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Scaling of solar wind ϵ and the AU, AL and AE indices as seen by WIND

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[1] We apply the finite size scaling technique to quantify the statistical properties of fluctuations in AU, AL and AE indices and in the ϵ parameter that represents energy input from the solar wind into the magnetosphere. We find that the exponents needed to rescale the probability density functions (PDF) of the fluctuations are the same to within experimental error for all four quantities. This self-similarity persists for time scales up to ~4 hours for AU, AL and ϵ and up to ~2 hours for AE. Fluctuations on shorter time scales than these are found to have similar long-tailed (leptokurtic) PDF, consistent with an underlying turbulent process. These quantitative and model-independent results place important constraints on models for the coupled solar wind-magnetosphere system. *INDEX TERMS:* 2784 Magnetospheric Physics: Solar wind/magnetosphere interactions; 2788 Storms and substorms; 2704 Auroral phenomena (2407); 2159 Interplanetary Physics: Plasma waves and turbulence; 3250 Mathematical Geophysics: Fractals and multifractals. **Citation:** Hnat, B., S. C. Chapman, G. Rowlands, N. W. Watkins, and M. P. Freeman, Scaling of solar wind ϵ and the AU, AL and AE indices as seen by WIND, *Geophys. Res. Lett.*, *29*(22), 2078, doi:10.1029/2002GL016054, 2002.

1. Introduction

[2] Recently, there has been considerable interest in viewing the coupled solar wind-magnetosphere as a complex system where multi-scale coupling is a fundamental aspect of the dynamics (see [*Chang*, 1992; *Chapman and Watkins*, 2001] and references therein). Examples of the observational motivation for this approach are i) bursty transport events in the magnetotail [*Angelopoulos et al.*, 1992] and ii) evidence that the statistics of these events are self-similar (as seen in auroral images [*Lui et al.*, 2000]). Geomagnetic indices are of particular interest in this context as they provide a global measure of magnetospheric output and are evenly sampled over a long time interval. There is a wealth of literature on the magnetosphere as an input-output system (see for example, [*Klimas et al.*, 1996; *Sitnov et al.*, 2000; *Tsurutani et al.*, 1990; *Vassiliadis et al.*, 2000; *Vörös et al.*, 1998]. Recent work has focussed on comparing some aspects of the scaling properties of input parameters such as ϵ [*Perreault and Akasofu*, 1978] and the AE index [*Davis and Sugiura*, 1966] to establish whether, to the lowest order, they are directly related [*Freeman et al.*, 2000; *Uritsky et al.*, 2001]. Although these studies are directed at understanding the coupled solar wind-magnetosphere in the context of Self-Organized Criticality (SOC), a comprehensive comparison of the scaling properties of the indices, and some proxy for the driver (ϵ) also has relevance for the predictability of this magnetospheric "output" from the input. Importantly, both "burstiness" (or intermittency) and self-similarity can arise from several processes including SOC and turbulence. Indeed, SOC models exhibit threshold instabilities, bursty flow events and statistical features consistent with the "scale-free" dynamics such as power law power spectra. It has been proposed by *Chang* [1992] that magnetospheric dynamics are indeed in the critical state or near it. Alternatively, *Consolini and De Michelis* [1998] used the Castaing distribution — the empirical model derived in *Castaing et al.* [1990] and based on a turbulent energy cascade — to obtain a two parameter functional form for the Probability Density Functions (PDF) of the AE fluctuations on various temporal scales. Turbulent descriptions of magnetospheric measures also model observed statistical intermittency, i.e., the presence of large deviations from the average value on different scales [*Consolini et al.*, 1996; *Vörös et al.*, 1998]. An increased probability of finding such large deviations is manifested in the departure of the PDF from Gaussian toward a leptokurtic distribution [*Sornette*, 2000].

[3] In this paper we will quantify both the intermittency and the self-similarity of the AU, AL, AE and ϵ time series using the technique of finite size scaling. This has the advantage of being model independent, and is also directly related to both turbulence models such as that of Castaing [*Castaing et al.*, 1990] and a Fokker-Planck description of the time series. The method was used in *Hnat et al.* [2002] where the mono-scaling of the solar wind magnetic energy density fluctuations was reported. We will find that fluctuations in all four quantities are strongly suggestive of turbulent processes and by quantifying this we can compare their properties directly.

[4] The AL, AU and AE indices data set investigated here comprises over 0.5 million, 1 minute averaged samples from January 1978 to January 1979 inclusive. The ϵ parameter defined in SI units as:

$$\epsilon = v\frac{B^2}{\mu_0} l_0^2 \sin^4(\Theta/2) \qquad (1)$$

where $l_0 \approx 7R_E$ and $\Theta = \arctan(|B_y|/B_z)$ is an estimate of the fraction of the solar wind Poynting flux through the dayside magnetosphere and was calculated from the WIND spacecraft key parameter database [*Lepping et al.*, 1995; *Ogilvie et al.*, 1995]. It comprises over 1 million, 46 second averaged samples from January 1995 to December 1998 inclusive. The data set includes intervals of both slow and

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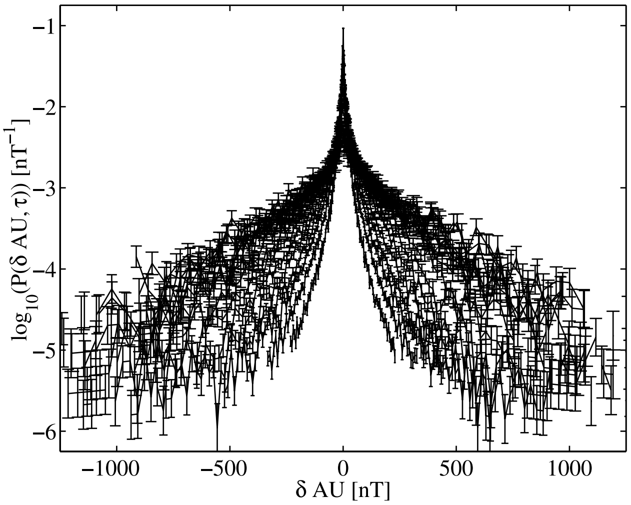

Figure 1. Unscaled PDFs of the AU index fluctuations. Time lag τ assumes values between 60 seconds and about 36 hrs. Standard deviation of the PDF increases with τ. Error bars on each bin within the PDF are estimated assuming Gaussian statistics for the data within each bin.

fast speed streams. The time series of indices and that of the ϵ parameter were obtained in different time intervals and here we assume that the samples are long enough to be statistically accurate.

2. Scaling of the Indices and ϵ

[5] The statistical properties of complex systems can exhibit a degree of universality reflecting the lack of a characteristic scale in their dynamics. A connection between the statistical approach and the dynamical one is given by a Fokker-Planck (F-P) equation [*van Kampen*, 1992] which describes the dynamics of the PDF and, in the most general form, can be written as:

$$\frac{\partial P(x,t)}{\partial t} = \nabla(P(x,t)\gamma(x)) + \nabla^2 D(x)P(x,t), \quad (2)$$

where $P(x, t)$ is a PDF of some quantity x that varies with time t, γ is the friction coefficient and $D(x)$ is a diffusion coefficient which in this case can vary with x. For certain choices of $D(x)$, a class of self-similar solutions of (2) satisfies a finite size scaling (in the usage of *Sornette* [2000], pg. 85, henceforth "scaling") relation given by:

$$P(x,\tau) = \tau^{-s} P_s(x\tau^{-s}). \quad (3)$$

This scaling is a direct consequence of the fact that the F-P equation is invariant under the transformation $x \to x\,\tau^{-s}$ and $P \to P\tau^{s}$. If, for given experimental data, a set of PDFs can be constructed, on different temporal scales τ, that satisfy relation (3) then a diffusion coefficient and corresponding F-P equation can be found to represent the data. A simple example is the Brownian random walk with $s = 1/2$, $D(x)$ = constant and Gaussian PDFs on all scales. Alternatively one can treat the identification of the scaling exponent s and, as we will see, the non-Gaussian nature of the rescaled PDFs (P_s) as a method for quantifying the intermittent character of the time series. Practically, obtaining the rescaled PDFs involves finding a rescaling index s directly from the integrated time series of the quantity X [*Hnat et al.*, 2002; *Sornette*, 2000].

[6] Let $X(t)$ represent the time series of the studied signal, in our case AU, AL, AE or the ε parameter. A set of time series $\delta X(t,\tau) = X(t + \tau) - X(t)$ is obtained for each value of non-overlapping time lag τ. The PDF $P(\delta X, \tau)$ is then obtained for each time series $\delta X(t, \tau)$. Figure 1 shows these PDFs for the δAU. A generic scaling approach is applied to these PDFs. Ideally, we use the peaks of the PDFs to obtain the scaling exponent s, as the peaks are the most populated parts of the distributions. In certain cases, however, the peaks may not be the optimal statistical measure for obtaining the scaling index. For example, the B_z component in (1) as well as the AU and AL indices are measured with an absolute accuracy of about 0.1 nT. Such discreteness in the time series and, in the case of the ϵ fluctuations, the large dynamical range introduce large errors in the estimation of the peak values $P(0, \tau)$ and may not give a correct scaling. Since, if the PDFs rescale, we can obtain the scaling exponent from any point on the curve in principle, we also determine the scaling properties of the standard deviation $\sigma(\tau)$ of each curve $P(\delta X, \tau)$ versus $\delta X(t, \tau)$.

[7] Figure 2 shows $P(0,\tau)$ plotted versus τ on log-log axes for $\delta X = \delta\epsilon$, δAE, δAU and δAL. Straight lines on such a plot suggest that the rescaling (3) holds at least for the peaks of the distributions. On Figure 2, lines were fitted with R^2 goodness of fit for the range of τ between 4 and 136 minutes, omitting points corresponding to the first two temporal scales as in these cases the sharp peaks of the PDFs can not be well resolved. The lines suggest self-similarity persists up to intervals of τ = 97 – 136 minutes. The slopes of these lines yield the exponents s and these are summarized in Table 1 along with the values obtained from analogous plots of $\sigma(\tau)$ versus τ which show the same scale break. We note that, for the ϵ parameter, the scaling index s obtained from the $P(0,\tau)$ is different from the Hurst exponent measured from the $\sigma(\tau)$. This difference could be a result of the previously discussed difficulties with the ϵ data. However, it

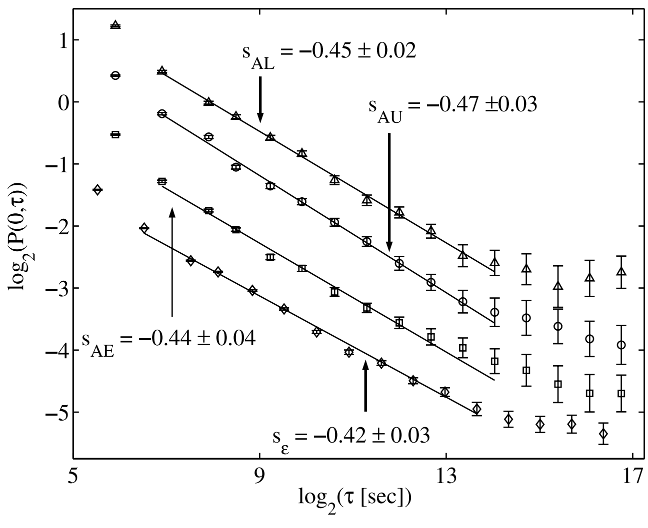

Figure 2. Scaling of the peaks of the PDFs for all quantities under. investigation: ◇ corresponds to ϵ, ○ AU index, △ AL index and □ the AE index. The plots have been offset vertically for clarity. Error bars as in Figure 1.

Table 1. Scaling Indices Derived From $P(0, \tau)$ and $\sigma(\tau)$ Power Laws

Quantity	$P(0,\tau)$ Scaling Index	$\sigma(\tau)$ Scaling Index	τ_{max}
ϵ	−0.42 ± 0.03	0.33 ± 0.04	4.5 hrs
AE-index	−0.44 ± 0.03	0.43 ± 0.03	2.1 hrs
AU-index	−0.47 ± 0.03	0.47 ± 0.02	4.5 hrs
AL-index	−0.45 ± 0.02	0.45 ± 0.02	4.5 hrs

does appear to be a feature of some real time series (see *Gopikrishnan et al.* [1999] for example). Indeed, such a difference between index s and H_σ is predicted in the case of fractional Lévy motion [*Chechkin and Gonchar*, 2000]. We see that, for the ϵ as well as the AL and AU indices, there is a range of τ up to 4.5 hours for which $P(0,\tau)$ plotted versus τ is well described by a power law τ^{-s} with indices $s = 0.42 \pm 0.03$ for the ϵ and $s = 0.45 \pm 0.02$ and $s = 0.47 \pm 0.03$ for the AL and AU indices, respectively. Thus the break in scaling at 4–5 hours in the AL and AU indices may have its origin in the solar wind, although the physical reason for the break at this timescale in epsilon is unclear. The break in the AE index, however, appears to occur at a smaller temporal scale of 2 hours, consistent with the scaling break timescale found in the same index by other analysis methods [*Consolini and De Michelis*, 1998; *Takalo et al.*, 1993]. This was interpreted by [*Takalo et al.*, 1993] as due to the characteristic substorm duration. *Takalo and Timonen* [1998] also reported a scaling break at the same 2 hour timescale for AL, in contrast to the 4–5 hour timescale found here. Indeed, one might have expected a substorm timescale to cause the same scaling break in both the AE and AL indices, because their substorm signatures are so similar in profile (e.g., Figure 2 of *Caan et al.* [1978]). The resolution may lie in the difference between analysis of differenced and undifferenced data [*Price and Newman*, 2001].

[8] Within this scaling range we now attempt to collapse each corresponding unscaled PDF onto a single master curve using the scaling (4). If the initial assumption of the self-similar solutions is correct, a single parameter rescaling, given by equation (3) for a mono-fractal process, would

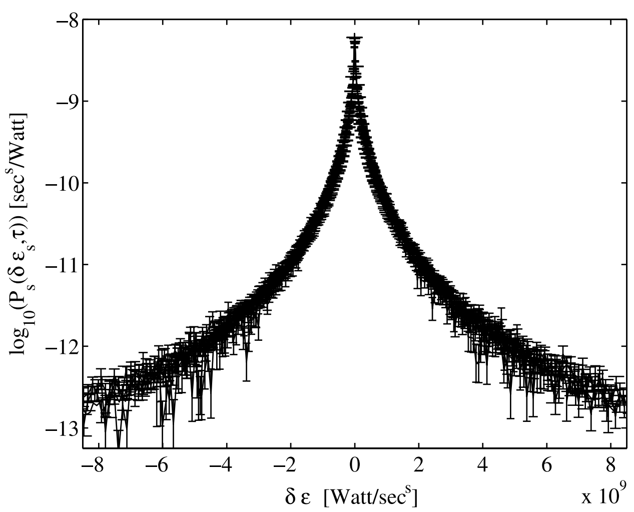

Figure 3. One parameter rescaling of the ϵ parameter fluctuations PDFs. The curves shown correspond to τ between 46 seconds and 4.5 hours. Error bars as in Figure 1.

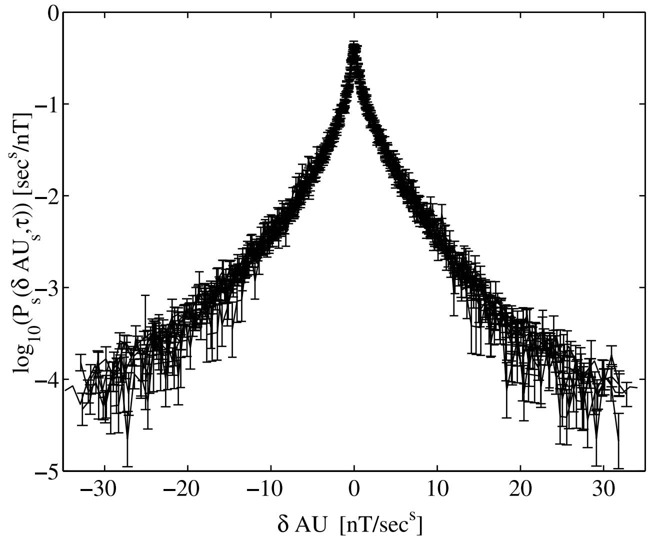

Figure 4. One parameter rescaling of the AU index fluctuation PDF. The curves shown correspond to τ between 46 seconds and 4.5 hours. Error bars as in Figure 1.

give a perfect collapse of PDFs on all scales. Practically, an approximate collapse of PDFs is an indicator of a dominant mono-fractal trend in the time series, i.e., this method may not be sensitive enough to detect multi-fractality that could be present only during short time intervals. Figures 3 and 4show the result of the one parameter rescaling applied to the unscaled PDF of the $\delta\varepsilon$ and the δAU index fluctuations, respectively, for the temporal scales up to ~4.5 hours. We see that the rescaling procedure (4) using the value of the exponent s of the peaks $P(0, \tau)$ shown in Figure 2, gives good collapse of each curve onto a single common functional form for the entire range of the data. These rescaled PDFs are leptokurtic rather than a Gaussian and are thus strongly suggestive of an underlying turbulent process.

[9] The successful rescaling of the PDFs now allows us to perform a direct comparison of the PDFs for all four quantities. Figure 5 shows these normalized PDFs $P_s(\delta X, \tau)$ for $\delta X = \delta\epsilon$, δAE and $\tau \approx 1$ hour overlaid on a single plot. The δX variable has been normalized to the rescaled standard deviation $\sigma_s(\tau \approx 1hr)$ of P_s in each case to facilitate this comparison. We then find that AE and ϵ fluctuations have indistinguishable P_s. The PDFs of δAU and δAL are asymmetric such that $-\delta AL$ fits δAU PDF closely (see insert in the Figure 5); when overlaid on the PDFs of the $\delta\epsilon$ and δAE these are also indistinguishable within errors. This provides strong evidence that the dominant contributions to the AE indices come from the eastward and westward electrojets of the approximately symmetric DP2 current system that is driven directly by the solar wind [*Freeman et al.*, 2000]. The mono-scaling of the investigated PDFs, together with the finite value of the samples' variance, indicates that a Fokker-Planck approach can be used to study the dynamics of the unscaled PDFs within their temporal scaling range.

3. Summary

[10] In this paper we have applied the generic and model independent scaling method to study the scaling of fluctuations in the ϵ parameter and the global magnetospheric

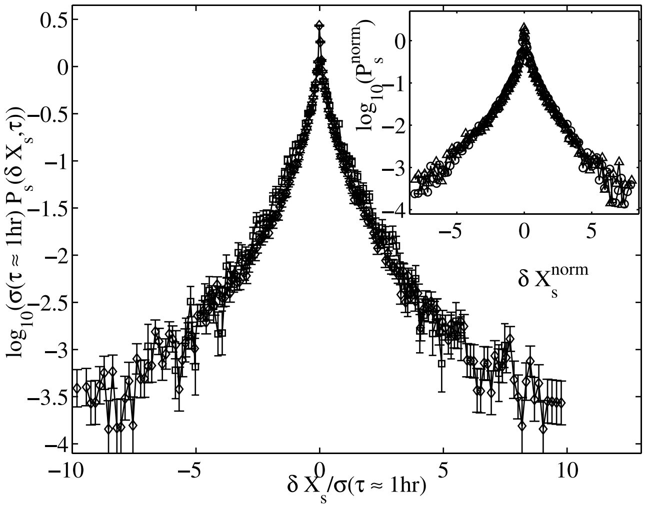

Figure 5. Direct comparison of the fluctuations PDFs for ϵ (◇) and AE index (□). Insert shows overlaid PDFs of AU(○) and $-AL(\Delta)$ fluctuations. Error bars as in Figure 1.

indices AU, AL and AE. The similar values of the scaling exponent and the leptokurtic nature of the single PDF that, to within errors, describes fluctuations on time scales up to τ_{max} in all four quantities provide an important quantitative constraint for models of the coupled solar wind-magnetosphere system. One possibility is that, up to $\tau_{max} \sim 4$ hours, fluctuations in AU and AL are directly reflecting those seen in the turbulent solar wind. The data also suggest that AE index departs from this scaling on shorter time scale of $\tau_{max} \sim 2$ hours. Importantly, identifying a close correspondence in the fluctuation PDF of ϵ, AE, AU and AL may simply indicate that fluctuations in the indices are strongly coupled to dayside processes and are thus weak indicators of the fluctuations in nightside energy output. The leptokurtic nature of the PDFs is strongly suggestive of turbulent processes, and in the case of AU and AL, these may then be either just that of the turbulent solar wind (and here ϵ) or may be locally generated turbulence which has an indistinguishable signature in its fluctuation PDF. In this case our results quantify the nature of this turbulence. We note, however, that certain classes of complex systems [*Chang et al.*, 1992a] are in principle capable of "passing through" input fluctuations into their output without being directly driven in the present sense [Chang, private communication, 2002]. Finally, the rescaling also indicates that a Fokker-Planck approach can be used to study the evolution of the fluctuation PDF. This raises a possibility of a new approach to understanding magnetospheric dynamics.

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