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Endogenous Choice of Electoral Rules in a Multi-party System with Two Dominant Parties[1]

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Abstract

We develop a model of endogenous choice of electoral rules in a multiparty system with two dominant parties, in an environment of uncertainty about the outcome of the election. Using quasi-lexicographic preferences over the number of seats necessary for a party to form a single-party government we explore the choice of the electoral law by the parties. We show that the minor parties never agree to an electoral reform that distorts the Proportional Representation system (PR). We also show that when the electoral competition among the two dominant parties is non-trivial there exists a unique and stable equilibrium: a unique new electoral rule is being adapted by the parliament in substitution of the PR rule. That is we show that when uncertainty about the outcome of the elections is present and if the dominant parties have a strong desire for single-party governments then strategic incentives to collude between them and distort the PR rule kick in. Hence, by colluding they also increase the probability that the winner will form a single-party government. The paper in effect shows that under an uncertain political environment the two dominant parties have an incentive to collude in favour of stability (single-party governments) by eliminating the effect of the third party in the formation of government. To conclude we also show that the equilibrium with the above characteristic is also unique. In an extension we use the timing of the electoral reform as a strategic variable.

1 Introduction

A crucial factor in most representative democracies is, not only the outcome of any electoral process, but also the realized distribution of seats in the legislative body based on the outcome of that procedure. It is obvious that the stability of any single-party government (or governmental coalition) critically depends on the number of seats allocated to the winning party, according to some previously chosen electoral law (Blais, 1991). One could go further and claim that it is this allocation of seats that matters the most: a party capturing say 51% of the popular vote and receiving 51% of the seats in the parliament, applying a proportional representation electoral rule, can form a less "stable" government than a party that capturing say 45% of the popular vote but according to some "favorably distortional" electoral rule (such as mixed or plurality electoral rules) occupies 60% of the available parliamentary seats (Blais, 1991). It can, of course, be argued that such electoral rules are deemed to be unfair by societies and hence, there is no need to worry about such incidences. Yet, a closer inspection of the real world reveals that in many countries, other electoral rules than Proportional Representation (PR hereinafter) are actually used to determine the allocation of parliamentary seats (Norris, 1997).

With this observation in mind, an interesting question arises: how and which electoral laws do parties choose given their expected share of the popular vote in the forthcoming electoral competition. That is to say, we shall try to endogenize the choice of electoral rules. Rather than arguing in a Duvergerian manner (Duverger, 1954) that it is some exogenous -possibly predetermined by some sort of constitutional arrangement- electoral rule that is responsible for shaping the political environment, we turn this assumption upside down (Colomer, 2005). In fact, we construct a model where the optimal choice of the electoral rule stems endogenously as a strategic choice of office-seeking parties, in their attempt to win the majority of the seats in the parliament and form single–party governments. We show that when uncertainty about the outcome of the elections is present, the larger parties have strong strategic incentives to collude and distort the PR in a direction favorable to the winning-party. We moreover show that, in case more than one parties are willing to distort the proportional rule, they completely agree on the level of the distortion caused to proportional representation by this new rule. That is, there exist a unique electoral rule, same for all, that maximizes the expected utility of each of these parties. The intuition behind this idea is that, when there is uncertainty about the outcome of the forthcoming elections, the strategic incentives of the dominant parties to control the parliament and pass legislation at-will align and allow them to collude and propose electoral rules that distort the PR rule. To put it in the most simple language possible, one can say that our model asserts the following: in bipartisan systems or in systems with two major dominant parties, the two major parties will collude, if there exist "enough" uncertainty about the electoral outcome and as a result, a majoritarian (non-PR) electoral rule is being chosen. In our context, the term "uncertainty" with respect to the outcome of the election, implies that the elections are contested by at least two parties that have a positive probability of winning (Andrews-Jackman, 2005).

Furthermore, it is interesting to derive conditions that determine the optimal choice of an electoral rule based on the expected outcome of the election. Another closely related

issue is that of stability of electoral rules. Does it exist an electoral rule that is stable? That is, once it is chosen by the parliament no other party will have an incentive to change that rule. Or even if it has the incentive to do so, it will not have the majority that is necessary to enact such a change. In our two-dominant party model we shall prove that this is in fact the case. And if such an electoral rule does exist, it is also the most efficient in terms of providing countries with stable, single-party governments (Blais, 1991). Finally, on the last section of this paper, we set out to explore the timing of the electoral rule reform as an extra dimension of strategic choice for the two dominant parties.

To address all these questions, we model a parliamentary democracy environment with parliamentary parties, where two of them are the dominant ones. That is, the two dominant parties have a positive probability of winning the election, whereas the rest are minority parties with zero probability of running first in the forthcoming election. Of course, we should clarify that winning an election in our model simply means collecting more votes compared to the other dominant party. This is something we diversify from the concept of wining enough votes to secure the majority of the seats in the next parliament. The transformation of popular vote into parliamentary seats is dependant on the implemented electoral rule, which is turn chosen in advance by the parties, through a parliamentary voting procedure. To capture all the degrees of proportionality distortion (from perfect proportionality to one-district majoritarian system) we shall introduce in the theoretical modeling the, so called, "bonus system". This electoral rule allocates most of the seats according to PR rule and the rest -known as the bonus seats- are given to the party that runs first in the election, regardless of the party winning the majority of the votes. Hence, by varying the amount of bonus seats we derive electoral rules with different degrees of proportionality distortion. But before entering into the specifics of the model in more detail, it is instructive to give a summary of previous results and some empirical observations.

2   Literature Review and Motivation

Recent literature in political science suggests that the existing variation in electoral rules and regimes across advanced democracies is due to the strategic decisions that current ruling parties make, anticipating the coordinating consequences of different electoral rules, to maximize their representation in the legislature (or even form a single party government) according to the following conditions: If the electoral arena is considered to be fairly stable and if the political environment and the existing electoral rule serves the current ruling parties, (it might be the case that two parties switch turns in office regularly) then they have limited or no incentives to modify the current electoral rule. However, if the political arena changes due to an anticipated entry of new voters (or change in their preferences) the ruling parties will consider changing the current electoral rule depending on two conditions: Firstly, the emergence of new parties and secondly, the coordinating capabilities of the old parties. If the new party is strong then the old parties shift from plurality/majority rules to PR only if they are locked into an equilibrium where no old party enjoys a dominant position (non-Duvergerian equilibrium), whereas they do not (they stick to majority/plurality rule) if one old party enjoys a dominant position (UK). Similarly, whenever the new entrants are weak, an non-PR electoral system is maintained regardless of the structure of the old

political system (USA). This argument is made in full clarity by Boix (1999), and is in accordance with the predictions of our theoretical model.

In spirit with this literature, Benoit (2004) adapts the argument made for endogenous choice of electoral rules and proposes a theoretical framework that examines patterns of electoral rule change. That is, he develops a model that studies the endogenous choice of electoral rules by the parties. After surveying the literature in order to group the patterns of electoral rules change, he proposes a model that derives conditions of endogenous electoral rule change by rational seat-maximizing political parties. His theoretical model predicts that electoral rule change occurs endogenously, when two conditions are met. First, if it exists a coalition of parties willing to agree to change the existing rule with another one, such that each of these parties is expected to score seat gains under the newly chosen electoral rule. And secondly, if the parties in the coalition can muster enough votes in the current parliament in order for this change to take place. Those results are very close to the theoretical predictions of our model but we diversify ourselves in two directions. Firstly, we explicitly state the conditions that allow the two dominant parties to collude for an electoral rule change from PR to a more majoritarian on. In particular, we identify that the incentives of the dominant parties to collude stem from their desire to form single-party governments and take full advantage of the "spoils of the office" and also, from the uncertainty over the outcome of the coming election. The latter allows those incentives to align and hence, gives enough room for collusion to take place. Secondly, our paper does not require that both parties, participating in the coalition that votes in favour of an electoral rule change, are part of the government. In fact, our paper states quite the opposite. It is the desire of the two dominant parties to form a single-party government that allows them to collude. The basic difference is that although in both models all parties in favour of the electoral rule change must score seat gains on expected terms, in our model this is not true in real terms when outcomes are realized. Our model predicts that the if the two parties collude to change the PR rule, in the end, only one of them will form a single-party government and enjoy the spoils of the office.

In a parallel attempt, Colomer (2005) presents and tests the hypothesis that it is in fact the number of parties that can explain the choice of electoral rules, rather than the other way around. He argues that already existing political parties tend to choose electoral system that allow them to "...crystallize, consolidate and reinforce..." the existing party system rather than change it dramatically. He also concludes that political systems that are dominated by a few parties tend to establish majority rule electoral systems, (or rules that distort the PR, in general) whereas multi-party systems already existed before the introduction of PR. Our paper formalizes this idea by showing explicitly how the strategic incentives of the two dominant parties align, in order to crystallize the existing two-dominant party system and increase the probability of occurrence of stable governments (that is, governments that have the majority of seats in the parliament). Furthermore, our model extends to show that the choice of electoral rule is a strategic choice of the parties, which is driven by their desire to increase their probability of forming a single-party government that has the ability to pass legislation at will - given, of course, some constitutional limitations. Hence, our paper builds upon those stylized facts and empirical evidence in order to create a simple theoretical model on how and why parties choose specific electoral rules. We also explore whether the choice of the electoral rule increases the stability of the political system and whether there

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exists a unique electoral rule (equilibrium) for a given political environment.

Finally, in the opposite direction, Ergun (2007) studies the change of electoral rule from plurality to the PR rule. He finds that for office motivated, rational, seat maximizing parties the following conditions have to be met for the change to take place. First, the government must be formed by a coalition, that is more than one party must share the spoils of the office. And secondly, the larger the number of parties and the more equitable the share of the spoils amongst them, the more likely is for the change to a PR rule to take place. That is, Ergun (2007) starting from the opposite direction compared to our paper where PR is the status quo rule, in examining the change of electoral rules echoes the same counter-Duvergerian approach. That it is the very nature of the political environment and the party structure that determine the choice of electoral rules. In his set up, a fractured, multi-party political system, where parties share the spoils of the office in an equitable way, makes the adaptation of PR rule more likely. In fact, this is the same link observed in the literature so far, just going the other way around.

At this point, it becomes imperative to equip our analytical arsenal with a fact that stems out from empirical observation. It is widely observed[1] that PR electoral rules go hand in hand with multi-member electoral districts, whereas Plurality rules require single-member districts (Norris 1999 and Blais 1991). Of course, mixed rules use a combination of single-member constituencies and larger multi-member districts. The reason that this diversification becomes crucial is that the size of an electoral district is positively correlated with the cost of running an electoral campaign. It has been empirically observed and argued by many political scientists (Blais, et al. 1991) that the cost of running elections increases with the size of the electoral district (since larger districts imply larger constituencies and hence, higher costs for candidates to reach out to citizens and gain votes for their parties). Moreover, apart from partisan competition with the other parties' candidates there is also a salient intra-party competition. In multi-member districts, many candidates from the same party compete with each other. Hence, they must also spend additional funds not only to persuade voters to vote for their party, but also to gain their first preference vote in order to get elected from the party list (with the implicit assumption that is highly unlikely with a PR rule for a party to get almost all the votes in that district, thus capturing all the available seats).

In sharp contrast with multi-member districts, under plurality electoral rules we observe single-member districts, where "the winner takes it all" rule simply implies that the candidate of the winning party wins the race. Given that electoral competition in single-member districts is less fierce and therefore less costly, we can assume that the cost parties have to suffer, in order to contest an election under a plurality system, is significantly smaller than with PR systems (Blais 1991). Of course, since mixed systems use a combination of multi- and single-member districts, we can expect that the cost of running elections under such rules must lie somewhere between those other two. And we can also hypothesize that the cost in mixed rules rises proportionally with the fraction of seats allocated under a PR

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[1]The only exception known to the authors is that of the US Presidential elections where plurality rule is implemented in multi-member districts and the winner of the plurality of the votes carries all the electors of that State for the Presidential election. Yet this pattern even inside the US is only observed in Presidential elections and it can be attributed to some part in the parliamentary history of the US. Even in Congressional and Senatorial elections US applies a plurality electoral law but in single-member electoral districts.

rule. In this paper, we abstain from making a more detailed analysis regarding the cost of running elections since, this is not the main focus of this work. For the time being, we shall only note that the cost of running elections rises with the increase in the degree of proportionality of the chosen electoral rule and vice versa.

As far as the two-major-party competition is concerned, Palfrey (1984) considers a two-dominant-party spatial competition model, where parties are in rational anticipation of a third party entry. But Palfrey's results relate to the position that the two dominant parties occupy in the political spectrum. Our analysis is distinct, since we focus on the choice of electoral rule made by the two dominant parties, given their expected vote share. This extra feature of the model might lead to some interesting explanations, concerning the prevalence of specific electoral laws in societies where there is significant political fragmentation and many old parties cease to exist or new entrants appear in the political scene (e.g. Israel).

After providing readers with the basic concepts and ideas around which this paper evolves, it is now appropriate to introduce the formal underpinnings of our simple model, before proceeding with a more extensive analysis and presenting our basic results.

3 The Model

3.1 Basic Ingredients

Formally we shall consider $N = \{1, 2, 3\}$ to be the set of parties involved in the electoral competition. All parties in N are assumed to be represented in the parliament. Each party $i \in N$ holds a proportion of seats in the preceding parliament s_i^0 , such that $\sum_{i=1}^{n} s_i^0 = 1$. Moreover, parties have information about the vote share that each party is about to receive in the forthcoming elections. Party 3, shall be assumed to be a "minority" party (minority referring to ideology or ethnicity), that is, its vote share shall never exceed the vote share of party 1 or this of party 2. The expectations on future vote shares are formed by information that is commonly available to all parties. The vote share of the "minority" party 3, shall be assumed to be fixed at a level v_3, whereas, the vote shares of the other two parties will be subject to uncertainty. Formally, the vote share of party 1 in the coming elections will be modeled as a random variable:

$$v_1 \sim \Phi_1,$$
where Φ_1 is a uniform distribution in $[a_1, b_1] \subset [0, 1]$.
Equivalently,
$$v_2 \sim \Phi_2,$$
where Φ_2 is a uniform distribution in $[a_2, b_2] \subset [0, 1]$.

Notice that these expectations need to satisfy $\sum_{i=1}^{n} v_i = 1$ and $v_3 \leq v_i$, $\forall i \in N$. That is, both a_1 and a_2 are bigger than v_3, both b_1 and b_2 are smaller than $1 - 2v_3$ and $a_2 = 1 - b_1 - v_3$ and $b_2 = 1 - a_1 - v_3$. The proportion of seats of party $i \in N$ in the new parliament will be defined as $s_i^l(v_i)$, where l will be the applied electoral law. As stated in the previous parts of the paper, the particular electoral laws that we are interested in are the first party bonus

systems. That is, l shall be the proportion of parliament seats that are allocated to the "first" party as bonus. Obviously, $l \in [0, 1]$ and, thereafter:

$$s_i^l(v_i) = v_i(1 - l) \text{ if } v_i < \tfrac{1-v_3}{2}$$
$$\text{and}$$
$$s_i^l(v_i) = v_i(1 - l) + l \text{ if } v_i > \tfrac{1-v_3}{2}.$$

Obviously, the third party given the assumptions stated above will never be entitled to the bonus as a result of never winning the election. The above seat allocation mechanism with respect to the popular vote is a standard application of the mixed electoral rule. The first component is the proportional allocation of the seats minus the bonus whereas the latter part is the bonus given to the winner of the election.

The utility of a party $i \in N$ shall be defined as:

$$u_i(v_1, v_2, v_3, l) = g s_i^l(v_i) + (1 - g),$$
$$\text{where } g \to 0 \text{ if } s_i^l(v_i) > 1/2 \text{ and } g = 1 \text{ if } s_i^l(v_i) \leq 1/2.$$

That is, parties in this environment are clearly office motivated. They care about the proportion of the seats they hold, only in the case that they are in opposition or when they win but cannot form a single-party government. When they can, they mainly receive rents from being in office. In a sense this type of preferences can be described as "quasi-lexicographic", if we think of the ability of forming a single-party government, by securing the majority of parliamentary seats, as a different "good" from just "simple" parliamentary seats that are below the necessary threshold. Hence, the two dominant parties mainly care to "consume" the former good, that is to form a single party government, and only if they cannot do that they care for extra parliamentary seats in order to increase their share of seats in the parliament. Put more formally, the marginal return of an extra parliament seat is infinitely larger when a party is in the opposition or has to negotiate with other parties to form a government than when the party has a complete control over the parliament.

This formulation of the utility function can be easily justified by following the political economy and political science literature. The formation of a single-party government allows the governing party to pass legislation at will, given of course some constitutional limitations, since it holds the majority of the seats in the parliament. Hence, every proposed piece of legislation, by the governing party to the parliament, will have secured in advance its acceptance by the majority of the parliament which supports the single-party government. Of course, there are some examples in recent political history, where single-party governments controlling the majority of seats in parliament were unable to pass some legislation without the support of the main opposition party (e.g. the Blair administration in its final years). Yet, it must be noted that those cases are just few exceptions from the general rule. When a party controls the majority of the seats in a parliament and subsequently is able to form a single-party government it faces no limits in passing legislation other than constitutional ones[2]. On the other hand, when a party is in opposition, or when it is first

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[2]We need to find some data to support the claim that single-party governments have the larger portion of their proposed legislation approved by the Parliament.

but not able to form a single-party government, then the number of seats that possesses in the parliament increases its bargaining position in the negotiations that follow for the formation of a coalition government. The more MP's a party has, the greater its role in the future government will be (e.g. more ministerial positions being awarded to its members). So, this formulation of preferences allows us to capture this very intuitive idea, which is commonly observed in current state of political affairs in most modern democracies.

3.2 The Game Structure

The game shall have the following stages.

(i) $l = 0$ (pure proportional system) and the party with the largest share of seats in the current parliament (either party one or party two) shall bear the role of the "proposer" of an electoral reform. That is, it shall propose $l \in [0, 1]$.

(ii) Parties shall vote on the "proposal" l and if the votes in favor of the reform surpass a given threshold $W \in [0, 1]$ (defined exogenously by the constitution, usually requiring a "super-majority"), then the electoral reform shall pass and the following elections will be conducted by the new law. In the opposite case, that is, if the proposal does not gather the necessary parliamentary support W, the electoral reform is cancelled and future elections are conducted according to the proportional system $l = 0$.

(iii) Elections take place, and each party, according to the results of public voting and the applied electoral law takes its new seat share and computes its utility.

3.3 Understanding the Proposer's Problem

For simplicity, let us for the rest of the paper assume that the proposer is always party 1. That is, we assume that $s_1^0 > s_2^0 > s_3^0$. Since the electoral law influences the seat shares of the parties and, thus, their utility after the coming elections, party 1 shall propose the electoral law $l \in [0, 1]$ that given the threshold $W \in [0, 1]$ maximizes its expected utility. That is, if $W \leq s_1^0$ party 1 will propose $l \in [0, 1]$ such that $l = \arg\max\{Eu_1(v_1, v_2, v_3, l)\}$. In other words, if the current seat shares of the proposer exceed the necessary amount of votes for an electoral reform, the proposer faces an unconstrained maximization program.

On the contrary, if $W > s_1^0$ the proposer needs the support of one (or more) of the other parties to proceed with a possible electoral reform. An obvious, but, nonetheless, useful corollary that can be stated at this point, demonstrates the behavior of the minority party 3 in such cases.

Lemma 1 *The minority party never consents to any electoral reform proposal*

Since the minority party expects to receive the bonus l with probability zero, it just expects utility losses from any distortion in the proportionality of the electoral system. Its expected utility from any electoral law is $Eu_3(v_1, v_2, v_3, l) = v_3(1 - l)$, which is, obviously decreasing in l.

Given the above observation, party 1 will have to secure party's 2 support in order to proceed with an electoral reform. That is it shall have to propose $l = \arg\max\{Eu_1(v_1, v_2, v_3, l)\}$

s.t. $Eu_2(v_1, v_2, v_3, l) \geq Eu_2(v_1, v_2, v_3, 0)$, facing, this time, a constrained (participation constrain of party 2) maximization program.

In general, since we have assumed that v_3 is fixed and common knowledge, and that $v_1 = 1 - v_2 - v_3$ the proposer faces one source of uncertainty (information about v_1 is equivalent to information about v_2). Thus, the proposer's expected utility shall be:

$$Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1 - a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l) + l]dv_1 + \int_{\frac{1/2-l}{1-l}}^{b_1} dv_1]$$

and, equivalently, party's 2 expected utility shall be:

$$Eu_2(v_1, v_2, v_3, l) = \frac{1}{(b_2 - a_2)}[\int_{a_2}^{(1-v_3)/2} v_2(1-l)dv_2 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_2(1-l) + l]dv_2 + \int_{\frac{1/2-l}{1-l}}^{b_2} dv_2].$$

Notice that there exist two critical vote shares. The first one, $(1 - v_3)/2$, defines the necessary vote share so as for one of the two parties to be first (and get the bonus l). The second, $\frac{1/2-l}{1-l}$, is the vote share that that first party needs to have a one party majority in the parliament given an electoral law l $(s_i^l(v_i) > 1/2)$. Obviously, if $(1 - v_3)/2 \geq \frac{1/2-l}{1-l}$ the first party will have the a majority of seats in the parliament independently of the exact vote share and, thus, the expected utility of party 1 shall be:

$$Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1 - a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{b_1} dv_1]$$

and party's 2:

$$Eu_2(v_1, v_2, v_3, l) = \frac{1}{(b_2 - a_2)}[\int_{a_2}^{(1-v_3)/2} v_2(1-l)dv_2 + \int_{(1-v_3)/2}^{b_2} dv_2]$$

As $\frac{1/2-l}{1-l}$ is decreasing in l, an increasing distortion in the proportionality of the electoral law (an increasing l) does not only affect the potential seat gains of a party in case it runs first in the elections but as well, increases the probability of this party gathering a majority of seats in the parliament and, hence, forming a single-party government.

3.4 Definitions

We will classify the results given the following definitions.

Definition 1 *The electoral reform process is trivial if $W \leq s_1^0$*

In case the proposer (Party 1) has at present a large enough proportion of parliament seats so as decide the electoral reform at will, then we shall consider that the reform process is a trivial one, as it will just depend only on the preferences of the proposer.

Definition 2 *The electoral reform is possible if $W \leq 1 - v_3$*

This comes directly from the implications of corollary 1. The minority party 3 shall never consent to an electoral reform, thus, if a reform is to take place, W must be such that the minority party cannot block the reform.

Definition 3 *The electoral competition is trivial if $a_1 > \frac{1-v_3}{2}$ or if $b_1 < \frac{1-v_3}{2}$*

The above definition just describes the case in which the probability of party 1 running first in the coming elections is either 1 or 0. In such cases the "winner" of the elections, and, thus, the party that will take the bonus l is not subject to uncertainty. On the other hand, when electoral competition is non-trivial, both parties 1 & 2 have a positive probability of running first in the elections and grabbing the "bonus" parliament seats.

Definition 4 *The proposer (Party 1) is the "leading" party if and only if $\frac{a_1+b_1}{2} > \frac{1-v_3}{2}$*

If a party is expected to run first in the elections, then it shall be called the leading party. Since $E(v_1) = \frac{a_1+b_1}{2}$ and $\frac{1-v_3}{2}$ the vote share threshold above which a party 1 is the winner of the elections, Party 1 is the leading party if and only if the above inequality holds. Alternatively, Party 2 is the leading one.

4 Results

Assuming that an electoral reform is possible ($W \leq 1 - v_3$) we can state the following.

Proposition 1 *When both the electoral reform process and the electoral competition are trivial and the proposer is: (i)the leading party, then $l^* \geq \max\{0, \frac{1/2-a_1}{1-a_1}\}$, (ii) not the leading party, then $l^* = 0$*

Here, the leading party is a sure winner and the proposer can implement any electoral reform without the consent of any other party. That is, if the leading party is the proposer of the electoral reform, the proportionality distortion introduced by the l^* bonus electoral rule will be such that it will guarantee to the winner a majority of seats in the new parliament. Otherwise, if the proposer is not the leading party (i.e. expected to lose the election) it proposes that the proportional representation rule is not amended. And since it holds the necessary majority in the parliament, the electoral rule does not change. This result can be viewed as the simplest case scenario.

The idea behind this proposition is very simple. Since the electoral reform process is trivial the proposer holds enough seats in the current parliament to pass any electoral rule, that is, it faces an unconstrained maximization program where there is no need to satisfy the participation constraint of Party 2. Hence, the proposer just chooses l^* in order to maximize expected utility. Given that the electoral competition is trivial Party 1 proposes $l^* = 0$ when it is not the expected winner (i.e. not the leading party) and proposes $l^* \geq \max\{0, \frac{1/2-a_1}{1-a_1}\}$ otherwise. That is, it proposes such a bonus that secures with certainty the majority of the seats in the new parliament when it is the leading party and expected to win the election. To see this, consider that the electoral competition is trivial and set $v_1 = a_1$ (that is the lowest possible outcome for party 1). Then solve for l^* such that $s_1^{l^*} = a_1(1 - l^*) + l^* > 1/2$.

The solution to this unconstrained maximization problem yields to the leading party, with certainty, the majority of the seats in the parliament, for every possible realization of the distribution Φ_1. Hence, by choosing this level of l^* the leading party guarantees it self the ability to form a single-party government and get the highest possible level of utility. On the other hand, if it is not the leading party any distortion of the PR rule will decrease its utility, since it will never get the bonus l^*, making its utility strictly decreasing on l^*. This completes the argument.

Proposition 2 *When the electoral competition is trivial but the electoral reform process is non-trivial, then $l^* = 0$[3].*

In this case, the two parties have to agree on the electoral reform and the proposer is either a certain winner, or a certain loser in the coming elections. In the first case, it is impossible to propose any bonus electoral rule such that Party 2 could consent in its implementation (since it just expects a utility loss from a potential distortion in the proportionality of the electoral rule, given that its utility is strictly decreasing in l^*). In the latter, the proposer is completely unwilling to distort the proportionality of the electoral rule because now, it is the proposer's utility that is strictly decreasing on l^*. The idea that drives the result is that in this case there is no room for collusion, since strategic incentives of the two dominant parties do not align, because there is no uncertainty over the outcome of the electoral competition. The leading party will always prefer a value of $l^* > 0$ but the other party will always reject this proposal because its utility is strictly decreasing in l^*. Hence, since the electoral reform is non-trivial and requires the consent of both dominant parties, it is obvious why no electoral rule reform is ever going to be accepted by this parliament. That is, we are stuck to the status quo ante and the proportional rule persists as the electoral rule (i.e. $l^* = 0$).

Proposition 3 *When the electoral competition is non-trivial then $l^* \in \{0, \frac{v_3}{1+v_3}\}$[4].*

This is the main result of the analysis. In an environment of uncertainty about the winning party of the coming elections, the distortion that the proposer might introduce in the proportionality of the electoral rule, by the means of a bonus system l, shall be such that it will guarantee that the party that will run first in the elections will have a majority of seats in the parliament. That is, in case party 1, alone or with the support of party 2, promotes a bonus electoral rule, it will be at this extent so as to create an artificial "two party system"and solidify the current shape of the political system. This result is a summary of the results in the next proposition, hence, we will restrict further analysis of this main result in the section that follows to combine Propositions 3, 4 and 5 together.

Next we will present two Propositions that build on the previous result and state with clarity the necessary and sufficient conditions for an electoral rule change to take place.

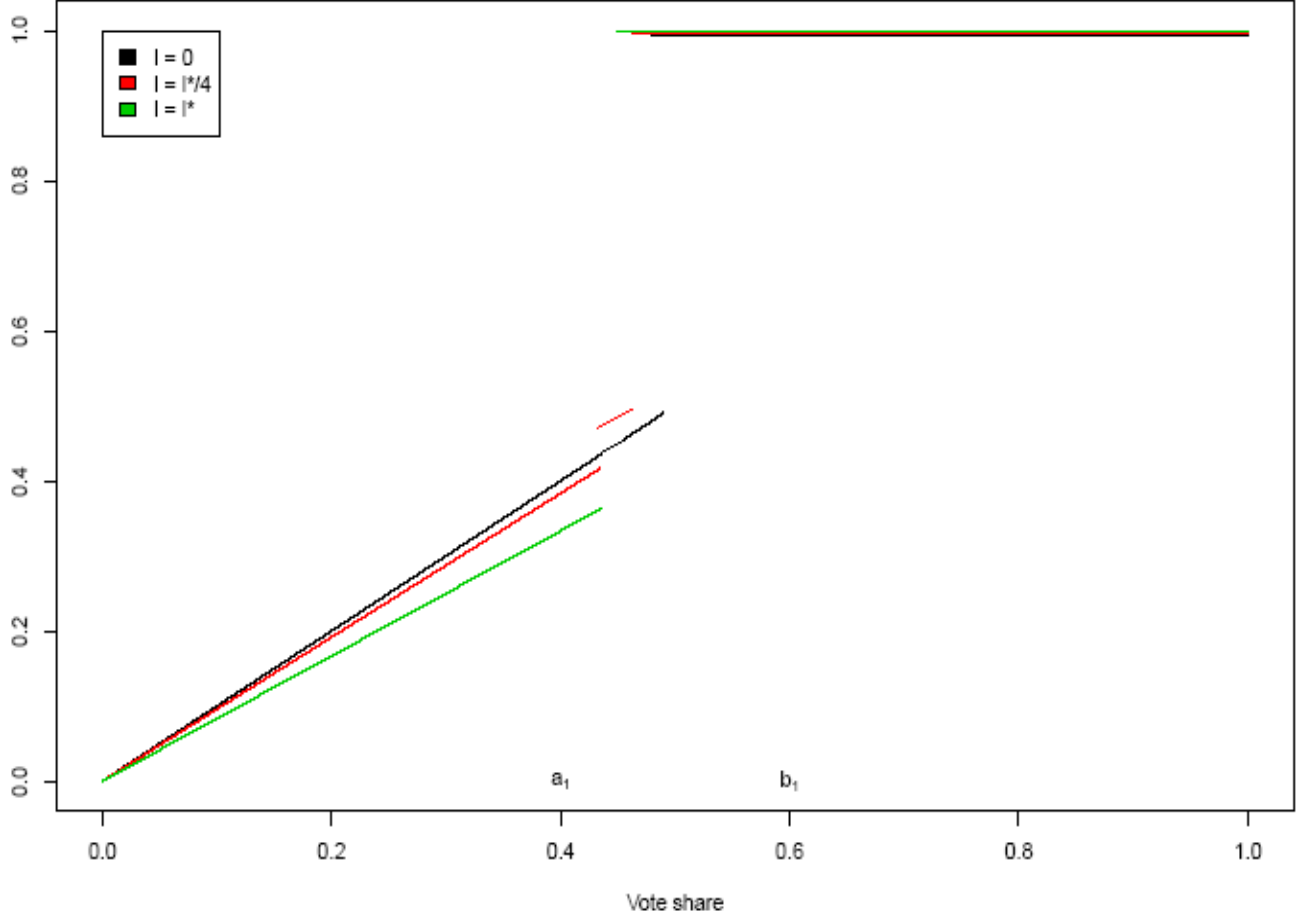**Proposition 4** *In specific, when the electoral competition is non-trivial but the electoral reform process is trivial, then the Parliament departs from PR ($l^* = 0$) and adapts $l^* = \frac{v_3}{1+v_3}$ if and only if $(b_1 - \frac{1-v_3}{2})(2 - b_1 - \frac{1-v_3}{2}) \geq (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_1)(\frac{1-v_3}{2} - a_1)$[5].*

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[3]Proof in the Appendix

[4]Proof in the Appendix.

[5]Proof in the Appendix

Utility function for a dominant party for different values of I

Legend:
- I = 0
- I = I*/4
- I = I*

Y-axis: 0.0 to 1.0
X-axis (Vote share): 0.0 to 1.0, with labels a_1 and b_1 marked

Proposition 5 *When both the electoral competition and the electoral reform process are non-trivial then in order for the Parliament to depart from PR ($l^* = 0$) and adapt $l^* = \frac{v_3}{1+v_3}$ there are two sets of both necessary and sufficient conditions: EITHER (i) the proposer is the leading party and $(b_2 - \frac{1-v_3}{2})(2 - b_2 - \frac{1-v_3}{2}) \geq (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_2)(\frac{1-v_3}{2} - a_2)$ is satisfied OR (ii) the proposer is not the leading party and $(b_1 - \frac{1-v_3}{2})(2 - b_1 - \frac{1-v_3}{2}) \geq (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_1)(\frac{1-v_3}{2} - a_1)$*[6].

Remark 1 *Here in presenting this result we have implicitly assumed that both b_1 and b_2 are less than $1/2$. In case that either b_1 and/or b_2 are greater than $1/2$ then one wanting to reproduce the above result of Proposition 4 simply substitutes b_1 and/or b_2 with the value $1/2$ and the result then follows in the same way as shown above.*

We will provide here an idea of the proof and a discussion for each case separately since this is the main result of the paper. Here the electoral competition is never trivial that is

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[6]Proof in the Appendix

the following conditions always hold: $a_1 < \frac{1-v_3}{2}$ and $b_1 > \frac{1-v_3}{2}$. First of all, we note that formally the proof is derived from the maximization program of the proposer. As shown in the appendix $Eu_i(v_1, v_2, v_3, l)$ for $i = \{1, 2\}$ is convex with respect to l (strictly convex for some values) for $l \in .(0, \frac{v_3}{1+v_3}]$ and decreasing for $l \in (\frac{v_3}{1+v_3}, 1]$. Hence, the only two candidates for an optimum, as we argued in the proof of proposition 3 are either $l = 0$ or $l = \frac{v_3}{1+v_3}$. In both cases, since the electoral reform process is trivial the proposer faces the unconstrained version of the maximization program, that is, it maximizes its expected utility $EU_i(v_1, v_2, v_3, l)$ by choosing $l^* s.t.$ $l^* = \arg\max EU_i(v_1, v_2, v_3, l)$ without having to satisfy the participation constraint of Party 2 (recall that by Lemma 1 the third party never agrees to accept any l^* other than the one that corresponds to the PR rule- that is $l^* = 0$). To continue with our proof, we will first argue why the two candidate equilibrium strategies (values of l^*) are either $l^* = 0$ or $l^* = \frac{v_3}{1+v_3}$. Technically speaking, those two equilibrium values of l^* are derived from the maximization program of the proposer (in fact they are corner solutions, see appendix). Nevertheless, the intuitive argument seems more clear if we refer to those values of l^* as equilibrium strategies. First, consider the case that $l \in (\frac{v_3}{1+v_3}, 1]$: in this case whenever $v_1 > \frac{1-v_3}{2}$ then the increase in the bonus does not give the proposer any extra utility since its utility is already 1 (this corresponds to the constant part of the Eu expression). That is, the value of l^* is such that $s_1^{l^*}(v_1) > \frac{1}{2}$. Hence, any further increase in the size of the bonus does not give the proposer any greater level of utility. On the contrary, consider the case when $v_1 < \frac{1-v_3}{2}$: in this case the strategy (proposal) with higher bonus than $l^* = \frac{v_3}{1+v_3}$ is strictly dominated by l^*. The reason is that in this case, the proposer runs second, hence, does not receive the bonus and its utility is strictly decreasing in l. This is exactly what we have shown in the proof, that for values of $l > \frac{v_3}{1+v_3}$ expected utility is decreasing. As a result, we have argued that any strategy to propose $l > l^*$ is weakly dominated by l^* for any value of v_1 and in particular, strictly dominated for values of $v_1 < \frac{1-v_3}{2}$. Hence, to conclude, strategy $l = 0$ and strategy $l^* = \frac{v_3}{1+v_3}$, by convexity of the Expected Utility with respect to l, as argued above, are the only two possible strategy proposals that maximize the proposer's expected utility. That is, in equilibrium the proposer proposes (plays) one of those two strategies (notice that in each case the equilibrium is unique).

Ideally, the proposer would prefer to propose the PR rule (i.e. $l = 0$) whenever is expected to run second and propose l^* otherwise. Yet in our environment, there exist uncertainty about the outcome of the election. Hence, the proposer has to compare the expected loss from proposing l^* and running second, (or put it differently the expected gain in utility from running first in the election and forming a single party government) with the expected loss in utility of winning the election and not being able to form a single party government because it proposed $l = 0$ (or equivalently the expected gain from running second and winning more seats). Those two equivalent statements are mathematically expressed by the preceding inequality. Given that parties 1 and 2 are symmetric the same analysis applies for the receiver of the proposal in deciding whether to accept or not the proposal. Now the results of the proposition follow: in case that the electoral reform process is trivial the proposer proposes l^* whenever the expected gain exceeds the expected loss and proposes $l = 0$ whenever the opposite is true. In case the electoral reform is non-trivial but the proposer is the leading party, which implies that he always prefers to propose l^*,since for Party 1 expected benefit always exceeds the expected loss. Yet now, the proposer faces a constrained maximization program and therefore it has to satisfy the participation

constraint of Party 2 (which is analogous and symmetric to the previous inequality for Party 1). Whenever the participation constraint of Party 2 is satisfied, the proposer (Party1) proposes l^* , otherwise Party 2 will reject any proposal of l^* other than $l = 0$ and as result the reform will not pass. Hence party one proposes $l = 0$. The case where the proposer is not the leading party is, in fact, the other side of the same coin. Now Party 2 is the leading party hence its participation constraint is always satisfied and it always accept a proposal of l^*. But now it is Party 1 that proposes l^*, even if it is not the leading party, whenever its expected gain exceeds its expected loss, and $l = 0$ otherwise. This completes the argument.

4.1 Discussion of the Results and Assumptions

In this section we will discuss in more detail the implications of the main results of this paper that are stated in Propositions 3, 4 and 5. The main implication of Proposition 3 is that in our model the two dominant parties endogenously choose an electoral rule such that they crystallize and solidify the two-party system. In fact, the optimal level of l^* is such that it completely eliminates the effect of the third party in the partisan competition. Of course the third party never stood any chance of winning the election, (i.e. running first) not to mention forming a single-party government. Yet, in the absence of a distorted electoral rule, (such as the bonus system) the third party played a role in the formation of coalition governments. In many instances it had the necessary amount of seats in the parliament in order to influence the formation of the government. Now in our model, we have shown by Proposition 3, (which is a generalized version of 4 and 5) that the two dominant parties have strategic incentives that align (under mild assumptions) in order to eliminate the political impact of the third party. Hence, they solidify the two-dominant party political environment by endogenously choosing the level of the bonus l^*. Our model therefore provides a theoretical expression for the reverse statement of Duverger's law, according to which, the electoral rule affects and shapes the political environment, the number of parties and the nature of the political competition. Here, starting upside down, we construct a model that shows the inverse, that is, it is parties that endogenously choose the electoral rule, in such a way, that they solidify the pre-existing political system (in our case one with two dominant parties). The drivers of these result are the strategic incentives of the dominant parties to collude, in an environment of uncertainty about the outcome of the election, (i.e. the electoral competition is non-trivial) and of course the desire of both dominant parties to form single-party governments, once they win the election. The latter, is what it creates the incentive to collude. They collude by proposing an electoral rule that distorts the PR in order to eliminate the impact of the third party. The former, (uncertainty over the outcome) is what it gives room for this collusion to take place. If there was no uncertainty over the outcome of the election, then there would have been no room for collusion. It is exactly this combination that allows the incentives to align under some mild conditions. Those conditions are the participation constraints of the two dominant parties. Regardless of being the leading party or not, each of the two will consent in a change of the electoral rule, only if in expected terms the benefits of the electoral reform (that distorts the PR into their favour as the bonus system does) are greater than the potential losses. In such a case, we can see that not only incentives align but also there is room for collusion. Otherwise,

as expected, no electoral rule reform takes place and hence, we are stuck with the PR rule. However, it must be noted that in the next section of paper we explore into greater depth the implications of those conditions that allow the transition from the PR rule to the bonus one.

Another equally important implication of Propositions 3, 4 and 5 is the fact that the equilibrium outcome is unique and stable. Once the two dominant parties decide to depart from the PR rule, there is a unique value l^* that is proposed and accepted. In fact, the value is such that it guarantees that the winner of the election will always form a single-party government. This can be viewed as a minimax behavior of the two dominant parties. Given that the expected gains from distorting the PR rule by adapting the bonus one exceed the expected loss, the two parties are faced with the problem which value of l to propose. The answer is unique and it is the value l^*, such that in the case each one of them is the winner of the election, it would be able to form a government on its own, but in case it loses, it would have minimized its maximum loss of seats (observe that the seat loss is minimized when we have the PR rule). That is they answer the following question: "which electoral rule guarantees me a single-party government in case I win but on the same time it minimizes my loss of parliamentary seats in the worst case scenario that I run second?". The answer in this question is a unique value of l, namely l^*, and this is an important feature of the model. Its implication is that not only parties have an incentive to agree and distort the PR rule in order to solidify the two-party political system, but they also agree on a unique new electoral rule. In the next section we will also examine the stability of the electoral reform process and check whether and under which conditions the change in the electoral rule is permanent.

On a final note, there is something that needs to be clarified with respect to the structure of the game and the absence of any bargaining process. One might argue that the results presented so far critically depend on the fact the structure of the game does not allow for any bargaining process between the two dominant parties. Whereas this statement would have been in general true in a generic context, in our particular set up the absence (or presence) of a bargaining process plays no role in obtaining the above results. The reason for this is the nature of the problem. As it was made clear by Proposition 3, in any case, there are 2 only two values of l that are candidates for an optimum. And it is also true by construction of the problem that one of the two parties will always be able to guarantee its most preferred outcome (either $l^* = 0$ or $l^* = \frac{v_3}{1+v_3}$). So the existence of a bargaining process, where Party 1 could have proposed an electoral rule reform and Party 2 could respond by a counter-proposal of a different value of l, would have been equivalent with our set-up and the results obtained under this new formulation would have been identical with our results. Hence, for simplicity but without any loss in generality, we refrain from adapting a bargaining approach and we stick to this simpler set-up of the problem.

4.2   Two step electoral reform

We shall now explore a particular environment, in which the electoral reform may be applied in the coming elections only if it gathers a support W_2 and may be applied in the first elections after the coming ones if it gathers a support $W_1 < W_2$. That is, now, there is an extra strategic consideration for both dominant parties apart from the particular bonus

magnitude; the exact time of the electoral reform application. If an electoral reform proposal gathers more than W_2 votes in the parliament, then we shall call it an "immediate electoral reform" and if it gathers more than W_1 votes but less than W_2 we shall call it a "two-step electoral reform"

To simplify the analysis, we shall assume that (i) the current electoral rule is $l = 0$, (ii) unless a single-party government is formed, elections are immediately repeated, (iii) the proposer either proposes $l = l^*$ or makes no electoral reform proposal (elections take place according to $l = 0$) and that (iv) $u_i(v_1, v_2, v_3, l) = 1$ if $s_i^l(v_i) > 1/2$ and $u_i(v_1, v_2, v_3, l) = 0$ if $s_i^l(v_i) \leq 1/2$.

To concentrate only in the non-trivial cases we shall furthermore assume that both b_1 and b_2 are larger than $1/2$.

Notice that in this framework the expected utility of parties not only depends on the exact electoral reform, but also on the implementation time of the reform. This is why we need to define a set of possible electoral reform scenarios $\Psi = \{0, L^*, \hat{L}^*\}$. The first element of the set Ψ indicates that no electoral reform takes place, the second element that an immediate electoral reform l^* is implemented and the last element of the set that a two step electoral reform is decided (first elections are performed according to rule $l = 0$ and those that follow according to rule $l = l^*$).

So, the expected utility of party one is formally defined as $E[u_1(v_1, v_2, v_3, l)|\psi]$, where $\psi \in \Psi$. In specific:

$$E[u_1(v_1, v_2, v_3, l)|0] =$$
$$\frac{b_1-1/2}{b_1-a_1} + \beta(1 - \frac{b_1-1/2}{b_1-a_1})(1 - \frac{b_2-1/2}{b_2-a_2})[\frac{b_1-1/2}{b_1-a_1} + \beta(1 - \frac{b_1-1/2}{b_1-a_1})(1 - \frac{b_2-1/2}{b_2-a_2})[\frac{b_1-1/2}{b_1-a_1} + ... =$$
$$= \frac{b_1-1/2}{b_1-a_1} + \beta(1 - \frac{b_1-1/2}{b_1-a_1})(1 - \frac{b_2-1/2}{b_2-a_2})\frac{b_1-1/2}{b_1-a_1} + \beta^2(1 - \frac{b_1-1/2}{b_1-a_1})^2(1 - \frac{b_2-1/2}{b_2-a_2})^2\frac{b_1-1/2}{b_1-a_1} + ... =$$
$$= \frac{b_1-1/2}{b_1-a_1}\frac{1}{1-\beta(1-\frac{b_1-1/2}{b_1-a_1})(1-\frac{b_2-1/2}{b_2-a_2})} =$$
$$= 2(a_2 - b_2)\frac{2b_1-1}{\beta - 2\beta a_1 - 2\beta a_2 - 4a_1 a_2 + 4a_1 b_2 + 4a_2 b_1 - 4b_1 b_2 + 4\beta a_1 a_2}$$

and
$$E[u_1(v_1, v_2, v_3, l)|L^*] = \frac{b_1 - \frac{1-v_3}{2}}{b_1 - a_1}$$

and

$$E[u_1(v_1, v_2, v_3, l)|\hat{L}^*] = \frac{b_1-1/2}{b_1-a_1} + \beta(1 - \frac{b_1-1/2}{b_1-a_1})(1 - \frac{b_2-1/2}{b_2-a_2})\frac{b_1 - \frac{1-v_3}{2}}{b_1-a_1}$$

where $\frac{b_1-1/2}{b_1-a_1}$ is the probability that party one gathers more than half of the votes in the elections, $\beta \in [0, 1]$ is the discount factor, $(1 - \frac{b_1-1/2}{b_1-a_1})(1 - \frac{b_2-1/2}{b_2-a_2})$ is the probability that none of the two dominant parties controls the parliament (and, thus, no single party government can be formed and we proceed to new elections) and $\frac{b_1 - \frac{1-v_3}{2}}{b_1-a_1}$ is the probability of party one being the winner of elections.

For simplicity and without loss of generality we shall consider that $\beta = 1$.

Lemma 2 *In this case dominant parties never agree on an electoral reform. Or in other words, if $s_1^0 < W_2$, no "immediate electoral reform" is ever observed.*

The validity of the above lemma is obvious if we consider the nature of the above expected utilities of the parties. Given that $\beta = 1$, then $E[u_1(v_1, v_2, v_3, l)|0]$ is just the probability that party one will at some point form a single-party government if no electoral reform takes place. Equivalently, $E[u_1(v_1, v_2, v_3, l)|L^*]$ is the probability that party one will form a single party government if an immediate electoral reform takes place. That is, $E[u_1(v_1, v_2, v_3, l)|0] + E[u_2(v_1, v_2, v_3, l)|0] = E[u_1(v_1, v_2, v_3, l)|L^*] + E[u_2(v_1, v_2, v_3, l)|L^*] = 1$. We can rewrite this as $E[u_1(v_1, v_2, v_3, l)|0] - E[u_1(v_1, v_2, v_3, l)|L^*] = E[u_2(v_1, v_2, v_3, l)|L^*] - E[u_2(v_1, v_2, v_3, l)|0]$. It, thus, becomes clear that if $s_1^0 < W_2$ and if party ones is for the reform $(E[u_1(v_1, v_2, v_3, l)|0] - E[u_1(v_1, v_2, v_3, l)|L^*] < 0)$ then party two will be against the reform $(E[u_2(v_1, v_2, v_3, l)|L^*] - E[u_2(v_1, v_2, v_3, l)|0] < 0)$. The immediate electoral reform cannot be an equilibrium of the present game. But what about the two-step electoral reform?

Proposition 6 *Consider that, $W_1 \leq s_1^0 < W_2$. If the proposer is the leading party then no electoral reform takes place ($l = 0$). If the proposer is not the leading party then a two-step electoral reform is applied (the proposer suggests $l = l^*$ and is the only party to support it)[7].*

We are implicitly assuming that parties act as a whole; that is, all the members of a party's parliamentary delegation vote the same. In this way, even if the current seats of a party exceed the super-majority threshold (W_2) and the party, say, prefers a two-step electoral reform to an immediate one, is compelled to act in favour of the later. This will not be of major significance, because, as the above proposition clearly indicates, we are especially interested in the equilibrium of a political environment in which no party has the adequate share of seats to apply an immediate electoral reform at will.

Unlike the main part of the analysis, where the proposer has a continuum of strategies to choose from, here the choice set is constrained to a binary choice (no proposal or proposal l^*). Yet this does not alter the set up dramatically since in Proposition 3 we have proven that in fact in equilibrium the optimal choice is a binary one (either $l = 0$ or $l = \frac{v_3}{1+v_3}$). Hence if the proposer offers l^* and $W_1 \leq s_1^0 < W_2$, then by lemma 2 party two shall vote against the reform. But since the seats of the proposer exceed the low threshold, the electoral reform will pass and will be applied in the elections after the coming ones. The intuition behind this result can be found in what we call the "confidence of the leader". The leading party knows that it is impossible to have an immediate electoral rule reform given that the second party will vote against it. But it also knows that if it proposes a bonus then in the unlikely case that in the end loses the elections it would have helped its opponent to win majority of seats in the parliament and form a single party government. So in expected terms it is worse off. Whereas if it sticks with the PR rule chances are that at some point after elections being repeated many times it would manage to secure the majority in the parliament and form a single party government. The crucial thing to note is that in when the proposal is being made the party who is considered to be the leading one for the upcoming election is also considered to be leading party on the continuation of the game should no party gain control of the parliament and hence new elections are called. This result can also be verified by actual election results. When a party is closer to the threshold of forming a single-party government but unable to do so the first time then as elections are repeated until some party

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[7]Proof in the Appendix.

takes control over the majority of the parliament, empirical observation says that citizens at some point after being tired from consecutive electoral procedures choose to give the majority of votes (and seats) to the leading party. This electoral behavior is what we call the "confidence of the leader". The opposite is true when the proposer is not the leading party. Then by reversing the previous argument we see that the proposer proposes l^* and by Lemma 2 is the only party that votes for it.

The best way to illustrate Proposition 6 is with an example. Imagine that the leading party is the well-prepared student whereas the other party is the ill-prepared one. On absolute terms, it is better if the exam questions are easy (or favorable) but in relative terms, even if it sounds counter-intuitive, the well-prepared student prefers a hard exam because this will help her diversify herself from the ill-prepared one and be the only to excel. That is, by choosing a hard exam it decreases the chances of the other student to pool with her and also excel. In general, in games of competition, such as elections, where the relative position matters the most, this argument makes much sense. The leading party prefers a "harder" or less favorable electoral law in order to decrease the other party's chances of winning the majority of seats in parliament and hence, forming a single-party government, being sure at the same time that even with the PR rule, at some point, after consecutive electoral procedures, it will finally secure the majority in the parliament. This is exactly the same with the confidence of the well-prepared student that no matter hard the exam is, in the end, she will be the best performing in the whole class.

5   Conclusions

To summarize, this paper is an attempt to shed some light in the way that parties choose the electoral rules with which elections take place. In this analytically tractable model, we try to explore the endogenous choice of electoral rules by the parties, in a setting where there exist two dominant parties. That is, we explore how the nature of the political competition, under an environment of uncertainty with respect to the outcome of the electoral process, affects the endogenous choice of the electoral rule. In recent political economy literature Duverger's law, that the electoral rule determines the nature of the political competition and the structure of the political system in general, has be reversed (that is the link between PR rules and multiparty systems as opposed to majority rules and bipartisanships). Here we present an analytical model that turns Duverger upside down in a more formal way than previous literature. This is not to say that we have proven Duverger wrong. Rather we argue that if the converse hypothesis is true, then our model provides a formal expression of this. That is to say, in our paper, we argue that the nature of political competition drives parties to choose endogenously the appropriate electoral rules in order to solidify and maintain the current political system. Hence, someone would expect that our model will predict that bipartisan systems will tend to choose majority rules, as opposed to multiparty systems, in which PR rule prevails.

This is exactly what our model theoretically predicts. Our main result states that in our setting, we have two dominant parties which, under some mild assumptions and conditions, have aligned incentives to collude and choose a majority electoral rule (a mixed system that uses PR and a bonus to the first party). The important feature of our model is the uniqueness

and stability of the equilibrium. Once the strategic incentives of the two dominant parties align, there is a unique proposal to reform the electoral rule and adapt a less proportional one, which is accepted by the two dominant parties. The key result of our paper is that when the electoral competition is non-trivial, regardless if the reform process is trivial or not, there is a unique proposal of a bonus electoral law that gets accepted by the parliament and is introduced as the new electoral rule. In fact, the value of the bonus l^* is such that it completely eliminates the political impact of the third party. If the PR rule was to remain, then in some instances the third minority party would have an important role to play, when either of the two major dominant parties was not able to occupy the majority of the seats in the parliament. In such instances, the third party would have an important role to play in the formation of a coalition government.

But in our model, the two dominant parties have incentives to eliminate this role of the third party. The strategic incentives to collude and reform the PR rule into a more majoritarian one come from the desire of the dominant parties to form a single-party government, once they win the elections. That is, they are office motivated. Hence, they have a strong preference of occupying the majority of the seats in parliament once they win elections. On the other hand, if there was no uncertainty about the outcome of the election, (that is the electoral game was trivial) then the room for agreement would be non existent. Hence, the second factor that allows the two dominant parties to agree upon a reform of the electoral rule into a more majoritarian one is the existence of uncertainty about the outcome of the electoral game. In such a case, each party tries to guarantee itself, by choosing / proposing the appropriate electoral rule reform, the majority of the seats in the forthcoming parliament and hence, the ability to govern alone and pass legislation at will. On the other hand, it tries to satisfy the participation constraint of the second party, so that for both parties the expected gain of the reform will exceed the expected loss. Otherwise, the second party will not accept the proposed change. This is not the same as saying that both parties agree on a distortion of the PR rule when they are the expected winners (leading parties as we refer to them in the main part of the analysis) because in that case they would have been no room for an agreement. Rather we prove that the two parties have incentives to collude and propose a unique electoral law reform that gets accepted by the parliament if the above condition is satisfied. The uniqueness of this equilibrium appears to have a very nice intuitive explanation: under the presence of electoral uncertainty, if it is the case that both parties would agree to distort the PR electoral rule into a more majoritarian one, then they will choose one that guarantees the formation of a single party government if the are the winners of the electoral game but on the other hand, it will minimize their maximum seat loss in the parliament (in comparison with the seats they would have occupied were the PR rule was not to be changed), if they are to run second. Hence, this minimax behavior of the two dominant parties is the third critical factor that combined with the other two (electoral uncertainty and desire for single-party governments) is responsible for the uniqueness of the equilibrium in our model.

In the model, we also examine a particular case where a super-majority is needed in order for the electoral rule reform to be accepted by the parliament. In this case, our results give an explanation as to why some times a party, even when it is the expected leader, is opposed to an electoral rule reform when in opposition. In particular, our result shows that when the necessary majority needed in order to change the electoral rule is more than

three fifths of the total seats in the parliament, then the opposition party (one of the two dominant ones) has incentives to block the proposed reform. We conclude our paper with a discussion on future extensions of the model on endogenous choice of electoral rules in multiparty systems with more than two parties that contest the elections and we provide a model that generalizes the result into a broader setting. Still the basic results confirm the intuition and the main result of the paper. It is the nature of political competition and the structure of the political environment with the rules of the electoral game that provide an adequate reasoning and explanation on how and why parties choose endogenously the electoral rule in order to solidify and crystallize the current status quo.

6   References

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7 Appendix

7.1 Proofs

Proposition 3.   Given the structure of the game, to show that when the electoral competition is non-trivial then $l^* \in \{0, \frac{v_3}{1+v_3}\}$, is quite easy. In the first part of the proof we shall demonstrate that, for both parties the exact bonus l that maximizes their expected utility is either 0 or $\frac{v_3}{1+v_3}$. Then given this result, we will offer a trivial argument to show that if both parties maximize their expected utility with a bonus $\frac{v_3}{1+v_3}$, this specific electoral reform takes place, and in case at least one maximizes its expected utility with $l = 0$ no electoral reform takes place.

For the first part of the proof we need to prove that the expected utility of party 1 is convex in $l \in [0, \frac{v_3}{1+v_3}]$, strictly convex in a subset of $[0, \frac{v_3}{1+v_3}]$ and decreasing in $(\frac{v_3}{1+v_3}, 1]$. The arguments are equivalent for party 2. Since the electoral competition is non-trivial, we have that $b_1 > \frac{1-v_3}{2}$. If $b_1 < 1/2$ then there exist $\hat{l} \in (0, 1)$ s.t. $\frac{1/2-\hat{l}}{1-\hat{l}} = b_1$. For $l \in [0, \hat{l}]$

we have that $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{b_1} [v_1(1-l)+l]dv_1]$, for

$l \in (\hat{l}, \frac{v_3}{1+v_3}]$ we have that $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l) +$

$l]dv_1 + \int_{\frac{1/2-l}{1-l}}^{b_1} dv_1]$ and for $l \in (\frac{v_3}{1+v_3}, 1]$ we have that $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-$

$l)dv_1 + \int_{(1-v_3)/2}^{b_1} dv_1]$. One may observe that $\frac{\partial^2 Eu_1(v_1,v_2,v_3,l)}{\partial l^2} = 0$ for $l \in [0, \hat{l}]$, $\frac{\partial^2 Eu_1(v_1,v_2,v_3,l)}{\partial l^2} > 0$

for $l \in (\hat{l}, \frac{v_3}{1+v_3}]$, and $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} < 0$ for $l \in (\frac{v_3}{1+v_3}, 1]$. Moreover, if $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} \geq 0$ for

$l \in [0, \hat{l}]$ then $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} > 0$ for $l \in (\hat{l}, \frac{v_3}{1+v_3}]$. That is, $Eu_1(v_1, v_2, v_3, l)$ is convex in $[0, \frac{v_3}{1+v_3}]$,
strictly convex in a subset of $[0, \frac{v_3}{1+v_3}]$ and decreasing in $(\frac{v_3}{1+v_3}, 1]$. The only candidates for
maximum are $\{0, \frac{v_3}{1+v_3}\}$. If $b_1 > 1/2$, for $l \in [0, \frac{v_3}{1+v_3}]$ we have that $Eu_1(v_1, v_2, v_3, l) =$

$\frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l)+l]dv_1 + \int_{\frac{1/2-l}{1-l}}^{b_1} dv_1]$ and for $l \in (\frac{v_3}{1+v_3}, 1]$ we have that

$Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{b_1} dv_1]$. Just, as before $\frac{\partial^2 Eu_1(v_1,v_2,v_3,l)}{\partial l^2} >$

0 for $l \in [0, \frac{v_3}{1+v_3}]$, and $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} < 0$ for $l \in (\frac{v_3}{1+v_3}, 1]$. That is, $Eu_1(v_1, v_2, v_3, l)$ is
strictly convex in $[0, \frac{v_3}{1+v_3}]$ and decreasing in $(\frac{v_3}{1+v_3}, 1]$. The only candidates for maximum are
$\{0, \frac{v_3}{1+v_3}\}$. This concludes the first part of the proof.

If both parties maximize their expected utility with $l = \frac{v_3}{1+v_3}$ then party one proposes
this electoral reform and party 2 votes for it. If the optimal bonus for the proposing party
1 is $l = 0$ then it does not propose any electoral reform. And when $l = 0$ maximizes the
expected utility of party 2 then it always votes against any electoral reform. ∎

Proposition 4.   We shall prove the result for values of $b_1 < \frac{1}{2}$. First let us note that
from Proposition 3 we have that when the electoral competition is non-trivial then there
are only two possible candidate values for an optimum, that is either $l^* = 0$ or $l^* = \frac{v_3}{1+v_3}$.
When the electoral rule reform is trivial then this implies that the proposer faces an un-
constrained maximization problem. Hence for a change of the electoral rule from $l^* = 0$ to
$l^* = \frac{v_3}{1+v_3}$ it suffices to show that $Eu_1(v_1, v_2, v_3, l = \frac{v_3}{1+v_3}) > Eu_1(v_1, v_2, v_3, l = 0)$. Assume

this condition holds. Then this implies that $\frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1 - \frac{v_3}{1+v_3})dv_1 + \int_{(1-v_3)/2}^{b_1} dv_1] >$

$\frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1 dv_1 + \int_{(1-v_3)/2}^{b_1} v_1 dv_1]$. After some algebraic manipulation this yields $\int_{(1-v_3)/2}^{b_1} dv_1 -$

$\int_{(1-v_3)/2}^{b_1} v_1 dv_1 > \int_{a_1}^{(1-v_3)/2} v_1 dv_1 - \int_{a_1}^{(1-v_3)/2} v_1(1 - \frac{v_3}{1+v_3})dv_1$, which implies $\int_{(1-v_3)/2}^{b_1} (1 - v_1)dv_1 >$

$(\frac{v_3}{1+v_3}) \int_{a_1}^{(1-v_3)/2} v_1 dv_1$. It is then easily checked that expanding the integrals yields the de-

sired inequality $(b_1 - \frac{1-v_3}{2})(2 - b_1 - \frac{1-v_3}{2}) \geq (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_1)(\frac{1-v_3}{2} - a_1)$ which completes

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the argument. Reversing the argument assume that the above inequality holds true but no electoral rule change takes place, that is $l^* = 0$. But then simple algebra yields $Eu_1(v_1, v_2, v_3, l = \frac{v_3}{1+v_3}) > Eu_1(v_1, v_2, v_3, l = 0)$ which implies $l^* = \frac{v_3}{1+v_3}$ a clear contradiction. Then the *only if* part follows. ∎

Proposition 5. In proving Proposition 5 we shall be using an equivalent argument with Proposition 4. Let us first note that here the duality of the conditions is due to the fact that the proposer faces the constrained version of the maximization problem (given that the electoral reform process requires the consent of both parties). That is for $l^* = 0$ to change to $l^* = \frac{v_3}{1+v_3}$ to take place both conditions $Eu_1(v_1, v_2, v_3, l = \frac{v_3}{1+v_3}) > Eu_1(v_1, v_2, v_3, l = 0)$ and $Eu_2(v_1, v_2, v_3, l = \frac{v_3}{1+v_3}) > Eu_2(v_1, v_2, v_3, l = 0)$ need to be satisfied simultaneously. It can be easily checked (as in Proposition 4) that the condition $(b_1 - \frac{1-v_3}{2})(2 - b_1 - \frac{1-v_3}{2}) \geq (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_1)(\frac{1-v_3}{2} - a_1)$, by an analogous argument, is both necessary and sufficient condition for Party 1 to support $l^* = \frac{v_3}{1+v_3}$. A directly analogous condition is also true for Party 2 by symmetricity of the problem. Hence we only need to check that the 'leading party condition' implies the above inequality for the other party as well. Note that $\frac{a_1+b_1}{2} > \frac{1-v_3}{2}$ implies $b_1 - \frac{1-v_3}{2} > \frac{1-v_3}{2} - a_1$. In turn this implies that $(b_1 - \frac{1-v_3}{2})(2 - b_1 - \frac{1-v_3}{2}) > (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_1)(\frac{1-v_3}{2} - a_1)$. By analogy this also holds true when Party 2 is the leading party and with a similar argument as in the proof of Proposition 4 it can be shown that this condition is both necessary and sufficient. This completes the argument. ∎

Proposition 6. Since $\beta = 1$, then $E[u_1(v_1, v_2, v_3, l)|0] = \frac{2(2b_1-1)(b_2-a_2)}{2a_1+2a_2-4a_1b_2-4a_2b_1+4b_1b_2-1}$ and $E[u_1(v_1, v_2, v_3, l)|\hat{L}^*] = \frac{b_1-1/2}{b_1-a_1} + (1 - \frac{b_1-1/2}{b_1-a_1})(1 - \frac{b_1-1/2}{b_1-a_1})\frac{b_1-\frac{1-v_3}{2}}{b_1-a_1}$. It is obvious that if $E[u_1(v_1, v_2, v_3, l)|0] > E[u_1(v_1, v_2, v_3, l)|L^*]$ then $E[u_1(v_1, v_2, v_3, l)|0] > E[u_1(v_1, v_2, v_3, l)|\hat{L}^*] > E[u_1(v_1, v_2, v_3, l)|L^*]$ and that if $E[u_1(v_1, v_2, v_3, l)|0] < E[u_1(v_1, v_2, v_3, l)|L^*]$ then $E[u_1(v_1, v_2, v_3, l)|0] < E[u_1(v_1, v_2, v_3, l)|\hat{L}^*] < E[u_1(v_1, v_2, v_3, l)|L^*]$. This holds for the second party as well. In other words if a party prefers an immediate reform to no-reform, then it also prefers a two step reform to no reform and the opposite. If the proposer (party one) is the leading party $(\frac{b_1-\frac{1-v_3}{2}}{b_1-a_1} > 1/2)$ then $E[u_1(v_1, v_2, v_3, l)|0] > E[u_1(v_1, v_2, v_3, l)|L^*]$ which means that $E[u_1(v_1, v_2, v_3, l)|0] > E[u_1(v_1, v_2, v_3, l)|\hat{L}^*]$ and that $E[u_2(v_1, v_2, v_3, l)|0] < E[u_2(v_1, v_2, v_3, l)|\hat{L}^*] < E[u_2(v_1, v_2, v_3, l)|L^*]$. Equivalently, if the proposer (party one) is not the leading party $(\frac{b_1-\frac{1-v_3}{2}}{b_1-a_1} < 1/2)$ then $E[u_1(v_1, v_2, v_3, l)|0] < E[u_1(v_1, v_2, v_3, l)|L^*]$ which means that $E[u_1(v_1, v_2, v_3, l)|0] < E[u_1(v_1, v_2, v_3, l)|\hat{L}^*]$ and that $E[u_2(v_1, v_2, v_3, l)|0] > E[u_2(v_1, v_2, v_3, l)|\hat{L}^*] > E[u_2(v_1, v_2, v_3, l)|L^*]$. ∎