Form-finding as a modelling tool for shaping mechanical components. A feasibility case study of an axial-flow compressor blade

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Abstract

This paper reports on an exploratory study to assess the capability of a novel, form-finding methodology for generating optimal shapes of shell-type structures subjected to complex load regimes, using an axial-flow, compressor blade as a case study. The methodology exploits the natural principle of 'form follows force', in which the structural form is shaped according to the forces acting on it. Such forms, or objects, which are found in nature, are known to have optimal strength and stiffness characteristics for a predominant load regime. Our methodology makes use of a direct relationship between form and force, offered by the Laplace-Young equation that describes shapes of minimal surface membranes, such as soap films. Unlike structural optimisation in which the form is modified within its initially prescribed shape, form-finding literally finds the shape within prescribed boundary conditions. This is the first application of the methodology to modelling mechanical components, such as a compressor blade. The blade shapes obtained by this method correspond, in the first instance, to a minimal form and, subsequently, a minimal form subjected to a known (empirically determined) pressure profile. The behaviour of the blades is studied under a variety of loads and includes modal analysis. In view of the methodology adopted, attention is directed to structural performance. The results, compared against a 'control' blade produced by a conventional design/optimisation method are very encouraging; they indicate that the proposed methodology has the potential to improve significantly the current blade design process.

Keywords: form-finding; minimal forms; compressor blade

1. Introduction

The aim of this exploratory study is to assess the potential of a new 'form-finding' methodology for generating optimal shapes of shell-type structures. In order to set up a severe enough test for the proposed method, we have chosen, as a case study, an axial-flow compressor blade design. Our form-finding approach is inspired by the observation that objects found in nature are highly optimised in terms of maximum stability, strength, and minimum weight. These objects are a consequence of a response to typical environmental forces[1,2] , so they conform to the principle of "form follows force". We have implemented this natural principle in the form-finding methodology, which employs an iterative computational process aimed at shaping the component according to a set of forces/pressures acting on it. The results of the study are compared against a 'control'- an optimised blade profile for which the design data, including the blade geometry and aerodynamic pressures, were made available by our industrial partner.

The shape of a gas turbine blade represents a complex balance between aerodynamic performance, structural stability and strength, and manufacturing requirements. The blade has to withstand the operational stress created by temperature, centrifugal, aerodynamic and impact loading. Recent failures due to impact loads e.g., bird strike, or runway stone projectiles have brought this type of loading to the forefront of blade design considerations[3,4].

Compressor blades are a subject of continual research aimed at providing novel solutions, such as a rotor to rotor configuration replacing the conventional rotor to stator[5], or attachment of composite blades in span-wise compression to a rotating casing[6]. Continual pressure to strengthen the competitiveness of the aero-engine industry by providing an improved performance at reduced weight and a reduced design time stimulated research into *value engineering*. Previously, the design approach has been to compute the performance of an aerodynamic profile first, as described by Denton[7,8]. However, with *value engineering*[9], the objective is to move from a CAD (Computer Aided Design) to the finished blade with a minimum number of manufacturing stages. In this respect, our proposed methodology offers a real advantage, in terms of the reduction in the number of iterations between aerodynamicists and structural engineers.

2. Research methodology/solution

2.1 General considerations

The form-force interaction is clearly visible in lightweight tension structures, such as sails, balloons, or soap films. Soap films are membrane structures, which, as a result of their low surface density and weight, plus the absence of significant shear stresses, have a constant tension field. Geometrically, they are characterised by a minimum surface area and equal and opposite curvatures at any point on the surface[10]. In a rigid structure, the equal and opposite curvatures contribute to uniform stiffness.

The relationship between inflation pressure, surface tension and curvature of a soap film surface is given by the Laplace-Young equation[11,12]:

$$T\,(1/R_1+1/R_2) = p \qquad (1)$$

where R_1 and R_2 are the radii of principal curvature, T is surface tension and p the pressure difference across the surface.

The above equation describes the transversal equilibrium for a homogeneous tension field. If p is not equal to zero, the equation describes a pneumatic membrane structure (a soap bubble).

The theoretical model adopts a soap film-type structure in finding the configurations of upper and lower surfaces of the blade and makes use of the relationship between pressure and surface curvature, as given by the principle of ‘minimality’ represented by the Laplace-Young equation. This equation is exploited in two ways:

(i) It is assumed that the pressure p is zero and hence, with T being non zero and constant, the mean curvature given as $½(1/R_1+1/R_2)$ is zero and, as a result, the so called minimal surface configuration is obtained (section 2.2). This configuration gives rise to the ‘minimal’ blade family of shapes.

(ii) It is assumed that the pressure p is non zero and, as a result, the so called ‘pressure’ surface configuration is obtained. Here, the ‘minimal’ blade profile is subjected to a known airfoil pressure, which gives rise to the ‘pressure’ blade family of shapes (section 2.3). Whether derived by a study of Joukowski aerofoil or by empirical determination of pressure distribution over an optimised shape, it is always found that the highest fluid speed over an airfoil, and therefore highest suction, correlates with the regions of more pronounced curvature, on the forward part of the surface. Consequently, applying a known pressure profile (with suitable smoothing and scaling of empirically determined data) to a flexible membrane, will, as a consequence of the Laplace-Young equation, produce a shape with appropriate curvature gradients.

In each of the two cases described above, the blade shape is ultimately generated by converting the top and bottom surfaces (which span the boundaries of their separate edge configurations) into rigid structures filled with a solid material.

The form-finding process begins with the adoption of top and bottom edge configurations extracted from the blade data provided by the industrial partner (Fig. 1). The data relates to a real blade, here referred to as the 'control', whose shape has been determined to achieve both structural and aerodynamic performance.

It is worth noting that, at the form-finding stage, surface idealisation does not consist of finite elements. The surface is described by bi-cubic splines, which makes it smooth and twice differentiable[13,14]. As a result, the geometry of the form-found surfaces represents a semi-analytical, 'whole surface solution' rather than a discrete set of numbers obtained from the usual finite element approaches.

Figure 1. Geometry of the 'control' blade

2.2. Form-finding of 'minimal' blades

The reason for this exercise was to examine the structural and vibrational performance of the blades characterised by equal and opposite curvatures on each of their surfaces. The numerical process of finding a minimal surface amounts to putting the pressure difference, p, in eqn (1) equal to zero and seeking a surface configuration, which, at every point, has the mean curvature, as well as the out-of-balance forces equal to zero. (Alternatively, it is also sufficient to impose the condition of constant tension in the searching algorithm and, provided the out-of-balance forces are zero, the conditions of minimum surface area plus zero mean curvature will be automatically satisfied).

At the start of the numerical procedure, a cloud of points is chosen to lie on a surface contained within a closed contour matching the boundaries of the 'control' blade. Using a smooth curvi-linear u-v grid, cubic splines are fitted to these points to define the initial surface. This initial surface does not match that of the 'control' blade.

The mean curvature is then calculated at the points on the u-v grid. As the initially assumed (guessed) surface is the first, crude approximation to the solution, this surface is unlikely to be minimal, i.e., its mean curvature given by the Laplace-Young equation, would not be zero. This gives rise to a pressure difference p. Integrating p over a small area surrounding each point on the surface grid produces forces operating normal the surface. These, combined with tangential forces arising from a prescribed constant surface tension at these locations, constitute the out-of-balance, or residual forces acting on the surface. An iterative, numerical procedure based on dynamic relaxation[13] (described below) is employed to bring these residual forces to zero. Iterations are necessary, because large changes in surface geometry create a geometrically non-linear problem. When the residual forces are reduced to zero, the resulting geometry is that of a minimal (soap-film) surface configuration.

The dynamic relaxation used in the form-finding process is a finite difference algorithm, which is particularly well suited to finding minimum potential energy configurations[15]. A simple demonstration of this, using a discrete system, is given in Ref [16]. In our case, the problem is well conditioned, since the solution has to satisfy the requirements of: constant surface tension and zero out-of balance forces. Provided the criterion for assured convergence[15] is met, there are no problems with numerical convergence.

Figure 2 shows surface configurations of a selected 'minimal' blades created by the above process. Amongst them, only the 'minimal' blade 1 has the same boundary contours as the 'control' blade; the others have their boundaries modified, as shown in Fig. 2. These modifications were guided by the need to improve the 'minimal' blade profiles, following the results of modal and load analyses, in which it became clear that due to excessive flexibilities, there was a need to increase blade thickness. The only way to do it when working with minimal surfaces is to change the boundary configuration. This was achieved by keeping the four original corner separations unaltered and giving the edge lines around the blade a shallow parabolic profile. A selection of the resulting shapes is presented in Fig. 2.

Figure 2. (a) Description of the 'minimal' blade family (b) Shape contours of selected 'minimal' blades versus the 'control'

Following the form-finding stage, the finite element, load and modal analyses, carried out on the 'minimal' blade family, showed that the 'minimal' blade 7 (which was not the heaviest) exhibited structural performance closest to that of the 'control' blade. The cross-sectional profiles of the blade, plotted against the 'control' blade, are shown in Figure 3. It can be seen that the shape of the' minimal' blade cross-section is significantly different to that of the 'control' blade and this difference in the material distribution is likely to affect vibrational characteristics.

Figure 3. 'Minimal' blade 7. Cross-sectional profiles (solid line), relative to the 'control' (dotted)

2.3 Form-finding of 'pressure' blades

The initial (top and bottom) surface configurations were assumed to be the same as for the 'minimal' blade 1. Starting from this minimal configuration and acknowledging that the shape of a minimal surface blade is not the result of typical loads acting on a compressor blade, the next step was to create a nett pressure profile from the data provided by the industrial partner and apply it as an inflation pressure to the top and bottom surfaces of the soap film model.

The values of the aerodynamic pressures supplied by the industrial partner were found to be varying gradually, except for consistent variations at the leading and trailing edges. The nett pressure is shown in Fig. 4(a) while Figure 4 (b) gives a fitted nett pressure profile, P.

Figure 4. Pressure data - untwisted blade. (a) Nett pressure (b) Fitted nett pressure

Assuming L is the length along the blade in mm measured from the root and W is the distance along the width of the blade in mm, measured from the centreline, a bi-linear trend was fitted to the nett pressure, P giving:

$$P = 5.44\times10^{-3} + W\times6.69\times10^{-5} - L\times1.14\times10^{-5}, \qquad (2)$$

in which the units are in N/m^2 (as supplied by the industrial partner) .

For form-finding purposes, the pressure P (or p in the Laplace-Young equation) acts on a purely notional surface tension, T, in a soap film material. The ratio of P/T affects the shape of the blades. In order to get realistic looking shapes, the value of the pressure P needs to be scaled and split between the top and bottom surfaces of the soap film model. This is achieved using two design parameters: s and k. Thus, the pressure functions are:

$P_T = k\times s \times P$ on the top surface,

and

$P_B = (1-k)\times s \times P$ on the bottom surface,

where $s=1/T$

The parameter k determines the proportion of the total pressure P that is applied to each of the two surfaces, after appropriate scaling of the total pressure using parameter s. A parametric study was carried out to establish appropriate values for k and s, so that the achieved profiles looked sensible. A value of s =1.4, and k in the range 0.7328 -0.7534, gave blade volumes of 93% - 97% of the volume of the 'control' blade. In effect, the mutually dependent parameters allow one to control the volume of the blade.

The process of finding the shape of a 'pressure' blade through an inflated membrane model involved, again, the use of the Laplace-Young equation within the iterative dynamic relaxation procedure. In this case, however, instead of the equilibrium surface being that which corresponds to zero pressure, it is that which corresponds to the applied pressures, P_T and P_B respectively, while maintaining constant surface tension.

Figure 5 shows shape contours of the family of 'pressure' blades: 9 to 11, and the 'control' blade.

Figure 5. Shape contours of the 'pressure' blades versus the 'control'

The finite element load and modal analyses, subsequently carried out for the family of 'pressure' blades made of real material (rather than the soap film), showed blade 10 to give a performance closest to that of the 'control' blade. The profile of the blade is shown in Figure 6.

Figure 6. 'Pressure' blade 10. Cross-sectional blade profiles (solid line), relative to the 'control' (dotted)

As can be seen, the cross-sectional profiles of the 'pressure' blade are very similar to those of the 'control' blade.

Figure 7 shows the variation in the volume of the material used by each member of the whole blade family, expressed as a fraction of the volume used by the 'control' blade. 'Pressure' blade 10 and 'minimal' blade 7 offer a similar saving in the volume of the material used but, as can be seen from Figures 6 and 3 respectively, the material in blade 7 is distributed quite differently.

Figure 7. Variations in volume of material in form-found blades relative to the 'control' blade volume of 30898.71 mm^3

3. Load analysis: finite element modelling

The behaviour of the blades under realistic load conditions was analysed in *Cosmosworks* 2008 FFEPlus (part of *Solidworks* 2008) using a solid model composed of 3-dimensional elastic finite elements working within large displacement theory. Using the 'control' blade, a successful validation step was performed to ensure consistency between the results from *Cosmosworks* and those obtained by in-house *SC03* FE solver used by the industrial partner.

The number of finite elements used in *Cosmosworks* was between 54,000 and 66,000, depending on the volume of the blade. The blades were assumed to be made of titanium.

Data preparation involved the creation of solid surfaces for use in *Solidworks*. Subsequently, this data was imported to *Cosmosworks* for the finite element analysis.

The families of 'minimal' and 'pressure' blades were studied by finite element analysis across four test cases:

Case 1. Point load at the tip (titanium)

Case 2. Centrifugal loading (titanium)

Case 3. Centrifugal loading + thermal loading (titanium)

Case 4. Modal (frequency) analysis (titanium)

<u>Case 1. Point loading for a titanium rotor</u>

In this case, static analysis was performed on a blade fully constrained at the root and subjected to a 30N load at the tip. The load was spread over a circular area 5 mm in diameter, as shown in Fig.8. This load case was chosen to simulate a foreign body striking the blade and it also provided a test case for experimental measurement.

Figure 8. Finite element modelling in Cosmosworks. (a) Static loading position (b) Loading direction and restraints. (Dimensions in mm)

The results, showing the maximum displacement for the full family of blades relative to the 'control' are shown in Fig. 9.

Figure 9. Case 1. Displacement of form-found blades relative to the 'control' blade displacement of 0.74 mm

For this load case, the results indicate that the displacements are significantly higher for the form-found blades, compared to the 'control' blade, with the lowest displacements coming from the 'pressure' blade family (blades 9-11). Being generally thicker, the 'pressure' blades have greater stiffness and hence lower displacement.

Data concerning maximum principal stresses P1 to P3 is presented in Fig. 10.

Figure 10. Case 1. Maximum values of principal stresses P1, P2 and P3, for form-found blades, relative to the 'control' blade with the corresponding stress values of: 1.88×10^{7} N/mm^{2} , 6.7×10^{6}N/mm^{2}, and 3.42×10^{6} N/mm^{2} respectively

It can be seen that the principal stresses are, on the whole, higher for the form-found blades compared to the 'control' blade. However, for the 'pressure' blade 10, the increase (of around 2%) is observed in the case of the principal stresses P1, while P2 and P3 are both lower than for the 'control' blade.

Figure 11 shows the pattern of stress and displacement for the form-found blades whose performance came close to that of the 'control' blade.

Figure 11. Case 1. Distribution of stresses and displacements in selected 'minimal' and 'pressure' blades versus the 'control'

Compared to the 'control' blade, each representative of the form-found family of blades has a slightly different stress distribution. However, there appears to be a more gradual stress variation in the form- found blades. A similar observation can be made with regard to displacements. Although the maximum displacement for the form-found blades is higher (Fig.9) than that of 'control' blade, the profile of displacements (Fig. 11) is more gradual. The effect of this on the aerodynamic performance remains to be explored.

Case 2. Static Centrifugal loading for a titanium blade

In this case, a static loading equivalent to a centrifugal force was applied. The force was calculated for a rotational speed of 7000 rpm for a blade attached to a rotor of 400mm radius. The rotational speed simulated a realistic loading this blade would experience during normal operation.

The results from the displacement analysis are shown in Fig. 12. In this case, the 'minimal' blade 7 produced the lowest value of displacement of the whole range.

Figure 12. Case 2. Displacement of form-found blades relative to the 'control' blade displacement of 4.71 mm

The maximum principal stresses relative to those of the 'control' blade are illustrated in Fig. 13. It can be seen that, in this case, a number of form-found blades ('minimal' blades 7-8, and 'pressure' blades 9-11) have lower values of maximum principal stresses than the 'control'.

Figure 13. Case 2. Maximum values of principal stresses P_1, P_2 and P_3, for form-found blades, relative to the 'control' blade with the corresponding stress values of: 3.93×10^8 N/mm^2, 1.87×10^8 N/mm^2, and 1.57×10^8 N/mm^2 respectively

Stress and displacement patterns for blades 7, 10, and the 'control' are given in Fig. 14. In this case, the stress and displacements are very similar between the 'pressure' blade 10 and the 'control' blade. Significantly different patterns of stress and displacements are observed in the 'minimal' blade 7.

Figure 14. Case 2. Distribution of stresses and displacements in selected 'minimal' and 'pressure' blades versus the 'control'

<u>Case 3. Static Centrifugal loading with thermal loading for a titanium blade</u>

This case is identical to Case 2, but with the addition of a 100^{o}C thermal loading placed on the blade. This brings the analysis a step closer to simulating realistic loading conditions. The results concerning displacements are given in Fig. 15, with the corresponding stresses shown in Fig. 16. Overall, stress and deformation patterns for blades 7, 10 and the 'control' blade are found to be similar to Case 2.

Figure 15. Case 3. Displacement of form-found blades relative to the 'control' blade displacement of 4.81 mm

Figure 16. Case 3. Maximum values of principal stresses P1, P2 and P3, for form-found blades, relative to the 'control' blade with the corresponding stress values of: 2.98×10^8 N/mm^2, 1.14×10^8 N/mm^2, and 9.24×10^7 N/mm^2 respectively

It can be seen that, in this case, the 'minimal' blade 7 has the lowest displacement, but principal stresses somewhat higher than the 'control' blade. The 'pressure' blade 10 has higher displacements and higher, but comparable stresses with those of the 'control' blade.

Case 4. Frequency/modal analysis for a titanium rotor blade

The blade was fully constrained at the root and no other loads were applied. This case provides the first 6 natural frequencies of the form-found blades, for a direct comparison with data obtained for the 'control' blade. The results are presented in Fig. 17, with the frequency analysis given in Fig. 18.

Figure 17. Modal shapes corresponding to the first six natural frequencies for:
(a)'Minimal' blade 7, (b) 'Pressure' blade 10, and *(c) 'Control' blade*

With reference to Fig.17, it can be seen that the 'minimal' blade 7 has distinctly different modal shapes compared to the 'pressure' blade 10 and the 'control' blade. Frequency analysis presented in Figure 18 indicates that the 'minimal' blade 7 has a lower natural frequency than the 'control' blade, and so do all form-found blades up to, and including, the third frequency. For the fourth frequency and above, the 'control' blade has a lower natural frequency than the 'pressure' blades.

Figure 18. First six natural frequencies for form-found blades versus the 'control'

In summary, the results of all the cases show that

- Displacements of the form-found blades are somewhat higher compared to the 'control' blade, but, as illustrated by the' minimal' blade 7, they follow a more gradual change in their profile
- Stresses in the form-found blades are higher than in the 'control' blade, but, for the Load case 2, they are actually lower in 'pressure' blade 10. Within the 'pressure' blade family, the displacements of blade 10 are consistently low (Figures 9, 12, and 15)
- Natural frequencies of the 'pressure' blades are slightly lower than in the 'control blade' for the first 3 modes, but becoming higher for modes 4, 5, and 6.

4. Summary and Conclusions

The paper reports on an exploratory, form-finding study of shell-type structures using an axial-flow compressor blade as a case study. The proposed methodology builds on the current knowledge of aerodynamic pressure profiles and boundary configurations for the blades, but the approach is radically different to the currently established design procedures; it involves a process in which the blade shape is found according to a prescribed loading regime, in which the Laplace-Young equation is used to obtain the relationship between ‘form’ and ‘force’ for the upper and lower surfaces of the blade. This approach differs from that of structural optimisation in that the geometric control applies only to the boundaries of the structure, and the found surface configurations represent a whole-surface solution, not a series of 2D profiles joined smoothly together.

It is found that the form-found ‘minimal’ blades have much reduced weight and exhibit a greater load compliance/flexibility compared to the ‘control’ blade. Their aerodynamic performance, however, would be poor on account of their cross-sectional profiles, flexibility, and low frequencies.

The adoption of aerodynamic pressure profile in form-finding and the achieved similarity of shape between the ‘pressure’ blades and the ‘control’ suggests that the obtained solutions represent plausible aerodynamic configurations for the ‘pressure’ blades, while delivering sound structural performance at reduced weight. To reinforce this conclusion, further investigation of aerodynamic characteristics of the form-found blades, linked to flutter calculations, is required.

The proposed ‘form follows force’ methodology represents a novel design tool capable of improving the design process by reducing, drastically, the number of iterations required between aerodynamicists and structural engineers engaged in optimisation of blade profiles. Future work will explore new shapes found by a load combination involving not only aerodynamic pressures, but also local ‘point’ loads.

Acknowledgment

The work was sponsored from the EPSRC standard research grant ref: GRS56016/1.

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