amongst sellers' types can help in our context is to see whether we can implement the efficient outcome if we do not require individual rationality and / or budget balance. Note, however, that incentive compatibility by itself requires that $\hat{p}^H \leq \hat{p}^L$ and this condition implies inefficiency whenever $s_H > s_L$. Therefore, the first-best outcome is unimplementable even if we are willing to give up budget balance and individual rationality and this shows that small correlation amongst sellers' types in unlikely to change the nature of our results.

4.5. **Continuum of Types.** Another interesting extension that one could consider is to allow for a continuum of quality levels: that is, each seller's object is of quality q where q lies in a compact interval $[\underline{q}, \overline{q}]$. Such an extended set-up with a continuum of sellers' types introduces two kinds of complications into our baseline model. First, the computation of the first-best level of surplus for a given number of sellers becomes complex because the optimal allocation depends on the net surplus function $s(q) = v(q) - c(q)$ about which nothing can be said *a priori*, where $v(q)$ and $c(q)$ respectively are a buyer's and a seller's reservations values for an object of quality q. Second, while one can extend the techniques introduced in Samuelson (1984) for the bilateral lemons context to setup the problem for the general case of many buyers and many sellers, solving the resulting problem analytically is not straightforward.

The main difference between a model with two types of sellers and one with a continuum of sellers' types is that incentive compatibility is a much more restrictive constraint in the latter. Intuitively, and very loosely, this suggests that the set of feasible mechanisms "shrinks" when we move from a two-type model to a continuum of types model. In our case, however, the main results that we have obtained

with a two-type model are essentially negative, and this suggests that introducing a continuum of types model is unlikely to change the nature of these results.

4.6. **Mechanisms involving Third Parties.** The combination of individual rationality and budget balance implies that there is no active role for third parties in the trading mechanisms that we consider. Mezzetti (2004) shows that if the active involvement of third parties is allowed, then it is possible to do better. Specifically, Mezzetti considers a mechanism of the following type: (a) In the first stage, the sellers report their types; (b) In the second stage, a decision is made as to which seller transfers her object to the buyer. No transfers are made at this stage; (c) After the object is transferred, the buyer and the sellers report their payoffs. The transfers are made on the basis of the reports of the buyer in the third stage. The key feature of this mechanism is that the trading decision is separated from the transfers whereas they are simultaneously determined in our mechanisms.

While this is an interesting mechanism and Mezzetti (2004) shows that it can implement the efficient outcome in a bilateral lemons context, it should be noted that the mechanism (i) requires budgets to be not balanced off-the-equilibrium path, (ii) requires an active third party (since budget is not always balanced off equilibrium), (iii) is vulnerable to collusion between the buyer and the third party. These aspects make it difficult to justify this mechanism as a "reasonable" trading mechanism in a market context.

5. Discussion and Related Literature

5.1. **Non-Monotonicity.** One aspect of our results, both in the benchmark and the extended models, is that relative efficiency — the "distance" between second-best and first-best expected surpluses — may be a non-monotonic function of the number of sellers. In the benchmark model, non-monotonicity arises when the probability that each seller owns a lemon is high enough so that the high quality object cannot be transferred with probability one even when all traders report that they have high quality objects.[13] When one adds additional sellers, the achievable expected surplus increases because the probability that all traders have high quality objects decreases. However, the first-best level of expected surplus also increases, and so it is not clear what happens to relative efficiency.

5.2. **Impact of Market Size.** Our analysis so far has involved holding the number of buyers constant while changing the number of sellers. In contrast, the literature in the private values case has considered the more traditional replication scenario,

[13]This is the case illustrated in Figure 3.

where the number of buyers and sellers are both varied while keeping the buyer-seller ratio constant. We have been able to derive limited results with regard to the impact on efficiency of changing the market size for our benchmark model and they are commensurate with the results that we obtain when only the number of sellers is changed.

If $N \leq M$ (the number of buyers is more than the number of sellers), then the analysis (in Appendix B) shows that replicating this economy has no impact on market efficiency. The more interesting case is when $M < N$. We can divide the analysis of this case into various sub-cases.

5.2.1. *The case in which $M < N$, $\alpha \leq \alpha^*$, and $s_H > s_L$.* Here, as can be seen from Figure 8 (in Appendix B), the "induced individual rationality constraint of the buyer" (given by $\tilde{p}^H[s_H - \alpha(v_H - c_L) + \tilde{p}^L(1-\alpha)s_L \geq 0$) does not bind. The first-best involves $\tilde{p}^H = N(1-\alpha)b^H, \tilde{p}^L = M - N(1-\alpha)b^H$ which is point F on the diagram. Hence, the first-best level of surplus is given by $E^M = N(1-\alpha)b^H s_H + [M - N(1-\alpha)b^H]s_L$. The achievable level of surplus (given by point B on the diagram) corresponds to $\tilde{p}^H = (1-\alpha)M, \tilde{p}^L = \alpha M$ and is given by $E^* = (1-\alpha)Ms_H + \alpha M s_L$. Hence, the level of market efficiency is given by

$$\frac{E^*}{E^M} = \frac{M[(1-\alpha)s_H + \alpha s_L]}{N(1-\alpha)b^H s_H + [M - N(1-\alpha)b^H]s_L}.$$

What happens when the market is replicated so that we have γM buyers and γN sellers where $\gamma > 1$? The level of market efficiency is now given by

$$\frac{E^*}{E^M}(\gamma) = \frac{M[(1-\alpha)s_H + \alpha s_L]}{N(1-\alpha)b^H(\gamma)s_H + [M - N(1-\alpha)b^H(\gamma)]s_L}$$

where $b^H(\gamma)$ is the value of b^H (given by (B.1)) when there are γM buyers and γN sellers. We are unable to show formally that $b^H(\gamma)$ is increasing in γ though simulation with a variety of parameters suggests it. Assuming that $b^H(\gamma)$ is increasing in γ, this means that market efficiency decreases as a function of market size. This is illustrated by the simulation in Figure 5.[14]

5.2.2. *The case of $M < N$, $\alpha > \alpha^*$, and $s_H > s_L$.* There are two possibilities here illustrated by Figures 9 and 10. In Figure 9, the optimal solution lies at point C which is above point A. Here, the situation corresponds to the previous subcase and market efficiency decreases as a function of market size because the distance between point C (the second-best) and point F (the first-best) increases as the market size grows. In Figure 10, the optimal solution is at point D. When the market

[14]The parameters used in Figure 5 are $\alpha = 0.35, v_H = 2.25, v_L = 1.15$ and in Figure 6, $\alpha = 0.75, v_H = 2.25, v_L = 1.1$. Note that all simulations have been done starting with an initial setting of $M = 1, N = 2$ and $c_H = 2, c_L = 1$.

size increases—at least, initially—market efficiency increases because the distance between point D and point F decreases. However, this cannot last because at some stage, the situation must revert to the one depicted in Figure 9 and at this point, market efficiency starts to decrease. Note that what is happening here is similar to what happens when we increase the number of sellers only. Figure 6 depicts a simulation corresponding to this sub-case.

5.2.3. *The case in which $M < N$ and $s_L > s_H$.* Here, if the first-best is achievable (point A in Figures 8–9) then we will continue to achieve the first-best even when we replicate this market. So, we start with a situation of full efficiency and continue to get it as the market is replicated. Figure 10, however, illustrates that the first-best (point A) may not always be achievable. Here, replicating the market can increase market efficiency as the simulation in Figure 7 shows. This is similar to what happened when we increased the number of sellers only. However, when we increase the number of sellers only, we also showed that asymptotic efficiency was always obtained. Since we are increasing the numbers of both sellers and buyers here, we might intuitively suspect that asympototic efficiency may not obtain. Figure 7 shows that this intuition is correct.[15]

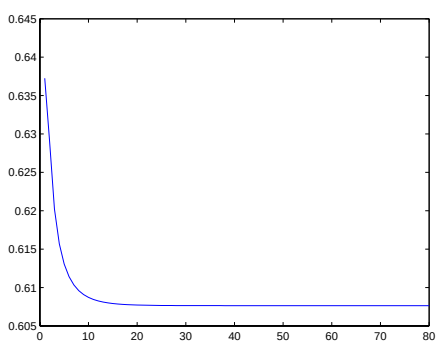

FIGURE 5. Market efficiency v/s Market size when $s_H > s_L$ illustrating decreasing efficiency with increasing market size.

5.3. **Related literature.** The literature on the market for lemons is too large to be summarized here; we confine ourselves to discussing those papers which have a direct bearing on our paper. The use of the mechanism design methodology can be regarded as a direct follow-up to the work of Samuelson (1984) who studied the

[15]The parameters used for the simulation depicted in Figure 7 are $v_L = 1.15$, $v_H = 2.05$ and $\alpha = 0.35$.

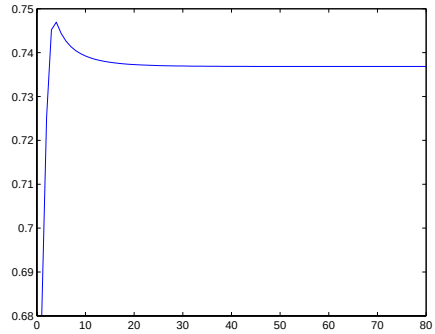

FIGURE 6. Market efficiency v/s Market size when $s_H > s_L$ illustrating non-monotonic behaviour of the efficiency ratio.

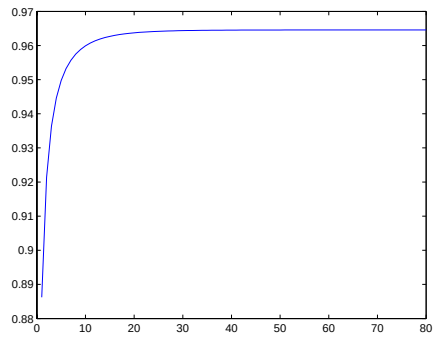

FIGURE 7. Market efficiency v/s Market size when $s_L > s_H$

bilateral lemons problem; our extension consists of analyzing the general case of finite but arbitrary number of sellers and buyers.[16]

The inefficiency pointed to by Akerlof (1970) has prompted economists to examine ways by which this inefficiency can be overcome. For instance, Klein and Leffler (1981) and Tirole (1996) have suggested that repeated interactions may overcome the adverse selection problem; on another dimension, the works of Hendel and Lizzeri (1999), Janssen and Roy (2002) and Hendel, Lizzeri and Siniscalchi (2004) suggest that particular features of the durable goods market, when taken into account, can overcome partially or even fully the inefficiency associated with the lemons market.

Our extended model is in the same spirit as these papers; however, there are two differences. Firstly, the other papers mostly use models with a continuum of agents; their models cannot therefore directly address the question of interest to us which is the impact of competition on market efficiency. In this context, note

[16]See also Evans (1989), Vincent (1989), Gul and Postlewaite (1992), and Manelli and Vincent (1995).

that in our extended model, we need private information on the buyers' side as well as a large number of sellers to obtain efficiency. Secondly, in contrast to the other papers which use the dynamic element, our extended model is still a static one and as such, closer to the basic Akerlof model. Our results thus show that it is possible to obtain asymptotic efficiency even in markets for non-durable goods and without using repeated game effects.[17]

APPENDIX A: OMITTED PROOFS

Proof of Lemma 1. The lemma can be established using standard techniques. Here is an outline of the argument:

(I) *Incentive Compatibility implies that* $\hat{p}^H \leq \hat{p}^L$: We can write (1) and (2) as

$$(\hat{p}^H - \hat{p}^L)c_H \leq \hat{t}^H - \hat{t}^L \leq (\hat{p}^H - \hat{p}^L)c_L.$$

The result follows since $c_H > c_L$.

(II) *For each type of seller, either her incentive compatibility constraint binds or her individual rationality constraints binds*: Suppose, to the contrary, that neither (1) nor (3) bind for the high-type seller. Then, one can increase the expected surplus by increasing $\hat{p}^H$ and lowering $\hat{t}^H$ without violating incentive compatibility or individual rationality. The same argument applies with respect to the low-type seller.

(III) *It cannot be the case that both incentive compatibility constraints bind*: Suppose, to the contrary, that this is the case. Then, neither individual rationality constraint can bind. We can thus increase the expected surplus by increasing $\hat{p}^H$ and $\hat{p}^L$ both by $\epsilon > 0$ and decreasing $\hat{t}^H$ and $\hat{t}^L$ both by $\delta > 0$. Note that these changes do not affect the incentive compatibility constraints. If ϵ and δ are small enough, then the individual rationality constraints are unaffected.

(IV) *It cannot be the case that the high-type seller's incentive compatibility constraint and the low-type seller's individual rationality constraint bind*: Suppose, to the contrary, that this is the case. Then, we have $\hat{t}^H = \hat{t}^L + (\hat{p}^H - \hat{p}^L)c_H = \hat{p}^L c_L + (\hat{p}^H - \hat{p}^L)c_H$. Therefore, $\hat{t}^H - \hat{p}^H c_H = \hat{p}^L c_L + (\hat{p}^H - \hat{p}^L)c_H - \hat{p}^H c_H = \hat{p}^L(c_L - c_H)$. Since $c_L < c_H$, this implies that we must have $\hat{p}^L = 0$, which in turn implies that $\hat{p}^H = 0$, and via the incentive compatibility constraints, that $\hat{t}^H = \hat{t}^L$. This implies that both incentive compatibility constraints bind, a contradiction.

[17]It is interesting to note that in the context of standard models of oligopolistic price competition but with the novel feature that consumers engage in costly search, Stahl (1989) and Janssen and Moraga-Gonzalez (2004) have shown that under certain conditions, increasing the number competing firms reduces welfare, and that the optimal market structure is one with a small number of firms. Furthermore, under some conditions there can be a non-monotonic relationship between welfare and the number of competing firms. These results, which are derived by bringing the standard oligopoly and search models together within a single framework, challenge the conventional wisdom that welfare is increasing in the number of firms. Although these results are derived in a world with symmetric information, they nonetheless offer an interesting parallel to the "similar" results obtained in the current paper in the context of lemons markets.

Hence, at a solution to (9), inequalities (2) and (3) will be binding constraints.

Proof of Lemma 2. Suppose that $(\hat{p}^H, \hat{p}^L, \hat{t}^H, \hat{t}^L)$ solves the mechanism design problem (9). Then, $\tilde{p}^H = N(1-\alpha)\hat{p}^H$ and $\tilde{p}^L = N\alpha\hat{p}^L$ satisfy (10)–(14), and so $E \leq E^*$. Now suppose that $(\tilde{p}^H, \tilde{p}^L)$ solves the reduced-form mechanism design problem (15). Define $\hat{p}^H = \tilde{p}^H/N(1-\alpha)$, $\hat{p}^L = \tilde{p}^L/N\alpha$, $\hat{t}^H = \hat{p}^H c_H$, and $\hat{t}^L = \hat{p}^L c_L + \hat{p}^H(c_H - c_L)$. It is straightforward to verify that $(\hat{p}^H, \hat{p}^L, \hat{t}^H, \hat{t}^L)$ satisfies (1)–(8), and hence $E^* \leq E$. Therefore $E = E^*$.

Proof of Propositions 1 and 2.[18] We conveniently break our argument into two main cases, depending on whether Z is negative or positive, where

$$Z \equiv s_H - \alpha(v_H - c_L)$$

is the coefficient of $\tilde{p}^H$ in (14). First consider the case when $Z \geq 0$ (i.e., $\alpha < s_H/(v_H - c_L)$).[19] In this case (14) can be rewritten as

$$\tilde{p}^H \geq \left[\frac{-(1-\alpha)s_L}{Z}\right]\tilde{p}_L,$$

and hence (since $Z \geq 0$) the feasible set of the maximization problem (15) is the shaded region in Figure 1. It thus follows that in this case the unique solution of (15) is at point B if $s_H > s_L$ and at point A if $s_H < s_L$, i.e.,

$$(\tilde{p}^L, \tilde{p}^H) = \begin{cases} (\alpha, 1-\alpha) & \text{if } s_H > s_L, \\ (1-(1-\alpha)^N, (1-\alpha)^N) & \text{if } s_H < s_L. \end{cases}$$

Now consider the case when $Z < 0$ (i.e., $\alpha > s_H/(v_H - c_L)$). In this case (14) can be rewritten as

$$\tilde{p}^H \leq \left[\frac{-(1-\alpha)s_L}{Z}\right]\tilde{p}_L.$$

Notice that in this case the line

$$\tilde{p}^H = \left[\frac{-(1-\alpha)s_L}{Z}\right]\tilde{p}_L \tag{A.1}$$

is positively sloped; whereas in the previous case when $Z \geq 0$, the line (A.1) was non-positively sloped. There are three subcases to consider here, depending on the relative position of the line (A.1).

If the slope of the line (A.1) is greater than or equal to $(1-\alpha)/\alpha$ — which is the case if and only if $\alpha \leq \alpha^*$ — then (A.1) lies above the line $\tilde{p}^H = [(1-\alpha)/\alpha]\tilde{p}^L$, and hence the feasible set of the maximization problem (15) in this case [when $\alpha \in (s_H/(v_H - c_L), \alpha^*)$] continues to be the shaded region in Figure 1. It thus follows that in this case the unique solution of (15) is the same as for the case above when $\alpha < s_H/(v_H - c_L)$.

[18]Since it is convenient to do so, the argument here establishes Propositions 1 and 2 which concern the baseline model in which $s_H > s_L$, but also establishes the corresponding results stated in subsection 4.1 that concern the scenario in which $s_L > s_H$.

[19]It may be noted that $s_H/(v_H - c_L) < \alpha^*$, and hence this case refers to Proposition 1.

Now suppose that $\alpha > \alpha^*$ — which means that the line (A.1) lies below the line $\tilde{p}^H = [(1-\alpha)/\alpha]\tilde{p}^L$. This is shown in Figures 2 and 3, depending on whether it intersects the line $\tilde{p}^H + \tilde{p}^L = 1$ to the left of (or at) point A or to the right of point A. After some simplification, it can be shown that the former is the case if and only if inequality (16) holds; and that the latter is the case if and only if (16) does not hold — notice that Proposition 2(i) concerns the former case while Proposition 2(ii) the latter.

When (16) holds, the unique solution of (15) lies, as shown in Figure 2, at point C if $s_H > s_L$ and at point A if $s_H < s_L$, i.e.,

$$(\tilde{p}^L, \tilde{p}^H) = \begin{cases} \left[\dfrac{-Z}{(1-\alpha)s_L - Z}, \dfrac{(1-\alpha)s_L}{(1-\alpha)s_L - Z}\right] & \text{if } s_H > s_L, \\ (1-(1-\alpha)^N, (1-\alpha)^N) & \text{if } s_H < s_L. \end{cases}$$

When (16) does not hold, then the unique solution of (15) lies, as shown in Figure 3, at point D, i.e.,

$$\tilde{p}^L = 1-(1-\alpha)^N \quad \text{and} \quad \tilde{p}^H = \left[\frac{-(1-\alpha)s_L}{Z}\right][1-(1-\alpha)^N].$$

Appendix B: Generalization to the Many-Buyers Case

The only difference between the mechanism design problem with $M = 1$, as stated in (9), and the problem with an arbitrary number M of buyers concern the three admissibility conditions. Suppose there are k sellers in all reporting that they are of type H. Let p_k^q denote the probability that a q type seller sells his product in this state of the world. Since symmetry implies that all q-type sellers must be treated identically, it follows that

$$p_k^H \leq \min\{1, M/k\} \text{ for } k > 0 \text{ and } p_k^L \leq \min\{1, M/(N-k)\} \text{ for } k < N.$$

Taking expectations across all possible realizations of k, these conditions imply that the mechanism must satisfy the following two conditions:

$$\hat{p}^H \leq \sum_{k=0}^{N-1} \binom{N-1}{k} (1-\alpha)^k \alpha^{N-1-k} \min\left\{1, \frac{M}{k+1}\right\} \quad \text{and} \tag{B.1}$$

$$\hat{p}^L \leq \sum_{k=0}^{N-1} \binom{N-1}{k} (1-\alpha)^k \alpha^{N-1-k} \min\left\{1, \frac{M}{N-k}\right\} \tag{B.2}$$

Furthermore, since the expected number of objects transferred to the buyers must be less than or equal to $\min\{M, N\}$, the mechanism must also satisfy

$$N[(1-\alpha)\hat{p}^H + \alpha\hat{p}^L] \leq \min\{M, N\}. \tag{B.3}$$

The mechanism design problem for an arbitrary M and arbitrary N is:

$$E \equiv \max_{\hat{p}^H, \hat{p}^L, \hat{t}^H, \hat{t}^L} \quad N\left[(1-\alpha)\hat{p}^H s_H + \alpha\hat{p}^L s_L\right] \tag{B.4}$$

subject to (1)–(5) and (B.1)–(B.3).

The Case of $\mathbf{M} < \mathbf{N}$. We solve the mechanism design problem (B.4) in this case in exactly the same manner as we did above for the case when $M = 1$. The only difference is that now the three admissibility constraints (B.1), (B.2) and (B.3) respectively become (using the same change of variables)

$$\tilde{p}^H \leq N(1-\alpha)b^H, \tag{B.5}$$

$$\tilde{p}^L \leq N\alpha b^L \quad \text{and} \tag{B.6}$$

$$\tilde{p}^L + \tilde{p}^H \leq M, \tag{B.7}$$

where b^H and b^L respectively denote the right-hand sides of (B.1) and (B.2). Hence, the mechanism design problem (B.4) for the case when $M < N$ can be solved by instead solving the following reduced-form problem:

$$E \equiv \max_{\tilde{p}^H, \tilde{p}^L} \quad \tilde{p}^H s_H + \tilde{p}^L s_L \tag{B.8}$$

$$\text{subject to (13)–(14) and (B.5)–(B.7)}.$$

With the aid of Figures 8-10 — which parallel Figures 1–3 — it is relatively easy to characterize the solution to (B.8) by using exactly the same arguments to those used in establishing Propositions 1 and 2.[20] In discussing the impact of the degree of competition on

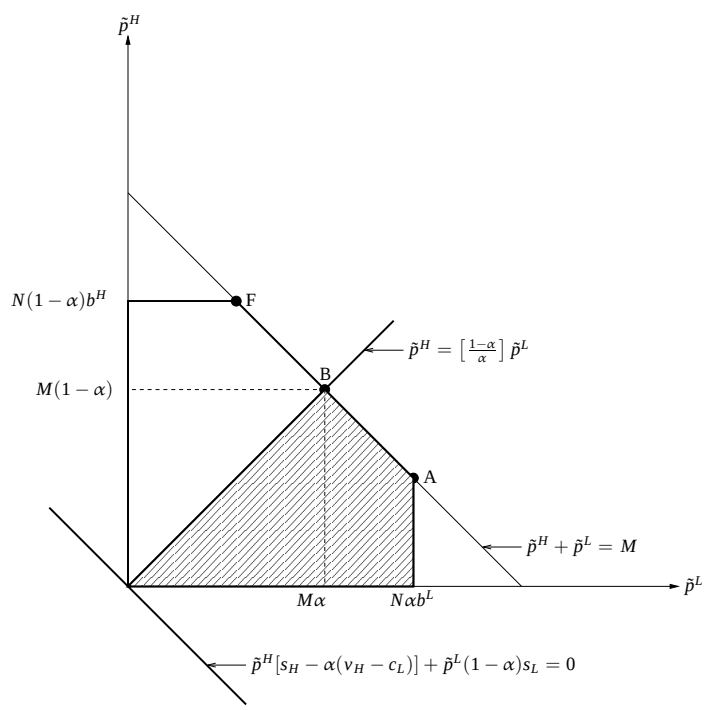

FIGURE 8. The feasible set when $M < N$ and $\alpha \leq \alpha^*$.

the second-best mechanism, we keep M fixed at some arbitrary level and allow N to vary

[20]It should be noted that both b^H and b^L are bounded from below by M/N. Furthermore, b^H and b^L are respectively bounded from above by $M(1-\alpha^N)/N(1-\alpha)$ and $M[1-(1-\alpha)^N]/N\alpha$.

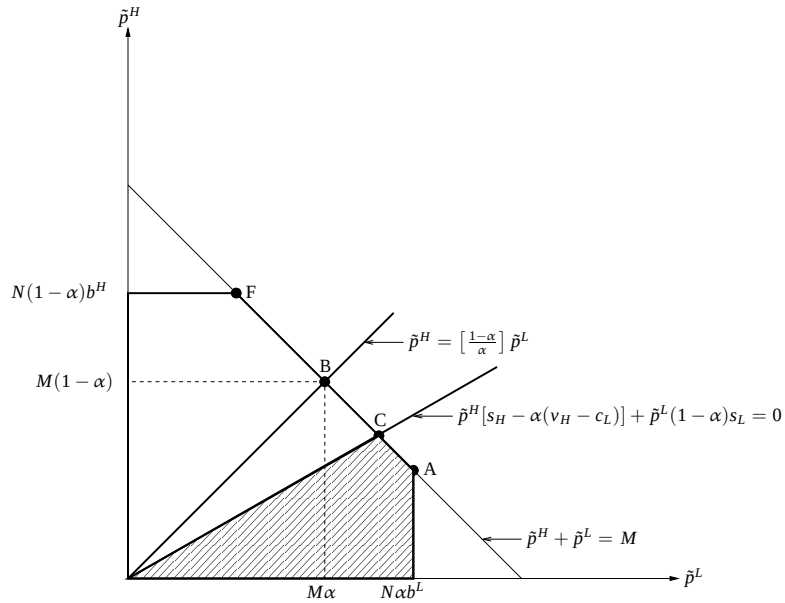

FIGURE 9. The feasible set when $M < N$, $\alpha > \alpha^*$ and point C lies above point A.

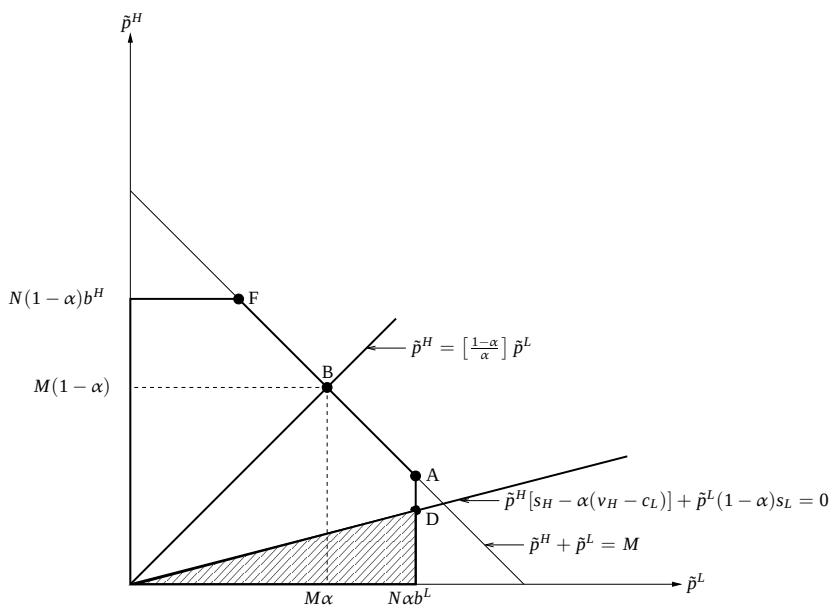

FIGURE 10. The feasible set when $M < N$, $\alpha > \alpha^*$ and point C, which is not shown, lies below point A.

over the set $\{M+1, M+2, \ldots\}$. First, consider the case when the buyers are soft. We first note that when $s_L > s_H$ then point A in Figure 8 depicts both the second-best outcome and the first-best outcome. On the other hand, when $s_H > s_L$ then point F depicts the first-best outcome while point B the second-best outcome. Since point B is unaffected by N while point F moves upwards along the $\tilde{p}^H + \tilde{p}^L = M$ line, where the former means that

the second-best expected surplus is independent of N while the latter means that first-best expected surplus is increasing in N, we obtain that relative efficiency is decreasing in N (for a fixed $M < N$).

Now consider the case when the buyers are tough. Just like in the single buyer case it can be shown that point C lies above point A (and hence Figure 9 applies) when $N \geq N^*$, where $N^* \geq 2$; and that C lies below A (and hence Figure 10 applies) when $N < N^*$.

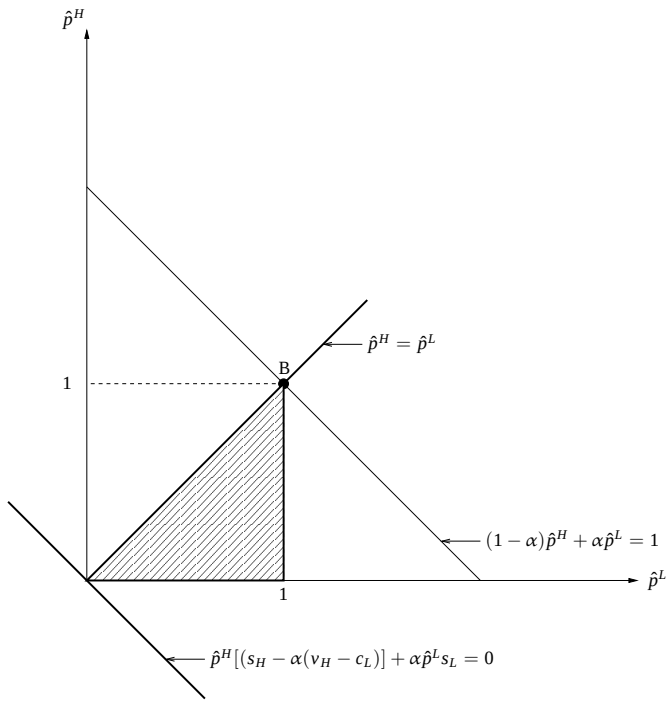

FIGURE 11. The feasible set when $M \geq N$ and $\alpha \leq \alpha^*$.

The Case of $\mathbf{M} \geq \mathbf{N}$. Since $M \geq N$, it follows that $b^H = b^L = 1$ (i.e., it is possible for all sellers to sell their objects since the total number of buyers exceeds the number of sellers). Hence, the admissibility conditions (B.1) and (B.2) respectively become

$$\hat{p}^H \leq 1 \quad \text{and} \tag{B.9}$$

$$\hat{p}^L \leq 1. \tag{B.10}$$

Since $M \geq N$ the admissibility condition (B.3) becomes:

$$(1-\alpha)\hat{p}^H + \alpha\hat{p}^L \leq 1 \tag{B.11}$$

Finally, since the analysis in section 3.1 applies, it follows that the relevant IC and IR conditions are [after undoing the change of variables]:

$$\hat{p}^H \leq \hat{p}^L \quad \text{and} \tag{B.12}$$

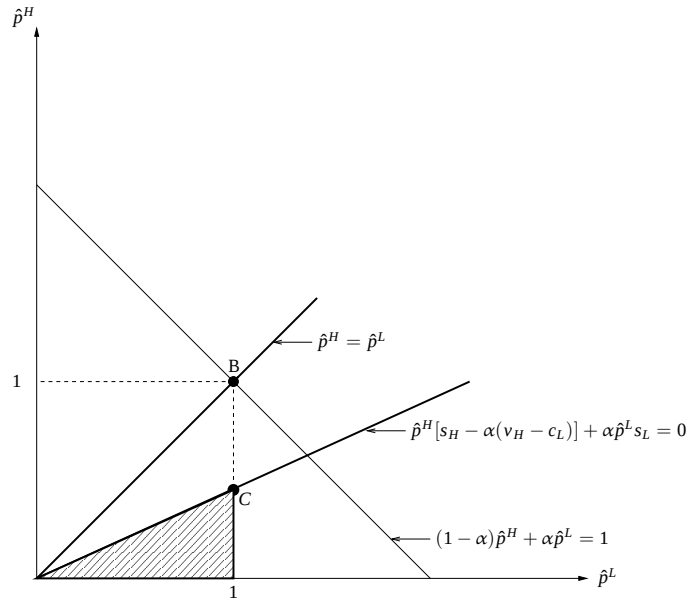

FIGURE 12. The feasible set when $M \geq N$ and $\alpha > \alpha^*$

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$$\hat{p}^H\left[s_H - \alpha(v_H - c_L)\right] + \alpha\hat{p}^L s_L \geq 0. \tag{B.13}$$

This means that the mechanism design problem (B.4) becomes:

$$E \equiv \max_{\hat{p}^H, \hat{p}^L, \hat{t}^H, \hat{t}^L} \quad N\left[(1-\alpha)\hat{p}^H s_H + \alpha\hat{p}^L s_L\right] \tag{B.4}$$

subject to (B.9)–(B.13).

Figures 11 and 12 respectively illustrate the feasible sets for the soft buyer and tough buyer cases. In both figures, the efficient point is defined uniquely by the intersection of the three admissibility constraints; this is point B in both diagrams. Figure 11 shows that when $\alpha \leq \alpha^*$, the first-best is achievable even with one seller (and at least one buyer). Increasing the number of sellers while maintaining the number of buyers at least equal to the number of sellers preserves efficiency. When $\alpha > \alpha^*$, on the other hand, Figure 12 shows that the first-best is no longer achievable. Increasing the number of sellers (while maintaining the number of buyers at least equal to the number of sellers) has no effect on the ability to achieve efficiency.

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