A Thesis Submitted for the Degree of PhD at the University of Warwick

http://go.warwick.ac.uk/wrap/3668

This thesis is made available online and is protected by original copyright. Please scroll down to view the document itself. Please refer to the repository record for this item for information to help you to cite it. Our policy information is available from the repository home page.
A Theory of Product Selection

(A Model of a NIC)

by

Il Houng Lee

Thesis submitted for the degree of Ph.D in
Warwick University

Department of Economics
University of Warwick

November 1989
To My Parents
Acknowledgement

I would like to thank my parents for their love and sacrifices on my behalf during the last few years, for their financial support as well as their constant encouragements through prayer, who have overstepped afar the duties of parents my supervisors, Prof. Marcus Miller and Dr. Norman Ireland, who have not spared their efforts and time for me with their mastery in their respective fields and great intellectual abilities, whom I have come to admire and respect deeply for their distinguished academic qualities as well as their gentle and kind personalities.

I would like to extend my gratitudes to Dr. N. Barr (LSE), Prof. J. Richmond (Essex) who have introduced me to this subject and Dr N. Schofield (Washington University) for showing me the boundlessness of mathematical economics the department of Economics, Warwick University, for its financial support, Prof. K. Wallis and Dr. P. Stoneman who were always ready to help, and to its members of staff, for their comments and helpful suggestions my sister and her husband, who have provided me and my wife many memorable vacations in the U.S. Miss H. Bateson, for her moral support and motherly encouragements.

Finally, but not least, I would like to thank my wife, Jee Yeon, who has been standing behind me with a gentle smile and without a single word of complaint, all these long days.
Contents

Introduction

1 The Objectives 1

2 Korea as a Newly Industrializing Country 5
   2.1 Real GDP Growth 6
   2.2 Debt Problem 9
   2.3 Unemployment 15
   2.4 Balance of Payment 21

3 The Korean Economy: A Survey 24
   3.1 Macroeconomic Aspects 24
      3.1.1 The Five Economic Development Plans 27
      3.1.2 Export-led Growth 32
   3.2 Microeconomic Aspects 35
      3.2.1 The Textile Industry 36
      3.2.2 The Electronic Industry 37
   3.3 The Financial Sector 40
      3.3.1 Deregulation in the Financial Market 41
      3.3.2 Two Sources of Inefficiencies 45

4 Conclusion 49

References
9 Consistent Conjectures

9.1 Uncertainty and Nash Equilibria
9.2 Rationality and Nash Equilibria
9.2.1 Definition of Consistent Conjectures
9.2.2 Parameterization and Consistent Conjectures
9.2.3 Dynamics and Consistent Conjectures
9.2.4 Infinite Regress of Expectation
9.3 Perry/Bresnahan's Consistent Conjectures Equilibrium
9.4 Cournot Equilibrium as the Consistent Conjectures Equilibrium
9.5 Conclusion

10 Competition for Market Leadership

10.1 First Stage Game given Cournot conjectures in the Marketing Stage
10.1.1 The Choice between staying a leader and becoming a follower
10.1.2 A Numerical Example
10.2 Folk Theorem and the Incumbent Firm
10.2.1 The Folk Theorem
10.2.2 Forced Leadership
10.2.3 A Credible Threat
10.3 First Stage Game given Bertrand Conjectures in the Marketing Stage
10.4 Production Cost Differences and Market Participation given c1 > c2
10.4.1 When a1 > a2
10.4.2 When a1 < a2
10.5 Conclusion

11. Licencing vs Independent Innovation

11.1 Licencing vs Independent Innovation
11.2 Licencing Fee per Unit Sold
11.2.1 a3 > D/(2A+D)
11.2.2 a3 < D/(2A+D)
11.3 Licencing Fee as a Lump Sum Cost
11.4 Licencing Fee charged on Quality Difference
11.5 Licencing Fee and Leapfrogging
11.6 Conclusion

12 Optimal Timing of Innovation

12.1 The Speed of Innovation Development
12.2 Single-Period Equilibrium
12.3 Level of Quality
12.4 Level of Profit
12.5 An increase in the Cost of Production
12.6 Multi-Period Competition
12.7 Conclusion
Lists of Tables

Chapter 2
Table 2-1 Real GDP Growth 7
Table 2-2 International Multipliers 8
Table 2-3 Gross Capital Formation 10
Table 2-4 Savings and Investment 11
Table 2-5 Indicators of external debt, nonoil developing countries 73-82 13
Table 2-6 Balance of Trade 14
Table 2-7 Total Debt 73-82 16
Table 2-8 Impact of exogenous shocks on external debt of nonoil developing countries 17
Table 2-9 Unemployment Rates 18
Table 2-10 Major Economic Indicators 20

Chapter 3
Table 3-1 Basic Economic Data 25
Table 3-2 Annual Foreign Investment 26
Table 3-3 The Five Economic Development Plans 28
Table 3-4 Korean Companies ranked in Fortune's largest 500 non-US companies 31
Table 3-5 Technology Development's Contribution to Economic Growth 34
Table 3-6 Electronics Export by Product Group and Company Classifications 39
Table 3-7 Interest Rates on Various Loans 42
Table 3-8 Types of Shareholders 46
Table 3-9 Share Holdings 47
List of Figures

Chapter 6
Figure 6-1 Consumer Indifference Curves as Functions of their Income 84

Chapter 7
Figure 7-1 Cournot-Nash Equilibria under various values of Qualities 93
Figure 7-2 Equilibrium values and Profit Levels for different Quality gaps 96
Figure 7-3 Cournot-Nash Equilibria and Utility Functions 98
Figure 7-4 Utility Levels under various values of Quality Levels 100
Figure 7-5 Reaction Functions with upper and Lower Bounds and zero cost of innovation 104
Figure 7-6 Reaction Function of Firm 1 with Positive Cost of Innovation 108
Figure 7-7 Reaction Function of Firm 2 with Positive Cost of Innovation 110
Figure 7-8 Reaction Functions derived from a Numerical Example 114
Figure 7-9 Two Possible Cournot-Nash Equilibria under each a different values of an Exogenous variables 116
Figure 7-10 The Effect of an increase in the Cost of Innovation by Firm 2 119

Chapter 8
Figure 8-1 Bertrand-Nash Equilibria under various Values of Quality Levels 126
Figure 8-2 Utility Levels under Bertrand competition for various Levels of Quality 128
Figure 8-3 Reaction Functions under Bertrand competition and zero Cost of Innovation 135
Figure 8-4 Reaction Functions with upper and lower Bounds on Quality Levels with zero Cost of Innovation 138
Figure 8-5 Reaction Functions with Positive Cost of Innovation 139
Figure 8-6 The Effect on Equilibrium as Cost of Innovation increases for each Level of Quality 146

Chapter 9
Figure 9-1 Nash Equilibria under Uncertainty of Firms' Strategic Behaviour 153
Figure 9-2 Nash Equilibria under Uncertainty of conjectures 155
Figure 9-3 Conjectures under various competition for each given values of β 164
Figure 9-4 Loci of Consistent Conjectures Nash Equilibrium 166

Chapter 10
Figure 10-1 Reaction Function of Firm 1 and Firm 2 before and after the changes in the Market Leadership 176
Figure 10-2 Simulation Result showing Reaction Functions of both Firms under changes of Market Leadership 182
Figure 10-3 Credible threats and Nash Equilibrium Strategy 187
Figure 10-4 Reaction Functions under Bertrand Conjectures in the Marketing Stage before and after changes in the Market Leadership 192
Figure 10-5 Possible values of Production Cost of Firm 1 given c1 > c2 196

Chapter 11
Figure 11-1 Nash equilibria under independent development 205
Figure 11-2 Reaction Function of firm 1 given licence fee received per unit sold 208
Figure 11-3 The Nash equilibrium given a fixed licence fee paid per unit sold 210
Figure 11-4 Reaction function of firm 1 given a licence fee paid as a function of quality difference 216
Figure 11-5 Loci of Nash equilibria 218
Figure 11-6 Paying licence fee or leap frogging 222

Chapter 12
Figure 12-1 Speed of Innovation vs Expenditure 228
Figure 12-2 Efficiencies in R&D and Nash Equilibria in Optimal Timing 235
Figure 12-3 Profit Contours of Firm 1 240
Figure 12-4 Zero Profit Contours of Firm 1 given IC1 241
Figure 12-5 Contract Curve under Collusion 243
Abstract

The objective of this work is to theoretically evaluate an important aspect of a Newly Industrializing Country (NICs): Korea. Namely, the behaviour of firms in Korea competing with firms in an industrialized country after all government intervention of the former is withdrawn. This aspect is considered in the main part while a descriptive introductory part introduces the Korean economy as a NIC.

We construct a simple asymmetric duopoly model where firms conjectures play an important role in deriving the Perfect Equilibrium for a two stage game. Different costs of production and first mover advantage form the basis of the asymmetry. We find that under Cournot conjectures assumption for the marketing stage and certain cost conditions, it is profitable for the incumbent firm to stay a leader and the follower to remain a follower. For some cost conditions and a credible threat at the disposal of the follower, the incumbent firm may be forced to stay a leader even though it is more profitable to become a follower.

We examine possible licensing rules the leader may propose to the follower. The dominant strategy, we find, is a licence fee that is a function of the quality difference between the top quality of the market leader and the level of quality it is selling to the follower. There will be a cost to the leader in terms of a lower licence fee to prevent possible leap forging. Once we allow for free copying, we find that the follower will copy closely the new product of the leadership. Under Bertrand conjectures assumption, we find that unless the firm with higher production cost remains the leader offering a higher quality product, it will be driven out of the market, i.e., either it has to innovate or die.
Introduction
1. The Objectives

The basic objective of this work is to develop a theoretical model to analyse aspects of industrial economics which are not only of close relevance to the characteristics of NICs but also very stimulating topics in themselves as regarding present and future strategic choices as well as economic theory. They are quality competition and market structure between firms in a newly industrializing country and a industrialized country.

Insofar as its relevance to the characteristics of NICs are concerned, the analysis will address a particular aspect of problems a country may encounter at its maturing stage. We shall take Korea as a country representative of NICs. The problems will evolve around the following question:

What is the optimal strategy for a firm in NICs when competing against firms in developed countries once all restrictions in trade has been abolished?

Industries in Korea have come into existence through extensive planning, financing and partial administration by the Government throughout the last three decades. The purpose of this quasi-laissez-faire approach was to promote an export led growth that would pull the economy out from an almost stagnant underdeveloped country into an
industrialized nation. Initially, investments were focused on textile industries since then required least skill and funds for take off. Then gradually, investments were diversified into semi-skilled manufacturing industries. During the second half of the 70s, heavy manufacturing industries were born leading finally to the electronics industries towards the end of the 70s. At present, Korea has reached a stage where it is now losing its comparative advantages, e.g., low wage rate, large number of semi-skilled workers and competitive exchange rate, to its neighbouring countries which include the potentially giant economy of China. Had it not been for the few large conglomerates in Korea which started to invest massively in the electronics industries a decade ago, the situation in which Korea might have found itself today would have been a very gloomy one. In addition to the loss of comparative advantages, its recent success in visible trade has meant that the argument for "Infant Industry", which served as a justification for high tariffs and import barriers became inappropriate and such protectionism has had to be abolished, at least gradually. Therefore, faced with the prospect of losing ground in the areas where economic growth has so far been enjoyed and having to compete without any shelters, Korean firms now have to engage in direct competition with firms in developed countries. In this competition, Korean firms still lag behind in terms of technology and know how. The question is whether a Korean firm in this situation should behave as a follower in markets of advanced technology or attempt to overtake the foreign incumbent firms if possible.
The electronics firms in Korea have been so far enjoying the fruit of being a follower in the world of electronics industry. This was mainly inevitable as technology was lagging too much behind to engage in quality competition and new product innovation. Nevertheless, the Korean electronics section has been able to take over 3% of the world electronics market and about one tenth that of Japan by producing colour TVs (7 million sets), VCRs (3.75 million sets) and PCs in 1988. On the other hand, active R&D projects are underway in Anyang Research establishments aiming at 16 mega bit d-ram, challenging the leaders of this market. This is a significant achievement for an economy that only two and a half decades ago was just able to produce a simple radio.

We shall be concerned with one particular aspect of competition: investment into R&D and quality competition. Under an asymmetric duopoly, i.e., a market structure consisting only of two firms, we shall examine the optimal strategy with respect to product (quality) selection under various conjectures and competition for market leadership. It is an asymmetric duopoly firstly because firms in Korea have dual objectives; profit and sales maximization made possible by Government policies of promoting export to obtain greater world market share due to foreign reserve shortages which in turn are used to finance the import of raw materials. Secondly, lower wage rates are available to Korean firms compared to that of advanced countries.
As a prelude, we briefly describe the Korean economy within the framework of the NICs. Then we survey the economy in her own right with emphasis on past performances, the present state and future prospects.
2. Korea as a Newly Industrializing Country

The criteria of categorizing a country as a NIC seem to be of a very imprecise nature but to involve some degree of industrialization, economic infra-structure, capacity level of production and current level of GNP. As these structural characteristics are in general difficult to measure, we show briefly the level of a few state variables of NICs that may distinguish them from the rest of the world.

Countries usually termed as an NIC range from those which still largely have the characteristics of developing countries as well as those whose economic performances, if not their international influence, are ranked together with Industrialized countries. Korea, Taiwan, Hong Kong and Singapore are usually known as the Asian NICs while Mexico, Brasil and Argentina as the American NICs. These countries are so diverse in nature that it is difficult to derive any common characteristics except that their GNP per capita are between a quarter to a half of those of an advanced country and the fact that they have enjoyed rapid economic growth for the last twenty years. It is inevitable, therefore, that not all NICs fit into all of the following characteristics we have listed below.
2.1 Real GDP growth

As it is obvious from the date shown in table 2-1, the NICs enjoyed a sustained growth in real GDP relative to those of the LDCs and the Industrialized countries. The comparison with LDCs may provide a more realistic picture of performances of NICs as a growth rate of 2% in an Industrialized country, with its large size of economy, may in fact be comparable with about 10% growth of a NIC with its relative smaller economy. In many countries, growth was partly export led. Exports in Korea, for example, were contributing between 20 to 30% of incremental growth each year during the 70s and early 80s. This consequently led many NICs to become heavily dependent on their foreign sector earnings. A study by Ezaki (1985) on the econometric link between various countries using international income multipliers is a good example which shows the high interdependent relationship among various economies (table 2-2). As it is clearly evident from the income multipliers between NICs and other regions of the world, figures listed in row 2 are relatively larger than the reverse effect shown in column 2, i.e., if NICs increase their autonomous expenditure by 1 unit, the impact of this change on other countries' GNP will be less than vise versa. This asymmetric mutual cross effect can be described as due to the fact that shares of foreign trade of NICs in proportion to their domestic market are much larger than the industrialized countries. However, within a time span of 8 years, both multipliers have increased implying a greater role being played by NICs than before.
Table 2-1

Real GDP Growth
(Annual averages, in percent)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Latin America</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Argentina*</td>
<td>2.5</td>
<td>2.8</td>
<td>0.5</td>
<td>4.3</td>
<td>-6.0</td>
</tr>
<tr>
<td>Brazil*</td>
<td>11.7</td>
<td>7.5</td>
<td>6.6</td>
<td>7.3</td>
<td>-1.7</td>
</tr>
<tr>
<td>Chile</td>
<td>1.9</td>
<td>-2.5</td>
<td>7.2</td>
<td>7.9</td>
<td>-2.6</td>
</tr>
<tr>
<td>Colombia</td>
<td>6.8</td>
<td>4.9</td>
<td>6.1</td>
<td>4.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Ecuador</td>
<td>12.9</td>
<td>6.0</td>
<td>7.4</td>
<td>4.9</td>
<td>2.9</td>
</tr>
<tr>
<td>Mexico*</td>
<td>7.0</td>
<td>5.9</td>
<td>5.3</td>
<td>8.7</td>
<td>4.5</td>
</tr>
<tr>
<td>Peru</td>
<td>6.1</td>
<td>5.1</td>
<td>0.0</td>
<td>3.5</td>
<td>2.3</td>
</tr>
<tr>
<td>Venezuela</td>
<td>5.3</td>
<td>6.0</td>
<td>6.1</td>
<td>-0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Asia</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indonesia</td>
<td>8.6</td>
<td>6.3</td>
<td>7.5</td>
<td>7.5</td>
<td>6.3</td>
</tr>
<tr>
<td>Korea*</td>
<td>9.9</td>
<td>7.6</td>
<td>12.3</td>
<td>0.1</td>
<td>6.0</td>
</tr>
<tr>
<td>Malaysia</td>
<td>8.7</td>
<td>4.6</td>
<td>8.7</td>
<td>8.6</td>
<td>5.2</td>
</tr>
<tr>
<td>Philippines</td>
<td>6.2</td>
<td>6.1</td>
<td>6.4</td>
<td>5.9</td>
<td>3.2</td>
</tr>
<tr>
<td>Taiwan*</td>
<td>11.6</td>
<td>3.8</td>
<td>12.4</td>
<td>7.3</td>
<td>4.7</td>
</tr>
<tr>
<td>Thailand</td>
<td>6.3</td>
<td>6.3</td>
<td>13.0</td>
<td>5.9</td>
<td>6.3</td>
</tr>
</tbody>
</table>

* NIC

### International Multipliers

<table>
<thead>
<tr>
<th>1970</th>
<th>Y&lt;sub&gt;i&lt;/sub&gt;</th>
<th>1978</th>
<th>ASEN</th>
<th>NICS</th>
<th>JAPN</th>
<th>USA</th>
<th>WEUR</th>
<th>OPEC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D&lt;sub&gt;j&lt;/sub&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ASEN</td>
<td>1.795</td>
<td>0.039</td>
<td>0.247</td>
<td>0.217</td>
<td>0.265</td>
<td>0.151</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.512</td>
<td>0.045</td>
<td>0.288</td>
<td>0.267</td>
<td>0.288</td>
<td>0.120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NICS</td>
<td>0.077</td>
<td>1.430</td>
<td>0.376</td>
<td>0.352</td>
<td>0.244</td>
<td>0.132</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.082</td>
<td>1.097</td>
<td>0.427</td>
<td>0.391</td>
<td>0.326</td>
<td>0.118</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JAPN</td>
<td>0.038</td>
<td>0.010</td>
<td>1.821</td>
<td>0.117</td>
<td>0.064</td>
<td>0.096</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.047</td>
<td>0.021</td>
<td>1.810</td>
<td>0.152</td>
<td>0.104</td>
<td>0.084</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEUR</td>
<td>0.008</td>
<td>0.005</td>
<td>0.022</td>
<td>0.104</td>
<td>2.053</td>
<td>0.117</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.012</td>
<td>0.012</td>
<td>0.046</td>
<td>0.133</td>
<td>2.160</td>
<td>0.073</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OPEC</td>
<td>0.004</td>
<td>0.004</td>
<td>0.027</td>
<td>0.083</td>
<td>0.180</td>
<td>1.260</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.015</td>
<td>0.017</td>
<td>0.111</td>
<td>0.204</td>
<td>0.461</td>
<td>1.073</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 2-2**

<table>
<thead>
<tr>
<th>MUL1</th>
<th>MUL2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.284</td>
<td>1.549</td>
</tr>
<tr>
<td>3.157</td>
<td>1.303</td>
</tr>
<tr>
<td>2.195</td>
<td>1.752</td>
</tr>
<tr>
<td>2.643</td>
<td>2.157</td>
</tr>
<tr>
<td>2.404</td>
<td>2.195</td>
</tr>
<tr>
<td>1.450</td>
<td>1.459</td>
</tr>
</tbody>
</table>

**Notes:**
- $D_j$: autonomous expenditure of the $j$-th country or region
- $y_i$: GDP of the $i$-th country or region
- MUL1: each country's simple multiplier
- MUL2: each country's trade multiplier

Source: M. Ezaki (1983)
A second reason for a relatively faster growth of GDP may have been the high rate of capital accumulation as can be observed from table 2-3. The average of Asian NICs amounts to about 32.2 as opposed to 26.6 for the rest of the Asian countries. Malkiel (1979) estimated the fixed investment ratio with respect to GNP in the U.S.A. between 1965 and 1974 to be 10.4 while in 1977 it fell to 9.3. He argued that one of the reasons for low investment ratio towards the end of 70s was the fall in Tobin's q ratio. It is defined as the market value of a company divided by the replacement value of its assets. A high q ratio implies that corporations will be encouraged to invest in new equipments whereas if assets sell for less than their replacement values, corporations selling new securities to buy new capital goods will create capital losses to their share holders.

Gross investment as a percentage of GNP for Korea in particular is shown in table 2-4. It shows that an average of 20 to 30 % investment ratio accompanied the GDP growth rate during this period. It is however, less clear whether the Tobin's q ratio would be able to provide explanations for such a ratio in NICs in absence of a well established stock market.

2.2 Debt problem
That domestic saving was unable to match the high investment level in most of Asian NICs can be seen from table 2-3. The
### Gross Capital Formation

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Burma</td>
<td>23.6</td>
<td>17.5</td>
<td>6.1</td>
<td>15.4</td>
<td>15.4</td>
</tr>
<tr>
<td>China (R.O)</td>
<td>29.5</td>
<td>31.7</td>
<td>-2.2</td>
<td>28.6</td>
<td>30.1</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>29.6</td>
<td>23.6</td>
<td>8.0</td>
<td>21.5</td>
<td>21.1</td>
</tr>
<tr>
<td>Indonesia</td>
<td>20.5</td>
<td>19.3</td>
<td>1.3</td>
<td>37.3</td>
<td>26.4</td>
</tr>
<tr>
<td>Korea (R.O)</td>
<td>27.4</td>
<td>22.5</td>
<td>4.9</td>
<td>36.4</td>
<td>28.1</td>
</tr>
<tr>
<td>Laos</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
<td>37.9</td>
<td>20.8</td>
</tr>
<tr>
<td>Malaysia</td>
<td>32.3</td>
<td>23.0</td>
<td>14.3</td>
<td>21.0</td>
<td>23.0</td>
</tr>
<tr>
<td>Philippines</td>
<td>29.7</td>
<td>24.9</td>
<td>4.8</td>
<td>19.7</td>
<td>23.0</td>
</tr>
<tr>
<td>Singapore</td>
<td>42.4</td>
<td>33.2</td>
<td>9.2</td>
<td>25.4</td>
<td>21.9</td>
</tr>
<tr>
<td>Thailand</td>
<td>27.9</td>
<td>21.8</td>
<td>6.1</td>
<td>25.4</td>
<td>24.2</td>
</tr>
<tr>
<td>India</td>
<td>23.8</td>
<td>20.9</td>
<td>3.0</td>
<td>12.5</td>
<td>20.2</td>
</tr>
</tbody>
</table>

### Savings and Investment

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1962</td>
<td>11.6</td>
<td>25.5</td>
<td>15.9</td>
</tr>
<tr>
<td>1963</td>
<td>16.4</td>
<td>48.0</td>
<td>20.4</td>
</tr>
<tr>
<td>1964</td>
<td>13.1</td>
<td>62.3</td>
<td>5.1</td>
</tr>
<tr>
<td>1965</td>
<td>14.1</td>
<td>45.1</td>
<td>-2.0</td>
</tr>
<tr>
<td>1966</td>
<td>19.9</td>
<td>54.5</td>
<td>12.5</td>
</tr>
<tr>
<td>1967</td>
<td>20.1</td>
<td>51.5</td>
<td>18.5</td>
</tr>
<tr>
<td>1968</td>
<td>23.3</td>
<td>58.3</td>
<td>28.5</td>
</tr>
<tr>
<td>1969</td>
<td>26.1</td>
<td>65.3</td>
<td>25.5</td>
</tr>
<tr>
<td>1970</td>
<td>24.5</td>
<td>64.7</td>
<td>26.9</td>
</tr>
<tr>
<td>1971</td>
<td>22.8</td>
<td>60.9</td>
<td>35.4</td>
</tr>
<tr>
<td>1972</td>
<td>20.6</td>
<td>72.5</td>
<td>16.6</td>
</tr>
<tr>
<td>1973</td>
<td>24.7</td>
<td>52.0</td>
<td>9.2</td>
</tr>
<tr>
<td>1974</td>
<td>27.6</td>
<td>66.0</td>
<td>36.1</td>
</tr>
<tr>
<td>1975</td>
<td>26.6</td>
<td>63.3</td>
<td>31.7</td>
</tr>
<tr>
<td>1976</td>
<td>24.9</td>
<td>50.6</td>
<td>4.5</td>
</tr>
<tr>
<td>1977</td>
<td>27.1</td>
<td>92.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>1978</td>
<td>30.2</td>
<td>84.7</td>
<td>7.4</td>
</tr>
<tr>
<td>1979</td>
<td>32.9</td>
<td>75.1</td>
<td>19.5</td>
</tr>
<tr>
<td>1980</td>
<td>28.6</td>
<td>63.2</td>
<td>29.8</td>
</tr>
<tr>
<td>1981</td>
<td>25.5</td>
<td>13.4</td>
<td>27.4</td>
</tr>
</tbody>
</table>

National Saving + Foreign Saving = 100 % (Gross Investment)
Foreign Saving - Net Borrowing = Net Transfer Payment from abroad

exception is Taiwan. This meant that NICs had to rely on borrowing from abroad. During the 70s, most of the external debt was long term borrowing as shown in table 2-5, such that it was expected that with the continuous growth and high investment ratio, NICs were able to pay back in due course. From the NICs’ point of view, the long term borrowings were of no real concern, at least in the 70s as they were favoured by high inflation rates, low world real interest rates initially and steady expansion of their volume of export with relative constant terms of trade. A stable nominal world interest rate accompanied by a high inflation rate led to the low real world interest rate while the increase in the volume of export meant that the debt services as against total export earnings were low. During the same period however, the sustaining inflation pushed up world nominal interest rates and added to the burden of interest payment in return for eroding the real value of outstanding debt. This inflationary acceleration became an acute problem by 81/2 as the inflation rate fell with nominal interest rate sluggish to adjust immediately. The world recession in the early 80s meant a reduction in their volume of exports (at least in the growth rate of export). This caused serious Balance of Payment problems (refer to table 2-6) and NICs had to rely on short term borrowing, not so much for investment but to pay for net interest payment of the long term loans as well as the current Balance of Payment deficit. Since approximately two-thirds of developing countries’ debt as of '84 was at floating interest rates tied to the London Interbank Offer Rates (LIBOR), higher current debt service had to be paid. The
Indicators of external debt, nonoil developing countries 1973-82
(billion dollars and percentages)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>External debt</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>130.1</td>
<td>160.8</td>
<td>190.8</td>
<td>228.0</td>
<td>278.5</td>
<td>336.3</td>
<td>396.9</td>
<td>474.0</td>
<td>550.0</td>
<td>612.4</td>
</tr>
<tr>
<td>Long-term</td>
<td>118.8</td>
<td>138.1</td>
<td>163.5</td>
<td>194.7</td>
<td>235.9</td>
<td>286.6</td>
<td>338.1</td>
<td>388.5</td>
<td>452.8</td>
<td>499.6</td>
</tr>
<tr>
<td>Exp. (Goods &amp; Services)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt/Export (%)</td>
<td>115.4</td>
<td>104.6</td>
<td>122.4</td>
<td>125.5</td>
<td>126.4</td>
<td>130.2</td>
<td>119.2</td>
<td>121.9</td>
<td>124.9</td>
<td>143.3</td>
</tr>
<tr>
<td>Debt Service/export</td>
<td>15.9</td>
<td>14.4</td>
<td>16.1</td>
<td>15.3</td>
<td>15.4</td>
<td>19.0</td>
<td>19.0</td>
<td>17.6</td>
<td>20.4</td>
<td>23.9</td>
</tr>
<tr>
<td>Debt/GDP (%)</td>
<td>22.4</td>
<td>21.8</td>
<td>23.8</td>
<td>25.7</td>
<td>27.4</td>
<td>28.5</td>
<td>27.5</td>
<td>27.6</td>
<td>31.0</td>
<td>34.7</td>
</tr>
</tbody>
</table>

Source: IMF, World Economic Outlook 1982, 1983
<table>
<thead>
<tr>
<th>Year</th>
<th>Argentina</th>
<th>Brazil</th>
<th>Japan</th>
<th>Korea</th>
<th>Mexico</th>
<th>Singapore</th>
</tr>
</thead>
<tbody>
<tr>
<td>1978</td>
<td>7836</td>
<td>14489</td>
<td>113950</td>
<td>17161</td>
<td>11424</td>
<td>13778</td>
</tr>
<tr>
<td></td>
<td>6048</td>
<td>21597</td>
<td>97150</td>
<td>18717</td>
<td>14789</td>
<td>14191</td>
</tr>
<tr>
<td>1979</td>
<td>9916</td>
<td>17998</td>
<td>126150</td>
<td>18530</td>
<td>16003</td>
<td>18648</td>
</tr>
<tr>
<td></td>
<td>10486</td>
<td>28493</td>
<td>133900</td>
<td>24120</td>
<td>21687</td>
<td>19349</td>
</tr>
<tr>
<td>1980</td>
<td>11202</td>
<td>23275</td>
<td>158230</td>
<td>22577</td>
<td>24628</td>
<td>25040</td>
</tr>
<tr>
<td></td>
<td>15999</td>
<td>36250</td>
<td>167450</td>
<td>28347</td>
<td>33065</td>
<td>26553</td>
</tr>
<tr>
<td>1981</td>
<td>11805</td>
<td>26923</td>
<td>189300</td>
<td>27269</td>
<td>30453</td>
<td>29130</td>
</tr>
<tr>
<td></td>
<td>16495</td>
<td>38872</td>
<td>182910</td>
<td>32416</td>
<td>44641</td>
<td>30451</td>
</tr>
<tr>
<td>1982</td>
<td>9755</td>
<td>23469</td>
<td>178750</td>
<td>28356</td>
<td>27674</td>
<td>30102</td>
</tr>
<tr>
<td></td>
<td>12097</td>
<td>39773</td>
<td>170520</td>
<td>31505</td>
<td>34188</td>
<td>31288</td>
</tr>
<tr>
<td>1983</td>
<td>9764</td>
<td>24341</td>
<td>183060</td>
<td>30383</td>
<td>28605</td>
<td>30940</td>
</tr>
<tr>
<td></td>
<td>12216</td>
<td>31286</td>
<td>160710</td>
<td>32581</td>
<td>23580</td>
<td>31747</td>
</tr>
<tr>
<td>1984</td>
<td>9909</td>
<td>30205</td>
<td>210390</td>
<td>33651</td>
<td>32326</td>
<td>31771</td>
</tr>
<tr>
<td></td>
<td>12406</td>
<td>30334</td>
<td>173610</td>
<td>35563</td>
<td>28736</td>
<td>32567</td>
</tr>
</tbody>
</table>

Source: Statistical Yearbook 1983/84
United Nations
deterioration of the debt crisis can be observed from table 2-7. The debt burden became an acute problem more so for NICs than the rest of the developing countries. As Simonsen (1983) noted rightly, unless export earnings grow at a higher rate than interest rate payments, the country's debt burden becomes worse. An interesting calculation by W.R. Cline (table 2-8) shows possible causes of debt increase in non-oil developing countries. Since most NICs except Mexico are non-oil producing countries and have a high dependence on foreign trade, these figures may apply more so on the NICs than the rest of the developing countries.

2.3 Unemployment

At least in the Asian NICs, the problem of unemployment was not serious even during the recessions between 81/2 (refer to table 2-9). This was mainly due to the combination of sustaining real GDP growth and wage policies (or absence of it as is illustrated later). As in most Asian NICs, the total population in Korea grew steadily after the second world war and the Korean war as the political and economic stabilities were reestablished. The consequence of this was that the size of the active working population will be increasing at a rate of 3% each year for the next decade or so. Therefore, unless economic growth keeps its minimum rate to accommodate population growth, there is bound to be a continuous rise in unemployment.
Total Debt 1973-82 (Billion US Dollars)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>6.4</td>
<td>8.0</td>
<td>7.9</td>
<td>8.3</td>
<td>9.7</td>
<td>12.5</td>
<td>19.0</td>
<td>27.2</td>
<td>35.7</td>
<td>38.0</td>
</tr>
<tr>
<td>Brazil</td>
<td>13.8</td>
<td>18.9</td>
<td>23.3</td>
<td>28.6</td>
<td>35.2</td>
<td>48.4</td>
<td>57.4</td>
<td>66.1</td>
<td>75.7</td>
<td>88.2</td>
</tr>
<tr>
<td>Chile</td>
<td>3.6</td>
<td>4.4</td>
<td>4.7</td>
<td>4.5</td>
<td>4.9</td>
<td>6.4</td>
<td>8.2</td>
<td>10.7</td>
<td>15.0</td>
<td>17.9</td>
</tr>
<tr>
<td>Indonesia</td>
<td>5.7</td>
<td>7.1</td>
<td>8.9</td>
<td>11.0</td>
<td>12.8</td>
<td>14.5</td>
<td>14.9</td>
<td>17.0</td>
<td>18.0</td>
<td>21.0</td>
</tr>
<tr>
<td>Korea</td>
<td>4.6</td>
<td>6.0</td>
<td>7.3</td>
<td>8.9</td>
<td>11.2</td>
<td>14.8</td>
<td>20.5</td>
<td>26.4</td>
<td>31.2</td>
<td>35.8</td>
</tr>
<tr>
<td>Mexico</td>
<td>8.6</td>
<td>12.8</td>
<td>16.9</td>
<td>21.8</td>
<td>27.1</td>
<td>33.6</td>
<td>40.8</td>
<td>53.8</td>
<td>67.0</td>
<td>82.0</td>
</tr>
</tbody>
</table>

Debt Service as percentage of export 1973-82

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>19.9</td>
<td>21.3</td>
<td>31.9</td>
<td>26.2</td>
<td>19.1</td>
<td>41.6</td>
<td>21.3</td>
<td>32.3</td>
<td>37.5</td>
<td>102.9</td>
</tr>
<tr>
<td>Brazil</td>
<td>36.7</td>
<td>36.0</td>
<td>40.8</td>
<td>45.3</td>
<td>48.7</td>
<td>59.3</td>
<td>65.6</td>
<td>60.8</td>
<td>66.9</td>
<td>87.1</td>
</tr>
<tr>
<td>Chile</td>
<td>35.1</td>
<td>37.4</td>
<td>42.7</td>
<td>41.7</td>
<td>45.9</td>
<td>49.7</td>
<td>44.4</td>
<td>41.3</td>
<td>61.0</td>
<td>60.4</td>
</tr>
<tr>
<td>Indonesia</td>
<td>3.4</td>
<td>2.1</td>
<td>6.2</td>
<td>7.2</td>
<td>8.3</td>
<td>9.7</td>
<td>7.4</td>
<td>4.9</td>
<td>5.2</td>
<td>11.3</td>
</tr>
<tr>
<td>Korea</td>
<td>11.5</td>
<td>11.8</td>
<td>12.5</td>
<td>9.8</td>
<td>10.2</td>
<td>12.0</td>
<td>13.9</td>
<td>17.3</td>
<td>18.8</td>
<td>21.1</td>
</tr>
<tr>
<td>Mexico</td>
<td>28.7</td>
<td>21.9</td>
<td>30.3</td>
<td>40.7</td>
<td>53.6</td>
<td>64.9</td>
<td>67.7</td>
<td>36.4</td>
<td>48.5</td>
<td>58.5</td>
</tr>
</tbody>
</table>

Impact of exogenous shocks on external debt of nonoil developing countries (billion dollars)

<table>
<thead>
<tr>
<th>Effects</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil price increase in excess of US inflation, 1974-82 cumulative (net oil importers only)</td>
<td>260</td>
</tr>
<tr>
<td>Real interest rate in excess of 1961-80 average: 1981 and 1982</td>
<td>41</td>
</tr>
<tr>
<td>Terms of trade loss, 1981-1982</td>
<td>79</td>
</tr>
<tr>
<td>Export volume loss caused by world recession, 1981-1982</td>
<td>21</td>
</tr>
<tr>
<td>Total</td>
<td>401</td>
</tr>
</tbody>
</table>

Memorandum items

<table>
<thead>
<tr>
<th>Total debt</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1973</td>
<td>130</td>
</tr>
<tr>
<td>1982</td>
<td>612</td>
</tr>
<tr>
<td>Increase</td>
<td>482</td>
</tr>
</tbody>
</table>

### Unemployment Rate (% of Total working population)

<table>
<thead>
<tr>
<th>Year</th>
<th>Argentina</th>
<th>Brazil</th>
<th>Hongkong</th>
<th>Japan</th>
<th>Korea</th>
<th>Singapore</th>
<th>U.K.</th>
<th>U.S.A.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>4.8</td>
<td>n.a.</td>
<td>n.a.</td>
<td>1.1</td>
<td>4.5</td>
<td>6.0</td>
<td>2.6</td>
<td>4.9</td>
</tr>
<tr>
<td>1975</td>
<td>2.3</td>
<td>n.a.</td>
<td>9.1</td>
<td>1.9</td>
<td>4.1</td>
<td>4.5</td>
<td>4.1</td>
<td>8.5</td>
</tr>
<tr>
<td>1977</td>
<td>2.8</td>
<td>2.3</td>
<td>4.3</td>
<td>2.0</td>
<td>3.8</td>
<td>3.9</td>
<td>6.2</td>
<td>7.1</td>
</tr>
<tr>
<td>1978</td>
<td>2.8</td>
<td>2.4</td>
<td>2.9</td>
<td>2.2</td>
<td>3.2</td>
<td>3.6</td>
<td>6.1</td>
<td>6.1</td>
</tr>
<tr>
<td>1979</td>
<td>2.0</td>
<td>2.8</td>
<td>2.9</td>
<td>2.1</td>
<td>3.8</td>
<td>3.4</td>
<td>5.7</td>
<td>5.8</td>
</tr>
<tr>
<td>1980</td>
<td>2.3</td>
<td>n.a.</td>
<td>3.8</td>
<td>2.0</td>
<td>5.2</td>
<td>3.0</td>
<td>7.3</td>
<td>7.1</td>
</tr>
<tr>
<td>1981</td>
<td>4.5</td>
<td>4.3</td>
<td>3.8</td>
<td>2.2</td>
<td>4.5</td>
<td>2.9</td>
<td>11.1</td>
<td>7.6</td>
</tr>
<tr>
<td>1982</td>
<td>4.8</td>
<td>n.a.</td>
<td>3.7</td>
<td>2.4</td>
<td>4.4</td>
<td>2.6</td>
<td>13.1</td>
<td>9.7</td>
</tr>
<tr>
<td>1983</td>
<td>4.2</td>
<td>n.a.</td>
<td>4.5</td>
<td>2.7</td>
<td>4.1</td>
<td>3.2</td>
<td>13.1</td>
<td>9.6</td>
</tr>
</tbody>
</table>

Source: Statistical Yearbook 1983/84
United Nations
Another important aspect to be noted is the set of characteristics of the labour market which may have kept the unemployment rate low and could keep it that way in whatever circumstances of the economy. As it is evident from the Korean example, the ratio of union member against the total number of employment in early 80s was still only 8% which was much lower than in most industrialized countries. The "Labor Union Law" and "Labour Disputes Adjustment Laws" which were revised extensively and repeatedly in the early 70s in Korea for example granted the Government the right to order a union to change its decision or make the decision null and void. Such strong legal grip by the Government assured that enough wage flexibility was introduced to keep its economy competitive and in full employment. This is evident in the growth rate of real wages between 1970 and 1986 in table 2-10 which exhibits no consistency. The real wage growth rate does not reflect a stable growth path with nominal wage rates adjusting for inflation. It is more closely correlated with GNP growth rate supporting the earlier claim that wages were raised or lowered according to the success or failure of economic performance. Since '86 however, there have been serious strikes which have put Korea into a period of transition in terms of Unionization, forcing the Government to adopt a much more liberal attitude towards the labour movement.
<table>
<thead>
<tr>
<th>Major Economic Indicators (Korea)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>GNP Growth rate</td>
</tr>
<tr>
<td>- 9.1  5.3  14.0  8.5  6.8  13.4  10.7  11.0  7.0  -4.8  6.6  5.4  11.9  8.5  5.4  12.5</td>
</tr>
<tr>
<td>Export Growth rate</td>
</tr>
<tr>
<td>34.2  27.8  52.1  98.6  38.3  13.9  51.8  30.2  26.5  18.4  16.3  21.4  2.8  11.9  19.6  3.6  14.6</td>
</tr>
<tr>
<td>Inflation (CPI)</td>
</tr>
<tr>
<td>15.9  13.5  11.7  3.1  24.3  25.3  15.3  10.1  14.4  18.3  28.7  21.3  7.2  3.4  2.3  2.5  2.3</td>
</tr>
<tr>
<td>Nominal Wages (g.r.)</td>
</tr>
<tr>
<td>26.9  16.2  13.9  18.0  35.3  27.0  34.7  33.8  34.3  28.6  22.7  20.1  14.7  12.2  8.1  9.9  9.1</td>
</tr>
<tr>
<td>Real Wages (g.r.)</td>
</tr>
<tr>
<td>9.3  2.4  2.0  14.3  8.8  1.4  16.8  21.5  17.4  8.7  -4.7  -2.6  6.9  10.4  5.7  7.3  6.7</td>
</tr>
<tr>
<td>Labour Productivity (KPCI)</td>
</tr>
<tr>
<td>12.7  9.6  8.8  8.8  11.4  11.6  7.5  10.5  11.9  15.9  10.6  18.1  7.8  13.6  10.5  7.1  13.6</td>
</tr>
<tr>
<td>Terms of Trade</td>
</tr>
<tr>
<td>100.0  99.2  98.7  93.7  76.3  68.7  78.5  83.9  87.4  85.5  74.2  72.6  75.8  76.5  78.1  78.6  85.1</td>
</tr>
<tr>
<td>Won/USS</td>
</tr>
<tr>
<td>310  347  393  398  405  484  484  484  484  484  607  681  831  776  806  870  882</td>
</tr>
</tbody>
</table>

KPCI: Korea Productivity Center Index, output per production worker

Source: Economic Planning Board
Bank of Korea
2.4 Balance of Payments

With few exceptions such as Hong Kong and Taiwan, most of the NICs suffered from Balance of Trade deficit (table 2-7) in spite of their rapid growth of export. This may have been the result of their large volume of import of petroleum and capital goods from abroad. In addition, Brazil and Mexico conducted an ambitious venture in the 70s investing heavily in reshaping their economy by constructing heavy industries with a long term perspective. Recession in the early 80s brought loss to the ambitious venture as the debt burdens of Brazil and Mexico show in table 6 while Taiwan and Hong Kong, with obviously smaller economic size, ventured much less ambitious investment and concentrated more on light manufacturing goods such that they were among the very few countries in NICs as well as in the rest of the LDCs who avoided the debt problem. Korea could be regarded as somewhere inbetween these two extreme cases. Another aspect of the Balance of Payment problem was the loss of terms of trade. The commodity export prices for NICs are sensitive to the business cycle of the industrialized countries in that in periods of recessions, export prices fall. This is due to the fact that NICs have to adjust their export prices with those of the industrialized countries so as to keep their export goods competitive and thereby keep their volume of export from falling. The import prices, however, do not always reflect the general inflation rate of the industrialized countries because the NICs with no real resources or advanced technology have to rely on their efficient and cheap labour force to import raw and intermediate materials and commodities, then manufacture and
assemble them for export. As the raw materials and intermediate commodities' prices do not necessarily reflect the inflation rate, the terms of trade worsen if inflation rate in industrialized countries are brought down sharply. This phenomena was felt even more so when there was the price rise of oil with falling inflation in industrialized countries in the early 80s.

Invisible trade has played a relatively insignificant role in NICs until the mid 80s when gradual deregulation in financial markets were observed. One of the main reason for this was that there was no stock exchange such as can be found in New York and London except in Hong Kong. Since Governments are more devoted in finding the right funds for industries in the exporting markets, industries never really felt the necessity to generate such a stock exchange. Foreign investors on the other hand saw too much risk in investing in NICs of which they had so little information. This lack of foreign investment has its advantages in that the exchange rate was responsive only to real changes such as net values of trade and not subject to volatility of the exchange rate as some industrialized countries experienced. In countries such as Korea and Taiwan, financial transfer to abroad was strictly controlled by the Government. In other words, the domestic interest rate was independent to world interest rates. The importance of the world interest rate lies in that it determines the cost of net debt service. Therefore, as rightly noted by M. Ezaki (1985), the only transmission channels are the actual trade volumes, export &
import prices, controlled financial flows and human and technological communications.
3. The Korean Economy: A Survey

3.1 Macroeconomic Aspects
Since the early 60s, the structure of the Korean economy has changed dramatically as is most evident from the changes in the sectoral production pattern. In 1961, 60.6% of the total working population were employed in the agricultural sector producing only 38.7% of the total national product, 8% and 28.2% were employed in the manufacturing and in the service sector producing 13.8% and 45.9% each. By the end of the 70s, these figures changed to 32.3% producing 18% of GNP, 21.7% producing 29.5% and 44.5% producing 59% of national product each. The GNP grew on average about 8.3% p.a. during the same period. There has been also a large growth in the foreign sector. Taking 1975 as the index year, imports increased from 5.8 in '62 to 357.7 while export increased from 1.1 to 417.5 during the same period. The ratio between import and export in '75 were 6.6:5 (table 3-1).

Another aspect to be noted is the role of foreign finance in Korea. There has always been a maximum ceiling of foreign direct investment in Korea which in the fourth plan it was $100m at current price and the ratio of foreign direct investment against borrowing from abroad never exceeded 10% (table 3-2). Since the late 70s, however, the GNP growth rate has been on average 6.6% p.a.. Export grew steadily reaching on average a growth rate of 13.6%. This steady growth meant that Korea was finally able to enjoy a trade
### Basic Economic Data
(Korea)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Land area (Sq. km)</td>
<td>98.431</td>
<td>98.431</td>
<td>98.477</td>
<td>98.969</td>
<td>98.971</td>
</tr>
<tr>
<td>Population (Million)</td>
<td>21.502</td>
<td>24.954</td>
<td>31.435</td>
<td>38.124</td>
<td>42.080</td>
</tr>
<tr>
<td>Labour Force</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployed</td>
<td>n.a.</td>
<td>8.2</td>
<td>4.5</td>
<td>5.2</td>
<td>3.1</td>
</tr>
<tr>
<td>Agriculture</td>
<td>n.a.</td>
<td>63.1</td>
<td>50.4</td>
<td>34.0</td>
<td>21.3</td>
</tr>
<tr>
<td>Manufacturing &amp; Mining</td>
<td>n.a.</td>
<td>8.7</td>
<td>14.3</td>
<td>22.6</td>
<td>27.5</td>
</tr>
<tr>
<td>Services</td>
<td>n.a.</td>
<td>28.2</td>
<td>35.2</td>
<td>43.4</td>
<td>52.2</td>
</tr>
<tr>
<td>Industrial Shares of GNP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agriculture</td>
<td>51.7</td>
<td>47.2</td>
<td>33.1</td>
<td>17.2</td>
<td>12.7</td>
</tr>
<tr>
<td>Manufacturing &amp; Mining</td>
<td>6.3</td>
<td>9.1</td>
<td>17.5</td>
<td>31.3</td>
<td>33.9</td>
</tr>
<tr>
<td>Services (1975 prices)</td>
<td>42.0</td>
<td>43.7</td>
<td>49.4</td>
<td>51.5</td>
<td>53.4</td>
</tr>
<tr>
<td>GNP current prices($bn)</td>
<td>1.353</td>
<td>1.948</td>
<td>7.834</td>
<td>56.460</td>
<td>97.527</td>
</tr>
<tr>
<td>Per Capita GNP ($)</td>
<td>67</td>
<td>80</td>
<td>243</td>
<td>1481</td>
<td>2320</td>
</tr>
<tr>
<td>Ratio of export/GNP</td>
<td>3.2</td>
<td>4.1</td>
<td>16.0</td>
<td>40.2</td>
<td>47.9</td>
</tr>
<tr>
<td>Ratio of import/GNP</td>
<td>9.8</td>
<td>12.7</td>
<td>25.3</td>
<td>50.4</td>
<td>43.5</td>
</tr>
</tbody>
</table>

Source: BOK National Income in Korea
EPB Major Statistics of Korean Economy
### Annual Foreign Investment (arrival)

**In Thousand Dollars**

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Capital Goods</th>
<th>Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>1962</td>
<td>575</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1963</td>
<td>2075</td>
<td>2075</td>
<td>-</td>
</tr>
<tr>
<td>1964</td>
<td>3050</td>
<td>3050</td>
<td>-</td>
</tr>
<tr>
<td>1965</td>
<td>10741</td>
<td>627</td>
<td>10114</td>
</tr>
<tr>
<td>1966</td>
<td>4822</td>
<td>88</td>
<td>4734</td>
</tr>
<tr>
<td>1967</td>
<td>12667</td>
<td>1190</td>
<td>11477</td>
</tr>
<tr>
<td>1968</td>
<td>14739</td>
<td>4466</td>
<td>10273</td>
</tr>
<tr>
<td>1969</td>
<td>6960</td>
<td>3131</td>
<td>3829</td>
</tr>
<tr>
<td>1970</td>
<td>25275</td>
<td>10385</td>
<td>14887</td>
</tr>
<tr>
<td>1971</td>
<td>36716</td>
<td>17551</td>
<td>19165</td>
</tr>
<tr>
<td>1972</td>
<td>61232</td>
<td>20022</td>
<td>41210</td>
</tr>
<tr>
<td>1973</td>
<td>158435</td>
<td>100626</td>
<td>57809</td>
</tr>
<tr>
<td>1974</td>
<td>162629</td>
<td>78069</td>
<td>84560</td>
</tr>
<tr>
<td>1975</td>
<td>69170</td>
<td>27836</td>
<td>41334</td>
</tr>
<tr>
<td>1976</td>
<td>105574</td>
<td>37407</td>
<td>68167</td>
</tr>
<tr>
<td>1977</td>
<td>102286</td>
<td>17236</td>
<td>85050</td>
</tr>
<tr>
<td>1978</td>
<td>100457</td>
<td>24612</td>
<td>75845</td>
</tr>
<tr>
<td>1979</td>
<td>126977</td>
<td>42200</td>
<td>84777</td>
</tr>
<tr>
<td>1980</td>
<td>96635</td>
<td>27361</td>
<td>69274</td>
</tr>
<tr>
<td>1981</td>
<td>105448</td>
<td>20793</td>
<td>84655</td>
</tr>
</tbody>
</table>

**Source:** Ministry of Finance  
EPB Major Statistics of Korean Economy  
1982, 1983
surplus since 1986 and an invisible trade surplus in 1987. The most dramatic change in the 80s was probably the development of the financial market in Korea. First came the deregulations on the commercial banks followed by establishments of new financial institutions funded by the private sector and the crack down on the curb market which led to an upsurge of the stock price index as funds were channelled into official financial markets. The inflation rate, which once reached as high as 287% in 1980 reflecting the political turmoil during that period, was kept down to below 3% after 1983.

3.1.1 The Five Economic Development Plans

It is a well established belief that the five Economic Development plans since 1962 have been the major driving force of the growth of the Korean economy. It is less clear, however, how the actual mechanism worked (refer to table 3-3).

The first Economic Development Plan was between 1962 to 1966. The main objectives of the plan was to establish some basis on which to build the economy. Investments were made in textile industries and radio manufacturing firms while imposing import bans on many foreign items.

The Second Plan (1967 to 1971) was based on three main principles; export-led strategy, import substitution and self-sufficient policy rather than an open economy. An
The Five Economic Development Plans (Planned and Actual)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>GNP in 1980 prices</td>
<td></td>
<td>10324</td>
<td>13746</td>
<td>23316</td>
<td>41592</td>
<td>66877</td>
<td>90000</td>
</tr>
<tr>
<td>$ US million</td>
<td></td>
<td>12607</td>
<td>18060</td>
<td>28717</td>
<td>36509</td>
<td>61010</td>
<td>97507</td>
</tr>
</tbody>
</table>

Industrial Structure

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>37.1</td>
<td>34.8</td>
<td>34.1</td>
<td>22.4</td>
<td>18.5</td>
<td>14.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>36.6</td>
<td>34.4</td>
<td>27.0</td>
<td>23.8</td>
<td>18.1</td>
<td>12.7</td>
<td></td>
</tr>
<tr>
<td>Manufacturing &amp; Mining</td>
<td>19.4</td>
<td>26.1</td>
<td>26.8</td>
<td>29.9</td>
<td>40.9</td>
<td>32.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>16.3</td>
<td>20.2</td>
<td>22.4</td>
<td>28.4</td>
<td>30.6</td>
<td>33.9</td>
<td></td>
</tr>
<tr>
<td>Services</td>
<td>43.5</td>
<td>39.1</td>
<td>39.2</td>
<td>49.7</td>
<td>40.6</td>
<td>52.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>47.1</td>
<td>45.4</td>
<td>50.6</td>
<td>48.6</td>
<td>51.3</td>
<td>53.4</td>
<td></td>
</tr>
</tbody>
</table>

Commodity Export US $ bn

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.66</td>
<td>1.38</td>
<td>5.50</td>
<td>35.1</td>
<td>202.4</td>
<td>530.0</td>
<td></td>
</tr>
<tr>
<td>0.54</td>
<td>2.09</td>
<td>9.74</td>
<td>47.30</td>
<td>210.0</td>
<td>422.5</td>
<td></td>
</tr>
</tbody>
</table>

Unemployment Ratio %

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>22.3</td>
<td>14.8</td>
<td>5.0</td>
<td>4.0</td>
<td>3.8</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>8.2</td>
<td>7.1</td>
<td>4.5</td>
<td>3.9</td>
<td>4.8</td>
<td>3.8</td>
<td></td>
</tr>
</tbody>
</table>


Source: EPB Fifth Five-Year Economic and Social Development Plan 1982
important issue during this period was whether to adopt domestic or foreign manufacturing machinery. The former would lead to boost of domestic machinery industry while the purchasers would have had to be contented with less efficient capital goods whereas the latter would have increased import but would have provided better quality. It turned out that the latter was favoured by most industries.

The goal during the Third Plan (1972 to 1976) was to achieve a self sufficient agricultural sector. The notion of harmony was introduced for the first time in the plan whereas only growth was the sole target. The harmony was referred to, at least in theory, as giving more attention to the overall economic performances as well as economic structure to include problems as income redistribution, welfare and sectoral discrepancies in income. The Government intervention on the allocation of resources, however grew even larger and investment in heavy and chemical industries were made. The share of investment to heavy and chemical industry as against light industry rose from 74% in 1976 to 82% in 1979. After the oil crisis in 1973, two plans were drawn up; export of manpower and a vigorous exploitation of new market. As a consequence, many trading companies were born during this period.

The Fourth Plan (1977 to 1981) focused its policy on eliminating foreign direct investment while domestic saving was hoped to exceed total investment. In fact, by 1977, net borrowing from the rest of the world was negative (table 2-
4). This was, however, a short lived hope as in 1979 and 1980, higher oil prices drove the economy to borrow more heavily than ever before except in the 1973 oil shock. The ratio of Research and Development to GNP was doubled to reach 0.9% of the GNP in 1980 while investment were channelled to more sophisticated products. The aim of this plan was i) to integrate into the world market excluding the foreign capital market, ii) to increase saving while keeping interest rate low for industrial investment and iii) to reduce Government involvement in the economy while making massive subsidies to industries under the Heavy and Chemical Industry Plan (1972). This plan identified six leading industries for export; steel, chemical, non-ferrous metal, machinery, shipbuilding and electrical products.

Imports were liberalized to a certain degree to avoid central banks issuing more currency as inflow of dollars remittance from overseas construction workers increased as a measure against domestic inflation. This meant that trade surplus goal had to be abandoned.

Due to high inflation in the late 70s (figure 2-10), the dominantly Keynesian type of government policies were weakened and a shift occurred to a more monetary approach. In 1980, interest rate were increased greatly while the Korean currency Won was devalued; it had been pegged to the U.S. Dollar since 1974.

During the Fifth Plan (1982 to 1986), priority was given to economic stability and the target for inflation was below
## Korean Companies ranked in Fortune's largest 500 non-U.S. Companies - Sales in 1,000s (1979)

<table>
<thead>
<tr>
<th>Company</th>
<th>Sales</th>
<th>Ranking 1979</th>
<th>Ranking 1978</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hyundai</td>
<td>4,303,841</td>
<td>80</td>
<td>78</td>
</tr>
<tr>
<td>Samsung</td>
<td>3,409,700</td>
<td>109</td>
<td>154</td>
</tr>
<tr>
<td>Daewoo</td>
<td>n.a.</td>
<td>-</td>
<td>155</td>
</tr>
<tr>
<td>Korean Oil</td>
<td>2,315,841</td>
<td>165</td>
<td>199</td>
</tr>
<tr>
<td>Lucky Group</td>
<td>1,760,362</td>
<td>227</td>
<td>134</td>
</tr>
<tr>
<td>Hyosung</td>
<td>1,558,439</td>
<td>258</td>
<td>-</td>
</tr>
<tr>
<td>POSCO</td>
<td>1,249,777</td>
<td>321</td>
<td>409</td>
</tr>
<tr>
<td>ICC</td>
<td>1,188,684</td>
<td>334</td>
<td>368</td>
</tr>
<tr>
<td>Ssangyong Cement</td>
<td>936,000</td>
<td>425</td>
<td>484</td>
</tr>
</tbody>
</table>

Source: Fortune Magazine
10% while the market mechanism was left without intervention to adjust the excess demand at that time. Structural reform of economic institutions and policies were called for; liberalization of the economic mechanisms, autonomous banking operations, efficient industrial incentive scheme and competitive market mechanism were encouraged. Import substitution was abandoned and only comparatively advantageous products were encouraged while the energy intensive industries were to be positively discouraged. R&D investment reached 2% of GNP in 1986.

3.1.2 Export-led Growth

The Korean economy as a whole was more or less centrally planned and controlled by the Government until recently. Imposing restrictions on import since the early 60s and devaluing the exchange rate in 1964 sheltered the Korean industries in their infant stage. A complicated list system was devised to allow rebates to export industries. The Ministry of Commerce and Industry was anxious to build up firms to a size which could compete with foreign multinationals. As a consequence, large conglomerates were born building new industries where the state dictated the nature, location of the plants and also provided the bulk of the investment (table 3-4). Export items were selected and a rationed credit system was conducted as a consequence of which the economy developed a dual structure; the export sector which was close to free trade and the domestic sector hemmed with restrictions. A maximum was reached in 1976.
when 80% of export was by top 20 items while in 1980, the figure went down to 70%.

Korean industries were advantaged not only by having been supported by Government but also by the fact that Korea was a late comer to world trade. By investing in new plants with the knowledge of those in the developed countries, they were able to overcome overmanning problems, e.g., the steel mill POSCO in Korea required manpower per ton half of that in the U.K. Hyundae, the largest conglomerate in Korea estimated that building a car plant of a certain size in Korea would cost only about 60% of a plant build by Volkswagen or Toyota.

The industries in Korea were initially also blessed with abundant availability of human capital and sufficient financial capital organized by the Government. Any inflationary effects on trade were continuously overcome by adjustment of the exchange rate. The biggest devaluation was probably in 1960 when the Won was devalued by 50%.

The favourable international situation during the 60s and early 70s meant continuous high demand of products in developed countries. Japan and the U.S. were looking for offshore basis for labour intensive manufacturing processes and found Korea an attractive place to invest. This helped the diffusion of technology from the advanced countries to the Korean economy (Table 3-5).
Technology Development's Contribution to Economic Growth

<table>
<thead>
<tr>
<th></th>
<th>Korea ('66-'76)</th>
<th>Japan ('53-'71)</th>
<th>U.S.A. ('48-'69)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic Growth Rate</td>
<td>9.7% 100</td>
<td>9.5% 100</td>
<td>8.8% 100</td>
</tr>
<tr>
<td>Labour</td>
<td>40</td>
<td>24</td>
<td>21</td>
</tr>
<tr>
<td>Capital</td>
<td>22</td>
<td>33</td>
<td>24</td>
</tr>
<tr>
<td>Others</td>
<td>32</td>
<td>30</td>
<td>33</td>
</tr>
<tr>
<td>Technology</td>
<td>6</td>
<td>13</td>
<td>22</td>
</tr>
</tbody>
</table>

Source: Long-term Prospect for Economic and Social Development (77-91) KDI
3.2 Microeconomic Aspects

There are two types of Korean Export industries; labour intensive industries employing predominantly women and the heavy industries (i.e., steel, iron and shipbuilding) producing with cheaper capital equipment and modern design.

Focus in this paper will be on the labour intensive industries, especially on the textile industry and the electronic industry. From table 3-5, we observe the high level of contribution of the labour force in economic development of Korea when compared with that of the United States and Japan. The reason why these Korean industries employ more female workers is because the average wage of a female worker doing the same job against a male worker is about 50 to 70%.

<table>
<thead>
<tr>
<th>% of female workers (1979)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>textile</td>
<td>69</td>
</tr>
<tr>
<td>clothing</td>
<td>77</td>
</tr>
<tr>
<td>foot wear</td>
<td>57</td>
</tr>
<tr>
<td>electric Machine</td>
<td>55</td>
</tr>
<tr>
<td>consumer electronics</td>
<td>65</td>
</tr>
<tr>
<td>domestic appliances</td>
<td>34</td>
</tr>
<tr>
<td>rest</td>
<td>28</td>
</tr>
<tr>
<td>total</td>
<td>42.9</td>
</tr>
</tbody>
</table>

Female workers in total produce about 46.8% of total exports implying that they are contributing about 17% more to exports than male workers.
3.2.1 The Textile Industry

The initial take off was by heavy investment by semi-government organizations which conducted a vigorous expansion program since 1956. The first problem they encountered was in 1961 of over capacity. Only 86% of spindles and 56% of looms were in operation while import of textile remained high. This was partly because cotton products were using the U.S. imported cotton P.L. 480 as its raw material. It was not until 1969 that export was able to attain the level of imports of textiles. The second problem was trade barriers. The U.S., which was importing the major part of the textiles Korea exported, imposed restrictive covenants on Korean cotton exports; any export earnings should be used to purchase further U.S. cotton P.L. 480. Furthermore, the Kennedy administration placed a quota on Korean Textile products in 1962. The third problem was that of silk exports. A considerable silk market in West existed but only if the price could be lowered. This required increases in productivity, new management techniques and lowering other running costs. However, increases in garment export produced with imported fabrics was profitable enough to outweigh the less successful cotton industry due to restrictions on exports. In the early 60s, domestic demand was dominant until early 70s for woollen and cotton, 1975 for silk, viscose, synthetic in fabrics and 1971 in chemical fibre at which time the export began the lead.
Cotton Industry
Product differentiation and decline in spinning capacities in advanced countries and constant low price contributed to gradual yarn export increases. Constant low price, however, was difficult to maintain as raw materials were imported and simple depreciation would not have cured the problem as would have made the imported raw materials even more expensive. However, continuous increase in productivity was able to keep the price constant and marginally escape unprofitability (table 2-10).

Chemical & Synthetic Fibres
In 1966, Rayon plant was created under the public ownership. Raw materials were imported from Japan. In order to replace the Japanese material, refining and petrochemical industry was created which replaced the import dependence from 83.8% in 1967 to 9.1% in 1979. However, since the domestic products were more expensive, they had to be subsidized by the Government.

3.2.2 The Electronics Industry
Gold Star Radio factory was built in 1959 at Pusan and import of small radios were banned in the following year. Two additional plants were built in '61 and '62 each with technical guidance of Japanese companies. The first exports of the small radios were made to New York and Hong Kong. However, low productivity, unskilled management, higher interest rate and higher electricity cost meant higher price and a lack of competitiveness in the foreign market.
In spite of these facts, export was possible through the Government making up for the higher cost. The U.S. regarded Korea as a good offshore assembly base and Motorola (1966), Signetics Fairchild Semiconductor, Data control (1967) moved into Korea followed by a Japanese joint venture with Korea.

As a result, 80.8% of total bonded process was in the hands of foreign enterprises in the late 60s. In 1969, Electronics Industry Promotion Law was introduced recognizing this industry as a "strategic export industry" with its own eight year plan.

One interesting phenomena (which was also observable in the textile industry) was that medium size firms employing 400 to 500 workers suffered from insufficient economies of scale. They were less efficient by 50% compared with bigger firms but at the same time 25% less compared with smaller firms. However, the total number of employees in the industry grew 919% in less than 10 years. Gradually, foreign enterprises withdrew due to higher labour cost and domestic investment grew with Government backing. (table 3-6)

Additional to its support in terms of finance and various other policies, the Government also created the Korea Institute of Electronics Technology in 1976 which develops and designs circuits and maintain a branch institute in Silicon Valley in the U.S.. In 1987, Korea's total output of electronic equipment and components amounted to $10.6 bn (ex-factory prices). It also claims to have about 20% of the US market for VCRs. The Government hopes to obtain 7%
Electronics Export by Product Group and Company Classification

(in million US dollars)

<table>
<thead>
<tr>
<th></th>
<th>Production Group</th>
<th>Company Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Consumer Electronics</td>
</tr>
<tr>
<td>1968</td>
<td>19.5</td>
<td>3.6</td>
</tr>
<tr>
<td>1972</td>
<td>142.1</td>
<td>27.9</td>
</tr>
<tr>
<td>1975</td>
<td>581.9</td>
<td>198.3</td>
</tr>
<tr>
<td>1976</td>
<td>1036.9</td>
<td>389.8</td>
</tr>
<tr>
<td>1977</td>
<td>1063.7</td>
<td>455.7</td>
</tr>
<tr>
<td>1978</td>
<td>1359.2</td>
<td>654.2</td>
</tr>
<tr>
<td>1979</td>
<td>1845.4</td>
<td>914.5</td>
</tr>
<tr>
<td>1980</td>
<td>2003.8</td>
<td>984.9</td>
</tr>
</tbody>
</table>

Source: KEB Industry in Korea 1981
of the world electronic equipment market by the end of next decade. Korea has already started building production plants abroad. Goldstar has a plant in the US making TVs, microwaves and VCRs. In West Germany, it produces TVs and VCR while Samsung will soon open a production plant in the U.K. for microwaves and possibly VCRs.

Stage 1: Assembly for local market
Stage 2: Direct Foreign Investment
Stage 3: Domestic mastery of existing process technology licencing
Stage 4: Producing at the frontier of existing technology.
If we use these stages to describe the development of the whole industry, Korean industries must be distributed between stages 2 and 3 facing now the real challenge of competing with firms in developed countries.

3.3 The Financial Sector
The Korean financial market consists of three different groups. The official banking sector consisting of the five commercial banks and special banks such as the Korean Exchange Bank and the Medium and Small Industry Bank. This official banking sector was highly controlled until 1982 when a gradual deregulation program was adopted. The commercial banks were given policy guidelines on which they assessed the worthiness of any loan to be made. This meant
that credit rationing was severe in some non-export oriented sectors leading to some general distortion of the economy. The second group consisting of Non-Monetary Financial institutions were less severely controlled. These comprise Development institutions like the Korea Development Bank and the Export-Import Bank of Korea, Savings Institutions like the Credit Unions, Mutual Savings and Finance Company, Life Insurance Companies, and finally Investment companies like the Korea Securities Finance Corporations. Thirdly, there exists a securities Market consisting of the Korea Stock Exchange and Securities companies. Unofficially, a fourth group exists in the Korean financial sector, the unorganized curb market. The securities market played little role until the mid 80s whereas the curb market and the Non-Monetary Financial institutions added much needed flexibility to the Korean financial market. Their respective interest rates are shown in table 3-7.

3.3.1 Deregulation in the Financial Market
Since 1982 when the Government embarked on gradual financial deregulation, partly forced by a financial scandal where high officials were found to be involved, the curb market shrunk while Non-Monetary Financial Institutions were greatly encouraged to grow. Only the official banking sector was left sitting with all the bad loans and ailing companies as a by product of Government intervention in the financial market. The insignificant role played by the Stock Market in Korea meant that private funds were
<table>
<thead>
<tr>
<th>Year</th>
<th>Curb market</th>
<th>Corporate bond</th>
<th>General</th>
<th>Selected policy loan</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Export</td>
<td>MIPF</td>
</tr>
<tr>
<td>1971</td>
<td>46.41</td>
<td>-</td>
<td>22.0</td>
<td>6.0</td>
<td>-</td>
</tr>
<tr>
<td>1972</td>
<td>38.97</td>
<td>-</td>
<td>19.0</td>
<td>6.0</td>
<td>-</td>
</tr>
<tr>
<td>1973</td>
<td>33.30</td>
<td>-</td>
<td>15.5</td>
<td>7.0</td>
<td>10.0</td>
</tr>
<tr>
<td>1974</td>
<td>40.56</td>
<td>-</td>
<td>15.5</td>
<td>9.0</td>
<td>12.0</td>
</tr>
<tr>
<td>1975</td>
<td>41.31</td>
<td>20.1</td>
<td>15.5</td>
<td>9.0</td>
<td>12.0</td>
</tr>
<tr>
<td>1976</td>
<td>40.47</td>
<td>20.4</td>
<td>17.0</td>
<td>8.0</td>
<td>13.0</td>
</tr>
<tr>
<td>1977</td>
<td>38.07</td>
<td>20.1</td>
<td>15.0</td>
<td>8.0</td>
<td>13.0</td>
</tr>
<tr>
<td>1978</td>
<td>41.22</td>
<td>21.1</td>
<td>18.5</td>
<td>9.0</td>
<td>15.0</td>
</tr>
<tr>
<td>1979</td>
<td>42.39</td>
<td>26.7</td>
<td>18.5</td>
<td>9.0</td>
<td>15.0</td>
</tr>
<tr>
<td>1980</td>
<td>44.94</td>
<td>30.1</td>
<td>24.5</td>
<td>15.0</td>
<td>20.0</td>
</tr>
<tr>
<td>1981</td>
<td>35.25</td>
<td>24.4</td>
<td>18.0</td>
<td>15.0</td>
<td>11.0</td>
</tr>
<tr>
<td>1982</td>
<td>33.12</td>
<td>17.29</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>1983</td>
<td>25.77</td>
<td>14.23</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>1984</td>
<td>24.84</td>
<td>14.12</td>
<td>10.5</td>
<td>10.0</td>
<td>10.5</td>
</tr>
</tbody>
</table>

MIPF: Machinery In. Promotion Fund  
NIF : National Investment Fund  
Source: BOK
channelled to highly speculative property markets. The rapid changes of ownership within the Korean property market not only provided an evidence of this speculative behaviour but also raised the price of properties to such an extend that it became very difficult for average income group workers to buy their own houses. This led the Government to impose strict regulations at various stages in the early 80s on the property market in an attempt to reduce distortion within the market. Policies included high taxes on ownership of a second property.

The reduction of the size of property market and the curb market meant that funds had to find another outlet; the official financial market. The bond market has been growing rapidly in recent years. The total listed public sector bonds rose from W 2496 bn in 1982 to W 8638 bn at the end of last year. Corporate bonds grew from W 3303 bn to W 10028 bn over the same period. The Government is also committed to expand the Korean equity market, even to the point of virtually forcing companies to go public. In 1986, 13 companies obtained new listings on the Korean Stock Exchange followed by another company in 1987. The stock price index rose from 79.85 in 1975 to 924.3 in March 1989. In fact, the stock price index in 1985 was only 163.37. It then increased to 272.61, 525.11 and 831.12 in the following consecutive years. Therefore, the actual significant take off of the stock price level began in 1986 when Korea began to experience significant trade surplus. The exchange rate (won/US $) went up from 880.2 in 1985 to 861.4 and 792.3 to reach 680.6 in January 1989. With the interest rate
relatively fixed at 10.0% (on time deposits), the improvement in international political situations, especially with Russia and China reducing the threat from the North, brought an optimism to the stock market in Korea. On top of these positive political circumstances, Korea announced that it had accepted the obligation of Article VIII, section 2, 3, and 4 of the IMF's Article of Agreement on Nov 1988. Members accepting the obligations of Article VIII undertake neither to impose restrictions on the making of payments and transfer for current international transactions nor to engage in discriminatory currency arrangements. Free international capital flow, gradually to be accepted in stages until 1992 implies that foreign investors may greatly increase their participation in the Korean stock market, raising its price index still further. This expectation has its roots in two grounds. First, the price earnings ratio (which is defined as the closing price of a stock divided by its earning usually of the most recent twelve months) is still much lower in Korean stock market (only 13.5) as against 60 in the Japanese stock market and about 20 in New York and London markets. Secondly, with Hong Kong's uncertain future, some funds may be channelled partly into the Korean stock market out of Hong Kong.

These inflows of capital will put even higher pressure on the Government monetary policy to keep down inflation. Already, the Government has issued bonds known as the "Monetary Control Bonds" and made mandatory sales to institutional investors. The Monetary Control Bonds already
amounts to US $6.8 bn in January 1989 and are expected to grow further with the continuing trade surplus.

3.3.2 Two Sources of Inefficiency

Two of the characteristics of the Korean stock market has been the existence of "Otte", a word derived from Japanese meaning "Big Hands" and Government intervention. The former is a group of investors who have unofficial links among themselves shifting jointly their portfolio holdings from one to the other such as to affect the price level. They buy heavily a particular type of shares only to resell quickly once the market is set in motion. Table 3-8 shows the different shareholders and their respective holdings while table 3-9 shows the distribution of share-ownership in size. We observe that a mere 1.3% of the total agents have almost 70% of the total market share. If there is some unofficial collective link even between some of the 10246 shareholders holding 10000 shares and more, we will observe a large change in the stock price level.

Now that the stock market has become an important financial institution in the economy and that free capital movement is expected in few years time, the level of Government intervention is reduced gradually the problem of Big Hand remains. As the volume of total wealth in the stock market increases, their influence will diminish. However, their existence in the stock market for the last two decades are bound to leave traces behind. Furthermore, investors who
### Types of Shareholders

(1985)

<table>
<thead>
<tr>
<th>Category</th>
<th># of shareholders</th>
<th># of shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gov &amp; Public Bodies</td>
<td>44 (0)</td>
<td>33788775 (0.41)</td>
</tr>
<tr>
<td>Banking Institutions</td>
<td>1086 (0.14)</td>
<td>569985934 (7.09)</td>
</tr>
<tr>
<td>Securities Companies</td>
<td>7590 (0.98)</td>
<td>590265769 (7.35)</td>
</tr>
<tr>
<td>Insurance Co. &amp; Others</td>
<td>9266 (1.19)</td>
<td>2408810765 (30.0)</td>
</tr>
<tr>
<td>Individuals</td>
<td>745363 (97.65)</td>
<td>4213837007 (52.48)</td>
</tr>
<tr>
<td>Foreign Holdings</td>
<td>122 (0.01)</td>
<td>211396615 (2.63)</td>
</tr>
<tr>
<td>Total</td>
<td>772471 (100)</td>
<td>8028084865 (100)</td>
</tr>
</tbody>
</table>

**Source:** Korea Stock Exchange, Securities Statistics Yearbook 1985
Table 3-9

<table>
<thead>
<tr>
<th>Share Holdings</th>
<th>100 shares and less</th>
<th>100 shares and more</th>
<th>1000 shares and more</th>
<th>10000 shares and more</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Shareholders</td>
<td>254222 (32.91)</td>
<td>237801 (30.77)</td>
<td>204984 (26.52)</td>
<td>65218 (8.43)</td>
<td>10246 (1.32)</td>
</tr>
<tr>
<td># of Shares</td>
<td>6564220 (0.08)</td>
<td>89467323 (1.1)</td>
<td>689949178 (8.58)</td>
<td>1722067106 (21.44)</td>
<td>5520037038 (68.75)</td>
</tr>
</tbody>
</table>

Source: Korea Stock Exchange, Securities Statistics Yearbook 1985
joined the stock market forced out of the highly speculative curb market and the property market will be determined to continue their old practices of making quick profit in a short time, in the stock market. These factors will push investors into myopic behaviour, highly speculative of what other investors will do at the next instance.
4. Conclusion

We first outlined four characteristics of NICs in terms of the state variables: real GDP growth, foreign debt, unemployment and Balance of Payment showing that Korea qualified as a representative country of NICs. NICs have enjoyed a relatively high GDP growth rate while being left with large foreign debt, low unemployment and a Balance of Payment deficit. In line with some other NICs, Korea enjoyed a rapid economic growth as a result of centrally planned and controlled Government policy and a firm grip of the authorities on limited financial resources. Export oriented firms were encouraged with low or even negative real interest payments on borrowing to finance new investments. Strategic industries were chosen, set up and promoted under various legislations. First target industry was textile, then light manufacturing industries, heavy and chemical industries and finally electronic industries were greatly encouraged. Each industry took about five to ten years to establish itself. The economy continued to change in the 80s. First came the deregulation of the financial market followed by a trade surplus in 1986. The former led to an extensive expansion of the financial market while the latter led to more open competition with foreign firms. The size of the economy has become too large for the Government to centrally control all aspects of the economy, and a trend towards a more laissez-faire attitude became inevitable.
This thesis attempts to provide a theoretical model on one aspect of the economy it encounters under this new trend. It will attempt to address the question of optimal strategy for a firm in competition against a firm in an Industrialized country. In particular, it will examine various licensing rules and possibility of leap frogging under different conjectures.
References

A World Bank Research Publication
John Hopkins University Press.

Calverley, J. (1982) "Korea, Exporting to Survive"
An Euromoney Special Study

Chapter 1, International Institute for Economics, MIT Press.

Harvard Institute of Economic Research
Harvard University # 1372.

Korea Development Institute (unpublished).

Ezaki, M. (1983) "Japan and the South East Asia: A Quantitative Appraisal of their Economic Relations"
SEAS # 117, Kyoto University.

Ezaki, M. (1985) "Economic Growth, Interdependence and Rivalry in South Asia"
SEAS # 119, Kyoto University.

SEAS # 118, Kyoto University.

Khan, M. S. (1986) "Exchange Rate Responses to Exogenous Shocks in Developing Countries"
World Bank Discussion Paper # 146.

Malkiel, B. G. (1979) "The Capital Formation Problem in the United States"


Economic Review, Seoul National University.

Korea Development Institute, Mimeo.
Korea Development Institute, Mimeo

Simonsen, M.H. (1983) "The Financial Crisis in Latin America"
Rio De Janeiro: Getulio Vargas Foundation

Tayor, L. (1980) "Models for Growth and Distribution for Brazil"
World Bank Research Publication, Oxford University Press

Bank of Korea Economic Reports (1960 - 1988)
Bank of Korea Economic Reviews (1960 - 1989)

Introduction to Science & Technology in the Republic of Korea (1984), Ministry of Science and Technology, ROK
Asymmetric Duopoly Competition
and
Product Selection
5. Strategy and Competition

The extensive study of the theory of oligopoly has not only been the result of an attempt to describe the real market structure but also the realization of the importance of individual firm's strategic decisions on the market equilibrium and the dynamics of evolution of the market structure itself. Based on such three-fold findings, the fundamental problem theorists encountered was what assumptions to make on an individual firm's conjecture on others, which variable to choose as the firms' strategic variable and how to accommodate the evolving market structure, thus intrinsically dynamic model, into a certain static model.

The two most frequently used strategic variables were quantity and price levels associated with the names of Cournot (1838) and Bertrand (1883) respectively. Their contribution was, however, not so much their particular choice of strategic variable but the introduction of the notion of firms' conjecture of other firms behaviour as a response to their own actions.

Using a symmetric duopoly\(^1\) model where goods are assumed to be homogeneous, Cournot assumes that firms determine their output level taking the prevailing output level of its rival as constant. In other words, a firm's
conjecture about its rival's reaction in response to its own action, expressed in terms of a conjectural variation, is zero. Bertrand, on the other hand believed that entrepreneurs controlled prices while using inventories to smooth sales. This means that each firm will try to undercut each other leading to a price war. The only stable equilibrium price level for both firms would be at their marginal cost level. When prices are equal to their marginal costs for a homogeneous commodity, one unit increase in the quantity supplied by one firm must lead to an one unit decrease in the quantity supplied by its rival. This may be denoted as a conjectural variation of minus one. This shows that in whatever strategic variable the value of the conjectural variation of a model is expressed, it may be translated in terms of another strategic variable yielding a different value of conjectural variation for the same model. It turns out to be that both Cournot and Bertrand competition are each extreme cases within noncooperative competition with respect to their choice of conjectural variation since the former assumes zero while the later assumes minus one implying a conjecture of one to one replacement of commodities by a firm in response to the other. Under Cournot competition, price level will be higher than the marginal cost with a smaller output supplied to the market. While under Bertrand competition, price level will equal to the marginal cost with a larger volume supplied to the market, given a downwards sloping demand curve. The reason for this difference is because under Cournot
competition, each firm behaves like a monopolist with given residual market share after deducting other firms' shares while under Bertrand competition, the "price war" leads to the same outcome as the perfectly competitive equilibrium.

Vives (1985) examined the efficiency of Bertrand and Cournot equilibria under a differentiated product market. His findings was that given a symmetric demand structure and unique Bertrand and Cournot equilibria, the Cournot competition yields a higher profit and price with a smaller quantity level. He also found that as the number of differential products grew, both equilibria converge to the efficient outcome. From a different aspect, Deneckere and Davidson (1985) argue that failure to explain industrial concentration arose from the almost exclusive attention given to quantity setting models rather than prices. The latter, yielding an upwards sloping reaction functions, provide a better theoretical justification as why merging may lead to higher price and profit levels. They state that the quantity setting models are useful only in that they provide a better insight into complex price setting model as shown by Kreps and Scheinkman (1983).

With the development of a wider usage of conjectures, the notion of Equilibrium had to evolve to accommodate the different circumstances of the market. A conjectural equilibrium is obtained when solving simultaneously the
first order conditions of firms' objective functions obtained under the assumption of profit maximization4).

The Cournot equilibrium is characterized by a vector of output levels upon which no single firm can unilaterally improve its profit. Therefore, a conjectural equilibrium is the Cournot equilibrium if conjectures in terms of output levels are zero.

The Nash equilibrium, on the other hand, is characterized by a vector of any of the possible strategic variables upon which no single firm can unilaterally improve its profit level by altering the strategic variable over which it maximized its profit in the first place. Therefore, a Nash equilibrium, obtained under the strategic variable of output level corresponds exactly to a Cournot equilibrium.

5.1 Consistent Conjectures
In an attempt to reduce this potentially many equilibria into a narrower band of equilibria, possibly into a unique equilibrium, rationality has been imposed on the behavioural assumptions of firms. This requires that a firm's conjecture about its rival's response to changes in its own decision variable must be equal to the actual responses of its rivals. Therefore a consistent (rational) conjectural variation is said to prevail if
this rationality assumption is imposed on firms behavioural assumption.

Laitner (1980) in an attempt to reduce the multiplicity of equilibria under conjectural models of oligopoly proposes two notions of rationality. The first notion, based on a static analysis is called rational at a certain output vector if predictions firms make on each others reactions in response to small changes in their own actions are satisfied. In other words, a firm will expect its rival to change its output level only in the direction of higher profit for itself. Therefore, the definition is satisfied as long as changes in each output levels brought increase to both profit levels. The second notion is based on a discrete dynamic model with positive cost of adjustment such that initial conditions as well as the evolution of output levels become important and nontrivial. A rational conjecture is said to prevail if the anticipation of a firm brought by an output change over time under profit maximizing objectives with the anticipation of its rival's dynamic output changes as a response to its own action is satisfied. Laitner proves, that under such a dynamic equilibria, multiplicity is further reduced but uniqueness of existence still not obtained.

Ulph (1983), on the other hand, claims that the problem of multiplicity of solutions under rational conjectures rests fundamentally on the weak specification of the definition of rationality. He begins his model by
defining a general notion of conjecture where an equilibrium is an output vector at which neither firm would be able to increase profit by changing their output given their "beliefs" about their rival's response. Since the equilibrium is based on beliefs, it leaves the possibility that it may be a false belief. In an attempt to verify their beliefs, each firm would have to engage in real changes to see the response of their rivals. An improvement in their belief of conjecture is obtained if firm i, say, can verify that firm j would be willing to produce the output level that firm i conjectures as a response to firm i changing its output level. This would only arise if firm j obtains at least as high a profit level by moving to the new output level given its conjecture about how firm i would then in turn respond. The implication of this definition is that firm j does not play strategically against i. If it were so, then firm j should have chosen a particular response which could lead i to hold conjectures favourable to j rather than respond passively. Therefore, if conjectures were to take such strategic considerations, the formation of conjectures would have to be altered and the definition of rationality correspondingly re-formulated.

The extreme side of the argument comes from Bresnahan (1981) and Perry (1982) who claim that the solution is unique under rationality assumption. Bresnahan, by equating the conjectural variation of a firm with the reaction function of its rival, showed that under
homogeneous product and constant marginal cost, Bertrand equilibrium is the unique consistent conjectures equilibrium (CCE). If on the other hand, the assumption of perfect substitutability is relaxed, CCE lies between Bertrand and Cournot but could still be unique. Therefore, under any given model structure, consistent conjecture will yield a unique equilibrium, although not identical in each case.

Perry (1982) examines consistent conjectural variation in a duopoly model with a homogeneous product. He defines consistency to prevail if the conjectural variation of one firm about its rival's response is equivalent to the derivative of its rival's reaction function with respect to its own output at equilibrium. He then examines the various cases of model structure with respect to cost and number of firms in the market: constant marginal cost, increasing and falling marginal cost and finally fixed and free entry situations. Not surprisingly, with identical definition of consistency as Bresnahan, he obtains the result of uniqueness too.

Kamien and Schwartz (1983) also derive the result that under homogeneous products and linear demand function, the Bertrand solution is the consistent solution in a symmetric duopoly model. Then, generalizing the demand functions, they show that each different formulation of a demand function yields a different conjectural variation with homogeneous products. This result is compatible with the result obtained by Perry and Bresnahan that
under each given unique model structure, there is only one conjectural variation which is consistent.

The problem evolving around consistent conjectures was not only restricted to multiplicity or uniqueness of equilibrium but also to the nature of the equilibrium itself. As just illustrated, unique consistent conjectures equilibrium is usually associated with Bertrand equilibrium. However, even this seem to be far from unanimous.

Daughety (1983) claims that there are actions other than the rational conjecture each firm can take, which will guarantee them higher profit. He claims that if firms are rational, then their rationality should be expressed in choosing an action which guarantees positive profit [e.g., Stackelberg follower\(^5\)] rather than zero profit which would result under consistent conjecture. His argument is based on his definition of consistent conjectures equilibrium. He defines a consistent conjectures equilibrium to be an equilibrium at which no individual changes in a decision variable is profitable. Therefore, the Bertrand conjecture is not a consistent conjectures equilibrium. By setting up a model where each firm has to solve an infinite regress problem, i.e., firm \(i\) maximizes its profit level subject to the maximizing behaviour of firm \(j\), which in turn will be based on the maximizing behaviour of firm \(i\) and so on. He then proves that the Cournot equilibrium is the consistent conjectures equilibrium.
5.2 Choice of Strategic Variable

Returning to the former part of the question as to which variable to select as the strategic variable, recent development in oligopoly theory have provided some degree of advances in this matter. There has been a general trend away from a single stage game and towards multi-stage games in the analysis of firms' behaviour under oligopolistic market conditions. A multi-stage game arises when firms have to consider more than one strategic variable when maximizing their objective functions. Each stage game usually involves determination of the level of one strategic variable. The outcome of one stage within a multi-stage game may influence the equilibrium of other stages as the decision in each stage of the remaining game will influence a firm's strategic position in the remaining stages in relation to other firms.

This general trend appears to have arisen due to the gradual realization of the complexity of the real structure of industries and the weakness of oligopoly theory to explain the behaviour of firms only in terms of a single stage competition involving the determination of only one strategic variable. Therefore, recent works have focused their analysis not only on the traditional strategic variables such as price or quantity level but also on the determination of capacity and the choice of product, one element of which is the quality level. This means that the timing of decisions and information
receptions are as important as the nature of the decisions themselves, leading to changes in the outcomes of duopoly analysis precisely because each stage of a multi-stage game is not independent of outcomes of other stages.

A multi-stage game gave rise to a new notion of equilibrium known as "Perfect Equilibrium". Since most multi-stage games used in oligopoly theory do not revise their planned strategy after each stage have been played, they are open loop strategies. Therefore, a Perfect Equilibrium in a open loop strategy is defined by Shaked and Sutton (1982) as follows:

"An n tuple of strategies is a Perfect Equilibrium, if after any stage, that part of the firms' strategies pertaining to the game consisting of those stages which remain, form a Nash Equilibrium in that game." (Shaked and Sutton (1982) P3)

The definition of Perfect Equilibrium requires that the last game be planned first, working recursively backwards and that it does not allow non-credible threats to be made. The equilibrium at each stage of the game, if Perfect Equilibrium is to prevail for the multi-stage game, is said to be sub-game Perfect Equilibrium. The difference between a single period game and multi stage game lies in that the former requires simultaneous equilibria in all relevant variables while the latter considers the equilibrium of each variable in sequence
such that the first stage game equilibrium serves as the initial condition of the next stage game. Therefore, a multi stage game equilibria may be regarded as refinement of Nash equilibrium of a single period game.

We have categorized some of the recent literatures using multi-stage games in their analysis of firm's behaviour into two groups. The first group assigns the capacity level as the strategic variable for the first stage of the multi-stage game while the later stage considers the levels of the product quality. But first, we outline a two stage game under imperfect information which differs from other works in its characterization of the different stages.

Among the literature on imperfect information assumptions (e.g., Schmalensee (1982), Wolinsky (1983), Shapiro (1982) and Klein & Leffler (1981)). Schmalensee is unique in introducing a two stage game. His work is based on empirical findings, in which later entrants to a market must be able to offer distinct benefits, and not just lower prices, if they are to overcome the pioneering brands. Unlike a standard duopoly analysis, he introduces two stages where each firm passively participates in turn, i.e., they lack active game theoretic interaction in product competition. The first firm introduces a pioneering brand in the first stage of the game while the second firm enters the market in the second stage. Both brands are assumed to have identical cost structures. It is the imperfect information of the
consumer on the quality of the product that plays a decisive role in giving the pioneering brand an advantage over the second brand. Since consumers are uncertain about the second brand being introduced into the market, they have strong inclination to keep to the pioneering brand. This result is obtained despite the fact that consumers have initially some subjective probability that a new brand may yield higher than their expected quality level for each price level. To be more precise, he categorizes products either as "work" or "don't work". Therefore, a high value of expected quality level for each price implies that a product "works". The fact that consumers have tested the quality of the pioneering product and found out that it works is sufficient for them to keep their initial choice of brand. Elimination of this assumption of imperfect information would lead the model in its second stage back to the standard duopoly equilibrium.

Other works have two strategic variables in their model, e.g., price and quality level. Instead of adopting a two stage game, their analysis are based on one period competition determining a combination of price/quality level. In particular, Wolinsky provides a model in which firms are capable of producing products at different quality levels. Consumers are assumed to have imperfect information. They have only partial knowledge of the quality of a firm's product. Then a notion of fulfilled-expectation equilibrium is introduced at which all price signals are such that each firm's profit maximizing
quality is what is being signalled by its price. This is attained because the availability of the information on the quality of firms, though imperfect, would enable some consumers to detect lower quality. A firm with sufficiently high mark up even under the fulfilled-expectation equilibrium condition would then want to avoid the consequent loss of sales by not providing a quality level signalled by its price level. Perfect information would reduce this model back to a standard vertically differentiated oligopoly market.

Under the perfect information assumptions, Kreps & Scheinkman (1983), Eaton & Grossman (1984) and Ireland (1986), introduce two stage games. It is a perfect information model in the sense that consumers know the quality level of the product while demand functions are fully revealed to firms. The former two articles assign the capacity level as determining the marginal cost (or productive efficiency) in the first stage, which in turn provides the basis of competitiveness in the next stage game of marketing the product. The last article, however, introduces the capacity level as a restriction on the potential quantity supplied at the next stage game of price competition.

Ireland (1986) introduces a two stage game in which firms decide on R&D expenditure in the first stage, the level of which will then determine the unit variable cost of production. At the second stage, product marketing takes place. He then argues that i) firms will choose price
level rather than quantity as their strategy variable, because of the assumption of unlimited capacities and differentiated product, ii) the possibility of a Nash equilibrium is minimal while a price-setting Stackelberg leader will emerge. The result of the second argument is obtained due to a specific assumption on consumers which give rise to a discontinuous demand system. He defines one group of consumers as r-class who simply buy the cheaper product irrespective of quality difference. A price war occurs as firms attempt to attract this r-class group until the firm with a slightly high cost function will opt for a smaller sales volume at a higher price. This will then lead other firm to raise the price level until another price war sets in, i.e., an Edgeworth cycle. This disequilibrium cycle of price competition prevails unless a certain condition, e.g., a particular pair of marginal costs, necessary for the existence of Nash equilibrium in this model is attained. Then, in a repeated game, a Stackelberg leader may emerge as this would benefit both firms compared to remaining in a disequilibrium cycle. Suppose one firm has a lower cost of production. The other firm will realize that once a price war sets in, it would not win due to the higher cost. Therefore, despite knowing that a slight reduction of its price would yield higher profit, it would not do so because of its conjecture of the other firm's reaction.

Eaton and Grossman (1984) present a model with two stages of game in which firms invest in a level of capacity
other than that which minimizes cost. This is attained as they assume that capacity investment behaviour in the first stage will determine the total cost of production in the second stage. Therefore, in order to alter the nature of the second stage competition, a particular level of investment is chosen which does not necessarily correspond to a minimizing cost level. The level of investment serves implicitly as a signal of each firm's strategy. Thus, they identify two motives for this divergent behaviour; to obtain a better strategic position at the next stage and to effect a reduction in capacity investment by rivals. They find that firms have incentives to overinvest as the conjectured response by their rivals in the marketing stage exceeds the actual response or if they believe that overinvestment itself will induce their rivals to opt for smaller capacity. As special cases of their work, conjecture based on Cournot-Nash will lead to overinvestment in capacity as shown by Brander and Spencer (1983) while a Bertrand conjecture will lead to an underinvestment as shown by Bulow, Geanakoplos and Klemperer (1983). Finally, only if both firms rightly perceive each other's conjectures will they revert back to the cost minimizing level of capacity investment.

Kreps and Scheinkman (1983) employ a two stage competition in an attempt to reinterpret the notion of Cournot as against Bertrand competition. By expressing their view that a realistic view of competition contains more than one stage of competition, they introduce the
capacity level as their strategic variable in the first stage followed by the second stage competition in price determination. Since firms cannot produce more than their capacity level decided upon in the first stage, the choice of capacity level determines the outcome of the second stage game. With an increasing cost function for capacity installation, each will attempt to set their price such as to sell nearest to its capacity level but at the same time, high enough to cover the capacity installation cost. With product homogeneity assumption, it follows that the firm with lowest price will sell the minimum of full capacity output and demand at its price level, while the other firm will sell the minimum of full capacity output and the maximum of zero and demand at its price level minus the full capacity output level of the other firm. The level of capacity is determined under zero conjectures. Unlike the standard Cournot assumption, however, the price level is not determined by an auctioneer, but by a Bertrand competition where actual production of products are assumed to take place at zero cost. They then prove that when capacities correspond to Cournot competition in the first stage, price levels in the second stage will in fact be the Cournot price, i.e., the price level they would have obtained under an auctioneer. As special cases of this work, Levitan & Shubick (1972) proved the existence of a subgame equilibrium for symmetric capacity levels while Dasgupta and Maskin (1982) established the existence of subgame equilibrium for all pairs of capacity levels. Kreps and Scheinkman did not escape criticisms, particularly with
respect to their strong findings that a subgame perfect equilibrium of their two stage game corresponds to an one stage Cournot competition. Eaton and Harrald (1988) points out that their result is sensitive to the rationing rule of the second stage game they employed. By comparing their analysis with that of Edgeworth, they show that under a linear demand function and small marginal cost relative to the price level at zero quantity, there is no rationing rule as such.

Brander & Spencer (1983) and Shaked & Sutton (1982) suggest the quality level of the product as their strategic variable for the first stage of the competition prior to the marketing stage game. The former adopts a Cournot approach while the later uses a Bertrand approach in their duopoly model at the final stage of the game. Although their approaches to the analysis of supply exhibit similarities, their objectives have a number of differences. Like the work by Eaton and Grossman, Brander and Spencer show that strategic use of R&D will increase the total amount of R&D undertaken. By introducing a symmetric two-stage Nash duopoly model, they also show that firms output will be higher, accompanied by a lower profit compared to a non-strategic use of R&D and simple cost minimizing behaviour of firms. If one firm alone uses R&D strategically, it can increase both its output and profit at the expense of other firms. However, if all firms attempt to influence the final outcome, there is a tendency for output to be
higher and profit and prices to be lower than in a corresponding industry without strategic R&D.

Shaked and Sutton (1982) presents an analysis based on a three stage non-cooperative game. Stage one involves decision of individual firm's entry. The second stage involves choice of quality level while the last game is on price determination. They apply the concept of a Perfect Equilibrium to the three stage game. They introduce a continuum of consumers homogeneous except in income $t$ while income itself is assumed to be uniformly distributed on some support $0 < a \leq t \leq b$. Each consumer is supposed to buy exactly one unit or nothing while firms produce one product quality each. Assigning a parameterized utility function $u(t,k) = u_k t$ to consumers where $u_k$ represents the utility level obtained by consuming product $k$ supplied by firm $k$, they prove that under the restriction $2a < b < 4a$, exactly two firms will have positive market share at the equilibrium which will exhibit two distinct product qualities and prices. This result for the third stage of the game (price competition) is obtained as competition between high quality products drives down the price level such that even the lowest income group would not purchase the lower quality products and the fact that consumers with income $t > t_k$ strictly prefer good $k$ at price $P_k$ to $k-1$ at $P_{k-1}$. Assuming subgame perfect equilibrium, the second stage game on quality competition is equivalent of finding a Nash Equilibrium in qualities on the basis of payoffs determined by the subsequent price level competition.
They show that firms will produce different quality product from each other by using two properties of the Revenue function: the revenue of the firm producing higher quality product is greater than the other and the revenue of both firms increase if the firm producing the higher quality product increases its quality level further. They then proceed to prove that for more than two firms in the market, there will only be one firm left producing top quality product at zero mark-up, i.e., zero because cost of production is assumed to be zero. The reason is that for more than two firms in the market, competition in the quality choice will drive all firms to set the same top quality while price will therefore be driven to zero. It follows that the most efficient firm will survive.

Hung and Schmitt (1988) continue the subject of vertical product differentiation and introduce additional features to Shaked and Sutton, namely production costs and sequential entry of firms. Their cost function are such that the first entry firm pays only a fixed amount \( F_1 \) to produce a product of a quality \( \theta_1 \). The second entry firm will have to pay \( F_2(\theta_1) \). \( F_2 \) is an increasing function of \( \theta_1 \). The existence of positive cost of entry and the quality choice implies that \( b > 2a \) is no more a sufficient condition because firms may not cover their costs and secondly, the first entry firm may deter entry of other firms. The degree of deterrence is determined in the second stage of the game. Under the same restriction of \( 2a < b < 4a \), existence of only two firms in the
market, quality level chosen by each firm is such that the second entry firm will have zero profit. For situations of more than two firms, the choice of quality by the second entry firm is to equate its revenue to its cost of entry. In other words, the quality choice of the second entry firm is such that no other firm can enter while the first entry firm can influence other followers' quality choice to its advantage. Furthermore, they prove that once the first entry firm opts for entry-accommodating equilibrium, (i.e., it chooses a level of quality such that other firms can afford to enter), it selects the highest product quality. Once threat of entry is introduced, they find that the highest product quality is not necessarily available on the market but depends on the cost structure of potential entry firms. This result is attained by examining conditions sufficient to adopt an entry-deterrence strategy by the first entry firm.

5.3 R&D and Market Structure

Developments in the evolutionary process of market structure were examined from various different aspects. They involved market structure changes due to new entry and exit of firms. As initially developed by Bain (1956), Modigliani (1958) and Sylos-Lambini (1962) with their Sylos Hypothesis based on the theory that the incumbent firm will behave in such a manner as to maintain a high pre-entry cost level for potential entry even if this may
not be a profit maximizing behaviour. Therefore, Sylos hypothesis implied an irrevocably-committed policy of fixed output. An alternative model, as developed by Wenders (1971) and Spence (1977) was based on the Excess Capital Hypothesis which allowed the incumbent firm to vary its output assuming that the potential entry firms base their decision on the capital employed by the incumbent firm whereas the incumbent firm adopts a capital level above the entry deterring level. More recent articles on this topic includes work by Hung & Schmitt (1988) as already discussed before.

Our attention is focused here, however, on the alternative approach to the evolution of market structure centering on investment into R&D leading to product or process innovation.

The analysis of innovation and market structure is a long standing problem dating back as far as Schumpeter (1943) with his remarks on evolutionary processes to include the statement:

"Capitalism, then, is by nature a form or method of economic change and not only never is but never can be stationary. ... The fundamental impulse that sets and keeps the capitalist engine in motion comes from the new consumers' goods, the new methods of production or transportation, the new markets, the new forms of industrial organization that capitalist enterprise create" (Schumpeter 1958, p 82-83)
During the last decades, many studies have concentrated on this issue, namely around the key concepts of preemptive patenting, market leadership and R&D competition. Until recently, studies in this area have been confined to a single period analysis, leaving market structure as endogenously determined by the behaviour of the incumbent firm(s) and (would be) followers.

For example, Scherer (1976) and Kamien & Schwartz (1975) explored the causal connection between the degree of concentration and R&D efforts on empirical findings. Galbraith (1973), on the other hand, examined the long run structural changes of industries to depend on technology of research and the nature of capital market endogenizing even the demand conditions. Dasgupta & Stiglitz (1980a) analysed theoretically the positive association between industrial R&D expenditure and the degree of concentration. They found that the market equilibrium sustains an unwarranted number of firms such that industry wide, R&D expenditure is excessive despite each firm undertaking less than the socially optimal level of R&D activity. In Dasgupta & Stiglitz (1980b), they develop their model further and examine the effect of competition in R&D on the level of research as well as the effect of R&D competition in the product market. They found that competition always leads to more research than in a pure monopoly and that a monopoly will persist by deterring new entry, if the R&D technology is also
available to the present monopolist in the product market.

Still on the same subject of R&D competition and first mover advantage, Reinganum (1981) adopts a more game theoretic approach to this topic. She develops a theory of optimal resource allocation to R&D under the assumption of uncertain technical advance and in the presence of game-playing rivals. In this framework, it is assumed that the firm which first perfects the innovation, takes the whole market share, i.e., becomes a monopolist. Uncertainty in the timing of successful innovation induces firms to make contingent plans for R&D investment. She then derives the conclusion that Nash equilibrium and the socially optimal rates of investment in R&D do not coincide.

As for the specific question of the incumbent firm dominating the market persistently, Gilbert and Newbery (1982) show that a firm can maintain a monopoly through preemptive activity even under many potential entrants. As long as the monopoly profit after introducing a new product is larger than the total industry profit given a rival patents, the monopoly profit from preemptive patenting will strictly exceed the monopolist's profit with entry of a rival. Given threat of entry, monopolist will preemptively patent a substitute and put it even into sleep, i.e., innovate but not use it in order to prevent potential firms entering the market and making use of the substitute.
John Vickers (1985), on the other hand, considers the case of several incumbent firms. He points out that the basis for preemptive patenting leading to persistence of monopoly lies in asymmetry of incentives between the incumbent and the potential entrant. He suggests that several incumbent firms may serve as a source for breaking this asymmetry of incentives for two reasons. Firstly, if there are more than one incumbent firm, the gain in preemptive patenting may not exceed the cost involved. Secondly, unless incumbent firms have some kind of a collusive agreement, there may be a tendency of firms opting for a free-rider as entry deterrence may be regarded as some sort of a public good.

Recent attempts to develop a sequential model of a patent race include works by Beath, Katsoulacos and Ulph (1987, 1988), Harris and Vickers (1985, 1987), Reinganum (1985) and Vickers (1986). Beath, Katsoulacos and Ulph (1988) in particular, claim that single period formulation is simply incapable of accommodating the dynamic process of market leadership or an exchange of leadership. They postulate that models which incorporate an explicit sequential structure could truly explain the dynamic market competition with respect to innovation. They argue, for example, that if a firm wins a succession of patents, it may then be able to operate as a monopolist whereas if it loses, the market may become more competitive. Anticipation of such an accumulation of market power will have a critical bearing on each firm's
valuation of any patent. If there is product innovation and only two firms engage in innovation, then increasing the patent protection to three periods can result in an equilibrium with persistent Bertrand competition while increasing the number of firms in innovation competition will lead back to persistent monopoly.

Reinganum (1985) develops a model based upon a sequence of innovations with finite period. Then solving recursively from the last sequence and assuming subgame perfect Nash equilibrium, she shows that there is no single persisting dominant firm. This result is obtained due to the finite sequence assumption. In the last sequence, the incumbent firm has less incentive to invest in R&D to speed up new innovation as it is enjoying a monopoly profit already. Its rivals, however, have nothing to loose when innovating earlier. This will lead to a change of leadership. Solving recursively under subgame perfect equilibrium then results in action/reaction of leadership.

Vickers (1986), on the other hand, points out that in Reinganum's (1985) model, the dynamics of the model does not play any role in altering market structure. He develops a model in which asymmetry of market structure gives rise to asymmetric incentives in each patent race. The outcome of each sequence determines the asymmetry of market structure of the following sequence. He derives a surprising result that greater degree of competitiveness in each sequence leads to persistence of monopoly. Since
innovation is assumed to arise in the process rather than in the product, i.e., cost saving innovation, the incumbent firm at one sequence has better position to win the next race due to a lower cost of production as this will enable the incumbent firm to change to a lower price level.

5.4 Modelling the Duopoly
In view of these recent developments in the theory of oligopoly, we adopt subgame perfect equilibrium of a two stage game to analyse the behaviour of firms in an asymmetric duopoly. Each firm is to represent a firm in NICs (Korea in particular) and OECD respectively. The lack of evidence concerning the behaviour of the Korean firm in particular with respect to its conjecture, lead to the view that an asymmetric Nash duopoly serves better to analyse the real world than a Stackelberg leader and follower analysis.

The asymmetry of the duopoly is based on the following findings. Firstly, firms in Korea have enjoyed a lower wage rate and production costs due to restrictive Government policies on union movements. Being a newcomer to the industries has meant that it has been able to construct plants in a more efficient manner. Secondly, heavy Government subsidies and incentives, such as almost free access to financial resources, suggest that Government actively encouraged firms to maximize revenue
rather than profit. The underlying objectives were not only to obtain a larger world market share but also to obtain foreign currency to finance imported raw materials which they needed to produce the export commodities in the first place. Thirdly, firms in Korea still lag behind in the level of technology and innovation. The firms we attempt to describe in this work may resemble most closely an electronics firm in Korea, perhaps established only one or two decades ago.

Therefore, in modelling the duopoly, we will assume that the firm in Korea has lower cost of production, dual objectives and a lower level of quality. Furthermore, the exchange rate is assumed to be fixed to unity such that price levels do not have to be converted in terms of the other nor to a common currency unit. Since some firms in Korea, especially those designated as export oriented firms, were discouraged or even restricted to supply to domestic market, we will assume that the market for both firms is restricted to consumers in the industrialized country.

In view of the circumstances firms face in Korea as illustrated in the introduction, we will analyse the problem of product (quality) selection of the firm in Korea and the behaviour of the firm in the industrialized country in response to a challenge to its market leadership. In the first stage game, we shall assume that firms engage in the selection of quality level corresponding to which they have to determine their level
of investment into R&D. In the second stage of the two stage game, we analyse the marketing competition between the two firms.

In chapter 6, we present a basic duopoly model based on Ireland (1987). At this stage, we assume the cost of development and innovation to be zero. Since we want subgame perfect equilibrium to prevail, we shall first analyse the outcome of the second stage of the game, namely marketing.

The problem we outlined before remains, i.e., the choice of strategic variable for the marketing stage. Although it may be true that Cournot competition, thus the choice of quantity as the strategic variable is the dominant strategy\(^8\), this does not mean that firms in the real world will adopt this.

The outcome of this marketing stage will be different depending sensitively on what conjectural variations we assume firms to believe. We therefore derive the outcome of the second stage game under Cournot competition in chapter 7. In the following chapter, we repeat the marketing stage in terms of Bertrand competition and compare the outcome. The first stage game is analysed in each section with and without positive cost of development. In chapter 9, we propose an alternative solution to this problem; the consistent conjectural assumption. We find that given full information and
rational firms, the Cournot conjectures equilibrium is likely to prevail after all adjustment has taken place.

In chapter 10, we consider the relationship between the leadership and quality competition. We show that the decision to stay a leader or a follower depends on certain cost conditions. Under Bertrand conjectures in the marketing stage, either the incumbent firm keeps its leadership or it will be driven out of the market due to its higher cost of production.

In chapter 11, as an extension of the previous chapter, we set out various licensing rules and derive a dominant strategy. Given such a strategy, we examine the possibility of lead frogging and the corresponding market structure.

Finally, the last chapter examines the optimal timing of new innovations still under the same topic of market leadership. We show that optimal timing depends on the initial levels of quality, efficiency in R&D techniques and the planning horizon of both firms.
6. An Asymmetric Duopoly Model

We consider a market adapted from Ireland (1987), where consumers have identical tastes and identical information on all relevant aspects of the market including the quality levels of all products. The only element that distinguishes consumers from each other is their level of income. Income is assumed to be uniformly distributed with a density of unity on the support defined by the interval $b \geq y \geq 0$ where $y$ denotes income. Therefore, the total number of consumers in the market is $b$. Furthermore, no two consumers have the same level of income while the difference between income levels in the ordered sequence of consumers is constant. The ordered continuum of consumers with respect to their income is such that the $i$-th consumer has a higher income to the $(i-1)$-th consumer. Consumers have perfectly inelastic demand functions at unity. In other words, each consumer buys only one or zero unit. All products are indivisible and can be ranked in terms of a commonly agreed quality parameter $a$ such that $a_1 > a_2$ implies product 1 of higher quality than product 2. A higher quality level may be obtained from a better design, performance or improved specification. Therefore, the market in question is a vertically differentiated product market.
6.1 The Consumers

We assign a utility function to each consumer as follows:

\[ w(\alpha, p; y) = \alpha(y - p) \]  \hspace{1cm} (6-1)

Thus, in the \((\alpha, p)\) space, the utility function is a rectangular hyperbolic. Note that under (6-1), we may interpret \((y-p)\) also as a composite commodity measured as a continuous variable.

We limit the number of firms in the supply side to two firms and exclude the possibility of potential entry of other firms. Each firm is producing only one type of product the quality of which may vary. Firm 1, to denote the firm in an industrialized country, maximizes profit while firm 2, a firm in Korea, has dual objectives: maximization of profit and revenue as already justified in the introduction. We endow firm 2 with a lower cost of production relative to that of firm 1. The cost functions are assumed to be linear in cost and quantity space with the marginal cost equal to the average cost for both firms.

The objective functions of both firms are thus defined as follows:

Firm 1 \hspace{1cm} \text{Max } \pi_1 = (p_1 - c_1) q_1 \hspace{1cm} (6-2)

Firm 2 \hspace{1cm} \text{Max } U_2 = s \pi_2 + (1 - s) R_2 \hspace{1cm} (6-3)

where \(p_i\): price level of firm \(i\)
\(c_i\): cost of production of firm \(i\)
\(q_i\): quantity supplied by firm \(i\)
\(0 \leq s \leq 1\)
\( \pi_2 = (P_2 - c'2)q_2, \quad R_2 = P_2q_2 \) and \( c_1 > c'2 \)

Rearranging (6-3) yields

\[ U_2 = s \left( \frac{1}{s} p_2 - c'2 \right) q_2 \]

or equivalently as

\[ U_2 = (p_2 - c_2) q_2 \]  \hspace{1cm} (6-4)

where \( c_2 = s \cdot c'2 < c_1 \)

A fall in \( s \) implies that firm 2 is shifting its priority to revenue maximization relative to profit. This has the same effect as scaling down the cost parameter in its objective function. While we assume \( c'2 < c_1 \) (and it follows that \( c_2 < c_1 \) because \( s \leq 1 \)), we endow firm 1 with a first mover advantage into the market such that it may produce its product at a higher quality level than its rival. This specification follows from the fact that firms in NICs are late comers into markets where firms in industrialized countries have already established themselves for a long time. At this stage, we assume the cost associated with quality improvement to be zero.

If \( a_1 \) represents the quality index of firm 1's product and \( a_2 \) that of firm 2's product, then for price levels \( (p_1, p_2) \), two consumers with income \( y_1 \) and \( y_2 \) such that \( y_1 > y_2 \), the following must hold:

1) If \( p_2 > p_1 \), both consumers will prefer \( q_1 \) to \( q_2 \) because
\[ w_1(a_1, p_1: y_1) > w_1(a_2, p_2: y_1) \] and \[ w_2(a_1, p_1: y_2) > w_2(a_2, p_2: y_2) \]. This is obvious under the particular specification of the utility function, namely that \( w_1(a_1, p_1: y_1) = a_1(y_1- \)
p1) and \( w_2(a_2, p_2 : y_1) = a_2(y_1 - p_2) \). Therefore, if \( p_2 > p_1 \), then \( w_1(a_1, p_1) > w_1(a_2, p_2) \) for any \( y \).

2) If a consumer with \( y_z \) prefers \( q_1 \), then a consumer with \( y_1 \) will prefer \( q_1 \) also. This follows from the fact that if \( a_1(y_z - p_1) > a_2(y_z - p_2) \), then small increase in \( y_z \) to \( y_1 \) will hold the inequality sign for \( a_1 > a_2 \).

3) If consumer with \( y_1 \) prefers \( q_2 \), then consumer with \( y_z \) will prefer \( q_2 \) also because if \( a_2(y_1 - p_2) > a_1(y_1 - p_1) \), then a fall in \( y_1 \) to \( y_z \) will hold the inequality sign for \( a_1 > a_2 \).

Finally, note that if there exists a continuum of consumers with respect to their income distributed uniformly with density one and the particular utility function as specified above, we may define an individual with an income of \( y^* \) such that he is indifferent between good 1 at \( (a_1, p_1) \) and good 2 at \( (a_2, p_2) \). In other words, the following holds:

\[
a_1(y^* - p_1) = a_2(y^* - p_2) \tag{6-5}
\]

The intuition behind this is that quality may be regarded as a general superior good such that higher income groups desire higher quality products. Refer to figure 6-1 where \( w_1, w_2 \) and \( w_3 \) represent one particular contour each of the utility functions of three consumers with income \( y_1, y^* \) and \( y_z \) respectively such that \( y_1 < y^* < y_z \). As these contours will become perfectly inelastic at their respective income levels, the contours with lower income will correspondingly be steeper. Suppose that good 1 and good 2 are characterized by \( (p_1, a_1) \) and \( (p_2, a_2) \) respectively. The
CONSUMER INDIFFERENCE CURVES AS FUNCTIONS OF THEIR INCOME
contours of the three consumers are chosen such that all three pass through \((a_2, p_2)\). Under (6-5), it follows that \(y^*\) must be such that \(w_2(y^*)\) passes through point \((a_1, p^*)\) as well. Note that the consumer with \(y_1\) would only choose good 1 if \(p_1 \leq p_1^1\) while consumer with income \(y_2\) would choose good 1 as long as \(p_1^2 \geq p^*\). Therefore, this tells us that for any combination of \((a_1, p^*)\) and \((a_2, p_2)\), there must always be some individual with income \(y^*\) such that (6-5) is satisfied.

6.2 The Demand Function

The endowment of a higher quality for firm 1 allows it to take up the share of the market consisting of the higher income group. Depending on its combination of quality and price level, there will be \(b-y^*\) consumers who would be willing to buy good 1 while the rest of the market is left for firm 2 depending on values of \((a_2, p_2)\).

In order to derive the demand functions, we first derive \(y^*\) from (6-5) which is given as follows:

\[
y^* = \frac{a_1 p_1 - a_2 p_2}{a_1 - a_2} \quad (6-6)
\]

Therefore, the sales of firm 1 is given as below:

\[
q_1 = b - \frac{a_1 p_1 - a_2 p_2}{a_1 - a_2} \quad (6-7)
\]
Note that \( b \) denotes the total number of consumers in the market. As for firm 2, which is left with a market share of \( y^* \), it will be able to supply \( q_2 \) for given values of \( \alpha_2 \) and \( p_2 \) where \( q_2 = y^* - p_2 \). This follows directly from the utility function in (6-1) where \( \alpha_2(y^* - p_2) \) must be weakly positive for consumers to participate in this particular market at all. Therefore, consumers with an income \( y \geq p_2 \) will participate in this market such that the demand for \( q_2 \) becomes as follows:

\[
q_2 = \frac{\alpha_1 p_1 - \alpha_2 p_2}{\alpha_1 - \alpha_2} - p_2
\]

For notational convenience, we define \( \beta = \frac{\alpha_2}{\alpha_1 - \alpha_2} \) such that the demand function (6-7) and (6-8) may be rewritten as below:

\[
q_1 = b - (1 + \beta) p_1 + \beta p_2 \quad (6-7')
\]
\[
q_2 = (1 + \beta) (p_1 - p_2) \quad (6-8')
\]

The inverse demand functions are accordingly given by:

\[
p_1 = b - q_1 - \frac{\beta}{1 + \beta} q_2 \quad (6-9)
\]
\[
p_2 = b - q_1 - q_2 \quad (6-10)
\]

Note that the inverse demand functions diverge by the difference in the quality level and that they will collapse to the same function as \( \alpha_2 \) approaches \( \alpha_1 \), i.e., \( \beta \to \infty \). If this occurs and \( c_1 = c_2 \), then we have the standard duopoly solution of either a Cournot\(^{10}\) or a Bertrand\(^{11}\) competition. Cournot solution will yield \( p^c > c \) and \( q^c \) where \( p^c = p_1 = p_2 \), \( c = c_1 = c_2 \) and \( q^c = q_1 = q_2 \) whereas

\[
\]
Bertrand solution will yield $p^B = c$ and $q^B$ such that $q^c < q^B$. We shall consider this case when $\beta$ is finite in the following two sections.

6.3 Two Stage Game

In particular, we shall consider a two stage non-cooperative game to analyse the outcome of this duopoly in which there are two strategic variables; price or quantity and quality level. In stage 1, firms decide on their value of $\alpha_i$ which will then provide them with different degrees of competitiveness for their next stage game of marketing. Since firms' competitiveness depend on the relative quality level as is evident from (6-10), we shall use the variable $\beta$ in the first stage of the game, which is equivalent to using $\alpha_1$ keeping $\alpha_2$ constant or using $\alpha_2$ keeping $\alpha_1$ constant.

As we propose a Perfect Equilibrium to prevail in the market, we require a Nash equilibrium under given strategies for the second stage competition after firms have determined their respective quality levels. We define a Nash equilibrium to exist if neither firms could increase their own payoffs by deviating from their equilibrium strategies given other players use their equilibrium strategies. Stage 2, will be considered first. In section III, firms will maximize their respective objective functions with respect to their quantity. We derive a non-cooperative Cournot-Nash equilibrium. Then the first stage is analysed where firms now predict the choice of quantity in the second stage as
the Nash outcomes of that game. In chapter 8, firms will maximize their objective function with respect to their prices in the marketing stage. A Bertrand non-cooperative Nash equilibrium is derived. Then, as in the Cournot case, the first stage is analysed. In both the Cournot and Bertrand model, the second stage is analysed under the assumption that $\beta$ has already been determined in the previous stage.

6.4 Conclusion

We have introduced a demand function derived from consumers differentiated only by their income level. By restricting the income distribution to a constant density function, we simplified the demand functions of firms. A non-constant unitary density function would have required the integral of the density function from $b$ to $y^*$ for firm 1 of which $b - y^*$ is a special case. By imposing different endowment characteristics to each firm, (higher cost function for firm 1 relative to firm 2, first mover advantage to firm 1 allowing first choice of market segment, we have derived the inverse demand functions for both firms.
The Quantity-Setting Model

The second stage of the game is considered first on the assumption that $a_1$, $a_2$ and thus $\beta$ are already determined. Firm 1 and firm 2 maximize $\pi_1$ and $U_2$ with respect to $q_i$ for $i = 1, 2$ respectively. We obtain the first order condition for firm 1 as follows:

$$\frac{d\pi_1}{dq_1} = (p_1 - c_1) + (\frac{dp_1}{dq_1}) q_1 = 0 \quad (7-1)$$

where $\frac{dp_1}{dq_1} = \frac{\delta p_1}{\delta q_1} + (\frac{\delta p_1}{\delta q_2})\frac{dq_2}{dq_1}$

and $\frac{dq_2}{dq_1}$ is firm 1's conjecture on firm 2's response

In other words, the net change in its own price level (as a response to a change in its quantity supplied) is equal to the sum of the change of its price level (as a response to its own quantity change), and the change in quantity of its rival (due to a change in its own quantity) multiplied by the change in its own price level (due to its rival's change in quantity), which it conjectures would be the rival's response to the change $dq_1$. Under a Cournot competition, $\frac{dq_2}{dq_1} = \frac{dq_1}{dq_2} = 0$ where superscript $c$ is used to denote Cournot conjectures. In other words, the conjectural variation expressed in terms of quantity is zero.
7.1 The Second Stage Game

7.1.1 The Demand Function

Substitute the inverse demand functions (6-9) and (6-10) into the objective functions (6-2) and (6-4). From the first order condition (7-1), we obtain for firm 1 the following:

\[[b-q_1-(\beta/1+\beta)q_2-c_1]q_1 = 0\]

Solving for \(q_1\) gives us the optimal response quantity by firm 1 for each given values of firm 2's output, i.e., the reaction function of firm 1. It is given by:

\[q_1 = \frac{1}{2} (b-c_1) - \frac{1}{2}(\beta/1+\beta) q_2\]  (7-2)

Likewise, for firm 2, we derive the first order condition of the objective function \(U_2\) and solve for \(q_2\) to obtain the reaction function given by:

\[q_2 = \frac{1}{2} (b-c_2) - \frac{1}{2} q_1\]  (7-3)

The reaction function of a firm is the profit maximizing output level defined as a function of its rival's output level and the slope of which represents the rate at which the firm's profit maximizing output will change as a response to a change in its rival's output.

An equilibrium will be established, given these two reaction functions, where \((q_1^c, q_2^c)\) satisfies both (7-2) and (7-3), i.e., \((q_1^c, q_2^c)\) is the solution to the two simultaneous equations. \((q_1^c, q_2^c)\) is the Cournot Nash
equilibrium as \( q_{1c} \) is the best reply given \( q_{2c} \) and vice versa. They are given as follows:

\[
q_{1c} = \frac{\beta(b-2c_1+c_2) + 2(b-c_1)}{4 + 3\beta}
\]

(7-4)

\[
q_{2c} = \frac{(1+\beta)(b+c_1-2c_2)}{4 + 3\beta}
\]

(7-5)

The asymmetry in the cost of production and the first mover advantage to firm 1 of this duopoly model are the causes of such asymmetric demand functions. If we restore the asymmetric assumptions to the standard duopoly model, i.e., \( a_1 = a_2, s = 1 \) and \( c_1 = c_2 = c \), then \( q_{1c} = q_{2c} = (b-c)/3 \) which is the well known solution to symmetric Cournot competition. Our asymmetric duopoly has infinitely many outcomes depending on the relative values of \( a_1 \) and \( a_2 \). We assume at this stage that the cost of innovation is zero.

The possible values of Nash Equilibrium for each value of \( \beta \) is shown in figure 7-1 as the line NN'. The line \( r_1 \) denotes the reaction function of firm 1. If the quality difference between the two firms is infinite (\( \beta = 0 \)), then the reaction function of firm 1 is perfectly horizontal implying that the optimal quality level for firm 1 is independent of that of firm 2 because of the infinitely large quality gap. Irrespective of how much firm 2 is willing to supply, firm 1 is not affected as all consumers will be trying to buy the product of firm 1 or else not participate in the market at all. On the
other hand, if the quality gap approaches zero, \((\beta = \infty)\), the slope of the reaction function of one firm will be equal to the inverse of the other. Perfect symmetry of reaction functions will not be attained as the different cost functions of production will yield different intercepts between the reaction functions and the quantity axis. Since \(c_1 > c_2\), we obtain \(q_2^c > q_1^c\) at the Nash equilibrium (point \(N'\)). \(r_2\) denotes the reaction function of firm 2. We note that it is not independent from the quality differences: the market share of firm 1 is determined by the value of \(y^*\) which in turn is determined by the values of \(a_1, a_2, p_1,\) and \(p_2\). Firm 2, on the other hand, is left with a market share determined by \(y^*\) or \(b - q_1\). Therefore, \(q_2\) is a function of \(b, q_1\) and \(p_2\). But \(q_1\) is determined by \(a_1, a_2, p_1,\) and \(p_2\). Thus, \(q_2\) is not only dependent on \(q_1\) but also dependent indirectly on the quality gap via \(q_1\). This is why in figure 7-1, it appears that \(r_2\) is independent of \(\beta\). This is so as \(\beta\) is expressed via \(q_1\) and therefore \(\beta\) does not appear as a shift parameter of \(r_2\). All points along the \(NN'\) line are possible equilibria for the second stage game depending on the actual outcome, \(a_1\) and \(a_2\), in the first stage game.

7.1.2 The Price Level

The corresponding price levels at each potential Nash equilibria are then determined by substituting into the Cournot quantity levels in (7-4) and (7-5) the inverse
Figure 7-1

COURNOT-NASH EQUILIBRIA UNDER VARIOUS VALUES OF QUALITIES
demand functions as given in (6-9) and (6-10) respectively. They are as follows:

\[ p_{1c}^c = \frac{\beta(b+c_1+c_2) + 2(b+c_1)}{(4 + 3\beta)} \tag{7-6} \]

\[ p_{2c}^c = \frac{\beta(b+c_1+c_2) + (b+c_1+2c_2)}{(4 + 3\beta)} \tag{7-7} \]

7.1.3 The Utility Levels

The utility levels of the firms at possible values of Nash equilibria are given by

\[ \pi_1 = (p_1-c_1)q_1 = \frac{[(\beta(b-2c_1+c_2)+2(b-c_1))]^2}{(4 + 3\beta)^2} \tag{7-8} \]

\[ U_2 = (p_2-c_2)q_2 = \frac{[(1+\beta)(b+c_1-2c_2)]^2}{(4 + 3\beta)^2} \tag{7-9} \]

Although it is not essential to the main argument, we illustrate below a convenient way of reading off the utility levels at each possible Nash equilibria (corresponding to each difference in the quality levels) in the quantity space. The utility levels can be viewed as functions of \( \beta \) and can also equally well be represented on the same diagram as firms' reaction functions. By substituting the indirect demand function (6-9) into the utility (profit) function (6-3), we obtain the utility level of firm 1 in terms of \( \beta, q_1 \) and \( q_2 \) given by

\[ \pi_1 = (b-q_1-(\beta/1+\beta)q_2-c_1)q_1 \tag{7-10} \]
Refer now to figure 7-2 where we assume an equilibrium value of \((q_1^o, q_2^o)\) at point A and a given constant value of \(b\). The quantity and price spaces are overlapped in such a way that the distance \(b-D\) is equal to \(D0q\). This does not convey any intrinsic meaning except that it simplifies the diagram in a sense we can map the sum of \(q_1\) and \(q_2\) into the \(P_1\) axis by means of a negatively sloped 45° line. In other words, we are trying to denote the term in bracket in (7-10) in terms of a distance in the \(P_1\) axis such that the area represented by that distance and the quantity level associated with that price level will represent the utility level of firm 1. Suppose \(\beta = \infty\) or \(\beta/1+\beta = 1\). This implies that changes in \(q_2\) are directly offset by a corresponding fall in \(P_1\) since \(P_1 = b - q_1 - q_2\). Therefore, the slope of AB line is to represent such a degree of impact on the price level. To have an infinitely large value of \(\beta\) implies that the quality between firm 1 and firm 2 are equal. Therefore, a change in \(q_2\) is offset by an equal amount of fall in \(P_1\). The price level of firm 1 given \(\beta = \infty\) is denoted by the distance \(B'0p\). The corresponding profit which is obtained by subtracting \(C1Op\) from \(B'0p\) and then multiplying it by \(E0q\) is represented by the shaded area \(\pi_{11}\). On the other hand, if the quality gap between two firm's product is infinite, then the line AB collapses to AE. The price level is now denoted by \(E'0p\). The corresponding price level is represented by the shaded area \(\pi_{12}\). Therefore, given an equilibrium value of \((q_1^o, q_2^o)\), the profit level of firm 1 may vary from \(\pi_{11}\) to \(\pi_{12}\) depending on the quality gap between two products.
Figure 7-2

EQUILIBRIUM VALUES AND PROFIT LEVELS FOR DIFFERENT QUALITY GAPS
The analysis changes slightly under the Cournot-Nash equilibria where \((q_1^c, q_2^c)\) themselves are functions of \(\beta\). However, using the figure (7-2), we may show the corresponding utility levels of firm 1 for different values of \(\beta\). As already obtained earlier, for \(\beta = 0\), the equilibrium is obtained at \((q_1^c, q_2^c)\) at point N in figure (7-3). For \(\beta = \infty\), equilibrium is obtained at \((q_1^{cc}, q_2^{cc})\) at point N'. Applying the same analysis as before, in figure (7-3), the corresponding utility levels at each point may be denoted by the area \(N'^{"BD}F\) and \(N'B'D'F'\) respectively. Therefore, firm 1's utility, at maximum attains \(NBDF\) if its quality level is infinitely superior to that of firm 2 while if the quality gap is reduced to zero, it will earn a profit of \(N'B'D'F'\). The same analysis may be carried out on firm 2.

We now focus our attention on how each utility level changes for different values of quality gap. The functions \(\pi_1\) and \(U_2\) for \(\beta \geq 0\) are characterized as below:

i) The slope of each function is given by

\[
\frac{d\pi_1}{d\beta} = \frac{-4[\beta(b-2c_1+c_2)+2(b-c_1)](b+c_1-2c_2)}{(4+3\beta)^3} < 0 \quad (7-11)
\]

\[
\frac{dU_2}{d\beta} = \frac{2(b+c_1-2c_2)^2 (1+\beta)}{(4+3\beta)^3} > 0 \quad (7-12)
\]
Figure 7-3

COURNOT-NASH EQUILIBRIA AND UTILITY FUNCTIONS (LEVELS)
We now assume that \( b > \frac{5c_1 - 2c_2}{6} \), a necessary condition for the following analysis to hold. Then we obtain the following inequalities:

\[
d^2\pi_1/d\beta^2 > 0; \quad d^2U_2/d\beta^2 < 0
\]

where

\[
d^2\pi_1 = \frac{8(b+c_1-2c_2)[6(b-2c_1+c_2)+(6b-5c_1-2c_2)]}{(4+3\beta)^4}
\]

and

\[
d^2U_2/d\beta^2 = -\frac{2(b+c_1-2c_2)^2(5+6\beta)/(4+3\beta)^4}{(4+3\beta)^4}
\]

ii) The limiting values of \( \pi_1 \) and \( U_2 \) are as follows:

\[
\lim_{\beta \to 0} \pi_1 = \frac{(b-c_1)^2}{4}
\]

\[
\lim_{\beta \to \infty} \pi_1 = \frac{(b-2c_1+c_2)^2}{3}
\]

\[
\lim_{\beta \to 0} U_2 = \frac{(b+c_1-2c_2)^2}{16}
\]

\[
\lim_{\beta \to \infty} U_2 = \frac{(b+c_1-2c_2)^2}{3}
\]

Therefore, we derive the curvature of utility levels of each firm with respect to \( \beta \) as in figure 7-4. For Firm 1, it is optimum if \( \beta \to 0 \) and for Firm 2 if \( \beta \to \infty \); i.e., widening and closing the gap of quality differentials respectively. Firm 1 wants to extend lead and firm 2 to catch up.

As already mentioned earlier, subgame perfect equilibrium in a two stage extended game is said to prevail if no firm wishes to revise its second stage strategy after the
UTILITY LEVELS UNDER VARIOUS VALUES OF QUALITY LEVELS
first stage has been completed. For any given values of \( a_1 \) and \( a_2 \), firms correctly anticipate the second stage output equilibrium, which is resolved as a Nash quantity game as described so far. The first stage equilibrium on the other hand is assumed to arise from a Nash game in \( a_1 \) and \( a_2 \). Therefore, we obtain a subgame perfect equilibrium for this two stage game if the choice of optimal strategy as given in (7-4) & (7-5) for the second stage game is not reversed depending on the outcome of the first stage game. In other words, the Cournot equilibrium in quantity will not be reselected after the first sub-game on quality choice has been completed.

7.2 First Stage Game

Consider now stage 1 of the game. We have already found that it is optimal for firm 1 if \( \beta \) tends to zero and to infinity for firm 2. A Non-cooperative Nash Equilibrium exists if the following four conditions are satisfied; (Friedman 1977, chapter 7)

A1) Number of firms is finite
A2) The strategy set for each firm is compact, convex subset of \( \mathbb{R}^n \)
A3) The scalar valued payoff functions are defined over all strategic space and is continuous, bounded everywhere
A4) The payoff functions are strictly quasi-concave with respect to strategy space
A1) is satisfied as we assume duopoly. A3) and A4) are satisfied as we have shown that by differentiating equation (8), we obtain as shown in equation (9) and (10) that payoff functions (utility levels) are strictly concave. A2), however, is not satisfied as the strategy set for each firm is from zero to infinity in $\mathbb{R}^1$. It violates the compactness. Therefore, without any further structural assumptions, there does not exist a Nash equilibrium for the first stage game.

7.2.1 The Reaction Functions

The reaction function of each firm is obtained from the first order conditions of the objective function as given in (7-8) and (7-9) with respect to $a_1$ and $a_2$ respectively. This is possible as subgame perfect equilibrium is assumed to prevail. These first order conditions are given by:

$$\frac{(A\beta + B)\beta}{(4+3\beta)^3(a_1-a_2)} = 0$$

(7-13)

$$\frac{2(1+\beta)D^2a_1}{(4+3\beta)^3(a_1-a_2)} = 0$$

(7-14)

where

$$A = b - 2c_1 + c_2$$

$$B = b - c_1$$

$$D = b + c_1 - 2c_2$$

The corresponding reaction functions of firm 1 is derived from 7-13 as $a_1 = [(-b+3c_1-2c_2)/(b-c_1)]a_2$ where $a_1$ is not equal to $a_2$, $a_1 \geq 0$, $a_2 \geq 0$. Otherwise, 7-13 is not
defined. Since $a_1 > 0$, we require $3c_1 - 2c_2 > b$ which is not satisfied given sufficiently large value of $b$. Therefore, the reaction function will be the $a_1$ axis in a $(a_1, a_2)$ space. Since $a_1 \neq a_2$, $a_1 \geq 0$, $a_2 \geq 0$. Therefore, the reaction function of firm 2 is the $a_2$ axis in the $(a_1, a_2)$ space. Since $a_1 \neq a_2$, there is no equilibrium as already stated above.

To solve for an equilibrium for the first stage, we introduce two alternative restrictions: limiting bounds on $a$ and positive cost of quality improvement.

7.2.2 Limiting Bounds on $a$
Suppose that there exist limits in potential development capabilities in technology at each given point in time such that $a_i \in [q_i, q_i]$ for $i = 1, 2$ where $q_1 > q_2 > a_1 > q_2$ and $a_1 > a_2$. This comes from the assumption that firm 1 has first mover advantage over firm 2 in terms of technology. As long as cost of innovation is zero, firm 1 will always choose $q_1$ while firm 2 will always choose $q_2$. In other words, as it costs nothing to produce higher quality product, and because relative higher quality product will always grant them higher payoff, each will choose a higher level. Therefore, the reaction function of firm 1 will be the vertical axis in figure 7-5 for $q_1 < a_1 < q_1$ and the horizontal lines at $q_1$ for $a_1 \geq q_1$ and at $q_1$ for $a_1 < q_1$ while the 45° line will be the reaction function of firm 2 for for $a_2 < a_2 < q_2$ and the vertical axis at $q_2$ and $q_2$ respectively. There exists a
REACTION FUNCTIONS WITH UPPER AND LOWER BOUNDS AND ZERO COST OF INNOVATION
Nash equilibrium at \( q \). This implies a positive finite value for \( \beta \), say \( Q \). Then the corresponding quality levels are represented by a point somewhere between \( N \) and \( N' \) in figure 7-1. Note that under subgame perfect equilibrium, removing the restrictions on firms' characteristics, first mover advantage and different cost structure, will lead to a standard duopoly result where profit levels are shown in (7-8) and (7-9) will revert to \((b-2c_1+c_2)/3\) and \((b+c_1-2c_2)/3\). For \( c_1 > c_2 \), and homogeneous product, both firms will set their price equal to \((b+c_1+c_2)/3\) while profit level of firm 2 will be higher than that of firm 1 due to a lower cost.

7.2.3. Positive Cost of Innovation

Suppose now that firms face cost functions of innovation specified as \( k_i = a_i a_i \) for \( i = 1, 2 \) where \( k_i \) is the total cost of innovation and \( a_i \) is the marginal cost where \( a_1 < a_2 \). This specification implies that cost depends on the absolute level of technology development and that there is no spill over effect, i.e., innovation of technology of one firm is not transmitted to the other firm and perfect patenting applies. We rewrite the cost function in terms of \( \beta \) as \( k_i = a_i (1+\beta/\beta) a_j \) for \( i, j = 1, 2 \) and \( i \neq j \).

Consider the optimal value of innovation for firm 1 first. We keep \( a_2 \) fixed while we examine the optimal level for firm 1. This then enables us to work in terms of \( \beta \) as it will be a scalar valued function of \( a_1 \). It is obtained at \( \beta^* \) (thus at \( a_1^* \) for some given value of \( a_2 \))
where the marginal cost of innovation is equal to the marginal benefit, i.e., \( \frac{d\pi_1}{d\beta} = \frac{dk_1}{d\beta} \).

Let 

\[
A = b - 2c_1 + c_2 \\
B = b - c_1 \\
D = b + c_1 - 2c_2
\]

as before, where \( A = D \) if \( c_1 = c_2 \).

The marginal condition yields the following:

\[
-4D[A\beta + 2B] = (3\beta + 4)^3(-a_1\alpha_2/\beta^2)
\]

which may also be expressed as

\[
(27a_1\alpha_2 - 4AD)\beta^3 + (108a_1\alpha_2 - 8BD)\beta^2 + 36a_1\alpha_2\beta + 64a_1\alpha_2 = 0
\]

For each given value of \( \alpha_2 \), there is an optimal value of \( \alpha_1 \) which will satisfy equation (7-15). In order to solve for \( \alpha_1 \), rearrange (7-15) as

\[
\alpha_1\alpha_2 = \frac{4D(A\beta + 2B)\beta^2}{(3\beta + 4)^3}
\]

The reaction function of firm 1 with positive cost of innovation is shown in figure 7-6. Below, we discuss the curvature of the reaction function.

For positive values of \( \beta \), \( \alpha_1\alpha_2 \) goes to zero as \( \beta \to 0 \) and from the coefficient of \( \beta^3 \), we note that it approaches \( 4DA/27 \) as \( \beta \to \infty \). Furthermore, note that differentiating equation (7-17) yields

\[
\frac{d\alpha_1}{d\beta} = \frac{4D\beta (E\beta + 16)}{(3\beta + 4)^4} > 0
\]

where \( E = 6b - 18c_1 + 12c_2 \)

The second derivative is obtained as
for positive values of $\beta$. The optimum value of $\beta$ for firm 1 depends on $a_1$ and $a_2$. Let $\beta^*_1(a_2, a_1)$ be the optimal value of $\beta$ (for firm 1) for each given value of $a_1$ and $a_2$. If we now keep $a_1$ fixed, then $\beta^*_1 = \beta^*_1(a_2)$ since $\beta^*_1(a_2) = a_2/(a_1-a_2)$. The reaction function of firm 1 is obtained by rearranging it as

$$a_1 = \frac{[\beta^*_1(a_2) + 1]/\beta^*_1(a_2)}{a_2}$$

It is no longer a vertical line. In fact, the slope of the reaction function gradually falls; it is a concave function with respect to $a_2$ between the interval $0 \leq a_2 \leq 4DA/27a_1$ as shown in figure 7-6. The concavity arises from the fact that $d^2a_1/da_2^2 = -[(d\beta^*)^2/da_1^2]a_2 - 2d\beta^*/da_1 < 0$. Intuitively, firm 1 will invest more in innovation to widen the quality gap as this yields higher profit even taking into account the cost of innovation. However, as firm 2 increases its quality level further, it becomes less profitable for firm 1 to increase its investment into innovation as the cost of innovation begins to offset the gain from innovation due to the higher $a_2$. The potentially maximum quality gap given the optimal behaviour of firm 1 for each given level of firm 2's quality level is obtained at $(a_1', a_2')$ where $a_2'$ satisfies $\beta' = a_2'/(a_1(\beta') - a_2')$, and $da_1/d\beta > d^2a_1/d\beta^2$. Then for values of $a_2' < a_2 < 4DA/27a_1$, we have $d^2a_1/da_2^2 < da_1/da_2$ while for values of $0 < a_2 < a_2'$, we have $d^2a_1/da_2^2 < da_1/da_2$. In other words, for each value of $a_2$ for $0 < a_2 < a_2'$, firm 1 invests more in its innovation as to widen the quality gap against its rival.
REACTION FUNCTION OF FIRM 1 WITH POSITIVE COST OF INNOVATION
The marginal increment in profit due to a small rise in the gap is greater than the marginal increment of innovation cost. For values of $a_2$ such that $a_2' < a_2 < 4DA/27a_1$, the situation is reversed.

Consider firm 2. Analogous to the case of firm 1, firm 2's marginal condition states:

$$2D^2(1+\beta)^3 = a_2a_1(4+3\beta)^3$$  \hspace{1cm} (7-20)

which may also be expressed as

$$(2D^2-27a_2a_1)\beta^3+(6D^2-108a_2a_1)\beta^2+(6D^2-36a_2a_1)\beta+2D^2-64a_2a_1=0$$  \hspace{1cm} (7-21)

In order to solve for $a_2$, rearrange 7-20 as follows:

$$a_2a_1 = \frac{2D^2(1+\beta)^3}{(3\beta+4)^3}$$  \hspace{1cm} (7-22)

Now, $a_1a_2$ approaches $2D^2/27$ as $\beta \to \infty$ and it approaches $2D^2/64$ as $\beta \to 0$ keeping $a_2$ constant. We obtain the optimum value $\beta^*2$ as a function of $a_1$.

Therefore, the reaction function of firm 2 can be derived as $a_2 = \{\beta^*_2(a_1)/[\beta^*_2(a_1)+1]\}a_1$ as shown in figure 7-7. For values of $0 \leq a_1 \leq 2D^2/64a_2$, $a_2$ is negative which is due to the fact that $a_2a_1 = 2D^2/64a_2$ at $\beta^*_2 = 0$ and that $\beta^*_2$ becomes negative for values of $a_2a_1$ below $2D^2/64a_2$.

The reaction function of firm 2 is a convex function with respect to $a_1$. Firm 2 does not invest anything in quality improvement of its product until its rival's quality level is $2D^2/64a_2$. This is due to the fact that for
Figure 7-7

\[ \alpha_2 = r_2(\alpha_1, \beta_2(\alpha_1)) \]

\[ \frac{2D^2}{64\sigma_2} \]

\[ \frac{2D^2}{27\sigma_2} \]

REACTION FUNCTION OF FIRM 2 WITH
POSITIVE COST OF INNOVATION
given values of $a_1 < 2D^2/64a_2$, we have a situation where the first mover firm 1 has too low a quality such that under $a_1 > a_2$ condition, it is not profitable for firm 2 to invest anything into innovation. The cost of innovation will outweigh the corresponding benefit.

We now prove the above statement algebraically. For $a_1 < 2D^2/64a_2$, we find that $dU_2/da_1 > dU_2/da_2 + dk_1/da_2$. In other words, a small increase in the quality of firm 1's product lowers the utility level of firm 2 by less than any compensatory movement by firm 2 in which it will incur cost of innovation.

Consider the following differentials

$$dU_2/da_1 = -2a_1D^2(1+\beta)/[(4+3\beta)^3(a_1-a_2)^2]$$

$$dU_2/da_2 = 2a_2D^2(1+\beta)/[(4+3\beta)^3(a_1-a_2)^2]$$

$$dk_2/da_2 = -a_2a_1^2/[(1+\beta)^2(a_1-a_2)^2]$$

Then

$$dU_2/da_1 - dU_2/da_2 = -2a_2D^2(1+\beta)(a_1+a_2)/[(4+3\beta)^3(a_1-a_2)^2]$$

(7-23)

We have to show that equation (7-23) is smaller than $dk_2/da_2$ (the cost of innovation for firm 2) at $a_2 = 0$.

$$-2a_2D^2(1+\beta)(a_1+a_2)/[(4+3\beta)^3(a_1-a_2)^2]$$

$$< -a_2a_1^2/[(1+\beta)^2(a_1-a_2)^2]$$

(7-24)

Multiply both sides by $(a_1-a_2)^2$ and write $\beta$ in terms of $a_1$ and $a_2$ to yield

$$2D^2(a_1+a_2)/a_2a_1^2 > [(4a_1-a_2)/a_1]^3$$

(7-25)

Substitute $a_2 = 0$ into (3-25) to obtain
Therefore, for values of $a_1 < \frac{2D^2}{64a_2}$, the cost of innovation for firm 2 is greater than any loss in the utility level caused by an increase in $a_1$.

For $a_1 > \frac{2D^2}{64a_2}$, firm 2 will gradually increase its investment into innovation for increasing values of its rival's quality level until it reaches $\frac{2D^2}{27a_2}$. At that point, firm 2's investment into innovation is equal to its rivals. The reason for its gradual increase can be explained by the same reason as above where for $\frac{2D^2}{64a_2} < a_1 < \frac{2D^2}{27a_2}$, it is more profitable for firm 2 to invest in innovation to improve its product quality so as to not widen the quality gap with firm 1. The investment into innovation is less costly than the loss of not doing so as $a_1$ increases steadily.

7.3 A Numerical Example

We assign arbitrary values to the exogenous variables satisfying condition $b > (5c_1 - 2c_2)/6$. The values are given as $b = 10$, $c_1 = 2$, $c_2 = 1$ and $a_1 = a_2 = 1$. This gives $A = 7$, $B = 8$, $D = 10$ and $4DA/27a_1 = 8.3$, $2D^2/64a_2 = 2$ and finally $2D^2/27a_2 = 4.8$. The following is the result of a simple simulation deriving the reaction functions:

Reaction Functions of

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_1$</td>
<td>$a_2$</td>
</tr>
<tr>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>0.6</td>
<td>0</td>
</tr>
<tr>
<td>1.1</td>
<td>0.1</td>
</tr>
<tr>
<td>1.6</td>
<td>0.3</td>
</tr>
<tr>
<td>2.1</td>
<td>0.5</td>
</tr>
<tr>
<td>2.6</td>
<td>0.8</td>
</tr>
<tr>
<td>3.1</td>
<td>1.2</td>
</tr>
<tr>
<td>3.6</td>
<td>1.5</td>
</tr>
<tr>
<td>4.1</td>
<td>1.9</td>
</tr>
<tr>
<td>4.6</td>
<td>2.4</td>
</tr>
<tr>
<td>5.1</td>
<td>3.0</td>
</tr>
<tr>
<td>5.5</td>
<td>3.6</td>
</tr>
<tr>
<td>6.1</td>
<td>4.3</td>
</tr>
<tr>
<td>6.6</td>
<td>5.0</td>
</tr>
<tr>
<td>7.1</td>
<td>5.9</td>
</tr>
<tr>
<td>7.6</td>
<td>6.8</td>
</tr>
<tr>
<td>8.0</td>
<td>7.8</td>
</tr>
<tr>
<td>8.3</td>
<td>8.3</td>
</tr>
</tbody>
</table>

They are plotted in figure 7-8 and approximated into a continuous line. This numerical example confirms our analytical findings on both the curvature as well as the terminal points of the two reaction functions.

### 7.4 Comparative Statics

By definition, a Non-cooperative Nash Equilibrium is attained at the point where reaction functions intersect.
Figure 7-8

REACTION FUNCTIONS DERIVED FROM A NUMERICAL EXAMPLE
The existence of Equilibrium is guaranteed if the following condition is satisfied

\[ \frac{4AD}{27a_1} > \frac{2D^2}{27a_2} \]  

(7-26)

In fact, for \( a_1 < a_2 \) by the assumption that firm 1 has the advantage in technology, (7-26) is satisfied. The proof is given as follows: Firstly, (7-26) may be rewritten as \( 2a_2A > a_1D \). Substitute for \( A \) and \( D \) to obtain \( 2a(b-2c_1+c_2 - a_1(b+c_1-2c_2) > 0 \). Rewrite it as \( (b-c_1)(2a_2-a_1) + 2c_2(a_2+a_1) > 0 \). Therefore, \( a_2 > a_1 \) is the sufficient condition for the existence of an unique Non-cooperative Nash Equilibrium for this game. One particular solution \( Z \) is shown in figure 7-8 where \( r_1 \) and \( r_2 \) represent reaction functions of firm 1 and firm 2 respectively. Below we consider the following cases where i) \( c_2 \) increases and ii) \( a_2 \) increases.

7.4.1 An Increase in the Cost of Production

If \( c_2 \) increases due to a genuine rise in the cost of production or as a result of a shift towards profit maximization relative to revenue, the value of \( D \) falls while that of \( A \) rises. Given that \( b > 5c_1 - 4c_2 \), the reaction function of firm 1 as shown in figure 7-9 will shift towards the origin from \( r_1 \) to \( r_1' \) because \( dD/dc_2 A + dA/dc_2 D < 0 \) while the reaction function of firm 2 will shift downwards from \( r_2 \) to \( r_2' \) because \( dD/dc_2 < 0 \). We note that the shape of both reaction functions guarantee that higher cost of production of firm 2 will result in narrowing the quality gap. The intuition of this result
TWO POSSIBLE COURNOT-NASH EQUILIBRIA
UNDER EACH A DIFFERENT VALUE OF AN
EXOGENOUS VARIABLE (COST OF PRODUCTION)
is that higher production cost of firm 2 will, given the
demand function as in (6-8), reduce the optimal level of
$q_2$ for each value of $\beta$ (refer to equation 7-5 where
dq2/dc2 > 0). The corresponding price level as given in
equation 7-7 rises, i.e., dP2/dc2 < 0. The utility level
of firm 2 falls. In order to compensate for the
reduction of its sale's volume, firm 2 increases the
quality level of its product for each given value of its
rival, i.e., $\beta$ increases. The price level corresponding
to this new quantity supplied will also increase. We may
infer this from the reaction function of firm 2 where for
each given value of $a_1$, we obtain a larger value of $a_2$
under $r'2$ than under $r_2$ in figure 7-9. Therefore, profit
of firm 2 will rise relative to zero change in the
utility increase. The marginal increase in the profit
level as a consequence of an increase in $a_2$ is larger
than the additional cost spent to raise $a_2$. For a given
value of $a_1$, this may be shown by the following result:

\[ \frac{d^2U_2}{dc_2d\beta} - \frac{dU_2}{dc_2} > 0 \]  \hspace{1cm} (7-27)

which is given as

\[ \frac{4(3\beta^3+7\beta+2)(1+\beta)(b+c_1-2c_2)}{(3\beta+4)^3} > 0 \]

Therefore, expenditure on improving $a_2$ will increase
until the marginal expenditure equals the marginal
increase in the quality level. Consider the effect of an
increase in $c_2$ on firm 1. As $q_2$ falls, the optimal value
of $q_1$ will rise. Note that the increase in the volume of
sales by firm 1 is less than the fall in $q_2$ as the
 corresponding new price level of firm 1 will be higher
than before. Furthermore, for each given value of $a_2$, firm 1 will lower its quality level as it could marginally increase its profit level using its improved competitiveness given the present gap in the quality level is greater than $\beta^*$ where $\beta^*$ satisfies the following equation:

$$3F\beta^* + 2(2F+G)\beta^* - 4F = 0$$  \hspace{1cm} (7-28)

$$F = \beta(b-2c_1+c_2)+2(b-c_1)$$

$$G = (b+c_1-2c_2)$$

(7-28) is obtained from $d^2\pi_1/dc^2\beta = d\pi_1/dc_2$.

For $\beta < \beta^*$, we have

$$d^2\pi_1/dc^2\beta - d\pi_1/dc_2 < 0$$  \hspace{1cm} (7-29)

Therefore, we obtain the result that the quality gap has narrowed due to both firms' maximizing behaviour as a response to an increase in the level of production cost of firm 2.

7.4.2 An Increase in the Cost of Innovation

Consider now the second situation where the cost of innovation increases. The reaction function of firm 2 will shift downwards as $2D^2/64a_2$ will fall if $a_2$ rises. The concavity property of firm 1 will ensure that the quality gap will widen while at the same time quality of both products will fall. In figure 7-10, this change in equilibrium may be represented as the original equilibrium $Z$ changing to $Z''$. The reason is that as firm 2 lowers its investment in innovation due to its higher cost of innovation, firm 1 finds itself in a better
THE EFFECT OF AN INCREASE IN THE COST OF INNOVATION BY FIRM 2
position to compete. Firm 1 will lower its quality level such as to cut back in cost of innovation, but only to such an extent as to leave a wider quality gap. In this way, firm 1 will be able to enjoy both the benefit of wider quality gap as well as lower cost of innovation due to the fall in absolute level of quality, and thus reduction in the cost of innovation.

7.5 Conclusion
Under the Cournot assumption, we have examined a two stage game where quality choice and marketing are considered sequentially. A Subgame Perfect Equilibrium was obtained as we considered the second stage game first. There were potentially many equilibria each corresponding to given values of $\beta$, i.e., the outcome of the first stage game. The equilibrium corresponding to $\beta = \omega$ was identical to a single period quantity game.

We found that the equilibrium of the second stage game did not exist without further structural restrictions as firm 1's optimal response in its quality choice as against that of firm 2 is to attain a large a quality gap as possible while firm 2 tries to close the gap. We introduced two alternative restrictions: potential upper and lower bounds of each quality level and positive cost of innovation. The Nash equilibrium under former restriction is attained at the upper limits of both firms potential quality levels. Under positive and increasing
cost of innovation with respect to quality levels, we required an additional assumption that the cost of innovation by firm 1 is less than that of firm 2 to attain an equilibrium.

We conducted comparative static exercises by varying each of two exogenous variables of firm 2. These were the cost of production, or a shift in the objectives from a simple profit maximization behaviour to a dual objective of profit and revenue and cost of innovation. These two variables were chosen as they best represent the empirical facts of firms in Korea; the problem of wage increases, ever increasing cost of innovation and Government policy in NICs to earn foreign reserves. We obtained the result that a higher wage rate in firm 2 will lead to a fall in both firms' quality level. However, the quality level of firm 2 will fall proportionally by less than its rival's, thus narrowing the quality gap. Price of both product will rise while firm 1 will increase its sale's volume. For an increase in the cost of innovation, we obtained that the quality gap will widen. At the same time, quality of both products will fall. Therefore, q2 falls while q1 rises.
Consider the model as outlined in chapter 6. We now examine this behaviour under a different form of firms' conjectural variation. In the previous chapter, we assumed that each firm conjectures that its rival’s output response to changes in its own quantity was zero. We now modify this assumption and propose Bertrand competition. The Bertrand model of duopoly describes a situation in which each firm conjectures that its rival's price level will not change in response to changes in its own price level. Avinash Dixit (1984) constructs a duopoly model with outputs as the choice variable and shows that equilibria other than Cournot are defined readily in the same analysis by defining appropriate forms of conjectural variation $dq_j/dq_i = v_i(q_i, q_j)$. $v_i$ is defined as $-(dp_j/dq_i)/(dp_j/dq_j)$ for Bertrand competition and as zero for the Cournot competition where $p_i$ and $q_i$ represent the price level and the quantity supplied by firm $i$ respectively.

8.1 The Second Stage Game

As before, we first consider stage two of the 2-stage game. In order to maximize the objective functions $\pi_i$ and $U_2$, we may either differentiate them with respect to $q_i$.
each and assume conjectures $v^1 = -1$ and $v^2 = -\beta/(1+\beta)$ for firm 1 and firm 2 obtained from (iv) of Appendix A respectively or differentiate them with respect to $p_1 p_2$ each and assume $v_j = 0$. 

8.1.1 The Demand Function

Below, we derive the demand function under Bertrand conjecture in two alternative ways to demonstrate that they yield the same result.

From the first order condition $d\pi_1/dq_1 = 0$ and $dU_2/dq_2 = 0$ as shown in ii) Appendix A, we may obtain the reaction functions for each firm in $(q_1, q_2)$ space as follows:

\[
q_1 = \frac{(1+\beta)(b-c_1)}{(2+\beta)} - \frac{\beta}{(2+\beta)} q_2 \quad (8-1)
\]

\[
q_2 = \frac{(1+\beta)(b-c_2)}{(2+\beta)} - \frac{(1+\beta)}{(2+\beta)} q_1 \quad (8-2)
\]

The corresponding quantities of sales in an equilibrium are obtained from (8-1) and (8-2) by solving simultaneously for $q_1$ and $q_2$.

\[
q^{B_1} = \frac{-(c_1-c_2)\beta^2+(2b-3c_1+c_2)\beta+2(b-c_1)}{3\beta+4} \quad (8-3)
\]

\[
q^{B_2} = \frac{(c_1-c_2)\beta^2+(b+2c_1-3c_2)\beta+(b+c_1-2c_2)}{3\beta+4} \quad (8-4)
\]
Substituting (8-3) and (8-4) into demand functions as given in equation (iii) Appendix A yield the following price levels for the Bertrand equilibria, as functions of $\beta$.

\[
\begin{align*}
P^B_1 &= \frac{(2c_1+c_2)\beta+2(b+c_1)}{3\beta+4} \\
P^B_2 &= \frac{(c_1+2c_2)\beta+(b+c_1+2c_2)}{3\beta+4}
\end{align*}
\] (8-5) (8-6)

Secondly, if we take $p_i$ as the strategic variable, we anticipate same equilibrium as obtained above. We differentiate the objective functions with respect to $p_i$, to obtain the first order conditions of the objective function for firm 1 as

\[
\frac{d\pi_1}{dp_1} = q_1 + (p_1 - c_1) \frac{dq_1}{dp_1} = 0
\] (8-7)

where \( \frac{dq_1}{dp_1} = \frac{\delta q_1}{\delta p_1} + \delta q_1 \delta p_2 \left( \frac{\delta p_2}{\delta p_1} \right) \) (8-8)

(\( \delta \) denotes partial differentiation)

Then \( \delta p_2/\delta p_1 = 0 \) under Bertrand assumption and \( \delta q_1/\delta p_1 = -(1+\beta) \) from equation iv), Appendix A. Therefore, (8-6) reduces to \( \frac{dq_1}{dp_1} = -(1+\beta) \). The first order condition (8-7) and substituting for $q_1$ gives the optimal value of $p_1$ as

\[
p_1 = \frac{b}{2(1+\beta)} + \frac{c_1}{2} + \frac{\beta}{2(1+\beta)} p_2
\] (8-9)

Likewise for firm 2, we obtain \( \frac{dq_2}{dp_2} = -(1+\beta) \) and the first order condition $q_2 - (p_2 - c_2) \frac{dq_2}{dp_2} = 0$. Therefore, the optimal price level for each given price level of its rival is given by
\[ p_2 = \frac{1}{2}(p_1 - c_2) \]  \hspace{1cm} (8-10)

Solving (8-9) and (8-10) simultaneously for \( p_1 \) and \( p_2 \) yields

\[ p_{B1} = \frac{(2c_1 + c_2)\beta + 2(b + c_1)}{3\beta + 4} \]  \hspace{1cm} (8-11)

\[ p_{B2} = \frac{(c_1 + 2c_2)\beta + (b + c_1 + c_2)}{3\beta + 4} \]  \hspace{1cm} (8-12)

Note that equation (8-11) and (8-12) are identical to equations (8-5) and (8-6) demonstrating the earlier statement that Bertrand solution may be obtained either by maximizing with respect to \( q_{is} \) or \( p_{is} \) as long as we adopt the correct conjecture. In appendix B, we illustrate the comparative static results obtained under both the Bertrand and Cournot conjectures.

8.1.2 Nash Equilibrium of Stage Two Game

The loci of possible Nash Equilibria in Non-cooperative Bertrand Competition is shown in figure 8-1 as the MM' curve. The figure shows four reaction functions in all. \( r_i \) represents firm i's reaction function if the quality gap between two firms is very large. The equilibrium is obtained at point M. Note that for \( c_1 = c_2 \) and \( b > c_1, p_1 > p_2 \). This is a necessary condition for firm 2 to attain positive market share. Otherwise, consumers would all prefer the higher quality product.

On the other hand, as the quality gap begins to get smaller (i.e., \( \beta \to \infty \)), \( q_1 \) initially increases only to
BERTRAND-NASH EQUILIBRIA UNDER VARIOUS VALUES OF QUALITY LEVELS
collapse to zero as \( \beta \) tends to infinity. We note that given the constraint that firm 1 has a higher quality, segmentation will not be defined if firms have same quality levels. It is apparent from (8-3) that for small values of \( \beta \), the coefficient of \( \beta \) dominates the coefficient of \( \beta^2 \). As the value of \( \beta \) increases further, the coefficient of \( \beta^2 \) begins to dominate. The intuition is that under a small quality difference, firm 2 with lower cost level will reduce its price level sufficiently low as to drive firm 1 out of the market. Even if \( c_1 = c_2 \), note from (8-3) and (8-4) that \( q_1 \) will be twice the amount of \( q_2 \). This is because under Bertrand competition, the price level will be equated to marginal cost. This means that given consumer demand as in chapter 6, the total market demand will be \( b-c \). Since firm 1 is a first mover into the market by assumption, it will take \( 2(b-c)/3 \) amount of the market share while the rest is left for firm 2. (This is only true if we assume consumers have perfect foresight of what each firm will charge irrespective of the time of product supply into the market).

8.2 Utility Levels

Consider the corresponding outcomes for firms' utilities of these equilibria. The utility levels can be graphed against \( \beta \) as shown in figure 8-2 given an additional assumption on the relative values of the exogenous variables, namely, \( b > 3c_1 - c_2 \). We assume this
Figure 3-2

UTILITY LEVELS UNDER BERTRAND COMPETITION
FOR VARIOUS LEVELS OF QUALITY
inequality to hold throughout the following analysis
where we derive the curvature of the utility levels.

Firstly, in order to express utility levels in terms of
\( \beta \), we substitute equation (8-3), (8-4) and (8-5), (8-6)
into the objective functions \( n_1 \) and \( U_2 \) in equations (7-8)
and (7-9) respectively to yield the following utility
levels:

\[
\pi_1 = \frac{f_1 \beta^3 - f_2 \beta^2 + f_3 \beta + f_4}{(3\beta+4)^2} \tag{8-13}
\]

where

\[
f_1 = (c_1-c_2)^2 > 0
\]

\[
f_2 = (c_1-c_2)(4b-5c_1+c_2) > 0
\]

\[
f_3 = 4(b-c_1)(b-2c_1+c_2) > 0
\]

\[
f_4 = 4(b-c_1)^2 > 0
\]

\[
U_2 = \frac{g_1\beta^3 + g_2\beta^2 + g_3\beta + g_4}{(3\beta+4)^2} \tag{8-14}
\]

where

\[
g_1 = (c_1-c_2)^2 > 0
\]

\[
g_2 = (c_1-c_2)+(2b+3c_1-5c_2) > 0
\]

\[
g_3 = (b+c_1-2c_2)(b+3c_1-4c_2) > 0
\]

\[
g_4 = (b+c_1-2c_2)^2
\]

Having derived the utility levels for firm 1 and firm 2
respectively for each given value of \( \beta \), we differentiate
the utility levels in terms of \( \beta \) to study their
curvatures more clearly.
The limiting values of \( \pi_1 \) and \( U_2 \) are given as:

\[
\lim_{\beta \to 0} \pi_1 = \frac{f_4}{16} > 0
\]

\[
\lim_{\beta \to 0} \pi_1 = \infty
\]

\[
\lim_{\beta \to 0} U_2 = \frac{g_4}{16} > 0
\]

\[
\lim_{\beta \to 0} U_2 = \infty
\]

The limits of the first derivatives are given as follows:

\[
\lim_{\beta \to 0} \frac{d\pi_1}{d\beta} = \frac{(2f_3-3f_4)}{32} < 0
\]

because \(-4(b-c_1)(b+2c_1-2c_2) < 0\)

\[
\lim_{\beta \to \infty} \frac{d\pi_1}{d\beta} = \frac{f_1}{9} > 0
\]

\[
\lim_{\beta \to 0} \frac{dU_2}{d\beta} = \frac{2g_3-g_4}{32} < 0
\]

because \(-(b+c_1-2c_2)(b-5c_1+6c_2) < 0\)

\[
\lim_{\beta \to \infty} \frac{dU_2}{d\beta} = \frac{g_1}{9} > 0
\]

Compare \( f_4 \) with \( g_3 \). Given \( b > 3c_1 - c_2 \), we know that \( f_4 < g_3 \).

The second order derivatives are given as follows:

\[
\frac{d^2\pi_1}{d\beta^2} = \frac{-36(f_1-f_2)\beta^2+9(5f_2+3f_3)\beta+6(8f_2-6f_3+9f_4)}{(3\beta+4)^3}
\]
\[
\frac{d^2 U_2}{d \beta^2} = \frac{3(16g_1-8g_2+3g_3)\beta+2(24g_1-18g_3+9g_4)}{(3\beta+4)^4} \quad (8-18)
\]

where \( f_1 - f_2 > 0 \)

\[
8f_2 + 3f_3 > 0
\]

\[
8f_2 - 6f_3 + 9f_4 > 0
\]

because \( 9f_4-6f_3 = 12(b-c_1)(b+c_1-2c_2) > 0 \)

and \( 16g_1- 8g_2 + 3g_3 > 0 \)

\[
24g_1 - 18g_3 + 9g_4 < 0
\]

because \( -(b+c_1-2c_2)(b+5c_1-6c_2) < 0 \)

These results imply that for small value of \( \beta \), the second derivatives of \( \pi_1 \) and \( U_2 \) are convex and concave respectively while for a large value of \( \beta \), their curvature changes to concave and to convex respectively.

We note two differences in the utility levels obtained as solutions between those under Cournot and Bertrand conjectures. Under the former, the maximizing value of \( \beta \) was \( \beta = 0 \) for \( \pi_1 \) and \( \beta = \infty \) for \( U_2 \). We now seem to have a situation where we attain maximum values of \( \pi_1 \) and \( U_2 \) at \( \beta = \infty \). However, under Bertrand competition, two restrictions on the value of \( \beta \) are required to guarantee firm 1 a positive market share and a non-negative mark-up on its cost of production. For \( q_1 > 0 \), we require that \( \beta < \beta \) where

\[
\beta = \frac{(2b-3c_1+c_2)+[(2b-3c_1+c_2)^2+8(c_1-c_2)(b-c_1)]^{1/2}}{2(c_1-c_2)} \quad (8-19)
\]
This is obtained from (8-3) where we solve for the value of \( \beta \) given \( q_1 = 0 \). As for positive mark up, \((p_1 - c_1) > 0\), we need \( \beta < \bar{\beta} \) where \( \bar{\beta} = \frac{2(b - c_1)}{(c_1 - c_2)} \). Suppose further that at the value of \( \beta = \beta' \), \( \pi_1 = 0 \).

8.3 A Numerical Example

In order to compare \( \beta' \), \( \bar{\beta} \) and \( \bar{\beta} \), we substitute simple numerical values. They are chosen such that they do not violate the assumption \( b > 3c_1 - c_2 \). We define the values of the exogenous variables as

- \( b = 10 \)
- \( c_1 = 2 \)
- \( c_2 = 1 \)

Then the coefficients of the profit function \( \pi_1 \) is given as below:

- \( f_1 = 1 \)
- \( f_2 = 31 \)
- \( f_3 = 224 \)
- \( f_4 = 256 \)

The values of \( \beta \) are given by

- \( \beta = 26.62 \)
- \( \bar{\beta} = 16 \)

Since \( \beta > \bar{\beta} \), \( \bar{\beta} \) is the binding constraint. In fact, for \( \beta = \bar{\beta} \), \( \pi_1 = 0 \). This implies that \( \beta' = \bar{\beta} \). This is easily explained by the fact that \( \pi_1 = (P_1 - c_1)q_1 \) and \( \pi_1 \) is zero if \( P_1 - c_1 = 0 \) or \( q_1 = 0 \). We know that \( \bar{\beta} \) is the only binding constraint on the profit function of firm 1 and that at \( \beta = \bar{\beta} \), \( P_1 = c_1 \). In other words, there were initially two possible reasons firm 1 could be forced out
of the market; zero market share or negative mark up. This simple numerical example shows however, that the price war under Bertrand conjecture will force firm 1 into a negative mark-up situation unless it succeeds to obtain an advantage in the quality of a product by $a_1 \geq (1 + \beta / \beta) a_2$ for each given value of $a_2$. Firm 1 attains its maximum profit level at $\beta = 0$, i.e., if the quality difference between the two products are very large. This result rests on the following reasons. The demand function is parameterized with respect to the price and the quality level. In order to sustain a fixed market share, a fall in the quality level must be accompanied by a corresponding fall in the price level. But the Bertrand conjectures assumption implies price competition and therefore the price level has a lower bound, i.e., constant cost of production level beyond which it must not fall. The only way that firm 1 can increase its profit level is by an increase in the quality level. Since the cost of innovation is assumed zero, firm 1 will be able to attain its maximum profit level as it increases the quality differences between the two products.

8.4 The First Stage Game
The optimal value of $\beta$, denoted as $\beta_1^*$, is obtained by the first order condition $d\pi_1/d\beta = (dP_1/d\beta) q_1 + (dq_1/d\beta)(P_1-c_1) = 0$. Thus $\beta_1^*$ satisfies the following condition:
\[ 3f_1\beta^3 + 12f_2\beta^2 - (9f_3+8f_2)\beta + 2(2f_3-3f_4) = 0 \quad (8-20) \]

In order to have an internal solution, \( \beta \) must be \( 0 \leq \beta \leq \beta_1' \) where \( \pi_1 \) at \( \beta_1' \) is zero. For a numerical solution for a value of \( \beta \), we substitute the same numerical values as given in the above example into (8-20) to yield
\[ 3\beta^3 + 372\beta^2 - 2264\beta - 640 = 0 \quad (8-21) \]

For \( \beta < \beta_1' = 16 \), the LHS of (8-21) is negative. Therefore, \( \beta_1^* > \beta_1' \) implies that \( \beta_1^* \) is not the optimizing value. In fact, the second order condition proves that \( \beta_1^* \) is the minimizing value. Therefore, as shown earlier, the optimum value of \( \beta \) is attained when the quality gap is infinite, i.e., \( \beta = 0 \). The reaction function of firm 1 in the first stage game is the vertical axis, i.e., whatever value of \( a_2 \), firm 1 will always wish to maximize the gap in the quality level. This is shown in Figure 8-3.

Similarly, firm 2's marginal condition is obtained from the first order condition of the utility function \( U_2 \).
\[
\frac{dU_2}{d\beta} = (dp_2/d\beta)q_2 + (dz/d\beta)(p_2-c_2) = 0 \quad (8-22)
\]

It may be rewritten as
\[ 3g_1\beta^3 + 12g_1\beta^2 + (8g_2-3g_3)\beta + (4g_3-2g_4) = 0 \quad (8-23) \]

Substituting our numerical values into 8-20 yields
\[ 3\beta^3 + 12\beta^2 - 31\beta - 280 = 0 \quad (8-24) \]

for \( g_1 = 1 \)
\( g_2 = 21 \)
\( g_3 = 120 \)
Figure 8-3

REACTION FUNCTIONS UNDER BERTRAND COMPETITION AND ZERO COST OF INNOVATION
We find that $\beta_2' \approx 1.2$ satisfies equation (8-24). For $\beta_2' \approx 1.2$, $d^2U_2/d\beta > 0$. Therefore, $\beta_2'$ is the minimizing value of $\beta$ where $U_2$ attains its minimum. In fact, from figure 8-2, we have shown that $U_2$ is maximized as $\beta \to \infty$. Therefore, the reaction function, which is given as $\alpha_2 = \beta_2^*/(\beta_2^*+1)\alpha_1$ where $\beta_2^*$ represents the maximizing value of $U_2$, approaches $\alpha_2 = \alpha_1$ as $\beta_2^* \to \infty$. Thus, the reaction function is the 45° line as shown in figure 8-3.

Consider the mark-up level of firm 2. It is given as

$$\frac{(c_1-c_2)\beta + (b+c_1-2c_2)}{(p_2-c_2)} = \frac{b+c_1-2c_2}{3\beta+4}$$

Note that for $c_1 = c_2$ and $\beta \to \infty$, we return to the symmetric duopoly solution under the Bertrand conjectures that firms are operating at zero mark up level. For $c_1 > c_2$, as the quality gap narrows, we have a positive mark-up.

8.5 The First Stage Game Equilibrium

Given no constraints on the value of $\alpha$s and no cost of innovation, the equilibrium is not attained as it violates the compactness assumption defined in chapter 7. As before, we impose a restriction on levels of quality
$a_1 \in [a_1, a_2]$ where $a_1 > a_2 > a_1 > a_2$. The equilibrium is attained at $z = (a_1, a_2)$ as shown in figure 8-4. If, on the other hand, the $a_1 = a_2$, then firm 1 will be driven out of the market as firm 2 will push its price level down such that the mark up of firm 1 becomes negative.

8.5.1 Positive Cost of Innovation

Suppose that the cost function of Innovation introduced in the Cournot model also applies to this Bertrand competition. Then for firm 1, the marginal condition, $d\pi_1/d\beta + d\pi_1/d\beta = 0$, states that

$$a_1a_2 = \frac{-[3f_1\beta^3 + 12f_2\beta^2 - (9f_3 + 8f_2)\beta + 2(2f_3 - 3f_4)]\beta^2}{(4 + 3\beta)^3} \quad (8-26)$$

As for firm 2, its marginal condition $dU_2/d\beta + dU_2/d\beta = 0$ yields

$$a_2a_1 = \frac{(1 + \beta)^2[3g_1\beta^3 + 12g_1\beta^2 + (8g_1 - 3g_3)\beta + (4g_3 - 2g_4)]}{(3\beta + 4)^3} \quad (8-27)$$

Equation (8-26) and (8-27) each show the optimal value of $a_1$ and $a_2$ for a fixed value of its rival's quality level. Therefore we obtain firms' reaction functions from these equation. Firm 1's reaction function now becomes $a_1 = \{[\beta^*1(a_2) + 1]/\beta^*1(a_2)\} a_2$ as shown in figure 8-5. The reaction function $r_1$ is concave to the origin and terminates at $a_1 = a_2(\beta)$.

Likewise, firm 2's reaction function $r_2$ becomes $a_2 = \{\beta^*2(a_1)/[\beta^*2(a_1) + 1]\} a_1$. Initiating from $(g_3-$
REACTION FUNCTIONS WITH UPPER AND LOWER BOUNDS ON QUALITY LEVELS WITH ZERO COST OF INNOVATION
REACTION FUNCTIONS WITH POSITIVE COST OF INNOVATION
$g_4)/32a_2$, it converges towards the 45° line. Comparing reaction functions derived under the Cournot conjectures assumption, we observe that $r_2$ is now convex to the origin while before, it was the concave to the origin. The convexity of the reaction function implies that as the value of $a_1$ rises, it is optimal for firm 2 to increase $a_2$ at a faster rate than the increase in $a_1$, i.e., $\beta$ increases. The reason is that the profit level of firm 2 increases at a slower rate than the increase in the cost of innovation with respect to an increase $a_2$. Therefore, for each incremental value of $a_1$, the proportionate increase in $a_2$ will fall.

The reason for the convexity of the reaction function of firm 2 can be obtained from the second derivatives of $U_2$ with respect to $\beta$.

\[
\frac{d^2U_2}{d\beta^2} = \frac{d^2P_2}{d\beta^2} \frac{d^2q_2}{d\beta^2} (P_2-c_2) + 2 \frac{dq_2}{d\beta} \frac{dP_2}{d\beta} + \frac{dq_2}{d\beta^2} + \frac{dP_2}{d\beta^2} \tag{8-28}
\]

The corresponding signs of the derivatives under Bertrand competition and Cournot competition are shown below as

<table>
<thead>
<tr>
<th></th>
<th>Bertrand</th>
<th>Cournot</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dq_2/d\beta$</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$dp_2/d\beta$</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$d^2q_2/d\beta^2$</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>$d^2p_2/d\beta^2$</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Note that under Cournot competition,
\[ \frac{d^2P_2}{d\beta^2} + \frac{d^2q_2}{d\beta^2} \leq 2 \left( \frac{dq_2}{d\beta} + \frac{dP_2}{d\beta} \right) < 2 \left( \frac{dq_2}{d\beta} + \frac{dP_2}{d\beta} \right) \] (8-29)

This is because the left hand side of 8-29 is negative as we can infer from the table above.

Under Bertrand competition, however, due to the relative low value of \( P_2 \), and large \( q_2 \), we obtain
\[ \frac{d^2P_2}{d\beta^2} + \frac{d^2q_2}{d\beta^2} > \left( \frac{dq_2}{d\beta} + \frac{dP_2}{d\beta} \right) + 2 \left( \frac{dq_2}{d\beta} + \frac{dP_2}{d\beta} \right) \] (8-30)

for small values of \( \beta \) while the inequality sign is reversed for large values of \( \beta \). Thus, we obtain the convexity of the utility function under the Bertrand competition.

Having demonstrated the difference in the reaction function of firm 2 under Cournot and Bertrand conjectures, we examine the intuition for the difference. Under the Cournot conjectures assumption, an increase in \( \beta \) is accompanied by an increase in the price level and the quantity supplied. The marginal increases, however, are falling. This implies that the utility level, with respect to \( \beta \) is increasing at a falling marginal rate. Under Bertrand conjectures assumption, however, the price level falls initially at a decreasing rate with respect to \( \beta \) and rises in an increasing rate while the quantity supplied falls after a brief initial rise yielding a utility function which is decreasing for small values of
and increasing for large values of $\beta$ while the marginal values are positive throughout the argument of $\beta$.

Since the marginal utility level under the Cournot conjectures is decreasing while that of Bertrand is increasing with respect to quality differences, an increase in the quality level of firm 1 will lead firm 2 to increase its quality level by more under Bertrand conjectures than it would under Cournot for low values of $a_1$. The Bertrand conjectures equilibria are less sensitive to changes in the quality differences than the Cournot conjectures, e.g., as they engage in "price war".

8.5.2 Upper Limits on Investment into R&D

The existence of the first stage game may not be sustainable, i.e., the intersection point of the two reaction functions may occur at a point at which profit levels of firms may be negative. Therefore, there is an additional restriction on possible values of quality level each firm can take. The cost of innovation cannot exceed the net profit, i.e., total profit net of the production cost. The restrictions for $a_1$ and $a_2$ are given as $a_1^*$ and $a_2^*$ where they each satisfy the following condition:

$$\pi_1[\beta(a_1, a_2)] - k_1[a_1, \beta(a_1, a_2)] = 0 \quad (8-31)$$

$$U_2[\beta(a_1, a_2)] - k_2[a_2, \beta(a_1, a_2)] = 0 \quad (8-32)$$

These restrictions are denoted in figure 8-5 as $t_1$ and $t_2$. The intersection point between $t_1$ and $t_2$ will be the
maximum value of \( a_1 \) firm 1 could choose while the intersection point between \( t_2 \) and \( r_2 \) will be the maximum value of firm 2. For a Nash equilibrium for the first stage to exist, the intersection points of the two reaction functions must be below the upper limit values of \( a_1 \) and \( a_2 \). Then a sustainable Nash equilibrium will exist, the precise nature of which will be determined by the specific values of the exogenous variables including \( a_1 \) and \( a_2 \). One particular equilibrium is shown in figure 8-5 denoted as \( Z \).

8.6 Comparative Static Analysis
We now examine the two cases as in the Cournot model; i) increase in \( c_2 \) due to an increase in the wage rate or in the shift of priorities and ii) increase in \( a_2 \).

8.6.1 An Increase in the Cost of Production
Consider the second stage of the game given an increase in the cost of production. We observe for (i):

\[
\begin{align*}
\frac{dp_1}{dc_2} &= \frac{\beta}{(3\beta+4)} > 0 \\
\frac{dp_2}{dc_2} &= \frac{(2\beta+2)}{(3\beta+4)} > 0 \\
\frac{dq_1}{dc_2} &= \frac{\beta(\beta+1)}{(3\beta+4)} > 0 \\
\frac{dq_2}{dc_2} &= \frac{-(\beta+1)(\beta+2)}{(3\beta+4)} < 0 \\
\frac{d\pi_1}{dc_2} &= \frac{12(c_1-c_2)\beta^3+2(2b-2c_1+c_2)\beta^2+4(b-c_1)\beta}{(3\beta+4)^3} \\
\text{and } \frac{d\pi_1}{dc_2} &> 0 \text{ for } \beta < \beta^* \text{ where }
\end{align*}
\]
\[
\beta'' = \frac{(2b-2c_1+c_2) + [(2b-2c_1+c_2)^2+8(c_1-c_2)(b-c_1)]^2}{2(c_1-c_2)} \tag{8-32}
\]
\[
dU_2/dc_2 = (dP_2/dc_2 - 1)q_2 + (P_2 - c_2)dg_2/dc_2 \tag{8-33}
\]
\[
= \frac{-[(\beta+1)/(3\beta+4)][q_2+(\beta+1)(P_2-c_2)]}{0}
\]

We obtain that firm 2 will raise its price level as to guarantee a positive mark up on the increase in production cost. The price increase means that quantity sold will decline. The net profit will fall. Firm 1, on the other hand, will increase its price level but only to such an extent as to leave the relative price level, \(P_1/P_2\) lower than before. This will increase its quantity being sold in the market. Therefore, firm 1 will be able to enjoy a rise in its profit level.

Consider now the effect on (i) in the first stage game. Firm 1's quality reaction function contracts downwards towards the origin. This implies that for each given value of \(c_2\) the value of \(c_1\) will be lower than before the change. The same argument applies as before when we considered the Cournot competition. As for firm 2, an increase in \(c_2\) implies a downwards shift of the reaction function. This is obtained due to
\[
2dg_3/dc_2 - dg_4/dc_2 < 0
\]

Therefore, both quality level will fall except that firm 1 will let its quality fall proportionately more than \(c_2\). The intuition is that as \(c_2\) increases, the price level of firm 2 increases while the investment on innovation falls, lowering quality level, in order to compensate for
the increase in the cost of production. Firm 1, on the other hand, will lower its product quality level as well, but only to such an extent as to leave the relative quality level $a_1/a_2$ higher than before. This will guarantee firm 1 a greater market share at a higher price level.

8.6.2 An Increase in the Cost of Innovation

Consider an increase in the cost of innovation by firm 2. The first stage game will yield a similar result as before. Firm 2's reaction function will shift downwards from $r_2$ to $r_2'$ as shown in figure 8-6 while that of its rivals will remain. Firm 2 will lower its level of quality. Firm 1 will thus have a greater advantage in terms of quality level. It will also lower its quality such as to gain from cost cuts which will offset the corresponding fall in the price level. The equilibrium of the first stage game will change from $Z'$ to $Z''$. A fall in $\beta$ implies a fall in $q_2$. The sale's volume of firm 1, however, depends on the present value of $\beta$. If $\beta$ was such that the present values of $q_1, q_2$ were given as between $M$ and $M''$ in figure 8-1, $dq_2/d\beta$ will be positive while between $M''$ and $M'$, it will be negative. The implication of this result is that an increase in the cost of innovation will lead both firms to lower their product quality levels.
THE EFFECT ON EQUILIBRIUM AS COST OF INNOVATION INCREASES FOR EACH GIVEN LEVEL OF QUALITY
8.7 Cournot vs Bertrand Equilibrium

Using the same numerical values used so far, we compare the results obtained under the Cournot and Bertrand conjectures. They are summarized in Appendix C.

8.7.1 The Price and the Quality Level
The price level of firm 1 is higher under quantity competition than under price competition for all argument of $\beta$. The price level of firm 2, however, is higher under quantity competition only for values of $\beta$ greater than 7/4. For smaller values of $\beta$, Bertrand competition attains a higher equilibrium price level for firm 2. The intuition is that under the Cournot competition, for small values of $\beta$ (i.e., large product quality gap), the sales volume of firm 1 is large. Since the price level is a linear function of both firm‘s sale‘s volume (as shown in equation 6-10), the price level of firm 2 will be low. A second reason for a low price level is that firm 2 is trying to maximize revenue with respect to quantity supplied. In order to attain the maximum quantity level with large quality gap, the only possible option is to lower its price level.

Under Bertrand competition, firm 2 will face a situation where it has to undercut the price level of firm 1 given a large quality gap. The price level need not, however, be as low as in the Cournot competition as it is maximizing with respect to the price level and does not have to attain any maximizing output level. As the quality difference is reduced, we obtain the general
result that price competition yield lower price level and a larger sales volume than under quantity competition.

8.7.2 The Utility Level
The utility level of firm 1 is always greater under Cournot competition for values of $\beta$ between 0 and 45. We found that the maximum permissible value of $\beta$ (under Bertrand competition) is 16 below which firm 1 is forced out by firm 2 and at which the profit level of firm 1 becomes negative. The profit level of firm 2 is not bound by the value of $\beta = 16$. The Cournot equilibrium yields a higher utility for firm 2 for $1 < \beta < 78$. For values of $\beta < 1$, i.e., a very large difference in the quality level, the Bertrand conjectures equilibrium yields higher utility level because of the low price level of firm 2 as explained above. For values of $\beta > 78$, i.e., as the quality difference is very small, firm 1 will be driven out of the market under Bertrand competition due to its higher cost of production (i.e., $q_1 = 0$).

8.8 Conclusion
We have shown that under Bertrand competition the Nash Equilibrium of the first stage game is not different from that of the Cournot competition under zero cost of innovation and difference in the cost of production. This result is obtained as we assumed identical upper and
lower bounds on potential quality levels to prevail in both the Cournot and Bertrand cases. Given positive cost of innovation, we observed one difference. The reaction function of firm 2 is now convex to the origin whereas before, it was concave. This was caused by the shape of profit function of firm 2. Under Cournot competition, the profit function was concave to the origin tending to a certain limit as $\beta \rightarrow \infty$. Now, we have a convex profit function with respect to $\beta$. This difference in the shape of firm 2's profit function has roots in the different assumptions of the strategic variables firms choose in the competition. An increase in the cost of production of firm 2 will result in an increase in $P_2$, then accordingly, a fall in $q_2$ will arise. However, this fall will be partly compensated by an increase in $a_2$. Firm 1, on the other hand, will increase its price level also while lowering its quality level such that it will lose some market share. The profit of firm 1 will have risen due to the rise in the price level and savings from the investment cuts. An increase in the cost of innovation, as it was the case under Cournot competition, will lead to a fall in both quality levels but $da_2 > da_1$. Finally, a fall in $s$, i.e., a shift in the objectives towards revenue maximization relative to profit, will have exactly the opposite result as when there was an increase in the cost of production.
9. Consistent Conjectures

We examine alternative approaches to the two second stage games adopted so far. We consider how different results are produced as we change the behavioural assumptions of firms, e.g., conjectural variations. In particular, we impose 'rationality' assumption on firms behaviour and show that Cournot equilibrium is a consistent conjectures equilibrium. We derive the consistent conjectures equilibrium according to the definition given by Perry (1980, 1982) and Bresnahan (1981), (to be denoted as P & B) and show that in absence of infinite regress problem, the consistent conjectures equilibrium will be between the Cournot and the Bertrand equilibrium for each given value of $\beta$. We then include the infinite regress problem as defined by Daughety (1983) and show that the consistent conjectures equilibrium obtained under the more general definition of Daughety attains the Cournot equilibrium as a solution.

One cannot justify theoretically, why firms should agree on choosing some common variable as their strategic variable and even less so if it comes to determining their conjectures. However, these two problems are not independent from each other. For whatever strategic variables firms choose, by an appropriate assumption on firms conjectures, any feasible outcome may be obtained.
In other words, the outcome of a competition with respect to a strategic variable based on some particular conjecture, may be sustained if one would change to another strategic variable by modifying the conjectures accordingly. Therefore, choice of strategic variable plays little significance if no rationale for some particular conjecture is given. Here, we assume that q is are the strategic variables and focus our attention on the values of conjectural variations.

The two cases of the second stage game considered so far differ in that we introduced two alternative assumptions on conjectural variations. Let $v_1$ represent the conjectural variation of firm 1 and $v_2$ that of firm 2. By definition, under the Cournot and Bertrand conjecture, the values of $v_1$ was given as 0 and -1 respectively while $v_2$ was given as 0 and $-\beta/1+\beta$ respectively.

9.1 Uncertainty and Nash Equilibria
Consider now the case of uncertainty where neither firm knows the conjectural variation of their rival when rationality and full information are not assumed. Below, we illustrate the multiplicity of Nash equilibria of the second stage game even if we restrict the possible behaviour of firms to be between Cournot and Bertrand conjectures. In other words, the possible conjectural variation is confined to some strict subset $[0, -1]$, i.e.,
This simply implies that a firm does not expect that a one unit change in its own output will lead to more than one unit change of its rivals. No positive reactions prevails which could be the case if products are complements. Then conjectural variations of firm 1 could be anywhere between 0 and -1 while firm 2 could choose between 0 and -\( \beta/(\beta+1) \). These values are derived from the definitions of Cournot and Bertrand competition as shown in equations v) and vi) in the Appendix A. We ignore cases of positive conjectures and collusion although Ireland and Hviid (1986) points out that even a positive conjecture may be consistent if the increase in optimism is sufficiently large within a model of demand uncertainty. Consider the extreme cases of \( \{0, -1\} \), \( \{0, -\beta/(\beta+1)\} \) for firm 1 and firm 2 respectively i.e., Cournot and Bertrand competition. The possible outcomes are as follows.

First, firm 1 conjectures that firm 2 will not respond to its own change in quantity, i.e., Cournot competition, while firm 2 conjectures that firm 1 will respond by exactly the same amount, but in opposite direction, as its own change in quantity, i.e., Bertrand competition. We then have an equilibrium for each given value of \( \beta \), which is obtained from mutually different beliefs on conjectures of firms. This is represented as the OS line in figure 9-1. As before, \( r_{iC} \) and \( r_{iB} \) denote the reaction
Figure 9-1

NASH EQUILIBRIA UNDER UNCERTAINTY OF FIRMS' STRATEGIC BEHAVIOUR
functions of firm i under Cournot and Bertrand conjecture respectively given $\beta = 0$. $r_{iC}$ and $r_{iB}$ are the same reaction functions except that they are obtained given $\beta = \infty$. Therefore, point $O$ is obtained for $\beta = 0$ while point $S$ is obtained for $\beta = \infty$.

Secondly, we could have the opposite case where firms' conjectures are each reversed. The possible outcomes are represented by the OR curve. Point $R$ is obtained for $\beta = \infty$ and point $O$ when $\beta = 0$. If we include the other two cases when firms both engage in Cournot and in Bertrand competition, we obtain in total four possible loci of non-cooperative Nash Equilibria as shown in figure 9-2. ON represents the Cournot-Cournot solution while OM represents the Bertrand-Bertrand solution. If we now allow all intermediary cases between the extreme four cases of C-V considered above, we obtain a whole area of possible Nash Equilibria as shown by the shaded areas in figure 9-2, i.e., it is a Convex Hull of all possible outcomes of the extreme cases.

9.2 Rationality and Nash Equilibria

If firms are rational, i.e., they have the capacity to analyse the market structure as well as infer the behaviour of other agents given that firms' rationality is a common knowledge, then the market must obtain a conditional rational Nash equilibrium at which their
NASH EQUILIBRIA UNDER UNCERTAINTY OF CONJECTURES

Figure 9-2
expectations of each other's conjectures must be equal to the actual conjectures. We define such conjectures as 'consistent conjectures' and the corresponding equilibrium as conditional Rational Non-cooperative Nash equilibrium. It is conditional in that all information must be correctly perceived by both firms. We now define consistent conjectural variation formally as \( v_i(q_i, q_j|\beta) \) = \( r_j(q_i, q_j|\beta) \) where \( v_i \) is firm i's conjecture on firm j's reaction in response to changes in \( q_i \) for each given values of \( \beta \) and \( r_j \) to be the actual reaction of firm j on changes in \( q_i \) for each given values of \( \beta \). In fact, \( r_j \) is the slope of the reaction function of firm j.

The notion of consistent conjectures first appeared as an attempt to give some rationale on the determination of firms' conjectures on each other. Initial attempts on this subject led to the uniqueness solution, whereas before, any feasible solution could be explained as a conjectures equilibrium by suitable choice of conjectures. These earlier findings were not without criticisms. Below, we illustrate the notion of consistent conjectures by P & B and examine the corresponding weaknesses.

9.2.1 Definition of Consistent Conjectures (P & B)
We define the objective function of a firm in oligopoly as
\[
\pi_i = p_i(q)q_i - c_i(q_i)
\]
where $p_i$, $q_i$ and $c_i$ represent the price level, the quantity level and the cost of production by firm $i$. The corresponding first order condition for a profit maximization is given by

$$q_i(\delta p_i/\delta q_i + \delta p_i/\delta q_j v^i) + p_i - \delta c_i/dq_i = 0 \quad (9-3)$$

(as before, $\delta$ represents partial differentiation)

where $v^i$ represents the conjectural variation of firm $i$ and is defined as $\delta q_j/\delta q_i$ for $i = 1, 2, \text{and } i \neq j$. Equation (9-3) is an implicit function of $q_i$ and $q_j$. We define it as

$$q_i = r_i(q_j) \quad (9-4)$$

Therefore, (9-4) solves (9-3). An oligopoly equilibrium is defined as $q^* = (q_1^*, q_2^*)$ where

$$q_1^* = r_1(q_2^*) \quad \text{and} \quad q_2^* = r_2(q_1^*) \quad (9-5)$$

A consistent conjectures equilibrium is a pair of quantities $q^*$ and of conjectures ($v^1$, $v^2$) such that

$$q_1^* = r_1(q_2^*), \quad q_2^* = r_2(q_1^*)$$

and for some $\tau > 0$,

$$v^1(q_1) = \delta r_1(q_1)/\delta q_1 \quad \text{for all } q_1^* - \tau < q_1 < q_1^* + \tau \quad (9-6)$$

$$v^2(q_2) = \delta r_2(q_2)/\delta q_2 \quad \text{for all } q_2^* - \tau < q_2 < q_2^* + \tau$$

In other words, a consistent conjectures equilibrium is attained at which the conjectures of firm $i$ on firm $j$ is equal to the actual behaviour of firm $j$. The uniqueness of consistent conjectures equilibrium is obtained under
certain assumptions on the specifications of demand and cost functions.

9.2.2 Parameterization and Consistent Conjectures
The crucial point to guarantee a unique equilibrium seems to rest on the way vis are derived. The generalization of demand and cost function will lead to a change in the way the vis are derived and thus eliminate the uniqueness result of P & B. Therefore, under full information assumption, i.e., each firm knows the values of all the exogenous variables and the objectives of firms (given that all information is a common knowledge), the specifications of demand and cost functions play a crucial role in determining the way the vis are derived. An example will illustrate this point.

Given such a full information structure, equation 9-6 may be rewritten as

\[ v_1(q_1, v^2) = \frac{\delta r_1(q_1, v^2)}{\delta q_1} \]  

\[ v_2(q_2, v^1) = \frac{\delta r_2(q_2, v^1)}{\delta q_2} \]  

The solution to this simultaneous equations are given as

\[ v_1 = \frac{\delta r_1[q_1, \delta r_2(q_2, v^1)/\delta q_2)]}{\delta q_1} \]  

\[ v_2 = \frac{\delta r_2[q_2, \delta r_1(q_1, v^2)/\delta q_1)]}{\delta q_2} \]

Thus, uniqueness is obtained for various possible specification of demand and cost functions as long as the
right hand side of (9-8) and (9-9) are both linear functions of $v^1$ and $v^2$ respectively.

9.2.3 Dynamics and Consistent Conjectures

One weakness of the P & B model is that it lacks any dynamic adjustment property. Under a dynamic framework, if firms start off from a position which may not be the consistent conjectures equilibrium as defined by P & B, the uniqueness result may not be attained as shown by Laitner (1980).

Ulph, on the other hand, acknowledging this finding, argues that to produce a properly formulated criterion for rationality would require a host of additional rationality criteria, which would require the correctness of conjectures about the consequences of output changes from points other than the equilibria relative to a given belief.

Most recent criticism on this aspect comes from Makowski (1987) where he adopts basically the same argument as Ulph's reinterpretation of the definition of consistent conjectures by P & B. He points out that the equations in (9-6) should be written as:

$$v^1(q) = \frac{dr_1(q)}{dq_1} = \frac{\delta r_1(q)}{\delta q_1} + \frac{\delta r_2(q)}{\delta q_2} v^2$$

$$v^2(q) = \frac{dr_2(q)}{dq_2} = \frac{\delta r_2(q)}{\delta q_2} + \frac{\delta r_1(q)}{\delta q_1} v^1$$

(9-10)
We note that the second term on the right hand side was assumed to be zero in P & B's definition.

The fundamental source that produces (9-10) as against (9-6) is that in the former, the function $r$ is given as $r_i = r_i(q_i, q_j)$ whereas in the later, it is given as $r_i = r_i(q_i, q_j(q_i(q_i(\ldots))))$ for $i = 1, 2$, and $i \neq j$. But then, the argument would be why the function $r_i$ should stop there instead of $r_i = r_i(q_i, q_j(q_i(q_i(q_i(\ldots))))))$ which would require an infinite number of repetition. In other words, not only is the initial starting position of firms important, but also how they come to be where they start off.

9.2.4 Infinite Regress of Expectation

The problem of infinite repetition just mentioned above, has not escaped attention in the past. Daughety (1985) solves an infinite regress problem where firm $i$ chooses an output level to maximize its profit subject to a model representing its rival $j$, which in turn maximizes its profit subject to a model of its rival, firm $i$'s model. This is repeated infinite times. Under a fundamentally the same definition of consistent conjectures to that of P & B, he shows that the Cournot equilibrium is the consistent conjectures equilibrium.

The strength of his approach lies in that there is no initial versus final position in the model but only one single move. Given full information about all the values
of the exogenous variables, the infinite regress is a thought experiment carried out in one move, thus eliminating the problem of how to determine the initial position as considered above.

It appears to be counter intuitive, first, that a Cournot equilibrium should be the consistent conjectures equilibrium. After all, the definition of consistent conjectures equilibrium requires that the expected response of one's rival is in fact the actual response. Cournot conjectures, however, assumes the rival's response to be zero. But this is exactly what a consistent conjectures equilibrium requires. Given a consistent conjectures equilibrium is attained, there is no reason why a firm should attempt to change its position, even less so as a response to its rival's change if it knows that the rival's move is contrary to profit maximization behaviour by the definition of consistent conjectures equilibrium.

Therefore, Cournot equilibrium is consistent with the definition of consistent conjectures equilibrium.

9.3 P & B Consistent Conjectures Equilibrium

We now examine the model presented in chapter 6 under consistent conjectures as defined by P & B. The reaction functions which are solutions to the profit maximizing first order conditions are given as follows:
\[ q_1 = \frac{(1+\beta)}{\beta(v^1+2)+2} (b-c1) - \frac{\beta}{\beta(v^1+2)+2} q_2 \] (9-11)

\[ q_2 = (2+v^2)^{-1} (b-c2) - (2+v^2)^{-1} q_1 \] (9-12)

For one particular value of \( \beta \), say \( \beta=1 \), (9-11) and (9-12) is given as follows:

\[ q_1' = \frac{2(b-c1)}{4+v^1} - \frac{1}{4+v^1} q_2' \] (9-13)

\[ q_2' = \frac{(b-c2)}{2+v^2} - \frac{1}{2+v^2} q_1' \] (9-14)

The equilibrium is obtained at values of \( q_1^* \) and \( q_2^* \) which are obtained from (9-13) and (9-14).

\[ q_1^* = \frac{[2(v^2+2)(b-c1)-(b-c2)]/[(v^1+4)(v^2+2)-1]}{(9-15)} \]

\[ q_2^* = \frac{[(v^1+4)(b-c2)-(b-c1)]/[(v^1+4)(v^2+2)-1]}{(9-16)} \]

We observe, as Laitner (1980) claims, that there are many possible values of \( v^1 \) and \( v^2 \) for each given value of \( q_1^* \) and \( q_2^* \).

Under rational conjecture, two restrictions are imposed as given in (5-6):

\[ v^1 = \delta q_2/\delta q_1 \quad \text{and} \quad v^2 = \delta q_1/\delta q_2 \] (9-17)

From (9-11) and (9-12), we obtain the value of \( dq_1/dq_j \) respectively.
Solving this simultaneously for \( v_1 \) and \( v_2 \) yield the followings

\[
\begin{align*}
    v_1 &= \frac{-(\beta+1)\pm(\beta+1)^{\frac{1}{2}}}{\beta} \quad (9-18) \\
    v_2 &= \frac{-(\beta+1)\pm(\beta+1)^{\frac{1}{2}}}{(1+\beta)} \quad (9-19)
\end{align*}
\]

Since the right hand side of equations in (9-17) are not linear functions of \( v_1 \) and \( v_2 \) respectively, we obtain in (9-18) and (9-19) two possible conjectures confirming our earlier findings. Let

\[
\begin{align*}
    v_1^+ &= \frac{-(\beta+1)+(\beta+1)^{\frac{1}{2}}}{\beta} \\
    v_1^- &= \frac{-(\beta+1)-(\beta+1)^{\frac{1}{2}}}{\beta}
\end{align*}
\]

The values of \( v_1^+ \) and \( v_1^- \) for each given value of \( \beta \) is represented in figure 9-3.

Substituting \( v_1 \) and \( v_2 \) back into the above reaction function (9-2) and (9-3), we obtain the following:

\[
\begin{align*}
    q_1 &= \frac{(1+\beta)}{(1+\beta)\pm(1+\beta)^{\frac{1}{2}}} (b-c_1) = \frac{\beta}{(1+\beta)\pm(1+\beta)^{\frac{1}{2}}} q_2 \quad (9-20) \\
    q_2 &= \frac{(1+\beta)}{(1+\beta)\pm(1+\beta)^{\frac{1}{2}}} (b-c_2) = \frac{(1+\beta)}{(1+\beta)\pm(1+\beta)^{\frac{1}{2}}} q_1 \quad (9-21)
\end{align*}
\]

The equilibrium for each given value of \( \beta \) is obtained from solving equations (9-20) and (9-21) simultaneously. They are given as
Figure 9-3

CONJECTURES UNDER VARIOUS COMPETITION FOR EACH GIVEN VALUES OF $\beta$
\[
q_1 = \frac{(1+\beta)((1+\beta)\beta(b-c_1)-\beta(b-c_2))}{[(1+\beta)(1+\beta)\beta]^2-(1+\beta)\beta}
\]  
(9-22)

\[
q_1 = \frac{(1+\beta)((1+\beta)\beta(b-c_2)-\beta(b-c_1))}{[(1+\beta)(1+\beta)\beta]^2-(1+\beta)\beta}
\]  
(9-23)

Note that in the limit \((\beta \to \infty, \beta \to 0)\), the consistent conjectural variation collapses to the Bertrand competition. Note that this uniqueness corresponds to the result obtained by Perry and Bresnahan for when \(\beta \to \infty\). This implies that under homogeneous product assumption, consistent conjecture results in uniqueness. As for \(\beta \to 0\), we get two solutions for each given value of \(\beta\) although the products are differentiated. The possible Nash equilibria for each given level of \(\beta\) is shown in figure 9-4. The curve OT and O'T' represent the Nash equilibria under \(v^i+\) and \(v^i-\) respectively for each given value of \(\beta\). ON and OM represent, as before, the Cournot-Nash and Bertrand-Nash equilibrium.

Below, we illustrate why the consistent conjectures equilibria under \(v^i+\) should be as shown in OT.

For \(0 \leq \beta \leq \infty\), compare the denominator of all the coefficients of the reaction functions obtained from Bertrand competition with consistent conjectural variation, i.e., \((2+\beta)\) and \((1+\beta)+(1+\beta)\beta\) respectively. Rewrite the former as \((1+\beta)+1\). Let \(x = 1+\beta\) and \(y = 1+x\). Then the former is given as \(y^1 = x+1\) while the latter as \(y^2 = x + x^{3/2}\). We note that \(dy^2/dx > dy^1/dx > 0\) for all
Figure 9-4

LOCi OF CONSISTENT CONJECTURES
NASH EQUILIBRIUM
argument of $x \geq 1$ since $\beta \geq 0$. Therefore, in spite of the converging limiting cases between Bertrand and consistent conjectural variations, the loci of Bertrand equilibria will lie above the loci of consistent equilibria. Therefore, one possible loci of equilibria of stage two game under consistent conjecture will lie between the Cournot and Bertrand competition.

9.4 Cournot Equilibrium as the Consistent Conjectures Equilibrium

So far, we have derived the consistent conjectures equilibrium of our model according to the definition of P & B. We found that there are two possible equilibria for each given value of $\beta$. There is, however, inconsistency in the definition as it stands. The reaction functions of firm 1 and 2 as given in equation (9-14) and (9-15). The slope of the reaction function should be equal to the conjectures. But in our case, they are not. Such inconsistency in the definition of consistent conjectures has led to other more refined notions as already discussed earlier. We also mentioned that solving infinite regress problem yield Cournot equilibrium. We now give the intuition for this result.

A consistent conjectures is defined at a point $(q_1, q_2)$ if firm 1's conjectures about the response of its rival to a small change in $q_1$ represent change in $q_2$ with which firm 2 would itself be satisfied. Therefore, if there is
a particular point \( q^* = (q_1^*, q_2^*) \) at which changes in \( q_1 \) away from \( q_1^* \) would not result in increase in firm 1's profitability for all feasible reaction from its rivals and vice versa, then \( q^* \) is defined as the consistent conjectures equilibrium. Feasible reaction means any change in \( q_2 \) as a response to \( q_1 \) which would lead to an increase in firm 2's profitability.

Given this definition, consider solving the infinite regress problem played in a single move. There may be more than one such point \( q^* \) at which the definition is satisfied. However, we consider one particular point, the Cournot equilibrium \( q^c = (q_1^c, q_2^c) \). We know that the Cournot equilibrium was obtained in chapter 7 under a zero conjecture. In other words, at \( q^c \), firm 1 will not change \( q_1 \) because it knows that firm 2's reaction within its feasible set will not improve firm 1's profitability. Likewise, firm 1 knows that firm 2 faces the same situation. Therefore, firm 1 conjectures that firm 2's conjectural variation is zero at \( q^c \) and that in the neighbourhood of this point, there is no reaction function to neither firm. Thus Cournot equilibrium satisfies the P & B as well as solves the infinite regress problem.
9.5 Conclusion

We have shown various possible outcomes of stage two game under uncertainty, full information and rationality. Under uncertainty of conjectural variations, there are infinitely many possible solutions as shown in figure 9-2. Any of those solutions may be obtained by an appropriate assumption of firms conjectures.

We then illustrate the various attempts in reducing this multiplicity of solutions. The common notion in question was termed as consistent conjectures. The controversy surrounding the definition included the derivation of vis, dynamics and infinite regress problems.

We first derived consistent conjectures equilibrium according to the definition of P & B. We found that for each given values of $\beta$, there were two equilibria. The dominant strategy for each given value of $\beta$ will be determined by the values of the exogenous variables, $b$, $c_1$ and $c_2$. We found that the consistent conjectural variation, however, differed from the slope of the reaction function, implying inconsistency in the definition. Thus adopting a more general definition by Laitner, Ulph and Daughety, we were able to locate at least one equilibria which was consistent with the definition of consistent conjectures equilibrium: the Cournot equilibrium. Although we did not prove the existence of other consistent equilibria, we found that the Cournot equilibrium is a solution under consistent conjectures.
Rationality is said to prevail when firms use all information they have including expectations on its rivals responses to its own changes in quantity when maximizing profit. Full information is defined as information about not only on its own firm but also that of its rival as well as knowledge of all exogenous variables. In this respect, given rationality prevails, Cournot equilibrium may be the unique solution to the duopoly competition.
10. Competition for Market Leadership

In chapter 7 and in 8, we have considered the Nash equilibrium under Cournot and Bertrand conjectures given that the market leadership in terms of quality level is held by firm 1. It is an interesting question as to whether in reality, a firm in NICs may overtake the leadership in some markets or even decide to remain a follower. In this chapter, we shall analyse these possibilities within the two stage game framework adopted so far.

Given an increase in the cost of firm 1 (both production cost and/or innovation cost), firm 1 may decide to give up the leadership and become a follower. In some cases, it may be forced to stay a leader. One of the concerns of industrial economists has been the effect of innovation on market structure. As already elaborated in the introductory chapter, a key question that has been constantly been addressed is persistent dominance as against action-reaction of firms changing leadership in turn. Various findings have been reported based on different model structure, e.g., single vs sequential game, product vs process innovation and finally, Bertrand vs Cournot competition in the marketing stage.

Initially, works by Dasgupta (1982) and Gilbert & Newbery (1982) were based on unconditional once and for all commitments of R&D efforts at the outset of a race lacking
behavioural dynamics. Fudenberg et al (1983) on the other hand introduced strategic interaction to yield a race for patents. Firms compete vigorously if they are nearly even while if one firm pulls far enough, the other drops out.

Harris and Vickers (1985) set up a model of asymmetric firms and of process innovation to explore strategic consequences of asymmetries between firms. Their asymmetries were firms' valuation of the price of winning, firms discount rates, efficiency at making progress and their initial distances from the finishing line. Their findings were that the behaviour of the winner of the race is often exactly as if he were the only player, except in the first stage of the race.

Reinganum (1985) introduces a model of sequential process innovation. Her finding was that the current incumbent firm has less incentive to innovate than his rivals, resulting in action-reaction. Vickers (1986) on the other hand, considers a model where firms not only take into account the immediate effects of the race, but also its indirect influences upon subsequent patent races. He shows that if the product market is competitive, e.g., Bertrand competition, there is an increase in the monopoly power of the incumbent firm whereas if it is not so competitive, e.g., Cournot competition, there will be action-reaction.

More recently, Beath, Katsoulaces and Ulph (1987) found that if the R&D investment is again for process innovation and if the marketing stage is competitive (Bertrand competition),
persistent dominance of the incumbent firm is observed. On the other hand, if investment is on product innovation, then even under Bertrand competition, we may either find persistent dominance or action-reaction. The former is guaranteed if the new innovation introduced does not provide too great a gap compared to the ex ante quality level so that monopoly is not observed.

Our concern here is to examine the possibility of the follower overtaking the leader within a model where the follower has lower cost of production and R&D investment is on product innovation. We shall examine under both the Cournot and Bertrand competition for the marketing stage. The incentives for firms to compete to become either a leader or a follower will be their relative profit levels. The maximum possible level of innovation is not fixed but will be determined by the profit maximizing firm given a positive cost of innovation.

In the following two chapters, as an extension to this subject, we shall examine firstly how such a race for market leadership may be affected by the introduction of various licencing rules and, secondly, the optimal investment into R&D and the timing of implementation of new innovations.

10.1 First Stage Game given Cournot-conjectures in the Marketing stage

The profit functions of firm 1 and firm 2 were given in (7-8) and (7-9) as follows:
We have established already the reaction functions of firm 1 and firm 2 in (7-17) and (7-22) respectively.

10.1.1 The Choice between staying a Leader and becoming a Follower

The profit function of firm 1, if it becomes the follower, may be obtained directly from the profit function of firm 2 as shown in (7-9). The profit level of firm 1 is given as

$$\pi_{1F} = \frac{[A(\Gamma+1)]^2}{[3+4\Gamma]^2} - a_1a_1'$$

(10-3)

(we define the quality level of firm 1 and firm 2 as $a_1'$ and $a_2'$ respectively if $a_2 > a_1$. The corresponding reaction function is given as

$$a_1' = \Gamma_1*(a_1')/[\Gamma_1*(a_1')+1] a_2'$$

(10-4)

where $\Gamma = a_1'/[a_2'-a_1']$ for $a_2' > a_1'$ and $\Gamma_1*(a_1')$ satisfies $a_1a_2 = 2A^2(1+\Gamma)^3/(3\Gamma+4)^3$.

Likewise, if firm 2 decides to become the leader, it would face a profit function given as
\[
U_{2L} = \frac{[2Ga_z' + (D-2G)a_1']^2}{4a_z'^2-a_1'^2} - a_2a_z' \tag{10-5}
\]

where \( G = b - c_2 \).

The corresponding reaction function is given as follows:

\[
a_2' = \frac{[\Gamma_2*(a_2') + 1]}{\Gamma_2} a_1' \tag{10-6}
\]

where \( \Gamma_2*(a_2') \) satisfies

\[
a_2a_1 = 4A(D\Gamma+2G)\Gamma^2/(3\Gamma+4)^3
\]

Analogous to the Cournot Nash equilibrium obtained in chapter 7, the equilibrium, given that market leadership changes, is shown in figure 10-1 as the point \( Z \). The point \( Z' \) is the original Cournot Nash equilibrium. \( r_i' \) and \( r_i \) denote the reaction function of firm \( i \) before and after the change in the market leadership respectively.

Note that \( D > A \). Therefore, we obtain that firm 2's reaction function as a leader is equal to that of firm 1 as a leader while firm 1's reaction function as a follower is more shrunk towards the origin when compared to that of firm 2 as a follower. The intuition is that as firm 1 has a higher production cost than firm 2 (given constant marginal and average cost), for each given value of its rival's quality level, it has to opt for a lower quality level to compensate the loss from the profit net of innovation cost by cutting down on its expenditure on innovation investment.

Furthermore, this asymmetry in firm 2's reaction function is a sufficient condition for the slope of the line \( OZ' \) to be less steep than the inverse of the slope of \( OZ \) due to the concavity of \( r_1' \) and \( r_2 \). In other words, the Cournot Nash
Figure 10-1

REACTION FUNCTIONS OF FIRM 1 AND FIRM 2
BEFORE AND AFTER THE CHANGES IN THE MARKET LEADERSHIP
equilibrium after the change in leadership is attained where the quality difference is larger relative to that before the change. Therefore, in the long run, i.e., it may take some time for initial conditions to weaken, given a follower overtakes a leader with a lower cost of production we will observe a widening quality gap although the absolute values in both firms will be smaller than in the case of no-overtaking.

Compare the profit levels of firm 1 between Z and Z'.

\[
\pi_1^L = \pi_1^F = \frac{[A\beta^*+2B]^2}{[3\beta^*+4]^2} - \alpha_1a_1^* - \frac{[A(\Gamma^*+1)]^2}{[3\Gamma^*+4]^2} + \alpha_1a_1^{**} \tag{10-7}
\]

where \( \beta^* = \frac{a_2^*/(a_1^*-a_2^*)} \)
\( \Gamma^* = \frac{a_1^{**}/(a_2^{**}-a_1^{**})} \)

and firm 1 attains Z and Z' at \((a_1^{**}, a_2^{**})\) and \((a_1^*, a_2^*)\) respectively.

Since we established that \(a_1^* > a_2^{**}\), \(a_1^* > a_2^*\) and \(a_2^* > a_1^{**}\), we know that \(\beta^* > \Gamma^*\). We need to establish the sign of equation (10-7) in order to find a dominant strategy for firm 1. We write (10-7) as

\[
\frac{[\Gamma(3B-2A)+A\beta+4(2B-A)]^2}{[(3\Gamma+4)(3\beta+4)]^2} - \alpha_1(a_1^* - a_1^{**}) \tag{10-8}
\]

where \(3B - 2A = D\) and \(2B - A = G\). Both the first and the second term will be positive, thus leaving the sign indeterminate. We do know, however, that an increase in the value of B will lead to an increase of the first term of (10-8) while an increase in the value of A will lead to an
increase of the value of the first term only if \( \beta < 2(\Gamma+2) \), which is indeterminate.

In order to obtain the sufficient condition for \( \pi_1^L > \pi_1^f \), we would require to solve \( \alpha^* \) and \( \Gamma^* \) in terms of the cost parameters. What we do know however, is that as the cost of production and/or cost of innovation increases, it becomes less profitable for firm 1 to defend its leadership. Note that an increase in \( c_1 \) implies a fall in the value of \( A, B \) and an increase in the value of \( D \). The Cournot Nash equilibrium will move towards the North-east direction as shown in figure 10-1 by the arrow starting from \( Z' \) as \( r_2' \) will shift upwards while \( r_1' \) is indeterminate. This will lead firm 1 to a lower profit contour.

Consider now the profit level of firm 2 at points \( Z \) and \( Z' \).

\[
U_{2F} - U_{2L} = \frac{[(1+\beta)D]^2}{(3\beta+4)^2} - a_2\alpha^* - \frac{[\Gamma+2\Gamma]^2}{(3\Gamma+4)^2} + a_2^{**} \quad (10-9)
\]

As before, we rewrite (10-9) to establish the dominant strategy for firm 2 as follows:

\[
a_2(a_2^{**}-a_2^*) - \frac{[2\Gamma\beta+\Gamma+2\beta(3\Gamma-2D)+4(2\Gamma-D)]^2}{[(3\Gamma+4)(3\beta+4)]^2} \quad (10-10)
\]

where \( 3\Gamma-2D = A \) and \( 2\Gamma-D = B \). As before in (10-8), the sign of (10-10) is indeterminate as both terms are positive. We now have a situation where an increase in values of \( A, B \) and \( D \) will lead to an increase of the second term in (10-10). The implication of this is that firm 2 will stay a
follower if the cost of innovation, i.e., $a_2$ is high or the relative cost of production $c_1/c_2$ is smaller than a certain value below which firm 2 may venture to overtake the leadership. For a precise value, we need to solve for $a^*$ and $r^*$ in terms of cost parameters.

We have established so far that the leader will defend its leadership if the cost of innovation is low and its cost of production is not too high relative to that of its follower. If, on the other hand, the cost of innovation increases to a very high level while the cost of production is fixed at a low level (or the cost of innovation is fixed at a low level and there is a sharp increase in the cost of production while that of its competitor remains unchanged), firm 1 will opt to become the follower reducing its quality level. This is because being a follower, firm 1 would not require a high quality level, thus saving from the reduction in the level of investment into innovation.

Analogous to firm 1, firm 2 will stay a follower if the cost of innovation is high while the relative cost of production is low. On the other hand, if the cost of production is very low, firm 2 may venture to overtake the leadership investing heavily in innovation which would now be possible from the savings obtained from the fall in the cost of production.

Suppose firm 2 copies technology from firm 1 without paying a licence fee and without independent innovation. This would be denoted as $a_2 = 0$. Then it pays firm 2 to become
the leader except that if it becomes a leader, its cost of innovation will be positive forcing it to become a follower again. Under such a case the follower may wish to approach the quality level of firm 1 very closely, but never actually take over until either a fall in firm 2's cost of innovation or a widening cost of production between the leader and the follower is observed. It could be that this is the reason why many electronics firms in Korea are following the quality levels of its rivals but, so far, no attempts have been made to overtake the leadership.

10.1.2 A Numerical Example
As before, we assign specific values for \( b \), \( c_1 \) and \( c_2 \) as follows to confirm our findings so far to include the curvatures of reaction functions as well as the values of each quality levels.

\[
\begin{align*}
\theta &= 10 \\
\alpha_1 &= 2 \\
\alpha_2 &= 1 \\
\beta_1 &= \beta_2 = 1
\end{align*}
\]

This implies that

\[
\begin{align*}
A &= 7 \\
B &= 8 \\
D &= 10 \\
2\theta^2/27\alpha_1 &= 3.6 \\
2\theta^2/64\alpha_1 &= 1.5
\end{align*}
\]
The result of a simulation deriving the reaction functions of both firms, given a change in leadership is obtained as follows:

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>( a_2 )</td>
</tr>
<tr>
<td>0.1</td>
<td>1.6</td>
</tr>
<tr>
<td>0.6</td>
<td>2.0</td>
</tr>
<tr>
<td>1.1</td>
<td>2.3</td>
</tr>
<tr>
<td>1.6</td>
<td>2.6</td>
</tr>
<tr>
<td>2.1</td>
<td>2.8</td>
</tr>
<tr>
<td>2.6</td>
<td>3.1</td>
</tr>
<tr>
<td>3.1</td>
<td>3.4</td>
</tr>
<tr>
<td>3.6</td>
<td>3.6</td>
</tr>
<tr>
<td>4.1</td>
<td>1.9</td>
</tr>
<tr>
<td>5.1</td>
<td>3.0</td>
</tr>
<tr>
<td>6.1</td>
<td>4.3</td>
</tr>
<tr>
<td>7.1</td>
<td>5.9</td>
</tr>
<tr>
<td>8.1</td>
<td>7.8</td>
</tr>
</tbody>
</table>

These values are plotted in figure 10-2 and approximated into a smooth curve. The values of \( r_1' \) and \( r_2' \) were already obtained from the numerical example in chapter 7. The equilibrium values are as follows:
Figure 10-2

SIMULATION RESULT SHOWING REACTION FUNCTIONS OF BOTH FIRMS UNDER CHANGES OF MARKET LEADERSHIP
\[ a_1^* = 5.5 \]
\[ a_2^* = 3.6 \]
\[ a_{1**} = 0.6 \]
\[ a_{2**} = 1.9 \]
\[ \beta^* = 1.89 \]
\[ \Gamma^* = 0.46 \]

This confirms our earlier findings that \( a_1^* > a_{2**}, a_1^* > a_{2**}, a_{2**} > a_{1**} \) and \( \beta^* > \Gamma^* \). The utility levels are given as follows:
\[ \pi_{1L} = 3.64 \]
\[ \pi_{1F} = 0.84 \]
\[ U_{2F} = 5.33 \]
\[ U_{2L} = 0.13 \]

Therefore, given \( b = 10, c_1 = 2 \) and \( c_2 = 1 \), we find that \( \pi_{1L} > \pi_{1F} \) and \( U_{2F} > U_{2L} \). In other words, the dominant strategies for each firm would be to stay the leader for firm 1 and to be a follower for firm 2.

10.2 Folk Theorem and the Incumbent firm

Earlier in this chapter, we derived the result that for \( \pi_{1L} > \pi_{1F} \) and \( U_{2F} > U_{2L} \) to hold, we would require certain conditions on the level of cost parameters. We required that the values of D, G and A to be high for \( \pi_{1L} > \pi_{1F} \) and D, A and B to be low for \( U_{2F} > U_{2L} \). For given values of \( b^* \) and \( c_2^* \), this implies that both firms will not attempt to change their market position as leader and follower as long
as there exists some value of $c_1^*$ that satisfies both (10-8) and (10-10). However, for certain values of $c_1$, we may have a situation where each firm would maximize its profit by becoming a follower. In other words, firm 1's profit maximizing strategy would be $a_1^*$ while that of firm 2 would be $a_2$ (as shown in figure 10-3). We consider this problem under an infinitely repeated game. In particular, we shall describe this situation under Folk theorem.

10.2.1 The Folk Theorem
The Folk Theorem states that the profit of Nash equilibrium strategies, i.e., $a_i$, in an infinitely repeated game are feasible, individually rational profits in an one-shot game providing that there is sufficiently little discounting of the future. The vector of profits, $\pi^*(a_1^*, a_2^*) = [\pi_1^*(a_1^*, a_2^*), \pi_2^*(a_1^*, a_2^*)]$ is feasible in an one shot game if it is a convex combination of pure strategy 2-tuples. The vector of profit $\pi^*(a_1^*, a_2^*)$ is individually rational if it pareto dominates the minmax outcome $\pi_{i\text{mm}}(a_1, a_2)$ for $i = 1, 2$. In other words, the minmax profit of a firm is the particular level below which its profit level can not be forced further down by its rival. We note that $(a_1^*, a_1^*)$ giving rise to $\pi^*$ are not restricted to be Nash equilibria for the one shot game, but generally are a larger set of outcomes.

Under such a setting of a game, firms' responses to their rival's deviant behaviour in the repetitive game strategy is implicitly inbuilt. For example, if firm 1 deviates from
\( \pi^*(a_1^*, a_2^*) \), firm 2 will react in the next stage such that \( \pi_1^*(a_1^*, a_2^*) \) becomes \( \pi_{\text{min}}(a_1, a_2) \) and will do so for ever. Given an infinite horizon and a discounting factor sufficiently small, such an one shot gain from deviation will be more than offset by their present discounted value of its future stream of profits at minmax level.

One problem with this notion of equilibrium is that it is not a perfect equilibrium as observed by Rubinstein (1979), i.e., it may not be in firm 2's best interest to respond with the minmax strategy for ever. This implies that the minmax strategy may not prove to be credible. One way of restoring subgame perfection is if an incentive in the form of a small increment to their average profitability is given to the firm not free riding. A proof of this last statement is given in Fudenberg and Maskin (1986).

10.2.2 Forced Leadership

Having outlined briefly the Folk Theorem, let us now return to our model where we assumed to have a situation where each firm finds itself to be better off becoming a follower.

In a one-shot game, the Nash equilibrium will be attained at the origin as this is the only point where the reaction functions of both firms intersect. This is because the reaction function of firm 1 is given as \( r_1 \) and that of firm 2 as \( r_2' \).
In an infinitely repeated game, however, the profit vector \( \pi^*(a_1^*, a_2^*) \) is one of the feasible and individually rational outcome. The reason being that any deviant behaviour by firm 1 other than \( a_1 \), firm 2 will adopt the minmax strategy. For example, suppose that firm 1 chooses \( a_1^" \) knowing that firm 2 will choose \( a_2^* \). The equilibrium for this period will be obtained at point \( Z^" \) still in figure 10-3. As a punishment, firm 2 will choose \( a_2^" \) for the rest of the infinitely repeated game where \( a_2^" \) is the minmax level of \( a_2 \) (at \( a_2^" \), \( \pi_1 = \minmax \pi_1(a_1, a_2) \)). This will force firm 1 to stay at \( a_1 \).

A formal treatment of this game is given as follows:

\[
\text{Firm 1: } \max_{a_1, a_2} \sum_{i=0}^{\infty} r^i \pi_1(a_1, a_2) \text{ with respect to } a_1|a_2 \\
\text{Firm 2: } \max_{a_2, a_1} \sum_{i=0}^{\infty} r^i U_2(a_1, a_2) \text{ with respect to } a_2|a_1
\]

where \( a_1|a_2 \) implies \( a_1 \) for each given value of \( a_2 \) and \( r \) is the discount rate.

One possible solution of these two maximization problem is given as \( a=(a_1, a_2) \). It is denoted as point \( Z' \) in figure 10-3. Note that firm 2 has no incentive to diverge from this equilibrium as it is maximizing its profit at \( Z' \) while firm 1 could increase its profit by lowering its quality choice. Knowing that firm 2 always chooses \( a_2 \), firm 1 could increase its profit if it were to diverge to \( a_1^" \). It would
Figure 10-3

CREDIBLE THREATS AND NASH EQUILIBRIUM STRATEGIES
then be on its reaction function \( r_1 \). Therefore, \( \pi_1(a_1, a_2) < \pi_1(a_1'', a_2'') \).

Once firm 1 diverges, firm 2 will punish firm 1 by moving to \( a_2'' \) which is the firm 1's profit minimizing value given \( a_1'' \). We know that \( a_2'' \) must lie to the right of \( a_2 \) because

\[
\frac{d\pi_1}{d\Gamma} \frac{d\Gamma}{da_2} = \frac{2A^2(\Gamma+1)}{(3\Gamma+4)^3} < 0
\]

Therefore, the profit level for firm 1 will be \( \pi_1(a_1'', a_2'') \) for the following periods. Note that for

\[
\pi_1(a_1'', a_2'') - \pi_1(a_1, a_2) < \sum_{i=0}^{\infty} r_i [\pi_1(a_1, a_2) - \pi_1(a_1'', a_2'')] \]

it is not optimal for firm 1 to diverge.

10.2.3 A Credible Threat

Suppose that the Government of NICs provide tax incentives as well as cheaper loans to encourage its firms producing commodities designated for export. The net effect of such provisions may be a lower cost of production (or firms in NICs). Firms in the industrialized countries may find that their cost of production is too high to compete against firms in NICs as incumbent firms and may find that to become a follower in terms of quality leadership may be more profitable. Since such a change in the market leadership implies higher innovation cost, firms in NICs will find it
less profitable. From the Governments of NICs, lower profit implies loss of foreign exchange earnings. This may promote the Governments in NICs to draw up contingent plans to subsidize their firms if needs arise. The contingent plan may be a promise to its firms to subsidize even a larger amount to counteract with a minimax strategy in case the foreign firm decides to become a follower. Although this would incur greater financial burden on the Government of a NIC, it would still carry out such a plan due to the limited earnings of foreign reserves which it needs to import the raw materials in the first place. Such a plan would be sufficient to provide credibility to threats their firms may impose on foreign firms. The existence of such a credible threat will then be sufficient for the equilibrium $Z'$ to be a sub game perfect equilibrium.

10.3 First Stage Game given Bertrand Conjectures in the Marketing stage

We shall consider the situation where a leader decides to opt out to become a follower, given Bertrand conjectures in the marketing stage. We first compare the profit levels of both firms in each case, where it is assumed that both firms engage in independent investment in innovation.

The profit function of firm 1, if it stays a leader is given as follows:
\[ \pi_1 = \frac{f_1\beta^3 - f_2\beta^2 + f_3\beta + f_4}{(3\beta+4)^2} - a_1a_1 \]  
\hspace{1cm} (10-11) 

where 
\[ f_1 = (c_1 - c_2)^2 \] 
\[ f_2 = (c_1 - c_2)(4b - 5c_1 + c_2) \] 
\[ f_3 = 4(b - c_1)(b - 2c_1 + c_2) \] 
\[ f_4 = 4(b - c_1)^2 \]

If it decides to become a follower, it would face a profit function given as

\[ \pi_1 = \frac{f_1\Gamma^3 - f_5\Gamma^2 + f_6\Gamma + f_7}{(3\Gamma+4)^2} - a_1a_1 \]  
\hspace{1cm} (10-12) 

where 
\[ f_5 = (c_1 - c_2)(2b + 3c_1 + 5c_2) \] 
\[ f_6 = (b + c_2 - 2c_1)(b - 4c_1 + 3c_2) \] 
\[ f_7 = (b + c_2 - 2c_1)^2 \]

The profit functions of firm 2, if it stays a follower, is given as

\[ U_2 = \frac{g_1\beta^3 - g_2\beta^2 + g_3\beta + g_4}{(3\beta+4)^2} - a_2a_2 \]  
\hspace{1cm} (10-13) 

where 
\[ g_1 = (c_1 - c_2)^2 \] 
\[ g_2 = (c_1 - c_2)(2b + 3c_1 - 5c_2) \] 
\[ g_3 = (b + c_1 - 2c_2)(b + 3c_1 - 4c_2) \] 
\[ g_4 = (b + c_1 - 2c_2)^2 \]

The profit function of firm 2, if it becomes a leader, is given as

\[ U_2 = \frac{g_1\Gamma^3 - g_5\Gamma^2 + g_6\Gamma + g_7}{(3\Gamma+4)^2} - a_2a_2 \]  
\hspace{1cm} (10-14) 

where 
\[ g_2 = (c_1 - c_2)(4b + c_1 - 5c_2) \]
\[ g_6 = 4(b - c_2)(b + c_1 - 2c_2) \]
\[ g_7 = 4(b - c_2)^2 \]

Given that firm 2 becomes the leader if firm 1 decides to become a follower, the Bertrand Nash equilibrium is attained at points \( Z \) as shown in figure 10-4. As before, \( Z' \) is the equilibrium given firm 1 is the follower and firm 2 the leader.

We recollect that the values of \( a_1'' \) and \( a_2'' \) were obtained in chapter 8 as the values satisfying (8-31) and (8-32) respectively. In other words, at \( a_1'' \), the profit level of firm 1 is zero while at \( a_2'' \), the profit level of firm 2 is zero. Note that an increase in \( c_2 \) shifts the reaction function \( r_2' \) downwards. This is because the value of \( 2g_3-g_4 \) falls and \( a_2'' \) falls as the value of \( c_2 \) rises. Since firm 1's cost of production is larger than that of firm 2, once firm 1 becomes a follower, the reaction function of firm 1 will be lower and shorter compared to that of firm 2 as a follower. Depending on the relative difference in the cost of production, firm 1's reaction function as a follower may either be \( r_1 \) or \( r_1'' \).

Consider the reaction function of firm 2. If \( c_1 \) falls, \( a_1'' \) will increase such that the reaction function of firm 1 stretches out further along the 45° line. Since \( c_2 < c_1 \), the reaction function of firm 2 as a leader will be larger and more stretched than that of firm 1 as a leader. Therefore, the reaction function of firm 2 as a leader will be \( r_2 \). This then implies that the relative quality level
REACTION FUNCTIONS UNDER BERTRAND CONJECTURES IN THE MARKETING STAGE BEFORE AND AFTER CHANGES IN THE MARKET LEADERSHIP
difference will be greater once the leadership changes hands. In absolute terms, the quality level of firm 2 will have risen while that of firm 1 fell such that the inverse of the slope of the line $OZ$ will be steeper than the slope of the line $OZ'$. 

If firm 1 as a follower faces a reaction function given as $r_1^*$, then firm 2, by increasing its quality level slightly above $a_2^{**}$, can drive out firm 1 completely out of the market. Therefore, firm 1 has a greater incentive to stay a leader than under the Cournot conjectures in the marketing stage. The implication of this is that either a firm 1 has to innovate continuously such as to cover its higher cost of production and thereby stay a leader, or it will be forced out of the market. Below, we shall consider this explicitly and show that for $c_1 > c_2$, the reaction function of firm 1 as a follower will always be of type $r_1^*$.

10.4 Production Cost Differences and Market Participation given $c_1 > c_2$.

10.4.1 When $a_1 > a_2$
Let us consider the case where $a_1 > a_2$ first. In order for firm 1 to earn a positive profit, it must guarantee itself both a positive market share and a positive mark up rate, i.e., $p_1^c - c_1 > 0$ and $q_1^c > 0$. Below, we derive conditions under various conjectures that satisfies these two conditions.
Under a Cournot conjectures assumption in the marketing stage, we know from equations (7-4) and (7-6) that these conditions require

\[
\beta(b - 2c_1 + c_2) + 2(b - c_1) \over (3\beta + 4) > 0
\]

(10-15)

which is satisfied if \( b > 2c_1 - c_2 \) and \( b > c_1 \) or more precisely, this implies that the production cost of firm 1, \( c_1 \), must satisfy the following:

\[
b(\beta + 2) + c_2\beta \\
c_1 < \frac{b(\beta + 2) + c_2\beta}{2(1+\beta)}
\]

(10-16)

Likewise, under a Bertrand conjectures in the marketing stage, given the equation (8-3) and (8-11), we require

\[
\beta(c_2 - c_1) + 2(b - c_1) \\
\over (3\beta + 4) > 0
\]

(10-17)

and

\[
(c_2 - c_1)\beta^2 + (2b - 3c_1 + c_2)\beta + 2(b - c_1) \\
\over (3\beta + 4) > 0
\]

(10-18)

This implies that the production cost of firm 1 must satisfy the following inequality even though this may not necessarily be the case in reality:

\[
2b + c_2\beta \\
c_1 < \frac{2b + c_2\beta}{\beta + 2}
\]

(10-19)

We consider below all potential values of \( c_1 \) given \( b > c_1 > c_2 \) for all values of \( \beta \) which will guarantee firm 1 a
positive mark-up and market share. Under Cournot conjectures in the marketing stage, we obtain \( c_1 < b \) as \( \beta \to 0 \) and \( c_1 < \frac{b}{2} + \frac{\beta}{4}c_2 \) as \( \beta \to \infty \). Therefore, for any given value of \( c_2 < c_1 \), firm 1 will stay in the market as long as \( c_1 < b \) for \( \beta \to 0 \) and \( c_1 < \frac{b}{2} + \frac{\beta}{4}c_2 \) for \( \beta \to \infty \).

Likewise, under the Bertrand conjectures in the marketing stage, we obtain \( c_1 < b \) as \( \beta \to 0 \) and \( c_1 = c_2 \) as \( \beta \to \infty \). This means that as the quality level of firm 2 approaches that of firm 1, the only way firm 1 can survive the marketing stage is if \( c_1 = c_2 \). The possible values for \( c_1 \) under these various cases is illustrated in figure 10-5.

10.4.2 When \( a_2 > a_1 \)

Consider now the case where firms may switch their market positions. Under such a case, Cournot conjectures in the marketing stage requires that

\[
\frac{(1+\Gamma)(b-2c_1+c_2)}{3\Gamma+4} > 0
\]  

(10-20)

This implies that the production cost of firm 1 has to satisfy

\[
c_1 < \frac{b+c_2}{2}
\]  

(10-21)

Under the case of the Bertrand conjectures, we require that

\[
\frac{(c_2+2c_1)\Gamma+(b+c_2+2c_1)}{3\Gamma+4} > 0
\]  

(10-22)

and
Figure 10-5

POSSIBLE VALUES OF PRODUCTION COST OF FIRM 1 GIVEN $c_1 > c_2$
This implies that
\[ b + (\Gamma+1)c_2 - c_1 < \frac{-c_1}{2(\Gamma+1)} \]  
(10-24)

This is stating that there is no possible value of \( c_1 \) within the argument \( b > c_1 > c_2 \) that will enable firm 1 to participate in the market once it becomes the follower. The reason is that under the Bertrand competition in the marketing stage, the prices are lower than those under the Cournot competition. Therefore, a firm with a high cost can only survive if its quality is sufficiently high enough to compensate for the high price level that is forced to charge due to the high production cost. This does not necessarily imply that the price level of the firm with higher production cost is higher than the firm with a lower production cost, but that the quality difference can not be sufficiently compensated by a lower price level.

To illustrate this point, consider the price level of firm 2 as a leader is given as follows:
\[ p_{2L} = \frac{(2c_2+c_1)\Gamma + 2(b+c_2)}{3\Gamma+4} \]  
(10-25)

The price level of firm 1 as a follower is given as
\[ p_{1F} = \frac{(2c_1-c_2)\Gamma + (b+c_1+c_2)}{3\Gamma+4} \]  
(10-26)

Then the difference in the price level \( p_{2L} - p_{1F} \) is given as:
The sign of (10-27) is indeterminate. Given that \( b > \frac{(7c_1-3c_2)}{3} \), (10-27) could even be positive. Therefore, even if the price level of the follower with higher production cost may be higher than that of a leader with a lower production cost, the cost conditions derived above states that it is not sufficient to compensate for the difference in the quality level.

10.5 Conclusion

We have examined the possibility that the market leadership may be challenged and found that under the Cournot conjectures assumption for the marketing stage, that it is more profitable for a firm with higher cost of production, to stay a leader, compensating its higher price level by providing a better quality product. Once the production cost reaches a very high level, it would benefit the leader to opt out to become a follower in which case it will be able to save from the lower expenditure in innovation investment. Once the market position changes, both firms will produce products at a lower quality while the new leader with a lower production cost will widen the relative quality difference such as to maximize its profit, which it can afford to do due to the lower cost of innovation.
We found that for given values of $b$ and $c_2$, the value of $c_1$ plays a crucial role in determining profit maximizing position, e.g., leader or follower, in the market. For some argument of $c_1$, it is profitable for each firm to be a follower. Under such a situation, an one shot game will lead to zero innovation strategy of both firms as the origin is the only point both reaction functions intersect. Under an infinite repetition, a situation may arise where the original incumbent firm with higher production cost may be forced to stay a leader. Such an outcome requires credible threat which may be provided by the Government of NICs where they may provide incentives to their firms in the form of drawn up continency plans, or in a more usual case where the Government is deeply involved in the management and financing of its strategically important industries that such plans are always implicitly there.

Finally, we examined the same situation of market leadership challenge under a Bertrand conjectures assumption for the marketing stage. We found that unless the firm with higher production cost remains a leader offering a high product quality to consumers, it will be driven out of the market. The implication of this for firm 1 is "Innovate or die" type survival unless its production cost is cut drastically such as to be able to compete in a Bertrand competition with firm 2. The decisions of firm 1 to innovate or close down will depend crucially on its relative cost of production and innovation. This result contradicts with the findings of Beath et al (1987) where under product innovation and Bertrand conjectures, we may observe either a persistent
dominance or action-reaction, both of which does not conform with 'innovation or die'.
11. Licencing vs Independent Innovation

Suppose that free copying of technology is not possible. It would then be to both firms' advantage if only one firm would invest in innovation while sharing the cost as well as the benefit by means of a licence fee. The obvious restrictions would be that the leader would not sell its technology level at a price more than the follower's independent innovation cost to provide an incentive to engage in such a deal.

From the follower's point of view, it may adopt either of two views: it may take its rivals profit level into consideration when determining optimal behaviour or it may not be concerned. Under the former case, it would not be satisfied with paying a licence fee that is just below the cost of independent innovation as it knows that firm 1, the leader, is gaining an extra additional profit, the licence fee. The actual magnitude would be determined by the bargaining power of both sides. It would, therefore, demand a share in the extra profit of firm 1 which would be expressed as paying a lower licence fee. Under the second view, it may continue to behave as before, regarding the licence fee as its cost of innovation. We assume that firm 2 adopts the latter view and examine below how under such a condition, the licence fee may be determined. Such a fee-taking behaviour may be more appropriate in our model of a
one shot game while in a multi-period game, the bargaining outcome may be more relevant as firms' decisions will be based not only on this period's outcome but also on sequences of outcomes, partially determined by this period's decision.

In this chapter, therefore, we allow the case where a follower (firm 2) may choose between paying a licence fee to the leader (firm 1) for the usage of some quality innovation or, as before, invest in its own innovation. Given such a situation, we look into the possibility of leapfrogging of the follower.

There are two types of licensing: ex ante and ex post licensing. The former offers the licensing advance right to the use of innovation from a specific R&D activity while the latter offers right to use an innovation of a specific technology. The ex ante licensee relies on the R&D activity of the licensor without its independent research activities while the ex post licensee carries out its own research except that the innovation of the licensor is superior to its own.

A fundamentally different means of innovation diffusion arises via imitations or free copying. A study by Mansfield, Schwartz and Wagner (1981) found in a study of 48 products that roughly 60 percent of patent bearing innovations were successfully imitated avoiding patent infringement within a period of four years.
One of the concerns of the literatures on licensing has been the effects of licensing on the level of research activities. Tandon (1982) finds that licensing at the profit maximizing fee is equivalent to producing as a monopolist and therefore concludes that licensing does not affect R&D activity. Contrary to Tandon, Salant (1984) finds that expectations of returns from future licensees encourage greater research by high cost firms in a symmetrical oligopoly. Gallini (1984) and Gallini and Water (1985) argue that the importance of ex ante licensing lies in deterring entry and limiting independent research capabilities of rivals. Thus given enough asymmetry in firms, licensing is likely to have an impact on the underlying R&D decisions.

We will, therefore, examine the effects of licensing by firm 1 on the level of innovation on both firms under a Perfect Nash equilibrium where the second stage is a Cournot Marketing stage.

We introduce three different types of licence fee firm 1 may adopt and examine their respective profitability for both firms as well as the corresponding market structure in terms of quality differences. By means of an example derived from simplifying the licencing rule, we then show how the market leadership may change under a licencing rule.
11.1 Licencing vs Independent Innovation

We define the objective functions of both firms as below:

Firm 1: \[ \text{Max } \pi_1 = (p_1-c_1)q_1 - k_1(a_1) + x \] (11-1)

Firm 2: \[ \text{Max } U_2 = (p_2-c_2)q_2 - k_2(a_2) - x \] (11-2)

As before, \( p_i, q_i \) and \( k_i \) denote the price level, quantity level and the cost of innovation by firm \( i \), and \( x \) denotes the licence fee. The three types of licencing fee we are to examine in this chapter is fundamentally a question as how to specify \( x \) while \( k_2(a_2) = 0 \).

The utility levels of firm 1 and firm 2, if they were to engage in independent innovation is given as follows:

\[
\pi_{1I} = \frac{[2Ba_1 + (A-2B)a_2^*]^2}{[4a_1^*-a_2^*]^2} - a_1a_1^* \quad (11-3)
\]

\[
U_{2f} = \frac{[Da_1^*]^2}{[4a_1^*-a_2^*]^2} - a_2a_2^* \quad (11-4)
\]

where firms attain the Cournot equilibrium \( Z \) at \( a_1^* \) and \( a_2^* \) as shown in figure 11-1. The utility levels of firm 1 and firm 2, given firm 2 pays a licence fee to firm 1 is given as follows:

\[
\pi_{1N} = \frac{[2Ba_1^0 + (A-2B)a_2^0]^2}{[4a_1^0-a_2^0]^2} - a_1a_1^0 + x(a_2^0|a_1^0) \quad (11-5)
\]

\[
U_{2N} = \frac{[Da_1^0]^2}{[4a_1^0-a_2^0]^2} - x(a_2^0|a_1^0) \quad (11-6)
\]
Figure 11-1

NASH EQUILIBRIA UNDER INDEPENDENT DEVELOPMENT
where firms attain a Cournot Nash equilibrium at \( a^{1*} \) and \( \alpha^{2*} \) and \( \alpha^{2*} \) is the amount firm 1 sells to firm 2 at a cost of \( x(\alpha^{2*}) \) given its own innovation level is \( a^{1*} \). Note that \( a^{i*} \) need not be equal to \( a^{i*} \) for \( i = 1, 2 \).

11.2 Licencing Fee per Unit Sold

Refer to equation (11-5) and (11-6). A licence fee per quantity sold implies that \( x \) is a function of \( q_2 \). But we note that given the second stage game as a Cournot marketing game, \( q_2 \) is defined as a function of \( a_1, a_2 \) and other exogenous parameters of the market. To be more precise, \( q_2 \) is an increasing function of \( a_2 \). Therefore, a licence fee designed to be a function of quantity sold is qualitatively equivalent to a fee designed to be a function of the level of quality. A little more complicated specification will enable even a quantitative equivalence between these two pricing mechanism. Therefore, we specify \( x \) as a function of \( a_2 \), e.g., we parameterize \( x \) as \( a_3 a_2 \), a limitation that constrains the equivalence.

Under this specification, the reaction function of firm 2 is unchanged from that under independent innovation. Firm 1, however, will now face a new reaction function. It will be derived from the values of \( a_1 \) and \( a_2 \) satisfying \( \beta^* \) where it is a solution to the marginal condition given as \( \frac{d\pi_1}{d\beta} = \frac{dk_1}{d\beta} - \frac{dk_2}{d\beta} \). It may be written as:
\[-4D[A\beta+2B] \quad [a_1a_2-a_3a_1] \quad \frac{\beta^2}{(3\beta+4)^3} \quad (11-7)\]

By substituting \(a_1 = [(1+\beta)/\beta]a_2\), the right hand side of (11-7) may be written as

\[-a_3[\beta(1-a)-a] \quad \frac{a_3}{\beta^2} \quad \text{where} \quad a = \frac{a_3}{a_1} \quad \text{where} \quad a = \frac{a_3}{a_1}\]

Therefore, equation (11-7) is given as

\[-4D(A\beta+2B)\beta^3 \quad a_1a_2 = \frac{\beta(1-a)-a)}{(3\beta+4)(\beta(1-a)-a)} \quad (11-8)\]

The corresponding reaction function is shown in figure 11-2 as \(r_1'\) given \(a_1 = 1\).

The reaction function is the 45° line for \(a_1 \in [0, S]\). For \(a_1 > S\), the reaction function becomes a curve converging to a line through the origin with a slope of \(1/a_3\). For \(a_1 < S\), firm 1 finds it more profitable to let \(a_2\) copy its latest technology at a price of \(a_3\) per quality level. The expenditure in innovation as well as the lower quantity sold by firm 1 due to the higher quality level of firm 2 is outweighed by its receipt of the licence fee from firm 2.

For \(a_1 > S\), however, the situation is reversed and firm 1 benefits more by increasing the quality difference and its sales volume than to be compensated by receiving the licence fee. Note that the point \(S\) is sensitive to the value of \(a_3\). For \(a_3 = 1\), i.e., firm 1 charges exactly the same amount as its cost of innovation, the reaction function will be the 45° line for all positive argument of \(a_1\). On the other as
REACTION FUNCTION OF FIRM 1 GIVEN LICENCE FEE RECEIVED PER UNIT SOLD
hand, for \( a_3 = 0 \), \( r_1' \) collapses to the original reaction function as obtained under independent innovation.

The reaction function of firm 2 remains unchanged except that \( a_2 \) is replaced by \( a_3 \). The former is an exogenous variable while the latter is a decision variable of firm 1. Without any loss of generality and to be consistent with earlier examples, we let \( a_2 = a_1 = 1 \) henceforth.

The subgame perfect equilibrium under this licence fee is obtained as shown in figure 11-3 as point \( Z' \). For \( Z' \) to be other than on the 45° line, \( a_3 \) must be less than \( D/(2A+D) \), a condition derived from below:

\[
\frac{2D^2}{27a_3} > \frac{4AD}{27(1-a_3)} \quad (11-9)
\]

As is apparent from figure 11-3, the left term of (11-9) is the point where \( r_2' \) begins to diverge from the 45° line. In other words, for smaller values of \( a_3 \), firm 1 would want to obtain a positive quality difference to increase its market share as to compensate for the lower licence fee. Below, we want to derive the profit maximizing level of \( a_3 \) for firm 1 in order to determine whether at that level, this licensing fee per unit sold is a dominant strategy over independent innovation. Since solving a maximization of this problem involves algebraic complications, we adopt the following route. Rather than solving for an optimal value of \( a_3 \), we simply examine whether there exists any value of \( a_3 \) that will render this licencing rule to be dominant. First, using numerical values, we examine for any value of \( a_3 \)
Figure 11-3

THE NASH EQUILIBRIUM GIVEN A FIXED LICENCE FEE PAID PER UNIT SOLD
larger than $D/(2A+D)$ that will result in a "licensing fee per unit sold" to dominate "independent innovation". If not, we look for values of $a_3$ less than $D/(2A+D)$ and repeat the process.

11.2.1 $a_3 > D/(2A+D)$

Let us now examine whether an increase in $a_3$ will lead to an increase in firm 1's profit level. We know from equation (7-8) and (7-9) in chapter 7 that for $\beta = \infty$, the profit levels of both firms are given as:

$$\pi_1^N = \pi_1(\alpha, \alpha) - \alpha(a_3) + a_3\alpha(a_3)$$

$$\pi_2^N = \pi_2(\alpha, \alpha) - a_3\alpha(a_3)$$

where $\alpha_1 = \alpha_2 = \alpha$.

Then $d\pi_1/da_3 = (a_3-1)\alpha_a/da_3 + \alpha(a_3)$

and $d\pi_2/da_3 = -\alpha(a_3) - a_3\alpha_a/da_3$

Since $\alpha(a_3) = 2D^2/27a_3$ for $\beta = \infty$ in our particular example, we have

$$da/da_3 = -2D^2/27a_3^2 < 0$$

Then substituting (11-12) into the first order condition gives

$$d\pi_1/da_3 = 2D^2/27a_3^2 > 0$$

$$d\pi_2/da_3 = 0$$
The reason that $dU_2/da_3 = 0$ is because as the cost of buying the new innovation rises, the lower quality firm 2 will be buying from firm 1 such that it does not affect the profit level of firm 2. It is obvious that firm 1 will impose as high a value of $a_3$ as possible given the constraint that firm 2 is not worse off as when innovating independently. The problem of firm 1 may be denoted as

$$\text{Max } \pi_1(a, a) + a(a_3)(a_3 - 1)$$

subject to $U_2(a, a) - a_3a(a_3) \geq U_2(a_1^*, a_2^*) - a_2^*$

Using our numerical example in (10.1.2), we know that the maximization problem cannot be solved as the constraint can never be satisfied. Given the numerical values, the left hand side yields 3.692, while the right hand side yields 5.33. Therefore, firm 2 will not engage in such a licensing rule and opt for independent innovation. For $a_3 > d/(2A+D)$, there is no value of $a_3$ that will entail this licensing rule to be dominant over independent innovation.

11.2.2 $a_3 < D/(2A+D)$

Having established that $a_3$ can not be larger than $D/(2A+D)$, we now examine whether for lower values of $a_3$, firm 1 would still be able to make a larger profit than under independent innovation which would prove the dominance of this licensing rule. Given numerical values, we know that $a_3$ must be less than 10/24 (i.e., $D/(2A+D)$). This implies that $a_2$ must be weakly greater than 17.78 (i.e., $2D^2/27a_3 \leq a_2$). Therefore, given $a_2 = 17.78$, the highest profit firm 2 can
guarantee itself is at $a_1 = a_2 = 17.78$. Note that firm 2 requires a profit level of at least 5.33 which it could obtain from independent innovation. Therefore, $a_2$ must be such that

$$U_2^I(a_1^*, a_2^*) \leq U_2^N(a_1, a_2) - a_3a_2 \quad (11-14)$$

In other words, $a_3 \leq 0.32$. However, we began our analysis assuming that $a_3 < 10/24$. Substituting $a_3 = 0.32$ yields $a_2 = 23$ which would require an even smaller value of $a_3$ than 0.32. Therefore, firm 2 will not engage in such a deal with firm 1 for whatever value of $a_3$. We conclude that the independent innovation is a dominant strategy.

11.3 Licence Fee as a Lump Sum Cost

If licence fee is charged as a lump sum cost, then the reaction function of the first stage of firm 1 does not change while that of firm 2 will become the 45° line. The reason for unchanged reaction function for firm 1 is plausible, namely that lump sum cost does not affect the marginal conditions. As for firm 2, for whatever amount of the licence fee, it knows that the closer the quality level to that of firm 1, the larger the profit level. Therefore, firm 2 will always choose the maximum quality level firm 1 can offer. In that respect, firm 2 will always choose $a_2' = a_1'$ as shown in figure 11-1.

Firm 1, knowing the behaviour of firm 2, will charge a lump sum fixed fee of $F$ where it is defined as
\[ F = U_2(a_1', a_2') - [U_2(a_1^*, a_2^*) - a_2^*] \quad (11-15) \]

As long as \( \pi_1(a_1^*, a_2^*) - a_1^* \) is smaller than \( \pi_1(a_1', a_2') - a_1' + F \), firm 1 will offer its highest quality to firm 2.

Note that a fixed lump sum license fee may be a dominant strategy for both firms than independent innovation if it makes firm 2 as well off as before while the profit level of firm 1 increases. Below, we examine in numerical values whether this licensing rule is a dominant strategy over independent innovation.

We know that \( a_1' = a_2' = 8.3 \) and that the profit level of firm 2 under independent innovation is 5.33. Since \( U_2(a_1', a_2') = 11.1 \), \( F \) is 5.78. Therefore, as long as firm 1 determines \( F \) equal or just less than 5.78, firm 2 will buy the technology from firm 1 than innovate it independently.

Consider now firm 1. The profit level of firm 1 under independent innovation was 3.64. Under this particular licensing rule, its profit level will become 2.88, which is less than it could have attained under independent innovation. Therefore, firm 1 will not want to engage in such an agreement. We conclude that independent innovation is still the dominant strategy.
11.4 Licencing Fee charged on Quality Difference

Suppose now that firm 1 allows firm 2 to freely copy its products but specifies a licence fee in such a way that the closer the quality between the two firms, the higher the licence fee. Firm 1 may devise such a system as to guarantee its market leadership. By specifying the licensing fee in such a way as to make it more profitable for the follower to buy the technology paying a licence fee than to engage in an independent innovation, the leader can benefit by moderating its own development speed as well as by receiving a licence fee from the follower. In fact, the IBM has imposed a similar deal on Korean electronics industries in 1988.

Such a structure of a licence fee would be obtainable by specifying \( x = a_4 \beta \). The marginal condition of firm 1 derived from its objective function will be as follows:

\[
\frac{4D(A\beta + 2B)\beta^2}{(3\beta + 4)^3} = a_4 \beta^2
\]

(11-16)

To be more precise, they become correspondence rather than functions. But we shall continue to call them functions in order to reduce complications. The corresponding reaction function is shown in figure 11-4 as \( r_1 \). For each given level of \( \alpha_2 \), there are two possible values of \( \alpha_1 \). For example, for a given value of \( \alpha_2 \) as shown in figure 11-4, there are two possible values of \( \alpha_1 \); \( \alpha_1 \) and \( \alpha_1 \). Firm 1 is indifferent between \( \alpha_1 \) and \( \alpha_1 \). The reason is that at \( \alpha_1 \), firm 1 is enjoying a higher profit level given the larger
Figure 11-4

REACTION FUNCTION OF FIRM 1 GIVEN A LICENCE FEE PAID AS A FUNCTION OF QUALITY DIFFERENCE
quality difference while receiving only a small amount from firm 2 as the licence fee. At \( g_1 \), however, firm 1 enjoys a lower profit level but receives a higher licence fee from firm 2 due to the smaller gap in the quality level.

Another interesting observation is that the reaction function becomes more blown up as the value of \( a_4 \) falls (shown in figure 11-4) and converges to the reaction function of firm 1 found under independent innovation. The intuition is clearly that as \( a_4 \) increases (the licence fee per quality difference increases), firm 1 finds that the licence fee quickly compensates for a lower quality level both in terms of that of firm 2 and in absolute level.

Firm 2, on the other hand, faces a marginal condition derived from its objective function given such a licensing rule is as follows:

\[
\frac{2D_2(1+\beta)}{(4+3\beta)^3} = a_4 \tag{11-17}
\]

The reaction function of firm 2 will be a straight line passing through the origin with a slope of \((1+\beta'')/\beta''\) where \( \beta'' \) satisfies the condition given in (11-11). There is an negative relationship between \( a_4 \) and \( \beta \) such that an increase in \( a_4 \) will lead to a fall in \( \beta \). For \( a_4 \) larger than some maximum value, say, \( a \), \( \beta \) becomes negative. Therefore, \( a \) is the maximum level of \( a_4 \) firm 1 can impose on firm 2. For values of \( a_4 \) within the argument of \( 0 < a_4 < a \), the loci of possible Nash equilibria is denoted as the curve OL in figure 11-5. We note that \( a_1' \) and \( a_2' \) are the same values
Figure 11-5

LOCi OF NASH EQUILIBRIa
those shown in figure 11-1, obtained under independent innovation.

Using our numerical values as before, we find that as the value of $a_4$ increases, (i.e., travelling towards the origin on the curve $OL$), we find that the profit level of firm 1 increases while that of firm 2 falls. Below is a table showing a few examples supporting this, where given our particular numerical values to the exogenous variables, 3.125 happens to be the limiting value of $a_4$.

<table>
<thead>
<tr>
<th>$a_4$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$\pi_1$</th>
<th>$U_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.125</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.76</td>
<td>0.616</td>
<td>0.056</td>
<td>14.74</td>
<td>6.26</td>
</tr>
<tr>
<td>1</td>
<td>3.23</td>
<td>1.76</td>
<td>8.48</td>
<td>7.18</td>
</tr>
<tr>
<td>0.5</td>
<td>3.84</td>
<td>2.6</td>
<td>8.35</td>
<td>8.4</td>
</tr>
<tr>
<td>0</td>
<td>8.3</td>
<td>8.3</td>
<td>5.4</td>
<td>11.1</td>
</tr>
</tbody>
</table>

We find that, even for $a_4$ close to 3.125, the profit level of firm 2 is higher than that under independent innovation. Firm 1 will therefore choose a value of $a_4$ close to 3.125 at which $U_2$ will be equal to 5.33. We conclude that this licensing rule is a dominant strategy for both firms as it increases both firms profit compared to other licensing rules.

Out of the four different cases, three types of licensing rules and independent innovation, we found that a licensing fee designed such that it is a function of quality difference will yield both firms higher profit. The reason is that both firms are able to cut back the expenses in
innovation investment while keeping a certain quality difference. The resulting market structure will be such that products will be of a much lower quality than under the case of independent innovation.

11.5 Licencing Fee and Leap Frogging

So far, we have examined different licencing rules given that the quality level of firm 1 itself is determined simultaneously with the quality level of firm 2. Here, we examine a stackelberg equilibrium by simplifying our model used so far by assuming that firm 1 has already determined its quality level at the subgame perfect equilibrium level \( a_1^* \) under independent innovation. Given this precommitted level, firm 2 will determine its quality level paying a licence fee. Firm 1's objective is to determine a licence fee, knowing the behaviour of firm 2, that will maximize its profit while not making firm 2 worse off. Given full information, firm 1 knows the objective function of firm 2, this is equivalent as firm 1 offering a package of \( a_2^* \), at a fixed fee. A maximization problem of firm 1 can be set up as follows:

\[
\text{Max } \pi_1(a_1^*, a_2) + [U_2(a_1^*, a_2) - [U_2(a_1^*, a_2^*) - a_2^*]] \quad (11-18)
\]

subject to \( a_2 \geq a_0 \)

\( a_2 \leq a_1^* \)

where \( \pi_1(a_1^*, a_2) = \frac{[2B(a_1^*) + (A-2B)a_2]_2}{(4a_1^* - a_2)^2} - a_1^* \)

\( U_2(a_1^*, a_2) = \frac{(Da_1^*)^2}{(4a_1^* - a_2)^2} \)
The fee is in fact the difference between the second and the third term of (11-19). The first order condition of (11-19) is given as:

\[
\frac{[2D^2 + 8B(2A - 3B)](a_1^*)^2 + 4(A - 2B)(2A - 3B)a_2a_1^*}{(4a_1^* - a_2)^3} = 0
\]

Therefore \( a_2 = \frac{[8B(3B - 2A) - 2D^2]}{(3B - 2A)(2B - A)} a_1^* \)

Given our numerical values as before, we obtain \( a_2 = 44/9 a_1^* \). Since \( a_2 \leq a_1^* \), the solution to this particular maximization problem is \( a_1 = a_2 = a_1^* \). The profit level of firm 1 net of the licence fee is \(-0.056\) while the licence fee amounts to 5.78 leaving a total profit of 5.72 to firm 1 while firm 2 is left with a profit level of 5.33.

The analysis above was restricted to \( a_2 \leq a_1^* \). However, for a high enough license fee, a follower may increase its profit level by overtaking the leader given the quality level of the leader. Note that the subgame perfect equilibrium under the licencing rule described above is at point T in figure 11-6. However, firm 2 not only has the choice of independent innovation, but also of leap frogging, i.e., it may decide to choose a quality level of \( a_2^{**} \). The profit levels of each firm at \((a_1^*, a_2^{**})\) are given as follows:

\[
\pi_1(a_1^*, a_2^{**}) = \frac{[A a_2^{**}]^2}{(4a_2^{**} - a_1^*)^2} - a_1^*
\]
Figure 11-6

PAYING LICENCE FEE OR LEAP FROGGING
where \( G = b - c_2 \) as before. The implication of this possibility imposes an additional restriction on the maximization problem of firm 1 as shown in (11-18). It is given as follows:

\[
U_2(a_1^*, a_2) - kF \geq U_2(a_1^*, a_2^{**})
\]

(11-19)

where \( 0 \leq k \leq 1 \) and \( F = U_2(a_1^*, a_2) - [U_2(a_1^*, a_2^*) - a_2^*] \)

In other words, the net profit level of firm 2 (after subtracting the licence fee) must be greater than or equal to the profit level it would have obtained had it overtaken the leadership by independent innovation.

The maximization problem may be restated as follows:

\[
\text{Max } \pi_1(a_1^*, a_2) + kF
\]

(11-20)

s.t. \( U_2(a_1^*, a_2) - kF \geq U_2(a_1^*, a_2^{**}) \)

(11-21)

\[
a_2 \geq a_0
\]

\[
a_2 \leq a_1^*
\]

The first order condition of (11-20) yields

\[
a_2 = \frac{8B(3B-2A) - 2kD^2}{(3B-2A)(2B-A)} a_1^*
\]

Since we found earlier that even for \( k = 1 \), that \( 8B(3B-2A) - 2kD^2 \geq (3B-2A)(2B-A) \) for the numerical values we used so
far, this inequality holds for all values of $k \in [0, 1]$. Therefore, irrespective of the values of $k$, the solution of $a_2$ is $a_2 = a_1^*$. Given this, we need to find the value of $k$ which will satisfy the constraint given in (11-21).

The value of $k$ is given as

$$\frac{U_2(a_1^*, a_2^*) - [U_2(a_1^*, a_2^*) - a_2^*]}{U_2(a_1^*, a_1^*)}$$

Therefore, given our numerical values as before, (11-22) is given as

$$\frac{6.25 - 5.33}{11.11} \leq 1$$

Therefore $k \leq 0.917$

Since firm 1 obtains higher profit for larger values of $k$, the value of $k$ will be determined at $k = 0.917$. Therefore, firm 1 has to be content with a lower licence fee in order to prevent firm 2 to overtake its market leadership by independent innovation, which would leave firm 1 much worse off than when both firms engaged in independent innovation with firm 1 keeping its market leadership.

11.6 Conclusion

We have introduced three different types of licencing rules, namely a fee that is determined as a function of the quantity of final product sold by the buyer of the technology, a fixed fee and finally a fee that is a function
of the quality difference between the top quality of the market leader and the level of quality it is selling to the follower. We obtained the result that the dominant strategy among the three different licencing rule including independent innovation is the last of the three licencing rules. Having established a dominant strategy, we examined the possibility of leap frogging by simplifying our model slightly. Rather than assuming that the optimal level of firm 1 is determined simultaneously with the quality level of the follower given a specific licencing rule, we assumed that the leader's quality level to be determined first at the subgame perfect equilibrium level, and derived a licence fee that would increase firm 1's profit while leaving firm 2 as well off as before. We found that firm 1 will sell its highest quality at hand, compensating its loss of profit from lower sales by the higher licence fee received from firm 2. On the other hand, allowing for the possibility of leap frogging of the follower, we found that firm 1 has to lower its licence fee while selling the same level of quality in order to prevent firm 2 from overtaking the market leadership which would leave firm 1 much worse off than independent innovation.
12. Optimal Timing of Innovation

Our objective here is to show the competition for market leadership focusing our attention on the speed of innovation development and on the timing of innovation implementation when there is just 1 point of implementation. We show that these two aspects are determined by the efficiencies with which innovation development is carried out as well as the responsiveness of profit changes to innovation implementations. We then examine briefly the market leadership under a sequence of innovations. We again assume asymmetric market structure from the outset of the model. Innovation will be product innovation as before, but like Vickers (1985), the outcome of one sequence will determine the structure of the market of the next period. In other words, the market structure is determined endogenously by the maximizing behaviour of both firms in each period. Although we do not derive the market share of each firm explicitly, this is conveyed implicitly in the level of quality increment of each period, i.e., a widening quality difference implies an increase in the share of market by firm 1 and vise versa. The structure of the model we present here assures that each sequence is not as drastic as in Reinganum (1985) in that the incumbent firm will hold less than 100% of the market share. This is because we assume Cournot competition for the marketing stage.
12.1 The Speed of Innovation Development

Suppose that the level of innovation $d\alpha$ is a function of time and investment into R&D represented as $d\alpha/dt=\mu(k)$ where $\mu(.)$ could be defined as the speed of innovation development and $k$ to be the constant expenditure for each value of $\mu$ per unit of time, (see for example Brock, 1982). The level of innovation, therefore, is a linear function of time. More R&D activity per unit of time yield greater total expenditure and more substantial innovation. We assume that $d\mu/dk < 1$ and $d^2\mu/dk < 0$ for positive values of $k$ which has similarities with the specification of the Penrose Effect as shown in Takayama (1974). This implies that $\mu(.)$ is a concave function with marginal increment of speed of innovation development strictly less than the average increment and decreasing with respect to $k$. Therefore, a firm attempting to speed up its innovation development will have to spend increasingly more to attain the same level of innovation development in a shorter time. Furthermore, we exclude cases of uncertainty arising from innovation successes or failures. We parameterize $\mu$ as $\mu = kr^i$, with $r < 1$, $i = 1, 2$. Note that $r_i > r_j$ implies that firm $i$ is more efficient in enhancing their technology than firm $j$ for a given level of investment into R&D per unit of time. Refer to figure 11-1 where for a given value of $\mu^*$, $k_2$ has to be larger than $k_1$ if $r_1 > r_2$. 
Figure 12-1

\[ \frac{d\alpha}{dt} = \mu \]

SPEED OF INNOVATION VS EXPENDITURE
12.2 Single Period Equilibrium

Suppose that firms engage in quality competition from \( t=0 \) to \( t=T^* \). We assume that only one implementation is carried out by each firm. We define \( T_1 \) and \( T_2 \) as the date of implementation of a new product by firm 1 and firm 2 respectively. The objective of firm 1 will be to maximize \( V_1(T_1, T_2) \) with respect to \( k_1 \) and \( T_1 \) where \( V_1(T_1, T_2) \) is given as follows:

\[
V_1(T_1, T_2) = \int_0^{T_1} \pi_1(0, 0) \, dt + \int_{T_1}^{T_2} \pi_1(T_1, 0) \, dt + \int_{T_2}^{T^*} \pi_1(T_1, T_2) \, dt - C(T_1)
\]  

(12-1)

\( \pi_1(0, 0) \) is the profit level of firm 1 prior to the introduction of new products by either firm. From chapter 7, we know that for given values of \( a_1 \) and \( a_2 \) and Cournot competition in the marketing stage, \( \pi_1(0, 0) \) is defined as

\[
\pi_1(0, 0) = \frac{[\beta(b-2c_1+c_2)+2(b-c_1)]^2}{(4+3\beta)^2}
\]

\( \pi_1(T_1, 0) \) represents the profit level of firm 1 ex post implementation of a new product at \( T_1 \) by firm 1 while firm 2 is still selling their old product. The increment of the quality level of the new product at \( T_1 \) as against its predecessor, is given as \( k_1r_1T_1 \). At this stage, we introduce an additional assumption to simplify the analysis, namely that for \( \delta \) sufficiently small, we can use the linear approximations:

\[
\pi_1(T_1, 0) = \pi_1(0, 0) + \delta_1 k_1 r_1 T_1 \]

(12-2)
where \( \delta_{11} = \frac{\partial \pi_1(0, 0)}{\partial a_1} = \frac{\partial \pi_1(0, 0)}{\partial \beta} \frac{\partial \beta}{\partial a_1} \)

\[
\frac{\partial \pi_1(0, 0)}{\partial \beta} = \frac{-4[\beta(b-2c_1+c_2)+2(b-c_1)]\left[b+c_1-2c_2\right]}{(4+3\beta)^3} \tag{12-3}
\]

and \( \frac{\partial \beta}{\partial a_1} = -\frac{\alpha_2}{(\alpha_1 - \alpha_2)^2} \)

Therefore, the profit level ex post introduction of the new product is equal to the ex ante profit level plus changes in the level of profit due to the higher quality level of the new product.

\( \pi_1(T_1, T_2) \) represents the profit level of firm 1 after both firm 1 and firm 2 have implemented new quality levels at \( T_1 \) and \( T_2 \) respectively. Here again, we make the following assumption:

\[
\pi_1(T_1, T_2) = \pi_1(0, 0) + \delta_{11}k_1r_1T_1 - \delta_{12}k_2r_2T_2 \tag{12-4}
\]

where \( \delta_{12} = \frac{\partial \pi_1(0, 0)}{\partial a_2} = \frac{\partial \pi_1(0, 0)}{\partial \beta} \frac{\partial \beta}{\partial a_2} < 0 \)

and \( \frac{\partial \beta}{\partial a_2} = \frac{\alpha_1}{(\alpha_1 - \alpha_2)^2} \)

Finally, the cost function \( C(T_1) \) is defined simply as

\[
C(T_1) = \int_0^{T_1} k \, dt = k_1T_1
\]

The problem firm 1 now faces is summarized below as:

\[
\max_{T_1, k_1} V_1(T_1, T_2)
\]

Then for \( T_1 \leq T_2 \)

\[
V_1(T_1, T_2) = T_1\pi_1 + (T_2-T_1)(\pi_1+\delta_{11}k_1r_1T_1) \\
+ (T^*-T_2)(\pi_1+\delta_{11}k_1r_1T_1 - \delta_{12}k_2r_2T_2) - k_1T_1 \tag{12-5}
\]

For \( T_1 \geq T_2 \), we have
\[ V_1(T_1, T_2) = T_2\pi_1 + (T_1 - T_2)(\pi_1 + \delta_{12}k_1r^2T_2) + (T^* - T_1) (\pi_1 + \delta_{12}k_2r^2T_2 - \delta_{11}k_1r^1T_1) - k_1T_1 \] (12-6)

We note that (12-5) and (12-6) are identical for the maximization problem with respect to \( k_1 \) and \( T_1 \). In other words, differentiating (12-5) with respect to \( k_1 \) yields the same result as when differentiating (12-6) with respect to \( k_1 \) as shown in (6-8). Likewise, differentiating (12-5) and (12-6) with respect to \( T_1 \) yield same results as shown in (12-7) below. Therefore, the maximization function is not restricted to the specific inequality between \( T_1 \) and \( T_2 \). The first order condition for the objective function are given as follows:

\[ \frac{dV_1}{dT_1} = -2\delta_{11}k_1r^1T_1 + T_2\delta_{11}k_1r^1 + T^*\delta_1k_1r^1 - T_2\delta_1k_1r^1 - k_1 = 0 \]

\[ k_1^{r_1} = \delta_{11}(T^* - 2T_1) \] (12-7)

\[ \frac{dV_1}{dk_1} = r_1(T_2 - T_1)\delta_{11}k_1r^1 - 1 + (T^* - T_2)\delta_1k_1r^1 - 1T_1r_1 - T_1 = 0 \]

\[ k_1^{r_1} = r_1\delta_{11}(T^* - T_1) \] (12-8)

From (12-7) and (12-8) we obtain the reaction function of firm 1 as \( T_1' = (1-r_1)/(2-r_1) \) \( T^* \), i.e., irrespective of the value of \( T_2 \), firm 1 will always implement a new innovation at \( (1-r_1)/(2-r_1) \) \( T^* \). The quality level of the new innovation will be higher than its predecessor by \( k_1r^1(1-r_1)/(2-r_1) \) \( T^* \) and the cost of this new product amounts to \( k_1(1-r_1)/(2-r_1) \) \( T^* \).

We assume that firm 2 faces the same structure in its objective function as firm 1 except for the values of
parameters, e.g., $\delta_{22}$, $\delta_{21}$, $k_2$ and $r_2$. For they are in turn defined as follows:

\[
\delta_{22} = \frac{dV_2(0,0)}{da_2} = \frac{dU_2}{d\beta} \frac{d\beta}{da_2} > 0
\]

\[
dU_2 = \frac{2(b+c_1-2c_2)^2(1+\beta)}{(4+3\beta)^3} \tag{12-9}
\]

and $\frac{d\beta}{da_2} = \frac{a_1}{(a_1-a_2)^2}$

Finally, $\delta_{21}$ is defined as $dV_2/da_1 = dV_2/d\beta \frac{d\beta}{da_1} < 0$. Note that the differences in the utility levels as shown here were obtained from a Cournot Nash competition in the second stage of a two stage game where firm 1 had the first mover advantage in the market enabling it to target its products to the high income group whereas firm 2 had the advantage of a lower cost of production. To be more precise, firm 1 had to opt for the higher income group as it would have ended up in a worse position had it aimed at the lower income group under Cournot competition. Under Bertrand competition, it would have been driven out of the market altogether due to obtaining a negative mark up as shown in the previous chapter.

The symmetry between the two firms with respect to the structure of the objective function, yields the reaction function of firm 2 as $T_2' = (1-r_2)/(2-r_2) T^*$. In other words, firm 2 will implement its new innovation at $(1-r_2)/(2-r_2) T^*$ at a cost of $k_2(1-r_2)/(2-r_2) T^*$ irrespective of firm 1's time of implementation. The quality level of its new product will be higher than its initial product by $k_2r_2(1-r_2)/(2-r_2) T^*$. 
The reason that both firms' reaction functions are independent of their competitor's timing of implementation is because their respective profit function are independent to each other's timing of new product implementation.

From (12-7) and \( T_1 = (1-r_1)/(2-r_1) \) \( T^* \), we obtain the optimal value of \( k_1 \) (constant expenditure into R&D per unit of time) as follows:

\[
k_{11-r1} = r_1\delta_11(T^* - T_1)
\]

Then

\[
k_{11-r1} = r_1\delta_11T^*/(2-r_1).
\]

Therefore

\[
k_1 = [(r_1\delta_11T^*)/(2-r_1)]^{1/(1-r_1)}
\]

The non cooperative Nash equilibrium of this single period competition is then given as

\[
T_1' = (1-r_1)/(2-r_1) \ T^*
\]

\[
T_2' = (1-r_2)/(2-r_2) \ T^*
\]

\[
k_1' = [(r_1\delta_11T^*)/(2-r_1)]^{1/(1-r_1)}
\]

\[
k_2' = [(r_2\delta_22T^*)/(2-r_2)]^{1/(1-r_2)}
\]

We note the followings:

\[
dT_1'/dr_1 = -1/(2-r_1)^2 < 0
\]

\[
dT_1'/dT^* = (1-r_1)/(2-r_1) > 0
\]

In other words, as the efficiency of innovation development increases, firms will implement their innovation at an earlier date whereas when their planning horizon increases, they will innovate at a later date.
Furthermore, we have $\frac{dk_i'}{dri} > 0$ because $\frac{d\log k_i'}{dri} > 0$

where

$$\frac{d\log k_i'}{dri} = \frac{1}{ri} \frac{d\delta_i}{dT} + \frac{1}{2-ri} + \frac{1}{ri(1-ri)} - \frac{1}{(1-ri)(2-ri)}$$

Therefore, not only do we observe an earlier date of implementation of a new product, the constant expenditure on innovation investment will be higher also.

For $T_1 = T_2$, we require $r_1 = r_2$ as shown in figure 12-2 and for $T_2 > T_1$, we require $r_1 > r_2$. Given $T_1 = T_2$ and $r_1 = r_2$, we obtain $k_1 > k_2$ if $\delta_1 > \delta_2$. But

$$\delta_1 - \delta_2 = \frac{(d\pi_1/d\beta - dU_2/d\beta \beta a_1/a_2)/(a_1-a_2)}{(11-10)}$$

Note that $|d\pi_1/d\beta| = |dU_2/d\beta| = 0$ for $\beta = \infty$. On the other hand, for $0 < \beta < \infty$, the sign of $d\pi_1/d\beta - dU_2/d\beta$ is indeterminate. Substitute (12-3) and (12-9) into equation (12-10) to obtain

$$\frac{2A[\beta(2a_2B-a_1A)+(2a_2C-a_1A)]}{(3\beta+4)^3(a_1-a_2)^2}$$

where $A = b + c_1 - 2c_2$

$B = b - 2c_1 + c_2$

$C = b - c_1$

As before, we assume that $b > 2c_1 - c_2$ such that $A, B, C > 0$.

For each given value of $a_2$, there will be a value of $a_1$ such that (12-11) is equal to zero. This may be expressed as $a_1$
EFFICIENCIES IN R&D AND NASH EQUILIBRIA
IN OPTIMAL TIMING
= g(a2). In other words, a1 = g(a2) satisfies Aa1^2 + 2a1a2C + 2(C-B)a2^2 = 0 which in turn was derived from equating equation (12-11) to zero. Then for any combination of positive (a1, a2) above g(a2) function in (a1, a2) space, would imply that (12-11) is negative and vice versa.

Suppose that the initial condition of the game was such that (a1, a2) was in fact above the g(.) function. Then we have a negative value for (12-11) and so an equal amount of increase in the quality level of both firms yields higher profits to firm 2 relative to firm 1. In other words, δ22 > δ11. This implies k1 < k2 for r1 = r2. Firm 2 will invest more in innovation to speed up research even if the new innovation is implemented simultaneously with firm 1. The reason for this asymmetry in behaviour is due to the difference in the profit level changes in response to quality level changes. It is profitable for firm 2 to invest more rigorously in research as per unit return in investment is higher than that of firm 1. We shall define the initial condition where (a1, a2) lie above g(.) as initial condition 1 (IC1) and the reverse as initial condition 2 (IC2). We shall assume, henceforth, that IC1 applies. The reverse of the results we obtained under IC1 will be true for IC2.

12.3 Level of Quality
Each firm's quality level of their new product over the quality of their old products are given as k1'r1(1-r1)/(2-r1)T* and k2'r2(1-r2)/(2-r2)T* respectively. Substituting
ki's with the optimal values obtained above, the increments of quality levels may be written respectively as

\[
(r_1 \delta_{11} T^* / 2 - r_1) r_1 / (1 - r_1) (r_1 - 1) / (r_1 - 2) T^* \quad (12-12)
\]

and

\[
(r_2 \delta_{22} T^* / 2 - r_2) r_2 / (1 - r_2) (r_2 - 1) / (r_2 - 2) T^* \quad (12-13)
\]

For \( T_1 = T_2 \) we require that \( r_1 = r_2 \), i.e., both firms are equally efficient in R&D. We note that the increment of the profit level of firm 2 with respect to quality improvement is larger than that of firm 1 under IC1, i.e., \( dU_1 / da_1 < dU_2 / da_2 \). Therefore, for \( T_1 = T_2 \), the quality gap is narrowing. In order to keep the quality gap constant, we require that (12-12) be equal to (12-13).

Since \( \delta_{22} > \delta_{11} \), we require \( r_1 > r_2 \) for constant quality gap. This implies \( T_1 < T_2 \). In other words, for firm 1 to overcome the disadvantage of smaller \( \delta_{11} \), firm 1 requires higher efficiency in R&D for each given level of expenditure. Given \( r_1 > r_2 \), firm 1 will, using this advantage of higher efficiency in R&D, introduce the new product later than its follower at a lower constant expenditure in R&D. To be more precise, consider the following. Since the change in the quality level is defined as \( k_1' r_1 T_1' \) and \( k_2' r_2 T_2' \) for each firm, given \( k_1' r_1 T_1' = k_2' r_2 T_2' \), it follows that \( k_1' r_1 > k_2' r_2 \). Let \( r_1' \) and \( r_2' \) be values satisfying \( k_1' = k_2' \) for each given values of \( \delta_{11} \) and \( \delta_{22} \) so that \( r_1' - r_2' > 0 \). For \( r_1 > r_1' \) and \( r_2 = r_2' \), or \( r_1 = r_1' \) and \( r_2 < r_2' \), we have a situation where firm 1 is spending less but for a longer period on R&D. In other
words, firm 1 is spending less money in R&D per unit of time but innovates later while firm 2, at a lower level of constant expenditure, innovates earlier attaining a constant quality gap with its leader. Therefore, under IC1, we require that $r_1 > r_2$ which will then lead to $T_1 < T_2$ to attain the constant quality gap.

Suppose that the firm in Korea attains larger profit relative to a firm in an industrialized country given they increase their respective quality levels by the same amount. In order for firms in the industrialized country to attain a constant quality difference with the firm in Korea, it must be more efficient in its innovation development. Once this is attained, the firm in the industrialized country will introduce a new innovation earlier than the Korean firm which will introduce its new innovation at a later date keeping a constant quality difference.

12.4 Level of Profit

The profit level $V_1(T_1, T_2)$ as defined in (12-5) was derived from adding the net profit made during this single period by implementing new innovation on the ex ante profit level $\pi_1(0, 0)$. Since we are interested in the net increment to profit in this period only, we define $W_1(T_1, T_2)$ as $V_1(T_1, T_2) - \pi_1T^*$. Then $W_1(T_1, T_2)$ is given as

$$W_1(T_1, T_2) = \delta_{11}k_1r_1T_1(T^*-T_1) - \delta_{12}k_2r_2T_2(T^*-T_2) - k_1T_1$$

(12-14)
To obtain the profit contours of firm 1 on $T_1$, $T_2$ space, we differentiate $W_1$ with respect to $T_1$ and $T_2$.

\[ \frac{dW_1}{dT_1} = \delta_{11}k_1r_1(T^*-2T_1) - k_1 = 0 \]
\[ T_1' = \frac{1}{2} T^* - k_1/2\delta_{11}k_1r_1 = (1-r_1)/(2-r_1) \]

\[ \frac{d^2W_1}{dT_1^2} = -2\delta_{11}k_1r_1 < 0. \] Therefore, $T_1'$ is the maximum.

\[ \frac{dW_1}{dT_2} = -\delta_{12}k_2r_2(T^* - 2T_2) = 0 \]
\[ T_2' = \frac{1}{2} T^* \]

\[ \frac{d^2W_1}{dT_2^2} = 2\delta_{12}k_2r_2 > 0. \] Therefore $T_2'$ is the minimum.

To obtain the slope of the contours, we totally differentiate $W_1$ to obtain

\[ \frac{dW_1}{dT_1} dT_1 + \frac{dW_1}{dT_2} dT_2 = 0 \]

\[ \frac{dT_2}{dT_1} = \frac{\delta_{11}k_1r_1(T^*-2T_1)-k_1}{\delta_{12}k_2r_2(T^*-2T_2)} \]

(12-15)

Therefore, $dT_2/dT_1$ is not defined at $T_2'$ and is infinite at $T_1'$. The profit level contours of firm 1 are shown in figure 12-3 where profit level increases as we move either up or down and the minimum is attained at $W_0$ where $T_2'$ becomes a tangent to the profit contour.

For firm 1 to have a zero net profit in this period, i.e., $W_1(T_1, T_2)=0$, we require that the contours of $W_1 = 0$ which passes through the origin, must also intersect the reaction function $T_1'$. As shown in figure 12-4, the reaction function of firm 2 must be a tangent to the profit contour $W = 0$. In order for this to be possible, a necessary condition is $T_1 < T_2$ or $r_1 > r_2$ confirming our earlier findings that under CI1, a necessary condition for both firms to attain equal
Figure 12-3

$W_3 > W_2 > W_1 > W_0$

PROFIT CONTOURS OF FIRM 1
ZERO PROFIT CONTOURS OF FIRM 1 GIVEN IC1
competitiveness was \( r_1 > r_2 \). Similarly, the profit level of firm 2 may be defined as

\[
W_2(T_1, T_2) = \delta_2 k_2 r_2 T_2 (T^*-T_2) - \delta_2 k_1 r_1 T_1 (T^*-T_1) - k_2 T_2 \tag{12-16}
\]

From the profit contours derived above, we find that there are other possible combinations of \( T_1 \) and \( T_2 \) that would yield higher profit to both firms. In fact, these points are represented by the aa curve in figure 12-5. This curve must pass through the origin as at the origin, both firm's profit level are zero. Therefore, one of the potentially many cooperative equilibrium is the origin, i.e., neither firm invest in R&D.

12.5 An increase in the cost of production

Consider finally the effect of an increase in the cost of production or a shift towards profit maximization relative to revenue. We note that \( \delta_11/\delta c_2 > 0 \) while \( \delta_22/\delta c_2 < 0 \). This means that \( g(.) \) function must shift downwards. In other words, for each given level of \( a_2 \), the minimum level of firm 1 to sustain its competitiveness, i.e., keep a constant quality difference, in the market falls with respect to an increase in \( c_2 \). The reason is that as \( c_2 \) increases, the ex post increment to profit of firm 2 falls relative to the ex ante level. We obtained earlier that the relative increments to profits due to changes in the quality levels were one of the variables that determined the level of investment into R&D. Therefore, as firm 2 now faces a lower
Figure 12-5

CONTRACT CURVE UNDER COLLUSION
increment to profit due to a change in the quality level compared to before the increment in c2, it will invest less in R&D. This will lead firm 1 to invest less in R&D.

An increase in the value of $\delta_{11}$ while a fall in the value of $\delta_{22}$ as a result of an increment in c2 implies that the market which was initially under IC1 may find itself in IC2. Under such a case, an increase in c2 must be accommodated by an increase in r2 relative to r1 if firm 2 is to sustain its competitiveness in the market with respect to the quality levels.

We have shown so far, that the optimum timing of innovation adoption and the corresponding level of expenditure will depend crucially on the initial condition with respect to quality levels of firms as well as on the values of the exogenous variables. The initial condition is important as this will determine the changes in the profit level in response to changes in the quality. Once we assume that initial quality levels are such that the profit of firm 1 increases by less than that of firm 2 with respect to an equal amount of an increase in each quality level, firm 2 will close down on the quality gap and reduce the profit level of firm 1 accordingly, unless firm 1 has a more efficient research and development technique. The actual amount firm 1 spends on R&D will depend on the relative differences in the R&D efficiency. It will adopt the new product earlier than its rival given that it has more efficient R&D technique such as to assure a constant quality gap. If, on the other hand, firms could collude, they would
be able to obtain mutually higher profit level along the contract curve. Finally, an increase in the cost of production by firm 2 leads to a change in the initial conditions. The \( g(.) \) function will shift downwards such that efficiency in R&D by firm 2 has to rise if it is to sustain its competitiveness with respect to its quality level.

12.6 Multi-Period Competition

Consider a situation where the single period game is played \( n \) times. Each period is played over a certain fixed time span of \( T^* \). The objective function for firm 1 under such a supergame may be defined as follows:

\[
\Phi_1(T_1, T_2) = V_1(T_{11}, T_{12}) + hV_1(T_{21}, T_{22}) + h^2V_1(T_{31}, T_{32}) + \ldots + h^{n-1}V_1(T_{n1}, T_{n2})
\]  

(12-17)

where \( 1-h \) is the discount rate and \( T_{ij} \) represents the time of implementation of a new innovation by firm 1 in the \( i \)th period. Since we have established earlier that \( V_1(T_{ij}, T_{j2}) = W_1(T_{ij}, T_{j2}) + \pi_1(0, 0)T^* \), we may rewrite (12-17) given \( h = 1 \) as follows:

\[
\Phi_1(T_1, T_2) = n\pi_1(0, 0)T^* + W_1(T_{11}, T_{12}) + W_1(T_{21}, T_{22}) + \ldots + W_1(T_{n1}, T_{n2})
\]  

(12-18)
We assume that Perfect equilibrium to prevail. Thus the subgame perfect equilibrium for the \( n^{th} \) period is obtained solving the following maximization problem:

Max \( W_1(T_{n1}, T_{n2}) \) subject to \( k_{n1} \) and \( T_{n1} \).

where \( k_{n1} \) represents the constant expenditure on innovation in period \( n \) by firm 1. The solutions, as already obtained under single period game, are given as follows:

\[
T_{n1}^* = \frac{(1-r_i)(2-r_i)T^*}{(2-r_i)} \\
k_{n1}^* = \frac{(r_i\delta_{ii}T^*/2-r_i)1/(1-r_i)}{(1-r_i)}
\]

for \( i = 1, 2 \).

Substituting these values back to the net increment to profit in period \( n \) yields

\[
W_1(T_1, T_2) = \delta_{11}a A - \delta_{12} \delta_{22}b B 
\]

(12-19)

where

\[
A = \frac{(r_1T^*T_{n1}^*[(r_1T^*/2-r_1)2r_1-1/1-r_1(T^*-T_{n1}^*)])}{(1-r_1)} \\
B = \frac{T_{n2}^*[(r_2T^*/2-r_2)2r_2-1/1-r_2(T^*-T_{n2}^*)]}{(1-r_2)} \\
a = \frac{1}{(1-r_1)} \\
b = \frac{r_2}{(1-r_2)}
\]

The solution to \( n-ith \) period for all \( i = 1, 2, \ldots, n-1 \) is identical to the \( n^{th} \) period as long as we assume that \( d\alpha_1 = d\alpha_2 \) and \( \delta_{ij} \) for \( i = 1, 2 \) and \( j = 1, 2 \) are time independent and \( d\delta_{ij}/d\alpha_l = 0 \). In other words, as long as the profit increment to a change in the quality level is constant with respect to the absolute level of the quality level and time. On the other hand, if the quality level changes are not
independent to the absolute level of quality, i.e., $d\delta_{i1}/da_1 > 0$, the quality level increase of firm 1 in one particular period will be by more than its previous level.

Since $\delta_{i1}$ is a function of $a_1$ and $a_2$, given that $da_1 = da_2$, $d\delta_{i1}/da_1 > 0$ will result in a higher level of quality increment than its previous level. The same analysis also applies to firm 2. Therefore, if $d\delta_{i2}/da_2 > 0$, the increment of present period's level of quality will be higher than that of the previous period. This then implies that for $d\delta_{i1}/da_1 > d\delta_{i2}/da_2$, the gap in the quality level will widen and vise versa.

A more crucial implication of this is the profit level. From (12-19), we know that

$$dW_1 = a(\delta_{i1})^{a-1}Ad\delta_{i1} - \delta_{i2}Bd\delta_{i2} - b\delta_{i2}\delta_{i2}b^{-1}Bd\delta_{i2}$$ \hspace{1cm} (12-20)

Therefore, given

$$aA\delta_{i1}^{a-1}d\delta_{i1} > b\delta_{i2}^{b}[d\delta_{i2}+b)(\delta_{i2})^{-1}d\delta_{i2}]$$ \hspace{1cm} (12-21)

we obtain the result that $V_1(T_{i1}, T_{i2}) < V_1(T_{i1}, T_{i2})$ without the discounting factor. If on the other hand the reverse is true, the profit level of firm 1 will be squeezed as quality level is increased at each period. If inequalities of both firm hold, then this implies that the market is expanding as both quality level are increased. If the inequality holds for firm 1 but not for firm 2, then the outcome of the supergame will create a favourable situation for firm 1 in that firm 1's profit level will increase in each period while firm 2's profit level will fall. The reverse also holds.
12.7 Conclusion

We have considered the optimal timing of innovation implementation as well as the constant expenditure that will maximize firms profits. For a single implementation case, we found that the optimal timing of both firms that maximize their profit depended on their planning horizon and their respective efficiency in innovation development. An increase in the efficiency of innovation development will not only lead both firms to implementing their new innovation earlier but also higher constant expenditure on investment innovation.

Under a particular case where firm 2's profit increment to a change in its profit level is larger than that of firm 1, we require that firm 1's efficiency in innovation development be greater than that of firm 2 if the difference is maintained, and thus, no change in the market structure.

We then considered the case when there is an increase in the cost of production by firm 2. This will increase and lower the profit increments of firm 1 and firm 2 respectively to a change in quality increase. For firm 2 to compensate for the increase in its cost of production, it has to increase its efficiency in innovation development if it is to maintain its relative quality level with firm 1.

Under a multi-period game, we find that the sub game perfect equilibrium is equal to the Nash equilibrium obtained under a single period game. The market structure will be
determined on how $\delta_{ij}$ change over time as $a_1$ and $a_2$ are increased in each period. The $\delta_{ij}$ trajectories with respect to $a_1$ and $a_2$ will then determine the market structure after $n$ periods given that the efficiency in R&D stays constant over time.
In the middle of 1988, the IBM in Korea has officially granted permission for the Korean electronics companies to freely copy their product (although limited to PCs), without prior consent, but to pay licence fee ex-post adoption. This move was the result of three basic circumstances in the market. First, the Korean computer companies are almost as advanced in producing personal computers as their rivals, but have been concentrating their efforts in diversifying the products such as to reach the extreme ends of the market whereas the IBM has focused more on innovation. The result of this led naturally to the second reason that the IBM has the primary market consisting of large firms and institutions requiring ever more sophisticated and powerful computers. The Korean computer companies, however, lagging behind by only a month the IBM in terms of technology, took the secondary market the demand of which consists largely of private users requiring less powerful but cheaper computers and for more diversified purposes. Finally, the third reason was that the Korean computer companies copied the IBM products anyway without paying the licence fee. The monitoring cost involved being large, it is difficult for the IBM to prevent free copying of its products.

This simple illustration of one aspect of the electronics market shows that the specification of the demand side of
our model was empirically consistent. Not only do we find that more expensive computers are made in the U.S.A. or in Japan and in many other fields of the electronics market to include video cameras and high quality stereo systems, but also that they attract the upper half of the market.

The assumption of first mover advantage of firm 1, i.e., the incumbent firm, was trivial and is justified by its relative advantages in technology. We also assumed that consumers have perfect information about the quality of products and firms have perfect information about the demand side. Furthermore, the exchange rate was assumed to be constant which under the present regime of free floating is hard to find anywhere. In spite of these weaknesses in the specifications of the real world, we have derived the following results.

A two-stage game under perfect equilibrium was considered first. As there is no empirical justification as to what variables firms choose (as under what principles firms' conjectures were determined) to maximize their objective functions, we analysed under two extreme forms of competition: Cournot and Bertrand. Under Cournot competition, we find that the subgame perfect equilibrium in the second stage game of marketing is characterised by quantities produced by both firms below that found for under Bertrand competition. We find that, under Cournot competition, output of firm 1 fell as against a rise in firm 2's output as the quality gap is reduced. The price levels of both firms are lower under Bertrand competition than under
Cournot. Under Bertrand competition, however, there exists a lower limit to the quality level of firm 1 for each given value of the quality level of firm 2 if it is to stay in the market at all. This is caused by the price war between two firms such that unless firm 1 attains a certain minimum quality gap, firm 2 will push its price level down far enough that the difference in the quality level can no longer be compensated for by the difference in the price level. We showed that under Bertrand competition, that firm 1 may be forced out of the market by negative mark-up rather than due to a complete loss of market share. The reason that firm 1 may not be forced out of the market under Cournot competition even if there is no quality gap is because of the first-mover advantage where firm 1 behaves as a price discriminating monopolist charging a higher price (due to higher production cost) and taking the upper group of the market in the face of a downwards sloping demand curve, while firm 2, at a lower price, behaves also like a monopolist in the market left over by firm 1. In other words, in Cournot competition, both firms act as monopolists given their residual demand curves.

For the first stage competition, we found under both Cournot and Bertrand competition for the marketing stage, that with zero cost of innovation and no upper and lower bounds on the level of quality, Nash-equilibrium did not exist. Since there is no cost of innovation, firm 2 will attempt to reduce the quality difference while firm 1 will attempt to push ahead leaving as wide a quality difference as possible. Imposing upper and lower bounds on the potential quality
levels yield the same result in both the Cournot and Bertrand competition for the marketing stage. Both firms will opt for the maximum potential quality levels. If the maximum quality levels are identical, then under Cournot competition, firm 1 will still stay in the market for the reason explained above while under Bertrand competition, it will be forced out of the market since firm 2 with its lower cost will push its price level down as far as to induce a negative mark up for firm 1.

Under a positive and increasing cost of innovation with respect to the quality level, we find that under certain conditions on the values of the exogenous variables, equilibrium exists under both regimes. Each equilibrium is obtained such that neither firm can improve their profit given the equilibrium strategy of their rivals. The nonlinearity of their reaction function arise from the trade off between higher quality leading to higher profit and higher cost of innovation. One difference between the reaction function between Cournot and Bertrand competition is that the reaction function of firm 2 was concave and convex to the origin respectively.

To analyse the consequences of wage rises or a shift in the emphasis within the objective function of firm 2 from revenue to profit maximization, we conduct a comparative static exercise where we assume an increase in the cost of production of firm 2 and a positive cost of innovation.
Under Cournot competition, the reaction functions of the quality levels of both firms will shift towards the origin due to the following reason. As the cost of production rises, quantity supplied by firm 2 will fall. This fall in the market will be partly compensated by an increase in the quantity supplied by firm 1 who will charge a slightly higher price than before without any loss of market share due to its price increase. The extent of firm 1's price increase will be such that the relative price level of firm 1 in relation to firm 2 will be lower than before. This marginal increase in competitiveness then enables firm 1 to lower its quality level as well, thus cutting the cost of innovation, so as to marginally increase its profit level. To sum up, an increase in the cost of firm 2 will lead to an decrease in firm 2's quantity supplied. Firm 1 will respond by increasing its quantity supplied accompanied by a combination of an increase in the price level and a fall in the quality level.

Under Bertrand competition, firm 2 will increase its price level as to guarantee itself a positive mark up as a response to an increase in its production cost. The quantity it supplies will fall. The reaction function of the quality level of firm 1 will now shrink towards the 45° line implying a fall in the optimal quality level for each given quality level of firm 2. The reason is that under Bertrand competition, both firms are pricing their products at their marginal cost level. The quality levels are then determined such as to maximize profit. As the price level of firm 2 rises, firm 1 is now able to afford a lower
quality level for each quantity supplied as higher marginal cost of production has meant a fall in the marginal profit level and thus a fall in the optimal quality level of firm 2. Therefore, as in the case of Cournot competition, both quality level will fall. These results may be plausible in the sense that as the cost of production of firm 2 rises, $a_2$ gets closer to that of firm 1, i.e., the degree of asymmetry is reduced. Therefore, the differences in the result of the competition must also be narrowed.

In order to relax the assumption on firms' conjectures, restricted either to Bertrand or Cournot, we considered all other possible outcomes corresponding to different conjectures. This produce infinitely many equilibria including possibilities where firm 1 may be forced out of the market if both firms engage in a price war under a narrow quality gap. To reduce such a multiplicity of possible equilibria, we propose consistency in conjectures. We defined consistent conjecture to prevail if firm 1 conjectures on firm 2's reaction function in response to changes in quantity supplied by firm 1 for each given values of quality difference, is in fact the actual reaction function of firm 2. Furthermore, it is attained at a point where it is not profitable for neither firm to deviate from this solution. We show that under such a restriction that the possible equilibria is reduced dramatically. For each given value of quality difference, we find that uniqueness of equilibrium prevails. Under an infinite regress of thought experiment by firms, such a consistent conjectures equilibrium is equal to a Cournot solution. Therefore,
given full information about market structure and values of exogenous variables as fixed, in the long run, we may observe a Cournot competition in the duopoly market consisting of rational profit maximizing firms.

Finally, we considered the question of persistent dominance of the incumbent firm in such a market described so far. We found an interesting result that under the Cournot conjectures assumption for the marketing stage and certain cost condition, it is more profitable for firm 1 to stay a leader while it is more profitable for firm 2 to stay a follower. The intuition is that firm 1 with higher fixed cost of production has to compensate for its higher price level by providing better quality product. On the other hand, for some changes in the costs of production, both firms would find it more profitable to become followers. To be more precise, if the cost of production of firm 1 increases such that the cost difference between both firms is further widened, firm 1 would benefit if it would change over to a follower, producing a lower quality product and thereby saving from cuts in its cost of innovation than to produce at even a higher quality level aiming at the remaining upper group consumers. Since it is profitable for firm 2 to stay also a follower, there will be a conflict of interest and the Nash equilibrium will be obtained where both quality levels will be zero. Under an infinitely repeated game, both firms will gain if firm 1 stays a leader while firm 2 remains a follower. Given that firm 2 has some credible threat to counteract any deviant behaviour of firm 1, the incumbent firm will be forced to stay a leader. Such
a credible threat may be provided by the Government of a NIC drawing up a contingent plan for its firm.

We introduced three different types of licensing rules into the general framework; a fee that is determined as a function of the quantity of final product sold by the buyer of the technology, a fixed fee and finally a fee that is a function of the quality difference between the top quality of the market leader and the level of quality it is selling to the follower. We concluded that the last of the three rules dominates others in terms of yielding highest profit to both firms. Having established a dominant strategy, we examined the possibility of leap frogging. We found that firm 1 will sell its highest quality at hand, compensating its loss of profit from lower sales by the higher license fee received from firm 2. To prevent leap frogging, firm 1 will have to lower its license fee selling the same quality level.

Once we allow for free copying, we found that firm 2 will be able to maximize its profit by copying closely the more advanced products from firm 1 but never actually take over the leadership as this would incur innovation costs.

Under Bertrand conjectures assumption for the marketing stage, we found that unless the firm with higher production cost remains a leader offering a higher product quality to consumers, it will be driven out of the market. The implication of this for firm 1 type survival is "Innovate or die".
Still under the same topic of persistent dominance and Cournot competition for the marketing stage, we examined the optimal timing of new innovation implementation. We analysed how the market structure is affected by different times of adoption of higher quality products. We assume the level of innovation to be the product of expenditure and time, allowing a version of the Penrose effect to take place, such that higher expenditure leads to a fall in the marginal speed of increment of development. Uncertainty is again ruled out. We find that the result of this single period game to depend crucially on the initial values of both quality levels. There exists some level of quality gap at which changes in the profit levels of firm 2 as a response to changes in the quality levels will be lower than that for firm 1. Thus, under such an initial condition, we obtain that both firms will innovate at the same time while firm 2 spends more on R&D expenditure such as to keep a constant quality gap. If, on the other hand, firms face an initial condition such that the quality level of firm 2 is higher than the level just described above, the only way that firm 1 can stay competitive is to have a higher efficiency in R&D process or technique. An increase in firm 2's cost of production or a shift in the priorities towards profits, will lead to the same result as if firm 2 had a fall in the efficiency in the R&D technique. Therefore, this will lead to a loss of profit of firm 2 and a widening gap of quality difference. We then briefly examine the multiperiod supergame and find that given the exogenous variables being constant over time and the initial
conditions such that the first period equilibrium is attained at which quality gap is kept constant, the same Nash-equilibrium is sustained throughout other repeated games.

The results we derived in this part of the thesis have the following implications to a firm in Korea that is to embark on a competition with firms in industrialized countries once any supportive action by its Government has been withdrawn and free copying becomes more difficult due to its image as firm in a newly emerging industrial country.

The shift in the emphasis from dual objectives of revenue and profit towards a single objective of profit has the same qualitative effect to firms as an increase in the wage rate, i.e., increase in the cost of production. Once this increase in the cost of production is observed, both firms will lower their quality levels relative to that prior to the cost increase. We shall observe a general increase in the price level of both products. These two results are true under both the Cournot and Bertrand conjectures for the marketing stage.

If firms are rational and full information is available freely, we may find that Cournot competition may prevail between the Korean firm and a firm in the industrialized country.

Given the present asymmetry in the cost of production and quality difference, we will observe that the Korean firm
will remain a follower as long as the cost of production of its rival does not increase to such an extent that its rival decides to cut its expenditure on innovation and become a follower too. If this is the case, a credible threat by the Korean firm will enable its to maintain its status as a follower and force its rival to remain a leader.

If the Korean firm's profit increment is more sensitive to quality improvements, the only way that the firm in the industrialized country can stay competitive is to have a higher efficiency in R&D technique. We will observe that the firm 2 will implement at a later date than its rival while maintaining a lower constant expenditure on innovation per unit of time.

Finally, if we observe a Bertrand competition for the marketing stage, the only way that the firm in the industrialized country can remain in the market is to innovate or die.
Reference

Bain, J. (1956) "Barries to New Competition"
Cambridge, MA, Harvard University Press

"Sequential Product Innovation and Industry Evolution"
Economic Journal 97 (Supplement)

"Sequential Product Innovation and Endogenous Market Structure"
Market Structure and Strategy Conference
University of Dundee, 22-25 August

Bertrand, J. (1983) "Revue de la théorie Mathématique de la richesse sociale et des recherches sur les principes mathématiques de la théorie des richesses"
Journal des Savants 499-508
Translated by J.W. Friedman in Daughety (1888) edited "Cournot Oligopoly", Cambridge Univ. Press

Bell Journal of Economics Vol 14 No 1

Bresnahan, T.F. (1981) "Duopoly models with consistent conjectures"
American Economic Review 71 P934-45

Bulow, J.I., Geanakoplos, J.D. & Klemperer, P.D. (1983) "Multimarket Oligopoly"
Cowles Foundation Discussion Paper 674
Yale University

Cournot, A. (1838) "Recherches sur les Principes Mathematiques do la Theori des Richesses"
Paris, Hachette (reprinted 1960)

Journal of Economics 1-28

Economic Journal 266-293

Draft, London School of Economics

Daughety, A.F. (1983) "Reconsidering Cournot: the Cournot Equilibrium is Consistent"
Rand Journal of Economics 6, No 6
Denecker, R. & Davidson, C. (1985) "Incentives to Form Coalitions with Bertrand Competition" Rand Journal of Economics 16 No 4 P473-86


Edgeworth, F. (1897) "La Teoria Puree del Monopolio" Giornale degli Economisti 13-31


Galbraith, J. (1936) "Monopoly Power and Price Rigidities" Quarterly Journal of Economics 466-468


Hviid, M. & Ireland, N.J. (1986) "Consistent conjectures with Information Transmission" Mimeo, University of Warwick


Kamien, M & Schwartz, N. (1975) "Cournot Oligopoly with Uncertain entry" Review of Economic Studies 125-131


Perry, M.K. (1982) "Oligopoly & Consistent Conjectural Variation"
Bell Journal of Economics 13 197-205

Reinganum, J.F. (1981) "Dynamic Games of Innovation"
Journal of Economic Theory 25 21-41

Reinganum, J.F. (1985) "Innovation and Industry Evolution"
Quarterly Journal of Economics 99 81-99

Salant, S., Switzer, S., Reynolds, R., (1983) "Losses from horizontal merger: the effects of an exogenous change in industry structure in Cournot-Nash Equilibrium"
Quarterly Journal of Economics 185-199

Schmalensee, R. (1982) "Product differentiation advantages of pioneering brands"
American Economic Review 72 pp349-65

Schumpeter, J. (1943) "Capitalism, Socialism and Democracy"
London, Allen and Unwin (5th edition with introduction)

Shaked, A. & Sutton, J. (1982) "Relaxed price competition through product differentiation"
Review of Economic Studies 49 3-13

Shaked, A. & Sutton, J. (1983) "Natural Oligopolies"
Econometrica 41 P1469-1484

Shapiro, C. (1982) "Consumer Information, Product Quality and Seller Reputation"
Bell Journal of Economics 13 20-35

Harvard Law Review 869

The Rand Journal of Economics 15 546-554

Spence, M. (1977) "Entry, Capacity, Investment and Oligopolistic Pricing"
Bell Journal of Economics 534-544

Sylos Labini, P. (1962) "Oligopoly and Technical Progress"
Cambridge, Mass, Harvard University Press.

Tandon, P. (1983) "Rivalry and the Excessive Allocation of Resources to Research"
Bell Journal of Economics 152-65
International Journal of Industrial Organization 1 131-154

Vickers, J.S. (1985a) "Pre-emptive Patenting, Joint Ventures, and the Persistence of Oligopoly"
International Journal of Industrial Organization 3 261-273

Vickers, J.S. (1985b) "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation"
Journal of Economic Theory 36 No 1 P166-175

Vickers, J.S. (1986) "The Evolution of Market Structure when there is a Sequence of Innovation"
Journal of Industrial Economics 35(1) 1-12

Wenders, J. (1971) "Excess Capacity as a Barrier to Entry"
Journal of Industrial Economics 14-19

Wolinsky, A. (1983) "Price as signals of product quality"
Review of Economic Studies 50 647-58
Footnotes

1. We define a duopoly to be symmetric if all exogenous variables within a model, to include both state and structural variables, are identical.

2. Avinash Dixit (1984) has formalized this notion within a compact algebraic structure.

3. A noncooperative competition occurs in the absence of any collusive agreement between firms. We may obtain a positive conjecture under cooperative behaviour or under a demand uncertainty as examined in Ireland & Hviid (1986) where each firm takes the decision of its rival as a signal (additional information to supplement the imperfect information) of the demand side.

4. The usual assumptions of the cost and demand functions and the concavity of the profit function are assumed to apply. The usual assumptions are: 1) the cost function is defined and continuous with output $x$ respect to and has a positive first derivatives while the second derivative is continuous 2) the demand function is defined and continuous for all output $x \geq 0$, it is closed and bounded given $x \geq x \geq 0$ (where $x$ is some upper limit) and has a negative first derivative while we assume the second derivative is constant. (refer to J.W. Friedman(1977) P19-20 for a detailed exposition)

5. This notion was first used by Stackelberg (1934), hence the name, who argued that in real world, there is usually one dominant firm in each industry. Other firms, which are relatively small, will have zero conjectures like in Cournot competition. The dominant firm, knowing its follower's strategy, maximizes with respect to its own output level as well as the known reaction of its followers.

6. See Selten (1975) for extensive analysis on the notion of Perfect Equilibrium. Here, we use the definition as rephrased by Shaked and Sutton (1982).

7. Edgeworth (1897), based on Bertrand's argument, carries it further and argues that a firm has incentive to undercut its rival's price level as long as it has unused capacity. The price war will stop only if they both have reached their full capacity production level. Their price level may be well below the marginal cost. Thus, one firm will set a higher price in the hope that its rival will follow. The rival would indeed follow, but not all the way, for by setting a price level just below its rival who has dared to put up the price level, it can undersell its rival. This will initiate another price war and the equilibrium will be somewhere between the initial price level corresponding to full capacity production level.
8. Dominant strategy is defined as the strategy vector 
\[ S^* = \{s_1^*, s_2^*, \ldots, s_n^*\} \] where \( n \) is the number of 
firms and \( s_i \in S \), a strategy set, such that it satisfies 
\[ \pi_i(S^*) \geq \pi_i(S) \] where \( S = \{s_1, s_2, \ldots, s_n\} \) for all \( i \).

9. He himself refers to the models developed by 

10. An oligopoly market where firms take their rivals' 
supply levels as given.

11. An oligopoly market where firms take their rivals' 
prices as independent of their own profit maximizing 
decisions.
Appendix A

Comparative Static Analysis on Firms Maximizing Behaviour
($\delta$ denotes partial differentiation)

i) Objective Functions

Firm 1: $\pi_1 = (P_1-c_1)q_1$
Firm 2: $U_2=(P_2-c_2)q_2$

ii) First Order Conditions

\[
\begin{align*}
\delta \pi_1/\delta q_1 &= 0 \\
(P_1-c_1) + q_1 \delta P_1/\delta q_1 &= 0 \\
\delta P_1/\delta q_1 &= \delta P_1 + q_2 \delta q_1 \\
\delta q_1 &= \delta q_1 + \delta q_2 \\
\delta q_2 &= \delta q_2 + \delta q_1
\end{align*}
\]

iii) Price Level

$P_1 = b - q_1 - (1/1+\beta)q_2$
$P_2 = b - q_1 - q_2$

iv) Quantity

$q_1 = b - (1+\beta)P_1 + \beta P_2$
$q_2 = (1+\beta)(P_1-P_2)$

v) Cournot Solution

$\delta q_2/\delta q_1 = 0$
$\delta q_1/\delta q_2 = 0$
$\delta P_1/\delta q_1 = -1$
$\delta P_2/\delta q_2 = -1$

\[
\begin{align*}
q_1 &= \frac{\beta}{(b-c_1) - \frac{b}{1+\beta} q_2} \\
q_2 &= \frac{\beta}{(b-c_2) - \frac{b}{1+\beta} q_1}
\end{align*}
\]

vi) Bertrand Solution

\[
\begin{align*}
\delta q_2/\delta q_1 &= -1 \\
\delta P_2/\delta q_1 &= -1 \\
\delta q_1/\delta P_2 &= -1 \\
\delta q_2/\delta P_2 &= -1
\end{align*}
\]

\[
\begin{align*}
q_1 &= \frac{1+\beta}{2+\beta} (b-c_1) - \frac{\beta}{2+\beta} q_2 \\
q_2 &= \frac{1+\beta}{2+\beta} (b-c_2) - \frac{\beta}{2+\beta} q_1
\end{align*}
\]

vii) Conjectural Variation

\[
\begin{align*}
\delta q_2/\delta q_1 &= v^1 \\
\delta q_1/\delta q_2 &= v^2
\end{align*}
\]

\[
\begin{align*}
q_1 &= \frac{(1+\beta)(b-c_1) - \beta q_2}{\beta(v^1+2) + 2} \\
q_2 &= \frac{(b+c_2) - q_1}{2 + v^2}
\end{align*}
\]
viii) Consistency

\[ v^1 = -\frac{1}{2 + v^2} \]
\[ v^2 = -\frac{1}{\beta (v^1 + 2) + 2} \]
\[ v^1 = \frac{-(1 + \beta) \pm (1 + \beta)^{\frac{1}{2}}}{\beta} \]
\[ v^2 = \frac{-(1 + \beta) \pm (1 + \beta)^{\frac{1}{2}}}{(1 + \beta)} \]

\[ \lim_{\beta \to \infty} v^1 = -1 \]
\[ \lim_{\beta \to 0} v^1 = -\infty \]
\[ \lim_{\beta \to \infty} v^2 = -1 \]
\[ \lim_{\beta \to 0} v^2 = -\infty \]
Appendix B

Comparative Static Results obtained under Bertrand Conjecture

\[ q_2 = \frac{(c_1-c_2)\beta^2 + (b+2c_1-3c_2)\beta + (b+c_1-2c_2)}{(3\beta+4)} \]

\[ p_2 = \frac{(c_1+2c_2)\beta + (b+c_1+2c_2)}{(3\beta+4)} \]

\[ \frac{dq_2}{d\beta} = \frac{(b + 5c_1 - 6c_2)}{(3\beta+4)^2} > 0 \]

\[ \frac{d^2q_2}{d\beta^2} = \frac{4(c_1-c_2) (3\beta+8)\beta - 2(b+7c_1-8c_2)}{(3\beta+4)^3} \]

\[ \frac{dp_2}{d\beta} = \frac{-(3b - c_1 - 2c_2)}{(3\beta+4)^2} < 0 \]

\[ \frac{d^2p_2}{d\beta^2} = \frac{2(3b - c_1 - 2c_2)}{(3\beta+4)^3} > 0 \]

Comparative Static Results obtained under Cournot Conjecture

\[ q_2 = \frac{(1+\beta)(b+c_1-2c_2)}{(3\beta+4)} \]

\[ p_2 = \frac{((b+c_1+c_2)\beta + (b+c_1+2c_2)}{(3\beta+4)} \]

\[ \frac{dq_2}{d\beta} = \frac{4(b + c_1 -2c_2)}{(3\beta+4)^2} > 0 \]
\[ \frac{d p_2}{d \beta} = \frac{(b + c_1 - 2c_2)}{(3\beta+4)^2} > 0 \]

\[ \frac{d^2 q_2}{d \beta^2} = \frac{-8(b+c_1-2c_2)}{(3\beta+4)^3} < 0 \]

\[ \frac{d^2 p_2}{d \beta^2} = \frac{-2(b+c_1-2c_2)}{(3\beta+4)^3} < 0 \]
Appendix C

Numerical Values of Equilibrium under Bertrand and Cournot Conjectures

\[
\frac{13\beta + 24}{3\beta + 4} > \frac{5\beta + 24}{3\beta + 4}
\]

\[
\frac{13\beta + 14}{3\beta + 4} > \frac{5\beta + 28}{3\beta + 4} \quad \text{for } \beta > \frac{7}{4}
\]

\[
\frac{7\beta + 16}{3\beta + 4} > \frac{-\beta^2 + 15\beta + 16}{3\beta + 4} \quad \text{for } 0 < \beta < 8
\]

\[
\frac{10\beta + 10}{3\beta + 4} > \frac{\beta^2 + 11\beta + 10}{3\beta + 4}
\]

\[
\frac{(7\beta + 16)^2}{(3\beta + 4)^2} > \frac{\beta^3 - 31\beta^2 + 224\beta + 256}{(3\beta + 4)^2} \quad \text{for } 0 < \beta < 45
\]

\[
\frac{(10\beta + 10)^2}{(3\beta + 4)^2} > \frac{\beta^3 + 21\beta^2 + 120\beta + 100}{(3\beta + 4)^2} \quad \text{for } 1 < \beta < 78
\]