The Design of Hybrid Stepping Motors Aided by Three Dimensional Finite Element Analysis

CLIFFORD MARK JOLLIFFE

Thesis submitted for the examination of the degree of Doctor of Philosophy

Department of Engineering
University of Warwick
Coventry, CV4 7AL

January 1999

9921568

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# TABLE OF CONTENTS

## LIST OF FIGURES

VIII

## GLOSSARY OF TERMS

XII

## ACKNOWLEDGEMENTS

XIV

## SUMMARY

XV

## CHAPTER 1

### INTRODUCTION

1

1.1 INTRODUCTION

1

1.2 BASIC VARIATIONS OF THE STEPPING MOTOR

1.2.1 VARIABLE RELUCTANCE MOTOR

2

1.2.2 PERMANENT MAGNET STEPPING MOTOR

4

1.3 OPERATION OF THE HYBRID STEPPING MOTOR

9

1.3.1 SIMPLE MOTOR EXAMPLE

9

1.3.2 CURRENT PATTERNS

13

1.3.3 MICROSTEP TING

14

1.4 STANDARD 200-STEP HYBRID STEPPING MOTOR

15

1.4.1 WINDINGS

16

1.5 STEPPING MOTOR DRIVES

19

1.5.1 UNIPOLAR DRIVE

19

1.5.2 R-L DRIVES

20

1.5.3 BIPOLAR DRIVES

21

1.5.4 POWER DUMPING

22

1.6 STEPPING MOTOR CONTROL SYSTEMS

22

1.6.1 EXAMPLE OF OPEN LOOP CONTROL (FIGURE 1.25)

22

1.6.2 EXAMPLE OF CLOSED LOOP CONTROL (FIGURE 1.26)

23

1.7 WHY INVESTIGATE THE HYBRID STEPPING MOTOR

24

1.8 FORMAT OF THIS THESIS

25

## CHAPTER 2

### THE HYBRID STEPPING MOTOR: ASPECTS OF CONSTRUCTION, TESTING, AND MODELLING

27

2.1 MANUFACTURE OF MODERN HYBRID STEPPING MOTORS

27

2.1.1 STATOR CONSTRUCTION

28
2.1.2 ROTOR CONSTRUCTION .............................................................................................................. 29
2.1.3 ROTOR AND STATOR JOINING .................................................................................................... 30
2.2 EXPERIMENTAL ARRANGEMENTS ............................................................................................. 31
2.2.1 OPEN CIRCUIT BACK EMF TESTING (FIGURE 2.1) ...................................................................... 31
2.2.2 STATIC TORQUE ........................................................................................................................ 31
2.2.3 DYNAMIC TORQUE .................................................................................................................... 33
2.3 TEST RIG APPARATUS.................................................................................................................. 34
2.3.1 CONTROL AND DISPLAY ............................................................................................................ 35
2.4 SPECIALISED TESTS ..................................................................................................................... 36
2.4.1 CURRENT WAVEFORMS AND INDUCTANCE CALCULATIONS [18].................................................. 36
2.4.2 FLUX LINKAGE MEASUREMENT [21] ........................................................................................... 38
2.4.3 THEORY OF FLUX-LINKAGE MEASUREMENT ............................................................................... 39
2.4.4 EXPERIMENTAL MEASUREMENT METHOD .................................................................................. 41
2.5 PREDICTION OF STEPPING MOTOR CHARACTERISTICS ....................................................... 42
2.5.1 USE OF ELECTROMAGNETIC ANALYSIS SOFTWARE ............................................................... 43
2.5.2 WORK CARRIED OUT ON FINITE ELEMENT MODELLING OF HYBRID STEPPING MOTORS ........ 43
2.5.3 SIMPLER COMPUTATIONAL METHODS IN THE DESIGN OF STEPPING MOTORS ............. 44
2.6 DISCUSSION ................................................................................................................................. 45

CHAPTER 3

ELECTROMAGNETIC MODELLING OF THE HYBRID STEPPER MOTOR ......................................................... 46

3.1 INTRODUCTION TO ADVANCED NUMERICAL TECHNIQUES ............................................................ 46
3.2 FINITE ELEMENT ANALYSIS OF THE HYBRID STEPPING MOTOR .................................................. 48
3.2.1 THE NEED FOR THREE DIMENSIONAL MODELLING OF THE HYBRID STEPPING MOTOR ........... 48
3.3 TYPICAL FINITE ELEMENT SOFTWARE PACKAGE ............................................................................ 50
3.3.1 PRE-PROCESSOR ........................................................................................................................ 51
3.3.2 SOLVER ...................................................................................................................................... 51
3.3.3 POST-PROCESSOR ....................................................................................................................... 51
3.4 MODELLING OF A SIMPLIFIED HYBRID STEPPING MOTOR ............................................................ 52
3.4.1 TOOTH CONSTRUCTION .............................................................................................................. 52
3.4.2 HIDDEN DETAIL METHOD .......................................................................................................... 53
3.4.3 TWISTED EXTRUSION METHOD ................................................................................................... 55
3.4.4 ADVANTAGES AND DISADVANTAGES OF EACH METHOD ................................................................ 55
3.4.5 PREFERRED METHOD .................................................................................................................. 55
3.4.6 BOUNDARY CONDITIONS ........................................................................................................... 57
3.4.7 NEUMANN .................................................................................................................................. 57
3.4.8 DIRICHLET .................................................................................................................................. 57
3.4.9 HYBRID STEPPING MOTOR PERIODICITY ..................................................................................... 59
3.4.10 MAGNET DEFINITION ............................................................................................................... 59
3.4.11 CONSTRUCTION OF MODEL FOR THE SIMPLE STEPPING MOTOR ........................................... 61
3.4.12 RESULTS .................................................................................................................................... 64
3.5 IMPLEMENTING A MODEL OF AN ACTUAL HYBRID STEPPING MOTOR ............................................. 67
3.5.1 PHYSICAL MODEL ....................................................................................................................... 67
3.5.2 GEOMETRIC MODEL + NUMERICAL MODEL ............................................................................ 67
CHAPTER 4

DEVELOPING A STATIC MODEL OF THE HYBRID STEPPING MOTOR FROM 3-D FINITE ELEMENT ANALYSIS

4.1 THREE DIMENSIONAL ANALYSIS MODELLING

4.2 HYSTEP PC MODELLING

4.3 B-H CHARACTERISTICS

4.3.1 APPROXIMATE ANALYTICAL SOLUTION

4.3.2 EXAMPLE OF MODELLING B-H CURVES

4.4 FLUX-LINKAGE CHARACTERISTICS

4.5 DETERMINING PERMEANCES OF AIR-GAP REGIONS USING TWO DIMENSIONAL FEA

4.5.1 METHOD OF CALCULATING RELUCTANCE

4.6 ISOLATED TEETH ANALYSIS

4.7 DETERMINING PERMEANCES ALGEBRAICALLY

4.7.1 PERMEANCES CALCULATION BY FLUX LINES METHODS

4.7.2 FLUX VERSUS MMF RELATIONSHIPS

4.8 TABULATED DATA OR CONTINUOUS FUNCTION FORM

4.9 THREE DIMENSIONAL EFFECTS COMPARISONS

4.9.1 MAGNET

4.9.2 LAMINATIONS

4.9.3 PERPENDICULAR FLUX

4.9.4 CROSS LAMINATION EFFECTS

4.9.5 END EFFECTS

4.9.6 SUMMARY OF CONTRIBUTORY FACTORS TO THE MAGNETIC BEHAVIOR
LIST OF FIGURES

CHAPTER 1
FIGURE 1.1 - SIMPLE VARIABLE RELUCTANCE MOTOR ................................................................. 2
FIGURE 1.2 - POLE CASTELLATIONS AND ROTOR TEETH ............................................................. 3
FIGURE 1.3 (A,B) - PERMANENT MAGNET STEPPING MOTOR, CHANGE OF EXCITATION IN THE STATOR .......... 4
FIGURE 1.4 - HYBRID STEPPING MOTOR .................................................................................... 5
FIGURE 1.5 - TYPICAL HYBRID MOTOR STATOR ......................................................................... 6
FIGURE 1.6 - ROTOR MAKE UP OF HYBRID STEPPING MOTOR .................................................. 6
FIGURE 1.7 - POLE CASTELLATIONS AND ROTOR ................................................................. 7
FIGURE 1.8 - CROSS SECTIONAL VIEW OF MAGNETIC FLUX PATH ......................................... 8
FIGURE 1.9 - MULTI-STACK ROTOR ............................................................................................ 8
FIGURE 1.10 - SIMPLE TWELVE STEPS PER REVOLUTION HYBRID MOTOR ............................. 9
FIGURE 1.11(A,B,C,D) - FULL STEPPING, SINGLE PHASE ......................................................... 10
FIGURE 1.12 - FULL STEPPING, TWO PHASES ......................................................................... 12
FIGURE 1.13 - HALF STEPPING .................................................................................................... 13
FIGURE 1.14 - FULL STEPPING - TWO PHASE ......................................................................... 13
FIGURE 1.15 - HALF STEP CURRENT ......................................................................................... 14
FIGURE 1.16 - PROFILED HALF STEP CURRENT ........................................................................ 14
FIGURE 1.17 - MICROSTEEPPING ............................................................................................... 15
FIGURE 1.18 - AC SYNCHRONOUS MOTOR CIRCUIT ................................................................. 16
FIGURE 1.19 - BIFLAR-WINDING ............................................................................................... 16
FIGURE 1.20 - BIPOLAR EXCLUSIVE WINDING .......................................................................... 18
FIGURE 1.21 - TYPICAL PERFORMANCE CURVES FOR SERIES AND PARALLEL CONNECTIONS .... 19
FIGURE 1.22 - BASIC UNIPOLAR DRIVE .................................................................................... 20
FIGURE 1.23 - R-L DRIVE PRINCIPLE OF REDUCING CURRENT RISE TIME ............................ 20
FIGURE 1.24 - BIPOLAR BRIDGE ............................................................................................... 21
FIGURE 1.25 - OPEN LOOP MOTOR/DRIVE SYSTEM ................................................................. 23
FIGURE 1.26 - MOTOR DRIVE SYSTEM WITH CLOSED LOOP CONTROL .................................... 24

CHAPTER 2
FIGURE 2.1 - BACK EMF EXPERIMENTAL ARRANGEMENT ....................................................... 31
FIGURE 2.2 - (A) - STATIC TORQUE AGAINST POSITION (B) - STATIC TORQUE AGAINST CURRENT .... 32
FIGURE 2.3 - DYNAMIC TORQUE/SPEED CHARACTERISTICS ................................................ 34
FIGURE 2.4 - TEST RIG LAYOUT ................................................................................................ 35
FIGURE 2.5 - INDUCTANCE CALCULATION CIRCUIT ............................................................... 37

CHAPTER 3
FIGURE 3.1 - 8:6 SWITCHED RELUCTANCE LAMINATION USED FOR THE SIMPLE STEPPING MOTOR .... 52
FIGURE 3.2 - HIDDEN DETAIL METHOD FOR CREATING TEETH ............................................. 54
FIGURE 3.3 - TEETH CREATED BY HIDDEN DETAIL METHOD .................................................. 54
FIGURE 3.4 - TWISTED ROTOR METHOD ................................................................................... 56
FIGURE 3.5 - GEOMETRIC AND MAGNETIC SYMMETRY .......................................................... 59
FIGURE 3.6 - FRONT ON VIEW SHOWING ROTOR TEETH MISALIGNMENT ................................. 62
FIGURE 3.7 - LAYER DEFINITIONS ............................................................................................. 63
FIGURE 3.8 - SOLID 180° MODEL OF THE SIMPLE STEPPING MOTOR ........................................ 63
FIGURE 3.9 - SIMPLE STEPPER, UNDER VIEW .......................................................................... 65
FIGURE 3.10 - SIMPLE STEPPER, FULL VIEW ........................................................................... 66
FIGURE 3.11 - ROTOR TEETH ELEMENTS ................................................................................... 69
FIGURE 3.12 - TWO PHASE TORQUE REPRESENTATION ......................................................... 71
FIGURE 3.13 - COGGING TORQUE ASSUMPTION ...................................................................... 72
FIGURE 3.14 - REDUCTION TECHNIQUE FOR REDUCING 4 NODES TO 2 .................................. 72
FIGURE 3.15 - BROKEN BOUNDARY CONDITION ...................................................................... 73
FIGURE 3.16 - COMPLETED BOUNDARY LINE ........................................................................... 74
FIGURE 3.17 - HALF ROTOR MODEL ........................................................................................... 77
FIGURE 3.18 - 2-D JOINED ROTOR AND STATOR ...................................................................... 78
FIGURE 3.19 - COMPLETED HALF MODEL ................................................................. 79
FIGURE 3.20 - TWO PHASE 2-D (360° BY REFLECTION) COMPLETE MODEL .................................................. 80
FIGURE 3.21 - CLOSE UP 3-D COMPLETE MODEL ....................................................................................... 81
FIGURE 3.22 - ROTOR EXCITED BY MAGNET FLUX ........................................................................ 83
FIGURE 3.23 - MAGNET FLUX PATHS ................................................................................................... 85
FIGURE 3.24 - AIR-GAP CROSSING ........................................................................................................ 86
FIGURE 3.25 - PLOT OF FLUX DISTRIBUTION IN UNEXCITED STATOR POLE ........................................................................ 87
FIGURE 3.26 - CLOSE UP OF ROTOR TEETH AND STATOR POLE ........................................................................ 89
FIGURE 3.27 - FLUX DISTRIBUTION IN AN EXCITED POLE ........................................................................... 90
FIGURE 3.28 - COMPLETE FLUX DISTRIBUTION .............................................................................................. 91
FIGURE 3.29 - FLUX DISTRIBUTION WITH HIGH EXCITATION ........................................................................ 92
FIGURE 3.30 - BREAKDOWN OF MAGNET PATH .......................................................................................... 93
FIGURE 3.31 - OVERALL FLUX DISTRIBUTION IN TWO PHASE (180°) .......................................................... 95
FIGURE 3.32 - OVERALL FLUX DISTRIBUTION IN TWO PHASE (360°) .......................................................... 95
FIGURE 3.33 - TWO PHASE ROTOR TEETH ................................................................................................. 96
FIGURE 3.34 - TWO PHASE INNER STATOR ................................................................................................ 96

CHAPTER 4
FIGURE 4.1 - BASIC OVERVIEW OF Hystep ................................................................. 101
FIGURE 4.2 - KNEE OF FITTED B-H CURVE TO REAL DATA POINTS ......................................................................... 106
FIGURE 4.3 - FITTED B-H CURVE TO REAL DATA POINTS BY NEW EQUATION ......................................................................... 106
FIGURE 4.4 - FLUX-LINKAGE CURVES ...................................................................................................... 107
FIGURE 4.5 - OBTAINING UN-ALIGNED FLUX-LINKAGE CURVE ............................................................................. 108
FIGURE 4.6 - FRONT ALIGNED POSITION WITH BACK UN-ALIGNED ................................................................................. 109
FIGURE 4.7 - ALIGNED ROTOR 2-D TEETH MESH ........................................................................................... 110
FIGURE 4.8A,B,C - TYPICAL FLUX DISTRIBUTION OF A ROTOR/ STATOR POSITION ........................................................................ 112
FIGURE 4.9 - TYPICAL FLUX AND FLUX DENSITY DISTRIBUTION OF A ROTOR/ STATOR POSITION ........................................................................ 112
FIGURE 4.10 - RELUCTANCE CALCULATION FROM VECTOR POTENTIAL ........................................................................... 114
FIGURE 4.11 - EXAMPLE OF ELLIPTICAL FLUX TUBE ......................................................................................... 121
FIGURE 4.12 - SECTIONS IN A SIMPLE TOOTH GEOMETRY .................................................................................... 123
FIGURE 4.13 - FINDING MMF FOR PATHS ...................................................................................................... 126
TABLE 4.1 - FINDING MMF FOR PATHS ...................................................................................................... 127
FIGURE 4.14 - Bx, By, AND Bz PLOT FOR ROTOR ............................................................................................. 128
FIGURE 4.15 - Bx, BY, AND Bz PLOT FOR AIR-GAP ....................................................................................... 129
FIGURE 4.16 - Bx, BY, AND Bz PLOT FOR STATOR ......................................................................................... 129
FIGURE 4.17 - WINDOW FOR MAGNET CO-ENERGY ........................................................................................ 132
FIGURE 4.18 - MAGNET ZERO POTENTIAL .................................................................................................... 133
FIGURE 4.19 - LAMINATION STRUCTURE .................................................................................................... 135
TABLE 4.2 - THREE DIMENSIONAL EFFECTS ............................................................................................. 137
FIGURE 4.20 - ZERO POTENTIAL IN STATOR BACK IRON ........................................................................... 138
FIGURE 4.21 - FULL LUMPED MODEL OF THE HYBRID STEPPING MOTOR .......................................................... 139
FIGURE 4.22 - HALF LUMPED MODEL OF THE HYBRID STEPPING MOTOR .......................................................... 140
FIGURE 4.23 - REDUCED ELEMENTS NETWORK ........................................................................................... 140
FIGURE 4.24 - SINGLE PHASE NETWORK ...................................................................................................... 141
FIGURE 4.25 A/B - MAGNET AND BACK IRON FLUX VERSUS MMF CHARACTERISTICS ........................................................................ 144
TABLE 4.3 - EFFECT OF BACK IRON MMF DROP ON MAGNET OUTPUT ........................................................................ 145
FIGURE 4.26 - ALIGNED ROTOR TEETH FLUX/ MMF CHARACTERISTICS ........................................................................ 148
FIGURE 4.27 - FRONT VIEW OF MOTOR SHOWING ALIGNED POSITION .......................................................... 150

CHAPTER 5
FIGURE 5.1 - SINGLE PHASE WINDING EXCITATION ............................................................................................. 156
FIGURE 5.2 - TWO PHASE EXCITATION SYSTEM ............................................................................................. 158
FIGURE 5.3A/B - TWO-PHASE FLUX LINKAGE CURVES .................................................................................... 159
FIGURE 5.4 - MAGNET FLUX LINKAGE PLOT .................................................................................................... 160
FIGURE 5.5 - CO-ENERGY OF DETENT TORQUE ........................................................................................ 162
FIGURE 5.6 - DETENT TORQUE SHOWN AS 4TH HARMONIC OF SINGLE PHASE MOTOR TORQUE .................. 162
FIGURE 5.7 - EXPERIMENTAL FLUX LINKAGE CURVE WITH 3-D FEA PREDICTION .......................................................... 163
GLOSSARY OF TERMS

A is Potential

$A$ is the Cross-sectional area

$A_{cond}$ is the cross-sectional area of a conductor

$B$ is the flux density

$E_L$ is the linearised mesh energy per unit length

$f$ is the frequency of the supply.

$H$ is the field strength

$H_e$ is the field strength coercivity

$I$ is the current

$i$ is the instantaneous current

$i_j$ is the current in phase $j$, ($j=1,2$)

$i_{10}$ is the initial current in phase 1

$i_{20}$ is the initial current in phase 2

$J$ is the current density,

$L$ is the inductance

$l$ is the instantaneous inductance

$m$ is the number of phases

$N$ is the number of turns in a conductor

$n_r$ is the number of teeth on the rotor stack.

$R$ is the resistance

$S$ is the steps per revolution,

$T$ is the Torque

$t$ is time

$t_{contant}$ is the electrical time constant

$V$ is the voltage

$v$ is the instantaneous voltage

$W_e$ is the electrical input power

$W_{mech.}$ is the mechanical output power

$W_{field}$ is the energy stored in the field.

$\omega_{rpm}$ is the speed in revolutions per minute

$t$ is the mean flux path length

$\lambda$ is the flux-linkage

$\lambda_{(a,0)}$ flux-linkage in terms of position and current

$\lambda_{0}$ is the flux-linkage at time $t=0$

$\lambda_{(q, \theta, \theta)}$ is the flux linkage associated with the stator phase due to the permanent magnet at any position $\theta$. 

xii
$\text{MMF}_{\text{mag}}$ is the MMF produced by the magnet

$mmf$ is the instantaneous MMF

$\Phi_{\text{mag}}$ is the peak flux generated by the magnet

$\phi$ is the magnetic flux

$\mathcal{R}_i$ is the mesh reluctance per unit length

$\theta$ is the position of the rotor

$\mu_o$ is the permeability of free space

$\mu_r$ is the relative permeability of the magnetic circuit

$\text{EMF}_{\text{mag}}$ is the rotational EMF produced by the magnet

$emf_j$ is the instantaneous motional EMF induced in phase 1 by the permanent magnet rotor.

$Pf$ is the packing factor

$rps$ are the revolutions per second
ACKNOWLEDGEMENTS

I would like to take this opportunity to express my thanks to Dr. Charles Pollock, and Dr. Alex Michaelides, for their patience, encouragement, and supervision.

This project could not have been undertaken without the financial support of Stebon. Ltd., in particular Mr Peter Buonaquisti, managing director. Industrial supervision has been given in the highest calibre by Mr Dave Baker, and I thank him for this. Thank you to all my new colleagues at Stebon, especially to Brian Nutter and Jon Harding.

Gratitude should be made to both John Reeve, and Matts Persson (Hoganas), who individually supported part of this project.

I would like to thank the DTI/ SERC programme and the Department of Engineering for funding and the use of facilities. Also to Vector Fields Ltd. for their excellent support with their Finite Element software.

To my companions in the Power Electronics Lab. Cheers to Mike Barnes (hope all goes well at UMIST), Kevin Richardson (Up the VILLA!), Helen Pollock (great kids), John Wale (bell ringer extraordinare), Andrew Chu (Don’t you dare run that Saber simulation now!), T’Shen (It’s only a plastic snake), Alex Michaelides (Up with the Turks), and Andy Leeson (gladiator).

Thank you to the Wade family, for their love and support. Special thanks to Kevin Brant, for some excellent intercontinental adventures.

Lastly praise to my wife, who has had to put up with an untidy office, and no fitted wardrobes, but at least I have finished the pond! I still wonder if you know what a hybrid stepper motor is, but I devote this work to you.
SUMMARY

Though the hybrid stepping motor has a long and proven history, in terms of toughness, accuracy of position and the ability to operate in open loop, motor performance improvements can still be made in terms of the physical structure of the motor's components. It is impossible to build a complete solution of the hybrid stepping motor using simple analytical functions or equivalent circuit representations. This is due to the difficulties introduced by the motor's highly non-linear three dimensional magnetic structure, of which the doubly salient tooth structure, axial magnet, and back iron all complicate the situation. However, with the recent advances in three dimensional finite element software a comprehensive study of the motor has been achieved in this thesis. This has allowed improvements to simpler two dimensional based mathematical models, which allow faster computation of the motor's electromagnetic performance. To aid modelling, novel equations which accurately model today's high permeability steels have been developed. These are shown to be more accurate than the established Jiles-Atherton method. Inductance calculations of the steel's flux paths have been comprehensively improved by the use of elliptical functions. The thesis concludes with the design of two quite individual new machines. The first dramatically improves a motor's power output, smoothness, noise levels, and resonance by modifying the tooth structure. The second uses soft magnetic composite materials to provide an isotropic path for cross lamination flux which flows in a stator's back iron. Both new designs are shown to offer a significant improvements to the high speed torque capability of the hybrid stepping motor.
CHAPTER 1

INTRODUCTION

1.1 Introduction

The Hybrid stepping motor is an electromechanical device usually used in positioning applications, but also finds its uses in applications where a high torque and controllable speed are required. Its basic operation is to translate a received electrical pulse into a mechanical movement. The hybrid stepping motor is a member of a large family of stepping motors of which there are many design types available. These range from extremely simple AC synchronous two pole magnet types to the more elaborate in physical design, like the hybrid stepping motor. In this thesis the emphasis is focused on the latter. However it is beneficial that two other members from the family of stepping motors are briefly described so that the evolution of the hybrid stepping motor can be better comprehended.

1.2 Basic Variations of the Stepping Motor

Three basic rotary stepping motors are found in industrial applications. These are the variable reluctance, permanent magnet, and hybrid stepping motor. All are essentially positioning devices, and exhibit the benefits of low cost, ruggedness, simple construction, low maintenance, and open loop control.
The variable reluctance motor has a construction that is rugged and relatively simple [1]. As the name suggests the motor works on the variable reluctance principle, where the change in the rotor position relative to the stator poles relates to a corresponding change in reluctance. This change in reluctance directly affects the flux/MMF ratio, and hence the torque. In its simplest form each pole may be a single tooth (figure 1.1), but a more practical design of pole is shown in figure 1.2. Here the rotor has numerous teeth and each stator pole has a set of teeth castellations. It should be noted that the simpler tooth structure in figure 1.1, is found more commonly in the type of motor called the switched
reluctance motor, which is presently seeing growing interest due to its simple mechanical structure and improving drive electronics [2].

The simple structure in figure 1.1 will be used to briefly describe the motor’s construction. It can be seen in the figure that the stator has eight salient poles, whereas the rotor has six. The rotor and stator are made up of thin pieces of sheet metal called laminations. Each stator pole has a coil wound around it and each opposite pole is connected to produce a reverse magnetic polarity. The example in figure 1.1 has four phases A, B, C, and D, made up of coils 1-1’, 2-2’, 3-3’, and 4-4’ respectively. Rotation is caused by applying current into the phases in sequence that causes two rotor teeth to line up with the excited stator poles. The energisation of the stator is then switched and the rotor is pulled to its next position.
1.2.2 Permanent Magnet Stepping Motor

Sometimes referred to as the tin can or can-stack motor, the construction of the permanent magnet stepping motor is shown in figure 1.3 (A,B). This motor is a low cost, low torque, low speed device, and is found commonly in computer peripherals. The motor steps in relatively large angles, but its simplicity allows mass production of a low cost device. The rotor is made of permanent magnet material and has individual magnetic poles magnetised radially on the rotor. The stator has a phase winding which can induce a two pole magnetic field. A rotor north pole aligns with the stator's south pole and a rotor south pole aligns with the north pole of the stator. The stator poles are made of two triangular sections that allow the top and bottom to line up with poles of opposite polarity on the rotor. Changing the coil current polarity in the stator coil causes a rotation of one rotor pole pitch as shown in figures 1.3(A) and (B).
1.2.3 Hybrid Stepping Motors

A hybrid stepping motor shown in figure 1.4, is by far the most widely used stepping motor in industrial applications.

![Figure 1.4 - Hybrid Stepping Motor](Photograph Courtesy of Stebon Ltd.)

The name is derived because it combines the operating principles of the permanent magnet motor and the variable reluctance type. The stator (figure 1.5) is on the periphery of the motor and is similar to the multi-tooth variable reluctance stator. In the most common implementation the laminations of the stator make up a set of eight electromagnetic poles that are wound in two phases to produce a four pole magnetic field. The windings around the stator poles are used to encourage or discourage the flow of magnetic flux through the poles depending on the rotor position required. As with all stepping motors, the function of the hybrid type is to move the rotor through a precise angular step when the current in one of the windings is switched. The stator poles produce separate latching points or steps for the motor.
The rotor (figure 1.6) of the hybrid motor is the important distinction between the hybrid and variable reluctance type stepping motor. A cylindrical magnet is sandwiched between two rotor end-caps. The magnet is axially magnetised causing one of the end-caps to act as a series of north poles and the other end-cap to be a set of south poles. The end-caps of the rotor are a series of steel laminations which are very gear like in appearance. The teeth on the two end-caps are mis-aligned with respect to each other by half a rotor tooth pitch. When the rotor teeth of one end-cap of the rotor are aligned with teeth of a particular stator pole, the teeth on the other rotor end-cap are mis-aligned with the teeth of the same
stator pole (figure 1.7). These rear end-cap’s teeth are found to be aligned with another stator pole which is at 90° relative to the original stator pole.

The rotor’s magnetic field will generate flux that will travel from the end of the magnet to another through the stator core (figure 1.8). The lines of flux will leave the north pole of the magnet through the front rotor end-cap and travel across the air-gap into the stator. The position the flux enters the stator will be influenced by the lowest reluctance path available. This lowest reluctance path will depend on how much the rotor and stator teeth are aligned, and the strength and polarity of any excitation on the stator coils. The flux path then travels along the stator back iron to a similar position at the rear of the stator. The actual position is again governed by the reluctance of the path. The flux will cross the air-gap into the rear rotor end-cap. It can then rejoin the magnet. In different parts of the motor the magnetic flux has significant radial, axial or tangential components.
To increase the torque output for a particular frame size diameter, multi-stacking can be employed. The construction of each stack of a multi-stack motor is essentially the same as for a single stack. The stator is longer to cover the extension of the rotor. Instead of the rotor consisting of a single magnet and two end-caps, the rotor now has two or more of these rotor stacks, each separated by a non-magnetic spacer, figure 1.9. This gives an increase in torque due to the increased length of teeth and better utilisation of stator copper (less end windings). There is a marginal increase in inertia, however this is relatively small compared to increasing the rotor diameter.
1.3 Operation of the Hybrid Stepping Motor

1.3.1 Simple Motor Example

The operation of the hybrid motor is shown in figure 1.10, using a simple 12 step per revolution motor. The rotor of this machine consists of two pole pieces with three teeth on each, the construction is inherently the same as the structure of figure 1.4. In between the two rotor end-caps there is a permanent magnet axially magnetised, producing a north pole at the back and a south at the front. The stator has four poles with coils wound on each. Phase 1 is made of poles 1A and 1B, and Phase 2 is made of 2A and 2B.

With no current flowing the rotor will take one of a possible 12 positions. This is because of the detent effect of the magnet trying to minimise the reluctance to its own magnetic flux path. This occurs when a pair of north and south rotor teeth align with two of the stator poles.

![Figure 1.10 - Simple Twelve Steps per Revolution Hybrid Motor](image-url)
i Single Phase Full Stepping

If current now passes through one of the stator phase windings as shown in figure 1.11A, the resultant north and south stator poles will attract teeth of the opposite polarity on each end of the rotor stack. There are now three stable positions for the rotor, the same as the number of teeth on each rotor end-cap. The torque required to deflect the rotor from its stable position is now much greater, and is referred to as the holding torque.

By changing the current flow from the first to the second set of stator windings (B), the stator field rotates through 90° and attracts a new pair of rotor poles. This results in a rotor step of 30° referred to as one full step. Applying current in phase 1 again but in the
opposite polarity, will cause the field to shift by another 90° and the rotor takes another full step of 30°, figure 1.11.C. If phase two is energised with a negative polarity current figure 1.11.D position will occur, then the condition in figure 1.11.A can be repeated. In completing these four steps the rotor is said to have moved through one tooth pitch. This simple motor has therefore a 12 steps per revolution in single phase full stepping mode.

ii Two Phase Full Stepping

If two coils are energised simultaneously (figure 1.12), the motor is said to be operating under two phase excitation. The rotor has now an ability to take up intermediate positions, since it can be equally attracted to two stator poles. Greater torque can be achieved in this way as all the stator poles are influencing the rotor. The motor can be made to take a full step by reversing the current in one of the phases, producing a 90° movement in the stator field. This is the normal method of two phase full stepping.

1.3.1.3 Two Phase Half Stepping

By alternately energising one phase and then two (figure 1.13), the rotor moves through only 15° at each stage and the number of steps per revolution will be doubled. This is called half stepping, and many industrial applications make much use of this mode. There is some loss of torque as all the windings are not in constant use, but motion is smoother at low speeds, and there is less positional overshoot.
Chapter 1. Introduction

FIGURE 1.12 - Full Stepping, Two Phases.

FIGURE 1.13 - Half Stepping
1.3.2 Current Patterns

When the motor is driven in its full step mode by energising two phases at a time (figure 1.14), the torque available to each step will be the same. In half step mode, two phases are energised alternately and then only one (figure 1.15). This will cause greater torque to be produced when the two phases are on. The alternate steps will therefore be strong (two phase on) and then weak (single phase on). The available torque is limited by the weaker step, even though the motion will be smoother. It would be beneficial to have equal torque on each step. A different current sequence can be used, which increases the phase current when only one phase is on. This will not burn out the winding, as motors are generally rated for two phases on. With one phase on the same total power will be dissipated in the windings if the current is increased by 40%. Using higher current when the single phase is on produces approximately equal torque on alternate steps, figure 1.16.
1.3.3 Microstepping.

Energising two phases with equal current produces an intermediate step position half-way between the one-phase on positions. If the two phase currents are unequal, the rotor will be shifted towards the pole with higher energisation. This effect is utilised in the microstepping drive, which subdivides the basic motor step by proportioning the current in two windings. In this way, the step size is reduced and smooth rotation is obtained. The
current pattern in the windings resembles two sine waves with a 90° phase shift between them as in figure 1.17.

![Figure 1.17 - Microstepping](image)

### 1.4 Standard 200-Step Hybrid Stepping Motor

The common 200 step hybrid stepping motor described in section 1.2.3, operates in a similar way to the simple 12 step model, but with 50 teeth on each rotor end-cap, and multiple teeth on each stator pole. It produces a step angle of 1.8° (200 steps) in two phase on mode, equation 1.1, and 0.9° (400 steps) with half stepping.

\[ S = 2mn_r, \quad (1.1) \]

where \( S \) is the steps per revolution,
\( m \) is the number of phases, and
\( n_r \) is the number of teeth on the rotor stack.

There are as many detent positions as full steps per revolution of the motor. The hybrid stepping motor may also be driven direct from a single phase supply using a capacitor in
series with one of the phases (figure 1.18). In the UK the 50 Hz supply will produce a
rotation of 60 rpm, equation 1.2.

$$\omega_{\text{rpm}} = f \frac{60}{n_r},$$  \hspace{1cm} (1.2)

Where $\omega_{\text{rpm}}$ is the speed in revolutions per minute, and

$f$ is the frequency of the supply.

![AC Synchronous Motor Circuit](image)

**FIGURE 1.18 - AC Synchronous Motor Circuit**

1.4.1 Windings

![Bifilar-Winding](image)

**FIGURE 1.19 - Bifilar-Winding**

Traditionally motors have had bifilar windings, which means there are two sets of
identical windings on each pole, as shown in figure 1.19. These are wound as if they were
single coils, producing two coils that are almost electrically and magnetically identical.
Chapter 1. Introduction

The origins of this type of winding go back to the use of the unipolar drive, which reverses the polarity of the magnetic field by energising the other coil on the stator pole instead of reversing the current. This allowed a simpler drive to be constructed. Today's drive technology has improved and as a result the bipolar drive has nearly eliminated the unipolar drive.

Other types of wire are exclusively used with bipolar drives. Each of the eight stator poles has a single coil placed on it. Poles 1, 3, 5, and 7 (figure 1.20) have coils of one phase, whereas the remainder have the second phase. Each consecutive coil of a phase has an alternate polarity. The coils on poles 1 and 3, and 5 and 7, are connected internally in pairs in parallel or series. This allows 4 wires to be brought out per phase.

Both the bifilar and exclusively bipolar winding allows two types of speed/torque characteristics to be obtained by the use of the 8 leads that come from the two phases. These are referred to as parallel and series connections. Connecting the winding in series causes a phase current to flow through twice as many turns. This doubles the ampere turns or MMF and causes a corresponding increase in torque. However this doubles the effective number of turns and increases the inductance by a factor of 4 relative to a single winding. Inductance $L$ is proportional to the number of turns $N$ squared.

$$L \propto N^2,$$  \hspace{1cm} (1.3)
This causes the torque to drop off as the motor speed is increased, as the peak current level in the winding will fall. Series connections are used when low speed torque is required and may well produce low speed resonance because of the high torque produced in this region. Connecting in parallel allows the current to divide between the two coils. It does not change the inductance (relative to one of the coils) as the number of effective ampere turns remains the same. The resistance of the coils is however halved, which produces a lower power dissipation. The current will be able to run 40% higher than in a single winding. Parallel connections are generally preferred as they produce a flatter torque curve and greater shaft power. Typical series and parallel performance curves are shown in figure 1.21.

FIGURE 1.20 - Bipolar Exclusive Winding
1.5 Stepping motor drives.

The stepping motor drive delivers the electrical power to the motor in response to a low level signal sent from a controller. The applied voltage is significant in controlling the current and has a dramatic effect on the speed range of the motor. Input signals to the drive consist of step pulses and direction information; One pulse for every step the motor is to take. This is true regardless of the motor resolution chosen. The motor will therefore require 200 pulses for full stepping or 50,000 pulses to produce a revolution for a 50,000 high resolution drive. Speed control is achieved by the rate at which the digital pulse train is fed to the drive. The rotor is expected to increment steps at the same rate as the signal.

1.5.1 Unipolar Drive

The simplest type of drive is the unipolar drive as shown in figure 1.22. Current in the drive can only flow in one direction. To reverse the magnetic field of a stator pole, the drive must operate with bifilar wound coils where the excitation is switched to the second coil. Generally in this simple drive the current is determined by the motor phase resistance.
and the applied voltage. This drive responds well at low speeds, but torque rapidly decreases as speed increases due to the inductance of the phase windings.

**Figure 1.22 - Basic Unipolar Drive[3]**

**Figure 1.23 - R-L Drive Principle of Reducing Current Rise Time[3]**

### 1.5.2 R-L Drives

In the unipolar drive the current build up is determined by the voltage, winding resistance, and winding inductance, figure 1.23. To increase the build speed of the current the voltage
would need to be increased. The final current reading would also be higher. By adding a resistor in series the current level can be limited. The electrical time constant $t_{\text{constant}}$ can be calculated by:

$$t_{\text{constant}} = \frac{L}{R}$$

(1.4)

Where $R$ is the resistance.

Adding an additional resistor will cause increased power to be dissipated in the series resistance, producing a significant amount of heat. This extra power must come from the DC power supply.

### 1.5.3 Bipolar Drives

![Bipolar Bridge Diagram]

FIGURE 1.24 - Bipolar Bridge [3]

One drawback of the unipolar drive is its inability to utilise all the coils on the motor's stator poles. At any one time only half the coils on a pole are being used. If both coils
were being used there would be a 40% increase in amp turns for the same power dissipation. To achieve this higher performance a bipolar drive is used. One phase of this type of drive is shown in a simple form in figure 1.24, and can drive current in either direction through each motor phase. The standard arrangement used is called the bridge system, which uses four transistors per phase winding.

1.5.4 Power Dumping

Because the motor has a permanent magnet, the stepping motor can act as a generator if the shaft is driven by an external source. Therefore care must be taken when the energy associated with the inertia of the load is returned to the drive during retardation. A large increase in supply voltage may damage the drive transistors and a power dump circuit must be incorporated to dissipate the regenerated power [3].

1.6 Stepping Motor Control Systems

1.6.1 Example of Open Loop Control (Figure 1.25)

The operator interface allows the user to input some required characteristic of the system, such as speed, acceleration, and distance. The interface may be simply a thumb wheel control or a complicated computerised serial interface. The information is passed to the controller that converts the information into the pulse and direction signal that are fed to the drive. Conditioning and selection of the resolution may be applied to the signal from the information given by the user or may be pre-set in hardware or software options in the controller. The drive will use a form of pulse width modulation to control the current
delivered to the motor windings. The rotor will rotate in a condition dependent on the sequence and style of the energising of its phase windings. The current delivered at a speed is not load dependent. If the motor misses a step or stalls there is no feedback to the controller that this has happened.

![Diagram of Open Loop Motor/Drive System](image)

**FIGURE 1.25 - Open Loop Motor/Drive System**

### 1.6.2 Example of Closed Loop Control (figure 1.26).

In addition to the function blocks of the open loop system, this system incorporates a feedback signal to the controller. Typically an encoder or resolver is used. In their simplest form provide information on position and step integrity. However they can be used to provide information for phase commutation and precise current control to achieve servo performance [4,5].
1.7 Why Investigate the Hybrid Stepping Motor.

Though the hybrid stepping motor has a long and proven history, in terms of toughness, accuracy of position and the ability to operate in open loop [5], motor performance improvements are still to be made in terms of power drives, controllers, and the physical motor construction. There is now increased competition from brushless DC servo systems, which have been aided by the continual price reduction of positional devices, especially encoders and resolvers.

Motor constructors and design engineers are increasingly interested in how tooth configuration, mechanical geometry, and magnetic material compositions affect motor performance [6,7]. These all jointly affect the generation and distribution of flux and therefore torque.

Some novel designs of the hybrid stepper motor have come on to the market in recent years, notably 3 and 5 phase excitation schemes [8,9,10,11] and also the use of so called
flux focusing stator magnets [12,13,14]. Three and 5-phase design claim to have a better settling time and winding copper utilisation due to the increase number of phases. Referring to equation 1.1 it can be seen that 3 and 5 phase hybrid stepper motors will have smaller natural step angles. Flux focusing stator magnet motors have a neodymium magnet placed in the stator teeth troughs. It is claimed that these force the leakage flux into the tooth steel, allowing more flux to generate torque. This method produces approximately 20% increase in holding torque.

The hybrid stepping motor has what is considered to be an unconventional magnetic construction. The magnetic fields involved produce a truly 3-dimensional electromagnetic problem, as significant radial, axial and tangential components of magnetic flux occur throughout the motor. The complex magnetic flux pattern makes performance predictions difficult. Stepping motors have been traditionally designed using empirical methods, relying on experimentation and experience [5]. It is now possible to consider studying these complex magnetic problems with new software packages, because of recent advances in computing power [15,16]. Using tools such as three dimensional finite element analysis software, a designer can produce an accurate model of a hybrid stepping motor. They are able to view the behaviour of flux paths, effects of material types, and generation of torque, without having to construct a prototype.

1.8 Format of this Thesis

A key component of this thesis describes the use of finite element analysis with the power to overcome some of the traditional problems encountered with such analysis. The use of
Chapter 1. Introduction

finite element analysis for the stepping motor provides the foundation of this doctoral project. This will validate assumptions commonly made about the hybrid stepping motor. This work is described in chapters 2, and 3.

Using data and knowledge gained from the three dimensional finite element analysis, improved analytical techniques for prediction of motor performance will be developed. This will involve simpler but accurate models that will eventually be incorporated into a computer aided design package. The simpler models are necessary so that they can be easily modified during the design phase without the requirements for complex 3-D modelling. This is presented in chapter 4 and 5.

In chapter 6 and 7, the modelling observations from the various tools are used to design two new hybrid stepping motors, which are constructed and tested.

Chapter 8 completes the thesis with some conclusions.
This chapter will briefly detail the manufacturing process of the hybrid stepping motor and will then describe some of the hardware apparatus and tests that are used in both manufacturing and experimentation to confirm that an existing motor meets set performance criteria, or that a prototype motor conforms with design or software modelling. An introduction is given to some of the computational methods used in motor design.

2.1 Manufacture of Modern Hybrid Stepping Motors

The hybrid stepping motor has few dedicated manufacturers in the UK. This project was sponsored by Stebon Ltd. and hence the following section is based on the experience gained on the shop floor and test area at Stebon and the work detailed by Stebon's technical manager, Mr D. Baker[17].

As described in chapter 1, the motor consists of a stator pack, stator winding, and a rotor assembly. The assembly of each part will be described before discussing the complete motor assembly and testing.


2.1.1 Stator Construction

The major component of the stator is the core pack. The stator laminations are brought in pre-punched and annealed. The laminations are typically 0.35 mm Transil. To construct a core pack, half of the required laminations are placed one on top of each other on a jig. The laminations are held in the same position by use of a datum point on the lamination, customarily positioned between the middle slot of the castellations. A layer of adhesive is placed between each lamination. The adhesive layer contains typically two types of glue, each with a different temperature stability. After the first half has been assembled on the jig the stack is turned over and the second half added to allow for lamination punch curvature. The complete stack is compressed to improve the packing factor. The packing factor is the ratio of steel material to total stack length. The stack length is checked and more laminations may be added to match the specified length.

The completed stator lamination core packs may be placed into a steel shell, depending on the design and protection rating of the motor. Each lamination in the stator core pack has an inner diameter that is undersized. This infers that if an un-machined rotor lamination was placed on to a stator lamination there would be no air-gap as the inner diameter of the stator lamination and the outer diameter of the rotor lamination would overlap. The completed stator core is then machined (honed) to produce a concentric inner diameter.

Copper windings are pre-wound on bobbins and taped to hold their shape. A pair of windings is selected for the requested performance, determined by the desired torque/speed performance and thermal requirements. A pair of windings is placed on to two of the stator poles of one phase. A second pair of coils is placed on the remaining two poles.
so that each phase is made up of four coils in two electrical circuits. Each phase is terminated with four wires that are appropriate for either terminal box or flying lead termination, providing the ability for the coils to be connected in series or parallel. The stator packs are then impregnated with varnish to provide mechanical rigidity. The varnish is cured by placing the stator pack in an oven.

### 2.1.2 Rotor Construction

Rotor construction starts by producing two rotor end-caps both made up of oversized outer diameter laminations. Like the stator, the rotor laminations are held together by a combination of adhesives whilst being built up on a jig. Each rotor requires a pair of end-caps for each stack. For example a 3-stack motor requires 6 end-caps. One of the rotor end-caps is placed on a jig, and an un-magnetised permanent magnet is placed on top. The permanent magnet material is generally alnico, samarium cobalt, or neodymium iron boron. The choice of magnet material is dependant on the application, or the motor’s working environment. The second rotor end-cap is then placed on the jig, and is correctly positioned using a second datum. This ensures the top end-cap has its teeth lined up with the bottom end-cap’s troughs.

The rotor assembly is now continued by placing the end-caps and magnet components on a non magnetic stainless steel shaft. To make a multi-stack rotor construction, a spacer is placed on the shaft between the stacks. This spacer is typically made of aluminium.
Chapter 2. The Hybrid Stepping Motor; Aspects of Construction, Testing, and Modelling.

The outside rotor diameter and shaft configuration (keys etc.) are now machined along with the bearing seats to maintain concentricity. Bearings are then pressed onto the completed rotor assembly.

2.1.3 Rotor and Stator Joining

The rotor / stator air-gap will typically be 0.1 mm from stator to rotor tooth. The air-gap must be kept as small as possible to ensure maximum torque generation. The rotor is kept in the correct orientation and position relative to the stator and mated to the stator by the use of end brackets. Firstly, the rotor is pressed into the drive end bracket (front flange), where the shaft extends. The front flange is used to hold the motor in position on the motor's final application. The flange is typically square and is usually standardised to a NEMA (North-American Electrical Manufacturers Association) configuration that dictates the motor frame sizing, though other flanges may alternatively be specified. This front flange is joined to the stator assembly by lining up spigot holes and pressing the stator onto a lip, positioned by through bolts. Further spigots are used to join the stator and rotor assembly to a rear end bracket. The leads from the phase windings are pulled through this end bracket. They are either left as flying leads or are alternatively crimped with spade terminals for use with a terminal box. This rear end bracket has provision for the placement of a position verification device or an extended rear shaft.

The complete assembly is now placed in a high energised field that axially magnetises the rotor in situ. This magnetisation in situ offers a significant advantage over other permanent magnet motors as it allows the rotor to be assembled completely before it is magnetised.
Chapter 2. The Hybrid Stepping Motor; Aspects of Construction, Testing, and Modelling.

2.2 Experimental Arrangements

This section describes the experimental arrangements that are commonly used to verify that a constructed motor meets performance criteria. Testing arrangements are also used to confirm modelling techniques and the accuracy of design predictions that are used later in this chapter and thesis. Testing covers both static and dynamic arrangements.

2.2.1 Open Circuit Back EMF Testing (Figure 2.1)

A simple test to calculate the rotational EMF produced from a motor, is achieved by connecting another precise speed motor (typically another hybrid stepping motor), to the shaft of the motor to be tested. The driving motor is run at a set speed which causes the test motor to generate an EMF across its phase windings due to the rotating magnetic field. The RMS value of the EMF is recorded and the near-sinusoidal wave form may also be captured. This test is often used in manufacturing to check that the permanent magnet has been fully magnetised, the winding pattern is correct, and the rotor has concentricity.

![FIGURE 2.1 - Back EMF experimental Arrangement](image)

2.2.2 Static Torque

Static torque may be measured in terms of single phase or two phase torque. Here the normal drive circuitry is replaced by a controlled DC current, supplied into the phases.
With no load the motor will sit in an equilibrium position. If load is applied, the rotor will rotate to a new rest position. If the measurement system is a simple string and pulley the torque produced will be equal to the force applied multiplied by the radius of the pulley. The force is recorded along with angular displacement to produce the approximate sinusoidal curve for torque/ angular displacement. The torque will reach a maximum, close to mid-way between the un-aligned and aligned equilibrium points (figure 2.2A). The point of maximum torque is found to be typically just after the mid point slightly closer to the un-aligned equilibrium position. As the angle between equilibrium points is extremely small, the string and pulley measurement method can be very inaccurate and measurement on a special test rig is preferred.

The maximum static holding torque increases with current until the motor’s steel saturates. A representative curve is shown in figure 2.2B. Here the motor is shown to start at a value greater than zero. This is due to the detent torque produced by the permanent magnet. The motor torque rises linearly at low current and ‘flattens’ off as the current increases and the motor steel saturates.

**FIGURE 2.2 (A) - Static Torque against Position (B) - Static Torque against Current**
2.2.3 Dynamic Torque

In the industrial world of stepping motor applications the dynamic torque is often seen as the critical aspect of performance. Standardising dynamic performance of motors is difficult to achieve. For example there are dynamic differences between a test rig and a motor driving a conveyor belt system in a factory. The inertia differences of the systems affect starting (pull in) and stalling characteristics. Drive circuitry, and in particular differences in inductance, efficiencies, and pulse input circuitry, from manufacturer to manufacturer will also dictate the way a motor will perform. The choice of current step patterns, anti-resonance circuitry, and drive voltage further complicate the situation.

A set of illustrative performance curves are shown in figure 2.3. The curves shown represent both parallel and series motor connections. Since most drives have a pre-set current limit at low speeds the series connection can be seen to produce almost double the low speed torque as the active MMF is effectively doubled. However this torque drops off at a lower speed due to the rapid build up of back EMF. The effect of increasing voltage (dashed lines) can be seen to increase the speed range (figure 2.3). In figure 2.3, the effect of instability or resonance is shown. This may be due to motor construction/design, coupling and backlash effects of the measuring system, or motor/drive inductance instability. This can be reduced with the effect of microstepping or use of an anti-resonance circuit. An anti-resonance circuit may be either passive or active. A passive circuit will cap the peak current at low speeds. An active circuit monitors the current in the phase windings, and will reduce the current when instability is present.
2.3 Test Rig Apparatus

In an attempt to standardise testing, a test rig was constructed. The test rig was designed so that EMF, static, and dynamic torque could be measured on one platform. The basic construction is shown in figure 2.4. A motor is positioned at one end which can be used to drive the shaft, which in turn is mechanically coupled to the motor on test. This can be used to drive the motor on test and hence obtain back EMF information.

Between the motors is a powder brake dynamometer which acts as a load and speed torque transducer. This transducer will also feed-back the torque and speed to a measurement display. On the end of the shaft is an encoder which allows position feedback for static torque measurement. The encoder also allows stall detection for automated dynamic testing.
Chapter 2. The Hybrid Stepping Motor; Aspects of Construction, Testing, and Modelling.

A dual channel current probe is also fitted which allows the current waveform from the phase windings of the motor under test to be monitored and fed back to the control/display system. Whilst much of the hardware was constructed by Stebon Ltd. the author was responsible for all the test rig control and measurement software.

![Test Rig Layout](image)

**FIGURE 2.4 - Test Rig Layout**

### 2.3.1 Control and Display

The control and display system are made up of a direct feedback display and control panel for the speed/torque transducer. This is also routed through a personal computer that has suitable input, output, and data acquisition PC cards. This enables the speed, torque, motor controllers, current probes, and encoders to be read or fed by software running on the PC. The software that operated on this test rig was written in the Visual programming language Labview™.
2.4 Specialised Tests

2.4.1 Current Waveforms and Inductance Calculations [18]

The current in the phase winding can be captured by the PC on a Labview\textsuperscript{TM} virtual instrument that acts very much like a digital oscilloscope. This allows the two phase currents to be monitored, captured and then examined. The current waveforms are caught using a Hall effect current transducer. This tool is also useful for capturing the rise of the current following the input of a voltage step. This allows the inductance of a winding at a fixed rotor position to be found. Using the circuit of figure 2.5 and capturing the current (and voltage) waveform the inductance at a fixed rotor position can be calculated by the relationship between instantaneous current and inductance which is:

\[
v = iR + L \frac{di}{dt}, \tag{2.1}
\]

where \(v\) is the instantaneous voltage,

\(i\) is the instantaneous current,

\(L\) is the inductance of the circuit,

and \(R\) is the resistance of the coil.

The inductance can also be represented by equations 2.2 and 2.3.

\[
L = N \frac{\phi}{i}, \tag{2.2}
\]

\[
L = \mu_0 \mu_r N^2 \frac{A}{\ell}, \tag{2.3}
\]

where \(L\) is the inductance,
Chapter 2. The Hybrid Stepping Motor; Aspects of Construction, Testing, and Modelling.

\[ N \] is the number of turns linked by the magnetic field,

\[ \phi \] is the magnetic flux,

\[ i \] is the current in coil,

\[ \mu_0 \] is the permeability of free space,

\[ \mu_r \] is the relative permeability of the magnetic circuit,

\[ A \] is the Cross-sectional area,

and \( \ell \) is the mean flux path length.

It can be shown that as the rotor turns the reluctance of the magnetic flux path and hence the winding inductance will change. Whilst the inductance calculation is an important reading in the theory of motor testing and performance prediction it is non-linearly related to the flux linkage of the motor, a term commonly referred to in the design of all types of stepping motors [19,20]. It is therefore preferable to use the test rig to measure flux linkage rather than inductance.

![Inductance Calculation Circuit](image)

**FIGURE 2.5 - Inductance Calculation Circuit**
2.4.2 Flux Linkage Measurement [21]

In its most common form, an electrical machine converts energy from electrical to mechanical or vice versa via a coupling magnetic field. Flux linkage is a key to describing this coupling characteristic. Measurement of the flux linkage in an electrical machine is useful for validation of design models. The knowledge of the magnetic energy stored in the coupling field is useful for optimising machine design. The relationship between phase current, flux linkage and mechanical output can be seen as representative of the whole energy process in the motor.

The modelling of these relationships can be achieved by modern computing tools such as finite element analysis methods described later in this thesis. However accurate measurement of flux linkage is still essential to validate these models.

In a machine where the excitation system is only by current in the windings, such as for a variable reluctance motor, the measurement of flux linkage can be obtained by integrating a measurement of the phase winding voltage. In the hybrid stepping motor the measurement is complicated by the presence of another MMF source, the permanent magnet. The measurement of the magnetic field inside the motor is not easily obtained. Sensing coils or magnetic sensing probes could be added to the motor but these complicate the motor design and cannot be added to existing built motors. In any case data collection would be extremely troublesome due to the uneven distribution of flux in the air-gap and steel, making it difficult to obtain accurate flux linkages. One method to acquire the magnetic characteristics of a physical motor is to calculate the stator flux-
linkages indirectly by integrating measurements obtained from the phase winding circuit (figure 2.5) [22].

2.4.3 Theory of Flux-Linkage Measurement

One of the most common methods for measurement of flux-linkage is to apply a step voltage to a phase winding and record the resulting current. The measurement is based on the circuit equation described in equation 2.1. This may be expressed so to include flux-linkage \( \lambda(\theta, i) \), (flux-linkage in terms of position and current).

\[
v(t) = i(t)R + \frac{d\lambda(\theta, i)}{dt}, \tag{2.4}
\]

and integrating and rearranging equation 2.4,

\[
\lambda(\theta, i) = \int_{0}^{t} [v(t) - i(t)R]dt + \lambda(0), \tag{2.5}
\]

where \( \lambda(0) \) is the flux-linkage at time \( t=0 \).

The change in flux linkage over a period of can be obtained from the measurements of the winding voltage and current, if the phase winding resistance is known. Initial conditions have to be chosen carefully with a known initial flux linkage \( \lambda(0) \) at the time the integration process starts. In a system in which the current in the phase winding is the only source of flux, and providing the remnant flux can be neglected, the initial condition \( \lambda(0) \)
(integration constant) is conveniently chosen to be zero when the current in the phase winding is zero, regardless of rotor position $\theta$, i.e.

$$\lambda(0) = \lambda(\theta, i=0) = 0.$$  \hfill (2.6)

In the hybrid stepping motor system which has two independent current sources, and a permanent magnet, the flux linkage associated with a particular phase is now a function of the two current sources and position. The circuit equation of one phase can be written as:

$$v_1(t) = i_1(t)R_1 + \frac{d\lambda_1(\theta, i_1, i_2)}{dt},$$ \hfill (2.7)

where $i_j$ is the current in phase $j$, ($j=1,2$).

If it can be assumed that $i_2$ and $\theta$ are constant, integrating equation (2.7), provides

$$\lambda_1(\theta, i_1, i_2) = \int_{0}^{t} [v_1(t) - i_1(t)r_1]dt + \lambda_1(0),$$ \hfill (2.8)

where

$$\lambda_1(0) = \lambda_1(\theta=\theta_0, i_1 = i_{10}, i_2 = i_{20}),$$

$i_{10}$ is the initial current in phase 1,

$i_{20}$ is the initial current in phase 2.
\[ \lambda_m(\theta, i_1 = 0) = \int_{0}^{t} \text{emf}_1(t) \, dt + \lambda_m(\theta = \theta_0, \, i_1 = 0, \, i_2 = 0) \]  

(2.9)

where

\[ \lambda_m(\theta = \theta_0, \, i_1 = 0, \, i_2 = 0) \]  
is the initial condition at which the integration starts (arbitrary initial condition),

\[ \lambda_m(\theta, \, i_1 = 0, \, i_2 = 0) \]  
is the flux linkage associated with the stator phase due to the permanent magnet at any position \( \theta \),

\( \text{emf}_1(t) \) is the instantaneous motional EMF induced in phase 1 by the permanent magnet rotor.

The arbitrary initial condition \( \lambda_m(\theta = \theta_0, \, i_1 = 0, \, i_2 = 0) \), was chosen to be zero when \( \theta = \theta_0 \). It represented the position of minimum flux-linkage in stator phase 1. All the integration results were therefore relative to this initial value.

### 2.4.4 Experimental Measurement Method

The experimental method includes two separate tests.

1) Permanent magnet EMF.

2) Locked rotor current waveform test.

1) This test is described in section 2.2.1. The EMF waveform is then integrated using equation 2.9 to obtain the flux linkage due to the permanent magnet at each angle \( \theta \).
1) This test is described in section 2.2.1. The EMF waveform is then integrated using equation 2.9 to obtain the flux linkage due to the permanent magnet at each angle $\theta$.

2) This method follows the theory of section 2.4.3. The rotor is locked at known rotor positions. A short voltage pulse is then applied to the phase winding and the current and voltage waveforms are captured. The current waveform is then used in conjunction with the terminal voltage and winding resistance to calculate the change in flux linkage arising from the applied voltage using the initial condition for that position obtained from the magnet EMF test. This test was repeated for all the desired flux-linkage rotor positions with positive and negative phase current. The results obtained will be shown in chapter 5.

2.5 Prediction of Stepping Motor Characteristics

Stepping motors have been designed in the past by working on knowledge gained from experience, testing and experimentation. Engineers designing motors (and other electrical devices) wish to improve mathematical analysis without the use of time consuming experimentation. Traditional analysis involved simple geometry, assumptions and imagination. This approach has been outmoded although not replaced, as demands for faster and more accurate solutions for the complex geometry found in such motors as the hybrid stepping motor are sought.

The prediction of the performance of the hybrid stepping motor has been widely published by some key authors and researchers. Chai is a well-respected author for work concerning...
modelling, especially on permanence based models [6]. Many papers have been published on attempts to provide equivalent circuits for the motor [23-34]. These equivalent circuits rely on tooth and air-gap permanence models that link the mechanical geometry of the motor to the magnetic circuitry, by typically breaking the motor into key parts. Once established, the equivalent circuits attempt to predict the inductance, flux-linkages, and torque, for differing phase currents, drive voltages and motor speeds.

### 2.5.1 Use of Electromagnetic Analysis Software

The many factors that affect the performance of the motor have been rarely modelled together, due to the diversity of the effects which occur throughout the motor. A total solution lends itself to use of computerised calculations for electromagnetic fields. Methods which fall into these categories are Finite Element Analysis (FEA), Boundary Element Methods (BEM), Boundary Integral Methods (BIM), Finite Difference Methods (FDM), and time-domain methods [35,36,37].

### 2.5.2 Work carried out on Finite Element Modelling of Hybrid Stepping Motors

The number of publications specifically about the use of FEM in the analysis of the hybrid stepping motor is very small. Two notable works have been by B. Forghani of Montreal [38], and E.C.T. So and S.J. Yang of Heriot Watt, Edinburgh [39]. B. Forghani's paper describes a 3-D solution of the hybrid stepping motor. A brief discussion of the ability to use a 2-D method coupled to an equivalent circuit is given. However the three dimensional modelling is one of the only few attempts at an overall solution. The hybrid stepping motor's geometrical shape is simplified to reduce the computational effort.
required. The laminated nature of the motor is ignored and the motor is modelled as a non-laminated component. Because of simplifications made the detail of analysis is concentrated on the main flux path's in the motor. A method for calculating the torque-rotor position using the virtual work method is also discussed. The paper by E.C.T. So and S.J. Yang [39], though primarily concerned with stator vibration calculations, introduces methods for modelling a stator tooth and corresponding rotor teeth of a hybrid stepping motor. It includes points on simplifications made, boundary conditions applied and limitations of their meshed model. It gives a useful insight into the modelling of a stepping motor. However it does not obtain a complete solution for the motor.

2.5.3 Simpler Computational Methods in The Design of Stepping Motors.

Finite element software and other electromagnetic field software are still relatively expensive. Generally companies contract out design work when the modelling involves extensive electromagnetic field analysis, or because of the initial software/hardware and training costs. However from the knowledge gained by using FEA and by improving equivalent circuits, more complete and accurate models may be created. These can form the basis of a computer aided design environment that uses these improved mathematical algorithms to predict the behaviour of the motor. Similar attempts have been made by Wang et al [24], who have published a paper on the study of CAD for stepping motors. It addresses the establishment of what the authors call an improved CAD model. Optimal design problems associated with the motor are also discussed.
2.6 Discussion

The physical construction and the methods employed to manufacture the hybrid stepping motor must be understood before design methods can be usefully employed. The start of this chapter reviewed the typical processes of manufacturing the motor and described the methods used to maintain concentricity of the motor.

Experimental arrangements were discussed for validating models and measuring a motor's static and dynamic performance. All experimental results that are presented in subsequent chapters have been obtained by adopting the procedures described.

The chapter concluded with a brief introduction to the use of mathematical modelling for the prediction of the performance of the hybrid stepping motor. These include equivalent motor circuit models and commercially available computational products such as FEM software. Both are described more fully in the following chapters.

Through the use of three dimensional analysis, a full three dimensional representation of the hybrid stepper motor will be developed. This thesis will describe a simpler algebraic model that has been developed and verified with the use of 3-D FEA and experimental testing. This algebraic model aims to produce accurate solutions with the advantages of faster computation times and simpler data input.
CHAPTER 3

ELECTROMAGNETIC MODELLING OF THE HYBRID STEPPING MOTOR

3.1 Introduction to Advanced Numerical Techniques

Modern electrical machines must be designed to meet specified operating conditions that are set by the load demand, while achieving good efficiency and reliability. The power and torque output of the motor must be balanced by input power, motor volume and cost. In order to meet such requirements, accurate machine performance at the design stage must be established. It is difficult to build a complete solution of the hybrid stepping motor using simple analytical functions or equivalent circuit representations. This is due to the difficulties introduced by the motor's highly non-linear three dimensional magnetic structure, for which the doubly salient tooth structure, axial magnet, and back iron all complicate the situation.

In recent years numerical techniques have been developed that can overcome the limitations of linear and steady state problems and have provided accurate solutions to a wide range of electromagnetic problems. For example, the forces exerted on the stator poles of the hybrid stepping motor have been calculated, by taking into account the three

1 This Chapter is constructed from several papers published by the author; Presented at IEEE conference on Magnetics, Okayama, Japan, March 1997.[40] Proceedings of Vector Field User Meeting, September 1994, Blenheim Palace, Oxford [41]
dimensional geometry and saturation effects of the materials using the finite element method [39].

Finite element analysis is the most widely used numerical method for transient and steady state solutions to two and three dimensional electromagnetic problems. The enormous capabilities of this technique are largely due to considerable advances in desktop computers. These advances allow the use of three dimensional techniques to be considered for the modelling and analysis of the hybrid stepping motor.

The computer based numerical technique for solving partial differential equations is implemented by representing the area of the problem under consideration by a collection of finite elements. The nodes associated with each element are the points in space where the field values are calculated. Governing equations for each element are set up and subsequently combined in order to describe a global property.

The governing laws of electromagnetism can be concisely expressed by Maxwell’s equations [43]. Throughout the project the software used for the electromagnetic modelling of the hybrid stepping motor was supplied by Vector Fields Ltd [15,16,44]. The basic electromagnetic field theory on which the packages are based follows from the laws of electromagnetism and may be found in the Appendices.
3.2 Finite Element analysis of the Hybrid Stepping Motor

As reviewed in chapter 2 the amount of publications specifically on the use of three dimensional finite element methods (FEM) for the analysis of the hybrid stepping motor is small. A simplified geometry solution has been produced by B. Forghani where the fundamental flux paths in the motor have been shown [38]. E.C.T. So and S.J. Yang introduced methods for modelling a stator tooth and pole, and corresponding rotor teeth for vibration analysis [39]. The paper describes the simplifications made, boundary conditions applied and limitations of their meshed model. It gives a useful insight into the modelling of a hybrid stepping motor. Other papers, which discuss finite element analyses of stepping motors do so briefly or investigate the problem with 2-dimensional analysis. An excellent study of the hybrid stepping motor using 2-dimensional analysis is carried out in [45,46].

3.2.1 The Need for Three Dimensional Modelling of the Hybrid Stepping Motor

Finite element analysis (FEA) is considered to be highly suited to handle the modelling complexities introduced by the extremely saturated, doubly salient iron structure of the hybrid stepping motor. The question arises as to whether the hybrid stepping motor can be modelled, with sufficient accuracy, using a two dimensional code and an equivalent lumped circuit. As the amount of articles solely on three dimensional FEA on the hybrid stepping motor is limited, the variable reluctance stepping motor will be used to highlight common fundamental problems in two dimensional finite element modelling of doubly salient motors. Simkin and Trowbridge [47] reported that inductance calculations using a
two-dimensional computer program on a stepping motor had shown good agreement with measurements when the rotor teeth were aligned with the excited stator teeth. When the rotor was in the un-aligned position, values for computed and measured inductances disagreed. In the unaligned position the air-gap facing the excited stator poles is relatively large causing a significant axial component of flux to arise, due to end winding effects. As the axial direction cannot be modelled by two dimensional code the discrepancies were thought to be caused by this.

The hybrid stepping motor is a truly 3-dimensional electromagnetic problem with significant radial, axial and tangential components of magnetic flux occurring throughout the motor. Therefore it is impossible to obtain a complete solution by solely using a two dimensional package. Like most electrical machines, the hybrid stepping motor is usually laminated in order to minimise eddy current losses. Two dimensional analysis cannot model inter-lamination effects, which change the torque output of the motor. In a paper by Huard [46] it was discussed how the laminations made it extremely difficult to model a hybrid stepping motor with 2-D FEA and a comprehensive lumped model. Three dimensional FEA allows the user to input a packing factor for the steel laminations. This allows the program to calculate the effect of the laminations using anisotopic material properties, that is by assigning a relatively high permeability in the direction parallel to the laminations compared to a considerably lower permeability in the direction perpendicular to them.
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

The magnet characteristics cannot be included in two-dimensional FEM analysis because of its usual orientation, perpendicular to the laminations. Such effects must be modelled by lumped circuit models, in combination with two-dimensional models.

Despite these observations, finite element analysis packages that employ two-dimensional formulations are established as the primary tool in the design of the hybrid stepping motors. This is notably due to the complexity of the hybrid stepping motor as a complete model, but also due to computing power and man hours required for a three-dimensional model. Users of finite element analysis software packages have found that two-dimensional models require far less CPU time to solve and occupy little disk space in contrast to three-dimensional models. At one position at a particular excitation, three-dimensional FEA would require 8 hours on an IBM RS6000 workstation and require around 45 Mb of disk space for the solution. For a lumped element network solver the 2-D data required to solve for three positions and ten varying excitations at each would be around 25 Mb of disk space and 2 hours total CPU time. In addition most 2-dimensional codes are available on PC platforms and are relatively inexpensive compared to their more complicated and expensive workstation cousins.

3.3 Typical Finite Element Software Package

A typical finite element software package contains a Pre-processor, a Solver, and a Post-processor [15,16]. The Pre-processor is where the user defines the model. The Solver solves the system equations, and the post-processor allows the user to view and analyses the results.
3.3.1 Pre-Processor

The first step in modelling, is to draw what is similar to a 2-D orthographic front view of the model. Usually this consists of the motor lamination with a surrounding air boundary. This view is drawn and defined initially by use of points and construction lines. Once the points are created, they are used as corners to create areas (facets), which are the different sections of the model. Once all of the 2-D area has been sectioned into facets, further subdivision can be achieved by splitting the facets into elements. To create a 3-D model the 2-D face is extruded to develop volumes in the Z-direction. Again, these extrusion layers can be subdivided. Now instead of areas the model consists of volumes. Each volume can be defined as a different material, for example, air or steel.

3.3.2 Solver

Here the Algebraic model is created. The user selects certain conditions to define the type of solution. Definitions for the units and materials are also required, i.e. non-linearity, anisotropy, periodicity and iterations. The solver then computes the numerical solution, which can then be post-processed. Once the solver is running there is no user interface.

3.3.3 Post-Processor

A solution file is read into the post-processor. Here the model can be viewed with the field potentials drawn or coloured onto its surface. Line graphs and histograms can also be created to show field values across parts of the model. Torque can be computed by Maxwell stress analysis, or through measurement of co-energy at different positions.
3.4 Modelling of a Simplified Hybrid Stepping Motor

To ascertain the functionality and abilities of the finite element software this exercise was used to get familiar with the finite element software. The primary aim was to develop a model that would correctly solve the basic fundamentals of 3-D flux distribution in the hybrid stepping motor.

The motor to be modelled was to be based on the laminations for a 4 phase switched reluctance motor with eight teeth on the stator and six on the rotor (8:6). Therefore the motor lamination could be simply adapted to create a basic hybrid stepping motor. In the simple model there are no teeth castellations on any of the poles. The lamination is shown in figure 3.1.

![FIGURE 3.1 - 8:6 Switched Reluctance Lamination used for the Simple Stepping Motor](image)

3.4.1 Tooth Construction

A key issue of modelling the hybrid stepping motor is the ability to efficiently replicate the teeth. Significant issues for modelling are that the teeth on each end-cap are out of phase with the other, plus the number of teeth being numerous on each rotor end-cap.
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

Even though the latter does not directly apply with this particular exercise, it was a notable consideration. Because of these key issues an investigation was made where a pair of rotor teeth (front and back end-cap positions) were modelled separately. This introduces the problem of creating the front and back teeth on the rotor of a hybrid stepping motor which are displaced relative to each other by half a tooth pitch.

Two methods in which the teeth can be practically created in a three dimensional model were investigated, these were respectively;

(i) Hidden Detail Method,

(ii) Twisted Extrusion Method.

3.4.2 Hidden Detail Method

Figures 3.2 and 3.3, show the first method. In this method all detail is drawn on one layer, the base plane, figure 3.2. This layer includes all hidden detail. The base plane would practically be the same as the face view on a draughtsman’s orthographic drawing. The next stage is to subdivide the plane into 2-D elements. Once completed the plane is then extruded in layers in the Z-direction, to produce a model in three dimensional elements. To obtain a tooth structure as in figure 3.3, the elements in figure 3.2, are correctly associated with the material which is present in each layer. For example the elements which are solid on the top layer are defined as steel, the other elements are defined as air. This process is continued through all the layers and builds up the three dimensional structure of figure 3.3.
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

FIGURE 3.2 - Hidden Detail Method for Creating Teeth

FIGURE 3.3 - Teeth Created by Hidden Detail Method
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

3.4.3 Twisted Extrusion Method

It is possible to rotate or move points created on the base plane to different positions on later extrusion stages. A single tooth could be drawn on the first layer. On the next layer the mesh is twisted so that on the final tooth layer the tooth is in its correct position. Figure 3.4, is a rotor constructed by the second method. This uses the technique to twist the middle layer (permanent magnet) thus locating the teeth on the front and back rotor stacks in correct positions.

3.4.4 Advantages and Disadvantages of Each Method

The first method uses far more elements than the second because of the redundant air elements. This is a problem because the amount of elements needs to be minimised in modelling the motor. The second method creates problems because it twists and deforms the air gap surrounding the mesh and hence makes it extremely difficult to join the rotor to the stator in modelling. This also makes it highly impractical for changing the position of the rotor relative to the stator and it would be better suited to a fixed position model.

3.4.5 Preferred Method

The first method is used in all subsequent three dimensional modelling because it was much simpler to implement and it avoided corruption of the mesh. In more complex geometries such as a real hybrid stepping motor, the disadvantages of second method make it too complicated to reap the benefits of the reduced elements. The second method would be nearly impossible to use as there would difficulty matching element nodes on the rotor to those on the stator. The second method is best matched to modelling complete
models at specific positions. For different rotor positions, an individual model would need to be created.

FIGURE 3.4 - Twisted Rotor Method
3.4.6 Boundary Conditions

There are two main types of boundary conditions available in the Finite element software package. These are known as the Neumann and Dirichlet conditions and are used to constrain and define the model boundaries.

3.4.7 Neumann

Neumann boundary conditions are used to constrain the motor to within its own system. A real life motor's flux interacts in theory with an infinite amount of surrounding space. This cannot be modelled with the FEM software. Neumann boundary conditions can be placed on the outer boundary of a volume of air (three times the motor's volume is a good value as results do not significantly change with results greater than this[48]). This surrounds the motor and attempts to model this effect to a reasonable accuracy. The Neumann condition forces flux to be tangential to the boundary by setting the derivative of the potential to 0;

\[
\frac{\partial \phi}{\partial n} = 0, \quad (3.1)
\]

this implies flux is tangential to boundary and hence flux linkage is zero.

3.4.8 Dirichlet

Dirichlet or symmetry conditions provide a way of reducing the size of the finite element representation of symmetrical problems by setting the potential on the boundary to zero;
\[ \phi = 0, \quad (3.2) \]

which implies that flux is normal to the boundary.

When a symmetry boundary is set up, before analysis of the problem can be undertaken (field equation solver), the periodicity of the symmetry boundary is required to be defined. Symmetry boundaries may be either positive or negative. A negative boundary condition means the potential of one node is minus the potential of its pair.

An example of a simple stator/rotor structure is given to illustrate periodicity. Figure 3.5. Concentrating primarily on the stator, the stator has been subdivided by lines labelled A to E. The letter in boxes indicate whether a particular pole is excited as a north or south field. A pole pitch of the stator is defined as the section between A and C. Geometrically the section between A and B is symmetrical, but it is not magnetically. Negative periodicity occurs between section A and D. Here there is geometric symmetry, but the poles are of opposite polarity. Positive result between A and E, again there is geometric symmetry, and the magnetic polarity is the same. Concentrating now on the rotor which is magnetised on its front face as a south pole, geometric periodicity is found between a pole pitch (E to G) at F. At position H we would not find geometric symmetry. However at I we would again find both geometric symmetry and positive magnetic symmetry. Within the software this symmetry boundary is considered as a single line. It is assigned a periodicity condition and a centre of rotation. For this example both stator and rotor could allow the condition of symmetry to occur at 180°, which allows half the machine to be modelled.
3.4.9 Hybrid Stepping Motor Periodicity.

Because of the axially magnetised rotor magnet, flux travels down through the rotor and then in or out of the stator poles depending on the current direction in the windings. If one pole was pushing flux into the rotor, the directly opposite stator pole would be doing likewise. The flux would be travelling down the rotor and out through the stator poles that are attracting in flux. This type of action makes the positive periodicity correct for the hybrid stepping motor. By using positive periodicity half of the hybrid stepping motor needs to be constructed.

3.4.10 Magnet Definition

The material of the magnet in Stebon’s hybrid stepper motors is typically Alnico and has been used in this example. The FEA software requires the magnet material’s B-H curve to
be entered. From the second quadrant of the B-H curve, the software will calculate the permeability of the material by using the equation;

$$\mu = \frac{B}{(H-H_c)}$$  \hspace{1cm} (3.3)

Where $\mu$ is the permeability,

- $B$ is the flux density,
- $H$ is the field strength,
- and $H_c$ is the field strength coercivity.

The software starts with an initial value of $\mu$. For linear problems this is constant. For non-linear problems the initial value used is the value of $\mu$ at $B=0$, i.e. the gradient of the B-H curve at $B=0$. The software then solves the equations and calculates values of the fields in the elements. The value of $H$ is determined and $B$ calculated from equation 3.3, with $\mu$ still the same value. The values of $B$ and $H$ are then compared to the B-H curve given. If they do not lie close enough to the B-H curve, a new value of $\mu$ is estimated using a Newton-Raphson scheme. The solution process is repeated using the new value of $\mu$. The estimate of $\mu$ happens separately for every element in the mesh. This continues until all elements contain field solutions that match the specified B-H characteristics. This allows the load point of the magnet to be determined.
3.4.11 Construction of Model for the Simple Stepping Motor

The motor is constructed from the laminations used in a four phase switched reluctance motor as shown in figure 3.1. The rotor would be constructed by having two stacks of rotor laminations separated by an axially magnetised magnet. The rotor's teeth on each stack are shown to be mis-aligned by half a tooth pitch relative to each stack figure 3.6. Each layer is built up using the hidden detail method described in section 3.4.2. For this exercise the stator and rotor details are modelled together on the top pre-extrusion plane. This plane is then extruded to create several volumes. These volumes are defined as having particular electromagnetic behaviours in terms of materials and boundary constraints.

The rotor in this model is defined as having 5 distinct volume layers (figure 3.7). The air layers of 1, and 5, are outside boundaries, which simulate the air around the model. These limit the actual surrounding environment to a finite area which acts as a boundary to leakage flux. The limits of this area constrain the flux paths by the use of boundary conditions. In this case a ‘Neumann’ type boundary is set [48]. The stator has 3 layers defined, the second layer is the steel and has an identical inside node collection as that of the rotor's layers 2 to 4, figure 3.7.

The completed model consists of a rotor meshed by a common air-gap to the stator. Current carrying coils are positioned on the stator with the correct polarity. The complete model for solving is shown in figure 3.8.
FIGURE 3.6 - Front On View Showing Rotor Teeth Misalignment
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

FIGURE 3.7 - Layer Definitions

FIGURE 3.8 - Solid 180° Model of the Simple Stepping Motor

The Design of Hybrid Stepping Motors aided by Three Dimensional Finite Element Analysis
3.4.12 Results

The solution is obtained by running the TOSCA algebraic solver [16]. An initial set-up in the pre-processor is required for material conditions (i.e. anisotropicity, material type, linear or non-linear solution, packing factors, etc.) boundary conditions and units. Once the TOSCA solver is running there is no user interface.

The modelling of a simple stepping motor was undertaken as a qualitative exercise, to show that the flux paths are as expected. The post-processor results showed the flux paths within the motor by the use of flux vectors, (figures 3.9, 3.10.) The flux in the magnet travels south to north (figure 3.9). It then leaves the rotor by the aligned teeth of the rotor and stator. A south pole is induced in the stator poles aligned with the rotor poles in the north end of the rotor. The flux travels axially through the stator back iron to the front of the motor. Here it enters the rotor teeth at right angles to the lines of flux at the other end of the rotor (figure 3.10). The results for this simple stepping motor showed that the construction methods used in 3-D were correct. This methodology was used to model an actual hybrid stepping motor.
FIGURE 3.9 - Simple Stepper, under view
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

3.6 Implementing a Model of an actual Hybrid Stepping Motor

The simple stepping motor example previously discussthe differ in three-dimensional operation of the actual hybrid stepping motor. The motor uses a series of identical parts, each having the same mass and inertia. The motor uses a series of identical magnets, each having the same magnetic moment. The motor uses a series of identical coils, each having the same electrical resistance. The motor uses a series of identical windings, each having the same electrical resistance.

FIGURE 3.10 - Simple Stepper, Full view
3.5 Implementing a Model of an actual Hybrid Stepping Motor

The simple stepping motor examined previously does not differ in fundamental operation to the actual hybrid stepping motor. However the real motor has a rather more complex tooth geometry, and thus the modelling is rather more advanced and takes a greater amount of planning and effort. Several stages are undertaken to create a model of a practical design, ready for a solution. The motors used for such analysis were the Stebon 850 and 1100 series motors. Each stage of the construction produces a particular status of the model. This model is then redefined or appended to before it is ready for solving.

3.5.1. Physical Model

The problem is specified in terms of geometry, materials, lamination structure, etc. The motor design is a laminated single stack, (two rotor end caps, each with 50 teeth and one Alnico permanent magnet). The stator has eight poles each with five castellated teeth. The coils of the two phases are wound around alternate stator poles to create a four pole stator field. The lamination packing factor has been calculated at 0.975 for the stator and 0.985 for the rotor, from measurements made on several manufactured components.

3.5.2. Geometric Model + Numerical model

The physical model needs to be translated onto a two dimensional top plane. This plane is then sub-divided into 2-dimensional areas called facets. This plane is extruded into a three dimensional representation of the motor by creating volumes from each facet. These volumes are known as elements.
If computation time is not to be excessive, 100,000 elements is a sensible maximum limit and generally cannot be exceeded. For element numbers above 50,000, the original software required part of the model code to be modified outside the design environment. However the number of elements was still limited to 100,000 and hence simplifications to the model had therefore to be found which had a minimal effect on analysis. In the motor, areas of similar dimensions were made the same to reduce the number of elements. For example the magnet outer diameter was found to be 1 mm smaller than the diameter of the trough of the rotor teeth, and these were equated for ease of modelling. Elsewhere elements needed to be used effectively, allowing areas of greater interest or importance to be discretised (distribution of elements) finer. In particular it is essential that a fine mesh is used around the teeth and in the air-gap, where significant effects on the motor's characteristics occur.

3.5.3 Air-Gap and Tooth Discretisation

It is important that the rotor and stator be created as two separate models. This allows the rotor and stator to be joined together at different rotor positions. To achieve correct meshing between the stator and rotor the start of the model should use the same points definition in the centre of the air gap. This start of the mesh data file should be copied and be used as an identical start to both the rotor and stator definition files. This will allow error free meshing of the two models.

It is necessary to have at least two layers of elements in the air-gap and it is generally considered to be sensible to have no less than 4 layers of elements [48]. The dissection of the air gap radius should be done so that the nodes on the radius are at equal distances...
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

from each other. The nodes are best defined as polar co-ordinates. This allows rotation of the rotor, with correct mesh linking between the nodes.

The biggest problem of creating rotor and stator models and meshing them together is that there can be far more nodes on the rotor outer diameter than on the stator inner diameter. This is because the rotor nodes are produced from two rotor end-caps, and rotor teeth nodes are found around the complete perimeter of the rotor. The stator nodes are concentrated at the bottom of the eight stator poles and there is less need to have nodes at the air-gap surface of the slots. The high number of nodes required on the rotor is illustrated in figure 3.11. In the motor there would be 50 teeth on each rotor end-cap, in the finite element top plane model there would be twice this number. Figure 3.11 shows how each layer needs to represent both overlapping sections of the front and back rotor teeth.

FIGURE 3.11 - Rotor Teeth Elements
It is possible to simplify the number of points on the rotor but this may cause the angle of rotation to become unsuitable. For a particular angle of rotation, the nodes on the predefined centre air-gap file must still match. If the hybrid motor was to be analysed simply as a single phase machine, a significant angle of measurement would be 1.8° as this is where maximum torque approximately occurs. Fortunately single phase analysis can take advantage of the fact that the stator poles are 1.8° misplaced from each other. Therefore by selecting appropriate phase excitation an aligned (0°), half aligned (1.8°) and unaligned positions (3.6°) are easily calculated before any specific angle discretisation is required. In single phase analysis 1.2° degree discretisation is a good angle as this produces a low rotor to stator node ratio. It is relatively easy to implement and from it 0°, 0.6°, 1.2°, 1.8°, 2.4°, 3°, and 3.6° angles of rotation can be analysed. However this not completely convenient for two phase analysis. In two phase analysis the maximum torque approximately occurs at 0.9° rotation from an aligned top pole tooth position, as shown in figure 3.12.

If discretisation of the air-gap was 0.9° the rotor to stator node ratio would increase. The angles which could be analysed would be 0°, 0.9°, 1.8°, 2.7°, and 3.6°, a reduction from seven to five angles when compared to the single phase case. It is not really practical to take the angle of rotation any smaller because the ratio would increase significantly.
The modeller has a choice of either using a 1.2° angle rotation with sine wave interpolation to obtain intermediate results, or to use triangulated elements to obtain 0.9° rotation. The modeller could however if time permitted use two models. In the available software at the start of the project, problems arose with using a discretisation finer than 1.2°. A model created later in the project used 0.9° discretisation for improved two phase analysis when the software was more robust and gave an increased number of elements.

### 3.5.4 Cogging Torque.

Figure 3.13 shows the cogging torque approximation of a hybrid stepping motor. There are four cycles per tooth pitch. The position of equilibrium occurs 400 times a revolution, with 200 being stable points. If the 0.9° angle discretisation was used, then the torque analysis using Maxwell stress calculation would only indicate the positions of equilibrium. With the 1.2° angle maximum torque would not be directly calculated but other points could be calculated and with the use of Fourier point analysis a maximum torque could be predicted. The analysis would be better suited to a 0.45° rotor movement as this would
allow the 0.9° maximum torque angles to be approximated by a sinewave. The cogging torque indicates a fundamental problem of hybrid stepping motor analysis. Here the modeller needs to consider the excessive computing time, or a reduced analysis.

![Cogging Torque Assumption](image)

**FIGURE 3.13 - Cogging Torque Assumption**

### 3.5.5 Reduction Method for Elements

In some areas of the model there is a need to reduce the number of nodes. Figure 3.14 displays a technique for reducing the number of nodes. This element-grading technique can be used elsewhere in the model.

![Reduction Technique](image)

**FIGURE 3.14 - Reduction technique for reducing 4 nodes to 2**
3.5.6 Material Definition

Material definition assigns a name and a material B-H characteristic to a particular volume. It is wise to call the material by a location name as well as the material type, i.e. stator iron. The modeller may also choose between a linear and non-linear analysis. In the hybrid stepping motor the analysis is required to be non-linear as parts of the steel in the motor become highly saturated. The magnet material is defined by entering a B-H curve, from which the software calculates the magnet's working point, as described in section 3.4.10. This working point will change depending on the phase winding excitation and position of the rotor. The magnet material in the motor is axially orientated and requires a vector orientation to define this. Axial magnets are fortunately defined as having a vector orientation of 000. The numbers represent Euler angles [48].

![FIGURE 3.15 - Broken Boundary Condition](image-url)
3.5.7 Mathematical Model

Here the choice of solution potentials and boundary conditions are defined. The modeller can also take advantage of any symmetry in the model. The boundary and symmetry conditions which are used in the hybrid stepping motor have been explained in section 3.4.

A rotation of either the rotor or stator relative to the other can be easily achieved. However, when a 'half symmetry' model is used the rotation of a mesh will cause a discontinuity in the symmetry boundary. The modeller is required to complete the boundary line. The boundary discontinuity is shown in figure 3.15. The rotor has been rotated by 2.4°. The left hand side break of the model is by the X-axis '30' position and this is required to be completed by adding a symmetry condition on the rotor mesh.
Similarly the right hand side is completed by adding a symmetry condition on the stator mesh. The completed symmetry condition is shown in figure 3.16.

### 3.5.8 Conductor Model

This creates the current carrying conductor regions of the model. In the OPERA-3D package conductors are defined separately to the rest of the model. The current in a conductor is set by inputting a current density in the cross sectional area of a conductor. The current density $J$ is calculated by:

$$J = \frac{NI}{A_{\text{cond}}},$$  \hspace{1cm} (3.4)

Where $J$ is the current density,

$N$ is the number of turns,

$I$ is the current,

and $A_{\text{cond}}$ is the cross-sectional area of the conductor.

### 3.5.9 Complete Model

In general a complete model is made up of three separate files, one each for the stator, rotor and conductors. Typically the stator is loaded first, then the rotor and finally the conductors. However the rotor file may require modifications if a particular rotor rotation is required. The simplest method of achieving the preferred rotor angle is by editing the rotor file. Near the start of each file is a line which defines the co-ordinate system for the model. In its simplest form the line will be described as:

$x y$
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

Referring to an X-Y co-ordinate system. The rotor file should be edited and have the line replaced by:

\[ \text{new | corsys } #_x #_y #_z \theta_x \theta_y \theta_z \]

Where 'new' indicates a new co-ordinate system,

\[ \text{a system return code,} \]

\[ \text{corsys, a name for the new co-ordinate system,} \]

\[ #_x #_y #_z \] are the rotation in the X, Y, and Z axes of the new co-ordinate system relative to the old system,

and \( \theta_x \theta_y \theta_z \) which will be a position in degrees for the rotation.

An example to achieve a rotor angle of 1.2° clockwise, for a co-ordinate system called \( \text{ROTOR} \), the line would read:

\[ \text{new | Rotor } 0 0 0 0 1.2 0 \]

Once the rotor file has been read in the user is required to complete the broken symmetry line as described in section 3.5.7. A 180° rotor model is shown in figure 3.17. A two dimensional mesh of the joined stator and rotor is depicted in figure 3.18.

Once all files have been read in the pre-processor will have a complete model. Figure 3.19 shows 180° of a completed model, figure 3.20 a perspective view and 3.21 a close up of the motor.
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

FIGURE 3.17 - Half Rotor Model

The Design of Hybrid Stepping Motors aided by Three Dimensional Finite Element Analysis
FIGURE 3.18 - 2-D joined Rotor and Stator
FIGURE 3.19 - Completed Half Model

The Design of Hybrid Stepping Motors aided by Three Dimensional Finite Element Analysis
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

FIGURE 3.20 - Two phase 2-D (360° by Reflection) Complete Model

The Design of Hybrid Stepping Motors aided by Three Dimensional Finite Element Analysis
FIGURE 3.21 - Close up 3-D Complete Model
3.5.10 Algebraic Model

Before the field equation solver (TOSCA) [15] can be run for the calculation of the solution potential, the user is required to select certain factors that define the type and accuracy of the solution. These include iteration count, convergence tolerance, type of solution (Newton-Raphson / simple up-date), and the way the solver deals with materials (linear / anisotropy).

To solve for the hybrid stepping motor, a non-linear, laminated material solution was used. The convergence tolerance was set at 0.001 with Newton-Raphson solution type. The maximum number of iterations was higher than the default of 15 and was increased to 40. This was because when the iteration count was 15, the numeric solution had sometimes not converged.

3.6 Analysis of the Un-Excited Motor

When no current is in the motor phase windings, the flux circulating throughout the motor is generated solely by the rotor's permanent magnet.

The significant flux paths in the magnet excitation can be classified as a series of loops emanating from one source. Starting from the magnet the flux enters the rotor end-cap. The inner flux paths travel the longest lengths to the outer limits of the end-caps, whereas the outer flux vectors in the magnet travel the shortest distances. As the flux takes the most permeable path, the majority of flux crosses the air-gap closest to the magnet, hence avoiding crossing the laminations. This is demonstrated by the low value of flux density.
on the outer faces of the rotor end-caps. The flux will travel mainly across the air-gap at an aligned position. This position is quickly saturated, thus flux will take the next permeable option, flowing across the semi-aligned teeth. Some of the flux will cross the air-gap at the un-aligned position, but mainly towards the centre of the rotor, where the path reluctance is comparable, to travelling through the laminations. Once in the stator, the flux on the outer edges is confined to travelling up the laminations towards the back iron. It then gradually flows in the Z-direction to enter the rotor again at the appropriate aligned, mid or un-aligned position. This is again dependent on the saturation of the stator teeth and the MMF dropped by travelling across the laminations.

FIGURE 3.22 - Rotor Excited by Magnet Flux
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

Figure 3.22 is a three dimensional representation of the rotor showing distribution of flux density. Even though the flux is generated solely from the magnet, comparable high areas of flux density can be seen on the inner edges of the rotor end-caps. The highest areas are found on the teeth edges that are most aligned with the stator pole castellations. The lowest areas on the teeth are found where the teeth are totally unaligned and on the middle sections of the end-cap. Fringing is commonly found on the edges of the teeth. The semi-aligned teeth reveal a balanced flux flow between them and this is mirrored around the magnet interface. The section of rotor located between the stator pole castellations exhibit little leakage flux paths.

The centre of the rotor end-caps faces on the front and back of the motor show the lowest areas of flux density. Closer to the rotor end-cap /magnet interface, the flux density can be relatively large. This has been found to be because of the way flux flows internally in the rotor, which is described in the following paragraph.

3.6.1 Internal Rotor Flux Paths

Considering lines of flux are a useful way of visualising the flux distribution in the motor though it is appreciated that they do not have a physical meaning. The internal flux paths in the rotor follow helical shaped routes. The figure of 3.23 shows three typical paths. Route A starts at the inside edge of a rotor end-cap, on the centre tooth of an aligned pole. It has been found in the 3-D FEA that this flux path will have the shortest length. It therefore needs to reach a point close to the outer diameter of the permanent magnet. The flux path needs to obtain an equal flux length each side of the magnet so the path lengths
in either end-cap are balanced. In the magnet itself the flux line is confined to a straight line of the magnet orientation.

In path B, the flux path starts on the same tooth as path A, but on the outer edge of the end-cap. It consequently has a longer Z-direction distance to travel. Its position of contact with the magnet face is much nearer the centre. The longer the Z-direction distance the closer to the centre of the magnet the flux path will enter. If a flux vector analysis was taken half-way into a rotor end-cap on the X-Y plane, flux paths closer to the inner diameter would be highly Z-directionally biased.

Path C starts on the centre of a tooth which is aligned with a side end tooth on the stator. In this case the flux path needs to meet the magnet interface (X-Y) at approximately the centre. The flux path will flow to the equivalent tooth on the other end-cap. The flux will always flow to the near side corresponding air-gap crossing on the other end-cap. This is true of all flux paths.

FIGURE 3.23 - Magnet Flux Paths
3.6.2 Analysis of the Air-gap Region

Figure 3.24 shows a close up of an air-gap region. With magnet excitation alone the flux density reaches a relatively high value only on the tips of the teeth. The flux is constrained within the teeth and flows up through the stator pole. The majority of the flux flows into the corresponding aligned stator pole and the two poles either side. However a large proportion of the flux does flow through the middle aligned pole. On the back centre pole (which is unaligned) a significant amount of flux leaks back into the rotor. This indicates that the un-aligned path is still relatively permeable because the laminated stator has a significant reluctance in the axial direction.
To illustrate the flux distribution across the aligned stator refer to the graph in figure 3.25. This graph represents a Y-plane (constant height), where the Z-axis runs from points 1 to 4, and the X-axis is from 1 to 2. The Y-axis height is half way up the stator pole. Here a near sinusoidal distribution in terms of magnitude is illustrated. The flux density is highest on the magnet side of the rotor end-cap length. The polarity quickly reverses at the centre of the magnet. A fringing step is found at the outside edge of the stator, where the steel meets the air boundary. On other FE studies the smoothness of the distribution, and height of this outer step is highly affected by the packing factor of the steel.

Component: BY
Maximum = 1.141003, Minimum = -0.766317
Integral = 49.0054
3.7 Single Phase Analysis

At low excitation the single phase flux distribution is essentially similar to that of the magnet only analysis. However as the current in the phase is increased, certain characteristics of single phase flux distribution emerge. Figure 3.26 shows a close up on a stator/rotor section. Here flux is flowing from the magnet into the front end-cap and is pulled into the aligned teeth by a low current excited winding. The phase winding is not shown for clarity. Flux flowing directly back into the un-aligned position at the back is now opposed by the winding polarity, hence more flux will flow upwards toward the back iron.

As the excitation increases the sinusoidal magnet distribution of figure 3.25 becomes less sinusoidal, with the rear region magnitude reducing. Eventually the flux on the rear region will increase to zero, and then will become positive in magnitude as the winding current increases. This is shown in figure 3.27. Again the Y-plane is half way up the stator pole. Points 3 to 4 are the X-axis, and 3 to 2 the Z-axis.
FIGURE 3.26 - Close up of Rotor Teeth and Stator Pole

The Design of Hybrid Stepping Motors aided by Three Dimensional Finite Element Analysis 89
FIGURE 3.27 - Flux distribution in an Excited Pole
FIGURE 3.28 - Complete Flux Distribution
FIGURE 3.29 - Flux distribution with High Excitation
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

The stator pole that does not have any excitation from the windings directly placed on it is only slightly affected by the increased MMF on the other excited phase. This can be demonstrated by the figures of 3.28 and 3.29. Figure 3.28 shows a rated current excitation, whereas figure 3.29 is a winding excitation above the typical current rating for the number of turns in the phase winding. The colour of the two middle poles shows relatively no change in flux density. This suggests low mutual coupling in these poles.

Figure 3.28 gives a view of the flux paths in the motor. A general path that starts from the front central pole on the aligned teeth, travels to the rear aligned teeth on the stator poles on found on the sides of the figure.

Breakdown of the flux flowing through the magnet can be seen in the high excitation model in figure 3.28. Flux avoids travelling through the magnet. It can be seen that flux flows from one front aligned stator pole to the front of the next stator pole, which is un-aligned. This flux has directly arrived from the opposite polarity winding, by coming across an un-aligned tooth air-gap and then proceeding over the rotor end-cap lamination (Figure 3.30).
This effect is a contribution of the fact that the winding excitation pull becomes much stronger than that of the magnet. As the MMF of the winding increases the corresponding ‘negative’ MMF produced by the magnet increases also. At the same time the amount of flux flowing through the magnet will decrease. This corresponds with the B-H characteristics of the magnet, which shows that for increasing ‘negative’ field strength the flux density will fall.

3.8 Two Phase Analysis

Figure 3.31 shows the flux distribution for two phase excitation. Major flux loops are seen operating between opposite winding polarities. The front central aligned pole has an excitation polarity differing to that of its left hand side neighbour and flux will flow to this left hand pole. As this left hand pole has an equal steel permanence on the front and back the influencing factor is the operation of the magnet. This would act to direct the flux into the back part of the stator pole. However it can be clearly observed that the front of the stator pole has a significant amount of flux flowing directly into the front of the rotor. The MMF drop by flux travelling across the laminations in the Z-direction in the stator core is significant. This suggests that the MMF dropped by travelling across the laminations is comparable to the MMF produced by the magnet. The overall distribution can be again viewed in the sense of flux density distribution in figure 3.32.

The corresponding rotor teeth view is figure 3.33. The teeth that are close to the stator poles that have differing polarities of excitation to that of the magnet exhibit high levels of flux density. The bottom inner stator teeth are shown in figure 3.34.
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

FIGURE 3.31 - Overall Flux distribution in Two Phase (180°)

FIGURE 3.32 - Overall Flux distribution in Two Phase (360°)

The Design of Hybrid Stepping Motors aided by Three Dimensional Finite Element Analysis
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

FIGURE 3.33 - Two Phase Rotor Teeth

FIGURE 3.34 - Two Phase Inner Stator.
Chapter 3. Electromagnetic Modelling of the Hybrid Stepping Motor

3.9 Rotor End-Cap Core

In all the analyses, the central core of the rotor end-caps that are on the outside of the motor (z-axis) typically do not have a high value of flux density. Recently some manufacturers have removed from this a cylindrical section from this area to produce lower inertia rotors. This concept appears to be a worthwhile feature, with little disadvantage. However flux density does increase nearer the magnet interface as described in section 3.6.2. A better solution would be to remove a cone, with the pinnacle nearest the magnet.

3.10 Summary

This chapter forms a tutorial of techniques for modelling and analysing hybrid stepping motors using 3-D finite element software packages. A typical finite element package has been described. To understand the concepts for modelling the hybrid stepping motors with such 3-D finite element software a simplified model of a motor has been constructed. This model paid particular attention to basic modelling of the hybrid stepping motor and has allowed a greater understanding of the three dimensional flux paths in the hybrid type motor.

The chapter proceeds with using the concepts developed for the simplified motor to construct models for a practical hybrid stepper motor. It shows how different excitation schemes and values, for different positions can be solved. The chapter concludes with a qualitative analysis of the hybrid stepping motor with the use of flux distribution plots,
which has allowed much expanded comprehension of the fundamental elements and operation of the motor.
CHAPTER 4

DEVELOPING A STATIC MODEL OF THE HYBRID
STEPPING MOTOR FROM 3-D FINITE ELEMENT
ANALYSIS

Three dimensional analysis can provide a high accuracy solution for a hybrid stepping motor problem, at the expense of a long computing time. However it would be beneficial if a model was devised to achieve an accurate first solution or an indication of a new design’s capability without resorting to the expense of three dimensional finite element analysis (3-D FEA) computing.

4.1 Three Dimensional Analysis Modelling

Three dimensional finite element analysis gives a complete and informative insight into the electromagnetic behaviour of a motor. The software can deal with the highly complex non-linear representation of magnetic steel properties, revealing non-linear saturation effects in the motor. It has the benefits of taking anisotropy effects of the laminations into account when calculating a solution. This is important as the operation of the hybrid
stepping motor depends on flux crossing the lamination in an axial direction perpendicular to the lamination plane in the back iron of the motor.

Three dimensional FEA allows easy manipulation of the different materials found in a motor. This allows differing grades of steel and packing factors to be selected for particular areas in the motor. Magnet polarity orientation can be defined in any three directions. End core and axial fringing effects are three dimensional effects, which increase in influence when the machine has a small axial length [49]. These can only be viewed in a 3-D analysis. Torque calculations may be performed by using the virtual work method, or the use of Maxwell stress analysis [48].

4.2 HyStep PC Modelling

HyStep is a computer modelling and simulation program developed during this project, for hybrid stepper motors written in the Borland™ C++ language [50]. It is used to predict the motor performance under static operating conditions and allows the motor's operation under dynamic situations to be solved. In order to generate a modelling technique for HyStep static analysis using 3-D FEA was performed to identify the key electromagnetic parameters. The hybrid stepper motor was split into sections (regions), that could then be modelled by different methods and subsequently integrated, producing a simple and accurate representation of the hybrid stepping motor. The basic structure of HyStep is shown in the flow chart of figure 4.1. The user can input motor material data ranging from B-H characteristics to tooth geometry using a selection of input menus. Options for
calculation methods are also required, as HyStep may use either look-up tables or curve fitting techniques.
4.3 B-H Characteristics

The hybrid stepping motor is considered to be a non-linear system, due to its operation under saturated magnetic conditions. Other non-linear characteristics are introduced by the permanent magnet in the rotor. The output relationships such as voltage/ current, flux/ MMF, torque/ position, torque/ speed, are therefore likely to be non-linear. This is an example of the classical ‘saturation’ non-linearity which is frequently encountered in many branches of engineering. Linear circuit elements are relatively easy to analyse. Exact analysis of circuits containing non-linear elements is extremely difficult. There are certain techniques that exist to approximate the effects of non-linearity and to predict the behaviour of real elements within acceptable limits. HyStep allows the use of look-up tables and curve fitting techniques to model the B-H characteristics.

4.3.1 Approximate Analytical Solution

If the non-linear characteristic can be described mathematically then a mathematical equation may be used to obtain a complete or approximate solution. For example the power series, may be used to describe non-linear characteristics;

\[ y = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots \]  

If the first two terms are only considered, and powers of \( x \) higher than 1 are neglected, then this is a linear approximation of the non-linear characteristic. The higher the power, which is included, the more accurate the solution may become, but the analysis becomes
more difficult. There are approximations that are better suited to modelling B-H characteristics. These include Jiles-Atherton [51] and Hill type rational functions [52,53].

Modelling of the non-linear magnetic characteristics of steel using a single equation can offer many benefits for motor design and analysis packages. Design and analysis tools often use a B-H look-up table for circuits and devices, employing magnetic materials in their construction. Many curve fitting equations have been used and in particular the well known Jiles-Atherton model for the B-H magnetisation curve given in equation 4.2 may be used [51].

\[ B = M(\coth(H/\alpha) - (\alpha/H)), \quad (4.2) \]

where \( M \) and \( \alpha \) are parameters.

This has been employed in some commercial simulation packages [54]. However, it has been found that the modern grades of Silicon steel used in electrical machines have a very sharp 'knee' in their B-H magnetisation curves and this is not always modelled accurately by the Jiles-Atherton expression. Furthermore, the Jiles-Atherton expression can be inaccurate at very low values of flux density.
4.3.2 Example of Modelling B-H curves [55,56]

New algebraic equations for fitting a B-H curve have been devised. These equations give excellent accuracy in the linear (low flux) region, follow the ‘knee’ and fit the saturation part of the curve with a very low degree of error. The new equations have been compared to the Jiles-Atherton equation and with the use of look-up tables and have fitted well [51].

There are many cases where the analysis for some items would be best represented by a non-linear model. These models are defined as those which have non-linear dependencies on their parameters. The Hill function can be suited to representing the B-H curve for magnetic steel. The Hill function is given by:

\[ B = \frac{\alpha H^n}{\beta + H^n}, \quad (4.3) \]

\( \alpha, \beta, \) and \( n \) are real numbers.

Non-linear regression is based on determining the values of the parameters that minimise the sum of the squares of the residual errors. The solution must follow an iterative procedure for the non-linear case. Successful solutions are often highly dependent on good initial guesses for the parameters.

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2 To be Published in a future Journal of Facsimile Applications.
A B-H curve of 30 points would require in general an array of 30 times 2 elements for a curve of 30 discrete points. If this could be reduced to five or fewer parameters a dramatic reduction in memory space would result. The new equations that have been found to accurately represent B-H curves of Silicon steel are of the form:

\[
B = \frac{\alpha H^n + \kappa H^{n+1}}{\beta + H^n + \epsilon H^{n+1}},
\]

\[(4.4)\]

\[
B = \frac{\alpha H^n + \kappa H^{n+1}}{\beta + H^n},
\]

\[(4.5)\]

where \(\alpha, \beta, \epsilon, \kappa\) and \(n\) are real numbers.

Equations 4.4 and 4.5, are examples of rational functions that were devised to model the B-H curves of magnetic steel\(^3\) typically found in electrical machines and have been implemented into the HyStep programme. Figures 4.2 and 4.3, show typical B-H curves commonly used. Unlike traditional polynomial fits and interpolation they do not exhibit instability, and can intelligently interpolate past the last point of data.

\(^3\) A 'magnetic steel' is defined by British Standards BS EN10106:1996.
Chapter 4. Developing a Static Model of the Hybrid Stepping Motor from 3-D Finite Element Analysis

**FIGURE 4.2 - Knee of fitted B-H curve to real data points**

**FIGURE 4.3 - Fitted B-H Curve to Real Data Points by New Equation**

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*The Design of Hybrid Stepping Motors aided by Three Dimensional Finite Element Analysis*
4.4 Flux-Linkage Characteristics

The modelling of the hybrid stepping motor requires the production of flux-linkage to current characteristics at different positions. Figure 4.4 illustrates 3 curves on a flux-linkage diagram. These curves are known as the aligned, mid, and un-aligned curves respectively. To produce the aligned curve a stator pole that has its teeth aligned with the front rotor end-cap’s teeth need to be analysed. The rear rotor end-cap’s teeth will be un-aligned with the same stator pole teeth. Without any current the flux linking the phase winding will be that produced by the magnet. This is represented as the non zero value of flux-linkage at no current on the aligned curve of figure 4.4

To produce the mid curve of figure 4.4, the area of analysis would be the stator pole next to the one used in the aligned curve analysis. This stator pole’s teeth will be half aligned with the front and rear rotor end-cap’s teeth, but displaced by half a tooth pitch. Finally the un-aligned curve of figure 4.4, is found to be a stator pole with its teeth aligned with
the rear rotor end-cap's teeth, whereas the front rotor end-cap's teeth are now un-aligned. The un-aligned curve is practically the same as the aligned curve with the polarity of the magnet reversed. The same curve can be achieved from the aligned curve by reflecting its values in the Y-axis and then the X-axis of the graph, figure 4.5.

![Graph showing aligned and un-aligned curves](image)

**FIGURE 4.5 - Obtaining Un-aligned Flux-Linkage Curve**

### 4.5 Determining Permeances of Air-Gap Regions using Two Dimensional FEA

A detailed description of the hybrid stepping motor operation was presented in chapter 1. In the previous chapters it was demonstrated that the motor operation relies on the generation of complex three dimensional (i.e. axial and radial) electromagnetic flux paths. Therefore, two dimensional (2-D) FEA cannot provide a complete solution for the hybrid stepping motor, as only the axial or radial plane can be analysed at any one time.
However some sections (or regions) in the hybrid stepping motor can be modelled to a good degree of accuracy by using two dimensional methods. In particular the tooth /air-gap regions have a majority of their flux travelling in the X-Y direction. The design of the tooth/ slot air-gap region strongly affects the static torque characteristics. It also affects the dynamic characteristics as it controls the inductance of each phase.

To make the curves of figure 4.4 from a two dimensional analysis, three situations need to be analysed. These are an aligned rotor tooth position, an un-aligned rotor tooth position and a half aligned (mid position) rotor tooth position. Two of these are shown in figure 4.6. From FEA solutions of these two dimensional tooth structures the complete flux-linkage to current characteristics of each position can be calculated. For 2-D FE analysis two excellent pieces of work were undertaken by Huard [45,46].

![Figure 4.6 - Front Aligned Position with Back Un-aligned.](image)

High accuracy can be achieved by using 2-D FEA on the tooth regions. It is beneficial to construct the pole/ teeth profile on a computer aided drawing package and import the outline as a DXF or similar file into the FEM pre-processor. This will allow the profile to...
be seen as a series of construction lines and can then be easily adapted into a mesh for solving.

Figure 4.7 shows the rotor aligned with the stator pole. To solve the resulting mesh, and to obtain a useful electromagnetic distribution calculation as in figures 4.8A (half aligned), 4.8B (Aligned), and 4.8C (un-aligned), there are several steps that need to be undertaken.
Chapter 4. Developing a Static Model of the Hybrid Stepping Motor from 3-D Finite Element Analysis

FIGURE 4.8A - Typical Flux Distribution of an Aligned Rotor/ Stator Position

FIGURE 4.8B - Typical Flux Distribution of a Mid Rotor/ Stator Position
FIGURE 4.8C - Typical Flux Distribution of a un-aligned Rotor/ Stator Position

FIGURE 4.9 - Typical Flux and Flux Density Distribution of a Rotor/ Stator Position
4.5.1 Method of Calculating Reluctance [45]

Figure 4.9 shows a typical flux distribution in an aligned rotor position relative to the stator pole. Flux can be seen to travel from the top down into the rotor region. When flux enters the top pole it is assumed to be travelling perpendicular to the top boundary. To obtain this condition a Neumann type boundary is required as reported in chapter 3. The left and right sides of the stator pole are forced to having a differing potential. The left and right boundary are related to each other by:

$$A_{\text{left}} = A_{\text{right}} + A_{\text{constant}}$$ (4.6)

The $A_{\text{constant}}$ is the constant potential difference of the left boundary to the right boundary. If the $A_{\text{right}}$ is set to zero then the Potential on the left will equal $A_{\text{constant}}$. This implies that the difference between the potential at corresponding nodes on opposite sides of the mesh will be constrained to differ by some constant amount.

Therefore for a particular solution we need to calculate $A_{\text{constant}}$ for a corresponding flux density. To understand the calculation of the potential constant, equation 4.7 is used with reference to figure 4.10 (in vector form).

$$\oint_{c} P_{o} \cdot d\ell = \int \bar{B} \cdot d\text{Area}_{\text{int}} = \phi,$$ (4.7)

Where $\ell$ is the length of the closed contour, and $\text{Area}_{\text{int}}$ the area of integration.
This states that the closed contour integral of the vector potential field is equal to the flux enclosed by the closed contour. The contour integration is indicated by path C. Since the problem is two dimensional the potential will be perpendicular to the front surface. Thus, it will be perpendicular to contours C2 and C4, and parallel to contours C1 and C3. The result of the contour integration is given by equation 4.10.

\[ \phi = A_{\text{left}} d - A_{\text{right}} d = A_{\text{constant}} d, \]  

(4.10)

where \( d \) is the depth of mesh, and equal in length to \( C_1 \) and \( C_3 \).

In terms of Flux density \( B \) we can assign an average flux density at the top of the block to be;

\[ B_{\text{ave top}} = \frac{\phi}{A} = \frac{A_{\text{constant}}}{p \times d} = \frac{A_{\text{constant}}}{p}, \]  

(4.11)
where $B_{ave~top}$ is the average flux density,

and $p$ the front face length which is equal to $C_2$ and $C_4$.

This allows a mesh to obtain a flux field without any coil or magnet excitation in the mesh. The mesh now can be solved. Quadratic or linear elements may be used in the solution. Quadratic elements will give a higher accuracy, due to their solution nature at the expense of increased computational time [48].

The general equation to calculate reluctance is shown in equation 4.12. This equation is not easily implemented when the problem is of a complex shape. Instead we make use of the linearised energy calculation that is readily available from FEM software packages[44].

The energy contained in a solution is equal to flux flowing through the mesh squared multiplied by the reluctance, i.e.

$$E_L = \frac{1}{2} \phi^2 R_L, \quad (4.12)$$

where $E_L$ and $R_L$ are the linearised mesh energy per unit length, and mesh reluctance per unit length respectively.

The mesh reluctance can be calculated by rearranging equation 4.12 in the form of
\[ R_L = \frac{2E_L}{\phi^2} \quad (4.13) \]

The linearised energy is calculated in the finite element program by:

\[ E_L = \sum_e \frac{1}{2} \frac{B_e^2}{\mu_e} dA_e \quad (4.14) \]

where \( B_e \) is the flux density in element \( e \), \( \mu_e \) is the operating point permeability of element \( e \), and \( A_e \) is the area of element \( e \).

The linearised energy concept is used to calculate reluctance will be per unit length, therefore the previous equation does not need to include the length of the element, i.e. \( \ell_e \).

After obtaining the linearised energy, the reluctance per unit length for a particular flux level can be calculated by using equation 4.14. From the solutions, a table of MMF/flux relations can be generated, this should be scaled by the appropriate length of the stack under observation.

### 4.6 Isolated Teeth Analysis

In many papers on analyses of flux tube frames appearing on the teeth for rotor and stator poles, the individual teeth are assumed to operate as identical components. This implies that other teeth do not interact, thus assuming that the teeth can be summed together in parallel as a set of lumped equal teeth [45]. Finite element studies, recognise that the end
tooth is not equal to a mid tooth and therefore make allowances for its differing characteristics, such as end tooth side flux lines. Problems arise when the modeller assumes that the stator tooth components act as a complete stator pole. One does not know how flux will distribute once entering the top of the summed teeth components. Though some will argue that allowing analysis of individual teeth will give a higher degree of accuracy of individual components, personal studies have shown the accuracy is lost because of the unknown interaction between the teeth and stator poles.

Above the teeth is a region that is ignored in many analyses. This area allows the flux to flow into a particular tooth. It has critical importance in the analysis of the teeth/air-gap region as a whole as it accounts for correct flux distribution.

The stator pole is disregarded in some analysis where the modeller argues that unless the pole is thin the pole will not affect the results. Some papers will give two answers to an analysis, showing the discrepancies to be due to the stator pole saturation [33,57]. In the majority of cases the stator pole is shown to have a relatively high flux density concentration in relation to the rest of the motor, especially in the aligned teeth position. It is difficult to say when the pole width will affect the performance of the motor. When the pole width is smaller than the combined width of the teeth, the motor’s performance may be affected by losses associated with higher than necessary flux densities in the pole. These will affect the MMF drop in the motor which is magnified with decreasing pole width [57]. Coil area is also dependent on pole width and as coil area is directly related to coil MMF, it would seem sensible to include the stator pole in all examples.
In the work involved in this thesis it has been found that the overall analysis of this region can be achieved accurately by taking both the pole and teeth, plus a small area of the rotor into account. This method can be difficult and tedious to calculate. By using 2-D FEM, a study on a particular design of tooth profile has allowed a highly accurate and in-depth prediction of the flux paths in the aligned, half aligned and unaligned rotor position regions. These have been successfully modelled by elliptical methods [58,59]. This method has the advantage of being relatively simple to input dimensions and has a fast solution time. These methods are described in the following sections.

4.7 Determining Permeances Algebraically

It would be beneficial to obtain permeance relationship with the use of finite element analysis to achieve greater accuracy. This however may be time consuming. If unusual designs of teeth are to be modelled it would be essential to obtain these relationships from a finite element package. If finite element software is not used then HyStep may alternatively calculate the permeance data by predicting the paths of flux lines internally. Notable work in this area has been carried out by several authors [6,31,57].
4.7.1 Permeances Calculation by Flux Lines

Methods[66,67,5]

As the flux density in the air-gap can be relatively large and the air permeability is low, a large MMF drop is found in the air-gap region itself. Methods have been proposed which calculate the reluctance of the gaps by the means of flux tubes. These tubes are sections whose permeances are calculated and summed for an overall permeance. The crudest method assumes that the air-gap is the only region worth analysing. However it can be shown that the steel teeth surrounding the air-gap operate at high flux densities and therefore become like air as their permeability decreases. It is worth noting that many air-gap analysis methods that use flux tubes do so by predicting the tubes as a series of circular and straight lines [33]. This is inaccurate for the hybrid stepper motor as the tubes tend to follow more elliptical paths, as described in the following section.

The saturated, non-linear steel lamination characteristics described in section 4.3 present difficulties in producing analytical methods for prediction of the flux-linkage at varying rotor positions. Corda and Stephenson produced work for a similar problem in the switched reluctance motor [60]. This was later improved by Materu and Krishan [34] with intermediate positions.

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4 Partly taken from a paper presented at IEEE conference on Magnetics Okyama Japan 1996

5 Partly taken from a transactions paper published by the IEEE Magnetics society
A new analytical method for estimating the minimum, maximum, and intermediate positions of both the hybrid stepping motor and other salient pole structures has been developed. The proposed method uses a combination of ellipses to predict the behaviour of the flux lines in all position. The elliptical curve closely matches the actual flux line path.

The reluctance $\mathfrak{R}$ of flux tube in the steel or air-gap can be calculated by a combination of its permeability and physical size,

$$\mathfrak{R} = \frac{\ell}{\mu \times A}, \quad (4.15)$$

where $\ell$ is the length,

$A$ the area of the flux tube,

and $\mu$ the permeability.

If the path length follows an ellipse, such as in figure 4.11, the arc of the ellipse relates to the length $\ell_{\text{ellipse}}$. The governing equation for an ellipse is,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \quad (4.15)$$

The length $\ell_{\text{ellipse}}$ of an elliptical path can be calculated by taking the average of either side of the flux tube under analysis, for each boundary of the flux tube the length is,

$$\ell_{\text{ellipse}} = \int_{x_1}^{x_2} \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx, \quad (4.16)$$
Chapter 4. Developing a Static Model of the Hybrid Stepping Motor from 3-D Finite Element Analysis

Where $x_1$ and $x_2$ are the start and end positions respectively.

Equation 4.16, may be calculated using Simpson’s rule that states that an integral

$$\int f(x)dx$$

can be approximated as;

$$\frac{d}{3}(y_0 + y_{2n} + 4\sum_{r=1}^{2n} y_{2r-1} + 2\sum_{r=1}^{2n-2} y_{2r}), \quad (4.17)$$

where $y_0 \ldots y_n$ are the co-ordinates which divide the area under $f(x)$ into strips of equal width $d$. The reluctance of the flux tube can now be calculated using equation 4.15.

![FIGURE 4.11 - Example of elliptical flux tube](image)

4.7.2 Flux versus MMF Relationships

The flux versus MMF relationship are easily determined from 2-D FEA results as a reluctance is calculated at a set value of flux $\phi$. The MMF drop in the 2-D profile can be calculated by

$$MMF = \phi R, \quad (4.18).$$
However to calculate the flux versus MMF relationship using the reluctances of the sections calculated by methods described in the previous section. Each will need to be summed with its neighbours to produce a complete flux-linkage versus MMF relationship for each motor position. MMF may be calculated

\[ MMF = Ni = \phi l, \quad (4.19) \]

where \( N \) is the number of coil turns and \( \phi \) the flux flowing in the path. The inductance can be determined by,

\[ \lambda = N\phi = li, \quad (4.20) \]

where \( \lambda \) is the flux-linkage and \( l \) is the instantaneous inductance.

The flux versus MMF profiles calculated by HyStep for each position are dictated by the size of the air-gap and the magnetisation (B-H) curves of the steel used in the circuit. The magnetic circuits of each position are divided into sections of different lengths and areas. The sections are given an average area that is assumed constant over that particular area. A simplified example is given in figure 4.12.
For a maximum flux density, $B_{\text{max}}$, in the steel, the maximum flux, $\phi_{\text{max}}$, calculated in any section is limited by,

$$\phi_{\text{max}} = B_{\text{max}} \times A_{\text{small sect}}$$  \hspace{1cm} (4.21)

where $A_{\text{small sect}}$ is the section with the smallest cross-sectional area.

From this equation it can easily be determined that the flux density of each section is found by,
\[ B = \frac{\phi}{A} \quad (4.22) \]

The MMF drop in each section is then determined using the B-H curve of the steel and the length of the section. For a given flux, the flux density for the section is given by equation 4.21. From the B-H curve of steel, the field strength \( H \) of the steel can be found. The MMF drop in these sections is then calculated by multiplying the section length \( l_{\text{section}} \) by field strength, equation 4.23.

\[ \text{MMF} = H \times l. \quad (4.23) \]

For air sections the relationship is linear between flux density and field strength, equation 4.24.

\[ B_{\text{air}} = \mu_0 \times H_{\text{air}} \quad (4.24) \]

where \( \mu_0 \) is the permeability of air.

The sections which are in series have their MMF summed to create a total MMF for a given flux equation 4.25.

\[ \text{MMF}_{\text{series total}}(\Phi) = \sum \text{MMF}_{\text{series sections}}. \quad (4.25) \]

Now there is a set of parallel \( \Phi \) versus MMF solutions. To find the total \( \Phi \) versus MMF relationship of the position, the fluxes of each parallel section need to be summed, with the series sections holding the same value of MMF.
\[ \Phi_{\text{total}}(MMF) = \sum_{i=1}^{n} \Phi_{\text{path}_i}(MMF), \quad (4.26) \]

where \( n \) = number of parallel paths, and \( \Phi_{\text{path}_i} \) is the value of flux in the series path \( i \). From this equation a flux versus MMF table can be created for each position under analysis. The complete process is summarised in the flow chart of figure 4.13.

### 4.8 Tabulated Data or Continuous Function Form

HyStep has two methods of storing and manipulating data. The first is the use of tabulated data or look-up tables, the second is the use of curve fitting techniques. In general the software will compute with the method chosen for B-H curve input, but it is possible to define which stages of HyStep use curve fitting and which use look-up tables. For example having the B-H tables as look-up table and the rest of the programs solutions controlled by curve fitting. This is advantageous in a situation where an unusual B-H curve relation is used or in a situation where the curve fit will not converge or the fit error estimate is too high. Curve fitting methods are extremely useful when the user requires results that are between or past known data points. The two methods are reviewed in table 4.1.
Chapter 4. Developing a Static Model of the Hybrid Stepping Motor from 3-D Finite Element Analysis

PRE-CALCULATION STAGE

INPUT BH CURVE
1. POINTS
(2. EQUATION)

DIMENSIONS
INPUT

MAXIMUM
FLUX
DENSITY

MINIMUM AREA

FLUX TABLE CREATED
BY FINDING MAXIMUM
FLUX

UN-ALIGNED

MID

ALIGNED

LENGTHS AND AREAS
CALCULATED FOR EACH
POSITION'S PATHS

AIRGAP RELUCATANCE FOUND

FLUX - MMF TABLE FOUND FOR EACH
PATH

NEW MMF TABLE CREATED BY FINDING THE AVERAGE
MAXIMUM MMF OF ALL THE POSITION'S PATHS

NEW VALUES OF FLUX FOUND AT NEW
VALUES OF MMF FOR EACH POSITION'S PATHS

FOR EACH MMF VALUE, EACH PATH HAS ITS
FLUX VALUE SUMMED, TO GIVE AN MMF-FLUX
TABLE FOR THE WHOLE POSITION

FIGURE 4.13 - Finding Flux/MMF table for each Position
### Standard Tables Method

1. From the table of flux values, calculate B-flux density for the appropriate section by:

   \[ B = \frac{\phi}{A} \text{ TESLA} \]

2. Find the corresponding H-field strength from the B-H table by using linear interpolation.

3. Calculate value of MMF for the section by:

   \[ MMF = H \times \ell \text{ AMPS} \]

4. Sum all the section MMF's for each path.

5. Find the average maximum MMF of all the paths.

6. Create the final MMF table for the position by using the answer of 5 as the maximum value.

7. Get value from MMF table and use linear interpolation to find corresponding flux for each path.

8. For each MMF value sum the flux to obtain the total flux required for that position.

9. A FLUX/MMF table for the position is obtained.

### Curve Fitting Method

1. From the table of flux values, calculate B-flux density for the appropriate section by:

   \[ B = \frac{\phi}{A} \text{ TESLA} \]

2. Find the corresponding H-field strength from the B-H equation;

   \[ B = f(a_i, H) \]

   by using a root finding program, i.e. Newton Raphson interpolation or Secant method.

3. Calculate value of MMF for the section by:

   \[ MMF = H \times \ell \text{ AMPS} \]

4. Sum all the section MMF's for each path and obtain a FLUX/MMF equation for that table.

5. Find the average maximum MMF of all the paths using the maximum values calculated for each path.

6. Find values of flux using the paths FLUX/MMF equations in discrete value up to answer found in 5.

7. For each MMF value sum the resulting flux to obtain the total flux required for that position.

8. A FLUX/MMF equation can be fitted to the new points, for the position. The equation will be of the form; \( \phi = f(a_i, mmf) \) Webers.

---

**Table 4.1 - Finding MMF for Paths**
4.9 Three Dimensional Effects Comparisons.

Can the tooth /air-gap regions be modelled to a high enough degree of accuracy using a 2-D analysis. Figure 4.14 is an analysis for an aligned curve taken inside the rotor end-cap. The left hand side is the rear end-cap (un-aligned teeth), and the right hand side is the front end-cap (the aligned teeth). The $B_y$ (flux density in the Y direction) is clearly the largest throughout the front pole. The $B_x$ and $B_z$ components of flux density are relatively small in relation to the $B_y$.

In the air-gap region which is represented by figure 4.15, the results are similar to the rotor. The peak of $B_y$ is lower as there is a spreading of flux over the complete rotor. The $B_x$ and $B_z$ components are still relatively small. The dip in $B_y$ occurs over the magnet where the air-gap length increases.
Chapter 4. Developing a Static Model of the Hybrid Stepping Motor from 3-D Finite Element Analysis

FIGURE 4.15 - Bx, By, and Bz Plot for Air-Gap

FIGURE 4.16 - Bx, By, and Bz Plot for Stator
Chapter 4. Developing a Static Model of the Hybrid Stepping Motor from 3-D Finite Element Analysis

The flux density distribution half way up the stator pole (figure 4.16) shows the X-direction component of flux density ($B_x$) has remained small. However there is a significant increase in the $B_z$ component as more flux will now flow in the Z-direction in orientation. This is due to flux flowing into the stator back-iron. On the right hand side next to the air boundary, a second peak is produced by the end effect of the coil. The $B_y$ component then falls and then peaks again towards the centre of the stack.

Key parts in this analysis to note, are that the rotor iron sides generally confine the flux to the aligned teeth section. In the air-gap the flux does tend to fringe increasing the base area of crossing. In the stator pole the flux shifts into the centre of the stator pole as flux distribution now has a larger amount of flux travelling in the Z-direction. When 2D FEA is undertaken the analysis uses an average flux density, therefore an average area is required. Generally the length of the Z-direction dispersion changes, giving around 15% increase in depth. For a flux tube calculation method the areas can be individually increased for the differing lengths. Therefore in 2-D FEM analysis the height of the stator pole should not be too long.

4.9.1 Magnet

The magnet plays an important role in the hybrid stepper. It generates flux when there is no current excitation into all the regions of the motor. The magnet is a non-linear source, and hence its MMF and flux will change due to loading. For some magnet materials it may be possible to simplify the relationship between MMF and flux to a linear source, but for magnet types like Alnico there is a non-linear relationship between flux and MMF. Generally the magnet flux travels through the parts of the motor with the lowest reluctance.
and so the flux is distributed disproportionately. The flux from the magnet produces a near sinusoidal relationship of flux-linkage to position into a phase winding. The magnet has a direct effect on the torque output of the motor as the flux-linkage of the coil with position, produces the magnet back EMF effect. This back EMF induced into a single phase is estimated by:

\[ \text{EMF}_{mag} = -N \frac{d\phi}{dt} = -N \frac{d\phi}{d\theta} \frac{d\theta}{dt} = \omega n_r N \Phi_{mag} \sin(n_r \theta). \]  \hspace{1cm} (4.27)

Where \( \Phi_{mag} \) is the peak flux generated by the magnet, \( n_r \) number of teeth on the rotor.

The torque produced by a phase is;

\[ T = -\frac{\text{EMF}^{i}_{mag}}{\omega} + \frac{1}{2} i^2 \frac{dL}{d\theta}. \]  \hspace{1cm} (4.28)

The second term which is torque produced by the change of phase inductance, is sometimes considered to be relatively low compared to the magnet contribution. For example when the motor is running under low currents and is not saturated, it has been seen that even for low excitation, saturation occurs on the teeth profiles. Accordingly for general operation of the motor the main source of back EMF is from the magnet, and its magnitude is directly related to the peak flux linkage from the magnet. This would indicate that the stronger the magnet the greater the torque produced. The last statement would again be subject to saturation effects in the iron of the motor.
The holding torque characteristics of the motor can be calculated by treating the motor as two counteracting variable reluctance motors [61]. The co-energy area stored between the points;

\[
\text{point}_1 = \text{mmf}_{\text{coil}} + \frac{1}{2}\text{mmf}_{\text{mag}},
\]

\[
\text{point}_2 = \text{mmf}_{\text{coil}} - \frac{1}{2}\text{mmf}_{\text{mag}},
\]

\(\text{mmf}_{\text{coil}}\) and \(\text{mmf}_{\text{mag}}\) are the instantaneous MMF’s produced by the coil and magnet respectively.

To obtain the maximum torque for a given coil excitation the magnet needs to be producing an amount of MMF which places its value in the middle of the largest area (figure 4.17).

\[\text{FIGURE 4.17 - Window for Magnet Co-energy}\]
The magnet has a varying potential along its axial length, with a zero potential in the centre. This is a significant property of the magnet as it will allow symmetrical modelling of the magnet (figure 4.19).

### 4.9.2 Laminations

In 2-dimensional machines, i.e. where only a radial field distribution exists, laminations resist the flow of eddy currents. Unfortunately the hybrid motor's operation principles require flux to flow perpendicular to the laminations.
4.9.3 Perpendicular Flux

Concentrating on flux which travels parallel to the laminations, the effects of laminations added to the reluctance of the circuit is small (increasing permeability of the circuit). The following equation shows the change of permeability in the same plane of the laminations with the packing factor of the laminations [48].

\[
\mu_{\text{plane}} = pf \times \mu_{\text{iron}} + (1 - pf) \mu_0,
\]

(4.31)

where \( pf \) is the packing factor of the laminations and is defined as the percentage of a lamination stack, which can be considered as solid.

The increased permeability in the same plane as the laminations affects the value of flux by a small percentage and in most cases can be assumed negligible. In some cases the laminations constrain certain areas to behave more two dimensionally. Such effects assist our accuracy when applied to certain motor sections such as the rotor/stator tooth regions.

4.9.4 Cross lamination Effects

Cross lamination effects are difficult to analyse, as the inter lamination is unlikely to be uniform across a whole lamination stack. The biggest cross lamination effects take place in the stator back iron. This is where the flux is required to cross the lamination. In the stator poles at the front and back of the motor, close to the air-gap, the flux crosses in a mainly Y-directioned manner. As the flux goes up the stator pole it becomes more z-directional.
The same will happen in the rotor stacks. Figure 4.19 gives an understanding of what a lamination stack consists of.

![Lamination Structure Diagram]

**FIGURE 4.19 - Lamination Structure**

The lamination can be seen in its most basic form as a steel sheet separated by a non-conductive layer. It has to be accepted that the actual physical structure of the inter-lamination air-gap cannot be truly modelled by a uniform packing factor. This is because of the complications introduced from air spaces that lie between laminations and is a result of the laminations' surface roughness. This surface roughness on the lamination surfaces, causes the air space between the laminations to be non-uniform. Hence it is not known if the air-gap is greater or less than the lamination insulation layer. Figure 4.20 shows the lamination insulation thickness as much larger than the physical air thickness. The actual thickness of the air space could be much larger than the insulation thickness. The inter-
lamination air gap is highly reluctant, this layer will affect the performance of the motor because of the increased reluctance.

Many analyses discount the back iron as having negligible reluctance when it is not saturated [33,61,62]. It is true that the back iron has generally a low flux density distribution, but the large volume that may be involved makes it an important factor. If the back iron and inter-lamination effects are seen as a combined issue, it does not take a large value of flux density to cause a detrimental change in motor performance. Flux flowing across laminations will have to cross extra amounts of air and adhesive. The packing factor is used to give an estimate of the change in cross lamination permeability,

\[ \mu_{\text{cross}} = \frac{\mu_0 \mu_{\text{iron}}}{\mu_f \times \mu_0 + (1 - \mu_f) \mu_{\text{iron}}} \]

This generally translates to an increase in reluctance of over 10 times making this a very significant part of the model.

4.9.5 End Effects

Referring back to figure 4.16, which is a plot of the stator poles with an excited phase winding, this shows that flux in the pole appears to have an extra peak nearer the end of the machine (air region). This effect can be accounted for by considering the 3-dimensional effects that arise at the end of the machine stack [49]. The major effect is the end winding flux, that is the flux lines generated from the conductor area that are outside the machine.
stack length. These flux lines link the phase winding via the main magnetic circuit. The other end effect is axial fringing where the flux lines travel in the axial direction through the air from the stator to the rotor. The effect is greatest when the teeth are in the aligned position. These effects are generally only accounted for in three dimensional analysis. This effect is also noticeable on the rotor end-caps where the magnet boundary occurs.

### 4.9.6 Summary of Contributory Factors to the Magnetic Behaviour

All the above are contributors to the performance of the motor. Table 4.2 summarises the key issues and effects.

<table>
<thead>
<tr>
<th>Property</th>
<th>key effects</th>
<th>Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air-gap/ rotor/ stator interface</td>
<td>Determining the flux linkage of the motor.</td>
<td>Extremely</td>
</tr>
<tr>
<td>Magnet</td>
<td>Major MMF source of the motor at low excitations.</td>
<td>Important</td>
</tr>
<tr>
<td>Back iron/ inter lamination effects</td>
<td>Reduces the effectiveness of the magnet by adding additional MMF loses.</td>
<td>Important</td>
</tr>
<tr>
<td>End effects</td>
<td>Causes extra flux linkage in the circuit</td>
<td>Noticeable</td>
</tr>
</tbody>
</table>

**TABLE 4.2 - Three Dimensional Effects**
4.10 Deriving a Simpler Model of the Hybrid Stepping Motor

Two important features have been shown in the analysis of the motor in 3-D FEM. The first is that the magnet has a zero potential point half way along its axial length. The second is that it can be shown that a second zero potential exists along the centre of the back iron (figure 4.20). These zero potential points can allow a symmetrical model to be developed.

![Figure 4.20 - Zero Potential in Stator Back Iron](image)

**FIGURE 4.20 - Zero Potential in Stator Back Iron**
Taking the regions of importance, the hybrid stepping motor can be simplified to give figure 4.21, which shows a full lumped element of a hybrid stepping motor. The magnet is modelled from calculating a load point on its B-H curve. This is converted to a MMF/flux point using the dimensions of the magnet. The lumped elements and MMF supplies are represented in the figures as linear elements and linear MMF supplies. These elements and MMF supplies are modelled as non-linear elements and MMFs. Each branch is based on the permanence modelling discussed earlier in the chapter. Now using the zero potential points of the FEM analysis the centre of the magnet can be joined to the centre of the back iron. The magnet can be modelled as a source that produces half the amount of MMF per flux unit. The back iron reluctance will also become half of its original value. Figure 4.22 shows the new arrangement. It can be seen in figure 4.22 and 4.23, that each reluctance contains a parallel equivalent. We can simplify the circuit further by halving the reluctances as they are in parallel and of equal magnitude. Therefore we obtain the final simple circuit which gives fewer amounts of elements to calculate (figure 4.23).
Chapter 4. Developing a Static Model of the Hybrid Stepping Motor from 3-D Finite Element Analysis

FIGURE 4.22 - Half Lumped Model of the Hybrid Stepping Motor

BRANCH A   BRANCH B₁   BRANCH B₂   BRANCH C

Magnet/Back Iron Branch

FIGURE 4.23 - Reduced Elements Network
4.10.1 Single Phase Excitation Case

Single phase excitation is when only one phase is switched on at any one time. This does not mean that there is no current in the second winding as the inductance of the coil can cause the current in the commutated phase to decay slowly. However in this static analysis the current is assumed to be in the excited phase only. In the single phase case the circuit can be represented as in figure 4.24.

![Single Phase Network Diagram](image)

**FIGURE 4.24 - Single Phase Network**

It has four branches.

1) The magnet and back iron branch,

2) Branch A,

3) Branch B,

4) Branch C.
4.10.2 The Magnet and Back Iron Branch

The magnet is represented by an MMF source which varies according to its load point calculated from its B-H characteristics. These characteristics can either be represented by a table or an equation in the HyStep program.

Firstly the B-H characteristics are converted from B and H to Φ and MMF respectively.

\[ \Phi = B \times A_{mag}, \quad (4.33) \]

\[ H \rightarrow \text{MMF} \quad MMF_{1/2mag} = H \times \frac{1}{2} \ell_{mag}, \quad (4.34) \]

Where \( MMF_{1/2mag} \) is half the MMF produced by the full length magnet, \( A_{mag} \) is the magnet area, and \( \ell_{mag} \) is the magnet length.

The back iron / inter-lamination component of this branch is treated as a reluctance section with a modified length and permeability. The modified length is the distance that the flux will travel in the stator back iron, and the rotor end-caps across the lamination air-gaps. It is estimated by;

\[ \ell_{\text{apparent}} = \ell_{mag} + (K_z \times 4 \times \ell_{stk}), \quad (4.35) \]

\( \ell_{stk} \) is the rotor stack length.
Chapter 4. Developing a Static Model of the Hybrid Stepping Motor from 3-D Finite Element Analysis

The $K_z$ coefficient is an approximation of how much of the rotor stacks, the flux travels across the lamination air-gap. From finite element studies 30% was found to be a good estimate thus $K_z = 0.3$. The factor of four multiplier assumes that the travel by flux in the stator is directly related to that of the rotor end-caps.

The permeability of the stack is calculated by equation 4.31 and the B-H curve of steel.

From the original B-H curve steel, a particular flux density has a corresponding field strength $H$. The permeability of the iron is calculated by:

$$\mu_{\text{iron}} = \frac{B}{H}. \quad (4.36)$$

This $\mu_{\text{iron}}$ is entered into equation 4.32 along with the packing factor to produce the modified permeability $\mu_{\text{modified}}$, which is identical to $\mu_z$. The modified permeability is used to find the modified field strength $H_{\text{modified}}$, equation 4.37, which in turn can be used to find the increased MMF drop,

$$H_{\text{modified}} = \frac{B}{\mu_{\text{modified}}}. \quad (4.37)$$

For this branch, there is now a reluctance and a MMF source. It can be seen that the back iron / inter-lamination effect will reduce the effectiveness of the magnet as a MMF source.
This can be viewed like an internal resistance to a power source. The greater this reluctance the less of the magnet MMF can be dropped across the other branches.

The magnet MMF versus flux characteristics are non-linear and are shown in figure 4.25A. A modified back iron relationship (solid) is shown along side a non laminated structure (dashed) of the same dimensions in figure 4.25B. The relationships are typically linear because they are affected by low flux density which falls within the linear region of the steel's B-H curve. This would change if the back-iron was made of a low saturating steel or a back iron design where the flux-density was allowed to rise above the linear region.

To represent the branch as a single identity, we need a representation that includes both the magnet source and the back iron. The MMF of the branch is the magnet MMF.
produced minus the MMF dropped in the back iron. The MMF source of the magnet generally works within the second quadrant, so it has a negative value.

\[ \text{MMF}_{\text{branch}} = -(\text{MMF}_{\text{mag}} + \text{MMF}_{\text{back iron}}), \quad (4.38) \]

\[ \phi_{\text{branch}} = \phi_{\text{mag}} = f(\text{MMF}_{\text{mag}}) = \phi_{\text{back iron}} = f(\text{MMF}_{\text{back iron}}). \quad (4.39) \]

Both the back iron reluctance and the magnet MMF are a function of MMF. It is easier to create a new relationship for the whole branch than to try and make a combination of both. If the data is in tabular form then a linear interpolation method can be used. If the data is in the preferred form of an equation then the equations are summed to obtain the new results, table 4.3 shows how.

<table>
<thead>
<tr>
<th>Flux value (Wb)</th>
<th>Magnet MMF (I)</th>
<th>Back Iron MMF Drop (I)</th>
<th>New MMF (I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1000</td>
<td>0</td>
<td>-1000</td>
</tr>
<tr>
<td>1e-4</td>
<td>-800</td>
<td>20</td>
<td>-780</td>
</tr>
<tr>
<td>4e-4</td>
<td>-100</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>5e-4</td>
<td>0</td>
<td>170</td>
<td>170</td>
</tr>
</tbody>
</table>

**TABLE 4.3** - Effect of back iron MMF drop on magnet output

The dashed line in figure 4.25A shows the new relationship of the branch. It has the effect of shifting the peak flux more into the positive MMF region. We now have a relationship for the whole branch which can be stored as an equation or in table form.
4.10.3 The Equation of Magnet B-H characteristics

The MMF versus flux profile of the magnet and hence the total branch characteristics are represented by a negative MMF region. As described earlier, it has been shown that the B-H characteristics of steel can be fitted to equations of 4.3, 4.4, and 4.5. Likewise the MMF versus flux relationships may be modelled by similar equations, for example,

\[
\phi = \frac{\alpha \text{(mmf)}^n}{\beta + \text{(mmf)}^n}, \quad (4.40)
\]

or its related modified rational functions;

\[
\phi = \frac{\alpha \text{(mmf)}^n + \kappa \text{(mmf)}^{n+1}}{\beta + \text{(mmf)}^n}, \quad (4.41)
\]

\[
\phi = \frac{\alpha \text{(mmf)}^n + \kappa \text{(mmf)}^{n+1}}{\beta + \text{(mmf)}^n + \epsilon \text{(mmf)}^{n+1}}, \quad (4.42)
\]

An offset \((\text{of})\) is required by HyStep for the equation to represent a positive quadrant equivalent. This is simply achieved by taking the absolute value of MMF when the flux is less than zero. In the resulting equation the offset is required to be added before solving, i.e.

\[
\phi = \frac{\alpha \text{(mmf) + of}^n}{\beta + \text{(mmf + of)}^n}. \quad (4.43)
\]
4.10.4 Branch A

In single phase operation, branch A will include a phase winding and a rotor aligned position reluctance for calculating the aligned flux-linkage curve (in conjunction with branch C). The reluctance will be made up of two identical reluctances in parallel. The same combination is used to find the single phase mid flux-linkage curve but with the coil current set to zero. The reluctances have been found by two dimensional methods, either 2-D FEA or elliptical path methods. The MMF in the windings, needs to be converted from the current and the number of turns,

$$\text{MMF}_{\text{coil}} = iN,$$  \hspace{1cm} (4.44)

where $N$ is the number of turns on the particular pole.

Because the branch’s reluctance is actually two parallel reluctances the flux flowing will need to be doubled. The branches MMF versus flux relationship needs to modified by the MMF in the windings. The relationship for the branch flux is;

$$\phi_{\text{branch A}} = 2 \times f(MMF_{\text{REL}} - \text{MMF}_{\text{coil}}),$$  \hspace{1cm} (4.45)

where $f$ indicates the ‘funtion of’.

The subtraction of the coil MMF to the reluctance MMF indicates that a larger flux will result for an increase in winding current. This addition of MMF means that when the coil current is increased flux in that branch will rise. For example consider figure 4.26. This diagram shows a simplified aligned tooth flux versus MMF profile. The flux flowing in this branch will be dictated by the MMF produced by the magnet if no current is flowing in
the phase winding. For example if the MMF produced by the magnet was 30 Ampere turns then the flux flowing in the branch will be approximately \(0.82 \times 10^4\) Webers. If for this simple example the magnet MMF does not change but there is now a MMF produced by the phase winding of 20, the flux will now increase to approximately \(1.1 \times 10^4\) Webers. This is because the MMF equation of the complete circuit is;

\[
MMF_{mag} = MMF_{Branch A} = MMF_{Branch B} = MMF_{Branch C} \quad (4.46)
\]

which implies that the value of \(MMF_{REL}\) must equal 50 Ampere turns for a magnet MMF of 30 Ampere turns and a coil MMF of 20 Ampere turns, i.e. \(50 = 70 - 20\).

![Figure 4.26 - Aligned Rotor Teeth Flux/MMF Characteristics](image)

**FIGURE 4.26 - Aligned Rotor Teeth Flux/MMF Characteristics**

### 4.10.5 Branch C

Similar to the branch A, but the coil MMF is reversed, and the reluctance is made up of the un-aligned rotor teeth. When the current is positive the MMF opposes that of the magnet. The flux equation in turn is;

\[
\phi_{branch C} = 2 \times \int (MMF_{REL} + MMF_{coil}) \quad (4.47)
\]
As the current in the winding increases a negative flux may occur. HyStep calculates this negative flux by checking if the MMF is less than zero. If the MMF is less than zero then the flux will be governed by

$$\phi_{\text{branch}c} (\text{MMF}_{\text{REL}} \pm \text{MMF}_{\text{coll}} < 0) = -f(\text{absolute}(\text{MMF}_{\text{REL}} \pm \text{MMF}_{\text{coll}})) \quad (4.48)$$

### 4.10.6 Branch B

Branch B represents when the rotor teeth are half aligned with the stator teeth, this position is also known as the mid rotor position. In single phase operation finding the aligned flux-linkages due to branches A and C, does not require branch B to have an excitation in its phase winding. Branch B can then be seen as four parallel reluctances lumped into one reluctance. The flux value is modified to four times the value of a single position.

$$\phi_{\text{branch}B} = 4 \times f(\text{MMF}_{\text{REL}}) \quad (4.49)$$

### 4.11 Example of Circuit Set-Up to Find Aligned and Unaligned Single Phase Flux Linkages.

To clarify the last section an example is now given. Here the excitation scheme is single phase, i.e. branches A and C will have coil excitation. This example will allow the aligned flux linkage path to be found. The aligned position is represented in figure 4.27 which shows the front of a hybrid stepping motor. The rotor is sitting in the stable equilibrium position with the top stator pole A aligned with the rotor teeth. The rotor magnet flux is
assisting pole A’s MMF, when the current flowing in the coil is positive. The network circuit will be as figure 4.24, with branch A being the aligned rotor, branch B, the mid rotor position and branch C, the un-aligned rotor position relative to the pole stator teeth.

There will be no current in the branch B, the MMF in the coil in branch C will be equal in magnitude but opposite in polarity to that of branch A. The main equation of the network states that the flux flowing into the back iron branch is equal the sum of the fluxes flowing in Branches A, B and C;

$$\phi_{mag} = \phi_A + \phi_B + \phi_C, \quad (4.50)$$

where $\phi_{mag,A,B,C}$ are a function of MMF described by equations 4.38, 4.44, 4.46, and 4.48 respectively. This can be solved by a Newton Ralpson method as long as the relationships of the branches are known.

HyStep starts its calculations from negative maximum current and increases up to the maximum current. This generates a MMF in the coils which interacts with the magnet
Chapter 4. Developing a Static Model of the Hybrid Stepping Motor from 3-D Finite Element Analysis

MMF to cause flux to flow in the aligned and un-aligned rotor positions, Branches A and C.

The aligned flux linkage curve (refer to figure 4.3) is the combination of the aligned and un-aligned rotor position flux linkages. The 'aligned flux linkage' flux is

\[ \phi_{\text{AL}} = \phi_A - \phi_C. \] (4.51)

The aligned flux linkage curve can be generated by

\[ \lambda_{\text{AL}(i)} = N \phi_{\text{AL}(i)}, \] (4.52)

where \( i \) is the current value and \( N \) the number of turns in the phase.

4.11.1 Generation of Unaligned and Mid Flux Linkage Curves

The un-aligned curve (figure 4.4) can be easily generated from the aligned curve as described in section 4.4. The mid curve can be calculated by using the circuit in figure 4.23. The MMF produced by Branches A and C will be zero. The circuit can also be produced from figure 4.24. The Branch B reluctances of figure 4.24 are four parallel reluctances and need to be split into two branches of two parallel reluctance's. The coils on Branch A and C need to be switched to Branches B1 and B2 respectively. The equations of the new branches will be;

\[ \phi_{\text{branch B}_1} = 2 \times f(MMF_{\text{REL}} + MMF_{\text{coil}}), \] (4.53)

\[ f_{\text{branch B}_2} = 2 \times f(MMF_{\text{REL}} - MMF_{\text{coil}}), \] (4.54)
and

\[ \phi_{mag} = \phi_A + \phi_{B_1} + \phi_{B_2} + \phi_C. \]  

(4.55)

The 'mid flux linkage' flux can be found by

\[ \phi_{MID} = \phi_{B_2} - \phi_{B_1}. \]  

(4.56)

The Mid flux linkage curve can be generated by

\[ \lambda_{MID(1)} = N\phi_{MID(1)}. \]  

(4.57)

### 4.12 Discussion

The analysis provided by the three-dimensional finite element modelling has provided a useful insight into the static characteristics of the hybrid stepper motor. The motor has been broken down in regions which have allowed simplified models to be applied to particular issues associated with the motor. Combining these models has allowed a lumped circuit to be produced for the hybrid stepping motor. This lumped circuit has been converted into a computational simulation packaged named HyStep. HyStep is used for sizing and computing the fundamental areas of performance for the motor. The program is capable of calculating the motor magnetisation curves using internal (analytical) routines. However for non-standard tooth geometry suggestions for using an external 2-D finite element package have been suggested.

New rational functions for modelling B-H characteristics have been presented. Modelling of the flux line distribution by elliptical methods has been examined.
CHAPTER 5

MODELLING VERIFICATION AND DYNAMIC ANALYSIS

The previous chapter has described the static modelling procedure for the hybrid stepping motor using computational analysis, namely HyStep and finite element analysis. The accuracy of these procedures has been checked by modelling the Stebon 1101 type motor, and comparing the predictions with experimental results. The Stebon 1101 is a single stack, bipolar motor, with a 42 NEMA sized frame. This chapter will express the strengths and weaknesses of the particular design methods. Starting with torque calculations it follows on to dynamic verification by considering a dynamic model of the motor. The proposed method comes through characteristics observed in the 3-D static modelling of the hybrid stepping motor and experimental test results.

5.1 Torque Calculation

Torque can be calculated by using the flux-linkage curves. The area enclosed by the curves is named the co-energy of the system. Torque is given by the rate of change of co-energy with respect to the movement of the rotor, at a constant excitation. For single phase excitation, the hybrid stepper motor is not a singly excited system, as it includes a magnet source. The energy balance equation can be written as:

\[ (\text{Electrical and magnetic power in}) = (\text{mechanical output power} + \text{rate of increase in magnetic energy}), \]  

\[ (5.1) \]
where the electrical power in is due to the EMF induced in the phase along with the current flowing, and the magnetic power is the voltage generated by the magnet multiplied by the current flowing.

Equation 5.1 can also be written as

\[ dW_e = dW_{\text{mech}} + dW_{\text{field}}. \] (5.2)

Where \( dW_e \) is the change in electrical input power, \( dW_{\text{mech}} \) is the change in mechanical output power, and \( dW_{\text{field}} \) is the change in energy stored in the field.

Introducing Faraday's law, \( dW_{\text{field}} \) can be expressed as

\[ dW_{\text{field}} = i d\lambda - T d\theta, \] (5.3)

Where \( i \) is the current in a particular phase winding, and \( \lambda \) is the flux linkage in a particular phase; the term \( T \) represents (average) torque while \( d\theta \) is the angular displacement. However the \( d\lambda \) term is not caused solely by the current flowing through the phase inductance, as there will be flux linkage due to the magnetic. Therefore equation 5.3 can be rewritten as

\[ dW_{\text{field}} = i (\lambda_{\text{mag}} + \lambda_{\text{coil}}) - T d\theta \] (5.4)

Where \( \lambda_{\text{mag}} \) is the flux linkage in the single phase winding due to the magnet, and \( \lambda_{\text{coil}} \) is the flux linkage in phase one due to the current in the winding.
By application of the chain rule the increment in field energy can be expressed in terms of flux linkage $\lambda$, and rotor position $\theta$,

$$\frac{dW_{\text{field}}}{d\lambda_{\text{mag}}} \bigg|_{\theta,\lambda_{\text{coil}}=\text{const}} d\lambda_{\text{mag}} + \frac{dW_{\text{field}}}{d\lambda_{\text{coil}}} \bigg|_{\theta,\lambda_{\text{mag}}=\text{const}} d\lambda_{\text{coil}} + \frac{dW_{\text{field}}}{d\theta} \bigg|_{\lambda_{\text{coil}},\lambda_{\text{mag}}=\text{const}} d\theta. \quad (5.5)$$

Comparison of equation 5.4 and 5.5 reveals that the last term of equation 5.5 is related directly to the torque term of equation 5.4.

$$T = -\frac{dW_{\text{field}}}{d\theta} \left(\lambda_{\text{mag}(\text{const})}, \lambda_{\text{coil}(\text{const})}, \theta\right). \quad (5.6)$$

The co-energy of a system may be derived from,

$$W_c = (\lambda_{\text{mag}} + \lambda_{\text{coil}}) i - W_{\text{field}}. \quad (5.7)$$

In order to estimate the average torque produced in a single phase hybrid stepping motor system, it is essential to have the flux linkage relationship curves of the phase winding for the aligned and the mid positions. The change in co-energy of the current carrying phase is the trajectory of the flux linkage curve. To understand the change of co-energy with respect to current and rotor position, the current excitation pattern needs to be defined. In the following example (figure 5.1) the current in the phase is assumed as an ideal square current and the current is switched on when the rotor is in the half-aligned position.

Looking at the change in co-energy in the first 1.8 degrees. For the pole A, subject to current excitation, the rotor moves from the mid position to the aligned. The co-energy curve is the area enclosed from the mid curve to the aligned on the flux linkage diagram.
The co-energy repeats itself every 1.8 degrees. When the current becomes negative, the area of co-energy is between the un-aligned and mid curves, for negative current values. This area is equal in size to the aligned to mid region for positive currents. The average torque can be calculated by

\[
T = \frac{\Delta W_c \times (\text{number of steps per rev})}{2\pi} = \frac{\Delta W_c \times (360^\circ/18^\circ)}{2\pi} = \frac{\Delta W_c \times 100}{\pi}.
\]  

(5.8)

In this example torque will be calculated from the co-energy area enclosed by the mid curve to the aligned curve for a positive current.

### 5.1.1 Two Phase System

The magnetic equivalent circuit of figure 4.22 is used to perform a two phase analysis. Now each branch has the ability to carry current in its winding. The analysis is much the same as for the single phase case except that the 5 branches will be represented by equations 4.38, 4.45, 4.47, 4.49, 4.54, and 4.55 (from chapter 4).

### 5.1.2 Average Torque

The system under analysis is a two phase doubly excited system and is very similar to the single phase system in terms of deriving the average torque from equation 5.8. The field energy can be written in terms of \( i_1 \) and \( i_2 \).

\[
dW_{\text{field}} = i_1 d\lambda_1 + i_2 d\lambda_2 - Td\theta , \tag{5.9}
\]

where \( i_1 \) and \( i_2 \) are the currents in each phase winding, \( \lambda_1 \) is the flux linkage in phase one (due to both the current in the windings and the magnet flux), and \( \lambda_2 \) is the flux linkage in phase two (again due to currents in the windings and magnet flux). A similar torque equation to equation 5.5, can be developed by following the same method which obtains equation 5.4 from 5.3.

\[
T = -\frac{dW_{\text{field}}(\lambda_{1(\text{const})}, \lambda_{2(\text{const})}, \theta)}{d\theta}, \tag{5.10}
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the flux linkages in phases 1 and 2 respectively.

The co-energy of the two phase system may be derived from,

\[
W_c = \lambda_1 i_1 + \lambda_2 i_2 - W_{\text{field}}. \tag{5.11}
\]

The torque \( T \) is given as the rate of change of co-energy with respect to the rotor angle,

\[
T = \frac{dW_c(i_1, i_2, \theta)}{d\theta}, \tag{5.12}
\]
Again the flux linkages in both windings are required to be known. The current excitation scheme of figure 5.2, shows the rotor shift in relationship to each pole. (Current levels unequal for clarity).

From the centre of figure 5.2, and looking at phase B which is wound on pole B, the current rises from zero to rated current when the rotor is in the un-aligned position. The current stays at this rated current until the rotor is in the aligned position, where the current falls. The trajectory on the flux-linkage curve is therefore as in figure 5.3a. For pole A which is subject to phase A, the rotor is at mid position when the current is at rated current. The rotor moves to the aligned position where the current drops to negative rated current. The rotor moves to the next mid point at the same current. The total rotor movement is 3.6°. The trajectory on the flux linkage curve is as in figure 5.3b. The areas of the trajectories of figures 5.3a and 5.3b, are equal, as the positive current region of...
figure 5.3b is equal to the un-aligned to mid region of figure 5.3a. Therefore for 3.6° movement and ideal current pulses the co-energy will be twice the area between aligned and un-aligned, for one flux-linkage diagram (positive current only).

\[
\text{The average torque can therefore be calculated by an adaptation of equation 5.8,}
\]

\[
T = \frac{(\Delta W_{c1} + \Delta W_{c2}) \times \text{(number of steps per rev)}}{2\pi} \times \frac{360^\circ}{3.6^\circ} = \frac{\Delta W_{c1} \times 100}{\pi}.
\]

In this example this is for the co-energy enclosed by the aligned to un-aligned curves on the flux-linkage curves, for a positive current.

5.2 Magnet Detent Torque

The theory presented for calculating torque is specifically used for calculating average torque. For the three main positions on the flux-linkage curve (aligned, mid and unaligned) it can be shown that the net change in co-energy between the positions at zero amps is zero, as the working point of the magnet is the same. Therefore the average torque is zero,
which makes sense as the motor would be able to rotate by itself. However if the change in
angular position was smaller, then the co-energy method could be used to calculate
instantaneous torque. For the magnet it can be shown that a peak torque is produced at a
rotor angle of 0.9°. This can be related to the back EMF / flux linkage plot of the magnet
generated from FE analysis in figure 5.4. To find the change in co-energy, the flux and
branch MMF value for 0.9° rotor position at zero amps is required. Also the values for the
aligned 0° position are required.

![Figure 5.4 - Magnet Flux Linkage Plot](image)

The flux value required to calculate the torque, for the 0.9° position, can be obtained from
the single phase circuit from chapter 4, figure 4.21. In this circuit the flux can be
calculated for the 0° (aligned), 1.8° (mid), and 3.6°(un-aligned) positions. To calculate the
flux at 0.9° from the aligned position, the circuit could be solved by generating flux versus
MMF relation curves for the 0.9° and 2.7° positions. An easier method is to take
advantage of the fact that the flux of a complete pole can be very well approximated by a
cosine function. Figure 5.4 shows the points calculated from FEM analysis, and a cosine
function fitted to these points.

The Design of Hybrid Stepping Motors aided by Three Dimensional Finite Element Analysis
function plotted alongside. The relationship can be fitted by second order Fourier approximation such as equation 5.14.

\[ y = A_0 + A_1 \cos(\omega_b x) + A_2 \cos(2\omega_b x), \]

(5.14)

where \( \omega_b \) is the period conversion.

For the angle of 0.9°, the second term of equation 5.14 is zero therefore the flux can be calculated by;

\[ \phi = A_0 + A_1 \cos(\omega_b \theta), \]

(5.15)

where \( \omega_b \) is equal to \( \pi/3.6 \).

To fit the relationship only the aligned, mid and un-aligned flux need to be obtained. The value of branch MMF would be assumed the same as that of an aligned rotor position as the change in working point of the magnet is very small.

Adapting the co-energy method described before, the change in the co-energy will have flux on the Y-axis and the branch MMF on the X-axis as in figure 5.5. The area for 0.9° rotor position is subtracted from the 0° rotor position co-energy area. In this circumstance the 0.9° rotor movement is 1/400th of a revolution. It must be remembered though that the detent torque is a 4th harmonic component of overall torque, i.e. it occurs 4 times in one complete rotor movement (aligned to aligned, figure 5.6). Therefore we need to average the value obtained for one complete revolution by dividing it by four. This relationship is shown in figure 4.31 in comparison to a single phase scheme. The detent torque can therefore be calculated by an adaptation of equation 5.13.

\[
T = \frac{\Delta W_c \times (\text{number of steps per rev})}{4 \times 2\pi} = \frac{\Delta W_c \times (\frac{360\degree}{0.9\degree})}{4 \times 2\pi} = \frac{\Delta W_c \times 50}{\pi}. \quad (5.16)
\]

\[\phi\]
\[\Delta\theta\]
\[\Delta_{\text{mmf}}\]
\[\text{mmf}\]

FIGURE 5.5 - Co-Energy of Detent Torque

\[\text{Torque}\]
\[\theta\]

FIGURE 5.6 - Detent Torque shown as 4th Harmonic of Single Phase Motor Torque

Though the detent torque is in phase with the single phase excitation, it will be out of phase with two phase excitation. This phase can be shifted by altering the teeth width of the stator and rotor, in an attempt to move the general maximum phase torque to be in line with the detent torque.

5.3 Experimental and Three Dimensional Finite Element Results

In order to assess the accuracy of the computational methods a particular phase flux-linkage loop was obtained by the experimental method described in chapter 2. The Stebon 1101 motor was modelled by the 3-D FE methods described in chapter 3, and its flux-linkage curve obtained. The comparison is shown in figure 5.7.

![Experimental Flux Linkage Curve with 3-D FEA Prediction](image)

**FIGURE 5.7** - Experimental Flux Linkage Curve with 3-D FEA Prediction

The results show good correlation. The three dimensional FEA tends to yield a higher absolute value of flux-linkage including the magnet (zero current) offset. Above 8 Amps the error does increase as saturation occurs. However the overall results are considered to
be within acceptable tolerances for computational, measurement and manufacturing discrepancies. The performance of Alnico magnet materials is highly dependent on the amount of cobalt present. The amount of cobalt present may vary from sample to sample.

5.3.1 HyStep Algebraic Predictions

The 1101 motor had its flux-linkage curves computed using HyStep’s internal flux path generator and back iron correction methods.

![Flux-Linkage Curve Predictions](image)

**FIGURE 5.8 - Non-Linear Network Flux Linkage Curve Predictions (3D-FEA Reference)**

The HyStep assessment produces good correlation with the three dimensional analysis, especially at low current levels (figure 5.8). However, errors do increase at higher current levels as the mathematical interpolation used in HyStep becomes less reliable. Calculations undertaken by other non-linear network solvers offer results that are typically higher than the 3-D analysis, mainly due to the absence of back-iron effects and the use of linear interpolation between data points. HyStep offers good prediction when used directly with data obtained from a finite element package. Using HyStep solely or collectively with 2-
dimensional data is very attractive when considering the reduced computational time and significantly lower effort involved.

On closer analysis of the two dimensional data that was generated for use in the network solvers it can be seen that the reluctance of each position is relatively constant at low values of flux (figure 5.9) and all converge at higher values of flux as the iron portion saturates and becomes like air. This can be also seen from the more standard representation of flux against MMF for the tooth profiles in figure 5.10, which is generated from the flux/reluctance profiles calculated by the HyStep internal flux path calculations. It can be seen that for lower values of flux that the un-aligned position is dominated by its air content and the linear region of the steel. As the position changes and the area under excitation becomes more influenced by the steel content, the contour of the position curve becomes more non-linear. In a hybrid stepping motor all rotor positions put some parts of the motor into saturation. The development of the non-linear modelling technique presented in chapter 4, is therefore very valuable.

![Figure 5.9 - Tooth Profile Reluctance](image)
5.4 Static Torque Test

A DC current was applied directly to the windings of the Stebon 1101 motor. A torque was then applied, and the displacement from an equilibrium position was recorded. This test is outlined in chapter 2.

The static torque profile produced by the Hybrid stepping motor at a particular angle can also be obtained from 3-D FEA by computing the integral of the Maxwell Stress tensor over the surface of the inter stator/rotor air-gap. The 3-D FEA static torque prediction and experimental data are shown in figures 5.11 to 5.12. The peak static torque predictions are displayed in figure 5.13.
FIGURE 5.11 - Static Torque, Experimental and 3-D Finite Element analysis (single phase) 0° refers to the Aligned Position

FIGURE 5.12 - Static Torque, Experimental and 3-D Finite Element analysis (Two phase Excitation) 0° refers to the Aligned Position
FIGURE 5.13 - Peak Static Torque - 3-D Finite Element analysis (Single and Two phase)

<table>
<thead>
<tr>
<th></th>
<th>Single Phase Excitation</th>
<th>Two Phase Excitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-D FEA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Torque by</td>
<td>0.92</td>
<td>1.40</td>
</tr>
<tr>
<td>Maxwell Stress (Nm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(at 3.5 amps)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HyStep</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Torque by Co-</td>
<td>0.96</td>
<td>1.44</td>
</tr>
<tr>
<td>energy (Nm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(at 3.5 amps)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Torque (Nm)</td>
<td>0.89</td>
<td>1.32</td>
</tr>
<tr>
<td>(3.5 Amps)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-D FEA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Torque by</td>
<td>1.58</td>
<td>2.53</td>
</tr>
<tr>
<td>Maxwell Stress (Nm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(at 7 amps)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HyStep</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Torque by Co-</td>
<td>1.64</td>
<td>2.71</td>
</tr>
<tr>
<td>energy (Nm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(at 7 amps)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Torque (Nm)</td>
<td>1.54</td>
<td>2.41</td>
</tr>
<tr>
<td>(at 7 Amps)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1 - Averaged Torque versus Virtual Work Predictions

The Design of Hybrid Stepping Motors aided by Three Dimensional Finite Element Analysis
The results from this comparison are limited in this section to three dimensional finite element results, and experimental data. This is because the virtual work co-energy method used in HyStep predicts average torque, and not instantaneous torque. However the results of the virtual work co-energy method can be verified by averaging the values of the instantaneous static torque values. The predicted average torque from HyStep and 3-D FEA are shown in table 5.1, together with experimental average torque values.

5.5 Dynamic Testing

Three dimensional finite element analysis produces static results but cannot directly predict dynamic performance, a major requirement for an industrial applications motor designer. The situation is further complicated by the complex variety of stepping motor drives and controllers, each with very different dynamic characteristics. Apart from designs of machines that are specifically for microstepping and mini-stepping, a motor that performs well under full stepping will generally have similar performance characteristics under half stepping, and other increased current resolutions, neglecting situations that produce motor resonance and inductance which will always be most pronounced in full stepping.

5.5.1 The Distribution of Energy

Figure 5.14 shows how energy is distributed within the motor-drive system and how electrical energy is converted into mechanical work and provides a model applicable to any electric motor. The process for this highly controlled system is thus:
Assuming the rotor is restrained, when a voltage is applied to a phase winding the current builds up over a finite period determined by the inductance of the coil. A magnetic flux associated with this current also builds up, acting as a reservoir for potential energy by storing it in the field of the air-gap.

If the rotor is now released while the current remains constant, the potential energy stored in the magnetic field is released as mechanical work in moving the rotor to its desired equilibrium position.

If the rotor remains restrained, but the current is removed, the magnetic field will collapse and the energy stored therein will be released back into the electrical circuit.

The sum of the energy recovered and the mechanical energy converted will be less than the energy put in by an amount equivalent to the losses.

![Diagram of Energy Distribution](image)

**FIGURE 5.14 - The Distribution of Energy Within the Hybrid Step Motor**

The losses indicated in figure 5.14 can be separated into three main categories as shown in Table 5.2.

<table>
<thead>
<tr>
<th>Electrical</th>
<th>Electromagnetic</th>
<th>Mechanical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper losses ($i^2R$)</td>
<td>Eddy currents</td>
<td>Friction</td>
</tr>
<tr>
<td>Switching losses</td>
<td>Hysteresis</td>
<td>Windage</td>
</tr>
</tbody>
</table>

**Table 5.2 - Losses in the Hybrid Stepping Motor**

The conversion of electrical energy to mechanical work can be summarised by the general rule:

\[
\text{Electrical energy in} = \text{Mechanical work out} + \text{Increase in magnetic energy} + \text{Combined losses} \quad (5.17)
\]

The balance of this equation becomes far more important under dynamic conditions than static, as most losses increase with speed, with the main exception being copper losses. This is because the winding inductance prevents the build-up of current and hence the current delivered to the windings decreases with speed. It is evident that a purely analytical approach to the calculation of the motor dynamic torque will, at best, be approximate and complex due to the interdependence of variables of the motor in tandem with a drive.

Over each motor operation cycle each of the elements in equation 5.1 apart from the losses, may take on positive and negative values, as energy is interchanged between the supply, the field and the load.

5.6 Mathematical Model for a Phase Winding of a Two Phase Hybrid Stepping Motor

It would be useful and convenient if a dynamic model of the motor could predict performance based on information already calculated by HyStep. Though the hybrid stepping motor is a dynamic non-linear system, further assumptions and simplifications about its operation under dynamic conditions can be ascertained from three dimensional
finite element modelling. In the following sections a mathematical model is derived for the phase windings and simplified from observations from 3-D FEM.

Hysteresis and eddy current losses are neglected in the following modelling. This can be considered as a not too unrealistic assumption, particularly when the motor is run at low speeds. However these effects will reduce flux-linkage, and all results need to viewed in the light of iron losses.

A model of a phase winding of the motor can be achieved by applying Kirchoff's voltage law and Faraday's law of electromagnetism.

\[ V = iR + \frac{d\lambda}{dt}, \quad (5.18) \]

where \( V \) is the voltage across the phase winding, and \( R \) is the phase winding resistance.

The \( iR \) term is the voltage drop due to the winding resistance, the second term of the equation is the induced EMF due to the rate of change of flux linkage with time. For Faraday's rule to be met, the terms on the right hand side must equal the terminal voltage of the phase.

The flux-linkage term can be obtained by either 3-D FEA or HyStep. This has been shown to be a function of magnetic geometry, and the magnetomotive forces developed by the current in the phase windings and those generated by the permanent magnet. In full, the flux linkage of phase 1 is a function of the current in both phases, the rotor position, and the magnet MMF,

\[ \lambda_i = \lambda_i(i_1, i_2, \theta, MMF_{mag}) \], \quad (5.19)
where $\theta$ is the rotor position.

Thus for phase 1, equation 5.18, is rewritten as

$$V_1 = i_1 R_1 + \frac{d\lambda_1(i_1, i_2, \theta, MMF_{mag})}{dt}.$$  

(5.20)

A similar expression for phase winding 2 holds true. By use of the chain rule, equation 5.20, can be expanded to give equation 5.21.

$$V_1 = i_1 R_1 + \frac{d\lambda_1}{di_1} \frac{dt}{dt} + \frac{d\lambda_1}{di_2} \frac{dt}{dt} + \frac{d\lambda_1}{d\theta} \frac{dt}{dt} + \frac{d\lambda_1}{dMMF_{mag}} \frac{dt}{dt},$$  

(5.21)

The first term on the right is still the resistive drop, because of the current flowing in the windings. The second term is the induced voltage due to the rate of change of current, $i_1$, in phase 1, with respect to time. It is named the self inductive voltage drop. The third term is called the mutual inductive voltage drop, which arises when voltage is induced in phase 1, because of a change in current flowing in phase 2. The next term arises because of the voltage induced when the rotor changes position with respect to time, it is rotor speed dependent. This is commonly known as the back EMF. The final term is the voltage induced because of the rate of change in the MMF developed by the permanent magnet with respect to time. A similar equation can be formed for phase 2.

The axially magnetised permanent magnet found in the rotor of the hybrid stepping motor, is subject to demagnetising fields produced when current flows in the stator phase windings. Within the normal operating characteristics of the motor, the magnetic flux generated by the magnet may be considered approximately constant once magnetised. This indicates that the magnet MMF will also remain approximately constant because of the
stable load point on the magnetisation curve. If the current in the windings does not exceed its rating, and the motor is not driven high into saturation than this change in MMF may be less than 10%. Table 5.3 shows results obtained from 3-D FEA results with the magnet operating under a full range of currents. The rating current of the motor is 7 Amps.

<table>
<thead>
<tr>
<th>Current (Amps)</th>
<th>magnet MMF (Ampere turns)</th>
<th>magnet flux density (Weber e-6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>255</td>
<td>823</td>
</tr>
<tr>
<td>2</td>
<td>254</td>
<td>825</td>
</tr>
<tr>
<td>3</td>
<td>255</td>
<td>823</td>
</tr>
<tr>
<td>4</td>
<td>260</td>
<td>822</td>
</tr>
<tr>
<td>5</td>
<td>264</td>
<td>819</td>
</tr>
<tr>
<td>7</td>
<td>276</td>
<td>814</td>
</tr>
<tr>
<td>8</td>
<td>283</td>
<td>811</td>
</tr>
<tr>
<td>10</td>
<td>298</td>
<td>804</td>
</tr>
<tr>
<td>12</td>
<td>313</td>
<td>799</td>
</tr>
<tr>
<td>14</td>
<td>328</td>
<td>793</td>
</tr>
</tbody>
</table>

TABLE 5.3 - Magnet Flux and MMF Values versus Winding Current in One Phase Taken at the Aligned Position

For simplicity it will be assumed that the magnet MMF change with time is zero, on the understanding that this may be increasingly inaccurate at higher winding currents.

\[
\frac{d\text{MMF}_{\text{mag}}}{dt} = 0. \tag{5.22}
\]

This term is therefore eliminated from the equation 5.21.

The mutual flux-linkage between the phases is governed by the \( \frac{d\lambda_1}{dt} \frac{di_2}{dt} \) term in equation 5.21. Three dimensional finite analysis agrees with assumptions [63,64,65] that the flux linking a particular phase is dominated by the current flowing in that phase and that induced by the permanent magnet. This implies that the effect of current in another phase on the
phase winding under examination is negligible. The next figure (5.15) shows the increase in flux (and flux density) in a stator pole as the current rises in its phase winding. Also included in the figure is the rise of flux (and flux density) in the second phase winding which does not have any current excitation. The starting point at zero current shows the flux linkage due to magnet only. Gradually the current in one winding is allowed to rise and the flux-linkage may be calculated for both phase windings by multiplying the flux by the phase turns. The flux-linkage in the second un-excited phase is nearly constant especially under rated conditions (7 Amps). Therefore it can be assumed that the mutual flux-linkage is negligible.

\[
\frac{d\lambda_1}{di_2} = 0. \tag{5.23}
\]

**FIGURE 5.16 - Flux Density and Flux in Phase 1 and 2 as the Current in Phase 1 is Varied**

The self inductance of the phase winding can be seen as the gradient of the flux-linkage curve, shown in figure 5.16.

\[ \frac{d\lambda}{dl_1} = I, \quad (5.24) \]

where \( I \) represents instantaneous self inductance.

For the linear part of the flux-linkage curve this remains constant, even over a change in position. Rewriting equation 5.15, by eliminating the appropriate terms, the equation of voltage (with respect to previously discussed conditions) may be considered as;

\[ V_1 = I, R_1 + \frac{dI}{dt} + \frac{d\lambda_1}{d\theta} \frac{d\theta}{dt}. \quad (5.25) \]

The remaining position dependent term is the back EMF. Generally the open circuit back EMF can be considered to be approximately sinusoidal with position. This term can be viewed experimentally by the back EMF test in chapter 2. If the speed is varied a graph of voltage induced against speed can be plotted. Neglecting harmonics and assuming the rotational EMF to be sinusoidal, and presuming that the motor is operating at constant speed, the following can be proposed.

\[ \omega = \frac{d\theta}{dt}, \quad (5.26) \]

Phase one rotational EMF can be written as,

\[ EMF_{mag} = \frac{d\lambda_m}{d\theta} \frac{d\theta}{dt} = K_r \omega \sin(n, \theta), \quad (5.27) \]

where \( EMF_{mag} \) is the rotational EMF due to magnet, and

\( K_r \) is the rotational EMF constant.

The other phase on the motor is shifted by an electrical angle of \( \pi/2 \) radians. This causes the back EMF to be shifted by the same angle. Therefore, for the other phase the equation would be;

\[
EMF_{mg2} = \frac{d\lambda}{d\theta} K_{e} \omega \cos(n, \theta) .
\]  \hspace{1cm} (5.28)

The rotational EMF constant can be obtained by taking the gradient of the back EMF versus speed curve (figure 5.17).

![Open Circuit rotational EMF](image)

**FIGURE 5.17 - Open Circuit Rotational RMS EMF**

However as the data is obtained experimentally, it is not available in HyStep. HyStep can calculate flux-linkage versus position. If the sinusoidal relationship of the magnet's flux-linkage relationship is taken as \( \frac{d\lambda}{d\theta} \) then we can obtain the expression for the back EMF in phase 1,
where $\lambda_{\text{mag-max}}$ is the maximum flux-linkage in the phase due to the magnet at zero current. This will hold unless the motor is pushed high into saturation. The second phase has a similar equation:

$$EMF_{\text{mag2}} = \frac{d\lambda_{\text{mag}}}{d\theta} = n_r\lambda_{\text{mag-max}}(t)\omega \sin(n_r\theta).$$ (5.29)

This allows the equations for phase 1 and 2 to be simplified and expressed as:

$$V_1 = i_1R_1 + L \frac{di_1}{dt} + \omega n_r\lambda_{\text{mag-max}}(t)\sin(n_r\theta),$$ (5.30)

$$V_2 = i_2R_2 + L \frac{di_2}{dt} + \omega n_r\lambda_{\text{mag-max}}(t)\cos(n_r\theta).$$ (5.31)

All of these quantities can be calculated from the static flux linkage data available in HyStep.

### 5.6.1 Current Observation

A key element to generating torque is the current available to produce flux in the motor whilst rotating. The current control schemes adopted all produce differing results. Full stepping produces harder step action than, say, half stepping. Microstepping gives, generally, a torque output closer to full stepping with a smoother operation. It also provides a better torque speed range. Concentrating on full stepping, the current at low speeds, where current has time to rise through the winding inductance, will appear square, figure 5.18a. However as speed increases the current begins to distort (figure 5.18b).
The current in the winding can be estimated by the phase winding equations 5.30 and 5.31. Rearranging the equations of the phase voltages to be in a form that allows the current to be calculated, gives the expressions;

\[
\frac{di_1}{dt} = \frac{1}{L_i} \left[ V_1(t) - i_1 R_1 - \omega n R_i + \lambda_{mag-mag}(t) \sin(n, \theta) \right], \quad (5.32)
\]

\[
\frac{di_2}{dt} = \frac{1}{L_i} \left[ V_2(t) - i_2 R_2 - \omega n R_i + \lambda_{mag-mag}(t) \cos(n, \theta) \right]. \quad (5.33)
\]

These differential equations can be solved by numerical techniques to predict the current wave form. These will allow HyStep to predict the torque by re-calculating the co-energy transfer, with the predicted current in the corresponding phase.

A third order Runge-Kutta method [66] was used to predict the current. This was felt to give a good compromise between accuracy and speed. The equation is of the form;

\[
y_{i+1} = y_i + \left[ \frac{I}{6} (k_1 + 4k_2 + k_3) \right] h, \quad (5.34)
\]

where

\[
k_1 = f(x_i, y_i), \quad (5.35)
\]

\[ k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}hk_i), \quad (5.36) \]

\[ k_3 = f(x_i + h, y_i - hk_i + 2hk_2), \quad (5.37) \]

\( x_i, y_i \) are the initial conditions of \( x \) and \( y \),

\( h \) is the numerical step size of the period of integration.

To integrate equations 5.32 and 5.33 \( 'x' \) and \( 'y' \) of equation 5.34 are time and current respectively. The period of integration will be over a step interval of the motor. The period can be calculated from the number of rotor steps per revolution. For a 200 step resolution (full stepping) the time period will be related to the speed in revolutions per second by:

\[ \text{period} = \frac{\text{steps}}{200} \times \text{rps}, \quad (5.38) \]

Where \( \text{steps} \) are the number of motor steps under observation, and \( \text{rps} \) are the revolutions per second, where \( \text{rps} \) is related to \( \omega \) by

\[ \omega = 2\pi \text{ rps.} \quad (5.39) \]

\( 'h' \) in equation 5.34 will be a value that is an order of magnitude of this period, i.e. a time step.

The maximum current is limited by a maximum current value, otherwise the current will continue to rise. In this method the drive is assumed to be using ideal switches and to exhibit no overshoot. The rotor position is assumed to in phase with the voltage and running at constant speed.
5.6.2 Verification of Current Waveforms

A series of tests was carried out to validate the dynamic current prediction method used by the HyStep program. The HyStep prediction assumes that the motor is working in a near ideal system. The predicted and experimental torque/speed curve are obtained for a series connection, peak to peak current of 7 Amps. The current waveform characteristics are shown for speeds of 1 rps and 10 rps in figure 5.19 and 5.20 respectively, which allow the generation of the torque speed curve of figure 5.21.

![Figure 5.19 - Current Waveforms for 1 rps, 7 Amps Peak](image-url)

**FIGURE 5.20 - Current Waveforms for 10 rps, 7 Amps**

**FIGURE 5.21 - Torque/ Speed Curve (Full Stepping - 7 Amps Peak)**
5.6.3 Verification of Current Waveforms at Different Speeds

Once a current waveform has been predicted for a particular speed, a method of using the co-energy changes to predict the torque is used. In an ideal system it can be seen that at a certain point in time the rotor will be in a particular position, with the phase winding containing a value of current. Therefore at this point a new flux-linkage can be calculated. A new profile can be drawn on the flux-linkage current diagram, which is the actual profile followed by the motor winding. This allows a new value of co-energy to be obtained. This calculation of co-energy allows a new torque to be predicted.

5.6.4 Sources of Error

With all measurement systems that measure dynamic results, errors will arise due to calibration and system characteristics. Errors will also arise because of the assumptions made in simplifying the model. Therefore a margin of error must be allowed for measurements. The largest source of error is neglecting the iron losses as this will reduce fluxes. The prediction methods also assume an instantaneous voltage change and ideal switches producing a perfect square voltage pulse. The prediction method interpolates between the aligned, mid, and unaligned positions to obtain flux-linkage characteristics for use in the virtual work method. Again the area enclosed by the operating trajectory is integrated by an estimation method, which is limited by the discretisation of the integration steps. At speeds higher than 40 rps the current waveform prediction is more susceptible to instability. The motor generally was subject to resonance when the speed was greater than 25 rps. This is a limiting factor of most full stepping current drives. This resonance effect
was not attempted to be modelled in this simulation as the effect is highly dependent to by the motor shaft coupling, which would be difficult to model.

5.7 Discussion

Adaptation of the co-energy virtual work methods has been carried out to provide torque characteristics for single and two phase commutation. A new understanding of the magnet detent operation has been examined by using the virtual work method and an explanation of how the magnet interacts with the phase excitation is shown.

Verification of experimental static results from the use of three dimensional finite element studies and the mathematical model derived in chapter 4 has been shown. A model has been devised for predicting current in the windings under dynamic operation. The model uses information that was calculated by the HyStep program, hence it can be easily incorporated to allow predictions of the torque versus speed curves for full stepping motion. Despite the assumptions which have been made in the modelling, the results produced are consistent with experimental results.
6.1 Background

Recently manufacturers have been concentrating on improving the torque performance of hybrid stepping motors by changing the magnet material in the rotor rather than changing the historically rigid tooth designs [12]. Low speed torque has been dramatically improved, though unfortunately, many new designs have suffered from little improvements at mid to high speeds. Stebon Ltd., sponsors of this project specialise in high powered shaft output designs and a design brief was given to improve the 1101 size 42 motor at mid to high speeds without resorting to winding changes. Ideally the physical construction and the manufacturing process of the motor were not to be modified. The criteria pointed towards modifying the tooth structure on the lamination. The following investigation concentrated on the stator lamination, in particular the tooth design. This would avoid expensive retooling and a design could be constructed without any special equipment and training.

Historically the hybrid stepping motor tooth profile has been designed with a typically square (parallel) stator tooth. The stator teeth are generally larger in width than the rotor teeth to achieve improved low speed torque with high positional accuracy. It is well
known that the tooth profile has an effect on the torque output of the motor especially in variable reluctance motors. In the hybrid stepping motor altering the tooth profile changes the inductance, and thus can aid current rise at higher motor speeds. This in turn increases the co-energy area at the operating speed and allows a greater available torque to the motor.

6.2 Modelling a New Tooth Design

According to Singh [67], four sets of equations are required to adequately model a stepping motor. These are:

- Voltage equations of the electrical circuits
- The change of flux linkage and inductance relative to rotor position
- Developed torque, including stored energy and co-energy
- Dynamic equations of the rotor

The modelling by the three dimensional finite element package OPERA-3D offers a reasonable possibility to predict the first three accurately. The HyStep software uses assumptions and simplifications that degrade prediction, but has favourable computation time, file size, and memory attributes and can model all four sets of equations. Furthermore, HyStep’s internal flux path calculator, assumes a parallel sided tooth geometry, which makes it difficult to model vastly different tooth designs. However the tooth geometry can be studied using 2-D FEA, as described in chapter 4. This data can then be used in HyStep with the dynamic model described in chapter 5.
6.3 The Effect of Tooth Design on Torque Production

The typical design of tooth profiles is summarised in a few parameters (figure 6.1):

- stator slot depth - Generally around $\frac{1}{2}$ tooth pitch
- Tooth width/tooth pitch ratio - Found to be between 0.38 to 0.45 (Harris et al propose 0.42 is suggested as an optimum value [68]). Tooth pitch equals the summation of the tooth and slot width.
- Slot shape between teeth, typically semi-circular on the rotor, but can be found to be rectangular on the stator.
- Air gap length which should be kept as small as possible.

The ratio of stator tooth width/rotor tooth width is known to affect holding torque capability. The higher this ratio, the higher the holding torque as the lines of magnetic flux are held under tension, thus opposing rotor disturbance more rigidly. A balance will have to be obtained for dynamic torque.

![Figure 6.1 - Tooth and Slot depth Considerations](image)

The torque of a motor is inversely proportional to the square of the air gap and directly proportional to the stack length. It is desirable to have a long motor and it is desirable to
have smooth tooth faces, as surface irregularities increase the air gap length, thus reducing torque and impeding smooth operation.

Wagner et al [69] investigated the quantitative effects of tooth/ slot ratio and slot shape on the magnetic circuit. It was concluded that the choice of rectangular or semi-circular stator slots has little effect on influencing the performance of a motor, however the rounded slots were marginally more effective. In a further study Wagner et al [70], analysed the leakage flux between adjacent pole pieces of a DC motor and found within reasonable design structures, that the leakage loss was around 1% to 4% of total flux. D.Torney [71] presented data for doubly slotted structures. He suggested that the tooth slot depth to slot width ought to be higher than 0.4 and that a tooth slot to tooth width ratio ought to be as high as 1.5 to ensure low air gap permanence. However in the modelling in this thesis little difference is observed when making the slot too deep, as in the un-aligned position, flux lines from the edge of the stator teeth tend to fringe into the rotor teeth edges. Keeping the slot type as a semi-circular shaped recess proves marginally more beneficial over a rectangular slot, as long as the rotor teeth slot to stator teeth width ratio is higher than 1.5.

Akiyama and Sueki [72] proposed that the side profiles of the stepping motor tooth contributed to the way leakage flux effectively determines torque production in a variable reluctance motor. It was proposed that tapered teeth could contribute greater torque at higher excitations.
Although the last study was not directly aimed at the hybrid stepping motor it suggests that a new design may affect the dynamic performance of the motor. From the study of current wave forms the effect of the $\frac{di_1}{dt}$ term of the equation;

$$V_1 = i_1R_1 + L\frac{di_1}{dt} + \alpha n_1 \lambda_{mag-max} \sin(n, \theta), \quad (6.1)$$

is a major contribution in the effect of current rise time and hence mid/ high speed torque.

It has been suggested by Akiyama and Sueki [72] that a dove tail tooth profile would offer maximum holding torque while an inverted or tapered tooth a higher dynamic torque, figure 6.2a, and 6.2b. The idea of tapered teeth leads to the use of exponential shaped tooth profiles to improve motor performance.

![Figure 6.2a/b - Dove Tail and Tapered Tooth Profile](image)

### 6.4 Selection of a New Tooth Profile

To overcome problems that might be encountered with providing sufficient space for the copper conductors, the advantages that may have been obtained in altering the reluctance path in the back iron and stator pole width were disregarded to concentrate on the tooth profile effects. Before selecting a new profile the standard design of tooth profile was analysed.
6.4.1 Standard Configuration

Figure 6.3 - Standard Configuration Tooth Geometry

This design (figure 6.3) is the standard Stebon 1100 series hybrid stepping motor with square rotor and stator teeth. It provides the comparison data against which any changes will be measured.

Analysis was undertaken using the Opera 2-d finite element package, using the methodology presented in chapter 4. Flux densities were produced showing the magnitude of flux density across the tooth faces of the stator and rotor. These showed that the vast majority of aligned flux passes through the central portion of the stator teeth with little contribution from the corners. However, most unaligned flux passes through the corners.
Chapter 6. New size 42 Frame Single Stack Hybrid Stepping Motor Design For Dynamic Performance

Figure 6.4 shows the flux versus MMF relationship for several positions between the aligned and unaligned locations. From top to bottom, the lines represent fully aligned, $\frac{3}{4}$ aligned, semi-aligned, $\frac{1}{4}$ aligned and fully unaligned respectively. The area under each section was integrated to obtain changes in co-energy and these are presented in table 6.1.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Change in Co-energy (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aligned to unaligned</td>
<td>7.03</td>
</tr>
<tr>
<td>Aligned to semi-aligned</td>
<td>4.07</td>
</tr>
<tr>
<td>Semi-aligned to unaligned</td>
<td>2.96</td>
</tr>
</tbody>
</table>

Table 6.1 - Performance of the Standard Configuration
FIGURE 6.5 - Standard Tooth Configuration: Flux versus Angle for 3 values of Constant MMF (0° and 7.20° are Aligned Positions)

Figures 6.5 and 6.6 show the variation in flux and rate of change of flux respectively against angle. In the flux versus angle relationship the waveform resembles a sinusoid with some low harmonic content. This harmonic content is due to the square edges of the teeth creating sharp rates of change of reluctance. As a consequence, the induced EMF profile, which is important when designing control circuits and considering high speed operation, is not sinusoidal. This also indicates torque ripple over a full step, where a larger change of flux with angle is found with constant MMF. This effect is multiplied as the co-energy change is concentrated in the area before the knee of saturation in the flux/MMF curves.
From studies of flux paths across the teeth of the standard motor and inspection of the flux density and flux paths it was noted that the central area of the tooth carries most of the flux efficiently, it would appear impossible to improve aligned flux without, say, reducing the air-gap. However, the corners of the stator teeth which carry little aligned flux also carry most undesirable unaligned flux. Therefore, it is a modification of the corners of the teeth that has been concentrated upon.

6.4.2 Proposed Designs

An investigation was undertaken to find the methods of reducing the harmonic effects of the tooth edges, whilst maintaining a high aligned to un-aligned co-energy value. Two designs were selected from the investigation and are detailed in the following sections.
6.4.3 ‘0.7 mm Filleted’ Configuration

Figure 6.7 ‘0.7 Filleted’ Configuration Proposed Tooth Geometry

Figure 6.7 shows the alternative design which has a 0.7 mm filleted radius on the tooth edge. This attempts to produce a smoother transition between rotor positions. Variations of the filleted radius were attempted. Using a 0.7 mm radius appeared to give the most reasonable results. Reducing the radius of the fillet in this tooth design did not provide a sizeable advantage over the standard profile.

An examination of the flux path diagrams showed that some flux will still flow into the sides of the rotor teeth which will maintain holding torque performance, while unaligned flux is impeded by the length of the air-gap created and the corners of the rotor teeth saturating.
6.4.4 '0.5 mm Chopped' Configuration

This design (Figure 6.8) follows from reversing the filleted design. From flux path studies, this arrangement shows that little flux flows through the tooth corners in the aligned position, because of the removal of the steel. The sections removed are arcs of radius 0.5 mm centred on the corner of the original tooth. In this way, the aligned flux paths should be minimally affected, but unaligned flux will be greatly impeded by increasing the air-gap between the stator teeth and the rotor teeth. This design does not rely on saturation effects for its operation and is a relatively simple and robust modification to the standard tooth design.

![Figure 6.8 - 'Chopped' Configuration Proposed Tooth Geometry](image)

6.4.5 Comparisons of Electromagnetic Characteristics

The flux to MMF characteristics of the filleted design are shown in figure 6.9. Table 6.2 shows the comparable change in co-energy. For the filleted design significant improvements were found to be between the semi-aligned and unaligned positions where
Chapter 6. New size 42 Frame Single Stack Hybrid Stepping Motor Design For Dynamic Performance

The improvement is around 27%. From the aligned to un-aligned and semi-aligned, there is also a substantial increase in co-energy. This increase of co-energy is available to be transformed into torque.

The flux to MMF characteristics for the chopped design shown in figure 6.10 give the changes in co-energy which are summarised in table 6.2. The data demonstrates that, while not greatly affecting aligned flux, this design offers a substantial improvement over standard teeth. Three quarters of this gain is between the semi-aligned and un-aligned positions.

<table>
<thead>
<tr>
<th>Co-energy change</th>
<th>Standard</th>
<th>0.7 Filleted</th>
<th>% change</th>
<th>0.5 Chopped</th>
<th>% change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co-energy change Aligned to un-aligned</td>
<td>7.03</td>
<td>8.42</td>
<td>+20</td>
<td>8.51</td>
<td>+21</td>
</tr>
<tr>
<td>Co-energy change Aligned to semi-aligned</td>
<td>4.07</td>
<td>4.67</td>
<td>+15</td>
<td>4.56</td>
<td>+12</td>
</tr>
<tr>
<td>Co-energy change Semi-aligned to un-aligned</td>
<td>2.96</td>
<td>3.75</td>
<td>+27</td>
<td>3.96</td>
<td>+34</td>
</tr>
</tbody>
</table>

Table 6.2 - Comparison of the Co-energy in Joules of the '0.7 Filleted' and '0.5 Chopped' against 'Standard'
FIGURE 6.9 - Graph of Flux vs. MMF for the '0.7 Filleted' Configuration in (top to bottom) the Aligned, Semi-aligned and Unaligned Positions

FIGURE 6.10 - Graph of Flux vs. MMF for the 'Chopped' Configuration in (top to bottom) the Aligned, Semi-aligned and Unaligned Positions.

Figures 6.11 and 6.12 show the flux and rate of change of flux respectively for the '0.7 mm filleted' design plotted against angle. The flux variation has a reduced low order
harmonic content. This indicates that the torque profile over a full step would be smoother over the standard design. This is further illustrated by the rate of change of flux with angle shown in figure 6.11. Here a lower harmonic content is evident in the more sinusoidal variations. Comparison of figure 6.11 with figure 6.5 shows that the aligned flux is largely unaffected by the filletting but the un-aligned flux is reduced (3.6°).

**FIGURE 6.11** - '0.7 filleted' Configuration in Flux versus Angle for 3 values of Constant MMF MMF (0° and 7.2° are Aligned Positions)
Chapter 6. New size 42 Frame Single Stack Hybrid Stepping Motor Design For Dynamic Performance

FIGURE 6.12 - '0.7 filleted' Configuration Rate of Change of Flux versus Angle for 3 Levels of Constant MMF MMF (0° and 7.2° are Aligned Positions)

Figures 6.13 and 6.14 show the variation with angle for flux and rate of change of flux respectively for the '0.5 mm chopped design'. Again the torque profile over a full step is will be smoother, as a result of the more consistent sinusoidal characteristic.
Chapter 6. New size 42 Frame Single Stack Hybrid Stepping Motor Design For Dynamic Performance

FIGURE 6.13 - ‘Chopped’ Configuration in Flux versus Angle Graph for 3 Constant Values of MMF (0° and 7.2° are Aligned Positions)

FIGURE 6.14 - ‘Chopped’ Configuration Rate of Change of Flux versus Angle for 3 Constant Values of MMF (0° and 7.20° are Aligned Positions)
6.4.6 Chosen Tooth Profile

The results obtained for the ‘chopped’ configuration showed the most promising increase in possible co-energy change. The reduction in un-aligned inductance in the linear region was calculated to be 15% less for the chopped design. Furthermore the generated back EMF profile appears smoother in figure 6.14, containing a lower harmonic content than the standard design. It was therefore chosen that the 0.5 mm radius ‘chopped’ design would be used for the basis of a new lamination design.

6.5 Hybrid Stepping Stator Construction

The stator laminations were wire eroded from sheets of silicon steel. The choice of silicon steel is identical to the type employed in the construction of the rotor laminations. This grade is the low loss TRANSIL type, selected to limit eddy current losses. The laminations were 0.35 mm in thickness. Before construction a special wash was prepared to clean the laminations, so that the adhesive would adhere well. The motor was built to the specifications summarised in table 6.3.

This is a terminal box design, which can be configured for parallel or series winding arrangements, by the use of 8 terminals as described in chapter 1. Windings are made exclusively for bi-polar drives. The motor is enclosed by a steel shell that offers IP44 protection. The rotor is constructed as a single stack type incorporating an Alinico Hycomax-3 magnet. The motor's outside dimensions are shown in figure 6.15. The procedure of constructing the motor is described in chapter 2. The design met the design criteria of being able to be built without any major training or equipment change.
## Table 6.3 - Motor Specifications

<table>
<thead>
<tr>
<th>Winding Type</th>
<th>250-75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase Resistance</td>
<td>0.26 Ohms in parallel</td>
</tr>
<tr>
<td>Maximum Bipolar Amps per Phase</td>
<td>7 Amps (two phase on) per phase.</td>
</tr>
<tr>
<td>Rotor Inertia</td>
<td>3.65 Kgcm²</td>
</tr>
<tr>
<td>Total weight</td>
<td>4.8 Kg</td>
</tr>
<tr>
<td>Frame/ Flange Size</td>
<td>NEMA size 42</td>
</tr>
<tr>
<td>Step Angle</td>
<td>1.8 Degree Standard (200 steps per rev)</td>
</tr>
<tr>
<td>Winding Insulation</td>
<td>Class F to IEC 85 and BS2757</td>
</tr>
<tr>
<td>Turns per phase</td>
<td>96</td>
</tr>
<tr>
<td>Maximum supply voltage</td>
<td>400V DC</td>
</tr>
<tr>
<td>Bearing Loading</td>
<td>Axial, 180 N max.</td>
</tr>
<tr>
<td></td>
<td>Radial 330N max.</td>
</tr>
</tbody>
</table>
6.6 Static Torque Expectations

In the standard design the stator tooth width is greater than the rotor tooth. Generally this aids two-phase static torque as, when the rotor sits between the two phase poles, there is still a large steel path present. In the simplified stepping motor of figure 6.16, a two phase holding position is shown. If the 'A' phases are attracting the rotor teeth, and the 'R' are repelling the teeth, it can be understood that the larger the tooth width the greater the holding torque. A larger tooth width will produce a greater stable equilibrium torque, and a smaller unstable torque when a repelling action occurs.
Chapter 6. New size 42 Frame Single Stack Hybrid Stepping Motor Design For Dynamic Performance

With a smaller tooth width this situation reverses. The unstable equilibrium torque will increase, but the stable holding torque will reduce; the unstable torque being more advantageous in dynamic torque production.

6.6.1 Experimental Low Speed Torque

Table 6.5 shows the torque output at 1 rps. This data is also shown in figure 6.17. This particular test was done with the drive (SX8) supplying a sinusoidal current waveform with a 5000 step resolution into the parallel windings of the motor. If the test had been undertaken in series the current would have to be halved, but the effects of inductance would have been negligible at this test speed.

<table>
<thead>
<tr>
<th>Current (RMS AMPS)</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Design</td>
<td>3.76</td>
<td>3.44</td>
<td>3.06</td>
<td>2.63</td>
<td>2.1</td>
<td>1.67</td>
</tr>
<tr>
<td>Dynamic Design</td>
<td>3.65</td>
<td>3.3</td>
<td>2.94</td>
<td>2.57</td>
<td>2.06</td>
<td>1.61</td>
</tr>
</tbody>
</table>

Table 6.5 - Low Speed (1 rps) Torque Differences between Motor Designs
As the standard design has more steel present in the rotor to stator path in the holding position, it fairs better at high excitation currents. The top current is actually higher than the motor's rated value (7 Amps RMS). The torque reduction from the new design ranges between a 3 - 3.6 %.

6.7 Dynamic Testing

Dynamic testing of both the motors was undertaken on several drives. The drive used for the majority of the tests is known as the Parker-Compumotor SX8 [3]. This drive offers a wide speed range due to its 120 V DC bus and offers substantial current control in terms of choice of magnitudes and resolution. Maximum current is 8 Amps RMS with a resolution of 25,000 steps per rev. Testing was undertaken on the test rig described in chapter 2. The majority of the tests were with the motor windings in parallel, and the drive resolution set to 5000 steps per revolution. This would deliver an approximate sinusoidal current waveform into the motor.
6.7.1 Over Current (8 Amps RMS) Test Results

The graph in figure 6.18 shows the dynamic performances at 8 Amps RMS. At low speeds the new motor is considerably quieter and smoother in motion. This is due to the lower content of torque harmonics and ripple, plus a more sinusoidal back EMF. Even though a high step resolution was chosen, the standard motor exhibits a mid speed resonance, which is far less apparent in the new design. It appears that the new tooth profile has better anti-resonance characteristics. A high speed resonance occurs in both the motors, although this is not as severe in the new motor. As the new motor has a lower inductance it can achieve a higher operating speed. In this instance the new motor has an increase in speed of over 600 rpm, whilst achieving a torque which is equal to the standard design’s mid speed torque. This increased speed leads to an improved shaft power output. The changes can be seen (in figure 6.19) to be quite dramatic with over 25% increase in maximum power output.

FIGURE 6.18 - S8 Drive, 5000 steps per rev, 8 Amps RMS.
Chapter 6. New size 42 Frame Single Stack Hybrid Stepping Motor Design For Dynamic Performance

![Power vs Speed Graph](image)

**FIGURE 6.19 - Shaft Power Out at 8 Amps and 5000 Steps per Revolution Current Profile**

### 6.7.2 Rated Current (7 Amps RMS) Test Results

At rated current the performance characteristics become more explicit. The graphs of figure 6.20 show that the mid speed resonance has virtually been eliminated by the new design.

![Torque vs Speed Graph](image)

**FIGURE 6.20 - S8 Drive, 5000 steps per rev, 7 Amps RMS.**

Furthermore the high speed resonance appears to be pushed to a higher speed on the torque speed curve by the new design. This indicates that this resonance is possibly drive...
related, as on some other drives it does not appear. Again there is an increase in maximum power out as shown in figure 6.21. Rotational smoothness and quieter operation are again apparent with the new design. Noise observations made during the test indicated that the new motor was far quieter over the speed range, especially at lower speeds. At speeds between 1 to 5 rps the motor was close to being inaudible, which is uncommon in stepping motors.

![Power vs Speed Graph](Image)

**FIGURE 6.21 - Shaft Power Out at 7 Amps and 5000 Steps per Revolution Current Profile**

### 6.7.3 Low Current (3 Amps RMS) Test Results

At 3 Amps RMS both motors produce a lower torque (figure 6.22). The same mid speed resonance occurs on the standard motor but not on the new design. Both motors suffer from the high speed drive resonance as they do not have significant current in the windings to overcome this. This in turn produces rough power out curves, figure 6.23. Hence neither of the motors would be suitable to run at low currents on this drive.
6.8 Change of Resolution Tests Results

Figure 6.24 shows the test results of the motors running under full stepping operation. The new design of motor exhibits an increased torque output when the motor speed is over 15 rps. This resonance period affects both designs, with the new design responding more favourably. The new motor shows a significant increase in speed range.
Hybrid stepping motors do not typically run using full stepping current profiles. This is because of the harsher running operations, lower step resolution, increased overshoot and mechanical ringing, makes the drive more prone to resonance.

Despite these difficulties the new motor demonstrates less susceptibility to the resonance problems associated with full stepping.

![Torque vs Speed graph](image)

**FIGURE 6.24** - Full Stepping (200 steps per rev), 8 Amps RMS.

Figure 6.25 shows the data recorded during half stepping. Most industrial applications make use of this stepping mode. There is a slight loss of torque production, but this mode typically results in much better low speed running, reduced overshoot, and less ringing at the end of each step. Performance in this mode therefore is a good industrial performance measurement. The mid speed resonance in the standard motor at 17.5 rps would cause some applications to stall. The torque dip of the standard motor would be from around 3 Nm to the measured value of around 1.5 Nm, a 50% loss in torque. This same resonance,
still present in the new design, results in a drop from 3 Nm to 2.5 Nm. Some resonance occurs in both motors at 30 rps. The power out curve is recorded in figure 6.26.

**FIGURE 6.25 - Torque Output in Half Stepping (400 steps / rev), 8 Amps RMS.**

**FIGURE 6.26 - Power out in Half Stepping (400 steps/ rev) 8 Amps RMS**
Chapter 6. New size 42 Frame Single Stack Hybrid Stepping Motor Design For Dynamic Performance

Microstepping attempts to produce smoother rotation, less torque ripple, resistance to resonance, and an increased step resolution. The new motor has been found to have an excellent performance with high resolution drives. Figure 6.27 shows both motors running with a drive resolution of 20,000 steps per rev. The new motor produces a smooth torque versus speed curve with few deviations. Comparing the two motors shows that the new motor performs extremely well in the troublesome mid speed and high speed regions. The power out curve of figure 6.28 shows major improvements in the available shaft power. The new design of motor presents a very favourable microstepping performance.

![Graph showing Torque (Nm) vs. Speed (RMS)](image)

**FIGURE 6.27 - Microstepping (20000 steps/rev), Torque Output**
6.9 Discussion

Stepping motors were originally designed to be low speed positioning type motors, operating in the stop/start region. This was because the largest segment of the market was office machinery where slow speed point to point moves were necessary. Recent stepping motor designs have concentrated on low speed high torque output. These have been penalised for higher speed operation by their high inductances.

A large sector of industrial machinery requires higher throughput, with increased torque for higher speed point to point moves. This market has increasingly been attacked by brushed DC servo motors. The stepping motor described in this chapter does not cost any more in material and yet offers an improved shaft power output, suitable for such jobs.
Chapter 6. New size 42 Frame Single Stack Hybrid Stepping Motor Design For Dynamic Performance

The motor offers a smoother rotation, quieter operation, lower mid speed instability, and greater high speed torque. When coupled with a microstepping drive the previous advantages are further enhanced, along with higher acceleration, flatter speed response, and greater shaft power output. The design is considerably easier to manufacture than existing micro stepping skewed rotor designs discussed in chapter 1. The new motor should find acceptance in machinery requiring increased throughput, quiet operation, and low vibration.

The bus voltage of the drive is often the key issue in high speed performance. By using this lower inductance design, lower voltage or lower current drives can be utilised to obtain similar performances to the existing design. This will dramatically reduce system prices, with little change in the motor. Alternatively copper losses can be reduced by turning down the number of turns in each phase, and increasing the turn gauge, to obtain a similar performance with the added benefits of lower resonance, and smoother and quieter movement.

This motor is planned to go into production with Stebon Ltd. and results have been verified by Parker Digiplan of Poole UK, who also found that the motor had significant improvements in settling time and overshoot.
CHAPTER 7

Application of Soft Magnetic Composite Materials in the Structure of the Hybrid Stepping Motor

7.1 Background

It has been reported that soft magnetic composite materials offer design advantages when compared with steel laminations due to their isotropic nature and the relative ease of producing complex shapes by the use of powder metallurgy methods [73-77]. Iron losses are low at elevated frequencies as the eddy current loss becomes a small proportion of the total output.

Because of the claims about the isotropic nature of powdered iron, this material may provide an improvement to the cross lamination flux patterns and iron losses at high rotational speeds in the hybrid stepping motor. This chapter is a study as to how such materials could be utilised in the hybrid stepping motor.

7.2 Laminations

Magnetic cores used for electric motors are usually made of strips of electric steel called laminations. These laminations are used to suppress the losses related to changing magnetisation. Typically the machines are produced with 0.35, to 0.5 mm laminations. The motors in this project have used 0.35 mm laminations. The material used is typically

A silicon iron. In motors with radial flux paths only, the higher the frequencies, the thinner the laminations that are used to reduce eddy current losses. Laminations are insulated one from another with coatings of non-conducting paint, on one or both sides. Alternatively, the lamination surface insulation can be a natural oxidation layer, an organic enamel, or an inorganic surface treatment. This breaks the continuity of the eddy-current-paths when the flux changes with time in a direction parallel to the surface plane of the lamination and reduces the losses. In general, the thinner the laminations, the lower the losses, since the eddy-current losses increase with the thickness of the lamination.

In laminated structures, eddy currents are minimised by constraining the flux to flow parallel to the surface plane of the lamination. Unfortunately in axial flux machines, such as the hybrid stepping motor, where flux is required to cross from one lamination to another, it has to travel through an inter-lamination boundary, figure 7.1. This boundary will act similar to a small air-gap, but with undefined and random dimensions. Therefore there will be an MMF drop at each inter lamination air-gap reducing flux and hence torque. Consequently if the areas of the motor that tend to experience cross lamination flux are replaced by an isotropic material, the MMF drop across the laminations could be reduced.

Electrical sheet steels have been the overwhelming choice for the soft iron structure in electrical machines, subject to time varying magnetic frequencies. Stebon Ltd. use TRANSIL a steel which was selected to reduce losses at higher frequencies. The grade of steel and lamination thickness is selected to reduce losses associated with its operating frequency and eddy currents. For applications (for example aerospace) where the...
fundamental frequencies of operation may be high, laminations of 0.1 mm thickness silicon and cobalt steels may be found. As stated the hybrid stepper motor may operate with frequencies of over 1 KHz, where the iron losses become a large order of magnitude. Reducing the thickness of the lamination is not a viable option because of the losses associated with cross lamination fluxes. This is where the use of soft magnetic composites becomes attractive.

**FIGURE 7.1 - Close-up Representation of Two Adjacent Laminations**

### 7.3 Soft Magnetic Composite Materials

Soft magnetic composite materials (SMC), also commonly known as powdered irons, consist of metal powder particles of pure iron. The use of pure iron instead of alloying elements offers the highest saturation induction outside cobalt/iron composites. Alloying elements are often introduced to minimise eddy current loss. In the case of soft magnetic materials, an insulating layer on the surface of the iron particle reduces the eddy current...
loss in all directions. To obtain a high flux density at low field strengths, high compressibility iron powder is used with a high compacting pressure, in an attempt to achieve this. Production techniques require the addition of lubricants and resin to enhance the strength of the material. These therefore degrade the permeability and make the material less suitable for low frequency applications.

7.4 Magnetic Properties of Soft Magnetic Composites

7.4.1 Iron Loss

If the material is used at low frequency it is only competitive with low grade lamination steels. However at frequencies higher than 50 Hz, the laminated steel’s performance is reduced, because of elevated frequency dependent losses. This makes the stepping motor an attractive option as it naturally operates at high electrical frequencies. However there is still a concern because of the relatively low unsaturated permeability. Figure 7.2 shows the B-H curve differences between a high grade steel and a high grade powdered iron. This curve is for a soft composite material that does not contain strengthening resin and machining lubricants.

7.4.2 Unsaturated Permeability

The unsaturated permeability is not as high as typical magnetic steels, and is approximately two thirds that of the magnetic steel. This can become a particular problem with small machines, which are magnetised from the armature, and where a large coil
current is found. This suggests that the material may only be suited for larger machines, where there is more flexibility for the choice of electric and magnetic loading.

**FIGURE 7.2 - Comparisons between the B-H curves of High Grade Powder Iron and Silicon Steels**

### 7.4.3 Isotropy

As the material behaves equally in all directions, it is known as isotropic. This is important both thermally and magnetically. The bulk thermal capacity of the composite material is similar to laminated steel, in all directions, whereas the laminated steel has a greater thermal capacity in the plane of the lamination. This allows the heat transfer in a powdered iron to be isotropic, aiding cooling in all directions in the core.

The isotropic magnetic characteristics are more important. The machine types which appear to justify powdered iron consideration are those with three dimensional magnetic flux structures and those with high operating frequencies. Losses associated with field variations normal to the plane of steel laminations (combined with the back EMF effect) limit the maximum speed of a motor. With powdered iron the improvement of torque per unit volume has good potential, particularly at elevated flux frequencies.
7.5 Improving Characteristics of the Hybrid Stepping Motor with SMC materials

The iron losses of the stepping motor could be reduced for the higher frequencies of excitation. However at low speeds this is offset by the lower permeability of the materials, reducing the flux. The hybrid stepping motor which has the unusual structure of an axially magnetised magnet, placed within its centre core, causes the flux to cross laminations in the z-direction. As the motor has significant radial, axial and tangential components of flux, the motor is a truly three dimensional machine. If the reduced losses at high frequency are sufficient to overcome the lower permeability, a better high speed stepping motor may be produced. This all depends on the economics, ease of construction, and robustness of the materials to be used.

7.6 Analysis of Hybrid Stepping motors Using Finite Element Software

By using the static three dimensional finite element software, an investigation into the positive uses of soft material composites may be established. However three dimensional finite element software produces static analysis of the hybrid stepping motor. Unfortunately, dynamic non-linear finite element analysis of rotary machines would be extremely computationally expensive for each possible motor design. However if improvements in the static performance could be made by altering the machine design with powdered iron, benefits obtained in dynamic performance would naturally transpire.

The following results are for a Stebon 1101 type motor.
7.6.1 Total Replacement of Steel by SMC Materials

The simplest investigation is to totally replace all the motor steel with SMC material. This involves switching from an anisotropic material with a packing factor to an isotropic type with the revised magnetisation characteristic. Table 7.1 shows the results of switching the steel to a SMC type P1171 manufactured by SMP [Appendices]. Referring to Table 7.1, it can be seen that at higher currents the torque is higher when using the laminated steels due to the increased permeability. At low current levels the SMC material produces better torque. The isotropic nature of the powdered iron is seen to be beneficial. With two phase excitation there is more flux flowing through the back iron, again the reduced permeability of the SMC is seen. This causes the difference in torque at low excitation to be of a lower margin. The motor is unlikely to run at low current levels and in single phase, and this makes SMC unsuitable to wholly replace the lamination steel. There may be some indication that changing either the rotor or stator by the soft magnetic composite, may produce improved results, however because the teeth may be operating with a flux density of around two Tesla, they would quickly saturate.

Some parts of the motor may be successfully replaced by SMC. The ideal locations are where the flux density does reach over 1.5 Tesla and has a high component of axial flux crossing the laminations normal to the surface plane. Places of low flux-density in the size 42 motor are the back iron ring and the centre region of the stator above the air-gap. The latter is because the flux does not fully fringe to the centre as described in figure 7.3. This effect particularly noticeable when the magnet length is a large percentage of the stator stack length.

<table>
<thead>
<tr>
<th>Amps</th>
<th>Silicon Steel Laminations</th>
<th>SMC</th>
<th>SINGLE PHASE TORQUE (Nm)</th>
<th>TWO PHASE TORQUE (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>1.494</td>
<td>2.723</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2.415</td>
<td>4.647</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>1.582</td>
<td>2.755</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2.133</td>
<td>4.028</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.1 -Laminated structure versus replacement by Powdered Iron for 1101 Motor

7.6.2 Partial Replacement of Steel by SMC Materials

Areas where the flux flows in an axial direction are found mainly in the rotor end-caps and in the centre of the stator core. Even though the flux density may be relatively low in this region, the MMF drop in these regions may be considerable as the flux has to travel across the laminations. Further research was undertaken to change the centre piece of the stator to SMC.

FIGURE 7.3 - Low area of Flux Density in the Stator due to rotor Fringing

The Design of Hybrid Stepping Motors aided by Three Dimensional Finite Element Analysis

7.6.3 Using a SMC Ring to Replace the Central Stator Laminations

If the central section of the motor was considered as a single section of the core pack, it may prove easy to replace this part with a ‘thick’ lamination of the new soft magnetic composite material. This would aid the manufacturing process of the complete motor. Unfortunately the material to be considered would have to machined from standard slugs, if any complex design was to be included. This would include teeth. In a complete manufacturing process the design would be formed by pressing the SMC material.

Several designs were investigated to see the practicality of using a central ring of the SMC material in the centre of the stator core. These were based around the original dimensions of the Stebon 1101 motor. The stator pole representation is based upon figure 7.4.

Design 1; Stator central section ring, stack length section of 10 mm, which is equal to the magnet length. Packing factor on remaining stator laminations being 0.98. The geometry of the ring being identical to the standard steel lamination in the X-Y plane.

Design 2; Stator central section ring, stack length middle section of 10 mm width. Remaining steel stator laminations having a packing factor 0.98. Saliency is added by removing the teeth from the bottom of the powdered iron section. The triangular section above is still included.

Design 3; Stator central section ring, triangular pole section removed to achieve a straight pole similar to switched reluctance lamination. Width of section again 10 mm. Packing factor 0.98 for remaining steel laminations.

Design 4; Stator central section ring. Pole section removed. Only back iron ring kept. Again with 10 mm width, and remaining stator lamination packing factor of 0.98.

Design 5; Stator central section ring; As design 1 but with stack of length middle section increased to 20 mm width. Packing factor 0.98.

Design 6; All stator back iron and teeth of central section replaced with SMC; Section length of SMC being 10 mm. Steel laminations remaining being of packing factor 0.98. No castellations. Back iron throughout the stator is constructed of soft magnetic composite materials. Design difficult to construct.

FIGURE 7.4 - New Pole Design for Powdered Iron Ring.  
*Note the position of the teeth arcs on the steel laminations.*

The results for static torque are presented in table 7.2. The standard reference motor having a stator of steel laminations with a packing factor of 0.98.

The results show that with the exception of Design 6 most of the designs perform well at low excitation in a single phase capacity. However as the single phase excitation level increases only design 1 and 2 offer real improvements. Design 1 is aided by the central teeth not being pushed to higher levels of flux density in the central region of the motor's stator. Design 2 forces the flux to travel in an X-Y direction up into the teeth by adding saliency to the region of low flux-density. At higher levels of flux-density this may penalise design 2. When the excitation system is switched to two phase, again the only designs of promise are 1 and 2. Here at higher excitation, they are on par with the completely laminated steel design.

<table>
<thead>
<tr>
<th>Design</th>
<th>Single Phase (3.5 Amps)</th>
<th>Single Phase (7 Amps)</th>
<th>Two Phase (3.5 Amps)</th>
<th>Two Phase (7 Amps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>1.494</td>
<td>2.415</td>
<td>2.723</td>
<td>4.647</td>
</tr>
<tr>
<td>Design 1</td>
<td>1.606</td>
<td>2.463</td>
<td>2.831</td>
<td>4.649</td>
</tr>
<tr>
<td>Design 2</td>
<td>1.613</td>
<td>2.535</td>
<td>2.867</td>
<td>4.652</td>
</tr>
<tr>
<td>Design 3</td>
<td>1.483</td>
<td>2.19</td>
<td>2.75</td>
<td>4.134</td>
</tr>
<tr>
<td>Design 4</td>
<td>1.433</td>
<td>2.028</td>
<td>2.434</td>
<td>3.772</td>
</tr>
<tr>
<td>Design 5</td>
<td>1.538</td>
<td>2.342</td>
<td>2.618</td>
<td>4.39</td>
</tr>
<tr>
<td>Design 6</td>
<td>1.18</td>
<td>1.690</td>
<td>1.759</td>
<td>2.193</td>
</tr>
</tbody>
</table>

*Table 7.2 - Torque Results for Powdered Iron Sections Investigation*

Having established that designs 1 and 2 offer the best static performance in relation to the standard design, the option of which type to further research has to be made. The two dominant factors are the strength of the structure, and the flux density limits of the material.

As the excitation rises, the fringing flux from the rotor end-caps travelling into the centre of the stator increases. This will eventually degrade the performance of design 2, below that of design 1. However within the normal operating range of the motor (7 Amps), a difference in performance would be minimal.

The soft magnetic materials have a notorious history of being extremely brittle. Though the addition of resins and lubricants aid machine processes they also degrade magnetic performance. With the powdered iron positioned between two packs of steel lamination they would act as a form of mechanical shield. However there is doubt about the ability of a set of SMC teeth on the stator to withstand honing. Therefore because of this, and the simplified structure of design 2, it seemed more suitable to develop this. A three dimensional model of design 2 is shown in figure 7.5.

FIGURE 7.5 - Design 2 Ring Position in Stator

The Design of Hybrid Stepping Motors aided by Three Dimensional Finite Element Analysis
7.6.4 The Effect of Changing the Width of the SMC Ring

Having established the profile of the design to proceed with, the effect of varying the SMC length within a fixed length stator was examined. Presently this length is set at 10 mm chosen because it matched the length of the rotor magnet. The profile of design 2 was therefore evaluated at various thicknesses. The curves of figures 7.6 to 7.9 are drawn from polynomial fits to data collected by varying the lengths of the SMC to find the static torque output. In single phase low excitation, it appears that the length of the section could be increased, as the flux can be pushed into the steel teeth, without saturating. As the excitation increases there is a sudden drop of torque as the fringing flux from the rotor end-caps becomes much more abundant and saturates the iron. This is further demonstrated in the two phase excitation scheme as the length at which the SMC material produces increased torque reduces as excitation increases. It therefore appears that for the normal operating current range of the motor a slightly smaller length than the magnet length should be considered.

![Figure 7.6 - Torque at 3.5 Amps Single phase Relative to Increasing Length of Powdered Iron Sections](image-url)

**FIGURE 7.7** - Torque at 7 Amps Single phase Relative to Increasing Length of Powdered Iron Sections

**FIGURE 7.8** - Torque at 3.5 Amps Two phase Relative to Increasing Length of Powdered Iron Sections

**FIGURE 7.9** - Torque at 7 Amps Two phase Relative to Increasing Length of Powdered Iron Sections

7.6.5 Dynamic Advantages

The study presented has been on the static performance of the motor. The aim was to find the geometry of the SMC material which optimised the static performance despite the lower permeability of the SMC. The other key elements described previously were concerned with dynamic properties, such as improved thermal conductance and lower losses at higher frequencies. The finite element software used in the project unfortunately cannot model these dynamic effects, and thus the effects of reducing or increasing the section length cannot be found without experimental methods or new software. However it is anticipated that any improvements demonstrated in static conditions will be further enhanced under dynamic conditions.

7.7 Building a Stepping Motor with a Soft Magnetic Composite Section

There are only a few manufacturers of soft magnetic composite materials in the world. In Europe the two main suppliers are SMP of Germany, and Höganäs AB of Sweden [Appendices]. SMP material data was used in the previous finite element analysis. Neither company could supply a slug the size of the outer diameter of the size 42 frame. Höganäs AB indicated that within the near future this slug would be available, when they moved to a new factory site. Höganäs AB could supply a soft magnetic material composite slug that could be machined for a size 34 motor. The decision was made to build a size 34 frame motor based on the same design as design 2, in an attempt to understand some of the dynamic effects of the new material. The details of this material are found in the appendices.

The next stage was to obtain the slug from the Swedish plant. After obtaining the slug, it would need to be wire eroded to the appropriate shape. A company was found and the material was sent with the relevant drawing. The material was returned and unfortunately had not been packed well. The result was that the sections were broken, due to the brittle nature of the material. After trying to repair the sections it was decided that the process would have to begin again. Fortunately Höganäs AB were willing to supply the slug again and it was wire eroded again. However a heavy time penalty was incurred.

7.7.1 Revising the Design for a Size 34 Frame Motor

The plan was to build four 34-frame single stack motors. Again the lengths of the SMC sections were to be based around the magnet length. The basic dimensions were as follows: The rotor end-caps are of length 12 mm. The magnet being nominally 5.7 mm. With a stator core pack length of 30 mm. The lengths for the SMC material to be evaluated are 4, 5.7 and 10.5 mm. These represent a loss of 13, 19, and 35% of the effective tooth area from the stator core pack.

The motors were to be constructed to the same style as the larger 42 frame, design 2. However in the smaller motor the lower permeability may have a more significant effect, since there was a similar coil MMF in both the larger and smaller motors. Also because the SMC had a lubricant added for machining, the magnetic properties were further reduced.

At the same time as the smaller size 34 motors were being built, a new finite element model was created. This involved creating an 850 model from the beginning and solving for the useful average two phase torque prediction.

7.8 Experimental Results

The following results have been taken using a CD60 [3] drive operating under half stepping with a 85V DC bus voltage. The motors have been wound in parallel to allow testing at higher rotor speeds, where the benefits of the powdered iron should be more noticeable.

7.8.1 Normal Operating Current Results

The motors are rated at 4.5 Amps RMS two phase on. The drive was set to be just above this rating at 4.9 Amps. The results are shown in figure 7.10. All the motors produced lower torque than the standard laminated design at low speeds. The machine with the smallest length of SMC (4 mm), matches the torque output of the standard machine at around 12 rps. It then follows a slightly higher profile for the rest of the range. The motor with the middle length of material (5.7 mm), has a much flatter low speed curve, albeit with a decreased power out. At half the speed the motor produces power out which is equal to the standard motor and this continues to climb for the rest of the speed range. The motor with the largest section of powdered iron, produces far less torque, this is inherently due to loss of holding torque from the removal of teeth. From the power curve of figure 7.13 it does exhibit a very linear response. However this machine does exhibit the greatest power out at high speeds. The peak power out of this motor appears to be above 50 rps.
7.8.2 High Current Results

In the last test all the motors produced less torque at low speeds than the normal motor. The same result occurs for a higher test current test of 6 Amp RMS (figure 7.12). The Hysteresis loss for the SMC lowers the efficiency of the motors at higher currents and lower speeds. As the speed increases and current is reduced due to the inductance of the winding and the reduced permeability of the SMC material becomes less dominant.
motor with the shortest length of SMC component motor provides a similar performance to the laminated motor at relatively low speeds. It eventually produces slightly more torque at medium to high speeds, and then its torque falls off at a similar gradient to that of the fully laminated structure.

![Torque Speed for 4.9 Amps Excitation](image)

**FIGURE 7.12 - Torque Speed for 4.9 Amps Excitation**

The motors with the medium and longest lengths of SMC motors produce less torque at low to mid speeds. The motor with the longest length of SMC achieves equal torque at the maximum upper speed. The middle length of SMC material only gives equal torque to the standard design around mid speed. Referring to the shaft power graph of figure 7.13, the motor with the middle sized section gives an increased shaft power over the standard design. The largest section of SMC produced a flat power out curve, and at very high speeds may produce an enhanced output power.
7.8.3 Low Current results

In the previous tests, concern has been expressed about the high magnetising field produced by the high current, hence producing large hysterises losses that reduce low speed torque. The motors were now tested with a current of 3.8 Amps RMS. The low current experimental data (figure 7.14) is more favourable to the material for hysterises losses. The two shorter SMC component machines offer a similar low speed performance. The increased flux density of the smaller 34 frame motor is sufficient to offset the gain produced by the isotropic nature and the added saliency. They both produced a better mid speed torque output, which interprets as a greater shaft power output than the standard motor. The motor with the middle sized SMC section obtains a better high speed output due to the reduced eddy current losses. This has to be contrasted with the slightly lower holding torque output. The larger SMC sections offer a slightly lower performance, and do not appear to offer any advantage over the smaller sections. The resulting power out curves would be very similar.
7.9 Finite Element Study on 34-Frame for Comparison

The following section details observations made by a brief finite element study for the comparison of SMC component motors to the standard 34-frame type motor. Figure 7.15 shows a finite element representation of the 34-frame motor. The analysis used a SMC length of 5.7 mm, with the motor operating in two phase. The results are summarised in table 7.3.

<table>
<thead>
<tr>
<th>Current</th>
<th>FE (Standard Model) Average Torque</th>
<th>FE (5.7 SMC Middle core) Average Torque</th>
<th>Experimental Standard Motor Maximum Low Speed Torque (Nm)</th>
<th>Experimental 5.7 mm Middle SMC Core Maximum Low Speed Torque.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.8 Amps RMS.)</td>
<td>1.00</td>
<td>1.13</td>
<td>0.99</td>
<td>0.97</td>
</tr>
<tr>
<td>(4.9 Amps RMS.)</td>
<td>1.29</td>
<td>1.31</td>
<td>1.35</td>
<td>1.16</td>
</tr>
<tr>
<td>(6.0 Amps RMS.)</td>
<td>1.56</td>
<td>1.44</td>
<td>1.52</td>
<td>1.32</td>
</tr>
</tbody>
</table>

**TABLE 7.3 - FE to Experimental Data Results**
FIGURE 7.15 - FE Representation of the 34-Frame Motor

The results show that the finite element prediction for the average torque of the standard motor is relatively good. However the FEA results for the SMC ring are overoptimistic. For the middle excitation, the 4.9 Amps FE predicts a higher performance than the standard motor, but this was not found in experimental data. At higher currents the trend is correct but of the wrong magnitude. One problem lies in the difference between the supplied slug of soft magnetic composite material and that modelled. The supplied form of material was envisaged to be machined and hence had some lubricant and resin added to protect it from this process. This would degrade its magnetic performance. The SMC modelled by the finite element software was considered to be a press moulding type, retaining its magnetic properties. If the motor was to be manufactured the part would be pressed.

The flow of flux through the back iron can be seen in figures 7.16 and 7.17. Here it can be clearly seen that the flux prefers to flow in the z-direction in the SMC machine, due to the isotropic nature of the material.
7.10 Applications of Soft Magnetic Composite in Hybrid Stepping Motors

The key features of using these materials in the hybrid stepper motor which have been highlighted by this chapter are:

- Improved isotropic nature with press mouldings, giving the possibility of improvements in static and low speed torque.
- Smoother torque/speed performance curves, giving more uniform shaft power.
- Greater power out at elevated speeds due to the lower eddy current losses.
- Lower performance by machining parts rather than pressing.
- Reduced performance at higher saturation levels due to hysteresis losses.

If the motor has a reduction gearing mounted to its shaft, the motor will not typically run at low speed. The normal operating range would be at middle speed. The graphs of torque...
and power versus speed, indicate improvements in this particular region, making the motors with SMC suitable for adoption.

In pusher systems, such as wood sawing, the shaft power required is generally at higher speeds, where the motor is highly loaded. The lower speed range is used for accelerating a smaller load (such as the wood), and the motor requires the power when pushing the wood into the cutter at higher, more constant speeds.

The very flat (but much lower torque) torque/speed response of the motor with the large section of SMC material could be utilised where a fast or simple control method of speed and acceleration was needed, such as 'on-the-fly' applications. If a packaging machine had different length products, the acceleration required to process the products would be different for each length. With a flat torque speed capability, the complete acceleration profile would only be set with one value that would match the linear power out curve of the motor. This would offer a simple control system and one that would not have to recalculate the acceleration for each product. The lower torque could possibly be used to protect the products from damage during a jam. The motor would stall before excessive force could be delivered.

A standard motor could have its performance rewound for a higher speed by reducing the number of phase turns and hence lowering the inductance. If the finite element predictions for a pressed section of soft magnetic material are accepted, then the SMC motor design would offer an improvement in performance over the complete torque range. However, the gradient of shaft power to speed in the results shows a trend towards a flat power out
curve (figure 7.15). Once maximum power is reached, the torque falls off at a rate of $1/\omega$, whereas a standard rewound motor would have high speed torque fall off at a greater rate.

It can be concluded, therefore, that there are some distinct possibilities and advantages for the use of soft magnetic composite materials in hybrid stepping motors. The experimental results have shown that although, the designs have a lower low speed performance there is significant high speed improvement, due to reduced losses. The use of pressed parts would offer greater performance than machined parts.
CHAPTER 8

CONCLUSION

8.1 Conclusion

A stepping motor is a synchronous motor which is used in low cost positioning systems.
Within their dynamic error limits, accurate path following and precise point to point
can be achieved. Their inherent holding torque and stiffness make them ideal for
point to point positioning and for point to point motion applications. Stepping motors have
been traditionally used in the office machinery market, and designs are still based upon
types suitable for slow speeds, operating within in the stop start region.

The construction procedures of hybrid stepping motors can dictate the possible
improvements of the motor due to machining processes and geometry tolerances such as
air gap radius. This thesis includes a comprehensive study of the manufacturing process
for these motors and the quality test procedures to ensure consistent motor performance.
This study allowed the author to understand some of the ‘real world’ constraints on
designing these motors. The verification of motor performance can be achieved by a series
of tests that have been described that allow a designer to understand some of the
fundamental performance elements in the motor.

Traditionally the hybrid stepping motor has been designed and analysed using a
combination of experimental and empirical two dimensionalised studies. Two dimensional
finite element software has been available for some time that allows a designer to study
the air gap tooth regions of the motor. However there has been little study into complete three dimensional modelling of the hybrid stepping motor, due to the complexity of the motor and the limited power of early 3-D FEA packages. The author has presented to his belief the first comprehensive study of the motor using three dimensional finite element analysis. This has allowed the author to describe key fundamental properties with a new insight.

From the study of the motor in 3-D, properties of the electromagnetic structure of the motor, in particular the back iron and teeth (which in the past have been widely disregarded) have been noted. These have been incorporated in to a new lumped element model that runs on a personal computer.

The mathematical models include derived equations for fitting the characteristics of highly permeable modern electromagnetic steels, and permanent magnets. These have been shown to perform better than the established Jiles-Atherton equations.

A key feature of the mathematical model is a technique of analytical solutions to the paths of flux within the motor using ellipses. The ellipse has been shown to be a more accurate representation of flux lines in certain parts of doubly salient machines, compared to circular paths assumed by previous authors.

Recent designs of stepping motors have looked to maximise low speed torque, without consideration of dynamic effects such as inductance and hence high speed performance has not been necessarily been improved. Industrial machinery demands faster throughput,
requiring increased torque at higher speeds, maximising shaft power. In a market sector coming increasingly under attack from brushless DC servo systems, stepping motor designs must retain their low cost, and improve their power output.

The software and modelling technique developed in this project has allowed development of two quite different conceptual designs of high shaft power motors. The first described in chapter 6 involved a redesign of the stator tooth profile to produce smoother operation, with low vibration and acoustic noise across a wide range of speeds. Whilst there was a small drop in holding torque there was a significant increase in high speed power output. When coupled with a microstepping drive a further improvement in performance has been demonstrated. Such designs should find acceptance in machines requiring increased throughput, quiet operation and low vibration.

The second new design described in chapter 7 used soft magnetic composite materials in place of some laminations in the stator. Two motors were analysed by finite element analysis (a size 42 and 34 frame size), and predicted that a pressed moulding of the new lamination would offer increased static torque, due to the composite isotropic effect in the motor back iron. Due to the material available in terms of diameter size the smaller 34 frame motor was built. This demonstrated lower eddy current losses and improved shaft power at medium to high speeds.

For high speed performance voltage is generally the most important factor. High speed performance can be attained by using low inductance windings, requiring high current for
low speed torque. The new designs should allow higher speeds with lower voltages, and lower currents to be utilised. These high speed improvements are not voltage related.

8.2 Author’s Contribution to Knowledge

A large amount of work has been carried out by the author on the study of stepper motors using two dimensional and three dimensional finite element methods. This thesis offers the first comprehensive study of the hybrid stepping motor with three dimensional electromagnetic analysis. The effect of the laminations in parts of the motor has been demonstrated to be a hindrance in the torque production of the motor. The author has presented some possible ways of using soft magnetic composite materials to reduce the loss of energy due to eddy currents associated with axial flux paths. Design work on tooth profiling has been demonstrated that can produce greater shaft power and lower mid-speed resonance in the motor. Together these advantages will allow stepping motors to be manufactured for applications requiring higher speed positioning with low acoustic noise.

Completely new equations for modelling the new generation of highly permeable magnetic steels have been detailed. These new equations have also been used for modelling numerous characteristics of the motor. Elliptical path mapping of flux lines has been demonstrated to give a more accurate description of flux contours.

8.3 Future Work

It has been demonstrated in this project that two dimensional finite element methods are extremely useful for modelling the tooth / air gap region of the motor. With the recent development of two dimensional design environments which through parameterisation of
Chapter 8. Conclusion

the geometry allow the rapid creation of FE meshes, it would be worthwhile to incorporate such a link into the HyStep program.

As the project neared its completion, soft magnetic materials of the size 42 frame diameter became available. It would be particularly worthwhile to manufacture a size 42 frame motor with a SMC ring, since the simulations were very encouraging and should produce a better improvement in performance than was achieved with size 34 frame motors. Design of a rotor incorporating powdered iron would also be attractive and should be investigated to find how SMC behave in physically rotating axial component.

This project has shown two very different ways in which stepping motors can be improved, each giving at least 10% more high speed power output. Further work should be done to determine what improvements can be gained by the design of a motor which incorporates both the revised tooth profile and appropriately placed powdered iron sections. The potential for a very significant increase in high speed power exists.
REFERENCES


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45. S.R. Huard, 'A Finite Element Based Lumped Parameter Model for 1.8° Stepping Motors', Incremental Motion, Control, System, and Device Conference 1990, Champaign, Illinois, Pg. 62-74.


APPENDIX A

Motor Specifications

Details of Stebon SDT-1101-250-70 Motor Referred to in this thesis.

-Hybrid Design
-1.8° Step Angle (200 Steps for Revolution)
-Two phase Bipolar Winding
-Class F insulation to IEC 85 and BS2757
-Nema 42 Frame and Flange Size
-Bearing Loading, axial 180N maximum
-Bearing Loading, radial 330N maximum
-Maximum Detent Torque 0.12Nm
-Parallel Resistance per Phase 0.26 ohms
-Parallel Inductance per Phase 2.8 mH.
-Bi-polar Current per Phase 7 A
- Rotor Inertia 3.65 Kgm^2
-Motor Weight 4.8 Kg
-IP44 Rating

Details of Stebon SDL-851-250-45 Motor Referred to in this thesis.

-Hybrid Design
-1.8° Step Angle (200 Steps for Revolution)
- Two phase Bipolar Winding
- Class F insulation to IEC 85 and BS2757
- Nema 34 Frame and Flange Size
- Bearing Loading, axial 160N maximum
- Bearing Loading, radial 250N maximum
- Maximum Detent Torque 0.05 Nm
- Parallel Resistance per Phase 0.24 ohms
- Parallel Inductance per Phase 1.5 mH.
- Bi-polar Current per Phase 4.5 A
- Rotor Inertia 0.6 Kg\(\text{m}^2\)
- Motor Weight 2.2 Kg
- IP22 Rating.
Appendix B

Three Dimensional Algorithm (Opera-3-D / Tosca) [15, 16]

Stationary magnetic fields consist of both solenoidal and rotational components. The field produced by electric currents has a rotational component inside the volumes where current flows. In the exterior space the field is solenoidal but the scalar potential is multi-valued. The field produced by magnetised volumes is solenoidal. It is convenient to separate the total field into two parts in order to obtain a description of the field in terms of the scalar potential. The total field intensity \( \mathbf{H} \) may be expressed as the sum of the source field intensity \( \mathbf{H}_s \) and the reduced field intensity \( \mathbf{H}_m \).

\[
\mathbf{H} = \mathbf{H}_s + \mathbf{H}_m \quad (A.1)
\]

The source field can be obtained directly from Biot-Savart law by integration over the region \( \Omega_j \) containing the current

\[
\mathbf{H}_s = \int_{\Omega_j} \frac{\mathbf{J} \times \mathbf{R}}{|\mathbf{R}|^3} \, d\Omega. \quad (A.2)
\]

The field satisfies

\[
\nabla \times \mathbf{H}_s = \mathbf{J}, \quad (A.3)
\]

so that
\[ \nabla \times H_M = 0. \quad (A.3) \]

The reduced field intensity can now be represented using the reduced scalar potential, \( \phi \),

\[ H_M = -\nabla \phi. \quad (A.4) \]

The divergence of the flux density \( B \) is always zero. Introducing the permeability tensor, \( \mu \), and combining the expressions for the source and reduced field intensity, gives the partial differential equation for the reduced scalar potential.

\[ \nabla \cdot \mu \nabla \phi - \nabla \cdot \mu \left( \int_{\Omega_s} \frac{J \times R}{|R|^3} \, d\Omega_s \right) = 0 \quad (A.5). \]

This equation can be solved using the finite element method. However, in magnetic materials the two parts of the field \( H_M \) and \( H_s \) tend to be of similar magnitude but opposite direction. Therefore, cancellation occurs in computing the field intensity \( H \), that results in a loss of accuracy. The errors can be completely avoided by combining the total and reduced scalar potential representations. Hence, exterior to the volumes where currents flow the total field can be represented using the total scalar potential \( \Psi \).

\[ H = -\nabla \Psi, \quad (A.6) \]

where the total scalar potential satisfies
\[ \nabla \cdot \mu \nabla \Psi = 0. \quad (A.7) \]

The minimal combination consists of using the reduced scalar potential only inside volumes where current flows and the total potential everywhere else.
APPENDIX C

The following pages are the .CPP files from the HyStep modelling program. The .H header files are not included due to space restrictions.
//***BH dialog window function ***//

DEFINE_RESPONSE_TABLE(BHDialog, TDialog)
  EV_COMMAND(104, ResetButton),
  EV_COMMAND(107, DefaultButton),
  EV_COMMAND(108, Clarification),
END_RESPONSE_TABLE;

//*** dialog defaults ***
TBhDialog::TBhDialog(TWindow* parent, const char* name)
  : TDialog(parent, name),
  TWindow(parent)
{
}

//load BH data default curve
DEFe=cc=27;

DEFe[0] = 0;
DEFe[1] = 64;
DEFe[2] = 92;
DEFe[3] = 101;
DEFe[4] = 11;
DEFe[5] = 2;
DEFe[6] = 3;
DEFe[7] = 4;
DEFe[8] = 45;
DEFe[9] = 5;
DEFe[10] = 55;
DEFe[12] = 6;
for(int bh=13; bh<27; bh++)
  DEFe[bh] = DEFe[bh-1] + 0.55;

DEFe[0] = 0;
DEFe[1] = OSAm;
DEFe[2] = OSAm + 0.5;
DEFe[3] = OSAm * 2;
DEFe[4] = OSAm * 4;
DEFe[5] = OSAm * 8;
DEFe[6] = OSAm * 16;
DEFe[7] = OSAm * 32;
DEFe[8] = OSAm * 64;
DEFe[9] = OSAm * 128;
DEFe[10] = OSAm * 256;
DEFe[11] = OSAm * 512;
DEFe[12] = OSAm * 1024;
DEFe[13] = OSAm * 2048;
DEFe[14] = OSAm * 4096;
DEFe[15] = OSAm * 8192;
DEFe[16] = OSAm * 16384;
DEFe[17] = OSAm * 32768;
DEFe[18] = OSAm * 65536;
for (int bh=17; bh<27; bh++)
  DEFe[bh] = DEFe[bh-1] + (OSAm * 0.05);

//*** magnet BH data default **
DEFeMagpoint=17;

DEFeMag[0] = 0;
DEFeMag[1] = 155;
DEFeMag[2] = 27;
DEFeMag[3] = 35;
DEFeMag[4] = 43;
DEFeMag[5] = 5;
DEFeMag[6] = 57;
DEFeMag[7] = 63;
DEFeMag[8] = 66;
DEFeMag[9] = 7;
DEFeMag[10] = 74;
DEFeMag[11] = 775;
DEFeMag[12] = 8;
DEFeMag[13] = 86;
DEFeMag[14] = 87;
DEFeMag[15] = 88;
DEFeMag[16] = 9;

DEFeMag[bh] = 1600 * OSAm;
for (int bh=17; bh<27; bh++)
  DEFeMag[bh] = DEFeMag[bh-1] + (100 * OSAm);

} // end of default data ***

//***FUNCTIONS CONTAINED WITHIN DIALOG WINDOWS***/
// DEFAULT BUTTON:// void
TBDialog::DefaultButton() //LOADS DEFAULT CURVE INTO CURRENT CURVE SPACE
{
### HEADERS
```cpp
#include <os\geometry.h>
#include <wdude\os\gdiobjec.h>
#include "oom.h"
#include "bh.h"
#include "class.h"
#include "ecast.h"
#include "wchild.h"
#include "drawgeo.h"
#include "wmdmam.h"
```
DECLARE CHILD WINDOW FOR GRAPH

DEFINE RESPONSE TABLE
TGeoFrame, TGeoFrame(TWindow*, parent, const char* title)
TFramewndow(parent, title)

{ X=25,
Y=25,
W=450,
H=450,
Style=WS_VISIBLE | WS_CAPTION | WS_BORDER | WS_SYSMENU | WS_MINIMIZEBOX
WS_CLIPCHILDREN | WS_THICKFRAME,
}

FUNCTIONS FOR TDbWindow (main window) class *****
void TDbWindow::EvPamtO()

TPaintDC paintDC(HWND, pParent)

if (MbhFlag == 1 & MbhFlag == 1) // Bh CHK = 1
else if (MbhFlag == 2 & MbhFlag == 2) // Bh CHK = 2,
else (BhCHK = 0)

char blank = " ";
/** chareters to tell user what curves have been selected */
char cur[] = "CURVE";
// with what method
char pt[] = "POINTS";
char methodbh[10];
char methodmag[10];

char title[] = "BH curve in current use:");
paintDC.TextOut(0, 10, title, strlen(title));
char holdbh[25];
strcpy(holdbh, MbhName);
strcat(holdbh, methodbh);
paintDC.TextOut(175, 10, holdbh, strlen(holdbh));

char mfile[] = "Magnet in current use:");
paintDC.TextOut(0, 25, mfile, strlen(mfile));
char holdmag[25];
strcpy(holdmag, MbhName);
strcat(holdmag, methodmag);
paintDC.TextOut(175, 25, holdmag, strlen(holdmag));

if (BhCHK == 0){
drk chk MP[] = "BH data missing, or of differing techniques!",(chkMP);)
paintDC.TextOut(1, 46, chkMP, strlen(chkMP));
else{char ch1: TMP[] = l'Dimensions Entered';
paintDC.TextOut(1, 46, chkMP, strlen(chkMP));
}

char MXfd[] = "Maximun Flux Density is set at:");
strcat(MXfd, maxFD. s);
paintDC.TextOut(1, 85, MXfd, strlen(MXfd));

//** main window function *****

void TDbWindow::CmbBHsaveO

void TDbWindow::CmbBHsaveO

TOpenSaveDialog:IData data(OFN_HIDEREADONLY|OFN_OVERWRITEPROMPT,
"BH Files ('. BH)I", BHI,
10, "BH"
//--if file can be saved holds filename in data filename
if (TFileDialog->Execu, e_TOOK)
else

printf("BH could not be saved
");


```cpp
ofstream outs(FileName);
if (!out) MessageBox("Cannot Open file", "Error", MB_OK);
else{
    outs << Mclassbh; // because class is streamed
}
}

/// load BH **
void TDBWindow::CmBHloadO
{
    static TOpenFileDialog TData data; // displays only BH files
    OFN_HIDEREADONLY, // "BH Files (*.BH)" BH
    "BH"
    if ((new TFileOpenDialog(this, data))->Execute() == IDOK) {
        ifstream m(data.FileName);
        if (!m) MessageBox("Cannot Open file", "Error", MB_OK);
    }
}

/// FUNCTIONS FOR BH FILES ENDED

/// BH STEEL MANIPULATION
void TDBWindow::CmBH
{
    TBhDialog* the_pad=new TBhDialog(this, TResId(1)); // calls the Dialog window (ID 1)
    the_pad->steel_magma; // steel data is bang dealt with
    the_pad->SdTransferBuffa(&tb);
    if (the_pad->Execute() == IDOK) {
        strcpy(Mclassbh.MbhName, tbname);
        for (int j=0; j<100; j++) {
            Mclassbh.MbhPomt=the_pad->bh[j].bh;
            strcpy(Mclassbh.MbhName, the_pad->name);
            Mclassbh.MbhPomt=the_pad->point;
            Mclassbh.MbhFlag=1;
        }
    }

/// function to deal with steel equation data **
void TDBWindow::CmBHcurveO
{
    // creates a transfer buffer to put data directly into windows
    struct TransferBuffer{
        char a[51], b[15], n[15], e[15], k[15], name[20];
        strcpy(a, Mclassbh.MbhCurve[0].s);
        strcpy(b, Mclassbh.MbhCurve[1].s);
        strcpy(n, Mclassbh.MbhCurve[2].s);
        strcpy(e, Mclassbh.MbhCurve[3].s);
        strcpy(k, Mclassbh.MbhCurve[4].s);
        strcpy(name, Mclassbh.MbhName);
    } tb;
    TDBCurDialog* the_pad=new TDBCurDialog(this, TResId(4)); // creates new dialog window
    the_pad->steel_magma; // steel data is bang dealt with
    if (the_pad->Execute() == IDOK) {
        for (int j=0; j<100; j++) {
            strcpy(Mclassbh.MbhPomt=the_pad->bh[j].bh;
            strcpy(Mclassbh.MbhName, the_pad->name);
            strcpy(Mclassbh.MbhPomt=the_pad->point;
            Mclassbh.MbhFlag=1;
        }
    }
    // return values from dialog to main window data
    for (int j=0; j<100; j++) {
        Mclassbh.MbhCurve[0][j]=tof(Mclassbh.MbhCurve[0][j]);
        Mclassbh.MbhCurve[1][j]=tof(Mclassbh.MbhCurve[1][j]);
        Mclassbh.MbhCurve[4][j]=tof(Mclassbh.MbhCurve[4][j]);
    }
    Mclassbh.MbhFlag=0; // flag set to equation
    return;
}
```
```c
void TDWindow::CmMAGauve
{
TBhDialog* the_pad=new TBhDialog(this, TResld(1));
for (int loop=0; loop<100; loop++)

the_pad->window_map1;
strcpy(the_pad->name, Mlassmag MbhName);
the_pad->openMlassmag MbhFrem;
if (the_pad->ExecuteIDOK)

for(int j=0; j<100; j++)

Mlassmag Mbh[j] b, the_pad->bh[j] b
strcpy(Mlassmag MbhName, the_pad->name);
Mlassmag MbhFlag=1;
}
}

void TDWindow CmMaGauve()
{

struct TransferBuffer{
char a[15],b[15],c[15],d[15],e[15],f[15],g[15],h[15],i[15],j[15],k[15],l[15],
}
tb;

strcpy(tb. a, Mdassmag MbhCurve[0]. s);
strcpy(tb.bMdassmag MbhCurve[1]. s);
strcpy(tb. cMDassmag MbhCurve[2]. s);
strcpy(tb. dMlassmag MbhCurve[3]. s);
strcpy(tb. eMdassmag MbhCurve[4]. s);
strcpy(tb. name, Mdassmag MbhName);

TBhCurDiOG* the_pad=new TBhCurDiOG(this, TResld(4));
the_pad->window_map1;
the_pad->SetTransferBuffer(ab);
if (the_pad->ExecuteIDOK)

MDassmag MbhCurve[0]. n=atof(Mdassmag MbhCurve[0]. s);
MDassmag MbhCurve[1]. n=atof(Mdassmag MbhCurve[1]. s);
MDassmag MbhCurve[2]. n=atof(Mdassmag MbhCurve[2]. s);
MDassmag MbhCurve[3]. n=atof(Mdassmag MbhCurve[3]. s);
MDassmag MbhCurve[4]. n=atof(Mdassmag MbhCurve[4]. s);
Mlassmag MbhFlag=2;
}

//get offset
struct TransferBuffer{
char a[15],b[15],
}
tb;

strcpy(ab a. MdassGEO. offset); x);
MDassGEO. offset n=atof(MdassGEO. offset); x);
}
}

void TDWindow CmDim()
{
int loop;
do ( loop=0;

struct TransferBuffer{
char a[10],b[10],c[10],d[10],e[10],f[10],g[10],h[10],
}
j0;

strcpy(ab a. MdassGEO. ndIM[0] x); strcpy(tb.b. MdassGEO. ndIM[1] x);
```

/* *** Function to deal with flux density input window *** */
void TDWindow::CmDen()
{
    int loop;
    do ( loop=0;
        struct TransferBuffer( //set up transfer buffer
def FD[101];
        );
        //put function which copies
        strcpy(fd, MdassGEO.maxFD);
        TDDialog* dm_pad=new TDDialog(this, TRemD(2)); //ready to call dialog
        dm_pad->SetTransferBuffer(&fd);
        if (dm_pad->Execute()==IDOK) //if OK pressed
            MdassGEO.transfer(fd, h, b, n, d, m, f);
            //check to see if any dimensions missing or zero value
            int checka=MdassGEO.zechoFD(fd, &check); //call function to check
            if (checka<0) //if dimensions missing
                MessageBox("Flux Density value missing or of zero value!");
            loop=1;
    ) while (loop==0);
}

/* *** Function to deal with packing factor input window *** */
void TDWindow::CmPfO()
{
    int loop;
    do ( loop=0;
        struct TransferBuffer( //set up transfer buffer
        pF[101];
        );
        //put function which copies
        strcpy(pF, MdassGEO.PF);
        TDDialog* dm_pad=new TDDialog(this, TRemD(3));
        dm_pad->SetTransferBuffer(&pF);
        if (dm_pad->Execute()==IDOK) //when ok pressed
            MdassGEO.PF=eval(MdassGEO.PF, pF);
            //check to see if flux density missing or zero value
            int checka=MdassGEO.zechoPF(pF, &check); //call function to check
            if (checka<0) //if dimensions missing
                MessageBox("Packing factor value missing or of zero value!");
            loop=1;
    ) while (loop==0);
}

/* *** Function to deal with packing factor input window *** */
void TDWindow::CmpPath()
{
    if (FCHK!=0 || DimCHK!=0 || BHCHK!=0) //data ready
        MessageBox("Dimension, BH data, or maximum Flux Density value missing");

else{
    graphyes = 1;
    MessageBox("
            "Path lengths and areas calculations started. This may take a little time",
            "PATHS",
            MB_ICONINFORMATION);
    MessageBeep(1);
    //time
    CursorCounter = 1;
    SetCapture();
    SetCursor(LeadCursor(NULL, SystemCursors[CursorCounter]));
}

minAREA = condun &mdash (MdassGeo); //calls function to convert data to be read by
//paths
if (minAREA == 0)
    MessageBox("Wrong path", "PATHS", MB_ICONERROR);
if (DESTROY == 0) //contains calculation in loop at an attempt to save memory
    do
        pathLA aLApath1(condun); aBApath2(condun); aCApath3(condun); aDApath4(condun); aEApath1(condun);
        // calculate area and lengths of steel paths
        alApath1 = condun; aBApath2 = condun; aCApath3 = condun; aDApath4 = condun; aEApath1 = condun;
        // calculate area and lengths of airgap paths
        MessageBeep(1);
        alA = aLApath1; aBA = aBApath2; aCA = aCApath3; aDA = aDApath4; aEA = aEApath1;
        if (alAAGrel == 0) a1BAGrel == 0 a1CAGrel == 0 a1DAGrel == 0 a1EAGrel == 1
            MessageBox("Wrong gap", "PATHS", MB_ICONERROR);
    if (MclassGEO.maxFD != 0)
        MessageBox("Flux table", "PATHS", MB_ICONERROR);
        if (MclassGEO.table[DIS] == 0)
            MessageBox("Error", "PATHS", MB_ICONERROR);
    else
        //calculate mmf by curve fitting
        double store[DIS]; //store for passing arrays to curve fitting
        for (irrt s = 0; s < DIS; s++) store(s) = MclassGEO.table(s);
        double ALarray[5];
        ALarray[0] = alA = aLApath1; ALarray[1] = aBA = aBApath2; ALarray[2] = aCA = aCApath3;
        //find average
        if (AVE == 0)
            MessageBox("AVE", "PATHS", MB_ICONERROR);
    /send average to compute mmf table
    MclassGEO.mmfset(AVE);
    if (MclassGEO.table[DIS] == 0)
        MessageBox("Error", "PATHS", MB_ICONERROR);
    else
        //final table, replace table by flux readings.
        MclassGEO.finalupdate();
        if (MclassGEO.table[1] == 0)
            MessageBox("Error", "PATHS", MB_ICONERROR);
    } else {
        //calculate mmf by curve fitting
        double store[DIS]; //store for passing arrays to curve fitting
        for (irrt s = 0; s < DIS; s++) store(s) = MclassGEO.table(s);
        double ALarray[5];
        ALarray[0] = alA = aLApath1; ALarray[1] = aBA = aBApath2; ALarray[2] = aCA = aCApath3;
        //find average
        if (AVE == 0)
            MessageBox("AVE", "PATHS", MB_ICONERROR);
        /send average to compute mmf table
        MclassGEO.mmfset(AVE);
        if (MclassGEO.table[DIS] == 0)
            MessageBox("Error", "PATHS", MB_ICONERROR);
        else
            //final table, replace table by flux readings.
            MclassGEO.finalupdate();
            if (MclassGEO.table[1] == 0)
                MessageBox("Error", "PATHS", MB_ICONERROR);
    } else {
        ]
//find average //by putting maximum flux value in each of the paths equations and then averaging the results
//send average to compute mmf table
MsfL.mmnSet(AVE);

/*
  aLnewfux(MsfL,table,MsfL.tmpflux); /calculate the flux
  aLnewfux(MsfL,table,MsfL.tmpflux); /in each path at the
  aLnewfux(MsfL,table,MsfL.tmpflux); /final mmf, and sums
  aLnewfux(MsfL,table,MsfL.tmpflux); /the values in tmpflux

  aLnewfux(MsfL,table,MsfL.tmpflux); /calculate the flux
  aLnewfux(MsfL,table,MsfL.tmpflux); /in each path at the
  aLnewfux(MsfL,table,MsfL.tmpflux); /final mmf, and sums
  aLnewfux(MsfL,table,MsfL.tmpflux); /the values in tmpflux
  aLnewfux(MsfL,table,MsfL.tmpflux); /create the flux

  aLnewfux(MsfL,table,MsfL.tmpflux); /final table, replace table f by Impflux readings
*/
//send average to compute mmf table
MsfL.mmnSet(AVE);

/*
  aLnewfux(MsfL,table,MsfL.tmpflux); /calculate the flux
  aLnewfux(MsfL,table,MsfL.tmpflux); /in each path at the
  aLnewfux(MsfL,table,MsfL.tmpflux); /final mmf, and sums
  aLnewfux(MsfL,table,MsfL.tmpflux); /the values in tmpflux
  aLnewfux(MsfL,table,MsfL.tmpflux); /create the flux

  aLnewfux(MsfL,table,MsfL.tmpflux); /final table, replace table f by Impflux readings
*/
//send average to compute mmf table
MsfL.mmnSet(AVE);

/*
  aLnewfux(MsfL,table,MsfL.tmpflux); /calculate the flux
  aLnewfux(MsfL,table,MsfL.tmpflux); /in each path at the
  aLnewfux(MsfL,table,MsfL.tmpflux); /final mmf, and sums
  aLnewfux(MsfL,table,MsfL.tmpflux); /the values in tmpflux
  aLnewfux(MsfL,table,MsfL.tmpflux); /create the flux

  aLnewfux(MsfL,table,MsfL.tmpflux); /final table, replace table f by Impflux readings
*/
//send average to compute mmf table
MsfL.mmnSet(AVE);

/*
  aLnewfux(MsfL,table,MsfL.tmpflux); /calculate the flux
  aLnewfux(MsfL,table,MsfL.tmpflux); /in each path at the
  aLnewfux(MsfL,table,MsfL.tmpflux); /final mmf, and sums
  aLnewfux(MsfL,table,MsfL.tmpflux); /the values in tmpflux
  aLnewfux(MsfL,table,MsfL.tmpflux); /create the flux

  aLnewfux(MsfL,table,MsfL.tmpflux); /final table, replace table f by Impflux readings
*/
//send average to compute mmf table
MsfL.mmnSet(AVE);
// by putting maximum flux value in each of the paths equations and then averaging the results
double AVE=ALaray(0)+ALaray(1)+ALaray(2)+ALaray(3)+ALaray(4);

// send average to compute mmf table
MnFM.mmfset(AVE);

MnFM.newflux(MnFM.table, MnFM.tmpflux); // calculates the flux in each path at the mB. egnewflux(MnFM.table, MnFM.tmpflux); //final mmf, and sums
MnFM.newflux(MnFM.table, MnFM.tmpflux); //the values in tmpflux
MnFM.newflux(MnFM.table, MnFM.tmpflux);

// final table, replace table by tmpflux readings.
MnFM.flushdata();

// need an equation to model relationship
static double storem[DIS];
for (mt kk=0; kk<DIS, kk++) {
  storem[kk]=MnFM.table[kk].m;
}

// final update
MnFM. finalupdateO;

// need an equation to model relationship
MnFM. flushdata();

for (mt kk=0; kk<DIS, kk++) {
  MnFM.equwhart(storem, storem[kk]);
}

while (DESTROY! =0),
MessageBeep(1);
do 

// calculate un-aligned position */\\\///
pathl. A utu unB, tmGunD, unE, unF, wi(3, unH, unl, unl; // create 10 paths
w Upathl(oondw); tmB. Upath2(condun); W1C. UpadWoonmm), mD. Upath4(condun);
unE. Upath6(oondvn);

// calculate area and lengths of steel paths
unA. Utooth3(condim); unB. UtoothWoondim); imC. Utoothl(condun); unD. Utooth2(oondun)

// calculate area and maths of aogap paths
MessageBeep(4);
 tmp. aogapRELO; twB. augapRELO; unC. aitgapRELO; unD. augapRELO;
unE. airgapRELO;
unf. avgapRELO; unG. airgapRELO. unH. airgapRELQ;
unl. airgapRELO; UN. avgapREL. O

// calculate reluctance of airgap paths
// the flux table of class MASTER MnFM
MnFM. fluxset10.mmAREA, MnFM.geo.maxFD);

if (bHCHK=1), // calculate mmf by tables method
double UName[10];
UName[0]=Ua.cah=m(Mdaastb[Mh, MnFM. table, Mdaastb[Mbh, MbhPomt]);
UName[1]=Ua.cah=m(Mdaastb[Mh, MnFM. table, Mdaastb[Mbh, MbhPomt]);
UName[2]=Ua.cah=m(Mdaastb[Mh, MnFM. table, Mdaastb[Mbh, MbhPomt]);
UName[3]=Ua.cah=m(Mdaastb[Mh, MnFM. table, Mdaastb[Mbh, MbhPomt]);
UName[4]=Ua.cah=m(Mdaastb[Mh, MnFM. table, Mdaastb[Mbh, MbhPomt]);
UName[5]=Ua.cah=m(Mdaastb[Mh, MnFM. table, Mdaastb[Mbh, MbhPomt]);
UName[6]=Ua.cah=m(Mdaastb[Mh, MnFM. table, Mdaastb[Mbh, MbhPomt]);
UName[7]=Ua.cah=m(Mdaastb[Mh, MnFM. table, Mdaastb[Mbh, MbhPomt]);
UName[8]=Ua.cah=m(Mdaastb[Mh, MnFM. table, Mdaastb[Mbh, MbhPomt]);
UName[9]=Ua.cah=m(Mdaastb[Mh, MnFM. table, Mdaastb[Mbh, MbhPomt]);

// End average

// end average to compute mmf table
MnFM.mmfset(AVE);

MnFM. newflux(MnFM.table, MnFM.tmpflux); // calculates the flux in each path at the mB. egnewflux(MnFM.table, MnFM.tmpflux); //final mmf, and sums
MnFM. newflux(MnFM.table, MnFM.tmpflux); //the values in tmpflux
MnFM. newflux(MnFM.table, MnFM.tmpflux);
MnFM. newflux(MnFM.table, MnFM.tmpflux);
MnFM. newflux(MnFM.table, MnFM.tmpflux);
MnFM. newflux(MnFM.table, MnFM.tmpflux);
// final table, replace table for tmpflux readings.
MunFM.finalupdate();
MessageBox(1);

} //calcualte mmf by curve fitting
double store[DIS]; // store for passing arrays to curve fitting
for (int sO; s<DIS; s++) store[s]=MunFM.table[s];
for (int kk=0; kk<DIS; kk++)
    store[kk]=MunFM.table[kk];
MunFM.equazt(store, store, 2);
}
while (DESTROY!=0);
// timer
ReleaseCapture();
:: SetCursor(LoadCursor(NULL, IDC_ARROW));
MessageBox("Airgap Reluctance is computed
Anays Built", MB_ICONINFORMATION);

// final table, replace table by tmpflux readings.
MunFM.finalupdate();
MessageBox(1);
// need an equation to model relationship
static double atorem[DIS];
for (int sO; s<DIS; s++)
    atorem[s]=MunFM.table[s];
for (int kk=0; kk<DIS; kk++)
    atorem[kk]=MunFM.table[kk];
MunFM.equazt(atreom, atorem, 2);
/ Ant poc-store[O];
}
// find average
// by putting maximum flux value in each of the paths equations and then
// averaging the results
double AVE=(wanaY[0]*unY[1]*unwaY[2]*unarmY[I]*unuaY[4]+
// send average to compute mmf table
MunFM.minset(AVE);
// send average to compute mmf table
// MunFM.minset(AVE);

} //calcualte mmf by curve fitting
double store[DIS]; // store for passing arrays to curve fitting
for (int sO; s<DIS; s++) store[s]=MunFM.table[s];
for (int kk=0; kk<DIS; kk++)
    store[kk]=MunFM.table[kk];
MunFM.equazt(store, store, 2);
}
while (DESTROY!=0);
// timer
ReleaseCapture();
:: SetCursor(LoadCursor(NULL, IDC_ARROW));
MessageBox("Airgap Reluctance is computed
Anays Built", MB_ICONINFORMATION);

// final table, replace table by tmpflux readings.
MunFM.finalupdate();
MessageBox(1);
// need an equation to model relationship
static double atorem[DIS];
for (int sO; s<DIS; s++)
    atorem[s]=MunFM.table[s];
for (int kk=0; kk<DIS; kk++)
    atorem[kk]=MunFM.table[kk];
MunFM.equazt(atreom, atorem, 2);
/ Ant poc-store[O];
}
// find average
// by putting maximum flux value in each of the paths equations and then
// averaging the results
double AVE=(wanaY[0]*unY[1]*unwaY[2]*unarmY[I]*unuaY[4]+
// send average to compute mmf table
MunFM.minset(AVE);
// send average to compute mmf table
// MunFM.minset(AVE);

} //calcualte mmf by curve fitting
double store[DIS]; // store for passing arrays to curve fitting
for (int sO; s<DIS; s++) store[s]=MunFM.table[s];
for (int kk=0; kk<DIS; kk++)
    store[kk]=MunFM.table[kk];
MunFM.equazt(store, store, 2);
}
while (DESTROY!=0);
// timer
ReleaseCapture();
:: SetCursor(LoadCursor(NULL, IDC_ARROW));
MessageBox("Airgap Reluctance is computed
Anays Built", MB_ICONINFORMATION);

// final table, replace table by tmpflux readings.
MunFM.finalupdate();
MessageBox(1);
// need an equation to model relationship
static double atorem[DIS];
for (int sO; s<DIS; s++)
    atorem[s]=MunFM.table[s];
for (int kk=0; kk<DIS; kk++)
    atorem[kk]=MunFM.table[kk];
MunFM.equazt(atreom, atorem, 2);
/ Ant poc-store[O];
}
// find average
// by putting maximum flux value in each of the paths equations and then
// averaging the results
double AVE=(wanaY[0]*unY[1]*unwaY[2]*unarmY[I]*unuaY[4]+
// send average to compute mmf table
MunFM.minset(AVE);
// send average to compute mmf table
// MunFM.minset(AVE);

} //calcualte mmf by curve fitting
double store[DIS]; // store for passing arrays to curve fitting
for (int sO; s<DIS; s++) store[s]=MunFM.table[s];
for (int kk=0; kk<DIS; kk++)
    store[kk]=MunFM.table[kk];
MunFM.equazt(store, store, 2);
}
while (DESTROY!=0);
// timer
ReleaseCapture();
:: SetCursor(LoadCursor(NULL, IDC_ARROW));
MessageBox("Airgap Reluctance is computed
Anays Built", MB_ICONINFORMATION);

// final table, replace table by tmpflux readings.
MunFM.finalupdate();
MessageBox(1);
// need an equation to model relationship
static double atorem[DIS];
for (int sO; s<DIS; s++)
    atorem[s]=MunFM.table[s];
for (int kk=0; kk<DIS; kk++)
    atorem[kk]=MunFM.table[kk];
MunFM.equazt(atreom, atorem, 2);
/ Ant poc-store[O];
}
// find average
// by putting maximum flux value in each of the paths equations and then
// averaging the results
double AVE=(wanaY[0]*unY[1]*unwaY[2]*unarmY[I]*unuaY[4]+
// send average to compute mmf table
MunFM.minset(AVE);
// send average to compute mmf table
// MunFM.minset(AVE);
double Blmag(NOST bh[5], NOST mag[5], FluxMmf FMbunag[DIS], GEO MdassGEO, classBH Mdassmag)
{
    // get dimensions
    // length of stack
    // inside diameter
    // outside diameter
    // maximum flux density in back zone=1 T
    double AREAbact=pow(m AssGEO, nsDIM[18]. n/2)*(MdassGEO, nsDIM[18]. n/2)*
                (MdassGEO, nsDIM[19]. n/2)*(MdassGEO, nsDIM[19]. n/2));
    double kpee=0.75;
    double LENmag=-3*MsclassGEO, nsDIM[15]. n*ClassGEO, nsDIM[17]. n-
                   MdassGEO, nsDIM[15]. n);
    double OFFSETmag=Mdassmag. of sd. n;
    double AREAmag=1e-6*MdassGEO, nsDIM[16]. n;
    double M3MKfluxl=AREAbact;
    float FF=MsclassGEO.PF.n.
    FMbunag[0]. f=0;
    FMbimag[0]. m=1*OFFSETmag*LENmag;
    double BTMP;
    double oon;
    for (int i=0; i<DIS; i++)
    
    FMbimag[i]. f=5*M3MKflucl*AREAbact;
    float FF=MsclassGEO.PF.n.
    BTMP=FMbimag[i] AREAbact;
    // double x, y,
    // double xk, yr,
    int count=0;
    do{
        (bh[0]. n*power(x bh[2]. n)+bh[4]. n*power(x bh[2]. n+1))/
            (bh[1]. n*power(x bh[2]. n)+bh[3]. n*power(x bh[2]. n+1))=BTMP;
        
        (bh[0]. n*power(x bh[2]. n)+bh[4]. n*power(x bh[2]. n+1))/
            (bh[1]. n*power(x bh[2]. n)+bh[3]. n*power(x bh[2]. n+1))=BTMP;
    } while (count<>300);
    count=0;
    do{
        (mag[0]. n*power(m nu, mag[2]. n))/
            (mag[1]. n*power(m nu, mag[2]. n))=BTMP;
        
        (mag[0]. n*power(m nu, mag[2]. n))/
            (mag[1]. n*power(m nu, mag[2]. n))=BTMP;
    } while (mag[0]. n>mag[2]. n);
```c
if (count<300)
    count=300;
while (count<300);

  double mHcon=mH-OFFSETmag;

  FMMmag(i).m+=mHcon*LENmag*Fab(FMMimag[i].m);

  double value=Fab(FMMimag[i].m);
  FMMimag[i].m=value;
  return value;
```

//Find H from equation

//Convert to flux

//Use equation to generate new mu and hence mmf

//Obtain mmf

//Generate equation //Done

//Generate magnet mmf/flux characteristics

//Merge backiron and magnet

//Create new equation for relationship
#include "com.h"
#include "geo.h"
#include "math.h"

//new Windows classes

void classBH: charheadO {
    formt v=0; i<100; ++i) (Mbh[i]. b=mbh[i]. a=0;
    char blank["50"],
    for (i=0; i<5; ++i) (MbhCurve[i]. n=0,
    strcpy(MbhCurve[i]. a, blank);
    MbhFlag=0;
    strcpy(MbhName, blank);
    strcpy(offset. a, blank);
    offset.n=0;
}

IMPLEMENT_CASTABLE(dassBH);
IMPLEMENT_STREAMABLE(dassBH);

void *classBH. StreamRead(qstream &in, uint32) const
//replacing default read
//stores all of dass in stream
{
    for (int i=0; i<100; ++i) {
        in->GetObjectQ->Mbh[i]. b;
        in->GetObjectQ->Mbh[i]. h; //BH curve in use
        in->GetObjectQ->MbhName, [20], //BH name to print on main window
        in->GetObjectQ->MbhPoint; //number of points
        in->GetObjectQ->MbhFlag; //0-no data 1-tables 2-coefficients
        for (i=0; i<5; ++i) {
            in->GetObjectQ->MbhCurve[i]. n;
        }
    }
}

//replacing default write command
void dassBH: StreamWrite(qstream &out) const
{
    for (int i=0; i<100, ++i) {
        out<<GetObjectQ->Mbh[i]. b;
        out<<GetObjectQ->Mbh[i]. h; //BH curve in use
        out<<GetObjectQ->MbhPoint; //number of points
        out<<GetObjectQ->MbhFlag; //0-no data 1-tables 2-coefficients
        for (i=0; i<5, ++i) {
            out<<GetObjectQ->MbhCurve[i]. n;
        }
    }
}

void GEO: transfer //copy transfer buffer strings to main window
(char a[10], char b[10], char c[10], char d[10],
char e[10], char f[10], char g[10], char h[10],
char i[10], char j[10], char k[10], char l[10],
char m[10], char n[10], char o[10], char p[10],
char q[10], char r[10], char s[10], char t[10],
char u[10], char v[10], char w[10], char x[10],
char y[10], char z[10])
{
    strcpy(nsd[0]. s, a);
    strcpy(nsd[1]. s, b);
    strcpy(nsd[2]. s, c);
    strcpy(nsd[3]. s, d);
    strcpy(nsd[4]. s, e);
    strcpy(nsd[5]. s, f);
    strcpy(nsd[6]. s, g);
    strcpy(nsd[7]. s, h);
    strcpy(nsd[8]. s, i);
    strcpy(nsd[9]. s, j);
    strcpy(nsd[10]. s, k);
    strcpy(nsd[11]. s, l);
    strcpy(nsd[12]. s, m);
    strcpy(nsd[13]. s, n);
    strcpy(nsd[14]. s, o);
    strcpy(nsd[15]. s, p);
    strcpy(nsd[16]. s, q);
    strcpy(nsd[17]. s, r);
    strcpy(nsd[18]. s, s);
    for (int kl=0; kl<20; kl++)
    nsD[lkl]=malloc(nsd[lkl]. s);
}

void GEO: stringblankO //clear strings at startup
char temp["50"],
strcpy(nsd[0]. s, temp["50"],
strcpy(nsd[1]. s, temp["50"],
strcpy(nsd[2]. s, temp["50"],
strcpy(nsd[3]. s, temp["50"],
strcpy(nsd[4]. s, temp["50"],
strcpy(nsd[5]. s, temp["50"],
strcpy(nsd[6]. s, temp["50"],
strcpy(nsd[7]. s, temp["50"],
strcpy(nsd[8]. s, temp["50"],
strcpy(nsd[9]. s, temp["50"],
strcpy(nsd[10]. s, temp["50"],
strcpy(nsd[11]. s, temp["50"],
strcpy(nsd[12]. s, temp["50"],
strcpy(nsd[13]. s, temp["50"],
strcpy(nsd[14]. s, temp["50"],
strcpy(nsd[15]. s, temp["50"],
strcpy(nsd[16]. s, temp["50"],
strcpy(nsd[17]. s, temp["50"],
strcpy(nsd[18]. s, temp["50"],
int GEO::check() //sees if any dimensions musung
{
    int check=0;
    for (int k=0; k<20; k++)
    {
        if (DIM[k].n>0) check=1;
    }
    return check;
}

int GEO::checkFD(NOST maxFD) //sees if flux density (or p0) has been entered
{
    int check=0;
    if (maxFD.n>0) check=1;
    return check;
}

//master functions
void master::deathead()
{
    for (int k=0; k<DIS;k++)
    {
        table[k].f=table[k].m=flux[k]=0;
    }
}

void master::fluxset(float path, double minarea, NOST FD) //no. of path, min area, max flux density)
{
    table[0].f=0;
    for (int i=1; i<DIS; i++)
    {
        table[i].f=(maxflux/(DIS-1))*i;
    }
}

void master::xmmfset(double avmmf) //average flux
{
    table[0].m=0;
    for (int i=1; i<DIS; i++)
    {
        table[i].m=avmmf/(DIS-1)*i;
    }
}

//function to set up xmmf array
void master::finalupdate() //set flux in final table
{
    table[0].f=0;
    for (int i=1; i<DIS; i++)
    {
        table[i].f=table[i].m=flux[i];
    }
}
double pathLA::sequmm(NOST bb'|FluxmflNMMFLUXDIS) calculate mmf using curve
{
  //variables
  double maxmnmf;
  int ind=0; //position of path
  int now=1; //flux position
  MMF[0]=0;
  double FHi; //fluc intial
  double fii=1; //H initial
  double Hn; // H new
  double Hkeep[7]; //keeps values of H
  double dFHi; //derivative of FHi
  double BTMP; //flux density tmp
  int Fcon; //convergence flag
  double con; // convergence percent

  int safer;
  do{
    MMF[ind]=OMMFLUX[ind]*AGEdi; //init gap mmf
    if (Al[ind]==0.0) //if area not equal to zero
      (BTMP=OMMFLUX[ind]*AL[ind]); //find flux density
    else (BTMP=0.0);
    MMF[ind]=MMF[ind]+. Hi'Al[ind];
    Hkeep[ind]=Hi;
    while (Fcon<1e-4)
    {
      if (BTMP==0.0) MMF[ind]=Hi;
      Hn=Hi-dFHi;
      if (fabs((Hn-Hi)/Hn)<100;
        Hi=Hi;
      safer++;
      if (con<-le-4) Fcon=1;
      MMF[ind]=MMFLUX[ind]*AL[ind];
      Hkeep[ind]=Hi;
    }
    if (BTMP==0.0) Hkeep[ind]=Hkeep[ind-1];
    safer=0;
    ind=++; //flux for next position
    while (ind<7); //do until seven section complete
    if (ind==7) ind=0; //reset to first position
    do { //find lowest value of H for last round
      if (ind<7) ind++; //do until all fluxes have mmf calculated
      maxmnmf=maxmnmf; //returns highest value of mmf
    }
  if (ind<DIS) //do until all fluxes have mmf calculated
  { //find lowest value of H for last round
    if (ind<7) ind++; //do until all fluxes have mmf calculated
    maxmnmf=maxmnmf; //returns highest value of mmf
    return maxmnmf;
  }
}
void pathLA::newfluxb(Fluxmfl Mmmfm[DIS], double flux[DIS])
{
  for (int src=0; src<DIS; src++)
    for (int tar=0; tar<DIS; tar++)
      lax[src]++;
}
//calculation of path LA's mmf newfluxb
void pathLA::newfluxb(Fluxmfl Mmmf[DIS], double flux[DIS])
{
  for (int src=0; src<DIS; src++)
    for (int tar=0; tar<DIS; tar++)
      lax[src]++;
}
}
#include "com.h"
#include "pathdim.h"
#include "drawgeo.h"

const float DtoR=0.017453292;

float cirde(float a, float b)
{
    return b;
}

tvoid drawgeo(GEO dun)
{
    float ALPHA=360/dun.nsD[0];
    float x1=0.5*dun.nsD[1]+kag2*dun.nsD[14];
    float x2=0.5*dun.nsD[7];
    float x=a*sin(DtoR*2*ALPHA);
    float x1=(a+dun.nsD[8])*cos(DtoR*2*ALPHA);
    float x2=(a+dun.nsD[12])*cos(DtoR*2*ALPHA);
    float y=a*cos(DtoR*2*ALPHA);
    float y1=(a+dun.nsD[8])*sin(DtoR*2*ALPHA);
    float y2=(a+dun.nsD[12])*sin(DtoR*2*ALPHA);
    float xx=a*cos(DtoR*1*ALPHA);
    float x1=(a+dun.nsD[8])*cos(DtoR*1*ALPHA);
    float y1=(a+dun.nsD[8])*sin(DtoR*1*ALPHA);
    pA.x=OS.x+k*(x+xx);
    pA1.x=OS.x+k*(x2+xx);
    pB.x=OS.x-k*(x-xx);
    pB1.x=OS.x-k*(x1-xx);
    pJ.x=OS.x+k*(x+xx);
    pJ1.x=OS.x+k*(x1+xx);
    pI.x=OS.x+k*(x-xx);
    pI1.x=OS.x+k*(x1-xx);
    pA.y=OS.y-k*(y+yy);
    pA1.y=OS.y-k*(y2+yy);
    pB.y=OS.y+k*(y-yy);
    pB1.y=OS.y+k*(y1+yy);
    pC.y=OS.y-k*(y+yy);
    pC1.y=OS.y-k*(y1+yy);
    pD.x=OS.x-k*(0.3*dim.nsD[11]);
    pD1.x=OS.x-k*(0.5*nsD[11]);
    pE.x=OS.x+k*(0.5*nsD[11]);
    pE1.x=OS.x+k*(0.5*nsD[11]);
    pF.x=OS.y-k*(0.3*dim[8,1]);
    pF1.x=OS.x+k*(0.5*nsD[8]+dim[8]);
    pG.x=OS.x-k*(0.5*nsD[8]+dim[8]);
    pG1.x=OS.x-k*(0.5*nsD[8]+dim[8]);
    p3.x=px;
    p4.x=px;
\[ x = a \sin(D\times R\times 3\times \alpha); \]
\[ y = a \cos(D\times R\times 3\times \alpha); \]
\[ x = a \sin(D\times R\times 2\times \alpha); \]
\[ y = a \cos(D\times R\times 2\times \alpha); \]
\[ x = a \sin(D\times R\times \alpha); \]
\[ y = a \cos(D\times R\times \alpha); \]
 FluxLink getDENT
 {
  double mmfR;  
  double mmF=10;
  double mmfu=10;

  double fluxbR, fluxdR, fluxMAGu, fluxR27;

  int count=0, int erflag=0;
  double con=1;
  do {
    // BRANCH A (magnet and back-iron relationship)
    double MMFcorr = mmfR + (1.0 * mmfR);
    double fluxMAGu = flux(MmagBIMAG, MMFcorr);
    double MMFcorr = mmfR + (1.0 * mmfu);
    double fluxMAGu = flux(MmagBIMAG, MMFcorr);

    double A0u = (fluxbR + 2.0 * fluxdR + fluxdR) / 4;
    double A1u = 0.5 * (fluxbR - fluxdR);
    double A2u = 0.25 * (fluxbR - fluxcR + fluxdR - fluxcR);
    double F0u = A0u + A1u * cos(2.0 * PI / 0.972) + A2u * cos(4.0 * PI / 0.972);
    double F27u = A0u + A1u * cos(2.0 * PI / 2.727) + A2u * cos(4.0 * PI / 2.727);

    if (erflag==1){
      if (F0u < F27u) erflag=0; else mmfu += 50;
    }
    if (erflag==1){
      if (F0u < F27u) erflag=0; else mmfu += 50;
    }
    MMFcorr = mmfR + (1.0 * mmfR);
    double fluxMAGR = flux(MmagBIMAG, MMFcorr);

    double fluxbR = fluxbR + 2.0 * fluxR27 + fluxdR;
    double A0R = (fluxbR + 2.0 * fluxcR + fluxdR) / 4;
    double A1R = 0.5 * (fluxbR - fluxdR);
    double A2R = 0.25 * (fluxbR - fluxcR + fluxdR - fluxcR);
    double F0R = A0R + A1R * cos(2.0 * PI / 0.972) + A2R * cos(4.0 * PI / 0.972);
    double F27R = A0R + A1R * cos(2.0 * PI / 2.727) + A2R * cos(4.0 * PI / 2.727);

    double conM;
    if (F0R < F27R) conM = mmfu; else mmfu = mmfR;
    con = abs((mmfR - conM) / mmfR) * 100;
    count++;
  } while (count < 100 && con > 0.5);

  FluxLink detent;
  detent.m=fluxbR;
  detent.n=fluxdR;
  detent.mdu=fluxR27;
  detent.mag = mmfR;
  return detent;
}
// *** HEADERS ***
#include "com.h"
#include "bmb.h"
#include "duls.h"
#include "mast.h"

#include "windman.h"
cost float GIAsm=79.57747155; //Cerstes to A per metre

// Function
Curve hart(double xx[DISI, double yy[DIS], int path)
{
    cost int no=10,
    int COUNT;
// rows columns
// initial guess may do work on initial guess for fib function!!
Curve RESULT;
    double IALP //yy[DIS]1*1.1; yyy[1]*1.1; cout<<endl"Alpha"
ALP;
    double IEBT //yy[DIS]-1*1.1; cout<<endl"Beta"
EBT;
    double NEWALP=0;
    double NEWEBT=0;
    double NEWn=0;

    static double SUM[4];
    static double T1[no][4];
    static double T2[3][4],
    static double COEFF[3][2][4] //alpha beta
    COEFF[0][0][0]=1.5*x[9]; COEFF[0][1][0]=1.5*x[9];
    COEFF[0][0][1]=1.5*x[9]; COEFF[0][2][1]=1.5*x[9];
    COEFF[0][0][2]=1.5*x[9]; COEFF[0][1][2]=1.5*x[9];
    COEFF[0][0][3]=1.5*x[9]; COEFF[0][1][3]=1.5*x[9];

    Curve keeper(4);
    int move=0,
    do (jALP=COEFF[path][0][move]; IEBT=COEFF[1][0][move]; move++;)
    do (T1[0][0]=T1[2][0]-T1[1][0]; T1[0][1]=T1[2][1]-T1[1][1];
        T1[0][2]=T1[2][2]-T1[1][2]; T1[0][3]=T1[2][3]-T1[1][3];
        for (int no; no<4; no++)
            T1[0][no]=pow(T1[0][no],2)
            T1[1][no]=pow(T1[1][no],2)
            T1[2][no]=pow(T1[2][no],2)
            T1[3][no]=pow(T1[3][no],2)
            T1[0][no]=pow(T1[0][no],2)
            T1[1][no]=pow(T1[1][no],2)
            T1[2][no]=pow(T1[2][no],2)
            T1[3][no]=pow(T1[3][no],2)

        for (no=0; no<4; no++)
            T1[0][no]=pow(T1[0][no],2)
            T1[1][no]=pow(T1[1][no],2)
            T1[2][no]=pow(T1[2][no],2)
            T1[3][no]=pow(T1[3][no],2)
            T1[0][no]=pow(T1[0][no],2)
            T1[1][no]=pow(T1[1][no],2)
            T1[2][no]=pow(T1[2][no],2)
            T1[3][no]=pow(T1[3][no],2)

    // solve systems of equations for D.
    double EQI[4], EQ2[4];
    for (cost no=4, no++)
        EQ[3]=T1[0][3]*T1[2][0]-T1[0][0]*T1[2][3];
        for (cost no=4, no++)
            EQ[3]=T1[0][3]*T1[2][0]-T1[0][0]*T1[2][3];
```java
double EQ2[i][j] = EQ2[i][j] - EQ1[i][j];

double EQ3[i];
for (i = 0; i < 4; i++)
EQ3[i] = EQ1[i] - EQ2[i];

double EQ4[i];
for (i = 0; i < 4; i++)
EQ4[i] = EQ3[i] - EQ1[i];

double D; 
D[0] = EQ4[0];
D[1] = EQ4[1];
D[2] = EQ4[2];
D[3] = EQ4[3];

// minimum value of Q
double alp, bet, n, alpi, beti, ni;
alp = ALP + D[0];
bet = BET + D[1];
n = N + D[2];
alpi = ALP + 0.5*D[0];
beti = BET + 0.5*D[1];
ni = N + 0.5*D[2];
double Q0 = 0, double Q05 = 0, double Q1 = 0;
for (i = 0; i < no; i++)
Q0 += pow((x[i] - ((alp/ni)/(bet/n)))), 2);
Q0 += pow((v[i] - ((alpi/nii)/(beti/nii)))), 2);
if ((Q1 - 2*Q0 - Q05) < 0)
Q1 = -Q0 + Q05;
double Vmin = 0.5 + 0.25*(Q0 - Q1);

NEWALP = alpi + Vmin; 
NEWBET = beti + Vmin;
NEWN = ni + Vmin;

// find convergence percentage
double Ealp = ((NEWALP - alp)/alp); 
Ebetr = (NEWBET - bet)/bet;
En = (NEWN - n)/n;

// convergence check
if (Ealp > 0.2 & Ebetr > 0.2 & En > 0.2) COUNT = 55;

// sum of squares of residuals
if (ErroR = 1) (SumSQ = 0.0); 
else (SumSQ = 0.0);
for (mt r = 0; r < no; r++)
SumSQ += pow((Y[i] - (RESULT.a/pow(. x[j] + RESULT.n)))/RESULT.b, 2); 

// MENU COMMANDS LINKed to Functions
DEFINE_RESPONSE_TABLE(100, 100, "<NEWBET>>, A N N <<NEWN, 
ErroR = 1, COUNT = 55, );

// define dimensions input window
EV COMMAND(301, "CmDun");
```
**Initialization of main window**

### TDbWindow:: TDbWindow(TWindow* parent, const char* title)

```cpp
TWindow(parent, title)
```

### Define child window for graph

#### DEFINE_RESPONSE_TABLE(TGraphFrame, TFrameWindow)

```cpp
END_RESPONSE_TABLE;
```

#### TGraphFrame:: TGraphFrame(TWindow* parent, const char* title)

```cpp
TFrameWindow(parent, title)
```

### void TDbWindow::EvPaint()

```cpp
TPaintDC paintDC(HWindow); // paint main window
```

#### Assign menu items

```cpp
AssignMenu(); // assign the menu to the screen
```

#### Check for flux density

```cpp
if(MdassGEO.MbhFlag && Mclassmag.MbhFlag) (BhCHK=1, )
else if (MdassGEO.MbhFlag && Mclassmag.MbhFlag) (BhCHK=2; )
else (BhCHK 0; )
```

#### Display names of steel and magnet

```cpp
char title["BH curve in current use: ";
paintDC.TextOut(0,15, title, strlen(title));
char holdbh[25];
strcpy(holdbh, Mclassbh.MbhName);
strcat(holdbh, methodbh);
paintDC.TextOut(175,10, holdbh, strlen(holdbh));
```

#### Check if dimensions have been entered

```cpp
if(DimCHK = 0)
```

#### Check BH data

```cpp
char chkBMP["BH data entered"];
paintDC.TextOut(1,46, chkBMP, strlen(chkBMP));
```

#### Check if dimensions have been entered

```cpp
if(DimCHK = 0)
```

#### Check BH data

```cpp
char chkBMP["BH data entered"];
paintDC.TextOut(1,46, chkBMP, strlen(chkBMP));
```
### FUNCTIONS FOR BH FILES ENDED

#### BH STEEL MANIPULATION

#### loads data from main window

```
the_pad->steel_map=0; // informs dialog window we are dealing with steel data
for (int i=0; i<100; i++) // loads bh data to dialog window
    the_pad->bh[i].name=Mdashb, Mdashb[i].point;
```

#### when the user presses enter ** return the data back to the main window

```
if (the_pad->Execute()==IDOK){
    for(i=0; i<100; i++)
    {
        Mdashb[i].name=the_pad->bh[i].name;
        Mdashb[i].point=the_pad->point;
    }
}
```

#### function to deal with steel equation data

```
// creates a transfer buffer to put data directly into windows
struct TransferBuffer
{
    char s[15], s[15], s[15][201];
};
```
strcpy(tb.a,MclassBH.MbhCurve[0].s);
strcpy(tb.b,MclassBH.MbhCurve[1].s);
strcpy(tb.c,MclassBH.MbhCurve[2].s);
strcpy(tb.d,MclassBH.MbhCurve[3].s);
strcpy(tb.e,MclassBH.MbhCurve[4].s);
strcpy(tb.name,MclassBH.MbhName);

TBhCurDialog* the_pad=new TBhCurDialog(this, TRnd(4)); //create new dialog window
the_pad->SetTransferBuffer(&tb);
if (the_pad->Execute()==IDOK){ //return values from dialog to main window data
    strcpy(MclassBH.MbhCurve[0].s, tb.e);
    strcpy(MclassBH.MbhCurve[1].s, tb.b);
    strcpy(MclassBH.MbhCurve[2].s, tb.c);
    strcpy(MclassBH.MbhCurve[3].s, tb.d);
    strcpy(MclassBH.MbhCurve[4].s, tb.e);
    strcpy(MclassBH.MbhName, tb.name);
    MclassBH.MbhFlag=2; //Tag set to equation
}

//**** magnet data **** same as CmBH and CmMACurve


void
TDbWindow::CmMAGw


strcpy(tb.a,Mclassmag.MbhCurve[0].s);
strcpy(tb.b,Mclassmag.MbhCurve[1].s);
strcpy(tb.c,Mclassmag.MbhCurve[2].s);
strcpy(tb.d,Mclassmag.MbhCurve[3].s);
strcpy(tb.e,Mclassmag.MbhCurve[4].s);
strcpy(tb.name,Mclassmag.MbhName);

TBhCurDialog* the_pad=new TBhCurDialog(this, TRnd(4)); //create new dialog window
the_pad->SetTransferBuffer(&tb);
if (the_pad->Execute()==IDOK){ //return values from dialog to main window data
    strcpy(Mclassmag.MbhCurve[0].s, tb.e);
    strcpy(Mclassmag.MbhCurve[1].s, tb.b);
    strcpy(Mclassmag.MbhCurve[2].s, tb.c);
    strcpy(Mclassmag.MbhCurve[3].s, tb.d);
    strcpy(Mclassmag.MbhCurve[4].s, tb.e);
    strcpy(Mclassmag.MbhName, tb.name);
    Mclassmag.MbhFlag=2; //Tag set to equation
}

//**** magnet data **** same as CmBH and CmMACurve


void
TDbWindow::CmMAGv


strcpy(cb.a,Mclassmag.offset);


strcpy(tb.a,Mclassmag.offset);
strcpy(tb.b,Mclassmag.offset[0].a);
strcpy(tb.c,Mclassmag.offset[1].a);
strcpy(tb.d,Mclassmag.offset[2].a);
strcpy(tb.e,Mclassmag.offset[3].a);
strcpy(tb.name,Mclassmag.Offset);

TBhCurDialog* the_pad=new TBhCurDialog(this, TRnd(4)); //create new dialog window
the_pad->SetTransferBuffer(&tb);
if (the_pad->Execute()==IDOK){ //return values from dialog to main window data
    strcpy(Mclassmag.Offset[0].a, tb.e);
    strcpy(Mclassmag.Offset[1].a, tb.b);
    strcpy(Mclassmag.Offset[2].a, tb.c);
    strcpy(Mclassmag.Offset[3].a, tb.d);
    strcpy(Mclassmag.Offset[4].a, tb.e);
    strcpy(Mclassmag.Offset[5].a, tb.name);
    Mclassmag.Offset[6]=2; //get offset
    struct TransferBuffer{
        char a[15],b[15];
        strcpy(cb.a,Mclassmag.offset);
    }
    strcpy(tb.a,Mclassmag.offset);
    strcpy(tb.b,Mclassmag.offset[0].a);
    strcpy(tb.c,Mclassmag.offset[1].a);
    strcpy(tb.d,Mclassmag.offset[2].a);
    strcpy(tb.e,Mclassmag.offset[3].a);
    strcpy(tb.name,Mclassmag.offset);
    Mclassmag.Offset[0].a=strcmp(Mclassmag.Offset[0].a, tb.e);
    Mclassmag.Offset[1].a=strcmp(Mclassmag.Offset[1].a, tb.b);
    Mclassmag.Offset[2].a=strcmp(Mclassmag.Offset[2].a, tb.c);
    Mclassmag.Offset[3].a=strcmp(Mclassmag.Offset[3].a, tb.d);
    Mclassmag.Offset[4].a=strcmp(Mclassmag.Offset[4].a, tb.e);
    Mclassmag.Offset[5].a=strcmp(Mclassmag.Offset[5].a, tb.name);
    Mclassmag.Offset[6]=2;
}
TBhO®ialog the_padd++new TBhOffDialog(this, TResId(5));
if (the_padd->Exeداع(0==IDOK){
   strcpy(Mdassmsg offset_t.x,b);
}

}****Function to deal with dimension input window****//
void TDWindow::CmDim() {
   int loop;
do { loop=0;
       struct TransferBuffer { //set up transfer buffer
       char a[10],b[10],c[10],d[10],e[10],t[10],g[10],h[10],i[10],j[10],k[10],l[10];
   }; strcpy(b,MdassGEO.domDIM[0].s); strcpy(a,MdassGEO.domDIM[1].s);
   strcpy(c,MdassGEO.domDIM[2].s); strcpy(d,MdassGEO.domDIM[3].s);
   strcpy(e,MdassGEO.domDIM[4].s); strcpy(t,MdassGEO.domDIM[5].s);
   strcpy(g,MdassGEO.domDIM[6].s); strcpy(h,MdassGEO.domDIM[7].s);
   strcpy(i,MdassGEO.domDIM[8].s); strcpy(j,MdassGEO.domDIM[9].s);
   strcpy(k,MdassGEO.domDIM[10].s); strcpy(l,MdassGEO.domDIM[11].s);
   strcpy(m,MdassGEO.domDIM[12].s); strcpy(n,MdassGEO.domDIM[13].s);
   strcpy(o,MdassGEO.domDIM[14].s); strcpy(p,MdassGEO.domDIM[15].s);
   TDimDialog+dim_padd++new TDimDialog(this, TResId(2)),//ready to call dialog
   dim_padd->SetTransferBuffer(&tb);
   if(dim_padd->Execute()==IDOK){//if OK pressed
      MdassGEO.transfer(b,a,b.d.t,g,h,i,j,k,l); //transfer function of GEO class
      //check if any dimensions missing or zero value
      int checka=MdassGEO.domchkO; //Main window check
      if(checka==0) { //if dimensions missing
         MessageBeep(-1); //call error box and make user re-enter dialog
         MessageBox("Dimension missing or of =o value!");
         loop=1;
      }
   }
   while (loop==0);
}

}****Function to deal with flux density input window****//
void TDWindow::CmDen() {
   int loop;
do { loop=0;
       struct TransferBuffer { //set up transfer buffer
       char f[10];
   }; strcpy(f,MdassGEO.maxFD.s);
   TDimDialog+dim_padd++new TDimDialog(this, TResId(2));
   //if OK pressed
   MdassGEO.maxFD.n=stof(MclassGEO.maxFD.s);
   //check to see if flux density missing or zero value
   int checka=MdassGEO.zeroFD(0); //new function to check
   if(checka==0) { //if flux density missing
      MessageBeep(-1); //call error box and make user re-enter dialog
      MessageBox("Flux Density value missing or of zero value!");
      loop=1;
   }
   while (loop==0);
}

}****Function to deal with flux density input window****//
void TDWindow::CmPath() {
   if(FDCHK==0 && DimCHK==0 && FCHK==0) //if data ready
      MessageBox("Dimension, BH data, or maximum Flux Density value missing!");
   else { graphyes=1;
         MessageBox("Path lengths and areas calculations started! This may take a little tune");
         MB_ICONINFORMATION;
         MessageBeep(-1);
}
// paths
if (in_min == 0) MessageBox("minarea", "PATHS", MB_ICONINFORMATION);
int DESTROY_v = 0; run position calculation in loop at an attempt to save memory
do {********** calculate signed position ***************************/
    / A.Pathl (condim); A.Path2 (condim); A.Path3 (condim);
    / A.Path4 (condim); A.Path5 (condim);
/ calculate area and lengths of steel paths
A.Pathl (condim); A.Path2 (condim); A.Path3 (condim);
A.Path4 (condim); A.Path5 (condim);
/ calculate area and lengths of airgap paths
MessageBeep(3);
if (A.Area — 4 || A.Area == 0) MessageBox("area", "PATHS", MB_ICONINFORMATION);
/ calculate reluctance of airgap paths
if (classGEO. maxFD) MessageBox("flux density", "PATHS", MB_ICONINFORMATION),
classFM = classFM.slope(classGEO. maxFD);
if (classFM.table[DIS.I] == 0) MessageBox("flux table", "PATHS", MB_ICONINFORMATION);
if (classFM.table[DIS.I] == 0) MessageBox("mmf table", "PATHS", MB_ICONINFORMATION);
if (classFM.table[DIS.I] == 0) MessageBox("mmf table", "PATHS", MB_ICONINFORMATION);
// find average
if (AVERAGE == 0) MessageBox("average", "PATHS", MB_ICONINFORMATION);
// send average to compute mmf table
classFM.mmfset(AVERAGE);
if (classFM.table[DIS.I] == 0) MessageBox("error", "PATHS", MB_ICONINFORMATION);
else {********** send average to compute mmf table ***********/
    double store = 0;
    for (int p = 0; p < classFM.table[DIS.I]; p++)
        double ALay = classFM.table[ALay];
        ALay = ALay * classFM.table[ALay];
        ALay = ALay * classFM.table[ALay];
        ALay = ALay * classFM.table[ALay];
        ALay = ALay * classFM.table[ALay];
        ALay = ALay * classFM.table[ALay];
        // find average
        double AVERAGE = (ALay + ALay + ALay + ALay + ALay) / 5;
        if (AVERAGE == 0) MessageBox("average", "PATHS", MB_ICONINFORMATION);
        // send average to compute mmf table
        classFM.mmfset(AVERAGE);
        if (classFM.table[DIS.I] == 0) MessageBox("error", "PATHS", MB_ICONINFORMATION);
        else {********** send average to compute mmf table ***********/
            double store = 0;
            for (int p = 0; p < classFM.table[DIS.I]; p++)
                double ALay = classFM.table[ALay];
                ALay = ALay * classFM.table[ALay];
                ALay = ALay * classFM.table[ALay];
                ALay = ALay * classFM.table[ALay];
                ALay = ALay * classFM.table[ALay];
                ALay = ALay * classFM.table[ALay];
                // find average
                double AVERAGE = (ALay + ALay + ALay + ALay + ALay) / 5;
                if (AVERAGE == 0) MessageBox("average", "PATHS", MB_ICONINFORMATION);
                // send average to compute mmf table
                classFM.mmfset(AVERAGE);
        } // end average
    } // end loop
```c
// final table, replace table by tmpflux readings.
MaIFM. finalupdate();
// need an equation to model relationship
static double storem[DIS];
for (int kk = 0; kk<DIS; kk++){
    storem[kk]=MaIFM. table[kk]. m;
    store[kk]=MaIFM. table[kk]. f;
    // final table, replace table by tmpflux readings.
    MaIFM. equ Hart(storem, store);  
    len = store[0];
}
while (DESTROYIO) { 
    // print data to file
    printf("%d
", len);
    // calculate area and lengths of steel paths
    mA.Mpath1(condim); mB.Mpath2(condim); mC.Mpath3(condim);
    mD.Mpath4(condim); mE.Mpath5(condim);
    // calculate area and lengths of airgap paths
    MessageBeep(-1);
    mA.avgapRELO; mB.avgapRELO; mC.avgapRELO; mD.avgapRELO; mE.avgapRELO;
    // calculate reluctance of airgap paths
    // fill flux table of class MASTER MaIFM
    MmidFM. fluxset(6, mmAREA, Mdassbh. Mbh. MXFD);
    if (BhCHK-I) {// calculate mmf by tables method
        double ALarray[5];
        ALarray[O]-mA.calmmf(Mdassbh Mbh, Mdassbh. Mbh.Pomt);
        ALarray[1]-mB.calmmf(Mdassbh Mbh, Mdassbh. Mbh.Pomt);
        ALarray[2]-mC.calmmf(Mdassbh Mbh, Mdassbh. Mbh.Pomt);
        ALarray[3]-mD.calmmf(Mclassbh. Mbh, Mdassbh. Mbh.Pomt);
        ALarray[4]-mE.calmmf(Mclassbh. Mbh, Mdassbh. Mbh.Pomt);
    } // find average
    // send average to compute mmf table
    MmidFM. mmfset(AVE);
    mA.newflux(MmidFM. table, MaIFM. newflux)
    mB. newflux(MmidFM. table, MaIFM. newflux)
    mC. newflux(MmidFM. table, MaIFM. newflux)
    mD.newflux(MmidFM. table, MaIFM. newflux)
    mE. newflux(MmidFM. table, MaIFM. newflux)
    // final table, replace table by tmpflux readings.
    MaIFM. finalupdate();
    //MessageBeep(-1);
    } // class: maintain mmf by curve fitting
    // send average to compute mmf table
    MmidFM. mmfset(AVE);
    mA.newflux(MmidFM. table, MaIFM. newflux)
    mB. newflux(MmidFM. table, MaIFM. newflux)
    mC. newflux(MmidFM. table, MaIFM. newflux)
    mD.newflux(MmidFM. table, MaIFM. newflux)
    mE. newflux(MmidFM. table, MaIFM. newflux)
    // final table, replace table by tmpflux readings.
    MaIFM. finalupdate();
    // need an equation to model relationship
    static double storem[DIS];
    for (int kk = 0; kk<DIS; kk++){
        storem[kk]=MaIFM. table[kk]. m;
        store[kk]=MaIFM. table[kk]. f;
    }
    MaIFM. equ Hart(storem, store);  
    len = store[0];
} // while (DESTROYIO) { 
```
while (DESTROY != 0) {
  do 
    MessageBeep("I");
  
    calculate un-aligned position 
  
    pathA unA.setPath(o); pathB unB.setPath(o); pathC unC.setPath(o); unD.setPath(o);
    unA.setPath(o);
    unF.setPath(o);
  
  //calculate area and lengths of steel paths
  unA. calArea(path1, path2);
  unB. calArea(path2, path3);
  unC. calArea(path3, path4);
  unD. calArea(path4, path5);
  
  //calculate reluctance of steel paths
  unA. calRel(ph1, ph2);
  unB. calRel(ph2, ph3);
  unC. calRel(ph3, ph4);
  unD. calRel(ph4, ph5);
  
  //calculate MMF by tables method
  double UNarray[10];
  UNarray[0] = cmf(MF, table, 1);
  UNarray[1] = cmf(MF, table, 2);
  UNarray[2] = cmf(MF, table, 3);
  UNarray[3] = cmf(MF, table, 4);
  UNarray[4] = cmf(MF, table, 5);
  
  
  double newflux(MF, table, 1); //calculate new flux of each path
  
  newflux(MF, table, 2);
  newflux(MF, table, 3);
  newflux(MF, table, 4);
  newflux(MF, table, 5);
  
  //final table, replace table (by tmpflux readings.
  MF.setTable(AVE);
  MF.setTable(AVE);
  MF.setTable(AVE);
  MF.setTable(AVE);
  MF.setTable(AVE);
  MF.setTable(AVE);
  MF.setTable(AVE);
  MF.setTable(AVE);
  MF.setTable(AVE);
  MF.setTable(AVE);
}

}

send average to compute mmf table

MFM.mmfSet(AVE);

unA.avgwaveflux(MFM.table,MFM.tmpflux) //calculate the flux
unD.avgwaveflux(MFM.table,MFM.tmpflux) //in each path at the
unG.avgwaveflux(MFM.table,MFM.tmpflux) //final mmf and sums
unB.avgwaveflux(MFM.table,MFM.tmpflux) //the values in tmpflux
unH.avgwaveflux(MFM.table,MFM.tmpflux);

//final table, replace table f by tmpflux readings.
MFM.finalUpdate();
//need an equation to model relationship
static double storem[DIS];
for(int kk=0; kk<DIS; kk++)
{
  storm[kk]=MFM.table[kk].m;
  storem[kk]=MFM.table[kk].f;
}
while(DESTROY<>0);

if(graphYes:\={1}):
  MessageBox(\"Airgap Reluctance is computed \n\";\"PATHS\", \"MB_CONFIGURATION\";
  graphyes=0;
}

//displays results of graphs
void TDBWindow: CmplGraph()
{
  if(graphyes==..1) //via data ready
    MessageBox(FLUX / MMF DATA IS NOT COMPUTED=);,
  else
  {
    double maxX=maxY;
    double kflux=250/maxY; //auto-scaling flux
    double lanmf=250/maxX; //auto-scaling mmf

    TGraphFtame * ThisChild, ThisChild.new TGraphFrame(this, \"GRAPH\")
    ThisChild.Create;
    TCanvas dc*ThisChild\;/get this window

    //plot aligned graph
    dc.MoveTo(origin.x,origin.y)\;/move to start
    for(i=0; i<DIS; ++i)
    {
      point.x=MFM.table[i].m;
      point.y=MFM.table[i].f;
      dc.LineTo(origin.x+point.x*kknum, origin.y+point.y*kflux);
    }

    //plot mid graph
    dc.MoveTo(origin.x,origin.y)\;/move to start
    for(i=0; i<DIS; ++i)
    {
      point.x=MFM.table[i] m;
      point.y=MFM.table[i] f;
      dc.LineTo(origin.x+point.x*kknum, origin.y+point.y*kflux);
    }

    //plot un-aligned
dc. MoveTo(ongin. x, ongin. y); //move to start
for (int i=0; i<DIS; i++)
{
pontx=MaxFM. equa[i];
pointy=MaxFM. equa[i];
dc.LineTo(ongin. x+pointx, ongin. y+pointy)
}

//axis
dc. MoveTo(ongin. x, ongin. y); //move to start
dc.LineTo(ongin. x+15, ongin. y); //move to start
dc.LineTo(ongin. x, ongin. y+155);

char xaxis[15];
gevtxt(xaxis, xaxis);
dc.TextOut(ongin. x+5, ongin. y+5, xaxis);
char yaxis[15];
gevtxt(yaxis, yaxis);
dc.TextOut(ongin. x+5, ongin. y+255, yaxis);
}
dc. LineTo(ongin. x, ongin. y); //move to start
char maxy=150;
strcpy(maxy, maxy);

//get offset
char maxm[20];
dc.TextOut(ongin. x+5, ongin. y-255, maxm);
char yada[15];
gcvt(yada, yada);

//plot aligned graph/
dc. MoveTo(ongin. x, ongin. y); //move to start
for (int i=0; i<DIS; i++)
{
pontx=MaxFM. equa[i];
pointy=MaxFM. equa[i];
dc.LineTo(ongin. x+pointx, ongin. y+pointy);
}

//plot mid graph/
dc.MoveTo(ongin. x, ongin. y); //move to start
for (int k=0; k<DIS; k++)
{
pontx=MaxFM. equa[i];
pointy=MaxFM. equa[i];
dc.LineTo(ongin. x+pointx, ongin. y+pointy);
}

//plot ungaph/
dc.MoveTo(ongin. x, ongin. y); //move to start
for (int k=0; k<DIS; k++)
{
pontx=MaxFM. equa[i];
pointy=MaxFM. equa[i];
dc.LineTo(ongin. x+pointx, ongin. y+pointy);
}

thePad->SetTransferBuffer(&cb);
if (thePad->ExecArea()==1)
{
strcpy(maxm, "OK");
}

//sets up origin of graph
struct xy{double x; double y;};
struct xy origin; xy point; //origin and current point
onguur 50; opton"y-300;

//plot aligned graph/
dc. MoveTo(ongin. x, ongin. y); //move to start
for (int i=0; i<DIS; i++)
{ pointx=MaxFM. equa[i];
pointy=MaxFM. equa[i];
dc.LineTo(ongin. x+pointx, ongin. y+pointy);
}

//plot mid graph/
dc.MoveTo(ongin. x, ongin. y); //move to start
for (int k=0; k<DIS; k++)
{ pointx=MaxFM. equa[i];
pointy=MaxFM. equa[i];
dc.LineTo(ongin. x+pointx, ongin. y+pointy);
}

//plot ungaph/
dc.MoveTo(ongin. x, ongin. y); //move to start
for (int k=0; k<DIS; k++)
{ pointx=MaxFM. equa[i];
pointy=MaxFM. equa[i];
dc.LineTo(ongin. x+pointx, ongin. y+pointy);
}
```cpp
// with colour
TColor color=RGB(0,0,0);
TColor color=RGB(0,0.255);
TPoint p(50,50);
TPoint c(200,150);
TPoint c(200,150);
TPoint c(200,150);
do.SelectStockObject(GRAY_BRUSH);
// draw graphs
void TDbWin::CmGraph()
if (Status == -1) // data ready
Ma, &ageßox("FLUX / NMF DATA IS NOT COMPUTED!");
x2, y2, picK(x1,y1), picK(x1,y1), //
dc.MoveTo(picK(x1,y1), picK(x1,y1), //
dc.LineTo(picK(x1,y1), picK(x1,y1), //
dc.LineTo(picK(x1,y1), picK(x1,y1), //
dc.LineTo(picK(x1,y1), picK(x1,y1), //
```
```cpp
if (BifCHK==1)
    end max/MAX Y;
if (MidFM.table[DIS-1].max/MAX Y>MidFM.table[DIS-1].max/MAX Y)
    max/MAX Y=MidFM.table[DIS-1].max/MAX Y;
if (MidFM.table[DIS-1].max/MAX Y>MidFM.table[DIS-1].max/MAX Y)
    max/MAX Y=MidFM.table[DIS-1].max/MAX Y;
double maxX=max/MAX Y/autopacing flux
double kdn=2500/max/MAX Y/autopacing mmf
//set up origin of graph
struct xy (double x; double y; );//new structure for x and y
xy origin; xy point; //origin and current point
origin.x=50; origin.y=300;
TGraphFrame * ThisChild;
ThisChild = new TGraphFrame(this, "GRAPH");
ThisChild->Create();
TCanvasDC dc("ThisChild");get this window

//plot aligned graph/
dc.MoveTo(origin.x, origin.y) /*move to start*/
for (int i=0; i<DIS; i++)
{    
    point.x=MidFM.table[i].x;
    point.y=MidFM.table[i].y;
    dc.LineTo(origin.x+(point.x*mmf),
              origin.y-(point.y*mmf));
}

//plot mid graph/
dc.MoveTo(origin.x, origin.y) /*move to start*/
for (i=0; i<DIS; i++)
{    
    point.x=MidFM.table[i].x;
    point.y=MidFM.table[i].y;
    dc.LineTo(origin.x+(point.x*mmf),
              origin.y-(point.y*mmf));
}

//plot un-aligned
dc.MoveTo(origin.x, origin.y) /*move to start*/
for (i=0; i<DIS; i++)
{    
    point.x=MidFM.table[i].x;
    point.y=MidFM.table[i].y;
    dc.LineTo(origin.x+(point.x*mmf),
              origin.y-(point.y*mmf));
}

else{
    //get offset
    char max/MAX Y="V";
    struct TransferBuffer{
        char s[3];
    };
    strcpy(cb, max/MAX Y);
    TGraphMmDialog * the_padd =new TGraphMmDialog(this, TRand[0]);
    if (the_padd->Execute()==IDOK)
        strcpy(max/MAX Y, cb);
    double maxX=max/MAX Y/autopacing flux
    double kdn=2500/max/MAX Y/autopacing mmf
    //set up origin of graph
    struct xy (double x; double y; );//new structure for x and y
    xy origin; xy point; //origin and current point
    origin.x=50; origin.y=300;
    TGraphFrame * ThisChild;
    ThisChild = new TGraphFrame(this, "GRAPH");
    ThisChild->Create();
    TCanvasDC dc("ThisChild");get this window

    /plot aligned graph/
dc.MoveTo(origin.x, origin.y) /*move to start*/
```
for (int i=0; i<DIS; i++)
{
    point.x=(maxX/DIS-1))*i;
    point.y=0.1*MFM.equ.a.pow(point.x, MFM.equ.a);
    dc.LineTo(origin.x+(point.x*255),
              origin.y-(point.y*54));
}

// plot mid graph //
dc.MoveTo(origin.x,origin.y);move to start
for (int i=0; i<DIS; i++)
{
    point.x=(maxX/DIS-1))*i;
    point.y=0.1*MFM.equ.a.pow(point.x, MFM.equ.a);
    dc.LineTo(origin.x+(point.x*255),
              origin.y-(point.y*54));
}

// plot right graph //
dc.MoveTo(origin.x,origin.y);move to start
for (int i=0; i<DIS; i++)
{
    point.x=(maxX/DIS-1))*i;
    if(MFM.equ.a==1.(point.y=point.x*MFM.equ.a,)
    else{
        point.y=0.1*MFM.equ.a.pow(point.x, MFM.equ.a);
        dc.LineTo(origin.x+(point.x*255),
                  origin.y-(point.y*54));
    }
}

//static torque
void TDBWindow::CmStatO{
    int phase;
    for (int i=0; i<DIS; i++)
    {
        flux[i]=magbi[i]*MFM.equ.a;
        FluxLink ALIGNED/make make this global
        if(phase-I){
        //build aligned curve
        }
    }
}

//calculate magnet back iron relationship
FluxLink magb[DIS];
double flux[DIS];
double MFM.equ.a;
int points=MAXCURR/cINC;
float cur-0;
//Ido next curve depend on phase single
if(pbase-I)(
//build aligned curve
)
}
Afloat last proof
for (atk points; k-1; k-)
} cur->setCNC;
ALIGNED = getMacroBIMAG, MAFM equ, MmifEM equ, MnifEM equ, Mmifmag,MFmagnet,
MidasGEO mad[15] a, MidasGEO mad[16] a, Midasmag offset a, arm, NUTP.a;
addItem(ALIGNED);

double ALIGNI
if (k==0) (ALIGNED = UNALIGNED.un)
} else (double torqueSa = ENG(ciNC, NUIF Tiếp, ALIGNED.w, UNFALIGNED.w);
ALTORu2 = torqueSa;
MafMID = MIDTORA;)

} cur->setCNC;
MID = getMacroBIMAG, MaFM equ, MmifEM equ, MnifEM equ, Mmifmag,MFmagnet,
MidasGEO mad[15] a, MidasGEO mad[16] a, Midasmag offset a, arm, NUTP.a;
mid.add(mid);

double MAfMB;
if (lc=portRs) (MafMID = MIDTORA;)

dse (double torqueSa = ENG(ciNC, NUIF Tiếp, ALIGNED.w, UNFALIGNED.w));
MIDTORA = torqueSa;
MafMID = MIDTORA;)

} // plot torque graph/
} // MoveTo(103position, position)
// TorqueSp newtorque(ALTORu2, MIDTORA);

// TGraphFrame "ThisChild;"
// ThisChild = new TGraphFrame(this, "GRAPH"),
// ThisChild->Create();
// TCanvas DC *("ThisChild"); get this window

// for (ist position=0, position=points, position++)
// (double plot = TORQUESp. PLOT(position);
// TLineTo(position, position, position+100)
// double value = plot;

// two phase
// build aligned curve

double OFFSET;
FluxLink ALIGN2.make make this global
linklist UNALIGNED; // for this
LINKlist ALTORu2;

// float lastCNC=1;
for (int k=points, k=1; k-)
} cur->setCNC;
ALIGNED = getMacroBIMAG, MAFM equ, MmifEM equ, MnifEM equ, Mmifmag,MFmagnet,
MidasGEO mad[15] a, MidasGEO mad[16] a, Midasmag offset a, arm, NUTP.a;
addItem(ALIGNED);

double ALIGNI
if (k==0) (ALIGNED = UNALIGNED.un)
} else (double torqueSa = ENG(ciNC, NUIF Tiếp, ALIGNED.w, UNFALIGNED.w);
ALTORu2 = torqueSa;
MafMID = MIDTORA;)

} FluxLink UNALIGNED// make make this global
linklist UNALIGNED; // for this
LINKlist UNTORA;
for (k=0; k<points; k++)
{
    expDec *= expDec;
    for (i=0; i<points; i++)
    {
        expDec *= expDec;
        result[k][i] += expDec * expDec;
    }

    result[k][i] *= expDec;
}

for (k=0; k<points; k++)

double ALCUNf = 0;

else
{
    double torqueSa = MENG(cINC, MUL, UNALIGNED, aLUNALIGNED, aL);
    TORQUE2p += torqueSa;
}

// calculate co-energy use trapezium
LINEn = TORQUE2p;
TORQUE2p = torqueSa; UNALIGNED, aL, UNM=

// plot torque graph
// dc.MoveTo(10,250); // move to start
for (int position=0; position<points; position++)
{
    double plot = TORQUE2p.PLOT(position);
    // dc.LineTo(10+position, position); // move to start
    dc.LineTo(10+position, position); // move to start
    double value = plot;

    // Main WIndow ****
    void TDbApp::MainWindow()
    {
        MainWindow = new TDbWindow(0,"SYSTEP","Permanent");
    }

    // ** RUN APPLICATION **
    int OwMain(int argc, char* argv[])
    {
        TDbApp app;
        return app.Run();
    //}
Curve hart(double x[DIS], double y[DIS], int path)
{
    int n=10;
    double x[10], y[10];
    static double d[5][3][4];
    double r;
    for (int i=0; i<n; i++)
    
        d[0][0][i]=x[i];
        d[0][1][i]=y[i];
        d[1][0][i]=x[i];
        d[1][1][i]=y[i];
        d[2][0][i]=x[i];
        d[2][1][i]=y[i];
        d[3][0][i]=x[i];
        d[3][1][i]=y[i];
        d[4][0][i]=x[i];
        d[4][1][i]=y[i];
    
    for (int i=0; i<n; i++)
    
        curve[i]=d[0][0][i];
        curve[i+1]=d[0][1][i];
        curve[i+2]=d[1][0][i];
        curve[i+3]=d[1][1][i];
        curve[i+4]=d[2][0][i];
        curve[i+5]=d[2][1][i];
        curve[i+6]=d[3][0][i];
        curve[i+7]=d[3][1][i];
        curve[i+8]=d[4][0][i];
        curve[i+9]=d[4][1][i];
    
    int COUNT; // anal guess may do work on anal guess for holuh function!!
    Curve RESULT;
    double NEWALP=0;
    double NEWBET=0;
    double NEWN=0;
    state double Sum[4];
    state double Ti[4][4];
    state double T2[3][4];
    state double COEFF[3][3][4], alpha beta
    COEFF[0][0][0]=1.5*x[0]; COEFF[0][0][1]=1.5*x[1];
    COEFF[0][1][0]=1.5*x[0]; COEFF[0][1][1]=1.5*x[1];
    COEFF[1][0][0]=1.5*x[0]; COEFF[1][0][1]=1.5*x[1];
    COEFF[1][1][0]=1.5*x[0]; COEFF[1][1][1]=1.5*x[1];
    COEFF[2][0][0]=1.5*x[0]; COEFF[2][0][1]=1.5*x[1];
    COEFF[2][1][0]=1.5*x[0]; COEFF[2][1][1]=1.5*x[1];
    COEFF[3][0][0]=1.5*x[0]; COEFF[3][0][1]=1.5*x[1];
    COEFF[3][1][0]=1.5*x[0]; COEFF[3][1][1]=1.5*x[1];
    
    // solve systems of equations for D.
    double EQ1[4], EQ2[4], EQ3[4], EQ4[4];
    for (int i=0; i<n; i++)
    
        EQ1[i]=pow(d[1][0][i],3)*d[1][1][i] + pow(d[1][1][i],3);
        EQ2[i]=pow(d[1][0][i],2)*d[0][1][i] + pow(d[0][0][i],2) +
        pow(d[1][1][i],2) + pow(d[1][0][i],2);
double EQ3[4];
for (i=0; i<4; i++)
    EQ3[i] = EQ3[i] + EQ(I);

double EQ4[4];
for (i=0; i<4; i++)
    EQ4[i] = EQ4[i] + EQ4[I];

double D[4];
D[2] = EQ4[0] - EQ4[1];
D[1] = EQ2[0] - EQ2[1];
D[0] = EQ2[0] - EQ2[1];

// minimum value of Q

double alp, bet, n,
alp = ALP[0], bet = BET[2];

// find convergence percentage

elep = fabs(NEWALP - ALP);
if (elep < 1.0e-5) COUNT++;
RESULT = NEWALP;

while (COUNT < 40) {
    SumSQ = 0;
    for (i = 0; i < no; i++)
        SumSQ += pow((y[i] - (NEWALP[i]*pow(x[i], n))), 2);
    if (SumSQ < 1.0e-5) COUNT++;
    newALP = OLDALP - (elep/SumSQ) * (NEWALP - OLDALP);
    COUNT++;
}
#include "static.h"

//** TStaticDialog **//

DEFINE_RESPONSE_TABLE(TStaticDialog, TDialog)
    # EV_COMMAND(954, Radio1),
    # EV_COMMAND(955, Radio2),
END_RESPONSE_TABLE,

***** intial set up for static torque Dialog *****
TStaticDialog::TStaticDialog(TWindow* parent, const char* name)
    :TDialog(parent, name),
    TWindow(parent)
{
    //Rbutton=0; //radio button selected for no phase,
}

functions contained within static torque dialog

void TStaticDialog::Radio1()
{
    Rbutton=1;
}

void TStaticDialog::Radio2()
{
    Rbutton=2;
}

**
/// this header stores the path calculation file ///
#include "com.h"
#include "gmain.h"

const float MU=1.2566e-6; //permitivity of free space

void pathLA: Apath(pathfim geo)
{
    for (int inc=0; inc<9; inc++)
    {
        AL[inc]=geo.STpwd*geo.DEP*1e-6;
        double lenA, lenB;
        lenA=pythag(geo.STpw, geo.STtd);
        COn Ctmp;
        Ctmp.x=geo.STpw; Ctmp.y=geo.STtd;
        lenB=pythag(COn, geo.STtd);
        AL[1]=lenA*lenB*5.0e-3;
        lenA=pythag(geo.STtd, geo.STtd);
        AL[2]=lenA*lenB*5.0e-3;
        double LENGTH, AREA;
        LENGTH=geo.STpwd*1.027e-3;
        AREA=geo.STpwd*geo.DEP*0.835e-6;
        /\axis pathA:
        AL[0]=LENGTH;
        AL[1]=AREA;
        AL[2]=1.027*geo.STpwd*1e-3;
        AL[3]=geo.STpwd*geo.DEP*1e-6;
        AL[4]=geo.RTrad*1e-3;
        AL[5]=geo.RTrad*0.5e-3;
        double LENGTH, AREA;
        LENGTH=geo.STpwd*1.027e-3;
        AREA=geo.STpwd*geo.DEP*0.835e-6;
        /\axis pathA:
        AL[0]=LENGTH;
        AL[1]=AREA;
        AL[2]=1.027*geo.RTrad*1e-3;
        AL[3]=1.027*geo.RTrad*1e-3;
        AL[4]=geo.RTrad*geo.DEP*1e-6;
        AL[5]=geo.RTrad*1e-3;
        AL[6]=geo.RTrad*1e-3;
        }
void pathLA: ApathXpathdim geo)
{
AL[0]= geo.STwid* geo.DEP*1e-6;  
AL[1]= geo.STht*1e-3;

double lenA, lenB;
CoC A= CompA, CompB;
CompA.x= geo.STwid; CompA.y= geo.STht;
CompB.x= geo.STwid; CompB.y= geo.STht;
lenA= pythag(CompA, geo.STht);
lenB= pythag(CompB, geo.STht);
AL[0]= lenA+lenB*0.5e-3;
lenA= pythag(geo.STht, geo.STht);
AL[1]= lenA+geo.STht*0.5e-3;
AL[2]= M_PI*(geo.STht*1e-3);
AL[3]= 6.5*geo.STht*1e-6;
LENQfH= geo.STht*1e-3;
AREA= geo.STwid* geo.DEP*1e-6;

/axis pathA;
AL[3]= LENGTH;
AL[3]= AREA;
AL[4]= 0.027*geo.RThgt*1e-3;
AL[4]= 0.965*geo.RThgt*1e-6;
AL[5]= geo.RThgt*1e-6;
AL[6]= geo.RThgt*1e-6;
}

void pathLA: ApathI(pathdim geo)
{
//set up ellipse function
axs pathA;

/THIS COMPUTES THE PATH LENGTH AND DISTANCES
//TOOTH ar-gap tooth A
pathA= geo.AGlen+geo.RThgt*0.15;
pathA= 3.5*(geo.STwid+ geo.RThgt)/2;
float a= ellipse(pathA);
Ta[0]= (geo.AGlen- a)/2;
Ta[0]= pathA/0.1387*geo.RThgt;
Ta[1]= lenA+Ta[1]*0.5e-3;
Ta[1]= pathA/0.065*geo.RThgt*1e-6;
Ta[2]= geo.RThgt*1e-6;
Ta[3]= 0;
Ta[4]= 0;
Ta[5]= 0;
Ta[6]= 0;
Ta[7]= 0;
empty= 3;
}

void pathLA: Atooth2(pathdim geo)
{
//set up ellipse function
axs pathA, pathB;

/THIS COMPUTES THE PATH LENGTH AND DISTANCES
//TOOTH ar-gap tooth B and D
pathA= geo.AGlen+geo.RThgt*0.2195;
pathA= 0.8*geo.RThgt/2;
ellipse(pathA);
Ta[2]= 0.2195*geo.RThgt;
Ta[3]= 0;
Ta[4]= 0;
Ta[5]= 0;
Ta[6]= 0;
empty= 3;
void pathA[4][4](pathdim geo)
{
    // set up slope function
    axis pathB;

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth C
    pathA[0][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[0][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth D
    pathA[4][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[4][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth C
    pathA[0][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[0][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth D
    pathA[4][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[4][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth C
    pathA[0][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[0][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth D
    pathA[4][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[4][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth C
    pathA[0][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[0][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth D
    pathA[4][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[4][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth C
    pathA[0][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[0][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth D
    pathA[4][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[4][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth C
    pathA[0][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[0][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth D
    pathA[4][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[4][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth C
    pathA[0][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[0][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth D
    pathA[4][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[4][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth C
    pathA[0][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[0][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth D
    pathA[4][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[4][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth C
    pathA[0][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[0][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap tooth D
    pathA[4][0]=geo.AGlen+geo.RThgt*0.202;
    pathA[4][1]=0.158*(geo.STwid+geo.RTwid);
    float declineA=dispass(pathA);
Boat mean - ipa(pathA)

Ta1[l] r((eUa A+geoAGlm)'0. 3e3;
Ta1[1] A=0.1185*(geo.RTwid+0.096*(geo.RThgt+geo.RTrad))'geo.aDEP*0.5e-6;

path A = geo.AGlm+geo.RThgt+geo.RTrad)'0.1097;
path A b=0.337*(geo.STwid+geo.RTrad);

distA = diste(pathA)

Ta1[2] r((eUa A+gcoAGlm)'0. 3e3;
Ta1[2] A=(pathAb+0.11852'geo RTwid+(geo. RThgt+geo. RTrad))'0.0M'geoa1DEP'0. Sa6;
Ta1[3] I r((geoAGlm)+(eUen))'0.5e3;

Ta1[3] A-geo. R7hvid'0.925373'geoa]2DEP'1o-6;
Ta1[4] L-0;
Ta1[4] A-0;

Ta1[5] L=0;
Ta1[5] A=0;

copy 3;

} // end pathA::Ampath1(pathdim geo)

{ // set up ellipse function
  cos pathA;

  // THIS COMPUTES THE PATH LENGTH AND DISTANCES
  // TOOTH AIR-gap tooth E

  path A = geo.AGlm+geo.RThgt+geo.RTrad)'0.089;
  path A b=0.732*(geo.STwid+geo.RTrad)/2;
  dist A = diste(pathA);

  Ta1[0] L=geo.AGlm+0.05e-3;
  Ta1[0] A=(pathA b+0.11852'geo RTwid+(geo. RThgt+geo. RTrad))'0.0M'geoa1DEP'0. Sa6;
  Ta1[1] L-0;
  Ta1[1] A-0;

  Ta1[2] L-(geoAGlm+dlat)'0.5 -3;
  Ta1[2] A-(1.067'pathA. b+0.075"geo. RTwid+(geo. RThgt+geo. RTrad)'0.24)
  *geo aDEP*0.5e-6;
  Ta1[3] L=0;
  Ta1[3] A=0;

  Ta1[4] L=0;
  Ta1[4] A=0;

  Ta1[5] L=0;
  Ta1[5] A=0;

  empty 3;

}
double LENGTH, AREA;

LENGTH = geo. STHgt/(cos(geo.EPSILON)) * 1e3;
AREA = 0.6379 * geo. STWid * 0.22077 * geo. STHgt / 0.5 * geo. DEP * 1e6;

void pathA()
pathA.geo.StHgt, pathA.b = 0.1105 * geo. STWid;
pathA. SWidth, pathA.
AL[1] = geo. STHgt/(cos(geo.EPSILON)) * 5e-3 * LENGTH * 0.5,
AL[2] = 0.2847 * geo. STWid * geo. DEP * 1e6 / AREA;
LENGTH = 1.166667 * geo. RThgt * 1e3;
AREA = 0.7574 * geo. RThgt * 2 * geo. RThgt * 0.5 * geo. DEP * 1e6;
padA. geo.RThgt * 0.182, pathA. b = geo. RThgt;
them(geo. RThgt, geo. RThgt, geo. RThgt, geo. RThgt, geo. RThgt); AL[4] = ((1.166667 * geo. RThgt * 0.5e-3 * LENGTH) * 0.5,
AL[5] = 0.8422 * geo. RThgt * geo. DEP * 1e6 / AREA;

AL[9] = geo. RTad * 1e3;
AL[3] = geo. RTad * 0.41472 * geo. RTad /geo. DEP * 1e6;
AL[6] = geo. RTad * 1e3;
AL[4] = AL[5];

void pathLA:: Mpath2(pathdirn geo)
for (int in = 0; in < 9; in++) AL[in] = 0;
AL[9] = geo. STWid / geo. DEP * 1e6;
AL[1] = geo. STHgt * 1e3;

double lenA, lenB;
CoOr CompA, CoCompB;
CompA. geo. STWid = (geo. STWid / 5), CoCompA. geo. STWid =
CompB. geo. STWid = (geo. STWid / 10), CoCompB. geo. STWid =
lenA = pythag(CoCompA. geo. STWid),
lenB = pythag(CoCompB. geo. STWid);
AL[1] = lenA + lenB * 0.5e-3;
AL[0] = pythag(geo. STWid, geo. STWid);
AL[1] = lenA + lenB * 0.5e-3 * geo. DEP * 1e6;
AL[2] = 0.785398 * geo. BETA * (geo. STWid + 2 * geo. STWid / 2);stu
(geo. STWid * M_PI * 0.25 * (geo. STWid / (1 / 6))) * 0.5e-3;
AL[2] = 0.74 * geo. STWid / 20) * 0.9 * geo. DEP * 1e6;

AL[1] = 2;
AL[4] = 1;

Soal LENGTH = geo. STHgt / (cos(geo. EPSILON)) * 1e3;
Soal AREA = 0.6379 * geo. STWid / geo. DEP * 1e6;

void pathLA:: Mpath3(pathdirn geo)
for (int in = 0; in < 9; in++) AL[in] = 0;
AL[9] = geo. STWid / geo. DEP * 1e6;
AL[1] = geo. STHgt * 1e3;

double lenA;
CoOr CompA, CoCompB;
CompA. geo. mbSTp.x = CoCompA. geo. mbSTp.y = geo. STWid;
lenA = pythag(CoCompA. geo. STWid);
AL[11] = geo. STWid * 1e3;
AL[12] = AL[11];

Al[1] = 2;
AL[4] = 1;
Soal LENGTH = geo. STHgt / (cos(geo. EPSILON)) * 1e3;
Soal AREA = 0.6379 * geo. STWid / geo. DEP * 1e6;

void pathLA:: Mpath4(pathdirn geo)
for (int in = 0; in < 9; in++) AL[in] = 0;
AL[9] = geo. STWid / geo. DEP * 1e6;
AL[1] = geo. STHgt * 1e3;

double lenA;
CoOr CompA, CoCompB;
CompA. geo. mbSTp.x = CoCompA. geo. mbSTp.y = geo. STWid;
lenA = pythag(CoCompA. geo. STWid);
AL[11] = geo. STWid * 1e3;
AL[12] = AL[11];

Al[1] = 2;
AL[4] = 1;
Soal LENGTH = geo. STHgt / (cos(geo. EPSILON)) * 1e3;
Soal AREA = 0.6379 * geo. STWid / geo. DEP * 1e6;
A142. Ir((0.783398 + gw. EEfA)'(9w. ST: ad+3'geo. STwid/25) +
(geo. S rad'M PI'0.25) + (geo. Slrad'(1/3)))'O. Sa3;

AL[2]JA-((47.5'geo SfwW/25) + (7.3'geo. $tra&12))'0.5'geo. DEP"1e6;

lAL. (3J ; /AAL 41. iwl;

goat LEN GTH-geo. S bgV(oos(geo. EPSILON))" e-3,

goat AREA-(24'goo. STwid/39)"geo. DEP' 1a6;

)dc pa&

PathA. rgeo. SThgt Pathkb 0.25641"gao. S wid;

&d cUae-d4pu(pathA);

AL[3]J. Ir((geo. Sing /(cos(geo. EPSULON))Fdtm)-0.5O 3+LENGfHy2;

AL[3]A-0.2g'gao. S[rnd'geo. DEP" 1a6+AREA;

AL[4] -1.25"geo. RTwid/41;

AL[5]JA-0.90909'geo. RTwid'geo. DEP" 1e6;

AL[51. It1.1'geo RTrad"1o-3;

AL[5]A-0.954545'gao. RTrad'goo. DEP' 1o-6;

AL[6] L-geo. RTweb" 1o-3;

AL[sl. A-AL[sl. n;

uuuiiuiaiuianiun

void pathA:: Mpath4(pathdim geo)
{
    //for (int inc=0; inc<9; inc++) AL[inc] = 0;
    AL[0]A=geo. STpwyd"(geo. DEP*1e-5);
    AL[0]L=geo. STpwtg*1e-3;

double lenA, lenB;
    COmp.A, COmpB;
    COmpA.x=geo. abStP.x + (geo. STpwyd/5); COmpA.y=geo. abStPy;
    COmpB.x=geo. abStPy + (geo. STpwyd/10); COmpB.y=geo. abStPy;
    lenA=pythag(COmpA, geo. STx);
    lenB=pythag(COmpB, geo. STx);

    AL[1]=lenA+lenB*0.3e-3;
    lenA=pythag(geo. STx&y, geo. STx&y);

    AL[1]=lenA+lenB*0.3e-3;

    AL[2]=0.366666'M PI'geo. STpwyd*1o-3;
    AL[3]=78"geo. STpwyd/40*(8"geo. STpwyd/20)*0.5"geo. DEP*1e-4;

    AL[3]L-1.070*geo. STid;
    AL[3]A-0.514545*geo. STid;

    //for (int inc=0; inc<9; inc++) AL[inc] = 0;
    AL[0]A=geo. STpwyd"(geo. DEP*1e-5);
    AL[0]L=geo. STpwtg*1e-3;

double lenA, lenB, lenA pythag(geo. STx&y, geo. STx&y);
    COmp.A, COmpB;
    COmpA.x=geo. abStP.x + (geo. STpwyd/5); COmpB.y=geo. abStPy;

    AL[1]=lenA+lenB*0.3e-3;
    lenA=pythag(COmpA, geo. STx);

    AL[1]=lenA+lenB*0.3e-3;

    AL[2]=0.366666'M PI'geo. STpwyd*1o-3;
    AL[3]=78"geo. STpwyd/40*(8"geo. STpwyd/20)*0.5"geo. DEP*1e-4;

    AL[3]L-1.070*geo. STid;
    AL[3]A-0.514545*geo. STid;

    return;

    void pathA:: Mpath5(pathdim geo)

    //for (int inc=0; inc<9; inc++) AL[inc] = 0;
    AL[0]A=geo. STpwyd"(geo. DEP*1e-5);
    AL[0]L=geo. STpwtg*1e-3;

double lenA, lenB, lenA pythag(geo. STx&y, geo. STx&y);
    COmp.A, COmpB;
    COmpA.x=geo. abStP.x + (geo. STpwyd/5); COmpB.y=geo. abStPy;

    AL[1]=lenA+lenB*0.3e-3;
    lenA=pythag(COmpA, geo. STx);

    AL[1]=lenA+lenB*0.3e-3;

    AL[2]=0.366666'M PI'geo. STpwyd*1o-3;
    AL[3]=78"geo. STpwyd/40*(8"geo. STpwyd/20)*0.5"geo. DEP*1e-4;

    AL[3]L-1.070*geo. STid;
    AL[3]A-0.514545*geo. STid;

    return;
void pathLA::Mtooth1(pathum geo) {
    //set up epsilon function
    set pathA, pathB;

    //THIS COMPUTES THE PATH LENGTH AND DISTANCES
    //TOOTH as gap tooth A
    pathA = geo.mAGlai + geo.SThtg/4;
    pathA.b = 0.177 + geo.RThgt;
    float dim = dipege(pathA);
    Tal[0].L = geo.mAGlai + dim)*0.5e-3;
    Tal[0].A = (geo.SThhtg + 0.177 + geo.RThgt) + geo.ADEP + 0.5e-6;
    Tal[1].L = geo.mAGlai + dim)*0.5e-3;
    Tal[1].A = (geo.RThgt + 0.177 + geo.RTwid)* geo.ADEP + 0.5e-6;
    pathA = geo.Renda + geo.RThgt + geo.RTbgt + geo.RThgt + geo.ADEP + 0.5e-6;
}

void pathLA::Mtooth2(pathum geo) {
    //set up epsilon function
    set pathA, pathB;

    //THIS COMPUTES THE PATH LENGTH AND DISTANCES
    //TOOTH as gap tooth B
    pathB = geo.mAGlai + geo.SThtg/4;
    pathB.b = 0.177 + geo.RThgt;
    float dimB = dipege(pathB);
    Tal[1].L = dimB + dim)*0.5e-3;
    Tal[1].A = (geo.RThgt + 0.177 + geo.RThgt) + geo.ADEP + 0.5e-6;
    pathB = geo.RThgt + 0.177 + geo.RTwid + 0.32 + geo.RThgt + geo.ADEP + 0.5e-6;
}

}
typedef struct { float x, y; } pt;


// Compute the path length and distances
for (int i = 0; i < 5; i++) {
    path[i] = A.distance(B); // Compute the distance between A and B
    for (int j = i + 1; j < 5; j++) {
        path[i].distance[j] = path[i].distance[j] + path[i].distance[j + 1]; // Sum the distances
    }
}

// Compute the path length and distances
for (int i = 0; i < 5; i++) {
    path[i] = A.distance(B); // Compute the distance between A and B
    for (int j = i + 1; j < 5; j++) {
        path[i].distance[j] = path[i].distance[j] + path[i].distance[j + 1]; // Sum the distances
    }
}

// Compute the path length and distances
for (int i = 0; i < 5; i++) {
    path[i] = A.distance(B); // Compute the distance between A and B
    for (int j = i + 1; j < 5; j++) {
        path[i].distance[j] = path[i].distance[j] + path[i].distance[j + 1]; // Sum the distances
    }
}

// Compute the path length and distances
for (int i = 0; i < 5; i++) {
    path[i] = A.distance(B); // Compute the distance between A and B
    for (int j = i + 1; j < 5; j++) {
        path[i].distance[j] = path[i].distance[j] + path[i].distance[j + 1]; // Sum the distances
    }
}

// Compute the path length and distances
for (int i = 0; i < 5; i++) {
    path[i] = A.distance(B); // Compute the distance between A and B
    for (int j = i + 1; j < 5; j++) {
        path[i].distance[j] = path[i].distance[j] + path[i].distance[j + 1]; // Sum the distances
    }
}

// Compute the path length and distances
for (int i = 0; i < 5; i++) {
    path[i] = A.distance(B); // Compute the distance between A and B
    for (int j = i + 1; j < 5; j++) {
        path[i].distance[j] = path[i].distance[j] + path[i].distance[j + 1]; // Sum the distances
    }
}

// Compute the path length and distances
for (int i = 0; i < 5; i++) {
    path[i] = A.distance(B); // Compute the distance between A and B
    for (int j = i + 1; j < 5; j++) {
        path[i].distance[j] = path[i].distance[j] + path[i].distance[j + 1]; // Sum the distances
    }
}

// Compute the path length and distances
for (int i = 0; i < 5; i++) {
    path[i] = A.distance(B); // Compute the distance between A and B
    for (int j = i + 1; j < 5; j++) {
        path[i].distance[j] = path[i].distance[j] + path[i].distance[j + 1]; // Sum the distances
    }
}
void pathLA::Mtooti5(pad&m goo)
{
  //set up ellipse function
  set padA;

  //THIS COMPUTES THE PATH LENGTH AND DISTANCES
  //TOOTH arc-gap tooth E
  pathA=geo.mAG1m+0.3846"geo.SThtg;
  pathA.b=0.191011"geo.R7wtd;
  float elenA=(ellipse(pathA));
  T[0] L=(GeoM+1.076"geo.mAG1m)"0.5e-3;
  T[0] A=geo.SThptg"2.31875"0.175675"geo.R7wtd"0.5"geo.ADEP"1e-4;
  T[1] L=0.0385"geo.mAG1m"0.5e-3;
  T[1] A=geo.R7wtd"0.08868"geo.ADEP"1e-4;
  T[2] L=(GeoM+1.03)"1e-3;
  T[2] A=0.08152"geo.ADEP"1e-4;
  T[3] L=0.15044"geo.R7wtd+0.20947"geo.R7htg"0.5e-6"geo.ADEP;
  pathA=geo.mAG1m+geo.R7htg(M_PI39.60)="12.860";
  pathA.b=0.3288"geo.R7wtd;
  dimA=ellipse(pathA);
  if(M_PI39.60)="0.0071388"M_SI39.60)="0.3o-6;
  else
  
  
  }

void padd. &,: Upadil(pelidim 9w)
{
  AL[0]. A-(geo.STwtd+geo.DEP1"6/10);
  AL[0] L=(geo.SThtg)"1e-3;

double lenA, lenB;
  lenA=pythag(geo.abStp,geo.GSttd);
  CComp CComp;
  CComp)argos abStp x=geo.STwtd 7; CComp y=geo.abStp y;
  lenB=pythag(CComp geos STTd);
  AL[1] L=(0.3"geo.2A+(lenA+lenB))"1e-3;
  lenA=pythag(geo.SThtg,geo.STTb);
  AL[1] A=(0.3"geo.STwd10)"0.5"geo.DEP1"1e-4;
  CComp=0 (CComp y=0; CComp x=0.481481"geo.2A+geo.BETA));
  CComp y=1.145688"geo.STwtd")geo.STwtd;
  lenB=pythag(CComp,CComp);
  AL[2] L=(0.9022"geo.2A+geo.BETA)+lenB="1e-3;
  AL[2] A=(1.31"lenA+0.37"geo.STwtd45)"0.5"geo.DEP1"1e-4;

double LENGTH, LENGTHB;
  LENGTH=(0.264"geo.STwtd+0.5e-6"geo.ADEP);
  empty=0;

}
//..............................................................................
void pathLA::pathl((pathden geo)
{
  AL[0] A=geo.STwtd+(geo.DEP1"6/10);
  AL[0] L=(geo.SThtg)"1e-3;

double lenA, lenB;
lenA=pythag(geo.abStp,geo.GSttd);
CComp CComp;
CComp)argos abStp x=geo.STwtd 7; CComp y=geo.abStp y;
lenB=pythag(CComp geos STTd);
AL[1] L=(0.3"geo.2A+(lenA+lenB))"1e-3;
lenA=pythag(geo.SThtg,geo.STTb);
AL[1] A=(0.3"geo.STwd10)"0.5"geo.DEP1"1e-4;
CComp=0 (CComp y=0; CComp x=0.481481"geo.2A+geo.BETA));
CComp y=1.145688"geo.STwtd")geo.STwtd;
lenB=pythag(CComp,CComp);
AL[2] L=(0.9022"geo.2A+geo.BETA)+lenB="1e-3;
AL[2] A=(1.31"lenA+0.37"geo.STwtd45)"0.5"geo.DEP1"1e-4;

double LENGTH, LENGTHB;
  LENGTH=(0.264"geo.STwtd+0.5e-6"geo.ADEP);
  empty=0;

}

AL[3] L=(GeoM+1+81664"LENGTH)A+0.3117"geo.R7wtd"0.2e-3;
AL[3] A=(1.28883"geo.R7wtd+0.94388"geo.R7htg)"geo.DEP1"0.5e-6;

AL[5] A=2;

AL[4] L=geo.RTweb"1e-3;
AL[4] A=0.6333"LENGTH)A+geo.DEP1"1e-4;

AL[5] L=0;
AL[5] A=0;
```cpp
void pathLA::Upath2(pathd= geo)
{
    // Code...
}

void pathLA::Upath3(pathd= geo)
{
    // Code...
}
```

These functions appear to be part of a geometric library, possibly for path planning or similar tasks. The exact purpose of each function is not entirely clear without additional context, as it involves operations like calculating distances and angles using the Pythagorean theorem and basic arithmetic operations on geometric objects.
double LENGTHA;
LENGTHA=(2*geo.RTrad+geo.RTwd);
pathA=0.136*geo.RTwd; pathA->geo.RThgt;

circleA=ellipse(pathA);
CCompG=geo.RTwd; CCompJ=geo.RThgt;
lenA->pythag(CompA,CCompl);

AL(2J. L-I 0.3'(geo.RTrad+0.7446'geo.RTrad+0.373'M Pl'geo.RTrad)
AL[4]A=0.3837e-3'LENGTHA;
AL[4]L=geo.RTwd;1e-3;
AL[5]=0;
AL[6]=0;
AL[6][A]=0;
}

void pathLA::UpathA(pathdm geo)
{

AL[2J. L-I 0.3'(eDeM+3.43478'geo.RTrad+Irnu.0.3'geo.RThgtY4;

AL[3]A=0.5'geo.RTwd; geo.RThgt+0.3'M PI'geo.RTrad+
0.3488'LENGTHA)'geo.DEP'O.5e-6;

AL[4]A=0.3837e-3'LENGTHA;
AL[4]L=geo.RTwd;1e-3;
AL[5]=0;
AL[6]=0;
AL[6][A]=0;

}

void pathLA::UpathA(pathdm geo)
{

AL[2J. L-I 0.3'(eDeM+3.43478'geo.RTrad+Irnu.0.3'geo.RThgtY4;

AL[3]A=0.5'geo.RTwd; geo.RThgt+0.3'M PI'geo.RTrad+
0.3488'LENGTHA)'geo.DEP'O.5e-6;

AL[4]A=0.3837e-3'LENGTHA;
AL[4]L=geo.RTwd;1e-3;
AL[5]=0;
AL[6]=0;
AL[6][A]=0;

}
double LENGTHA;
LENGTHA=(4*geo.RTwd*geo.RTwd + geo.DEP*geo.DEP); 
pathA=0.2*geo.DEP; pathA = geo.RTwd;

//p&d A. r0.096'geo. STwid; pathA. b-geo. STrd.
pathB r0.8'geo. STmd; pathB. b-goo STmd;
float eDmA=dipse(pathA7.

second k**. puh6(pathdmn gec )
{ 

double A[6] = 

3.837* LENGTHA*geo.DEP;
A[5] = 0.136136*LENGTHA*geo.DEP; 
}

void padd k**. puh6(pathdmn gec )
{ 

double A[6] = 

3.837* LENGTHA*geo.DEP;
A[5] = 0.136136*LENGTHA*geo.DEP; 
}
void pathLA::UpshA(pathLkA geo)
{
    AL[0] = geo.STpwid"(geo.DEP*1e-6/10);
    AL[1] = geo.STphtg"1e-3;
    double lenA, lenB;
    COmp x=COmp1;
    CComp y=COmp2;
    CComp z=COmp3;
    CComp A=geo.shSTpx;
    CComp B=geo.shSTpy;
    CComp A=geo.mhSTpx;
    CComp B=geo.mhSTpy;
    lenA=pythag(CComp geo.STta);
    lenB=pythag(CComp geo.STti);
    AL[1] = (lenA + lenB)"lenA)*0.5e-3;
    lenA=pythag(geo.STta, geo.STti);
    AL[1] = (lenA + lenB)"lenA)*0.5e-3;
    geo pathA;
    pathA.x = 0.099"geo.STwid; pathA.y = geo.SThtg;
    geo dimA = sfpx(pathA);
    COmp x=geo.STta, COmp y=geo.STti;
    lenA=pythag(CComp, CComp);
    AL[1] = (dimA + lenA)"lenA)*0.5e-3;
    lenA=pythag(geo.STta, geo.STti);
    AL[1] = (dimA + lenA)"lenA)*0.5e-3;
    geo pathA, pathB;
    pathA = geo.STwid, pathB = geo.SThtg;
    geo dimA = sfpx(pathA);
    float lenA = sfpx(pathB);
    AL[1] = (lenA + lenA)"lenA)*0.5e-3;
    lenA = sfpx(geo.STta, geo.STti);
    AL[1] = (lenA + lenA)"lenA)*0.5e-3;
    float lenA = sfpx(geo.STta, geo.STti);
    AL[0] = geo.STphtg"1e-3;
    AL[1] = geo.STphtg"1e-3;
    double lenA, lenB;
    COmp x=COmp1;
    CComp y=COmp2;
    CComp z=COmp3;
    CComp A=geo.STpx;
    CComp B=geo.STpy;
    CComp A=geo.mhSTpx;
    CComp B=geo.mhSTpy;
    lenA=pythag(CComp geo.STs);
    lenB=pythag(CComp geo.STs);
    AL[1] = (lenA + lenB)"lenA)*0.5e-3;
    lenA=pythag(geo.STs, geo.STs);
    AL[1] = (lenA + lenB)"lenA)*0.5e-3;
    geo pathA, pathB;
    pathA = geo.STwid, pathB = geo.SThtg;
    geo dimA = sfpx(pathA);
    geo dimB = sfpx(pathB);
    AL[1] = (dimA + dimB)"dimB)*1e-3;
    AL[1] = (dimA + dimB)"dimB)*1e-3;
    double LENGTHA;
    void pathLA::UpshB(pathLkA geo)
    {
        AL[0] = go
void pathLA::Upath1(pathdim geo)
{
    AL[0].A=geo.STpdim*geo.DEP*1e-4/10;
    AL[0].L=geo.STpht+1e-5;
    double lenA, lenB;
    lenA=pythag(geo.STStp, geo.STStt);
    COOr COMpl, COmp1;
    COMpr.prene=STStp*(geo.STpdiv/1e5); COMpr wy=geo.STStp y;
    lenB=pythag(COMpr, geo.STStt);
    AL[1].L=(0.5*lenA+lenB)+lenA*0.5e-3;
    lenA=pythag(STStt, geo.STStt);
    AL[1].A=(0.5*lenA*(geo.STpdiv/1e5)+0.5*geo.DEP*1e-6;
    COmp.x=0; COmpl.y=0; COMpl.x=(0.4814*lenA*sgin(geo.BETA));
    COmp.y=0.1183*geo.STrad+geo.STwid;
    lenB=pythag(COMpr, COMpl);
    AL[1].L=(0.5*lenA*sgin(geo.BETA)+lenB)*0.5+geo.STrad/1e3;
    AL[1].A=(32*lenA/50*(1+geo.STrad/1e3)*0.5*geo.DEP*1e-6;
    double LENGTH, LENGTH;
    LENGTH=geo.RTrad+geo.RThgt;
    LENGTHB=geo.RTrad+geo.RTwed;

    AL[1].L=LENGTH; pathA. b=0.5*LENGTH;
    AL[1].L=0.216**LENGTH;
    float dimA=dispath(pathA);
    float dimB=dispath(pathB);
    AL[2].L=dimA+dimB+1.81666*LENGTH+0.5177*geo.RTrad/1e3;
    AL[3].A=(1.2878*geo.RTrad+0.9438*geo.RTwed+geo.RThgt)*geo.DEP*0.5e-6;
    //AL[9].A=2;
    //AL[4].A=1;
    AL[4].A=1.533*LENGTH*geo.DEP*1e-4;
    AL[5].L=0;
    AL[5].A=0;
    AL[6].A=0;
}


void pathLA::Utooth1(pathdim geo)
{
    //set up ellipse function
    float pathA.p, pathB.p;
    //THIS COMPUTES THE PATH LENGTH AND DISTANCES
    //TOOTH ar-gap tooth mid left
    float Cupy=geo.RTrad-0.5*geo.STwed;
    pathA.cpy=geo.RTrad+geo.STrad+geo.RTwed+geo.STrad/0.214;
    pathA.cpy=geo.RTrad+geo.STrad/0.214;
    float dimA=dispath(pathA);
    pathB.cpy=geo.RTrad+geo.RTwed/0.6111;
    pathB.cpy=geo.RTrad+geo.RTwed/0.6111;
    float dimB=dispath(pathB);
    pathC.cpy=geo.RTrad;
    pathC.cpy=geo.RTrad;
    float arc=dispath(pathC);
    Ta[0].A=(dimA+dimB-cpy)*0.5e-3;
    Ta[0].A=(0.525*geo.RTwed+0.9163*geo.STrad+0.3839*geo.STrad)
        *geo.DEP*0.5e-4;
    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    //TOOTH ar-gap tooth mid right
    float Cupy=geo.RTrad;
    pathB.bpy=geo.RTrad+geo.RTwed; pathB.bpy=geo.RTrad+geo.RTwed;
    pathB.bpy=geo.RTrad+geo.RTwed;
    float dimB=dispath(pathB);
    pathC.bpy=geo.RTrad;
    pathC.bpy=geo.RTrad;
    float arc=dispath(pathC);
    Ta[0].L=(dimA+dimB-cpy)*0.5e-3;
    Ta[0].L=(0.525*geo.RTwed+0.9163*geo.STrad+0.3839*geo.STrad)
        *geo.DEP*0.5e-4;
void pathLA::Utooth2(pathdun goo)
{
    // set up ellipse function
    // pathA, pathB,

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH arc-gap both left right
    float Gap=geo.RTad-0.5*geo.STwid;
    pathA=geo.uAGlan+geo.RThgt*0.0254+geo.STwid*0.01667*geo.RThgt;
p0 = geo.ADEP*0.5e-6;
    pathB = geo.STwid*0.1271+Gap;
    pathB=1.33333*geo.RThgt+geo.uAGlan;
    dimA=::ellipse(pathA);
    Talj2::L(dimA+dimB*0.5e-3);
    Talj3::L(geo.RThgt+0.101695*geo.STwid)
p0 = geo.ADEP*0.5e-6;
p0 = geo.STwid*0.322+Gap;
    pathA=1.33333*geo.RThgt+geo.uAGlan;
    dimA=::ellipse(pathA);

    arc-geo.RThgt*0.285;
    Talj4::L(dimA+dimB*0.5e-3);
    Talj4::A=0.18604*geo.STwid;
    0.64141*geo.RTad;
p0 = geo.ADEP*0.5e-6;

    Talj5::L=0;
    Talj5::A=0;
    empty=1;
}

//void pathLA::Utooth2(pathdun)
{
    // set up ellipse function
    // pathA, pathB,

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH arc-gap both left right
    float Gap=geo.RTad-0.5*geo.STwid;
    pathA=geo.uAGlan+geo.RThgt*0.0254+geo.STwid*0.01667*geo.RThgt;
p0 = geo.ADEP*0.5e-6;
    pathB = geo.STwid*0.1271+Gap;
    pathB=1.33333*geo.RThgt+geo.uAGlan;
    dimA=::ellipse(pathA);
    Talj2::L(dimA+dimB*0.5e-3);
    Talj3::L(geo.RThgt+0.101695*geo.STwid)
p0 = geo.ADEP*0.5e-6;
p0 = geo.STwid*0.322+Gap;
    pathA=1.33333*geo.RThgt+geo.uAGlan;
    dimA=::ellipse(pathA);

    arc-geo.RThgt*0.285;
    Talj4::L(dimA+dimB*0.5e-3);
    Talj4::A=0.18604*geo.STwid;
    0.64141*geo.RTad;
p0 = geo.ADEP*0.5e-6;

    Talj5::L=0;
    Talj5::A=0;
    empty=1;
}
```cpp
void pathLA::Utooth3(pathdim geo)
{

    //set up ellipse function
    int pathA, pathB;

    //THIS COMPUTES THE PATH LENGTH AND DISTANCES
    //TOOTH air-gap gap left and tooth left
    float Gap=geo.RTrad-0.5*geo.STwid;

    pathA.arm=geo.uAGlen+geo.STmdT*0.20833;
    pathA.b=Gap; geo.RTrad=0.5*geo.STwid;
    float pathA=ellipse(pathA);
    pathB.arm=geo.uAGlen+geo.STmdT*0.3;
    pathB.b=Gap; geo.RTrad=0.5*geo.STwid;
    float pathB=ellipse(pathB);

    Tal[0]=(-clenA+clenB)*0.5-3;
    Tal[0]=A=0.144*geo.STmdT+0.5714*geo.RTrad)
    "geo.ADEP":0.5e-6;

    float Len=lm(clenA*Gap)+
               (geo.STmdT*0.20833*geo.uAGlen+0.12*geo.RTrad)
               (geo.STmdT*0.3*geo.uAGlen+0.12*geo.RTrad);

    Tal[1]=(-clenA+clenB)*0.5-3;
    Tal[1]=A=0.714*geo.RTrad+0.5666*geo.STmdT
    "geo.ADEP":0.5e-6;

    float pathA=geo.RTrad*1.0625+geo.uAGlen;
    pathA=Gap+0.05714*geo.STwid;
    float pathA=ellipse(pathA);
    Tal[2]=(-clenA*Gap)*0.5-3;
    Tal[2]=A=0.833*geo.RTrad+
               (geo.uAGlen+1.5708*geo.RTrad*0.08163*geo.STmdT)
               "geo.ADEP":0.5e-6;

    float arc=0.187*geo.RTrad;
    Tal[3]=(-clenA+clenB*arc)*0.5-3;
    Tal[3]=A=0.044*geo.RTrad+
               (geo.uAGlen+1.5708*geo.RTrad)
               "geo.ADEP":0.5e-6;

    Tal[4]=0;
    Tal[4]=A=0;

    Tal[5]=0;
    Tal[5]=A=0;

    empty2;
}

void pathLA::Utooth4(pathdim geo)
{

    //set up ellipse function
    int pathA, pathB;

    //THIS COMPUTES THE PATH LENGTH AND DISTANCES
    //TOOTH air-gap gap left and tooth right
    float Gap=geo.RTrad-0.5*geo.STwid;

    pathA.arm=geo.uAGlen+geo.RTrad*0.74;
    pathA.b=Gap; geo.RTrad=0.5*geo.STwid;
    float pathA=ellipse(pathA);
    pathB.arm=geo.uAGlen+geo.RTrad*0.6111;
    pathB.b=Gap; geo.RTrad=0.5*geo.STmdT;
    float pathB=ellipse(pathB);
    float arc=0.1707*geo.RTrad;

    Tal[0]=(-clenA+clenB*arc)*0.5-3;
    Tal[0]=A=0.2474*geo.STmdT+0.9948*geo.RTrad
    "geo.ADEP":0.5e-6;

    float Len=geo.uAGlen+1.5708*geo.RTrad+0.3625*geo.RTrad*arc)
    "geo.ADEP":0.5e-3;

    Tal[1]=(-clenA+clenB)*0.5-3;
    Tal[1]=A=0.2474*geo.STmdT+0.9948*geo.RTrad
    "geo.ADEP":0.5e-6;

    float Len=geo.uAGlen+1.5708*geo.RTrad+0.3625*geo.RTrad*arc)
    "geo.ADEP":0.5e-3;
```
Tal[1].A=(geo.RTrad*0.3133+0.875*geo.RThgt*0.15464*geo.STwd)
*geo.ADEP*0.5e-6;

//
Lem=sgi*(Gap*(geo.uAGlen+0.13*geo.RThgt))
*geo.ADEP*0.5e-6;
Tal[2].L=(L lem+dimA)*0.5e-3;
Tal[2].A=(0.375*geo.RThgt*0.05925*geo.STwd+0.03333*geo.STht)
*geo.ADEP*0.5e-6,
pathB=(0.266*geo.RThgt*0.26)/Gap;
pathB=(0.07246*geo.RThgt*0.131*geo.RThgt*0.15464*geo.STwd)
*geo.ADEP*0.5e-6;
Tal[4].A=(0.2*geo.RThgt*0.07246*geo.STwd)
*geo.ADEP*0.5e-6;

void pathLA::Utooth5(pathdun goo) {
//set up ellipse function

//THIS COMPUTES THE PATH LENGTH AND DISTANCES
//TOOTH air-gap right end tooth right
float Gap=geo.RTrad*0.3*geo.STwd;
pathA=(geo.uAGlen*geo.RThgt)*0.3;
pathB=(geo.RThgt*0.48*geo.RTrad);
float dimA=ellipse(pathA);
pathB=(geo.uAGlen*geo.RThgt)*0.11;
pathB=(geo.RThgt*0.146*geo.RTrad);
float dimB=ellipse(pathB);

Tal[0].L=(dimA+dimB)*0.5e-3;
Tal[0].A=(0.1 96*geo.RThgt*geo.RTrad*0.33227)
*geo.ADEP*0.5e-6;

Tal[2].L=0;
Tal[2].A=0;
empty=1;
}
void pathLA:: Utooth6(pathdim gao)
{
    //set up ellipse function
    axis pathA, pathB;

    // THIS COMPUTES THE PATH LENGTH AND DISTANCES
    // TOOTH air-gap right and tooth left
    float Gap=gao.RTad-0.5*geo.STwid;
    float arcw=0.148*geo.RTrad;
    pathA.a=geo.uAGlen-geo.RHgt+geo.RTrad*1.19;
    pathA.b=Gap+0.061*geo.STwid;
    float elI=ellipse(pathA);
    pathB.a=geo.uAGlen+0.19*geo.RTrad;
    pathB.b=Gap+0.16*geo.RTrad;
    float elII=ellipse(pathB);
    float arcB=0.043*geo.STrad;

    Tal[0].L=(elI+elII-arcw-arcB)*0.5e-3;
    Tal[0].A=(0.225*geo.S7wid+1.047*geo.RTrad)*geo.ADEP*0.5e-3;

    pathA.a=geo.uAGlen+0.5*geo.RHgt;
    pathA.b=Gap+0.04*geo.STwid;
    elI=ellipse(pathA);
    Tal[1].L=(elI+elII)*0.5e-3;
    Tal[1].A=(geo.RTrad*0.18+0.5*geo.RHgt+0.12698*geo.SFwid)*geo.ADEP*0.5e-3;

    pathB.a=(0.4318*geo.RHgt+geo.uAGlen);
    pathB.b=(0.088*geo.RTrad+Gap);
    elII=ellipse(pathB);
    Tal[2].L=(elII+elI)*0.5e-3;
    Tal[2].A=(0.088*geo.RTrad+0.49*geo.RHgt+0.5454*geo.SFwid)*geo.ADEP*0.5e-6;

    pathA.a=(geo.RTrad/3)+Gap/10;
    pathB.b=(0.4257*geo.RTrad+geo.uAGlen+geo.RHgt/10);
    elI=ellipse(pathA)*10;
    float arcw=0.115*geo.STrad;
    Tal[3].L=(elI+elII)*0.5e-3;
    Tal[3].A=(0.2233*geo.RTrad+0.5454*geo.SFwid)*geo.ADEP*0.5e-3;

    Tal[4].L=10;
    Tal[4].A=0;

    empty=1;
}

//function to calculate reluctance
void pathLA::argsgRELQ //calculate air gap reluctance
{
    double INVtmpR=0;
    for (int te=0; te<empty; te++)
    {
        INVtmpR += Tal[te].A*MUX/Tal[te].L; //sum the permanence
    }
    AGrd=1/INVtmpR;
}
EVENTUALLY put a check so that STwid>RTwid
if (geo.STwid>geo.RTwid) {
  Mwid=geo.STwid;
  Mhgt=geo.SThgt;
  mwid=geo.STwid;
  mwidgeo SThgt;
}
else {
  Mwid=geo.RTwid;
  Mhgt=geo.RThgt;
  mwid=geo.RTwid;
  mwidgeo RThgt;
}

This could be in the end of the dimension def,
therefore these commands would have to be geo.Mwid..etc.
```c
#include "com.h"
#include "math.h"
Curve main(double x[DIS],double y[DIS])
{
    int COUNT=0;
    //rows columns
    //initial guess may do work on initial guess for brach function!
    Curve RESULT;
    int mid=DIS/2; //mid point
    double iALP=pow(x[0],2)/pow(y[0],2); cout<<"Alpha"<<iALP;
    double iBET=pow(y[0],2); cout<<"Beta"<<iBET;
    double N=1;
    for (int p=0; p<DIS, p++)
    {
        SumSQ=pow(y[i],2);
        (iALP*pow(y[i],2))/pow(iBET+y[i],2);
    }
    if (fabs(iALP-iALP<DIS/2) && fabs(iBET+iBET<DIS/2))
        double T1[4][4];
    double T2[4][4];
    ///
    double EQ1[4];
    double EQ2[4];
    for (i=0;i<4, i++)
    {
        EQ1[i]=T1[2][2][0][0]T2[2][2][0][0];
        EQ2[i]=T1[1][1][0][0];
    }
    double EQ3[4];
    for (i=0; i<4, i++)
    {
        EQ3[i]=T1[1][1][0][0]T2[1][1][0][0];
    }
    double EQ4[4];
    for (i=0; i<4, i++)
    {
        EQ4[i]=T1[1][1][0][0]T2[1][1][0][0];
    }
    /// solve systems of equations for D.
    double EQ1[4], EQ2[4];
    for (i=0, i<4, i++)
    {
        EQ1[i]=T1[2][2][0][0]T2[2][2][0][0];
        EQ2[i]=T1[1][1][0][0];
    }
    double EQ3[4];
    for (i=0, i<4, i++)
    {
        EQ3[i]=T1[1][1][0][0]T2[1][1][0][0];
        EQ4[i]=T1[1][1][0][0];
    }
    double EQ4[4];
    for (i=0; i<4, i++)
    {
        EQ4[i]=T1[1][1][0][0]T2[1][1][0][0];
    }
    double D[4];
    D[4]=EQ1[4]*EQ2[4];
    D[1]=EQ2[1]*EQ2[1];
    D[0]=EQ1[0]*EQ1[0];
    //
    try
    {
        double alp1, bet1; alp1=pow(x[1],2)/pow(y[1],2); bet1=pow(y[1],2); cout<<"Alpha"<<alp1;
        double Q=0; double Q=0; double Q=0;
        for (i=0; i<DIS, i++)
        {
            Vmin=(0.5*2*(Q-Q1)*(Q1-Q))<0;
        }
    }
    catch (e)
    {
        cout<<"Error: "<<e<<endl;
    }
```
double NEWALP = iALP + Vmin * D[1];
double NEWBET = iBET + Vmin * D[1];
double NEWN = N + Vmin * D[2];

//sum of squares of residuals
SumSQ = 0;
for (i=0; i<DIS; i++) {
    SumSQ += pow(YG1d[NE WALP* pow(x[i], NEWN)] / (NEWBET + pow(x[i], NEWN))), 2);
}

//End convergence percentage
double Ealp = (NEWALP - iALP) * 100; Ealp = fabs(Ealp);
double Ebet = (NEWBET - iBET) * 100; Ebet = fabs(Ebet);
double En = (NEWN - iN) * 100; En = fabs(En);

//converge check
if (Ealp < 1e-5 && Ebet < 1e-5 && En < 1e-5) COUNT = 55;

RESULT = NEWALP; RESULT3 = NEWBET; RESULT = NEWN;
//while (COUNT < 50);
return RESULT;

#include "com.h"

// LINKlist

double LINKlist::PLOT(int position)
{
    LINK* current=first;
    double value;
    for (int i=0; i<position; i++)
    {
        current=current->next;
    }
    value=current->data;
    return value;
}

void LINKlist::addTail(FluxLink d)
{
    LINK* newlink=new LINK;
    newlink->data=d;
    newlink->next=first;
    first=newlink;
}

void LINKlist::addAdd(double t)
{
    LINK* newlink=new LINK;
    newlink->data=t;
    newlink->next=first;
    first=newlink;
}

double LINKlist::detent()
{
    LINK* current=first;
    double mmf=current->data.mag;
    return mmf;
}

double LINKlist::moment(int NUP, double cNC, double offset, double Mx)
{
    double total=0;
    double b=0;
    double x=0;
    LINK* current=first;
    while (current != NULL && a != Mx)
    {
        a=-(cNC/2);
        b=-(current->data.al)*(current->data.un)+offset;
        current=current->next;
        x=x+1;
        total+=b*x*(x*x+2);  
    }
    return total;
}

void LINKlist::finaltorque(LINKlist A, LINKlist B)
{
    double total;
    double keep=0;
    LINK* currentA=A.first;
    LINK* currentB=B.first;
    while (currentA != NULL && currentB != NULL)
    {
        total+=100*(currentA->data.currentB->data)/M_PI;
        keep=total;
        LINK* newlink=new LINK;
        newlink->data=keep;
        newlink->next=first;
        first=newlink;
        currentA=currentA->next;
        currentB=currentB->next;
    }
}
double flux(Curve, double); // function to calculate flux

double flux(Curve equ, double MMF)
{
    double fluxi = equ.a * pow(MMF, equ.n) / (equ.b + pow(MMF, equ.n));
    return fluxi;
}

double dflux(Curve equ, double MMF)
{
    double fluxi = equ.a * pow(MMF, equ.n) / (equ.b + pow(MMF, equ.n));
    return fluxi;
}

double coENG
(float width, int turns, double AL1, double UN1, double AL2, double UN2,
int reg, double offset)
{
    if (reg == 1) { a = 1; b = 0; }
    double AREA = (0.5 * width * turns * (AL1 - UN1 + offset) + (AL2 - UN2 + offset));
    return AREA;
}

 FluxLink get(Curve BIMAG, Curve MMFM, Curve midFM, Curve unfFM,
double MMFMag, Curve magnet, double MMFMagDouble, double areaMAG, double Ho,
float current, mt tutus mt pot)
{
    double ornltlums;  
    double coilb;
    double coilcl;
    double coilc2;
    double coild;
    double coilbm;
    double coilcn;
    int flagb=0;
    int flagc=0;
    int flagd=0;
    double mmf=200;
    double mmfn;
    double con.
    do(
        // BRANCH A (magnet and backiron relationship)
        double MMFoorr = MMFMag - (-1 * mmf);
        double flmdNAG = flux(BIMAG, MMFoorr);  
        double dfluxMAG = aux(BIMAG, MMFoorr).  
        // BRANCH B (so called aligned branch) //MmFM. equ
        if (pos == 2) { coilb = coil; else coilb = 0; }  
        if (((mmf > 0) || flagb) == 1) { flagb = 0; 
            double mmfc = fabs(mmf + coilb);  
            double fluxb = flux(MFM, mmfc);  
            if (flagb == 0) fluxb = fluxb * -1;  
            double dfluxb = flux(MFM, mmfc);  
        }  
        // BRANCH C (so called mid branch 1) //MmFM. equ
        if (pos == 2) { coilc1 = coil; else coilc1 = 0; }  
        if (((mmf > 0) || flagc) == 1) { flagc = 0; 
            double mmfc1 = fabs(mmf + coilc1);  
            double fluxc1 = flux(MFM, mmfc1);  
            if (flagc == 0) fluxc1 = fluxc1 * -1;  
            double dfluxc1 = flux(MFM, mmfc1);  
        }  
        // BRANCH C2 (so called mid branch 2) //MmFM. equ
        if (pos == 2) { coilc2 = coil; else coilc2 = 0; }  
        if (((mmf > 0) || flagc) == 1) { flagc = 0; 
            double mmfc2 = fabs(mmf + coilc2);  
            double fluxc2 = flux(MFM, mmfc2);  
            if (flagc == 0) fluxc2 = fluxc2 * -1;  
            double dfluxc2 = flux(MFM, mmfc2);  
        }  
        // BRANCH D (so called un branch) //MmFM. equ
        if (pos == 2) { coild = coil; else coild = 0; }  
        if (((mmf > 0) || flagd) == 1) { flagd = 0; 
            double mmfd = fabs(mmf + coild);  
            double fluxd = flux(MFM, mmfd);  
            if (flagd == 0) fluxd = fluxd * -1;  
            double dfluxd = flux(MFM, mmfd);  
        }  
    } while (1);
void patLA::egnewfun(FuncMmf Fm[DIS], double addfflux[DIS])
{
    int inc=0;
    double bh[1];
    bh[0]=alp
    bh[1]=beta
    bh[2]=m
    bh[3]=e
    bh[4]=k

    do {
        H=F[inc].m;
        addfflux[inc] = ((bh[0]*pow(H,bh[2])+bh[1]*pow(H,bh[2]+1))/
        (bh[1]-pow(H,bh[2])+bh[3]*pow(H,bh[2]+1)),
        inc++;
    }while (inc<DIS);
}
# Branch A (magnet and back-wound relationship)

```c
double MMFcorr = MMagMF * (1 - mmfu);
double fluxMag = flux(MaveBIMAG, MMFcorr);

double fluxBL = flux(MmagMF, mmfu);
double fluxBU = flux(MmagMF, mmfu);

double AB2 = fluxBL + 2 * fluxBU + fluxBU;
double AB4 = 0.5 * (fluxBL - fluxBU);

double fluxBLMag = flux(MaveBIMAG, MMFcorr);

do {  
  if (flag == 0) {  
    double mmfu = 50;  
  } else {  
    double mmfu = (MMFcorr - MMFcorr) / (fluxBL - fluxBU);  
  }

  double fluxMAG = flux(MaveBIMAG, MMFcorr);

  if (fluxMAG < 0) {  
    mMag = fluxMAG + fluxMAG;  
  } else {  
    mMag = fluxMAG - fluxMAG;  
  }

  count++;  
  } while (count < 100 & & con > 1e-5);  
```

Details:
- BRANCH A:
  - Flux Linkage Calculation
  - Magnet and Back-wound Relationship
  - Iterative Process
  - Flux Magnitude Calculation
  - Threshold Conditions
if(flagd==1) fluxd=fluxd'-1;
  double dfllux=dx(magnFMe,mmfd);

  // find magnet mmf // not important
  // flux flowing through magnet
  double MMFcoa=MbimagMMF+(-1*mmf);
  double fluxMAG=flux(M=veBIMAG, MMFcoa);
  count=0;
  while (count<100 && count>1.5);
  //find magnet mmf // not important
  // flux flowing through magnet
  double iguessH=2*mmfl(1 o-3'magMAG))FHo; //H guess using mmf calculated

  // BRANCH B (so called aligned branch) //MbmFM equs
  if (pos=0 || pos=2) coilb-coil, else coilb-0;
  if((mmf+coilb)<o) flagl'l; else flagb-0;
  double mmfb=fabs(mmf+coilb);
  double fluxb=flux(M=veFMe, mmfb);
  if (flagb=1) fluxb=fluxb'-1;
  if (pos=0 || pos=2) details ahßuxb. else details midl-fuxb;

  // BRANCH C1 (so called mid branch 1) //MbmFM equs
  if (pos=1 || pos=2) coilc=coil, else coilc=0;
  if((mmf+coilc)<o) flagc'=1; else flagc=0;
  double mmfc1=fabs(mmf+coilc);
  double fluxc=flux(M=veFMe, mmfc1);
  if(flagc=1) fluxc=fluxc'-1;
  if (pos=1) details ahfluxc.else details mid2-fuxc;

  // BRANCH C2 (so called mid branch 2) //MbmFM equs
  if (pos=1 || pos=2) coilc2=-1*coil, else coilc2=0;
  if((mmf+coilc2)<o) flagc2=1; else flagc2=0;
  double mmfc2=fabs(mmf+coilc2);
  double fluxc2=flux(M=veFMe, mmfc2);
  if(flagc2=1) fluxc2=fluxc2'-1;
  if (pos=0) details ahfluxc.else details mid2-fuxc;

  // BRANCH D (so called un branch) //MbmFM equs
  if (pos=0 || pos=2) coilc=coil, else coilc=0;
  if((mmf+coilc)<o) flagd=1; else flagd=0;
  double mmfd=fabs(mmf+coilc);
  double fluxd=flux(M=veFMe, mmfd);
  if(flagd=1) fluxd=fluxd'-1;
  if (pos=0 || pos=2) details ahfluxc.else details mid2-fuxc;
  return details;
}
APPENDIX D

Data sheets for soft magnetic composite materials used in chapter 7.
### Powder Composite Material—1172

#### Physical Properties

- **Density**: 7.37 g/cm³
- **Electrical resistivity**: 0.5 – 10⁴ ȍ cm
- **Young's modulus**: 40–70 kN/mm²
- **Compression strength**: 240 N/mm²
- **Bending strength**: 50 – 100 N/mm²
- **Coeff. of linear expansion**: 12–14 10⁻⁶/K (20° bis 100°C)
- **Thermal conductivity**: 6–15 W/m·K
- **Permissible operating temperature**: max. 150 °C

#### Magnetic Properties

- **Permeability (40 mT)**: 115
- **Max. permeability**: 190
- **Saturation flux density**: 1.90 T
- **Coercivity**: 4.10 A/cm
- **Specific core losses**: 23 μWs/cm³ (0.1 T; 1 kHz)
- **Core losses (1 T; 50 Hz)**: 60 mW/cm³

#### Data Sheets of Other Materials

Data sheets of other materials upon request!
ABM100.32 is an insulated iron powder developed for soft magnetic applications for frequencies < 10 kHz. The powder's high compressibility and durable insulation gives high induction and low losses in combination with constant permeability over a broad frequency range.

The special insulation also allows annealing which gives very low losses at frequencies < 1000 Hz. The distributed air gap in the material combined with the cost-effective and three-dimensional shape making capability of P/M technology offers flexibility for designers.

**MIXES**
1. ABM 100.32 + 0.5% Kenolube + 0.5% Phenolic Resin
2. ABM 100.32 + 0.5% Kenolube

**TYPICAL DATA**
- Apparent density 3.2 g/cm³
- Flow 22.6 %/s

<table>
<thead>
<tr>
<th>Mix 1</th>
<th>Mix 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>GD(600 MPa), g/cm³</td>
<td>7.18</td>
</tr>
<tr>
<td>CD(600 MPa), g/cm³</td>
<td>7.09</td>
</tr>
</tbody>
</table>

**HEAT TREATMENT**
- Curing 150°C, 1 h in air
- 150°C, 1 h in air
- Annealing 500°C, 0.5 h in air

<table>
<thead>
<tr>
<th>Mix 1</th>
<th>Mix 2</th>
<th>Mix 1</th>
<th>Mix 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength (TRS), N/mm²</td>
<td>27</td>
<td>20</td>
<td>63</td>
</tr>
<tr>
<td>Dimensional change, %</td>
<td>-0.025</td>
<td>-0.002</td>
<td>0.03</td>
</tr>
<tr>
<td>Bₘₐₓ</td>
<td>800 MPa</td>
<td>225</td>
<td>290</td>
</tr>
<tr>
<td>600 MPa</td>
<td>265</td>
<td>365</td>
<td></td>
</tr>
</tbody>
</table>

ABM 100.32

Höganäs

The Difference in Knowledge
Manufacturing Conditions

Powder
ABM100.32 is an insulated sub 150 micron powder.

Manufacturing
The test toroids were manufactured from mixes of ABM100.32, lubricant and phenolic resin as indicated.
Compacting pressure: 600 MPa and 800 MPa.

Heat Treatment
Curing
All test specimens were cured at 150°C for 1 hour in air.
Annealing
After curing an annealing was performed at 500°C for 30 minutes in air.

Testing Conditions
Physical properties were determined in accordance with applicable ISO standards.
Magnetic properties were measured on toroids with following dimensions
Outer diameter 55 mm
Inner diameter 45 mm
Height 5 mm

Core Loss (Frequency)

Core Loss (Frequency) at 50 Hz

Höganäs
The Difference is Knowledge
Höganäs AB, S-263 83 Höganäs, Sweden
Tel. +46-42 330000, Fax +46-42 331150