AN ANALYSIS OF SOME PROBLEMS IN ADVERTISING AND QUALITY
COMPETITION WITH SPECIAL REFERENCE TO
CONSUMER DURABLES MARKETS

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ABSTRACT OF THESIS

The thesis examines advertising behaviour and quality-setting behaviour at the firm level. In both cases economic theory is used to discover theoretically optimal behaviour patterns which may then be compared with the behaviour patterns exhibited by firms operating in the real world.

The 'neoclassical' economic model of advertising is reviewed and a general version embodying the 'marketing mix' concept is developed. Possible means of testing for optimal advertising behaviour at the firm level are discussed. The usual method of testing for optimal advertising behaviour was shown to rely on a method which provided no information about the behaviour of firms, the usual test relies on a 'snapshot' comparison of values of the firm's discretionary variables and parameters of the demand function facing the firm. An alternative method of testing is developed the use of a stock-adjustment approach in conjunction with an 'optimality rule' allows the construction of a test which views firms' behaviour. The test is applied to advertising data for the five major U.K. motor manufacturers during the period 1958-68.

The 'quality' problem is analysed at the model or variety level. The problem of defining 'quality' is discussed, and it is suggested that if 'quality' is suitably defined there will be a useful relationship between the prices and 'qualities' of a varieties of a given product. The possible theoretical bases a price-quality relationship (and hence the 'Hedonic' technique) are analysed and shown to indicate different forms for the price-quality relationship. Appropriate methods of estimating the price-quality relationship are suggested. A model of variety demand allowing for quality differences by incorporating the residuals from the estimated price-quality relationship in the demand function is proposed. Price-quality relationships and demand functions are estimated using data for U.K. passenger cars.
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A central problem in 'industrial' economics and the 'theory of the firm' is the analysis of the product differentiation activities of an individual firm taking into account that firm's existence within a particular market structure. Naturally what is meant by 'product differentiation' depends crucially on the definition of 'product' which is employed. In traditional microeconomic analysis a 'product' seems to have been defined in terms of substitutability. Two objects were the same product if they were perfect substitutes for one another. Thus a green freezer was only the same product as (an otherwise identical) white freezer if the consumer considered them perfect substitutes; i.e. if the variable 'colour' was one to which the consumer was completely indifferent. The inability of traditional analysis to tackle the realistic situation where several similar brands of essentially the same product (e.g. medium sized family saloon cars) coexist stems from the restrictive definition of 'product' which is employed. Clearly there are two dimensions to the satisfying of wants, quantity and quality. Thus a freezer is desired for its ability to preserve food, to make ice and to fit into a particular kitchen colour scheme. A theory which attempts to explain the buying and selling of almost any 'product' must take account of desires for both quantity and quality.

There is a third dimension of competition which is present in all 'imperfect' market structures, i.e. advertising. Advertising may take many forms and may be conducted in different regions and different media; i.e. there is an allocation problem as well as the problem of deciding upon
It is customary in economic analysis to deal only with the magnitude of the advertising budget, and to leave suboptimisation problems to areas more concerned with the internal management of the firm.

Thus finding the optimum position for the firm (in terms of some set of objectives) requires optimising at least four sets of discretionary variables, the prices of the products, outputs, total advertising expenditures on the different products and the levels of the different 'qualities' embodied in the products. We may reduce the number of sets of variables to three by equating sales to production, and making sales per unit of time a function of price, advertising and 'quality'. This is a reasonable approach provided we are interested only in the profits and/or sales of the firm and not in its balance sheet, that is if we are willing to ignore inventory problems.

Economic analysis nearly always deals with aggregates in some form or other. Even in the analysis of the individual firm consumers are usually treated in the aggregate. The firm is often said 'to face a demand function', that is the consumers are treated as a 'black box'. The firm stimulates consumers (and rival firms) by offering a product at some price-quality-advertising combination. How the 'black box' (i.e. consumers in the aggregate) reacts is assumed to be a function of its contents, and the mathematical expression of its responses to the various stimuli is known as the demand function. Thus in its simplest form the pattern of reactions and stimuli in a given market can be represented by the simple flow diagram:

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    FIRM A ↔ --- → CONSUMERS
      \       \     /  \\
       \     COMPETING FIRMS
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(Black arrows represent stimuli and dotted arrows responses.)

The two problems are not necessarily independent.
Firm A offers a price-quality-advertising mix to the consumers who react according to the demand function faced by firm A. The oligopoly problem is more complex. Firm A knows how consumers will react in any given situation (i.e., a situation where competing firms' product mixes are known with certainty), that is he knows the demand function which faces him. If firm A knew exactly how competing firms operated he would be able to determine his optimum product mix. Several possibilities admit themselves:

(i) each firm knows exactly how the others will react in any given situation.

(ii) each firm assumes that the other firm will follow a specific behaviour pattern in any given time period; i.e. firm A can always enter competing firms' product mixes as parameters in his demand function. In both cases (i) and (ii) the firms can be said to be operating under conditions of certainty; case (ii) behaviour being Cournot type behaviour.

(iii) each firm can assign some probability to each of the other firms' possible reactions. In this case the firm is operating under conditions of risk.

(iv) no firm knows how the others will react, but each firm knows what the others' possible reactions are. In this case the firms are said to be acting under uncertainty.

In all these cases the firms take account of the fact that they are inter-dependent, the essential conceptual difference between the cases is the amount of knowledge each firm has, or believes it has, about the actions and reactions of other firms. It is largely within this kind of framework that the problem of optimising the values of any single firm's discretionary
variables is of interest to the economist.

ADVERTISING AND THE FIRM.

Advertising is a subject of interest to both economists and management scientists. Roughly speaking economists are usually interested in the overall problem of optimising the total advertising budget, whereas the management scientist is interested in the suboptimisation problems of allocating the budget to the different media, in addition to the aggregate appropriations problem. In recent years, however, such a division has become blurred, and a literature of the 'mathematical analysis of marketing' has grown up. Management scientists, operations researchers and economists have all contributed to this literature. The existence of a widespread interest in a quantitative approach to marketing problems provides a compelling reason for studying it, since if we are seeking to explain the behaviour of firms, and if managers (and marketing men in particular) are developing this approach to business problems, then an analysis of the literature on the mathematical analysis of marketing should yield some clues to the behaviour of firms. Whether or not the rigorous, analytical approach to marketing which is provided by modern management science textbooks has yet had its influence on large company decision-making is another matter.

Advertising and promotional expenditures have become significant in the marketing of many products. The extent of advertising has been pointed out by Doyle (1968a), Kaldor and Silverman (1948), the Economists Advisory Group (1967) and Treasure (1971) amongst others. The marketing manager faces many decisions with respect to advertising. Decisions must be made with regard to the goals of advertising, the size of the overall
budget and the choice of media, appeals and copy. If these decisions are interrelated, for example if the appropriations decision is not independent of the choice of media, we may not (as economists) be justified in considering the overall advertising budget (the appropriation) as the only variable. As Treasure (1971) puts it:

'The effectiveness of an advertising campaign is a function of three variables: the creative content; the media mix; and the appropriation. Thus one cannot measure the sales effectiveness of an advertising campaign for a brand simply in terms of changes in the size of expenditure. This is a point which most economists and outside students cannot bring themselves to believe.'

Treasure probably misrepresents the views of most economists on this point. It is possible to accept his argument, but the economist usually tackles problems where it is reasonable to assume optimising behaviour at the distributional level. Nevertheless when economists look at a specific point in time then obviously there is an oversimplification in the economists usual approach to the problem of advertising. It might also be true that many economists would accept Treasure's point, but are hampered in tackling such detailed studies of advertising by lack of data. At the very minimum an economist would require data brand by brand, medium by medium, but unfortunately such disaggregated data is a rarity. Thus the firm (or the marketing department of the firm) will have to reach a large number of decisions with respect to its advertising policy, but the single decision which is of most interest to economists is the size of the appropriation.

It is of some interest to note the methods which firms in fact employ to set their advertising budgets. The Institute of Practitioners in Advertising (I.P.A.) have listed commonly employed methods of budgeting for industrial advertising. Briefly these are as follows:

(1) set the budget as a percentage of last year's turnover

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1 but see Doyle (1968a)
2 Institute of Practitioners in Advertising (1968)
(2) set the budget as a percentage of next year's anticipated sales
(3) set the budget as a percentage of profit
(4) set the budget to keep up with competitors
(5) a 'points system'
(6) the 'task method'

Methods 1, 2 and 3 may be characterised as 'rules-of-thumb'. Method 4 is only possible if there is a market 'leader' whom the firm can 'follow'. Method 3 recognises the notion that liquidity problems may exert an influence on the size of the advertising budget. Methods 1 and 2 will yield similar results where demand is inelastic and production capacity inflexible. The practice of working out appropriations by systems of points allocation suffers from the weakness that the allocation of points depends on the arbitrary judgement of an individual. The I.P.A. clearly favours the task method, which relates advertising directly to marketing objectives. The objectives of the advertising are established and the cost of achieving each objective assessed. The grand total of these costings shows the overall appropriation required. The task method means itemising and building up in contrast to the practice of starting with a lump sum appropriation and breaking it down to the various channels of advertising only so far as it will go. The relevance of economic analysis in a situation where the task method is employed depends on the objectives of the advertising. Fortunately these objectives are often stated in terms which are familiar to economics. An American survey (McNiven ed. (1969)) stated:

'For product or service advertising, the objectives most frequently mentioned are associated with marketing plans or expectations, which may be expressed in terms of volume, revenue, income, profits, earnings, market shares or other measures.'

Jastram has suggested a distinction between 'mechanistic' and 'non-mechanistic
appropriations policies. (see Jastram (1950)) A 'mechanistic' policy is one which makes the advertising appropriation depend on some other variable within the firm's operations. Once the value of that variable is 'known', the size of the advertising budget is 'known'. A 'non-mechanistic' policy encompasses all other appropriation methods. Jastram's data shows that in the 1930's two thirds of the (American) companies surveyed used mechanistic methods. A more recent British study (Taplin (1959)) confirms the popularity of mechanistic approaches. Taplin notes an exception in the case of declining brands, where firms frequently take corrective action. Mechanistic methods are most frequently employed in the advertising of established brands. The two approaches most frequently observed by Taplin were the practices of a) setting advertising appropriations as a fairly stable percentage of sales, and b) treating advertising expenditure as a discretionary deduction from profits. Despite the apparent simplicity of the decision rules described above, the problem of deciding on an advertising appropriation is a complex one.

This thesis examines the problem of advertising at the firm level. In particular it is concerned with advancing various economic models of optimal advertising behaviour at the firm level and testing these models against reality. Thus the economic models are used to generate predictions about the optimal level of the discretionary variable advertising, and these predictions are then compared with empirically observed levels of advertising. The thesis does not attempt a comprehensive survey of all possible contributions of economic analysis to advertising problems, but concentrates on extending and refining the traditional techniques of microeconomics in the field of advertising behaviour.

†see Taplin (1963), Chapter 7.
'QUALITY' AND THE FIRM.

'Quality' competition has been much more neglected as an area of study than has advertising (or price) competition. This is not altogether unexpected when we recall that the classical definition of 'product' does not allow the same 'product' to have different collections of characteristics. Nevertheless the marketing division of a firm may be faced with a greater array of problems connected with product characteristics (or 'qualities') than it is with advertising. The problems are particularly pronounced where the firm markets a consumer durable, where the physical characteristics of the product are (in part) discretionary variables in the promotional mix. Paralleling the advertising problem is the question of the optimal levels of all the different 'qualities' of a given product. Again it is possible to discover theoretically optimum levels of 'quality', and compare these to actual levels of quality chosen by firms.

However there is a (perhaps) more practical reason for applying microeconomic analysis to quality competition in consumer durables. Microeconomic theory may be able to aid us in a search for a method of predicting brand or model shares in a consumer durable market. The efficient estimation of demand curves is a necessity both for the prediction of model shares, and as a prelude to the application of economic analysis in evaluating firms' behaviour. The estimation of demand at the model level is crucial to the firm for planning and investment purposes. An efficient method of demand estimation requires that we not only take into account the prices of, and advertising expenditures on, all competing products, but the quality mix of all competing products as well. The thesis attempts to develop a method of estimating brand or model market shares on the basis of microeconomic theory.
The method is then tested using data referring to the U.K. market for small and medium sized passenger cars. The remaining chapters of the thesis are as follows:

**CHAPTER 2** presents a discussion of the economic theory of advertising in the monopoly case. The simple static profit-maximising model is extended in various directions.

**CHAPTER 3** generalises the model of chapter 2 in two important ways. Firstly a competitive industry is considered and secondly the treatment of promotional competition is generalised to allow competition of a 'marketing mix' type. Various empirical approaches to testing the models against firms' behaviour, or vice versa, are discussed. A stock-adjustment approach to the problem is proposed and developed.

**CHAPTER 4** extends the work of Chapters 2 and 3 by considering some alternative theories of firm behaviour. A particular area of concern is the role which oligopolistic interdependence plays in determining competitive strategies with respect to advertising and price ('quality' is dealt with in Chapter 6).

**CHAPTER 5** tests the stock-adjustment model of Chapter 3 using data relating to passenger cars.

**CHAPTER 6** discusses the problems raised when 'quality' is to incorporated into a model of demand. Existing work in the field is discussed and a definition or 'quality' is sought. The relationship between price and 'quality' is analysed; in particular the 'hedonic' technique proposed by Griliches (1961) is discussed. The nature and limitations of 'quality' competition in an oligopolistic industry are discussed.

**CHAPTER 7** attempts to provide theoretical underpinnings for an intuitive method of incorporating quality differences into a model of demand at the brand or model level. A method for testing optimal quality-setting behaviour at the firm level is proposed.
respect to the different marketing variables, particularly price, (above-the-line) advertising and quality.

CHAPTER 8 specifically examines pricing behaviour. A dynamic model of demand is developed from the discussion of chapters 4 and 6, an optimal pricing path is derived from it. Estimates of various market parameters are obtained and used to examine the theoretically optimal path and compare it with the actual pricing path.

CHAPTER 9 tests the theory of chapters 5 and 7. The proposed forms of the price-quality relationship are estimated using the passenger car data.

CHAPTER 10 estimates the proposed model demand functions of chapter 7.

CHAPTER 11 provides a summary and conclusions.

At some points the links between the treatment of advertising and quality competition are very strong. At the extreme theoretical level of Chapter 3 advertising and quality variables are treated identically, they are merely different elements of the 'marketing mix'. A direct descendant of this kind of treatment is the model presented in the Appendix to Chapter 7 which allows a test for optimal quality-setting behaviour to be developed; that test is very similar to the tests of optimal advertising behaviour which occur in the literature (e.g. Cowling (1972), Lambin (1970b)) and as developed in the early part of this thesis. Optimal quality-setting behaviour has received little theoretical, and virtually no, empirical attention in the past. Nevertheless treating advertising and quality variables in the same way does have drawbacks. Advertising and quality competition are in many ways different, and Chapters 4, 6 and 7 examine these differences; they are an attempt to draw out the distinguishing features of different promotional variables. For example it is suggested that advertising levels can be varied quickly whilst 'qualities' can only be altered over a longer period. Hence at some points in the thesis advertising and quality
sit easily together whilst at other times they are deliberately separated. There is nothing wrong with this approach; existing theory is developed and exploited where this fruitful, and new theory is suggested where existing theory has shortcomings. The progression (in simple terms) is from the familiar theory of advertising competition to the more general marketing mix approach and then (abstracting from all that has gone before) to some considerations of the 'theory of quality competition'.

Chapters 2, 3, 4, 6 and 7 attempt to build a strong theoretical foundation for the empirical work of the remaining chapters. That is not to say that all the empirical work is valueless in the absence of the theory advanced in the text. Only the empirical work of Chapter 5 is necessarily closely identified with a particular theoretical model advanced in this thesis. Chapters 8-10 are of particular interest even without the associated theoretical background as it is advanced here. The results reported in those chapters are susceptible to commonsense interpretations as well as to interpretation in the light of a particular body of theory, and perhaps this is a major attraction of such empirical work. The price that is paid for such general interest is perhaps that the theoretical justification for the work may appear to be less tight.

Much of the data used in this thesis was collected by Keith Cowling and John Cubbin of the University of Warwick in connection with their own research. I am extremely grateful for its use. Although data was available for the whole of the period 1956-68 inclusive my own checking of the data revealed some doubts about the accuracy of the 1956 data. The research reports here uses data referring to the period from 1957 onwards. The data consisted of the specifications, list prices and sales for individual models of cars listed in Appendix 9.1. Although some additional data on larger cars was also available it was not used. In addition to the data already mentioned the advertising shares for the major U.K. motor manufacturers and industry above-the-line advertising expenditures as used in Chapter 5 was made available to me from the same source. At points where this
data is used acknowledgement is made. Other data used was collected from primary sources as detailed in the text.

I am grateful to John Cubbin for commenting on an earlier version of Chapter 4. Dennis Leech gave me advice on the interpretation of results in Chapter 5. Norman Ireland read and commented on an earlier version of Chapter 8. I repeat my thanks to my supervisor for help received. Charles Rowley and John Cable commented on an earlier version and I have benefited greatly from their suggestions. Any errors which remain are mine alone.
NOTES TO CHAPTER 1.

1. 'Product' is here defined on an ad hoc basis, but nevertheless in a commonsense fashion.

2. Some economists might argue that some minimum level of expenditure required to ensure perfect information in the market is consistent with perfect competition.

3. 'Qualities' are to be defined later, but may be considered to be those attributes of a product for which the product itself is desired, for example passenger room in a car.

4. For an alternative classification of cases see McGuire (1964), Chapter 6. We have assumed that consumers cannot influence firms without acting through the market. In fact consumers may be able to express or 'voice' their opinions without actually 'consuming'. However it is the purpose of this thesis to study actual demand, not the protestations of consumers.

5. Clearly where there has been a tradition in a firm of appropriating advertising expenditures by rule-of-thumb (for example making advertising expenditures a proportion of immediate past sales or retained profits), and the decision is taken at a high level, then only the suboptimisation problems are of interest to the firm's marketing experts. Rule-of-thumb advertising appropriation behaviour is described by Keuhn (1961) and Taplin (1959).

6. The first contribution was, I believe, the excellent book by Bass et al. (1961).

7. As a particular example of this we may postulate a model in which the effectiveness of an advertising campaign, E, is a function of the number of units of advertising used in each of the n media.
If the prices of the n media, \( p_1, \ldots, p_n \), are given, and the size of the advertising appropriation is fixed at \( A \), then the marketing manager is faced with the problem of maximising \( E \), subject to the constraint \( \sum p_i a_i = A \), for \( i = 1, \ldots, n \). The \( n + 1 \) first-order conditions for the solution to this problem are

\[
\frac{\partial E}{\partial a_i} + \lambda p_i = 0 \quad i = 1, \ldots, n
\]

where \( \lambda \) is the Lagrangian multiplier, and

\[
\sum p_i a_i - A = 0
\]

If the effectiveness function \( E \) is homogeneous to degree \( k \), then by Euler's theorem,

\[
\sum a_i \frac{\partial E}{\partial a_i} = kE
\]

Multiplying each of the first \( n \) first-order conditions by \( a_i \), \( i = 1, \ldots, n \), and summing,

\[
\sum a_i \frac{\partial E}{\partial a_i} + \lambda p_i a_i = 0
\]

Combining with Euler's theorem we have

\[
\lambda A = kE
\]
Thus the optimum effectiveness of the campaign is uniquely determined by the size of the appropriation alone. We are therefore (in this model) justified (as economists) in treating the appropriation as the only advertising variable of interest, provided we assume optimisation at the level of the problem solved above. The parallels between the effectiveness function $E$ and production functions are obvious. Homogeneity in such a function may not be an unreasonable assumption. On the other hand a businessman might argue that effectiveness is not merely determined by the number of units of advertising purchased in each medium. Approaches to measuring advertising 'effectiveness' or 'performance' are given in Montgomery and Urban (1969), Chapter 3, Murdick (1971), Chapter 32 and Narver and Savitt (1971), Chapter 12.

8. An obvious objection to the method is that the objectives may no longer be optimal when achievement costs are taken into account.

9. If the controlling interest in a company were to change hands, and out of sheer pride the new owners insisted on product name changes which were heavily publicised, economics would not provide a suitable framework for analysing the situation.

10. The empirical work contained in the thesis refers to a consumer durable, passenger cars.

11. A similar method is employed by Cowling and Cubbin (1971a), Cowling (1972).
CHAPTER 2.

THE ECONOMIC THEORY OF ADVERTISING: THE MONOPOLY CASE.

INTRODUCTION.

This chapter reviews the application of microeconomic theory to problems connected with advertising at the firm level. In particular the classical case of pure monopoly is considered in detail. It is shown how changing the assumptions of the model alters the basic results of the simple model of advertising behaviour.

STATIC ANALYSIS.

The basic reference work in the static analysis of optimal pricing and advertising behaviour is the article by Dorfman and Steiner (1954). The demand function facing the monopolist is taken to be a continuous, (twice) differentiable function of price \( p \) and advertising expenditure \( A \);

\[
2.1 \quad q = q(p, A)
\]

Total costs assumed to be given by:

\[
2.2 \quad TC = c(q) + A
\]

where \( c(q) \) is a continuous differentiable function of output \( q \). Profits \( \Pi \) are then:

\[
2.3 \quad \Pi = p q(p, A) - c(q(p, A)) - A
\]
The two first-order conditions for the maximisation of profits are:

2.4 \[ \pi_p = q + p q_p - c'(q) q_p = 0 \]

2.5 \[ \pi_A = p q_A - c'(q) q_A - 1 = 0 \]

Elimination of the term \((p - c'(q))\) from equations 2.4 and 2.5 yields the optimising condition:

2.6 \[ -\frac{1}{q} q_p = q_A \]

Defining the price elasticity of demand \(\eta_p\) as \(\frac{p}{q} q_p\) and the advertising elasticity of demand \(\eta_A\) as \(\frac{p}{q} q_A\), 2.6 may be rewritten:

2.7 \[ -\frac{\eta_A}{\eta_p} = \frac{A}{p q} \]

2.7 is a restatement of the theorem proved by Dorfman and Steiner. It states that for a profit-maximising monopolist the optimal level of advertising intensity (the advertising-sales ratio) is equal to the modulus of the ratio of the advertising to the price elasticities of demand. Appendix 2.1 gives the second-order conditions for the problem. Equation 2.7 gives a basic result of great importance. It refers to the simplest possible case, but fortunately it is easily generalisable to other situations.

As a first generalisation consider a utility-maximising firm. Let the managerial utility function have profits \((\pi)\) and sales revenue \((R)\) as arguments. The firm is assumed to maximise its utility subject to the constraint of the accounting relation:
Form the Lagrangian function:

\[ L = U(\pi, R) + \lambda [pq - c(q) - A - \pi] \]

where \( \lambda \) is the Lagrangian multiplier and \( R = pq \). The first-order conditions for a constrained maximum on \( U \) are

\[ 0 = U_{\pi}p + U_{R} \frac{\partial R}{\partial p} + \lambda [q + p \frac{\partial q}{\partial p} - c(q) \frac{\partial q}{\partial p} - \pi p] \tag{2.10} \]

\[ 0 = U_{\pi}A + U_{R} \frac{\partial R}{\partial A} + \lambda [p \frac{\partial q}{\partial A} - c'(q) \frac{\partial q}{\partial A} - 1 - \pi A] \tag{2.11} \]

\[ 0 = U_{\pi} - \lambda \tag{2.12} \]

Equation 2.10 may be written:

\[ U_{\pi} (p - c'(q)) \frac{\partial q}{\partial p} = - [U_{R}(q + p \frac{\partial q}{\partial p}) + U_{\pi}q] \tag{2.13} \]

Equation 2.11 may be written:

\[ U_{\pi} (p - c'(q)) \frac{\partial q}{\partial A} = - [U_{R} p \frac{\partial q}{\partial A} - U_{\pi}] \tag{2.14} \]

Combining equations 2.13 and 2.14 and simplifying yields the optimisation condition:

\[ \frac{n_{A}}{n_{p}} = - \frac{A}{pq} \frac{U_{\pi}}{U_{R} + U_{\pi}} \tag{2.15} \]
We can see immediately from 2.15 that where sales revenue enters the managerial utility function as an argument the optimum advertising-sales ratio will be higher than it would be if profits were the only argument in the managerial utility function. 5,6

The result 2.15 is similar to the one which obtains in the case where the firm has some preference for advertising. For example if the firm has an expense preference for advertising, then advertising will be taken beyond the point at which it maximises its contribution to profits. We may conceive of this situation in a generalised sense by taking advertising expenditure to be an argument in the firm's utility function. Following the method of equations 2.8 - 2.15 the optimising condition can easily be shown to be

\[ \frac{nA}{np} = \frac{A}{p} \frac{U_{II}}{U_{II}} - \frac{U_{A}}{U_{II}} \]

a result which is qualitatively similar to 2.15. 7

If we feel uneasy about treating advertising in the simple way outlined above, it is easy to generalise the model by including more than one advertising variable. Let \( A_i \) be expenditure on advertising in medium \( i, i = 1, \ldots, n \). Demand is a function of the price charged and the advertising appropriations in the \( n \) media. The total advertising appropriation is \( \sum_{i=1}^{n} A_i \). Profits are given by :

\[ \Pi = p q(p, A_1, \ldots, A_n) - c(q) - \sum A_i \]

The \( n + 1 \) first-order conditions for a maximum on \( \Pi \) are :
2.18 \[ [p - c'(q)] \frac{\partial q}{\partial p} + q = 0 \]

2.19 \[ [p - c'(q)] \frac{\partial q}{\partial A_i} - 1 = 0 \] \quad i = 1, \ldots, n

2.18 and 2.19 yield the familiar optimisation condition

2.20 \[- \frac{p}{q} \frac{\partial q}{\partial p} = p \frac{\partial q}{\partial A_i} \] \quad i = 1, \ldots, n

\( p \frac{\partial q}{\partial A_i} \) (\( = \mu_i \)) is the marginal product of advertising in medium \( i \).

Summing the \( n \) conditions 2.20 we obtain:

2.21 \[- \eta_p = \frac{1}{n} \sum_{i=1}^{n} \mu_i \]

An alternative method of writing the usual Dorfman - Steiner condition (equation 2.7) is:

2.22 \[- \eta_p = p \frac{\partial q}{\partial A} = \mu \]

But in a situation where advertising is supplied by a competitive market and where the monopolist is maximising profits, the marginal products of the \( n \) media are all equal at the optimum. Conditions 2.21 and 2.22 are equivalent. Condition 2.20 may also be written:

2.23 \[- \frac{\eta_{A_i}}{\eta_p} = \frac{A_i}{pq} \] \quad i = 1, \ldots, n

where \( \eta_{A_i} \) is the elasticity of demand with respect to advertising expenditures in medium \( i \) (\( = \frac{A_i}{q} \frac{\partial q}{\partial A_i} \)). Summing the \( n \) conditions 2.23 yields the condition

2.24 \[- \frac{\sum \eta_{A_i}}{\eta_p} = \frac{\bar{A}_i}{p q} \]
An interesting alternative analysis of the monopoly situation has been proposed by Brems (1967). He recognises the point that the decision-maker has to know how far to employ each advertising medium, not just the optimum total expenditure on selling effort. Actually this question is already answered (by equation 2.23). Brems employs the techniques of input-output analysis to arrive at the same solution. Whilst the Brems approach is an interesting one it has the same major defects as the neoclassical analysis of equations 2.17 to 2.24. Demand and cost functions must still be assumed to be continuous and differentiable with respect to all dimensions of selling effort. Brems reaches the same results by a longer route with no less a restrictive set of assumptions.

A feature of many markets is the multiplicity of brands of a similar product, where each brand has a significant cross-elasticity with respect to every other brand. A monopolist may choose to sell more than one brand of a given product for several reasons. Firstly it may be true that the total size of the market is a function of the number of alternative brands available. Thus a monopolistic cigarette manufacturer might find it profitable to market filter and non-filter cigarettes in a variety of sizes, qualities and packages. By building advertising campaigns around specific brands 'brand loyalty' may be induced. Clearly where the costs of introducing new brands are low there will be a greater tendency towards proliferation. Secondly the common ownership of several brands may be an entry preventative mechanism. Consider the simplest possible Markov-switching situation where there are n brands. If x per cent of consumers tend to switch in any period and redistribute themselves equally over all other
brands, then a new brand can expect to gain \( \frac{x}{n+1} \) per cent of the market in its initial period. The greater is the number of existing brands \((n)\) the smaller is the market share a new entrant can expect to gain both initially and ultimately. Thirdly since there are diminishing returns to advertising for each brand it will be profitable at some point to stop advertising existing brands and 'launch' a new brand. Consider a firm producing and selling \( n \) brands. The prices of the \( n \) brands are \( p_j \), \( j = 1, \ldots, n \), and the advertising budgets attached to them \( A_j \), \( j = 1, \ldots, n \).

The demand function for brand \( j \) is:

\[
q_j = q_j(p_1, \ldots, p_n ; A_1, \ldots, A_n)
\]

and the total cost function facing the firm is:

\[
TC = c(q_1, \ldots, q_n) + \sum_{j=1}^{n} A_j
\]

The profit function is:

\[
\Pi = \sum_{j=1}^{n} p_j q_j(p_1, \ldots, p_n ; A_1, \ldots, A_n) - c(q_1, \ldots, q_n) - \sum A_j
\]

The problem of maximising profits is formally identical with finding the profit-maximising collusion solution for a group of oligopolists. The \( 2n \) marginal conditions for a maximum on \( \Pi \) are:

\[
0 = \sum_{k=1}^{n} p_k \frac{\partial q_k}{\partial p_j} + q_j - \sum_{k=1}^{n} \frac{\partial c}{\partial q_k} \cdot \frac{\partial q_k}{\partial p_j} \quad j = 1, \ldots, n
\]

\[
0 = \sum_{k=1}^{n} p_k \frac{\partial q_k}{\partial A_j} - \sum_{k=1}^{n} \frac{\partial c}{\partial q_k} \cdot \frac{\partial q_k}{\partial A_j} - 1 \quad j = 1, \ldots, n
\]

Consider the brand \( j \), for optimality:

\[
2.30 \quad (p_j - \frac{3c}{\alpha q_j} \frac{\alpha q_j}{\alpha p_j}) = -q_j - \sum_{k \neq j} \left( p_k - \frac{3c}{\alpha q_k} \frac{\alpha q_k}{\alpha p_j} \right)
\]

\[
2.31 \quad (p_j - \frac{3c}{\alpha q_j} \frac{\alpha q_j}{\alpha A_j}) = 1 - \sum_{k \neq j} \left( p_k - \frac{3c}{\alpha q_k} \frac{\alpha q_k}{\alpha A_j} \right)
\]

If \( \eta_{A_j} \) and \( \eta_{p_j} \) are the own advertising and price elasticities,

\[
2.32 \quad \frac{\eta_{A_j}}{\eta_{p_j}} = \frac{A_j - A_j \sum_{k \neq j} \left( p_k - \frac{3c}{\alpha q_k} \frac{\alpha q_k}{\alpha A_j} \right)}{p_j q_j - p_j \sum_{k \neq j} \left( p_k - \frac{3c}{\alpha q_k} \frac{\alpha q_k}{\alpha p_j} \right)}
\]

Since \( \frac{\alpha q_k}{\alpha A_j} < 0 \) and \( \frac{\alpha q_k}{\alpha p_j} > 0 \) for substitute goods it follows from 2.32 that at the optimum

\[
2.33 \quad \frac{A_j}{p_j q_j} < \frac{\eta_{A_j}}{\eta_{p_j}}
\]

If the goods were complementary then \( \frac{\alpha q_k}{\alpha A_j} > 0 \) and \( \frac{\alpha q_k}{\alpha p_j} < 0 \) and the inequality in 2.33 would be reversed. However, at the firm level

\[
2.34 \quad \sum_{j} \frac{A_j}{p_j q_j} > \frac{\sum \eta_{A_j}}{\sum \eta_{p_j}}
\]

The precise relationship between advertising intensity at the firm level and the individual brand elasticities will depend on (amongst other factors) the relative market shares of the \( n \) brands.

We may also generalise the Dorfman - Steiner theorem to take
account of the fact that the monopolist may be acting under conditions of uncertainty. Suppose the firm faces the stochastic demand function

\[ q = q(p, A) + u \]

where \( u \) is a continuous random variable with density \( g(u) \) and expected value 0. Further suppose that its variance is:

\[ \int_{\alpha}^{\infty} u^2 \, g(u) \, du > 0 \]

where \( \alpha \) is the lower bound of \( u \). If we know something about the attitude towards risk adopted by the firm we may be able to reach some conclusions regarding optimal policy. Horowitz (1970a) assumes the model of equations 2.35 and 2.36, and that management's attitudes towards risk are summarised in a Neumann - Morgenstern risk preference schedule expressed solely in terms of profits. The basic conclusions of Horowitz's analysis are of interest. The Dorfman - Steiner conditions are inviolate for a price-setting, quantity-taking monopolist who is linear in risk, the elasticities can simply be interpreted as expected values. In other situations (risk-aversion or risk-taking) where the optimum price and advertising levels are to be determined simultaneously, no unambiguous predictions can be made about the equilibrium conditions. Nevertheless, at any given price risk aversion encourages the firm to advertise more than it would if acting under certainty, or if the firm were linear in risk or risk-taking. The increase in the advertising budget raises the expected demand at the given price, and reduces the variance in revenues, costs and profits.
We may also note in the current context that uncertainty may sometimes be eliminated from the situation, but at some cost. Stigler (1961) provides a model in which advertising is seen as an "instrument for the elimination of ignorance". The usual marginal cost - marginal revenue equality emerges as an equilibrium condition, but in addition Stigler shows that the monopolist will advertise more the higher the "death rate". The "death rate", b say, is defined in the following terms. In any given period some fraction, b, of potential consumers will be "born" and "die" in the population. "Death" includes not only physical departure from the population (or potential market) but also forgetting the seller. Stigler suggests that b will be large for seldom purchased, expensive commodities. Thus the more 'uncertain' is the population (potential market) the greater the level of advertising the monopolist will adopt. Uemsetz (1964) rightly points out that promotional activities (including advertising) are, in part, joint products with the things they promote; information is being sold along with the advertised good. Where product quality etc. is 'regulated' by some external agency or by consumers, the activities of branding, labelling and advertising reduce the costs of policing the regulations. Promotional activity may (in part) reduce the transactions costs which the consumer would otherwise face.

over......
DYNAMIC ANALYSIS.

Static analysis has been employed to show how the Dorfman - Steiner theorem is modified when the basic model of a uniproduct profit-maximising monopolist is extended to take into account other (monopoly) situations. However it has long been recognised that there are dynamic effects of advertising. Current demand is affected by both current and past advertising expenditures, and current advertising expenditures affect both current and future demand. Several theoretical models have been advanced to take account of the dynamic effects of advertising, notably by Nerlove and Arrow (1962), Jacquemin (1971), Jacquemin and Thisse (1972), and Schmalensee (1972). We may take the Jacquemin (1971) model as being representative. The demand function facing the monopolist is assumed to be:

\[ q = q(p, A, K, t) \]

where \( A \) is current advertising expenditures and \( K \) is the level or "stock" of product differentiation, a variable which summarises the sole effect of current and past advertising on current demand. We assume

\[ \frac{\partial q}{\partial p} < 0 \]

\[ \frac{\partial q}{\partial A} > 0 \quad \frac{\partial^2 q}{\partial A^2} < 0 \]

\[ \frac{\partial q}{\partial K} > 0 \quad \frac{\partial^2 q}{\partial K^2} < 0 \]

We assume that \( K \) depreciates over time at the constant proportional rate \( \beta \); i.e. :
The firm's profit function at time $t$ is

$$
\pi(t) = pq(p, A, K, t) - c[q(p, A, K, t)] - A
$$

The firm attempts to maximise the present value of the stream of profits $(V)$:

$$
V = \int_0^\infty e^{-\rho t} \pi(p, A, K, t) \, dt
$$

given the transition equation \(2.38\). $\rho$ is the discount rate, taken to be positive and constant. Using the Maximum Principle the relevant Hamiltonian is

$$
H(t) = e^{-\rho t} \{ pq(p, A, K, t) - c[q(p, A, K, t)] - A + \lambda(t)(\alpha A - \beta K) \}
$$

The first-order conditions for a maximum are:

$$
\frac{\partial H}{\partial p} = (p - c'(q)) \frac{\partial q}{\partial p} + q = 0
$$

$$
\frac{\partial H}{\partial A} = (p - c'(q)) \frac{\partial q}{\partial A} - 1 + \alpha \lambda(t) = 0
$$

2.42 and 2.43 yield the optimising condition

$$
\frac{A}{pq} = -\frac{\eta_A}{\eta_p(1 - \alpha \lambda)}
$$

where $\eta_A = \frac{A}{q} \frac{\partial q}{\partial A}$ and $\eta_p = \frac{p}{q} \frac{\partial q}{\partial p}$. $\frac{\eta_A}{1 - \alpha \lambda}$ may be considered to be the long-run advertising elasticity of demand. The long-run advertising elasticity is greater than the short-run advertising

\cite{Intriligator1971}.
elasticity when $\alpha < 1$, and the optimum advertising intensity is less than that predicted by the static analysis. When there is no long-run effect of advertising ($\lambda = 0$), (2.44) becomes the static Dorfman - Steiner condition.

This analysis takes no account of the fact that the long-run price elasticity may not be the same as the short-run price elasticity. One way in which this might be taken into account is as follows. Define a variable $S$ which summarises the sole effect of current and past sales on current demand. The sign of $\frac{3q}{3S}$ is ambiguous, for some habit-forming products we clearly expect $\frac{3q}{3S} > 0$, but for some durable goods we might expect $\frac{3q}{3S} < 0$. Knowing the transitional equation $\dot{S} = f(R, S)$ where $R$ is current sales, we could find a maximum for the present value of current and future profits. There would however be two state variables ($K$ and $S$) and two transitional equations. The solution will be in stage form. In the first stage the firm will devote its resources to building up either sales goodwill ($S$) or advertising goodwill ($K$). In the second stage the other type will be built up. This process will be repeated until eventually the firm is able to maintain both types of goodwill at, or near, the optimum. It is interesting to note that this pattern implies that at any point in time the firm may be pursuing optimising behaviour, but that advertising intensity can range above or below the predicted optimum depending on the stage the firm is going through.

Finally we may note that the situation where there are 'bandwagon' or 'snob' effects in demand has been analysed by Eeckhoudt (1972) and Ireland (forthcoming). If demand in period $t$ is given by:

\[
q(t) = e^{x(t)} q[p(t), A(t)]
\]
then we have a bandwagon effect if \( x(t) > 0 \) and a snob effect if \( x(t) < 0 \). If \( x(t) \) is some function of past sales, then clearly past pricing and advertising policies affect current demand. In this situation we would not expect the Dorfman - Steiner theorem to hold, and this is indeed the conclusion reached by Ireland.

Advertising may also raise barriers to entry. The monopolist (or oligopolist for that matter) must concern himself not only with existing competition but also potential competition by raising barriers to entry in the industry. Thus the monopolist would adopt a higher level of current advertising than suggested by the model given above when the potential competition reducing effect of promotional expenditure is taken into account. Advertising may be seen as reducing the probability of new entry given the existing market structure (as suggested by Baron (1973)), or raising the barriers to entry, i.e. raising the 'limit price'. (see Schupack (1972)).
APPENDIX 2.1.

The Second-Order Conditions for the Dorfman - Steiner Theorem.

The demand function facing the monopolist is taken to be a continuous, (twice) differentiable function of price (p) and the advertising appropriation (A) :

\[ q = q(p, A) \]

The total cost function is :

\[ TC = c(q) + A \]

where \( c(q) \) is continuous and twice differentiable.

Profits (\( \Pi \)) are :

\[ \Pi = pq(p, A) - c[q(p, A)] - A \]

The first-order conditions for a stationary value of \( \Pi \) are :

\[ \Pi_p = 0 = q + p \frac{\partial q}{\partial p} - c'(q) \frac{\partial q}{\partial p} \]

\[ \Pi_A = 0 = p \frac{\partial q}{\partial A} - c'(q) \frac{\partial q}{\partial A} - 1 \]

For \( \Pi \) to be a relative maximum :

\[ \Delta = \Pi_{pp} \Pi_{AA} - (\Pi_{pA})^2 > 0 \]
where

\[
\begin{align*}
\pi_{pp} &= p \frac{a^2 q}{a p^2} - c'(q) \frac{a^2 q}{a p^2} + 2 \frac{a q}{a p} - c''(q) \left( \frac{a q}{a p} \right)^2 \\
\pi_{AA} &= p \frac{a^2 q}{a A^2} - c'(q) \frac{a^2 q}{a A^2} - c''(q) \left( \frac{a q}{a A} \right)^2 \\
\pi_{PA} &= p \frac{a^2 q}{a p a} - c'(q) \frac{a^2 q}{a p a} + \frac{a q}{a A} - c''(q) \frac{a q}{a A} \frac{a q}{a p}
\end{align*}
\]

Substituting for \([p - c'(q)]\) from equation 2.1.4 in 2.1.7 we have:

\[
\frac{a q}{a p} \pi_{pp} = - q \frac{a^2 q}{a p^2} + 2(\frac{a q}{a p})^2 - c''(q)(\frac{a q}{a p})^3
\]

Substituting for \([p - c'(q)]\) from equation 2.1.5 in 2.1.8 we have:

\[
\frac{a q}{a A} \pi_{AA} = \frac{a^2 q}{a A^2} - c''(q)(\frac{a q}{a A})^3
\]

Since \(\frac{a q}{a A} > 0\) and \(\frac{a q}{a p} < 0\), two necessary second-order conditions for a maximum of \(\pi\) are:

\[
\begin{align*}
\frac{a q}{a p} \pi_{pp} &> 0 \\
\frac{a q}{a A} \pi_{AA} &< 0
\end{align*}
\]

Conditions 2.1.12 are:

\[
\begin{align*}
- q \frac{a^2 q}{a p^2} + 2(\frac{a q}{a p})^2 - c''(q)(\frac{a q}{a p})^3 &> 0
\end{align*}
\]
2.1.14 \[ \frac{\partial^2 q}{\partial A^2} - c''(q) (\frac{\partial q}{\partial A})^3 < 0 \]

If marginal costs are constant or rising \((c''(q) \geq 0)\), 2.1.13 and 2.1.14 are satisfied if

2.1.15 \[ \frac{\partial^2 q}{\partial p^2} < 0 \]
2.1.16 \[ \frac{\partial^2 q}{\partial A^2} < 0 \]

Condition 2.1.16 states that there must be diminishing returns to advertising expenditure.

If marginal costs are falling \((c''(q) < 0)\), additional conditions will have to be met to ensure a relative maximum. These conditions are of a complex nature.\(^{15}\)
NOTES TO CHAPTER 2 AND APPENDIX 2.1.

1. A useful reprinting of the article, with discussion and alternative proofs of the Dorfman-Steiner theorems is contained in Bass et al. (1961).

2. "Managerial" theories of the firm (such as those postulated by Baumol (1959), Marris (1963), (1964), and Williamson (1963), (1964) ) have been reformulated as problems of utility maximisation subject to constraints. The necessary links between the proposed arguments in the utility function and theories of the sources of managerial utility have been developed by these and other writers.

3. In order for the second-order conditions to be satisfied the utility function $U(\pi, R)$ would have to yield indifference curves convex to the origin. Azariadis et al. (1972) have shown that "all separable utility functions provided that both marginal utilities, $U_\pi$ and $U_R$, are non-negative, non-increasing functions of $\pi$ and $R$ respectively" yield the desired indifference curves. The first writer to apply a partial utility approach to the sales maximisation hypothesis seems to have been Peston (1959).

4. Peel (1973) considers the less general case of a firm maximising utility where only profits and output are arguments. The case considered by Peel produces the rather odd-looking (but easily shown) result that inclusion of output as an argument in the utility function has no effect on the size of the optimum advertising-sales ratio. Output is, however, higher and profit smaller than in the pure profit maximising case.

5. Although this need not always be the case. In a three-way discussion (Kafoglis (1970), Hawkins (1970) and Haveman and De Bartolo (1970) ) of the strict Baumol model it was shown that under some conditions revenue maximisation could lead to higher production expense and lower price than profit maximisation. However this was not put forward as
a common case, nor was a partial utility approach used. Baumol's own views on revenue maximisation are put forward in Baumol (1958), (1959) and (1961).

6. We may also demonstrate what happens when payments to one factor of production also enters the utility function (as say payments to managers might). Suppose that the only arguments of the utility function are profits ($\Pi$), sales revenue ($R$) and payments to factor $X_j, p_j X_j$. Utility is then maximised subject to the constraints $q_S = q(X_1, \ldots, X_n)$ the production function, $q_D = q(p, A)$ the demand function and $\Pi = pq - \sum_{i=1}^{n} p_i X_i - A$, the account identity. Equilibrium ($q_S = q_D$) is assumed. The Lagrangian function is

$$L = U(\Pi, R, p_j X_j) + \lambda_1[q - q(X_1, \ldots, X_n)] + \lambda_2[q - q(p, A)] + \lambda_3[\Pi - pq + \sum p_i X_i + A]$$

The equilibrium conditions are:

$$0 = U_{\Pi} + \lambda_3$$

$$0 = U_R \frac{2R}{2p} - \lambda_2 \frac{2q}{2p} - \lambda_3 \left[p \frac{2q}{2p} + q\right]$$

$$0 = U_R \frac{2R}{2A} - \lambda_2 \frac{2q}{2A} + \lambda_3 \left[1 - p \frac{2q}{2A}\right]$$

$$0 = p_j U_{p_j X_j} - \lambda_1 \frac{2q}{2X_j} + \lambda_3 p_j$$

$$0 = - \lambda_1 \frac{2q}{2X_k} + \lambda_3 p_k \quad k = 1, \ldots, n \quad k \neq j$$

Eliminating the Lagrangian multipliers from the equilibrium conditions we have:
i.e. the factor $j$ is paid a price higher than would normally be the case. A feature of the managerial discretion models discussed by Williamson (1964, Chapter 4) is that the firm sets output and price in accordance with the conventional $MC = MR$ rule, but costs are 'inflated' by the higher usage of, and payments to, staff. Thus some of the available possible profit is diverted from owners' to managers, who receive it in terms of higher salaries and discretionary payments. Extension of the model here will confirm the Williamson results.

7. Firms might have expense preferences for advertising because advertising can create a barrier to entry; a firm may be able to consolidate or maintain its market position by advertising heavily. Alternatively managers may like to associate themselves with a widely known product; or the association of a given firm with a highly advertised product may make recruitment easier. In essence firms may have an expense preference for advertising because they feel that advertising yields greater sevurity.

8. \( \frac{\partial q}{\partial A_i} \) is also the marginal productivity of advertising in medium $i$ since the marginal cost of advertising is 1. We may also note that the model assumes that 'advertising' in the different media can be treated as independent activities. If there is interdependence between advertising activity in the different media then we must include interaction terms of the form \( \frac{\partial q}{\partial A_i} \frac{\partial A_j}{\partial A_i} \) \( i = 1, \ldots, n; j = 1, \ldots, n; j \neq i \); in 2.19. If we wish to relax the assumption of independence of
advertising activity, then an example of an appropriate model is given in note 7 to Chapter 1. Alternatively we may include the interaction terms in the model or equations 2.17 to 2.24. Such a change would necessitate the redefinition of the advertising elasticities $n_{A_i}$ to take account of the interaction terms.

9. $n$ may or may not be the optimum number of brands.

10. Thus we are concerned with the model due to Horowitz. See Horowitz (1970a) and (1970b).

11. If one believes that 'large' firms (i.e. those with some market power) are likely to be conservative, that is risk-averting, then the analysis is of little practical help. Much of the literature on large firms and the objectives of their managers would seem to support the notion that the management of such companies are likely to act in a conservative risk-averting fashion. (see for example Marris (1964) and his discussion of the 'security motive' of managers and Fellner (1949) on the asymmetry of rewards to managers.)

12. Increased advertising raises the probability of selling what has been produced.

13. The Horowitz analysis assumes a cost function which is continuous and differentiable. Many economists feel that a linear programming approach (assuming discrete production processes) is more realistic in many cases. Liviatan (1971) has examined a linear programming model in the context of uncertain demand. The firm is assumed to face a random exogeneous demand requirement which is specified by some known and bounded density function. Liviatan's major conclusion is that uncertain demand leads to a 'smoothing' of the isoquants, the production process in effect becomes continuous.
14. Ireland (forthcoming) in fact chooses the function

\[ X(t) = \int_0^t \alpha[p(t), A(t)] \, dt \]

that is \( X(t) \) depends on cumulative past sales.

15. The 'proof' given by Bass et al. (1961) that the first-order conditions always give a relative maximum is false. They correctly derive an expression for \( \pi_{pp} \), but then in effect partially differentiate the equation \( \pi_p = 0 \) and substitute the resulting expression into the (correctly) derived expression for \( \pi_{pp} \), showing that \( \pi_{pp} = 0 \). Continued application of this method would yield \( \Delta = 0 \), in which case the method for determining the nature of the stationary value fails.
THE ECONOMIC THEORY OF ADVERTISING: COMPETITIVE SITUATIONS.

INTRODUCTION.

Chapter 2 considered the economic analysis of advertising applied to the monopoly case. This chapter extends that analysis to industries where the number of competitors is small enough to ensure that some degree of mutual interdependence exists, i.e. oligopolies. Not all the situations of Chapter 2 are generalised since it is clear what the end results of those generalisations would be.

A GENERALISED STATIC MODEL.

We assume that the demand function facing the firm may be written as the definitional equation

\[ q = s \cdot Q \]  

where \( s \) is the market share of the firm and \( Q \) is industry demand. We further assume that:

\[ s = s(m, M) \]

and

\[ Q = Q(m, M, Z) \]

where \( m \) is an \( n \)-vector of marketing variables controlled by the firm.
M is an n-vector of marketing variables controlled by rival firms.

Z is a vector of variables exogeneous to the firm which influence industry demand.

Thus m includes the firm's price and the advertising expenditures in the different media, as well as other possible items of promotional expenditure. M refers to the values of marketing variables adopted by rival firms, and hence the marketing share function s(m, M) is a function of the prices and advertising allocations of all firms in the market. The implicit assumption of equations 3.1 to 3.3 is that the vector of variables Z does not affect the 'balance' between the firms in the market. Thus if there is a rise in real incomes, we can expect the overall size of the market to increase (if the goods in the market are superior), but the respective market shares to remain the same, ceteris paribus. Although this is a frequently employed assumption, and a very useful one, it is as well to remember that it has been made. It might be violated in a market where the good is superior and the different brands cover a wide range of prices and the change in Z involves an increase in income. In this case we might expect a substitution of dearer for cheaper brands, provided dearer brands were of a higher quality. Thus the demand function facing the firm is:

\[ q = s(m, M) Q(m, M, Z) \]

Suppose that the competitive interaction between firms can be expressed by the vector of marketing reaction functions:

\[ M = f(m) \]
Often the restriction is placed on reaction functions of type 3.5 that variations in one promotional variable attract reactions in terms of that variable only; the rivals are assumed to react to a price change by changing price and to an advertising increase by altering their own advertising budgets etc. That restriction need not be placed on reaction functions 3.5. Let the total cost function of the firm be:

\[ TC = c(q, m) \]

Profits are then:

\[ \Pi = pq - c(q, m) \]

and the m first-order conditions for a profit maximum are:

\[ 3.8 \quad 0 = \frac{\partial \Pi}{\partial p} = q + p \frac{\partial q}{\partial p} - c'(q) \frac{\partial q}{\partial p} \]

\[ 3.9 \quad 0 = \frac{\partial \Pi}{\partial m} = p \frac{\partial q}{\partial m} - c'(q) \frac{\partial q}{\partial m} - \frac{\partial c}{\partial m} \]

which yield the \((m - 1)\) optimisation conditions:

\[ 3.10 \quad - \frac{n_m}{n_p} = \frac{m \frac{\partial c}{\partial m}}{pq \frac{\partial q}{\partial m}} \]

where \(n_m\) are the elasticities of demand with respect to the m marketing elements (\(= \frac{m \frac{\partial q}{\partial m}}{q \frac{\partial q}{\partial p}}\)) and \(n_p\) is the price elasticity of demand (\(= \frac{p \frac{\partial q}{\partial p}}{q \frac{\partial q}{\partial p}}\)). In the case where m is measured in terms of expenditure on a promotional activity,\( \frac{\partial c}{\partial m} = 1 \) as in the usual advertising case. 3.10 is merely a restatement of the usual Dorfman - Steiner theorem, but now the elasticities \(n_m\) and \(n_p\) take into account market-share effects, aggregate market effects...
and the \((m \times n)\) competitive interaction effects which take place. (In the case where we are considering the one elasticity \(\eta_m\) (rather than the vector of elasticities \(\eta_m\)) there are only \(n\) possible reactions to take into account.) The elasticities \(\eta_m\) can be broken down into these effects as follows:\(^3\) since

\[
3.11 \quad q = s(m, M) Q (m, M, Z)
\]

then

\[
3.12 \quad \frac{\partial q}{\partial m} = \frac{\partial s}{\partial m} Q + \frac{\partial s}{\partial M} \frac{\partial M}{\partial m} Q + \frac{\partial Q}{\partial m} s + \frac{\partial Q}{\partial M} \frac{\partial M}{\partial m} s
\]

If \(D_m\) is a \((n \times n)\) diagonal matrix of the elements of vector \(n\), then multiplying both sides of 3.12 by \(D_m/q\) yields the equation:

\[
3.13 \quad \frac{D_m}{q} \frac{\partial q}{\partial m} = \frac{\partial s}{\partial m} D_m Q + \frac{\partial s}{\partial M} \frac{\partial M}{\partial m} D_m Q + \frac{\partial Q}{\partial m} D_m s + \frac{\partial Q}{\partial M} \frac{\partial M}{\partial m} D_m s
\]

or

\[
3.14 \quad \frac{D_m}{q} \frac{\partial q}{\partial m} = \frac{D_m}{s} \frac{\partial s}{\partial m} + \frac{D_m}{Q} \frac{\partial Q}{\partial m} + \frac{\partial M}{\partial m} \left[ \frac{D_m}{s} \frac{\partial s}{\partial m} + \frac{D_m}{Q} \frac{\partial Q}{\partial m} \right]
\]

or in matrix notation

\[
3.15 \quad \eta_{q,m} = [I + R][\eta_{Q,m} + \eta_{s,m}]
\]

where

- \(I\) is the identity matrix
- \(R\) is the matrix of reaction terms \(\frac{\partial M}{\partial m}\)
- \(\eta_{q,m}\) is the elasticity of demand with respect to \(m\)
- \(\eta_{Q,m}\) is the elasticity of aggregate (market) demand with respect to \(m\)
- \(\eta_{s,m}\) is the elasticity of market share with respect to \(m\).
In the case of one particular dimension of marketing effort, $m_i$, we may write

$$\eta_{q,m_i} = \left[ E_i + \frac{\partial M}{\partial m_i} \right] [\eta_{s,m_i} + \eta_{Q,m_i}]$$

where $E_i$ is the vector $[0, \ldots, 1, \ldots, 0]$ with 1 in the $i^{th}$ column, and $\frac{\partial M}{\partial m_i}$ is a row vector of interaction terms. When the marketing variable under consideration is price, $3.16$ becomes

$$\eta_{q,p} = \left[ E_i + \frac{\partial M}{\partial p} \right] [\eta_{s,p} + \eta_{Q,p}]$$

$3.15$ and $3.16$ may be simplified in a number of commonly encountered cases. Consider equation $3.16$ in the following cases:

(i) Monopoly:
$$q = Q, \quad \frac{\partial M}{\partial m_i} = 0 \quad \text{for all } i$$

$3.16$ becomes

$$\eta_{q,m_i} = \eta_{Q,m_i}$$

(ii) Cournot oligopoly where market size is fixed:
$$\frac{\partial M}{\partial m_i} = 0, \quad \eta_{Q,m_i} = 0 \quad \text{for all } i$$

$3.16$ becomes

$$\eta_{q,m_i} = \eta_{s,m_i}$$

(iii) Cournot oligopoly:
$$\frac{\partial M}{\partial m_i} = 0, \quad \text{for all } i$$

$3.16$ becomes
3.20 \[ n_{q,m_i} = n_{s,m_i} + n_{Q,m_i} \]

(iv) Oligopoly where market size is fixed;

\[ n_{Q,m_i} = 0 \quad \text{for all } i \]

3.16 becomes

3.21 \[ n_{q,m_i} = \left[ 1 + \frac{aM}{\bar{m}_i} \right] n_{s,m_i} \]

(v) Oligopoly;

3.16 remains.

Equations 3.16 and 3.17 in conjunction with equation 3.10 yield a general statement of the Dorfman - Steiner theorem where the firm is assumed to be a profit-maximiser.

A DYNAMIC MODEL OF COMPETITIVE ADVERTISING BEHAVIOUR.

When we attempt to formulate a dynamic model of marketing behaviour, that is we recognise that current demand is affected by both current and past levels of marketing variables, we are forced to sacrifice much of the generality of the model outlined in equations 3.1 to 3.17. We are simply not able to cope with the situation where several variables are to be 'controlled' through time. Instead we concentrate on one such variable, advertising expenditures. The problem then is to discover what the optimum level of current advertising expenditures should be, given that advertising has a 'stock' effect. One of the first contributions to the area was that of Vidale and Wolfe (1957), however the work was designed for more limited ends. Like that of the Vidale and Wolfe article, the Sasieni (1971) approach is designed for more 'practical' ends. Nevertheless the Sasieni article illustrates an earlier contention, that where the sub-
ject of advertising is concerned, management scientists and economists are not necessarily distant from one another. Both models attempt to determine what the optimal rate of advertising expenditure should be when the decision variables are linked by the relationship \( \dot{S} = g(S, A, t) \) where \( S = S(t) \) is the sales rate at time \( t \), and \( A = A(t) \) is the rate of advertising expenditures at time \( t \). The classic economic contribution is that of Nerlove and Arrow (1962). Using a variational approach they were able to show that at the optimum (for a monopolist)

\[
3.22 \quad \frac{K}{pq} = -\frac{n_A}{n_P(d+\delta)}
\]

where \( K \) is advertising 'goodwill', i.e. a variable which summarises the effects of current and past advertising outlays on demand, \( n_A \) and \( n_P \) the (short-run) elasticities with respect to advertising and price, \( \rho \) is the discount rate and \( \delta \) is the rate at which 'goodwill' depreciates over time. Several writers have since presented generalisations of the Nerlove - Arrow type of model. In particular control theory has been applied to the problem. The Nerlove - Arrow model has been criticised on the grounds that if the actual stock of goodwill is below the optimum, then the optimal policy is to advertise infinitely for an instant, that is, the optimal policy is an instantaneous jump. In a practical sense the Nerlove - Arrow model tells us nothing about the path the firm should adopt to reach the long-run optimum, but merely outlines the nature of the long-run optimum.

Consider a variant of the model employed by Jacquemin (1971), (1972a). Let the demand function facing the firm be written

\[
3.23 \quad q = q(p, p_r, A, A_r, K, t)
\]
where $p$ is the price charged

$p_r$ is the vector of rivals' prices

$A$ is current advertising expenditures by the firm

$A_r$ is the vector of rivals' current advertising expenditures

$K$ is the stock of goodwill, a variable summarising the effect of past advertising outlays on current demand

with

$$\frac{\partial q}{\partial p} < 0 \quad \frac{\partial q}{\partial p_r} > 0$$

$$\frac{\partial q}{\partial A} > 0 \quad \frac{\partial^2 q}{\partial A^2} < 0$$

$$\frac{\partial q}{\partial A_r} < 0$$

$$\frac{\partial q}{\partial K} > 0 \quad \frac{\partial^2 q}{\partial K^2} < 0$$

$$\frac{\partial^2 q}{\partial K \partial A_r} < 0$$

Oligopoly theory suggests that

3.24 \hspace{1cm} p_r = g(p) \\
\hspace{1cm} A_r = f(A)

with \hspace{1cm} \frac{\partial A_r}{\partial A} > 0

$$\frac{\partial p_r}{\partial p} > 0$$

The level of goodwill, $K$, is assumed to depreciate over time at the constant rate $\delta$ such that

3.25 \hspace{1cm} K = A - \delta K, \quad \delta > 0
The level of goodwill in the initial period is known \( (K_0) \). The firm is assumed to maximise present-valued profits \( \int_0^\infty e^{-\rho t} \pi(t) \, dt \) subject to the transition equation 3.25. \( \rho \) is the discount rate, taken to be positive and constant. Using the Pontryagin Maximum Principle\(^*\) we define the Hamiltonian function, \( H(t) \), as

\[
H(t) = e^{-\rho t} \{ \pi(t) + \lambda(t)(A - \delta K) \}
\]

where

\[
\pi(t) = pq(p, p_r, A, A_r, K, t) - c(q) - A
\]

and \( \lambda(t) \) may be interpreted as the shadow price of a unit of goodwill at time \( t \). The first-order conditions for maximising \( H(t) \) are

\[
\begin{align*}
\frac{\partial H}{\partial p} &= 0 \\
\frac{\partial H}{\partial A} &= 0 \\
\frac{\partial H}{\partial K} &= -\lambda e^{-\rho t}
\end{align*}
\]

Conditions 3.28 and 3.29 yield the equations

\[
\begin{align*}
(p - c'(q)) \left( \frac{\partial q}{\partial p} + \frac{\partial q}{\partial p_r} \frac{\partial p_r}{\partial p} \right) &= -q \\
(p - c'(q)) \left( \frac{\partial q}{\partial A} + \frac{\partial q}{\partial A_r} \frac{\partial A_r}{\partial A} \right) &= 1 - \lambda(t)
\end{align*}
\]

Define \( \eta_A \), the advertising elasticity of demand taking into account reaction, as \( \frac{A}{q} \left[ \frac{\partial q}{\partial A} + \frac{\partial q}{\partial A_r} \frac{\partial A_r}{\partial A} \right] \) and \( \eta_p \), the price elasticity of demand taking into account reaction, as \( \frac{p}{q} \left[ \frac{\partial q}{\partial p} + \frac{\partial q}{\partial p_r} \frac{\partial p_r}{\partial p} \right] \), then equations 3.31 and 3.32 yield

\[
\frac{A}{pq} = -\frac{\eta_A}{\eta_p(1-\lambda)}
\]

\(^*\) see Intriligator (1971).
3.33 is a condition which relates to the optimum level of current advertising expenditures. We may also derive a condition for the optimum stock of goodwill, $K$. Condition 3.30 yields the equation

$$\frac{\partial H}{\partial K} = \rho e^{-\rho t} - \lambda e^{-\rho t}$$

where

$$\frac{\partial H}{\partial K} = e^{-\rho t} \left[ (p - c'(q)) \frac{\partial q}{\partial K} - \lambda \delta \right]$$

3.34 and 3.35 yield

$$\frac{\partial q}{\partial K} = \frac{\lambda (\rho + \delta) - \lambda}{p - c'(q)}$$

Defining the goodwill elasticity of demand ($n_K$) as $\frac{K}{q} \frac{\partial q}{\partial K}$, 3.36 and 3.31 yield

$$\frac{K}{pq} = - \frac{n_K}{\eta_p} \frac{1}{\lambda (\rho + \delta) - \lambda}$$

In the special case where $\lambda$ (the shadow price of a unit of goodwill) is equal to the price of a unit of advertising, $p_a$, 3.37 may be written

$$\frac{K}{pq} = - \frac{n_K}{\eta_p} \frac{1}{p_a (\rho + \delta) - p_a}$$

3.38 is the result given by Tsurumi (1972). One advantage of using the control theory approach is that it allows us to say something about the path the firm should adopt to reach the optimum. Indeed, we could solve the model in order to obtain the path of $K$. Jacquemin (1972a) shows that the optimal policy when $K(0) < K^*$, where $K^*$ is the desired level of $K$, is to advertise more heavily in the initial periods and gradually decrease the level of $A(t)$ as $K^*$ is approached.
A STOCK-ADJUSTMENT MODEL OF ADVERTISING EXPENDITURES.

The results 3.37 and 3.38 may be considered representative of the results derived from all similar models of firm behaviour with respect to advertising. All the models derive conditions which relate some measure of the firm's behaviour (usually the goodwill intensity, $\frac{K}{pq}$, or the advertising intensity, $\frac{A}{pq}$) to observable parameters of the firm's situation (elasticities, the price of advertising, the discount rate etc.). One of the problems of such conditions is the difficulty of measuring the stock of advertising goodwill $K$. We may avoid this problem by incorporating the results of 3.37 and 3.38 into a stock-adjustment model of advertising behaviour. Assume that the firm 'knows' what the optimum level of $K (= K^*)$ is, and that $K^*$ is defined by equation 3.37 or equation 3.38, or the optimality condition resulting from some other model of advertising behaviour. Suppose that $K_t^*$ can only be approached in the simple fashion

$$3.39 \quad \Delta K_t = \beta \left[ K_t^* - K_{t-1} \right]$$

where $\Delta K_t$ is the net goodwill investment in period $t$ and $0 < \beta < 1$. Current advertising expenditures are given by the discrete version of equation 3.25, i.e.

$$3.40 \quad A_t = \Delta K_t + \delta K_{t-1} \quad \text{with} \quad 0 < \delta < 1$$

Combining 3.39 and 3.40 gives

$$3.41 \quad A_t = \beta K_t^* + (\delta - \beta) K_{t-1}$$

We 'know' what $K_t^*$ is, but we have to measure $K_{t-1}$. 

Given the identity

3.42 \[ K_t = \Delta K_t + K_{t-1} \]

and equation 3.40 we obtain

3.43 \[ K_t = A_t + (1 - \delta) K_{t-1} \]

and hence

3.44 \[ K_{t-1} = A_{t-1} + (1 - \delta) K_{t-2} \text{ etc.} \]

Successive applications of 3.44 yields the equation

3.45 \[ K_{t-1} = A_{t-1} + (1 - \delta) A_{t-2} + (1 - \delta)^2 A_{t-3} + \ldots \]
\[ \ldots + (1 - \delta)^L A_{t-L} + (1 - \delta)^L K_{t-L-1} \]

where period t-L is the first period for which data is available. 3.45 may be more compactly written

3.46 \[ K_{t-1} = \sum_{i=1}^{L} (1 - \delta)^{i-1} A_{t-i} + (1 - \delta)^L K_{t-L-1} \]

Substituting from 3.46 in 3.41 we have

3.47 \[ A_t = B K_t^* + (\delta - \beta) \left\{ \sum_{i=1}^{L} (1 - \delta)^i A_{t-i} + (1 - \delta)^L K_{t-L-1} \right\} \]

Application of the Koyck transformation to 3.47 yields, in period t-1,

3.48 \[ (1 - \delta) A_{t-1} = B K_{t-1}^*(1 - \delta) + (\delta - \beta) \sum_{i=1}^{L} (1 - \delta)^i A_{t-i} \]
\[ (\delta - \beta)(1 - \delta)^L K_{t-L-1} \]

Subtraction of 3.48 from 3.47 yields

* See Koyck (1954).
From equation 3.38 we have

\[ K_t^* = - \frac{n_K}{n_p} R_t \frac{1}{p_{at}(\rho+\delta)-p_a} \]

where \( R_t = p_t q_t \). Substitution of 3.50 in 3.49 yields

\[ A_t = \beta c_1 \frac{R_t}{p_{at}} \frac{1}{(\rho+\delta)-p_a} - \beta(1-\delta)c_1 \frac{R_{t-1}}{p_{at-1}} \frac{1}{(\rho+\delta)-p_a} + \]

\[ + (1-\delta)A_{t-1} \]

where \( c_1 = - \frac{n_K}{n_p} \), i.e. we assume that the elasticities \( n_K \) and \( n_p \) are constant through time. 3.51 can be estimated by 'gradient' methods, and this is done in Chapter 5. The stock-adjustment model can be used in conjunction with other optimality rules for advertising behaviour, provided a technique is available for estimating the resulting equation. Form 3.51 has the advantage that one of the independent variables is the price of advertising, which is known. Alternative specifications which include the terms \( \lambda(t) \) or \( \dot{\lambda} \) in place of \( p_a(t) \) and \( \dot{p}_a \) cannot be so easily estimated.

Equation 3.51 cannot be seen as an advertising appropriations relationship. Rather 3.51 is an expression of the form of behaviour which would be observable, ex post, if we could identify a firm which conformed to the assumptions of the model, or acted so that the end result were that it conformed to the assumptions of the model. It could be used as an advertising appropriations relationship provided the firm had some independently or simultaneously derived estimates of \( R_t \) and \( p_{at} \). A possible simultaneous model would contain both equations 3.51 and a function for \( R_t \).
based on the demand function facing the firm. Ex post, 3.51 may be consistent with the firm following other patterns of behaviour. In particular 3.51 may possibly be observed where the firm adopts rule-of-thumb behaviour with regard to setting advertising appropriations. In this latter case, however, the parameters of the model may not fall in the expected ranges derived from the profit-maximising model. 16

A potentially serious criticism of the model outlined in equations 3.23 to 3.51 is raised in footnote 13 to this chapter. Equation 3.39 can be said to be a constraint on the firm's advertising behaviour and that as such it should be recognised in the Hamiltonian 3.26. If the firm is unaware of a constraint such as 3.39 then no objection to the model is raised. On the other hand recognition of 3.39 by the firm would lead it to set artificially high target values of \( K_t \) for itself; in falling short of the 'high' target value of \( K \) it could reach its truly desired target value as given by condition 3.38. The artificially high target value of \( K \) the firm would choose is \( \frac{1}{\beta}(K^* - K_{t-1}) + K_{t-1} \). The end result of such a choice of target would be the same as having \( \beta = 1 \) and sticking to the original \( K^* \) as target. The firm can follow such a procedure only if the setting of artificially high targets is costless; if there are costs of adjustment attached to changes in the advertising stock (other than the costs of the advertising itself) then the firm is no longer 'free' to adopt such a strategy. The appropriate theoretical approach to such a problem would be to incorporate adjustment costs into the control model of equations 3.23 to 3.38.

Alternatively, 3.39 may not be an active constraint at all. Adjustment to new equilibrium levels of \( K_t \) may be possible within a period; i.e. \( \beta = 1 \), and the lagged advertising term in 3.51 disappears. Only if the price of advertising is also constant, i.e. \( p_{at} = p_{at-1} \) and hence \( \beta_a = 0 \), will 3.51 become a simple equation linking current advertising expenditures to
the change in sales and lagged sales. Use of the stock-adjustment procedure permits the derivation of 3.51 which retains most of its interest and information even if $\beta = 1$. Hence if the firm believes that $\beta = 1$, or $\beta$ is actually equal to unity no problems are raised by the use of the stock-adjustment procedure in conjunction with the control model of equations 3.23 to 3.38. For 3.51 to be consistent with the control model we would require $\beta = 1$ (or $0 \leq \beta \leq 1$ if the firm only believes that $\beta = 1$); $0 \leq \delta \leq 1$; $0 \leq \rho \leq 1$; $0 \leq \eta_K \leq 1$ etc.

So even though the stock-adjustment model is essentially a model without a theory, inasmuch as it says nothing about the derivation of the optimal value of the advertising stock, we must at least concern ourselves with the question of whether the stock-adjustment approach can be consistent with any given theory of determination of $K^*$. In the current case this depends on the assumption that $\beta$ can be treated as if it were equal to unity, because that is actually the case, or the firm believes that it is actually the case, or the firm acts so that the end result is that $\beta = 1$. This latter case is not unlikely if one views the firm as a hierarchical structure. The firm's decision makers may frequently set targets for lower levels of the hierarchy which are unrealistic in the sense that the decision makers do not expect them to be achieved, the realistic and desired target (to which only the decision makers are privy), being less than the stated target. We may also note that 3.38 implies not only that the firm is optimising but also that it is in equilibrium, i.e. $K_t = K^*_t$. For consistency between the control model and the stock-adjustment model it is essential that the stock-adjustment model permits equilibrium to be not only attained, but also maintained. This requirement again implies that $\beta = 1$; equation 3.51 will be unaffected except for the fact that $\beta$ takes on the value one.

Use of the stock-adjustment model has one potential advantage over other methods which has not yet been pointed out. The model does not require us to know what the values of the elasticities $\eta_K$ and $\eta_P$ are, we need only make the
assumption that they are constant. Thus the model does not rely on an assumption of any given kind of firm behaviour, for example Cournot behaviour, estimates of the elasticities, $c_i$, can be obtained whatever the form of the demand function 3.23. Estimates of $c_i$ can be compared with the 'known' values of the advertising stock-sales ratio since an estimate of $\delta$ can also be derived from equation 3.51.

Condition 3.50 unambiguously defines $K^*_t$. 3.50 is only a valid condition if $(\rho + \delta) > \frac{p_a}{p_{at}}$, that is the sum of the discount rate and the advertising decay rate should be greater than the proportional change in the price of advertising. Thus if the price of advertising is increasing rapidly 3.50 will no longer indicate a 'sensible' value of the advertising stock. Whether or not the price of advertising is likely to increase at such a rate as to render 3.50 unrealistic as a predictive rule is a question of expectations. Where the estimation of an equation such as 3.51 is concerned the question is simply answered by inspection of the relevant price series for advertising. 3.51 will be satisfactory as a description of the behaviour of $A_x$ as long as $(\rho + \delta)$ is greater than $\frac{\dot{p}_a}{p_{at}}$ for all $t$. 
NOTES TO CHAPTER 3.

1. A perhaps more important example of the importance of this restriction is that the cross-elasticities of demand between any brand included in the demand function and any particular excluded product must be equal. Thus if the demand function refers to margarine, and the price of butter is one of the exogeneous variables $Z$, the assumption is that a change in the price of butter will affect all the brands of margarine equally.

2. Thus we are assuming that the firm is a Stackelberg-type 'leader'. The pattern of marketing reaction functions will be different for a 'follower'. The model includes Cournot-type behaviour. Stackelberg disequilibrium may exist, all firms can think of themselves as leaders; the model takes the standpoint of an individual firm.

3. The proof given here is the same as that given by Lambin (1973).

4. A similar set of cases is discussed by Lambin (1973) and Lambin et al. (1973)

5. For a discussion of the Cournot model see Henderson and Quandt (1971), Chapter 6; Stigler (1968a), Chapter 4, Addendum 1; and (1966), Chapter 12; Fellner (1949), Chapter 2. Mathematical expositions of the Cournot model are given in Intriligator (1971); Cohen and Cyert (1965) and Hadar (1971).

6. That is, market demand depends only on the vector of variables $Z$, which are assumed to be exogeneous to all firms in the industry. Thus no firm can expand market demand through its own efforts.

7. More accurately, 3.16 remains only for a Stackelberg leader-type firm. A follower would fit into categories (ii) and (iii).

8. See Gould (1970); Jacquemin (1971), (1972a); Jacquemin and Thisse
Plessner (1972) gives an interesting alternative approach.

The model is continuous, which many would claim is a serious disadvantage. However, discrete models are rare and discrete models which yield testable predictions even rarer, but see the papers by Ireland and Jones (1972) and Tsurumi (1972).

For a discussion of the meaning of the first-order conditions see Jacquemin (1972a).

Tsurumi used a variational approach to obtain this result. If the specification of the advertising variable in the model of equations 3.23 to 3.38 is altered so that $A$ and $A_r$ are measured in physical units and the expenditure on advertising at time $t$ is $p_a(t) A(t)$, then the optimality condition with respect to advertising becomes

$$\frac{A}{pq} (p_a - \lambda) = -\frac{\eta A}{\eta p}. \quad (*)$$

When the stock of goodwill, $K$, is optimal and $\lambda = p_a$ then the optimal level of advertising is undetermined. Equation 3.38 was derived by Tsurumi in the context of a model where the price of advertising was allowed to vary through time. This specification of the model (where advertising is measured in physical units and the advertising is admitted as an independent variable) is pursued in Chapter 5.

This kind of policy applies in a variety of market situations other than advertising. See Jacquemin (1972b).

A potentially serious objection can be raised to this approach. If the firm maximises profits subject to 3.39 then the optimal level of the stock variable, $K^*$, may no longer be given by 3.38. Three possible 'escapes' exist. Firstly, the firm may not know that it is restricted by 3.39, it may believe that the optimal stock, once known, can be
attained within a time period. Secondly, 3.39 will be consistent with 3.38 if $\beta = 1$, that is if in fact the optimal stock can indeed be attained within the time period. Thirdly, if the firm realises it is restrained in the manner indicated by 3.39 it will adopt a higher level of $K^*$ than indicated by 3.38 and aim for that, knowing that it cannot reach it but in falling short may fulfil 3.38. (Provided setting high but 'artificial' targets is costless.) In this case $K^*$ will no longer be given by 3.38, but the actual optimal path of advertising expenditures through time may be unaffected.

14. Methods which test for optimal advertising behaviour directly by attempting to measure the elasticities $\eta_K$ and $\eta_P$ usually have to employ the same assumption.

15. $P_{at}$ may be known ex ante from the 'price list' of suppliers of advertising to the firm.

16. Thus if the firm is adopting rule-of-thumb behaviour, and 3.51 provides a good fit, then there is no reason to expect $0 < \beta < 1$, $0 < \delta < 1$, $0 < \rho < 1$, $0 < \eta_K < 1$, etc.
CHAPTER 4

ALTERNATIVE EXPLANATIONS OF FIRMS' BEHAVIOUR.

INTRODUCTION.

Previous chapters have analysed the situations of monopoly and non co-operative oligopoly. We now turn to a further set of situations which are of some importance in the real world, as even casual observation will confirm: co-operative or collusive solutions to the competitive situation. The variety of available collusive arrangements is varied and complex, Scherer (1971) notes:

"The variety of collusive pricing arrangements in industry is limited only by the bounds of human ingenuity." %

Scherer's observation applies not only to collusion over prices, but also to collusion over the discretionary variables, for example promotional activities, quality, outputs of specific products, geographical location and extent of markets and so on. We are not interested here in discussing explicit collusive agreements, or collusion in general. On the other hand, it may be possible to identify behaviour patterns marked by the degree of inter-firm co-operation which are common to several industries; price-leadership is a frequently discussed example. Since no precise definition of co-operative or collusive behaviour can be given we follow the practice of discussing a few situations which might be classed under the 'co-operative' heading in the hope that results emerge which can be distinguished from the results that emerge when non co-operative behaviour is assumed. Co-operation is seen as one solution to the oligopoly interdependence problem. Implicit co-operative behaviour arises because in certain circumstances it may pay all firms in an industry to follow the same rules in setting the

% Scherer (1971), p. 158.
values of their own discretionary variables. For example, each firm may recognise the group's common interest in orderly pricing, and hence each firm may adhere to the existing price structure. The existing price structure would have to be satisfactory in the eyes of all firms, and the threat of entry must be small at the existing set of prices. If it is not there is a significant probability that the composition of the group will alter to the detriment of some or all members of the group. Adherence to the accepted price structure has two important features. Firstly, it permits price stability in the short run and secondly, each firm is able to remove a great deal of uncertainty from its expectations. Balanced against this is the fragile nature of oligopolistic co-operation. 'Signals' from one firm to another can easily be misinterpreted. Firms may then only alter prices in response to clear-cut changes in industry demand and cost conditions, when there is little fear of misinterpretation of a price change. The approaches to facets of the oligopoly problem given here are not meant to exhaust the possibilities. Rather, an attempt is made to discuss some of the more common solutions to the interdependence problem in the dual hopes of casting more light on, and integrating (at least in part), the approaches of this chapter into the framework of Chapters 1-3.

PRICE LEADERSHIP.

The price-leadership model is developed through the medium of conjectural variations in the setting of an industry producing differentiated products. Analytically, it may make no difference whether all firms in an industry explicitly agree to follow one another's price increases, or whether over some period of time all firms in an industry happen to find it expedient to follow the price changes instigated by some members of the industry. The important point is that we may be able to make reasonable assump-
tions about the firms' expectations concerning rivals' reactions to price changes.

Price-leadership can arise in many ways. It might be the product of industry structure as in the "dominant firm" case. The industry is structured such that one firm supplies a large section of the market, and a 'fringe' of smaller firms supplies the rest of the market. Fringe firms are considered individually too small to influence the overall industry price level. Dominant firm price-leadership is perhaps the least interesting case, and is not further discussed. Situations other than the dominant firm type have sometimes been classified under the headings of "collusive" and "barometric". Here we dispense with that distinction and attempt to analyse the problem as a whole.

Consider an industry of \( n \) firms each producing one variety of a (differentiated) product. If the \( i \)th firm knows its current price and the prices of its \( n-1 \) rivals, and if the \( i \)th firm can predict the prices its rivals would charge, given that it charged some other price, on the basis of the given set of prices alone, then we have price leadership where the \( i \)th firm is the leader. Such a situation will occur if the firms \( j = 1, \ldots, n \), \( j \neq i \), have no alternative (the dominant firm case) or if the following firms find it in their interests to follow the leader's change. The latter cases may be roughly divided into three sub-sets.

Firstly, price-leadership may emerge essentially from the structure of the industry. Firm \( i \) may alter its price knowing that if it does so and all other firms in the industry follow, then all firms in the industry will be in a better position after the change in price. Price-leadership may in effect lead the industry towards a position of industry profit maximisation,
or the joint maximisation of whatever set of objectives is important to the individual firms. Structural elements are important for the same reasons that they are important in cartelisation. All firms must have similar costs and similar objectives, otherwise some firms will not wish to follow the leader's move and the system will tend to break down. This situation corresponds fairly closely to the 'collusive' price-leadership situation, described by Scherer as "...that type of price-leadership especially apt to facilitate monopolistic price solutions."

Secondly, all firms within an industry may face the same set of circumstances which make it desirable for any given firm to alter price, provided all other firms in the industry make a similar price change. The circumstances which lead to such a situation may be only indirectly related to industry structure. Examples are an industry-wide fall in demand; rising input costs common to all firms in the industry; the emergence of a substitute product produced outside the industry; the availability of a new technique, and so on. Appendix 4.1 illustrates the second of these examples with reference to the reaction of U.K. motor companies to the 1968 steel price increase. A pre-condition for the emergence of price-leadership in such a situation is that all firms must be aware of the circumstances, and reach a common conclusion as to their effect. A second necessary condition for price-leadership to emerge is that the leader must accurately reflect the changed state of the industry through his price change. This second case of price-leadership corresponds roughly to the 'barometric' firm case. As Stigler has neatly put it, the barometric firm "commands adherence of rivals to his price only because, and to the extent that, his price reflects market conditions with tolerable promptness."†

The most common pattern of responses is that prices are lowered when market

† Scherer (1971), p. 166
‡ Stigler (1947), pp. 445-446
conditions are depressed, and raised when demand and cost conditions 'support' the higher level. In a world where market conditions are rapidly changing and industrial concentration is high, the emergence of such price-leadership should be unsurprising. The maintenance of such price-leadership requires that all firms in the industry have similar objectives. The third sub-set of cases is simply an amalgamation of the first two. We might reasonably expect that in the majority of cases of price-leadership both structural and circumstantial factors will be at work. Indeed, for price-leadership to emerge as a solution to the problems raised by changing circumstances, the industry structure must be 'right'.

Consider a simple static formal model of price-leadership, where firm 1 is the leader. Let the demand function facing firm 1 be

\[ q_1 = q_1(p_1, \ldots, p_n; A_1, \ldots, A_n; Z_1) \]

where \( q_i, i = 1, \ldots, n \) are the outputs of \( n \) differentiated products, \( A_1, \ldots, A_n \) are advertising outlays of the \( n \) companies and \( Z_1 \) is a vector of exogeneous variables which affects the demand for \( q_1 \). The profit function for firm 1 is

\[ \pi_1 = p_1 q_1 - c_1(q_1) - A_1 \]

and for a profit maximum

\[ \frac{\partial \pi_1}{\partial p_1} = 0 \]

\[ \frac{\partial \pi_1}{\partial A_1} = 0 \]
Assuming competitive interdependence with respect to both advertising and price variables 4.3 and 4.4 may be written

\[ (p_1 - \frac{\partial c_1}{\partial q_1}) \left[ \frac{\partial q_1}{\partial p_1} + \sum_{i=2}^{n} \frac{\partial q_1}{\partial a_i} \frac{\partial a_i}{\partial p_1} \right] = -q \]  

4.6 \[ (p_1 - \frac{\partial c_1}{\partial q_1}) \left[ \frac{\partial q_1}{\partial A_i} + \sum_{i=2}^{n} \frac{\partial q_1}{\partial A_i} \frac{\partial A_i}{\partial p_1} \right] = 1 \]

Consider price-leadership and assume that the price-leadership takes the form of maintaining constant ratios between different prices, that is if firm 1 raises its price by x per cent the others will do likewise, hence

\[ p_i = k_i p_1, \quad i = 2, \ldots, n \]

and

\[ \frac{\partial p_i}{\partial p_1} = k_i, \quad i = 2, \ldots, n \]

Appendix 4.1 provides some evidence that at least over part of the period 1967 - 71 the U.K. motor manufacturers seemed to behave (outwardly, at least) approximately in this fashion. 4.5 may be rewritten as

\[ (p_1 - \frac{\partial c_1}{\partial q_1}) \left[ \frac{p_1}{q_1} \frac{\partial q_1}{\partial p_1} + \frac{p_1}{q_1} \sum_{i=2}^{n} \frac{\partial q_1}{\partial p_i} k_i \right] = -p_l \]

or

\[ (p_1 - \frac{\partial c_1}{\partial q_1}) \left[ \eta p_1 + \sum_{i=2}^{n} \frac{p_i}{q_1} \frac{\partial q_1}{\partial p_i} \right] = -p_l \]

where \( \eta p_l \) is the own price elasticity of demand for product 1 and \( \frac{p_i}{q_1} \frac{\partial q_1}{\partial p_i} \) is the cross-elasticity between demand for product 1 and the price of product \( i \). Equation 4.6 may be written

\[ (p_1 - \frac{\partial c_1}{\partial q_1}) \left[ \eta A_1 + \frac{A_1}{q_1} \sum_{i=2}^{n} \frac{\partial q_1}{\partial A_i} \frac{\partial A_i}{\partial p_1} k_i \right] = \frac{A_1}{q_1} \]
If there is 'advertising-leadership' similar to the price-leadership of above, i.e. if

\[ A_i = h_i A_1 \quad i = 2, \ldots, n \]

and hence

\[ \frac{\partial A_i}{\partial A_1} = h_i \quad i = 2, \ldots, n \]

then 4.11 becomes

\[ (p_1 - \frac{\partial c_1}{\partial q_1}) \left[ \eta A_1 + \sum_{i=2}^{n} A_i \frac{\partial q_i}{\partial A_i} \right] = \frac{A_1}{q_1} \]

where \( \eta A_1 \) is the own advertising elasticity of demand for product 1 and \( \frac{A_i}{q_1} \frac{\partial q_1}{\partial A_i} \) is the cross-elasticity between demand for product 1 and the advertising outlay on product \( i \). Consider the following four cases:

1) the firm acts as a Cournot oligopolist with respect to both price and advertising. In this case

\[ \frac{\partial p_i}{\partial p_1} = 0 \quad \text{for } i = 2, \ldots, n \]

\[ \frac{\partial A_i}{\partial A_1} = 0 \quad \text{for } i = 2, \ldots, n \]

and the optimality condition is

\[ - \frac{\eta A_1}{\eta p_1} = \frac{A_1}{p_1 q_1} \]

2) the firm acts as a leader with respect to price and a Cournot oligopolist with respect to advertising. In this case

\[ \frac{\partial p_i}{\partial p_1} = k_i \quad \text{for } i = 2, \ldots, n \]

\[ \frac{\partial A_i}{\partial A_1} = 0 \quad \text{for } i = 2, \ldots, n \]
and the optimality condition is

\[ - \frac{\eta A_1}{(n \pi_1 + \text{PCE})} = p_1 q_1 \]

where PCE is the sum of the \( n-1 \) price cross-elasticities

\[ \frac{\partial q_1}{\partial q_i} = \pi_{ij}, \quad 1 = 2, \ldots, n \]

3) the firm acts as a Cournot oligopolist with respect to price and as a leader with respect to advertising. In this case the optimality condition is

\[ - \frac{\eta (A_1 + \text{ACE})}{n p_1} = \frac{A_1}{p_1 q_1} \]

where ACE is the sum of the \( n-1 \) advertising cross-elasticities of demand \( \frac{A_i}{q_1} \frac{\partial q_1}{\partial A_i} \), \( i = 2, \ldots, n \).

4) the firm acts as a leader with respect to both price and advertising. In this case the optimality condition is

\[ - \frac{\eta (A_1 + \text{ACE})}{n p_1} = \frac{A_1}{p_1 q_1} \]

The results will be unaltered if firm 1 is a 'follower' rather than the 'leader'. Suppose firm j is the leader, then

\[ p_i = \frac{Z_i p_j}{i = 1, \ldots, n, \ i \neq j} \]

then

\[ p_1 = \frac{Z_1 p_j}{p_1 = \frac{Z_1 p_j}{p_1 = \frac{Z_1 p_j}{k = 1, \ldots, n, \ k \neq j}} \]

and hence

\[ p_1 = \frac{Z_1}{Z_k p_k} \]
i.e. the form of the price reaction functions is maintained. Since the products 1, ..., n are substitutes, the price cross-elasticities will be positive and the advertising cross-elasticities negative. Thus optimal advertising intensity will be higher in case 2 than in case 1, and lower in case 3 than in case 1. The net effect of the cross-elasticities on optimal advertising intensity in case 4 is a priori unknown. Whether co-operation over price or non-price variables is more likely is open to question. On the one hand a firm may easily escape detection of its 'price changes' by making changes which do not represent alterations in list price. Equally, it may be easier for firms to monitor one another's advertising activities. In this situation case 3 seems most likely. On the other hand we might argue that in some industries prices are closely monitored by rival firms, by consumers and the press. Thus price changes in the new car market are widely known, as Appendix A.1 shows. Equally, promotional activities in the new car market may be very difficult to monitor because of the widespread nature of the retail motor trade. Case 2 would then be most appropriate. This topic is discussed further below. Cases 1 - 4 listed above are, of course, a subset of the general cases (1) - (v) (equations 3.18 - 3.21) of Chapter 3. In particular we are restricting ourselves in cases 1 - 4 to the situation where reaction takes place in kind, i.e. one where price changes attract only retaliatory price changes and so on.

RULE-OF-THUMB PRICING.

There seems to be little doubt that a large number of firms make their pricing decisions with the aid of 'rules-of-thumb'. On the surface, at least, such rules-of-thumb do provide the necessary basis for oligopolistic co-operation. The major difference between the commonly encountered rules-of-thumb pricing policies and the traditional marginalist approach is that the use of rules-of-thumb does not take into account the possible demand conditions facing the firm. The most typical such rule is
the 'full-cost' or 'cost-plus' pricing principle in which a 'normal' or desired profit margin (or percentage return on invested capital) is added to the estimated unit costs in order to calculate the unit price of the product. Thus if all firms in an industry have similar costs and employ similar cost-plus pricing schemes, the uncertainty which clouds oligopoly is lessened, rivals' reactions are more predictable. Cost-plus pricing linked with price-leadership is a very powerful weapon for reducing oligopolistic uncertainty. Whether or not full-cost pricing takes into account demand conditions and is consistent with the notion of a firm maximising some set of objectives depends on how the margin is determined. Some light may be shed on this question by briefly considering a sophisticated version of the full-cost pricing principle. The firm adopts some long-term objective, say earning 15% after taxes on invested capital over the years. Since the firm is unsure of demand in the coming period, costs are calculated on the basis of some assumption about volume; for example, operation of the available plant at 80% of conservatively rated capacity. A 'price' is calculated by adding to the calculated average cost a sufficient margin to ensure that the stated objective is met. This 'price' is then used as a basis for deciding the actual price, taking into account factors other than costs. Whether or not this procedure is consistent with profit (or utility) maximisation depends on how the adopted margins vary between products, and how the margin on any given product varies through time in response to changing demand conditions. Marginalist approaches do relate the mark-up over marginal cost to market parameters, for example the price-elasticity of demand. If, over the range of output in which we are interested, average costs are approximately constant, then the mark-up of marginalist theory is the same as the mark-up in full-cost pricing rules. The full-cost rule will then be qual-
itatively consistent with marginalist economic theory if the adopted margins vary between products in the same way as do the predicted margins under marginalist theory. For example, if a firm chose a higher margin on a product with few close substitutes than on a product with many close substitutes (i.e. one where demand was relatively elastic) then the pricing behaviour of the firm would be qualitatively consistent with the predictions of marginalist theory.

THE KINKED DEMAND CURVE.5

The kinked demand curve theory does not attempt to explain how an individual oligopolist sets price, rather it seeks to explain why oligopoly prices tend to be 'rigid', that is tend not to alter in the short-run in response to short-run fluctuations in cost and demand conditions. The kinked demand curve theory adds weight to the notion that prices might be rigid in the short-run by imputing expectations about rivals' reactions to the individual oligopolist. Essentially, the original Sweezy proposition6 assumed that an individual oligopolist would adopt a 'pessimistic' view of rivals' reactions, that if he raised price no rival would follow, but if he lowered price all rivals would follow with the possible consequence of a price war. No individual oligopolist would then find it worthwhile to alter price unless the ruling industry price (the price at the 'kink') was above the price which would maximise industry profits.

The opposite behaviour pattern is also possible, illustrated by the so-called 'reverse kink'. The implication is that if an individual oligopolist raises price all rivals will follow, but if he lowers

5 see Sweezy (1939).
price no rival will follow. This situation may occur where all firms are operating at or near full capacity, and hence may be less keen to increase sales than to increase profits. Increasing costs will add weight to this common desire to increase prices but not decrease them. Again Appendix 4.1 provides some evidence for the 'reverse kink' inasmuch as nearly all price increases were followed, but the only price decrease reported was not followed.

Two major problems come to the fore in any discussion of kinked demand curves. Firstly, there is the question of how the ruling price was initially arrived at, and secondly, the fact that change in the ruling price will have to occur as changing demand and cost conditions render the existing price obsolete. The first question is somewhat irrelevant, it has some of the properties of the chicken and egg paradox. The second question tends to answer itself. The kinked demand curve theory is an explanation of short-run price rigidity; the kink will only exist if the majority of firms in the industry are happy with the price at the kink. When the ruling price becomes obsolete, and the majority of firms would prefer a new price, then there is little doubt that a new price will be found. How it is found the kinked demand curve theory does not say, but some form of barometric price-leadership would seem to be the answer. The kinked demand curve theory and the notion of barometric price leadership have the common element that some price changes will be followed whilst others will not. Recent experience in the U.K. motor industry, as described in Appendix 4.1, is consistent with both barometric price leadership and reverse kink explanations, due to a combination of rising costs and reasonably high levels of demand. Analytically, the two situations are identical given that price rises are always followed whilst price decreases are never followed and hardly ever occur.

† see Efryomson (1955) for discussion of the 'reverse kink'.
ADVERTISING.

As a description of the real world the marginalist model of advertising is clearly misleading. However, we have not necessarily tried to put forward the marginalist model as a description of firm behaviour, but rather we have used it to provide 'optimality' rules relating to some of the firm's discretionary variables. We may briefly turn away to ask the question of whether the likely actual behaviour of firms is capable of being qualitatively consistent with theoretically optimal behaviour.

At the outset it was noted that many firms adopt some form of rule-of-thumb to set advertising budgets. The most commonly employed rule-of-thumb is that of setting the advertising budget as a fraction of anticipated sales. The theoretical models presented above have the common property that the advertising intensity be proportional to the advertising elasticity of demand and inversely proportional to the price elasticity of demand. The rule-of-thumb method of setting advertising appropriations will be qualitatively consistent with the marginalist model if the chosen advertising intensity varies between products in the same way that the advertising elasticity and the inverse of price elasticity do. Whether or not the 'task method' of setting advertising budgets is consistent with optimising behaviour depends very much on what objectives are chosen for the task method. If the objectives are stated in terms of variables which are of interest to the economist, for example profits, sales or market shares, then consistency at the qualitative level exists. Other rules-of-thumb, for example setting advertising expenditures as a residual from profits are clearly not consistent with optimising behaviour.

\(\text{\textsuperscript{1}}\text{see Chapter 1.}\)
For example, one way of moving from a loss-making to a profit-making position may be to increase expenditure on advertising, hence increasing sales and moving the firm to a lower average cost position. The residual-from-profits method would have the firm do the opposite.

The preceding three sections have considered the possibility of co-operative pricing behaviour emerging in the absence of explicit agreements. We may profitably consider the same problem in relation to promotional expenditures in general and above-the-line advertising in particular. It is clear that some firms do set advertising budgets defensively, that is they set their advertising budgets to keep up with competitors. They may do this because of their attitude toward promotional competition or because they have very little discretion in the matter (i.e. they are followers after a dominant firm). Perhaps the most interesting question is whether oligopolists are more likely to co-operate over price or non-price variables, or both. We may put forward two major supporting reasons for the assertion that co-operation is more likely over price. Firstly, co-operation over price is more effective in preserving monopoly profits than is co-operation over non-price variables. Secondly, co-operation over price may be much more easily accomplished than co-operation over non-price variables. The former reason, which may alternatively be given as that price competition is much more effective in increasing output and reducing profits than non-price competition, is a common belief amongst economists and others. In the context of a very simple model, Stigler (1968b) was able to show that in order for this belief to be true marginal costs of production must not rise so rapidly as the marginal costs of non-price variables. Stigler suggests that such a state of affairs is very plausible. The second reason, that co-operation over price may be easier to achieve
than co-operation over non-price variables, follows from the variety of non-price variables. Leaving aside quality competition of a major kind, non-price competition encompasses changes in above-the-line advertising, below-the-line promotional activities, packaging changes and so on.

The opposite argument may be advanced. Co-operation over non-price variables may be held to be more likely than co-operation over price. Firms may easily be able to escape detection or price changes by making various changes which do not represent changes in list price. In particular, as Stigler (1964) points out, the 'price' we are concerned with is not the easily observed list price, but the private transactions price between buyer and seller. On the other hand, advertising, to be effective, has to be readily available to all, and hence monitoring is easy. Clearly, which argument we accept depends on the particular features of the market under consideration. Some markets may be characterised by a great deal of promotional activity at the buyer-seller level, which is very difficult to monitor. For such markets the sales force is an important element in the promotional mix. The retail car trade may be an example of this type of market.

Above-the-line advertising is easily detected by rivals, indeed there may even be considerable advance notice of changes in advertising policy via the trade journals of the advertising industry. (Above-the-line) advertising-leadership might be expected to emerge where all firms in an industry face a set of circumstances which make it desirable for any given firm to alter advertising outlays provided all other firms do likewise. For example, firms in an industry where advertising intensity is high may all be faced with a fall in demand which squeezes profits; all firms may then wish to reduce their advertising outlays, and advertising leadership can emerge as a solution.
SUMMARY.

The first part of Chapter 3 presented a generalised (static) model of promotional activity at the firm level. Chapter 4 has outlined some situations which may be considered special cases of the model in Chapter 3. The cases were selected because there is good reason to believe that the behaviour implied by them is encountered in the real world. The first section of the Chapter (on price-leadership) and Appendix 4.1 suggest that price-leadership is a possible explanation of the pricing behaviour of motor manufacturers in recent years. It was also suggested that 'price-leadership' was more likely to occur (in the context of the motor industry) than 'advertising-leadership', implying a higher 'optimal' advertising-sales ratio than that predicted by the usual version of the Dorfman-Steiner theorem. The 'kinked' demand curve explanation of behaviour was also considered and it was suggested that it was consistent with the notion of price leadership in the context of the motor industry.

The existence of rules-of-thumb for setting prices and advertising budgets was also considered. It was concluded that the fact that rules-of-thumb were frequently used by firms in practice was not necessarily a bar to the attainment of theoretically optimal positions as described by the theory of Chapters 2 and 3.
APPENDIX 4.1


In this appendix the price-changes initiated by the "big four" U.K. motor manufacturers are listed. Both the extent of the increases and the date of their announcement has been reported. Some other illustrative items of information have been reported where they seemed relevant. The manufacturers operate in more than one market segment, but only the price changes affecting family saloon cars are reported. The object of the exercise is not an attempt to prove that the U.K. motor industry was characterised by price-leadership or any other explanation of pricing behaviour, but merely to report the outward signs of "big four" pricing behaviour during this period. We may note that this period contains the devaluation of the pound, a rise in the price of steel and sharply rising (money) wages. The information was obtained from editions of "The Times", except where stated. The dates of the relevant editions are underlined.

January 18, 1967  British Leyland announce price increases to take effect from January 30th. Increases: basic price of Mini de luxe up by £25, basic prices of 1800 range up by £29.

February 25, 1967  Ford increase basic prices of all Anglia and Cortina models by £25 from March 1st. Corsair models (except 2000E) to go up by £10.


British Leyland, Ford, Vauxhall and Chrysler (Rootes Group).

† price net of purchase tax.
April 1, 1967  Statement of March 31st. reported by "The Times":  "The price of some models of Vauxhall cars is to be increased", Mr. M. B. Marr, manager of Vauxhall Motors, Luton, said last night. The increases to be announced next week "will probably be in the same range as other makers have made", he said.

________________________

March 18, 1968  Vauxhall prices increased by 3-8%, 15 out of 17 models in range to go up, the other two having only just been repriced. Increases partly in response to purchase tax increases.

March 20, 1968  In addition to purchase tax increases, Rootes increase prices by 7%.

May 6, 1968    B.M.C. increase prices by 3-5%.

May 11, 1968    Ford increase prices of all models by an average of 4%.

(Standard - Triumph raised prices by 2½-3% on July 1st.)

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July 10, 1968    Rootes lower prices by an average of 7% on five medium saloons. It was also reported that Rootes were holding very big stocks and suffering from falling sales.

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September 3, 1969  Vauxhall increase prices of all cars except Victor and Ventura range by an average of 4% from September 1st.

September 11, 1969  British Leyland increase prices of Mini saloons and Minor 1000 saloons by 4% from September 15th.

† "The Times" July 1, 1968.
October 1, 1969  Ford raise prices on some models by 3-5% from September 30th.

October 13, 1969  British Leyland raise Austin-Morris prices (except Mini and Minor models) by 5%.

February 21, 1970  British Leyland announce price increases of 2½-6%. "The Times" reported that the increases "surprised the motor industry which had been expecting something far higher." The report went on to say, "It was understood that at least one company was having to reconsider its own revisions which it had hoped to announce next week."

April 1, 1970  Prices of Ford cars go up by 3-4%.

April 2, 1970  Rootes announced on April 1st. increases of 2-4% on basic prices of most Rootes cars.

April 30, 1970  "The Times" reports: "Prices of Vauxhall cars are to be increased. An announcement from the company is believed to be imminent and the increases are expected to be about 5 per cent."

August 20, 1970  Vauxhall is to increase prices of seven of its models in the Victor, Ventura and VX 4/90 ranges by 4-7%.

September 25, 1970  Ford announce a new, more powerful and more expensive range of Escorts. Capri prices are also increased by 3-4%, but a modified interior is introduced.

September 30, 1970  Some British Leyland prices increased by 4% from 5th October. "The Times" reports: "Ford and Vauxhall have
already announced similar rises and Chrysler is expected to follow shortly.

October 1, 1970
Chrysler increase prices by an average of 4%.

October 5, 1970
Vauxhall increase prices of larger cars by 3-9% from 8 October.

January 1, 1971
Chrysler cars are to go up in price by an average of 6%.

January 9, 1971
British Leyland are to raise some prices by 2-5% from January 11. "The Times" reports: "Although Ford and Vauxhall would not comment further, it was understood last night that despite the companies' normal practice of raising prices in unison, neither group has any short-term plans to increase prices."

March 25, 1971
"The Times" reports: "British Leyland are to raise the prices of all their "bread and butter" cars early next month..."

"The rises are on all Austin-Morris division models and other specialist models not included in the January price rises. They average between 5 and 7½ per cent..."

"Other motor groups are expected in the near future to follow Leyland, who blamed the rises on "continuing cost inflation"."

April 10, 1971
The Ford Motor Company is to raise prices from April 16th. Rises range from £37.86p (5%) on a 2 door Escort to £106 (6%) on a Zodiac.

May 1, 1971
Vauxhall increase prices of all their cars by an average of 7%. 
May 25, 1971 Chrysler announce increases of 2% on several models including the Avenger.

December 9, 1971 Ford raise prices of Escorts, Cortinas, Capris and Zodiacs by an average of 3.3%.

December 17, 1971 Vauxhall cars to cost an average of 3½% more from January 1st. 1972.

(British Leyland prices went up in the week 15-22 January 1972 by approximately 3½-4% on all cars in the Austin-Morris range. Chrysler raised prices by approximately the same amount in late February.)

We may summarise the information given above in the form of Table 4.1.1. The rounds of price changes referred to are groups of similar price changes which take place within a short space of each other. Column 5 gives the time elapsing between the first company's announcement of a price increase (Column 1) to the last company's announcement of a price increase (Column 4). The seventh block of increases January 1st. 1971 - May 25th. 1971 contain two separate announcements by both British Leyland and Chrysler. The announcements have been considered complementary.

Ford, British Leyland and Vauxhall seem to act very much in concert, whilst Chrysler in one period (rounds 3 and 4) acted in a contrary fashion to the other three companies. Ford and British Leyland together accounted for approximately 70 per cent of the market during the period. The lags between similar price changes between Ford and British Leyland were much shorter than those stated in the fifth column of Table 4.1.1.
TABLE 4.1.1.

'Rounds' of price increases in the U.K. motor industry

<table>
<thead>
<tr>
<th>'ROUND'</th>
<th>1st COMPANY TO ALTER PRICES</th>
<th>2nd COMPANY</th>
<th>3rd COMPANY</th>
<th>4th COMPANY</th>
<th>TIME SPAN +</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>British Leyland</td>
<td>Ford</td>
<td>Rootes</td>
<td>Vauxhall</td>
<td>74</td>
</tr>
<tr>
<td>2</td>
<td>Vauxhall</td>
<td>Rootes</td>
<td>British Leyland</td>
<td>Ford</td>
<td>56</td>
</tr>
<tr>
<td>3</td>
<td>Rootes</td>
<td>(downward change)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Vauxhall</td>
<td>British Leyland</td>
<td>Ford</td>
<td></td>
<td>41</td>
</tr>
<tr>
<td>5</td>
<td>British Leyland</td>
<td>Ford</td>
<td>Chrysler</td>
<td>Vauxhall</td>
<td>69</td>
</tr>
<tr>
<td>6</td>
<td>Vauxhall</td>
<td>Ford</td>
<td>British Leyland</td>
<td>Chrysler</td>
<td>48</td>
</tr>
<tr>
<td>7</td>
<td>Chrysler</td>
<td>British Leyland</td>
<td>Ford</td>
<td>Vauxhall</td>
<td>126</td>
</tr>
<tr>
<td>8</td>
<td>Ford</td>
<td>Vauxhall</td>
<td>British Leyland</td>
<td>Chrysler</td>
<td>73 approx.</td>
</tr>
</tbody>
</table>

+i.e. time elapsed between first company announcing changes and last company announcing changes in days.
Although it is not intended to discuss the results given in this Appendix in the context of a formal model of firm pricing behaviour, the results do tend to suggest some 'models' rather than others. One feature of the pattern of price changes outlined above is clearly that little or no list price competition is evident where 'established' models are concerned. In addition the general direction of price change has been upward, and upward price changes are frequently associated with rising input costs. Thus the evidence of this Appendix may be seen as consistent with both the 'barometric price-leadership' and 'reverse kink' explanations of pricing behaviour. In many practical situations the two 'theoretical' explanations may be indistinguishable.
NOTES TO CHAPTER 4 AND APPENDIX 4.1.

1. A fuller account of collusion is given by Scherer (1971).

2. Scherer (1971), Chapter 6, discusses price-leadership using the collusive-barometric distinction.


5. For the original statement of the 'theory' see Sweezy (1939). Alternative views are given by Stigler (1947) and Efryomson (1955).

6. Information obtained by comparing weekly price lists of cars given in the magazine "Motor".
EMPIRICAL ASPECTS OF THE ADVERTISING MODEL.

CHAPTER 5

INTRODUCTION.

The perspective adopted so far has been a micro-analytic one. The (not uncommon or particularly original) argument is advanced that if we wish to know more about advertising effects and about factors affecting the level of advertising then a logical place to look is the brand level of competition. The micro-analytic approach is not capable of taking into account all the problems encountered when considering advertising; on the other hand it is not entirely devoid of predictive content, particularly when quantitative information is available. The chapter discusses the methods used to give the approach a quantitative basis, and applies the model developed in Chapter 3.

'DIRECT' METHODS OF TESTING THE THEORY.

Consider the Dorfman - Steiner theorem as an example of the kind of result generated by the theory. The Dorfman - Steiner theorem may be tested 'directly' by deriving estimates of the appropriate elasticities and comparing those estimates with actual values of firms' decision variables. Divergence of actual behaviour from the apparent optimum can then be explained, at the quantitative level at least, in terms of differences in the degree of competitive interdependence, differing attitudes to risk, and so on. The studies by Lambin are an excellent example. Two relationships turn out to be of great importance in the Lambin studies, and in all similar studies. The first has already been extensively explored, that the

modulus of the ratio of the advertising and price elasticities of demand, appropriately defined given the assumptions of the model, should equal the advertising-sales ratio.\textsuperscript{2} The second relationship is simply derived from the optimisation conditions of the usual model. Let the firm's profit function be

\[ \Pi = pq - c(q) - A \]

where \( q = q(p) \). The first-order conditions for a maximum are:

\[ \frac{\partial \Pi}{\partial p} = (p - MC) \frac{\partial q}{\partial p} + q = 0 \]

\[ \frac{\partial \Pi}{\partial A} = (p - MC) \frac{\partial q}{\partial A} - 1 = 0 \]

Equations 5.2 and 5.3 may be written

\[ \frac{p - MC}{p} = \frac{1}{\eta_p} \]

\[ \frac{p - MC}{pq} = \frac{A}{pq} \frac{1}{\eta_A} \]

where \( \eta_p \) and \( \eta_A \) are defined as before. The term \( \frac{p - MC}{p} \) is the gross profit margin, provided we assume constant costs over the range of output we are considering, that is marginal cost is equal to average variable cost. If \( \eta_A \) is the short-run elasticity with respect to advertising, the long-run condition corresponding to 5.5 is

\[ \frac{p - MC}{p} = A \frac{(1-\lambda)}{pq} \frac{1}{\eta_A} \]

where \( \frac{\eta_A}{1-\lambda} \) is the long-run advertising elasticity and \( \lambda \) is the
advertising decay rate. Estimates of $\eta_A$, $\eta_p$, and $\lambda$ can be derived and compared with known values of measures of the firm's performance, i.e. the gross profit margin and the ratio of advertising expenditures to sales. A more complex approach to the problem would be to estimate the same demand function in different ways, thus obtaining estimates of differing elasticities for the same product. Thus if a demand function were estimated in both absolute sales and market share form, the difference between the two derived elasticities could be attributed to competitive interaction, provided industry demand was inelastic. The limits and pitfalls of the econometric approach to the study of advertising have frequently been pointed out. In particular a simultaneity problem arises in the estimation of demand functions when advertising expenditure is included as a variable. Current advertising expenditures influence current sales, and if the implications of the analysis are accepted, current advertising expenditures are set with reference to current sales. Recognition of this simultaneity should lead us to adopt an estimation method which takes it into account, for example two-stage least squares. A suitable model would then consist of, say, the demand function and an advertising appropriations relationship consistent with optimising behaviour. Such a method has been employed by Cowling (1972). On the other hand it may be that this simultaneity problem does not exist. It is possible for the firm to set the advertising appropriation on some basis other than the level of current sales or the expected level of sales in the period when the advertising is to take place. In this case our interest is in whether or not the firm in fact appears to act in an optimal fashion, ex post, or at least how far the firm's actual behaviour is in accord with the predictions of the theoretical model, without undue regard for the question of whether the firm 'fits' the assumptions

1 see Quandt (1964).
of the theoretical model. If such a set of circumstances prevails then single equation estimation methods are acceptable. Thus there are two distinct viewpoints we may take. We can attempt to build a model which is in accord with what we believe firms actually do, in which case the model must be operational and the problem of simultaneity must be faced, or we may simply compare estimated parameters of the market situation with actual behaviour, asking only the question whether or not actual behaviour is consistent with optimal behaviour. The advantage of the first method is that it tells us something about how firms actually behave, its disadvantage is that it is a difficult method of investigation, requiring the construction of a model of the firm. The second method tells us nothing about the way firms behave, particularly with regard to the actual method of appropriating advertising expenditures, but is relatively simple to apply to specific markets. This second method will be applied to the U.K. passenger car market as part of this thesis.

Typically researchers have considered non-durables markets, and results have been quite good in the sense that estimated parameters of the market situation have been of the expected magnitude, and often actual policies have appeared to be quite close to the suggested optimal policies. Cowling (1972) reports measured values of the ratio $\frac{e_A}{e_p} A_i / R_i$ (where $e_A$, $e_p$ are the estimated elasticities of market share with respect to advertising share and relative price for the average firm in the market and $A_i / R_i$ is the market average advertising-sales ratio) of 1.41 for margarine, 4.15 for coffee and 0.78 for toothpaste. The 'expected' value of the ratio is unity. For the two durables markets studied by Cowling the values of this 'optimality ratio' are 13.7 (cars) and 10.88 (tractors). Several reasons may be advanced for the failure of the
method where durables are concerned. Firstly, there is bound to be a serious error of omission problem; much promotional activity in the two markets discussed by Cowling takes place at point of sale or is dealer (rather than manufacturer) instigated. The measure of advertising used was expenditure on national press and television advertising. Secondly, advertising may be linked to new model introductions giving measures of the advertising share elasticity an upward bias. Thirdly, the essential effect of advertising may be on the timing of purchases rather than the level of purchases within a period. Lambin (1970a) reports values of the marginal profit contribution per dollar of advertising expenditure in the range 10 - 28 cents for three brands of a "small electrical appliance whose rate of ownership among households is as high as 75 per cent." Lambin (1970b) reports another result for a consumer durable good (unspecified in the paper). In this case the opposite finding to the Cowling one emerges, that the firm seemed to overspend on advertising, although only to a fairly small extent. Expressing Lambin's result in terms of a Cowling-type optimality ratio gives a value of 0.736 when short-run market share elasticities are employed. Peles (1971b) reports a net rate of return to advertising expenditures of 10 per cent for a "beer firm" and a net rate of return of approximately 50 per cent for cigarette firms. Both results assume a period of influence of advertising expenditures of 1.75 years and a rate of amortisation of advertising expenditures of 40 per cent. Samuels (1972) gives estimated advertising elasticities in the fruit squash market of approximately 0.065 (September 1967), 0.265 (June 1968) and 0.18 (August 1968).

AN ALTERNATIVE TO THE DIRECT METHODS: THE STOCK-ADJUSTMENT MODEL.

The 'direct' methods of testing for optimality in firms' advertising behaviour present a paradox. We are interested in firms' behaviour, but we examine a relationship which is determined by the behaviour of consumers, the demand function facing the firm, in order to test for optimality. Recognition of the simultaneity problem does not help much, since any advertising appropriations relationship incorporated into the model is picked without any reference to the model of optimal behaviour. At best an advertising appropriations relationship which is general enough to include optimising behaviour must be chosen. An alternative method is derived in Chapter 3. In general the method is based on discovering optimality rules for the optimal stock of advertising goodwill, and incorporating those optimality rules into a stock-adjustment model to yield an estimating equation which relates the firm's current advertising expenditures to current and/or past values of other measures of the firm's performance. The particular model developed in Chapter 3 yielded the estimating equation

\[
A_t = \beta c_1 \frac{R_t}{P_{A_t}} \left( \frac{1}{(p+\delta) \Delta P_{A_t}} \right) - \beta (1-\delta) c_1 \frac{R_{t-1}}{P_{A_{t-1}}} \left( \frac{1}{(p+\delta) \Delta P_{A_{t-1}}} \right) + \frac{1}{P_{A_{t-1}}} + (1-\beta)A_{t-1}
\]

where \( c_1 = -\frac{n_K}{n_p} \)

\( n_K \) is the elasticity of demand with respect to the stock of advertising goodwill

\( n_p \) is the price elasticity of demand taking into account competitive reaction
\( p \) is the discount rate

\( \delta \) is the goodwill decay rate

\( p_{At} \) is the price of advertising in period \( t \)

\( R_t \) is the value of sales in period \( t \)

\( A_t \) is advertising expenditure in period \( t \) deflated by the price index of advertising \( (p_{at}) \)

The advantages of testing for optimal behaviour by estimating equation 5.7 are firstly, that firms' behaviour is examined to test for optimality and secondly, that the model from which 5.7 is derived deals specifically with the advertising stock. In addition, 5.7 has the particular advantage that the price of advertising is incorporated as an explanatory variable.

The disadvantages of the model are that it still only allows an ex post view of firm behaviour, and that it requires non-linear methods of estimation.

One other feature of the model is of interest. Provided we assume that \( \eta_K \) and \( \eta_p \) are constant (i.e. that the demand function is log-linear in \( K \) and \( p \)) it is not necessary to estimate them. The advantages of this are two-fold. Firstly, \( \eta_K \) is very difficult to estimate and anything which circumvents this problem is desirable. Secondly, \( \eta_p \) may be defined as widely as we like, in particular we may define \( \eta_p \) to take into account reaction by competitors. The disadvantage is clear: the supposed constancy of \( \eta_K \) and \( \eta_p \) is a large assumption.

**ESTIMATION OF THE STOCK-ADJUSTMENT MODEL.**

Estimation of 5.7 requires the use of non-linear methods. The particular method chosen was probably the simplest. 5.7 was estimated for different values of \((p + \delta)\) which were chosen on theoretical and
practical grounds. A 'point' estimate was obtained by choosing that value of \((\rho + \delta)\) which minimised the residual (unexplained) sum of squares (R.S.S.). Given this value of \((\rho + \delta)\) 5.7 is exactly identified. An estimate of \(\beta\) can be obtained from the estimated coefficient of \(A_{t-1}\); knowing \(\beta, c_1\) can be obtained from the coefficient of the current sales term, and hence knowing \(\beta\) and \(c_1\), an estimate of \(\delta\) (and thus of \(\rho\)) can be obtained from the coefficient of the immediate past sales term. Some problems are raised by the fact that the non-linearity in 5.7 occurs in the denominator of two variables. Thus 5.7 breaks down when 
\[
(\rho + \delta) = \frac{\Delta p_{A_t}}{p_{A_t}}. 
\]
We would expect that as 
\[
(\rho + \delta) - \frac{\Delta p_{A_t}}{p_{A_t}} \rightarrow 0, 
\]
R.S.S. \(\rightarrow \infty\). Thus we would expect the function linking \((\rho + \delta)\) to the residual sum of squares to have \((n + 1)\) discontinuities, where \(n\) is the number of periods for which data is available. The consequences of this fact are not serious provided we expect the global minimum value of the residual sum of squares to occur at a value of \((\rho + \delta)\) outside the range where such discontinuities occur. However, preliminary investigations using the data intended for estimation of 5.7 showed that this could not be safely assumed.

Initially investigations were undertaken for the range of values \(0.1 < (\rho + \delta) < 0.75\). In this range no complications arise since the maximum value of \(\frac{\Delta p_{A_t}}{p_{A_t}}\) was 0.0636 for the period under consideration. 

(see Table 5.1). If the estimates of equation 5.7 were to be consistent with the model advanced in Chapter 3 we would require that the minimum value of the R.S.S. should occur for a value of \((\rho + \delta)\) greater than the maximum value of \(\frac{\Delta p_{A_t}}{p_{A_t}}\) recorded in the period of observation. It was apparent
as a result of the preliminary investigations that this was not to be the case, and that further investigation of equation 5.7 was necessary. The usual "hill-climbing" routines for estimating non-linear relationships could not be used since the non-linearities in the estimating equations occurred in the denominators of the independent variables and could therefore give rise to discontinuities in the R.S.S. (given that initial explorations showed that the minimum values of the R.S.S. did not lie in the expected positive range).* It was therefore decided to adhere to the estimation method outlined above. Careful choice of values of \((\rho + \delta)\) bearing in mind the known values where discontinuities would occur permitted a useful mapping of values of the R.S.S.

In order to obtain a complete mapping of the values of the residual sum of squares a large number of values of \((\rho + \delta)\) had to be tried. In each case 87 values of \((\rho + \delta)\) in the range

\[-0.9 < (\rho + \delta) < 0.95\]

were used. 60 of the chosen values occurred within, or close to, the range where all the discontinuities occurred;

\[-0.0386 < (\rho + \delta) < 0.0636.\]

17 of the chosen values occurred in the range

\[0.1 \text{ to } 0.95\]

and 10 in the range

\[-0.9 \text{ to } -0.1.\]

The range

\[0.1 < (\rho + \delta) < 0.95\]

is consistent with the profit-maximising model, given that reasonable values for the discount rate, \(\rho\), might lie in the range

\[0.05 \text{ to } 0.35\]

and reasonable values for the decay rate of the advertising stock might be in the range

\[0.05 \text{ to } 0.6\]

(evidence on the likely size of the decay rate can be found in Peles (1971b) and Cowling et al. (1973)).

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*I am indebted to Dennis Leech of the University of Warwick for advice on this point.*
THE DATA.

5.7 was estimated using data for the (then) five major U.K. motor manufacturers for the period 1958-68 inclusive. The data is annual, giving a total of eleven observations for each company. The values of the McGuinness (1973) price index for advertising and the terms \( \frac{\Delta p_A}{p_A} \) and \( \frac{\Delta p_A}{p_A} / \frac{p_A}{p_{A-1}} \) are shown in Table 5.1. The values in the right-hand columns of the table are those at which 5.7 is undefined, i.e. the values of \((\rho + \delta)\) for which the R.S.S. becomes infinite.

**TABLE 5.1.**

The McGuinness Price Index for Advertising: index based on the average cost per 1000 readers in national newspapers.

<table>
<thead>
<tr>
<th>Year</th>
<th>( p_A )</th>
<th>( \frac{\Delta p_A}{p_A} )</th>
<th>( \frac{\Delta p_A}{p_A} / \frac{p_A}{p_{A-1}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1956</td>
<td>1.1932</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1957</td>
<td>1.1925</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1958</td>
<td>1.2735</td>
<td>0.0636</td>
<td>-0.0059</td>
</tr>
<tr>
<td>1959</td>
<td>1.2915</td>
<td>0.0139</td>
<td>0.0636</td>
</tr>
<tr>
<td>1960</td>
<td>1.2880</td>
<td>-0.0027</td>
<td>0.0139</td>
</tr>
<tr>
<td>1961</td>
<td>1.3090</td>
<td>0.0160</td>
<td>-0.0027</td>
</tr>
<tr>
<td>1962</td>
<td>1.3685</td>
<td>0.0435</td>
<td>0.0160</td>
</tr>
<tr>
<td>1963</td>
<td>1.3417</td>
<td>-0.0200</td>
<td>0.0435</td>
</tr>
<tr>
<td>1964</td>
<td>1.4264</td>
<td>0.0524</td>
<td>-0.0200</td>
</tr>
<tr>
<td>1965</td>
<td>1.4545</td>
<td>0.0193</td>
<td>0.0524</td>
</tr>
<tr>
<td>1966</td>
<td>1.4382</td>
<td>-0.0113</td>
<td>0.0193</td>
</tr>
<tr>
<td>1967</td>
<td>1.3751</td>
<td>0.0386</td>
<td>-0.0113</td>
</tr>
<tr>
<td>1968</td>
<td>1.4463</td>
<td>0.0492</td>
<td>0.0386</td>
</tr>
</tbody>
</table>
RESULTS.

The results of the estimation of equation 5.7 are summarised in Table 5.2 and figures 5.1 to 5.10 inclusive. Figures 5.1 to 5.10 plot the derived values of the residual (unexplained) sum of squares against the chosen values of \((\rho + \delta)\). Table 5.2 reports the regression results where a global minimum value of the residual sum of squares was reached for each of the five companies. Thus for each company two graphs and one regression result are reported. The first graph in each company pair shows values of the R.S.S. for values of \((\rho + \delta)\) in the range \(-0.9 < (\rho + \delta) < 0.95\), but excluding those values of \((\rho + \delta)\) which fall within the range of the values of \((\rho + \delta)\) where it is predicted that discontinuities will occur. The two dotted lines indicate the bounds of this range. The second graph in each company pair shows values of the R.S.S. for values of \((\rho + \delta)\) within the range where it is predicted that discontinuities will occur. The dotted lines indicate the critical values of \((\rho + \delta)\). Between any pair of dotted lines we would expect the plotted paths to form a U-shape. Inspection shows that this does not always seem to be the case, although further points would have to be plotted before we can be certain that the results do not conform to expectations. In a few cases the expected U-shape emerges very clearly. The one regression result reported for each company is that one containing the value of \((\rho + \delta)\) which gives the minimum value of the R.S.S. for all the values of \((\rho + \delta)\) tried (called above the global minimum value of the R.S.S.). The regressions are well determined, but in none of the cases do the results conform to the predictions of the model. In order for the reported results to be consistent with the proposed stock-adjustment model we would (at the very least) require that the estimated coefficient on the current sales term be positive.
and that the estimated coefficient on the immediate past sales term be negative. Confidence intervals for \((p + \delta)\) may be derived using a method proposed by Goldfeld and Quandt (1972). The confidence interval for \((p + \delta)\) may be derived from the R.S.S. using the formula

\[
5.8 \quad n \log \frac{R.S.S. (p+\delta)}{n} - n \log \frac{R.S.S. (p+\delta)}{n} < \frac{1}{2} \chi^2_p (\alpha)
\]

where

- \(n\) is the number of observations
- \(R.S.S. (p+\delta)\) is the value of the R.S.S. at the bounds of the confidence interval
- \(R.S.S. (p+\delta)\) is the value of R.S.S. at the global minimum
- \(p\) is the number of restrictions placed on the estimating equation
- \(\alpha\) is the level of significance.

In the current example a five per cent level is chosen and \(p = 1\). The test implied by 5.7 is only applicable where the function linking \((p + \delta)\) and the R.S.S. is continuous. Since the global minimum values of the R.S.S. for B.M.C. and Ford lie within the range of discontinuities, no confidence interval can be derived for \((p + \delta)\) in these two cases. For Standard-Triumph the C.I. is \(0.08 < (p + \delta) < 0.22\). For Vauxhall a lower bound to the C.I. is not available, since it falls outside the range of observations; the upper bound of the C.I. is \(-0.07\). For Rootes an upper bound is not available, the lower bound being \(0.19\).
The t-values reported in Table 5.2 indicate confidence intervals for the parameter estimates conditional upon a particular value of \((\rho + \delta)\) being used in estimation. Normally, of course, all parameters are estimated jointly, hence the t-values (and D-W statistic values) quoted in Table 5.2 do not permit the usual interpretation. They can however be used as a guide to the degree of determination of the individual parameter estimates. We might expect the t-values to be larger than they would be in the (hypothetical) case of joint estimation of \((\rho + \delta)\) along with the other parameters, and thus they should be treated with caution. Caution is particularly indicated given the width of the confidence intervals on the estimates of \((\rho + \delta)\). Thus the results for Vauxhall and Rootes in Table 5.2 might well be interpreted as if the regressions were not significant, despite the 'reasonable' F-scores reported.

The results reported in Table 5.2 are difficult to assess. Discounting the results in the Vauxhall and Rootes columns since they are best interpreted as being not significant leaves the results for B.M.C., Ford and Standard-Triumph to be explained. In none of these cases do the sizes of the coefficients correspond with the sizes predicted by the model of Chapter 3. In particular all of the coefficients on \(A_{t-1}\) are significantly different from zero, in two cases the estimate of \(\beta\) (1 minus the coefficient on \(A_{t-1}\)) is itself not significantly different from zero. On the basis of Chapter 3 we would expect the sign on the current sales term to be positive and the sign on the immediate past sales term to be negative. In none of the three cases is this prediction borne out. In addition to the sign problem the coefficients themselves are very small. Were the signs 'correct', advertising-stock/sales ratios of considerably less than one thousandth would be indicated in all three cases.

Clearly the results given in Table 5.2 and in Figures 5.1 to 5.10 are not consistent with the model in Chapter 3. Nor do the results seem to be consistent with any other theory of advertising behaviour. In particular
when \((p + 6)\) falls in the range of the discontinuities a very erratic path for \(A_t\) can be produced. Clearly such an erratic path is an unsatisfactory prediction and reflects on the results given in Table 4.2. For example the firms do not seem, ex post, to be adopting the 'rule' that advertising expenditures should be a constant proportion of sales. Table 5.3 illustrates this contention by giving the means, standard deviations and ranges of advertising intensity for the five companies over the period under investigation (1958-68 inclusive).

Table 5.3 might not tell the whole story. The theory defines ex ante advertising intensity and Table 5.3 examines actual advertising intensity. Thus planned advertising intensity could deviate from actual advertising intensity, particularly due to variations in actual sales revenue. We should therefore examine the variance in advertising and sales revenue separately. One way of doing this and hence testing for rule-of-thumb behaviour with respect to advertising appropriations is to regress advertising on current sales and/or immediate past sales. This may be done in absolute form or first difference form. Table 5.4 reports results for the five companies. The negative coefficients are difficult to explain, but the regressions in which they feature are poorly determined. Except in the case of B.M.C., who appear to be behaving roughly in accord with the constant advertising intensity rule-of-thumb, the regressions fail to yield any fresh information.

Although the empirical results must cast doubt on the theoretical model, it would be foolish to reject the model without any reference to the quality of the test of it. The results of Table 5.2 represent a potentially poor test of the model or Chapter 3 given the size of the estimated coefficients and the nature of the car market. Above-the-line advertising does not seem to be an important promotional variable in the car market (except for new models), advertising intensities are typically low, and a previous study (Cowling (1972)) has indicated that car manufacturers may not be adopting
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profit-maximising behaviour. It would be useful to repeat the procedure of this Chapter using data referring to a product market where it is known that above-the-line advertising is an important promotional variable.\textsuperscript{14} We must not forget however that the results of Table 5.2 can be used as evidence of non-optimising behaviour by car manufacturers.

**SUMMARY.**

The stock-adjustment model of Chapter 3 was estimated using data referring to the five major U.K. motor manufacturers over the period 1958-68. Whilst the results cast grave doubts on the theoretical model it was suggested that the test was not a good one. It would be of considerable interest to apply the model to a non-durable market.

An alternative approach which maintains the standpoint of examining firms' (rather than consumers') behaviour would be to derive an optimal path for advertising at the firm level from a model of optimising behaviour.\textsuperscript{15} There is no guarantee, however, that such a method will yield an equation which is any easier to estimate, or more empirically useful than 5.7. The results of Tables 5.2, 5.3 and 5.4 do not seem to support any of the hypotheses concerning firms' advertising behaviour which have been mentioned so far.
Regression results for values of \((p + \delta)\) which produce a
global minimum value of the Residual Sum of Squares.

<table>
<thead>
<tr>
<th>(p + \delta)</th>
<th>B.M.C. (x10^{-6})</th>
<th>FORD (x10^{-6})</th>
<th>STANDARD TRIUMPH (x10^{-4})</th>
<th>VAUXHALL (x10^{-3})</th>
<th>ROOTES (x10^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>3.7975</td>
<td>-3.558</td>
<td>-1.2135</td>
<td>-5.8475</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.1969)</td>
<td>(3.2261)</td>
<td>(1.8443)</td>
<td>(0.8875)</td>
<td></td>
</tr>
<tr>
<td>0.047</td>
<td>-1.5511</td>
<td>-3.6245</td>
<td>-5.8475</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.6245)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>2.4631 (x10^{-6})</td>
<td>2.0316 (x10^{-6})</td>
<td>8.5957 (x10^{-4})</td>
<td>-0.7316 (x10^{-3})</td>
<td>10.5043 (x10^{-3})</td>
</tr>
<tr>
<td></td>
<td>(2.4466)</td>
<td>(0.8983)</td>
<td>(6.7372)</td>
<td>(1.0492)</td>
<td>(1.7635)</td>
</tr>
<tr>
<td>-0.3</td>
<td>1.0225 (x10^{-6})</td>
<td>1.0221 (x10^{-6})</td>
<td>0.5937 (x10^{-4})</td>
<td>-0.0222 (x10^{-3})</td>
<td>0.3240 (x10^{-3})</td>
</tr>
<tr>
<td></td>
<td>(2.9085)</td>
<td>(23.2274)</td>
<td>(4.2403)</td>
<td>(0.6476)</td>
<td>(0.8503)</td>
</tr>
<tr>
<td>0.7</td>
<td>1.0225</td>
<td>1.0221</td>
<td>0.5937</td>
<td>-0.0222</td>
<td>0.3240</td>
</tr>
<tr>
<td></td>
<td>(2.9085)</td>
<td>(23.2274)</td>
<td>(4.2403)</td>
<td>(0.6476)</td>
<td>(0.8503)</td>
</tr>
<tr>
<td>d.f.</td>
<td>3, 8</td>
<td>3, 8</td>
<td>3, 8</td>
<td>3, 8</td>
<td>3, 8</td>
</tr>
<tr>
<td>F.</td>
<td>290.671</td>
<td>190.768</td>
<td>90.4042</td>
<td>17.1097</td>
<td>12.9163</td>
</tr>
<tr>
<td>D-W</td>
<td>2.1214</td>
<td>2.3006</td>
<td>2.5441</td>
<td>1.7847</td>
<td>2.5381</td>
</tr>
</tbody>
</table>

\(t\) - values in brackets.
Table 5.3

Means, Standard Deviations and Ranges of the Advertising Intensities ($A_t/R_t$, where $A_t$ is measured in expenditure terms) for the period 1958 - 68 inclusive.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.M.C.</td>
<td>0.005091</td>
<td>0.001150</td>
<td>0.003071 - 0.007496</td>
</tr>
<tr>
<td>FORD</td>
<td>0.005823</td>
<td>0.001628</td>
<td>0.003477 - 0.008279</td>
</tr>
<tr>
<td>STANDARD TRIUMPH</td>
<td>0.032703</td>
<td>0.01839</td>
<td>0.012247 - 0.082344</td>
</tr>
<tr>
<td>VAUXHALL</td>
<td>0.007474</td>
<td>0.003753</td>
<td>0.001612 - 0.01638</td>
</tr>
<tr>
<td>ROOTES</td>
<td>0.009443</td>
<td>0.006226</td>
<td>0.002641 - 0.02894</td>
</tr>
</tbody>
</table>
Regression of $\Delta A_t$ on $\Delta R_t$ and $\Delta R_{t-1}$ for the period 1959 - 1968 inclusive.

<table>
<thead>
<tr>
<th></th>
<th>B.M.C.</th>
<th>FORD</th>
<th>STANDARD TRIUMPH</th>
<th>VAUXHALL</th>
<th>ROOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta R_t$</td>
<td>0.001912</td>
<td>0.009453</td>
<td>0.002355</td>
<td>0.001471</td>
<td>-0.01505</td>
</tr>
<tr>
<td></td>
<td>(1.6809)</td>
<td>(0.6618)</td>
<td>(0.2544)</td>
<td>(0.3524)</td>
<td>(1.4560)</td>
</tr>
<tr>
<td>$\Delta R_{t-1}$</td>
<td>0.001975</td>
<td>0.017133</td>
<td>-0.002046</td>
<td>-0.001321</td>
<td>0.003804</td>
</tr>
<tr>
<td></td>
<td>(1.7635)</td>
<td>(0.8939)</td>
<td>(0.2165)</td>
<td>(0.2996)</td>
<td>(0.3743)</td>
</tr>
<tr>
<td>d.f.</td>
<td>2, 8</td>
<td>2, 8</td>
<td>2, 8</td>
<td>2, 8</td>
<td>2, 8</td>
</tr>
<tr>
<td>F.</td>
<td>1.7274</td>
<td>0.4080</td>
<td>0.04149</td>
<td>0.1048</td>
<td>1.06173</td>
</tr>
<tr>
<td>D-W</td>
<td>2.5053</td>
<td>1.0437</td>
<td>2.1478</td>
<td>3.3920</td>
<td>3.0277</td>
</tr>
</tbody>
</table>

$t$ - values in brackets.
FIGURE 5.1
Graph of Residual Sum of Squares (RSS) against values of \((\rho + \delta)\)
B.M.C.
FIGURE 5.2
Values of Residual Sum of Squares (RSS) for values of
\((\rho + \delta)\) in the range \(-.04 \leq (\rho + \delta) \leq .07\)
B.M.C.
Graph of Residual Sum of Squares (RSS) against values of $(\rho + \delta)$.
FIGURE 5.4
Values of Residual Sum of Squares (RSS) for values of $(\rho + \delta)$ in the range $-0.04 < (\rho + \delta) < 0.07$
Graph of Residual Sum of Squares (RSS) against values of \((p + \delta)\)

Standard Triumph
Values of Residual Sum of Squares (RSS) for values of $(\rho + \delta)$ in the range $-.04 < (\rho + \delta) < .07$

Standard Triumph
FIGURE 5.7
Graph of Residual Sum of Squares (RSS) against values of \((p + 6)\)

Vauxhall
FIGURE 5.8
Values of Residual Sum of Squares (RSS) for values of 
$(\rho + \delta)$ in the range $-0.04 \leq (\rho + \delta) \leq 0.07$

Vauxhall
FIGURE 5.9
Graph of Residual Sum of Squares (RSS) against values of (p + δ)

Rootes
FIGURE 5.10
Values of Residual Sum of Squares (RSS) for values of $(\rho + \delta)$ in the range $-.04 \leq (\rho + \delta) \leq .07$
NOTES TO CHAPTER 5.

1. At the qualitative level, however, the model is largely 'empty'. The reasons for this are now well known, and were extensively discussed by Archibald (1961), (1964) and Lancaster (1962). (A replay of the Chicago - Archibald debate is contained in Rowley (1972).) Archibald (1961) has written of this kind of model "...significant qualitative predictions cannot be obtained without quantitative information."

2. The long-run elasticity of demand with respect to advertising may be easily derived empirically. Suppose that 'goodwill' is given by

\[ \sum_{i=0}^{\infty} \lambda^i A_{t-1} \]

where \( 0 < \lambda < 1 \). Use of this definition of goodwill in the demand function and application of the Koyck transformation yields an estimable demand function. See for example Lambin (1969). The general result of such a treatment is that the long-run elasticity is equal to the short-run elasticity divided by \((1 - \lambda)\). \( \lambda \) (the decay rate) has itself been the subject of many studies, for example Palda (1964); Lambin (1969), (1972); Peles (1971b) and McGuinness and Cowling (1972). Other lag functions may be applied, giving different results. Geometrically declining lags are, however, the most commonly assumed type, and give good results empirically.

3. Lambin (1973) uses this approach extensively. When the demand function is estimated in absolute sales form, the independent variables are expressed in absolute units also. Similarly a market share function has independent variables expressed as shares.

4. This problem has recently been extensively discussed by Schmalensee (1972).

5. Thus Cowling (1972) adopts an advertising appropriations relationship of the form

\[ A'_{r_t} = \gamma_0 S'_{r_t} S'_{r_{t-1}} A'_{r_{t-1}} (1-\rho) \]
where $A_{it}$ is the firm's share of industry advertising in period $t$ and $S_{it}$ is the firm's market share in period $t$. This appropriations relationship is held to be consistent with both theoretical optimality rules and rule-of-thumb behaviour of the type described by Taplin (1959) and Kuehn (1961).

6. Thus we embrace the positive approach to methodology espoused by Friedman (see Friedman (1953)).

7. Clearly one has to assume something about the relationship between unmeasured promotion and the measured advertising input. A possible solution to this problem was given in Chapter 1, note 7.

8. Evidence from an advertising appropriations relationship estimated by Cowling suggest that retaliatory advertising was important. Allowing for this in the car market reduces the observed optimality ratio by about half.

9. Samuel's model is, however, open to severe criticism. Other studies which examine the relationship between advertising and sales, but are not necessarily directed toward discovering whether actual firm behaviour is optimal or not from the economic point of view are Vidale and Wolfe (1957); Palda (1964); Telser (1962b); Kitchener and Rowland (1971); Sexton (1972) and Schnabel (1972).

10. Indeed it is this feature of the model which allows us to estimate it at all. Most models assume that the price of advertising is constant, a highly unrealistic assumption. (See McGuinness in Cowling et al. (1973)).

11. As pointed out earlier 5.7 suffers from the simultaneity problem if it is to be treated as an advertising appropriations relationship.

12. Clearly when 
\[ (\rho + \delta) - \frac{\Delta P_{A_t}}{P_{A_t}} \approx 0 \]
small changes in the chosen value of \( \rho + \delta \) will result in very large changes in the value of the term \( \frac{R_t}{P_{A_t}} \frac{1}{(\rho + \delta) - \Delta P_{A_t}} \) resulting in the level of determination of
the regression altering very rapidly as \((p + \delta)\) approaches \(\Delta p_{A_t}/p_{A_t}\). \((n + 1)\) discontinuities will occur because of the
lagged variable \(\frac{R_{t-1}}{p_{A_t-1}} \frac{1}{(p+\delta)-\Delta p_{A_t-1}}\).

13. British Motor Corporation, Ford, Vauxhall, Rootes and Standard Triumph. Over the period of observation British Motor Corporation and Standard Triumph merged. Rootes has been taken over by Chrysler since 1968. Sales data was provided by the companies involved. Advertising data was obtained from *Statistical Review of Press and T.V. Advertising*, Legion Publishing Co. The price index of advertising is from McGuinness in Cowling et al. (1973). The advertising and sales data was originally collected for other purposes, notably papers by Keith Cowling and John Cubbin in connection with research undertaken with finance from the Centre for Industrial, Economic and Business Research, University of Warwick. I am extremely grateful for the use of this material.

14. It would be of interest to apply the model to a market which on the basis of other tests seemed to be characterised by 'optimal' advertising behaviour on the part of its constituent firms. The margarine, instant coffee and toothpaste markets may fall into this category. (Cowling (1972).)

15. Such an equation is proposed by Jacquemin (1971).
CHAPTER 6.

THE ANALYSIS APPLIED TO QUALITY.

INTRODUCTION.

This chapter attempts to explore the problems which arise when quality differences between similar products are integrated into a model of demand. Previous work in the field is reviewed, and an attempt to define 'quality' is made. The possible relationship between prices and 'qualities' in a group of competing products is discussed, and in particular the so-called 'hedonic' technique is considered.

DEFINITIONS.

Marketing men and management scientists often speak of product 'quality' as being an important element in the marketing mix. Another, but similar, usage of the term appears in the term 'quality control'. Economists often refer to the concept of 'quality competition'. The consumer takes 'quality' into account when making a purchasing decision, or when comparing current to past products. Thus the housewife may complain that the 'quality' of potatoes or milk is not as 'good' as it was. Clearly the word 'quality' is being used differently in these examples. It is therefore helpful to have a definition of 'quality' before proceeding to incorporate the term into a theory of demand.

Given a product, which is already specified in the sense that its characteristics have been decided upon, its 'quality' may vary because the quality of the raw materials and labour inputs incorporated in its

- see Griliches (1961).
manufacture may vary. Hence variation in 'quality' occurs between different units of the same variety or 'brand'. Firms recognise this and usually employ some checking procedure to ensure that the finished product is up to a minimum standard. On a different level the housewife may well claim that potatoes are of 'poorer quality' because a greater percentage of them are damaged or inedible, are difficult to scrape, or become mushy when boiled. She will frequently be prepared to pay a higher price for potatoes which do not have as many faults. In this case the housewife is concerned with the 'quality' differences which occur between different brands or varieties of the same product. It is this latter source of quality differentiation, the inter-variety type, which is of importance to economics. 'Quality' control, where 'quality' differences are of the within brand or variety type, is of importance to production managers and those who wish to optimise at the production line level.

We may briefly distinguish four different views of 'quality' which appear in the economics literature. The simplest view occurs in the early attempts to incorporate quality into the marginalist framework; an example of this approach is contained in the work of Dorfman and Steiner (1954). A continuous index of quality is assumed, where quality is defined as "....any aspect of a product, including the services included in the contract of sales, which influences the demand curve." Any number of different quality indexes can be incorporated into the model provided the demand function is a continuous and differentiable function of price and each quality index. One extra marginal condition can be derived for each quality index added to the demand function. A graphical analysis of the problem, with only one quality index, but which nevertheless contains some useful insights, has been given by Abbott (1953, 1955).
Brems (1967) argued that some aspects of product quality are non-quantitative (and could not be incorporated into a continuous index of the Dorfman - Steiner type). He therefore proposed a model which borrowed the methods of input-output analysis. The model is not discussed here since it turns out to be marginalist in practice and the results are directly comparable to those derived by Dorfman and Steiner.

Lancaster (1966), amongst others, has recognised products as 'bundles of characteristics'. Drawing on this concept he has presented a theory of consumer behaviour based on activity analysis postulates. This conception of 'product' is useful and is retained for this thesis; however, considerable attention will have to be paid to the question of what is meant by 'characteristics'.

Griliches (1961) and Griliches and Adelman (1961) were concerned with the failure of price indexes to take full account of quality changes through time. The problem was tackled on a theoretical and a practical level. Griliches (1961) points out that the reason why different varieties of the same product sell at different prices "must be due to some differences in their properties, dimensions, or other 'qualities', real or imaginary."¹ The idea of the so-called 'Hedonic Price Principle' is thus quite simple: to derive implicit specification prices from cross-sectional data on the price of various varieties or 'models' of the particular product.¹ The discussion of 'quality' presented below takes in ideas from Abbott, Griliches and Lancaster.

Following Abbott (1955) we may make a distinction between 'vertical' quality differences and 'horizontal' quality differences. An

¹ Griliches (1961, p. 175)
attempt is made to make the Abbott distinction more rigorous by defining the two types of quality differences on the basis of the preference functions associated with them.

Vertical quality differences exist when one variety of a product can be said to be "better" or "worse" than another. For example, if we have two tyres which are identical in the services they provide to the consumer except that A gives more mileage than B, then we can say that A is "better" than B. More strictly we can say that for all rational consumers for whom mileage enters into the utility function (assumed to be a monotonic increasing function of mileage) in a positive way $A \succeq B$. If $\mu$ is taken to be some increasing index of mileage then $A(\mu) > B(\mu) \Rightarrow A \succeq B$, ceteris paribus. More generally consider any two varieties of the same product, A and B say, which are identical except for the level of the vertical quality $\mu$. If $P_A$, $P_B$, $Q_A$ and $Q_B$ are the prices and sales in units of varieties A and B respectively and all consumers are rational, then

$$A(\mu) > B(\mu) \Rightarrow A \succeq B \quad \text{for all consumers}$$

and if

$$A(\mu) > B(\mu) \quad \text{and} \quad P_A < P_B, \quad \text{then} \quad Q_B = 0.$$ 

In the second case if consumers are rational only A will be produced. If $A(\mu) > B(\mu)$ and both A and B are produced, we expect $P_A > P_B$. We expect therefore to observe an increasing one-to-one correspondence between levels of $\mu$ and prices for different varieties of the same product. An implication of the above is that if the level of vertical quality in a given variety of a given product is increased, then the demand curve for that variety will shift to the right. Also the total production cost is an increasing function of the level of vertical quality. Hence if both
Consider two varieties of a product, A and B, which are identical except in one aspect. If ARB for some consumers and BαA for some others (irrespective of the cost differences between A and B) then the difference between A and B is a horizontal quality difference. Examples might be the different paint finishes on cars, or the choice between automatic and manual transmission on cars.

PROBLEMS.

The two-way classification proposed above does not have the desirable property of completeness. There are quality differences which conform to neither of the preference patterns given above. Three examples of the problems which can arise when attempts are made to use the distinction given above are given together with possible solutions.

(i) The differences in cost between mild and bitter beer are not incidental; mild usually costs less to produce and is often cheaper to buy. Some consumers prefer mild to bitter and vice versa. There may even be some consumers who would switch from one to the other if the price differential were altered. The same argument can be applied to similar journeys by coach and train: that is, the same preference pattern can be identified. For many purposes, for example explaining the market shares of various brands, we can consider mild and bitter as being separate products, that is
having separate market demand curves. If, on the other hand, we are directly interested in the share of passengers carried by coach and rail from C to D, then coach and rail journeys should be considered as different varieties of the same product. The definition of product needs to be modified in the light of the particular problem under consideration.

(ii) Consider two cars which are identical except that one is self-coloured and the other is a two-tone model. The two-tone version may cost more, but there will be consumers who prefer a plain coloured car to a two-tone car even if both were offered at the same price. Provided the two-tone version is offered as an option common to all varieties of the product at roughly the same price, and provided the proportion of consumers who prefer two-tone cars is the same for each variety, then no practical problems are created in the explanation of demand by vertical quality differences. The sales of the two-tone variety may be added to the sales of the plain coloured variety. This technique for dealing with multi-specification varieties (optional extras on cars etc.) has been widely used. (See Cowling and Cubbin (1971a), (1971b); Cowling and Rayner (1970); Griliches and Ohta (1973).)

(iii) The third example poses more difficult questions. Consider two tables which are identical except that one is constructed of pine and the other of oak. The oak table may be very much more expensive, but some consumers may purchase the pine table even if it were more expensive than the oak table. Again the nature of the problem under investigation dictates the solution. In many cases the analysis would have to be carried out for the two types of table separately, with the condition that the measures of quality employed are the same in both cases.

The three examples given above illustrate the practical importance of defining 'quality' closely, and of attempting to define the differ-
ent types of quality difference with reference to the assumed preference patterns attached to those qualities differences. A consideration of individual preferences is of help when dealing with aggregates since it allows us to shed some light on some of the tricky problems of defining 'product'.

The distinction between vertical and horizontal quality differences proposed above relies on the idea of monotonicity. It may be argued that the "axiom of monotonicity" applied in the goods (rather than characteristics) case is one of the least acceptable elements of consumer theory. We must therefore be wary before attempting to define types of characteristics on the basis of monotonic or non-monotonic types of preference patterns. As set out above the proposed distinction is intended to help in the problem of selecting the relevant quality characteristics of goods for inclusion in a model of variety demand. From that point of view we must be reasonably sure that the distinction is going to be helpful in the actual job of selecting characteristics. What matters from the theoretical point of view is that the distinction does indeed identify those characteristics which influence demand in a particular way common to all the selected characteristics. Thus the proposed distinction must avoid the rather unhelpful definition of quality as anything which influences the demand curve but is not otherwise taken account of by the demand function.

We may advance two major reasons why few, if any, characteristics will exhibit the properties of a 'vertical' quality. Firstly it may be argued that in practice other things never are equal and hence for whichever characteristic is under consideration more of it will have bad as well as good effects. Thus a longer car may well be roomier, but it will also be more
difficult to park. Secondly it may be argued that monotonicity must eventually break down. Just as a surfeit of champagne may have unpleasant effects, a car which stops too quickly or goes too fast may be very dangerous. This second problem is fairly easily overcome. It makes little sense to consider, for example, all cars as varieties of the same product. For many consumers estate cars and high-powered sports cars may not be substitutes. By defining the product group more closely the problem can be avoided. Thus the empirical work in this thesis applies to only small and medium family saloon cars at the lower end of the market. Nevertheless a large portion of the car market is covered by such varieties.

The first objection is more difficult to overcome. Nevertheless in theory it may well be possible to identify vertical quality differences on the basis of the distinction given above. Consider the variable 'length' in cars. Within reasonable bounds it seems sensible to suggest that a longer car will always be chosen given equal prices, provided that all other things really are equal, including the turning circle and ease of parking. It is alright to treat length as a vertical quality difference if 'parkability' is included in the price-quality regression, allowing the ceteris paribus assumption to be satisfied, statistically at least. If 'parkability' is not included we have to recognise that there is a problem. Thus the distinction does not rely on our actually being able to identify two models which, in the real world, conform to the ceteris paribus requirement; all we require is that in theory if the ceteris paribus assumption was satisfied we may then reasonably assume that the given pattern of preferences would be observed. By contrast the variable 'weight' is not a vertical quality characteristic of cars since it is not possible to assume that all consumers will prefer a heavier (or lighter) car given that all other characteristics were held equal. In practical terms the question really depends on how many other characteristics the ceteris paribus
assumption embraces. It is reasonable to include petrol consumption as a vertical quality if some measure of other aspects of engine performance are also included. With a variable such as length rather more variables will have to be covered by the ceteris paribus assumption before we can admit length as a vertical quality difference.

THE PRICE - QUALITY RELATIONSHIP.

More strictly this section is concerned with the relationship between the set of prices and the set of vertical quality differences which exists between different varieties of the same product. Horizontal quality differences are ignored; however, it may be argued that horizontal quality differences, possibly because of their incidental cost, are least important in those product groups where vertical differences are most important (consumer durables, houses, some services, etc).

Within the same product several vertical quality differences may exist. The problem is that the different vertical qualities are not commensurable. The discussion above indicates that differences in the vertical qualities attached to varieties of the same product are revealed in prices. By estimating the function

6.1 \[ p_i = f(X_{ji}) \]

where \( p_i, i=1,...,n \) say, are the prices of the \( n \) varieties, and \( X_{ji}, j=1,...,k, i=1,...,n \), are the levels of the \( k \) qualities embodied in the \( n \) varieties, incommensurable quality differences can be converted into commensurable price differentials. However, vertical quality differences may not be the only factors affecting price, so 6.1 may be more
accurately defined as

\[ p_i = g(x_{ji}, z_i, U_i) \]

where \( z_i \) is a vector of variables affecting prices other than vertical quality differences and \( U_i \) is a random disturbance term. Equation 6.2 contains the essence of the so-called Hedonic Price Principle re-advanced by Griliches (1961), but originally appearing in Court (1939). Stigler has called the idea embodied in equation 6.2 "...a fairly straightforward extension of the basic logic." (1966, p. 78)

Equation 6.2 is derived from observation of the real world, the technique used being regression analysis. Regression analysis has two advantages for this purpose. Firstly, proxy variables may be used in place of vertical quality variables for which data is unobtainable (for example, 'comfort' in cars). Secondly, the scales on which the quality levels are measured need only be ordinal, since ordinal scales may be described by a series of dummy variables, but at some cost in loss of degrees of freedom.

Griliches (1961) suggests that the existence and usefulness of a function such as 6.2 is an empirical rather than a theoretical question, and that 6.2 "...can always be made into a tautology by specifying enough factors or qualities." (Griliches (1961, p. 175, footnote)). Both these statements are misleading. If the arguments \( x_{ji} \) of 6.2 refer only to vertical quality differences, then the existence of a relationship such as 6.2 is clear from the arguments given above, and further 6.2 will not be tautologous unless the variables \( z_i \) are specified to allow it to be so. Care has to be exercised in the job of selecting the variables \( x_{ji} \) (and \( z_i \)). Based on the discussion above a potential variable \( x_{ji} \) will be admitted
if it satisfies the following criteria:

(i) the variable must be a vertical quality difference, or bear a constant relationship to a vertical quality difference, where 'vertical quality difference' is defined as above.

(ii) 'options' (as in example (ii) above) must not be included. The question of 'options' and intra-variety variation is discussed in Appendix 6.1.

(iii) no vertical quality variable should be included which is not in principle applicable to all the varieties in the product group. If some quality was specific to a sub-set of the available varieties there is a possibility of incommensurability. If we wished to construct a hedonic price series for the commodity 'buckets', a dummy variable indicating whether metal buckets were galvanised or not would be inadmissible since plastic buckets cannot be galvanised. The relevant variable to include might be the "ability to resist rust formation". These criteria are somewhat more demanding than those which have been previously employed, and would exclude some variables which have been used in previous studies (see the discussion concerning the variable "weight" in studies of automobile sales below). Griliches and Adelman (1961) state that "...naturally, only those quality dimensions whose coefficients are statistically significant should be retained for the quality index." A quality dimension might not have a significant coefficient because not all varieties exhibited that quality dimension, although in principle they could all do so. 5

An example of what happens when a variable which is inadmissible by the above criteria is actually included in the price-quality regression as an independent variable is provided by Triplett (1969). He considered
the use of the variable "weight" to explain new (U.S.) passenger car prices. Use of the weight variable leads to good results in terms of explanatory power, but for confusing reasons. The desired underlying relationship (in the Triplett study at least) was expressed as

6.3 \[ p' = Zh \]

where \( p' \) is some transformation of the prices of the varieties (e.g. \( \log_{e} p \)), \( Z \) is a vector of 'quality levels' and \( h \) is a vector of the implicit prices of those qualities. Weight, \( w \), is included in the vector \( Z \). But weight is also a function of the quality levels, since any increase in engine size and power, luxury, provision of optional extras, etc. adds to the weight of the automobile. Hence there exists a relationship

6.4 \[ w = Zg \]

where \( g \) is a vector of coefficients on the quality levels. Thus Triplett argues that when weight is included as a variable in the hedonic price series the actual equation estimated is of the form

6.5 \[ p' = c + dw \]

where the coefficient on the weight variable, \( d \), reflects a complex relationship between the \( h \)'s, the implicit prices of the true quality attributes. The use of weight as a quality variable is particularly naive if intertemporal comparisons are to be undertaken. It is undoubtedly true that modern technology permits lighter and stronger automobile bodies and frames than was hitherto possible. We might expect weight to explain price to an extent in a product like television sets, but no one wishing to buy a portable set would think extra weight a desirable attribute, ceteris paribus.
The discussion given above would be of very little practical concern if the explanatory power of hedonic price series was in practice normally poor. Fortunately, the hypothesis that a large proportion of the variance between prices of coexisting models can be explained by the variance in the levels of the characteristics embodied in these models is well supported by the available evidence. The "state of the art" with respect to the hedonic price hypothesis has been reviewed by Griliches (1967, 1971). Typically, writers report high values for $R^2$ and correspondingly low standard errors. The (hedonic price) hypothesis has most frequently been examined for automobiles. Results for U.S. cars have been reported by Griliches (1961); Griliches and Adelman (1961); Griliches and Ohta (1973); Triplett (1969); and Dhrymes (1967, 1971). Cowling (1972) and Cowling and Cubbin (1971a, 1971b, 1972) report results for the U.K. car industry. The tractor market has been analysed for the U.S. by Fettig (1963) and for the U.K. by Cowling and Rayner (1970); Rayner and Cowling (1970); and Rayner (1968). The investigations of refrigerators in the U.S. by Dhrymes (1967, 1971) illustrate the extensive use of dummy variables to describe ordinal quality scales. An interesting application of the hedonic approach applied to a regional housing market has been supplied by Cubbin (1970). Most of the studies cited employ the functional form

$$\ln p_i = \alpha + \sum_j b_j x_{ij} + u_i$$

6.6
to estimate the price-quality relationship, where $p_i$ is the price of the $i$th variety and $x_{ij}$ is the level of the $j$th quality embodied in the $i$th variety. 

* see also Triplett (1966).
A MODEL OF DEMAND INCORPORATING VERTICAL QUALITIES.

Much of the work using the hedonic price principle has been directed towards the problem of adjusting cost-of-living indexes for the quality changes which take place through time. Thus the problem (recurrent in the literature) of trying to discover whether a change in quality has taken place, and what the effects of that change are, need not concern us here. In particular we need not worry unduly about the problems associated with the introduction of "new commodities". At any point in time the varieties of a particular product available have all survived the competitive environment. Two potential problems are evident with any price-quality relationship. Burstein (1961) has shown that if a variety of a product incorporates a new feature, and is sold in an imperfect market, the inference cannot be drawn that the new feature is more highly valued by all consumers than the cost differential attributable to that feature. This is merely a reflection of the fact that in an imperfect market the partial elasticity of demand with respect to any quality dimension is less than infinite. Dhrymes (1967) points out that different firms may employ different pricing policies, so that the same (vertical) quality difference is differently evaluated by different firms, hence there may be some doubt concerning the quality contribution of a particular vertical quality to the price of a product. Whether or not these two problems are important depends on the interpretation given to the price-quality relationship, and the interpretation given depends on the use to which it is put. We have argued above that at any point in time there exists a relationship between prices and vertical qualities, and that the relationship will be monotonic and increasing with respect to each of the vertical qualities (except for a disturbance term). Neither of the potential problems mentioned by Burstein

* see Nicholson (1967).
The discussion above indicates that in theory there exists a perfect monotonic increasing relationship between prices and vertical qualities. Its observations consist of data relating to different varieties of the same product, all of which have survived the competitive environment at some point in time. Estimation of the price-quality relationship for any particular product yields a residual for each variety, denoted in 6.2 by $U_i$, which following Cowling et al. we may call the "quality-adjusted price." Since the $U_i$'s are derived from regression analysis, $U_i = 0$ for the average observation.* A negative residual indicates that when vertical quality differences are taken into account, the particular variety is priced lower than average; in everyday terms it is a "good buy". A positive residual indicates the reverse: if only vertical quality differences are taken into account, then the particular variety appears as a "bad buy", given the existing array of prices and associated vertical quality levels. We might expect that in industry equilibrium all the quality-adjusted prices would be equal to zero. This will only be true if all firms in the industry face identical demand curves, and by implication identical cost curves, since by the time equilibrium is reached all high cost producers will have been driven out of the market. We will be able to observe non-zero quality adjusted prices because of the distribution in tastes or because firms have different costs.

Typically, markets will be in disequilibrium and a range of quality-adjusted prices will be observed.† Quality-adjusted prices may be incorporated as arguments in a demand function since they indicate whether varieties are "good" or "bad" buys. This approach has been used by Cowling and see the articles referred to above.

* provided that the usual assumptions about the error term are met.
† but see the arguments given below.
his co-workers. Their method does however have problems. Typically, the observations incorporated into the price-quality relationship are weighted by sales or market share, the argument being that such weighting makes the relationship "more demand oriented". If the resulting value of quality-adjusted price is in its turn used to estimate variety demand, then demand or market share appears on both the left and right hand sides of the estimating equation. It does not help if we derive the weight to be applied by looking at the same range of varieties during some other time periods when tastes were the same. A heavily advertised variety which would have a positive quality-adjusted price in an unweighted regression may sell quite well, but would be weighted spuriously. If we are attempting to adjust a price index for quality changes through time then weighting the observations by sales is a desirable thing to do. But we are attempting to explain sales on the basis of vertical quality differences, and hence an unweighted price-quality relationship of the sort described above is indicated. We might reasonably argue that weighted observations reflect the pricing policies of firms whilst unweighted observations reflect genuine consumption choice.

At this stage of the discussion three possible methods of incorporating vertical quality differences into a variety level demand function present themselves.

(i) We may follow the reasoning of Cowling and include quality-adjusted price in the demand function directly. We would expect the partial relationship between quality-adjusted price \( (U_i) \) and sales \( (q_i) \) to have the form shown in figure 6.1. 

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\( ^4 \) see Cowling and Rayner (1970).
\( ^+ \) Cowling (1972) and also Cowling and Cubbin (1971a).
A possible relationship between quality-adjusted price and demand at the variety level.

Varieties which have a positive $U_i$ would still be expected to sell in the disequilibrium situation because of the existence of brand loyalty or horizontal quality differences. The approach is particularly useful when the focus of attention is not quality competition but advertising competition, since it provides a neat way of allowing the demand function to make reference to qualities without including them explicitly.

(ii) (i) may be extended by including both list price (or some transformation of it) and quality-adjusted price as independent variables. No econometric problems are presented since list price and quality-adjusted price (the residuals from the estimated price-quality relationship) are uncorrelated by assumption. A variety with a relatively high list price may exhibit low sales because it is beyond the means of a large section of the market; there may be an income effect at work. (i) is only an appropriate method if we assume that income does not have any differential effects on demand. The relationship between list price and demand will clearly depend on the shape of the Engel curve for that commodity.

* In practice the degree of correlation between $U_i$ and $P_i$ will depend on the specification of the price-quality relation.
(iii) We could adopt the method used by number compilers, that is to estimate a price-quality relationship using a 'large' number of vertical qualities as independent variables, and then incorporate directly into the demand function only those which turned out to be statistically significant. Many writers have suggested that even for a complex product the number of such variables will not exceed six or seven. For some consumer durables three or four variables may suffice. Use of this method will allow the derivation of quality elasticities and thus provide some of the necessary information for a test of the Dorfman - Steiner theorem as it relates to qualities. This point is taken up in Chapter 6. A potential disadvantage is that a high degree of multicollinearity is likely to exist between the qualities, and good estimates of the quality elasticities hard to obtain.

Economists usually deal with equilibrium prices and quantities in their attempts to employ demand analysis, although typically most markets will be in disequilibrium. As indicated above there is no real reason to believe that all quality-adjusted prices will be zero for markets in equilibrium. The fact that different firms face different cost functions, that there is a distribution of tastes across consumers and that product information is not a free good to consumers are all factors which permit the existence of a range of quality-adjusted prices in equilibrium. In a less than perfect world there are doubtless other reasons as well. In this thesis I have followed the practice of previous writers in relating values of quality-adjusted price to levels of market share. Of course it could be argued that if in theory and/or in practice quality-adjusted prices are a measure of market disequilibrium then values or quality-adjusted price should be related to changes in market share in an attempt to explain market adjustment processes. It is difficult to see, however, why quality-adjusted prices (measured in the way

* in the sense of estimates with minimum possible variance. The existence of a high degree of multicollinearity will lead to erroneous attribution of influence on the dependent variable amongst the independent variables.
suggested above) should reflect disequilibrium in the market. For example if there is general excess demand in the market then this should be reflected in the prices of all varieties in the market, the price of a given variety will be higher than it would otherwise be, but that is true of all varieties. All that might happen is that the origin around which the quality-adjusted prices are distributed will have shifted. In addition the 'shape' of the distribution may well be altered, but that is not important unless we are attempting intertemporal comparisons of quality-adjusted price; such comparisons are only permissible in limited circumstances and under special assumptions (as in the Simple Repackaging Hypothesis of Chapter 7). If there is excess demand for particular varieties then the model may be inappropriate.

A REVIEW OF SOME PREVIOUS STUDIES.

The only existing studies which attempt to analyse demand relationships using the hedonic technique as shown above are those by Cowling and Rayner (1970) and Cowling and Cubbin (1971a). One feature of the work (taken together) is an attempt to test the Dorfman - Steiner theorem. Cowling (1972), (a development of the work contained in Cowling and Rayner (1970) and Cowling and Cubbin (1971a)) reports results for five U.K. markets: cars, tractors, margarine, instant coffee, and toothpaste. The results for the three non-durables markets were consistent with the Dorfman - Steiner predictions given the assumptions of the model. For the two durables markets the results were far from satisfactory in this respect. Cowling (1972) suggests several possible reasons why this difference might occur. With so few observations it is impossible to generalise the result that advertising appears highly sub-optimal in the two durables markets
(cars and tractors) to other durables markets. It may well be that advertising plays a different role in durables markets to that which it performs in non-durables markets. The studies for the non-durables markets did not use the hedonic technique.

List price, quality-adjusted price and advertising may not be the only variables which influence demand at the brand or variety level. Brand loyalty or horizontal quality differences may also have an effect on demand. The use of dummy variables to distinguish brands in the estimation of the demand function could pick up some of these differences. The systematic differences which may exist between brands but which are not captured by differences in levels of the included quality variables, called "make effects" by Griliches and Ohta (1973), are clearly potentially important determinants of demand. ("Make effects" measure omitted variables.) Griliches and Ohta (1973) confirm the existence of "make effects" in the U.S. automobile market. The use of one dummy variable for each variety is clearly impossible in a cross-sectional study. However, "make effects" (including brand loyalty) may accrue to the products of a particular manufacturer rather than to a particular variety. Thus a person who has previously owned a B.M.C. Mini may demonstrate brand loyalty by buying a B.M.C. 1300. The number of dummy variables that would have to be included might then be fairly small. Cowling (1972) did not use dummy variables to distinguish manufacturers, but preferred to exclude "makes" where such effects were likely to be greatest.¹³ The coefficients on included dummy variables may often be interpreted in the light of observed facts about the market.

This chapter has dealt with two major topics so far. Firstly an attempt is made to define 'quality' and secondly a method of incorporating 'quality' differences into a model of demand was suggested. It seems
logical to move on to a consideration of how the introduction of quality variables affects our ideas on inter-firm competition.

**THE NATURE OF QUALITY COMPETITION.**

In line with the preceding discussion we restrict our attention to consumer durables markets. In such markets there can be little doubt that the desire for variety exhibited by consumers in the aggregate is a very genuine one. A private motorist may desire a car for several reasons: to go to work, to take the family on holiday, to transport goods, to facilitate shopping, etc. The particular combination of transportation services required dictates the choice of model. The focus of attention in this section is not, however, on consumption; rather, we wish to consider 'quality competition' in the context of an oligopoly. In particular the effects of quality competition on oligopolistic interdependence are considered.

The car market illustrates the problems of quality competition very well. It is oligopolistically structured, products are differentiated, innovation is important, the product is complex and industry demand fluctuates. We may reasonable suppose that each of the big firms in the U.K. car market recognises their mutual interdependence. In particular, we may assume that if one firm attempts to increase its market share in the short-run by some means, then rival firms will retaliate in some way.

Price and quality competition are linked to a much greater extent than price and advertising competition. Quality change may thus provide a means of implicitly changing price, or masking a price change.
A manufacturer may alter an existing model or replace an existing model by a new one, thus altering both price and quality. Such price changes differ from price changes in the absence of specifications changes in two important ways:

1) they take longer to accomplish. Whereas the price of an existing model can be altered very quickly, quality changes can take much longer from inception to maturity.¹⁴

2) rivals (and consumers) may well be uncertain about the extent of the implicit price change.

Both features render reaction unlikely in the short-run. In the former case this is simply a result of the lags between a change being made by one manufacturer and retaliation by another manufacturer. In the second case a hurried assessment of a complex situation may lead to a needless or inappropriate price change. Retaliation is likely to be postponed until the market signals that it is needed; that is, until there is a perceptible shift in sales or market shares. It is difficult to forecast what the nature of retaliation to an implicit price/quality change will be. Retaliation in the form of a new model introduction may be impossible unless a new model has already been substantially developed and can be produced with the existing plant. Reductions in the prices of existing models is possible, but it is a strategy which runs the risk of further reaction in the form of reductions in prices. Thirdly, a manufacturer may 'revamp' existing models as a substitute for the introduction of a new model, or as a stop-gap policy until a new model is in production. The extent to which existing models can be revamped may be limited, and there will still be some retaliatory lag before the revamped model reaches the market.
Thus the fact of physically differentiated products in a given market mitigates some of the important aspects of oligopolistic interdependence. Two testable predictions follow from this argument. Firstly, the demand for new models should be less price elastic than the demand for existing models, because they offer a new mix of characteristics and have fewer close substitutes than existing models. Secondly, a price index adjusted for quality changes should show more flexibility (i.e. greater variation about trend), particularly in a downward direction, than an index derived solely from list price changes. Cowling and Cubbin (1971b) tested this prediction for the U.K. car market and found it to be upheld for the period 1956-68.15

It is clear that the role played by new models is crucial not only in quality competition, but also price competition. We might also note that advertising strategies often seem to be linked to new model introductions. New models differ from established models in three respects. Firstly, new models incorporate cost reducing innovations. This point is discussed above. Secondly, new models may have fewer close substitutes than existing models. Thirdly, buyers are less able to make an accurate evaluation of new models. It is because new models incorporate cost reducing innovations that the argument that new model introductions leave the way open for implicit price reductions has so much force. In the opposite situation, where the prices of existing models have to be held fairly stable in an environment of rapidly rising costs, new model introductions allow implicit price-cost margin increases.16 The hypothesis that new models have fewer close substitutes than existing models relies for its appeal on several factors. New models incorporate the latest technology and in style-type goods (e.g. cars) the latest styling. Insofar as
new technology renders the new product superior to existing models, the effect is to reinforce the effect of modern styling, to make the new model markedly different from existing models. In some sense new models belong in a different vintage. If the models in question have a fairly long market life we might expect there to be substantially fewer new models than existing models. 17

From a theoretical standpoint the brief discussion in this section carries the implication that the Cournot model may well provide a reasonable approximation to reality where quality competition is concerned. The twin difficulties faced by would-be retaliators of evaluating a quality change and responding to it given existing production capabilities are the major reasons why retaliation may be seen as unlikely. The model of Appendix 7.1 implicitly assumes Cournot-type reaction to quality changes.

SUMMARY.

This Chapter has discussed the nature of quality competition and attempted to derive an operational classification of types of quality differences. It was suggested that the relationship between the prices of a set of varieties of a given product and the vertical quality levels embodied in them is monotonic and increasing with respect to each quality level of the vertical type. In all everyday situations the relationship is a stochastic rather than an exact one. The possibility of explaining demand by quality-adjusted prices (the residuals from the estimated price-quality relationship) was explored. Finally brief consideration was given to the nature of quality competition and it was concluded that the Cournot hypothesis was applicable to quality competition situations.
APPENDIX 6.1.

'WITHIN MODEL' QUALITY DIFFERENCES, DUMMY VARIABLES, 'STRIPPING', AND THE SELECTION OF OBSERVATIONS.

Some products, especially consumer durables, come in a large number of makes, brands, models, variations, sizes, etc. We may distinguish two types of quality differences between the different varieties of a given product. "Within model" quality differences are those quality differences which are exhibited within the range of variations on the same basic model. For example, in December 1971 twelve different variations of the Ford Escort were available from new. Apart from two estate variations and two high performance variations, the remaining eight variations differed only in terms of engine size, the number of doors and the trim provided (and, of course, price). "Between model (or variety)" quality differences are those described in Chapter 5, for example those quality differences which distinguish a Ford Escort from a Vauxhall Viva. Data limitations often force us to confine our attention to between model differences, the major such limitation being that sales data is not normally available for the different variations of the same basic model. Neglect of within model quality differences may not be a tremendous loss provided manufacturers' pricing policies are consistent across the variations of a particular model. In fact the pricing policy of manufacturers seems to be to price the basic variation in a given range of cars and base the prices of the different variations on the basic variation price. Some evidence is presented for British Leyland cars and Ford cars. For British Leyland list prices were regressed on data for the Mini, 1100/1300, Marina, Maxi and 1800 ranges. The British Leyland results are:
\[ \hat{\beta}_1 = 521.58 + 0.127 X_i + 23.77 D_1 \]
\[ + 78.26 D_2 + 59.75 D_3 + 157.08 D_4 \]
\[ + 50.19 D_5 + 32.76 D_6 + 38.72 D_7 \]
\[ + 87.78 D_8 + 172.46 D_9 + 168.05 D_{10} \]
\[ n = 28 \quad R^2 = 0.985 \]

where figures in brackets are standard errors

\[ \hat{p}_1 = \text{list price in pounds} \]
\[ X_i = \text{engine cubic capacity in 100's of c.c:s} \]
\[ D_1 = 0 \text{ if a two door variant, 0 if a four door variant} \]
\[ D_2 = 1 \text{ if variant is a 'Clubman', 0 otherwise} \]
\[ D_3 = 1 \text{ if variant is an estate, 0 otherwise} \]
\[ D_4 = 1 \text{ if variant is a 'GT', 0 otherwise} \]
\[ D_5 = 1 \text{ if variant is in 1100/1300 range, 0 otherwise} \]
\[ D_6 = 1 \text{ if variant is 'deluxe', 0 otherwise} \]
\[ D_7 = 1 \text{ if variant is 'Super', 0 otherwise} \]
\[ D_8 = 1 \text{ if variant is in the Marina range, 0 otherwise} \]
\[ D_9 = 1 \text{ if variant is 'TC', 0 otherwise} \]
\[ D_{10} = 1 \text{ if variant is a Wolseley, 0 otherwise} \]
Model ranges included in the Ford results are all Escort variants (except the two high performance variants), and all Cortina and Capri variants. The results are as follows:

\[
\hat{\beta}_1 = 632.04 + 15.07 X_i + 26.79 D_1 \\
(84.4) \quad (16.66) \quad (3.01)
\]

\[+ 44.88 D_2 + 76.39 D_3 + 91.24 D_4 \\
(4.73) \quad (8.2) \quad (8.02)
\]

\[+ 230.33 D_5 + 123.4 D_6 + 153.87 D_7 \\
(16.43) \quad (14.33) \quad (14.64)
\]

\[+ 108.11 D_8 \\
(10.5)
\]

\[n = 23 \quad R^2 = 0.990\]

where figures in brackets are standard errors

\[\hat{\beta}_1 = \text{list price of the } i^{\text{th}} \text{ variant in pounds}\]

\[X_i = \text{engine cubic capacity in 100's of c.c.'s}\]

\[D_1 = 0 \text{ if variant has 2 doors, } 1 \text{ if variant has 4 doors}\]

\[D_2 = 1 \text{ if variant has luxury interior trim, } 0 \text{ otherwise}\]

\[D_3 = 1 \text{ if variant is in the Cortina range, } 0 \text{ otherwise}\]

\[D_4 = 1 \text{ if variant has luxury interior and exterior trim, } 0 \text{ otherwise}\]

\[D_5 = 1 \text{ if variant is 'GXL', } 0 \text{ otherwise}\]

\[D_6 = 1 \text{ if variant is an estate, } 0 \text{ otherwise}\]

\[D_7 = 1 \text{ if variant is in the Capri range, } 0 \text{ otherwise}\]
\[ D_g = 1 \text{ if variant is a 'GT', } = 0 \text{ otherwise.} \]

Differences between the pricing policies of the two manufacturers do exist. Ford charge over twice as much for an estate version on average than do B.L.M.C. The possibility arises of an Escort saloon being a better buy than a 1300 saloon, but a 1300 estate being a better buy than an Escort estate. On the other hand, both manufacturers charged about £25 extra for a four door saloon. We might then adopt the approach of including both the standard saloon and standard estate versions in the price-quality relationship, but exclude other variants. The results indicate some of the differences which arise when the available data is not sufficiently disaggregated to distinguish between versions of a model.

Another related problem exists. Within a product group some varieties may have features as standard which other models do not, (but might offer as optional extras). Two solutions are available. Dummy variables may be included to cover the features, or the prices of those varieties which do not include features available as standard on other varieties can be adjusted by adding on the supposed prices of the missing features. This second method is (paradoxically) known as 'stripping' and has been successfully used by Cowling and Rayner (1970) in their study of tractors and Griliches and Ohta (1973) in their study of the U.S. automobile market. Both methods are needed to minimise specification error. If the feature involved is, say, a power braking system, the correct variables to be employed in the price-quality relationship might be stopping distance, ease of operation of the brakes, etc. To include such measures is normally impossible, and hence some other method of allowing for the problem is called for. The method chosen for dealing with the problem will normally depend on the feature under consideration. Inclusion of a dummy variable for the feature is preferable,
but in many cases wasteful in terms of degrees of freedom. 'Stripping' may then be a useful technique when the missing feature is available as an extra at relatively small cost.
1. Or at least Griliches (1961) holds that it is that simple. Even Griliches and Ohta (1973) dismisses most of the problems associated with the Hedonic Price Principle and maintains the 1961 line. However, other writers feel that the H.P.P. is beset by quite difficult problems and many of these are discussed in Chapters 6 and 7.

2. In common with the Lancaster (1966) system of classifying products, the current system does not require a consideration of 'new' products as such. 'New' products are seen as offering a different combination of existing vertical qualities, but incorporating 'innovational quality changes'. We may roughly define an innovational quality change as a change which is considered an improvement by some buyers in spite of what additional cost is involved (if additional cost is involved) so that the new variety of the product is bought along with existing varieties. The new variety may displace one or more of the existing varieties. The introduction of a new variety will alter the relationship between the set of prices and the set of quality levels; the monotonic nature of the relationship is however preserved.

3. The particular problems exhibited here are only a subset of the older collection of problems encountered when trying to define 'product'. All three examples point out particular "breaks" in the substitution pattern between potential varieties of the same product. Houthakker (1951) makes the same point when he writes "... it is hard to say exactly what makes an item of consumption a quality of some more comprehensive commodity, rather than a commodity by itself". (p. 155)

4. In nearly all practical cases we assume either $Z_i = 0$ for all $i$, or $\frac{\partial x_{ij}}{\partial Z_i} = 0$ for all $j$ and all $i$.

5. The discussion leans towards including only 'performance' variables in the price-quality relationship. The question of whether perform-
ance variables or physical characteristics variables are more appropriate is taken up in Chapter 7.

6. The equations 6.3 and 6.4 might be more accurately written
$$ p' = Z'h $$ and
$$ w' = Z'g $$ where $$ w' $$ and $$ Z' $$ are some transformations of $$ w $$ and $$ Z $$. 6.3 and 6.4 will then still permit the use of linear regression techniques, but the estimating equations are more generalised. 6.5 would then be
$$ p' = c + dw' $$.

7. Griliches (1971) contains several of the papers subsequently referred to, as well as a comprehensive bibliography of work performed up to 1971.

8. A full discussion of the appropriate functional form for the price-quality relationship is contained in Chapter 7.

9. 'Industry' is here defined as that group of firms or divisions of firms which produces the varieties used as data in the price-quality relationship.

10. Further evidence to support the use of unweighted price-quality relationships is presented in Chapter 7.

11. We are here implicitly assuming that the price-quality relationship is linear, yielding residuals $$ U_i $$ such that $$ -\infty < U_i < +\infty $$ with $$ \overline{U_i} = 0 $$ . For price-quality relationships of other functional forms the argument can easily be appropriately altered.

12. For evidence see Cubbin (1970); Cowling and Cubbin (1971b, 1972); Cowling and Rayner (1970); Dhrymes (1967); Griliches (1961, 1967, 1971); Griliches and Adelman (1961); Lancaster (1967); Rayner (1968); Rayner and Cowling (1967); Triplett (1966, 1969). Although fourteen references are cited some papers are based on the same (or very similar) estimates of the price-quality relationship. The papers by Cowling and Cubbin refer to an estimated relationship for U.K. passenger cars; by Cowling and Rayner to tractors; and by Griliches (1961, 1967) and Griliches and Adelman (1961) to U.S. passenger cars.
13. In particular Jaguars and Rovers were excluded from the sample partly because of the high reputation of Jaguars for "excellence" and Rovers for "reliability". Both "excellence" and "reliability" were omitted quality variables.

14. There may be specifications changes which can be rapidly executed. A car manufacturer could decide to offer radial tyres instead of cross-ply tyres as standard on a given model, thus implicitly reducing price. Retaliation could then be immediate.

15. Cowling and Cubbin felt that their results were consistent with the notion of a firm facing a kinked demand curve with respect to list price changes. The kink could be avoided by changes in quality. Appendix 4.1. refers to the period 1967-71 and in the latter part of this period the reverse kink seems to offer a better explanation.

16. In periods of severe cost inflation manufacturers may be prevented from putting up the prices of existing models as much as they would like, either for reasons of losing goodwill or legal reasons (as in periods of "price restraint"). New model introductions then provide a means of restoring price-cost margins, although the extent to which margins can be restored will depend on prices of existing models. Thus in periods when there are limits to the rate of increase of prices we would expect a quality-adjusted price index to be more flexible in an upward direction than a crude price index.

17. Precisely when a 'new' model becomes 'old' is difficult to say, but presumably the transfer depends more on the rate of introduction of new models rather than time per se.

18. The data refers to specifications as at December 1st. 1971 and was taken from the publication "Motorists Guide to New and Used Car Prices", Blackfriars Press Periodicals Ltd.
19. The insignificance of the cubic capacity variable, $X_i$, is probably due to the fact that the dummy variables taken together almost completely determine the engine capacity. This is particularly true for the B.M.C. observations where brand names almost completely determine engine size. Again the brand names largely determine engine size in Ford cars. In particular the smallest cars are all Escort variants. The terms GXL and GT do not necessarily imply larger engine capacities.

20. It is highly unlikely that advertising data will be so disaggregated since one piece of 'copy' is frequently best employed promoting an entire model range.
CHAPTER 7.

THEORETICAL FOUNDATIONS FOR THE HEDONIC TECHNIQUE.

INTRODUCTION.

The chapter considers two distinct areas. Firstly, a consideration is given to the possible sources in microeconomic theory of a theoretical underpinning for the hedonic technique presented in Chapter 6. Such a consideration will throw some light on three questions which frequently appear in the literature on hedonic techniques. The questions are:

(i) what are the appropriate weights to be used in the price-quality regression?

(ii) what is the appropriate functional form for the price-quality relationship?

(iii) should the qualities in the price-quality relationship be 'performance variables' or 'physical characteristics'? Performance variables enter the consumer's utility function, but not the cost function and vice versa for physical characteristics.

The second area considered is that of incorporating information gained by application of the hedonic technique into a model of optimal quality-setting behaviour at the firm level. To this end the Dorfman - Steiner theorem is restated in terms of implicit prices on qualities. This second topic is dealt with in Appendix 7.1.
THEORETICAL FOUNDATIONS OF THE "HEDONIC TECHNIQUE" 1

We may identify three possible theoretical models underlying the hedonic technique.

(A) The household production model.

This model is strictly neoclassical in spirit. It combines elements from both the marginalist models of consumption and production to produce a model of 'household production'. The idea was first fully worked out by Muth (1966) who stated the central hypothesis of the household production model as "...commodities purchased on the market are inputs into the production of goods within the household." Such production was characterised by conventional production functions. The goods produced, in turn, were arguments of a conventional utility function of the household. The household purchases market commodities \(x_1, \ldots, x_m\) in order to jointly produce the goods \(z_1\) and \(z_2\) which yield utility according to

\[ U = U(z_1, z_2) \]

We assume that only two goods are produced for the sake of simplicity of exposition; such an assumption does not restrict any of the results to be proved. Suppose that the transformation function between commodities and goods is

\[ F(x_1, \ldots, x_m, z_1, z_2) = 0 \]

and that 7.2 is homogeneous in both commodities and goods and has the usual neoclassical properties of a convex production possibility frontier given the levels of the inputs (commodities) and concave isoquants given

1 Muth (1966, p. 699)
the levels of the outputs \((Z_1, Z_2)\). Assume that the household faces the budget constraint

\[ y = \sum p_i x_i \quad i = 1, \ldots, m \]

The household maximises utility subject to the budget constraint 7.3. The solution to the utility maximisation problem is two-stage. Firstly, the cost of producing any given bundle of goods is minimised, the result will be a cost function with commodity prices and goods as arguments. Utility can then be maximised subject to the constraint of the (derived) cost function. The solution has been fully worked out by Muellbauer (1972). However, here we are only interested in deriving a demand curve on the basis of the model. We proceed as follows. For the first part of the solution we minimise

\[ C = \sum p_i x_i \]

subject to the transformation function

\[ F(x_1, \ldots, x_m, Z_1, Z_2) = 0 \]

The solution to this part of the problem is the cost function

\[ C = \pi_1 Z_1 + \pi_2 Z_2 \]

where

\[ \pi_j = \frac{\partial C}{\partial Z_j} \quad j = 1, 2 \]

The second part of the solution yielding the derived demand curves for the goods \(Z_1\) and \(Z_2\) is derived by maximising the utility function 7.1
subject to the cost function 7.5. The demand functions for $Z_1$ and $Z_2$ are then the solutions to the equations

7.7 \[ \frac{\partial U}{\partial Z_1} + \lambda \frac{\partial c}{\partial Z_1} = 0 \]

7.8 \[ \frac{\partial U}{\partial Z_2} + \lambda \frac{\partial c}{\partial Z_2} = 0 \]

7.5 \[ \pi_1 Z_1 + \pi_2 Z_2 - c = 0 \]

where $\lambda$ is the value of the Lagrangian multiplier. In order to consider the neoclassical household production model outlined above as a basis for the hedonic technique, some far-reaching assumptions have to be made. Take, for example, the problem of applying the model to the market for cars. In order to make sense of the model the commodities $x_1, \ldots, x_m$ must be taken as physical characteristics and the goods $Z_1$ and $Z_2$ as performance variables. The justification for the hedonic approach is then supposed to be the cost function 7.5. It is easy to see that 7.5 can only provide a justification for the hedonic approach if the household technology is non-joint and exhibits constant returns to scale. These two conditions will ensure the following necessary results:

(i) the cost function 7.5 will be linear, i.e. the cost function will be

7.9 \[ c = c_1 Z_1 + c_2 Z_2 \quad c_1, c_2 \text{ constants} \]

(ii) $C = c$ is equal to the 'income' $y$ of the household.

Let us take the second condition first. We suppose that the household operates under a two-tier system in deciding its consumption behaviour. It first of all decides how much it wishes to spend on a car, thus determining its budget constraint for the second tier of the process (described above).
If there is a large number of different varieties of car available the household should be able to find one particular variety which meets its requirements; that is, one which embodies the desired levels of the performance variables at the correct price. Note that an essential assumption in this argument is that all available cars satisfy the relationship

$$P = \sum p_i x_i$$

where the price of the car is $P$ and $p_i$ are the $m$ prices of the $m$ physical characteristics $x_i$, $i = 1, \ldots, m$. The first condition above is crucial, since if and only if it is met will consumers having the same household technology but different tastes be facing the same shadow costs. In short, it is essential for the hedonic technique that the $p_i$'s be independent of $Z_1$ and $Z_2$, of $U$ and the household's income. It is easy to show intuitively how 7.9 arises from the conditions of non-jointness and constant returns to scale in the household technology. Consider the output combination $Z_1 = l$, $Z_2 = 0$. Let the minimum cost of producing this combination be $\sum p_i x_i = c_1$, where $x_i$, $i = 1, \ldots, m$, are the optimal levels of the inputs required to produce the given output combination. If there are constant returns to scale the optimal input levels to produce the output combination $Z_1 = Z_1^*, Z_2 = 0$ will be $x_1 x_1^*, i = 1, \ldots, m$, with minimum cost $c_1 Z_1^*$. The same argument can be applied to the output combinations $Z_1 = 0$, $Z_2 = 1$ and $Z_1 = 0$, $Z_2 = Z_2^*$, where $c_2 = \sum p_i x_2 i$. If there is non-jointness in the production of $Z_1$ and $Z_2$ the minimum cost of producing any combination of $Z_1$ and $Z_2$ must be $c_1 Z_1 + c_2 Z_2$.

It is not possible on the basis of this model alone to derive a demand curve for cars in general or a particular model of car. However,
in a perfect world all available models would conform to the condition

\[ P_k = c_1Z_{1k} + c_2Z_{2k} \]

where \( P_k \) is the price of the \( k^{th} \) model and \( Z_{jk}, j = 1, 2 \) are the levels of the performance variables embodied in the \( k^{th} \) model. The world is not perfect however, and hence relationship 7.11 will not be exact. We may therefore argue that demand for specific varieties will be related to the residuals from an estimation of 7.11 in the manner suggested in Chapter 6. Thus the price-quality relationship indicated on the basis of this model has the following features:

(i) the appropriate estimation technique is unweighted linear regression;
(ii) the appropriate functional form is linear;
(iii) the independent variables should be specified as performance variables.

The objections to the use of this model as a justification for the hedonic hypothesis are clear. Perhaps the least restrictive of the assumptions embodied in the model is that of constant returns to scale in the household production function. On the other hand the assumption of non-joint technology is harder to swallow. For example, the performance variables 'fuel economy', 'acceleration' and 'top speed' are clearly the results of joint production. The corollary of the non-joint, constant returns case is that if one of the assumptions is broken then even if all consumers have the same utility function the shadow costs \( (\Pi_1 \text{ and } \Pi_2) \) will vary according to the household's income, and the model will no longer yield an equation consistent with the hedonic hypothesis.
(B) The Lancaster Model.

In general suppose that we can write the household production relation as

\[ Z_1 = f_1(x_1, \ldots, x_m) \]

\[ Z_2 = f_2(x_1, \ldots, x_m) \]

The consumer then maximises utility subject to the twin production constraints \( \text{7.12} \) and the budget constraint. The Lagrangian function is

\[ L = U(Z_1, Z_2) + \lambda_1(f_1(x_1, \ldots, x_m) - Z_1) + \lambda_2(f_2(x_1, \ldots, x_m) - Z_2) + \pi(y - \Sigma p_i x_i) \]

In the absence of corner solutions, the marginal conditions are

\[ \pi p_i = \lambda_1 \frac{\partial f_1}{\partial x_i} + \lambda_2 \frac{\partial f_2}{\partial x_i} \quad i = 1, \ldots, m \]

\[ \lambda_1 = \frac{\partial U}{\partial Z_1}, \quad \lambda_2 = \frac{\partial U}{\partial Z_2} \]

The solution implies that the ratio of the marginal utilities of \( Z_1 \) and \( Z_2 \) must be equal to the ratio of their shadow costs of production (\( \pi_1 \) and \( \pi_2 \)), i.e.

\[ \frac{\lambda_1}{\lambda_2} = \frac{\pi_1}{\pi_2} \]

Further, at the optimum the marginal utility of any good divided by its price (shadow cost) should be equal to the marginal utility of income; i.e.

\[ \frac{\lambda_1}{\pi_1} = \frac{\lambda_2}{\pi_2} = \pi \]
Thus the marginal conditions 7.14 may be written

\[ p_i = \pi_1 \frac{\partial f_1}{\partial x_i} + \pi_2 \frac{\partial f_2}{\partial x_i} \quad i = 1, \ldots, n \]

A specific example of this kind of model is that due to Lancaster (1966). In this model we have:

\[ Z_1 = \sum b_{1i} x_i \]
\[ Z_2 = \sum b_{2i} x_i \]

that is, each unit of market commodity \( x_i \) is composed of a fixed amount \( b_{1i} \) of \( Z_1 \) and \( b_{2i} \) of \( Z_2 \). Figure 7.1 shows the production possibility frontier corresponding to a given income and given market prices where there are three market commodities.

Figure 7.1. Production Possibility Frontier in the Lancaster Model.
The shadow prices $\pi_1$ and $\pi_2$, corresponding to the linear programming problem of minimising the cost of purchasing a given bundle $Z_1$, $Z_2$ subject to $x_i \geq 0$ for all $i$ and the given prices, can be calculated provided there is divisibility and more market commodities than $Z$'s (goods). For purchased goods

$$7.20 \quad p_i = \pi_1 b_{1i} + \pi_2 b_{2i}$$

7.20 corresponds to 7.18 for this particular problem. If the $b_{ji}$'s (the amounts of the $j^{th}$ good embodied in the $i^{th}$ market commodity) are observed, linear regression can be used to estimate the shadow costs given that imperfections in the market prevent 7.20 from being an exact relationship. Thus the Lancaster model may be seen as a particular example of the household production model, which although it imposes constraints on the forms of the production functions for $Z_1$ and $Z_2$ (they must be linear and additive) does overcome the problem of jointness in the production of $Z_1$ and $Z_2$.

If consumers' indifference curves are similar enough in shape, they will all choose points on the same ray from the origin, and for a producer to sell anything he must price according to 7.20. The arguments of Chapter 6 may then be applied to the problem of choosing a suitable demand function.

If the indifference curves are dissimilar, different consumers will choose points on different rays from the origin. It may be reasonable, however, to define the market commodity group fairly closely with respect to prices, and to weight the observations of 7.20 by value shares thus picking up some form of median relationship.
The model is immediately applicable without the need to redefine the \( x \)'s as physical characteristics. A potentially serious objection is that we will wish to apply the model to market commodities which are not divisible, for example cars. It is likely that any model we attempt to construct would fall foul of this objection. It is possible to view the household as optimising as though it would choose the perfect car, an amalgam of features from existing cars, and then picking the available variety closest to the optimum. The price-quality relationship indicated on the basis of the Lancaster model has the following features:

(i) the appropriate estimation technique is linear regression, unweighted or weighted by value shares according to the assumptions made about the likely distribution of tastes;
(ii) the appropriate functional form is a linear one;
(iii) the independent variables should be specified as performance variables.

Both the household production model as described under heading (A) and the Lancaster model as described under heading (B) indicate a linear form for the price-quality relationship. In practice the most frequently encountered empirical form is the semi-log function

\[
\log p_i = \sum b_j x_j^6
\]

It is easy to show that such a relationship cannot result from the household production model. Suppose the production functions are of the form

\[
Z_j = f_j(b_j, \ldots, b_m; x_1, \ldots, x_m) \quad j = 1, 2
\]

From 7.18 we have
but 7.21 implies that

\[ p_i = e^{\frac{\partial f_1}{\partial x_i} + \frac{\partial f_2}{\partial x_i}} \]

Thus the production functions must satisfy the relationship

\[ \prod_i b_1^i + \prod_i b_2^i = e^{\frac{\partial f_1}{\partial x_i} + \frac{\partial f_2}{\partial x_i}} \]

which is impossible except for trivial versions of \( f_j(i) \).

(C) The Simple Repackaging Hypothesis.

One (and probably the only) alternative to the household production model as an underlying model of the hedonic technique is the "simple repackaging hypothesis". This hypothesis was advanced by Fisher and Shell (reprinted in Griliches (1971)), and states that a quality improvement in a particular good is equivalent to more of the old good. Hence market goods of a particular kind can be aggregated; the aggregate is simply the sum of the quality indices weighted by the number of units of each good purchased. Formally we write the utility function

\[ U = U(x_1^*, x_2, \ldots, x_n) \]

where \( x_1^* \) is the aggregate of goods of type \( x_1 \) as described above and \( x_2, \ldots, x_n \) are other market goods. \( x_1^* \) is the sum of the quality indices weighted by the number of units of variety \( x_1 \) purchased, i.e.

\[ x_1^* = \sum_{j=1}^{m} a_j x_{1j} \]
where \( m \) varieties of product \( x_1 \) are purchased and

\[
a_j = g(b_{1j}, b_{2j})
\]

where \( b_{1j} \) and \( b_{2j} \) are the amounts of the characteristics \( b_1 \) and \( b_2 \) embodied in variety \( j \). Clearly for 7.26 to be a rationale for the hedonic technique (in the sense of the term as used in this thesis) the \( a_j \)'s must be prices, then the variable \( x^*_1 \) is defined as the total amount spent on varieties of product \( x_1 \). Alternatively (since it is not very helpful to just define arguments of the utility function as expenditures on various products) we must assume that some relationship exists between prices of varieties of product \( x_1 \) and the \( a_j \)'s. The advantage of the simple repackaging hypothesis is that it gives a rationale for adjusting price indices for the quality-changes which take place through time. 8 Nothing could be simpler than regarding quantities of existing, improved goods as more of the goods which existed in some previous period. The simple repackaging hypothesis does not yield a price-quality relationship which can be incorporated into a theory of demand. At best the simple repackaging hypothesis suggests a relatively stable (through time) technological relation between goods and prices. The form of the price-quality relation is a priori unknown. Simple repackaging implies an unchanging technological relationship of the form of 7.26 through time. An appropriate estimation technique would then be pooled time series cross-section regression using time dummies to pick up price changes common to all the varieties of the product. By contrast, the Lancaster hypothesis implies the use of single-year regressions, since even if tastes remain the same through time, the set of varieties available will change and there will be no guarantee that the implied price-quality relation will be stable through time.
Chapter 6 essentially discusses the empirical approach to the problem of specifying price-quality relations, although an attempt is made to apply some basic theory to the question of selection of independent variables. Chapter 7 discusses the possible theoretical backgrounds for the price-quality relationship. In the course of Chapters 6 and 7 the further distinction was made between "physical characteristics" and "performance variables".

Both the empirical and theoretical approaches to determining the form of the price-quality relationship have advantages and disadvantages, given the current state of knowledge. Ideally we would like to base the specification on a sound theoretical model. However it has to be admitted that the three models proposed above are somewhat limited; indeed several misgivings about each model are cited above. Of the three the simple repackaging hypothesis is the least specific and leaves several specification problems to be settled by the empirical approach. The objections to the household production model are so severe that it is not considered further as a 'live' alternative. The Lancaster model results in a linear price-quality relationship only because linearity is imposed on the transformation functions 7.12. It may be argued that the linearity restriction on functions 7.12 is undesirable, but the fact remains that only the linear additive production functions 7.12 yield a potentially useful (in the sense of simple and being capable of estimation) price-quality relationship.†

The balance of the available empirical evidence seems to be in favour of a semi-log form for the price-quality relationship (see Chapter 6).

† see footnote 7 to this chapter and section (B) above.
although some writers have successfully employed other functional forms. If the theoretical evidence of this Chapter points to any functional form for the price-quality relationship in particular, then it suggests that the linear one might be the most useful. The empirical work in this thesis concentrates (with some exceptions) on the use of the linear price-quality relationship for two major reasons. Firstly, little empirical work has so far been done employing linear price-quality relationships and, secondly, despite some misgivings, I believe it does have desirable theoretical properties which derive both from its inherent simplicity (it yields constant shadow prices on qualities for example) and from the arguments given above. Favouring a linear price-quality relationship here is not meant as a criticism of work which has employed other functional forms.

Neither is the choice (in empirical work) between the use of physical characteristics and performance variables as explanatory variables in the price-quality relationship as clear cut as the arguments of this Chapter might suggest. In practice physical characteristics and performance variables will be causally related, indeed that is the nature of the transformation functions 7.2 (in the Household Production model) and 7.12 (in the Lancaster model). Hence in practical terms it will make little difference whether performance variables or physical characteristics are used in empirical work; physical characteristics may often stand as proxy performance variables.

None of the above means that we may ignore the theoretical approach completely. After all an empirical procedure without an underlying theory (as the Hedonic technique in its Griliches (1961) form is) must always be regarded with some suspicion even if, a priori, it does seem highly plausible. At the very minimum the theory of this Chapter does shed some light on the specification problem, and suggest specifications which might otherwise have been ignored.
SUMMARY.

The discussion of Chapters 6 and 7 as it relates to the price-quality relationship is summarised in Table 7.1. The argument presented in Chapter 6 is characterised by the term 'local approximation theory'. The neoclassical household production model is not included because of its limited applicability. The only space which has not been previously dealt with is the estimation technique column of the local approximation theory row. Clearly whether we regard the price-quality relationship as being reasonably stable through time is a matter of assumptions. Stability will obviously depend on the state of technology, manufacturers' pricing policies and the stability of the industry. There are good reasons why we might opt for the pooled cross-sectional regressions in testing the local approximation theory. Consumers may not only consider the current array of available varieties, but also the varieties available in previous time periods, given that price levels for all goods may not be the same currently as in the past. This assumption will be particularly valid if we are dealing with a repeat purchase. Such intertemporal comparisons are only valid if the price-quality relationship is fairly stable. In particular innovation may render intertemporal comparisons difficult.

The local approximation theory suggests that consumers take the array of prices and qualities set by firms as given; any interaction between firms and consumers takes the form suggested in Chapter 6. Price-quality relationships derived from theories of household technology take the opposite point of view: firms are assumed to take the price-quality relationship preferred by consumers as given. The simple repackaging hypothesis is based on the assumption of a fixed technology through time and the derived price-quality relationship must come from firms. Although the specification of the price-quality relationship is derived from either firms or consumers exclu-
### TABLE 7.1. Summary of theoretical implications for the price-quality relationship.

<table>
<thead>
<tr>
<th></th>
<th>Functional Form of P-Q Relationship</th>
<th>Estimation Technique</th>
<th>Weighting</th>
<th>Specification of Independent Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local Approximation Theory</td>
<td>Unspecified</td>
<td>(i) Single period regressions (ii) Pooled cross-sectional regression with time dummies (if P-Q relation is assumed stable through time)</td>
<td>None</td>
<td>Vertical qualities +</td>
</tr>
<tr>
<td>Lancaster Hypothesis</td>
<td>Linear</td>
<td>Single period regressions</td>
<td>None if tastes assumed similar Value shares if tastes dissimilar</td>
<td>Performance variables</td>
</tr>
<tr>
<td>Simple Repackaging Hypothesis</td>
<td>Unspecified</td>
<td>Pooled cross-sectional regressions with time dummies</td>
<td>None</td>
<td>Physical characteristics</td>
</tr>
</tbody>
</table>

**Notes:**
- " or market shares in absolute units if prices of all varieties are approximately equal.
- + and possibly dummies for manufacturers to pick up 'make' effects.
ively it will be modified by the normal longer-run market processes. Var-
ieties which do not fit into a price-quality relationship derived from con-
sumption behaviour will die, births (i.e. new varieties) which are success-
ful will alter the entire price-quality relationship no matter whether it
is derived from a theory of consumption or production.
APPENDIX 7.1.

AN EXTENSION OF THE OPTIMALITY CONDITIONS TO PROVIDE A TEST OF OPTIMAL QUALITY SETTING AT THE FIRM LEVEL.

Suppose that the demand function facing the firm may be written

\[ q_i = q_i(p_i; k_{l_1}, \ldots, k_{l_m}; x_{l_1}, \ldots, x_{l_n}; Z) \]

where \( x_{l_1}, \ldots, x_{l_n} \) are the levels of the \( n \) qualities which can be measured along a continuous scale embodied in variety \( l \), \( k_{l_1}, \ldots, k_{l_m} \) are the levels of the variables other than price and the \( x \)'s which influence demand and are under the firm's control and \( Z \) is a vector of exogeneous variables which influence demand. If total cost is a continuous twice differentiable function of output and the continuous quality dimensions, the optimality conditions with respect to the continuous quality dimensions may be written:

\[ \frac{x_{ji}}{R_i} \frac{ac/ax_{ji}}{np_i} = -\frac{n_{x_{ji}}}{np_i} \]

where

- \( R_i \) is sales revenue
- \( \frac{ac}{ax_{ji}} \) is the marginal cost of raising the level of quality dimension \( j \)
- \( n_{x_{ji}} \) is the elasticity of demand with respect to the level of quality dimension \( j \)
- \( np_i \) is the price elasticity of demand

7.1.2 is merely a restatement of the generalised theorem of Chapter 2. The problem with attempting to test for optimal behaviour with respect to setting the levels of the \( x_{ji} \)'s is that the costs of increasing quality
are unobservable, that is \( \frac{\partial C}{\partial x_{ji}} \) is unknown. Analysis of a model of demand like that proposed in Chapter 6 will provide a solution to this problem. Suppose that the demand function for variety \( i \) may be written:

\[
q_i = q_i(U_i, k_{i1}, \ldots, k_{i9}, Z)
\]

where \( U_i \) represents quality adjusted price and is determined by \( x_{ji}, i=1, \ldots, n \) and some of the variables \( k_{n1}, \ldots, k_{n9} \). \( U_i \) is strictly the residual from the price-quality regression, thus \( p_i \) and \( U_i \) are related by the function

\[
p_i = p_i(U_i; \hat{\beta}_{i1}, \ldots, \hat{\beta}_{i9}; x_{1i}, \ldots, x_{ni}; k_{i1}, \ldots, k_{i9})
\]

where \( \hat{\beta}_{i1}, \ldots, \hat{\beta}_{i9} \) are the estimated coefficients on the independent variables in the price-quality regression:

\[
\hat{\beta}_i = \hat{\beta}_i(\hat{\beta}_{i1}, \ldots, \hat{\beta}_{i9}; x_{1i}, \ldots, x_{ni}; k_{i1}, \ldots, k_{i9})
\]

The actual specification of the residual depends on the functional form of the price-quality regression, thus

\[
U_i = U_i(p_i, \hat{\beta}_i)
\]

and thus

\[
\frac{\partial U_i}{\partial x_{ji}} = \frac{\partial U_i}{\partial p_i} \frac{\partial p_i}{\partial x_{ji}} + \frac{\partial U_i}{\partial \hat{\beta}_i} \frac{\partial \hat{\beta}_i}{\partial x_{ji}}
\]

where

\[
\frac{\partial p_i}{\partial x_{ji}} = 0
\]
7.1.9 \[ \frac{\partial U_i}{\partial p_i} + \frac{\partial U_i}{\partial x_{ji}} = 0 \]

\[ \frac{\partial U_i}{\partial x_{ji}} \] is the estimated implicit price of \( x_{ji} \)

If the cost function for variety \( i \) is

7.1.10 \[ TC_i = c_i(q_i; x_{i1}, \ldots, x_{in_i}; k_{i1}, \ldots, k_{if}) \]

The profit function for variety \( i \) is

7.1.11 \[ \pi_i = p_iq_i(U_i; k_{i1}, \ldots, k_{if}; z) - c_i(q_i; x_{i1}, \ldots, x_{in_i}; k_{i1}, \ldots, k_{if}) \]

Examining only those profit-maximising conditions which relate to price and the continuous qualities we have

7.1.12 \[ \frac{\partial \pi_i}{\partial p_i} = 0 = (p_i - \frac{\partial c_i}{\partial q_i}) \frac{\partial q_i}{\partial U_i} \frac{\partial U_i}{\partial p_i} + q_i \]

7.1.13 \[ \frac{\partial \pi_i}{\partial x_{ji}} = 0 = (p_i - \frac{\partial c_i}{\partial q_i}) \frac{\partial q_i}{\partial U_i} \frac{\partial U_i}{\partial x_{ji}} - \frac{\partial c_i}{\partial x_{ji}} \]

Division of 7.1.12 and 7.1.13 will yield

7.1.14 \[ \frac{\partial c_i}{\partial x_{ji}} = -q_i \frac{\partial U_i/\partial x_{ji}}{\partial U_i/\partial p_i} \]

And using 7.1.9 we derive the condition

7.1.15 \[ \frac{\partial c_i}{\partial x_{ji}} = q_i \frac{\partial p_i}{\partial x_{ji}} \]

7.1.16 is the usual marginal condition of price theory, that the level of
should be determined where the marginal cost of the additional increment of \( x_{ji} \) is just equal to the (estimated) marginal gain in revenue attributable to that increase. 7.1.15 may be used to eliminate the unknown marginal cost from 7.1.2 to yield the condition

\[
x_{ji} \frac{q_i \hat{\beta}_i / \alpha x_{ji}}{R_i} = -\frac{n x_{ji}}{n p_i}
\]

or

\[
x_{ji} \frac{\hat{\beta}_i / \alpha x_{ji}}{p_i} = -\frac{n x_{ji}}{n p_i}
\]

where \( \frac{\alpha x_{ji}}{x_{ji}} \) is the portion of the price which is accounted for by quality \( j \). The implicit price is derived from the estimated price-quality relation, whilst the elasticities may be obtained by estimating the demand function in the form 7.1.1.

Several points need to be made about this optimality test. Firstly, it is clearly most applicable when the qualities are being defined as physical characteristics and least appropriate when a performance variable definition is being employed. Secondly, the method as described is applied only to qualities which can be measured on a continuous scale. We might argue that it could be applied to qualities which are measured nominally or ordinally in the price-quality relation since ex ante (i.e. at the planning stage) such qualities will be continuously variable. Such might be the case where 'trim' on cars is concerned, but not where the available technology permits only discrete choices, for example between possible braking systems. Thirdly, we are assuming optimality to test for optimality. In this the method is no worse than that used to test for optimality in firms' advertising behaviour. The only difference between the quality and advertising cases is that in one case the price of the discret-
ionary variable is known and in the other it has to be estimated. Fourthly, the method is nothing more than a test, *ex post*, for optimal quality-setting behaviour. Integrating price and quality into a model of firm decision making using quality-adjusted price misses some of the important elements of the price-quality competition problem. A price change is not really symmetric with a change in quality since the changes have quite different characteristics. Quality changes usually require investment and hence the retaliatory lag is quite different to those incurred when price is changed. Differences will also occur in consumption. Although in equilibrium the effect of a price cut will be identical to the effect of a quality increase, both will change quality-adjusted price; one would expect the consumer to reach equilibrium faster in response to a price cut than to a quality change, because of the difficulty in assessing the change and the utility to be derived from it. Although the test itself does not require the estimation of a demand function incorporating quality-adjusted price, the theory behind the test does assume the existence of such a demand function.
NOTES TO CHAPTER 7 AND APPENDIX 7.1.

1. The only work which seems to have been done in this area is that by Muellbauer (1972). The discussion here is based on that paper.

2. The Muth (1966) rather than Muellbauer (1972) terminology is being used here. Commodities are things which are purchased on the market and as such yield no direct utility. They are purchased because jointly they can provide goods which are direct sources of utility and are defined in a neoclassical sense. Thus the purpose of buying an automobile and petrol (commodities) is to yield transport services (goods).

3. Muellbauer (1972) proves this theorem rigorously.

4. Which requires homotheticity and similar tastes, or if tastes are non-homothetic, similar tastes and similar incomes.

5. Or since the prices of market products are all approximately equal, by absolute shares.

6. An examination of the papers contained in Griliches (1971) will confirm this.

7. The Lancaster version of the household production model may be easily applied given other (probably less plausible) production functions to generate alternative price-quality relationships. For example consider the production functions

\[ Z_j = e^{\sum b_{ji} x_i} \quad j = 1, 2 \]

then applying equation 7.18 we immediately generate the price-quality relationship

\[ p_i = \pi_1 b_{1i} e^{\sum b_{1i} x_i} + \pi_2 b_{2i} e^{\sum b_{2i} x_i} \]

Only the linear additive production functions yield a potentially useful price-quality relationship.
8. Even here it has severe limitations, since most quality change will not be of the simple repackaging type. Modern cars may stop more quickly than previous models. This feature augments the safety of the product, but can hardly be considered as yielding 'more' of the old car.

9. 7.1.9 is the condition for a linear price-quality relationship. For a log-linear price-quality relationship \( U_i = \log P_i - \log \hat{P}_i \)

\[
\frac{\partial U_i}{\partial P_i} = 1 \quad \text{and} \quad \frac{\partial U_i}{\partial \hat{P}_i} = -1 \quad \text{given} \quad \frac{\partial P_i}{\partial \hat{P}_i} = \frac{\partial \hat{P}_i}{\partial P_i} = 0.
\]

Incorporation of these relationships into the model of Appendix 6.1 yields the optimality condition \( \frac{\partial \hat{P}_i}{\partial P_i} \frac{\partial x_{ji}}{\partial x_{ji}} = \eta_{ji} \). The difference between this condition and 7.1.17 (the replacement of \( P_i \) with \( \hat{P}_i \)) reflects the different specification of the quality-adjusted price variable.
CHAPTER 8.

A DYNAMIC PRICING MODEL.

INTRODUCTION.

This chapter extends the theme developed at the end of Chapter 6 and attempts to develop a theory concerning the 'life-cycle' of a variety of a product. An optimal pricing path is derived from the theory and compared with actual pricing paths adopted by manufacturers.

DEMAND OVER THE 'LIFE' OF A VARIETY.

We may identify four separate factors which change or shift the demand curve for a given variety through time. We may call these the "age effect", the "competitive reaction effect", the "stock effect" and the "advertising stock effect". The age effect has already been partly dealt with in Chapter 6. It was suggested that new varieties of a consumer durable incorporated style changes and the latest technology. In addition, the bundle of characteristics presented by a new variety should be "more relevant" in market terms than the bundles embodied in existing varieties. That is, given that tastes change, new varieties should embody bundles of characteristics closer to the theoretically optimal bundle than do the varieties which have been on the market for some time. When the variety has been on the market for some time, it will itself gradually become an 'old' variety, and other newer varieties will have entered the market, depressing demand for the variety in question. Two possible predictions follow from such an argument. Firstly, there is the hypothesis of Chapter 6, that demand for new models is less price elastic than demand for old models. Secondly, we
might expect the demand curve for a new model to shift downwards through time, i.e. as the variety "ages".

The "competitive reaction effect" inevitably influences the age effect since it relates to the rate of competitive introduction of new varieties. If the new variety introduced by the firm is successful in sales terms we would expect reaction by rivals to be more likely than if the new model were relatively unsuccessful in sales terms, and hence made little inroad into rivals' sales. That is, we postulate that competitive reaction is prompted by market share or sales considerations rather than with the profitability of the new variety. We assume that rivals have some latitude where their planned new model introductions are concerned. At the very minimum, manufacturers will be able to "revamp" existing models and hence alter the bundles of characteristics they embody. At best, manufacturers will be able to bring planned new model introduction times substantially forward to combat the success of some other firm's new model. Hence, the innovating firm is faced with a dilemma. If the firm introduces the new variety at a relatively low price and it gains a substantial market share, then reaction in the form of retaliatory new model introduction will take place more quickly than if a higher price were charged. The result will be that the age effect will be accelerated and the demand curve will shift downward. If, on the other hand, the firm charges a higher initial price, current demand will suffer but future demand will be higher since the age effect will be less influenced by competitive reaction. There is a trade-off between current and future demand. ²

The "stock effect" reinforces the age effect. The stock effect exists because we are concerned with a durable good. As stocks of the particular variety increase, current demand is depressed. For simplicity we
assume that stocks are a function of cumulative sales. Acting in an opposite fashion to the stock effect is the influence of replacement demand. Replacement demand might exert its influence later in the life of the product than the stock effect.

The "advertising stock effect" works in the opposite direction to the age effect. Old models have an advertising stock associated with them, which is in part reflected by 'brand loyalty'. New models may be more difficult to sell than old models because there are barriers induced by the advertising stock associated with existing models. Williamson (1963a) presents a model in which the cumulative selling expense associated with a variety influences the demand curve for that variety. Schupack (1972) has developed a model in which advertising raises the limit price (by raising entry barriers) and influences the speed at which rivals enter the market. In the current model we again assume that cumulative sales are important, that as more units of the new variety are sold, the new variety becomes more familiar to consumers. The brand loyalty barriers of other models are broken down and the stock of knowledge concerning the new variety increases.

COSTS OVER THE 'LIFE' OF A VARIETY.

Unit costs may be influenced by cumulative output. If the variety is a variety of a complex product, then experience of production may lead to lower unit costs as the production process becomes "debugged"; that is, it may become more efficient through time. Clearly, the notion of falling unit costs as experience grows is similar to the 'learning-by-doing' hypothesis advanced by Arrow (1962) and extended by Sheshinski (1967), although in this case experience is measured by cumulative gross investment.
We assume that at any point in time output and sales are equal. Cumulative sales then exert three distinct influences on the profit position of the firm. Firstly, cumulative sales acts to depress current and future demand via the age effect, competitive reaction effect and stock effect. Secondly, replacement demand and the advertising stock effect are both increased by cumulative sales and tend to bolster demand. Thirdly, unit costs at any given level of output are lower for higher levels of cumulative output (= sales) via an "experience effect".

THE FORMAL MODEL.

We define the variable $r(t)$ such that

$8.1 \quad r(t) = \int_0^t q(t) \, d(t)$

that is, $r(t)$ is cumulative output at time $t$. Let the demand function facing the firm at time $t$ be

$8.2 \quad q(t) = h[r(t)] \, q(p, p_r, Z)$

where

- $p$ is the price charged by the firm
- $p_r$ is a vector of rivals' prices or some index of rivals' prices
- $Z'$ is a vector of variables other than prices and the 'effects' given above which influence demand. They may be endogenous to the firm (e.g., advertising) or exogeneous.

$h[r(t)]$ summarises the effects exerted on the demand function through time by cumulative output and $h[r(0)] = h_0$. 
We might expect that early in the life of the variety $h'[r(t)] > 0$ as the advertising stock is built up; during the "middle life" of the variety $h'[r(t)] < 0$ as the age, competitive reaction and stock effects exert their influence; and finally, $h'[r(t)] = 0$ as replacement demand and the other effects even out. Oligopoly theory suggests that

$$8.3 \quad p = p(P_r)$$

For convenience we write demand function 8.2 as

$$8.4 \quad q(t) = h[r(t)] q^*(p)$$

The path of sales through time is described by the derivative of 8.4 with respect to time, i.e.

$$8.5 \quad \dot{q} = h'[r(t)] \dot{q}^*(p) + h[r(t)] \frac{\partial q^*}{\partial p} \dot{p}$$

where from 8.1

$$8.6 \quad \dot{r} = q(t)$$

We assume that at any point in time unit costs are constant and given by

$$8.7 \quad C(t) = C_L + g[r(t)]C_0$$

where

- $C_L$ is the lower bound of unit costs
- $C_0$ is constant
- $g[r(o)] = g_o$
\begin{align*}
g[r(\infty)] &= 0 \\
g'[r(t)] &< 0
\end{align*}

Thus total costs at any given point in time are given by

\begin{equation}
TC(t) = q(t) c(t)
\end{equation}

The firm maximises the present value of the stream of profits, $V$, where $V$ is given by

\begin{equation}
V = \int_0^T \left[ p(t) - c(t) \right] q(t) e^{-\rho t} dt
\end{equation}

where $\rho$ is the discount rate (taken to be positive and constant) subject to the transition equation

\begin{equation}
\dot{r} = q(t)
\end{equation}

and the boundary conditions $h[r(o)] = h_0$ and $g[r(o)] = g_0$. Using the Pontryagin Maximum Principle\,\(^\dagger\) we form the Hamiltonian function, $H$,

\begin{equation}
H = e^{-\rho t} \left[ (p - C_L - g(r)c_0) h(r)q^*(p) + \lambda h(r)q^*(p) \right]
\end{equation}

after dropping time subscripts. The first-order conditions for an interior maximum of $V$ are\,\(^3\)

\begin{align*}
\frac{\partial H}{\partial p} &= 0 \\
\frac{\partial H}{\partial r} &= -(e^{-\rho t}\lambda) \\
\lambda(T) &= 0 \\
\dot{r} &= q(t)
\end{align*}

\(\dagger\)see Intriligator (1971).
8.11 may be written (after cancellation of the term $e^{-\rho t}h(r)$)

\[ 0 = \left[ p - C_L - g(r)C_0 + \lambda \right] \frac{\partial q^*(p)}{\partial p} + q^*(p) \]

or

\[ \lambda = \frac{-q^*(p)}{\partial q^* / \partial p} - \left[ p - C_L - g(r)C_0 \right] \]

Condition 8.12 may be written (after cancellation of the term $e^{-\rho t}$)

\[ \rho \lambda - \dot{\lambda} = \left[ p - C_L - g(r)C_0 + \lambda \right] h'(r)q^*(p) - g'(r)C_0 h(r)q^*(p) \]

Differentiating 8.15 with respect to time we obtain

\[ \dot{\lambda} = -\left\{ 2 - \frac{q^*(p)}{(3q^*/3p)^2} \frac{\partial^2 q^*}{\partial p^2} \right\} \dot{p} + g'(r)\dot{r}C_0 \]

Substituting for $\lambda$ (from 8.15) and $\dot{\lambda}$ (from 8.17) into 8.16 we obtain the trajectory for $p$ given by

\[ \{ -2 + \frac{q^*(p)}{(3q^*/3p)^2} \} \dot{p} = -\rho \frac{q^*(p)}{\partial q^* / \partial p} - \rho \left[ p - C_L - g(r)C_0 \right] + \frac{h'(r)q^*(p)^2}{\partial q^* / \partial p} \]

(since $\dot{r} = q(t)$).

Given an expression for $\dot{p}$ we may find the trajectory for $q$ by substitution into 8.5. In many practical situations 8.18 will in fact be a good deal simpler. In particular, if the demand function is separable and linear, semi-log linear or log linear with respect to $q$ and $q^*(p)$, then the term $\frac{q^*(p)}{(3q^*/3p)^2} \frac{\partial^2 q^*}{\partial p^2}$ will be a constant. The sign of $\dot{p}$ cannot be unambiguously determined on the basis of qualitative information concerning costs, demand and the discount rate alone. Some quantitative information,
particularly in relation to the demand function, is required in order to determine the sign of, and eventually the path of, $\dot{p}$.

AN APPLICATION OF THE FORMAL MODEL.

The formal model attempts to provide a 'theory' of demand over the life of a particular variety of a product. Several implicit assumptions are made, but one of the most important is that the variety should not change substantially through its life. Thus if major specifications changes are made to the variety then, for the purposes of this model, the variety must be deemed changed. The formal model is here applied to four models of passenger cars. All four cars were small four seat saloons which had a substantial market life, and were unchanged with respect to body shape and style and basic mechanical details during the period under study. Annual data for the four models was available for the period 1957-68 inclusive. Again the data used was made available to me by Keith Cowling and John Cubbin. The only available advertising data was at the company, rather than brand or model, level. Above-the-line advertising does not seem to be an important determinant of demand where the passenger cars are concerned. The index of rivals' prices used was one which was adjusted for quality. It is calculated in Chapter 9. Model A had a very long market life, which is not entirely captured by the data. By 1957 it was already well established, and any advertising stock effects should have ceased to play a major role in determining demand for model A. It was eventually taken off the market shortly after 1968. Model B was introduced in 1959 and is still (Spring 1974) on the market, and looks like being so for some time. Models C and D were both on the market during the period 1959-67 inclusive.

†see Chapter 4 above and Cowling (1972).
RESULTS.

Several different specifications of demand function 8.2 were tried. The results reported in Table 8.1 are for the estimated demand function

\[ MS_i_t = a_0 \left( \frac{P_{i,t}}{P_{r,t}} \right)^{\alpha_1} h[r(t)] V_t \]

where

- \( MS_i_t \) is the market share of variety \( i \) in period \( t \)
- \( P_{i,t} \) is the list price of variety \( i \) in period \( t \)
- \( P_{r,t} \) is the value of the quality-adjusted price index in period \( t \)
- \( V_t \) is a random disturbance term

Using market share as the dependent variable is consistent with the formal model, given some assumptions about the term \( \frac{\partial Q}{\partial q_i} \). For the purposes of this model we assume that \( \frac{\partial Q}{\partial q_i} = 0 \); i.e. that total market size is invariant with respect to the sales of variety \( i \). Given \( \frac{\partial Q}{\partial q_i} = 0 \), multiplicative separable nature of 8.19, and the Cournot assumption

\[ \frac{\partial P_{r,t}}{\partial P_{i,t}} = 0 \]

8.18 becomes (after dropping subscripts)

\[ - \frac{(\alpha+1)}{\alpha} \dot{p} = - \frac{\alpha}{\alpha} p - \rho \left[ p - c_L - c_0 g(r) \right] + \frac{h'(r)}{\alpha} qp \]

A semi-log form of 8.19 yielded results which were very little different from those reported in Table 8.1. Including list price and the value of
the price index separately as independent variables led to higher F-scores in some cases, but by and large wrong signs on price variables. Inclusion of only the price relative reduces multicollinearity.

Two forms of the function \( h[r(t)] \) were applied:

\[
\begin{align*}
8.22 & \quad \ln h[r(t)] = a + b[r(t)] + c[r(t)]^2 \\
8.23 & \quad \ln h[r(t)] = a' + b'[r(t)]
\end{align*}
\]

8.23 is theoretically more appropriate in the case of model A where all the cumulative output effects could be expected to work in the same direction. Results for models B, C and D are reported for the functional form 8.22.

**DISCUSSION OF RESULTS.**

Some of the results of Table 8.1 are encouraging. The price-elasticities of demand, given the Cournot assumption \( \frac{\partial \delta}{\partial r} = 0 \), are of the correct sign and in the cases of models A, B and C of approximately the correct magnitude. On the other hand the coefficient on the price variable is poorly determined. Part of this must be due to the fairly high degree of multicollinearity which exists between the independent variables. The correlation matrices showing the extent of this are given in Table 8.2.

The function \( h[r(t)] \) shows the assumed pattern. For model A \( h'(r) < 0 \) in all years; for models B, C and D, \( h'(r) > 0 \) in the early years and \( h'(r) < 0 \) in later years. Model B was introduced in late 1959 and the turning point of the function \( h[r(t)] \) occurred during 1964. Model C was introduced in 1959 and the turning point occurred in 1960.
Model D followed the same pattern as model C.

Demand information alone is not sufficient to determine the optimal trajectory for price. As 8.21 shows, cost information is also required. However, provided we assume that at no point in time will it be optimal for the firm to charge a price lower than unit cost, we may be able to say something more about $p'$. From 8.21 we have

\[ p' = \frac{c}{a+1} p + \frac{c_0}{a+1} [p - c_L - c_0 g(r)] - \frac{h'(r)}{a+1} q p \]

where

\[ a < -2 \]

\[ 0 < [p - c_L - c_0 g(r)] \]

\[ p > 0 \]

Clearly, the sign of $p'$ depends in part on the sign of $h'(r)$. In the early stages of a model's life it is possible for $p' > 0$ when $h'(r) > 0$. In the later stages of a model's life when $h'(r) < 0$, $p' < 0$ seems more likely. The optimal trajectory for $q$ inacts similar problems: from 8.5 the trajectory for $q$ is given by

\[ q' = h'(r)q q^*(p) + h(r) \frac{dq^*}{dp} p \]

When $p$ and $h'(r)$ have the same sign, which seems likely, the sign of $q'$ is undetermined, given $\frac{dq^*}{dp} < 0$.

It is fairly easy to show that only in the cases of models B and C will the available data yield a likely sounding result. From 8.24 we may derive an expression for the proportional rate of change of $p$:

\[ \frac{p'}{p} = \frac{c}{a+1} + \frac{c_0}{a+1} \left( \frac{p - c_L - c_0 g(r)}{p} \right) - \frac{h'(r)}{a+1} q \]
Given likely values of the discount rate, \( \rho \), and the gross mark-up
\[
\frac{p-C_L-C_g(r)}{p}
\]
the first two terms on the right-hand side of 8.26 will be small. In the case of model A the maximum value of the term
\[-\frac{h'(r)q}{\alpha+1}\]
is approximately \(-0.22\), and hence this term is likely to dominate the expression for \( \frac{\dot{p}}{p} \) and yield the prediction of large and ever-increasing proportional rates of fall of price. In the case of model D values of the term \(-\frac{h'(r)q}{\alpha+1}\) are very high. In the year of introduction of model D (1959) the value of the term was approximately 3. For model B values of \(-\frac{h'(r)q}{\alpha+1}\) lie in the range \(-0.251 \rightarrow 0.043\). For model C the range is \(-0.581 \rightarrow 0.090\). The negative values occur at the end of the model's life and the positive values at the beginning.

**SUMMARY.**

The model of demand proposed early in the chapter was tested using data pertaining to four models of passenger cars. The 'effects' identified theoretically had the predicted result on the demand curve. The estimated price elasticities had the 'correct' sign and were approximately of the correct magnitude. Although it was not possible to construct a theoretically optimal pricing path (because of the lack of cost data) and compare it with the actual pricing path, it was shown that the predicted values of parameters of the demand situation did not lead to a priori "sensible" values for the proportional rate of change of price. However, it would be too much to expect that such a simple model would predict believable changes in decision variables. Two particular comments may be made. Firstly, the specification of the demand function used con-
strains the price-elasticity of demand for a model to be constant through time, whilst the theoretical suggestion was made that this would not be the case. Secondly, if consumers were able to predict firms' behaviour then their behaviour might be altered. Thus if a firm reduces a model's price this might lead to the prediction of further reductions by consumers, leading to a smaller initial demand. Not including this might lead the model to predict large falls in price over time.
Regression results for dynamic pricing model. Dependent variable \( \ln MS_i \).

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MODEL A</th>
<th>MODEL B</th>
<th>MODEL C</th>
<th>MODEL D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>2.9268</td>
<td>0.1500</td>
<td>2.7306</td>
<td>6.5450</td>
</tr>
<tr>
<td></td>
<td>(0.8254)</td>
<td>(0.0265)</td>
<td>(0.8356)</td>
<td>(0.3781)</td>
</tr>
<tr>
<td>( \frac{p_{it}}{p_{rt}} )</td>
<td>-2.9048</td>
<td>-2.1703</td>
<td>-3.2074</td>
<td>-5.8960</td>
</tr>
<tr>
<td></td>
<td>(1.1683)</td>
<td>(0.4904)</td>
<td>(1.4290)</td>
<td>(0.5374)</td>
</tr>
<tr>
<td>( r(t) )</td>
<td>-3.9715</td>
<td>4.5355</td>
<td>4.3514</td>
<td>2.2356</td>
</tr>
<tr>
<td></td>
<td>( \times 10^{-6} )</td>
<td>( \times 10^{-6} )</td>
<td>( \times 10^{-7} )</td>
<td>( \times 10^{-6} )</td>
</tr>
<tr>
<td></td>
<td>(6.1490)</td>
<td>(1.9696)</td>
<td>(0.3286)</td>
<td>(0.2672)</td>
</tr>
<tr>
<td>( r(t)^2 )</td>
<td>-5.6421</td>
<td>-1.7797</td>
<td>-1.2567</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \times 10^{-12} )</td>
<td>( \times 10^{-12} )</td>
<td>( \times 10^{-11} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.8763)</td>
<td>(1.0768)</td>
<td>(0.5561)</td>
<td></td>
</tr>
<tr>
<td>d.f.</td>
<td>2, 9</td>
<td>3, 6</td>
<td>3, 5</td>
<td>3, 5</td>
</tr>
<tr>
<td>F</td>
<td>95.8554</td>
<td>4.2833</td>
<td>5.9493</td>
<td>1.7203</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.9552</td>
<td>0.6817</td>
<td>0.7812</td>
<td>0.5079</td>
</tr>
<tr>
<td>D-W</td>
<td>1.2808</td>
<td>1.4766</td>
<td>1.8570</td>
<td>2.9987</td>
</tr>
</tbody>
</table>

* t-values in brackets
TABLE 8.2
Correlations between the independent variables for the regression given in Table 8.1.

**MODEL A**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(t) )</td>
<td>( 0.8553 )</td>
</tr>
<tr>
<td>( \ln (p/p_r) )</td>
<td></td>
</tr>
</tbody>
</table>

**MODEL B**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(t) )</td>
<td>( 0.9422 )</td>
</tr>
<tr>
<td>( \ln (p/p_r) )</td>
<td>( 0.8580 )</td>
</tr>
<tr>
<td>( r(t)^2 )</td>
<td>( 0.9701 )</td>
</tr>
</tbody>
</table>

**MODEL C**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(t) )</td>
<td>( 0.4455 )</td>
</tr>
<tr>
<td>( \ln (p/p_r) )</td>
<td>( 0.4291 )</td>
</tr>
<tr>
<td>( r(t)^2 )</td>
<td>( 0.9733 )</td>
</tr>
</tbody>
</table>

**MODEL D**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(t) )</td>
<td>( 0.7396 )</td>
</tr>
<tr>
<td>( \ln (p/p_r) )</td>
<td>( 0.6041 )</td>
</tr>
<tr>
<td>( r(t)^2 )</td>
<td>( 0.9714 )</td>
</tr>
</tbody>
</table>
1. The discussion here is related to a consumer durable, but much of what is said can equally well be applied to other types of product.

2. The problem has elements in common with the 'limit pricing' problem. In both cases entry into the market by rivals is related to current aspects of the firm's policy. In particular, the model given here is quite closely related to the limit pricing models of Pashigian (1968) and Phelps and Winter (1970). See also the more general articles by Jacquemin (1972b) and Jacquemin and Thisse (1972).

3. Here, as elsewhere, we assume that the second-order conditions are satisfied.

4. 8.13 is identical to the instantaneous profit-maximisation condition except for the multiplier \( \lambda \).

5. Minor improvements and modifications were made to the models through time, but these modifications tended to be common to all vehicles and largely unadvertised. Thus model A in this study received many minor improvements, including better side lighting, stronger shock absorbers, etc.

6. Hence advertising stock is not included as an independent variable in the estimated demand functions. When it is included its estimated coefficient has the wrong sign in the majority of cases.

7. A sufficient condition for \( p > 0 \) when \( h'(r) > 0 \) is \( h'(r)q > p \). Maximum values for the term \( h'(r)q \) are as follows:

   - Model A = -0.423 approx.
   - Model B = 0.05 approx.
   - Model C = 0.199 approx.
   - Model D = 20.77 approx.

8. They are, however, insignificantly different from zero.
CHAPTER 9.

ESTIMATES OF THE PRICE-QUALITY RELATIONSHIP.

INTRODUCTION.

Chapters 6 and 7 proposed three major alternative bases for the price-quality relationship: the local approximation approach, the Lancaster hypothesis and the simple repackaging hypothesis. The three approaches suggested different underlying forms for the price-quality relationship. This chapter contains estimates of some of these forms of the price-quality relationship and discusses the implications of the results derived from those for the competing hypotheses. The data used refers to U.K. passenger cars during the period 1957-68.

HYPOTHESES TO BE TESTED.

The three underlying theories noted above are not necessarily competing hypotheses. The local approximation approach and the Lancaster hypothesis are both essentially theories of consumption whilst the simple repackaging hypothesis is an assumption about technology, although it could be argued to imply a particular pattern of consumption behaviour. It was shown in Chapter 7 that only the Lancaster hypothesis implied a very definite functional form for the price-quality relationship. In particular it is possible for the simple repackaging hypothesis and the local approximation approach to co-exist. It might not be possible to settle on any one hypothesis no matter how sophisticated the hypothesis testing technique.

Chapters 6 and 7 suggested a theoretical distinction between vertical qualities, performance variables and physical characteristics, and associated these different definitions of 'quality' with the local approximation approach, the Lancaster hypothesis and the simple repackaging
hypothesis respectively. The data available for this study does not, in practice, allow for such sophisticated distinctions to be employed in testing. In fact the same set of independent variables is employed whatever the form of the price-quality regression estimated. Although this is a serious shortcoming, we may justly argue that the set of variables chosen satisfy the vertical qualities definition well and are at least reasonable proxies for a set of performance variables. This point is amplified in the section below.

The data limitations restrict the available distinctions between the theories to questions of functional form of the price-quality relationship. Figure 9.1 shows the different functional forms estimated in this chapter. It also underlines the point that it is not necessarily possible to choose between the simple repackaging hypothesis and the local approximation theory. Only two out of the (large) possible set of functional forms are used, linear and log-linear. The popular semi-log form is ignored on the grounds that it has been well tried by other researchers. The small number of observations in each of the years makes the inclusion of manufacturer dummies in single-year regressions impossible.

DESCRIPTION OF THE DATA.

The available data refers to the 28 saloon cars listed in Appendix 9.1. The list includes all the domestically produced smaller cars available during all or part of the period, but no imported cars. In practical terms 'model' is difficult to define, 'model range' is perhaps a more useful concept. The available data was not sufficiently disaggregated to distinguish the sales of some models within certain ranges. No problems are presented if we aggregate the data to the model range level, provided all the ranges are comparable in 'breadth'. The only model which contravenes this requirement to any extent is perhaps the Mark 2 Cortina (available during the years 1966-68). The major problem is that some observations include variants with different see Chapter 6 and Muellbauer (1972a).
FIGURE 9.1: Relationship of regression forms to theories underlying price-quality relation.

1. **Choice of Theory**
   - **Lancaster Hypothesis**
     - Linear functional form for single-year x-sectional data
     - All consumers have similar tastes?
       - Yes
       - Use unweighted observations by market share
       - No
     - Functional form unspecified for pooled x-sectional data
       - Make effects potentially important?
         - Yes
         - Use manufacturer dummies
         - No
   - **Simple Repackaging Hypothesis**
     - Functional form unspecified for single-year x-sectional data
     - Tastes stable through time?
       - Yes
       - No

2. **Local Approximation**
erent engine sizes (Heralds, Escorts, Vivas etc.) whereas others do not. The data reported treats model ranges as if they were the most popular low-priced variant within the range. One consequence of defining the product fairly narrowly and aggregating the data to the model range level is that in any given year the number of observations is small. Where a given model was available for only a small part of a year, either because it was introduced or discontinued in that year, and consequently had low sales, then the model was not included for that year. In some cases the specifications of a model changed during the course of a year, and the available data did not distinguish sales between the differing sets of specifications. In such cases the data refers to the variant which had the greater market life during the year. In general such changes in specification took place late in the year.

The variables used in the study are:

- the dependent variable: list price
- the independent variables:
  - brake horse-power of model (B.H.P.)
  - length of model in inches
  - the petrol consumption of the model in miles per gallon (M.P.G.)
  - dummy variable for 'gears' = 1 if model has 4 or more forward gears
    = 0 otherwise
  - leg room in the rear of the car in inches
  - dummy variable for 'disc brakes' = 1 if model has disc brakes
    = 0 otherwise
  - dummy variable for 'power brakes' = 1 if the model has power-assisted brakes
    = 0 otherwise

Weights used in some regressions are market shares.

† see Table 9.1.
All the independent variables satisfy the vertical qualities criteria outlined in Chapter 6. One is a performance variable (fuel consumption), B.H.P. cannot be properly considered a performance variable since it does not enter the utility function per se, but it is closely related to those performance variables connected with the power of the model. The remaining five variables are all physical characteristics of the models. The major shortcoming evident in the independent variables is the lack of performance variables, nevertheless it may be argued that the independent variables listed above stand as proxies for the unmeasured performance variables. (see Chapter 7).^8

RESULTS.

The results are summarised in tables 9.1 to 9.6. Tables 9.1 and 9.2 have a direct bearing on the Lancaster hypothesis. Tables 9.3 and 9.4 relate to both the simple repackaging hypothesis and the local approximation theory. Table 9.5 relates to the local approximation theory alone. Table 9.6 points out some of the limitations of the single-year regression results in general. It is convenient to consider the results as they relate to the specific theories as shown in figure 9.1.

(i) The Lancaster Hypothesis.

Tables 9.1 and 9.2 show three features of the results for a possible test of the Lancaster hypothesis. Firstly the coefficients on the variables are in general not significant. Secondly the signs on many of the coefficients are not as theoretically expected. Thirdly, and perhaps most importantly, the coefficients show great variability over time. A fairly natural response to these objections would be to aggregate the observations across years, but the Lancaster hypothesis rules this out. The Lancaster hypothesis predicts that the coefficients will vary through time, but the extent and unpatterned variation shown by the estimated coefficients in tables 9.1 and 9.2 is worrying.

Table 9.6 amplifies the data given for 1966 in table 9.1. The
regression for 1966 was the one which was best determined and is taken as
the example. Clearly confidence intervals for shadow prices in other years
will be wider still. Table 9.6 shows that there is little evidence in table
9.1 for the hypothesis that shadow prices change through time. The structure
of the multicollinearity between independent variables in 1966 is also shown
in table 9.6. The surprisingly low partial correlation coefficient between
length and leg room is peculiar to 1966 only in that it is negative. The
average for the other eleven years is 0.2855. Originally three 'dimensions'
variables were used. The variable measuring width in front of the model has
been omitted from the reported results. It was highly correlated with length
and by and large poorly determined. It would obviously be helpful to reduce
the degree of multicollinearity between the quality variables. Two possible
methods of doing this are available. Firstly we might combine (or eliminate)
those quality variables which occur in fixed proportions. Cowling and Cubbin
(1972) combined the 'dimensions' variables to yield the single quality var-
iable "passenger room". Their results were disappointing; the estimated co-
efficients usually took on the wrong sign and were poorly determined. Here
we have opted to eliminate the width variable. It is also worthwhile consid-
ering combining some of the mechanical performance variables with some dim-
ensions varaibles. Thus performance may be more closely related to the power-
weight ratio than to B.H.P., M.P.G., or length. Secondly we might correct
list price by standardising according to qualities where 'option' prices are
available; in essence we may use the technique of "stripping" discussed in
in Appendix 6.1. This technique has been applied where heaters are concerned,
but unfortunately prices are not available for any other options (e.g. disc
brakes). Results for approximately the same market were reported by Cowling
and Cubbin (1972). Their single-year cross-sections used weighted data and
a wider set of observations, including some much larger models. Although
their regressions were better determined , individual parameter estimates
were still poor. In particular the coefficients on M.P.G. and "passenger
area" usually took on the 'wrong' sign (negative).
We cannot reject the Lancaster hypothesis on the basis of tables 9.1 and 9.2, rather we should look for a more suitable test of it. Any such test would require the two features of more data and the use of performance variables as independent variables. The Lancaster hypothesis is a theory of consumption, i.e. it provides a basis for predicting choice between the available alternatives for consumption. As such it should include choice between new and second-hand varieties of a good, provided there is a well organised second-hand market. Such is the case where cars are concerned. A consumer with sufficient money to purchase a new Mini may well (rationally) prefer to purchase a second-hand Maxi. Immediately we have a new source of data for testing the Lancaster hypothesis. A suitable test of the Lancaster hypothesis where tastes are homothetic over a reasonable range of incomes would be to estimate linear single-year cross-sectional regressions for all new and second-hand cars available in a given price range using performance variables as data. No such estimates of price-quality relationships have been carried out.

(ii) The Local Approximation Theory.

Tables 9.2 and 9.5 present estimates of price-quality relationships consistent with the local approximation theory where intertemporal comparisons are not permitted. Clearly exactly the same objections can be levelled against the results of table 9.5 as were levelled against the results of tables 9.1 and 9.2. When intertemporal comparisons are taken into account the story is somewhat different. The results which are applicable to the intertemporal case are discussed in the following section.

(iii) The Simple Repackaging Hypothesis/Local Approximation Theory (Intertemporal Case.)

A perfectly good case can be made for the argument that the S.R.H. provides a theory of determination of the price-quality relationship from the supply side whilst the local approximation theory provides an explanation
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### TABLE 9.2  
Price-quality regressions: linear; single-year cross-sections; observations weighted by market share.  
t-values in brackets.

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TABLE 9.4 Price-quality regression: linear; pooled data with time and manufacturer dummies; 1957-68 inclusive.

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Rootes dummy variable: 4.5488, t-value: 0.5578
Standard-Triumph D.V.: 27.7114, t-value: 2.8562
Vauxhall D.V.: 5.7925, t-value: 0.4752

n: 171
F: 66.8948
R²: 0.9041
D-W: 2.2507
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<td>0.2289</td>
<td>0.3402</td>
</tr>
<tr>
<td>Dummy Variable for 'gears'</td>
<td>0.0145</td>
<td>0.0623</td>
<td>0.0769</td>
<td>0.0749</td>
<td>0.0512</td>
<td>0.0435</td>
<td>-0.0125</td>
<td>0.0230</td>
<td>-0.0218</td>
<td>-0.0089</td>
<td>0.0050</td>
<td>-0.0130</td>
</tr>
<tr>
<td>Dummy Variable for 'disc brakes'</td>
<td>0.3151</td>
<td>-0.3596</td>
<td>0.2284</td>
<td>0.2422</td>
<td>0.2904</td>
<td>-0.1276</td>
<td>0.0959</td>
<td>0.4468</td>
<td>0.1885</td>
<td>0.2036</td>
<td>0.3621</td>
<td>-0.3802</td>
</tr>
<tr>
<td>Dummy Variable for 'power brakes'</td>
<td>0.0276</td>
<td>0.0144</td>
<td>0.0500</td>
<td>0.0359</td>
<td>0.0061</td>
<td>-0.0027</td>
<td>-0.0431</td>
<td>-0.8335</td>
<td>-1.0833</td>
<td>-1.4571</td>
<td>-1.4571</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>n</th>
<th>11</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>15</th>
<th>16</th>
<th>18</th>
<th>17</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>R²</td>
<td>0.9218</td>
<td>0.9055</td>
<td>0.9159</td>
<td>0.9111</td>
<td>0.8657</td>
<td>0.8199</td>
<td>0.8577</td>
<td>0.8287</td>
<td>0.9367</td>
<td>0.9270</td>
<td>0.9461</td>
<td>0.9272</td>
</tr>
<tr>
<td>D-W</td>
<td>1.5324</td>
<td>1.4516</td>
<td>1.3190</td>
<td>1.1088</td>
<td>0.9593</td>
<td>1.3711</td>
<td>1.9740</td>
<td>1.9730</td>
<td>2.3628</td>
<td>2.3724</td>
<td>2.2326</td>
<td>2.9622</td>
</tr>
</tbody>
</table>
### TABLE 9.6.

Confidence intervals for shadow quality prices; 1966 data.

<table>
<thead>
<tr>
<th>QUALITY</th>
<th>95% C.I.</th>
<th>YEARS IN WHICH SHADOW PRICE SIGNIFICANTLY DIFFERENT FROM 1966 PRICE.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.H.P.</td>
<td>$3.9056 \pm 3.8653$</td>
<td>1968</td>
</tr>
<tr>
<td>Length</td>
<td>$0.2190 \pm 0.8379$</td>
<td>1957, 1958, 1962, 1963, 1965.</td>
</tr>
<tr>
<td>Gears</td>
<td>$-8.7936 \pm 97.94$</td>
<td>1963</td>
</tr>
<tr>
<td>Leg room</td>
<td>$3.4754 \pm 14.357$</td>
<td></td>
</tr>
<tr>
<td>Disc brakes</td>
<td>$47.476 \pm 56.76$</td>
<td></td>
</tr>
</tbody>
</table>

Partial correlation coefficients between qualities; 1966 data.

<table>
<thead>
<tr>
<th></th>
<th>Length</th>
<th>M.P.G.</th>
<th>Gears</th>
<th>Leg room</th>
<th>Disc brakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.H.P.</td>
<td>.6201</td>
<td>-.8288</td>
<td>-.0637</td>
<td>.2486</td>
<td>.6895</td>
</tr>
<tr>
<td>Length</td>
<td>-.4764</td>
<td>-.0698</td>
<td>-.0119</td>
<td>.4068</td>
<td></td>
</tr>
<tr>
<td>M.P.G.</td>
<td></td>
<td>.3000</td>
<td>.0412</td>
<td>-.4709</td>
<td></td>
</tr>
<tr>
<td>Gears</td>
<td></td>
<td></td>
<td>.0924</td>
<td>.2425</td>
<td></td>
</tr>
<tr>
<td>Leg room</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.3118</td>
</tr>
</tbody>
</table>
of how that relationship may be related to consumption behaviour. The two theories may be seen as complementary rather than competing. Tables 9.3 and 9.4 present estimates of the price-quality relationship consistent with both the above theories. The regressions are well determined. Only the leg room coefficient is insignificant. The negative coefficient on fuel economy (M.P.G.) does not imply that, ceteris paribus, a more economical model will be sold for less. Fuel economy is negatively correlated with some of the included quality variables (e.g. B.H.P. and length) and negatively correlated with omitted quality variables, particularly those which would add weight to the car (e.g. optional extras, more padding in seats etc.). Approximately half the coefficients on the 'year' dummy variables are significant in table 9.4. Both the coefficients identifying Ford models and Standard-Triumph models are significant.

The S.R.H. implies a reasonably stable technology through time, whilst the local approximation approach requires reasonably stable tastes. Some evidence supporting these two assumptions is provided by the fact that one model of car included in the sample appeared virtually unchanged with respect to specifications in all twelve years. Several other models appeared in at least nine of the twelve years.

**A QUALITY-ADJUSTED PRICE INDEX FOR CARS.**

We may construct a quality-adjusted price index for cars from the results of table 9.4. Given that the regression hyperplane passes through the point of means, the coefficients on the year dummy variables show the change in price which occurs for the average car (the model with the average price and average bundle of characteristics) each year. The derived price index then relates to the price of this 'average' model during the period. The mean price of all models included in the sample is £527 68p. In 1963 the
identical model would have cost £527 68p. less £16 77p., the latter figure being the estimated coefficient on the 1963 time dummy in table 9.4. Converting this information into an index with a value of 100 in 1957 yields the quality-adjusted price index shown in column 1 of table 9.7. Column 2 of table 9.7 gives the values of a quality-adjusted price index for cars calculated by Cowling and Cubbin (1972). Their method of constructing a quality-adjusted price index relied on estimating single-year price-quality regressions and thus deriving the price of a quality-constant car from those regressions for each year. Column 3 of table 9.7 gives values for an "established models" index calculated by Cowling and Cubbin. The sample of models used in period t was restricted to those models which also appeared in period t-1. The index does not take into account changes in quality-adjusted price brought about by the introduction of new models or improvements in old models. The three indexes are shown in Figure 9.2.

Both the quality-adjusted price indexes are much more flexible downwards than the established models index. This result is in line with the hypothesis put forward in Chapter 7, that in a situation of oligopolistic interdependence list prices would tend to be rigid whilst the probability of rapid reaction could be largely removed by making simultaneous changes in prices and specifications. The comparison with the established models index is ideal, since the established models index reflects changes in list price only. Its remarkable stability during the period 1957-64 (reflecting list price rigidity) contrasts markedly with the variation about trend shown by the two quality-adjusted price indexes during the same period. Comparison of the two quality-adjusted price indexes shows the index calculated above (the 'Morris' index) to be more stable than the Cowling-Cubbin index, at least

† Cowling and Cubbin (1972).

<table>
<thead>
<tr>
<th>YEAR</th>
<th>MORRIS 1957 = 100</th>
<th>CHAIN HEDONIC INDEX COWLING-CUBBIN 1956 = 100</th>
<th>ESTABLISHED MODELS INDEX 1956 = 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>100.00</td>
<td>110.24</td>
<td>107.83</td>
</tr>
<tr>
<td>1958</td>
<td>99.94</td>
<td>110.85</td>
<td>100.47</td>
</tr>
<tr>
<td>1959</td>
<td>100.22</td>
<td>109.49</td>
<td>106.49</td>
</tr>
<tr>
<td>1960</td>
<td>100.30</td>
<td>110.11</td>
<td>107.51</td>
</tr>
<tr>
<td>1961</td>
<td>99.14</td>
<td>111.06</td>
<td>107.51</td>
</tr>
<tr>
<td>1962</td>
<td>96.82</td>
<td>103.11</td>
<td>107.13</td>
</tr>
<tr>
<td>1963</td>
<td>94.99</td>
<td>97.92</td>
<td>107.67</td>
</tr>
<tr>
<td>1964</td>
<td>95.03</td>
<td>95.91</td>
<td>109.43</td>
</tr>
<tr>
<td>1965</td>
<td>96.39</td>
<td>99.57</td>
<td>114.27</td>
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<tr>
<td>1966</td>
<td>96.36</td>
<td>99.21</td>
<td>114.20</td>
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<tr>
<td>1967</td>
<td>100.70</td>
<td>107.10</td>
<td>120.52</td>
</tr>
<tr>
<td>1968</td>
<td>104.22</td>
<td>111.10</td>
<td>120.83</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>YEAR</th>
<th>1957 = 100</th>
<th>1957 = 100</th>
<th>1957 = 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>1958</td>
<td>99.94</td>
<td>100.55</td>
<td>98.73</td>
</tr>
<tr>
<td>1959</td>
<td>100.22</td>
<td>99.31</td>
<td>98.75</td>
</tr>
<tr>
<td>1960</td>
<td>100.30</td>
<td>99.88</td>
<td>99.70</td>
</tr>
<tr>
<td>1961</td>
<td>99.14</td>
<td>100.74</td>
<td>99.70</td>
</tr>
<tr>
<td>1962</td>
<td>96.82</td>
<td>93.53</td>
<td>99.35</td>
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<tr>
<td>1963</td>
<td>94.99</td>
<td>88.82</td>
<td>99.85</td>
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<tr>
<td>1964</td>
<td>95.03</td>
<td>87.00</td>
<td>101.48</td>
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<td>1965</td>
<td>96.39</td>
<td>90.32</td>
<td>105.97</td>
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<tr>
<td>1966</td>
<td>96.36</td>
<td>98.99</td>
<td>105.90</td>
</tr>
<tr>
<td>1967</td>
<td>100.70</td>
<td>97.15</td>
<td>111.76</td>
</tr>
<tr>
<td>1968</td>
<td>104.22</td>
<td>100.78</td>
<td>112.05</td>
</tr>
</tbody>
</table>
FIGURE 9.2 Price Indexes for Cars 1957-68.
up to 1967. This observation must be in part explained by the fact that the Morris index uses pooled cross-sectional data whilst the Cowling-Cubbin index relies on the use of estimates derived from single-year cross-sections with their attendant problem of great variability of shadow prices through time. 10

SUMMARY.

Single-year cross-sectional estimates of the price-quality relationship for U.K. cars were poor. It was suggested that this was due to the limited number of observations in each time period. The price-quality regressions yielded no evidence for or against the Lancaster hypothesis. A possible alternative method of testing the Lancaster hypothesis was advanced. Any new investigation of the price-quality relationship in cross-section could most profitably be conducted along these lines. Pooling the available data improved the level of determination of the price-quality regression and the shadow prices on qualities. 'Make effects' also appeared to be of some importance. A quality-adjusted price index for cars was constructed. The index agreed much more closely with another quality-adjusted price index than with an index relating only to list price changes of established models. The downward flexibility of the index supported the hypothesis of Chapter 6.
APPENDIX 9.1.

MODELS OF PASSENGER CAR INCLUDED IN THE SAMPLE USED TO ESTIMATE THE PRICE-QUALITY RELATIONSHIP.

AUSTIN
A35
A40
A55

B.L.M.C.
Mini
1100/1300
1800

FORD
Anglia
Consul
Classic
Corsair
Cortina
Escort
Popular
Prefect
Zephyr 4

HILLMAN
Imp
Hunter
Minx
Super Minx

MORRIS
Minor 1000
Oxford

STANDARD
8/10

TRIUMPH
Herald
1300
<table>
<thead>
<tr>
<th>VAUXHALL</th>
<th>Victor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Victor 101</td>
</tr>
<tr>
<td></td>
<td>Victor 2000</td>
</tr>
<tr>
<td></td>
<td>Viva</td>
</tr>
</tbody>
</table>
NOTES TO CHAPTER 9.
1. Probably a serious omission in view of the success of the Volkswagen 'Beetle' and the Renaults 4 and Dauphine during the period.
2. Market share data was provided by the firms involved, and was provided at different levels of aggregation by different firms.
3. Some of the bottom priced variants in the model ranges had very low sales, in these cases the data refers to the lowest priced variant having substantial sales. Such a decision only has to be made where the available data has to be aggregated. Appendix 5.1 suggests that the errors introduced by the process of aggregating the data to the model range level may be fairly small.
4. Information on the dates of introduction and discontinuation of models was obtained from the publication Motorist's Guide to New and Used Car Prices. Four models were affected in this way.
5. Often specification changes would be announced by manufacturers at the Motor Show during the autumn. This practice has largely ceased since 1968.
6. Values for price and quality variables were obtained from test reports in the magazines Autocar and Motor. The data was collected by John Cubbin of the Department of Economics, University of Warwick.
7. List price includes the price of a heater where this is not standard equipment. No other standardisation of the price data was possible, but in fact the only models which incorporated features into the standard price which were extras on some models were classified as 'luxury' models. A dummy variable for 'luxury' models was not significantly different from zero when included in the price-quality regression.
8. It was suggested in Chapter 6 that 'length' could only be validly considered a vertical quality difference if all other variables which a change in length might be associated were also taken into account in the price-quality relationship. Clearly in practical terms such a requirement may be impossible to meet and we will always face an omitted variable problem. Cowling and Cubbin (1972) have suggested that length is positively associated with stylishness (without giving any hard evidence). On a priori grounds such a suggestion would seem highly unlikely. Admittedly some models may be longer than is strictly required by engineering constraints allowing more freedom in styling, but to suggest that a longer car is more 'stylish' than a shorter car as far as a large proportion of consumers are concerned seems very odd. Alternative methods of dealing with the problem of styling are suggested later.

9. Models for which all four manufacturer dummy variables take on the value 0 are B.M.C. models. Dhrymes (1967) reports results for the U.S. automobile industry which have a similar implication. By regressing price on qualities for the three major manufacturers separately he was able to show that coefficients on qualities varied between manufacturers, and in many cases the differences were significant. In addition inclusion of the number of units sold as an explanatory variable improved the regressions, quantity had a significant negative coefficient. Thus Dhrymes argued that the estimated coefficients in his price-quality relationship reflected the 'cost plus mark-up' pricing policies of the manufacturers, not implicit prices in the sense of an implicit consumer valuation. The negative coefficient on quantity may merely reflect the fact that there are more potential buyers for low-priced models. Griliches and Ohta (1973) also supports the significance of 'make effects'.
10. Cowling and Cubbin recognise that this problem exists, but rightly argue that their interest is in estimates of predicted model price rather than shadow prices on qualities. Estimates of predicted price will be comparable between time periods if the structure of the multicollinearity between independent variables is the same during each time period (see Johnston (1963)).
CHAPTER 10.

THE RELATIONSHIP BETWEEN 'QUALITIES' AND VARIETY DEMAND.

INTRODUCTION.

This chapter considers methods of incorporating quality variables into the demand function at the model, brand or variety level. Chapter 6 suggested two basic methods of accomplishing this. Firstly the residual from the estimated price-quality relationship could be used as an explanatory variable taking the place of prices and qualities. Secondly quality variables could be introduced into the demand function explicitly. Both methods are used in this chapter. Optimality tests for quality levels (as described in Appendix 7.1) are conducted using estimates of quality elasticities derived from the second method of demand estimation, and estimates of shadow quality prices derived in Chapter 9.

INCORPORATING 'QUALITY-ADJUSTED PRICE' INTO A MODEL OF VARIETY DEMAND.

Chapter 6 argued that demand could be related to the residual from an estimated (linear) price-quality relationship in a specific way. Chapter 6 went on to provide some theoretical foundation for the price-quality relationship postulated in Chapter 6. The crucial point is that some of the theories of the price-quality relationship implied that the shadow (implicit) prices on the qualities changed through time (the Lancaster hypothesis and the local approximation theory without intertemporal comparisons), whilst others implied that the shadow prices remained constant through time (the Simple Repackaging Hypothesis and the local approximation theory with intertemporal comparisons). Only if the shadow prices remain constant through time, and hence the residuals from the price-quality relationship may be compared between time periods, may we pool the available data
in the estimation of the variety demand function. Intertemporal comparisons of quality-adjusted prices are only valid where the price-quality relationship uses pooled cross-sectional data. Given that the variety demand functions are to be estimated in market share form, Figure 10.1 shows the appropriate form of market share function incorporating quality-adjusted price. In all cases we require that the market share functions be homogeneous to degree zero in prices and qualities. This condition is implicitly met in the case of the Lancaster hypothesis where tastes are assumed to be non-homothetic since that case requires that all 'incomes' and hence prices paid be roughly constant. Linear price-quality relationships are homogeneous to degree 1, and hence the inclusion of the residuals from such a relationship alone will not ensure homogeneity to degree 0 in the demand function. The appropriate variable to stand for prices and qualities would then be the residual divided by the predicted price.

Given the poor nature of the estimated single-year price-quality relationships reported in Chapter 9, there is little point in attempting to estimate single-year market share functions. Demand functions corresponding to the bottom right-hand cell of Figure 10.1 are, however, worth estimating. Three additional points may be made concerning the theoretical form of the market share function. Firstly there will be an omitted variable problem due to the fact that the price-quality relationship does not take into account styling variables. In their work on the tractor market Cowling and Rayner (1970) compensated for this by including the number of years since the model was introduced as an explanatory variable in the variety market share function. We may usefully adopt the same strategy here, i.e. we implicitly assume that 'stylishness' is determined by the age of the variety. Secondly it was suggested in Chapter 6 that even though the market was being defined fairly narrowly in price terms, some buyers may still be prevented from purchasing some varieties by income constraints. We could segment the market by using dummy variables to distinguish varieties in various sectors of the
**FIGURE 10.1** Relationship between possible market share functions and 'theories' of the price-quality relationship.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Lancaster Hypothesis, or 'Local Approximation' (\Rightarrow) Shadow Prices (\Rightarrow) changing through time.</td>
<td>If incomes roughly the same, O.K. If incomes different, require homogeneity to degree 0 in prices and qualities.</td>
<td>No, since derivation of price-quality relationship implies that choice is made on a different basis each year.</td>
</tr>
<tr>
<td>Simple Repackaging Hypothesis (\Rightarrow) constant technology (\Rightarrow) constant shadow prices</td>
<td>S.R.H. alone is not a theory of consumption, so cannot say anything about the formulation of the market share function.</td>
<td></td>
</tr>
<tr>
<td>'Local Approximation' with intertemporal comparisons (\Rightarrow) reasonably constant tastes (\Rightarrow) constant shadow prices.</td>
<td>No, since derivation of price-quality relationship implies that choice is made on the same basis through time.</td>
<td>Yes, if market share function is homogeneous to degree 0 in prices and qualities.</td>
</tr>
</tbody>
</table>
price range, but such a method is rather clumsy. Here we use the variable list price relative to the mean of list price in each of the years of the sample in quadratic form. Thirdly the problem over advertising data still exists. The available data is at the company, rather than model, level. Advertising is not therefore included as an explanatory variable in the market share function for the twin reasons that the data is at an inappropriate level of aggregation, and that above-the-line advertising does not seem to be an important competitive weapon in the U.K. motor industry.

QUALITY-ADJUSTED PRICES AND MODEL DEMAND.

We assume that market share at the model or variety level is determined by three variables; quality-adjusted price, the length of time since the model was introduced, and list price relative to other models. Data for quality-adjusted prices is derived from the residuals and predicted variety prices from the regression reported in Table 9.4. The quality-adjusted price variable used is the residual divided by the predicted price. The length of time since introduction, or 'life', of the model was measured in years, and defined to be 0 in the year of introduction. Results for two forms of market share function are reported. Firstly we assume that adjustment to price and quality changes is rapid, and hence only current quality-adjusted price influences demand. The appropriate market share function is

\[ \ln q'_{it} = \beta_0 + \beta_1 QAP_{it} + \beta_2 l_{it} + \beta_3 P_{it} + \beta_4 P_{it}^2 + u_{it} \]

where
- \( q'_{it} \) is the market share of the ith model in period t
- \( QAP_{it} \) is the quality-adjusted price of the ith model in period t
- \( l_{it} \) is the 'life' of the ith model in period t

† see Chapter 5.

† i.e. we allow for the possibility that the cheapest model does not come in the most popular segment of the price range.
\( P_{it} \) is the list price of the \( i \)th model in period \( t \), relative to mean list price of models in period \( t \). List price is defined as in Chapter 9.

\( u_{it} \) is a random disturbance term.

Equation 10.1 is consistent with the possible shape of demand curve shown in figure 6.1. Secondly we may suggest that adjustment to price and quality changes does not take place rapidly, and that both current and past levels of quality-adjusted price influence demand. Assume that the market share function is

\[
\ln q'_{it} = \beta_0 + \beta_1 \left[ \sum_{t=0}^{T} \lambda^{T} QAP_{it-T} \right] + \beta_2 l_{it} + u_{it}.
\]

where \( 0 \leq \lambda \leq 1 \).

period \( t-L \) is the first period for which data is available.

In period \( t-1 \) 10.2 becomes

\[
\ln q'_{it-1} = \lambda \beta_0 + \beta_1 \left[ \sum_{t=1}^{T} \lambda^{T} QAP_{it-T} \right] + \lambda \beta_2 l_{it-1} + \lambda u_{it-1}.
\]

after multiplication of both sides by \( \lambda \).

Subtraction of 10.3 from 10.2 yields

\[
\ln q'_{it} = \lambda(\beta_0 + \beta_2) + \beta_1 QAP_{it} + \beta_2(1 - \lambda) l_{it} + \lambda \ln q'_{it-1} + u_{it} - \lambda u_{it-1}.
\]

since \( \text{cor}(u_{it}, u_{it-1}) = 0 \) and

\[
\beta_2 l_{it} - \lambda \beta_2 l_{it-1} = \beta_2 l_{it} - \lambda \beta_2 (l_{it} - l).
\]

Results are given in Table 10.1. The first column relates to form 10.1. All the coefficients are well determined except that on the variable of central interest, quality-adjusted price. That coefficient takes on the 'wrong' sign.
TABLE 10.1.

Estimated market share functions for the U.K. passenger car market, 1958-68.

Dependent variable In q$_{it}$

<table>
<thead>
<tr>
<th>Variable</th>
<th>No adjustment lag assumed</th>
<th>Adjustment lags assumed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-3.6542 (-2.3671)</td>
<td>-4.7911 (-3.0367)</td>
</tr>
<tr>
<td>QAP$_{it}$</td>
<td>0.9897 (1.2177)</td>
<td>-0.3828 (-0.3969)</td>
</tr>
<tr>
<td>l$_{it}$</td>
<td>-0.0425 (-3.4562)</td>
<td>-0.0105 (-0.7445)</td>
</tr>
<tr>
<td>P$_{it}$</td>
<td>6.7015 (2.2030)</td>
<td>6.8254 (2.1534)</td>
</tr>
<tr>
<td>P$_{it}^2$</td>
<td>-4.5005 (-2.9884)</td>
<td>-4.6144 (-2.9412)</td>
</tr>
<tr>
<td>Models</td>
<td>-0.0769 (-3.7790)</td>
<td>-0.0946 (-3.8365)</td>
</tr>
<tr>
<td>ln q$_{it-1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>F</td>
<td>20.0111</td>
<td>19.7513</td>
</tr>
<tr>
<td>R$^2$</td>
<td>0.3938</td>
<td>0.3376</td>
</tr>
<tr>
<td>D-W</td>
<td>1.9731</td>
<td>1.8099</td>
</tr>
</tbody>
</table>

* t values in brackets
but is, however, not significantly different from zero. The signs on the 'life' and 'price' variable coefficients are as expected and the coefficients are significant. The poor nature of the relationship between market share and quality-adjusted price is illustrated by the fact that the gross correlation coefficient between ln (market share) and quality-adjusted price is only -0.0230. Examination of the residuals from the estimated equation in the first column of table 10.1 shows that significantly more negative residuals occurred during the last five years of the period than during the first six. Table 10.2 illustrates this.

**TABLE 10.2.**

Pattern of residuals from regression reported in the first column of Table 10.1.

<table>
<thead>
<tr>
<th>Period</th>
<th>Sign of residuals</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-ve</td>
<td>+ve</td>
</tr>
<tr>
<td>1957-63</td>
<td>29 (39)</td>
<td>49 (41)</td>
</tr>
<tr>
<td>1964-68</td>
<td>49 (39)</td>
<td>33 (41)</td>
</tr>
<tr>
<td>Totals</td>
<td>78</td>
<td>82</td>
</tr>
</tbody>
</table>

\[ x^2 = 8.2501; \quad x^2_{0.01,1d.f.} = 6.635. \]

Expected frequencies are given in parentheses.

Negative residuals in the later period indicate that observed market shares are considerably less than predicted on the basis of the regression model. The simplest explanation of this phenomenon is that the number of competing models tended to increase during the period 1958-68. Two alternative ways of dealing with this feature of the market were tried; firstly a simple time trend variable was included, and secondly a variable measuring the number...
of different models available ('models') was included as an explanatory variable. The results were slightly better in the latter case and only these are reported. The same pattern of residuals occurred whenever the 'models' or trend variable were not included. Inclusion of the 'models' variable reduced the $\chi^2$ value calculated on the same basis as in Table 10.2 (i.e. for the second column regression of Table 10.1) to 0.4.

The results of Table 10.1 cast doubt on the hypothesis that quality recognition by buyers takes the form postulated above. There are three main reasons why the results might not support the original theory. Firstly, buyers may not recognise quality (at least those quality variables included in the price-quality regression) as an important variable in the car purchasing decision. Secondly, the quality-adjusted price variable may be poorly chosen, a different measure of quality-adjusted price may perform better. Thirdly, we may not have adequately taken account of the changes in market demand parameters which have occurred, for example the changes in price elasticities at the variety level through time or the greater availability of information in later time periods. Before we accept the first explanation we must be sure that the other two are not sufficient to explain the results of Table 10.1. Evidence from the only other studies which study demand in this way would seem to point to the second explanation. Cowling and Rayner (1970) obtained well determined model market share relationships for tractors with significant quality-adjusted price variables. Their method was somewhat different to the one adopted here. They used weighted linear single-year regressions to obtain values of quality-adjusted price, but pooled data for quite a long period (1948-65) to estimate the market share function. Surprisingly the addition of a deflator for quality-adjusted price did not improve the results. One feature of the Cowling-Rayner model is the very simple, but effective, price-quality relationship which is employed; list price is very largely explained by the two variables belt horse-power and type of engine (petrol or diesel). Given the type of engine, quality-
adjusted price merely indicates the horse-power of the model relative to other models at the same price. The quality-adjusted price variable in the model market share equations is really measuring the shadow price of a unit of horse-power for the particular model. Where price-quality relationships are more complex such simple explanations of the role of quality-adjusted price in the market share function are not possible. Cowling (1972) presents some results for the U.K. car market. By averaging the estimated model residuals for a given manufacturer's set of models a manufacturer or firm quality-adjusted price variable was obtained. This variable was of the 'correct' sign and was significant in explaining market share at the firm level. Again weighted single-year price-quality regressions were used to derive values of quality-adjusted prices which were then incorporated into a model of demand using pooled data (for the period 1956-68). The reasons for the success of the Cowling model are unclear.

One feature of the price-quality regression used to obtain quality-adjusted prices is its use of dummy variables to distinguish the manufacturer of a given model and hence pick up hypothesised 'make effects'. Neither of the two studies cited do this. It was suggested that the dummy variables measured (albeit crudely) omitted quality variables associated with particular manufacturers. There is, however, no guarantee that the dummy variables in fact do this. The positive coefficient on the Standard-Triumph variable in Table 9.4 may merely indicate that Standard-Triumph prices are higher than other manufacturers given the same quality mix, residuals derived from the regressions in Table 9.4 would then yield low predicted prices where Standard-Triumph models were concerned, leading to a possible predominance of negative residuals attached to Standard-Triumph models in the demand function of Table 10.1. The spuriously low quality-adjusted prices would lead to high market share predictions for Standard-Triumph models. On the other hand the omission of the manufacturer dummy variable from the price-quality regression may fail to take into account some attributes of Standard-Triumph models which are important in determining their 'correct'
quality-adjusted prices. Some evidence can be gained by looking at the residuals from the regressions reported in Table 9.4, an identical regression which omits the four manufacturer dummy variables, and the estimated market share function reported in column 2 of Table 10.1. In the Standard-Triumph case where 'make effects' were not included eleven (out of 14) models had positive residuals. Inclusion of 'make effects' reduced this number to five, and use of this set of derived quality-adjusted prices led to Standard-Triumph models having twelve (out of 13) negative residuals in the estimated market share function. In the case of Ford models the opposite pattern is observed. In the 'without 'make effects'' price-quality regression 38 Ford observations had negative residuals; when 'make effects' were measured this number dropped to 20 (out of 43). Twenty-five (out of 39) Ford models took on positive residuals in the market share function. The data thus seems to support the hypothesis that Fords are relatively underpriced and Standard-Triumph models relatively overpriced, and that manufacturer dummy variables in the price-quality relationship are capturing this, rather than measuring omitted quality variables.

Table 9.3 reports results for an estimated log-linear price-quality relationship which does not include manufacturer dummy variables. In this case the residuals may be included in the market share function without modification since the residual is equal to log $P_{it}$. Homogeneity to degree 0 of the demand function in prices and qualities is therefore guaranteed by the inclusion of the residual. Several possible market functions were estimated incorporating this measure of quality-adjusted price. All yielded negative coefficients on quality-adjusted price, but the only form which yielded a coefficient on quality-adjusted price significantly different from zero was
\[ \log q_{it}^9 = -0.5975 - 1.5772 QAP_{it} - 0.0103 l_{it} \]
\[ (-3.7143) \quad (-1.9840) \quad (-1.5174) \]
\[ - 0.0405 \text{ Models} + 0.0013 \log q_{it-1} \]
\[ (-3.8276) \quad (1.1181) \]

\[ n = 160 ; F = 5.1189 ; R^2 = 0.1166 ; D-W = 2.2875. \]

\( t \)-scores in brackets.

It would seem that the measure of quality-adjusted price derived on the basis of a price-quality regression which did not incorporate manufacturer dummies is better fitted to the task of explaining model demand.

Clearly the measure of quality-adjusted price chosen is crucial. We are unable, therefore, to reject the notion that quality recognition by buyers, as it has been described earlier, is not an important determinant of demand at the model level. A further consequence of the results given in Table 10.1 and equation 10.6 is that only current values of quality-adjusted price are important in explaining current model demand. In none of the regressions is the coefficient on lagged market share significantly different from zero. Again this result conflicts with those of Cowling and Rayner (1970) and Cowling (1972).

DEMAND FUNCTIONS INCORPORATING QUALITIES DIRECTLY.

It was suggested in Chapter 6 that we might estimate model demand by including qualities as independent variables explicitly. One objection to this technique is that any such market share regressions will be bedevilled by multicollinearity between the independent variables. This does not matter provided we are only interested in obtaining predicted values for market shares, but part of our interest is to obtain quality elasticities so that we can test for optimal quality setting behaviour by firms along the lines proposed in Appendix 7.1. We may attempt optimality tests
TABLE 10.3.

Estimate of demand function for cars using qualities as explanatory variables; 1958-68.
Dependent variable $\log q_{it}$.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>COEFFICIENT</th>
<th>t SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.7344</td>
<td>-4.9504</td>
</tr>
<tr>
<td>$\log P_{it}$</td>
<td>-1.9075</td>
<td>-3.0827</td>
</tr>
<tr>
<td>$\log P_{it-1}$</td>
<td>-0.0001</td>
<td>-0.0061</td>
</tr>
<tr>
<td>$\log B.H.P.$</td>
<td>-1.0280</td>
<td>-2.0919</td>
</tr>
<tr>
<td>$\log length$</td>
<td>2.0448</td>
<td>2.5646</td>
</tr>
<tr>
<td>$\log M.P.G.$</td>
<td>0.1847</td>
<td>0.3496</td>
</tr>
<tr>
<td>$\log leg room$</td>
<td>3.8038</td>
<td>3.7664</td>
</tr>
<tr>
<td>Gears</td>
<td>0.0628</td>
<td>0.9356</td>
</tr>
<tr>
<td>Disc Brakes</td>
<td>0.0403</td>
<td>0.7185</td>
</tr>
<tr>
<td>Life</td>
<td>-0.0211</td>
<td>-3.5255</td>
</tr>
<tr>
<td>Models</td>
<td>-0.0353</td>
<td>-3.4100</td>
</tr>
<tr>
<td>$\log q_{it-1}$</td>
<td>0.0010</td>
<td>0.6555</td>
</tr>
<tr>
<td>n</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>11.1231</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.4526</td>
<td></td>
</tr>
<tr>
<td>D-W</td>
<td>2.0532</td>
<td></td>
</tr>
</tbody>
</table>
where the coefficients take on the 'correct' sign and are reasonably well determined.

Results for a market share function incorporating qualities directly are given in Table 10.3. The quality variables are defined as in Chapter 9, except that they are expressed as relative to the mean values of those variables in each year, and other variables as in the preceding section.\(^\text{10}\) It has been assumed that both current and past quality levels affect demand, but that adjustment to qualities takes place with identical lag structures. The resulting estimating equation is derived in Appendix 10.1. Again a feature of the results is that only current quality levels seem to affect current demand, the coefficient on the lagged market share term is not significantly from zero. The 'life' and 'models' variables are both of the 'correct' sign and significant. The indicated price elasticity is of the kind of magnitude we might expect. The significantly determined quality variables have coefficients which are either of the 'wrong' sign or much larger than expected. We may again put this down to the multicollinearity which exists between the explanatory variables. The extent of this is shown in Table 10.4.

**TABLE 10.4.**

Correlation coefficients between selected explanatory variables used in the regression of Table 10.3.

<table>
<thead>
<tr>
<th></th>
<th>log B.H.P.</th>
<th>log length</th>
<th>log M.P.G.</th>
<th>log leg room</th>
</tr>
</thead>
<tbody>
<tr>
<td>log P(_t)</td>
<td>0.8859</td>
<td>0.7526</td>
<td>-0.7869</td>
<td>0.2590</td>
</tr>
<tr>
<td>log B.H.P.</td>
<td>0.7969</td>
<td>-0.8766</td>
<td>0.2796</td>
<td></td>
</tr>
<tr>
<td>log length</td>
<td></td>
<td>-0.8247</td>
<td>0.2246</td>
<td></td>
</tr>
<tr>
<td>log M.P.G.</td>
<td></td>
<td></td>
<td>-0.0840</td>
<td></td>
</tr>
</tbody>
</table>
Clearly we may only 'conduct' optimality tests for quality levels on the lines proposed in Appendix 7.1 at the most superficial level. Appendix 7.1 suggests that the proportion of price "accounted for" by any given quality should be equal to the ratio of that quality elasticity to the price elasticity. Clearly this requires that the quality elasticity be less than the modulus of the price elasticity. In the two cases where the results of Table 10.3 yield estimates of quality elasticities which are both significant and positive, they are much too high.

SUMMARY.

Estimates of market share using quality-adjusted price as the only independent variable relating to prices and qualities showed very little relationship between quality-adjusted price and market share. This result might arise from the use of inappropriate measures of quality-adjusted price. The only comparable study (Cowling and Rayner (1970)) employed a very simple price-quality relationship. An estimate of a market share function using qualities per se as explanatory variables was quite well determined. The derived price elasticity was of the expected sign and order of magnitude, but the existence of multicollinearity between the other explanatory variables (qualities) prevented good estimates of the quality elasticities being obtained. Optimality tests on the basis of Appendix 7.1 were therefore not possible. All the results of Chapter 10 suggested that only current model price and quality levels had a significant effect on current model demand. The age of the model, in terms of time since market introduction, was significant in determining model demand. This latter observation suggests that styling and "up-to-dateness" is an important determinant of model demand. The use of manufacturer dummy variables in the price-quality relationship was shown not to pick-up 'make effects' in the sense of manufacturer specific quality variables. Rather such dummy variables tended to reflect the general level of prices (given quality levels) of the different manufacturers.† excepting equation 10.6.
APPENDIX 10.1

A generalised derivation of the estimating equation used in the model of demand incorporating 'qualities' directly. Let the market share function for the \( i \)th variety be

\[
q_{it} = \alpha_0 + \alpha_1 p_{it} + \sum_{j=1}^{k} \beta_j x_{ijt}^*
\]

where \( q_{it} \) is the market share of variety \( i \) at time \( t \), or some transformation of it.

\( p_{it} \) is the relative price of the \( i \)th variety in period \( t \), or some transformation of it.

\( x_{ijt}^* \) summarises the effects of current and past levels of the relative amount of the \( j \)th quality embodied in the \( i \)th variety on current demand for the \( i \)th variety, or some transformation of it, the same transformation being applied to each of the \( k \) qualities.

\( x_{ijt}^* \) is defined by the equation

\[
x_{ijt}^* = \sum_{T=0}^{L} \lambda_j^T x_{ijt-T}
\]

where \( 0 < \lambda_j < 1 \), \( j = 1, \ldots, k \).

period \( t-L \) is the first period for which data is available.

\( x_{ijt-T}^* \) measures the relative amount of quality \( j \) embodied in variety \( i \) in period \( t-T \), or the same transformation of it as the transformation of \( x_{ijt}^* \).

(i.e. if \( x_{ijt}^* \) stands for \( \log x_{ijt}^* \), then \( x_{ijt-T}^* \) stands for \( \log x_{ijt-T}^* \).)
Applying the Koyck transformation to \( X_{i,j_t}^* \) we have from 10.1.2

10.1.3
\[
X_{i,j_{t-1}}^* = \frac{1}{\lambda_j} \sum_{T=1}^{T-1} \lambda_j^{T-1} X_{i,j_{T-1}}, \quad j = 1, \ldots, k
\]

and hence

10.1.4
\[
\lambda_j X_{i,j_{t-1}}^* = \frac{1}{\lambda_j} \sum_{T=1}^{T-1} \lambda_j^{T-1} X_{i,j_{T-1}}, \quad j = 1, \ldots, k
\]

Subtraction of 10.1.4 from 10.1.2 yields

10.1.5
\[
X_{i,j_t}^* - \lambda_j X_{i,j_{t-1}}^* = X_{i,j_t}^*, \quad j = 1, \ldots, k
\]

Substituting for \( X_{i,j_t}^* \) in the market share function 10.1.1 we have

10.1.6
\[
q_{i_t} = \alpha_o + \alpha_1 p_{i_t} + \sum_j \beta_j X_{i,j_t} + \lambda \sum_j \beta_j X_{i,j_{t-1}}^*, \quad j = 1, \ldots, k
\]

If \( \lambda_j = \lambda \) for all \( j=1, \ldots, k \) 10.1.6 becomes

10.1.7
\[
q_{i_t} = \alpha_o + \alpha_1 p_{i_t} + \sum_j \beta_j X_{i,j_t} + \lambda \sum_j \beta_j X_{i,j_{t-1}}^*
\]

and from the market share function (10.1.1) lagged by one period

10.1.8
\[
\lambda \sum_j \beta_j X_{i,j_{t-1}}^* = \lambda q_{i_{t-1}} - \lambda^2 \alpha_o - \lambda \alpha_1 p_{i_{t-1}}
\]

and substituting into 10.1.7 we obtain the estimating equation

10.1.9
\[
q_{i_t} = \alpha_o + \alpha_1 p_{i_t} + \alpha_1 p_{i_{t-1}} + \sum_j \beta_j X_{i,j_t} + \lambda q_{i_{t-1}}
\]

where \( \alpha_o = \alpha_o - \lambda \alpha_o \)

\( \alpha_o = -\lambda \alpha_1 \)

† see Koyck (1954).
NOTES TO CHAPTER 10.

1. The argument may be easily applied to other functional forms of the price-quality relationship with correspondingly defined residuals.

2. A doubling of all prices and qualities would lead to a doubling of the residual.

3. Use of the variable $U_i / \hat{p}_i$ rather than just $U_i$ necessitates a small adjustment to result 6.1.17, since $\hat{p}_i = \hat{i} (p_{ij})$, $j = 1, ..., n$, including

$$\frac{X_{ji}}{\partial \hat{p}_i} \frac{\partial \hat{p}_i}{\partial x_{ji}} = - \frac{nX_{ji}}{n_{p_i}}$$

4. If the shadow prices change from year to year and hence the demand functions change from year to year, then the model cannot be used to predict demand for new varieties. A model which implies constant shadow prices can be used to predict demand for new varieties.

5. A model was deemed to be 'new' if a major body restyling had taken place, that is the model had to actually "change shape" to be deemed 'new'. Thus data is included on two different models of Cortina in two different (consecutive) time periods. In 1959 Ford introduced a model named the Anglia. The pre-existing model bearing that name was renamed the Prefect and the pre-existing Prefect model was discontinued. The variable 'life' is thus measured from the date of introduction of that particular body shape and style rather than identifying name. Although the Prefect changed in 1959, the 'new' (1959) model had a body style dating from well before 1959.

6. Although there were great advantages to be gained by pooling data to estimate the price-quality relationship, there seem to be very few to be gained from pooling observations to estimate the demand curve. Clearly many other variables than quality-adjusted price will be needed to explain demand (market share). In particular the struct-
ure of the industry may well change through time. Cowling and Rayner (1970) are the only writers to have previously attempted to estimate model demand in this way. In their work on the U.K. tractor market they used a conventional trend variable to take account of the tendency for model demand to become more elastic over time.

7. A deflator is indicated where list prices are rising.

8. Only 13 Standard-Triumph observations and 39 Ford observations are included in the market share functions since they were estimated for the period 1958-68. Price-quality relationships refer to the period 1957-68.

9. The coefficient on $QAP_{it} (= \log P_{it})$ may be interpreted as the list price elasticity of market share provided $\hat{aP}_{it} = 0$.

10. Again the residuals showed the pattern described in the previous section when the 'models' variable was omitted. The $\chi^2$ test yielded a value of 9.1076 with 1 d.f.

11. It was suggested earlier that buyers were able to make intertemporal comparisons when (subjectively) evaluating the price-quality relationship. If they can do this then the notion that they are not capable of fully adjusting to changes in quality-adjusted price through time has very little force. In essence the intertemporal comparisons, and hence the adjustment process, are undertaken at a different level in the model.

Equation 10.6 omits the relative list price variables $P_{it}$ and $P_{it}^2$, whereas Table 10.1 includes these. Part of the "improvement" evident in the results of equation 10.6 may be due to this change; the roles played by the quality-adjusted price and list price variables in ex-
plaining demand may be partially overlapping. Thus whilst the results of Table 10.1 are acceptable in that they yield a maximum value of $P_{it}$ of approximately 0.74 in all cases (indicating that, ceteris paribus, a model having a list price about three-quarters of the mean list price of models in any given period will have the greatest market share), equation 10.6, which omits list price variables, is 'preferable' to all the results of Table 10.1.

12. Hence multicollinearity between the qualities was not a problem encountered by them.
CHAPTER II.

SUMMARY AND CONCLUSIONS.

INTRODUCTION.

The approach of the thesis has been to analyse the roles which advertising and quality variables have on demand at the model, brand or variety level (excepting Chapter 4 which analysed a company level model of advertising). The neoclassical microeconomic view of the firm has been taken as the benchmark. Optimal behaviour patterns for the firm can be deduced from the economic model and these derived optimal patterns of behaviour compared with actual patterns of firm behaviour. An important question arises out of this type of comparison. Given that firms do not act literally in the fashion suggested by the economic model, we must analyse whether the actual behaviour of firms allows the theoretically optimal behaviour pattern to be adopted. This particular problem is discussed in Chapter 7. The theoretically derived optimal behaviour patterns were compared to behaviour exhibited by U.K. passenger car manufacturers during the period 1957-68. Although the data was not ideally suited to testing advertising models, the a priori assumed importance of physical quality differences between models rendered the market particularly suited to a study of the role of quality in the competitive process. Three major topics were considered and these are discussed below.

ADVERTISING COMPETITION.

Chapters 2 and 3 developed the neoclassical model of advertising, taking the Dorfman-Steiner theorem as the cornerstone of the analysis. The Dorfman-Steiner theorem predicts that when the monopolistic firm

\footnote{Dorfman and Steiner (1954).}
is maximising profit it will set price and the advertising budget so that

\[
- \frac{\eta_A}{\eta_p} = \frac{A}{pq}
\]

The model was extended to several other situations, the most important being ones where sales revenue was a determinant of managerial utility and where the monopolist was uncertain about the demand function. A proof of the Nerlove-Arrow condition (for optimal advertising under dynamic conditions) using control theory was given. All the situations examined altered the basic Dorfman-Steiner and Nerlove-Arrow results, in most cases the indicated optimal advertising-sales ratio was higher than in the simple Dorfman-Steiner case.

The Dorfman-Steiner model can be extended to cover situations where there are more marketing variables (including quality variables) and where inter-firm competition exists. The precise results of such a generalisation depend on what is assumed about the competitive interactions between rival firms. Some special cases are discussed in Chapter 4. The most convenient "reference model" to take is the Cournot model, i.e. one where the firm assumes that changes in one of its decision variables will not prompt retaliatory changes in decision variables by other firms. In the Cournot case the simple Dorfman-Steiner rule re-emerges. The results of applying other assumptions about competitive reactions can then be compared to the basic Dorfman-Steiner result. In particular a price-leadership model is examined. The possibility of advertising-leadership emerging as a competitive strategy is considered, but it was concluded (on theoretical grounds) that price-leadership was more likely to occur than advertising-leadership. Evidence is presented (Appendix 4.1) to suggest that the U.K. car industry conforms to a price-leadership pattern, at least in respect to list prices of established models. Chapter 4 also recognises that the use of rules-of-thumb to aid in actual price and advertising decisions is widespread. However it is concluded that the
use of some rules-of-thumb, for example the practices of mark-up pricing and setting the advertising budget as a percentage of estimated sales, did not preclude optimising behaviour by firms where optimality is defined in terms of the theoretical models of Chapter 3.

Dynamic models of advertising were considered. Chapter 3 presents a fairly general model. Previous attempts to test for the adoption of 'optimal' advertising policies by firms were discussed. Usually these rested on a somewhat paradoxical method of testing; values of estimated parameters of the demand function (elasticities with respect to discretionary variables) were compared with observed values of firms' promotional variables, i.e. some measure of advertising expenditure relative to sales, per unit price, or profit. Thus writers tested for optimal behaviour on the part of firms by estimating a relationship largely determined by the behaviour of consumers (the demand function). An alternative to this method of testing was proposed. The conditions relating to the optimal stock of advertising derived from the model of Chapter 3 were incorporated into a stock-adjustment model, and a condition relating current advertising expenditures to current and past levels of sales revenue and advertising prices, and to past advertising expenditures was derived. Although this relationship (3.51) could not be interpreted as an advertising appropriations relationship, its estimation did permit the testing for optimal advertising behaviour, ex post, by examining firms' behaviour directly.

The dynamic model of competitive advertising behaviour considered in Chapter 3 is of special interest because it is more general than the Nerlove-Arrow model and explicitly considers the price of advertising as a variable which determines the optimal path of advertising through time. The theoretical prediction of the dynamic model of advertising behaviour are not susceptible to testing by the same methods that are used to test the static Dorfman-Steiner model. This is partly because the parameters (discount rate, advertising stock decay rate etc.) of the condition for the optimal advertising stock - sales
ratio are much more complicated than they are in the simple Dorfman-Steiner case. It was suggested that a possible method of circumventing this problem would be to incorporate the optimality condition for advertising stock into a stock-adjustment model. The estimating equation derived from using this approach (3.51) had the advantages that all unobservable parameters of the theoretical model (including the price and advertising stock elasticities) were estimated in the process of applying regression techniques to the estimating equation. The fact that prior knowledge of the elasticities was not required meant that no particular assumption about competitive interactions needed to be made. The regression technique used was necessarily an iterative one, but bar the volume of work involved in estimating the equation no special problems existed. The results of estimating the stock-adjustment equation were very poor. The estimated parameters bore little relationship to the predicted values of those parameters when data on company advertising in the U.K. motor industry was used for testing. Alternative interpretations of the regression results were of no help. What looked to be a promising approach turned out to be of little help when applied to the data used in this thesis. Some possible reasons why this could be the case were given. One particular difficulty may be inherent in the method of testing chosen. The stock-adjustment approach assumes that the optimal stock can only be attained with time (except in the limiting case where the adjustment coefficient is unity); the control theory approach used to generate the optimal level of stock assumes that adjustment can be carried out instantaneously (or in practical terms in the space of one period). Thus for the two models to be mutually consistent the estimated value of the adjustment coefficient should be unity. This was not the case.

Empirical conclusions on the basis of this work are hard to come by, except that the initial results using this approach are discouraging. Use of data for other more heavily advertised products might reverse this feeling but there are still theoretical difficulties to overcome. An earlier study of the U.K. market (Cowling (1972)) suggests that company advertising is far from the theoretically optimal level. It should not surprise us that results
on a test which is much more specific than the Cowling test should also be poor. The problem of testing dynamic models of optimal advertising behaviour remains, but at least the work done here identifies those problems more clearly and explores the directions in which the solution might lie. Improvement of the method and more appropriate data are called for.

THE RELATIONSHIP BETWEEN PRICES, 'QUALITIES' AND MODEL DEMAND.

Previous writers have made little attempt to provide a definition of 'quality'. Chapters 6 and 7 made an attempt to rectify this omission. It was shown that any relationship between the prices of a set of similar goods and their 'qualities' had to take into account both the form of the relationship and the definition of 'quality' employed. This is a departure from the opinions of previous researchers who, by and large, have regarded the choice of functional form and independent variables of the price-quality relationship as an empirical matter.

Most existing work on price-quality relationships embraces the so-called "hedonic principle". Chapter 6 reviewed this work. Chapter 7 attempted to provide some theoretical determination for the price-quality relationship, and in the process some guidance in the matter of choice of functional form and explanatory variables. It was suggested that there were only two serious candidates for the role of 'theoretical basis for the hedonic principle'. Both these (the Lancaster hypothesis concerning consumption behaviour and the Simple Repackaging Hypothesis, an assumption about technology) were explored and shown to lead to quite different price-quality relationships.

Following Cowling and Rayner (1970) and Cowling (1972) the possibility of incorporating the residuals from estimated price-quality relationships into a model of demand was examined. This topic was introduced in Chapter 6 and extended in Chapter 10. It was postulated that a negative residual (or "quality-adjusted price") should indicate that a particular model had a relatively low price, given its quality mix, whilst
a positive residual would indicate a relatively high price after qualities had been taken into account. We would therefore expect quality-adjusted price to be negatively related to demand at the model level. Appendix 7.1 gives a method of testing for optimal quality-setting behaviour at the model level using data derived from estimated price-quality relationships and estimated quality elasticities.

Tests of the hypotheses contained in Chapters 6 and 7 were conducted in Chapters 9 and 10. Only estimates of price-quality relationships conforming to the Simple Repackaging Hypothesis and the theory proposed in Chapter 6 (the "local approximation theory") proved satisfactory. It was suggested that this was due to the fact that the S.R.H. implied a price-quality relationship using pooled cross-sectional data, whereas the Lancaster hypothesis required the use of single-year cross-sections. The lack of observations in any given year, coupled with the fairly large number of (multicollinear) variables used to explain price, meant that individual parameter estimates on qualities were very poorly determined in the single-year cross-sectional regressions. A possible alternative test of the Lancaster hypothesis was proposed; a test which requires that 'qualities' be defined as "performance variables" and that data on second-hand cars be incorporated.

Estimates of price-quality relationships using pooled cross-sectional data present a fairly novel departure from the bulk of established practice. Estimates of shadow quality prices derived from such regressions were well determined, and usually of the expected signs and order of magnitude. The use of dummy variables to distinguish data relating to the different years of the sample permitted the construction of a quality-adjusted price index for cars over the period. This index (reported in Chapter 9) compared favourably with another (differently calculated) quality-adjusted price index for cars. Dummy variables for manufacturers were also included (Table 9.4) in an attempt to measure manufacturer specific omitted quality variables; two out of the four were significant. A further examination of the
role of these variables (see Chapter 10) showed, however, that they did not seem to be measuring omitted quality variables, but rather they were reflecting the differences in the general level of prices between manufacturers, after quality differences had been taken into account. Attempts to relate the residuals from the estimated price-quality regressions of Chapter 9 to model market share showed very little relationship between quality-adjusted price and market share.

The major implications of Chapters 6, 7, 9 and 10 are as follows. Firstly the form of the price-quality relationship is not merely a question of empirics, there are strong theoretical reasons for favouring some forms more than others. Secondly a great deal of good data is required to estimate single-year cross-sectional price-quality relationships. Where this is not available the technique of pooling the available data and including year dummies in the price-quality regression may be a useful alternative. We must be aware, however, that use of this technique implicitly assumes that either tastes, or technology, or both are reasonably constant over the period of the pooling. Thirdly it is advisable to take every available step to reduce the inevitable multicollinearity which exists between quality variables. Fourthly there must be some doubt cast on the hypothesis that there is a relationship between quality-adjusted price and market share at the model level. If there is such a relationship it is very sensitive to the definition of quality-adjusted price employed. Despite the fact that the price-quality regressions from which the quality-adjusted prices were derived were well determined, estimated coefficients on quality-adjusted price in the estimated model market share functions were rarely significantly different from zero. Fifthly the most profitable area for future study would seem to be based on the Lancaster hypothesis. Chapters 7 and 9 discuss this.
QUALITY COMPETITION.

Chapters 6 and 8 consider some hypotheses concerning the nature of quality competition. The basic tenet of the discussion was that reaction by firms to simultaneous changes in price and specifications to a model by a rival was almost impossible. Alteration of the price-quality mix of a model is hence an important competitive weapon since such action circumvents some of the awkward aspects of oligopolistic interdependence. Such competition is characterised by new model introductions. Chapter 8 develops the theme introduced in the last section of Chapter 6 and suggests how the model demand curve alters over the market life of the model. It is postulated that the demand curve for a particular model shifts in a predictable way through time. The effect of time will be to initially shift the demand curve to the right as consumer knowledge of the new variety spreads, and then to depress the demand curve as the model becomes older in market terms. Using this theory an optimal pricing path is derived via the techniques of optimal control theory. This optimal path is then compared with the actual paths of price exhibited by four models of car. The most encouraging result is that even on the basis of such a simple model the hypothesised demand cycle is well supported by the empirical evidence.

A further prediction of Chapter 7 is that a price index which is unadjusted for quality changes should show greater flexibility about trend than a price index which relates to list prices only. A manufacturer, since he faces no reaction, is able to adjust prices in his desired direction much more easily for new models than for established models (where alteration of list prices attracts rapid retaliation). A quality-adjusted price index for cars during the period 1957-68 was constructed using data given in Chapter 9, and the prediction of Chapter 7 is confirmed.
Chapter 10 reports empirical attempts to explain variety demand using the residuals from the estimated price-quality relationships ("quality-adjusted prices"). In nearly all cases the estimated coefficient on quality-adjusted price was not significantly different from zero. In only one case (equation 10.6) was the quality-adjusted price variable well determined, significantly different from zero and of the expected sign and magnitude. 10.6 was constructed in response to objections (raised in the text) to earlier models of Chapter 10 and in many ways it is theoretically the most satisfactory estimating equation. The results of Chapter 10 suggested that only current price and current quality levels had a significant influence on model demand. This result may be regarded as somewhat surprising, but might be explained by the difficulties involved in merely comparing the current array of prices and qualities without invoking intertemporal comparisons as well. This method of incorporating quality differences into a model of variety or model demand does not look as successful in this thesis as it has done in previous studies (Cowling (1972), Cowling and Rayner (1970)). Results are very sensitive to the definition of quality-adjusted price employed, where the definition depends on the form of the estimated price-quality relationship. There seem to be no a priori grounds for preferring one definition to another.

More work, both theoretical and empirical, needs to be done in the field of demand for durable goods. An understanding of the factors which influence the demand for durables is prior to the development of techniques of model demand estimation. The availability of such techniques would be of great practical help. This thesis has (I hope) at least pointed out some of the problems involved and attempted to solve them. In particular it has tried to use microeconomic theory as an aid to solution. Surprisingly such an approach to the question appears to be fairly novel.
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