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Optimal sigmoidal tuning curves for intensity encoding sensory neurons with quasi-Poisson variability

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Background

Rate-coding neurons are often characterized by their tuning curve, that is, the average firing rate, $T(x)$, as a function of stimulus intensity, x . However the substantial natural variability in firing rate that often occurs for a fixed stimulus provides a limitation on the fidelity of firing rate encoding of stimuli. Consequently, stimulus-dependent variance in firing rate, $V(x)$, is crucial in studies of tuning curve optimality. Information theory can be used to quantify such limits and to address the question of finding the tuning curve that maximizes information rate [1].

Firing activity is often modeled as a Poisson point process, such that $V(x) = T(x)$. However, this assumption can break down for intensity encoding neurons with monotonically non-decreasing (e.g. *sigmoidal*) tuning curves, such as primary afferent auditory nerve fibers, where refractoriness can cause firing rate saturation. As the rate nears this point, variability decreases, and to a first approximation becomes binomial rather than Poisson, so that $V(x)$ varies quadratically with $T(x)$. Such neurons are sometimes called *quasi-Poisson*.

Results

We have derived a sufficient condition for achieving maximum Shannon mutual information between stimulus intensity and firing rate when the variability is quasi-Poisson such that $V(x) = s^2T(x)(1-T(x))$, and s is small [2]. The sufficient condition leads to analytical expressions for two ways to achieve maximize mutual information: (i) an optimal monotonically non-decreasing tuning curve for

any given stimulus distribution and (ii) an optimal stimulus for any given monotonically non-decreasing tuning curve [2].

The optimal tuning curve for a stimulus with cumulative distribution function $F_x(x)$ is $T^o(x) = 0.5 - 0.5\cos(\pi F_x(x))$, while for a tuning curve $T(x)$, the optimal probability density function of the stimulus is $f_x^o(x) = (dT(x)/dx)/(\pi(T(x)(1-T(x))))^{0.5}$. Our derivation also provides an expression for the reduction in mutual information when the tuning curve and stimulus distribution are not optimally matched [2]. This expression is a function of the relative entropy between the stimulus distribution, and a distribution known as Jeffrey's prior. The derivation makes use of a relationship between Shannon mutual information and Fisher information discussed, for example, in [1].

Discussion

Unlike neurons with a 'preferred stimulus' (unimodal tuning curves), optimality conditions for neurons where firing rates increase monotonically with stimulus intensity (e.g. sigmoidally) have received little attention. A notable exception is [3,4], which maximizes Fisher information, and considers only Poisson variability. In contrast, we maximize mutual information, and consider quasi-Poisson variability. This leads to a very versatile analytical solution that allows for refractoriness. A limitation to be addressed in future work is how well the quadratic relationship $V(x) = s^2T(x)(1-T(x))$ compares with measured variability. Finally, while we assume small s , our solution provides a lower bound to the achievable

mutual information for larger s , and is hence a worst-case scenario.

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