Quantifying scaling in the velocity field of the anisotropic turbulent solar wind

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[1] Solar wind turbulence is dominated by Alfvénic fluctuations with power spectral exponents that somewhat surprisingly evolve toward the Kolmogorov value of −5/3, that of hydrodynamic turbulence. We analyze in situ satellite observations at 1AU and show that the turbulence decomposes linearly into two coexistent components perpendicular and parallel to the local average magnetic field and determine their distinct intermittency independent scaling exponents. The first of these is consistent with recent predictions for anisotropic MHD turbulence and the second is closer to Kolmogorov-like scaling. **Citation:** Chapman, S. C., and B. Hnat (2007), Quantifying scaling in the velocity field of the anisotropic turbulent solar wind, *Geophys. Res. Lett.*, *34*, L17103, doi:10.1029/2007GL030518.

1. Introduction

[2] The solar wind provides a unique laboratory for the study of Magnetohydrodynamic (MHD) turbulence with a magnetic Reynolds number estimated to exceed $\sim 10^5$ [*Matthaeus et al.*, 2005]. In situ satellite observations of bulk plasma parameters suggest turbulence via the statistical properties of their fluctuations [*Tu and Marsch*, 1995; *Goldstein*, 2001]. Quantifying these fluctuations is also central to understanding both the transport of solar energetic particles and galactic cosmic rays within the heliosphere, and solar wind evolution with implications for the mechanisms that accelerate the wind at the corona.

[3] Numerical and analytical studies of incompressible MHD, where the cascade is mediated by Alfvénic fluctuations, provide different predictions for scaling exponents, depending upon the strength of the turbulence, the strength of the background magnetic field, and anisotropy. Iroshnikov and Kraichnan's [*Iroshnikov*, 1964; *Kraichnan*, 1965, hereinafter referred to as IK] original isotropic, weak (random phase) phenomenology leads to a $\sim k^{-3/2}$ spectrum (see also *Dobrowolny et al.* [1980] for the asymmetric case). Introducing anisotropy in the weak case leads to a $\sim k_\perp^{-2}$ spectrum for fluctuations perpendicular to the background magnetic field [e.g., *Galtier et al.*, 2000]. In contrast, strong turbulence phenomenology [see *Goldreich and Sridhar*, 1997, and references therein] and anisotropy [see *Higdon*, 1984, and references therein] yields a $\sim k_\perp^{-5/3}$ spectrum [hereafter referred to as GS]. This symmetric case where the fluxes of oppositely directed Alfvén waves are equal does not however strictly apply to the solar wind [*Goldreich*

and Sridhar, 1997], where the fluxes are observed to be asymmetric. Recent numerical simulations [*Müller and Grappin*, 2005], and analysis [*Boldyrev*, 2006, hereinafter referred to as SB] obtain a $\sim k_\perp^{-3/2}$ spectrum for the case of a strong local background magnetic field, this −3/2 exponent, combined with the anisotropy of the fluctuations, is in contradiction with both IK and GS phenomenologies.

[4] Alfvénic fluctuations dominate the observed power in the solar wind with propagation principally away from the sun implying solar origin [e.g., *Horbury et al.*, 2005]. Somewhat surprisingly then, the power spectra [e.g., *Tu and Marsch*, 1995; *Goldstein*, 2001] suggest an exponent evolving toward the Kolmogorov-like [*Kolmogorov*, 1941, hereinafter referred to as K-41] value of $\sim -5/3$, that of hydrodynamic turbulence. Intervals can be found where different magnetic field and velocity components simultaneously exhibit scaling consistent with −5/3 and −3/2 spectra [e.g., *Veltri*, 1999], indeed, this scaling can be difficult to distinguish in low order moments [*Carbone et al.*, 1995]. The flow is also observed to be intermittent, this has been suggested to account for the 'anomalous' −5/3 power spectra in terms of incompressible MHD [*Carbone*, 1993]. Alfvénic fluctuations, when isolated by the use of Elsasser variables [e.g., *Horbury et al.*, 2005] and decomposed by considering different average magnetic field orientations that occur at different times, are found to be multicomponent [*Matthaeus et al.*, 1990], and coupled [*Milano et al.*, 2004]. This observationally inspired picture, of an essentially incompressible, multicomponent Alfvénic turbulence suggests that a significant population of Alfvénic fluctuations evolve to have wave vectors almost perpendicular to the background magnetic field, leading to a 'fluid-like' phenomenology, and the −5/3 power spectral slope. However, fluctuations in solar wind density are not simply proportional to that in the magnetic field [*Spangler and Spiller*, 2004] and show non-trivial scaling [*Tu and Marsch*, 1995; *Hnat et al.*, 2003] suggesting that the turbulence is compressible [*Hnat et al.*, 2005]. The role of compressibility is thus an open question. An important corollary is that the full behaviour cannot be captured by models which describe the observed Alfvénic properties in terms of fluctuating coronal fields that have advected passively in the expanding solar wind [see, e.g., *Giacalone et al.*, 2006]. There is however also evidence in non-cascade quantities, such as magnetic energy density, of a signature within the inertial range that shows scaling that correlates with the level of magnetic complexity in the corona [*Hnat et al.*, 2007; *Kiyani et al.*, 2007].

[5] Here, we will quantify scaling exponents of velocity fluctuations in the anisotropic turbulent solar wind in order to make comparisons both with the theoretical predictions above, and with the scaling seen in quantities which reflect

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the compressibility, as well as the Alfvénicity of the turbulent flow.

2. Dimensional Analysis and Scaling Exponents

[6] We will discuss the statistical properties of fluctuations in components of velocity w.r.t. the local background magnetic field, by considering ensemble averages. Fluctuations in the velocity field can be characterized by the difference in components, or in the magnitude, $\delta v = v(r + L) - v(r)$ at two points separated by distance L. The dependence of δv upon L is determined in a statistical sense through the moments $\langle \delta v^p \rangle$, where $\langle \ldots \rangle$ denotes an ensemble average over r. Statistical theories of turbulence then anticipate scaling $\langle \delta v_L^p \rangle \sim L^{\zeta(p)}$.

[7] To fix ideas, we first introduce a notation for the scaling exponents in terms of dimensional analysis. A fluctuation δv on lengthscale L transfers kinetic energy δv^2 on timescale $T \sim L/\delta v$, implying an energy transfer rate $\epsilon_L \sim \delta v^2/T \sim \delta v^3/L$. If the statistics of the fluctuations in the energy transfer rate are independent of L, its p moments $\langle \epsilon_L^p \rangle \sim \epsilon_0^p$ where the constant ϵ_0 is the average rate of energy transfer. This is consistent with the K-41 scaling $\langle \delta v_L^p \rangle \sim L^{p/3}$.

[8] In practice, hydrodynamic flows deviate from this simple scaling. This intermittency [e.g., *Frisch*, 1995] is introduced through a lengthscale dependence of the fluctuations in energy transfer rate so that $\langle \epsilon^p{}_L \rangle \sim \epsilon_0^p (L/L_0)^{\mu(p)}$, where L_0 is some characteristic lengthscale and $\mu(p)$ is the intermittency correction. The scaling for the moments then becomes $\langle \delta v_L^p \rangle \sim L^{\zeta(p)}$ with the K-41 exponents $\zeta(p) = p/3 + \mu(p/3)$. For incompressible MHD turbulence, Alfvénic phenomenology modifies the energy transfer time $T \sim (L/\delta v)(v_0/\delta v)^\alpha$, (where v_0 is a characteristic speed, i.e., the Alfvén speed) then $\alpha = 1$ for IK. For anisotropic phenomenology where L refers to a magnetic field perpendicular lengthscale, GS for example implies $\alpha_\perp = 0$, and SB for example $\alpha_\perp = 1$. The above dimensional argument then gives an energy transfer rate $\epsilon_L \sim \delta v^{(3+\alpha)}/L$ so that $\langle \delta v^p \rangle \sim L^{\zeta(p)}$ now with $\zeta(p) = p/(3 + \alpha) + \mu(p/(3 + \alpha))$. In the absence of intermittency this corresponds to a power spectrum along L of $E(k_L) \sim \langle \delta v^2 \rangle / k_L \sim k_L^{-(5+\alpha)/(3+\alpha)}$.

[9] The experimental study of turbulence then centres around measurement of the $\zeta(p)$. A full description requires the (difficult to determine) intermittency correction, the $\mu(p)$. If the system is in a homogeneous steady state, the average energy transfer rate is uniform so that $\langle \epsilon_L \rangle = \epsilon_0$ and $\mu(1) = 0$ so that for MHD flows, this simple dimensional argument implies that $\zeta(3 + \alpha) = 1$, independent of the intermittency of the flow. This is not exact in the sense of K-41 for which we have $\zeta(3) = 1$ (the "4/5" law) [e.g., *Frisch*, 1995]. A determination of the lower order moments that is sufficiently accurate to distinguish $\alpha = 0$ and $\alpha = 1$ is possible for in- situ observations of the solar wind and we present this here.

3. Structure Function Analysis

[10] We now consider time series from a single spacecraft so that the ensemble averages that we will consider will be over time rather than over space, the spatial separation above being replaced by a time interval τ- the Taylor

hypothesis [*Matthaeus et al.*, 2005]. Consistent with almost all experimental studies of turbulence we consider generalized structure functions of a given parameter x: $S_p(\tau) = \langle |x(t + \tau) - x(t)|^p \rangle$ [see, e.g., *Chapman et al.*, 2005, and references therein].

[11] Solar wind monitors such as the ACE spacecraft spend several-year long periods in orbit about the Lagrange point sunward of the earth. We analyze 64 s averaged plasma parameters from ACE for the interval 01/01/1998–12/31/2001, this consists of $\sim 1.6 \times 10^6$ samples and is dominated by slow solar wind.

[12] We consider vector velocity fluctuations which we will decompose w.r.t. the direction of the magnetic field. In the turbulent flow, the magnetic field also fluctuates, but we can consider a local background value by constructing a running average of the vector magnetic field over the timescale τ'. For each interval over which we obtain a *difference* in velocity $\delta \mathbf{v} = \mathbf{v}(t + \tau) - \mathbf{v}(t)$ we also obtain a vector *average* for the magnetic field direction $\hat{\mathbf{b}} = \overline{\mathbf{B}}/|\overline{\mathbf{B}}|$ from a vector sum of all the observed vector values between t and $t + \tau'$, $\overline{\mathbf{B}}(t, \tau') = \mathbf{B}(t) + \ldots + \mathbf{B}(t + \tau')$, with τ' centred on τ. We choose the interval $\tau' = 2\tau$ here as the minimum (Nyquist) necessary to capture wavelike fluctuations. Velocity differences $\delta \mathbf{v}$ which are Alfvénic in character will then have the property that the scalar product $\delta \mathbf{v} \cdot \hat{\mathbf{b}}$ will vanish. This does not filter out compressive fluctuations. This condition filters out all those fluctuations which generate a velocity displacement perpendicular to the local magnetic field, and is thus distinct from the Elsasser [*Horbury et al.*, 2005] variables which select propagating pure Alfvén waves.

[13] Figure 1 shows the procedure for extracting the scaling exponents from the data. We plot the structure functions of the quantity $\delta v_\parallel = \delta \mathbf{v} \cdot \hat{\mathbf{b}}$, that is, $S_p = \langle |\delta \mathbf{v} \cdot \hat{\mathbf{b}}|^p \rangle$ versus τ, (inset) and the corresponding scaling exponents, the $\zeta(p)$ for the region where $S_p \sim \tau^{\zeta(p)}$ (main plot). The inset panel shows the structure functions of fluctuations for $p = 1-4$. There is scaling over timescales of minutes up to a few hours, the timescale for large scale coherent structures and the onset of strong variation in the Alfvén ratio. The scaling exponents, that is, the $\zeta(p)$, where $S_p(\tau) \sim \tau^{\zeta(p)}$, are the gradients of these scaling regions, and these are shown in the main plot. The error bars provide an estimate of the uncertainty in the gradients of the fitted lines (linear regression error). Finite, experimental data sets include a small number of extreme events which have poor representation statistically and may obscure the scaling properties of the time series. One method [*Veltri*, 1999; *Chapman et al.*, 2005] (for other approaches, see, e.g., *Katul et al.* [1994], *Horbury and Balogh* [1997], and *Kiyani et al.* [2006]) for excluding these rare events is to fix a (large) upper limit on the magnitude of fluctuations used in computing the structure functions. Importantly, this limit is varied with the temporal scale τ to account for the growth of range with τ in the time series. The figure shows the exponents computed for a range of values for this upper limit $[5-20]\sigma(\tau)$, where $\sigma(\tau) = S_2^{1/2}$. We see that the scaling exponents are not strongly sensitive to the value of the upper limit and are thus reliable. Above $10\sigma(\tau)$ this process eliminates less than 1% of the data points.
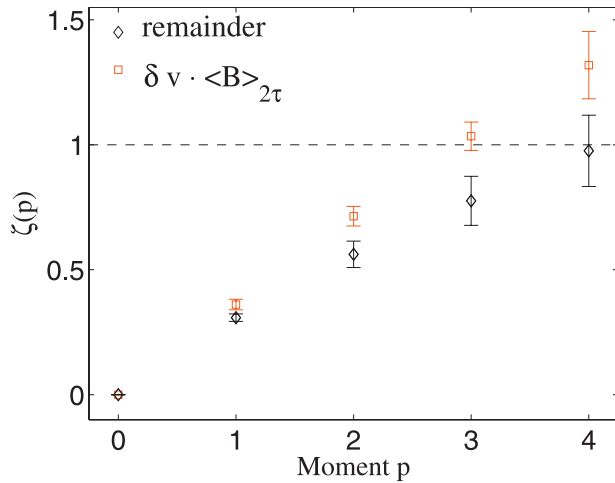
[14] In Figure 2 we compare these exponents with those obtained for the remaining signal, $\delta v_\perp =$
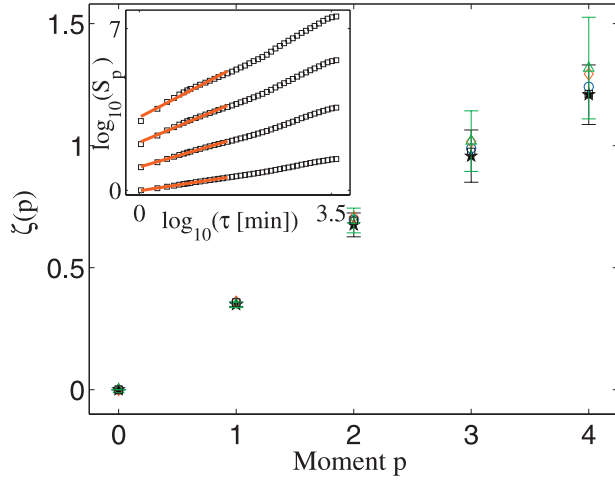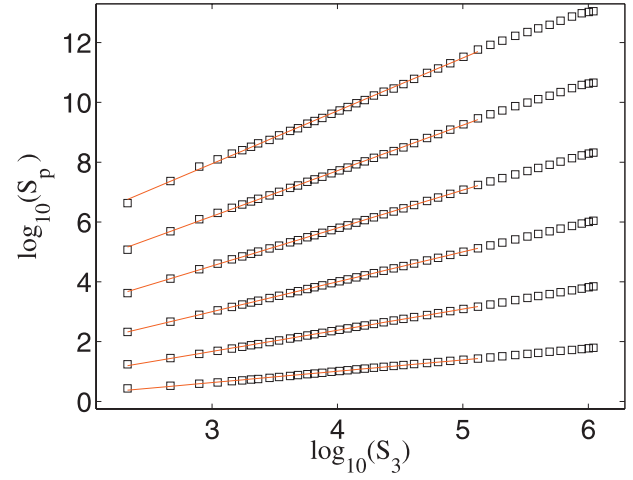
Figure 1. Structure function analysis of $\langle|\delta\mathbf{v}\cdot\hat{\mathbf{b}}|^p\rangle$. Inset: structure functions versus differencing interval (traces offset for clarity). Main plot: scaling exponents computed from the raw data (stars), and applying an upper limit to fluctuation size of $20\sigma(\tau)$ (stars), $15\sigma(\tau)$ (circles), $10\sigma(\tau)$ (triangles) and $5\sigma(\tau)$ (diamonds).

Figure 3. Structure functions S_p versus S_3 for $p = 1$-6 for $S_p = \langle|\delta\mathbf{v}\cdot\hat{\mathbf{b}}|^p\rangle$. Traces offset for clarity.

$\sqrt{\left(\delta\mathbf{v}\cdot\delta\mathbf{v}-\left(\delta\mathbf{v}\cdot\hat{\mathbf{b}}\right)^2\right)}$. We can see that both these quantities show a clear scaling range (which we will verify) with scaling exponents $\zeta(3)$ and $\zeta(4)$ close to unity for $\delta v_{\parallel}$ and $\delta v_{\perp}$ respectively.

[15] This result is consistent with the fluctuations in velocity being a simple linear superposition close to: (i) parallel to the local background magnetic field with $\alpha_{\parallel}\sim 0$, and (ii) perpendicular to the local background magnetic field with $\alpha_{\perp}\sim 1$ scaling. From the figure we can also see that the scaling in $\delta v_{\perp}$ is clearly multifractal (that is, convex with p) whereas that in $\delta v_{\parallel}$ is closer to self-affine (that is, almost linear with p). A multifractal signature in the velocity field is highly suggestive of intermittent turbulence [*Frisch*, 1995].

[16] We verify that these quantities indeed show an extended scaling region by means of ESS [*Frisch*, 1995]. If the scaling is such that the $S_p \sim S_q^{\zeta(p)/\zeta(q)}$ then a plot of S_p versus S_q will reveal the range of the underlying power law dependence with τ. If, as here, one of the $\zeta(p)$ are close to unity, the ESS plot will in addition provide a better estimate of the $\zeta(p)$. Figures 3 and 4 show S_p versus S_3 for $\delta v_{\parallel}$ and versus S_4 for $\delta v_{\perp}$ respectively, and we see that there is scaling over several orders of magnitude.

4. Conclusions

[17] We have decomposed the solar wind velocity fluctuations in the inertial range into components parallel to and perpendicular to the local background magnetic field direction. The characteristic nature of the signal is revealed to be a coexistence of two signatures which show an extended scaling range. The first of these, seen in the perpendicular velocity component is consistent with predictions for an-

Figure 2. Scaling exponents $\zeta(p)$ versus p for the structure functions of $\langle|\delta\mathbf{v}\cdot\hat{\mathbf{b}}|^p\rangle$ and of the remaining signal. Note that $\zeta(3)\approx 1$ and $\zeta(4)\approx 1$ respectively for these quantities.
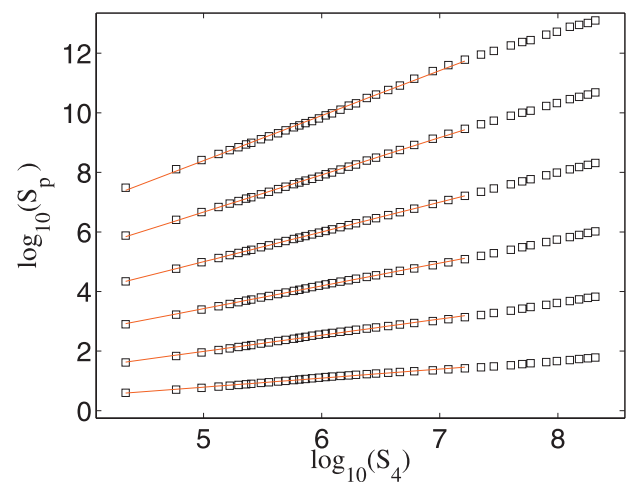

Figure 4. Structure functions S_p versus S_4 for $p = 1-6$ for $S_p = \langle|\sqrt{\left(\delta\mathbf{v}\cdot\delta\mathbf{v}-\left(\delta\mathbf{v}\cdot\hat{\mathbf{b}}\right)^2\right)}|^p\rangle$. Traces offset for clarity.

isotropic Alfvénic turbulence in a background field. The second is seen in the parallel velocity component with roughly "K-41-like" scaling. Intriguingly, the latter scaling is also that found in fluctuations in the density [*Hnat et al.*, 2005]. This clearly elucidates the previously proposed multicomponent nature of solar wind turbulence and may suggest one of two scenarios. One is that the turbulent solar wind is comprised of two weakly interacting components-one (seen in $\delta v_\parallel$) from the process that generates the solar wind at the corona and the other (seen in $\delta v_\perp$) that evolves in the high Reynolds number flow. Alternatively, the two components both arise from anisotropic compressible MHD turbulence in the presence of a background field, in which case this determination of their scaling properties points to potential development of theories of MHD turbulence. Given recent evidence [*Kiyani et al.*, 2007] for a self-affine signature in magnetic energy density that may be of solar origin that shows both solar cycle and latitudinal dependence, further work may unravel the interplay between the signatures of scaling generated at the corona, and by the evolving turbulence.

References

Boldyrev, S. (2006), Spectrum of magnetohydrodynamic turbulence, *Phys. Rev. Lett.*, *96*, 115002, doi:10.1103/PhysRevLett.96.115002.

Carbone, V. (1993), Cascade model for intermittency in fully developed magnetohydrodynamic turbulence, *Phys. Rev. Lett.*, *71*, 1546.

Carbone, V., P. Veltri, and R. Bruno (1995), Experimental evidence for differences in the extended self similarity scaling laws between fluid and magnetohydrodynamic turbulent flows, *Phys. Rev. Lett.*, *75*, 3110.

Chapman, S. C., B. Hnat, G. Rowlands, and N. W. Watkins (2005), Scaling collapse and structure functions: Identifying self-affinity in finite length time series, *Nonlinear Processes Geophys.*, *12*, 767.

Dobrowolny, M., A. Mangeney, and P. Veltri (1980), Fully developed hydromagnetic turbulence in interplanetary space, *Phys. Rev. Lett.*, *45*, 144.

Frisch, U. (1995), *Turbulence: The Legacy of A. N. Kolmogorov*, 136 pp., Cambridge Univ. Press, Cambridge, U. K.

Galtier, S., S. V. Nazarenko, A. C. Newell, and A. Pouquet (2000), Anisotropic turbulence of shear Alfven waves, *J. Plasma Phys.*, *63*, 447.

Giacalone, J., J. R. Jokipii, and W. H. Matthaeus (2006), Structure of the turbulent interplanetary magnetic field, *Astrophys. J.*, *641*, L61, doi:10.1086/503770.

Goldreich, P., and S. Sridhar (1997), Magnetohydrodynamic turbulence revisited, *Astrophys. J.*, *485*, 680.

Goldstein, M. L. (2001), Major unsolved problems in space plasma physics, *Astrophys. Space Sci.*, *277*, 349.

Higdon, J. C. (1984), Density fluctuations in the interstellar medium: Evidence for anisotropic magnetogasdynamic turbulence. I. Model and astrophysical sites, *Astrophys. J.*, *285*, 109.

Hnat, B., S. C. Chapman, and G. Rowlands (2003), Intermittency, scaling, and the Fokker-Planck approach to fluctuations of the solar wind bulk plasma parameters as seen by the WIND spacecraft, *Phys. Rev. E.*, *67*, 056404, doi:10.1103/PhysRevE.67.056404.

Hnat, B., S. C. Chapman, and G. Rowlands (2005), Compressibility in solar wind plasma turbulence, *Phys. Rev. Lett.*, *94*, 204502, doi:10.1103/PhysRevLett.94.204502.

Hnat, B., S. C. Chapman, K. Kiyani, G. Rowlands, and N. W. Watkins (2007), On the fractal nature of the magnetic field energy density in the solar wind, *Geophys. Res. Lett.*, *34*, L15108, doi:10.1029/2007GL029531.

Horbury, T. S., and A. Balogh (1997), Structure function measurements of the intermittent MHD turbulent cascade, *Nonlinear Processes Geophys.*, *4*, 185.

Horbury, T. S., M. A. Forman, and S. Oughton (2005), Spacecraft observations of solar wind turbulence: An overview, *Plasma Phys. Controlled Fusion*, *47*, B703, doi:10.1088/0741-3335/47/12B/S52.

Iroshnikov, P. S. (1964), Turbulence of a conducting fluid in a strong magnetic field, *Sov. Astron.*, *7*, 566.

Katul, G. G., J. D. Albertson, C. R. Chu, and M. B. Parlange (1994), Intermittency in atmospheric surface layer turbulence: The orthonormal wavelet representation, in *Wavelets in Geophysics*, edited by E. Foufoula-Georgiou and P. Kumar, p. 81, Academic, New York.

Kiyani, K., S. C. Chapman, and B. Hnat (2006), Extracting the scaling exponents of a self-affine, non-Gaussian process from a finite-length time series, *Phys. Rev. E.*, *74*, 051122, doi:10.1103/PhysRevE.74.051122.

Kiyani, K., S. C. Chapman, B. Hnat, and R. M. Nicol (2007), Self-similar signature of the active solar corona within the inertial range of solar wind turbulence, *Phys. Rev. Lett.*, *98*, 211101, doi:10.1103/PhysRevLett.98.211101.

Kolmogorov, A. N. (1941), Local structure of turbulence in an incompressible viscous fluid at very high Reynolds numbers, *C. R. Acad. Sci.*, *30*, 301.

Kraichnan, R. H. (1965), Inertial range spectrum of hydromagetic turbulence, *Phys. Fluids*, *8*, 1385.

Matthaeus, W. H., M. L. Goldstein, and D. A. Roberts (1990), Evidence for the presence of quasi-two dimesional nearly incompressible fluctuations in the solar wind, *J. Geophys. Res.*, *95*, 20,673.

Matthaeus, W. H., S. Dasso, J. M. Weygand, L. J. Milano, C. W. Smith, and M. G. Kivelson (2005), Spatial correlation of solar wind turbulence from two point measurements, *Phys. Rev. Lett.*, *95*, 231101, doi:10.1103/PhysRevLett.95.231101.

Milano, L. J., S. Dasso, W. H. Matthaeus, and C. W. Smith (2004), Spectral distribution of the cross helicity in the solar wind, *Phys. Rev. Lett.*, *93*, 155005, doi:10.1103/PhysRevLett.93.155005.

Müller, W.-C., and R. Grappin (2005), Spectral energy dynamics in magnetohydrodynamic turbulence, *Phys. Rev. Lett.*, *95*, 114502, doi:10.1103/PhysRevLett.95.114502.

Spangler, S. R., and L. G. Spiller (2004), An empirical investigation of compressibility in magnetohydrodynamic turbulence, *Phys. Plasmas*, *11*, 1969.

Tu, C.-Y., and E. Marsch (1995), MHD structures, waves and turbulence in the solar wind: Observations and theories, *Space Sci. Rev.*, *73*, 1.

Veltri, P. (1999), MHD turbulence in the solar wind: Self- similarity, intermittency and coherent structures, *Plasma Phys. Controlled Fusion*, *41*, A787.

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