Staggered Wages and Monetary Policy: A Dynamic General Equilibrium Approach

by

Guido Ascari

Thesis submitted in fulfilment of the requirements of the degree of Doctor of Philosophy to the University of Warwick

Department of Economics

University of Warwick

August 1998

to Francesca

Acknowledgments

I am immensely grateful to Neil Rankin, without whose guidance it would have been impossible to complete this thesis. In the many discussions we had during these years at Warwick, I had the opportunity to enlarge a great deal my knowledge of economics. Not only, his teaching, his support, his constant and helpful presence whenever I needed, his effort in supervision and his precision provide an example that will have forever an enormous influence on me. I am proud of being able to say I have been one of his student.

I want to thank Juan Garcia and Berthold Herrendorf with whom I discussed a lot of economic ideas and projects. With the first one in particularly I started what I hope will be a fruitful and long-lasting collaboration. Juan has only one defect: he continuously comes up with too many new ideas with respect to the number I can handle. He is however an excellent flat-mate and Mr. Fixit. Discussing with Berthold I learned greatly about economics and about the profession.

I am also indebted with a great number of other people for helpful discussions and comments during these years: Antoine d'Autume, Jean-Pascal Bénassy, Giuseppe Bertola, Russell Cooper, John Driffill, Michel Juillard, Hubert Kempf, Marcus Miller, Andrew Oswald, Franck Portier, Michele Santoni, Peter Smith, Alan Sutherland, John Taylor, Mike Wickens and participants to seminars and conferences. I am also grateful to the Economics

Department at Warwick for having provided an excellent working environment.

A particular thank goes to Hubert Kempf and to the all the people who made me feel welcome at Paris I - M.A.D. during my visiting period, particularly Alexis, Fred, Emanuelle and Etíenne.

Last but not least a big, big thank to the many people who help make my life enjoyable at Warwick. Apologising to the ones I will inevitably forget in this long list: Ales, Alessandra, Anna, Antonio, Eric, Fabrizio, Gaelle, Giovanna, Juan M., Jesus, Karim, Lorenzo, Marco, Michele, Tuomas, Silvia S., Silvia V. and particularly to Ema and my 'compagna di banco' Silvia.

Financial support by the European Commission TMR Programme (TMR Fellowship: ERBFMBICT961073) is gratefully acknowledged.

DECLARATION

From Chapter 2, we derived a paper titled "Superneutrality of money in staggered wage setting models", forthcoming in *Macroeconomics Dynamics*, vol.2 (3). The paper has also been presented at the following conferences: 23-25 May 1997, Théories et Méthodes de la Macroéconomie, Louvain-La-Neuve, Belgium; 31 August - 2 September 1997, European Economic Association Conference, Tolouse, France.

From Chapter 3, we derive a paper titled "Staggered wages and disinflation dynamics: what can more microfoundations tell us ? ", joint with my supervisor Dr. Neil Rankin, and circulated as CEPR Discussion Paper No. 1476 and Institute of Economics, University of Copenhagen Discussion Papers No. 97-16. The paper has also been presented at 27 - 30 August 1997, European Econometric Association Conference, Tolouse, France.

From Chapter 4, we derive a paper titled "Optimising agents, staggered wages and persistence in the real effects of money shocks", circulated as Warwick Economics Department Research Paper No. 486. The paper has also been presented at the following conferences: 10 - 12 September 1997, Money, Macro and Finance Conference, Durham, U.K; 7-10 January 1998, Econometric Society Winter Meeting, Liblice, Prague, Czech Republic; 23 January 1998, Monetary Policy and the Business Cycle CEPR - ESRC Workshop, Centre for Economic Policy Research, London, UK; 31 March - 3 April 1998,

Young Economist Sessions, Royal Economic Society Conference, University of Warwick, UK.

From Chapter 5, we derive a paper titled "Relative wage concern: the missing piece in the contract multiplier?", joint with my colleague Juan Angel Garcia. The paper has also been presented at the following conferences: 23-25 May 1997, Théories et Méthodes de la Macroéconomie, Louvain-La-Neuve, Belgium; 2-5 September 1998, European Economic Association Conference, Berlin, Germany.

From Chapter 6, we derive a paper titled "On price/wage staggering and persistence", joint with my colleague Juan Angel Garcia.

The papers have also been presented in various seminars at CEPREMAP (France), University of Lugano (Switzerland), Paris I (France), Pavia (Italy), Southampton (UK), Warwick (UK), York (UK).

SUMMARY

In the first chapter, first we review the famous Taylor (1979, 1980a) model of staggered wage setting and then we present original work in describing the structure of a dynamic general equilibrium model with staggered wage setting *à la* Taylor. This model is central to the thesis since the results presented in chapters 2, 3 and 4 are based on it. Moreover, also the models in chapters 5 and 6, while somewhat different, originate from it.

Chapter 2 addresses the issue of superneutrality of money using the model presented in the previous chapter. It demonstrates that, once staggered wages are introduced in an optimising framework, a mild permanent change in the rate of growth of money could have substantial effects on the steady state aggregate level of output and welfare. Previous studies fail to reproduce these results because they consider restrictively simple utility and production functions. The model exhibits high costs of inflation and provides a rationale for the pursuit of price stability observed in western countries.

Chapter 3 studies analytically the output costs of a reduction in monetary growth in the dynamic general equilibrium model with staggered wages of the previous chapters. We show that the introduction of microfoundations helps to resolve the puzzle recently raised by Laurence Ball (1994), namely that disinflation in staggered pricing models causes a boom. In our model disinflation, whether unanticipated or anticipated, unambiguously causes a slump.

The analytical results are restricted to the tractable case (log-linearisation of the model around a zero steady state inflation), but a long appendix checks the robustness of these results through non-linear simulations.

Chapter 4 investigates whether staggered wages could induce a high degree of persistence in the real effects of money shocks. We show how the parameters of Taylor's model depend upon the microeconomic fundamentals and the conduct of monetary policy. We conclude that high persistence is an unlikely outcome. Either sensible values of the microeconomic parameters or a moderate rate of underlying inflation imply a low degree of persistence. This is the persistence puzzle we referred to above. Furthermore, we show that: (i) the model is highly non-linear; (ii) the conduct of monetary policy affects the structural parameters of Taylor's wage setting equation, providing a clear example of the Lucas critique; (iii) the inertia of the system is inversely related to the level of average inflation.

In Chapter 5 we incorporate explicit relative wage concern on the part of wage-setters into the dynamic general equilibrium model with staggered wages developed in the previous chapters. We then investigate the effects of money shocks on both inflation and output. In contrast to previous models of staggered wages/prices, output and inflation persistence are a robust finding of the model. Moreover, they hold for all the sensible parametrisations. Given the empirical evidence on relative wage concern, we conclude that this

may be the missing piece in the money shocks persistence puzzle.

Chapter 6 presents a unifying framework to analyse the ability of price versus wage staggering to generate persistence. The results are fairly general in that they derive from a stylised log-linear model which encompasses most of the microfounded models of price/wage staggering, found recently in the literature. The results highlight the importance of the underlying economic structure for the ability of staggered price/wage models to generate persistence of the real effects of money shocks.

Contents

List of Tables

List of Figures

Introduction

The first staggered wage models appeared in the literature at the end of the '70's and they immediately occupied the front stage of the macroeconomic debate of the time. The so-called New Classical Macroeconomics had just demonstrated, through the famous *Lucas aggregate supply curve*, that systematic demand-management policies are of no use to stabilise output, since only unanticipated policies could have an effect (and only a temporary one) on output levels - the so called "policy ineffectiveness proposition" of Sargent and Wallace (1975). This critique of the Keynesian theory of active stabilisation policies was based on two main cornerstones: market clearing and rational expectations. The first reaction of the Keynesian school to this attack rests on the sticky wage models of Fischer (1977) and Gray (1976). One cornerstone of the New Classical theory, i.e., market clearing, was removed by recognising that workers engage in long-term labour contracts leading to sticky wages. The purpose was to demonstrate that the "policy ineffectiveness proposition" was due to the flexible price assumption rather than to

the rational expectations one. Fischer (1977) and Gray (1976) assumed that wages were set one (or more than one) period in advance, such that the labour market *cleared in expectations,* in the sense that the expected quantity of labour supplied equalled the expected quantity of labour demanded. In the case when a shock occurred, then supply and demand would have differed from what was expected and it was assumed that demand determined employment in this case. Fischer (1977) then showed that in a model with rational expectations and sticky wages, monetary policy can play a role in stabilising the economy. The focus on sticky wages was quite a natural one, deriving from simple casual observation. In the US, formal labour contracts prevail in heavily unionised industries like steel, automobiles, rubber, etc.. Many of these contracts extend for more than one year (most for three years). Moreover, wage contract negotiations in unionised industries are likely to influence the level of wages in all industries, since they tend to be imitated elsewhere, and even in the non-unionised industries nominal wages are mostly fixed for one year.

However, these first sticky wage models provided no explanations of the persistence of the real effects of monetary shocks. In particular, the persistence could not last longer than the duration of the nominal wage contracts. Taylor (1979, 1980a), in trying to build an empirical model which could mimic the fluctuations in U.S. time series, overcame this problem by removing the

assumption of synchronisation of wage setting decisions. Taylor observed not only that wages are negotiated discretely in time, but also that contracts are not renewed all at the same time, that is, contract negotiations are staggered. The staggering of wage decisions creates a rational expectations nominal propagation mechanism which propagates shocks over time. In such a set up, Taylor (1980a) demonstrated that, following supply shocks, systematic demand-management policies could help in stabilising output. In particular, in a staggered wage economy policymakers face a trade-off between the variance of prices and that of output, a so-called second-order Phillips curve. While Taylor's original article focused on the optimal response of monetary policy to supply shocks, most of the followers (e.g., Taylor (1983), Blanchard (1983), Blanchard (1986), West (1988), Phaneuf (1990)) focused on the effects of monetary disturbances. They showed that the nominal propagation mechanism generated by staggered wage models could be particularly appealing for the study of the role of monetary disturbances in the business cycle. In fact, it can help explain how monetary shocks could generate the type of output fluctuations observed in actual data, particularly the persistence of the real effects of money shocks, or, more generally, of demand-management policies. As a proof of how much these early staggered wage models have been influential in the literature, it is probably enough to say that nowadays every textbook presents or, at least, mentions them.

Apart from synchronisation vs. staggering, there is another important difference between the type of wage contracts in Fischer (1977), Gray (1976) and Taylor (1979, 1980a). Fischer (1977) studied the implication of *predetermined* multi-period wage contracts for the efficacy of stabilisation policies. The Fischer-type wage contracts are called predetermined (see Blanchard and Fischer (1989)) because they allow agents to set different wage rates in the different future periods of the contract, even if all the wages specified in the contract have been negotiated in the period of renewal of the contract (and cannot be renegotiated in the future periods for which the wage contract will last). This contract structure should correspond in practice to multi-year labour contracts. For example, if unions sign today contracts for more than one year, say three, then they would probably call for predetermined wage rate increases each year (for example, to take into account the expected inflation). Gray (1976) explicitly considered the question of the optimal degree of indexation of multi-period nominal wage contracts. However, multi-year wage contracts are not the general rule outside heavily unionised industries. Taylor (1979, 1980a) instead analysed the implications of a different kind of contracts, called *fixed* staggered wage contracts. In this case, the wage rate is not allowed to vary in the different periods of the contract. Workers sign a contract that specifies a fixed wage rate for each period for which the contract will last, that is, the wage rate has to be the same in each period of the

contract. This contract structure should correspond in practice to one year labour contracts, where the periods can be interpreted as semesters, quarters or months.[1]

Actually, most of the wage contracts in the economy are generally renewed every year and fixed within the year. Moreover, obviously different workers negotiate the contracts in different periods of the year. After the '80's, especially in the US and UK, the activity of the unions, both in terms of number of members and of the amount of coverage, has been progressively narrowed. This diminished the importance of multi-year labour contracts in the whole economy. Indeed: *"... wages [are almost always fixed between adjustments] outside the North American union sector. More than 80 percent of US wages are set for one year or less with no time-variation; [...] in many countries, such as the United Kingdom, virtually* ***all*** *wages are set for one year or less with no time-variation"* (Ball (1994), p. 288, emphasis as in the original). Taylor (1998) reviews price and wage setting behaviour in market economies based on direct and indirect evidence. He concludes that: (1) since not everyone sets prices or wages at the same time, wage and price

[1] Another very popular staggered wage/price model in the literature is the model by Calvo (1983a,b). This is a continuous time model where the duration of the contract is stochastic and governed by a Poisson process. Even if very elegant from a formal point of view, its empirical relevance seems doubtful (see Taylor (1998)). Calvo's model will be analysed in Chapter 2 and 3.

setting appears to be staggered, such that contract periods overlap with each other; (2) wages and prices are set at fixed values for fairly long periods of time and are frequently, though not always, non-contingent on events that occur during the contract period; (3) most of the wages, though not all of them, are negotiated annually. This suggests that Taylor-type of contract structure is the most relevant in economies nowadays and quantitatively it seems a good approximation to take one year as the duration of the contract. For this reason, we will focus on this type of contract structure in this thesis.

Even if the first generation of staggered wage models were consistent with certain observed features of wage setting behaviour, they have been seriously criticised for being *ad hoc* because of lacking rigorous theoretical foundations. Particularly, they left open three key questions: (i) why we observe wage contracts fixed for so long; (ii) why we observe staggering of wage setting decisions; (iii) why, given the constraint due to the staggering structure, the wage was not chosen optimally. From the mid '80's a huge literature, the so-called New Keynesian literature, has been devoted to these issues, which are however still somewhat unsettled. Good surveys and discussions of these points are provided by Blanchard and Fischer (1989), Romer (1996), and, more recently, by Taylor (1998), and here we sum up very briefly some results from those references.[2]

[2]Most of the New Keynesian literature actually focused on sticky prices rather than on sticky wages for reasons summarised, for example in Mankiw (1990). However, *"Econo-*

With respect to (i) a first major puzzle is why contracts are not indexed to all relevant information. Gray (1976) provides part of the answer, showing that full indexation with regard to a single variable such as the price level is not optimal in the presence both of demand and supply shocks. In a real word with many types of different shocks, the practical answer probably rests on complexity, asymmetric information and measurement problems. A second major issue concerns the distinction between *time-dependent* and *state-dependent rules.* While the first type of rule (as in the models discussed above) takes the time interval between subsequent wage/price adjustments as exogenous and the size of the adjustment as endogenous, the latter does vice versa. That is, the difference between the actual and the desired level of wage/price triggers the adjustment after a certain level. Sheshinski and Weiss (1977), Sheshinski and Weiss (1983), and Benabou (1989) investigate the optimal *(S,s)* rule for a firm under different conditions.[3] In the aggregate

mists differ about whether they view these criticisms [of nominal wage contracting models] *as serious"* (Mankiw (1990), p. 1657). For a survey of theories of price rigidities see Andersen (1994).

[3]Such rules are called *(S,s)* rules, because they take the following form: when the difference between the actual and the desired optimal price exceeds an upper bound $S > 0$ or become less than a lower bound $s < 0$, the actual price is changed and set equal to the optimal one. *(S,s)* models are the dynamic versions of the first static menu costs models of Akerlof and Yellen (1985a,b) and Mankiw (1985). They assume a cost of physically changing the price (printing new catalogue or menu in a restaurant, from which the name

not every firm will change its price every period and therefore we have a sort of state-dependent staggering. Caplin and Spulber (1987) demonstrate the surprising, and much discussed, result that if firms follow *(S,s)* rules, money can be completely neutral in the aggregate. However, Caplin and Spulber's (1987) result seems not to be robust, as shown by Benabou (1988), Caplin and Leahy (1991), Caballero and Engel (1991, 1993a,b), Tsiddon (1991, 1993), and Conlon and Liu (1997). The fact is that the aggregation over the whole economy of *(S,s)* rules is particularly difficult, and not always possible; thus these models are not very tractable and need to rest on special assumptions to be solved. Hence, they have not been used so far in quantitative models or in dynamic general equilibrium models (a notable exception as a first attempt in this direction is Dotsey, King, and Wolman (1996)). In reality, probably both time-dependent and state-dependent elements are present in wage and price contracts. Which are the most important will depend on the kind of contract. There are two main costs in adjusting prices: the first is to understand the state of the economy (and then calculate the optimal price or negotiate a wage contract, i.e., negotiation costs) and the second one is to physically change the price in accordance with the state of the economy (printing new catalogue, the so-called "menu costs"). If the first one is the higher, than a time-dependent rule would probably be optimal, while if the

"menu cost" models). Hence the price would be changed only if the benefit exceeds the (menu) cost.

second one is higher then a state-dependent rule would probably be optimal. It follows that time-dependent rules *à la* Taylor are probably a very good approximation for wage contracts (as in this thesis), while for final good prices, a state-dependent rule would presumably be a better approximation.

Several papers have been devoted to the second question, (ii), that is, why we observe staggering in wage/price decisions. Fethke and Policano (1984, 1986) demonstrate that staggering can arise as a stable equilibrium when there are sector specific shocks, while Ball and Romer (1989) do the same assuming asymmetric seasonal shocks. Ball and Cecchetti (1988) show that a staggering equilibrium can be supported as an equilibrium because it allows price-setter agents to obtain information about the prices of the others, before choosing their own prices. Maskin and Tirole (1988), Lau (1996), Fraja (1993) show that staggering can arise endogenously in oligopoly models because of strategic considerations. Very recently, Bhaskar (1998) proves that staggering can be an equilibrium in a model with many heterogeneous firms, which have stronger strategic complementarity within-industry than across-industry. This result is particularly important since it does not rest on strategic considerations between 'few large' price-setters, but it arise in a model with 'many small' firms, as in the usual monopolistic competition macromodels.

This last point brings us to (iii) and specifically to the role of monop-

olistic competition. First, nominal rigidities logically require price-setting agents and hence imperfect competition. If agents are price setters, then they would fix the price optimally. Imperfect competition then delivers an optimal wage/price setting rule, in contrast to the *ad hoc* expected-market-clearing approach of the first generation of sticky wage/price models, where we were left with the question of who was actually setting the wage/price. Second, among the different types of imperfect competition market structures, monopolistic competition is usually employed in macromodels, because it avoids strategic interactions between different price-setting agents. Third, monopolistic competition provides theoretical foundations for a demand-determined output. Given the monopolistic distortion, firms are pricing above marginal costs and hence are willing to satisfy the extra demand at given prices (at least up to a point), following a shock. Monopolistic competition thus solves some inconsistencies of the early expected-market-clearing nominal rigidity models. The pioneering works investigating the macroeconomic consequences of sticky prices and monopolistic competition in a general equilibrium model are Svensson (1986) and Blanchard and Kiyotaki (1987). A first attempt to introduce dynamics and staggering in these models is Blanchard and Fischer (1989), but the model is not intrinsically dynamic, because dynamics is actually superimposed *ad hoc* on a simplified version of the static model of Blanchard and Kiyotaki (1987).

In the '80's, another school of thought antithetical to the New Keynesian started to develop a different approach to the study of business cycle fluctuations, following the work of the Nobel prize winner Robert E. Lucas. According to this approach, modern macroeconomics, and particularly business cycle research, requires us to build explicitly dynamic models, with full microfoundations and intertemporally optimising agents in a dynamic general equilibrium context. Obviously such models are much more complicated than *ad hoc* models and early attempts at building them were bound to be stylised. The pioneering works in this area are Kydland and Prescott (1982) and Long and Plosser (1983). From then onwards, the title of this latter work has been used to indicate a new branch of the literature: the Real Business Cycle literature.

Early real business cycle models tried to reproduce actual business cycle features as the response of optimising agents to exogenous real shocks in a purely real economy and under Walrasian market clearing. Macroeconomic fluctuations were explained only with technological shocks, while demand and nominal shocks were absent from these purely real models. During the '80's the Real Business Cycle and the New Keynesian practitioners did not talk constructively to each other, and instead engaged in a fierce debate (see e.g., Prescott (1986) and Summers (1986) or Plosser (1989) and Mankiw (1989)). However, during the '90's convergence between the two approaches

starts to develop. Clearly early Real Business Cycle models were too stylised and their restrictions could not last for long. Researchers of both approaches began to introduce different features in the benchmark Real Business Cycle model, such as imperfectly competitive markets (e.g., Rotemberg and Woodford (1993, 1995)), money (e.g., Cooley and Hansen (1989), Christiano and Eichenbaum (1992)) and nominal rigidities (e.g., Hairault and Portier (1993), Kimball (1995)). As Bénassy (1995, p. 304) noted: *"... a number of researchers [...] have convincingly argued that the consideration of price, and especially wage rigidities, in monetary economies subject to real and monetary shocks allowed to substantially improve the capacity of these business cycle models to match a number of stylized facts in actual economies."* Particularly important works are the ones by Hairault and Portier (1993) and Bénassy (1995). Hairault and Portier (1993), following Kydland and Prescott (1982), is the among the first works that numerically simulates and evaluates a dynamic general equilibrium model with money and nominal rigidity, in the form of adjustment costs in price changes. Bénassy (1995), instead, following the analytical approach of Long and Plosser (1983), inspects the analytical mechanism of a dynamic general equilibrium model with preset wages. In the recent years an increasing number of papers have been devoted to enlarging the stylised framework of the early real business cycle models and a good early survey of this literature is provided by Cooley (1995).[4] It seems that

[4] Given the rapid development of this literature, especially with respect to money, nom-

a new 'hybrid' paradigm for macroeconomic modelling has emerged combining the elegant methodology of the Real Business Cycle paradigm with the elements of realism of the New Keynesian paradigm. In particular, these hybrid models introduce money, sticky prices and monopolistic competition in an otherwise standard Real Business Cycle framework. In order to do that, they necessarily leave aside more fundamental questions about the microfoundations of money and sticky prices/wages to concentrate on their macroeconomic implications.

In this thesis we will develop further this new hybrid approach and we will combine the two strands of the literature mentioned above. In particular, we will introduce the idea of staggered multi-period nominal wages in a dynamic general equilibrium framework. Following the 'hybrid' approach, we will therefore not consider questions (i) and (ii) above. Hence, even if our wage setting rule is derived by intertemporally optimising agents, we would acknowledge from the start that our microfoundations are not complete. Specifically, we superimpose Taylor's (1979) staggered wage structure on a dynamic general equilibrium framework, without providing - within the model - a justification of why this structure should exist in an optimising

inal rigidities and the monetary transmission mechanism, the several works of Christiano, Eichenbaum and Evans are a more up-to-date reference (see, as a late references, Christiano, Eichenbaum, and Evans (1997) and Christiano, Eichenbaum, and Evans (1998))

framework.[5] In other words, we look at the macroeconomic implications of something we observe in reality, i.e., staggered wages, in a dynamic general equilibrium framework.[6] We then will analyse the effects of changes in monetary policy in such a framework. The motivation for this work is therefore given by the fact that modern business cycle research is almost entirely carried out within the context of dynamic general equilibrium macromodels. In this approach, the role of monetary shocks in generating the output fluctuations observed in actual data is still controversial. Monetary dynamic general equilibrium macromodels need to incorporate some forms of nominal rigidities to generate short-run monetary non-neutralities. At the beginning of the research which led to this thesis, we thought that the overlapping contracts model of Taylor (1979, 80a) could have some prominent role to play in this approach. As explained above, the reason is that such contracting schemes bring in not only the nominal rigidity necessary for the impact effect of the monetary innovation, but also provide a nominal propagation mechanism in a framework otherwise lacking endogenous propagation mechanisms.

The dynamic general equilibrium model described in Chapter 1 has indeed proved to be more fruitful than expected. Almost all the work of this

[5] Actually this staggered wage structure acts as an additional constraint and hence can not be optimal in our model, that is, households would be better off without it.

[6] As it appears from what was discussed above, there is now quite a huge parallel literature dealing with the issue of why staggering is observed.

thesis is actually based on that model and on its modifications, providing consistency to the chapters of this thesis. In particular, in Chapter 2, we investigate the steady state properties of the model addressing the superneutrality issue. Chapters 3 and 4 look at the effects in such a model of changes in monetary policy, in the form respectively of a disinflation and of a money shock. Chapters 5 and 6 are respectively based on a modification of the basic model and on a generalisation of it. That is, there is in practice one basic model which runs all the way through the thesis.

A further point to note is that this model is, to my knowledge, the first dynamic general equilibrium model to include wage staggering and monopolistic unions.[7] During the years I devoted to this research project, some closely related studies appeared in the literature, but almost all of them focused on price staggering. A very small sample of early dynamic general equilibrium models with staggered prices focusing on various issues and problems are: Ireland (1995), Woodford (1996), Yun (1996), Chari et al. (1996). The latter has already been quite influential and has already generated a very lively debate. Our thesis fits exactly into this very recent debate. Chapter 4, developed contemporaneously and independently from Chari et al. (1996),

[7] Cho and Cooley (1995) is an earlier attempt, but monopolistic competition is absent from the analysis. The hypothesis on wage setting is therefore the expected-market-clearing one as in the old *ad hoc* model. As explained above, this approach does not seem to be coherent with a microfounded general equilibrium model.

reproduces their main result in a wage staggering economy. This result generates a puzzle about the degree of persistence of the real effects of money shock in dynamic general equilibrium staggered wage (Chapter 4) or price (Chari et al. (1996)) models. Chapter 5 proposes a solution to this puzzle, introducing explicit relative wage concern on the part of the workers. This chapter is hence among the recent contributions which suggest reasons why this persistence puzzle may arise. The debate, still under way, is at the heart of the attempt to build a quantitative macroeconomic model describing the monetary policy transmission mechanism which could be used to analyse the effects of monetary policy changes and to design optimal monetary policy rules (e.g., Rotemberg and Woodford (1997)). Main contributors to this debate are: Erceg (1997), Kiley (1997), Rotemberg and Woodford (1997), Andersen (1998a,b), Bergin and Feenstra (1998), Jeanne (1998). Chapter 6 provides a unifying framework for the already quite large literature on this topic and it thus may be useful for interpreting and understanding the current debate.

More specifically, the thesis is organised as follows. In the first chapter, first we review the famous Taylor (1979, 1980a) model of staggered wage setting and then we present original work in describing the structure of the basic model of the thesis: a dynamic general equilibrium model with staggered wage setting *à la* Taylor.

Chapter 2 addresses the issue of superneutrality of money using the model presented in the previous chapter. It demonstrates that, once staggered wages are introduced in an optimising framework, a mild permanent change in the rate of growth of money could have substantial effects on the steady state aggregate level of output and welfare. Previous studies fail to reproduce these results because they consider restrictively simple utility and production functions. The model exhibits high costs of inflation and provides a rationale for the pursuit of price stability observed in western countries.

Chapter 3 studies analytically the output costs of a reduction in monetary growth in the dynamic general equilibrium model with staggered wages of the previous chapters. We show that the introduction of microfoundations helps to resolve the puzzle recently raised by Laurence Ball (1994), namely that disinflation in staggered pricing models causes a boom. In our model disinflation, whether unanticipated or anticipated, unambiguously causes a slump. The analytical results are restricted to the tractable case (log-linearisation of the model around a zero steady state inflation), but a long appendix checks the robustness of these results through non-linear simulations.

Chapter 4 investigates whether staggered wages could induce a high degree of persistence in the real effects of money shocks. We show how the parameters of Taylor's model depend upon the microeconomic fundamentals and the conduct of monetary policy. We conclude that high persistence is an

unlikely outcome. Either sensible values of the microeconomic parameters or a moderate rate of underlying inflation imply a low degree of persistence. This is the persistence puzzle we referred to above. Furthermore, we show that: (i) the model is highly non-linear; (ii) the conduct of monetary policy affects the structural parameters of Taylor's wage setting equation, providing a clear example of the Lucas critique; (iii) the inertia of the system is inversely related to the level of average inflation.

In Chapter 5 we incorporate explicit relative wage concern on the part of wage-setters into the dynamic general equilibrium model with staggered wages developed in the previous chapters. We then investigate the effects of money shocks on both inflation and output. In contrast to Chapter 4, output and inflation persistence are a robust finding of the model. Given the empirical evidence on relative wage concern, we conclude that this may be the missing piece in the money shocks persistence puzzle.

Chapter 6 presents a unifying framework to analyse the ability of price versus wage staggering to generate persistence. The results are fairly general in that they derive from a stylised log-linear model which encompasses most of the microfounded models of price/wage staggering, found recently in the literature. The results highlight the importance of the underlying economic structure for the ability of staggered price/wage models to generate persistence of the real effects of money shocks.

Chapter 1

A Dynamic General Equilibrium Model with Staggered Wage Setting

1.1 Introduction

This first chapter introduces the tools needed to tackle the main issues that will be treated in the following chapters. In particular, we describe two models that will continually be coming up in the following analysis.

Firstly, we review the famous Taylor (1979, 1980a) model of staggered wage setting. Even though the model is very well known, the description of the model will be quite extended given the central role that, for comparison

purposes, the literature based on it will have all through the thesis.

Secondly, we start to present original work in describing the structure of a dynamic general equilibrium model with staggered wage setting *à la* Taylor. This model is central to the thesis since the results presented in chapters 2, 3 and 4 are based on it. Moreover, also the models in chapters 5 and 6, while somewhat different, originate from it.

1.2 Taylor's (1979, 1980a) Staggered Wage Model

In Taylor's (1979) model the economy is divided into two sectors, say sectors A and B. In each sector the nominal wage is negotiated every two periods and it is kept fixed between the two periods. Moreover, contract decisions are staggered in the sense that wage negotiations are not made at the same time in the two sectors. In other words, sector A fixes the wages in periods t, $t+2$, $t+4$..., while sector B in period $t-1$, $t+1$, $t+3$... .[1] Let us call x_t the 'new' nominal wage contract which is negotiated in period t.[2] Then, the

[1] *"To make things simple suppose that wage contracts last one year and that decision dates are evenly staggered: half of the contracts are set in January and half in July."* (Taylor (1979), p. 109)

[2] Hence $x_t, x_{t+2}, x_{t+4}...$ are the wage contracts negotiated in sector A, while $x_{t-1}, x_{t+1}, x_{t+3}...$ are the ones negotiated in sector B.

fundamental equation of Taylor's model is the following wage setting rule

$$x_t = b\ x_{t-1} + d\ E_{t-1}x_{t+1} + \gamma\ (b\ E_{t-1}y_t + d\ E_{t-1}y_{t+1}) \tag{1.1}$$

where p = price level, x = 'new' nominal wage; y = output and all the variables are expressed in terms of log-deviation from an initial trend. E_{t-1} is the expectation operator and in front of a variable represents its conditional expectation based on the information available at the beginning of period t (or end of period $t-1$). Hence, the wage contract are signed at the beginning of the period, before the realisation of period t shock, that is, based on period $t-1$ information set. As Taylor explains:[3]

> *Equation* [(1.1)] *states the assumption that the contract wage set at the start of each semiannual period depends on three factors: the contract wage set in the previous period, the contract wage expected to be set in the next period, and a weighted average of excess demand expected during the next two periods. Since, by assumption,* x_t *will prevail for two periods, firms and/or unions contemplating a wage adjustment in period t will be concerned with wage rates which will be in effect during periods t and t+1.* (Taylor (1979), p. 109)
>
> *...Most theories of wages adjustment suggest that labor market conditions will influence wages and, in particular, that wages will be bid up relative to the prevailing wage during periods when the unemployment rate is low, and conversely when the unemployment rate is high. This, for example, is the explicit assumption used in my 1980 article. [...] The behavioral equations reflect a relative wage concern on the part of the workers.* (Taylor (1983), p. 987-988)
>
> *Let us assume that* $b + d = 1$ *so that the current contract decision is homogenous of degree 1 in these lag and lead contracts. If* $b = d =$

[3]Since actually Taylor's model is central to all the thesis, this long quotation from Taylor will turn out to be very useful in what follows. In particular, it helps to see what new interpretations of Taylor's reasoning microfoundations make possible.

1/2 then the lag and lead distribution is symmetric. This has been the parametric assumption of my previous work and reflects the plausible assumption that current negotiations weight other contracts according to the number of periods that they overlap with the current contract. In this sense, when b and d are equal to 1/2, contract decisions are unbiased. Wage setters look forward to the same degree they look backward. However, I will allow for the possibility of biased weights in this paper by permitting b and d to differ from 1/2. This permit a spectrum of contract determination hypotheses between the extremes of pure backward looking (b = 1) and pure forward looking (d = 1). As will be demonstrated below the size of b vs. d is important for the dynamic behavior of contracts, and for the sensitivity of wage behavior to excess demand. (Taylor (1979), p. 109) *...more backward-looking wage determination increases the persistence or the inertia of the aggregate wages.[...] more forward-looking contract determination increases the impact of aggregate demand policy on wages.* (Taylor (1979), p. 110-111)

The model is closed by the following two equations

$$p_t = \frac{1}{2}(x_{t-1} + x_t) \tag{1.2}$$

$$y_t = m_t - p_t \tag{1.3}$$

where m is the log-deviation from trend of the money supply. Equation (1.2) is simply a mark-up equation, which states that the aggregate price level is given by an average of the existing nominal wage contracts. This equation implicitly assumes constant returns to labour. Equation (1.3) is just a static aggregate demand equation.

In Taylor's words above, *"the behavioral equations reflect a relative wage concern on the part of the workers"*. However, substituting equation (1.2) into (1.1), it yields

$$x_t = b\ p_t + d\ E_{t-1}p_{t+1} + \frac{\gamma}{2}\ (b\ E_{t-1}y_t + d\ E_{t-1}y_{t+1})\ . \tag{1.4}$$

As already noted by Buiter and Jewitt (1981) and Blanchard (1990), this equation shows that Taylor's wage setting rule can have actually a different interpretation. In setting the wage, workers care only about their absolute real wage. Hence they look at the general price level in the two periods in which the wage is fixed (plus the labour market conditions in the two periods). It follows that since the price level depends on the wage in the other sector of the economy, then one can write the wage setting rule as (1.1). In other words, workers care about the wages in the other sectors only through the effect these wages have on the price level and, in turn, on their own absolute real wage. Hence, there is no actual relative wage concern *per sé* in Taylor's model. In Blanchard's (1990, note 19, p. 805) words: *"It is sometimes argued that the Taylor model depends on the assumption that workers care directly about their wages in comparison to other wages [...] this is not the case."* This remark will be important for Chapter 5, where, following probably the original intention of Taylor (1979), we will explicitly introduce relative wage concern on the part of the workers in the dynamic general equilibrium model presented in the following section of this chapter.

Taylor in his original articles (1979, 1980a) focused on real shocks and on the optimal monetary policy response to such shocks. However, the subsequent literature focuses on monetary shocks (e.g., West (1988), Ambler and Phaneuf (1989), Phaneuf (1990)). Thus, the money supply is assumed to follow an exogenously given stochastic process. For a given expected path of the money supply, the model exhibits the following saddle path solution

$$x_t = \lambda_s x_{t-1} + \sum_{i=0}^{\infty} \frac{(\varphi - 1)\lambda_s}{b} \left(\frac{1}{\lambda_u}\right)^i [bE_{t-1}(m_{t+i}) + dE_{t-1}(m_{t+1+i})] \quad (1.5)$$

where

$$\lambda_s = \frac{\varphi - \sqrt{\varphi^2 - 4d(1-d)}}{2d}; \quad \lambda_u = \frac{\varphi + \sqrt{\varphi^2 - 4d(1-d)}}{2d}; \quad \varphi = \frac{1+\frac{\gamma}{2}}{1-\frac{\gamma}{2}} . \tag{1.6}$$

λ_s and λ_u are respectively the stable and the unstable root of the saddle equilibrium. Now suppose m_t follows a random walk. Then, (1.5) becomes

$$x_t = \lambda_s x_{t-1} + (1-\lambda_s) m_{t-1} \tag{1.7}$$

and the dynamics of output are given by

$$y_t = \lambda_s y_{t-1} + (m_t - m_{t-1}) + \frac{1}{2}(1-\lambda_s)(m_{t-1} - m_{t-2}) . \tag{1.8}$$

The first important thing to note therefore is that the model exhibits persistence in the real effect of money shocks. This issue regarding the persistence of money shocks in staggered wage models will be one of the main focuses of the thesis and will be discussed in depth in Chapters 4, 5 and 6.

A second point to note is that *in the long run* while money is naturally neutral in Taylor's model, it is *not* superneutral, unless $b = d = 1/2$. In the above model when the variables are all at their initial steady state level, the log-deviations are equal zero and the equations are trivially satisfied. Suppose a permanent step increase in the money supply such that in the new steady state, instead of having $m_t = 0$, we have $m_t = \overline{m} \neq 0$, $\forall t$. Then, in the new steady state $p^{ss} = x^{ss} = \overline{m}$ and $y^{ss} = 0$, that is, the nominal variables increase as much as the money supply and money is neutral in the long run. Now suppose a change in the trend such that the money supply grows forever at a different rate than the initial one. Then, in the new steady state $m_t - m_{t-1} = \phi \neq 0$ and the nominal variables will grow in steady state such that $p_t^{ss} - p_{t-1}^{ss} = \phi \neq 0$, $x_t^{ss} - x_{t-1}^{ss} = \phi \neq 0$, $\forall t$. It is thus trivial to show

that the equations of the model imply $y^{ss} = \frac{b-d}{\gamma}\phi$. Then, output is constant in steady state, but it depends on the new rate of growth of money, that is, money is not superneutral in steady state. This occurs unless $b = d = 1/2$.[4]

Finally, the third and most important point is that Taylor's model is an *ad hoc* log-linear structural model in which the behavioural equations are exogenously specified at the outset. It lacks microfoundations and intertemporal optimisation. Taylor (1979) openly acknowledges the need for microfoundations.[5] In the model presented in the following section, we incorporate staggered wage setting *à la* Taylor (1979) in an optimising dynamic general equilibrium framework. One aim is to open the 'black box' of the structural ad hoc parameters of Taylor's famous wage setting equation to show how these parameters depend upon the microeconomic fundamentals and the conduct of monetary policy. Besides, intertemporal optimisation adds new features to the model due to the intertemporal links missing in the simple Taylor model.

Such a dynamic general equilibrium model with staggered wage setting will allow us to address several issues: (i) the superneutrality of money in the next chapter; (ii) the adjustment dynamics following disinflationary policies in Chapter 3; (iii) the persistence in the real effect of money shocks in Chapter 4.

---

[4]The issue of the non-superneutrality of money in staggered wage models will be discussed in the next chapter.

[5] *"Unfortunately, the assumed contract formation behavior is not explictly derived from a utility maximization model. [...] the micro foundations of the staggered contract model presented here are far from complete."* (Taylor (1979), p. 111) *"The microfoundations of such models need to be developed more rigourously"* (Taylor (1979), p. 112).

1.3 A Dynamic General Equilibrium Model with Staggered Wage Setting

The model introduces staggered wage setting *à la* Taylor (1979) in the framework presented in Rankin (1998). The economy consists of a continuum of industries indexed by $i \in [0, 1]$, and of a continuum of industry-specific household-unions.[6] Every industry produces a single differentiated perishable product and the goods market in each industry is competitive. Since labour is not allowed to move across industries, the household-union has monopoly power in the labour market. Preferences are CES over consumption goods which are gross substitutes. All firms have the same technology and households have the same preferences. The symmetry of the economy is broken by supposing staggered wages. The economy is divided into two sectors of equal size: industries $i \in [0, \frac{1}{2}]$ and industry-specific household-unions $j \in [0, \frac{1}{2}]$ compose sector A, while industries $i \in (\frac{1}{2}, 1]$ and industry-specific household-unions $j \in (\frac{1}{2}, 1]$ compose sector B. Exactly as in Taylor's model structure in the previous section, in each sector every two periods household-unions set nominal wages which are fixed and constant between the two periods. Then, staggering is introduced by assuming that sector A fixes the wages in even periods, while sector B in odd periods.

Furthermore, we assume no uncertainty in the model. Actually this hy-

[6] A continuum of industries means that no imperfectly competitive agent is 'large' relative to the economy as a whole. The 'household-union' should be thought of as an aggregate of all the households which work in the industry, who collude in the setting of the wage.

pothesis is only made for reasons of simplicity. It is in fact not crucial for the results concerning the dynamics of the model, since the latter will be studied log-linearising the model around the deterministic steady state.[7] The expectations operator could be straightforwardly incorporated in the model, as the model in Chapter 5 will show.

Demands for output and labour in the two sectors

All the household-unions have the same utility function

$$U_j = \sum_{i=0}^{\infty} \beta^t u(C_{jt}, M_{jt}/P_t, L_{jt}) \tag{1.9}$$

where $0 < \beta < 1$. C_{jt} is a consumption index defined by the CES function

$$C_{jt} = \left[\int_0^1 C_{jit}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \tag{1.10}$$

where the elasticity of substitution θ is bigger than 1. Here, M_{jt}/P_t represents the real money balances held at the end of period t. Real money balances enter the utility function because of the liquidity services that money provides. The last term in the utility function, L_{jt}, is the quantity of labour supplied by household j during period t. The CES preferences give rise to the standard demand functions for good i by household j

$$C_{jit} = \left[\frac{P_{it}}{P_t} \right]^{-\theta} \frac{E_{jt}}{P_t} \tag{1.11}$$

where P_t is the price index defined as

$$P_t = \left[\int_0^1 P_{it}^{1-\theta} di \right]^{\frac{1}{1-\theta}} \tag{1.12}$$

and E_{jt} is total goods expenditure of household j. As a consequence, the maximised sub-utility C_{jt} is equal to E_{jt}/P_t.

[7] Hence the dynamic model will be linear and certainty equivalence holds.

Firms are price-takers in both the goods and labour markets and they all have the following production function $Y_{it} = \alpha L_{it}^{\sigma}$, $0 < \sigma \leq 1$. The nominal wage is fixed by the monopolistic household-unions, before firms choose employment. Thus, firms maximise profits given the nominal wage W_{it}. The demands for labour and the output of firm i are

$$L_{it} = \left[\frac{1}{\alpha\sigma}\frac{W_{it}}{P_{it}}\right]^{\frac{1}{\sigma-1}}; \qquad Y_{it} = \alpha\left[\frac{1}{\alpha\sigma}\frac{W_{it}}{P_{it}}\right]^{\frac{\sigma}{\sigma-1}} . \tag{1.13}$$

When choosing the wage, the union realises that its behaviour influences the price of the output i, and therefore the demand for labour. Given the total demand for industry i's output (i.e., $C_{it} \equiv \int_0^1 C_{jit}dj$), simply by imposing the equilibrium condition on goods market, $C_{it} = Y_{it}$, the following relation between the labour demand and the nominal wage is found

$$L_{it} = K_t W_{it}^{-\varepsilon} \tag{1.14}$$

where

$$\varepsilon \equiv \frac{\theta}{\sigma + (1-\sigma)\theta}; \qquad K_t \equiv (\sigma\alpha)^{\varepsilon}\left[\frac{E_t}{\alpha P_t^{1-\theta}}\right]^{\frac{\varepsilon}{\theta}}; \qquad E_t \equiv \int_0^1 E_{jt}dj . \tag{1.15}$$

This is the demand function faced by the monopoly union in industry i and it exhibits a constant money-wage elasticity, ε, which is a function of the underlying parameters of preferences and technology. Moreover, since industry i has measure zero in the economy as a whole, aggregate expenditure, E_t, and the price index, P_t, are considered as given by the union. Thus, the term K_t is parametric to the union.

In period t, unions in sector A (if t is even), set their wage for the next two periods. Although they act independently of each other, the complete symmetry within each sector implies that all sector-A unions will set the same wage, W_{At}, and likewise in sector B. As before, let us denote the 'new

wage' for periods t and $t+1$, X_t, so that $W_{At} = W_{At+1} = X_t$. Meanwhile sector B unions are locked into the wage they set one period before, so $W_{Bt} = W_{Bt-1} = X_{t-1}$. Therefore $X_t, X_{t+2}, X_{t+4}, \ldots$, are the wages fixed by sector A, and $X_{t+1}, X_{t+3}, X_{t+5}, \ldots$, are the wages fixed by sector B. If P_{At} is the common price charged in all sector-A industries, and likewise P_{Bt} in sector B, then the supplied output levels of a typical industry in each of the two sectors in period t are

$$Y^s_{At} = \alpha \left[\frac{1}{\alpha\sigma}\frac{X_t}{P_{At}}\right]^{\frac{\sigma}{\sigma-1}}; \qquad Y^s_{Bt} = \alpha \left[\frac{1}{\alpha\sigma}\frac{X_{t-1}}{P_{Bt}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{1.16}$$

The demands for the outputs of a typical industry in each of the two sectors in period t are

$$Y^d_{At} = \left[\frac{P_{At}}{P_t}\right]^{-\theta}\frac{E_t}{P_t}; \qquad Y^d_{Bt} = \left[\frac{P_{Bt}}{P_t}\right]^{-\theta}\frac{E_t}{P_t}; \tag{1.17}$$

where $P_t = \left[\frac{1}{2}P_{At}^{1-\theta} + \frac{1}{2}P_{Bt}^{1-\theta}\right]^{\frac{1}{1-\theta}}$.

In equilibrium, aggregate nominal output is equal to aggregate nominal expenditure on consumption

$$\frac{1}{2}P_{At}Y_{At} + \frac{1}{2}P_{Bt}Y_{Bt} = E_t = P_tC_t = \frac{1}{2}P_tC_{At} + \frac{1}{2}P_tC_{Bt} = \frac{1}{2}E_{At} + \frac{1}{2}E_{Bt} \tag{1.18}$$

where (for example) $E_{At} \equiv P_tC_{At}$ is expenditure by a typical sector-A household. Note that $P_{At}Y_{At} \neq P_{At}C_{At}$, in general. This is for two reasons: households which work in sector A receive profits also from sector B; and households in sector A can borrow from or lend to households in sector B.

The intertemporal behaviour of the household-union

Every period the household-union j chooses the level of consumption and the quantities of money and bonds it will transfer to next period; while every alternate period it chooses the level of the money wage. Each household

enters period t with a predetermined level of wealth, given by money M_{jt-1} and by the gross interest on bonds $I_{t-1}B_{jt-1}$, where $I_t = (P_{t+1}/P_t)R_t$ and R_t is the gross real interest rate. During period t it receives a common lump-sum transfer T_t, wage income $W_{jt}L_{jt}$, and an equal share in every firm's profits, totalling Π_t. In certain periods it may also receive (or make, if negative) an insurance payment, H_{jt} (this is explained below). Its budget constraint is therefore given by

$$P_tC_{jt} + M_{jt} + B_{jt} = M_{jt-1} + I_{t-1}B_{jt-1} + W_{jt}L_{jt} + \Pi_t + T_t + H_{jt} \ . \quad (1.19)$$

Since the nominal wage is fixed for two periods, at the beginning of period t the household-union decides the nominal wage, X_{jt}, to be charged in t and in $t+1$. After two periods the problem faced is again the same. The household hence maximises the utility function with respect to consumption, real balances and labour subject to the sequence of the following constraints: the budget constraints (1.19), the labour demands (1.14), and the additional constraint that nominal wages have to be fixed for two successive periods: $X_{jt} = W_{jt} = W_{jt+1}$, for $t = 0, 2, 4...$. Deriving the first-order conditions and rearranging, we obtain

$$u_C(j,t) = R_t\beta u_C(j,t+1) \quad (1.20)$$

$$u_{M/P}(j,t) = (1 - 1/I_t)u_C(j,t) \quad (1.21)$$

$$X_{jt} = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left[-\frac{u_L(j,t) + \beta u_L(j,t+1)\frac{K_{t+1}}{K_t}}{u_C(j,t)\frac{1}{P_t} + \beta u_C(j,t+1)\frac{K_{t+1}}{K_tP_{t+1}}}\right] \quad (1.22)$$

where subscripts indicate partial derivatives and (j,t) is shorthand for the

arguments of household j's utility in period t. (1.20) is a standard consumption Euler equation for optimal intertemporal consumption choice. (1.21) equates the marginal utility of real balances to the marginal consumption opportunity cost of holding money.

Equation (1.22) characterises this model and deserves a few comments. First, (1.22) provides us with the optimal wage charged by the monopoly household-union. This is given by a fixed mark-up $\varepsilon/(\varepsilon-1)$ over the quantity in the square brackets. The latter is a ratio between weighted averages of the marginal disutility of labour and the marginal utility of consumption over the two periods, that is, a kind of average over the optimal flexible wages of the two periods. These average values are weighted by the discount factor β and by the coefficient K. Second, note that, since X_{jt} is received by the household-union both in period t and $t+1$, in period t the wage is set looking at what will happen tomorrow, while in period $t+1$ the wage, already set, depends on what happened yesterday. This is the source of the inertia in nominal variables induced by staggering. Third, Gray's (1976) type of nominally rigid labour contract is set so as to clear the labour market 'in expectation' and therefore the assumption that employment is always on the labour demand curve is inconsistent with optimisation. This is not true in our model. Here, since the wage is above the competitive one, ex post it is optimal for the household-union to satisfy an unexpected increase in

labour demand.[8] Alternatively, in Fischer's (1977) or Taylor's (1979) types of contract, the wage is set in order to achieve an ad hoc target wage level, while, here, explicit optimising foundations for wage setting are provided.

To find an explicit solution of the model, in the following chapters we need to assume an explicit form of the utility function. Before turning to investigate the properties of the deterministic steady state of this model in the next chapter, a few comments on the insurance scheme assumed are needed. We have presented the model assuming no uncertainty. However, this could seem at odds with the intention to study the response of the model to unexpected disinflationary policies (i.e., Chapter 3) and/or money shocks (i.e., Chapter 4). Nevertheless, since the model will be log-linearised around the deterministic steady state, the expectation operator could be straightforwardly incorporated in the model, without changing the results presented in Chapters 3 and 4. The reason why we assume an insurance scheme between households is that we want to abstract from distributional complications.[9] To solve the model we need to solve an aggregation problem. By definition of the output and consumption indices, $E_t = P_t C_{At}/2 + P_t C_{Bt}/2 = P_{At} Y_{At}/2 + P_{Bt} Y_{Bt}/2$. What is missing is a relation between the aggregate nominal expenditure E_t and the nominal expenditure by a typical household of each sector, $E_{At} \equiv P_t C_{At}$, $E_{Bt} \equiv P_t C_{Bt}$.

[8] Obviously this is true until the real wage is equal to the competitive one. In what follows we suppose that this is never the case. That is, no shock occurs which is so big that the real wage equals the competitive one, hence households never wish to ration an unexpected increase in labour demand.

[9] In a fully stochastic model, the assumption of complete markets plays the role of the insurance scheme assumed here.

We know that expenditure by a household depends on its lifetime wealth. In general this wealth could differ between the sectors, depending on the past history of the economy. Moreover, since households are infinitely-lived and have identical preferences, the long-run distribution of wealth between sectors is not tied down by the steady-state conditions. Any change in the distribution, following a shock (both permanent, i.e., disinflation, or temporary, i.e., money shock), will persist indefinitely: that is, an individual household or sector's wealth has a "unit root" property. One consequence of this is that, in the absence of any redistribution mechanism, if households start out with equal wealth and then an unanticipated shock occurs, a permanent asymmetry between the sectors will be introduced. This is because, in the period in which it occurs, the shock hits the sectors in a different way: some can negotiate the wage contract, some others can not. Households in one sector therefore will suffer a disproportionate loss of income, which will spill over into their lifetime wealth. From then on, the economy will evolve in a lopsided way, because the permanent wealth difference will be reflected in a permanent labour supply, and therefore output, difference between the sectors. However interesting these distributional complications are, we shall abstract from them in this thesis, both because they are a digression from the main argument and because it is very difficult to deal with them. To do this we assume that there exists an insurance scheme between the two sectors. Suppose that a shock is a random event, with a known, but very small probability of occurring. Then it will be rational for risk-averse households to take out insurance (through a competitive insurance industry, which in equilibrium will make zero profits). In other words, households will pool their resources to shield themselves against the possibility of a shock. Given that

all the households are identical (same preferences and same initial wealth), it is known that the optimal insurance scheme is such that in every period the marginal utility of consumption will be the same for each household. Hence,

$$u_C(j,t) = u_C(i,t) \qquad \forall i,j \in [0.1] \text{ and } \forall t \tag{1.23}$$

that is

$$u_C(A,t) = u_C(B,t) \qquad \forall t \tag{1.24}$$

This additional relation (1.24) will then allow us to close the model.[10]

We turn now to the description of the deterministic steady state of this model.

[10] I thank Giuseppe Bertola for suggesting this argument.

Chapter 2

Superneutrality of Money in Staggered Wage Setting Models

2.1 Introduction

Staggered wage/price setting models have been used widely in the literature to investigate the dynamic response of the economy following a monetary policy shock and/or to assess the impact of disinflationary policies (see Ball (1994) and references therein). However, quite surprisingly, very little has been said about the influence of the rate of growth of money on long-run output and welfare. In this chapter, we address the issue of *superneutrality* of money in the steady state using the dynamic general equilibrium model with staggered wage setting presented in the previous chapter.

Previous log-linear staggered wage/price models already acknowledged that money could be non-superneutral. However, this issue was believed to

be a minor one. The reason is that in naive log-linear staggering models, money growth rates can affect steady state output only if the intertemporal rate of discount is different from zero. Consider, for example, the model of Calvo (1983a,b). The key equation is the wage/price setting rule:[1]

$$x(t) = (\rho + r) \int_t^{\infty} (p(s) + \gamma y(s)) e^{-(\rho+r)(s-t)} ds \tag{2.1}$$

where $\beta > 0$, $\rho > 0$ and $x(t)$ = wage/price set on contracts renewed; $p(t)$ = average price level; $y(t)$ = level of output; ρ = parameter governing the Poisson process of wage/price changes; r = intertemporal rate of discount. Under Calvo's (1983a,b) pricing structure, prices are reset at random times which arrive with probability ρ. (2.1) gives the reset price as a weighted average of future price and demand levels. We have introduced a positive pure time preference rate, i.e., r, into this equation, by adding it to ρ to obtain the discount factor of the future price-and-demand index. Calvo (1983a,b), sets r to zero. According to Calvo (1983a, p. 238, footnote 5) the reason for this is: *"With a non zero real interest rate,* [(2.1)] *could also naturally incorporate a factor to reflect it. This addition would, however, make steady states (in the ensuing analyses) sensitive to permanent changes in the rate of devaluation or expansion of money supply. Without denying the existence, and maybe importance, of such effects, we will stick to form* [(2.1)] *for the sake of simplicity."* If $r = 0$, then, steady state output would not depend on the rate of growth of money supply. However, in its standard text-book log-linear presentation (e.g., Blanchard and Fischer (1989)), Calvo's (1983a,b) model has been used to study the effects of a disinflation, but we are left with

[1] The full Calvo's (1983a,b) model will be presented and analysed in the next chapter. Here, the wage/price setting equation is sufficient for the argument.

the question of why the policy-maker wants to disinflate given that money is superneutral and that there are short-run costs of disinflating.

The same question becomes even more puzzling if $r \neq 0$ (as usually supposed in economic models). Even if the possibility of incorporating such a parameter has been acknowledged (in footnotes) by Calvo, most commonly it goes unmentioned[2], and even those authors who do mention it nevertheless proceed to ignore it, arguing that it is close to zero in practice. We shall show that, even though r may be small, it still plays an important role. In the case $r \neq 0$, in fact, steady state output is an ever increasing function of the rate of growth of money. Romer (1990, p. 208, footnote 5) provides a clear intuitive explanation for this result "[...] *if the real interest rate is positive, higher trend inflation increases mean output (given* $[\rho]$*). The source of this effect is that trend inflation causes the expected profit-maximising price to be rising over time and that a positive real interest rate causes firms to put relatively greater weight on current rather than future optimal prices; thus they charge less than the weighted average expected profit-maximising price.*"[3] A positive r, then, by decreasing x, lowers the average level of prices and hence raises the level of output. However, it is evident that a level of output ever increasing in the rate of money growth is not a very desirable feature of a model built with

[2] As, for example, in Blanchard and Fischer (1989) and Ball (1994).

[3] Note that the same is true for a Taylor-type of model, with variables expressed in levels. On one hand, the equivalent to the hypothesis $r = 0$ in Calvo's model is $b = d = 1/2$ for Taylor's model. As stated in the previous chapter, in this case money would be superneutral. On the other hand, the equivalent to $r > 0$ in Calvo's model, is $b > d$ for a Taylor-type of model in levels. Then the steady state level of output would be ever-increasing in the steady state rate of growth of money in such a model.

the purpose of studying disinflation. Given that, and also because the effect of the intertemporal rate of discount is supposed empirically to be rather small, the original simplifying, but unjustified, assumption $r = 0$ has become an established and natural one in log-linear staggered wage/price models.

In contrast, in dynamic general equilibrium monetary models, there is always a strong reason to disinflate, since a positive rate of inflation forces agents to economise on real balances. Indeed, this is exactly what happens in the more complete version of Calvo's model, i.e., Calvo (1983b), in which the model is embedded in a utility-maximising framework. However, Calvo (1983b) assumes the existence of a costless "price-regulation mechanism" to ensure that each consumer pays the same whatever the firm at which she realises her purchases. Although this hypothesis permits a very neat and elegant analysis of the short-run dynamics, it deletes all of the interesting effects due to the interrelation of different firms charging different prices. However, as we will show in this chapter, these interrelations are precisely the source of strong non-superneutrality effects. Once the "price-regulation mechanism" hypothesis is made, from the point of view of the effects of the rate of growth of money on the steady state, the model is exactly equivalent to a flexible price one. Indeed, steady state output is independent of the rate of growth of money and the Friedman rule turns out to be optimal, since it satiates the demand for real money balances.[4] In Calvo (1983b), the welfare costs of a positive rate of inflation, therefore, are just the cost of the inflation tax on real money balances. An estimate of this cost is provided by

[4]It is probably worth recalling that in Calvo (1983b) leisure does not enter the utility function and the labour supply is exogenous.

Cooley and Hansen (1989). They build a general equilibrium cash-in-advance monetary model with flexible prices and estimate the cost of the inflation tax of a 10% rate of inflation to be around 0.4% of the GNP.

In a recent paper, Ireland (1995) analyses disinflationary monetary policies in a general equilibrium monetary model with staggered prices. His model exhibits staggered price setting decisions and explicitly takes into account that different firms charge different prices. It turns out that the optimal monetary policy requires a trade-off between the zero inflation tax and productive efficiency. In fact, on one hand, the Friedman rule requires a negative rate of growth of the money supply, whereas, on the other hand, productive efficiency requires holding the money supply fixed. The optimal monetary policy would balance these two effects, hence determining a negative rate of inflation, but bigger than the one necessary for zero inflation tax.[5] However the model of Ireland (1995) implies only very mild effects of the rate of growth of money on steady state output and welfare, the order of magnitude being similar to the one of Cooley and Hansen (1989). As shown below, the explanation of this result lies in the simplified structure of Ireland's (1995) model in which the production function is linear in labour, as is the utility function, and moreover the elasticity of substitution between consumption

[5]Note that in a model with these features, the reason why the policy maker wants to disinflate is immediately evident. However, the long-run gains have to be compared with the short-run costs of disinflating, due to the staggered price setting structure. Therefore, Ireland (1995) analyses the very interesting issue of the optimal disinflationary path, that is, how monetary policy can disinflate optimally, balancing the short-run costs and the long-run gains.

goods equals one. In conclusion, neither Calvo's (1983b) model, with the "price-regulation mechanism" hypothesis, nor Ireland's (1995) model is able to catch the importance of non-superneutrality in a staggered wage/price model.

We study the properties of the steady state of the model presented in the previous chapter with respect to changes in the rate of growth of money. The model is similar to that of Ireland (1995), but has three distinct features. First, it enables us to explicitly demonstrate the role of the intertemporal discount factor in the wage setting process and to show the same effect explained above by Romer (1990). Second, the model is more articulated in that it allows for more general utility and production functions. In particular, it introduces the usual non-linearities in technology and preferences: decreasing returns to labour, increasing marginal disutility of labour and an elasticity of substitution among goods bigger than one.[6] These non-linearities prove to be the channel through which the rate of growth of money strongly affects the steady state output and welfare. Third, the model exhibits staggered wages instead of prices, as advocated by Ireland (1995).[7] Hence, it provides a rationale for the price stability-oriented policies observed in western countries. We do observe staggered wage setting behaviour, we do observe a

[6] When we use the expression "usual non-linearities in utility and production functions", we mean precisely the three points just mentioned.

[7] "Cho and Cooley (1992) *find that nominal wage setting models do a better job than nominal price setting models in matching a variety of correlations that appear in the data. While their work indicates that it would be useful to consider the nature of optimal monetary policy in economies with wage rigidities as well as price rigidities, this is left as a task for future research.*" (Ireland (1995), p. 1432)

relatively low inflation rate in western countries, and, notwithstanding high unemployment, the latter are still seeking price stability. We demonstrate that, in the presence of staggered wage setting, an inflation rate of the magnitude recently registered in developed economies (5%) can cause very high costs both in output and in welfare, giving strong support to the pursuit of price stability. The same argument would be much more difficult to sustain in a model with nominal rigidities in the form of fixed staggered price, since this feature, unlike fixed-staggered wages, is not observed in modern western economies.

The chapter is organised as follows. In section 2, we describe the steady state of the model showing why money is not superneutral in the steady state. In section 3, some simple numerical examples suggest that varying the steady state money growth rate could have a strong impact on steady state output and welfare. Therefore, in the presence of staggered adjustment, superneutrality of money turns out to be a very important, rather than a minor, issue.

2.2 Steady State Analysis

To find an explicit solution of the model the following particular form of the instantaneous utility function is assumed:[8]

$$u_{jt}(C_{jt}, M_{jt}/P_t, L_{jt}) = \delta \ln C_{jt} + (1-\delta) \ln M_{jt}/P_t - \chi L^e_{jt} \qquad (2.2)$$

[8]Rankin (1998) and many others in the literature (see references therein) use a similar utility function.

where $e > 1$. This is a partially logarithmic utility function that exhibits increasing marginal disutility of labour. The first order conditions of the model now are given by

$$C_{jt+1} = \beta R_t C_{jt} \tag{2.3}$$

$$C_{jt} = \frac{\delta}{1-\delta}\left(1 - \frac{1}{I_t}\right)\frac{M_{jt}}{P_t} \tag{2.4}$$

$$X_{jt} = \left[\left(\frac{\varepsilon}{\varepsilon - 1}\right)\frac{\chi e}{\delta}\frac{K_t^e + \beta K_{t+1}^e}{\frac{K_t}{P_t C_{jt}} + \beta\frac{K_{t+1}}{P_{t+1}C_{jt+1}}}\right]^{\frac{1}{1+\varepsilon(e-1)}} . \tag{2.5}$$

First note that, as said previously, the optimal nominal wage is a ratio between average values of the disutility from labour and of the utility from consumption over the two periods, that is, a sort of average between the optimal flexible wages of the two periods. Note further that this average values are weighted by the discount factor and by the coefficient K and that, since $\beta < 1$, the variables of the second period are discounted. Hence, they are given a lower weight in calculating the average value. This is exactly the source of the Calvo-Romer effect, described in the introduction.

Secondly, since the marginal utility of consumption will be the same across households in each period and given the separable specification of the utility function, households will consume the same in each period. That is, the consumption of sector A is the same as the consumption of sector B in every period. Thus, imposing $C_{At} = C_{Bt} = C_t = E_{At}/P_t = E_{Bt}/P_t = E_t/C_t$, it is straightforward to solve for the steady state.

Macroeconomic equilibrium

Let's firstly concentrate on the demand side of the economy. We can sum (2.3) and (2.4) across all households j, and then use $I_t = (P_{t+1}/P_t)R_t$ to

eliminate I_t and R_t. This yields the following relationship between aggregate money and consumption demands

$$\frac{M_t}{P_t C_t} = \frac{1-\delta}{\delta} + \beta \frac{M_t}{M_{t+1}} \frac{M_{t+1}}{P_{t+1} C_{t+1}} \qquad \text{or} \qquad Z_t = \frac{1-\delta}{\delta} + \beta \Phi_{t+1} Z_{t+1} \quad (2.6)$$

where $C_t \equiv \int_0^1 C_{jt} dj$, $M_t \equiv \int_0^1 M_{jt} dj$. The alternative version of the equation follows from imposing money market equilibrium, and letting $\Phi_{t+1} = M_t/M_{t+1}$ denote the rate of decrease of the money supply. This is a first-order linear difference equation in $Z_t \equiv M_t/P_t C_t = M_t/E_t$, the ratio between real balances and real consumption, that is, the inverse of the consumption-velocity of circulation of money, or the so-called 'Cambridge k'.

(2.6) is a key equation of the model and will be the focus of the next chapter. Since $\beta < 1$, and $\Phi < 1$ if there is positive monetary growth, it is clearly unstable in the forward dynamics. However Z_t is not a predetermined variable, since neither P_t nor C_t are predetermined. Therefore instability is, in this case, a welcome property because it allows us, by ruling out all divergent paths, to select a unique solution (this is the usual "saddlepath" argument). Here, this solution corresponds to the steady state. The steady state value of Z is[9]

$$Z = \frac{1-\delta}{\delta(1-\beta\Phi)} \, . \qquad (2.7)$$

Note that Z is a decreasing function of the steady state rate of growth of money (i.e. of $1/\Phi$). The higher the rate of money growth, the higher

[9]Note that we must have $\Phi < 1/\beta$ otherwise Z is negative. In fact if $\Phi = 1/\beta$ (which is Friedman's well-known optimum quantity of money rule), the gross nominal interest rate would be unity in the steady state. In this case individuals would be satiated with money which implies, given that our utility function does not have a satiation point, infinite real money balances and hence infinite Z.

the inflation tax on real balances and the lower is the ratio between real balances and consumption, since households choose to economise on their money holdings.

Aggregating (2.4) and substituting into (2.6) for Z_t and Z_{t+1}, yields the corresponding dynamic equation for I_t

$$I_t = \frac{I_{t+1}(1+\beta\Phi)-1}{\beta\Phi I_{t+1}} \ . \tag{2.8}$$

(2.8) exhibits two stationary values for I: 1 and $1/\beta\Phi$. The first one is locally stable and the second one locally unstable. Since we obviously require $I_t > 1$, again a unique path can be selected by ruling out divergent paths. Restricting Φ to be less than 1, then the unique non-divergent path satisfying $I_t > 1$ for all t is given by

$$I_t = 1/\beta\Phi \qquad \text{for all } t \ . \tag{2.9}$$

An increase in the rate of growth of the money supply, lowering Φ, clearly increases the steady state value of the nominal interest rate.

An important thing to note with regard to (2.6) and (2.8) is that these dynamic equations are totally independent of the supply side of the economy. In other words, they hold independently of whether wages are flexible, fixed, or predetermined but time-varying, and of their synchronisation. (2.6) and (2.8) derive just from the demand side of the economy and so the dynamics of Z_t are independent of the supply side. This point will be very important in the analysis of the disinflation dynamics of the next chapter.

We can now look at the supply side of the model to solve for X. We are now able to express all the variables of the model, i.e. sectors' outputs and prices and aggregate price and output, as functions of M_t, Z_t, X_{t-1} and X_t.

Given the definition of Z_t, substituting $E_t = M_t/Z_t$ into (1.17), we get

$$Y_{At}^d = \left[\frac{P_{At}}{P_t}\right]^{-\theta} \frac{M_t}{Z_t P_t}; \qquad Y_{Bt}^d = \left[\frac{P_{Bt}}{P_t}\right]^{-\theta} \frac{M_t}{Z_t P_t}. \tag{2.10}$$

Imposing the equilibrium conditions on the goods market in the two industries (i.e., $Y_{At}^d = Y_{At}^s$ and $Y_{Bt}^d = Y_{Bt}^s$) and using

$$P_t = \left[\frac{1}{2}P_{At}^{1-\theta} + \frac{1}{2}P_{Bt}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{2.11}$$

allow us to express prices and outputs as functions of M_t, Z_t, X_{t-1} and X_t, as follows (when t is an even number)

$$Y_{At} = \left(\frac{1}{2}\right)^{1-\sigma} \alpha \left[\frac{\sigma M_t/Z_t X_t}{1 + (X_{t-1}/X_t)^{1-\varepsilon}}\right]^{\sigma} \tag{2.12}$$

$$Y_{Bt} = \left(\frac{1}{2}\right)^{1-\sigma} \alpha \left[\frac{\sigma M_t/Z_t X_{t-1}}{(X_t/X_{t-1})^{1-\varepsilon} + 1}\right]^{\sigma} \tag{2.13}$$

$$P_{At} = \frac{1}{\alpha}\left(\frac{X_t}{\sigma}\right)^{\sigma} \left[\frac{2M_t/Z_t}{1 + (X_{t-1}/X_t)^{1-\varepsilon}}\right]^{1-\sigma} \tag{2.14}$$

$$P_{Bt} = \frac{1}{\alpha}\left(\frac{X_{t-1}}{\sigma}\right)^{\sigma} \left[\frac{2M_t/Z_t}{(X_t/X_{t-1})^{1-\varepsilon} + 1}\right]^{1-\sigma} \tag{2.15}$$

$$P_t = \frac{1}{\alpha}\left(\frac{M_t}{Z_t}\right)^{1-\sigma} \left(\frac{X_t}{\sigma}\right)^{\sigma} \left[\frac{1}{2} + \frac{1}{2}\left(\frac{X_{t-1}}{X_t}\right)^{1-\varepsilon}\right]^{\sigma/(1-\varepsilon)} \tag{2.16}$$

Given the definition of the aggregate price level, aggregate output is defined, as in normal national income accounting, as

$$Y_t \equiv \frac{\frac{1}{2}P_{At}Y_{At} + \frac{1}{2}P_{Bt}Y_{Bt}}{P_t} = \frac{E_t}{P_t} = C_t \tag{2.17}$$

Then

$$Y_t = \frac{M_t}{P_t Z_t} \tag{2.18}$$

In period $t+1$ the above expressions for sectors' prices and outputs are slightly different, since $t+1$ is an odd number, whence the expressions (1.16) for supplies must be modified. In fact $Y_{At+1}, Y_{Bt+1}, P_{At+1}, P_{Bt+1}$ can be obtained by substituting X_{t-1} by X_{t+1}, M_t by M_{t+1} and Z_t by Z_{t+1}.

Moreover, it is immediate to find

$$\frac{P_{At}}{P_{Bt}} = \left[\frac{X_t}{X_{t-1}}\right]^{\frac{\sigma}{\sigma+\theta(1-\sigma)}}; \qquad \frac{P_{At+1}}{P_{Bt+1}} = \left[\frac{X_t}{X_{t+1}}\right]^{\frac{\sigma}{\sigma+\theta(1-\sigma)}}; \tag{2.19}$$

$$\frac{Y_{At}}{Y_{Bt}} = \left[\frac{X_{t-1}}{X_t}\right]^{\sigma\varepsilon}; \qquad \frac{Y_{At+1}}{Y_{Bt+1}} = \left[\frac{X_{t+1}}{X_t}\right]^{\sigma\varepsilon}; \tag{2.20}$$

$$\frac{Y_{At}}{Y_{At+1}} = \left[\frac{M_t}{M_{t+1}} \frac{1+\left(\frac{X_{t+1}}{X_t}\right)^{1-\varepsilon}}{1+\left(\frac{X_{t-1}}{X_t}\right)^{1-\varepsilon}}\right]^{\sigma} \tag{2.21}$$

These equations show how relative sectors' behaviour is fully characterised by the difference between the money wages in force. Trivially, if the two sectors exhibit the same money wage they will exhibit also the same price and hence the same level of output. Whenever the wages prevailing in the sectors are different, instead the ratio between the wages determines the ratio of sectors' prices and output. It is hence very easy to grasp what would happen in a steady state when the money supply is growing at a given rate:[10] X will be growing at the same rate over time and thus the sectoral output will undergo a two-period cycle, as we will see below.

To solve for X, we can aggregate (2.5) as

$$X_t = \left[\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi e}{\delta}\frac{K_t^e + \beta K_{t+1}^e}{\frac{K_t}{P_t C_{At}} + \beta\frac{K_{t+1}}{P_{t+1}C_{At+1}}}\right]^{\frac{1}{1+\varepsilon(e-1)}} \qquad for\ t = 0,\ 2,\ 4, ...; \tag{2.22}$$

[10]Note however, that the equations above also hold outside the steady state.

while for $t = 1, 3, 5, ...$, the expression is the same but with C_B replacing C_A. Recalling the insurance argument, we may now impose $C_{At} = C_{Bt} = C_t = Y_t$ and substitute for K_t, P_t and Y_t as respectively given by (1.15), (2.16) and (2.18), to obtain

$$X_t = \left(\frac{\varepsilon}{\varepsilon - 1}\frac{\chi e}{2\delta\sigma}\right)^{\frac{1}{e}} \left(2\sigma\frac{M_t}{Z_t}\right) \left[1 + \left(\frac{X_{t-1}}{X_t}\right)^{1-\varepsilon}\right]^{\frac{1-e}{e}}$$

$$\times \left\{ \frac{1 + \beta\left(\frac{Z_t M_{t+1}}{Z_{t+1} M_t}\right)^e \left[\frac{1+\left(\frac{X_{t-1}}{X_t}\right)^{1-\varepsilon}}{1+\left(\frac{X_{t+1}}{X_t}\right)^{1-\varepsilon}}\right]^e}{1 + \beta\left[\frac{1+\left(\frac{X_{t-1}}{X_t}\right)^{1-\varepsilon}}{1+\left(\frac{X_{t+1}}{X_t}\right)^{1-\varepsilon}}\right]} \right\}^{\frac{1}{e}} \tag{2.23}$$

This highly non-linear equation gives the dynamics for X_t as a function of the exogenous money supply path (recall that the dynamics of Z_t depends only on the exogenous money supply path). We shall now concentrate instead on the steady state of the model, leaving the dynamics and the investigation of (2.23) to the next chapters.

In a steady state, the money supply is growing at the constant rate of $((1/\Phi) - 1)$, where Φ is defined as (M_t/M_{t+1}). Unsurprisingly, the money wage X_t and the aggregate price level P_t are found to grow through time at the same rate as the money supply. The steady state is characterised by the following expressions[11]

$$X_t = \left(\frac{\varepsilon}{\varepsilon - 1}\frac{\chi e}{2\delta\sigma}\right)^{\frac{1}{e}} \left(2\sigma\frac{\delta}{1-\delta}(1 - \beta\Phi)M_t\right) \left[1 + \Phi^{1-\varepsilon}\right]^{\frac{1-e}{e}} \left\{\frac{1 + \beta\Phi^{-e\varepsilon}}{1 + \beta\Phi^{1-\varepsilon}}\right\}^{\frac{1}{e}} \tag{2.24}$$

[11] $Y_n = \alpha\left[\frac{\varepsilon\chi e}{(\varepsilon-1)\delta\sigma}\right]^{\frac{-\sigma}{e}}$ is the steady state output in the flexible wage/price version of this model.

Period t	Period $t+1$	Period $t+2$	Period $t+3$
Y_{At}	$Y_{At+1} = Y_{At}\Phi^{-\sigma\varepsilon}$ $Y_{At+1} = Y_{Bt} > Y_{At}$	$Y_{At+2} = Y_{At}$	$Y_{At+3} = Y_{At+1}$
Y_{Bt}	$Y_{Bt+1} = Y_{Bt}\Phi^{\sigma\varepsilon}$ $Y_{Bt+1} = Y_{At} < Y_{Bt}$	$Y_{Bt+2} = Y_{Bt}$	$Y_{Bt+3} = Y_{Bt+1}$
P_{At}	$P_{At+1} = \Phi^{\varepsilon(\sigma-1)}P_{At}$ $P_{At+1} > P_{At}$	$P_{At+2} = \Phi^{\varepsilon(\sigma-1)-2\varepsilon\sigma/\theta}P_{At+1}$	$P_{At+3} = \Phi^{\varepsilon(\sigma-1)}P_{At+2}$
P_{Bt}	$P_{Bt+1} = \Phi^{\varepsilon(\sigma-1)-2\varepsilon\sigma/\theta}P_{Bt}$	$P_{Bt+2} = \Phi^{\varepsilon(\sigma-1)}P_{Bt+1}$	$P_{Bt+3} = \Phi^{\varepsilon(\sigma-1)-2\varepsilon\sigma/\theta}P_{Bt+2}$

Table 2.1: Steady State Sectors' Output and Price Behaviour

$$Y_{At} = Y_n\left(\frac{1}{2}\right)^{1-\frac{\sigma}{e}}\left[1+\Phi^{1-\varepsilon}\right]^{-\frac{\sigma}{e}}\left\{\frac{1+\beta\Phi^{-e\varepsilon}}{1+\beta\Phi^{1-\varepsilon}}\right\}^{-\frac{\sigma}{e}} \tag{2.25}$$

$$Y_{Bt} = Y_{At}\Phi^{-\sigma\varepsilon} \tag{2.26}$$

$$P_{At} = Y_n^{-1}2^{1-\frac{\sigma}{e}}\left(\frac{\delta}{1-\delta}(1-\beta\Phi)M_t\right)\left[1+\Phi^{1-\varepsilon}\right]^{\frac{\sigma}{e}-1}\left\{\frac{1+\beta\Phi^{-e\varepsilon}}{1+\beta\Phi^{1-\varepsilon}}\right\}^{\frac{\sigma}{e}} \tag{2.27}$$

$$P_{Bt} = P_{At}\Phi^{1-\varepsilon(1-\sigma)} \tag{2.28}$$

$$P_t = Y_n^{-1}2^{-\frac{\theta}{1-\theta}-\frac{\sigma}{e}}\left(\frac{\delta}{1-\delta}(1-\beta\Phi)M_t\right)\left[1+\Phi^{1-\varepsilon}\right]^{\frac{\theta}{1-\theta}+\frac{\sigma}{e}}\left\{\frac{1+\beta\Phi^{-e\varepsilon}}{1+\beta\Phi^{1-\varepsilon}}\right\}^{\frac{\sigma}{e}} \tag{2.29}$$

Since X_t is growing over time, the sector that sets X_t will exhibit a higher money wage than the sector that has already set the money wage, X_{t-1}, in period $t-1$. In every period, therefore, there will be two different money wages in the economy, and so, two different prices. The symmetry between sectors' behaviour is evident from Table 2.1.

If money grows at a positive rate, then $\Phi < 1$. The steady state behaviour of each sector is characterised by a slump in the period the wage is set,

followed by a boom in the next period, when the other sector fixes the wage. More precisely, $Y_{At} = Y_{Bt-1} = Y_{slump}$ and $Y_{At+1} = Y_{Bt} = Y_{boom}$. Therefore, each sector exhibits the boom in the period in which the other sector fixes its money wage.

The ratio between the slump and the boom in each sector is given by $Y_{At}/Y_{At+1} = Y_{Bt-1}/Y_{Bt} = \Phi^{\sigma\varepsilon}$. Intuitively, two effects come into play. First, since the money wage is fixed for two successive periods, it is a kind of average of the optimal one. As a result, it is higher than the optimal one in the first period and lower in the second period. This is a supply-side effect: the wage is set higher than the optimal in the slump period. The same is also true for prices determining a second demand-side effect which derives from the interrelation between relative prices and demand. Because there are two sectors charging different prices, the composition of demand would change as the ratio of the prices in the two sectors changes. Thus, the amplitude of the cycle depends not only on σ, but also on θ. In fact, ε is an increasing function of θ: the bigger the elasticity of substitution, the larger the size of the cycle. In the period in which sector A fixes the money wage, sector B is still locked in the one fixed a period before. As a consequence, sector A exhibits not only a higher money wage, but also a higher real wage (and price) than sector B. This has two negative effects on the output of sector A. The first one is for supply reasons since the real wage is higher than the

optimal one. The second one acts through demand, given that the price of sector B is lower and the goods are imperfect substitutes.[12] Note that these effects, because of the assumption of the "price-regulation mechanism", are ignored in Calvo's (1983b) model. Moreover, they are partially present in Ireland's (1995) model, but their potential importance is simply choked off at the very outset, by putting a priori $\sigma = e = \theta = 1$ and thereby cancelling the "non-linearities".

Sector prices are growing over time, but the rate of growth is different, depending on whether the sector money wage has changed or not. Besides, the aggregate price index P_t is growing at the same rate as the money supply.

Households are paid a higher wage, but they work less in a slump than in a boom. They receive less income in a slump than in a boom. More precisely: $L_{At}X_t/L_{Bt}X_{t-1} = \Phi^{\varepsilon-1} < 1$; therefore, the difference between income levels also is an increasing function of θ. However, with regard to welfare, since households of both sectors enjoy the same level of consumption and real

[12]Imagine this economy without staggering, that is, household-unions fix the money wage for two periods, but they all reset it in the same period. Then, its behaviour would be characterised by a rather artificial oscillation in aggregate output between boom and slump. In fact, given the complete symmetry of the economy, only this first supply-side effect would be present, and the ratio of output between the boom and the slump would be equal to Φ^{σ}. In the staggered model, the second effect enlarges the amplitude of the cycle.

money balances, it follows immediately that households are better off in a slump simply because they work less ($L_{At}/L_{Bt} = \Phi^{\varepsilon}$).

What about real aggregate output? Given the definition of real aggregate output in (2.17), we have

$$Y_{ss} = Y_n 2^{\frac{\sigma}{e}+\frac{\theta}{1-\theta}} \left[1+\Phi^{1-\varepsilon}\right]^{\frac{\theta}{\theta-1}-\frac{\sigma}{e}} \left\{\frac{1+\beta\Phi^{-e\varepsilon}}{1+\beta\Phi^{1-\varepsilon}}\right\}^{-\frac{\sigma}{e}} \tag{2.30}$$

Real aggregate output, therefore, is constant over time, but it depends on Φ, that is, money is not superneutral in steady state. If $\Phi = 1$, then $Y_{ss} = Y_n$. In the case of a constant money supply, in fact, staggering has no effect in steady state, because nominal wages and all of the nominal variables are constant over time. Therefore, there would be complete symmetry between the two sectors that would produce the same level of output and charge the same price. In a steady state, the economy would behave as a flexible wage one.

The derivative of Y_{ss} with respect to Φ is equal to

$$\frac{\partial Y_{ss}}{\partial \Phi} = Y_{ss}\left[\left(\frac{\theta}{\theta-1}-\frac{\sigma}{e}\right)\frac{1-\varepsilon}{\Phi^{\varepsilon}+\Phi} - \frac{\beta\sigma}{e}\left(\frac{1-\varepsilon}{\Phi^{\varepsilon}+\beta\Phi} - \frac{\varepsilon e}{\Phi(\Phi^{e\varepsilon}+\beta)}\right)\right] \tag{2.31}$$

and the square bracket can be written as

$$\frac{\sigma(\varepsilon-1)}{e}(\Phi^{e\varepsilon}+\beta)(1-\beta)\Phi^{\varepsilon-1} + \sigma\varepsilon(\Phi^{\varepsilon-1}+\beta)(\beta\Phi^{\varepsilon-1}-\Phi^{e\varepsilon}) \ . \tag{2.32}$$

Then, it follows that $\partial Y_{ss}/\partial\Phi$ is non-positive for $\Phi = 1$, and the condition $\beta > \Phi^{1+\varepsilon(e-1)}$ is sufficient for $\partial Y_{ss}/\partial\Phi$ to be non-negative. This implies

that Y_{ss} reaches a maximum at a point $\Phi^* \in (0,1)$ such that $\beta^{\frac{\sigma+(1-\sigma)\theta}{\sigma+\theta(e-\sigma)}} \leq \Phi^* \leq 1$. Hence, to maximise the level of steady state output, the system requires a positive level of inflation.[13] This is, however, only due to a positive intertemporal rate of discount. If the latter is equal to zero, i.e., $\beta = 1$, then Y_{ss} is maximised for $\Phi = 1$. The reason for this result is exactly the same as explained by Romer (1990).

Nevertheless, unlike Calvo's model, Y_{ss} is not ever increasing in the rate of growth of money and then it is obvious that there are other effects coming into play. The intuition is straightforward given the zigzag behaviour of sectors' output in steady state. Because of the non-linearities characterising the production and utility functions, households generally would prefer to produce the same amount of output in each period. A rate of inflation different from zero and the staggered adjustment structure, however, impose

[13]This is quite a strong result by itself. There is a widespread belief that a little bit of inflation is good for the economy, and if commentators look at GDP to judge what is good and what is bad for the economy, then the model would justify this belief. To our knowledge, only Dazinger (1988) gets a similar result in dynamic general equilibrium models comparable to ours, but looking at welfare and not at output. Other models (e.g. Cooley and Hansen (1989), Ireland (1995)) look at a cash-in-advance economy. In this case: *"When money is supplied according to a constant growth rate rule that implies positive nominal interest rates, individuals substitute leisure for goods, output and investment fall"* (Cooley and Hansen (1989) p. 735). Therefore, if prices are flexible, output level is maximised in correspondence with the Friedman rule.

a cycle on sectors' output. The bigger the cycle, on average, the lower is the output. These non-linearity effects add to the intertemporal rate of discount effect and actually dominate it, making Y_{ss} decline as the rate of inflation grows.

Suppose that $\theta = 0$ and $e = \sigma = 1$, then

$$\frac{Y_{ss}}{Y_n} = \frac{2(1+\beta\Phi)}{(1+\Phi)(1+\beta)} \tag{2.33}$$

and only the intertemporal rate of discount effect is present here.[14] As Figure 2.1 shows, Y_{ss} is ever increasing in the rate of growth of money (rgm in the graphs) and the lower β, the bigger is this effect. Intuitively, if $\theta = 0$, the goods are no longer gross substitutes and each sector just receives half of the real demand. The output of the sectors is just the same in every period. This case corresponds to Calvo's (1983a) model. If θ is close to 1, then, as Figure 2.2 shows, the effect of β is still present, but there is also a weak effect resulting from the substitution among different goods. The latter effect eventually would overtake the former one as the rate of money growth rises. This case corresponds to Ireland's (1995) model.

Figure 2.3 shows the behaviour of Y_{ss} for what can be considered plausible values of the parameters.[15] As easily can be seen, the non-linearity effects

[14]By hypothesis, θ should be bigger than one; otherwise, as is well known in this Dixit-Stiglitz (1977) kind of framework, the model is not well defined. However, this extreme case is considered as an aid to intuition.

[15]In this base case, the value of θ is set to 6 as in Hairault and Portier (1993). More

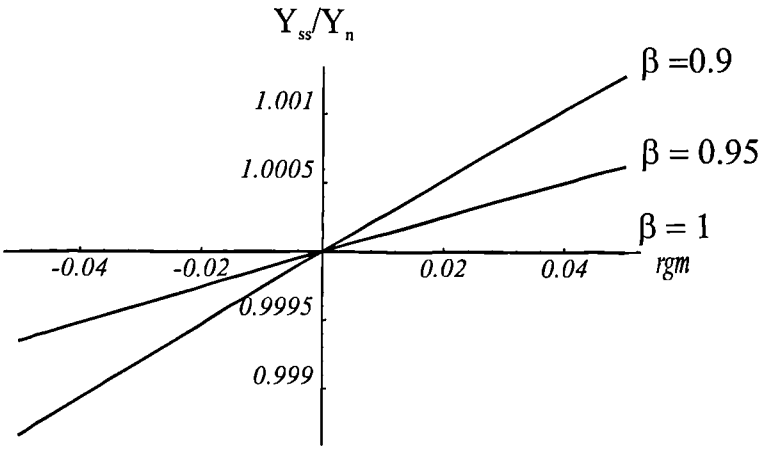

Figure 2.1: *Steady State Output:* $\theta = 0, e = \sigma = 1$

make the curve bend. The maximum still occurs at a positive (0.2%) rate of growth of money (because $\beta < 1$), but then the curve bends very quickly as the money growth rate increases. Moreover, note that our model reproduces the empirical negative correlation between inflation rates and employment rates reported by Cooley and Hansen (1989, section IIIC).

The welfare level in the aggregate is a function of Φ.[16] In particular,

problematic is to find a sensible value for e. Following Macurdy (1981), e should be set equal to 4.3. However, most Pencavel (1986) estimations place e between 3.2 and infinity. As a conclusion, the elasticity of intertemporal substitution in labour supply should be low. Therefore, in the base case, e is set equal to 4.5, but there are virtually no reasons for not choosing a higher value of e. Then, $\sigma = 1$ and $\beta = 0.95$. In the next section, some numerical examples are shown for different values of these parameters. (Moreover, $\chi = 0.01$, $\delta = 0.99$ and $\alpha = 1$, but these three parameters are not important for next section results).

[16]The welfare is defined as the weighted sum of the welfare level of the households

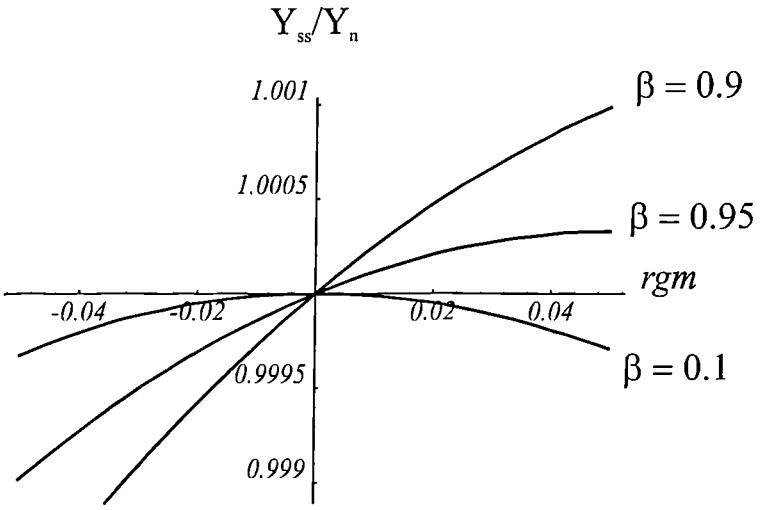

Figure 2.2: *Steady State Output:* $\theta = 1.0000001, e = \sigma = 1$

welfare tends to infinity as Φ tends to $1/\beta$. As a result, in this model the optimal money rule also corresponds to the Friedman rule. In this case, real money balances, and thus welfare, tend to infinity and the model is undetermined.

Figure 2.4 and 2.5 show the behaviour of welfare in Ireland's (1995) case (i.e., $e = \sigma = 1$, θ close to 1) and in the base case, respectively. In the first case, welfare is decreasing in the money growth rate simply because real balances are decreasing as the latter rises. Since output does not change very much, because the intertemporal rate of discount is small, then the real balances effect dominates. In the second case, instead, the non-linearity effects

of the two sectors with weights equal to (1/2) which is the size of the sectors. Hence: $W = U_A/2 + U_B/2$.

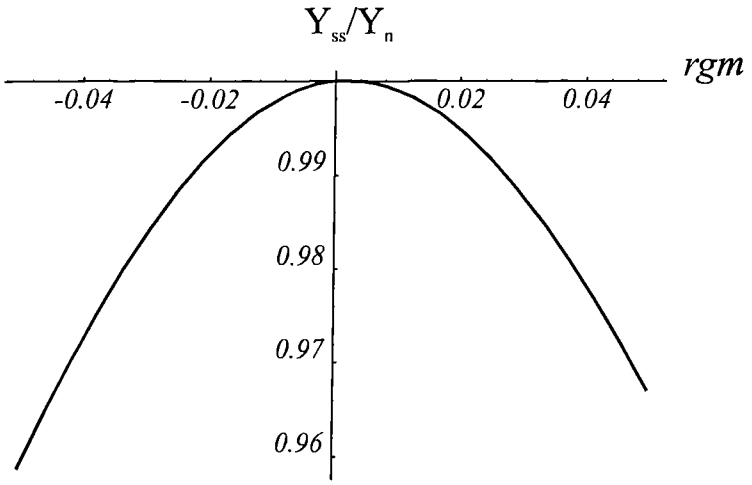

Figure 2.3: *Steady State Output:* $\theta = 6, e = 4.5, \sigma = 1, \beta = 0.95$

clearly dominate. Further to the left in Figure 2.5, welfare starts to increase and tends to infinity as rgm approaches $\beta - 1 = -0.05$ (off the graph). Our utility function, chosen for tractability and simplicity reasons, does not imply a satiation point for real balances. However, this is highly unrealistic. We can suppose the existence of a satiation point, simply assuming that for all $M/P >$ some $(M/P)^*$, the second term in the utility function (i.e., ln (M_t/P_t)) will be replaced by a constant k.[17] If k is not unrealistically high, then the effects on the real part of the utility function would dominate.[18] In

[17]This hypothesis would not change the results presented here for $(M/P)^*$ sufficiently large.

[18]See Obstfeld and Rogoff (1995, 1996) and King and Wolman (1996) for a similar argument.

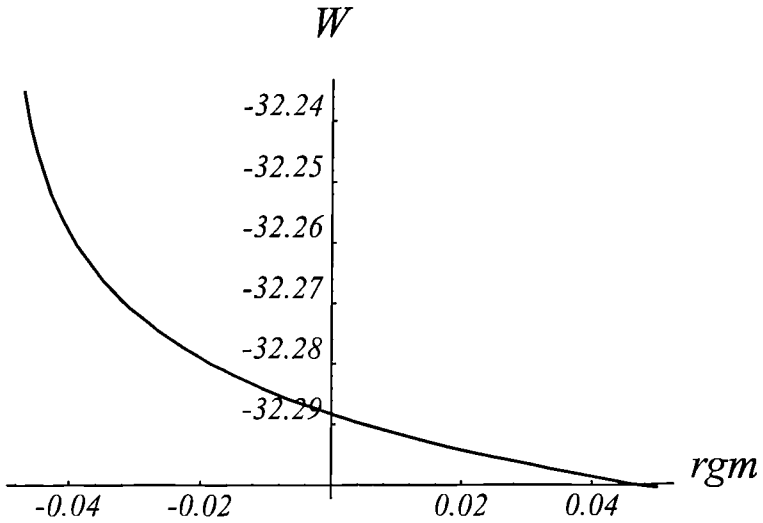

Figure 2.4: *Steady State Welfare:* $\theta = 1.0000001, e = \sigma = 1$

Figure 2.5, then welfare would be maximised at rgm = -0.45%.[19]

As the graphs show, staggered wage setting calls for price stability because it gives rise to high cost of inflation.

[19]Moreover, note that if the intertemporal rate of discount is relatively high, then welfare could be maximised in correspondence to a positive rgm (e.g. if $\beta = 0.9$, then $argmax\{W(rgm)\} = 0.1\%$).

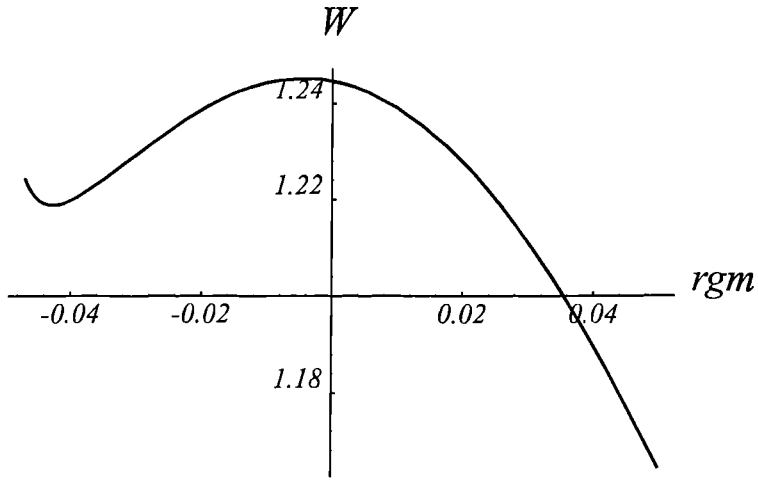

Figure 2.5: *Steady State Welfare:* $\theta = 6, e = 4.5, \sigma = 1, \beta = 0.95$

2.3 How Big Are the Effects of a Change in Money Growth ?

In this section some numerical examples are presented to give an insight into how big the effects due to non-linearities can be.[20] What has been computed is how the steady state of the system changes, when the steady state rate of money growth goes from 5% to zero. The results are summarised in Table 2.2. $\triangle Y$ gives the percentage change in aggregate output; $\triangle W$ gives the change in welfare. This is measured in "consumption equivalent" gain, that is, how much consumption must be given to the households (holding other

[20]Even if similar to Cooley and Hansen (1989, section IIIB), this is not a calibration exercise. The exercise remains theoretical and close to the spirit of the exercise of, e.g., Blanchard and Kiyotaki (1987).

arguments of their utility function constant) to make them enjoy the same level of welfare as in the case of zero money growth.[21]

Just from a quick look at the table, it is immediately evident that a wide range of possibilities arises. In particular, the results depend heavily on the values of the non-linearity parameters θ, σ, and e. Since the amplitude of the sectors' relative outputs is $\left(\frac{1}{1+rgm}\right)^{\sigma\varepsilon}$, the bigger θ and σ, the bigger is the amplitude of the cycle and the more the curve bends. In the base case in which $\sigma = 1$, $e = 4.5$, and $\theta = 6$, then a change from 5% to 0 in the rate of money growth leads to a change in steady state output of 3.41%. The change in welfare is equal to 8.44% increase in consumption. The variation in welfare due only to the changes in consumption and labour levels is equivalent to 6.96% gain in consumption, while the rest of the gain is due to the change in real balances. These numbers are somewhat impressive, given that 5% money growth is very moderate inflation consistent with the experience of most western countries. In the absence of non-linearities, as in Ireland's (1995) model, there is actually a loss in output, since only the intertemporal rate of discount effect is present. This is, however, somewhat

[21] It is straightforward to show that the equivalent consumption variation as a percentage of initial consumption level is given by

$$\triangle W = \left[\exp\left(\frac{W(0) - W(5\%)}{\delta}\right) - 1\right] 100$$

$\sigma = 1$	$\theta = 1$	$\theta = 6$	$\theta = 10$	$\theta = 20$
$e = 1$	$\Delta Y = -0.03$ $\Delta W = 1.24$	$\Delta Y = 0.12$ $\Delta W = 1.56$	$\Delta Y = 0.23$ $\Delta W = 1.84$	$\Delta Y = 0.48$ $\Delta W = 2.35$
$e = 4.5$	$\Delta Y = 0.071$ $\Delta W = 1.46$	$\Delta Y = 3.41$ $\Delta W = 8.44$	$\Delta Y = 8.70$ $\Delta W = 19.94$	$\Delta Y = 25.13$ $\Delta W = 59.40$
$e = 10$	$\Delta Y = 0.23$ $\Delta W = 1.79$	$\Delta Y = 7.40$ $\Delta W = 17.02$	$\Delta Y = 15.82$ $\Delta W = 36.29$	$\Delta Y = 36.05$ $\Delta W = 88.70$
$\sigma = 0.8$	$\theta = 1$	$\theta = 6$	$\theta = 10$	$\theta = 20$
$e = 1$	$\Delta Y = -0.026$ $\Delta W = 1.26$	$\Delta Y = 0.0213$ $\Delta W = 1.38$	$\Delta Y = -0.035$ $\Delta W = 1.41$	$\Delta Y = -0.049$ $\Delta W = 1.44$
$e = 4.5$	$\Delta Y = 0.057$ $\Delta W = 1.43$	$\Delta Y = 0.683$ $\Delta W = 2.71$	$\Delta Y = 0.977$ $\Delta W = 3.32$	$\Delta Y = 1.33$ $\Delta W = 4.06$
$e = 10$	$\Delta Y = 0.187$ $\Delta W = 1.69$	$\Delta Y = 1.73$ $\Delta W = 4.86$	$\Delta Y = 2.39$ $\Delta W = 6.27$	$\Delta Y = 3.16$ $\Delta W = 7.89$
$\sigma = 0.5$	$\theta = 1$	$\theta = 6$	$\theta = 10$	$\theta = 20$
$e = 1$	$\Delta Y = -0.016$ $\Delta W = 1.28$	$\Delta Y = -0.006$ $\Delta W = 1.30$	$\Delta Y = -0.004$ $\Delta W = 1.31$	$\Delta Y = -0.003$ $\Delta W = 1.32$
$e = 4.5$	$\Delta Y = 0.036$ $\Delta W = 1.38$	$\Delta Y = 0.13$ $\Delta W = 1.58$	$\Delta Y = 0.147$ $\Delta W = 1.61$	$\Delta Y = 0.163$ $\Delta W = 1.64$
$e = 10$	$\Delta Y = 0.117$ $\Delta W = 1.55$	$\Delta Y = 0.359$ $\Delta W = 2.05$	$\Delta Y = 0.404$ $\Delta W = 2.14$	$\Delta Y = 0.444$ $\Delta W = 2.21$

Table 2.2: Output and Welfare Steady State Changes from 5% to 0 Money Growth

negligible (- 0.032). Nonetheless, the gain in welfare is equal to 1.24% of consumption, but this is entirely due to the rise in real balances. In fact, the variation in welfare due only to a change in output and labour levels is equal to a loss of -0.066%.

In the extreme case in which $\theta = 20$, $e = 10$, and $\sigma = 1$, the change in output amounts to 36.05% and the one in welfare to 88.70% equivalent variation in consumption.[22] The equivalent variation in consumption due only to the variation of output and labour levels amounts to 85.1% of the initial consumption level. In this case, the effects are therefore of enormous magnitude.

The results are very sensitive to all of the non-linearity parameters and especially to σ. If σ is low then, the effects are reduced greatly, though still of some importance. Intuitively, $\sigma = 1$ corresponds to the maximum degree of nominal rigidity. In this case, in fact, each sector can afford to satisfy all of the received demand without changing its price. If instead σ <1, then the higher the output produced by the sector, the higher is the price to be charged because of the diminishing returns to labour. This fact obviously reduces the gap between the prices of the two sectors and therefore their

[22] Given what has been discussed in footnote 15 about the intertemporal elasticity of labour supply and given that $\theta = 20$ has been used in the literature for this kind of numerical examples (e.g. Blanchard and Kiyotaki (1987)), this case is actually not as extreme as it might seem.

output gap in each period.

Figures 2.6, 2.7, 2.8 show very neatly how the curve bends more as the non-linearity parameters increase.[23]

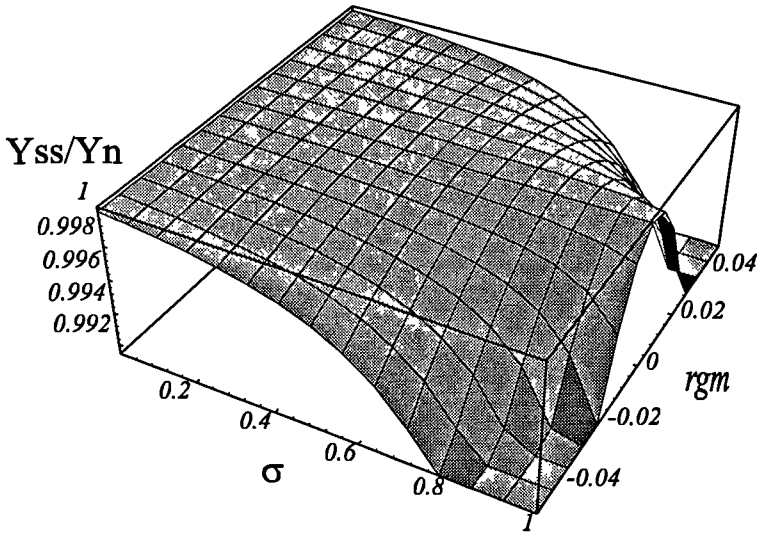

Figure 2.6: *Steady State Output:* $\theta = 6, e = 4.5, \beta = 0.95$

2.4 Conclusions

The numerical results suggest that, in staggered adjustment models, superneutrality is far from being the minor issue that has been thought so

[23]What is actually plotted on the vertical axis is the ratio Y_{ss}/Y_n, which is the ratio between the aggregate output in the staggering model and the one in the flexible wage version of the same model. That is why the Figure 2.6 is decreasing in sigma. The aggregate output Y_{ss} is actually increasing in sigma (as Y_n), but the higher sigma, the higher is the effect described in the paper and the lower is the ratio between Y_{ss} and Y_n.

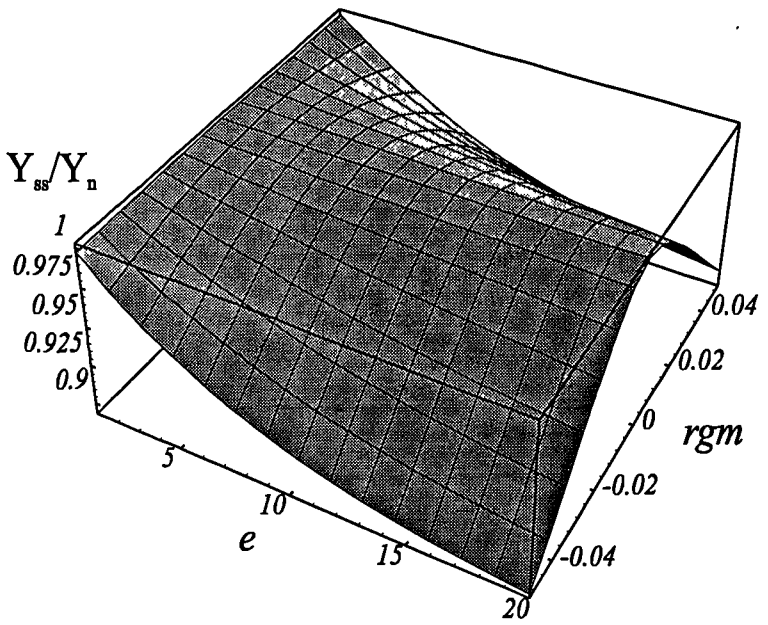

Figure 2.7: *Steady State Output:* $\theta = 6, \sigma = 1, \beta = 0.95$

far. This chapter actually demonstrates that it very well can be the case that a mild permanent change in the rate of growth of money could have substantial effects on aggregate output and welfare. Previous models with staggered wage/price behaviour fail to acknowledge this fact. In the steady state, Calvo's (1983b) model behaves as a flexible price one, because of the peculiar hypothesis of a price-regulation mechanism. Ireland's (1995) model is too simple in its structure to detect strong effects of the rate of growth of money on the steady state output and welfare. In particular, because of the linearity in the production and utility functions in labour and the elasticity of substitution among goods equal to one, Ireland's (1995) model does not capture the effects due to the usual non-linearities in technology and pref-

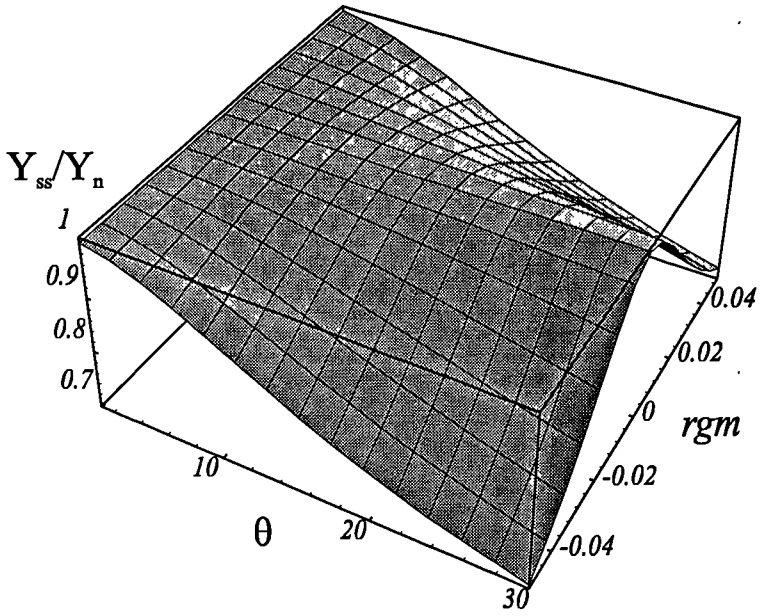

Figure 2.8: *Steady State Output:* $e = 4.5, \sigma = 1, \beta = 0.95$

erences: decreasing return to scale to labour, increasing marginal disutility of labour and elasticity of substitution among goods bigger than one. Once these effects are taken into account, it turns out that staggered wage setting behaviour induces strong non-superneutrality properties and high costs of inflation. Hence, given that staggered wages are observed in western countries, we can easily explain high costs of inflation and provide a rationale for the pursuit of price stability in western countries.

Chapter 3

Staggered Wages and Disinflation Dynamics

3.1 Introduction

Ever since the work of Gray (1976), Fischer (1977) and Taylor (1979), staggered, preset wages or prices have been the mainstream Keynesian explanation of why monetary policy is able to have significant and persistent effects on real output in the presence of rational expectations. For example, Blanchard and Summers (1988, p. 182) argued: *"On the alternative "Keynesian view", even credible disinflation is likely to increase unemployment for some time, because of the inflationary momentum caused by overlapping price and wage decisions."* However, Ball (1994) challenged this common view. In particular, he argued that, while staggered preset pricing is the most likely source

of persistent negative output effects of a permanent reduction in the level of the money supply (a "deflation"), the same effects do not follow a permanent reduction in the money supply's rate of growth (a "disinflation").[1] We call this the "disinflation puzzle". This puzzle is most starkly expressed by Ball (1994), though it was noted before this by others, e.g. Blanchard (1983), Buiter and Miller (1985). Ball presents a model with the surprising result that *"with credible policy and a realistic specification of staggering, quick disinflations cause booms"*. For this reason, Ball and others (e.g. Miller and Sutherland (1993), Driffil and Miller (1993)) have gone on to conclude that essential to the explanation of why disinflations cause prolonged recessions in practice is that policy lacks "credibility".

Ball and the other authors who arrive at the same conclusion, however, use macromodels which are directly postulated rather than explicitly derived from microfoundations. In this chapter we use our model of staggered wages to address the disinflation puzzle. The question we then ask is whether introducing such microfoundations still leads to Ball's and others' conclusion, or whether, on the other hand, it could help to resolve the disinflation puzzle without needing to appeal to lack of credibility.

The advantages of the more microfounded approach are that it leads to an internally consistent model. Directly postulated models tend to be well

[1] In this chapter we are concerned with the second part of Ball's claim. The analysis of the effects of money shocks will be the focus of the next chapters.

developed in certain sectors and sketchily developed in others, implying inconsistency between different aspects of the same agents' behaviour. In the models used so far in the disinflation puzzle literature, such an inconsistency commonly exists between the forward-looking behaviour in the pricing sector and the myopic behaviour in the consumption and money demand sectors. Moreover these models do not allow for dynamic optimising behaviour even in the pricing sector. Such limitations may turn out to be justifiable simplifications, but we can only know this by first building a more microfounded model and making a comparison. The analysis contained in this chapter will suggest that the direct postulation approach may have led earlier researchers to dismiss certain features as unimportant too readily, and to overlook certain key parameter restrictions which the microfounded model implies ought to hold in directly postulated models.

An important insight which underlies our investigation is that Ball's (1994) paradoxical result is due to an element of preannouncement in the policy experiment he considers. Disinflation in his analysis consists of putting a linear time trend into the rate of monetary growth, to bring it down continuously from its initial level to zero by a known date. A similar but simpler policy experiment would be to preannounce that at some future date the monetary growth rate will be discontinuously reduced from its current level to zero. In general we may note that such a preannounced disinflation has

two conflicting effects on short-run output. First, anticipation of lower future inflation raises the demand for current real balances. Since the path of the money supply has not yet changed, and since the current price level is sticky, the money market can then only clear if current output falls, to push the demand for real balances back down. This is the contractionary effect of the announcement: it stems from a fall in the desired velocity of circulation of money. Second, anticipation of lower future inflation induces price-setters to begin lowering their prices in advance of the policy change. The price reduction boosts the supply of real balances, which stimulates the demand for goods and thus output. This is the expansionary effect of the announcement. Ball, like many other authors, assumes away the first effect by postulating a very simple aggregate demand equation based on a constant velocity of circulation, and as a result the second effect prevails: this is the hidden source of his "disinflationary boom".

With an immediate and unanticipated cut in monetary growth, on the other hand, price reductions cannot precede the implementation of the policy, so that the second, expansionary, effect just mentioned is absent. Moreover, it would seem reasonable to suppose that, since some fraction of prices are predetermined when the policy begins, the fact that the money supply soon takes a lower growth path would reduce real balances, causing a slump. However, as other authors have shown, this is not necessarily the case. It depends

on the staggering structure: if prices are set for a fixed period, as in Taylor's (1979) structure, then the argument is correct; but if the period is of random length (determined by a Poisson process), as in Calvo's (1983a,b) structure, then the argument turns out to be false. Here we have another variant of the "disinflation puzzle": although a sudden, discontinuous slowdown in monetary growth does not produce a boom, with Calvo-type staggering it may nevertheless not produce a slump either, so that immediate and costless disinflation appears to be possible. This, indeed, was the variant of the puzzle noticed by Buiter and Miller (1985). However, here also, costless disinflation only results if the simple aggregate demand equation is assumed; if instead we allow money demand to respond to anticipated inflation then the velocity effect mentioned above will still be present, and the expected slump will be obtained.

The use by these earlier authors of the simple form of aggregate demand was not because they were unaware that a more general form could explain a slump, but because they believed that this effect was likely to be weak, so that to omit it was a reasonable simplification. One of our key findings from the microfounded model is that this is not a reasonable simplification. Not only does our model in general imply that the aggregate demand simplification used by these earlier authors is an extreme case, but it shows that there is a connection between this property of the model and another property

about which simplifying assumptions are typically made. The latter is the long-run effect of disinflation on output, which is assumed to be zero in directly-postulated models. We show that the condition for this to be zero in our log-linearised model is that agents' time preference rate be zero; but if this is assumed, the effect of anticipated inflation on aggregate demand tends to infinity, which is the opposite of the simple aggregate demand property which earlier authors have imposed. More microfoundations thus reveal a contradiction between two of the simplifying assumptions found in earlier models.

Perhaps our most notable finding, however, is that preannounced disinflation cannot cause a boom, in opposition to Ball's (1994) result. We show that, in response to the announcement of a future disinflation, output falls, and along its subsequent time path it never exceeds its original level prior to the announcement. By contrast the same policy experiment in a standard directly postulated model containing Ball's simplifications, implies that following the announcement output gradually rises, reaching a peak at the date the disinflation is implemented, and then gradually falling back again to its original level. The microfounded model thus banishes the apparently crazy behaviour found in the basic directly-postulated model. The reason for this difference in behaviour is closely linked to the contradiction between the two common simplifying assumptions referred to above. The directly postu-

lated model can be made to deliver a similar time path to the microfounded model, but only if a particular parameter restriction is respected, and this condition is in fact violated by the common simplifying assumptions. Reasoning without microfoundations is thus likely to lead us to unnecessarily puzzling conclusions.

Other authors, who have also recently look at staggered prices or wages in a dynamic general equilibrium framework (e.g., Cho and Cooley (1995), Kimball (1995), Woodford (1996), Sutherland (1996), Yun (1996), Chari et al. (1996)) have not been concerned with the disinflation question, however; mostly they have been concerned with numerical simulations in order to find a good quantitative match with business cycle stylised facts as represented by statistical co-movements, in the tradition started by the real business cycle literature. As already mentioned in the previous chapter, one dynamic general equilibrium paper which does look at disinflation is by Ireland (1995). Ireland looks at the question of optimal disinflation, and is again especially focused on calibration. In this chapter we take an analytical rather than a numerical approach, seeking to understand more about the basic mechanics of the disinflation process. We also wish to maintain a link to the literature based on directly postulated models, and so devote significant space to a comparison with such models.

The structure of the chapter is as follows. Section 3.2 presents the log-

linearised version of our model on which the subsequent analysis is based. Section 3.3 uses it to examine the effects of a disinflation, both unanticipated and anticipated. Section 3.4 provides a comparison with Ball's and others' results, based on directly-postulated models, using Calvo's model. Section 3.6 concludes.

3.2 The Log-linearised Model

To study the dynamics of the model following a disinflationary policy, we now take a log-linear approximation to the model about the zero-inflation steady state. Lower-case letters are used to denote log-deviations of variables from their steady state values: for example, $y_t \equiv \ln Y_t - \ln Y^*$ (where * indicates the zero-inflation steady-state value), while $\phi_t \equiv \ln \Phi_t$ (since $\Phi^* = 1$) $\equiv m_{t-1} - m_t$.

It should be born in mind that the type of disinflation with which we are henceforth concerned is therefore one relevant to moderate rates of inflation, i.e. in which inflation remains in the appropriate neighbourhood of zero. Given the complexity of the model, it turns out that this is the only analytically tractable case. That is, the log-linearised model around a steady state in which $\Phi \neq 1$ is not analytically tractable and we need to resort to simulations. Since we want to take an analytical rather than a numerical approach, seeking to understand more about the mechanisms of the disin-

flation process, and comparing it to the *ad hoc* postulated models, we are forced to confine ourselves to this particular case. However, in the Appendix 3.7.3 some simulation results are provided to check the robustness of our analytical findings. These simulations are performed with the package for non-linear models DYNARE, elaborated by Michel Juillard at CEPREMAP (see Juillard (1996)).

Demand Side

The dynamics of the demand side of the economy are described by the following two equations

$$y_t = m_t - p_t - z_t \tag{3.1}$$

$$z_t = z_{t+1} + (m_t - m_{t+1}) \ . \tag{3.2}$$

(3.1) follows from the definition of the 'Cambridge k', i.e., $Z_t \equiv M_t/P_tC_t$ and from the goods market equilibrium condition $C_t = Y_t$. (3.2) from the dynamic equation for Z_t, i.e. (2.6). It is important to stress again that these dynamic equations are totally independent of the supply side of the economy. In other words, (3.1) and (3.2) derive just from the demand side of the economy and so the dynamics of Z_t are independent of the supply side and of whether wages are flexible, fixed, or predetermined but time-varying, and of their synchronisation.

The interpretation is straightforward. Log-linearising the first-order con-

dition (2.4) yields

$$y_t = m_t - p_t + \kappa i_t \tag{3.3}$$

where $\kappa \equiv \beta/(1-\beta) = 1/(I^* - 1) > 0$. Then, given (3.1), we have

$$z_t = -\kappa i_t \ . \tag{3.4}$$

Therefore z_t represents the effect of the nominal interest rate on aggregate demand. (3.3) (or equivalently (3.4)) looks very familiar. It describes a Cagan-type demand-for-money effect: the higher the inflation rate, the higher the nominal interest rate; hence the higher the opportunity cost of holding money and the larger the velocity of circulation of money (or the lower 'Cambridge k', i.e. z_t). Given m_t and p_t, this means the higher is aggregate demand. Equation (3.4) states explicitly that velocity is an increasing function of the nominal interest rate. Though similar to a static LM or aggregate demand equation, such as is commonly used in directly postulated models, here (3.1) is paired with equation (3.2), which gives us the forward-looking dynamics deriving from intertemporal optimisation.

Supply Side

The dynamics of the supply side of the economy are given by

$$x_t = \frac{1}{1+\beta}[p_t + \gamma y_t] + \frac{\beta}{1+\beta}[p_{t+1} + \gamma y_{t+1}] \tag{3.5}$$

$$p_t = \frac{1-\sigma}{\sigma} y_t + \frac{1}{2}(x_t + x_{t-1}) \tag{3.6}$$

where $\gamma \equiv [e + (1-\sigma)(\theta - 1)]/[\theta e - \sigma(\theta - 1)]$. (3.5) derives from the log-linearisation of the first order condition for the optimal wage, i.e., (2.5), while

(3.6) is obtained by log-linearising the aggregate price level equation, i.e., (2.16). There is a striking similarity between this model and Taylor's (1979) model. However, here: (i) the equations are derived from an optimisation process; (ii) the Taylor parameters are no longer *ad hoc*, but depend on preferences and technology; (iii) the dynamics of demand, obtained from intertemporal optimisation, are taken into account.[2]

Substituting (3.1) and (3.6) into (3.5) we obtain

$$-bx_{t-1} + (h+1)x_t - dx_{t+1} = h[b(m_t - z_t) + d(m_{t+1} - z_{t+1})] \tag{3.7}$$

where $d \equiv 1 - b \equiv \beta/(1+\beta)$ and $h \equiv 2e/(e-1)(\varepsilon - 1)$. (3.2) and (3.7) constitute a third-order dynamical system. However, since the dynamics in (3.2) do not depend on the supply side, the system has a recursive structure. Following a change in monetary policy, the forward-looking variable z_t is governed only by (3.2), and its path then feeds into (3.7). Therefore we can treat the right-hand side of (3.7) as an exogenous forcing variable and solve for the dynamic response of the x variable as a function of the right-hand side terms.

The Effects of a Deflation

Although our interest is in the effects of a *disinflation* (a reduction in monetary growth), it is useful for comparison, and to illustrate the basic dynamics of the system, first to describe briefly the effects of a *deflation* (a

[2] A full comparison with Taylor's model is provided in the next chapter.

reduction in the level of the money supply). A key point is that a deflation (provided it is unanticipated) has no effect on velocity, $-z_t$. This is apparent from (3.2), since the expected rate of monetary growth, $m_{t+1} - m_t$, on the right-hand side, is always zero in a deflation. Therefore z_t remains constant at its initial value of zero. The dynamic response of the economy hence depends only on the wage-setting dynamics given by (3.7). This makes the dynamics of a deflation the same as Taylor's (1979) dynamics. The second-order equation (3.7) has one stable eigenvalue λ_s between 0 and 1, and one unstable eigenvalue λ_u greater than 1 (this is shown in the Appendix 3.7.1). The system hence satisfies the normal 'saddlepath' requirement for a unique perfect foresight solution, since (3.7) has precisely one naturally given initial value (namely the lagged value of the reset wage, x_{t-1}). As in Taylor (1979), we can then show that the deflation causes output to fall on impact. Over time it returns monotonically to its original level, according to $y_t = \lambda_s y_{t-1}$. In the case of a deflation, there is hence no doubt that the outcome is a persistent slump: this is true both in the microfounded model and in directly-postulated models. The only room for debate is over the degree of persistence, i.e. the size of λ_s. This issue will be the focus of the next chapters.

3.3 Disinflation Dynamics

In this section, we use the microfounded model to look at, first, an unanticipated disinflation; and, second, a preannounced disinflation. In the next section we compare our results with those from directly postulated models.

3.3.1 Unanticipated Disinflation

The complete linearised model, we recall, consists of equations (3.2) and (3.7). Since nominal variables such as x_t are ever-growing in steady states with positive inflation, it is now helpful to rewrite (3.7) using x_t normalised by the money supply, i.e., $v_t \equiv x_t - m_t$. Further, we can convert the second-order equation to two first-order equations by defining $w_t \equiv v_{t-1}$, which will enable us to depict the dynamics on a phase diagram. The system thus transformed becomes

$$z_t = \beta z_{t+1} + \beta \phi_{t+1} \tag{3.8}$$

$$v_{t+1} = \frac{h+1}{d} v_t - \frac{b}{d} w_t + \left[2\frac{hb}{d} z_t + \phi_{t+1} - \frac{b}{d}\phi_t \right] \tag{3.9}$$

$$w_{t+1} \equiv v_t \tag{3.10}$$

The sub-system (3.9)-(3.10) yields the phase diagram drawn in Figure 3.1 below. We can show that this sub-system has the same eigenvalues (λ_s, λ_u) as (3.7). The slope of the saddlepath is thus given by λ_s (<1), and of the unstable separatrix by λ_u (>1).

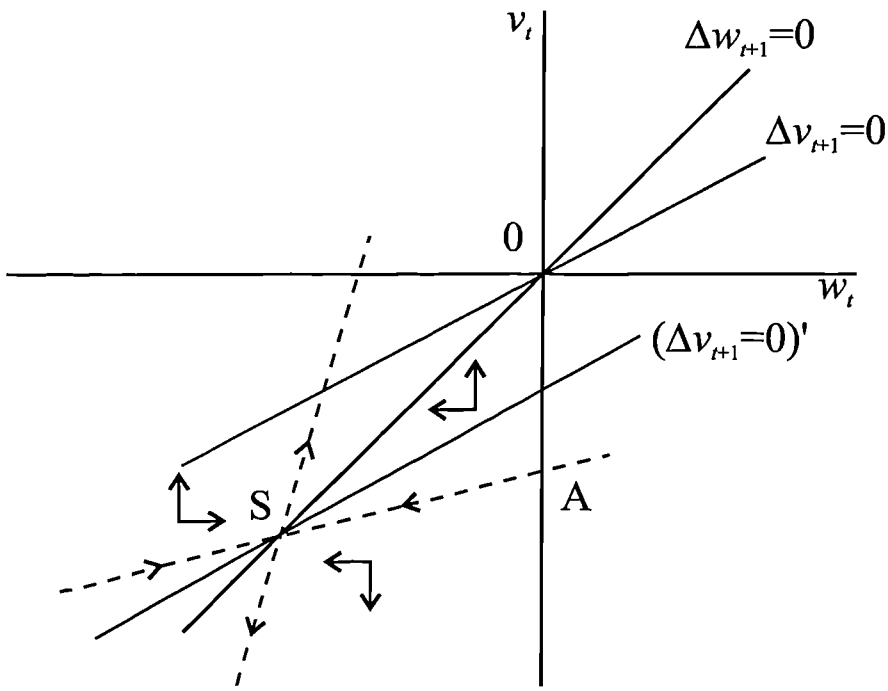

Figure 3.1: *Phase Diagram for the Microfounded Model*

Now consider the effect of a permanent, unanticipated disinflation. We suppose that ϕ is initially zero, and then undergoes a permanent increase[3]. Unlike in the case of a deflation discussed above, the policy change here does affect z_t. From the purely 'forward-looking' equation (3.8), we see that z_t must jump up from zero to its new steady-state value of $\beta\phi/(1-\beta)$. This corresponds to a jump down in the nominal interest rate from zero to $-\phi$, and is the effect of lower inflation in lowering velocity, as noted earlier. Turning at-

[3] Although this means we are actually looking at a shift from zero to negative monetary growth, this is just a normalisation: the effects would be the same for any decrease in monetary growth in the linearised model.

tention from (3.8) to (3.9)-(3.10), we can then see that the composite forcing variable of the system (the term in square brackets) undergoes a once-and-for-all change. This implies a once-and-for-all shift in the $\triangle v_{t+1} = 0$ locus in Figure 3.1. To determine the sign of the shift, consider the expression for the steady-state value of v

$$v = \frac{(1-\beta)^2 - 2\beta h}{(1+\beta)(1-\beta)h}\phi \ . \tag{3.11}$$

The numerator of this expression is positive for β sufficiently close to zero, and negative for β sufficiently close to one. Hence, whether v rises or falls in the long run is ambiguous, but since it is generally believed that β is close to one, a fall seems the most likely. In the diagram, this corresponds to a downward shift in the $\triangle v_{t+1} = 0$ locus. The time path of v in Figure 3.1 is thus a jump down from 0 to A at time t, followed by a further gradual fall as the economy converges along the saddlepath AS.

The dynamic behaviour of v is not very interesting in itself, but it helps to reveal the behaviour of the main variable of interest, output. The relationship of output to v is given by (combining (3.1) and (3.6) and the definition of v)

$$y_t = -\sigma\left[\frac{1}{2}(v_t + v_{t-1} + \phi_t) + z_t\right] \tag{3.12}$$

In the case of β close to zero (i.e., converse to that illustrated), where v jumps upwards on impact and then continues rising over time, we see unambiguously from (3.12) that output falls on impact, and then falls further

as time progresses. In the case of β close to one, there at first appears to be ambiguity, so it is helpful to refer in addition to the solution for steady-state output, readily computed as

$$y = -\sigma(1-\beta)\frac{2+h}{2(1+\beta)h}\phi \ . \tag{3.13}$$

This shows that output still falls in the long run when β is close to 1, provided that β does not actually equal 1.[4] Since v is in this case decreasing along the adjustment path, we may deduce from (3.12) that output is rising, and hence that it must overshoot its long-run value in the short run. To reinforce this, consider the following explicit solution for short-run output (which is derived from (3.12), using the facts that $v_t - v = \lambda_s(v_{t-1} - v)$ along the saddlepath, and $v_{t-1} = 0$ initially)

$$y_t = -\sigma\frac{2 + \frac{(1-\lambda_s)(1-\beta)^2}{h} + 2\beta\lambda_s}{2(1+\beta)(1-\beta)}\phi \ . \tag{3.14}$$

As β tends to one, y_t as given by (3.14) tends to minus infinity[5], while the steady-state solution (3.13) tends to zero: hence there must be overshooting

[4]Recall from the previous chapter that if $\beta < 1$, then $rgm = 0$ is always on the left of the maximum level of steady state output in Figures 2.1, 2.2, 2.3 (i.e., steady state output is increasing in rgm (decreasing in Φ) in that point). If instead $\beta = 1$, then, $rgm = 0$ (i.e., $\Phi = 1$) coincides with the maximum steady state output.

[5]Although λ_s is also a function of β, which must be taken into account when considering the limit of (3.14), in the Appendix 3.7.1 we show that as β varies between 0 and 1, λ_s remains within a strict subinterval of [0,1].

for β sufficiently large. In fact, it is clear from (3.11) that the condition for overshooting is $(1-\beta)^2 < 2bh$.

We may summarise these findings by the following proposition:

Proposition 3.1. *In the microfounded model with a strictly positive time preference rate (i.e. $\beta < 1$), an unanticipated disinflation always produces a slump (a level of output lower than the original level) in the short, medium and long run. As the time preference rate tends to zero (i.e. $\beta \to 1$), the size of the long-run slump tends to zero and the size of the short-run slump tends to infinity. Output overshoots its long-run level in the short run iff $(1-\beta)^2 < 2bh$.*

The possible dynamic responses may thus be sketched as follows:

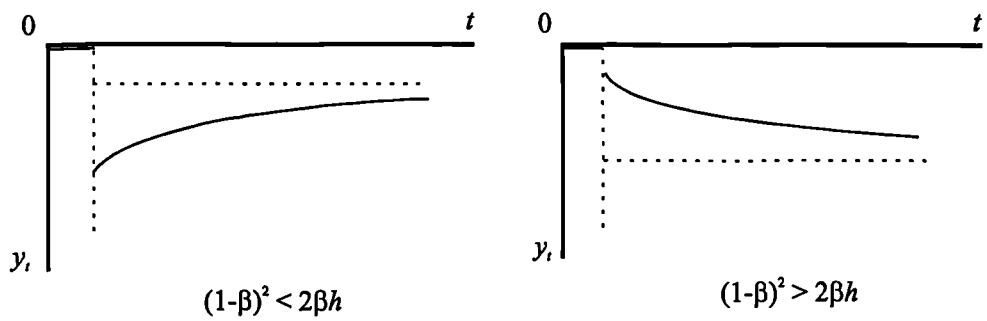

Figure 3.2: *Unanticipated Disinflation: Possible Dynamic Responses*

What are the forces causing the slump? In the short run there are two: first, prices are sluggish to adjust, so that the lower path for the money supply lowers real balances and thus goods demand; and, second, the reduction in the nominal interest rate raises the demand for real balances, i.e. lowers

the velocity of circulation, which further depresses goods demand. These effects can be seen in (3.12), where they correspond to a fall in m_t and a rise in z_t, respectively. In the long run, the reduced output is caused by a strictly positive time preference rate, as we know from the Calvo-Romer effect, analysed in the previous chapter.

3.3.2 Preannounced Disinflation

Now let us consider an increase in ϕ which is announced in period τ and implemented in period $T > \tau$. The relevant model is still (3.8)-(3.10) with output determined by (3.12).

First, it is a simple matter to solve (3.8) forwards to show that

$$z_t = \frac{\beta^{T-t}}{1-\beta}\phi \qquad for\ t = \tau, \tau+1, ..., T-1\ . \tag{3.15}$$

This tells us that when the policy is announced, money demand per unit consumed, z_t, rises (and correspondingly the velocity, -z_t, and the nominal interest rate falls, recalling (3.4)), and it continues to rise over time as the implementation date approaches. By $T-1$, z_t has reached its new steady state level, $\beta\phi/(1-\beta)$, where it remains thereafter.

The rise in z_t ahead of the implementation of the policy is the source of the short-run contractionary effect of an anticipated disinflation which we referred to in the Introduction. We can see this contractionary effect easily from (3.12): for given (x_t, x_{t-1}), the higher z_t implies a lower y_t. The

forward-looking nature of portfolio behaviour, as represented by (3.8), is clearly crucial in causing this. However, we cannot yet be sure that the overall short-run effect is a slump, because x_t jumps downwards in anticipation of its lower long-run value. (3.12) shows that this has the opposite effect on y_t. This lowering of the wage is the cause of the short-run expansionary effect of an anticipated disinflation which we referred to in the Introduction. The next task is therefore to solve for the path of x_t (or v_t) and so to evaluate the relative strengths of these two conflicting forces.

The path of v_t is governed by the system (3.9)-(3.10), and having determined the path of z_t, we can now work out the path of the composite forcing variable of this system, given by the term in square brackets. For $t \geq T$, this variable is constant at the same value as in the case of unanticipated disinflation; but for $t < T$ it is time-varying, since z_t is time-varying. This means that it is not feasible to draw a phase diagram for this case, since the locus $\triangle v_{t+1} = 0$ would be continually shifting. In the Appendix 3.7.2 we show how to solve the system algebraically. Once a solution for v_t has been obtained, this and (3.15) may be substituted into (3.12) to yield a solution for y_t. Doing this, we obtain the following expressions for y_t

$$y_\tau = -\frac{\sigma}{2h}\frac{1-\beta}{1+\beta}\left\{\frac{2h(1+\lambda_s)}{(1-\beta)^2}\beta^{T-\tau+1} + (h+1-\lambda_s)\left(\frac{1}{\lambda_u}\right)^{T-\tau}\right\}\phi \quad for\ t=\tau \tag{3.16}$$

$$y_t = -\frac{\sigma}{2h}\frac{1-\beta}{1+\beta}\left\{\frac{2\beta h(1+\lambda_s)}{(1-\beta)^2}\beta^{T-\tau}\lambda_s^{t-\tau} + \frac{h+1-\lambda_s}{\lambda_u-\lambda_s}\right.$$

$$\left[(1+\lambda_u)\lambda_u^{t-\tau} - (1+\lambda_s)\lambda_s^{t-\tau}\right]\lambda_u^{\tau-T}\Big\}\,\phi \qquad for\ t=\tau+1,...,T-1 \quad (3.17)$$

$$y_t = -\frac{\sigma}{2h}\frac{1-\beta}{1+\beta}\left\{\left[\frac{2\beta h}{(1-\beta)^2}\beta^{T-\tau} - \frac{h+1-\lambda_s}{\lambda_u-\lambda_s}\left(\frac{1}{\lambda_u}\right)^{T-\tau}\right.\right.$$

$$\left.\left.+\frac{h+1-\lambda_u}{\lambda_u-\lambda_s}\left(\frac{1}{\lambda_s}\right)^{T-\tau}\right](1+\lambda_s)\lambda_s^{t-\tau} + (2+h)\right\}\phi \qquad for\ t=T,T+1,....$$

(3.18)

(3.16)-(3.18) encapsulate the main results of this chapter. As far as the impact effect is concerned, we see straight away from (3.16) that y_τ is negative. The impact effect is hence unambiguously a slump: the contractionary effect of higher z_t turns out to outweigh the expansionary effect of the lower v_t. To see whether a boom might nevertheless develop as time progresses, consider next (3.17). (3.17) is clearly negative, since the term in square brackets is positive (recalling that $\lambda_s < 1 < \lambda_u$ and $t \geq \tau+1$). This tells us that there cannot be a boom during the pre-implementation phase, however long it is. This is our most significant finding, since it is during this phase that a boom may develop in the directly postulated model, as we will see below. Finally, consider the post-implementation phase. Since output converges monotonically on its new, negative, steady state value during this phase, it will always be negative here if it is negative at $t = T$. Now, we can show that (3.17) evaluated at $t = T$ (i.e., one period after the date at

which it is in principle last valid) equals (3.18) evaluated at $t = T$, indicating that the two segments of the time path exactly match up at the switch point. Since (3.17) was seen to be negative for all $t > \tau$, it then follows that output is negative at $t = T$. Thus there can be no boom during the post-implementation phase, either. We may summarise these findings as:

Proposition 3.2. *In the microfounded model, a preannounced disinflation causes a slump (a level of output lower than the original level) when the announcement is made, and in no period thereafter does output exceed its original level. This is true no matter how far in advance the announcement is made.*

Sketches of two possible time paths are given in Figure 3.3.

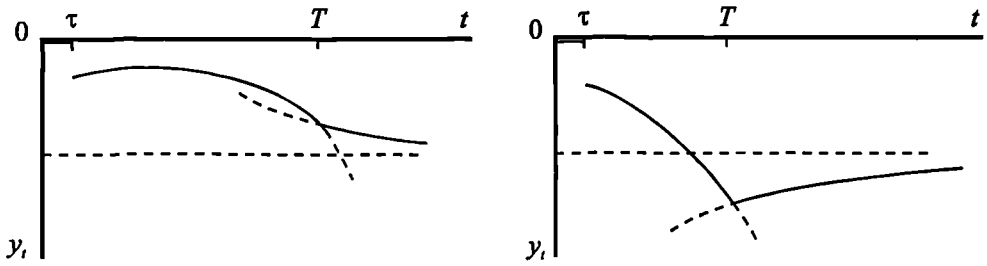

Figure 3.3: *Preannounced Disinflation: Possible Dynamic Responses*

The main point to note is that y_t never rises above its original level at any point along the path. Nevertheless, if the preannouncement period is long enough, after the initial slump there will be a 'recovery' phase during which output rises for a time, before sinking back down again: this is the case

illustrated on the left. On the other hand, if the preannouncement period is short, we could observe overshooting of the long-run level, as illustrated on the right.

3.4 Comparison with Directly Postulated Models

Our microfounded model has suggested that the effects of disinflation on output under staggered pricing are not particularly puzzling: when it is unanticipated, disinflation has a potentially large output cost in the short run, and also a small output cost in the long-run; while when it is preannounced, the short-run output cost is likely to be smaller but still not zero, and certainly not negative. As we noted in the Introduction, this contrasts with the impression obtained from other authors' analyses based on directly postulated models. In this section we want to illustrate this contrast by referring to a very commonly used version of a directly postulated staggered pricing model. Specifically, we shall take the version of Calvo's (1983b) staggered pricing model presented in Blanchard and Fischer (1989, Chapter 10). We first briefly recap how this model delivers apparently paradoxical results under the simplifications commonly imposed on it; and then show how respecting a particular parameter condition - which is violated by the common

simplifications - restores its behaviour to something more like that of the microfounded model. The microfounded model thus suggests that if we are going to continue to use the standard directly postulated model, we should revise our view of what are acceptable simplifications of it.

The directly postulated model is described by the equations[6]

$$x(t) = (\rho + r) \int_t^{\infty} (p(s) + \gamma y(s)) e^{-(\rho+r)(s-t)} ds \tag{3.19}$$

$$p(t) = \rho \int_{-\infty}^{t} x(s) e^{-\rho(t-s)} ds \tag{3.20}$$

$$y(t) = a[m(t) - p(t)] + k\pi \tag{3.21}$$

(3.19) has been already introduced in the Introduction of the previous chapter. Note that it is clearly similar to (3.5) in the microfounded model. As before, we generalise the usual text-book presentation by introducing a positive pure time preference rate, r, into this equation, and by adding it to ρ to obtain the discount factor of the future price-and-demand index. We have already shown its importance analysing the superneutrality issue, but we shall show that again r plays an important role in the disinflation dynamics (as correspondingly β plays a fundamental role in the disinflation dynamics of the microfounded model, as we have just seen). Equation (3.20)

[6] Apart from the generalisation already introduced in the previous chapter (a positive time preference rate), this is exactly the model from Blanchard and Fischer (1989), Chapter 10, pp. 548-551.

gives the current price level as an average of firms' outstanding prices: it is the counterpart of (3.6) in the microfounded model. Lastly, (3.21) is the aggregate demand equation. This postulates that goods demand depends positively on real balances and on the inflation rate. Such a relationship could be derived from standard IS and LM equations, provided that it is the real interest rate which enters the IS and the nominal interest rate which enters the LM. (3.21) is similar to (3.3) in the microfounded model. A very common simplification of (3.21) is to set $k = 0$, thus imposing a constant velocity of circulation of money.[7] However, this is a strongly 'monetarist' assumption for which no very good justification exists, and we shall see that it has a lot to do with the "disinflation puzzle".

Some manipulation reduces (3.19)-(3.21) to the following differential equation pair

$$\dot{\pi} = r\pi - \rho(\rho + r)\gamma y \tag{3.22}$$

$$\dot{y} = a\phi - (a - kr)\pi - k\rho(\rho + r)\gamma y \ . \tag{3.23}$$

These may be used to plot a phase diagram in (y, π)-space as in Figures 3.4 and 3.5. Although p is a predetermined variable in this model, π is not, and nor is y. Hence when an unexpected shock occurs, (y, π) both jump. The locus along which they jump is given by (3.21) (in which we note m and p are both predetermined). This locus has the slope $1/k$, and is depicted as

[7] See, for example, Taylor (1979), Ball (1994), Miller and Sutherland (1993).

the line OO below.

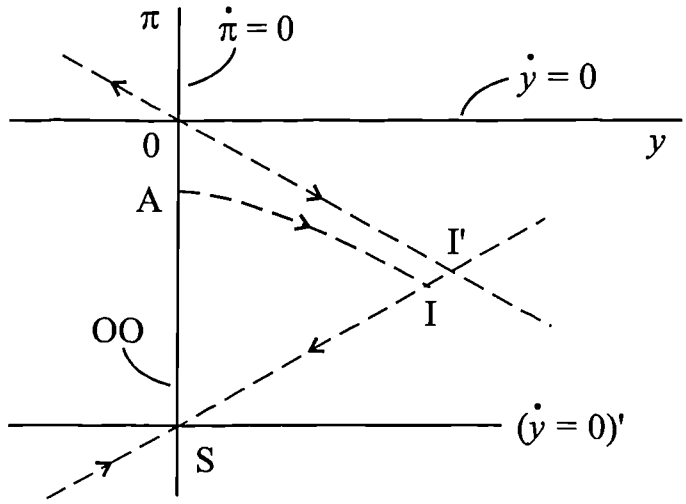

Figure 3.4: *Calvo's Model with Common Simplifications:* $k = r = 0$

First consider the model under the two common simplifications $k = r = 0$ (see Figure 3.4). The stationary loci $\dot{y} = 0$ and $\dot{\pi} = 0$ are then respectively vertical and horizontal, and OO is also vertical. Suppose the economy is in an initial steady state at 0, and there is an unanticipated disinflation, i.e. ϕ is raised above zero. This shifts the $\dot{y} = 0$ locus down, moving the steady state vertically down to S. Since OO is also vertical, the economy is in this case able to jump straight to the new steady state, with no change in output. This is the form of the "disinflation puzzle" noted by Buiter and Miller (1985): disinflation can be instantaneous and costless despite staggered prices. Now suppose the fall in monetary growth is preannounced: in this case, the economy jumps down to A upon announcement; it then travels

along AI, reaching I upon implementation, and thereafter converges along the saddlepath to S. In this case we get a version of Ball's (1994) stronger form of the disinflation puzzle: disinflation causes a boom.

One way out of the first puzzle, as has been acknowledged by these authors, is to let $k > 0$: OO then passes through O with a positive slope and intersects the saddlepath 'south-west' of S, so that output drops on impact and then recovers gradually.[8] $k > 0$ also helps us with the second puzzle, inasmuch as there will now be a slump upon announcement. However, it does not rule out the possibility of a boom farther along the adjustment path. Indeed, for a sufficiently long preannouncement, the model still implies that a boom is inevitable. This follows from the fact that as the preannouncement period tends to infinity, the path of the economy during this period tends closer and closer to the segment of the unstable separatrix OI'. For long preannouncements, $k > 0$ hence does not help very much with Ball's (1994) puzzle. To deal with this it is necessary to relax in addition the second simplifying assumption, i.e. to let $r > 0$.

Let us then turn to the general case where both k and r are non-zero. $r > 0$ implies that the $\dot{\pi} = 0$ locus now has a positive slope. It also implies that the $\dot{y} = 0$ locus becomes ambiguous in slope. There are two possibilities, depending on the sign of $a - kr$. When $a - kr$ is positive, $\dot{y} = 0$ is downward

[8]This case is illustrated in Blanchard and Fischer (1989).

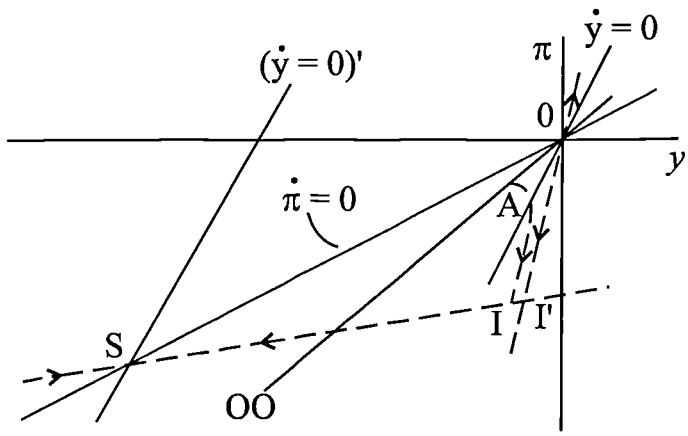

Figure 3.5: *Calvo's Model:* $a - kr < 0$

sloping, and the unstable separatrix remains downward-sloping. Sketching the phase diagram for this case, it is readily shown that a sufficiently pre-announced disinflation continues to cause a boom. When $a - kr$ is negative, however, $\dot{y} = 0$ is upward-sloping, and the unstable separatrix becomes upward-sloping. This is the situation illustrated in Figure 3.5. Repeating the earlier reasoning, a typical time path for a preannounced disinflation is AIS. We see that output is never higher than its original level along this path. Moreover, as the preannouncement period tends to infinity so that the path tends to 0I'S, it remains true that no boom emerges.

We therefore see that, even in the basic directly postulated model, it is possible to avoid the puzzle of disinflationary booms. To do this we need to abandon both of the common simplifying assumptions $k = 0$ and $r = 0$, and moreover to abandon them by a sufficient margin, such that $kr > a$. It does

not appear to have occurred to earlier authors to look at this case, owing to their prior beliefs that k and r were small. However our investigation of microfoundations suggests that this case is, indeed, the relevant one. In the microfounded model, we saw that β, which is the counterpart of $1/(1+r)$ here, also determines the sensitivity of aggregate demand to the nominal interest rate κ (see again 3.3), i.e. the counterpart of k here; and that as β tends to unity, κ tends to infinity. This gave rise to our finding that as β tends to unity, the short-run slump caused by an unanticipated disinflation becomes arbitrarily large. The microfoundations therefore suggest that in the directly postulated model k and r should not be regarded as independent: k should be seen as a decreasing function of r. They moreover suggest that it is inconsistent to set both k and r to zero: k and r should be jointly bounded away from zero. From what we have just seen of the macroeconomic behaviour, we can now suggest that the appropriate joint bound is, in fact, $kr > a$.

3.5 Non-Linear Simulations

As said above, given the complexity of the model, it turns out that the case $\Phi = 1$ is the only analytically tractable. Since we wanted to take an analytical rather than a numerical approach, we were forced to confine ourselves to this particular case in the main text. In fact, when the model is log-linearised

around a steady state in which $\Phi \neq 1$, we need to resort to simulations. That is what we do in the Appendix 3.7.3. We thus provide some simulation results to check the robustness of our analytical findings. These simulations are performed with the package for non-linear models DYNARE, written by Michel Juillard at CEPREMAP (see Juillard (1996)).[9] This package solves discrete, deterministic, dynamic systems with forward-looking variables using an algorithm described in Laffargue (1990) and Boucekkine (1995), and using GAUSS as the main software. The algorithm rests upon Newton-Raphson iterations and the triangulation of a large matrix by Gauss's elimination. The advantage is that it does not log-linearise the model, but it simulates its non-linear dynamics. The algorithm is thus very appropriate for our purposes. In fact a disinflationary policy involves a transitional dynamics that move from one steady state to another one. Hence, unless the two steady state are 'infinitesimally' close, approximating the model dynamics by log-linearisation could turn out to be misleading. Furthermore, the fact that the algorithm works only for deterministic model does not create any problems, given the particular policy experiment we are concerned with: a one–and-for-all change in policy (i.e., permanent change in the rate of growth of money, anticipated and not).

In the Appendix 3.7.3 we tackle two issues. First, we will check the accu-

[9] I would like to thank Michel Juillard for the time he devoted to me to help me with these simulations, while I was visiting Paris I-M.A.D..

racy of the log-linear approximation. Second, we will check the robustness of the main results in propositions 3.1 and 3.2 with respect to a starting point different from $\Phi = 1$. With respect to the first issue, we conclude that all the results stated in the two propositions for the log-linearised model holds also for the non-linear dynamics and for relevant size of disinflations. Hence, the approximation implied by the log-linearisation is pretty accurate. With respect to the second issue, we were able to find counterexamples to proposition 3.1.and 3.2. Indeed, as we know from the previous chapter, if the initial inflation rate is positive, the output long-run gains of a disinflation can be positive. That is, in the long run output will be above its initial value, hence a boom. In the case of an unanticipated disinflation, the simulation results show the following: (i) after an unanticipated disinflation from positive inflation, the short-run dynamics induce a slump which will be offset by the long-run gains; (ii) an unanticipated disinflation will never cause 'overshooting' in the sense of output levels bigger than the final one (i.e., the new steady state). Nevertheless, this could be the case for preannounced disinflations, if the anticipation period is sufficiently long. In the two cases shown in the Appendix (i.e., from 5% to 3% and from 10% to 5%), for example, we need at least 8 periods preannouncement to get such behaviour. However, these preannouncement periods are rather long. Indeed, if we suppose that nominal wages are fixed for one year, then one period in the model would correspond

to six months. Hence, 8 periods are 4 years. Moreover, since preannouncement smoothes the transitional dynamics the size of the overshooting is really marginal.

3.6 Conclusions

In this chapter we have used the dynamic general equilibrium model with staggered wages presented in the first two chapters to study analytically the effects of a reduction in the rate of monetary growth (a *disinflation*). We find that the result of disinflation is a recession in the short and medium run, and that output will be slightly lower in the long run, too. This is true both when the disinflation is unanticipated and when it is announced in advance.

Our particular motivation was the puzzling finding of Ball (1994), in a directly postulated model, that disinflations cause booms. We first noted that this finding is associated with the element of preannouncement in the policy assumed by Ball. More microfoundations tell us that Ball's paradox is mainly due to simplifying assumptions regarding the time preference rate and the formulation of the aggregate demand equation. These simplifications are inconsistent with microfoundations - at least, with the particular rather standard set of microfoundations introduced here. The microfounded model produces a reaction to a disinflation (a reduction in monetary growth) which is not, after all, sharply different from the standard reaction to a deflation (a

reduction in the level of the money supply) found in Taylor's (1979) model. Hence, in contrast to what several authors have recently concluded, it does not appear necessary to appeal to lack of policy credibility in order to explain why disinflations cause slumps.

3.7 Appendix

3.7.1 Properties of λ_s , λ_u

The characteristic equation of the difference equation (3.7) may be written as (recalling $b = 1 - d = 1/(1-\beta)$)

$$\lambda^2 = \frac{(h+1)(1+\beta)}{\beta}\lambda - \frac{1}{\beta} \tag{3.24}$$

We plot the left-hand side (LHS) and right-hand side (RHS) of this below.

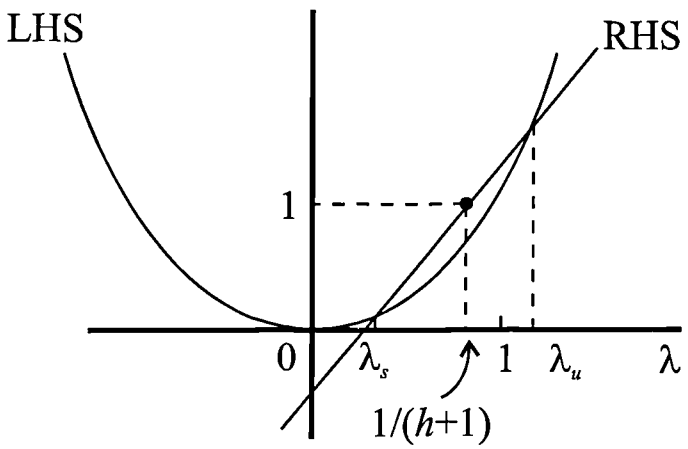

Figure 3.6: *Properties of the Eigenvalues*

The LHS is a parabola, while the RHS is a line with positive slope and

negative intercept. The RHS passes through the point $(1/[h+1], 1)$, which lies unambiguously above the parabola. As β goes from 0 to 1 the line pivots clockwise around the point, going from vertical to a slope of $2(h+1)$. From this we can immediately see that:

(i) The smaller eigenvalue, λ_s, lies strictly between 0 and 1, for all values of β and h.

(ii) The larger eigenvalue, λ_u, is strictly greater than 1, for all values of β and h.

Further, since $\lambda_s \lambda_u = 1/\beta$ (from the characteristic equation), it follows from (i) that $\lambda_u > 1/\beta$.

3.7.2 The Time Path under Anticipated Disinflation

Our aim is to find the time path of v_t. The general form of the difference equation for v_t is (combining (3.9) and (3.10))

$$-bv_{t-1} + (h+1)v_t - dv_{t+1} = -2hbz_t - d\phi_{t+1} + b\phi_t \tag{3.25}$$

where

$$\phi_t = \begin{cases} 0 & \text{for } t \leq T-1 \\ \phi > 0 & \text{for } t \geq T \end{cases} \quad \text{and} \quad z_t = \begin{cases} 0 & \text{for } t \leq \tau - 1 \\ \frac{\beta^{T-t}}{1-\beta}\phi & \text{for } t = \tau, ..., T-2 \\ \frac{\beta}{1-\beta}\phi & \text{for } t \geq T-1 \end{cases}$$

Substituting ϕ_t, z_t out, we have three versions of the difference equation

in v_t

$$-bv_{t-1}+(h+1)v_t-dv_{t+1} = -\frac{2hb\beta^T}{1-\beta}\phi(1/\beta)^t \qquad \text{for } t=\tau,...,T-2 \tag{3.26}$$

$$-bv_{t-1}+(h+1)v_t-dv_{t+1} = -\frac{2hb\beta}{1-\beta}\phi+(b-d)\phi \qquad \text{for } t \geq T \tag{3.27}$$

$$-bv_{T-2}+(h+1)v_{T-1}-dv_T = -\frac{2hb\beta}{1-\beta}\phi-d\phi \qquad \text{for } t=T-1 \tag{3.28}$$

The indefinite solutions to (3.26) and (3.27) are

$$v_t = A_1\lambda_s^t + A_2\lambda_u^t - \frac{2\beta^T}{1-\beta^2}\phi(1/\beta)^t \qquad \text{for } t=\tau-1,...,T-1 \tag{3.29}$$

$$v_t = B_1\lambda_s^t + B_2\lambda_u^t + v \qquad \text{for } t \geq T-1 \tag{3.30}$$

where the eigenvalues λ_s, λ_u are determined as in 3.7.1 above. (3.26) has a time-varying 'constant' term: its solution hence involves a time-varying particular integral: see, e.g., Chiang (1974). A_1, A_2, B_1, B_2 are constants of integration, to be determined below. Note the ranges of t for which these equations hold: this is because they must be satisfied by all instances of v_t to which (3.26) and (3.27) apply.

We now seek to solve for A_1, A_2, B_1, B_2 using the known boundary conditions on the time path. First, since $\lambda_u > 1$, convergence from date T onwards clearly requires $B_2 = 0$: this is the usual saddlepath condition. Next, $v_{\tau-1} = 0$ in the initial steady state, so (3.29) must satisfy this

$$0 = v_{\tau-1} = A_1\lambda_s^{\tau-1} + A_2\lambda_u^{\tau-1} - \frac{2\beta^T}{1-\beta^2}\phi(1/\beta)^{\tau-1} \tag{3.31}$$

Further, writing out (3.29) for the last two periods in which it holds, and (3.30) for the first two periods in which it holds, we have

$$v_{T-2} = A_1\lambda_s^{T-2} + A_2\lambda_u^{T-2} - \frac{2\beta^T}{1-\beta^2}\phi(1/\beta)^{T-2} \tag{3.32}$$

$$v_{T-1} = A_1\lambda_s^{T-1} + A_2\lambda_u^{T-1} - \frac{2\beta^T}{1-\beta^2}\phi(1/\beta)^{T-1} \tag{3.33}$$

$$v_{T-1} = B_1\lambda_s^{T-1} + v \tag{3.34}$$

$$v_T = B_1\lambda_s^{T} + v \tag{3.35}$$

(3.31)-(3.35) together with (3.28) provide us with six equations in the six unknowns $(A_1, A_2, B_1, v_{T-2}, v_{T-1}, v_T)$. Since they are linear in the unknowns, we can solve them explicitly. For A_1, A_2, B_1 we get, after some work

$$A_1 = \left[\frac{2\beta h}{(1-\beta)^2}\beta^{T-\tau} - \frac{h+1-\lambda_s}{\lambda_u-\lambda_s}\left(\frac{1}{\lambda_u}\right)^{T-\tau}\right]\lambda_s^{1-\tau}\frac{1}{h}\frac{1-\beta}{1+\beta}\phi \tag{3.36}$$

$$A_2 = \frac{h+1-\lambda_s}{\lambda_u - \lambda_s}\lambda_u^{1-T}\frac{1}{h}\frac{1-\beta}{1+\beta}\phi \tag{3.37}$$

$$B_1 = \left[\frac{2\beta h}{(1-\beta)^2}\beta^{T-\tau} - \frac{h+1-\lambda_s}{\lambda_u - \lambda_s}\left(\frac{1}{\lambda_u}\right)^{T-\tau}\right.$$

$$\left. + \frac{h+1-\lambda_u}{\lambda_u - \lambda_s}\left(\frac{1}{\lambda_s}\right)^{T-\tau}\right]\lambda_s^{1-\tau}\frac{1}{h}\frac{1-\beta}{1+\beta}\phi \tag{3.38}$$

(Here, some simplification has been achieved by making use of the characteristic equation, and also of the relations, i.e., $\lambda_s\lambda_u = 1/\beta$, $\lambda_s + \lambda_u = (h+1)(1+\beta)/\beta$, which it implies.) Substituting these values back into (3.29) and (3.30) then completes the solution for v_t.

3.7.3 Some Numerical Simulations

Accuracy of the log-linear approximation

We now check if the results obtained for the log-linear model still hold when the full non-linear dynamics is taken into account. First, with respect to unanticipated disinflations, proposition 3.1 states that: (i) output is always lower than the original level; (ii) as $\beta \to 1$, the size of the long-run slump tends to zero and the size of the short-run slump tends to infinity. We simulate the non-linear dynamics of the model for a disinflation from 0 to -2%.[10] The parameter values we used correspond to the 'base case' described

[10]This change is sufficiently big to question the validity of a log-linear approximation.

in Chapter 2, see footnote 15 therein.[11] In Figure 3.7 β is set equal to 0.9, in Figure 3.9 $\beta = 0.6$ and in Figure 3.8 $\beta = 0.3$.

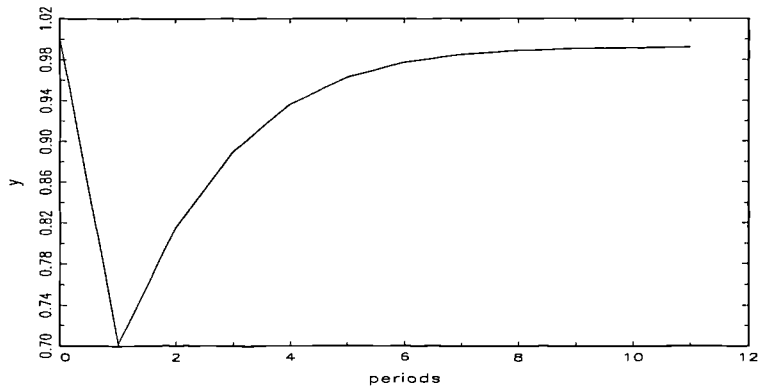

Figure 3.7: *Disinflation from 0 to -2%,* $\beta = 0.9$

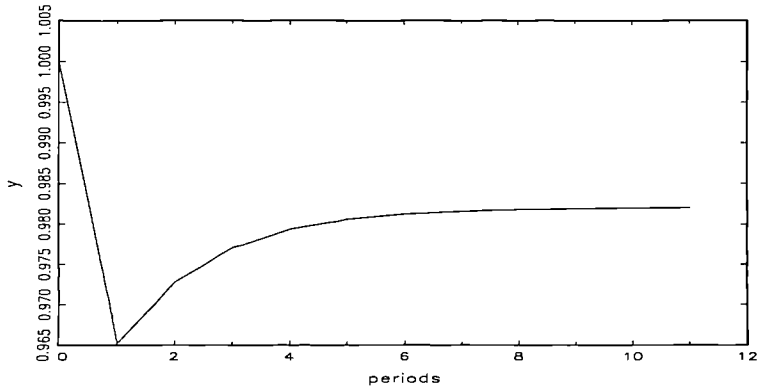

Figure 3.8: *Disinflation from 0 to -2%,* $\beta = 0.6$

Moreover, the simulation results do not change qualitatively for disinflation of even bigger size.

[11] The only exeption is the parameter α which is set such that the starting steady state output level is normalised to 1 in all the simulations presented in this appendix.

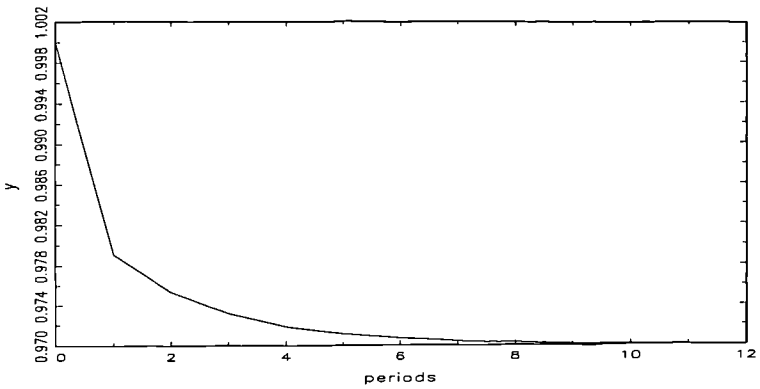

Figure 3.9: *Disinflation from 0 to -2%,* $\beta = 0.3$

Both the results in proposition 3.1 are confirmed. As we know from Chapter 2, if the time preference rate is positive (i.e., $\beta < 1$), then the long-run effect of a disinflation from 0 to -2% is negative. Moreover, all along the path of the transitional dynamics between the two steady states, output never exceeds its original level. Besides, the smaller β, the smaller the size of the short-run effect and the larger the size of the long-run effect. Furthermore, Figures 3.7 and 3.9 correspond to the left picture of Figure 3.2 in the main text, while Figure 3.8 to right one, since $(1-\beta)^2 > 2\beta h$ in this latter case.[12]

Second, with respect to preannounced disinflations, proposition 3.2 states that: (i) the announcement causes a slump on impact; (ii) output is always lower than the original level all along the transitional path. Figure 3.10 corresponds to the left picture of Figure 3.3. The next two Figures show

[12] Given our parameter values $h = \frac{2e}{(e-1)(\varepsilon-1)} = \frac{2*4.5}{3.5*5} = 0.514$.

that even for very long preannouncement output never exceeds its starting level. Moreover, they visualise the effect of the preannouncement: anticipation unsurprisingly smooths the transitional dynamics, in the sense that it diminishes both the negative impact effect and the subsequent recovery. Indeed, the transitional dynamics in Figure 3.12 is basically flat in comparison with the others, as can readily be checked by looking at the scale of the vertical axis.

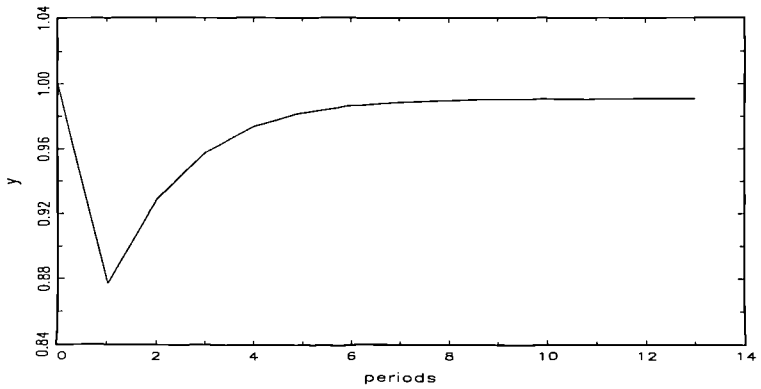

Figure 3.10: *From 0 to -2%, 2 Periods Preannouncement,* $\beta = 0.9$

In conclusion, all the results stated in the two propositions for the log-linearised model holds also for the non-linear dynamics and for relevant size of disinflations. Hence, the approximation implied by the log-linearisation is pretty accurate.

Robustness of the main results

What happens if the starting level of steady state inflation is different from 0? Can we find any counterexamples in the transitional dynamics?

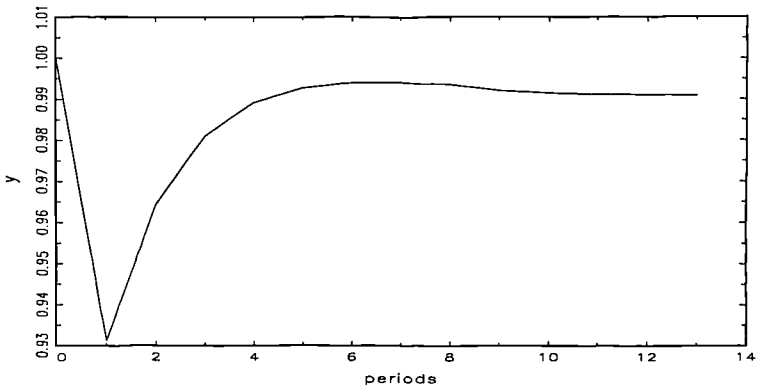

Figure 3.11: *From 0 to -2%, 8 Periods Preannouncement,* $\beta = 0.9$

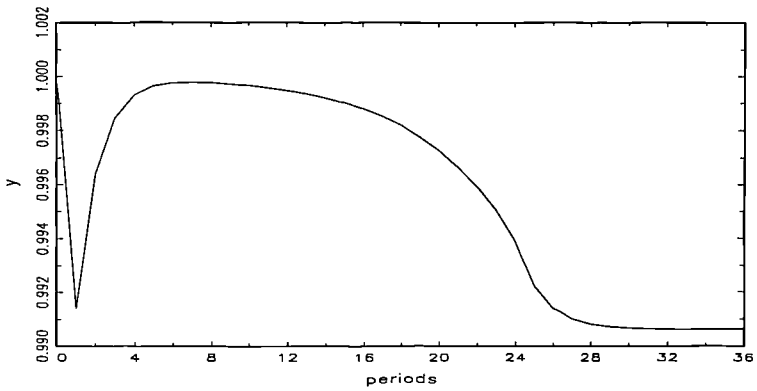

Figure 3.12: *From 0 to -2%, 24 Periods Preannouncement,* $\beta = 0.9$

Analytically we cannot answer this question, but simulations can. Obviously, one can perform all sorts of simulations for all of the possible combination of admissible parameter values. Hence, we need guidance and again we stick with the parameter values of footnote 15 in Chapter 2. We have seen that if the starting level of inflation, from which disinflation starts, is zero, then disinflation will induce a slump in the short, medium and long-term. Would these features still hold if a disinflation starts from a positive inflation rate,

as after all happens in the real world? The short answer is only in the short-run. The intuition is very simple and actually we would not need any simulation at all. From Chapter 2, we know already that the long-run effect is likely to be positive, for positive level of inflation, as Figure 2.3 showed. If we are at the right of the maximum Y_{ss} in Figure 2.3, then a disinflation would have a positive long-run effect on output. Moreover, in the previous chapter we also showed that these effects (the non-superneutrality effects) are likely to be big, and the bigger, the higher the starting level of inflation. Output will hence rise in the long-run, and the bigger these long-run gains, the faster the output level will recover from the slump on impact, as Figures 3.13 and 3.14 show.[13] Summing up, following an unanticipated disinflation from positive inflation, the short-run dynamics induce a slump which will be offset by the long-run gains. However, an unanticipated disinflation will never cause 'overshooting' in the sense of output levels bigger than the final one (i.e., the new steady state).

Nevertheless, this could be the case for preannounced disinflations, if the anticipation period is sufficiently long. In the two cases above (i.e., from 5% to 3% and from 10% to 5%), for example, we need at least 8 periods preannouncement to get such behaviour (see Figures below). However, these

[13]The velocity of adjustment of the transitional dynamics depends on the root of the dynamic equations. The next chapter will deal with the issue of the dependence of the velocity of adjustment (i.e., persistence) on the starting level of steady state inflation.

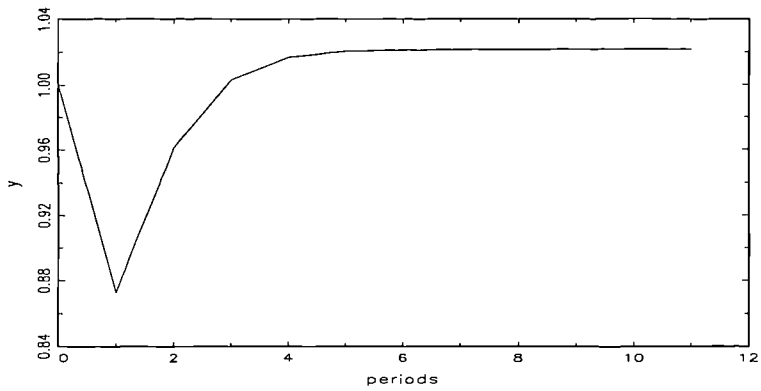

Figure 3.13: *From 5% to 3%, Base Case, No Anticipation*

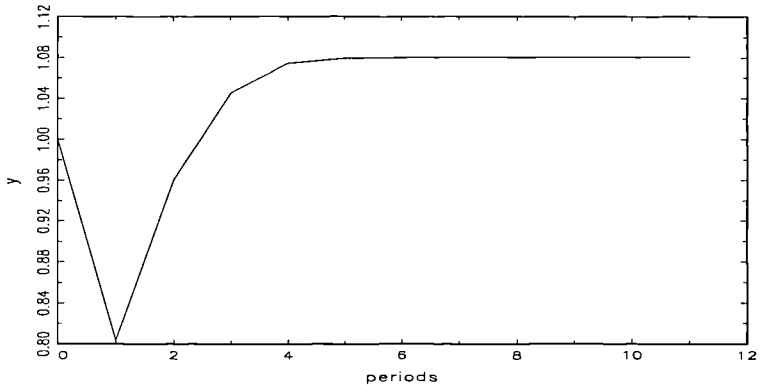

Figure 3.14: *From 10% to 5%, Base Case, No Anticipation*

preannouncement periods are rather long. Indeed, if we suppose that nominal wages are fixed for one year, then one period in the model would correspond to six months. Hence, 8 periods are 4 years. Moreover, since preannouncement smoothes the transitional dynamics the size of the overshooting is really marginal.

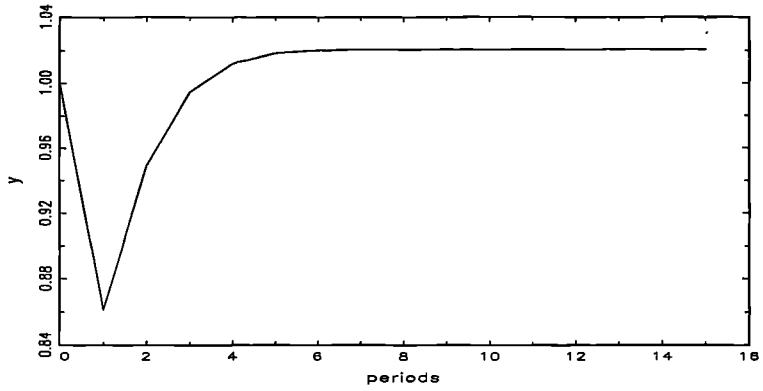

Figure 3.15: *From 5% to 3%, 4 Periods Preannouncement*

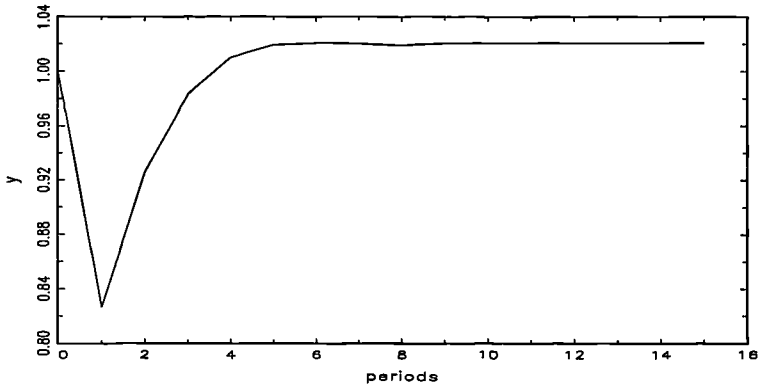

Figure 3.16: *From 5% to 3%, 8 Periods Preannouncement*

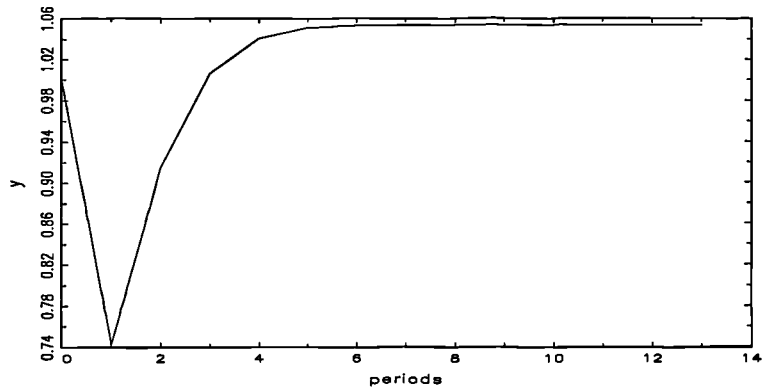

Figure 3.17: *From 10% to 5%, 4 Periods Preannouncement*

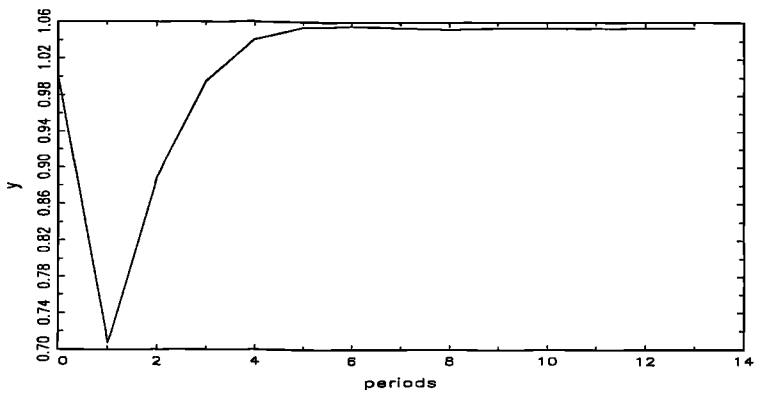

Figure 3.18: *From 10% to 5%, 8 Periods Preannouncement*

Chapter 4

Staggered Wages and Persistence in the Real Effects of Money Shocks

4.1 Introduction

In the previous two chapters we have used the dynamic general equilibrium model described in Chapter 1 to look at the effects of changes in the *rate* of money growth, both in the long-run (Chapter 2) and in the short-run (Chapter 3). In this chapter, instead, we turn to look at the effects of changes in the *level* of the money supply. This brings about the very important issue of the persistence of the real effects of money shocks, which will be the focus

of the remaining three chapters of the thesis.

The evidence that the GDP process contains a unit root has been interpreted as rejecting traditional theories of economic fluctuations which assume the business cycle to be mainly driven by temporary nominal disturbances. In their seminal article, Nelson and Plosser (1982) claim that their finding *"gives an important role to real factors in output fluctuations and places limits on the importance of monetary theories of the business cycle."* (Nelson and Plosser (1982), p. 161) The same conclusion was supported by the analysis of Campbell and Mankiw (1987), even if these authors were more cautious in drawing any definite conclusion.[1] However, following the interpretation of Nelson and Plosser (1982), numerous researchers started to develop what is already a massive branch of the literature: the real business cycle literature.

Recent papers by West (1988) and Phaneuf (1990) try to challenge this view. They both use Taylor's (1979, 1980a) staggered wage model with a feedback monetary policy rule and with monetary shocks only. They show that, for plausible values of the parameters, the model is able to generate high persistence of money shocks. It seems that staggered wage models could induce a near-random walk behaviour in GDP, that is, an autoregressive root of about 0.8, statistically indistinguishable from a unit root in finite sample.

However, we already stressed in Chapter 1 that Taylor's (1979, 1980a)

[1] *"A conclusion as extreme as that of Nelson and Plosser is of course not necessary."* (Campbell and Mankiw (1987), pp. 876-877)

model is an *ad hoc* log-linear structural model. Wage setting rules are exogenously specified at the outset and the parameters are likely not to be policy-invariant (see Taylor (1980b)). Taylor (1979) openly acknowledges the need for microfoundations. In the first two chapters, we developed a dynamic general equilibrium model on which we imposed exactly Taylor's (1979) staggered wage structure. In this chapter, the aim is to use this model in order to open the "black box" of the structural *ad hoc* parameters of the famous Taylor wage setting equation. We can then ask whether the microfounded model confirms previous findings by West (1988) and Phaneuf (1990). Notwithstanding the apparent simplicity of Taylor's model (1979), it turns out that a log-linearised version of our model exactly coincides with it. We are therefore able to show how the parameters of Taylor's wage setting equation depend upon the microeconomic fundamentals and the conduct of monetary policy.

Our main finding is that high persistence of the real effects of money shocks in staggered wage models is an unlikely outcome. Consequently, our result refutes the earlier view of West (1988) and Phaneuf (1990) for almost any reasonable values of the microeconomic parameters and of the underlying rate of inflation.

In more detail: first, the model log-linearised around a zero inflation steady state illuminates the role of the microeconomic parameters in gen-

erating persistence. Blanchard (1990) stressed that a low responsiveness of nominal wages to the business cycle conditions was a key factor to generate high persistence of money shocks in staggered wage models. We provide new findings for staggered wage models regarding the interrelation among the responsiveness of nominal wages to the business cycle, the income effect on labour supply and the intertemporal elasticity of substitution of labour. In particular, it is shown that a staggered wage model can generate significant persistence, even if far from near-random walk behaviour in GDP, in a zero steady- state inflation economy for some values of the underlying parameters. However, in accordance with previous findings (see Blanchard and Fischer (1989) and Chari et al. (1996)), we show that a high degree of persistence, in the sense of near-random walk behaviour in GDP, can only arise from a low income effect on labour supply and a high intertemporal elasticity of substitution of labour.

Second, and most importantly, our intertemporal microfounded model reveals that the conduct of monetary policy affects the structural parameters of Taylor's wage setting equation, providing a clear example of the Lucas critique. Specifically, it is shown that non-zero steady state inflation modifies the degree of forward and backward looking in Taylor's wage setting equation, hence altering the degree of persistence implied by the model. As a consequence, even in the cases where the microfounded parameters are such

that they could generate persistence in a zero inflation economy, a positive, but still low, steady state rate of inflation diminishes persistence sharply. Moreover, as a consequence: (i) the model is highly non-linear; (ii) the inertia of the system and the short-run output-inflation trade-off is inversely related to the level of average inflation as found by Ball et al. (1988).

We conclude that either sensible values of the microeconomic parameters, or a moderate rate of underlying inflation as observed in western economies, or both, cut down persistence far below near random-walk behaviour.

Some of our results confirm those already suggested by Blanchard and Fischer (1989) and Romer (1996). Indeed in section 4.3, we will develop as an example a comparison between our model and Blanchard and Fischer's (1989) one. However, these text-book models are a hybrid between *ad hoc* and microfounded models. For example, the Blanchard and Fischer (1989) model derives from a simplified version of Blanchard and Kiyotaki's (1987) static model. On this simplified version they superimpose *ad hoc* dynamics due to Taylor's staggered structure of price decisions. Thus, while in their models the dynamics are superimposed on a static model in a somewhat *ad hoc* way, the present model is truly intrinsically dynamic, since it is derived from an explicit intertemporal optimisation process. Consequently, while our model is somewhat similar to the Blanchard and Fischer (1989) and Romer (1996) models when log-linearised around a zero inflation economy,

similarities stop when a more realistic positive rate of underlying inflation is considered. In other words, by construction, the Lucas critique point cannot be addressed by the hybrid models of Blanchard and Fischer (1989) and Romer (1996).

Our analysis is also very related to a recent contemporaneous and independent contribution by Chari et al. (1996) with whom we share the same main result.[2] However, despite the similarities between the two analyses, the two studies have distinct features. (i) While Chari et al. (1996) build a dynamic general equilibrium model with staggered price setting, the model presented in this chapter is closer in spirit to Taylor's original (1979) model in that it explicitly considers the labour market and the optimal wage setting rule. As we will see, this actually makes a difference in a zero inflation economy. (ii) Our analysis is analytically oriented and tries to find explicit solutions and comparisons with previous results to explain the mechanism at work. On the other hand, Chari et al. (1996) rely heavily on calibration and simulation techniques. This enables them to simulate quite a number of different versions of their basic model, providing robustness checks of their main finding. (iii) Our analysis focuses on the important issue of the relation

[2]It is maybe important to note that this main finding seems to have already generated a debate in the literature where most try to oppose to it (e.g., Andersen (1998a,b), Erceg (1997), Jeanne (1998), Kiley (1997)), while others reinforce it (e.g., Ellison and Scott (1998)).

between the policy behaviour and agents' response, while Chari et al. (1996) do not mention this point. Thus, the two works seem to be complementary.

We also develop later a brief comparison with the recent Rotemberg and Woodford (1997) intertemporal model of staggered prices, explaining some features (and casting some doubts on the robustness) of their results.

The chapter is organised as follows. The next section compares the model presented here with Taylor's (1979) one and briefly with the ones in Chari et al. (1996) and Rotemberg and Woodford (1997). Section 4.3 compares the model with the one in Blanchard and Fischer (1989), Chapter 8. This seems useful to fully understand the relation between the level of persistence and the microeconomic fundamentals. Section 4.4 studies how the degree of persistence varies as the microeconomic fundamentals vary, shows the importance of non-linearity and how the Lucas critique applies very much to Taylor's model. Section 4.5 concludes.

4.2 The Optimal Wage Setting Rule

Taylor's (1979) model and related literature

First we need to recall very briefly Taylor's model, as presented in the first section of Chapter 1. Taylor's model consists of the following three equations

$$x_t = b\ x_{t-1} + d\ E_{t-1}x_{t+1} + \gamma\ (b\ E_{t-1}y_t + d\ E_{t-1}y_{t+1}) \tag{4.1}$$

$$p_t = \frac{1}{2}(x_{t-1} + x_t) \tag{4.2}$$

$$y_t = m_t - p_t \tag{4.3}$$

where, as explained in Chapter 1, p = price level, x = 'new' nominal wage; y = output; m = money supply and all the variables are expressed in terms of log-deviation from trend. E_{t-1} represents the conditional expectation of a variable based on the information available at the beginning of period t (or end of period $t-1$).

For the argument in this and above all in the next chapter, it is worth recalling also that in Taylor's words: *"the behavioral equations reflect a relative wage concern on the part of the workers"*. However, substituting equation (4.2) into (4.1), it yields

$$x_t = b\, p_t + d\, E_{t-1}p_{t+1} + \frac{\gamma}{2}\left(b\, E_{t-1}y_t + d\, E_{t-1}y_{t+1}\right) . \tag{4.4}$$

As already noted by Buiter and Jewitt (1981) and Blanchard (1990), this means that in setting the wage, workers care only about their absolute real wage. In other words, workers care about the wages in the other sectors only through the effect these wages have on the price level. In contrast to Taylor's words, there is no actual real wage concern *per sé* on the part of the workers. This point is explicitly demonstrated in a dynamic general equilibrium setting in this chapter. Indeed, our workers, who are "neoclassically" concerned only about their absolute real wage, give rise to the same equations as in Taylor's model. We want to stress this point because the next chapter will investigate the possible role that relative wage considerations can play in explaining

persistence in staggered wage models.

For a given expected path of the money supply, the model exhibits the following saddle path solution

$$x_t = \lambda_s x_{t-1} + \sum_{i=0}^{\infty} \frac{(\varphi - 1)\lambda_s}{b} \left(\frac{1}{\lambda_u}\right)^i [bE_{t-1}(m_{t+i}) + dE_{t-1}(m_{t+1+i})] \quad (4.5)$$

where

$$\lambda_s = \frac{\varphi - \sqrt{\varphi^2 - 4d(1-d)}}{2d}; \quad \lambda_u = \frac{\varphi + \sqrt{\varphi^2 - 4d(1-d)}}{2d}; \quad \varphi = \frac{1 + \frac{\gamma}{2}}{1 - \frac{\gamma}{2}} . \quad (4.6)$$

λ_s and λ_u are respectively the stable and the unstable root of the saddle equilibrium. If $d = b = 1/2$, then λ_s reduces to $\lambda_s = \varphi - \sqrt{\varphi^2 - 1} = \frac{1-\sqrt{\gamma/2}}{1+\sqrt{\gamma/2}}$.

Now suppose m_t follows a random walk. Then, (4.5) becomes

$$x_t = \lambda_s x_{t-1} + (1 - \lambda_s) m_{t-1} \quad (4.7)$$

and the dynamics of output are given by

$$y_t = \lambda_s y_{t-1} + (m_t - m_{t-1}) + \frac{1}{2}(1 - \lambda_s)(m_{t-1} - m_{t-2}) . \quad (4.8)$$

The model therefore exhibits persistence in the real effects of money shocks. Persistence of money shocks then basically depends on two parameters: the degree of forward looking behaviour (i.e., d) and the degree of sensitivity of the money wage to business cycle conditions (i.e., γ). The higher d, the lower the inertia of the aggregate wages. However, d is thought to be relatively uninteresting and it is often put equal to $\frac{1}{2}$ in the literature,

which focused the attention on γ as the crucial parameter to determine the degree of persistence.[3] The intuition of the key importance of γ is the following: given (4.3), money shocks can have significant and prolonged effects on output only if the price level adjusts slowly and, given (4.1) and (4.2), the higher γ, the higher the sensitivity of the nominal variables to movements in output, the faster the adjustment of prices. It is easy to show, in fact, that λ_s is a decreasing function of γ. For the US, Taylor (1980b) estimated γ to be between 0.05 and 0.1, while Sachs (1980) estimated γ to be between 0.07 and 0.1.

Another implication of this model is that it exhibits a Phillips-curve-type output-inflation trade-off. Following a positive monetary shock, prices adjust sluggishly and output is temporarily above its natural level. This trade-off would depend on λ_s (and hence on d and γ): the higher λ_s, the flatter the Phillips curve.

West (1988) and Phaneuf (1990) investigate whether this model could generate near random-walk behaviour in output. Phaneuf (1990) closed the model with the following equation, representing a feedback policy rule[4]

$$m_t = ap_t + vy_t \tag{4.9}$$

[3]See, for instance, Driskill and Sheffrin (1986), DeLong and Summers (1986), Ambler and Phaneuf (1989, 1992).

[4]Taylor (1979, 1980a,b) sets $v = 0$.

Incorporating this equation, the solution of the system changes slightly, since now $\varphi = \left(\frac{1+(1-c)\gamma/2}{1-(1-c)\gamma/2}\right)$, where $c = (a-v)/(1-v)$. The parameter c is a policy parameter which indicates the degree of accommodation of monetary policy to price changes: the higher c, the more accommodative is monetary policy. Analytically, the role of c parallels the one of γ and a lower value of c causes, *ceteris paribus*, a lower degree of persistence (as c approaches one, φ approaches unity). In other words, introducing (4.9) in the model is analytically equivalent to multiplying γ by $1-c$. Phaneuf (1990) took estimated values of γ and c for Canada, Germany, Italy, United Kingdom and US from some previous studies in the literature. He found γ to lie between 0 and 0.32 and c to lie between 0.71 and 0.91. Despite the fact that γ was on average found to be bigger than Taylor's (1980b) and Sachs' (1980) estimates, Phaneuf (1990) found that countries associated with higher values of γ exhibited also higher values of c. This observation led him to determine λ_s to be between 0.65 and 1 and to conclude : *"...the evidence of a unit root in the real GNP process of many countries is not necessarily inconsistent with a contract-based approach to the business cycle. [...] the asynchronization of wage contracts can potentially play the role of an important dynamic propagation mechanism and can contribute substantially to the persistence of output fluctuations."* (Phaneuf (1990), p. 590) Moreover, he acknowledged the potential role of d in generating persistence: the lower d, the higher

the backward looking bias in Taylor's wage setting rule and the higher the persistence. However: *"Given that no evidence is currently available on the direction of this bias, if any, it would seem that one important item on the agenda of future research should be to determine empirically the value* [of d]." (Phaneuf (1990), pp. 590- 591)

West (1988) considered two different monetary policy rules: one targeted the interest rate and the other targeted the money supply. Then he simulated Taylor's model in these two different cases, choosing values of γ between 0.01 and 0.1. He concluded that in an economy characterised by overlapping nominal contracts money shocks could induce near random-walk behaviour in output.

In what follows we will use the log-linearised version of our model to investigate the same issue of Phaneuf (1990) and West (1988). With respect to Phaneuf (1990), we can detect the micro determinants of d and hence of the forward vs. backward looking bias. With respect to West (1988), we demonstrate that the parameters of Taylor's wage setting equation are likely to depend heavily on the monetary policy rule. Moreover, by the same token, we also provide an explanation of Phaneuf (1990) finding that countries in which the monetary policy is more loose exhibit a higher value of γ.

The optimal wage setting rule

Let us recall the first order condition of our model for a general utility

function, that is

$$u_C(t) = R_t \beta u_C(t+1) \tag{4.10}$$

$$u_{M/P}(t) = (1 - 1/I_t) u_C(t) \tag{4.11}$$

$$X_t = \left(-\frac{\varepsilon}{\varepsilon - 1}\right) \left[\frac{u_L(t) + \beta u_L(t+1)\frac{K_{t+1}}{K_t}}{u_C(t)\frac{1}{P_t} + \beta u_C(t+1)\frac{K_{t+1}}{K_t P_{t+1}}}\right] \tag{4.12}$$

where $u_s(t) = \frac{\partial u(C_t,(M/P)_t,L_t)}{\partial s_t}$, for $s = C, M/P, L$. We have already commented on these equations in Chapter one. However, even at the cost of repeating ourselves, a further comment on the optimal wage setting rule is useful. Contrary to the hybrid textbook models of Blanchard and Fischer (1989) and Romer (1996), the wage setting rule (4.12) results from intertemporal optimisation. As a consequence, it is not simply given by the average (with weights equal to 1/2) of the optimal wages (derived from a static model) in the two periods. In (4.12), the weight on 2nd-period values is the composite term K_{t+1}/K_t and the discount factor β. Moreover, K_{t+1}/K_t evaluated in an inflationary steady state is equal to $(1 + rgm)^{\varepsilon}$. Hence, once the wage setting rule is explicitly derived from intertemporal optimisation: (i) the endogenous weights determine the degree of backward versus forward looking behaviour; (ii) the parameters of the log-linearised version of (4.12) are not structural, but depend on policy. The analysis of this third first-order condition and the comparison between it and Taylor's wage setting equation is the focus of the present chapter.

Log-linearising (4.12), around a steady state we get the following expression

$$x_t = b_1 p_t + d_1 p_{t+1} + b_2 y_t + d_2 y_{t+1} + b_3 m_t + d_3 m_{t+1} \tag{4.13}$$

where again lower case letters are used for variables as log-deviations from steady state. b_i and d_i , for $i = 1, 2, 3$, depend upon technology and preferences parameters and upon the steady state rate of money growth, as shown in the Appendix 4.6.

(4.13) corresponds to (4.4). Therefore, as we stressed above, while Taylor justifies his wage setting rule by arguing that there is a "Keynesian" relative wage concern on the part of the workers, equation (4.4) shows his model to be analytically equivalent to one in which workers are "neoclassically" concerned about their real wage, as in the model illustrated here.

Log-linearising the aggregate price level equation, i.e., (2.16) we obtain the following expression

$$p_t = \left(\frac{1-\sigma}{\sigma}\right) y_t + q x_t + (1-q) x_{t-1} \qquad q = \left(\frac{1}{1+\Phi^{1-\varepsilon}}\right) \tag{4.14}$$

The parallel between equation (4.14) and equation (4.2) is obvious. Expression (4.14) is just more general since it allows both the returns to scale to be lower than one and the parameters to depend upon the steady state rate of growth of the money supply (i.e., Φ).

Substituting (4.14) in (4.13), we get

$$x_t = b_4 x_{t-1} + d_4 x_{t+1} + b_5 y_t + d_5 y_{t+1} + b_6 m_t + d_6 m_{t+1} \tag{4.15}$$

which corresponds to the Taylor's wage setting rule. The comparison is then between:[5]

TAYLOR ('79, '80)	MICROFOUNDED MODEL
$x_t = bx_{t-1} + dx_{t+1} + \gamma(by_t + dy_{t+1})$	$x_t = b_4x_{t-1} + d_4x_{t+1} + b_5y_t + d_5y_{t+1} + b_6m_t + d_6m_{t+1}$
$p_t = \frac{1}{2}(x_t + x_{t-1})$	$p_t = \left(\frac{1-\sigma}{\sigma}\right) y_t + qx_t + (1-q)x_{t-1}$
$x_t = bp_t + dp_{t+1} + (\gamma/2)(by_t + dy_{t+1})$	$x_t = b_1p_t + d_1p_{t+1} + b_2y_t + d_2y_{t+1} + b_3m_t + d_3m_{t+1}$

Simply adding to the system (4.13), (4.14) and (4.15) above the static aggregate demand equation supposed by Taylor: $y_t = m_t - p_t$,we can reproduce Taylor's result about the persistence of money shocks. We are ready now to compare the model with Taylor's and we can derive some results which are proved in the Appendix 4.6.[6]

Proposition 4.1. *If the utility function is additively separable in real money balances, that is, if* $u_{L,M/P} = \frac{\partial^2 u(t)}{\partial L_t \partial (M/P)_t} = 0$ *and* $u_{C,M/P} = \frac{\partial^2 u(t)}{\partial C_t \partial (M/P)_t} =$

[5] For simplicity we drop the expectation operator in Taylor's model. Note that the expectation operator can straightforwardly be incorporated in the model of this paper, without affecting the results we are concerned with.

[6] It is probably worth noting that, despite being analytically equivalent, the numerical order of the equation reflects the different interpretation of the wage setting rule in the two models. In Taylor's, workers directly care about relative wages (see (4.1)) and indirectly about real wages through (4.2) (see (4.4)). In the microfounded model, instead, workers care directly about real wages (see (4.13)) and indirectly about relative wages through (4.14) (see (4.15)).

0, *then nominal balances do not appear in (4.13) and (4.15).*

Therefore, Taylor's wage setting rule could be justified only if the underlying utility function is additively separable in real money balances or if one is inclined to think that the above cross derivatives are of negligible magnitude. Otherwise, both (4.1) and (4.4) should include real money balances.

Proposition 4.2. *The sums* $b_2 + d_2 = g = \frac{\eta_{LC}-\eta_{CC}+(\varepsilon/\theta)(\eta_{LL}-\eta_{CL})}{1+\varepsilon(\eta_{LL}-\eta_{CL})}$ *and* $b_3 + d_3 = \frac{\eta_{L,M/P}-\eta_{C,M/P}}{1+\varepsilon(\eta_{LL}-\eta_{CL})}$ *are both independent of* Φ *(i.e., independent of the policy rule). Moreover, the "no-money-illusion" constraint* $b+d = 1$, *imposed ad hoc by Taylor, naturally holds in the microfounded model, since both* $b_4 + d_4 = 1$ *and* $b_1 + d_1 = 1$.

We define $\eta_{rs} = \frac{u_{rs}s}{u_r}$ which represents the elasticity of the marginal utility of r with respect to s, for $r, s = C, M/P, L$. Both in (4.13) and in (4.15) the coefficient of the nominal variables, respectively prices and wages, sum to 1, as supposed by Taylor. This would suggest that those parameters could be interpreted as d and b in Taylor's model, that is, the degree of backward and forward looking behaviour. However, since the expressions (4.13) and (4.15) cannot be factorised as can the corresponding Taylor's equations (4.4) and (4.1), we are not able to identify the crucial parameter γ in this general formulation.

Proposition 4.3. *All the parameters in (4.13), (4.14) and (4.15) depend upon the policy rule. In particular, as the rate of growth of money tends to*

infinity then b_i's , for $i = 1, .., 6$, tend to zero and d_i's , for $i = 1, .., 6$, tend to finite values, and viceversa as the rate of growth of money tends to -100%.

It is immediately evident what the last two propositions imply for the wage setting rule (4.13). As the monetary trend increases, more weight is put on the future variables and less on the present one. In fact, we know from proposition 4.2 that the sum of a coefficient of a variable in t (b_i) plus the coefficient of the same variable in $t+1$ (d_i) is constant. Then, as the rate of money growth increases, b_i decreases while d_i increases. In other words, as the monetary trend increases, more weight is put on the future variables and less on present ones. As a matter of fact, the higher the inflation trend, the more forward-looking is the wage setting equation and thus the lower is persistence. Recall the comment on equation (4.12). The future variables in the optimal wage setting rule are weighted by K_{t+1}/K_t which, when evaluated in steady state, is equal to $(1 + rgm)^{\varepsilon}$. The weight is hence increasing in the steady state rate of growth of money. As advocated by Phaneuf (1990), we are therefore able to provide an explanation of the magnitude of the backward versus forward looking bias. This very important issue, concerning the dependence of *ad hoc* Taylor's parameters on average inflation, will be thoroughly discussed in section 4.4.

Proposition 4.4. *If the utility function is additively separable in real money balances and if money supply is constant in steady state, then the*

microfounded model exactly coincides with Taylor's model, where $b = 1 - d = \left(\frac{1}{1+\beta}\right)$ *and* $\gamma = 2\left(g + \frac{1-\sigma}{\sigma}\right)$.

As a result, Taylor's wage setting rule (4.1) can be interpreted as the log-linear approximation of a monopolistic household-union optimal wage setting rule around a steady state with constant money supply. In fact, if $\Phi = 1$ in steady state and $\eta_{L,M/P} = \eta_{C,M/P} = 0$, (4.13), (4.14) and (4.15) respectively become

$$x_t = \left(\frac{1}{1+\beta}\right)[p_t + gy_t] + \left(\frac{\beta}{1+\beta}\right)[p_{t+1} + gy_{t+1}] \tag{4.16}$$

$$p_t = \left(\frac{1-\sigma}{\sigma}\right) y_t + \frac{1}{2}(x_t + x_{t-1}) \tag{4.17}$$

$$x_t = \left(\frac{1}{1+\beta}\right)[x_{t-1} + \gamma y_t] + \left(\frac{\beta}{1+\beta}\right)[x_{t+1} + \gamma y_{t+1}] \tag{4.18}$$

which precisely matches equations (4.4), (4.2) and (4.1).[7]

The striking feature of those three equations is not only that they perfectly parallel Taylor's assumptions, but they above all provide an extremely

[7] Actually, despite the successive approximations used to get here, the model of this paper is still a little bit more general than Taylor's in that it allows σ to be different from 1. Thus, the mark-up equation (4.14) incorporates a decreasing returns to scale effect. As a consequence, the persistence root of the model is given by $\lambda_s = \frac{1-\sqrt{\frac{\gamma\sigma}{2}}}{1+\sqrt{\frac{\gamma\sigma}{2}}} = \frac{1-\sqrt{1+\sigma(g-1)}}{1+\sqrt{1+\sigma(g-1)}}$.

natural interpretation of Taylor's *ad hoc* structural parameters.[8] The backward and forward-looking parameters, respectively d and b in Taylor's model, very simply depends only on the rate of time preferences. If the intertemporal rate of discount is zero then $\beta = 1$ and $b = d = \frac{1}{2}$ and *"contract decisions are unbiased"*. If agents naturally discount the future, then they are biased backward, in the sense that present variables have higher weight than future variables. Given that $\beta \leq 1$, then follows that $d \leq b$.[9] Recall that the degree of persistence is a decreasing function of the forward-looking parameter d. Then, the degree of persistence in Taylor's model is an increasing function of the intertemporal rate of discount.

The elasticity of the money wage to the business cycle conditions is given by $\gamma = 2\left(g + \frac{1-\sigma}{\sigma}\right)$ where, as in proposition 4.2, $g = \frac{\eta_{LC} - \eta_{CC} + (\varepsilon/\theta)(\eta_{LL} - \eta_{CL})}{1 + \varepsilon(\eta_{LL} - \eta_{CL})}$. Now suppose additive separability between consumption and labour then $\eta_{LC} = \eta_{CL} = 0$ and g becomes: $g = \frac{-\eta_{CC} + (\varepsilon/\theta)\eta_{LL}}{1 + \varepsilon\eta_{LL}}$. It is standard to assume

[8] We now do some comparative statics, investigating the effects of the various technology and preferences parameters on persistence. In doing so, we assume that η_{CC} and η_{LL} are constant parameters (as in most utility functions employed in macroeconomic studies). If this was not the case, then a change in a parameter (i.e., β, θ, σ) would change the steady state of the model changing η_{CC} and η_{LL}. I thank Antoine d'Autume for drawing my attention to this. Besides, if the utility function is additively separable, η_{CC} represents the income effect on labour supply.

[9] Proposition 5 should however have already warned the reader that the degree of backward and forward looking depends also on the steady state monetary policy.

both an increasing marginal disutility of labour and a decreasing marginal utility of consumption, hence g is always positive.

First, γ is a decreasing function of θ, that is, the bigger θ, the bigger the persistence. Following a positive money shock, the new money wage will be set higher than the one already fixed in the previous period by the other sector. However, the bigger θ, the bigger the loss in demand a sector will face fixing the new level of money wage, and hence of price, bigger than the one of the other sector. Therefore, the unions will tend to fix the new wage close to the existing one inducing more price level inertia.[10]

Second, while both g and γ are decreasing functions of σ, persistence increases with σ if and only if $g < 1$. Simple intuition would suggest that $\sigma = 1$ corresponds to the maximum degree of nominal rigidities an thus that persistence would be increasing in σ. As it is immediately evident from equation (4.14), for example, if a positive money shock raises output, then, firms could satisfy the excess demand without changing their prices only if

[10]This result appears to be counterintuitive, since we might have expected a more competitive economy to exhibit a lower degree of price inertia. Note that the model presented in Blanchard and Fischer (1989), Chapter 8, which will be analysed in the next section, exhibits the same kind of effect, sharing the same feature.The result is mainly due to the institutional assumption about the fixed length of the contract. Presumably the more competitive the economy, the lower is the length of the contracts and the more flexible are the prices.

$\sigma = 1$. However, this is true only if $g < 1$, that is if the cost (i.e., wage) does not increase too much as more output has to be produced.[11] Moreover:

Proposition 4.5. *(i) The persistence is increasing (decreasing) in the intertemporal elasticity of substitution of labour, $(-1/\eta_{LL})$, if and only if the intertemporal elasticity of consumption, $(-1/\eta_{CC})$, is bigger (lower) than the elasticity of substitution in consumption goods, θ; (ii) A low income effect together with a high intertemporal elasticity of substitution of labour causes a low value of γ and hence a high degree of persistence.*

The second result is actually very intuitive and already present in Blanchard and Fischer (1989), Chapter 8. Note that γ is an increasing function of $|\eta_{CC}|$. The higher the elasticity of marginal utility of consumption, the more the marginal utility of consumption is going to fall for a given increase in output (= consumption, in equilibrium), the more wages are pushed up since households would prefer to exchange consumption for more leisure at the margin. In other words, the lower the intertemporal elasticity of consumption (that is just the inverse of the elasticity of marginal utility of consumption), the more households would like to substitute an increase in consumption with an increase in leisure within the period, leading to a raise in the wages.

[11] Therefore, since in the *ad hoc* Taylor's model g is usually supposed to be very low, Taylor's hypothesis of constant returns to scale, implicitly embedded into (4.2), actually favours persistence.

Note that if the utility over consumption is linear, then this income effect on labour supply is absent. With a zero income effect, wages are not responsive to changes in the marginal utility of consumption. Hence the real wage just depends on the marginal utility of labour. Then, the less elastic is the latter (i.e., the lower is η_{LL}) the lower is the pressure on the wages for a given increase in the labour demand. If utility over consumption and labour is linear (i.e., $\eta_{CC} = \eta_{LL} = 0$) then $g = 0$. However, γ is still different from zero unless $\sigma = 1$.

The first result of proposition 4.5 is a novelty and fairly general as our analysis suggests. It follows from simple algebra that

$$\frac{\partial g}{\partial \eta_{LL}} \gtreqless 0 \qquad \Longleftrightarrow \qquad (-\eta_{CC}) \lesseqgtr \frac{1}{\theta} \tag{4.19}$$

Therefore the direction of the effect of a higher elasticity of marginal utility of labour critically depends on the relative values of η_{CC} and θ. Straightforward intuition would lead to presume a positive relation between γ and η_{LL}: the higher the intertemporal elasticity of substitution of labour (i.e., the lower η_{LL}), the lower the sensitivity of wages to output, i.e., γ, as found in Blanchard and Fischer (1989) and in Chari et al. (1996). Instead, this result shows that this is true only if the intertemporal substitution of consumption is bigger than θ. However, the intertemporal substitution of consumption is usually assumed to be low and generally around one, while θ is, by hypothesis

bigger than one.[12] Therefore, the relation between γ and the intertemporal elasticity in labour is the opposite of what intuition would suggest, unless we are willing to assume unrealistically low income effects. Figure 4.1 graphically shows the result summarised in the last proposition.

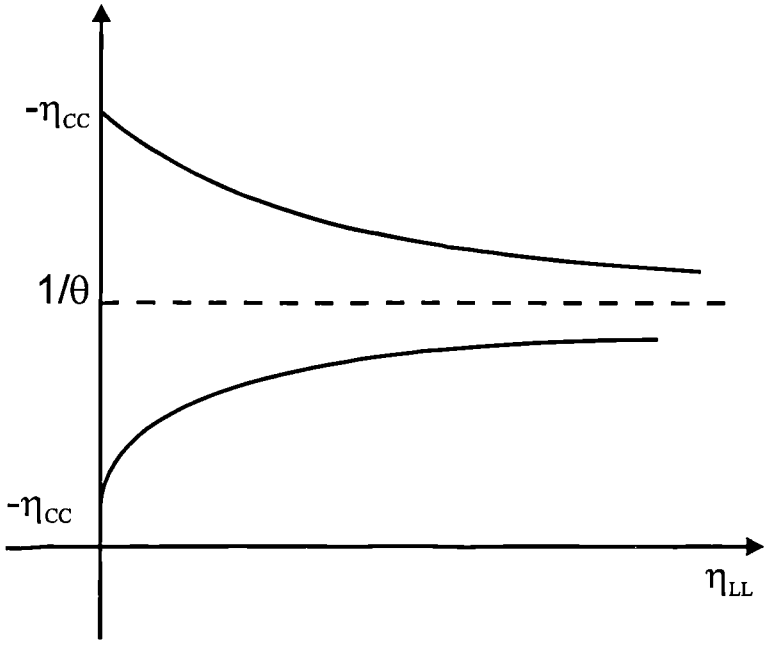

Figure 4.1: *Persistence as a Function of the Microeconomic Parameters*

Proposition 4.5 has important implications with respect to other recent works on microfounded models with staggering. Considering the case $\sigma = \beta = 1$, Chari et al. (1996) shows that in their price staggering model the

[12] Blanchard and Fischer (1989), p. 44, wrote : *"Substantial empirical work has been devoted to estimating $[(-\eta_{CC})]$ under the assumption that is indeed constant [...] Estimates of $[(-\eta_{CC})]$ vary substantially but usually lie around or below unity".*

sensitivity of the real wage to output is: $g = \eta_{LL} - \eta_{CC}$.[13] Thus, they conclude that, since the empirical evidence on labour supply surveyed by Pencavel (1986) suggests that η_{LL} should be at least 1, $g > 1$ (see Chari et al. (1996), p. 15). That is, even assuming zero income effects, g is too high to generate persistence. On the contrary, this is not the case in our model. Assuming $\sigma = \beta = 1$, then $g = \frac{-\eta_{CC}+\eta_{LL}}{1+\theta\eta_{LL}}$. If $\eta_{CC} = 0$, g is always lower than $1/\theta$. Besides, unless one assumes unbelievably high income effect, $g < 1$. This would suggest that staggered wage models are more likely to deliver persistence than staggered price models.[14] Next, note that when $\sigma = 1$ our model coincides with a yeoman-farmer model where now η_{LL} should be interpreted as the elasticity of the marginal disutility of production. The model hence encompasses the Blanchard and Fischer (1989) yeoman-farmer model. However, as said before proposition 4.5, Blanchard and Fischer (1989) obtain the same result as Chari et al. (1996), i.e., a positive relation between η_{LL} and g. This is only due to the fact they use a particular utility function with zero income effects, hence placing their model in the lower part of Figure 4.1, as we will show in the next section. Finally, our model relates closely

[13] Under the assumption $\sigma = \beta = 1$, apart from the different expression for g, the two models coincide and the solution is given by $\lambda_s = (1 - \sqrt{g})/(1 + \sqrt{g}) = (1 - \sqrt{\gamma/2})/(1 + \sqrt{\gamma/2})$.

[14] However, this is not true. See Chapter 6 which is devoted to the analysis of the different implications of wages vs. price staggering on persistence.

to the Rotemberg and Woodford (1997) yeoman-farmer model, in the sense that the sensitivity of the staggered variable with respect to output is given by the same formula for g.[15] Their benchmark calibration is the following: $(-\eta_{CC}) = 0.16$, $\eta_{LL} = 0.47$ and $\theta = 7.88$, which delivers a low value of $g = 0.134$.[16] They acknowledge the fact that such a low value of η_{LL} is difficult to believe.[17] However, they stress that their results do not rely on high labour supply elasticity. We can show that the reason for this is the

[15]This is quite a strong claim but it should be thought as limited to the issue we are concerned with here (i.e., the sensitivity of the real wage with respect to output and persistence). In reality, apart from this point the two models are very much different. Rotemberg and Woodford (1997): (i) have a Calvo-type of structure in price staggering; (ii) have other rigidities to help the model match the impulse response function of an unrestricted VAR; (iii) are mainly concerned with another issue: computing optimal monetary policy in an optimising framework. In order to match the data, however, the model should generate some persistence. As far as persistence is concerned, a key parameter in their model is κ which formula (see p. 316) exactly coincides with our g, if one abstracts from the terms due to the particular definition of the staggered variable and to the Calvo-type of structure (respectively $(1-\alpha)/\alpha$ and $1-\alpha\beta$ in their notation at p. 316).

[16]Which given footnote 13, yields $\lambda_s = 0.46$ in our model. However, given what said in the previous footnote, we cannot directly compare the implied value of persistence in our model and in theirs.

[17]It must be stressed again that in a yeoman-farmer model η_{LL} is no more simply the inverse of the intertemporal elasticity of labour supply (see Chapter 6). In their benchmark calibration the elasticity of labour supply is equal to 9.5 (again too high to be believed).

calibration of the income effects parameter which is extremely low.[18] Indeed with a value of θ around 7.7, then $1/\theta = 0.13$. Then we are in the upper part of Figure 4.1, but the vertical distance between $(-\eta_{CC})$ and $1/\theta$ is squeezed and equal to 0.03, making the value of η_{LL} basically unimportant. Virtually any value of η_{LL} is compatible with $g = 0.134$, just changing θ marginally.[19] All this analysis holds when the model is log-linearised around a particular steady state: zero money growth. This is exactly what Rotemberg and Woodford (1997) do. Indeed, when the model is log-linearised around a steady state for a general Φ results are quite substantially affected. Although not acknowledged by the literature, this point is crucial as section 4.4 will show.

In this section we have carried out a thorough comparison between our microfounded model and Taylor's model and other models in the recent literature. While some of the result are in line with the intuition and previous findings, some others are not, showing that previous findings do not hold in general. We reckon that to understand fully where these results come from,

[18] They acknowledge this (see Rotemberg and Woodford (1997), pp. 321-322) and hence it seems that their model remains subject to the critique of Chari et al. (1996) and the present model: empirical persistence of output responses to monetary shocks can be reproduce only for implausible parameter values.

[19] For example, if $\eta_{LL} = 4.77$ (i.e., labour supply elasticity = 0.3) and $\theta = 7.51$, then $g = 0.134$.

the example of the next section would help.

4.3 A Useful Comparison

In this section we briefly compare our model and the one in Blanchard and Fischer (1989), (B/F in what follows), Chapter 8, to provide a better understanding of the above results. The B/F model derives from a simplified version of Blanchard and Kiyotaki's (1987) static model. In particular, the B/F analysis removes the labour market assuming that each household is at the same time producer and consumer (i.e., yeoman-farmer assumption). On this simplified version they superimpose dynamics due to Taylor's staggered structure of price decisions. There are two basic differences between the setting of their model and of the one presented here: (i) staggering in wages vs. staggering in prices; (ii) while in their model the dynamic is somewhat *ad hoc* overimposed to a static model, the presented model is truly intrinsically dynamic, since it is derived from an explicit intertemporal optimisation process. However, we want here to focus on another crucial difference between the two: the utility function specification. B/F's utility function of agent i is the following: $u_i = (C_i/f)^f[(M_i/P)/(1-f)]^{1-f} - (\chi/e)Y_i^e$, where $e \geq 1$. This specification implies no income effect on labour supply.[20] Then,

[20]The utility function is borrowed from Blanchard and Kiyotaki (1987). In footnote 7 at p.650, Blanchard and Kiyotaki wrote: *"The assumption that utility is homogenous*

the degree of persistence of money shocks is given by

$$\lambda_s = \frac{e + \theta(e-1) - 2\sqrt{[(e-1)(1+\theta(e-1)]}}{1 + (e-1)(\theta - 1)} \tag{4.20}$$

where θ is again the elasticity of substitution among goods. The degree of persistence is a decreasing function of $e - 1$, that is, an increasing function of the intertemporal substitution in effort in production (which corresponds to the one in labour supply if constant returns to labour are assumed).[21] If $e = 1$, then $\lambda_s = 1$ and money shocks have permanent effects on production. It is easy to see why. The price setting rule in B/F is

$$x_t = \frac{1}{2}\{hx_{t-1} + (1-h)E[m_t|t]\} + \frac{1}{2}\{hE[xt+1|t] + (1-h)E[m_{t+1}|t]\} \tag{4.21}$$

where $h = \frac{1+(\theta-1)(e-1)}{1+(\theta+1)(e-1)}$. Using the log-linear static aggregate demand equation $y_t = m_t - p_t$ to substitute out m_t and m_{t+1} in (4.21) we can get Taylor's price setting rule

$$x_t = \frac{1}{2}\{x_{t-1} + 2\left(\frac{1-h}{1+h}\right)E[y_t|t]\} + \frac{1}{2}\{E[x_{t+1}|t] + 2\left(\frac{1-h}{1+h}\right)E[y_{t+1}|t]\} \tag{4.22}$$

---

of degree one in consumption and real money balances, as well as additively separable in consumption and real money balances on the one hand, and leisure, on the other, eliminates income effects on labor supply. Under these assumptions, competitive labor supply would just be a function of the real wage..."

[21] As anticipated, note that the effect of θ on λ_s is the same as in our model.

where $\gamma_{B/F} = 2\left(\frac{1-h}{1+h}\right) = 2\frac{e-1}{1+\theta(e-1)}$ has the same role as γ in Taylor's model.[22] Given that there are no income effects on labour supply ($\eta_{CC} = 0$), then we are in the bottom part of Figure 4.1. In fact, the bigger the intertemporal elasticity of substitution of labour supply (i.e., $1/\eta_{LL} = 1/(e-1)$), the lower $\gamma_{B/F}$, then the bigger the stable root and the persistence.

In order to make our model more similar to the one in Blanchard and Fischer (1989), we use the utility function employed in Chapter 2, that is

$$u_{jt}(C_{jt}, M_{jt}/P_t, L_{jt}) = \delta \ln C_{jt} + (1-\delta)\ln M_{jt}/P_t - \chi L_{jt}^e \tag{4.23a}$$

and we further suppose $\sigma = \beta = 1$. In this case, $\lambda_s = \frac{1-\sqrt{g}}{1+\sqrt{g}}$ and $g = \gamma/2 = \frac{e}{1+\theta(e-1)}$. The simple comparison between g and $\gamma_{B/F}$ can be summarised in the following proposition.

Proposition 4.6. *Comparing our model with the one in B/F we can conclude:*

(i) while in our model the degree of inertia of nominal variables is a decreasing function of the intertemporal elasticity of substitution of labour, the

[22] In fact the stable root $\lambda_s = \frac{1-\sqrt{1-h^2}}{h} = \frac{1}{h} - \sqrt{\frac{1}{h^2} - 1}$ which delivers equation (4.20) (see B/F, p. 395). Then $1/h$ corresponds to φ in (4.5), since B/F suppose $b = d = 1/2$ (that is, $\beta = 1$ in our model). Given the definition of $\varphi = \frac{1+\gamma/2}{1-\gamma/2}$, then putting $\varphi = 1/h$ we get exactly $\gamma = 2\frac{1-h}{1+h}$. Alternatively, putting $\varphi = 1/h$, the stable root in (4.5) can be written as:

$$\lambda_s = \frac{1-\sqrt{\frac{1-h}{1+h}}}{1+\sqrt{\frac{1-h}{1+h}}} = \frac{1-\sqrt{\gamma_{B/F}/2}}{1+\sqrt{\gamma_{B/F}/2}}.$$

There is therefore perfect correspondence between Taylor's model and B/F's one.

contrary is true in B/F's one;

(ii) the elasticity of nominal wages to fluctuations in output, i.e., γ, is however always lower in B/F and hence the degree of inertia of nominal variables is always bigger in B/F's model, with respect to the one here presented.

Proposition 4.6 just basically restates proposition 4.5 applying it to the two proposed examples. Let's develop the intuition. Recall that the optimal wage setting rule of the model, given by (4.12), is strictly linked to the optimal rule in the flexible wage case

$$\frac{X_t}{P_t} = \left[-\frac{\varepsilon}{\varepsilon - 1}\right]\left[\frac{u_L(t)}{u_C(t)}\right] = (constant)Y_t^g \tag{4.24}$$

where g is exactly the elasticity of the optimal wage with respect to output. In fact, as we saw above, the simplifying assumptions $\Phi = \beta = 1$, basically remove any asymmetry among the weights of the log-linearised version of (4.12) and not surprisingly g appears as the elasticity of the nominal wage in (4.13) with respect to both y_t and y_{t+1}. (4.24) can be rewritten as

$$X_t = (constant)\left[\frac{P_t^{1+\theta(e-1)}Y_t^{e-1}X_t^{-\theta(e-1)}}{1/Y_t}\right] \tag{4.25}$$

An increase in Y_t has two effects: (i) K_t increases shifting the demand for labour curve upwards; (ii) the marginal utility of wealth (that, given (4.23a) is equal to the one of consumption) decreases since an increase in consumption is expected. Both these effects go in the same direction and the money wage has to rise. Then, there is a third effect: the union realises that

an increase in the money wages causes the demand for labour, and hence the marginal disutility of labour, to decrease proportionally to the parameter θ (which equals ε if $\sigma = 1$). How much should the money wage increase? Taking into account the three effects then the elasticity of X_t with respect to Y_t is exactly g. In B/F's case the second effect is absent, since the marginal utility of wealth is constant. Therefore the elasticity of X_t with respect to Y_t is exactly $\gamma_{B/F}/2$. The absence of the second effect makes$\gamma_{B/F}/2$ lower than g.

Alternatively, we can reason in the following way. The optimal price choice in B/F can be thought as the optimal decision of a monopolist which maximises profits given the demand curve. The profits are given by the following indirect utility function

$$U_i = (P_i/P)Y_i - (\chi/e)Y_i^e - (M_i/P) \tag{4.26}$$

where the last term is given for the agent. The demand function is

$$Y_i = (P_i/P) - (M/P) \qquad \text{where} \qquad (M/P) = (constant)Y \tag{4.27}$$

The optimal rule is then simply found by equating the marginal cost and

the marginal revenue of the monopolist, that is[23]

$$MR_{B/F} = (constant)P_i^{-\theta}P^{\theta-1}Y = MC_{B/F} = (constant)P_i^{-1-\theta e}P^{\theta e}Y^e \tag{4.28}$$

In our model the monopolist union equals

$$MR = (constant)X_t^{-\theta}P_t^{\theta-1} = MC = (constant)X_t^{-1-\theta e}P_t^{\theta e}Y_t^e \tag{4.29}$$

Comparing (4.28) with (4.29) again the above argument is reproduced. While the marginal cost is exactly the same, the marginal revenue is different. One unit of income spent in consumption and real balances produce a constant level of utility in B/F. In our model, instead, the utility produced by one unit of income decreases with the level of consumption (or income, since in equilibrium $C = Y$). Hence, $MR = MR_{B/F}/Y$, where $1/Y$ is just the marginal utility of wealth.

What happens when output rises? Look at Figure 4.2. Both MC and MR are increasing function of P_i and MC is steeper than MR. Moreover, the higher e, the steeper MC. In Figure 4.2 we have drawn two marginal cost curves: MC corresponds to a high value of e, while $\underline{MC}$ to a low one. Firstly note the difference between the two models. Suppose we are at point A. If Y increases then in B/F both MC and MR shifts to the right, while in

[23]Note that MR and MC are negative since they do not correspond to the usual textbook definitons. In fact, MR and MC are the partial derivatives of revenues and costs respectively, with respect to price rather than quantity.

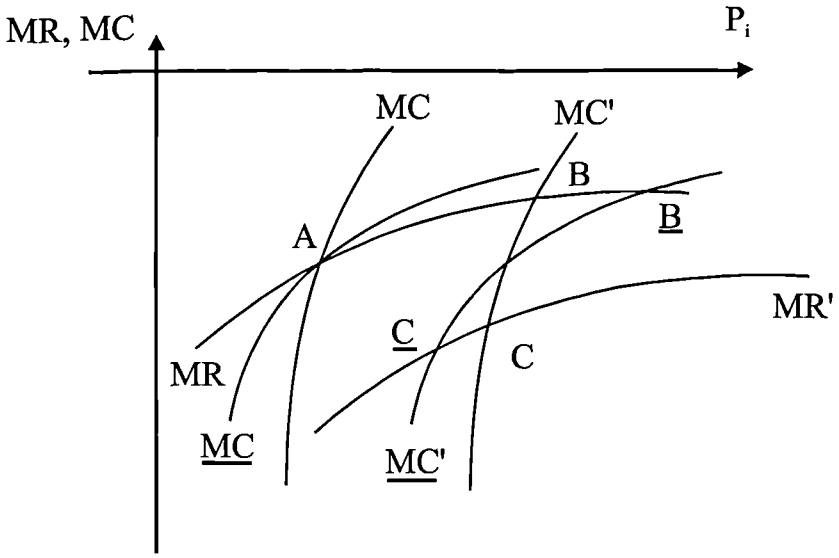

Figure 4.2: *Comparison between our model and the one in Blanchard and Fischer (1989)*

the model presented here only MC shifts. Therefore, for a given increase in Y, P_i is bigger in our model than in B/F's one (compare B with C, and/or $\underline{B}$ with $\underline{C}$). That is, the elasticity of P_i with respect to Y is always lower in B/F's model. Moreover, the flatter MC, the bigger the difference between the level of the new price/wage in the two models (compare the difference between B and C, with the one between $\underline{B}$ and $\underline{C}$). In other words, the lower e, the bigger the difference between the elasticity of P_i with respect to Y in the two models, exactly as in Figure 4.1. Secondly, consider the effect of e on this elasticity in the two models. In B/F, the lower e, the flatter the MC curve, the lower the elasticity of P_i with respect to Y (compare C with $\underline{C}$). In our model, the opposite is true (compare B with $\underline{B}$).

To conclude, in this section we presented a comparison between the dynamic general equilibrium model with staggered wages of the previous chapters and the one in Blanchard and Fischer (1989), Chapter 8, in order to illustrate the relation between income effects, intertemporal substitution of labour and persistence of money shocks. This enabled us to explain in a very plain and intuitive way the apparent puzzle of point (i) in proposition 4.5, summarised in Figure 4.1.[24]

4.4 Persistence, Non-linearity and the Effects of Average Inflation

So far we have explored the analytical mechanism of the model comparing it with Taylor's and B/F's models. We are then ready to numerically evaluate the degree of persistence of money shocks implied by the model. Let's slightly modify the utility function in (4.23a). In order to allow the intertemporal elasticity of consumption to be different from one, we simply substitute $[\ln C_t]$ with $\frac{C_t^{1-a}-1}{1-a}$ in (4.23a), then $\eta_{CC} = -a$.

In Table 4.1 we report three cases for a zero money growth policy rule.

[24]Moreover note that the results of this section about the elasticity of price/wage to output are generally valid, in the sense that they do not depend on the assumption of staggered wages/prices. Figure 4.1 has been in fact drawn looking at the optimal flexible price/wage rule.

In the first case both consumption and labour enter linearly in the utility function, whence $\eta_{CC} = \eta_{LL} = 0$.[25] Then the money wage is completely inelastic with respect to output changes and money shocks have permanent effects on the level of output. The second case is our "base case", given the estimate of microeconomic parameters used in the calibration literature.[26] γ is found to lie just outside the range of estimates in Phaneuf (1990) and to be much higher both than the estimates of Taylor (1980b) for the US and of the values used by West (1988).[27] The value of the stable root is in this case far from inducing a near-random walk behaviour in output following a monetary shock. However, even in the case of non-negligible income effects, the degree of persistence could be increased by raising the value of θ and e. The third case in Table 4.1 shows that for (not completely implausible) values of the parameters the model is able to generate a substantial degree of persistence ($\lambda_s = 0.6$). Nonetheless, only for extreme and unrealistic values

[25] If $\eta_{CC} = \eta_{LL} = 0$ the model actually breaks down, since the labour demand and supply curves are both horizontal and they do not intersect. This case must then be interpreted as a limiting one.

[26] As discussed in Chapter 2, footnote 15 the following values are used as indicative: $\theta = 6$, $e = 4.5$, $\sigma = 1$, $\beta = 0.95$, $\chi = 0.01$, $\delta = 0.99$ and $a = 1$ (for the latter see footnote 12).

[27] Chari et al. (1996) calibrate to be well above one. However, it should be stressed that ours is not a calibration exercise. We are more interested in a kind of robustness exercise to assess whether persistence can be a likely outcome and, if so, under which conditions.

	γ	λ_s
$\theta = 6; \sigma = 1; e = 1; a = 0$	0	1
$\theta = 6; \sigma = 1; e = 4.5; a = 1$	0.409	0.3868
$\theta = 15; \sigma = 1; e = 15; a = 1$	0.1422	0.5936

Table 4.1: Value of γ and λ_s as microeconomic parameters vary: 3 cases

of these parameters (e.g., $\theta = 42$, $e = 20$) can we obtain Taylor's estimate of γ ($= 0.05$) and consequently a near-random walk degree of persistence ($= 0.745$).

In conclusion, it seems that the results of West (1988) and Phaneuf (1990) can only be supported by extreme values of the microeconomic fundamentals. In particular, we need either a zero income effect and virtually infinite elasticity of substitution of labour supply[28], or, for more realistic values of η_{CC}, implausibly high values of θ and e. Note that a high degree of persistence could nevertheless occur in the *ad hoc* model because of a very accommodating feedback monetary policy rule (i.e., high value of c).However, we can get a substantial degree of persistence for fairly plausible values of the parameters, even if far from near-random walk behaviour.

Nevertheless, it remains to carefully consider the important issue of the relation between the structural parameters of Taylor's wage setting rule and

[28]Note that in the case $a = 0$ and $e = 1.03$, we get exactly $\gamma = 0.05$, as Taylor. However, a slight increase of e above one has a dramatic effect on persistence. Only for value of e lower than 1.05, we can still get a degree of persistence bigger than 0.7. This suggests a higly non-linear relation between e and persistence in this case.

$rgm \rightarrow$ $parameters \downarrow$	-4%	0	$+5\%$	$+10\%$
$\theta = 6; \sigma = 1$ $e = 1; a = 0$	$\lambda_s = 1$ $b_4 = 0.575$ $d_4 = 0.425$ $b_5 = 0.0205$ $d_5 = -0.0205$	$\lambda_s = 1$ $b_4 = 0.513$ $d_4 = 0.487$ $b_5 = 0$ $d_5 = 0$	$\lambda_s = 0.822$ $b_4 = 0.439$ $d_4 = 0.561$ $b_5 = -0.025$ $d_5 = 0.025$	$\lambda_s = 0.6343$ $b_4 = 0.3614$ $d_4 = 0.639$ $b_5 = -0.051$ $d_5 = 0.051$
$\theta = 6; \sigma = 1$ $e = 4.5; a = 1$	$\lambda_s = 0.5476$ $b_4 = 0.771$ $d_4 = 0.229$ $b_5 = 0.332$ $d_5 = 0.105$	$\lambda_s = 0.3868$ $b_4 = 0.513$ $d_4 = 0.487$ $b_5 = 0.210$ $d_5 = 0.199$	$\lambda_s = 0.1291$ $b_4 = 0.204$ $d_4 = 0.796$ $b_5 = 0.098$ $d_5 = 0.347$	$\lambda_s = -0.0167$ $b_4 = 0.002$ $d_4 = 0.998$ $b_5 = 0.040$ $d_5 = 0.494$
$\theta = 15; \sigma = 1$ $e = 15; a = 1$	$\lambda_s = 0.8335$ $b_4 = 1.042$ $d_4 = -0.042$ $b_5 = 0.201$ $d_5 = 1.9 \bullet 10^{-5}$	$\lambda_s = 0.5936$ $b_4 = 0.513$ $d_4 = 0.487$ $b_5 = 0.073$ $d_5 = 0.069$	$\lambda_s = -0.0394$ $b_4 = -0.047$ $d_4 = 1.047$ $b_5 = 3.9 \bullet 10^{-6}$ $d_5 = 0.217$	$\lambda_s = -0.043$ $b_4 = -0.057$ $d_4 = 1.057$ $b_5 = 1.82 \bullet 10^{-10}$ $d_5 = 0.356$

Table 4.2: Value of persistence and of the parameters of Taylor's wage setting rule as microeconomic fundamentals and rate of growth of money vary: 3 cases

the steady state rate of money growth. Looking at Table 4.2[29] it is evident that the conclusions in the previous paragraph are not robust when monetary policies other than a constant money supply are considered.

Firstly, look at the value of the stable root. In every case, the degree of persistence is the higher, the lower the rate of growth of money (i.e., rgm). Besides, the sensitivity of λ_s with respect to changes in rgm is the higher, the bigger the microeconomic parameters. If money decreases at the rate of 4% in steady state, then, with respect to the case of zero money growth, the persistence rises from 0.3868 to 0.5476 in the "base case" and from 0.5936 to 0.8335 in the third case. On the other hand, if money grows at a 10% rate, in both cases the persistence is virtually nil. The degree of sensitivity of λ_s

[29] If $rgm \neq 0$ there is no single measure for γ and that is why we report all the parameters of the wage setting rule in Table 4.2.

on rgm is indeed somewhat impressive. Also in the first case, the persistence strongly diminishes as the rate of growth of money arises and at a 10% rgm, λ_s is far from a near-random walk behaviour. Figure 4.3 visualises these effects.

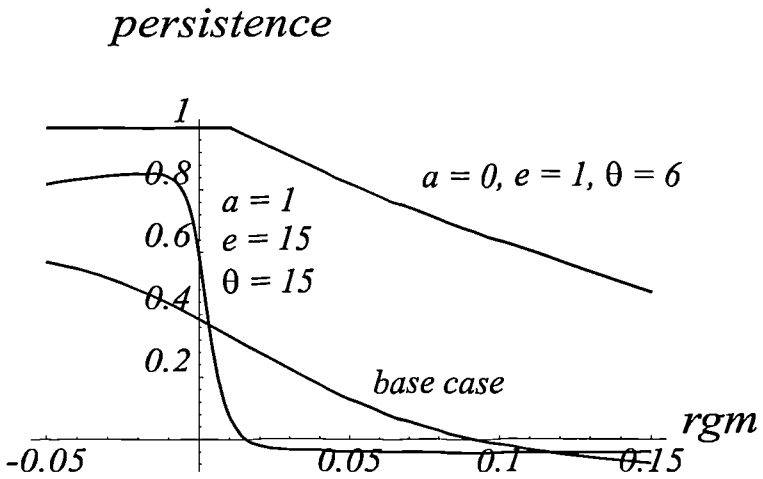

Figure 4.3: *Persistence as a Function of the Steady State Rate of Money Growth*

To understand this result now look at the other parameters of Taylor's wage setting rule (4.15). b_i refers to the backward-looking variables, while d_i to the forward-looking ones; the subscripts 4 and 5 respectively refer to the nominal wage and to output. As the rgm increases, the b's decrease and the d's increase: the higher the rgm, the higher the weights on the forward-looking variables, and the lower the persistence. In other words, a high rate of underlying inflation causes Taylor's wage rule to collapse into a pure forward-looking equation where only the future variables are taken into

account: the inertia due to the backward-looking behaviour in Taylor's rule vanishes, and so does persistence.

This striking finding raises a number of interesting points.

(i) It demonstrates that, once microfoundations are taken into account, the importance of non-linearity becomes evident and can no longer be neglected. The response of the system to a money shock heavily depends on the starting point, that is on the underlying rate of inflation, in a non-linear manner. The *ad hoc* log-linear models are therefore quite misleading.[30]

(ii) The previous point is very much related to the famous Lucas critique. As Sargent puts it: *"Robert E. Lucas (1976) criticised a range of econometric policy evaluation procedures because they used models that assumed private agents' decision rules to be invariant with respect to the laws of motion that they faced. Those models took as structural [...] private agents' decision [...*and*] violated the principle that an optimal decision rule h(x) is a function of the law of motion* $g(x_t,\ u_t,\ \varepsilon_t)$*."* (Sargent (1987), pp. 40-41) Once the model is microfounded and the fully optimising decision process of the agents is explicitly taken into account, the policy behaviour enters the *ad hoc* structural parameters. And that occurs in a clear and intuitive way in this model. A high underlying rate of inflation would demolish the inertia in the system,

[30] This is particularly important when transitional dynamics between two different steady states are analysed, as in the previous chapter. Note however, that a log-linear approximation seemed to be quite accurate as we concluded in Appendix 3.7.3.

making of Taylor's wage setting equation a pure forward looking one. Quite a large number of researchers have suggested that fixed staggered contracts represent a good approximation of reality only in economies displaying fairly stable prices. On the basis of the Lucas critique, they observed that contracts would not survive in an environment without stable prices simply because agents would take that into account and adjust their behaviour. This is exactly what the present analysis suggests. Parameters are policy-dependent such that a high rate of underlying inflation would actually make the contracts irrelevant, dramatically changing the way they are set and reducing Taylor's wage setting equation to a purely forward looking one.

(iii) Following further the Lucas critique argument, one would expect that a higher level of average inflation would shorten the length of the contracts. Since the latter is fixed by hypothesis, this is not possible in the model. However intuition strongly suggests it would happen if the model allowed for that. This is in line with the intuition of Ball et al. (1988). Ball et al. (1988) suggested that high inflation lubricates the frictions in price adjustment. The higher is the inflation rate, the more often firms adjust their prices to keep up with the price level, so the faster the adjustment in the aggregate price level and the smaller the real effects following an aggregate demand disturbance. This would imply a negative relation between the real effects of aggregate demand disturbances and the average level of inflation. Ball et al.

(1988) tested this implication and they concluded: *"A robust finding is that this trade-off is affected by the average rate of inflation. In countries with low inflation, the short-run Phillips curve is relatively flat - fluctuations in nominal aggregate demand have large effects on output. In countries with high inflation, the Phillips curve is steep".* (Ball et al. (1988), p. 59) Our model has the same empirical implication and, as explained above, the intuition is somewhat similar. Recall that Taylor's model implies that the slope of the Phillips curve is inversely related to λ_s. Then, the higher is the rate of average inflation, the more forward looking is Taylor's wage setting rule, the lower λ_s and the steeper the Phillips curve. *"Traditional Keynesian models, such as textbook models of price adjustment or the staggered contracts models of Fischer and Taylor, do not share the key predictions of our model. These older theories treat the degree of nominal rigidity (for example, the length of labor contracts or the adjustment speed of the price level) as fixed parameters; thus they rule out the channel through which average inflation affects the output-inflation trade-off."* (Ball et al. (1988), p. 29) Instead, we were able to show that, once microfoundations are explicitly considered, the adjustment speed of labour contracts do depend upon average inflation. Thus, the Lucas critique goes through even if the length of the contracts is fixed by hypothesis. We are however aware that a more satisfactory model should allow for changes in the length of the contracts.

(iv) West (1988) simulates Taylor's model under different monetary policy rule for given values of the structural parameters of the wage equation. Given the above results, one may question the theoretical validity of analyses such as those carried out in West (1988). The structural parameters are not policy invariant. It would be very interesting to study how these parameters and persistence vary as different feedback monetary policy rules are implemented.

(v) By the same token, the model suggests an explanation for Phaneuf's (1990) empirical finding that, across countries, higher values of γ are associated with a higher degree of monetary accommodation. This is because it is probably reasonable to think that countries more prone to accommodate monetary shocks are also those more likely to display relatively higher underlying inflation.

(vi) Moreover, this suggests a further interpretation. There is a widely confirmed positive relationship between average inflation and the variance of nominal output. It seems that countries exhibiting low inflation rates are strictly controlling their monetary policy and will seldom be affected by substantial monetary shocks. Therefore, staggered contracts could potentially induce a fairly high degree of persistence only when a monetary shock is unlikely to occur. In other words, staggered contracts could generate high persistence of monetary shocks only when the staggering structure does not matter, since monetary policy is tightly controlled.

Before concluding a final remark follows. From proposition 4.4, we know that, when $rgm = 0$, $\gamma = 2\left(g + \frac{1-\sigma}{\sigma}\right)$. The hypothesis $\sigma = 1$, implicit in Taylor's analysis, actually induces, ceteris paribus, the maximum degree of persistence in our model since $g < 1$. Is this hypothesis the best one? Probably not. In fact, the analysis is concerned with short-run adjustment. Labour is the only input in the production function. It is therefore sensible to interpret the production function as a reduced form of a short-run Cobb-Douglas one, in which capital is simply fixed and embodied in the constant term. Following this interpretation, then calibrated studies would suggest the labour's share of national income, i.e., 0.7, to be a good approximation for σ. But as the formula at the beginning of the paragraph shows, γ and hence λ_s tends to be particularly sensitive to σ. In fact, with constant steady state money supply (i.e., Table 4.1), if $\sigma = 0.7$, in the "base case" γ jumps to 1.37 and persistence falls to 0.19, while, in the case $a = 0$ and $e = 1$, γ rises to 0.86 and persistence drops to 0.3. Moreover, the lower σ, the lower the sensitivity of the structural parameters of Taylor's equation to the underlying rate of inflation (see Figure 4.4 and 4.5). If someone was puzzled about the excessive sensitivity of these parameters to average inflation, then he will be inclined to think $\sigma = 0.7$ to be the relevant case.

To conclude, whatever sensible values one assigns to the parameters, either a moderate rate of underlying inflation such as is observed in western

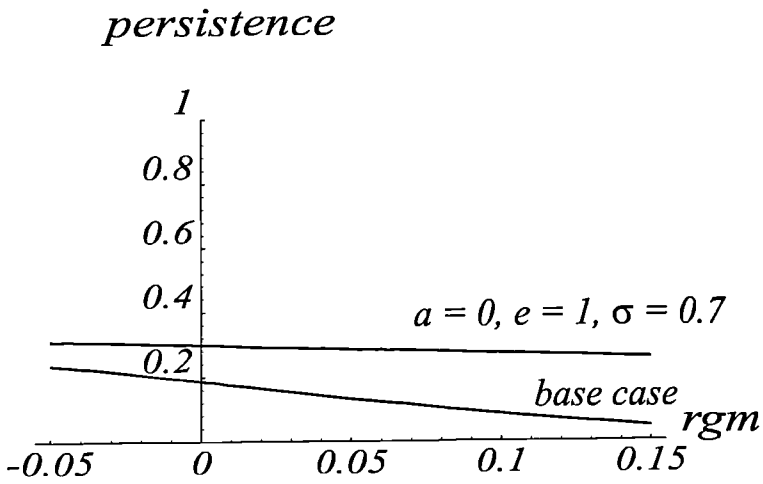

Figure 4.4: *Persistence as a Function of the Steady State Rate of Money Growth when* $\sigma = 0.7$

economies, or a value of σ equal to the share of labour in output, or both, cut down persistence not only far below near random-walk behaviour, but also below any level notably different from zero.

4.5 Conclusions

In this chapter we used the dynamic general equilibrium model with optimising agents and staggered wages à la Taylor (1979) developed in previous chapters to look at the persistence of the real effects of money shocks. If, as were West (1988) and Phaneuf (1990), we had been looking for results to corroborate the view that staggered wage models could induce a high degree of persistence of money shocks, the microfounded model does not seem to pro-

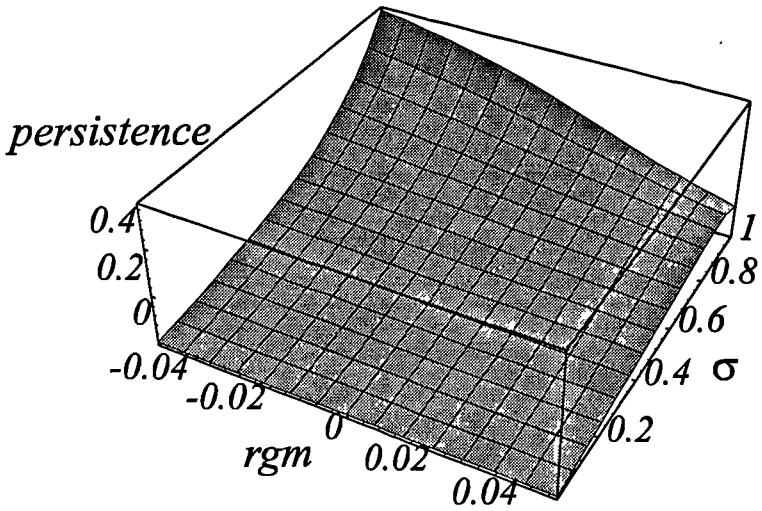

Figure 4.5: *Persistence as a Function of the Steady State Rate of Money Growth and of σ*

vide them. On the contrary, it confutes that view. The model demonstrates that for a large range of reasonable parameter values a notable degree of persistence is an unlikely outcome. Moreover, even for parameter values such that the model generates persistence, a moderate rate of underlying inflation cuts down persistence sharply. In conclusion, sensible values of the microeconomic parameters and/or a moderate rate of underlying inflation such as we observe in western economies cut down persistence not only far below near random-walk behaviour, but also below any level notably different from zero. Moreover,n through investigating the microeconomic fundamentals of the ad hoc Taylor wage rule, the model emphasises the role of non-linearity and of the Lucas critique. In brief, the model shows that staggered wages alone are

not able to explain a significant degree of persistence of the real effects of money shocks.

4.6 Appendix

Substitute in (4.12) K_t, as given by (1.15) and impose the equilibrium condition $Y_t = C_t$. Then, log-linearising (4.12) around a steady state we obtain

$$x_t = a_1 y_t + a_2 y_{t+1} + a_3 l_t + a_4 l_{t+1} + a_5 m_t + a_6 m_{t+1} + a_7 p_t + a_8 p_{t+1} \quad (4.30)$$

where now the variables are expressed as log-deviations from steady state and where:

$$a_1 = \left(\frac{\eta_{LC} + \frac{\varepsilon}{\theta}}{1 + \beta \frac{u_L(t+1)}{u_L(t)} \Phi^{-\varepsilon}} \right) - \left(\frac{\eta_{CC} + \frac{\varepsilon}{\theta}}{1 + \beta \Phi^{1-\varepsilon}} \right) \quad (4.31)$$

$$a_2 = \beta \left\{ \left(\frac{\eta_{LC} + \frac{\varepsilon}{\theta}}{\frac{u_L(t)}{u_L(t+1)} \Phi^{\varepsilon} + \beta} \right) - \left(\frac{\eta_{CC} + \frac{\varepsilon}{\theta}}{\Phi^{\varepsilon-1} + \beta} \right) \right\} \quad (4.32)$$

$$a_3 = \left(\frac{\eta_{LL}}{1 + \beta \frac{u_L(t+1)}{u_L(t)} \Phi^{-\varepsilon}} \right) - \left(\frac{\eta_{CL}}{1 + \beta \Phi^{1-\varepsilon}} \right) \quad (4.33)$$

$$a_4 = \beta \left\{ \left(\frac{\eta_{LL}}{\frac{u_L(t)}{u_L(t+1)} \Phi^{\varepsilon} + \beta} \right) - \left(\frac{\eta_{CL}}{\Phi^{\varepsilon-1} + \beta} \right) \right\} \quad (4.34)$$

$$a_5 = \left(\frac{\eta_{L,M/P}}{1 + \beta \frac{u_L(t+1)}{u_L(t)} \Phi^{-\varepsilon}} \right) - \left(\frac{\eta_{C,M/P}}{1 + \beta \Phi^{1-\varepsilon}} \right) \quad (4.35)$$

$$a_6 = \beta \left\{ \left(\frac{\eta_{L,M/P}}{\frac{u_L(t)}{u_L(t+1)}\Phi^{\varepsilon} + \beta} \right) - \left(\frac{\eta_{C,M/P}}{\Phi^{\varepsilon-1} + \beta} \right) \right\} \tag{4.36}$$

$$a_7 = \left(\frac{\varepsilon}{1 + \beta \frac{u_L(t+1)}{u_L(t)} \Phi^{-\varepsilon}} \right) - \left(\frac{\varepsilon - 1}{1 + \beta \Phi^{1-\varepsilon}} \right) \tag{4.37}$$

$$a_8 = \beta \left\{ \left(\frac{\varepsilon}{\frac{u_L(t)}{u_L(t+1)}\Phi^{\varepsilon} + \beta} \right) - \left(\frac{\varepsilon - 1}{\Phi^{\varepsilon-1} + \beta} \right) \right\} \tag{4.38}$$

Note that $a_1 + a_2$, $a_3 + a_4$, $a_5 + a_6$, $a_7 + a_8$ do not depend on Φ.

Then from (1.15) we know that the amount of labour in each of the two periods of the contract is given by: $L_{it} = K_t X_{it}^{-\varepsilon}$ and $L_{it+1} = K_{t+1} X_{it}^{-\varepsilon}$. Substitute for K and express the variables as log-deviations, to obtain

$$l_t = (\varepsilon/\theta) y_t + \varepsilon(p_t - x_t) \qquad \text{and} \qquad l_{t+1} = (\varepsilon/\theta) y_{t+1} + \varepsilon(p_{t+1} - x_t) \tag{4.39}$$

Use these two expressions to substitute out labour in (4.30) and get

$$x_t = b_1 p_t + d_1 p_{t+1} + b_2 y_t + d_2 y_{t+1} + b_3 m_t + d_3 m_{t+1} \tag{4.40}$$

which is equation (4.13) in the main text and where

$$b_1 = \frac{a_7 + a_3 \varepsilon}{1 + \varepsilon(a_3 + a_4)} \tag{4.41}$$

$$d_1 = \frac{a_8 + a_4 \varepsilon}{1 + \varepsilon(a_3 + a_4)} \tag{4.42}$$

$$b_2 = \frac{a_1 + a_3 \frac{\varepsilon}{\theta}}{1 + \varepsilon(a_3 + a_4)} \tag{4.43}$$

$$d_2 = \frac{a_2 + a_4 \frac{\varepsilon}{\theta}}{1 + \varepsilon(a_3 + a_4)} \tag{4.44}$$

$$b_3 = \frac{a_5}{1+\varepsilon(a_3+a_4)} \tag{4.45}$$

$$d_3 = \frac{a_6}{1+\varepsilon(a_3+a_4)} \tag{4.46}$$

Proof of proposition 4.1.

If $u_{L,M/P} = \frac{\partial^2 u(t)}{\partial L_t \partial (M/P)_t} = 0$ and $u_{C,M/P} = \frac{\partial^2 u(t)}{\partial C_t \partial (M/P)_t} = 0$, then $\eta_{C,M/P} = \eta_{L,M/P} = 0$ and then $a_5 = a_6 = b_3 = d_3 = 0$ and real balances do not appear in (4.13) and (4.15).

Proof of proposition 4.2.

Since $a_1 + a_2$, $a_3 + a_4$, $a_5 + a_6$, $a_7 + a_8$ do not depend on Φ, then it follows immediately that the same holds for $b_1 + d_1$, $b_2 + d_2$, $b_3 + d_3$. Thus simple algebra shows that $b_1 + d_1 = 1$, $b_2 + d_2 = g\frac{\eta_{LC}-\eta_{CC}+(\varepsilon/\theta)(\eta_{LL}-\eta_{CL})}{1+\varepsilon(\eta_{LL}-\eta_{CL})}$ and $b_3 + d_3 = \frac{\eta_{L,M/P}-\eta_{C,M/P}}{1+\varepsilon(\eta_{LL}-\eta_{CL})}$.

Proof of proposition 4.3.

Recall the definition of Φ_t: $\Phi_t = M_{t-1}/M_t$, that is the rate of decrease of money supply in t. The parameters of the log-linearised equations must be evaluated in a steady state, that is, Φ_t is constant for all t. Supposing the ratio $\frac{u_L(t+1)}{u_L(t)}$ remain limited as Φ goes to zero or infinity. Then the result follows.

Proof of proposition 4.4.

If money supply is constant in steady state, then Φ is equal to one. As we know from Chapter 2, all of the nominal variables are constant in steady state, too. The level of output is therefore the same in each sectors and constant

over time, in steady state. Hence also the amount of labour supplied by the households of the two sectors is the same and constant over time. Then, $\frac{u_L(t+1)}{u_L(t)} = 1$. Given that, substitute $\Phi = \frac{u_L(t+1)}{u_L(t)} = 1$ in the above expressions and the result follows.

Chapter 5

Relative Wage Concern: The Missing Piece in the Contract Multiplier?

5.1 Introduction

In the previous chapter we have cast serious doubts on the explanatory power of staggered wage setting in accounting for output persistence. Chari et al. (1996) (CKM henceforth) have done the same for a price staggering model. In other words, it seems that, once embedded in a dynamic general equilibrium framework, staggered wage/price model cannot generate what Taylor called

the "contract multiplier".[1]

However, this is a fundamental issue for any monetary DGE macromodel: any theoretical interpretation of the business cycle assigning a serious role to monetary disturbances must allow for substantial persistence of the real effects of money shocks to mimick actual data. Persistence of the real effects of money shocks requires *endogenous stickiness* in the sense that price-setting agents choose not to change their prices/wages by a large amount when they reset them.

This chapter reconsiders the existence of a contract multiplier. We we will argue that the wage setting process is better represented as the result of the combination of small nominal *and* real rigidities, in contrast with the simpler approach of the previous chapter or of Chari et al. (1996).

Some contributions have already highlighted that the combination of small nominal rigidities and real rigidities has the potential to generate endogenous stickiness (Ball and Romer (1990), Blanchard (1990), Romer (1996), Jeanne (1998)), and consequently output persistence. Here we investigate that conjecture. Our source of real rigidity in the labour market arises from an explicit account of relative real wage concern in the bargaining process. We review below some strong empirical evidence pointing at relative

[1] In Taylor's (1980a, p.2) words: *"In effect, each contract is written relative to other contracts, and it causes shocks to be passed on from one contract to another – a sort of "contract multiplier".*

wages as a fundamental factor in the wage setting process. As we stressed several times in this thesis, Taylor's model was thought to incorporate a "Keynesian" component of relative wage concern on the part of the workers. However, his model is analytically equivalent to one in which workers are ("neoclassically"), only concerned about the level of their own real wages (see Buiter and Jewitt (1981), Blanchard (1990) and Chapter 4). Relative wage concern considerations have been therefore left out of the analysis so far. That omission seems to be a serious weakness of the contracting specification assumed in Taylor's model, as Buiter and Jewitt (1981) suggest. In a recent contribution very related to ours, Fuhrer and Moore (1995) (FM henceforth) have also pointed to the lack of inflation persistence generated by Taylor's staggering wage model as a major empirical failure arising from the contracting scheme assumed in the model.

Our analytical framework is based on the model of the previous chapters. It is thus kept as close as possible to those of the previous studies which have highlighted the weaknesses of the Taylor contracting specification, namely CKM, FM and Chapter 4. By incorporating relative wage concern we aim at enriching the analysis of wage-setting with respect to those studies. Moreover, we try to capture the spirit of the original work by Taylor since it was (arguably) aimed at considering relative wage considerations. We then assess the analytical and quantitative importance of staggered wage setting on

output and inflation persistence in our model.

The quantitative version of the model provides strong support for the existence of a substantial contract multiplier. Two features of the model strengthen the importance of our result. First, the wage contracting specification is the only mechanism through which the effects of nominal shocks are propagated in our model. We refrain from introducing capital accumulation, adjustment costs, endogenous mark-ups, or any other possible factor which enhances the nominal propagation mechanism derived here. Second, as in previous analyses of staggered wage setting, our results also highlight the potential role of high intertemporal elasticities of substitution of consumption and labour supply in favoring persistence, but *by no means* rely on them to generate a substantial degree of persistence. This latter point is evident from our "conservative" parameter choices in the calibration exercise. Notwithstanding these features of the model, we find that output persistence is a likely outcome.

We also provide analytical results that highlight the intuition behind the sharp contrast between our results and those of the recent literature. We log-linearise our wage setting equation around the steady state and compare it to those of CKM and Chapter 4. The key difference is the elasticity of wages with respect to business cycle conditions. In our model, relative wage concern on the part of workers lowers that sensitivity. A sensible calibration

of the parameters governing relative wage considerations generates a powerful contract multiplier and thus substantial persistence in both inflation and output.

The remainder of this chapter is organized as follows. In section 5.2 we present some empirical evidence and our formalisation of relative wage concern on the part of wage-setters. Our benchmark economy is presented in section 5.3. We study the analytical implications of relative wage concern in section 5.4, and compare our findings to previous studies of staggered wage/price models. We then proceed to analyse the quantitative implications. Section 5.5 describes the calibration of the model and reports our simulation results. Those results are complemented by the sensitivity analysis carried out in Section 5.6. Finally, Section 5.7 concludes.

5.2 The Case for Relative Wage Concern

The existence of relative wage concern on the part of workers/ relative comparisons has a long tradition in economics, starting from Adam Smith (1976). However, the most influential account of relative wage concern and its implications came undoubtedly from John Maynard Keynes (1936) (p.14):

> *Though the struggle over money-wages between individuals and groups is often believed to determine the general level of real wages, it is, in fact, concerned with a different object. Since there is imperfect mobility of labour, and wages do not tend to an exact equality of net advantage in different occupations, any individual or group of individ-*

> *uals, who consent to a reduction of money-wages relatively to others, will suffer a* **relative** *reduction in real wages, which is sufficient justification for them to resist it. [...] In other words, the struggle about money-wages primarily affects the* **distribution** *of the aggregate real wage between different labour groups, and not its average amount per unit of employment, [...] . The effect of combination on the part of a group of workers is to protect their* **relative** *real wage.*[2]

Relative wage concern is not a new topic for more recent economic literature either. Relative wage considerations have recently been introduced with reference to fair wage determination and the impact on effort and unemployment (Frank (1984), Summers (1988), Akerlof and Yellen (1990)). In this chapter, we instead focus on the implications of relative wage concern for the wage setting process and the contract multiplier.

5.2.1 Some Empirical Evidence

Labour economists long ago pointed out the interdependence between trade union's wage claims as a stylized fact in the bargaining process. Union "rivalry" and inter-union "jealousy" have been studied in Oswald (1979), Gylfason and Lindbeck (1984), Risager (1992), among many others. Furthermore, in recent years a new source of empirical evidence has received considerable attention by economists: surveys on self-reported levels of satisfaction of workers, which already form the fundamental material of study for a large empirical literature in social psychology. Such data has been used as proxy for utility data.[3] Capelli and Sherer (1988) use data from a major U.S. airline;

[2]Emphasis is as in the original.

[3]As pointed out by Clark and Oswald (1996a), footnote 4: *"It might be argued, in the extreme, that these are random numbers merely made up by survey respondents. Psycholo-*

Clark and Oswald (1996a) from the British Household Panel Survey. Both studies report measures of the importance of relative wages for individual workers. Their results:

(i) point to "market" relative wages as *the* fundamental factor (and statistically strongly significant on regressions of job satisfaction) for individual workers

(ii) more surprisingly, the level of the own real wage/income plays a minor role, if any at all. Moreover, its coefficient is found statistically insignificant. This finding provides strong support for utility functions that allow for relativities in wage setting.

Risager (1992) found also strong evidence in his investigation of the wage rivalry hypothesis using Danish data for skilled and unskilled workers. His analysis of wage setting behaviour:

(ii) identifies a very strong "following" behaviour in wage setting;[4]

(iii) finds the level of unemployment/business cycle conditions statistically insignificant.[5]

---

gists, who are at least as aware of this possibility as economists, have long since abandoned such a view."

[4] *"There is a high degree of wage interdependence, even in the short term. The reaction function for skilled men shows that the wage for skilled men increases by 5.8% in response to a 10% wage increase for unskilled men. Unskilled men's reaction function shows a 12.4% wage increase in response to a 10% wage increase for skilled men"* (Risager (1992), p. 550-551).

[5] *"...has the persistently high unemployment exerted a direct downward pressure on real wages? The answer is negative since both the current and lagged unemployment rates are highly insignificant in the two wage equations. To find out whether unemployment has a*

Moreover, Campbell and Kamlani (1997) (Table II) report results from other studies which suggest that relative wage concern is very significant in heavily unionized firms (Agell and Lundborg (1995)). Individual survey data reported above also provide a strong justification for the behaviour of unions from the personal preferences of their potential members.

5.2.2 Relative Real Wage Concern and Staggered Wage Setting

The structure of wage setting in the model is defined by two features: (i) staggered wage setting; (ii) relative real wage concern.

(i) *Staggered Wage Setting*[6]

Wage contracts, denoted by X, are negotiated in nominal terms. The economy is divided into N distinct sectors. Each sector is composed of $1/N$ industries (indexed by i) and their corresponding unions (indexed by j). Contracts specify the nominal wages that will be held fixed for N periods. That is, for a union setting the nominal wage in period t, $X_{jt+k} = X_{jt}$ for $k = 0, ...N-1$. Furthermore, unions indexed $j \in [0, 1/N]$ set their wages in periods 0, N, $2N$, unions indexed $j \in [1/N, 2/N]$ do so in periods 1, $N+1$, $2N+1$, etc. Note that complete symmetry among households is broken by the fact that in each of the sectors the wage is set in a staggered and overlapping fashion. However, unions belonging to the same sector will set the same

temporary effect, we included the current and lagged rate of change in unemployment [...], but both variables are highly insignificant." (Risager (1992), p. 553)

[6]This is just a generalisation of the staggering structure of the model in previous chapters.

wage.[7] Crucial to the interpretation of the model is the idea that in any period t there are N different contracts in effect. The existing contracts can be indexed by "j", because each sector sets its wage in a specific period.

(ii) *Relative Real Wage Concern*

Taylor's model was originally thought to incorporate a "Keynesian" component of relative wage concern on the part of the workers. However, as demonstrated in the previous chapter, his model is analytically equivalent to a model in which workers are "neoclassically" concerned only about the level of their real wages. Instead, relative wage considerations seem to be a fundamental part of the story in the wage setting process. Then, two questions are of primary interest: (i) how to introduce relative wage concern on the part of workers in the model; (ii) the definition of what actually households compare.

With respect to the first question, we model relative wage concern by including an additional argument in the utility function of the representative household. This approach runs against the deeply-rooted resistence to modifying the structure of preferences of agents.[8] Similar kind of preferences

[7] Let us call the new wage set in period t in industries $i \in [0, 1/N]$, X_t. Then, unions indexed $j \in [1/N, 2/N]$ will set their new nominal wage in period $t+1$, unions indexed $j \in [2/N, 3/N]$ will set their new nominal wage in period $t+2$, and so on. Therefore X_t, X_{t+N}, X_{t+2N}...are the wages fixed by the sector which comprises industries $i \in [0, 1/N]$, $X_{t+1}, X_{t+1+N}, X_{t+1+2N}$...the wages fixed by the sector that comprises industries $i \in [1/N, 2/N]$ and so on.

[8] As Akerlof (1997, p. 1005) puts it: *"Traditional economics has been based on methodological individualism. Until quite recently, with some rare exceptions, it has not been appreciated that this method can be, or perhaps I should say,* ***should be****, extended in de-*

have been recently proposed as an explanation for some puzzles in asset pricing (Abel (1990), Gali (1994), Campbell and Cochrane (1995)), consumption (Carrol and Weil (1994)), and growth (Carrol et al. (1997)).[9] More generally, in recent years a growing literature has emerged encompassing economic and social elements, and in particular status concern (see Frank (1984, 1985) and references therein, Baxter (1988), Kandel and Lazear (1992), Clark and Oswald (1996a,b) or Akerlof (1997)). Nevertheless, despite the available empirical evidence on unions' behaviour, and sociological and psychological considerations, it can be seen as an *ad hoc* unjustifiable short-cut. However, the purpose of this chapter is to consider the implications of relative wage concern for output and inflation persistence in a DGE framework. By introducing relative wage considerations explicitly we aim at: (i) an unambiguous identification of the analytical implications; (ii) establishing whether sensible values of the key parameters governing relative wage concern can explain output and inflation persistence.

Second, with respect to the definition of the reference wage, we denote the relative wage argument in the utility function of the representative household-union j, $RW_t{}^j$. Following FM[10], we define the *contract price in period s*, CP_s^j,

---

scribing social decisions to include dependence of individuals' utility on the utility or the actions of others. Except under rare circumstances, such interactions produce externalities." (Emphasis is as in the original.)

[9]Depending on the particular specification they are referred to as "interdependent preferences", "external habit formation", "Keeping up (or catching up) with the Joneses" or "relative income hypothesis".

[10]In what follows we keep the notation as close as possible to that of FM. Our definition of the cases B and C below also follows theirs. We present a brief comparison of our model

as the value of the contract signed by the union j in period s. To clarify the definitions note that in this subsection we use the index t to refer to the period in which the comparison takes place. s instead refers to the period in which the contract is signed. Recall that for a union setting the nominal wage in period s, $X_{s+k} = X_s$ for $k = 0, ...N-1$. Workers compare the value of the contract they sign in period s, that is CP_s, to the *index of contract prices, V*. Crucial to the modelling of the relative wage concern is the choice of the reference wage index for comparison purposes. We define V_t as the average of the contract prices of the workers in the *other* sectors in effect in period t, that is, the average of the contracts negotiated by the *other* unions. We believe that "outward comparison" specification to be the most relevant in the real world.[11] Thus, ${RW_t}^j$ is defined as the ratio between the value of the contract signed by union j and still valid in t to the index of contract prices in the relevant period t.

Because of staggering, at any period t there are N different contracts in effect, therefore N different CP_s and N different RW_t^j, one for each representative sectorial union. To highlight the mechanism through which relative real wage concern will influence wage setting, consider the problem faced by a union that sets the nominal wage in period t. Assume that the contract lasts for four periods ($N = 4$). The decision of the union then takes into account

with FM's one in Section 5.4.3.

[11]The term "outward comparison" follows a recent work by Carrol et al. (1997), and it is employed to highlight the absence of the representative individual's variables in the definition of the reference stock for comparison purposes. Specifically, in our setting "our contract price" does not enter the index of contract prices to which it is compared in the bargaining process.

that by setting X_s, it also fixes CP_s for the next four periods (hence, we have indexed it by s). The optimal X_s is thus set by comparing the current price contract the union is negotiating (that is CP_s) to the indexes of real contract prices V for periods t to $t+3$. The RW the workers will face in period t and in the following three periods as a result of the wage they are negotiating are then given by

$$RW_t^j = \left(\frac{CP_s}{V_t}\right) = \frac{CP_s}{(1/3)(CP_{s-3}+CP_{s-2}+CP_{s-1})}; \quad RW_{t+1}^j = \left(\frac{CP_s}{V_{t+1}}\right) = \frac{CP_s}{1/3\ (CP_{s-2}+CP_{s-1}+CP_{s+1})}$$

$$RW_{t+2}^j = \left(\frac{CPs}{V_{t+2}}\right) = \frac{CP_s}{1/3\ (CP_{s-1}+CP_{s+1}+CP_{s+2})}; \quad RW_{t+3}^j = \left(\frac{CP_s}{V_{t+3}}\right) = \frac{CP_s}{1/3\ (CP_{s+1}+CP_{s+2}+CP_{s+3})}.$$

Note that, because of the "outward comparison" specification, the V_t terms are not just updated symmetrically in the four periods of duration of the contract.[12]

To fully explore the implications of relative wage concern, we consider three different definitions of the value of a contract and hence of RW. We also drop the distinction between the indexes s and t introduced before for explanatory purposes. We highlight the differences on the RW_t^j faced by the union j arising from the three cases so we also drop the superscript j.

Case A: *Current Value Relative Real Wage Concern*

[12]Future variables are replaced by their expected values. We drop the expectation operator for convenience. Note also that the RW terms are different for each household-union in different cohorts, depending on the period in which they set their wage.

In this case agents compare the real wage they earn in period t with the average of the real wages earned by the other workers *in period t*. Thus all the nominal wages are deflated by the same price index P_t. It follows that the price level cancels out in the definition of RW_t and we are left only with nominal wages. Hence, in every period the wage-setters behave as if comparing their "money wage" with the average "money wage" of the *other* sectors. Therefore,

$$CP_t = X_t \; ; \quad RW_t = \frac{X_t}{(1/3)(X_{t-3} + X_{t-2} + X_{t-1})} \quad .$$

Case B*: Simplified Relative Real Wage Concern*

Workers care about the relative real wage unions manage to attain in the negotiation period. CP is therefore defined as the money wage deflated only by the aggregate price level in the period the wage was negotiated, that is[13]

$$CP_t = \frac{X_t}{P_t} \; ; \quad RW_t = \frac{X_t/P_t}{(1/3)\left(\frac{X_{t-3}}{P_{t-3}} + \frac{X_{t-2}}{P_{t-2}} + \frac{X_{t-1}}{P_{t-1}}\right)} \quad .$$

[13]Suppose a union negotiates in period t and succeeds to get a real wage X_t/P_t in period t. Then, in the next period, i.e., $t+1$, another union will negotiate a new wage. This union does not want to leave the negotiation table with a real wage for that period lower than the one negotiated last period by the previous union. In other words, the real wage the unions obtain in the negotiation is seen by the members as a sign of their bargaining power. This approach to the wage bargaining process implies a degree of myopic behaviour from the union since the wage contract lasts four periods. Even if theoretically unsatisfactory, this behavioural hypothesis: (i) could be interpreted as a simplified case of the one considered below; (ii) it is probably not far from actual unions' behaviour.

Case C: *Theoretically Preferable Relative Real Wage Concern*

In this case we suppose that the workers are concerned with their average real wage over the life of the contract. Accordingly CP is defined as the money wage deflated by a weighted average of the price level in the four periods in which the contract lasts. Hence: $CP_t = X_t/\overline{P}_t$, where $\overline{P}_t = \frac{P_t+\beta P_{t+1}+\beta^2 P_{t+2}+\beta^3 P_t}{1+\beta+\beta^2+\beta^3}$. Agents therefore calculate the average $\overline{P}_t$ discounting the future price levels by the preference discount factor β. They then compare the value of their contract, i.e. CP_t, with an average of the ones that overlap with it.

$$CP_t = \frac{X_t}{\overline{P}_t}\ ; \quad RW_t = \frac{X_t/\overline{P}_t}{(1/3)\left(\frac{X_{t-3}}{\overline{P}_{t-3}} + \frac{X_{t-2}}{\overline{P}_{t-2}} + \frac{X_{t-1}}{\overline{P}_{t-1}}\right)}\ .$$

To sum up, in Case A workers are comparing their real wage period by period, in Case B they compare the real wage they manage to attain at the time they negotiated it and in Case C they compare their real wage over the whole life of the contract.

5.3 The Benchmark Economy

The model is based on the framework of the previous chapters. There are three types of agents in the economy: firms, households and the government. The economy consists of a continuum of industries indexed by $i \in [0, 1]$, and

a continuum of industry specific unions.[14] Every industry produces a perishable product and comprises a continuum of firms. Hence the goods market in every industry is competitive. All households have the same preferences. Household j consumes a composite good, defined by a CES index over consumption goods of each industry, i.e. $C_{jt} = \left[\int_0^1 C_{jit}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$. The elasticity of substitution among goods, θ, is assumed strictly greater than one. This specification gives rise to the following demand function

$$C_{jit} = \left[\frac{P_{it}}{P_t}\right]^{-\theta} \frac{E_{jt}}{P_t} \tag{5.1}$$

where E_{jt} is household's total nominal expenditure on goods, and P_t is the aggregate price index defined as $P_t = \left[\int_0^1 P_{it}^{1-\theta} di\right]^{\frac{1}{1-\theta}}$.

5.3.1 Firms

All firms have the same technology, given by $Y_{it} = \alpha L_{it}^{\sigma}$, where labour is the only factor of production. We refrain from introducing capital into the model, because we want to focus on the degree of persistence of the real effects of money shocks induced by our relative-wage-concern contracting scheme.[15] Firms within each industry are price takers both in the goods

[14] Since there is a continuum of industries, no union is 'large' relative to the economy as a whole.

[15] Capital accumulation alone, however, has been shown to be a very weak propagation mechanism (see Cogley and Nason (1995)). On the other hand, a recent contribution by

and in the labour market. Hence firms maximise profits period by period given the nominal wage X_{it} , set by the industry-specific union. The labour demand and output of firm i are given by

$$L_{it} = \left[\frac{1}{\alpha\sigma}\frac{X_{it}}{P_{it}}\right]^{\frac{1}{\sigma-1}}; \qquad Y_{it} = \left[\frac{1}{\alpha\sigma}\frac{X_{it}}{P_{it}}\right]^{\frac{\sigma}{\sigma-1}} . \tag{5.2}$$

Imposing the equilibrium condition in the goods market, given by

$$C_{it} = \int_0^1 C_{jit}dj = Y_{it} \qquad \forall i \in [0,1], \tag{5.3}$$

yields the following relation between the labour demand and the nominal wage

$$L_{it} = K_t X_{it}^{-\varepsilon} \quad \text{where} \quad \varepsilon = \frac{\theta}{\sigma + (1-\sigma)\theta} \quad \text{and} \quad K_t = \sigma^{\varepsilon}\left[\frac{E_t}{P_t^{1-\theta}}\right]^{\frac{\varepsilon}{\theta}} . \tag{5.4}$$

This is the constant money-wage elasticity labor demand function faced by the monopolistic household-union in industry i. The elasticity is equal to ε which depends on technology and preference parameters. K_t is parametric to the union which takes aggregate variables as given, since industry i is of measure zero in the economy as a whole.

Andersen (1998b) suggests that the combination of staggered nominal wage contracts and real propagation mechanisms such as capital accumulation could substantially enhance persistence.

5.3.2 Households

The two fundamental features of the households' behaviour are: first, their monopoly power in nominal wage setting, and second, their relative wage concern.

The industry-specific household-unions enjoy monopoly power because labour is not allowed to move across industries. In period t the household maximises a utility function of the form

$$U_j = \sum_{k=0}^{\infty} \beta^k E_t \left[u(C_{t+k}, (M_{t+k}/P_{t+k}), L_{t+k}, RW_{t+k}) \right] \tag{5.5}$$

The arguments in the utility function C_{t+k}, M_{t+k}/P_{t+k} and L_{t+k}, are respectively the consumption of the composite good, the end-of-period real money balances and the labour supply of the households. The utility function satisfies the standard conditions $u_C(\cdot) > 0$, $u_{M/P}(\cdot) > 0$, $u_L(\cdot) < 0$, $u_{cc}(\cdot) < 0$, $u_{M/P,M/P}(\cdot) < 0$, $u_{LL}(\cdot) < 0$, where $u_r(t)$ denotes the first partial derivative of the instantaneous utility function and $u_{rr}(\cdot)$ the second, with respect the argument r. The specification of the relative wage argument RW_{t+k} has been discussed in Section 5.2.2. The utility function satisfies $u_{RW}(\cdot) > 0$, $u_{RWRW}(\cdot) < 0$. The purpose of the chapter is to study the effects of the introduction of relative wage concern in the utility function, on inflation and above all output persistence.

The choice variables for the household are the level of consumption, the quantities of money and bonds it will transfer to next period and the level of

the nominal wage that must be fixed for N periods. The household's budget constraint evolves according to

$$P_t C_{jt} + M_{jt} + \sum_{s_{t+1}} Q(s^{t+1} \mid s^t) B_j(s^{t+1}) \leq M_{jt-1} + B_{jt} + W_{jt} L_{jt} + \Pi_{jt} + T_{jt} \tag{5.6}$$

where $Q(s^{t+1} \mid s^t)$ is the stochastic discount factor equal to the money value of a contingent claim in state s^t to one dollar in state s^{t+1}.[16] M_{jt} denotes money holdings at the end of period t, B_{jt} the quantity of bonds in period t, T_{jt} the nominal lump-sum transfer received by the household from the government, Π_{jt} the profits distributed by firms and $L_{jt} W_{jt}$ the labour income. Each household maximises its expected lifetime utility subject to the sequence of budget constraints (5.6), the sequence of labour demand curves (5.4), and the additional constraint that the nominal wage will be fixed for N periods. The first order conditions for this problem can be expressed as follows (the index j is dropped to lighten notation),

$$\frac{u_{M/P}(t)}{u_C(t)} = \frac{R_t - 1}{R_t} \tag{5.7}$$

[16] Following CKM, let s^t denote the state of the world in period t. Denote with $\Pr(s^{t+1} \mid s^t)$ the probability that in the next period the state of the world will be s^{t+1}, conditional to the state s^t in period t. To lighten notation and avoid indexing each variable with respect the state of the world, we use the expectation operator and the dating of the variables. Then, $\Theta_t = \Theta(s^t)$ and $E_t(\Theta_k) = \sum_{s^k} \Pr(s^k \mid s^t)\Theta(s^k)$, where Θ_t is whatever variable or function of variables, s^k is the state in period $k > t$ and the sum is calculated on all the possible future states s^k.

$$u_C(t) = \beta R_t E_t \left(\frac{u_C(t+1)P_t}{P_{t+1}} \right) \tag{5.8}$$

$$\sum_{s^{t+1}} Q(s^{t+1} \mid s^t) = \frac{\beta E_t(\lambda_{t+1})}{\lambda_t} = \beta E_t \left(\frac{u_C(t+1)P_t}{u_C(t)P_{t+1}} \right) = \frac{1}{R_t} \tag{5.9}$$

$$X_t = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \left\{ \frac{E_{t-1}\left[\sum_{r=o}^{N-1} \beta^r \left(-u_L(t+r)K_{t+r}\right)\right]}{E_{t-1}\left[\sum_{r=o}^{N-1} \beta^r \left(\frac{u_C(t+r)K_{t+r}}{P_{t+r}}\right)\right]} + \frac{E_{t-1}\left[\frac{1}{\varepsilon} u_{RW}(t+r)\frac{\partial RW(t+r)}{\partial X_t}\right]}{E_{t-1}\left[\sum_{r=o}^{N-1} \beta^r \left(\frac{u_C(t+r)K_{t+r}}{P_{t+r}}\right)\right]} \right\} \tag{5.10}$$

where λ_t is the multiplier attached to the budget constraint in period t. The first three equations are standard: (5.7) represents the optimal choice between consumption and money; (5.8) is the Euler equation for consumption and (5.9) gives the gross nominal interest rate R_t.[17]

Equation (5.10) gives the optimal nominal wage set by the monopolistic household-union for N periods. Firstly, note that we assume the wage to be set before the realisation of period t shock, hence based on $t-1$ information set. The optimal wage is given by a fixed mark-up $\varepsilon/(\varepsilon-1)$ over the quantity in the curly brackets. That expression is composed of two terms. The first term represents a ratio between expected weighted averages of the marginal disutility of labour and the marginal utility of consumption over N periods.

[17]Note that $\sum_{s^{t+1}} Q(s^{t+1} \mid s^t)$ is the current value of a nominal bond that gives one unit of money for sure in the next period. On the other hand, $Q(s^{t+1} \mid s^t) = \beta \Pr(s^{t+1} \mid s^t) \left(\frac{u_C(s^{t+1})P_t}{u_C(t)P(s^{t+1})} \right)$ is the current price of a claim of one unit of money contingent on the realisation of state s^{t+1} in the next period.

In other words, the first component is a weighted average of the optimal flexible wages of the N periods, exactly as in the model of the previous chapter (just generalised for N periods, instead of only two). The second term is an expected weighted average of the relative wage concern components over the N periods. In other words, now we have this extra term in the optimal wage rule with respect to the model in the previous chapters. This latter term is positive; hence, a relative wage concern on the part of the workers tends to increase their wage.[18] In both terms the weights are defined by β, K_{t+i}, P_{t+i} and ε.[19]

[18]Indeed, it will turn out that the steady state money (real) wage of this model is higher that the steady state money (real) wage of the same model, but without a relative-wage term in the utility function (that is, the model of the previous chapters).

[19]It is probably worth noting again that, given (5.10), it is ex-post optimal for the unions to satisfy an unexpected increase in labour demand. Unions are obviously ex-post willing to satisfy extra demand for labour until the real wage is equal to the competitive one. In what follows we suppose that never to be the case. The fact that employment is always on the labour demand curve is hence consistent with optimisation in this case, in contrast to the old style Gray-Fischer-Taylor models in which the wage was set in accordance with a target level that cleared the labour market in expectation.

5.3.3 Government

The role of the government is limited to providing lump-sum transfers through which money is introduced in the economy. These transfers satisfy

$$T_t = M_t - M_{t-1} \tag{5.11}$$

and the nominal money supply process is described by

$$M_t = \mu_t M_{t-1} \tag{5.12}$$

where μ_t follows a stochastic process (to be specified below).

The resource constraint for this economy is obtained by aggregating (5.6) over all households and imposing equilibrium conditions on the money and bond market

$$\int_0^1 P_t C_{jt} dj \leq \int_0^1 (W_{jt} L_{jt} + \Pi_{jt}) dj \tag{5.13}$$

while the equilibrium condition on goods markets (5.3) implies

$$\int_0^1 P_t C_{it} di = \int_0^1 P_t Y_{it} di = P_t Y_t \tag{5.14}$$

where $Y_t = \frac{\int_0^1 P_{it} Y_{it} di}{P_t} = C_t$ is real aggregate output, defined as in national income accounting.

An equilibrium for this economy is described by a vector of allocations $\{C_{jt}, M_{jt}, B_{jt}, X_{t-k}, L_{jt}, Y_{it}, P_{it}, P_t, Y_t, R_t, Q(s^{t+1} \mid s^t)\}$ for $k = 0, ..., N-1$ such that: (i) taking other sectors' variables and aggregate variables as given, consumer allocations solve the consumer's problem $\forall j$, that is, (5.7), (5.8),

(5.9) and (5.10) hold $\forall j$; (ii) taking the nominal wage as given, firms' output and labour demand maximise profits according to (5.2) and (5.4); (iii) the transfers and the money supply process satisfy (5.11) and (5.12); (iv) the resource constraint (5.13) and the goods market equilibria ((5.3) and (5.14)) are satisfied.

To solve for the dynamics, we first calculate the steady state of the model. We then apply the standard Blanchard and Kahn (1980) methodology to the log-linearised model around the steady state, using GAUSS codes. Some details on the solution and the GAUSS codes for Case C are provided in the Appendix 5.8.

5.4 Analytical Implications of Relative Wage Concern

5.4.1 The "γ-puzzle"

The literature building upon the original works by Taylor (1979, 1980a) has focused on the ability of the "contract multiplier" induced by staggered wage-setting to propagate monetary shocks and mimic the persistence properties exhibited by US. data. As we know already from the previous chapter, those models can be summarised by a wage setting equation, a price level equation and a static aggregate demand equation, that is

$$x_t = \frac{1}{2}(p_t + E_t p_{t+1}) + \frac{\gamma}{2}(E_t y_t + E_t y_{t+1}) \tag{5.15}$$

$$p_t = \frac{1}{2}(x_t + x_{t-1}) \tag{5.16}$$

$$y_t = m_t - p_t \tag{5.17}$$

We also know already that fluctuations of output will have a small impact on prices *if and only if* the elasticity of wages with respect to output, Taylor's γ, is low. For the US, Taylor (1980b) estimates γ to be between 0.05 and 0.1, while Sachs (1980) estimates it to be between 0.01 and 0.07. In his numerical investigation of persistence properties of Taylor's (1980a) model, West (1988) uses two possible values for γ: 0.01 and 0.1. More recently, Phaneuf (1990) takes estimated values for γ for Canada, Germany, Italy, UK and US. He finds γ to lie between 0 and 0.32 and hence Ambler and Phaneuf (1992) calibrate $\gamma = 0.15$. Jeanne (1998) suggests that γ should lie between 0.05 and 0.2. Furthermore, in an important recent contribution from the labour literature, Blanchflower and Oswald (1994), using microdata from household statistics, provides estimates of the effects of unemployment on wages in more than 10 countries. Their estimates are consistently around -0.1. Therefore, the empirical literature suggests a value of γ around 0.1, which is consistent with the existence of a contract multiplier.

As seen in the previous chapter, recent research on output persistence incorporates the staggered wage/price models into a DGE framework. Sound microfoundations for the wage/price equation under staggering are then pro-

vided, opening the "black box" of the *ad hoc* parameters. A log-linearisation of the wage setting rule around a deterministic steady-state with constant money supply ($\overline{\mu} = 1$) and constant returns to scale ($\sigma = 1$) implies that the fundamental parameter γ depends on the labour supply decision. It is in turn determined by the elasticities of the marginal utilities of consumption (with respect to consumption) (η_C)[20] and of labour (with respect to labour) (η_L), evaluated at steady state. Given the existing evidence from microdata on the intertemporal elasticities of substitution of consumption ($-1/\eta_C$) and of labour supply ($1/\eta_L$), a sensible calibration of γ gives a value too high to generate persistence. For example, in CKM a sensible calibration gives a value of $\gamma \geq 1$. In conclusion, the calibration of γ based on well-established microevidence becomes incompatible with the empirical estimates from macrodata, as we concluded in the previous chapter. This is what we called the "γ-puzzle".

[20] In this chapter we will use η_s for the elasticity of the marginal utility of s with respect to s, instead of η_{ss} as in the previous chapter to lighten notation. Thus, for example, now the elasticity of the marginal utility of consumption with respect to consumption is denoted by η_C instead of η_{CC} as in the previous chapter. This is possible now since in our analytical section we only consider additively separable utility functions and hence the cross elasticities are zero.

5.4.2 Effects of Relative Wage Concern

Can the model with relative wage concern solve the "γ-puzzle"? We argue that this is the case. The intuition is as follows. A negative η_{RW} causes a "following" behaviour in wage setting by the agents.[21] Suppose there is a negative shock to the rate of growth of money. Agents want to keep their real wage in line with the existing ones. Under staggering, it generates a slower adjustment in nominal variables, that is, a degree of endogenous stickiness, which leads to persistence of the real effects of money shocks. Specifically, relative real wage concern influences wage setting decisions by critically lowering the sensitivity of nominal variables to the business cycle conditions. The following proposition formalises the intuition.

Proposition 5.1. *Let the utility function be separable in all its argument and the RW_t term be a linear function of X_t. Then, log-linearising the resulting wage setting rule around the steady-state with $\overline{\mu} = 1$, the equivalent in our model to Taylor's γ is:*[22]

$$\gamma_{RW} = \left\{ \frac{\left\{ \frac{\frac{\varepsilon}{\theta}\eta_L - \eta_C}{\varepsilon\eta_L + 1} \right\} - \left\{ \left[\frac{\frac{\varepsilon}{\theta} + \eta_C}{\varepsilon\eta_L + 1} \right] \left[\frac{U_{RW}(.)}{-\varepsilon\, U_L(.)\, K_t\, X_t^{-\varepsilon}} \right] \right\}}{1 - \left\{ \left[\frac{\eta_{RW} + \varepsilon}{\varepsilon\eta_L + 1} \right] \left[\frac{U_{RW}(\cdot)}{-\varepsilon\, U_L(.)\, K_t\, X_t^{-\varepsilon}} \right] \right\}} \right\} \tag{5.18}$$

[21] See Clark and Oswald (1996b).

[22] With respect to the model of this chapter, this proposition is basically the equivalent of proposition 4.4 and γ_{RW} corresponds to g in Chapter 4. The model of this chapter is however closer to FM's one than to Taylor's one, as the next section shows (see 5.21 and 5.22).

Therefore, γ_{RW} is decreasing in the absolute value of η_{RW}.

The first term in curly brackets in the numerator corresponds to the γ arising simply from staggered wages, i.e., g in Chapter 4. In this model, it is complemented by additional terms incorporating the marginal utility of the relative wage term, $U_{RW}(\cdot)$, and its elasticity η_{RW}. The inconsistency of the microfounded wage setting equations and the empirical estimates can then be resolved. The presence of $(-\eta_{RW})$ increasing the denominator of the expression is crucial. It lowers the sensitivity of wages to the business cycle conditions, allows for endogenous stickiness and thus makes output persistence a likely outcome. Its quantitative implications are the focus of the remaining sections of this chapter.[23]

5.4.3 A Comparison with FM Specification

This section completes the description of the analytics of the model. Its purpose is twofold. First, we discuss the log-linearisation of the money-wage setting rule. Our log-linearised model is somehow close to the FM contract specification. Hence, a brief comparison between the two specifications clarifies further the mechanisms at work in the model. Second, the log-linearised specification of the wage-setting rule will play a fundamental role in the cal-

[23]Furthermore, it is worth noting that the effect of the intertemporal elasticity of substitution in labour supply, i.e. η_L, is ambiguous.

ibration of the parameters in Section 5.5.2.

We parameterise the instantaneous utility function as follows

$$u\left(C, \frac{M}{P}, L, RW\right) = \frac{1}{\nu} \ln \left[bC^{\nu} + (1-b)\left(\frac{M}{P}\right)^{\nu}\right] - \chi L^{e} + \frac{\psi}{1-\tau}\ (RW)^{1-\tau}. \tag{5.19}$$

Note that the crucial η_{RW} is simply equal to $-\tau$.

A log-linearisation of (5.10) around the steady state with $\overline{\mu} = 1$ and $\beta = 1$ yields

$$\Omega\ x_t = \frac{1}{4}\Omega \sum_{i=0}^{3} E_t p_{t+i} + \frac{1}{4}\ \Lambda \sum_{i=0}^{3} E_t(v_{t+i} - cp_t) + \frac{1}{4}\Gamma \sum_{i=0}^{3} E_t y_{t+i} \tag{5.20}$$

where lower case letters denote log-deviations from steady state values and

$$\Omega = \tfrac{\sigma(\varepsilon-1)[1+\varepsilon(e-1)]-\psi e\varepsilon}{\sigma(\varepsilon-1)}; \qquad \Lambda = \tfrac{\psi(\tau-1)}{\sigma(\varepsilon-1)}; \qquad \Gamma = \tfrac{\sigma(\varepsilon-1)[\theta+\varepsilon(e-1)]-\psi e\varepsilon}{\theta\sigma(\varepsilon-1)}.$$

The intuition behind it can be easily explained.[24] Ω represents the weight on the own real wage. Γ represents the sensitivity of the nominal wage with respect to the business cycle conditions, exactly as in Taylor's model. Λ captures the importance of relative wage concern. That term in the wage setting rule is the novelty of the chapter. Traditional staggered wage models, like Taylor (1980a), CKM and Chapter 4, impose $\Lambda = 0$.

Our log-linearised model is close to FM. Indeed, our model could be thought as a microfounded version of their model. They present and estimate

[24] For standard parameter values Ω, Λ and Γ are non-negative.

an *ad hoc "...contracting model, in which agents are concerned with relative real wages, that is data consistent"* (FM, abstract). They suppose that agents set nominal wages such that CP equals the average real contract price index expected to prevail over the life of the contract, adjusted for excess demand conditions. That is

$$cp_t = \sum_{i=0}^{3} f_i E_t(v_{t+i} + \gamma y_{t+i}) \tag{5.21}$$

They present two cases which correspond to our cases B and C in Section 5.2.2. In particular, in the *theoretically preferable case* (Case C in Section 5.2.2) equation (5.20) can be written as

$$cp_t = \frac{1}{4}\frac{\Lambda}{\Lambda+\Omega}\sum_{i=0}^{3} E_t v_{t+i} + \frac{1}{4}\frac{\Gamma}{\Lambda+\Omega}\sum_{i=0}^{3} E_t y_{t+i} \tag{5.22}$$

There are two important differences between our microfounded wage setting equation and the *ad-hoc* one of FM.[25] First, FM define the v_{t+i} terms as the average of the existing real contract prices *including* the real contract price of the sector currently negotiating the wage. As explained in Section 5.2.2, we believe that our outward comparison better replicates actual relative wage concern. Second, the coefficient on the v_{t+i} is not necessarily equal to unity in our model, so one-to-one following behaviour is not imposed.

[25] A third minor difference highlights the additional insights obtained from microfoundations. FM impose the weights f_i to be decreasing linearly and estimate the slope parameter. Instead, without imposing $\beta = 1$, in our model the equivalent to the f_i terms are decreasing and have a very intuitive interpretation: they depend naturally on the discount factor β (similarly to d and b in the previous chapter).

For equation (5.22) to match FM's formulation we need to impose $\Omega = 0$. This implies: (i) setting equal to zero the standard 'neoclassical' real wage concern on the part of the workers; (ii) imposing a one-to-one following behaviour in wage setting, since a 10% change in v_{t+i} leads then to a 10% change in CP_t.

5.5 Quantitative Implications of Relative Wage Concern

5.5.1 Model Calibration

We assume that contracts last for four quarters ($N = 4$). The rate of growth of money follows the stochastic process

$$\ln \mu_t = \rho \ln \mu_{t-1} + (1-\rho) \ln \overline{\mu} + \xi_t \tag{5.23}$$

where ξ is a normally distributed i.i.d. mean zero shock with standard deviation σ. Following CKM, we calibrate $\overline{\mu} = 1.06^{\frac{1}{4}}$ and $\rho = 0.57$.[26]

Since households can exchange contingent claims, they perfectly insure

[26] Since we are just interested in the persistence properties of the model, we actually focus only on impulse response functions to money shocks. Hence, the standard deviation of the rate of growth of money process does not play any role. In addition, in what follows, we calibrate the model as closely as possible to CKM in order to make possible the comparison with their results.

themselves against fluctuations in income, pooling resources. Given that households are *ex-ante* identical, they will therefore enjoy the same marginal utility of consumption in every period. Given (5.19), they will enjoy the same level of consumption and real money balances in each period. Moreover, given (5.19), (5.7) implies the following money demand equation

$$\ln\left(\frac{M_t}{P_t}\right) = -\frac{1}{1-v}\ln\left(\frac{b}{1-b}\right) + \ln C_t - \frac{1}{1-v}\ln\left(\frac{R_t-1}{R_t}\right) \tag{5.24}$$

which is exactly the same as equation (43) in CKM. Following CKM, we use Mankiw and Summers' (1986) money demand regressions, and obtain $\nu = -17.52$ and $b = 0.73$.

The parameter e determines the intertemporal elasticity of labour supply ($1/\eta_L = 1/(e-1)$). Macurdy (1981) suggests $e = 4.3$, while most Pencavel's (1986) estimations place e between 3.2 and infinity. We calibrate $e = 6$ (which implies a low intertemporal elasticity of substitution of labour supply of 0.2).

For the discount factor we choose the standard value from business cycle literature, i.e. $\beta = 0.96^{\frac{1}{4}}$. We interpret our production function as a short-run production function where the level of capital is fixed. Hence, the labour share of output, i.e. σ, is set equal to 0.67, as it is standard in this literature. We calibrate $\theta = 6$, as Hairault and Portier (1993).[27] Finally we calibrate χ

[27]There is no parameter corresponding to our θ in CKM. Since they use a CES function as technology for producing final goods from intermediate goods, it follows that their CES

such that the number of average aggregate hours of work in steady state is equal to 1/3 and α (which is just a scaling factor) such that aggregate output is equal to one.

5.5.2 Calibration of the Relative Wage Concern Parameters

Crucial for the analysis are the values of the parameters of the relative wage concern argument in the utility function, i.e. ψ and τ. To our knowledge, there are no microestimates to be used as reference values for these parameters in the labour literature.

Traditional staggered wage models (as Taylor (1980a), CKM or Chapter 4) impose $\Lambda = 0$. As said in 5.2.1, empirical evidence however suggests that wage setting behaviour is better characterized by pure relative wage considerations and strong following behaviour, with the level of absolute real wage playing a minor role, if any at all. We therefore impose $\Omega = 0$ and employ the estimates in FM to calibrate ψ and τ. Specifically, looking at (5.22), we use the constraint $\Omega = 0$ to pin down ψ[28]; then use FM's estimate of γ to determine τ. The estimated value of γ in FM is 0.00109 for the theoret-

parameter is a technology parameter which gives the elasticity of substitution in input demand.

[28]Note that τ enter only in the expression for Λ and not in the ones for Ω and Γ.

ically preferable specification and 0.00435 for the simplified specification.[29] We obtain a value for ψ of 0.76.[30] However the value of τ implied by the estimate of FM is sky-high, equal to 844!! With $\tau = 844$ the model generates a ridiculous degree of persistence, as Figure 5.1 shows. The level of output remains above its steady state value for more than 60 periods!!

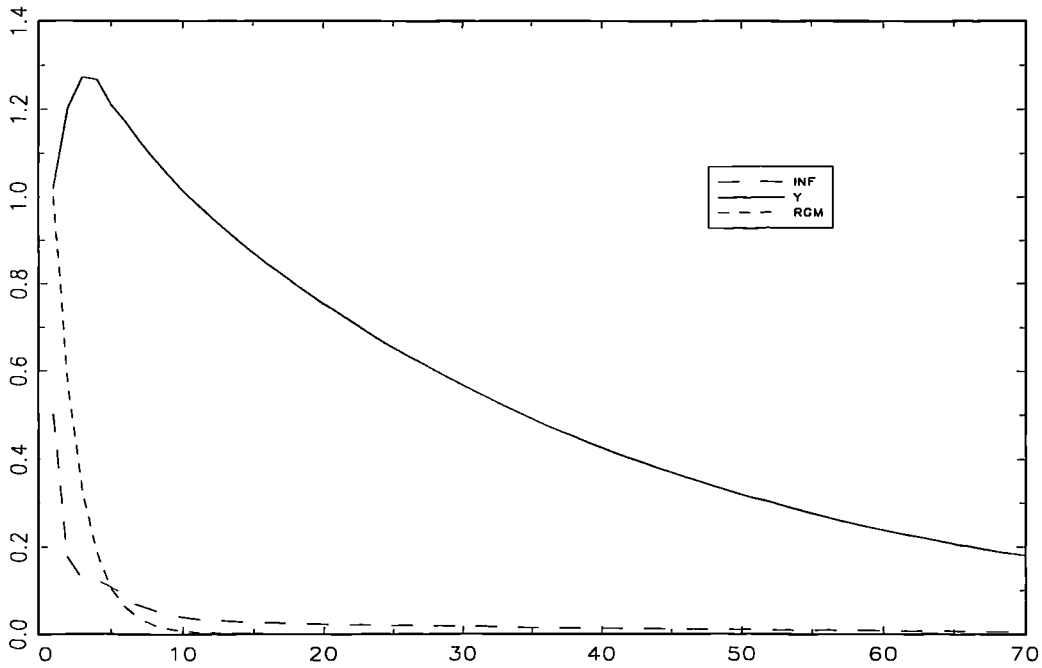

Figure 5.1: *Theoretically Preferable Relative Wage Concern. 1% Money Shock: Output and Inflation.* $\tau = 843.9689$, $\psi = 0.7588$

[29]These values are extremely low and only marginally significant. The t-ratio is 1.54 for the theoretically preferable specification and 2.3 for the simplified one. The results are therefore in line with Risager (1992).

[30]The steady state of the model imposes an upper bound on the value of ψ equal to $\overline{\psi} = 0.84$, otherwise the nominal wage is negative.

Preferences	$\beta = 0.96^{\frac{1}{4}}$; $\nu = -17.52$; $b = 0.73$; $\theta = 6$; $e = 6$
Technology	$\sigma = 0.67$
Money Growth Process	$\overline{\mu} = 1.06^{\frac{1}{4}}$; $\rho = 0.57$
Relative Wage Concern	$\psi = 0.76$; $\tau = 10.2$

Table 5.1: Calibrated Parameter Values

FM estimates of γ are however substantially lower than the results in the empirical literature discussed in Section 5.2.1 and 5.4.1. That literature suggests a value around 0.1. We have shown in Section 5.4.2 that such value is not in principle incompatible with sound microfundations once relative wage concern is taken into account. We therefore consider as a benchmark case a value of γ equal to 0.1. The implied value of τ is 10.2. Table 5.1 summarises the calibration of the model parameters.

5.5.3 Simulation Results

Figures 5.2, 5.3 and 5.4 show the impulse response functions for output and inflation, following a 1% money shock, as implied by the three models, respectively Cases A (Current value), B (Simplified) and C (Theoretically Preferable), described in Section 5.2.2. The impact effect is the same in all models since we assume the money wage to be fixed before the realisation of the

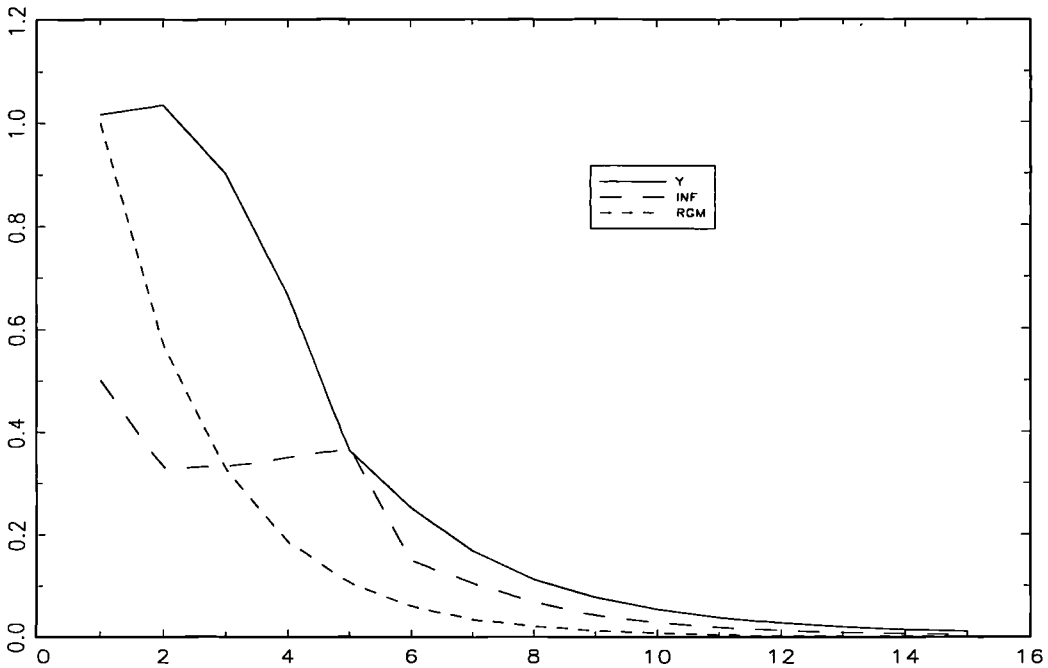

Figure 5.2: *Current Value Relative Wage Concern. 1% Money Shock: Output and Inflation.* $\tau = 10.1884$, $\psi = 0.7588$

shock.[31] Output jumps on impact of 1.02%. Note that in all cases the model is able to mimic the hump-shaped response in the trend reverting component of output as shown by Blanchard and Quah (1989), Cochrane (1994) and Cogley and Nason (1995). Moreover, in all cases persistence both in output and in inflation is substantial.[32] The current value real wage concern

[31] Hence, in the first period the money wage is at its steady state level and is not affected by the money shock.

[32] To measure the degree of persistence we take the quarter in which the log-deviation of output from steady state falls and remains thereafter below 0.05% in absolute value.

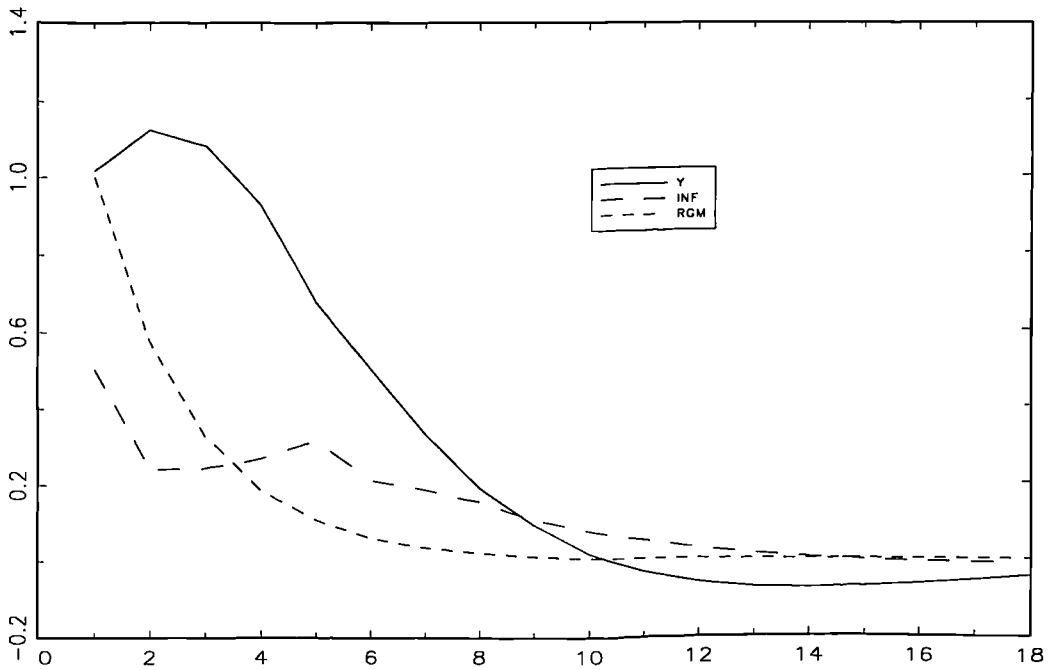

Figure 5.3: *Simplified Relative Wage Concern. 1% Money Shock: Output and Inflation.* $\tau = 10.1884$, $\psi = 0.7588$

case (Case A) exhibits the lowest degree of output persistence equal to 11 quarters. Persistence increases to 18 quarters in the simplified relative real wage concern case (Case B).[33] In the theoretically preferable case (Case C) the effects on output last for 3 years. There is an intuitive reason for these differences. The current value real wage concern case implies the lowest order of dynamics in the model, since the price level is absent from CP. In other words, agents look backward the same degree they look forward, but both these degrees are limited with respect to the two other cases. That

[33] After ten quarters, output actually falls below the steady state value.

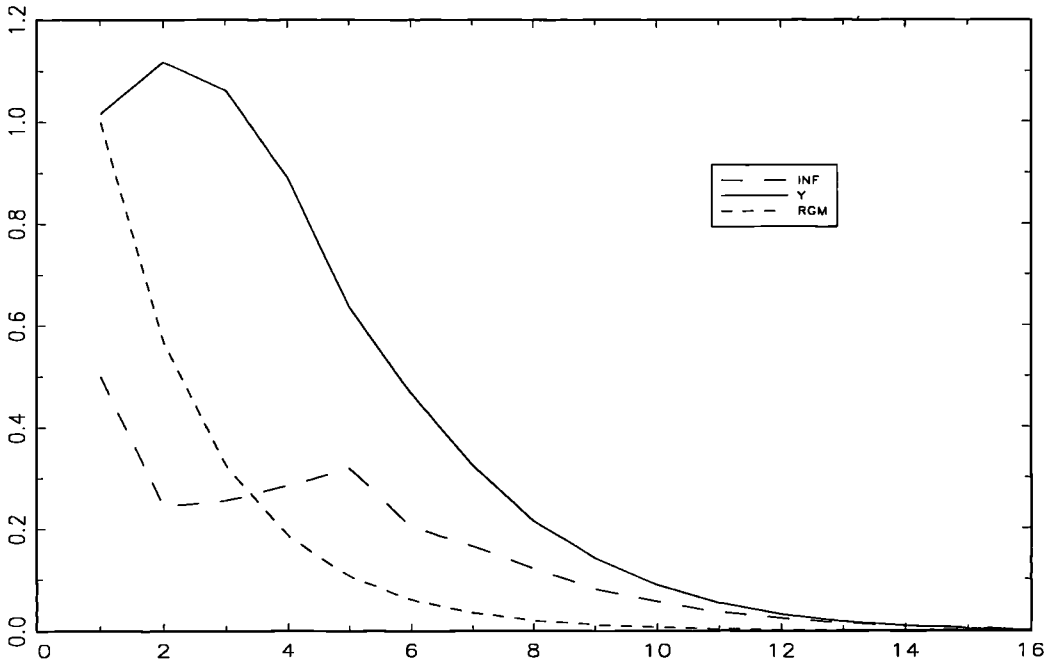

Figure 5.4: *Theoretically Preferable Relative Wage Concern. 1% Money Shock: Output and Inflation.* $\tau = 10.1884$, $\psi = 0.7588$

is, substituting the definitions of CP and the equation for the price level[34] in equation (5.20), the highest lagged nominal wage term is x_{t-3}, while the highest lead nominal wage term is x_{t+3}. The dynamics instead goes from x_{t-6} to x_{t+3} in the simplified case (B) and from x_{t-6} to x_{t+6} in the theoretically preferable case (C).[35] In fact in the simplified case, the price level enters the

[34]The log-linearised formula for the price level is $p_t = \left(\frac{1-\sigma}{\sigma}\right) y_t + \sum_{i=0}^{3} q^i x_{t-i}$ where $q^i = \left(\frac{\mu^{i(\varepsilon-1)}}{\sum_{i=0}^{3} \mu^{i(\varepsilon-1)}}\right)$.

[35]The same holds if we express (5.20) in terms of inflation, because we get: $F(\pi_{t-3}, ..., \pi_{t+3}, \widetilde{y}_t) = 0$ in case A; $F(\pi_{t-6}, ..., \pi_{t+3}, \widetilde{y}_t) = 0$ in case B and $F(\pi_{t-6}, ..., \pi_{t+6}, \widetilde{y}_t) = 0$ in case C.

specification of CP and hence, since V_t includes CP_{t-3}, X_{t-6} enters equation (5.20). However, since future prices do not enter the specification of CP_t, V_{t+3} brings in only p_{t+3} and hence x_{t+3}. In the theoretically preferable case, instead, agents are less myopic and CP includes future prices through $\overline{p}$. It follows that v_{t+3} depends on $\overline{p}_{t+3}$ and hence x_{t+6}. To sum up, in Case A agents basically care about their relative nominal wages over the length of the contract and hence the order of the dynamics is limited with respect to the other two cases, since the price level does not enter CP. In Case B agents are only concerned about the real wage attained in the negotiation period and hence they myopically look backward more than they look forward. In Case C instead agents compare relative real wages over the whole length of the contract and hence look backward the same degree they look forward. This implies an higher degree of inertia in Case B with respect to Case C and hence an higher degree of persistence, as shown in the Figures.[36]

In all the cases therefore, the model is able to generate a fairly substantial amount of persistence both in output and in inflation, without needing a gigantic value of τ. Our analysis hence suggests that staggered wage setting

[36]Higher dynamics do not necessarily imply higher persistence. It mainly depends on the relative weights on backward vs. forward looking variables. Hence, it seems that the relative weight of backward and forward looking variables is not the same in the three models. This suggests that the different specifications do not simply spread the same relative weights over higher order dynamics.

incorporating a relative real wage concern on the part of the workers might be a very important mechanism through which monetary shocks are propagated in the economy. Previous results may have thus failed to account for output persistence in a microfounded model with staggering because of their oversimplified modelling of the wage setting decisions.

5.6 Some Sensitivity Analysis

In this section we provide some sensitivity analysis of the two key parameters τ and ψ. We focus on the theoretically preferable specification.

5.6.1 Sensitivity of Persistence with respect to τ.

Figures 5.5, 5.6 and 5.7 plot the impulse response functions for values of τ of 31.63, 19.38 and 5.59, corresponding to values of γ of 0.03, 0.05 and 0.2 respectively. Unsurprisingly, the degree of output persistence consistently decreases with τ. With $\tau = 31.36$, the effects of money shocks on output die away after 21 quarters, if $\tau = 19.38$ after 4 years and if $\tau = 5.59$ after 9 quarters.

In CKM's model: *"the persistence properties of output are highly nonlinear in γ, so that increasing γ to a small amount above 0.05 reduces persistence sharply. [...] even with values of γ as low as 0.25 output movements are not very persistent."* (CKM, p. 15). Values of γ higher than 0.25 also de-

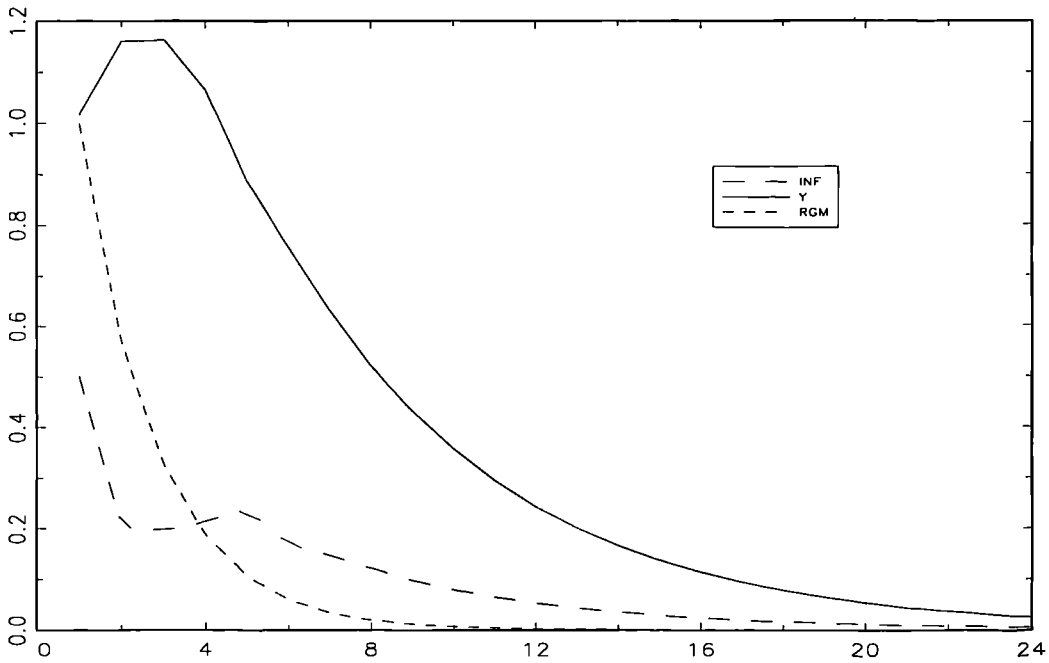

Figure 5.5: *Theoretically Preferable Relative Wage Concern. 1% Money Shock: Output and Inflation.* $\tau = 31.6279$, $\psi = 0.7588$

crease persistence in our model. Nevertheless, the perspective should change: even with values of γ as **high** as 0.25, our staggered wage model is still able to generate output persistence. As discussed in Section 5.4.1, empirical estimates put 0.25 among the highest possible values for γ. CKM consider instead their calibration value $\gamma = 1.22$ the actual reference point. Indeed, they argue that only values of γ greater than one are compatible with sound microfundations in staggered wage models. However, that is not necessarily the case, as we proved in Section 5.4.2. On the contrary, our model suggests

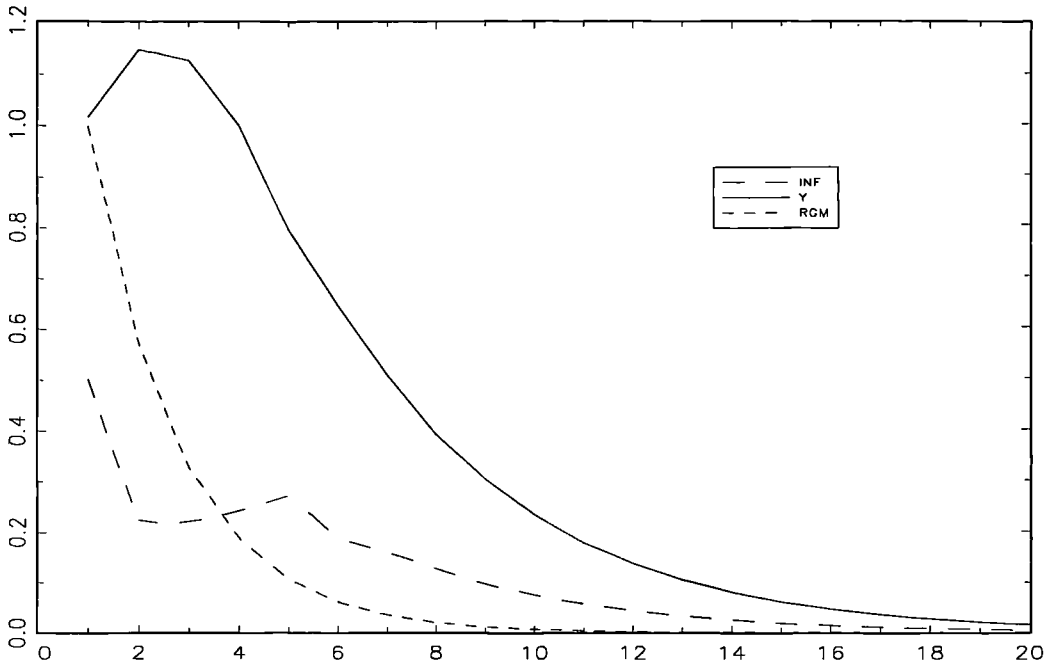

Figure 5.6: *Theoretically Preferable Relative Wage Concern. 1% Money Shock: Output and Inflation.* $\tau = 19.3767$, $\psi = 0.7588$

that their analysis omitted fundamental features of the wage setting.[37] Once the relative wage concern on the part of the workers is incorporated, it solves the data inconsistency of microfunded staggered wage models with respect to the calibration of γ. We investigate further the relationship between γ and the key parameter τ. Figure 5.8 shows the trade-off between the values

[37]In fact, some of the results they report are quite puzzling: *"It turns out that if we assume a labor supply elasticity large enough to get γ down to 0.05, the model generates ridiculously large output effects in the impact period. [...] following a shock which raises the growth rate of money supply by 1% after one year [...] output rises of 30%."* (CKM, p. 16). This is instead not true in our model.

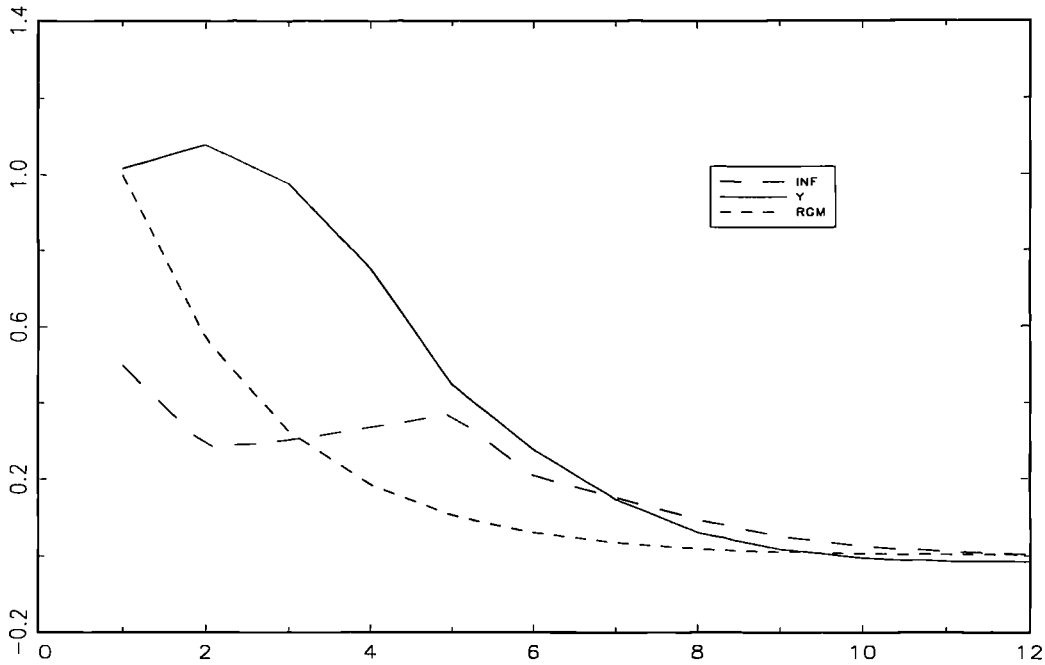

Figure 5.7: *Theoretically Preferable Relative Wage Concern. 1% Money Shock: Output and Inflation.* $\tau = 5.5942$, $\psi = 0.7588$

of γ and τ. This relationship is highly non-linear. It implies that fairly small departures from our conservative parameter choices can increase persistence sharply.

Inflation persistence is, on the other hand, very little sensitive to changes in τ. The effects of money shocks on inflation die away in all cases after 10/12 quarters, as in the base case.

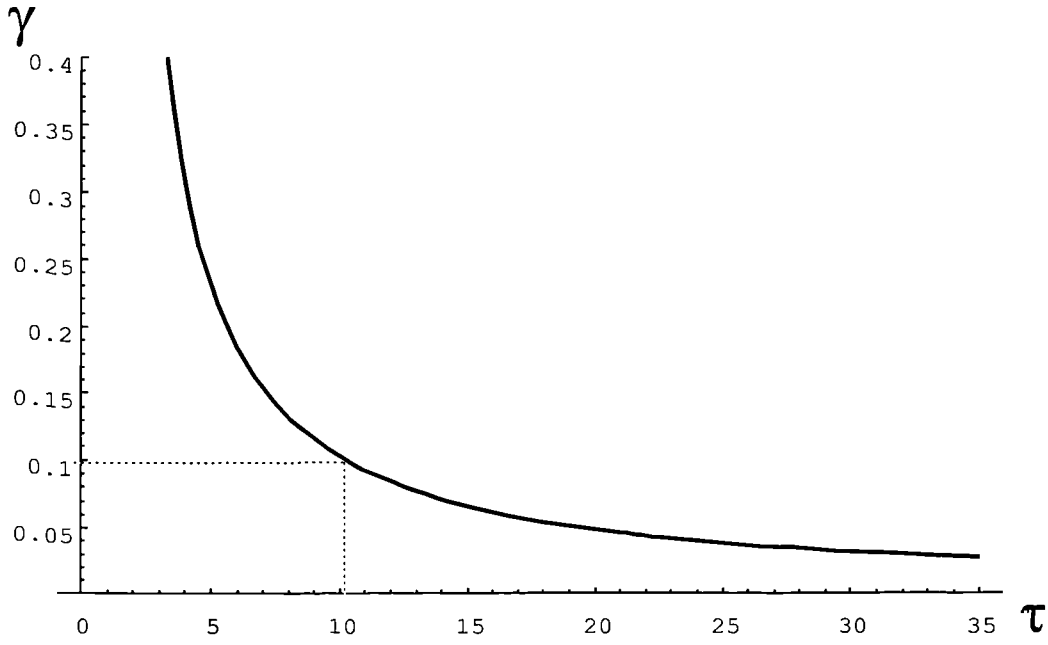

Figure 5.8: γ *and* τ *Trade-off*

5.6.2 Sensitivity of Persistence with respect to ψ

In the previous section, we set $\Omega = 0$ in our wage setting rule and calibrated ψ to be 0.76. Empirical evidence presented in Section 5.2.1 points at that case as the most relevant one. However, our money-wage setting equation (5.20) incorporates two elements: (i) the absolute real wage concern (weighted by Ω); (ii) the relative wage concern (weighted by Λ). In this section we analyse the implications of both relative wage and level of own real wage considerations for wage setting decisions.

Recall that in the *theoretically preferable case* equation (5.20) can be written as (5.22). We consider two alternative cases. In the first case $\Lambda = 3\Omega$.

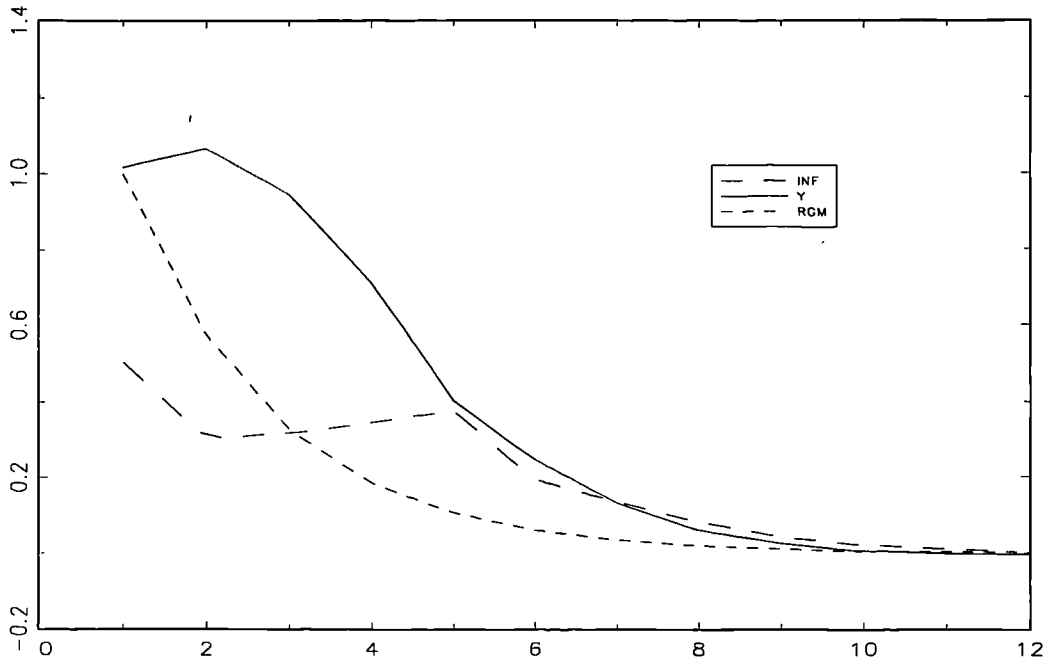

Figure 5.9: *Theoretically Preferable Relative Wage Concern. 1% Money Shock: Output and Inflation.* $\tau = 10.1884$, $\psi = 0.6192$

The parameter on the indexes of real wages in the other sectors ($\sum_{i=0}^{3} E_t v_{t+i}$) in equation (5.22) above is equal 3/4. Thus, there is no more one-to-one following behaviour: a 10% increase in the sum of the future indexes of real contract prices leads to a 7.5% in the current contract price, CP. The implied value for ψ in this case is 0.62.[38] Output and inflation persistence decreases

[38] χ is the weight of the labour supply term and ψ that of the relative wage concern in the utility function. χ is calibrated to produce an average level of hours worked in the economy equal to 1/3, as standard in this literature. For the benchmark case $\psi = 0.76$, $\chi = 3.4$. Note however that as ψ decreases, then χ has to increase to maintain the average aggregate labour hours at 1/3. Specifically, χ in this case becomes 10.77. This tends to

to 9 quarters (Figure 5.9). In the second case we set $\Lambda = \Omega$. There is then equal weighting of the absolute and the relative real wage considerations in wage setting. Now a 10% increase in the sum of the future indexes of real contract prices leads only to a 5% in the contract price, CP. The implied value for ψ in this case is extremely low and equal to 0.2.[39] Persistence in both inflation (2 years) and output (7 quarters) decreases further (Figure 5.10).

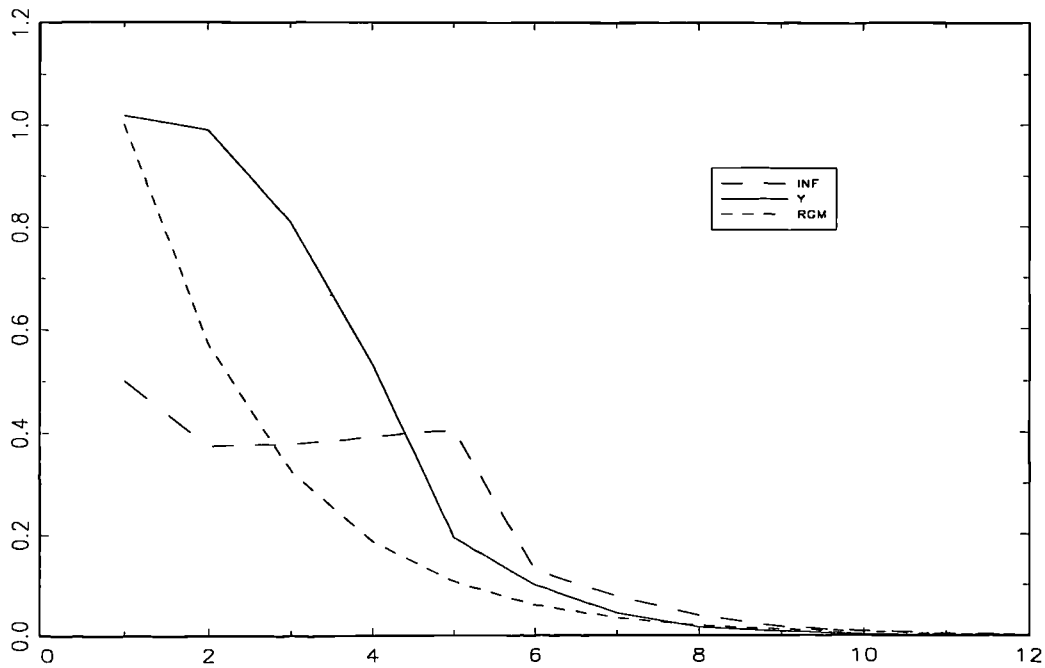

Figure 5.10: *Theoretically Preferable Relative Wage Concern. 1% Money Shock: Output and Inflation.* $\tau = 10.1884$, $\psi = 0.2032$

make more costly any marginal increase in the supply of labour.

[39]The implied value of χ is 32.7!!

Both output and inflation persistence therefore decrease with ψ. The intuition is simple. If $\Omega = 0$ wage setting is mainly influenced by relative wage considerations. Persistence is then a very likely outcome. As Ω increases, note that we get back to Taylor's model, that we already know cannot generate neither output nor inflation persistence.

5.6.3 Sensitivity of Persistence with respect to σ

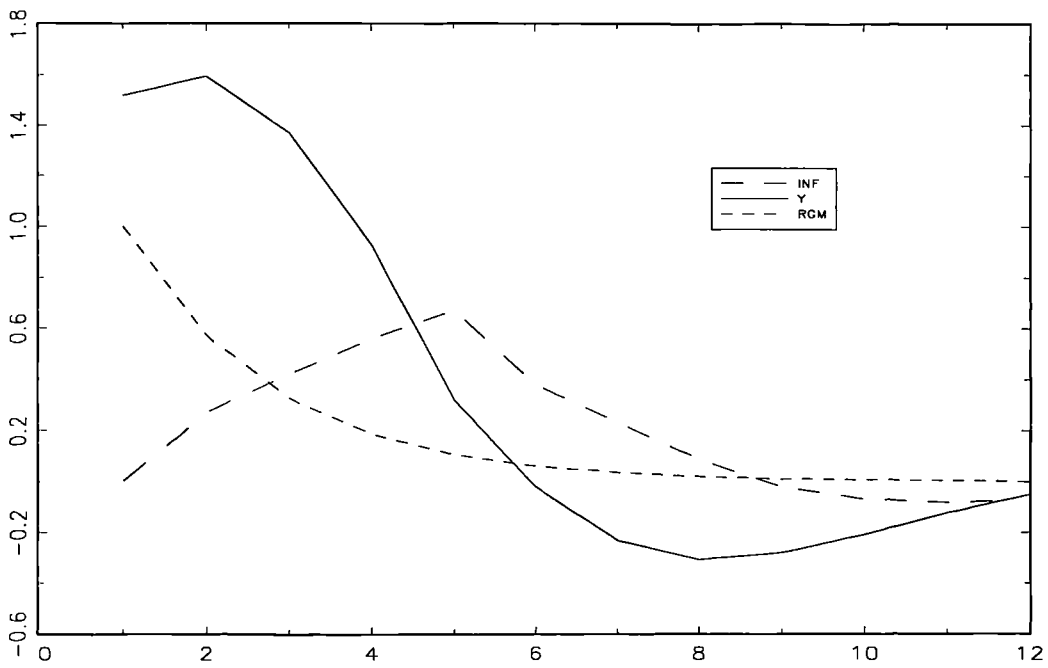

Figure 5.11: *Theoretically Preferable Relative Wage Concern. 1% Money Shock: Output and Inflation.* $\sigma = 1$, $\tau = 10.7$, $\psi = 4.2$, $\chi = 10.7$

A final remark concerns the sensitivity of output and inflation response to σ. We consider our stylised production function as a short-run production

function where capital is fixed and calibrate $\sigma = 0.67$. This implies $(1 - \sigma)/\sigma \simeq 0.5$. Hence a 10% increase in output automatically leads to a 5% increase in prices (see footnote 34). However, factor hoarding and inventory stocks may limit the impact of increased output on prices by allowing for constant returns to scale in the short-run, that is, $\sigma \simeq 1$. Our model does not incorporate any factor hoarding. However, for illustrative purposes, Figure 5.11 shows the impulse responses of output and inflation for $\sigma = 1$.[40] Inflation becomes much more sluggish: it peaks after 5 quarters and then gradually returns to its steady state level. As a result, the shape of the impulse response function for output also changes: after 6 quarters from the shock the economy would enter a recession which peaks after 8 quarters. This shows how this model, if allowed to incorporate some factors hoarding, can generate strong inflation persistence.

5.7 Conclusions

Here we have reconsidered the presence of a strong contract multiplier as a fundamental nominal propagation mechanism in staggered wage economies.

[40]In this case, some values of the parameters change: $\tau = 10.67$ (to keep $\gamma = 0.1$), $\psi = 4.2$ (to keep $\Omega = 0$) and $\chi = 10.72$ (to keep steady-state working hours equal to 1/3). Note that also the upper value on ψ changes, i.e. $\overline{\psi}$ is now equal to 4.82; the value of ψ above is thus still consistent.

The previous chapter and other recent works has questioned the existence of such a multiplier. Those staggered price/wage models have failed to generate persistence of the real effects of money shocks. Our model does. We add relative wage concern on the part of the workers to the model analysed in the previous chapter. This provides a combination of nominal and real rigidities capable of generate a substantial amount of endogenous stickiness, even with a very inelastic intertemporal elasticity of labour supply. As a result, output and inflation persistence are a likely outcome in our framework.

The relative wage concern on the part of the workers is the key feature of the model. The notion of relative wage concern is not new for economists and goes back a long way, at least to J.M. Keynes. Moreover, a great deal of applied studies provide overwhelming evidence for a relative wage concern on the part of the workers. Furthermore, an increasing number of works in any field of the literature started considering status and sociological considerations to be able to explain various puzzles that standard economic framework could not explain. Introducing a relative wage concern in the analysis, by adding a term to the utility function, places our work within this growing economic literature. Our results show that failing to account for this specific source of real rigidity might be an important weakness of previous staggered wage models, responsible for their negative results concerning output and inflation persistence.

Our analysis also highlights the mechanism by which our specific combination of nominal and real rigidities contributes to the presence of endogenous stickiness. We analyse this mechanism by focusing on the elasticity of the wages with respect to the business cycle conditions, i.e., the famous parameter γ in Taylor's wage setting rule specification. Only for relatively low values of that parameter does output persistence arise (in the order of 0.05, the benchmark Taylor's estimate). From a log-linearized version of the wage setting equation around a deterministic steady-state with constant money supply, CKM and Chapter 4 have proved the dependence of γ on the intertemporal elasticity of labour supply and intertemporal elasticity of consumption. According to well-established micro evidence, CKM calibrated $\gamma = 1.22$ for their price-staggering model, in sharp contrast to numerous empirical studies that place it around 0.1. They conclude by discarding completely staggered wages as relevant propagation mechanism *"...because γ is necessarily greater than 1"*. In our staggered-wage model, from a log-linearized version of the wage setting rule once relative wage concern is introduced in the analysis, we instead show that $\gamma \geqslant 1$ *is* ***not*** *necessarily the case.* Nor are the estimated values of the empirical literature incompatible with sound microfoundations at all. High values of γ may well arise, however, from an oversimplified account of the wage setting decisions.

Our model delivers a substantial amount of persistence both in output and

inflation. This result is very robust to different calibrations/specifications of the model. Moreover, we derive a simple relationship between the key parameter and the value of γ. This relationship is highly non-linear. It implies that fairly small departures from our conservative parameter choices can increase persistence sharply. Given the substantial amount of empirical evidence supporting a relative wage concern on the part of workers, our analysis leads us to conclude that this may well be the missing piece in the money shocks persistence puzzle.

5.8 Appendix: The Solution Method and the GAUSS Codes

The procedure used to simulate the model is quite standard in the business cycle literature. We would like to thank Dr. Morten Ravn who gave a course at Warwick University on simulation methods. Our codes are based on the ones he supplied during the course.

The procedure rests on the following steps: (i) solve for the steady state of the model; (ii) linearise the model around the steady state;[41] (iii) build the dynamic system distinguishing among control, state, and costate variables;

[41] Given that the model deals with nominal variables and that the money supply follows a certain rate of growth, we need to make the system stationary. We did that dividing all period t variables by M_{t-1}.

(iv) apply the Blanchard and Kahn (1980) methodology for solving linear dynamic system with forward-looking variables to the dynamic system just built; (v) simulate the model to produce impulse response functions. The last three stages correspond to the three GAUSS codes below which are the codes we used to simulate Case C.[42]

The first code implements (iii) and part of (iv). The dynamic system for this particular version of our model is made up by 29 control variables (Y_t, Y_{t+1}, Y_{t+2}, Y_{t+3}, Y_{t+4}, Y_{t+5}, P_t, P_{t+1}, P_{t+2}, P_{t+3}, P_{t+4}, P_{t+5}, Z_t, Z_{t+1}, Z_{t+2}, Z_{t+3}, Z_{t+4}, $\overline{P}_t$, $\overline{P}_{t+1}$, $\overline{P}_{t+2}$, CP_t, CP_{t+1}, CP_{t+2}, CP_{t-3}, CP_{t-2}, CP_{t-1}, $\overline{P}_{t-3}$, $\overline{P}_{t-2}, \overline{P}_{t-1}$), 6 state variables ($P_{t-3}$, P_{t-2}, P_{t-1}, X_{t-3}, X_{t-2}, X_{t-1}), 7 costate variables (X_t, X_{t+1}, $X_{t+2}X_{t+3}$, X_{t+4}, X_{t+5}, Z_{t+5}) and one exogenous variable (μ).[43] The code is then divided in parts (as can be seen by the comments in the codes which should help the reader; comments are between @@): (i) define the parameter values and all of the auxiliary variables we find convenient to build; (ii) define the steady sate formulas and other useful variables

[42] Even if what distinguishes Case A and Case B from Case C is only the definiton of CP, the dynamic systems are quite different in their order. The different definitions of CP imply different lag and lead structure and hence a different number of state and costate variables in the system. However, the codes for Case A and Case B are very similar to the codes for Case C, once the correct log-linearised dynamic system is built. Given that, these codes are not presented.

[43] All the variables are normailsed according to what reported in footnote 41.

based on the steady state values; (iii) define the dynamic system, defining the equations for the control, state and costate variables plus other variables one wants to build for interest (e.g., inflation) or for checking purposes (relationships between variables that should be satisfied in each period); (iv) transform the model according to Blanchard and Kahn.

The second code calculates the optimal decision rules for the perfect foresight model, that is the solution of the perfect foresight model. In other words how all the variables depend upon the predetermined (state) variables and the exogenous variables. Here we need to solve a problem peculiar to our model. We suppose that households decide about the wage before the realisation of the shock, while the other decisions are taken after the realisation of the shock. One way to solve the system is implemented in this second code. Since wages are set before the realisation of the shock, we can easily calculate the impact effect of a shock. Then, for the period thereafter, we can use the optimal decision rules to calculate the adjustment dynamics of the model from the point in which the system is pushed to by the impact effect. Note that since we are just interested in the impulse response function of the model, this procedure is feasible. In fact, the impulse response function gives the response of the model after a one-for-all shock. Hence, after the shock is realised the system behaves like a perfect foresight model. However, if, as in the real business cycle tradition, one wants to calculate correlations

this way it is not feasible (since in this case random shocks are continuously occurring in each period). In that case, another way of proceeding should be implemented. This is done by the fourth and fifth code presented below. In particular, first the optimal decision rules for a perfect foresight model are found, by running the first and the second code. Then, we can pass the expectation operator through the decision rules to find the expressions for the expected variables. Then a new system is built where the expectations are taken into account (this is done running the fourth code). Then one can run again the second code to solve for the optimal decision rules of this new dynamic system with expectations.

The third code finally calculates and plot the impulse response functions. The fifth code does the same for the system with expectations.

FIRST CODE

```
CLEAR ALL;
@@ DIMENSION OF CONTROL SPACE (NC), PREDETERMINED (NK),AND NON-PREDETERMINED
VECTORS (NS),EXOGENEOUS STATE VECTOR (NN) @@
@@ ORDERING OF VARIABLES:
PREDETERMINED=ENDOGENOUS STATE=K :
PIII | PII | PI | XIII | XII | XI
NON-PREDETERMINED = COSTATES = L:
X | X1 | X2 | X3 | X4 | X5 | Z5
EXOGENOUS VARIABLES=N:
RGM
FLOWS=CONTROLS=C:
Y | Y1 | Y2 | Y3 | Y4 | Y5 | P | P1 | P2 | P3 | P4 | P5 | Z | Z1 | Z2 | Z3 | Z4 | AP | AP1 | AP2 | CP
| CP1 | CP2 | CPIII | CPII | CPI | APIII | APII | API @@
@@ DIMENSION AND NAME OF CONTROL SPACE (NC), PREDETERMINED (NK), AND NON-
PREDETERMINED VECTORS (NL), EXOGENEOUS STATE VECTOR (NN) @@
NC=29;NK=6;NL=7;NN=1;
NAMEC="Y"|"Y1"|"Y2"|"Y3"|"Y4"|"Y5"|"P"|"P1"|"P2"|"P3"|"P4"|"P5"|
"Z"|"Z1"|"Z2"|"Z3"|"Z4"|"PA"|"PA1"|"PA2"|"CP"|"CP1"|"CP2"|
"CPIII"|"CPII"|"CPI"|"APIII"|"APII"|"API"; @@ CONTROLS @@
NAMEK="PIII"|"PII"|"PI"|"XIII"|"XII"|"XI"; @@ ENDOGENOUS STATES @@
```

```
NAMEL="X"|"X1"|"X2"|"X3"|"X4"|"X5"|"Z5"; @ COSTATES @
NAMEE="RGM"; @ EXOGENOUS STATES (SHOCKS) @
@ ========================================== @
@ ECONOMIC PARAMETER VALUES @
@ ========================================== @
@TRANSITION MATRIX@
RAA=0.57;
RHO=ZEROS(NN,NN);
RHO[1,1]=RAA;
RGMBAR=1.06^(0.25);
NUBAR=1/RGMBAR;
T=6;
E=6;
GAM=0.1;
S=0.67;
B=0.73;
V=-17.52;
BETA=0.96^(0.25);
EPS=T/(S+(1-S)*T);
ZBAR=((B*(1-BETA*NUBAR))/(1-B))^(1/(V-1));
U1=1+((1-B)/B)*ZBAR^V;
ZNU1=((B*(1-BETA))/(1-B))^(1/(V-1));
ASS=1+((1-B)/B)*ZNU1^V;
PSI=(S*(EPS-1)*(EPS*(E-1)+1))/(EPS*E*ASS);
TAU=1+((EPS*E*(T-1))/(GAM*T*(EPS*(E-1)+1)));
@@case gamma=3omega@@
@@psi=3*(s*(eps-1)*(1+eps*(e-1)))/((tau-1)*ASS+3*e*eps*ASS);@
@@case gamma=omega@@
@@psi=(s*(eps-1)*(1+eps*(e-1)))/((tau-1)*u1+3*e*eps*u1);@
G0=(NUBAR+NUBAR^2+NUBAR^3)^(-1);
G1=(1+NUBAR^2+NUBAR^3)^(-1);
G2=(1+NUBAR+NUBAR^3)^(-1);
G3=(1+NUBAR+NUBAR^2)^(-1);
VOBAR=1;
V1BAR=1;
V2BAR=1;
V3BAR=1;
FNU=(VOBAR)^(1-TAU)+BETA*(V1BAR)^(1-TAU)+
BETA^2*(V2BAR)^(1-TAU)+BETA^3*(V3BAR)^(1-TAU);
A11=1+BETA*NUBAR^(-E*EPS)+(BETA*NUBAR^(-E*EPS))^2+(BETA*NUBAR^(-E*EPS))^3;
A12=1+BETA*NUBAR^(RO)+(BETA*NUBAR^(RO))^2+(BETA*NUBAR^(RO))^3;
A1=A11/A12;
A2=FNU/A12;
A3=1+NUBAR^RO+NUBAR^(2*RO)+NUBAR^(3*RO);
A4=4*S*(EPS-1)-PSI*A3*A2*U1;
PUB=4*S*(EPS-1)/(A3*A2*U1);
A7=1+RGMBAR^EPS+RGMBAR^(2*EPS)+RGMBAR^(3*EPS);
FBN=1+(BETA/NUBAR)+(BETA/NUBAR)^2+(BETA/NUBAR)^3;
JB0=FBN^(-1);
JB1=BETA/(NUBAR*FBN);
JB2=((BETA/NUBAR)^2)/FBN;
JB3=((BETA/NUBAR)^3)/FBN;
JB4=(JB1+JB2+JB3)*RAA+(JB2+JB3)*(RAA^2)+JB3*(RAA^3);
PIP=((4/3)^(-E))*(A7^E)/(A3*A1);
DI=(A4/(E*EPS*U1))*PIP;
A5=(DI*E*EPS*U1)/(A4);
ALPHA=(4^(S-S/RO))*(A5^(S/E))*(A3^(S/RO+S*(1-E)/E))*(A1^(S/E));
EBAR=RGMBAR/ZBAR;
@ ========================================== @
@ STEADY STATE CALCULATIONS @
@ ========================================== @
@ AGGREGATE VARIABLES @
XBAR=4*S*(A5^(1/E))*EBAR*A3^((1-E)/E)*A1^(1/E);
PBAR=ALPHA^(-1)*S^(-S)*(0.25)^(S/RO)*EBAR^(1-S)*XBAR^S*A3^(S/RO);
YBAR=EBAR/PBAR;
RBAR=1/(BETA*NUBAR);
LAMBAR=1/(PBAR*YBAR*U1);
APBAR=PBAR*FNU*FBN;
CPBAR=XBAR/APBAR;
@ SECTORS' VARIABLES @
XIBAR=XBAR*NUBAR;
XIIBAR=XIBAR*NUBAR;
```

```
XIIIBAR=XIIBAR*NUBAR;
X1BAR=XBAR*RGMBAR;
X2BAR=X1BAR*RGMBAR;
X3BAR=X2BAR*RGMBAR;
X4BAR=X3BAR*RGMBAR;
X5BAR=X4BAR*RGMBAR;
PIBAR=PBAR*NUBAR;
PIIBAR=PIBAR*NUBAR;
PIIIBAR=PIIBAR*NUBAR;
P1BAR=PBAR*RGMBAR;
P2BAR=P1BAR*RGMBAR;
P3BAR=P2BAR*RGMBAR;
P4BAR=P3BAR*RGMBAR;
P5BAR=P4BAR*RGMBAR;
APIBAR=APBAR*NUBAR;
APIIBAR=APIBAR*NUBAR;
APIIIBAR=APIIBAR*NUBAR;
AP1BAR=APBAR*RGMBAR;
AP2BAR=AP1BAR*RGMBAR;
XPBAR=XBAR/PBAR;
WBAR=(XBAR+XIBAR+XIIBAR+XIIIBAR)/(4*PBAR);
PABAR=(4*EBAR/A3)^(1-S)*ALPHA^(-1)*S^(-S)*XBAR^S;
PBBAR=PABAR*NUBAR^(EPS*S/T);
PCBAR=PABAR*NUBAR^(2*EPS*S/T);
PDBAR=PABAR*NUBAR^(3*EPS*S/T);
YABAR=ALPHA*A5^(-S/E)*A3^(-S/E)*A1^(-S/E);
YBBAR=YABAR*RGMBAR^(EPS*S);
YCBAR=YABAR*RGMBAR^(2*EPS*S);
YDBAR=YABAR*RGMBAR^(3*EPS*S);
LABAR=A5^(-1/E)*A3^(-1/E)*A1^(-1/E);
LBBAR=LABAR*RGMBAR^EPS;
LCBAR=LABAR*RGMBAR^(2*EPS);
LDBAR=LABAR*RGMBAR^(3*EPS);
LBAR=(1/4)*LABAR*A7;
/*
@ PARAMETERS MODEL AND WAGE RULE @
@ LHS WAGE RULE @
*/
K1=-RO/A12;
K2=BETA*K1*NUBAR^RO;
K3=BETA^2*K1*NUBAR^(2*RO);
K4=BETA^3*K1*NUBAR^(3*RO);
K5=((EPS/T)-1)/A12;
K6=BETA*K5*NUBAR^RO;
K7=BETA^2*K5*NUBAR^(2*RO);
K8=BETA^3*K5*NUBAR^(3*RO);
K9=-K2-K3-K4;
K10=-K3-K4;
K11=-K4;
K29=-(V*(1-BETA*NUBAR)*ZBAR/(U1*A12));
K30=BETA*NUBAR^(1-EPS)*K29;
K31=BETA*NUBAR^(1-EPS)*K30;
K32=BETA*NUBAR^(1-EPS)*K31;
@ RHS WAGE RULE@
OM1=DI*E*EPS*(ALPHA*S)^(E*EPS)*ALPHA^(-E*EPS/T);
OMBAR1=OM1*PBAR^(E*EPS)*YBAR^(E*EPS/T)*A11;
OMBAR2=XBAR^(E*EPS)*PSI*FNU;
OMBAR=OMBAR1+OMBAR2;
K12=(OM1*E*EPS*PBAR^(E*EPS)*YBAR^(E*EPS/T))/OMBAR;
K13=K12*BETA*NUBAR^(-E*EPS);
K14=K12*BETA^2*NUBAR^(-2*E*EPS);
K15=K12*BETA^3*NUBAR^(-3*E*EPS);
K16=K12/T;
K17=K13/T;
K18=K14/T;
K19=K15/T;
K20=-K13-K14-K15;
K21=-K14-K15;
K22=-K15;
K23=(1-TAU)*OMBAR2/OMBAR;
```

```
K24=(XBAR^(E*EPS)*PSI*(TAU-1)*VOBAR^(1-TAU))/OMBAR;
K25=(XBAR^(E*EPS)*PSI*(TAU-1)*BETA*V1BAR^(1-TAU))/OMBAR;
K26=(XBAR^(E*EPS)*PSI*(TAU-1)*BETA^2*V2BAR^(1-TAU))/OMBAR;
K27=(XBAR^(E*EPS)*PSI*(TAU-1)*BETA^3*V3BAR^(1-TAU))/OMBAR;
K28=E*EPS*OMBAR2/OMBAR;
@@OTHER USEFUL COSTANTS@
J0=1/A3;
J1=(NUBAR^RO)/A3;
J2=(NUBAR^(2*RO))/A3;
J3=(NUBAR^(3*RO))/A3;
B1=-1+(ZBAR*V*(1-BETA*NUBAR)/(1+ZBAR*(1-BETA*NUBAR)))+
((V-1)*(1-BETA*NUBAR)/(BETA*NUBAR));
B2=1-(ZBAR*V*(1-BETA*NUBAR)/(1+ZBAR*(1-BETA*NUBAR)));
SS=PABAR*YABAR+PBBAR*YBBAR+PCBAR*YCBAR+PDBAR*YDBAR;
SSS=PBAR*YBAR;
@@NOTE: MUST BE SS=SSS=EBAR @@
SS1=PABAR*YABAR/SS;
SS2=PBBAR*YBBAR/SS;
SS3=PCBAR*YCBAR/SS;
SS4=PDBAR*YDBAR/SS;
@@SSVAL=ENDSTATES|COSTATES|EXSTATES|CONTROLS|EXTRACONTROLS @
SSVAL=PIIIBAR|PIIBAR|PIBAR|XIIIBAR|XIIBAR|XIBAR|XBAR|X1BAR|X2BAR|
X3BAR|X4BAR|X5BAR|ZBAR|
RGMBAR|YBAR|YBAR|YBAR|YBAR|YBAR|PBAR|P1BAR|P2BAR|P3BAR|P4BAR|P5BAR|
ZBAR|ZBAR|ZBAR|ZBAR|ZBAR|APBAR|AP1BAR|AP2BAR|CPBAR|CPBAR|CPBAR|
CPBAR|CPBAR|CPBAR|APIIIBAR|APIIBAR|APIBAR|
PABAR|PBBAR|PCBAR|PDBAR|YABAR|YBBAR|YCBAR|YDBAR|
LABAR|LBBAR|LCBAR|LDBAR|LBAR|RBAR|LAMBAR|
0|0|0|0|0|0|0|0|0;
@@ ========================================= @@
@@BUILDING THE DYNAMIC SYSTEM: EQUATIONS FOR CONTROLS, (CO)STATES AND
AUXULIARY (FLOW) VARIABLES@@
@@ ========================================= @@
@@ MATRICES IN CONTROL SYSTEM: MCC*C(t)=MCS*S(t)+MCE*E(t)+ MCEI*E(t-1)+MCEII*E(t-
2) @@
MCC=ZEROS(NC,NC);
MCC[1,1]=1;
MCC[1,7]=1;
MCC[1,13]=1;
MCC[2,2]=1;
MCC[2,8]=1;
MCC[2,14]=1;
MCC[3,3]=1;
MCC[3,9]=1;
MCC[3,15]=1;
MCC[4,4]=1;
MCC[4,10]=1;
MCC[4,16]=1;
MCC[5,5]=1;
MCC[5,11]=1;
MCC[5,17]=1;
MCC[6,6]=1;
MCC[6,12]=1;
MCC[7,7]=1;
MCC[7,1]=-(1-S)/S;
MCC[8,8]=1;
MCC[8,2]=-(1-S)/S;
MCC[9,9]=1;
MCC[9,3]=-(1-S)/S;
MCC[10,10]=1;
MCC[10,4]=-(1-S)/S;
MCC[11,11]=1;
MCC[11,5]=-(1-S)/S;
MCC[12,12]=1;
MCC[12,6]=-(1-S)/S;
MCC[13,13]=B1;
MCC[13,14]=B2;
MCC[14,14]=B1;
MCC[14,15]=B2;
```

```
MCC[15,15]=B1;
MCC[15,16]=B2;
MCC[16,16]=B1;
MCC[16,17]=B2;
MCC[17,17]=B1;
MCC[18,18]=1;
MCC[18,7]=-JB0;
MCC[18,8]=-JB1;
MCC[18,9]=-JB2;
MCC[18,10]=-JB3;
MCC[19,19]=1;
MCC[19,8]=-JB0;
MCC[19,9]=-JB1;
MCC[19,10]=-JB2;
MCC[19,11]=-JB3;
MCC[20,20]=1;
MCC[20,9]=-JB0;
MCC[20,10]=-JB1;
MCC[20,11]=-JB2;
MCC[20,12]=-JB3;
MCC[21,21]=1;
MCC[21,18]=1;
MCC[22,22]=1;
MCC[22,19]=1;
MCC[23,23]=1;
MCC[23,20]=1;
MCC[24,24]=1;
MCC[24,27]=1;
MCC[25,25]=1;
MCC[25,28]=1;
MCC[26,26]=1;
MCC[26,29]=1;
MCC[27,27]=1;
MCC[27,7]=-JB3;
MCC[28,28]=1;
MCC[28,7]=-JB2;
MCC[28,8]=-JB3;
MCC[29,29]=1;
MCC[29,7]=-JB1;
MCC[29,8]=-JB2;
MCC[29,9]=-JB3;
MCS=ZEROS(NC,(NK+NL));
MCS[6,13]=-1;
MCS[7,4]=J3;
MCS[7,5]=J2;
MCS[7,6]=J1;
MCS[7,7]=J0;
MCS[8,5]=J3;
MCS[8,6]=J2;
MCS[8,7]=J1;
MCS[8,8]=J0;
MCS[9,6]=J3;
MCS[9,7]=J2;
MCS[9,8]=J1;
MCS[9,9]=J0;
MCS[10,7]=J3;
MCS[10,8]=J2;
MCS[10,9]=J1;
MCS[10,10]=J0;
MCS[11,8]=J3;
MCS[11,9]=J2;
MCS[11,10]=J1;
MCS[11,11]=J0;
MCS[12,9]=J3;
MCS[12,10]=J2;
MCS[12,11]=J1;
MCS[12,12]=J0;
MCS[17,13]=-B2;
```

```
MCS[21,7]=1;
MCS[22,8]=1;
MCS[23,9]=1;
MCS[24,4]=1;
MCS[25,5]=1;
MCS[26,6]=1;
MCS[27,1]=JB0;
MCS[27,2]=JB1;
MCS[27,3]=JB2;
MCS[28,2]=JB0;
MCS[28,3]=JB1;
MCS[29,3]=JB0;
MCE=ZEROS(NC,NN);
MCE[1,1]=1;
MCE[2,1]=1+RAA;
MCE[3,1]=1+RAA+RAA^2;
MCE[4,1]=1+RAA+RAA^2+RAA^3;
MCE[5,1]=1+RAA+RAA^2+RAA^3+RAA^4;
MCE[6,1]=1+RAA+RAA^2+RAA^3+RAA^4+RAA^5;
MCE[13,1]=RAA;
MCE[14,1]=(RAA^2);
MCE[15,1]=(RAA^3);
MCE[16,1]=(RAA^4);
MCE[17,1]=(RAA^5);
@@ MATRICES IN STATE EQUATIONS @
@@ MSS0*E(S(t+1))+MSS1*S(t)=MSC0*E(C(t+1))+MSC1*C(t)+MSE0*E(E(t+1)) + MSE1*E(t)
@@
MSS0=ZEROS((NK+NL),(NK+NL));
MSS0[1,1]=1;
MSS0[2,2]=1;
MSS0[3,3]=1;
MSS0[4,4]=1;
MSS0[5,5]=1;
MSS0[6,6]=1;
MSS0[7,7]=1;
MSS0[8,8]=1;
MSS0[9,9]=1;
MSS0[10,10]=1;
MSS0[11,11]=1;
MSS0[13,13]=B2;
MSS1=ZEROS((NK+NL),(NK+NL));
MSS1[1,2]=-1;
MSS1[2,3]=-1;
MSS1[4,5]=-1;
MSS1[5,6]=-1;
MSS1[6,7]=-1;
MSS1[7,8]=-1;
MSS1[8,9]=-1;
MSS1[9,10]=-1;
MSS1[10,11]=-1;
MSS1[11,12]=-1;
MSS1[12,7]=1+EPS*(E-1)-K28;
MSS1[13,13]=B1;
MSC0=ZEROS((NK+NL),NC);
MSC0[12,23]=K27/3;
MSC1=ZEROS((NK+NL),NC);
MSC1[3,7]=1;
MSC1[12,1]=K16-K5;
MSC1[12,2]=-K6+K17;
MSC1[12,3]=-K7+K18;
MSC1[12,7]=K12-K1;
MSC1[12,8]=-K2+K13;
MSC1[12,9]=-K3+K14;
MSC1[12,10]=-K4+K15;
MSC1[12,4]=-K8+K19;
MSC1[12,13]=-K29;
MSC1[12,14]=-K30;
MSC1[12,15]=-K31;
```

```
MSC1[12,16]=-K32;
MSC1[12,21]=K23;
MSC1[12,22]=(K25+K26+K27)/3;
MSC1[12,23]=(K26+K27)/3;
MSC1[12,24]=K24/3;
MSC1[12,25]=(K24+K25)/3;
MSC1[12,26]=(K24+K25+K26)/3;
MSE0=ZEROS((NK+NL),NN);
MSE0[13,1]=(RAA^5);
MSE1=ZEROS((NK+NL),NN);
MSE1[1,1]=-1;
MSE1[2,1]=-1;
MSE1[3,1]=-1;
MSE1[4,1]=-1;
MSE1[5,1]=-1;
MSE1[6,1]=-1;
MSE1[7,1]=-1;
MSE1[8,1]=-1;
MSE1[9,1]=-1;
MSE1[10,1]=-1;
MSE1[11,1]=-1;
@ AUXILIARY FLOW VARIABLES@
NXF=24; @ DIMENSION AND NAMES OF EXTRA FLOWS @
NAMEXC="PA"|"PB"|"PC"|"PD"|"YA"|"YB"|"YC"|"YD"|
"LA"|"LB"|"LC"|"LD"|"L"|"R"|"LAM"|"C1"|"C2"|"C3"|"C4"|
"C5"|"C6"|"C7"|"C8"|"C9";
@ MF*XC(t) = MFC*C(t) + MFKE*[K(t)|E(t)]' + MFL*L(t) @
MF=ZEROS(NXF,NXF);
MF[1,1]=1;
MF[2,2]=1;
MF[2,1]=-1;
MF[3,3]=1;
MF[3,1]=-1;
MF[4,4]=1;
MF[4,1]=-1;
MF[5,5]=1;
MF[5,1]=S/(S-1);
MF[6,6]=1;
MF[6,5]=-1;
MF[6,1]=-T;
MF[6,2]=T;
MF[7,7]=1;
MF[7,5]=-1;
MF[7,1]=-T;
MF[7,3]=T;
MF[8,8]=1;
MF[8,5]=-1;
MF[8,1]=-T;
MF[8,4]=T;
MF[9,9]=1;
MF[9,1]=1/(1-S);
MF[10,10]=1;
MF[10,2]=1/(1-S);
MF[11,11]=1;
MF[11,3]=1/(1-S);
MF[12,12]=1;
MF[12,4]=1/(1-S);
MF[13,13]=1;
MF[13,9]=1/A7;
MF[13,10]=(RGMBAR^EPS)/A7;
MF[13,11]=(RGMBAR^(2*EPS))/A7;
MF[13,12]=(RGMBAR^(3*EPS))/A7;
MF[14,14]=1/(RBAR-1);
MF[15,15]=1;
MF[16,16]=1;
MF[16,5]=1;
MF[16,1]=T;
MF[17,17]=1;
```

```
MF[17,6]=1;
MF[17,2]=T;
MF[18,18]=1;
MF[18,7]=1;
MF[18,3]=T;
MF[19,19]=1;
MF[19,8]=1;
MF[19,4]=T;
MF[20,20]=1;
MF[20,1]=SS1;
MF[20,5]=SS1;
MF[20,2]=SS2;
MF[20,6]=SS2;
MF[20,3]=SS3;
MF[20,7]=SS3;
MF[20,4]=SS4;
MF[20,8]=SS4;
MF[21,21]=1;
MF[21,14]=1;
MF[22,22]=1;
MF[22,6]=1;
MF[22,2]=S/(S-1);
MF[23,23]=1;
MF[23,7]=1;
MF[23,3]=S/(S-1);
MF[24,24]=1;
MF[24,8]=1;
MF[24,4]=S/(S-1);
CAZ=INV(MF);
MFC=ZEROS(NXF,NC);
MFC[1,1]=1-S;
MFC[1,7]=1-S;
MFC[14,13]=V-1;
MFC[15,1]=-1;
MFC[15,7]=-1;
MFC[15,13]=-V*(U1-1)/U1;
MFC[16,1]=1;
MFC[16,7]=T;
MFC[17,1]=1;
MFC[17,7]=T;
MFC[18,1]=1;
MFC[18,7]=T;
MFC[19,1]=1;
MFC[19,7]=T;
MFC[20,1]=1;
MFC[20,7]=1;
MFC[21,2]=1;
MFC[21,1]=-1;
MFC[21,7]=-1;
MFC[21,8]=1;
MFC[21,13]=-V*(U1-1)/U1;
MFC[21,14]=V*(U1-1)/U1;
MFKE=ZEROS(NXF,NK+NN);
MFKE[1,4]=-(1-S)*RO*J3;
MFKE[1,5]=-(1-S)*RO*J2;
MFKE[1,6]=-(1-S)*RO*J1;
MFKE[2,6]=EPS*S/T;
MFKE[3,5]=EPS*S/T;
MFKE[4,4]=EPS*S/T;
MFKE[10,6]=1/(S-1);
MFKE[11,5]=1/(S-1);
MFKE[12,4]=1/(S-1);
MFKE[22,6]=S/(S-1);
MFKE[23,5]=S/(S-1);
MFKE[24,4]=S/(S-1);
MFL=ZEROS(NXF,NL);
MFL[1,1]=(1-S)*RO*(1-J0)+S;
MFL[2,1]=-EPS*S/T;
```

```
MFL[3,1]=-EPS*S/T;
MFL[4,1]=-EPS*S/T;
MFL[5,1]=S/(S-1);
MFL[9,1]=1/(S-1);
@ FVC LINKS EXTRA CONTROLS TO FUNDAMENTAL CONTROLS @
FVC=(INV(MF))*MFC;
@ FVC LINKS EXTRA CONTROLS TO ENDOGENOUS STATES AND EXOGENOUS STATES
@
FVKE=(INV(MF))*MFKE;
@ FVL LINKS EXTRA CONTROLS TO COSTATES @
FVL=(INV(MF))*MFL;
@ ============================================== @
@ FUNDAMENTAL STATE-COSTATE DIFFERENCE EQUATION @
@ ============================================== @
MSss0 = MSS0 - MSC0*(INV(MCC))*MCS;
MSss1 = MSS1 - MSC1*(INV(MCC))*MCS;
MSsc0 = MSE0 + MSC0*(INV(MCC))*MCE;
MSsc1 = MSE1 + MSC1*(INV(MCC))*MCE;
W = -(INV(MSss0))*MSss1;
R = (INV(MSss0))*MSsc0;
Q = (INV(MSss0))*MSsc1;
@ ========================================== @
@ EIGENVECTOR-EIGENVALUE DECOMPOSITION OF STATE TRANSITION MATRIX @
@ ========================================== @
@ FIRST WE FIND THE REAL PARTS OF THE EIGENVALUES (X1)
AND EIGENVECTORS (X3) @
{X1,X3}=EIGV(W);
X11=REAL(X1);
AMU=ABS(X11);
@ SECOND WE ORDER THE EIGENVALUES@
IN=SORTC(AMU,1);
IND1=INDNV(IN,AMU);
I=1;
DO UNTIL I>(NK+NL)-1;
IF IND1[I,1] == IND1[I+1,1];
IND1[I+1,1] = 1+IND1[I,1];
ENDIF;
I=I+1;
ENDO;
@ THIRD WE ORDER THE COLUMNS OF THE EIGENVECTORS (X3) BY THE
INDICATOR RESULTING FROM THE ORDERING OF THE EIGENVALUES@
P=ZEROS((NK+NL),(NK+NL));
I=1;
DO UNTIL I>(NK+NL);
P[1:(NK+NL),I]=X3[1:(NK+NL),IND1[I,1]];
I=I+1;
ENDO;
@ FINALLY WE FORM A DIAGONAL MATRIX (MU) IN WHICH THE DIAGONAL HAVE THE
EIGENVALUES IN ASCENDING ABSOLUTE VALUE@
MU=ZEROS((NK+NL),(NK+NL));
I=1;
DO UNTIL I>(NK+NL);
MU[I,I]=X1[IND1[I,1],1];
I=I+1;
ENDO;
@ WE NOW HAVE P AND MU FOR WHICH WE KNOW THAT P*MU*P^-1=W (ALSO
X3*DIAG(X1)*X3=W) @
@ ========================================= @
@ PARTITIONING THE MATRICES @
@ ========================================= @
MU1=MU[1:NK,1:NK];
MU2=MU[NK+1:NK+NL,NK+1:NK+NL];
P11=P[1:NK,1:NK];
P12=P[1:NK,NK+1:NK+NL];
P21=P[NK+1:NK+NL,1:NK];
P22=P[NK+1:NK+NL,NK+1:NK+NL];
PS=INV(P);
PS11=PS[1:NK,1:NK];
PS12=PS[1:NK,NK+1:NK+NL];
PS21=PS[NK+1:NK+NL,1:NK];
PS22=PS[NK+1:NK+NL,NK+1:NK+NL];
RKE=R[1:NK,1:NN];
RLE=R[NK+1:NK+NL,1:NN];
```

```
QKE=Q[1:NK,1:NN];
QLE=Q[NK+1:NK+NL,1:NN];
@ ==================================== @
@ COMPOSITE EXPRESSIONS @
@ ==================================== @
SP1=-(INV(MU2))*(PS21*RKE+PS22*RLE);
SP2=-(INV(MU2))*(PS21*QKE+PS22*QLE);
KLK=P11*MU1*(INV(P11));
KTL=(P11*MU1*PS12+P12*MU2*PS22)*(INV(PS22));
NAME=NAMEK|NAMEE|NAMEL|NAMEC|NAMEXC;
```

SECOND CODE

```
@ ==================================== @
@ COMPUTATION OF DECISION RULES @
@ ==================================== @
@ IN THIS PROGRAM WE WILL COMPUTE MARKOV DECISION RULES (MDR)
FOR THE LINEAR DYNAMIC MODEL@
FL = SP1*RHO + SP2;
IRHO=EYE(ROWS(RHO));
I=1;
LEE=ZEROS(NL,NN);
DO UNTIL I>NL;
Q=FL[I,1:NN];
MU2I=1/MU2[I,I];
DSUM=INV((IRHO-MU2I*RHO));
LEE[I,1:NN]=Q*DSUM;
I=I+1;
ENDO;
@ ==================================== @
@ STATE DECISION RULES @
@ ==================================== @
KEC=RKE*RHO+QKE+KTL*LEE;
ULE=(INV(PS22))*LEE;
ULK=-(INV(PS22))*PS21;
@ ================================== @
@ SYSTEM DECISION RULES @
@ ================================== @
MKE=ZEROS(ROWS(KLK)+NN,COLS(KLK)+COLS(KEC));
MKE[1:ROWS(KLK),1:COLS(KLK)]=KLK;
MKE[ROWS(KLK)+1:ROWS(KLK)+NN,1:NK]=ZEROS(NN,NK);
MKE[1:ROWS(KEC),COLS(KLK)+1:COLS(KLK)+COLS(KEC)]=KEC;
MKE[ROWS(KEC)+1:ROWS(KEC)+ROWS(RHO),COLS(KLK)+1:COLS(KLK)+COLS(KEC)]=RHO;
@ ================================== @
@ INCORPORATION OF SHADOW PRICE, CONTROLS AND OTHER FLOWS @
@ ================================== @
LKE=ZEROS(NL,NK+NN);
LKE[.,1:COLS(ULK)]=ULK; LKE[.,COLS(ULK)+1:COLS(ULK)+COLS(ULE)]=ULE;
Z=(INV(MCC))*MCS;
M0CK=Z[1:NC,1:NK];
M0CL=Z[1:NC,NK+1:NK+NL];
M0CE=(INV(MCC))*MCE;
M0CKE=(M0CK+M0CL*ULK)~(M0CE+M0CL*ULE);
FKE=FVC*M0CKE+FVKE+FVL*LKE;
@RRKE=(ULK-ULK*KLK)~(ULE-(ULK*KEC+ULE*RHO));@
H=LKE|M0CKE|FKE;
@==============================@
@ THE IMPACT PERIOD @
@==============================@
B3=(-B2/B1);
@ SHOCK @
RGMT=0.01;
ZT=RGMT*RAA/(B1*(1-B3*RAA));
ELT=ZEROS(2,1);
ELT[1,1]=RGMT;
ELT[2,1]=ZT;
```

```
@ VARIABLES: Y|P|L|R|LAM @
VIP=ZEROS(5,2);
VIP[1,1]=S;
VIP[1,2]=-S;
VIP[2,1]=1-S;
VIP[2,2]=-1+S;
VIP[3,1]=1;
VIP[3,2]=-1;
VIP[4,2]=(RBAR-1)*(V-1);
VIP[5,1]=-1;
VIP[5,2]=1-V*(U1-1)/U1;
IP=VIP*ELT;
@ IP = Y|P|L|R|LAM COLUMN VECTOR @
MKE=REAL(MKE);
LKE=REAL(LKE);
M0CKE=REAL(M0CKE);
H=REAL(H);
FKE=REAL(FKE);
LOCATE 1,1;
FORMAT /LDS 4,3;
OUTPUT ON;
OUTPUT FILE=CKTPFM.OUT RESET;
"                                   ---  ---------  -- ";
" IMPACT EFFECT ";
"                                                      ";
"                                                      ";
"              RGM(t)              -";
"Z ";;ZT;
"Y ";;IP[1,1];
"P ";;IP[2,1];
"L ";;IP[3,1];
"R ";;IP[4,1];
"LAM ";;IP[5,1];
WAIT;
"                                          ---         ";
" NEAR STEADY STATE DYNAMICS AFTER FIRST PERIOD";
"                                                      ";
"                                   ---                ";
"              PI'S              -";
" ";;$NAME[1:NK+NN,1]';
I=1;
DO UNTIL I>NK;
$NAME[I,1];;MKE[I,.];
I=I+1;
ENDO;
I=1;
DO UNTIL I>NL;
$NAME[I+NN+NK,1];;LKE[I,.];
I=I+1;
ENDO;
I=1;
DO UNTIL I>NC;
$NAME[I+NK+NL+NN,1];;M0CKE[I,.];
I=I+1;
ENDO;
I=1;
DO UNTIL I>NXF;
$NAME[I+NK+NL+NN+NC,1];;FKE[I,.];
I=I+1;
ENDO;
"                                                      ";
WAIT;
FORMAT 4,4;
"                                                      ";
"                    - PARAMETERIZATION                  --  ";
"                                                      ";
"                                                      ";
" PARAMETERS";
"                                                 -";
"DISCOUNT FACTOR ";;BETA;
"B ";;B;
"V ";;V;
"ALPHA ";;ALPHA;
```

```
"DI ";;DI;
"PSI ";;PSI;
"RELATIVE WAGE RISK AVERSION (TAU) ";;TAU;
"SIGMA ";;S;
"THETA ";;T;
"INTERTEMPORAL ELASTICITY OF LABOUR SUPPLY ";;1/(E-1);
"TREND IN THE MONEY SUPPLY PROCESS ";;RGMBAR;
"AUTOCORRELATION MONEY SUPPLY PROCESS ";;RAA;
WAIT;
"                                                    ";
" STEADY STATE: AGGREGATE VARIABLES";
"                                                    ";
"STEADY STATE OUTPUT ";;YBAR;
"STEADY STATE VELOCITY OF MONEY ";;1/ZBAR;
"STEADY STATE PRICE LEVEL ";;PBAR;
"STEADY STATE AGGREGATE EMPLOYMENT ";;LBAR;
"STEADY STATE NOMINAL INTEREST RATE ";;RBAR;
"SS REAL INTEREST RATE ";;RBAR/RGMBAR;
"SS LAMDBA ";;LAMBAR;
WAIT;
"                                                    ";
" STEADY STATE: SECTORS' VARIABLES";
"                                                    ";
"STEADY STATE WAGE SECTOR A ";;XBAR;
"STEADY STATE WAGE SECTOR B ";;XIBAR;
"STEADY STATE WAGE SECTOR C ";;XIIBAR;
"STEADY STATE WAGE SECTOR D ";;XIIIBAR;
"STEADY STATE OUTPUT SECTOR A ";;YABAR;
"STEADY STATE OUTPUT SECTOR B ";;YBBAR;
"STEADY STATE OUTPUT SECTOR C ";;YCBAR;
"STEADY STATE OUTPUT SECTOR D ";;YDBAR;
"STEADY STATE PRICE SECTOR A ";;PABAR;
"STEADY STATE PRICE SECTOR B ";;PBBAR;
"STEADY STATE PRICE SECTOR C ";;PCBAR;
"STEADY STATE PRICE SECTOR D ";;PDBAR;
"STEADY STATE EMPLOYMENT SECTOR A ";;LABAR;
"STEADY STATE EMPLOYMENT SECTOR B ";;LBBAR;
"STEADY STATE EMPLOYMENT SECTOR C ";;LCBAR;
"STEADY STATE EMPLOYMENT SECTOR D ";;LDBAR;
"                                                    ";
"                                                    ";
OUTPUT OFF;
```

THIRD CODE

```
@STARTING POINT AFTER THE IMPACT EFFECT @
S=-RGMT|-RGMT|IP[2,1]-RGMT|-RGMT|-RGMT|-RGMT|RGMT*RAA;
@ FORECAST HORIZON @
NIR=11;
@ GENERATING IMPULSES @
TI=SEQA(1,1,NIR+1);
IR=ZEROS(ROWS(MKE)+ROWS(H),NIR);
I=1;
DO UNTIL I>NIR;
IR[1:ROWS(MKE),I]=S;
IR[ROWS(MKE)+1:ROWS(IR),I]=(II*S);
S=MKE*S;
I=I+1;
ENDO;
IR=REAL(IR);
IRI=ZEROS(ROWS(IR),1);
IRI[7,1]=RGMT;
IRI[15,1]=IP[1,1];
IRI[21,1]=IP[2,1];
IRI[27,1]=ZT;
```

```
IRRI=ONES(4,1);
IRI[44:47,1]=IP[2,1]*IRRI;
IRI[48:51,1]=IP[1,1]*IRRI;
IRI[52:55,1]=IP[3,1]*IRRI;
IRI[56,1]=IP[3,1];
IRI[57,1]=IP[4,1];
IRI[58,1]=IP[5,1];
@_____________________________________________@
@ THE GROSS INFLATION SERIES @
@_____________________________________________@
INF=ZEROS(1,NIR+1);
INF[1,1]=IP[2,1];
INF[1,2]=IRI[7,1]+IR[21,1] -IP[2,1];
INF[1,3:NIR+1]=IR[7,1:(NIR-1)]+IR[21,2:NIR]-IR[21,1:(NIR-1)];
IRII=IRI~IR;
IR=IRII|INF;
IR=IR*100;
@ PLOT IMPULSE RESPONSES @
LIBRARY PGRAPH;
GRAPHSET;
_PLEGCTL={2,3,6,4.5};
_PCOLOR={15};
_PLTYPE={1,6,3};
_PDATE="";
_PLEGSTR=
"INF\000"\
"Y\000"\
"RGM\000";
XY(TI,IR[68,.]'~IR[15,.]'~IR[7,.]');
```

FOURTH CODE

```
@RUN BOTH FIRST AND SECOND CODES BEFORE THIS FILE AND RUN AGAIN THE FIRST
ONE AFTER THIS FILE BEFORE THE FIFTH CODE@
@ DIMENSION OF CONTROL SPACE (NC), PREDETERMINED (NK), AND NON-PREDETERMINED
VECTORS (NS), EXOGENEOUS STATE VECTOR (NN) @
@ ORDERING OF VARIABLES: PREDETERMINED=ENDOGENOUS STATE=K :
PIII | PII | PI | XIII | XII | XI | ERGM
NON-PREDETERMINED = COSTATES = L:
Z
EXOGENOUS VARIABLES=N:
RGM
FLOWS=CONTROLS=C:
EY | EY1 | EY2 | EY3 | EP | EP1 | EP2 | EP3 | EZ | EZ1 | EZ2 | EZ3 | ECP | ECP1 | ECP2 |
ECPIII | ECPII | ECPI | ECP3 | X | Y | P | @
@ DIMENSION AND NAME @
NC=22;
NK=7;
NL=1;
NN=1;
NAMEC="EY"|"EY1"|"EY2"|"EY3"|
"EP"|"EP1"|"EP2"|"EP3"|
"EZ"|"EZ1"|"EZ2"|"EZ3"|"ECP"|"ECP1"|"ECP2"|
"ECPIII"|"ECPII"|"ECPI"|"ECP3"|"X"|"Y"|"P"; @ CONTROLS @
NAMEK="PIII"|"PII"|"PI"|"XIII"|"XII"|"XI"|"ERGM"; @ END STATES @
NAMEL="Z"; @ COSTATES @
NAMEE="RGM"; @ EXOGENOUS STATES (SHOCKS) @
@SSVAL=ENDSTATES|EXSTATES|COSTATES|CONTROLS|EXTRACONTROLS @
SSVAL=PIIIBAR|PIIBAR|PIBAR|XIIIBAR|XIIBAR|XIBAR|RGMBAR|
RGMBAR|
ZBAR|
YBAR|YBAR|YBAR|YBAR|PBAR|P1BAR|P2BAR|P3BAR|
ZBAR|ZBAR|ZBAR|ZBAR|CPBAR|CPBAR|CPBAR|
CPBAR|CPBAR|CPBAR|CPBAR|XBAR|YBAR|PBAR|
```

```
PABAR|PBBAR|PCBAR|PDBAR|YABAR|YBBAR|YCBAR|YDBAR|
LABAR|LBBAR|LCBAR|LDBAR|LBAR|RBAR|LAMBAR|
0|0|0|0|0|0|0|0|0;
@ MATRICES IN CONTROL SYSTEM: MCC*C(t)=MCS*S(t)+MCE*E(t) @
MCC=ZEROS(NC,NC);
MCC[1,1]=1;
MCC[2,2]=1;
MCC[3,3]=1;
MCC[4,4]=1;
MCC[5,5]=1;
MCC[6,6]=1;
MCC[7,7]=1;
MCC[8,8]=1;
MCC[9,9]=1;
MCC[10,10]=1;
MCC[11,11]=1;
MCC[12,12]=1;
MCC[13,13]=1;
MCC[14,14]=1;
MCC[15,15]=1;
MCC[16,16]=1;
MCC[17,17]=1;
MCC[18,18]=1;
MCC[19,19]=1;
MCC[20,20]=1+EPS*(E-1)-K28;
MCC[20,1]=-K16+K5;
MCC[20,2]=K6-K17;
MCC[20,3]=K7-K18;
MCC[20,5]=-K12+K1;
MCC[20,6]=K2-K13;
MCC[20,7]=K3-K14;
MCC[20,8]=K4-K15;
MCC[20,4]=K8-K19;
MCC[20,9]=K29;
MCC[20,10]=K30;
MCC[20,11]=K31;
MCC[20,12]=K32;
MCC[20,13]=-K23;
MCC[20,14]=-(K25+K26+K27)/3;
MCC[20,15]=-(K26+K27)/3;
MCC[20,16]=-K24/3;
MCC[20,17]=-(K24+K25)/3;
MCC[20,18]=-(K24+K25+K26)/3;
MCC[20,19]=-(K27)/3;
MCC[21,21]=1;
MCC[21,22]=1;
MCC[22,22]=1;
MCC[22,21]=-(1-S)/S;
MCC[22,20]=-J0;
MKEI=KEC|RHO;
MCS=ZEROS(NC,(NK+NL));
MCS[1:4,1:NK]=M0CKE[1:4,1:NK];
MCS[5:8,1:NK]=M0CKE[7:10,1:NK];
MCS[9:12,1:NK]=M0CKE[13:16,1:NK];
MCS[13:18,1:NK]=M0CKE[21:26,1:NK];
MCS[19,1:(NK-1)]=M0CKE[23,1:(NK-1)]*KLK;
MCS[19,NK]=M0CKE[23,.]*MKEI;
MCS[21,8]=-1;
MCS[22,4]=J3;
MCS[22,5]=J2;
MCS[22,6]=J1;
MCE=ZEROS(NC,NN);
MCE[21,1]=1;
@ MATRICES IN STATE EQUATIONS @
@ MSS0*E(S(t+1))+MSS1*S(t)=MSC0*E(C(t+1))+MSC1*C(t)+MSE0*E(E(t+1)) + MSE1*E(t)@
MSS0=ZEROS((NK+NL),(NK+NL));
MSS0[1,1]=1;
MSS0[2,2]=1;
```

```
MSS0[3,3]=1;
MSS0[4,4]=1;
MSS0[5,5]=1;
MSS0[6,6]=1;
MSS0[7,7]=1;
MSS0[8,8]=B2;
MSS1=ZEROS((NK+NL),(NK+NL));
MSS1[1,2]=-1;
MSS1[2,3]=-1;
MSS1[4,5]=-1;
MSS1[5,6]=-1;
MSS1[8,8]=B1;
MSC0=ZEROS((NK+NL),NC);
MSC1=ZEROS((NK+NL),NC);
MSC1[3,22]=1;
MSC1[6,20]=1;
MSE0=ZEROS((NK+NL),NN);
MSE0[8,1]=1;
MSE1=ZEROS((NK+NL),NN);
MSE1[1,1]=-1;
MSE1[2,1]=-1;
MSE1[3,1]=-1;
MSE1[4,1]=-1;
MSE1[5,1]=-1;
MSE1[6,1]=-1;
MSE1[7,1]=RAA;
@@ ==================================== @
@@ RELATIONS LINKING FLOW VARIABLES (EXTRA CONTROLS) TO
FUNDAMENTAL CONTROLS AND STATES @
@@ ==================================== @
@@ THE ORDER OF VARIABLES IS: @
NXF=24; @@ DIMENSION AND NAMES OF EXTRA CONTROLS @
NAMEXC="PA" |"PB" |"PC" |"PD" |"YA" |"YB" |"YC" |"YD" |"LA" |"LB" |"LC" |"LD" |"L"
|"R" |"LAM" |"C1" |"C2" |"C3" |"C4" |"C5" |"C6" |"C7" |"C8"| "C9";
@ MF*XC(t) = MFC*C(t) + MFKE*[K(t)|E(t)]' + MFL*L(t) @
MF=ZEROS(NXF,NXF);
MF[1,1]=1;
MF[2,2]=1;
MF[2,1]=-1;
MF[3,3]=1;
MF[3,1]=-1;
MF[4,4]=1;
MF[4,1]=-1;
MF[5,5]=1;
MF[5,1]=S/(S-1);
MF[6,6]=1;
MF[6,5]=-1;
MF[6,1]=-T;
MF[6,2]=T;
MF[7,7]=1;
MF[7,5]=-1;
MF[7,1]=-T;
MF[7,3]=T;
MF[8,8]=1;
MF[8,5]=-1;
MF[8,1]=-T;
MF[8,4]=T;
MF[9,9]=1;
MF[9,1]=1/(1-S);
MF[10,10]=1;
MF[10,2]=1/(1-S);
MF[11,11]=1;
MF[11,3]=1/(1-S);
MF[12,12]=1;
MF[12,4]=1/(1-S);
MF[13,13]=1;
MF[13,9]=1/A7;
MF[13,10]=(RGMBAR^EPS)/A7;
MF[13,11]=(RGMBAR^(2*EPS))/A7;
```

```
MF[13,12]=(RGMBAR^(3*EPS))/A7;
MF[14,14]=1/(RBAR-1);
MF[15,15]=1;
MF[16,16]=1;
MF[16,5]=1;
MF[16,1]=T;
MF[17,17]=1;
MF[17,6]=1;
MF[17,2]=T;
MF[18,18]=1;
MF[18,7]=1;
MF[18,3]=T;
MF[19,19]=1;
MF[19,8]=1;
MF[19,4]=T;
MF[20,20]=1;
MF[20,1]=SS1;
MF[20,5]=SS1;
MF[20,2]=SS2;
MF[20,6]=SS2;
MF[20,3]=SS3;
MF[20,7]=SS3;
MF[20,4]=SS4;
MF[20,8]=SS4;
MF[21,21]=1;
MF[21,14]=1;
MF[22,22]=1;
MF[22,6]=1;
MF[22,2]=S/(S-1);
MF[23,23]=1;
MF[23,7]=1;
MF[23,3]=S/(S-1);
MF[24,24]=1;
MF[24,8]=1;
MF[24,4]=S/(S-1);
MFC=ZEROS(NXF,NC);
MFC[1,21]=1-S;
MFC[1,22]=1-S;
MFC[15,21]=-1;
MFC[15,22]=-1;
MFC[16,21]=1;
MFC[16,22]=T;
MFC[17,21]=1;
MFC[17,22]=T;
MFC[18,21]=1;
MFC[18,22]=T;
MFC[19,21]=1;
MFC[19,22]=T;
MFC[20,21]=1;
MFC[20,22]=1;
MFC[21,2]=1;
MFC[21,21]=-1;
MFC[21,22]=-1;
MFC[21,6]=1;
MFC[21,10]=V*(U1-1)/U1;
MFC[1,20]=(1-S)*RO*(1-J0)+S;
MFC[2,20]=-EPS*S/T;
MFC[3,20]=-EPS*S/T;
MFC[4,20]=-EPS*S/T;
MFC[5,20]=S/(S-1);
MFC[9,20]=1/(S-1);
MFKE=ZEROS(NXF,NK+NN);
MFKE[1,4]=-(1-S)*RO*J3;
MFKE[1,5]=-(1-S)*RO*J2;
MFKE[1,6]=-(1-S)*RO*J1;
MFKE[2,6]=EPS*S/T;
MFKE[3,5]=EPS*S/T;
MFKE[4,4]=EPS*S/T;
```

```
MFKE[10,6]=1/(S-1);
MFKE[11,5]=1/(S-1);
MFKE[12,4]=1/(S-1);
MFKE[22,6]=S/(S-1);
MFKE[23,5]=S/(S-1);
MFKE[24,4]=S/(S-1);
MFL=ZEROS(NXF,NL);
MFL[14,1]=V-1;
MFL[15,1]=-V*(U1-1)/U1;
MFL[21,1]=-V*(U1-1)/U1;
FVC=(INV(MF))*MFC;
FVKE=(INV(MF))*MFKE;
FVL=(INV(MF))*MFL;
MSss0 = MSS0 - MSC0*(INV(MCC))*MCS;
MSss1 = MSS1 - MSC1*(INV(MCC))*MCS;
MSsc0 = MSE0 + MSC0*(INV(MCC))*MCE;
MSsc1 = MSE1 + MSC1*(INV(MCC))*MCE;
W = -(INV(MSss0))*MSss1;
R = (INV(MSss0))*MSsc0;
Q = (INV(MSss0))*MSsc1;
{X1,X3}=EIGV(W);
X11=REAL(X1);
AMU=ABS(X11);
IN=SORTC(AMU,1);
IND1=INDNV(IN,AMU);
I=1;
DO UNTIL I>(NK+NL)-1;
IF IND1[I,1] == IND1[I+1,1];
IND1[I+1,1] = 1+IND1[I,1];
ENDIF;
I=I+1;
ENDO;
P=ZEROS((NK+NL),(NK+NL));
I=1;
DO UNTIL I>(NK+NL);
P[1:(NK+NL),I]=X3[1:(NK+NL),IND1[I,1]];
I=I+1;
ENDO;
MU=ZEROS((NK+NL),(NK+NL));
I=1;
DO UNTIL I>(NK+NL);
MU[I,I]=X1[IND1[I,1],1];
I=I+1;
ENDO;
MU1=MU[1:NK,1:NK];
MU2=MU[NK+1:NK+NL,NK+1:NK+NL];
P11=P[1:NK,1:NK];
P12=P[1:NK,NK+1:NK+NL];
P21=P[NK+1:NK+NL,1:NK];
P22=P[NK+1:NK+NL,NK+1:NK+NL];
PS=INV(P);
PS11=PS[1:NK,1:NK];
PS12=PS[1:NK,NK+1:NK+NL];
PS21=PS[NK+1:NK+NL,1:NK];
PS22=PS[NK+1:NK+NL,NK+1:NK+NL];
RKE=R[1:NK,1:NN];
RLE=R[NK+1:NK+NL,1:NN];
QKE=Q[1:NK,1:NN];
QLE=Q[NK+1:NK+NL,1:NN];
SP1=-(INV(MU2))*(PS21*RKE+PS22*RLE);
SP2=-(INV(MU2))*(PS21*QKE+PS22*QLE);
KLK=P11*MU1*(INV(P11));
KTL=(P11*MU1*PS12+P12*MU2*PS22)*(INV(PS22));
NAME=NAMEK|NAMEE|NAMEL|NAMEC|NAMEXC;
```

FIFTH CODE

@REMEMBER THE ORDER OF FILES RUNNING:1) FIRST = PERFECT FORESIGHT SYSTEM; 2) SECOMND = PERFECT FORESIGHT DECISION RULES; 3) FOURTH = EXPECTATION SYSTEM (GIVEN THE PF DECISION RULES); 4) SECOND = EXPECTATION SYSTEM DECISION RULES; 5) FIFTH = IMPULSE RESPONSE FUNCTION OF THE EXP SYSTEM@

```
@SHOCK THE EXOGENOUS VARIABLE@
S=0|0|0|0|0|0|0|0.01;
@ FORECAST HORIZON @
NIR=20;
@ GENERATING IMPULSES @
TI=SEQA(1,1,NIR);
IR=ZEROS(ROWS(MKE)+ROWS(H),NIR);
I=1;
DO UNTIL I>NIR;
IR[1:ROWS(MKE),I]=S;
IR[ROWS(MKE)+1:ROWS(IR),I]=(H*S);
S=MKE*S;
I=I+1;
ENDO;
IR=REAL(IR);
INF=ZEROS(1,NIR);
INF[1,1]=IR[31,1];
INF[1,2:NIR]=IR[8,1:(NIR-1)]+IR[31,2:NIR]-IR[31,1:(NIR-1)];
IR=IR|INF|W;
@ PLOT IMPULSE RESPONSES @
LIBRARY PGRAPH;
GRAPHSET;
_PLEGCTL={2,3,5,4.5};
_PLEGSTR="RGM\000"\
"Y\000"\
"INF\000";
XY(TI,IR[8,.]'~IR[30,.]'~IR[56,.]');
```

Chapter 6

On Price/Wage Staggering and Persistence: A Unifying Framework

6.1 Introduction

In the previous chapters we have seen that some models (i.e., Chari et al. (1996) and Chapter 4) have seriously questioned the explanatory power of staggered price/wage setting in accounting for output persistence for reasonable parameter values. On the other hand, Rotemberg and Woodford (1997) and Erceg (1997) claim that their model can match the observed degree of persistence. In the literature it hence seems there is no consensus.

Besides, the comparison among existing models is far from straightforward since Chari et al. (1996) focus on price staggering, Chapter 4 on wage staggering, Erceg (1997) on both and Rotemberg and Woodford (1997) employ the yeoman-farmer hypothesis.

In this final chapter, we provide a unifying framework to analyse the issue of output persistence in staggered wage/price models, an issue to which a large part of this thesis has been devoted. Our aim is to clarify it by: (i) highlighting the differences between price and wage staggering; (ii) analysing which features of the underlying economy with superimposed price/wage staggering are crucial for generating output persistence; (iii) ranking the different potential specifications according to their ability to generate output persistence. In order to do so, we build a stylised log-linear model that encompasses (most of) the existing microfounded models of price/wage staggering.

Our results highlight that: (i) the difference between the persistence properties of price and wage staggering models derives from the underlying economic structure, not merely from the fact of price staggering rather than wage staggering. In particular, the substitutability between goods and/or labour types plays the major role in generating persistence. In models with only substitutability between goods, then price staggering naturally delivers higher persistence than wage staggering, while in models with substitutabil-

ity between labour types, the opposite is true. (ii) The distinction between free mobility and no mobility of labour is fundamental. No-mobility-of labour models (both "industrial" and "craft" union models) bring in new mechanisms that increase persistence. (iii) While in price/wage staggering models, a substantial (in the sense of near random walk behaviour) degree of persistence is an unlikely outcome, two models can deliver significant persistence: the yeoman farmer model and the "craft" union models with wage staggering. (iv) Ceteris paribus (i.e., for realistic values of all the other parameters), these conclusions do not depend on the particular value assigned to the intertemporal elasticity of labour supply, which instead has been so far the focus of this literature.

Note that the first result is at odds with Andersen's (1998a) one that wage staggering models deliver higher persistence than price staggering one. However, our analysis shows that Andersen's (1998a) finding is not due to an intrinsic difference between wage and price staggering models, but to the particular assumptions in the two cases there presented. The results are useful not only for interpreting the existing literature but also for those who might consider undertaking further research in this area. Specifically, the combination of nominal and real rigidities has recently received renewed attention (e.g., Jeanne (1998), Andersen (1998b), Kiley (1997), Bergin and Feenstra (1998), Chapter 5). That line of research is likely to continue in the

near future. We therefore think that our analysis of the effects of the deep parameters of the underlying economy on the degree of persistence will prove enlightening.

6.2 Reduced Form Staggered Price/Wage Models

In this section we briefly review some results from the previous chapters to concisely sum up the differences between the original Taylor (1979) model and the corresponding models which can be obtained in a microfounded framework. This will help us to understand the argument in the next sections of the chapter.

6.2.1 Taylor's Original (1979) Model

Taylor's (1979) model (ignoring discounting and the expectation operator)[1] is

$$x_t = \frac{1}{2}(x_{t-1} + x_{t+1}) + \frac{1}{2}\,\gamma\,(y_t + y_{t+1}) \tag{6.1}$$

$$p_t = \frac{1}{2}(x_{t-1} + x_t) \tag{6.2}$$

$$y_t = m_t - p_t \quad . \tag{6.3}$$

As Buiter and Jewitt (1981) and Blanchard (1990) noted (6.1) and (6.2) imply

$$x_t = \frac{1}{2}(p_t + p_{t+1}) + \frac{1}{2}\,g\,(y_t + y_{t+1}) \tag{6.4}$$

where $g = \frac{\gamma}{2}$. The persistence root is

$$\lambda = \frac{1-\sqrt{g}}{1+\sqrt{g}} = \frac{1-\sqrt{\frac{\gamma}{2}}}{1+\sqrt{\frac{\gamma}{2}}} \quad . \tag{6.5}$$

Comments:

(i) Effect of γ (the only parameter in this model). The higher γ, the lower the persistence. $\gamma = 2$ is the threshold value between negative and positive persistence. If $\gamma = 0$, any temporary money shock has a permanent effect on

[1] We have seen in Chapter 4 that: (i) Discounting does not have a controversial effect. The effect of β is very clear because it only changes (1/2) to something different, but it does not enter the other parameters of the reduced form staggered model (i.e. γ, a). Moreover, its quantitative effect (since β is thought to be very close to one) seems unimportant. (ii) Uncertainty is not going to change the persistence properties of the models. Because these are what we are interested in here, for simplicity it is better to leave uncertainty out.

output. Hence, interesting values of γ are $\gamma \in [0, 2]$. Moreover, to get a value of $\lambda = 0.8$, γ should be as low as 0.025.

(ii) Since (6.2) does not include an output term, then constant returns to labour are assumed, i.e., $\sigma = 1$ (in the productiona function $Y = L^{\sigma}$) and γ is simply $2g$.

6.2.2 Taylor's Generalised (1979) model

Taylor's (1979) model can be generalised to include decreasing return to scale to labour. When $\sigma \neq 1$, then the aggregate supply equation becomes

$$p_t = \frac{1}{2}(x_{t-1} + x_t) + ay_t \tag{6.6}$$

where a is positive (and equal to $\frac{1-\sigma}{\sigma}$).

Now suppose a model composed of (6.1), (6.6) and (6.3). Now γ and σ are the primitive parameters. The degree of persistence for this model is

$$\lambda = \frac{1 - \sqrt{\frac{\gamma\sigma}{2}}}{1 + \sqrt{\frac{\gamma\sigma}{2}}} \quad . \tag{6.7}$$

Substituting (6.6) in (6.1) yields (6.4) with $g = \frac{\gamma}{2} - a$.

Comments:

(i) γ and g no longer coincide (apart from the scaling factor 1/2) as in the other case. Moreover, they can have (and are likely to have) different signs. Recall that γ is the primitive, while g is derived by substitution of (6.6) into (6.1). Suppose γ is positive and low, as originally intended by Taylor. Then, for $\sigma = 0.67$, $a \simeq 0.5$ and g is negative whenever γ is lower than 1.

(ii) It follows that the Buiter and Jewitt (1981) and Blanchard (1990) statement does not hold anymore when $\sigma \neq 1$, because it will be difficult to interpret (6.4) as a directly postulated wage rule (or, equivalently as a labour supply curve) when g is negative. The model is a true 'relative wage concern' model and there is no longer a direct correspondence between a relative wage concern (6.1) and a real wage concern (6.4). In other words, recalling (5.20), prices do not feedback into (6.1) and wages do respond only to other wages and output.[2] That is exactly why g can be negative. Once (6.1) is expressed in terms of prices, as in (6.4), prices already incorporate the increase in output due to ay_t and this should be subtracted from $\gamma/2$.

(iii) Look at (6.7). Given that γ is a primitive object, the effect of σ is *always* counterintuitive: the higher σ, the lower the persistence.[3] This is because only output affects wages. The higher σ, the higher the response of output (and lower the one of prices) to a money shock, then the higher the response in wages and quicker the adjustment. Therefore there is a trade-off between the size of the impact effect on output and its persistence, induced by σ.

---

[2]Such a model is hence very similar to the one in the previous chapter in Case A with $\Omega = 0$.

[3]Simple intuition would suggest that $\sigma = 1$ corresponds to the maximum degree of nominal rigidities (because firms could satisfy any excess demand without changing their prices) and thus that persistence would be increasing in σ.

6.2.3 A reduced form 'microfounded' model

In standard microeconomic reasoning, agents care about their own real wage and not about their relative wage. Hence, a standard microfounded model would deliver a model composed of (6.4), (6.6), (6.3). As said in (ii) before, this model and the one presented before, *even if analytically equivalent, are radically different in their economic interpretation.* Now g and a are the primitive parameters and $\gamma = 2(g + a)$ is derived. The solution is

$$\lambda = \frac{1 - \sqrt{1 + \sigma(g - 1)}}{1 + \sqrt{1 + \sigma(g - 1)}} \quad . \tag{6.8}$$

Comments:

(i) Since g is positive, calibrating $a = 0.5$ would deliver a value of γ always bigger than 1.

(ii) The effect of σ is now ambiguous and depends on the value of g. If $g > 1$, then persistence is decreasing in σ and viceversa. The intuition is as follows. Suppose a money shock m_t occurs after the wages are set. Then, in the impact period $y_t = \sigma m_t$ and $p_t = (1 - \sigma)m_t$. The wage set next period will be influenced by two components: (a) the first equal to $(1-\sigma)m_t$, because prices have increased; (b) the second equal to $g\sigma m_t$, because output has increased. A change $\triangle\sigma$ in σ diminishes (a) by -$\triangle\sigma$ and (b) by $g\triangle\sigma$, with a net effect of $(g - 1)\triangle\sigma$, as (6.8) shows.

(iii) Moreover, if $g \geq 1$, then $\lambda \leq 0$. It follows that for this model to deliver persistence g must *necessarily* be lower than one and in this case

persistence is decreasing in σ.

6.3 A Perfectly Flexible Wage and Price Microfounded Model

In this section we sketch a perfectly flexible wage/price log-linear model which can be thought as derived from a log-linearised version of a microfounded model. The model is very general in its formulation and could be easily derived as a log-linearised version of most models of monopolistic competition such as the one in Blanchard and Kiyotaki (1987) or the different versions presented by Dixon and Rankin (1994)in their survey. Even if all the equations are log-linear, therefore, the model is not an *ad hoc* model; we will keep referring to the underlying microeconomic structure of the model and of its several versions in the next section. The general framework is similar to the one of the previous chapters. The size of the whole economy is normalised to 1 and thus the economy consists of a continuum of industries indexed by $i \in [0, 1]$.[4] Every industry produces a single differentiated perishable prod-

[4]The fact that we have a continuum (a 'large' number) of agents means that each firm and household takes the aggregate variables as given and there is no strategic interaction among them. Readers need to keep this in mind. We often just present the formulas for sectors, which are obtained simply aggregating across typical firms or households belonging to the sector. However, it is not like having only two agents (one for each sector). If that

uct. Households are indexed by $j \in [0,1]$ and they live forever. All firms have the same technology and households have the same preferences. Preferences are CES over consumption goods which are gross substitutes. In order to ease the introduction of staggering in the next section, the supply side of the economy is divided in two sectors: A and B, for simplicity of equal size (one half). The aggregate demand is given by a standard aggregate demand equation[5]

$$y_t = m_t - p_t \quad . \tag{6.9}$$

As usual, lower case variables denote log-deviations from steady state. The aggregate price level is just the average of the two sectors' prices (since firms

was the case we would need to take into account strategic interactions.

[5]Note that we impose a constant-velocity-of-circulation aggregate demand function on the model, because we want to focus on the supply side of the model. Even if this aggregate demand looks as a static one, (6.9) can be derived from intertemporal optimisation in two cases. First, as Bénassy (1995) shows, the velocity of circulation of money is constant in an intertemporal optimising model, whenever money is injected only through interest payments on bonds. Second, given the utility function of Chapter 4, we have seen that the log-linearised aggregate demand is equal to $y_t = m_t - p_t + z_t$, where the last term evolves according to a pure forward-looking equation. Following a one, temporary i.i.d. money shock, then z_t jumps immediately to the steady state, that is, it stays constant. Then, if money shocks are temporary and not autocorrelated z_t does not have any effect on persistence and we can just impose (6.9) on the model, as we did in Chapter 4.

belonging to the same sectors will set independently the same price)

$$p_t = \frac{1}{2}(p_{A_t} + p_{B_t}) \tag{6.10}$$

where p_{X_t} = price of sector X for $X = A, B$. Firms are identical and labour is the only factor of production. From a short-run production function (in levels) of the form $Y_t = \alpha L_t^{\sigma}$, the sectors' supply functions are

$$p_{A_t} = w_{A_t} + a y_{A_t} \tag{6.11}$$

$$p_{B_t} = w_{B_t} + a y_{B_t} \tag{6.12}$$

where $a = \frac{1-\sigma}{\sigma}$ and w_{X_t} = nominal wage in sector X; y_{X_t} = sector's X real output. Note that (6.11) and (6.12) hold regardless of whether the goods market is competitive or monopolistic, as long as each firm faces a demand curve with constant elasticity.[6] The only difference between the two cases, in fact, is the presence of a constant mark-up (given that the elasticity of demand is constant) in the equation in levels for the monopolistic goods markets; but then in the log-linearised version the constants disappear. The two cases then have the same log-linear formula for the supply functions of a typical firm.

Industries produce differentiated goods and θ is the elasticity of substitution among goods.[7] Industries' demands are derived from a log-linearisation

[6]This actually is the usual assumption in monopolistic competition macromodels.

[7]Despite the fact that industries produce differentiated goods, we can still regard a

of the standard Dixit-Stiglitz formula

$$y_{A_t} = \theta(p_t - p_{A_t}) + y_t \tag{6.13}$$

$$y_{B_t} = \theta(p_t - p_{B_t}) + y_t \quad . \tag{6.14}$$

Note that (6.13), (6.14) and (6.10) yield

$$y_t = \frac{1}{2}(y_{A_t} + y_{B_t}) \tag{6.15}$$

which simply states that aggregate output is given by the weighted average of outputs in a typical industry of each sector.

Households are also divided in two groups of equal size (one half), C and D. The standard first-order condition for labour supply (in levels) equates the ratio between the marginal disutility of labour and the marginal utility of consumption to the real wage $\left(\text{i.e., } \frac{W_t}{P_t} = \frac{-U_l}{U_c}\right)$. Assuming an additively separable utility function, we can write

$$w_{C_t} = \eta_{ll} l_{C_t} + \eta_{cc} c_{C_t} + p_t \tag{6.16}$$

$$w_{D_t} = \eta_{ll} l_{D_t} + \eta_{cc} c_{D_t} + p_t \tag{6.17}$$

where w_{S_t} = nominal wage in group S, for $S = C, D$; l_{S_t} = amount of labour supplied by a typical household in group S ; c_{S_t} = consumption of a

competitive goods market as encompassed by these formulas. This would be the case in the framework of the previous chapters, where in each industry there are a 'large' number of firms, and not only one.

typical household in group S ; $\eta_{ll} = \frac{U_{ll}\, l}{U_l} > 0$ is the elasticity of the marginal disutility of labour with respect to labour; $\eta_{cc} = \frac{-U_{cc}\, c}{U_c} > 0$ is (minus) the elasticity of the marginal utility of consumption with respect to consumption. Throughout, we will assume that η_{ll} and η_{cc} are constant (as for most of the utility functions used in macromodels). Given that we have assumed an additively separable underlying utility function, then η_{ll} and η_{cc} are the inverses of the intertemporal elasticity of substitution in labour supply and in consumption respectively. Moreover, η_{cc} coincides with the income effect on labour supply.[8]

Whenever there is some heterogeneity across infinitely-lived households in macromodels, this creates an analytical problem. As discussed in Chapter 1, the usual way to deal with heterogeneity and to avoid any distributional complication is to assume complete markets. Agents can then completely insure themselves against idiosyncratic shocks or unpredictable fluctuations. This implies the marginal utility of consumption to be equalised across households each period. Hence, given an additively separable utility function, households consume the same in each period. Since the goods market equilibrium implies $y_t = \frac{1}{2}(c_{C_t} + c_{D_t})$, then each of the two groups consumes half of the real

[8] Parallel to the supply functions for the goods market, i.e., (6.11) and (6.12), the supply functions in the labour market, i.e., (6.16) and (6.17) hold both in a competitive labour market and in a labour market characterised by monopoly unions, provided labour demand functions have a constant elasticity.

output each period, that is

$$c_{C_t} = y_t \tag{6.18}$$

$$c_{D_t} = y_t \quad . \tag{6.19}$$

So far we have presented a rather general perfectly flexible wage/price log-linear model whose equations are consistent with virtually any monopolistic competition macromodel. However, we have a perfectly flexible wage/price model with 12 variables (i.e., y_t, p_t, p_{A_t}, p_{B_t}, w_{A_t}, w_{B_t}, y_{A_t}, y_{B_t}, l_{C_t}, l_{D_t}, c_{C_t}, c_{D_t}), one exogenous variable, i.e. m_t, and 10 equations ((6.9), (6.10), (6.11), (6.12), (6.13), (6.14), (6.16), (6.17), (6.18), (6.19)). The two missing equations are the sectors' labour demands, l_{C_t} and l_{D_t}. These two latter equations will be different according to the type of labour market structure we assume. As we will see, different labour market structures turn out to be crucial for the persistence properties of the staggered version of the model.

To introduce staggering in the model we proceeds in the following way. The first step is to close the perfectly flexible wage/price model, that is to write down the two equations for the sectors' labour demands according to the different labour market structures we will analyse in the next section. The second step is to find the optimal rule in the perfectly flexible wage/price model for the nominal variable we want to stagger (price or wage), given the labour market structure. The third step transforms the perfectly flexible wage/price model in a staggered one, by assuming that the nominal variable

of interest is set for two periods in staggered fashion, and it is simply given by the average of today's and tomorrow's optimal rules (which we have just found for the perfectly flexible wage/price model in step two). Thus, our approach is very similar to Blanchard and Fischer (1989), Chapter 8.[9] We investigate the persistence properties of several different models according to the type of labour market we consider and to the nominal variable (price or wage) we stagger. We show how each of these different models corresponds to a model in the literature and can be expressed in the same reduced form as one of the three models of the previous section. The implications of the alternative economic structures for persistence will then be immediately evident.

6.4 Staggered Models

In this section we present different wage/price staggering models, derived from the perfectly flexible wage/price model of the previous section. The discussion and the comparison among all these different models are postponed to section 6.5.

[9]The same result can be obtained by staggering directly the equation for the nominal variable of interest in the static model and then substitute out to get a reduced form alike one of the models in the previous section. That is, we can first make step three and then step two. This is just saying that it does not matter the order in which step two and three are performed.

6.4.1 Staggered Prices and Perfect Labour Mobility: Chari et al. (1996)

First, we need to close the perfectly flexible wage/price model. If labour is completely mobile across sectors, households supply labour to both sectors in the economy and the wage is equalised across households and firms, i.e. $w_{A_t} = w_{B_t} = w_{C_t} = w_{D_t}$. Given (6.16), (6.17), (6.18) and (6.19) then the two groups of households supply the same amount of labour. Thus

$$l_{C_t} = l_{D_t} = l_t = \frac{1}{\sigma}\, y_t \tag{6.20}$$

and this gives the two missing sectors' labour demands in the perfectly flexible wage/price model.

Second, we find the optimal rule in the perfectly flexible wage/price model for the variable we want to stagger, that is sectors' prices. Substituting (6.20) in (6.16) and (6.17) and using (6.18) and (6.19), yields

$$w_{C_t} = w_{D_t} = w_t = p_t + \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_t \equiv p_t + \widetilde{\gamma}\, y_t \quad . \tag{6.21}$$

Use this expression in (6.11) and (6.12) and substitute out sectors' output making use of (6.13) and (6.14), to obtain

$$p_{A_t} = p_{B_t} = p_t + \left(\frac{\widetilde{\gamma} + a}{1 + a\theta}\right) y_t \tag{6.22}$$

which is the optimal pricing rule of sector-A firms as a function of p_t and y_t.

Third, we introduce staggering. We then suppose that each firm in a given sector, acting independently, sets the price for two periods in a staggered

fashion. That is, while firms in sector A fix the price in even periods, the ones in sector B fix it in odd periods. Thus

$$p_{i_{t+s}} = p_{i_{t+s+1}} = \frac{1}{2}(p_{t+s} + p_{t+s+1}) + \frac{1}{2}\left(\frac{\widetilde{\gamma} + a}{1 + a\theta}\right)(y_{t+s} + y_{t+s+1}) \qquad (6.23)$$

$$for\ i \in A \text{ and } for\ s = 0, 2, 4...$$

$$p_{i_{t+s-1}} = p_{i_{t+s}} = \frac{1}{2}(p_{t+s-1} + p_{t+s}) + \frac{1}{2}\left(\frac{\widetilde{\gamma} + a}{1 + a\theta}\right)(y_{t+s-1} + y_{t+s}) \qquad (6.24)$$

$$for\ i \in B \text{ and } for\ s = 0, 2, 4... \ .$$

Aggregating across sectors and denoting the staggered variable by x_t, the reduced form of the model can be written as[10]

$$x_t = \frac{1}{2}(p_t + p_{t+1}) + \frac{1}{2}\,\overline{\gamma}\,(y_t + y_{t+1})$$

$$p_t = \frac{1}{2}(x_{t-1} + x_t)$$

[10] As explained in footnote 9 above, we can reverse the order of steps two and three. That is, the same expressions can be obtained by staggering at the outset (6.11) and (6.12) for a typical firm in each sector. That is, for a typical firm i in sector A

$$p_{i_t} = p_{i_{t+s+1}} = \frac{1}{2}(w_{t+s} + w_{t+s+1}) + \frac{a}{2}(y_{i_{t+s}} + y_{i_{t+s+1}}) \qquad for\ i \in A \text{ and } for\ s = 0, 2, 4...$$

and then aggregate across sector A and substitute out for w and y_A using (6.21) and (6.13).

$$y_t = m_t - p_t$$

where $\overline{\gamma} = \left(\frac{\widetilde{\gamma}+a}{1+a\theta}\right)$.[11] The model then corresponds to the one in 6.2.1 with $\overline{\gamma} = g$, and its solution for the root governing output thus is

$$\lambda = \frac{1-\sqrt{R_1}}{1+\sqrt{R_1}} \quad ; \quad \text{where} \quad R_1 \equiv \overline{\gamma} = \frac{\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma}{\sigma + \theta\,(1-\sigma)} \quad . \tag{6.25}$$

Given the staggered version of (6.11), the equation for the staggered variable is

$$x_t = \frac{1}{2}(w_t + w_{t+1}) + \frac{a}{2}(y_{A_t} + y_{A_{t+1}}) \tag{6.26}$$

which can be expressed as

$$\begin{aligned} x_t &= \frac{1}{2}(x_{t-1} + x_{t+1}) + (\omega_t + \omega_{t+1}) + a(y_{A_t} + y_{A_{t+1}}) = \\ &= \frac{1}{2}(x_{t-1} + x_{t+1}) + \left(\frac{1}{1+a\theta}\right)(\omega_t + \omega_{t+1}) + \left(\frac{a}{1+a\theta}\right)(y_t + y_{t+1}) \end{aligned} \tag{6.27}$$

where $\omega_t = (w_t - p_t)$ represents the real wage. If $\sigma = 1$ (constant returns to labour), then $a = 0$ and equation (6.27) exactly matches equation (46) in Chari et al. (1996), p. 13. Moreover, in this case $\overline{\gamma} = \widetilde{\gamma} = \eta_{ll} + \eta_{cc}$, exactly as their γ at page 15.

The model is also a generalisation of the *ad hoc* price staggering model of Andersen (1998a), which is obtained setting $\sigma = 1$.

[11] To avoid confusion, it is better to stress that we denote: $\widetilde{\gamma}$ as the sensitivity of real wage to aggregate output ($w_{X_t} - p_t = \widetilde{\gamma} y_t$) and $\overline{\gamma}$ as the sensitivity of sectors' price to aggregate output ($p_{X_t} - p_t = \overline{\gamma} y_t$).

6.4.2 Staggered Prices and No Mobility of Labour: the Yeoman-Farmer Model: Blanchard and Fischer (1989) and Rotemberg and Woodford (1997)

In this case, households of group C work for firms in sector A, while households of group D work for firms in sector B. The model is equivalent to the yeoman-farmer model where each household produces a differentiated good and there is no labour market. Thus, the two missing equations in the perfectly flexible wage/price model are: $l_{C_t} = l_{A_t} = \frac{y_{A_t}}{\sigma}$, $w_{A_t} = w_{C_t}$ and $l_{D_t} = l_{B_t} = \frac{y_{B_t}}{\sigma}$, $w_{B_t} = w_{D_t}$. The wage equation in the perfectly flexible wage/price model is given by

$$\begin{aligned} w_{A_t} &= \eta_{ll}\frac{y_{A_t}}{\sigma} + \eta_{cc}y_t + p_t = \\ &= p_t + \left[\frac{\eta_{ll} + \eta_{cc}(1+\theta a)\sigma}{\sigma(1+\theta a) + \theta\eta_{ll}}\right] y_t = p_t + \widetilde{\gamma}\, y_t \quad . \end{aligned} \tag{6.28}$$

(6.28) corresponds to (6.21) in the previous model, and we can proceed following the same steps as before to get the optimal pricing rule for the farmer. However, apart from the different expression for $\widetilde{\gamma}$, the model is analytically equivalent to the previous one. Thus, we know the solution is $\lambda = \frac{1-\sqrt{\overline{\gamma}}}{1+\sqrt{\overline{\gamma}}} \equiv \frac{1-\sqrt{R_2}}{1+\sqrt{R_2}}$ where $\overline{\gamma} = \frac{\widetilde{\gamma}+a}{1+a\theta}$. Again, the persistence properties of this model depend only on

$$R_2 = \overline{\gamma} = \frac{\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma}{\sigma + \theta\,(1-\sigma) + \theta\eta_{ll}} \quad . \tag{6.29}$$

The model encompasses two recent yeoman-farmer models: Blanchard

and Fischer (1989) and Rotemberg and Woodford (1997). Normally a standard yeoman-farmer model includes in the utility function a term $V(y_i)$ to represent the disutility from producing for the farmer. The standard first order condition that gives the optimal price in a farmer model is (in levels)

$$\frac{P_i}{P} = \left(\frac{\theta}{\theta - 1}\right)\left(\frac{V'(y_i)}{U'(c_i)}\right) \quad . \tag{6.30}$$

It states that the optimal ratio between the price of the good produced by the farmer and the aggregate price level is given by a constant mark-up over the ratio between the marginal disutility from effort in production and the marginal utility of consumption. Log-linearising (6.30) for sector A households, it yields

$$p_{A_t} - p_t = \eta_{yy} y_{A_t} + \eta_{cc} y_t \tag{6.31}$$

where $\eta_{yy} = (V_{y_i y_i} y_i / V_{y_i}) > 0$ is the elasticity of the marginal disutility of production. Given our production function, i.e., $Y_t = \alpha L_t^{\sigma}$, simple algebra shows that $\eta_{yy} = (\eta_{ll}/\sigma + a)$. Substituting (6.13) in (6.31) gives

$$p_{A_t} - p_t = \left(\frac{\eta_{yy} + \eta_{cc}}{1 + \theta\eta_{yy}}\right) y_t = \left(\frac{\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma}{\sigma + \theta\,(1 - \sigma) + \theta\eta_{ll}}\right) y_t = \overline{\gamma} y_t \tag{6.32}$$

which is the optimal pricing rule for the perfectly flexible wage/price model. Performing then step three, that is considering (6.32) under staggering, shows our model to be equivalent to Rotemberg and Woodford (1997) (see their formula for κ at p.316).[12]

[12] With respect to Rotemberg and Woodford (1997) model, this claim is subject to the caveats of footnote 15 in Chapter 4.

Our model is also a generalisation of Blanchard and Fischer (1989) model, whose specification of preferences implies zero income effect on labour supply (i.e., $\eta_{cc} = 0$). Then

$$p_{A_t} = p_t + \left[\frac{\eta_{yy}}{1+\eta_{yy}\theta}\right] y_t = \left[\frac{1+(\theta-1)\eta_{yy}}{1+\eta_{yy}\theta}\right] p_t + \left[\frac{\eta_{yy}}{1+\eta_{yy}\theta}\right] m_t \quad (6.33)$$

which is equivalent to equation (9) of Blanchard and Fischer (1989), p. 385.[13]

6.4.3 Staggered Wages and Perfect Labour Mobility: the *ad hoc* models

The combination of wage staggering and perfect labour mobility is unusual in microfounded models. The reason is very simple: there are no convincing microfoundations for this case. Indeed, if wages are staggered, it means that someone must have the power to set them. That is, staggered wages should go together with monopoly unions which set the wages. The unions therefore must enjoy market power and there can not be perfect labour mobility or a competitive labour market. However, this case is of interest to illustrate the difference between price and wage staggering. Further and most importantly, this is the so called "expected-market–clearing-case" employed by the *ad hoc* literature of the 70's and 80's on staggered wage models (e.g., Gray (1976),

[13]They use the following utility function for the yeoman farmer: $U_i = \left(\frac{C_i}{d}\right)^d \left(\frac{M_i/P}{1-d}\right)^{1-d} - \left(\frac{f}{\beta}\right) Y_i^{\beta}$. Hence, the elasticity of the marginal disutility of production is given by $(\beta - 1)$. Substituting η_{yy} with $(\beta - 1)$ equation (6.33) exactly match theirs.

Fischer (1977) and Taylor (1979)). Gray's (1976), Fischer's (1977) or Taylor's (1979) types of nominally rigid labour contract are set in order to achieve an *ad hoc* target wage level, which is the one that clears the labour market 'in expectation'.[14] In terms of our microfounded perfectly flexible wage/price model, this assumption implies that in each firm the workforce is equally divided between the two groups of workers. Then the reference wage for all the firms in each period is equal to $(1/2)\,(w_{A_t} + w_{B_t})$. Moreover, all the firms would charge the same price and produce the same level of output. Hence the one-period nominal wage contract will simply be

$$w_t = p_t + \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_t = p_t + \widetilde{\gamma}\, y_t \tag{6.34}$$

and under staggering

$$x_t = \frac{1}{2}(p_t + p_{t+1}) + \frac{1}{2}\widetilde{\gamma}(y_t + y_{t+1}) \quad . \tag{6.35}$$

The aggregate price level is given by

$$p_t = p_{A_t} = p_{B_t} = \frac{1}{2}(x_{t-1} + x_t) + a y_t \quad . \tag{6.36}$$

The model then corresponds to the one in 6.2.3 with $g = \widetilde{\gamma}$. The solution is hence given by $\lambda = \frac{1-\sqrt{1+\sigma(g-1)}}{1+\sqrt{1+\sigma(g-1)}} \equiv \frac{1-\sqrt{R_3}}{1+\sqrt{R_3}}$, where

$$R_3 = \sigma\widetilde{\gamma} + 1 - \sigma = \eta_{ll} + \sigma\eta_{cc} + 1 - \sigma \quad . \tag{6.37}$$

[14]Therefore the assumption that employment is always on the labour demand curve is inconsistent with optimisation. This is not true in monopoly union models. There the wage is above the competitive one, and ex post it is optimal for the household-union to satisfy an unexpected increase in labour demand.

This model also corresponds to the staggered wage *ad hoc* model in Andersen (1998a) when $\widetilde{\gamma} = 0$ is imposed. This restriction is at the root of Andersen's (1998a) result that wage staggering models are potentially able to deliver persistence while the price staggering ones are not. The intuition provided by Andersen (1998a) states that in wage staggering models the adjustment burden is borne by the prices, while wages do not react to excess demand conditions. While this is correct as the description of the impact effect of a money shock when wages are preset, the argument can not be automatically transferred to the dynamic model. In other words, when workers renegotiate a new wage, they will take into account the expected (labour) demand conditions (hence $\widetilde{\gamma}$ should be different from 0).

6.4.4 Staggered Wages and No Mobility of Labour: Chapter 4 Model

Here households of group C only work for firms in sector A, while households of group D work for firms in sector B. Then, equation (6.28) holds and staggering yields

$$x_t = \frac{1}{2}(p_t + p_{t+1}) + \frac{1}{2}\widetilde{\gamma}(y_t + y_{t+1}) \tag{6.38}$$

where now $\widetilde{\gamma} = \left[\frac{\eta_{ll}+\eta_{cc}(1+\theta a)\sigma}{\sigma(1+\theta a)+\theta\eta_{ll}}\right]$. The model is then analytically equivalent to the previous one, and the solution just depends on

$$R_4 = 1 + \sigma(\widetilde{\gamma} - 1) = \frac{[\sigma + \theta(1-\sigma)]\,[\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma]}{\sigma + \theta(1-\sigma) + \theta\eta_{ll}} \quad . \tag{6.39}$$

$\widetilde{\gamma}$ can be written as

$$\widetilde{\gamma} = \left[\frac{\eta_{ll} + \eta_{cc}(\sigma + \theta(1-\sigma))}{(\sigma + \theta(1-\sigma)) + \theta\eta_{ll}}\right] = \left[\frac{\frac{\varepsilon}{\theta}\eta_{ll} + \eta_{cc}}{1 + \varepsilon\eta_{ll}}\right] \tag{6.40}$$

which corresponds to the expression for g in Chapter 4 (p. 136).

6.4.5 Staggered Wages and Craft Unions: Blanchard and Kiyotaki (1987), Erceg (1997)

Another very widely used hypothesis to depict the labour market in micro-founded models is the one of Blanchard and Kiyotaki (1987) and recently used in works focused on the persistence issue (e.g., Erceg (1997), Kim (1996)). The labour market is there assumed to be composed of a large number of households that supply differentiated labour inputs. Firms regard each household's labour services as an imperfect substitute for the labour services of other households. Then, households who provide a particular labour service group together as a union and act as wage-setters in the labour market. This labour market structure is sometimes called a "craft" union structure, while the one presented in the previous sections is named "industrial" union structure (see e.g., Dixon and Rankin (1994)). Indeed, in the first case unions are organised by labour skills, while in the other case unions are characterised as specific to the industry to which its members supply (the only type of) labour. Note further that both cases imply a different kind of labour immobility. In a "craft" union labour market structure labour can not move across

skills, while in a "industrial" union labour market structure workers can not move across sectors.

With respect to the perfectly flexible wage/price model of section 6.3, the production function for a firm i is now a CES, that is (in levels)

$$Y_{i_t} = \left[\int_j L_{ij_t}^{\frac{\vartheta-1}{\vartheta}} \right]^{\frac{\sigma\vartheta}{(\vartheta-1)}} \tag{6.41}$$

where ϑ is the elasticity of technical substitution between different types of labour inputs.[15] This production function yields the following constant elasticity demand for labour type j of firm i

$$L_{ij_t} = \left[\frac{W_{j_t}}{W_t} \right]^{-\vartheta} Y_{i_t}^{\frac{1}{\sigma}} \tag{6.42}$$

where $W_t = \left[\int_j W_{j_t}^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}}$ is the wage index (which exactly parallels the standard Dixit-Stiglitz price index for differentiated goods). Note that since all the firms face the same wage index, they will produce the same level of output. We can aggregate across firms and then log-linearise (6.42), to obtain the labour demand for labour type j in the whole economy (in log-deviations)

$$l_{j_t} = \vartheta(w_t - w_{j_t}) + \frac{1}{\sigma} y_t \quad . \tag{6.43}$$

We can then aggregate across households who fix the wage in the same periods, realising that within each cohort a symmetric equilibrium holds such

[15] Note that we use θ for the elasticity of substitution between consumption goods and ϑ for the elasticity of technical substitution between different types of labour inputs.

that households in the same cohort will fix the same wage. Hence, the missing equations for labour demand to close the perfectly flexible wage/price model are

$$l_{C_t} = \vartheta(w_t - w_{C_t}) + \frac{1}{\sigma} y_t \tag{6.44}$$

$$l_{D_t} = \vartheta(w_t - w_{D_t}) + \frac{1}{\sigma} y_t \tag{6.45}$$

where

$$w_t = \frac{1}{2}(w_{C_t} + w_{D_t}) \quad . \tag{6.46}$$

Equation (6.44) matches equation (30) in Erceg (1997).

Substituting (6.44) and (6.45) respectively in (6.16) and (6.17), we get the optimal wage setting rule for households of group C and D, that is

$$w_{C_t} = \left(\frac{1}{1+\vartheta\eta_{ll}}\right)\left[\vartheta\eta_{ll} w_t + \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_t + p_t\right] \tag{6.47}$$

$$w_{D_t} = \left(\frac{1}{1+\vartheta\eta_{ll}}\right)\left[\vartheta\eta_{ll} w_t + \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_t + p_t\right] \tag{6.48}$$

and this is the equation we need to consider for the staggering model.[16] Before doing that it is convenient to substitute out the wage index using (6.46) into (6.47) and (6.48) to get

$$w_{C_t} = \left(\frac{1}{1+\frac{\vartheta\eta_{ll}}{2}}\right)\left[\frac{\vartheta\eta_{ll}}{2} w_{D_t} + \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_t + p_t\right] \tag{6.49}$$

[16] Note that unsurprisingly (6.47) and (6.48) immediately imply that in the static model the two groups of households set the same wage, that is a symmetric equilibrium holds. This obviously is not the case in the wage staggering model, because the two cohorts of households set the wage in different periods.

$$w_{D_t} = \left(\frac{1}{1+\frac{\vartheta\eta_{ll}}{2}}\right)\left[\frac{\vartheta\eta_{ll}}{2}w_{C_t} + \left(\frac{\eta_{ll}}{\sigma}+\eta_{cc}\right)y_t + p_t\right] \quad . \tag{6.50}$$

Now suppose we use these two formulas to derive the staggered wage model, then we can write the supply side as

$$\begin{aligned} x_t &= \left(\frac{1}{1+\frac{\vartheta\eta_{ll}}{2}}\right)\frac{1}{2}\left\{\left[\frac{\vartheta\eta_{ll}}{2}x_{t-1} + \left(\frac{\eta_{ll}}{\sigma}+\eta_{cc}\right)y_t + p_t\right]\right. \\ &\quad \left. + \left[\frac{\vartheta\eta_{ll}}{2}x_{t+1} + \left(\frac{\eta_{ll}}{\sigma}+\eta_{cc}\right)y_{t+1} + p_{t+1}\right]\right\} \end{aligned} \tag{6.51}$$

$$p_t = p_{A_t} = p_{B_t} = w_t + ay_t = \frac{1}{2}(x_t + x_{t-1}) + ay_t \tag{6.52}$$

which, substituting out for x_{t-1} and x_{t+1} in (6.51), delivers the following reduced form of this model

$$x_t = \frac{1}{2}\left(p_t + p_{t+1}\right) + \frac{1}{2}\left[\frac{\frac{\eta_{ll}}{\sigma}+\eta_{cc}-a\vartheta\eta_{ll}}{1+\vartheta\eta_{ll}}\right](y_t + y_{t+1}) \tag{6.53}$$

$$p_t = w_t + ay_t = \frac{1}{2}(x_t + x_{t-1}) + ay_t$$

$$y_t = m_t - p_t$$

where now $\widetilde{\gamma} = \left[\frac{\frac{\eta_{ll}}{\sigma}+\eta_{cc}-a\vartheta\eta_{ll}}{1+\vartheta\eta_{ll}}\right]$. As did the two previous ones, the model again corresponds to the one in 6.2.3 with $g = \widetilde{\gamma}$ and the solution hence is

$$R_5 = 1 + \sigma(\widetilde{\gamma} - 1) = \frac{\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma}{1 + \vartheta\eta_{ll}} \quad . \tag{6.54}$$

First, as Erceg (1997) noted, the group of households adjusting its wage upward following a positive money shock realises that it will experience some reduction in relative demand for its labour skill, according to the elasticity ϑ. This will make it to choose a smaller nominal wage increase. The intuition is

confirmed by the following equation, obtained by substituting out the price level in (6.53)

$$x_t = \frac{1}{2}\left(x_{t-1} + x_{t+1}\right) + \left[\frac{\frac{\eta_{ll}}{\sigma} + \eta_{cc} + a}{1 + \vartheta\eta_{ll}}\right]\left(y_t + y_{t+1}\right) \tag{6.55}$$

which is the equivalent of equation (31) in Erceg (1997). As $\vartheta \to \infty$, then the coefficient on the aggregate demand term, i.e., γ, tends to zero and persistence tends to a unit root.

Second, note that (6.51) actually resembles equation (5.20), in Case A in Chapter 5. The "craft" union model makes a sort of relative wage concern arise endogenously. However, the interpretation is radically different. Here, workers are concerned with losing demand for their labour, while in Chapter 5 workers are concerned with status and sociological considerations.

Finally, if $\vartheta = 0$, we are back to the "expected-market–clearing-case" of section 6.4.3. This explicitly demonstrates why the "expected-market–clearing-case" can not be supported by any sensible microfoundations. In fact, a necessary condition for interior solution of the "craft" union model is $\vartheta > 1$. If $\vartheta = 0$, then the monopoly union would face a demand curve with zero elasticity and in this case it would fix an infinite wage.

If we use the Blanchard and Kiyotaki (1987) labour market framework to derive implications for persistence in the price staggering case, we are obviously back to the case in section 6.4.1. In fact, if all the households reset the wages in each period, then a symmetric equilibrium holds and they will

all choose the same wage. Wages will be equalised across different "crafts". Analytically it is like having just one type of labour and the parameter ϑ becomes irrelevant for the dynamics of the model. The optimal wage rule in the perfectly flexible wage/price model will simply be $w_{C_t} = w_{D_t} = w_t = p_t + \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_t$ as in (6.21) and we get the same reduced form model as in section 6.4.1.

6.4.6 Liquidity Constraints

So far we have assumed the existence of complete markets which implies households to consume always half of aggregate real output. In this section we make instead the other extreme assumption: workers are completely liquidity constrained. This assumption is actually going to make a difference only in the "industrial" union - no labour mobility cases. Then, the liquidity constraint hypothesis would imply that households consume the output of the sector to which they supply labour (i.e., $c_{Ct} = y_{At}$, $c_{Dt} = y_{Bt}$).[17]

In the yeoman farmer model then farmers will consume what they produce. Basically, what changes is that the marginal effort cost in production is now equal to

$$w - p_t = \left(\frac{\eta_{ll}}{\sigma} + \eta_{cc}\right) y_{At} = \varphi \; y_{At} \tag{6.56}$$

[17] A further assumption is actually needed: households only receive profits from firms in their own sector.

	Price Staggering	Wage Staggering
Perfect labour mobility	$R_1 = \frac{\eta_{ll}+\sigma\eta_{cc}+1-\sigma}{\sigma+\theta\ (1-\sigma)}$	$R_3 = \eta_{ll} + \sigma\eta_{cc} + 1 - \sigma$
No mobility of labour ("Industrial" unions)	$R_2 = \frac{\eta_{ll}+\sigma\eta_{cc}+1-\sigma}{\sigma+\theta\ (1-\sigma)+\theta\eta_{ll}}$	$R_4 = \frac{[\sigma+\theta(1-\sigma)][\eta_{ll}+\sigma\eta_{cc}+1-\sigma]}{\sigma+\theta\ (1-\sigma)+\theta\eta_{ll}}$
Liquidity constraint	$R_6 = \frac{\eta_{ll}+\sigma\eta_{cc}+1-\sigma}{\sigma+\theta\ (1-\sigma)+\theta\eta_{ll}+\theta\sigma\eta_{cc}}$	$R_7 = \frac{[\sigma+\theta(1-\sigma)][\eta_{ll}+\sigma\eta_{cc}+1-\sigma]}{\sigma+\theta\ (1-\sigma)+\theta\eta_{ll}+\theta\sigma\eta_{cc}}$
"Craft" unions	$R_1 = \frac{\eta_{ll}+\sigma\eta_{cc}+1-\sigma}{\sigma+\theta\ (1-\sigma)}$	$R_5 = \frac{\eta_{ll}+\sigma\eta_{cc}+1-\sigma}{1+\vartheta\eta_{ll}}$

Table 6.1: Persistence Properties of the Different Models

while the rest of the model stays the same. Thus, repeating steps one to three, it yields

$$R_6 = g = \left[\frac{a+\varphi}{1+(a+\varphi)\theta}\right] = \frac{\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma}{\sigma + \theta\ (1-\sigma) + \theta\eta_{ll} + \theta\sigma\eta_{cc}} \tag{6.57}$$

Similarly, in the wage staggering case instead, we have

$$R_7 = \frac{[\sigma + \theta(1-\sigma)]\,[\eta_{ll} + \sigma\eta_{cc} + 1 - \sigma]}{\sigma + \theta\ (1-\sigma) + \theta\eta_{ll} + \theta\sigma\eta_{cc}} \tag{6.58}$$

6.5 Wage/Price Staggering and Persistence

6.5.1 Analytical Results

Table 6.1 reports the different solutions delivered by the models according to $\lambda = \frac{1-\sqrt{R_i}}{1+\sqrt{R_i}}$.

Proposition 6.1. *Look at Table 6.1 by rows. In all the cases but the last one, wage staggering models always deliver lower (or at most equal) output*

persistence than price staggering models. With regard to the "craft" union case the condition for $R_1 < R_5$ *is:* $\vartheta < \frac{(\theta-1)(1-\sigma)}{\eta_{ll}}$.

Specifically, apart from the "craft" union case, given a particular model of price staggering $R_{ws} = [\sigma + \theta(1-\sigma)]R_{ps}$, that is, the corresponding model of wage staggering exhibits a value of R which is $[\sigma + \theta(1-\sigma)]$ times bigger than the corresponding price staggering model. Given that $0 < \sigma \leq 1$ and $\theta > 1$, then $[\sigma + \theta(1-\sigma)] \geq 1$ and this implies $R_{ws} \geq R_{ps}$. Note that the wage staggering - "craft" union case contains the parameter ϑ which is missing from all the other cases. Its analytical comparison with the other models therefore is obviously going to result in ambiguous statements.

Corollary 6.1.1. *Under constant returns to scale to labour, i.e.* $\sigma = 1$, $[\sigma + \theta(1-\sigma)] = 1$ *and hence* $R_{ws} = R_{ps}$. *That is:* $R_1 = R_3$, $R_2 = R_4$ *and* $R_6 = R_7$. *Hence, apart from the "craft" union case, if* $\sigma = 1$, *wage staggering and price staggering generate the same degree of output persistence.*

Proposition 6.2. *Look at Table 6.1 by columns.*

(i) Whichever the staggered nominal variable, prices or wages, persistence is the lowest if there is perfect labour mobility. That is, constraints on labour mobility (both "industrial" and "craft" union cases) tend to increase persistence.

(ii) Liquidity constraints tend to increase persistence. Indeed, models with liquidity constraints exhibit a lower value of R *with respect to the same*

models without such constraints.

Moreover, models with free mobility of labour, price or wage staggering, are likely to fail the necessary condition $R < 1$ for monotonic convergence - i.e., they are likely to deliver a negative root.

Additional results

(1) Effect of η_{ll}.

$$\begin{aligned}
\frac{\partial R_i}{\partial \eta_{ll}} &> 0 && for\ i = 1,3,6,7\ ; \\
\frac{\partial R_i}{\partial \eta_{ll}} &\gtrless 0 \iff \frac{1}{\theta} \gtrless \eta_{cc}\ , && for\ i = 2,4; \\
\frac{\partial R_5}{\partial \eta_{ll}} &\gtrless 0 \iff \frac{1}{\vartheta} \gtrless 1 + \sigma(\eta_{cc} - 1)\ . &&
\end{aligned}$$

The effect of η_{ll} is an interesting and delicate issue, as we know from the previous chapters. Simple intuition suggests that $\uparrow \eta_{ll} \Longrightarrow \uparrow \gamma \Longrightarrow \uparrow R \Longrightarrow \downarrow$ *persistence* as found by Blanchard and Fischer (1989) and Chari et al. (1996). Thus they conclude that a low value of η_{ll} (i.e., a high intertemporal elasticity of substitution in labour supply) is necessary to generate persistence. Here we show that depending on the particular set up of the model the intuition might or might not hold. In particular, (i) it holds for models with perfect labour mobility; (ii) it is likely not to hold for standard calibration values in the no-labour mobility cases (both in "industrial" and in "craft" union models); (iii) it holds again when liquidity constraints are added to these latter models. We want to stress that again it is the underlying economic structure chosen and *not* the difference between price and wage

staggering that matters.[18]

(2) Effect of η_{cc}.

$$\frac{\partial R}{\partial \eta_{cc}} > 0 \qquad \textit{for all the models.}$$

This suggests that the specification of preferences with high intertemporal elasticity of substitution in consumption (low income effect on labour supply) are a promising route to generate output persistence, as already suggested by Chari et al. (1996) or in Chapter 4. However, the likely magnitude of these derivatives changes from model to model being particularly low for liquidity constraint models.

(3) Effect of σ.[19]

$$\begin{aligned}
\frac{\partial R_1}{\partial \sigma} &\gtrless 0 \iff \eta_{cc} \gtrless \frac{1}{\theta} - \left[\frac{\theta - 1}{\theta}\right] \eta_{ll}; \\
\frac{\partial R_2}{\partial \sigma} &\gtrless 0 \iff \eta_{cc} \gtrless \frac{1}{\theta}\ ; \\
\frac{\partial R_i}{\partial \sigma} &\gtrless 0 \iff \eta_{cc} \gtrless 1 \quad for\ i = 3, 5 \\
\frac{\partial R_4}{\partial \sigma} &< 0 \impliedby \eta_{cc} \leq 1\ ;
\end{aligned}$$

Simple intuition would suggest that $\sigma = 1$ is the maximum degree of

[18]Note that Blanchard and Fischer (1989) in a yeoman farmer model arrive with respect to the effect of η_{ll} at the same conclusion of Chari et al. (1996) in their price staggering model and free mobility of labour. This is because Blanchard and Fischer (1989) use a particular utility function with zero income effects on labour supply (i.e., $1/\theta > \eta_{cc} \quad 0$.

[19]The cases for R_6 and R_7 are not presented because the conditions are very complicated and meaningless expressions.

nominal rigidity, hence this case would deliver the maximum degree of persistence. On the contrary, in staggered price models persistence is decreasing in σ for realistic parameter values. It is more difficult to reach any definite conclusion for staggered wage models. However, $\eta_{cc} \leqslant 1$ is a sufficient, but not necessary condition for persistence to be increasing in σ for case 4 and the overall condition for $\partial R_4/\partial\sigma$ to be negative is very likely to be satisfied.[20]

(4) Effect of θ.

$$\begin{aligned} \frac{\partial R_i}{\partial \theta} &< 0 \qquad for\ i = 1,2,4,6,7 \\ \frac{\partial R_i}{\partial \theta} &= 0 \qquad for\ i = 3,5 \end{aligned}$$

An increase in θ therefore tends to increase persistence. The effect is the same one described in Chapter 4. Following a positive money shock, the new price (money wage) will be set higher than the one already fixed in the previous period by the other sector. However, the bigger θ, the bigger the loss in demand for goods (for labour) that firms (unions) in the sector will face, fixing the new level of price (money wage) bigger than the one of the other sector. Therefore, firms (unions) will tend to fix the new price (wage) close to the existing one inducing more price (wage) level inertia.[21]

[20] The condition is:

$$\frac{\partial R_4}{\partial \sigma} < 0 \Leftrightarrow \eta_{cc} < 1 + \frac{\theta\eta_{ll}(\theta-1)(\eta_{ll}+\sigma\eta_{cc}+1-\sigma)}{[\sigma+\theta(1-\sigma)][\sigma+\theta(1-\sigma)+\theta\eta_{ll}]}$$

which substituting standard calibration values (see below) gives: $1 < 11.4$.

[21] Note that, apart the "craft" union case, the only case in which this effect is absent is the

The same effect acts in the wage staggering - "craft" union model, but now the relevant parameter is ϑ, the elasticity of substitution among different types of skills. Hence: $(\partial R_5/\partial\vartheta) < 0$. In the price staggering - "craft" union model, instead, these effect is absent since all the unions renegotiate the wage in each period, setting the same wage.

6.5.2 Quantitative Results

In this section we want to address the following question: *is any of these models likely to deliver high persistence?*

Firstly note that persistence is rapidly decreasing for low values of R: only values of R very close to zero could deliver some notable persistence. For example, if we quite arbitrarily define a significant degree of persistence to be a value of at least 0.5, then R should not be higher than 0.11.

Given the calibration literature, as indicative benchmark values, we take: $\sigma = 0.67$, $\theta = 6$, $\eta_{cc} = 1$, $\eta_{ll} = 5$ and $\vartheta = 10$.[22] For our benchmark case, the values of R in the different models presented here and the implied values

one which corresponds to the *ad hoc* models, i.e., wage staggering and competitive labour market. In this case, in fact, θ does not play any role since both groups of households are equally employed by all the firms.

[22]This latter value is the one used by Erceg (1997). Moreover, we already know that η_{ll} is actually quite difficult to tie down and that, given Pencavel's (1986) results, it could range from 1 to infinity.

for persistence are the following:

$$R_1 = 2.264 \qquad \lambda_1 = -0.24$$

$$R_2 = 0.184 \qquad \lambda_2 = 0.4$$

$$R_3 = 6 \qquad \lambda_3 = -0.42$$

$$R_4 = 0.487 \qquad \lambda_4 = 0.18$$

$$R_5 = 0.118 \qquad \lambda_5 = 0.49$$

$$R_6 = 0.164 \qquad \lambda_6 = 0.42$$

$$R_7 = 0.43 \qquad \lambda_7 = 0.21$$

As underlined in the previous section, while there are some slight differences between price staggering models and wage staggering ones, the critical difference arises from the assumption on labour mobility. In fact, models with free mobility of labour are likely to deliver a *negative* root for output persistence. In "industrial" and "craft" union cases, there seems to be however a quantitative difference between price and wage staggering models. In "industrial" union models, price staggering delivers more persistence, while in "craft" union models wage staggering delivers more persistence. As a conclusion *only two class of models can deliver a substantial degree of persistence: the yeoman farmer model (i.e., price staggering and no labour mobility across sectors ("industrial" unions)) and the model with wage staggering and no labour mobility across skills ("craft" unions).* Moreover, even if liquid-

ity constraints do increase persistence, their quantitative importance seems negligible.

6.5.3 Discussion

Given our unifying framework, we now review the models in the recent literature. With respect to Chari et al. (1996) model of staggered prices and perfect labour mobility, our results can explain why they conclude that by no means a staggered price model could deliver any notable persistence. In their Section 4, Chari et al. (1996) shows that in their staggered price model, putting $\sigma = 1$, the sensitivity of the real wage to output is: $R_1 = \overline{\gamma} = \widetilde{\gamma} = \eta_{ll} + \eta_{cc}$. Hence, they conclude that, since η_{ll} should be at least 1, then $\overline{\gamma} \geqslant 1$. That is, even assuming zero income effects, $\overline{\gamma}$ is too high to generate any persistence at all. It is worth noting that, since $\partial R_1/\partial\sigma$ is likely to be positive, Chari et al. (1996) in their section on 'intuition', assuming $\sigma = 1$, actually presented a case biased against persistence (obviously, only with regard to σ). In other words, their $\overline{\gamma}$ is biased upwards. Nevertheless, their main argument goes through since their model exhibits staggered prices and perfect labour mobility. Central to the argument is the fact that persistence ($\overline{\gamma}$) is increasing (decreasing) in the intertemporal elasticity of labour supply, which is agreed to be very low.

However, as stated by *(1)*, the argument is likely to be reversed for models

with no mobility of labour. Indeed, the no-labour-mobility models reach a minimum for R when η_{ll} tends to infinity, which goes exactly against the critics of nominal rigidity propagation mechanism models. Nevertheless, in contrast with what has been suggested so far by the literature, in the no-labour-mobility models, *the degree of persistence seems extremely insensitive to the value of the intertemporal elasticity of substitution of labour supply.* Consider just plausible values for η_{ll}: $\eta_{ll} \in [1, \infty)$. Then, *ceteris paribus*, in the yeoman farmer model R_2 varies from 0.23 to 0.17, that is, $\lambda_2 \in [0.35, 0.42]$, in the staggered wage - "industrial" union model R_4 varies from 0.61 to 0.44, that is, $\lambda_4 \in [0.12, 0.2]$ and in the staggered wage - "craft" union model R_5 varies from 0.18 to 0.1, that is, $\lambda_4 \in [0.4, 0.52]$. In other words, *ceteris paribus* (i.e., for realistic values of the other parameters of the model), the intertemporal elasticity of labour supply is NOT a key parameter of the model with respect to its ability of generating persistence; η_{ll} alone can not change substantially the persistence properties of these models.

An extreme example of this implication is the calibration of Rotemberg and Woodford (1997) . The benchmark calibration of their yeoman farmer model is the following: $\eta_{cc} = 0.16$, $\eta_{yy} = 0.47$, $\sigma = 0.75$ and $\theta = 7.88$, which delivers a low value of $\overline{\gamma} = 0.134$ in (6.32).[23] However, they stress that their

[23]They acknowledge the fact that such a low value of η_{yy} is difficult to believe, since they suggest it implies an intertemporal substitution in labour supply of 9.5 (i.e., $\eta_{ll} = 0.105$). However, they calculate $\eta_{ll} = \sigma(\widetilde{\gamma} - \eta_{cc})$, which they calibrate $\eta_{ll} = 0.75(0.3 - 0.16) =$

results do not rely on high labour supply elasticity (i.e., low value of η_{yy}). Indeed, look at the formula for $\overline{\gamma}$, i.e., (6.32). With such values of η_{cc} and θ, if $\eta_{yy} \in [0, \infty)$ then $\overline{\gamma} \in [\eta_{cc} = 0.16, 1/\theta = 0.13]$. That is, the value of η_{yy} (and hence of η_{ll}) is basically unimportant, as far as persistence is concerned, since $\overline{\gamma}$ does not change very much (alternatively, very marginal changes in the value of θ can keep $\overline{\gamma} = 0.134$).[24]

This example shows not only that the value of η_{ll} can be unimportant, but also that instead the interrelation between η_{ll} and η_{cc} is important. For example, in the constant returns to scale case, as shown in Chari et al. (1996) and in Chapter 4, particular (and peculiar) assumptions on the form of the utility function could make all the models deliver very high persistence. Specifically, a high intertemporal elasticity of labour supply (i.e., $\eta_{ll} \to 0$) and of consumption (i.e., $\eta_{cc} \to 0$) make $R_i \to 0$ and $\lambda_i \to 1$ for all the models.

From the above we can draw the following conclusion. Reading the literature, one gets the feeling that everything rests on the value of the in-

0.105. The formula for η_{ll}, however, is consistent with a competitive labour market as (6.21) shows. Simple algebra, instead, shows that in a yeoman farmer model the consistent formula for η_{ll} as a function of $\widetilde{\gamma}$ is: $\eta_{ll} = \sigma(1 + \theta a)(\eta_{cc} - \widetilde{\gamma})/(\theta\widetilde{\gamma} - 1)$ which given their parameter values yields a negative (?) η_{ll}.

[24]Figure 4.1 actually visualises this effect: the vertical distance between η_{cc} and $1/\theta$ is squeezed to 0.03 and hence the curve in Figure 4.1 is basically flat.

tertemporal substitution in labour supply. The above results show that to focus only this parameter can be misleading. In models with staggering and labour mobility, persistence is decreasing with η_{ll}, while the contrary is true for models with staggering and no-labour-mobility. However, this is not the reason why the first class of models can not deliver any persistence, while the second one can. Changes in η_{ll} alone do not substantially change the capability of the models of generating persistence.[25] In other words, if one thought that the no-labour-mobility models were able to generate some persistence simply because persistence was increasing in η_{ll} which is very high, this would be wrong.

Moreover, the difference in the capability of the models in generating persistence does not only depend upon one parameter (i.e., η_{ll}), but it is deeper and given by the structure of the model. In particular, the elasticities of substitution between goods or skills are the key parameters in all the staggering models. The intuition is straightforward. Endogenous nominal stickiness arises if price (wage)-setting agents choose not to change their prices (wages) by a large amount when they reset them, following a money shock. In the above models, they would be willing to do so for only one reason: to preserve demand. They recognise that when, following a positive

[25] Note that, in the Chari et al. (1996) case, we could have also written: since $R_1 = g = \widetilde{\gamma} = \eta_{ll} + \eta_{cc}$ and η_{cc} is around one 1, then $g \gtrapprox 1$; that is, even assuming infinite elasticity of labour supply, g is too high to generate persistence.

money shock, they reset a higher price (wage) they will lose demand for goods (labour) with respect to the other firms (workers) locked into the contract already signed one period before. Then, depending on the structure of the model, the goods market (and hence the elasticity of substitution between goods, i.e., θ) or the labour market (and hence the elasticity of substitution between skills, i.e., ϑ) plays the pivotal role. In the first case, price staggering would naturally deliver more persistence than wage staggering and vice versa in the latter case.

Consider the first three model structures in Table 6.1. Looking at them by rows, we saw that, given a particular model of price staggering $R_{ws} = [\sigma + \theta(1-\sigma)]R_{ps}$, and this implies $R_{ws} \geq R_{ps}$. This just derives from the different relevant elasticity in the different cases. The focus is here on the good markets, since there are no substitution between different labour skills. In the price staggering models, firms will face a demand for goods whose elasticity is θ, while the relevant elasticity for wage setters is the elasticity of the demand for labour with respect to the money wage, that is $\varepsilon = \frac{\theta}{\sigma+\theta(1-\sigma)}$.[26] In words, the elasticity of demand faced by price setters is $[\sigma + \theta(1-\sigma)]$ bigger than the elasticity of demand faced by wage setters, and, as a result, R of the price staggering models is $[\sigma + \theta(1-\sigma)]$ lower than the one of the

[26] Again the exception is the wage staggering and competitive labour market case, where θ does not play any role since both groups of households are equally employed by all the firms.

corresponding wage staggering model.

Now look at the first three model structures in Table 6.1 by columns and particularly at the important difference between the perfect labour mobility case and the no labour mobility one. In the no labour mobility models, the workers internalise the fact that demanding higher wages affects the price of the good produced by the industry. This in turn affects the demand (through θ) for the good and subsequently the demand for labour. Algebraically, this difference is highlighted by the term $\theta\eta_{ll}$ in the denominator of R_2 and R_4. The fact that in setting the wage the workers internalise this effect is what distinguishes the two classes of models, making marginal cost rises much slower in the no labour mobility case. Indeed, given the likely magnitudes of θ and η_{ll}, this internalisation quantitatively makes a big difference, as shown above. Again, however, since the focus is on the goods market, price staggering delivers more persistence than wage staggering models, as explained in the previous paragraph.

The story in the "craft" unions model is instead radically different. Here substitutability between labour types is added to the substitutability between goods. However, this is going to make a difference only if wage decision are not synchronised, hence only in the wage staggering case. As explained in the previous two paragraphs, as far as the goods market is concerned, R_1 is $[\sigma + \theta(1 - \sigma)]$ bigger than R_5, but in the wage staggering case there is

an additional effect which is absent in the price staggering case. Workers realise that they are facing a labour demand curve where elasticity has now an extra term, ϑ. Then, as for the no labour mobility case, $\vartheta\eta_{ll}$ appears in the denominator of R_5.

The quantitative difference between the two models which are able to deliver significant persistence (namely, the yeoman farmer model and the wage staggering model with skills substitutability) is thus going to rest on the quantitative difference between the relevant elasticities, i.e., θ and ϑ. If we calibrate $\theta = 10$,[27] then in the yeoman farmer case, we get $R_4 = 0.11$ and $\lambda_4 = 0.5$, basically equivalent the benchmark case for the R_5 and λ_5.[28]

A final remark is needed. The perfectly flexible wage/price model presented here is quite stylised, but also quite general, as we tried to show above. Indeed, since it encompasses most of the microfounded models with staggering in the literature, it can be thought as derived from the log-linearisation of a more general microfounded model. However, implicitly

[27] Often in the literature (e.g., Chari et al. (1996)) the CES function is used to describe the technology for producing final goods from intermediate goods. It follows that, even if the elasticity of demand for intermediate goods is given by θ, the latter is basically a technology parameter which gives the elasticity of substitution in inputs. Chari et al. (1996) calibrate it equal to 10.

[28] Moreover, this would suggest that combining price *and* wage staggering in a "craft" union model could deliver substantial persistence. This intuition is developed by Erceg (1997)

the log-linearised model presumes another not innocuous assumption. The model is log-linearised around a particular steady-state with constant money supply (i.e., zero inflation steady state). In fact, the policy parameters (or the inflation trend) do not appear in the model. However, in Chapter 4 we showed the degree of persistence to be considerably decreasing in the steady state inflation trend. This point has to be take into account and to be combined with the results above.

6.6 Conclusions

We have derived a stylised log-linear model which encompasses most of the microfounded models of price/wage staggering. We have shown the importance of the underlying economic structure for the ability of staggered price/wage models to explain the persistence of the real effects of money shocks. The main conclusions are:

(i) Qualitative implications

- *The difference between the persistence properties of price and wage staggering models derives from the underlying economic structure.* In particular, the substitutability between goods and/or labour types plays the major role in generating persistence. In models with only substitutability between goods, then price staggering naturally delivers higher persistence than wage staggering, while in models with substitutability between labour types, the

opposite is true. It follows than only in "craft" union models *à la* Blanchard and Kiyotaki (1987), wage staggering generates higher persistence than price staggering.

- *The distinction between free mobility and no mobility of labour is fundamental.* No-mobility-of labour models (both "industrial" and "craft" union models) bring in new mechanisms that increase persistence. In "industrial" union models the mechanism rests on the fact that workers internalise the effect of higher wage claims on the industry good price and hence on demand. In "craft" union models, it is the substitutability between labour skills that generates wage adjustment inertia.

- *Liquidity constraints tend to (marginally) increase persistence.*

(ii) Quantitative implications

- *While in these models, a substantial (in the sense of near random walk behaviour) degree of persistence is an unlikely outcome, two models can deliver significant persistence: the yeoman farmer model and the "craft" union models with wage staggering.* It is not by chance then that the two models in the literature which claim to be able to generate a contract multiplier are Rotemberg and Woodford (1997) and Erceg (1997). The first one is a yeoman-farmer model, the latter one is a "craft" union model.[29]

[29] Some other works actually share the same claim, but in a somewhat non-standard framework and hence our model cannot encompass their ones. For example, Andersen (1998b) considers an *ad hoc* utility function of a monopoly union, Chapter 5 of this thesis

- *Ceteris paribus (i.e., for realistic values of all the other parameters), these conclusions do not depend on the particular value assigned to the intertemporal elasticity of labour supply,* which has been so far the focus of this literature. This suggests that the importance of the intertemporal elasticity of labour supply in generating persistence in staggered wage/price models may have been somewhat overstated.

- All the quantitative results are subject to the *caveat* that the framework we used can be obtained by a *log-linearisation around a zero inflation steady state.* It is very likely, given our results in Chapter 4, that the quantitative results change once a log-linearisation around a positive inflation steady state is considered.

---

consider the existence of relative wage concern, Bergin and Feenstra (1998) consider a translog form for preferences.

Conclusions

In this thesis we have analysed the effects of monetary policy in staggered wage models *à la* Taylor (1979, 1980a). In the first four chapters of the thesis we have exploited the dynamic general equilibrium model with staggered wage setting *à la* Taylor, built in Chapter 1, to address different issues concerning monetary policy changes: superneutrality, dynamic effects of a disinflation and dynamic effects of money shocks. The latter has given rise to the persistence puzzle, since it has demonstrated that staggered wages are not able to explain a high degree of persistence of money shocks once they are embedded in a dynamic general equilibrium framework. We share this main result of Chapter 4 with a contemporaneous, and already very influential, paper by Chari et al. (1996). Despite the fact that this latter paper has not been published yet, it has already generated a very lively debate in the literature about the persistence puzzle. We have participated in this debate with the last two chapters of the thesis. While Chapter 5 has proposed a solution to the puzzle by taking into explicit consideration relative wage concern on

the part of the workers, Chapter 6 has presented a unifying framework which encompasses most of the microfounded model of staggering in the literature and it thus may be very useful to interpret and understand the current debate on the subject. The debate, still under way, is at the heart of the attempt to build a quantitative macroeconomic model describing the monetary policy transmission mechanism in order to analyse the effects of monetary policy changes and to design optimal monetary policy rules (e.g., Rotemberg and Woodford (1997)). Thanks to the numerous contributors some steps forward have been made, but there is still a great deal of exciting work to be done. We immodestly hope that this thesis may be seen as a little stick to help the steps.

The remainder of the conclusions summarises the main finding of each chapter.

The first chapter has introduced the tools needed to tackle the main issues that have been treated in the following chapters. In particular, we have described two models that have continuously come up in the following analysis. First, we have reviewed the famous Taylor (1979, 1980) model of staggered wage setting. Second, we have presented original work in describing the structure of a dynamic general equilibrium model with staggered wage setting *à la* Taylor. This model is central to the thesis since the results presented in chapters 2, 3 and 4 are based on it. Moreover, also the models

in chapters 5 and 6, while somewhat different, have originated from it.

Chapter 2 has addressed the issue of superneutrality of money using the model presented in the Chapter 1. It has demonstrated that, once staggered wages are introduced in an optimising framework, a mild permanent change in the rate of growth of money could have substantial effects on the steady state aggregate level of output and welfare. The numerical results have thus suggested that, in staggered adjustment models, superneutrality is far from being the minor issue that has been thought so far. Previous models with staggered wage/price behaviour have failed to acknowledge this fact. In the steady state, Calvo's (1983a) model behaves as a flexible price one, due to the peculiar hypothesis of a price-regulation mechanism. Ireland's (1995) model is too simple in its structure to detect strong effects of the rate of growth of money on the steady state output and welfare. In particular, because of the linearity in the production and utility functions in labour and the elasticity of substitution among goods equal to one, Ireland's (1995) model has not been able to capture the effects due to the usual non-linearities in technology and preferences: decreasing return to scale to labour, increasing marginal disutility of labour and elasticity of substitution among goods bigger than one. Once these effects have been taken into account, it has turned out that staggered wage setting behaviour induces strong non-superneutrality properties and high costs of inflation. Hence, given that staggered wages are

observed in western countries, we could easily explain high costs of inflation and provide a rationale for the pursuit of price stability in western countries.

In Chapter 3 we have used the dynamic general equilibrium model with staggered wages presented in the first two chapters to study analytically the effects of a reduction in the rate of monetary growth (a *disinflation*). We have found that the result of disinflation is a recession in the short and medium run, and that output is slightly lower in the long run, too. This is true both when the disinflation is unanticipated and when it is announced in advance. Our particular motivation was the puzzling finding of Ball (1994), in a directly postulated model, that disinflations cause booms. We have first noted that this finding is associated with the element of preannouncement in the policy assumed by Ball. More microfoundations have told us that Ball's paradox was mainly due to simplifying assumptions regarding the time preference rate and the formulation of the aggregate demand equation. These simplifications were inconsistent with microfoundations - at least, with the particular rather standard set of microfoundations has been introduced here. The microfounded model produces a reaction to a disinflation (a reduction in monetary growth) which is not, after all, sharply different from the standard reaction to a deflation (a reduction in the level of the money supply) found in Taylor's (1979) model. Hence, in contrast to what several authors have recently concluded, it does not appear necessary to appeal to lack of policy

credibility in order to explain why disinflations cause slumps.

Chapter 4 has looked at the issue of the persistence of the real effects of money shocks. If, as were West (1988) and Phaneuf (1990), we had been looking for results to corroborate the view that staggered wage models could induce a high degree of persistence of money shocks, the microfounded model does not seem to have provided them. On the contrary, it has confuted that view. The model has demonstrated that for a large range of reasonable parameter values a notable degree of persistence is an unlikely outcome. Moreover, even for parameter values such that the model generated persistence, a moderate rate of underlying inflation cut down persistence sharply. In conclusion, sensible values of the microeconomic parameters and/or a moderate rate of underlying inflation such as we observe in western economies cut down persistence not only far below near random-walk behaviour, but also below any level notably different from zero. Moreover, investigating the microeconomic fundamentals of the *ad hoc* Taylor wage rule, the model has emphasised the role of non-linearity and of the Lucas critique. In brief, the model has shown that staggered wages alone are not able to explain a notable degree of persistence of the real effects of money shocks.

Chapter 5 has proposed a solution to the persistence puzzle which had arisen from the analysis of the previous chapter, which had questioned the existence of a contract multiplier. We have added explicit relative wage concern

on the part of the workers to the model analysed in the previous chapter. This has provided a combination of nominal and real rigidities capable of generating a substantial amount of endogenous stickiness, even with a very inelastic intertemporal elasticity of labour supply. As a result, output and inflation persistence are a likely outcome in our framework. The relative wage concern on the part of the workers is the key feature of the model. The notion of relative wage concern is not new for economists and goes back a long way, at least to J.M. Keynes. Moreover, a great deal of applied studies have provided overwhelming evidence for a relative wage concern on the part of the workers. Furthermore, an increasing amount of work in many fields of economics has started considering status and sociological considerations in order to be able to explain various puzzles that the standard economic framework could not explain. Introducing a relative wage concern in the analysis, by adding a term to the utility function, has placed our work within this growing economic literature. Our results have shown that failing to account for this specific source of real rigidity might be an important weakness of previous staggered wage models, responsible for their negative results concerning output and inflation persistence. Our model delivers a substantial amount of persistence both in output and inflation. Given the substantial amount of empirical evidence supporting a relative wage concern on the part of workers, our analysis has led us to conclude that this may be the missing

piece in the money shocks persistence puzzle.

Chapter 6 has presented a stylised log-linear model which encompasses most of the microfounded models of price/wage staggering and has focused on the persistence of the real effects of money shocks generated by the different versions of the basic model. We have shown the importance of the underlying economic structure for the capability of staggered price/wage models to generate persistence. In particular, the no mobility of labour assumption seems crucial for these kinds of models to be able to generate a notable degree of persistence.

As said above, future research has still a great deal of work to do in this relatively new field of the literature. With respect to monetary policy issues, the next step would probably be to try to design the optimal policy rule in this general equilibrium framework (a first attempt in this direction is Rotemberg and Woodford (1997)). A step further would consist in trying to endogenise the policy bringing in time consistency considerations. However, apart from monetary policy, there are other features that need to be considered and introduced into the model. Fiscal policy, for example, is still too stylised in this models. Moreover, since in most of this models Ricardian equivalence holds, most of the interesting effects of fiscal policy are simply washed away. Another important extension is considering open economies (Obstfeld and Rogoff (1995) is one the first dynamic general equilibrium model of open

economies with nominal rigidities, but their model do not have staggering). The road to a quantitative macroeconomic model with rigorous theoretical microfoundations, which can be used for policy analysis is still long, but the literature seems to know the direction for this long journey.

Bibliography

Abel, A. (1990). Asset prices under habit formation and catching up with the Joneses. *American Economic Review 40*, 38–42.

Agell, J. and P. Lundborg (1995). Theories of pay and unemployment: Survey evidence from swedish manufacturing firms. *Scandinavian Journal of Economics 97*, 295–307.

Akerlof, G. A. (1997). Social distance and social decision. *Econometrica 65*, 1005 1027.

Akerlof, G. A. and J. L. Yellen (1985a). Can small deviations from rationality make significant differences to economic equilibria? *American Economic Review 75*, 708–721.

Akerlof, G. A. and J. L. Yellen (1985b). A near-rational model of the business cycle, with wage and price inertia. *Quarterly Journal of Economics 100*, 823–838.

Akerlof, G. A. and J. L. Yellen (1990). The fair-wage effort hypothesis and unemployment. *Quarterly Journal of Economics 105*, 255–284.

Ambler, S. and L. Phaneuf (1989). Accomodation, price flexibility and fluctuations. *Economics Letters 30*, 185–190.

Ambler, S. and L. Phaneuf (1992). Wage contracts and business cycle models. *European Economic Review 36*, 783–800.

Andersen, T. M. (1994). *Price Rigidity. Causes and Macroeconomic Implications.* Oxford, Clarendon Press.

Andersen, T. M. (1998a). Persistency in sticky price models. *European Economic Review, Papers and Proceedings 42*, 593–603.

Andersen, T. M. (1998b). Staggered wage setting and output persistence. Mimeo, University of AArhus.

Ball, L. (1994). Credible disinflation with staggered price-setting. *American Economic Review 84*, 282–289.

Ball, L. and S. Cecchetti (1988). Imperfect information and staggered pricing. *American Economic Review 78*, 999–1018.

Ball, L., G. N. Mankiw, and D. Romer (1988). The new keynesian economics and the output-inflation trade-off. *Brookings Papers on Economic Activity 1*, 1–65.

Ball, L. and D. Romer (1989). The equilibrium and optimal tiiming of price changes. *Review of Economic Studies 56*, 179 198.

Ball, L. and D. Romer (1990). Real rigidities and the non-neutrality of

money. *Review of Economic Studies 57*, 183 203.

Baxter, J. L. (1988). *Social and Psychological Foundations of Economic Analysis*. New York, Harvester Wheatsheaf.

Benabou, R. (1988). Search, price setting and inflation. *Review of Economic Studies 55*, 353–376.

Benabou, R. (1989). Optimal price dynamics and speculation with a storable good. *Econometrica 57*, 41 80.

Bénassy, J. P. (1995). Money and wage contracts in an optimizing model of the business cycle. *Journal of Monetary Economics 35*, 303 315.

Bergin, P. R. and R. C. Feenstra (1998). Staggered price setting and endogenous persistence. NBER Working Paper no. 6492.

Bhaskar, V. (1998). On endogenously staggered prices. University of St. Andrews Discussion Paper Series No. 9806.

Blanchard, O. J. (1983). Price asynchronization and price level inertia. In R. Dornbush and M. H. Simonsen (Eds.), *Inflation, Debt and Indexation*, pp. 3–24. Cambridge, MA, The MIT Press.

Blanchard, O. J. (1986). The wage price spiral. *Quarterly Journal of Economics 101*, 543–565.

Blanchard, O. J. (1990). Why does money affect output? A survey. In B. M. Friedman and F. H. Hahn (Eds.), *Handbook of Monetary Eco-*

nomics, pp. 779–835. Amsterdam, North-Holland.

Blanchard, O. J. and S. Fischer (1989). *Lectures on Macroeconomics.* Cambridge, MA, The MIT Press.

Blanchard, O. J. and C. M. Kahn (1980). The solution of linear difference models under rational expectations. *Econometrica 48*, 1305–1311.

Blanchard, O. J. and N. Kiyotaki (1987). Monopolistic competition and the effects of aggregate demand. *American Economic Review 77*, 647–666.

Blanchard, O. J. and D. Quah (1989). The dynamic effects of aggregate demand and supply disturbances. *American Economic Review 79*, 655 673.

Blanchard, O. J. and L. H. Summers (1988). Beyond the natural rate hypothesis. *American Economic Review, Papers and Proceedings 78*, 182–187.

Blanchflower, D. G. and A. J. Oswald (1994). *The Wage Curve.* Cambridge MA, The MIT Press.

Boucekkine, R. (1995). An alternative methodology for solving nonlinear forward-looking models. *Journal of Economic Dynamics and Control 19*, 711–734.

Buiter, W. and I. Jewitt (1981). Staggered wage setting with real wage rel-

ativities: Variations on a theme by Taylor. *The Manchester School 49*, 211–228.

Buiter, W. and M. H. Miller (1985). Costs and benefits of an anti-inflationary policy: Questions and issues. In V. Argy and J. Neville (Eds.), *Inflation and Unemployment: Theory, Experience and Policy Making.* London, Allen and Unwin.

Caballero, R. J. and E. M. R. A. Engel (1991). Dynamic (S,s) economics. *Econometrica 59*, 1659–1686.

Caballero, R. J. and E. M. R. A. Engel (1993a). Heterogeneity and output fluctuations in a dynamic menu-cost economy. *Review of Economic Studies 60*, 95 119.

Caballero, R. J. and E. M. R. A. Engel (1993b). Microeconomic rigidities and aggregate price dynamics. *European Economic Review 37*, 691–711.

Calvo, G. A. (1983a). Staggered contracts and exchange rate policy. In J. A. Frenkel (Ed.), *Exchange Rates and International Macroeconomics*, pp. 235–252. Chicago, University of Chicago Press.

Calvo, G. A. (1983b). Staggered prices in a utility-maximising framework. *Journal of Monetary Economics 12*, 383–398.

Campbell, C. M. and K. S. Kamlani (1997). The reasons for wage rigidity:

Evidence from a survey of firms. *Quarterly Journal of Economics 112*, 759 789.

Campbell, J. Y. and J. H. Cochrane (1995). By force of habit: A consumption-based explanation of the aggregate stock market behavior. NBER Working Paper no. 4995.

Campbell, J. Y. and G. N. Mankiw (1987). Are output fluctuations transitory? *Quarterly Journal of Economics 102*, 857–880.

Capelli, P. and P. D. Sherer (1988). Satisfaction, market wages and labour relations: An airline studies. *Industrial Relations 27*, 56–73.

Caplin, A. S. and J. Leahy (1991). State-dependent pricing and the dynamics of money and output. *Quarterly Journal of Economics 106*, 683 708.

Caplin, A. S. and D. F. Spulber (1987). Menu costs and the neutrality of money. *Quarterly Journal of Economics 102*, 703–725.

Carrol, C. D., J. Overland, and D. N. Weil (1997). Comparison utility in a growth model. NBER Working Paper no. 6138.

Carrol, C. D. and D. N. Weil (1994). Saving and growth: A reinterpretation. *Rochester Conference Series on Public Policy 40*, 133–192.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (1996). Sticky prices models of the business cycle: Can the contract multiplier solve the

persistence problem? NBER Working Paper no. 5809.

Chiang, A. C. (1974). *Fundamental Methods of Mathematical Economics.* 2nd edition, Tokyo, McGraw-Hill Kogakusha.

Cho, J.-O. and T. F. Cooley (1992). The business cycle with nominal contracts. Mimeo, Queens University, Kingston, Ontario.

Cho, J.-O. and T. F. Cooley (1995). The business cycle with nominal contracts. *Economic Theory 6*, 13–33.

Christiano, L. J. and M. Eichenbaum (1992). Liquidity effects and the monetary transmission mechanism. *American Economic Review 82*, 346 353.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (1997). Sticky price and limited partecipation models of money: A comparison. *European Economic Review 41*, 1201–1249.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (1998). Monetary policy shocks: What have we learned and to what end? In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics.* Amsterdam, North-Holland.

Clark, A. E. and A. J. Oswald (1996a). Satisfaction and comparison income. *Journal of Public Economics 61*, 359 381.

Clark, A. E. and A. J. Oswald (1996b). Status risk-aversion and following

behaviour in social and economic setting. Warwick Economic Research Papers no. 476, forthcoming *Journal of Public Economics*.

Cochrane, J. H. (1994). Permanent and transitory components of GNP and stock prices. *Quarterly Journal of Economics 109*, 241 265.

Cogley, T. and J. N. Nason (1995). Output dynamics in real-business-cycle models. *American Economic Review 85*, 492–511.

Conlon, J. R. and C. Y. Liu (1997). Can more frequent price changes lead to price inertia? Nonneutralities in a state-dependent pricing context. *International Economic Review 38*, 893–914.

Cooley, T. F. (1995). *Frontiers of Business Cycle Research.* Princeton, New Jersey, Princeton University Press.

Cooley, T. F. and G. D. Hansen (1989). The inflation tax in a real business cycle model. *American Economic Review 79*, 733–748.

Dazinger, L. (1988). Costs of price adjustment and the welfare economics of inflation and disinflation. *American Economic Review 78*, 633–646.

DeLong, B. J. and L. H. Summers (1986). Is increased price flexibility stabilizing? *American Economic Review 76*, 1031–1044.

Dixit, A. and J. Stiglitz (1977). Monopolistic competition and optimum product diversity. *American Economic Review 67*, 297 308.

Dixon, H. and N. Rankin (1994). Imperfect competition and macroeco-

nomics: A survey. *Oxford Economic Papers 46*, 171–199.

Dotsey, M., R. G. King, and A. L. Wolman (1996). State dependent pricing and the dynamics of the business cycle. Mimeo.

Driffil, J. E. and M. H. Miller (1993). Learning and inflation convergence in the ERM. *Economic Journal 103*, 369–378.

Driskill, R. A. and S. M. Sheffrin (1986). Is price flexibility destabilizing? *American Economic Review 76*, 802–807.

Ellison, M. and A. Scott (1998). Sticky prices and volatile output: Or when is a phillips curve not a phillips curve. CEPR discussion paper no. 1849.

Erceg, C. (1997). Nominal wage rigidities and the propagation of monetary disturbances. Mimeo, Board of Governors of the Federal Reserve System.

Fethke, G. and A. Policano (1984). Wage contingencies, the patterns of negotiation and aggregate implications of alternative contract structures. *Journal of Monetary Economics 14*, 151–170.

Fethke, G. and A. Policano (1986). Will wage setters ever stagger decisions? *Quarterly Journal of Economics 101*, 867–877.

Fischer, S. (1977). Long-term contracts, rational expectations, and the optimal money supply rule. *Journal of Political Economy 85*, 191–205.

Fraja, G. D. (1993). Staggered versus synchronised wage setting in oligopoly. *European Economic Review 37*, 1507–1522.

Frank, R. H. (1984). Interdependent preferences and the competitive wage structure. *Rand Journal of Economics 15*, 510–520.

Frank, R. H. (1985). *On Choosing the Right Pond: Human Behavior and the Quest for Status.* Oxford, Oxford University Press.

Fuhrer, J. and G. Moore (1995). Inflation persistence. *Quarterly Journal of Economics 110*, 127–159.

Gali, J. (1994). Keeping up with the Joneses: Consumption externalities, portfolio choice and asset prices. *Journal of Money, Credit and Banking 26*, 1–8.

Gray, J. A. (1976). Wage indexation: a macroeconomic approach. *Journal of Monetary Economics 2*, 221–235.

Gylfason, T. and A. Lindbeck (1984). Union rivalry and wages: An oligopolistic approach. *Economica 51*, 129–139.

Hairault, J.-O. and F. Portier (1993). Money, new keynesian macroeconomics and the business cycle. *European Economic Review 37*, 1533–1568.

Ireland, P. N. (1995). Optimal disinflationary paths. *Journal of Economic Dynamics and Control 19*, 1429–1448.

Jeanne, O. (1998). Generating real persistent effects of monetary shocks: How much nominal rigidity do we really need? *European Economic Review 42*, 1009–1032.

Juillard, M. (1996). Dynare: A program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm. CEPREMAP Working Paper No. 9602.

Kandel, E. and E. P. Lazear (1992). Peer pressure and partnerships. *Journal of Political Economy 100*, 801–817.

Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money.* New York, Harcourt Brace.

Kiley, M. T. (1997). Staggered price setting and real rigidities. Mimeo, Board of Governors of the Federal Reserve System.

Kim, J. (1996). Monetary policy in a stochastic equilibrium model with real and nominal rigidities. Mimeo, Yale University.

Kimball, M. S. (1995). The quantitative analytics of the basic neomonetarist model. *Journal of Money, Credit and Banking 27*, 1241–1277.

King, R. G. and A. L. Wolman (1996). Inflation targeting in a St. Louis model of the 21st century. NBER Working Paper no. 5507.

Kydland, F. E. and E. C. Prescott (1982). Time to build and aggregate fluctuations. *Econometrica 50*, 1345 1370.

Laffargue, J.-P. (1990). Résolution d'un modèle macroéconomique avec anticipations rationnelles. *Annales d'Économie et de Statistique 17*, 97–119.

Lau, S.-H. P. (1996). Aggregate pattern of time-dependent adjustment rules, I: Game theoretic analysis of staggered versus synchronized wage setting. *Economic Journal 106*, 1645–1658.

Long, J. B. and C. I. Plosser (1983). Real business cycle. *Journal of Political Economy 91*, 39–69.

Lucas, R. E. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy 1*, 19–46.

Macurdy, T. E. (1981). An empirical model of labor supply in a life-cycle setting. *Journal of Political Economy 89*, 1059–1085.

Mankiw, G. N. (1985). Small menu costs and large business cycles: A macroeconomic model of monopoly. *Quarterly Journal of Economics 100*, 529–539.

Mankiw, G. N. (1989). Real business cycle: A new keynesian perspective. *Journal of Economic Perspectives 3*, 79–90.

Mankiw, G. N. (1990). A quick refresher course in macroeconomics. *Journal of Economic Literature 28*, 1645–1660.

Mankiw, G. N. and L. H. Summers (1986). Money demand and the effects

of fiscal policies. *Journal of Money, Credit and Banking 18*, 415–429.

Maskin, E. and J. Tirole (1988). A theory of dynamic oligopoly, II: Price competition, kinked demand curves and edgeworth cycles. *Econometrica 56*, 571–599.

Miller, M. H. and A. Sutherland (1993). Contracts, credibility and common knowledge: Their influence on inflation convergence. *IMF Staff Papers 40*, 178–201.

Nelson, C. R. and C. I. Plosser (1982). Trends and random walks in macroeconomic time series. *Journal of Monetary Economics 10*, 139–162.

Obstfeld, M. and K. Rogoff (1995). Exchange rate dynamics redux. *Journal of Political Economy 103*, 624–660.

Obstfeld, M. and K. Rogoff (1996). *Foundations of International Macroeconomics*. Cambridge, MA, The MIT Press.

Oswald, A. J. (1979). Wage determination in an economy with many trade unions. *Oxford Economic Papers 31*, 369–385.

Pencavel, J. (1986). Labor supply of men: A survey. In O. C. Ashenfelter and R. Layard (Eds.), *Handbook of Labor Economics*, pp. 3–102. Amsterdam, North-Holland.

Phaneuf, L. (1990). Wage contracts and the unit root hypothesis. *Canadian Journal of Economics 23*, 580–592.

Phelps, E. S. (1978). Disinflation without recession: Adaptive guideposts and monetary policy. *Weltwirtschaftliches Archiv 114*, 783–809.

Plosser, C. I. (1989). Understanding real business cycles. *Journal of Economic Persepctives 3*, 51–77.

Prescott, E. (1986). Theory ahead of business cycle measurement. *Federal Reserve Bank of Minneapolis Quarterly Review (Fall)*, 9–22.

Rankin, N. (1998). Nominal rigidity and monetary uncertainty. *European Economic Review 42*, 185–199.

Risager, O. (1992). Wage rivalry and insider-outsider relations: Evidence for skilled and unskilled men in Denmark. *Scandinavian Journal of Economics 94*, 543–559.

Romer, D. (1990). Staggered price setting with endogenous frequency of adjustment. *Economics Letters 32*, 205–210.

Romer, D. (1996). *Advanced Macroeconomics.* New York, McGraw-Hill.

Rotemberg, J. J. and M. Woodford (1993). Dynamic general equilibrium models with imperfectly competitive product markets. NBER Working Paper no. 4502.

Rotemberg, J. J. and M. Woodford (1995). Dynamic general equilibrium models with imperfectly competitive product markets. In T. F. Cooley (Ed.), *Frontiers of Business Cycle Research*, pp. 243 293. Princeton,

New Jersey, Princeton University Press.

Rotemberg, J. J. and M. Woodford (1997). An optimization-based econometric framework for the evaluation of monetary policy. In J. J. Rotemberg and B. S. Bernanke (Eds.), *NBER Macroeconomics Annual 1997*, pp. 297–346. Cambridge, MA, The MIT Press.

Sachs, J. (1980). The changing cyclical behaviour of wages and prices: 1890-1976. *American Economic Review 70*, 78–90.

Sargent, T. J. (1987). *Dynamic Macroeconomic Theory.* Cambridge, MA, Harvard University Press.

Sargent, T. J. and N. Wallace (1975). Rational expectations, the optimal monetary instrument, and the optimal money supply rule. *Journal of Political Economy 83*, 241–254.

Sheshinski, E. and Y. Weiss (1977). Inflation and costs of price adjustment. *Review of Economic Studies 44*, 287–303.

Sheshinski, E. and Y. Weiss (1983). Optimum pricing policies under stochastic inflation. *Review of Economic Studies 50*, 513–529.

Smith, A. (1976). *A Theory of Moral Sentiments.* Oxford, Clarendon Press.

Summers, L. (1986). Some skeptical observations on real business cycle theory. *Federal Reserve Bank of Minneapolis Quarterly Review (Fall)*, 23–27.

Summers, L. H. (1988). Relative wages, efficiency wages, and keynesian unemployment. *American Economic Review, Papers and Proceedings 78*, 383–388.

Sutherland, A. (1996). Financial market integration and macroeconomic volatility. *Scandinavian Journal of Economics 98*, 521–539.

Svensson, L. E. O. (1986). Sticky goods prices, flexible asset prices, monopolistic competition, and monetary policy. *Review of Economic Studies 52*, 385–405.

Taylor, J. B. (1979). Staggered wage setting in a macro model. *American Economic Review 69*, 108–113.

Taylor, J. B. (1980a). Aggregate dynamics and staggered contracts. *Journal of Political Economy 88*, 1–23.

Taylor, J. B. (1980b). Output and price stability: An international comparison. *Journal of Economic Dynamics and Control 2*, 109–132.

Taylor, J. B. (1983). Union wage settlement during a disinflation. *American Economic Review 73*, 981 993.

Taylor, J. B. (1998). Staggered price and wage setting in macroeconomics. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics.* Amsterdam, North-Holland.

Tsiddon, D. (1991). On the stubborness of sticky prices. *International*

Economic Review 32, 69–75.

Tsiddon, D. (1993). The (mis)behavior of the aggregate price level. *Review of Economic Studies 60*, 889–902.

West, K. D. (1988). On the interpretation of near random walk behavior in GNP. *American Economic Review 78*, 202–209.

Woodford, M. (1996). Control of the public debt: A requirement for price stability? NBER Working Paper no. 5684.

Yun, T. (1996). Nominal price rigidity, money supply endogeneity and business cycle. *Journal of Monetary Economics 37*, 345–370.

Errata Corrige

This few lines are an amendment to an imprecision in Chapter 4. In particular equation (4.13) on p. 131 should be written as:

$$x_t = b_1 p_t + d_1 p_{t+1} + b_2 y_t + d_2 y_{t+1} + b_3 \widetilde{m}_t + d_3 \widetilde{m}_{t+1} \tag{4.13}$$

where $\widetilde{m}_t = m_t - p_t$, that is, real money balances in period t.

Accordingly equation (4.15) on the same page should be written as:

$$x_t = b_4 x_{t-1} + d_4 x_{t+1} + b_5 y_t + d_5 y_{t+1} + b_6 \widetilde{m}_t + d_6 \widetilde{m}_{t+1} \tag{4.15}$$

The same two equations are reported in the subsequent page (p.132) and also there they should be amended. Accordingly Proposition 4.1 pp. 132-133 should be (the correction is bold):

Proposition 4.1. *If the utility function is additively separable in real money balances, that is, if* $u_{L,M/P} = \frac{\partial^2 u(t)}{\partial L_t \partial (M/P)_t} = 0$ *and* $u_{C,M/P} = \frac{\partial^2 u(t)}{\partial C_t \partial (M/P)_t} = 0$, *then* ***real money*** *balances do not appear in (4.13) and (4.15).*

According to this amendment also equation (4.30) on p. 163 (in the Appendix of Chapter 4) should be written as:

$$x_t = a_1 y_t + a_2 y_{t+1} + a_3 l_t + a_4 l_{t+1} + a_5 \widetilde{m}_t + a_6 \widetilde{m}_{t+1} + a_7 p_t + a_8 p_{t+1} \tag{4.30}$$

and so also equation (4.40) on p. 164, that is:

$$x_t = b_1 p_t + d_1 p_{t+1} + b_2 y_t + d_2 y_{t+1} + b_3 \widetilde{m}_t + d_3 \widetilde{m}_{t+1} \tag{4.40}$$

which is just equation (4.13) above.