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Performance Analysis of Spectrum Sensing With Multiple Primary Users

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Abstract

The effect of multiple primary users on the spectrum sensing performance is investigated. Different models for the primary user traffic are considered. The effects of different system parameters on the sening accuracy are examined. Numerical results show that the spectrum sensing performance is significantly degraded by the primary user traffic, and that the degradation decreases when the number of primary users increases.

Index Terms

Cognitive radio, primary user traffic, spectrum sensing.

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I. INTRODUCTION

Spectrum sensing is a critical functionality of cognitive radio [1]. It enables unlicensed users, referred to as cognitive radio (CR) users hereafter, to find the "spectrum holes". Many works have been conducted on spectrum sensing [2] - [4]. Among them, energy detector is the most widely used method. All these previous works assume that the primary user is either absent or present during the whole sensing period. However, in practice, the primary user may arrive or leave during the sensing period. The effect of the primary user traffic on the sensing performance has been analyzed in [5] for the case when only one primary user occupies the licensed spectrum at a time. In [6], the energy detection was improved to reduce the effect from the primary user traffic when only one primary user is present. However, in many widely used code division multiple access (CDMA) systems, such as 3G and WiMAX, the systems are designed to have several users operating in the same frequency band simultaneously. The "spectrum holes" also include vacant unlicensed bands. In this case, several unlicensed systems, such as Wi-Fi, Bluetooth and DECT, will share the same band without coordination, giving the scenario where multiple primary users may occupy the same band. All these realistic applications motivate a general investigation of the effect of primary user traffic on the sensing performance with multiple primary users.

In this letter, the effect of primary user traffic on the performance of energy detection is evaluated by considering the general case when multiple primary users arrive or leave during the sensing period. Different models for the primary user traffic are considered. Numerical results show that the performance of energy detection is significantly degraded when the primary user status changes during the sensing period, and that the degradation decreases when the number of primary users increases.

II. SYSTEM MODEL

In the energy detection, the output of a band-pass filter with bandwidth W is squared and integrated over the observation interval T. Let the time-bandwidth product $TW = m$, and assume that m is an integer. The total number of samples is $2m$. Then, the output of the energy detector is $\mathcal{Y} = \sum_{n=1}^{2m} Y_n^2$, where $Y_n = Z_n$ when the n-th sample does not contain the primary signal and $Y_n = S_n^{(u)} + Z_n$ when the n-th sample does contain the primary signal, Z_n are independent samples of the additive white Gaussian noise (AWGN) with mean zero and variance α^2, and $S_n^{(u)}$ are samples of the signals from u primary users. It is assumed that each primary user signal is independent and identically distributed with average power P. Thus, the average SNRs for one primary user and u primary users are $\gamma = P/\alpha^2$ and $u\gamma$, respectively. In the case when the primary user signal is non-identically distributed, γ and $u\gamma$ can be replaced by γ_i for the i-th primary signal and $\sum_{i=1}^{u} \gamma_i$ in the following results, respectively.

Each primary user has two status: busy or idle. The holding time of busy or idle is assumed to be random and has cumulative distribution functions (CDFs) $F_\lambda(x)$ or $F_\mu(x)$, respectively. Denote the mean holding times of busy and idle as λ and μ, respectively. Therefore, at any time instant, a primary user is busy with probability $p_b(\lambda, \mu) = \frac{\lambda}{\mu+\lambda}$, and idle with probability $p_i(\lambda, \mu) = 1 - p_b(\lambda, \mu)$. Assume that a primary user is idle at the beginning of the sensing period, and then becomes busy after the k-th sample. Then, the last sample of the idle period is the k-th sample. The probability mass function (PMF) for the case when the primary user's status changes from idle to busy after the k-th sample is derived as [9]

$$p_\mu(k) = F_\mu(kT_s) - F_\mu((k-1)T_s) \tag{1}$$

where T_s is the sample interval. Similarly, the PMF for the case when the primary user's status changes from busy to idle after the k-th sample is derived as

$$p_\lambda(k) = F_\lambda(kT_s) - F_\lambda((k-1)T_s). \tag{2}$$

Note that this alternating renewal process model has been verified by real traffic data [7] [8] and has been used in different works [9]- [11]. Therefore, our analysis based on this model applies to these practical cases [7]- [11]. As well, since the analysis is based on a very general traffic model in (1) and (2), it is valid for any model of the primary network with a specific PMF. In the numerical examples, several typical models will be examined but study of each primary user's activitity in different practical systems is beyond the scope of the paper.

III. Performance Analysis

Assume that the state of each primary user changes at most once during the sensing period. This is the case when the sensing period is at the same level of the holding time but also the case when the sensing period is shorter than the holding time but the primary user happens to change status during the sensing period. We consider the case of two primary users first. In this case, at any time instant, the channel can be idle with probability $p_I(\lambda, \mu) = p_i^2(\lambda, \mu)$, or be occupied by one primary user with probability $p_{B1}(\lambda, \mu) = 2p_i(\lambda, \mu)p_b(\lambda, \mu)$, or be occupied by two primary users with probability $p_{B2}(\lambda, \mu) = p_b^2(\lambda, \mu)$. Since the state of each primary user changes at most once during the sensing period, the channel state can change up to twice during the sensing period. Then, the binary hypothesis testing problem in the conventional energy detector given by [2] can be decomposed into a ten-hypothesis testing problem as

$$\mathcal{Y} = \begin{cases} \sum_{n=1}^{2m}(S_n^{(2)} + Z_n)^2, & \mathcal{H}_{1,1} \\ \sum_{n=1}^{k_1}(S_n^{(1)} + Z_n)^2 + \sum_{n=k_1+1}^{2m}(S_n^{(2)} + Z_n)^2, & \mathcal{H}_{1,2} \\ \sum_{n=1}^{k_1}(S_n^{(2)} + Z_n)^2 + \sum_{n=k_1+1}^{2m}(S_n^{(1)} + Z_n)^2, & \mathcal{H}_{1,3} \\ \sum_{n=1}^{2m}(S_n^{(1)} + Z_n)^2, & \mathcal{H}_{1,4} \\ \sum_{n=1}^{k_1} Z_n^2 + \sum_{n=k_1+1}^{2m}(S_n^{(1)} + Z_n)^2, & \mathcal{H}_{1,5} \\ \sum_{n=1}^{min(k_1,k_2)} Z_n^2 + \sum_{n=min(k_1,k_2)+1}^{max(k_1,k_2)}(S_n^{(1)} + Z_n)^2 + \sum_{n=max(k_1,k_2)+1}^{2m}(S_n^{(2)} + Z_n)^2, & \mathcal{H}_{1,6} \\ \sum_{n=1}^{min(k_1,k_2)}(S_n^{(1)} + Z_n)^2 + \sum_{n=k_1+1}^{k_2} Z_n^2 + \sum_{n=k_2+1}^{k_1}(S_n^{(2)} + Z_n)^2 & \\ + \sum_{n=max(k_1,k_2)+1}^{2m}(S_n^{(1)} + Z_n)^2, & \mathcal{H}_{1,7} \\ \sum_{n=1}^{2m} Z_n^2, & \mathcal{H}_{0,1} \\ \sum_{n=1}^{k_1}(S_n^{(1)} + Z_n)^2 + \sum_{n=k_1+1}^{2m} Z_n^2, & \mathcal{H}_{0,2} \\ \sum_{n=1}^{min(k_1,k_2)}(S_n^{(2)} + Z_n)^2 + \sum_{n=min(k_1,k_2)+1}^{max(k_1,k_2)}(S_n^{(1)} + Z_n)^2 + \sum_{n=max(k_1,k_2)+1}^{2m} Z_n^2, & \mathcal{H}_{0,3} \end{cases} \quad (3)$$

where k_1 represents the number of samples after which the first primary user's status changes, k_2 represents the number of samples after which the second primary user's status changes, k_1 and k_2 are determined by the primary user traffic and $k_1, k_2 \in [1, 2m]$, $S_n^{(1)}$ is the primary user signal for one primary user, $S_n^{(2)}$ is the primary user signal for two primary users, Z_n is defined as before, and $\sum_{i=a}^{b}(\cdot) = 0$ when $a > b$. One sees that the conventional sensing model in [2] corresponds to the hypotheses of $\mathcal{H}_{1,4}$ and $\mathcal{H}_{0,1}$ in (3), and the sensing model for one primary user in [5] corresponds to the hypotheses of $\mathcal{H}_{1,4}$, $\mathcal{H}_{1,5}$, $\mathcal{H}_{0,1}$ and $\mathcal{H}_{0,2}$ in (3).

The probabilities of detection and false alarm can be derived as

$$P_d(\lambda,\mu) = \frac{1}{P(\mathcal{H}_1)}\{P(\mathcal{H}_{1,1},\lambda,\mu)\cdot P(\mathcal{H}_1|\mathcal{H}_{1,1}) + P(\mathcal{H}_{1,4},\lambda,\mu)\cdot P(\mathcal{H}_1|\mathcal{H}_{1,4}) \tag{4}$$
$$+ \sum_{k_1=1}^{2m}[P(\mathcal{H}_{1,2},\lambda,\mu,k_1)\cdot P(\mathcal{H}_1|\mathcal{H}_{1,2},k_1) + P(\mathcal{H}_{1,3},\lambda,\mu,k_1)\cdot P(\mathcal{H}_1|\mathcal{H}_{1,3},k_1)$$
$$+ P(\mathcal{H}_{1,5},\lambda,\mu,k_1)\cdot P(\mathcal{H}_1|\mathcal{H}_{1,5},k_1) + \sum_{k_2=1}^{2m}(P(\mathcal{H}_{1,6},\lambda,\mu,k_1,k_2)\cdot P(\mathcal{H}_1|\mathcal{H}_{1,6},k_1,k_2)$$
$$+ P(\mathcal{H}_{1,7},\lambda,\mu,k_1,k_2)\cdot P(\mathcal{H}_1|\mathcal{H}_{1,7},k_1,k_2))]\}$$

and

$$P_f(\lambda,\mu) = \frac{1}{P(\mathcal{H}_0)}\{P(\mathcal{H}_{0,1},\lambda,\mu)\cdot P(\mathcal{H}_1|\mathcal{H}_{0,1}) + \sum_{k_1=1}^{2m}[P(\mathcal{H}_{0,2},\lambda,\mu,k_1)\cdot P(\mathcal{H}_1|\mathcal{H}_{0,2},k_1) \tag{5}$$
$$+ \sum_{k_2=1}^{2m} P(\mathcal{H}_{0,3},\lambda,\mu,k_1,k_2)\cdot P(\mathcal{H}_1|\mathcal{H}_{0,3},k_1,k_2)]\},$$

respectively, where

$$P(\mathcal{H}_1) = P(\mathcal{H}_{1,1},\lambda,\mu) + P(\mathcal{H}_{1,4},\lambda,\mu) + \sum_{k_1=1}^{2m}[P(\mathcal{H}_{1,2},\lambda,\mu,k_1) + P(\mathcal{H}_{1,3},\lambda,\mu,k_1) \tag{6}$$
$$+ P(\mathcal{H}_{1,5},\lambda,\mu,k_1) + \sum_{k_2=1}^{2m}(P(\mathcal{H}_{1,6},\lambda,\mu,k_1,k_2) + P(\mathcal{H}_{1,7},\lambda,\mu,k_1,k_2))]$$

is the probability that the channel is occupied, and

$$P(\mathcal{H}_0) = P(\mathcal{H}_{0,1},\lambda,\mu) + \sum_{k_1=1}^{2m}[P(\mathcal{H}_{0,2},\lambda,\mu,k_1) + \sum_{k_2=1}^{2m} P(\mathcal{H}_{0,3},\lambda,\mu,k_1,k_2)] \tag{7}$$

is the probability that the channel is idle, $P(\mathcal{H}_{1,1},\lambda,\mu)$, $P(\mathcal{H}_{1,2},\lambda,\mu,k_1)$, $P(\mathcal{H}_{1,3},\lambda,\mu,k_1)$, $P(\mathcal{H}_{1,4},\lambda,\mu)$, $P(\mathcal{H}_{1,5},\lambda,\mu,k_1)$, $P(\mathcal{H}_{1,6},\lambda,\mu,k_1,k_2)$, $P(\mathcal{H}_{1,7},\lambda,\mu,k_1,k_2)$, $P(\mathcal{H}_{0,1},\lambda,\mu)$, $P(\mathcal{H}_{0,2},\lambda,\mu,k_1)$ and $P(\mathcal{H}_{0,3},\lambda,\mu,k_1,k_2)$ are defined in Appendix A. Note from (4)-(7) that the probability that the primary user leaves or arrives during the sensing period is given by $\tilde{P} = (P(\mathcal{H}_1) - P(\mathcal{H}_{1,1},\lambda,\mu) - P(\mathcal{H}_{1,4},\lambda,\mu)) + (P(\mathcal{H}_0) - P(\mathcal{H}_{0,1},\lambda,\mu))$. This result is general and applies to all applications. The specific value of $\tilde{P}$ could be low or high, depending on the specific sensing period and primary mean holding time in the interested applications.

The above analysis can be specialized to the primary networks with $\lambda << \mu$ by setting $p_{B1}(\lambda,\mu) = 0$ and $p_{B2}(\lambda,\mu) = 0$ in the equations, as $p_b(\lambda,\mu) \approx 0$ and $p_i(\lambda,\mu) \approx 1$. It applies to the case of two primary users. Using similar methods, one can extend it to the case of more primary users. The complexity grows exponentially with the number of primary users. Thus, it does not lead to a tractable analysis for a large number of primary users. On the other hand, a simplified special case exists when λ equals μ. One can let k_1 and k_2 span from 0 to $2m$ and define $p_\mu(0) = 1 - F_\lambda(T)$, $p_\lambda(0) = 1 - F_\mu(T)$. Then, one has the case of N primary users as

$$\mathcal{Y} = \begin{cases} \sum_{n=1}^{k_1} Z_n^2 + \sum_{n=k_1+1}^{k_2} (S_n^{(1)} + Z_n)^2 & \\ \quad + \sum_{n=k_2+1}^{k_3} (S_n^{(2)} + Z_n)^2 + ... + \sum_{n=k_N+1}^{2m} (S_n^{(N)} + Z_n)^2, & \mathcal{H}_{1,1} \\ \vdots & \vdots \\ \sum_{n=1}^{k_1} (S_n^{(i-1)} + Z_n)^2 + \sum_{n=k_1+1}^{k_2} (S_n^{(i-2)} + Z_n)^2 + \cdots & \\ \quad + \sum_{n=k_{i-2}+1}^{k_{i-1}} (S_n^{(i-(i-1))} + Z_n)^2 + \sum_{n=k_{i-1}+1}^{k_i} Z_n^2 + \sum_{n=k_i+1}^{k_{i+1}} (S_n^{(1)} + Z_n)^2 & \\ \quad + \sum_{n=k_{i+1}+1}^{k_{i+2}} (S_n^{(2)} + Z_n)^2 + \cdots + \sum_{n=k_N+1}^{2m} (S_n^{(N-(i-1))} + Z_n)^2, & \mathcal{H}_{1,i} \\ \vdots & \vdots \\ \sum_{n=1}^{k_1} (S_n^{(N-1)} + Z_n)^2 + \sum_{n=k_1+1}^{k_2} (S_n^{(N-2)} + Z_n)^2 + \cdots & \\ \quad + \sum_{n=k_{N-1}+1}^{k_N} Z_n^2 + \sum_{n=k_N+1}^{2m} (S_n^{(1)} + Z_n)^2, & \mathcal{H}_{1,N} \\ \sum_{n=1}^{k_1} (S_n^{(N)} + Z_n)^2 + \sum_{n=k_1+1}^{k_2} (S_n^{(N-1)} + Z_n)^2 + \cdots & \\ \quad + \sum_{n=k_{N-1}+1}^{k_N} (S_n^{(N-(N-1))} + Z_n)^2 + \sum_{n=k_N+1}^{2m} Z_n^2, & \mathcal{H}_0 \end{cases} \tag{8}$$

where $\mathcal{H}_{1,i}$ represent the hypothesis that the channel is occupied by $N-(i-1)$ primary users at the end of the sensing period, $\mathcal{H}_0$ represent the hypothesis that the channel is idle at the end of the sensing period, $k_1, \cdots k_i, \cdots k_N \in [0, 2m]$ represents the number of samples after which the primary user status changes with $k_1 \le \cdots \le k_i \le \cdots \le k_N$, and $\sum_{i=a}^{b}(\cdot) = 0$ when $a > b$.

The probabilities of detection and false alarm in this case are derived in Appendix B as

$$P_d(\lambda,\mu) = \frac{1}{P(\mathcal{H}_1)}[\sum_{k_1=0}^{2m}\sum_{k_2=0}^{2m}\cdots\sum_{k_N=0}^{2m} P(\mathcal{H}_{1,1},\lambda,\mu,k_1,\cdots,k_N)\cdot P(\mathcal{H}_1|H_{1,1},k_1,\cdots,k_N)+\cdots$$
$$+P(\mathcal{H}_{1,N},\lambda,\mu,k_1,\cdots,k_N)\cdot P(\mathcal{H}_1|H_{1,N},k_1,\cdots,k_N)] \tag{9}$$

and

$$P_f(\lambda,\mu) = \frac{1}{P(\mathcal{H}_0)}[\sum_{k_1=0}^{2m}\sum_{k_2=0}^{2m}\cdots\sum_{k_N=0}^{2m} P(\mathcal{H}_0,\lambda,\mu,k_1,\cdots,k_N)\cdot P(\mathcal{H}_1|\mathcal{H}_0,k_1,\cdots,k_N)] \tag{10}$$

respectively, where

$$P(\mathcal{H}_1) = \sum_{k_1=0}^{2m}\sum_{k_2=0}^{2m}\cdots\sum_{k_N=0}^{2m} P(\mathcal{H}_{1,1},\lambda,\mu,k_1,\cdots,k_N)+\cdots+P(\mathcal{H}_{1,N},\lambda,\mu,k_1,\cdots,k_N) \tag{11}$$

is the probability that the channel is occupied,

$$P(\mathcal{H}_0) = \sum_{k_1=0}^{2m}\sum_{k_2=0}^{2m}\cdots\sum_{k_N=0}^{2m} P(\mathcal{H}_0,\lambda,\mu,k_1,\cdots,k_N) \tag{12}$$

is the probability that the channel is idle, and the expressions of $P(\mathcal{H}_{1,1},\lambda,\mu,k_1,\cdots,k_N)$, $\cdots$, $P(\mathcal{H}_{1,N},\lambda,\mu,k_1,\cdots,k_N)$ and $P(\mathcal{H}_0,\lambda,\mu,k_1,\cdots,k_N)$ are given in Appendix B. Then, the error probability can be calculated as

$$P_e(\lambda,\mu) = [1-P_d(\lambda,\mu)]P(\mathcal{H}_1)+P_f(\lambda,\mu)P(\mathcal{H}_0). \tag{13}$$

Note that the above results assume the same traffic load for all primary users. It can be easily extended to the case when different primary users have different loads by replacing λ and μ with λ_i and μ_i, respectively, for the i-th user.

IV. Numerical Results and Discussion

In this section, numerical examples are presented. In the Neyman-Pearson (NP) criterion, η is calculated by assigning a predetermined value β to the probability of false-alarm derived [12]. In the minimum error-probability (ME) criterion, η is calculated by minimizing the probability of error [12]. We set $T_s = 0.00125$ s in all the examples. Exponential distribution [13], Gamma distribution [14] and lognormal distribution [15] are used to model the primary user traffic. Also, assume $\lambda = \mu$. Other relationships between λ and μ for other network conditions can also be examined for two primary users. For more than two primary users, this assumption has to be used but it still gives very useful and important insights on the sensing performance, which serves the purpose of this paper.

Fig. 1 compares the simulation results and the analytical results for the probability of detection P_d. Two primary users are considered with exponential traffic, $\gamma = -5$ dB, and the NP criterion for $\beta = 0.01$. One sees that the simulation results agree with the analytical results well for all the cases. Comparing different values of μ, it can be seen that, the larger the value of μ is, the higher the probability of detection will be, under the same conditions. This is due to the fact that, the larger the mean holding time is, the less the probability that the primary user status changes during the sensing time will be, which improves the detection performance.

Fig. 2 shows the receiver operating characteristics (ROC) curves for different models of the primary user traffic based on the NP criterion. In the calculation, the detection threshold η is derived from (5) numerically, by varying P_f from 10^{-3} to 1. Also, we have $T = 0.05$ s and $\gamma = 0$ dB. Comparing the ROC curves for the same distribution with different variances, one sees that a smaller variance gives a higher P_d for the same P_f. This is because when the mean holding time is larger than the sensing time, a smaller variance makes the probability of a primary user status changes during the sensing period smaller and therefore, the sensing performance is better. Comparing the ROC curves for different distributions, it is seen that the sensing performance for the lognormal distributed holding time is more sensitive to the variance than that for the Gamma distributed holding time. For the same system, in Fig. 3, we take the exponential holding time as an example to show the effect of the mean holding time on the spectrum sensing performance. One can see that a smaller mean holding time results in a lower P_d for the same given P_f. This is due to the fact that a smaller mean holding time makes it more likely for the primary user status to change during the sensing period and to degrade the sensing performance. Fig. 4 shows the ROC curves for different relationships between μ and λ for two primary users when $\lambda = 0.2$. When λ is fixed to 0.2, it can be seen from the figure that a smaller value of μ gives a better ROC performance. However, the performance gain is smaller when μ is smaller.

Fig. 5 shows the error probability versus the number of samples for different models of the primary user traffic. Two primary users are considered. The mean holding time in this figure is determined by the ratio $R = \lambda/T$, and we set $R = 3$ in this comparison. We have $\gamma = 0$ dB, and the ME criterion is used. It can be observed that a smaller variance results in a lower error probability, and the error probability for the lognormal distributed holding time is more sensitive to the variance than that for the Gamma distributed holding time. It can also be shown that a larger R results in a lower error probability. This is because that, the larger the value of R is, the smaller the probability that a primary user status changes during the sensing period will be. Also one can show that the error probability for a larger R is more sensitive to the number of samples than that for a smaller R.

Fig. 6 shows the ROC curves for different numbers of primary users. The NP criterion is used with P_f varying from 10^{-3} to 1. We set $T = 0.01$ s, $\gamma = 0$ dB, and the holding time is exponentially distributed with means 0.01 s and 0.02 s. As expected, a larger number of primary users results in a higher probability of detection under the same conditions. Comparing the performance gains achieved by multiple primary users for different values of mean holding time, it is seen that a larger mean holding time increases the performance gain.

Fig. 7 shows the error probability versus the SNR of the primary signal for different numbers of primary users. We set $T = 0.01$ s, and the mean holding time in this figure is also determined by the ratio $R = \lambda/T$, which is set at 3 and 6 in this comparison. It is seen that the decreasing rate of the error probability for $R = 3$ is smaller than that for $R = 6$. This is also due to the fact that a smaller R results in a higher probability that the primary users arrive or leave during the sensing period. Also, as expected, a larger number of primary users results in a lower error probability under the same conditions.

V. Conclusions

The effect of the primary user traffic on the performance of spectrum sensing has been analyzed for the case when multiple primary users arrive or leave during the sensing period. Numerical results have shown that the performance of spectrum sensing will be degraded by the primary user traffic and the degradation decreases when the number of primary users increases. This analysis tells us how spectrum sensing will perform for a given traffic and a given number of primary users. However, knowledge of the traffic distribution and the number of primary users is not required in the energy detection. Although this paper extends the single-user case in [5] using a common method, to the best of the authors' knowledge, the result is new and has not been obtained in the literature. Due to the exponential complexity for an arbitrary number of primary users, this paper only presents a simplified result for $\lambda = \mu$. Although this result is useful and important, future research will derive general closed-form expressions for any values of mean holding times by considering approximations to the hypotheses-testing problem.

APPENDIX A
DERIVATIONS OF (4) AND (5)

Based on the traffic model given in (1) and (2), and assuming that two primary users are independent, the probability for each channel state can be calculated as

$$\begin{aligned}
P(\mathcal{H}_{1,1}, \lambda, \mu) &= p_{B2}(1 - F_\lambda(T))(1 - F_\lambda(T)) \qquad (14)\\
P(\mathcal{H}_{1,2}, \lambda, \mu, k_1) &= p_{B1} p_\mu(k_1)(1 - F_\lambda(T))\\
P(\mathcal{H}_{1,3}, \lambda, \mu, k_1) &= 2p_{B2}(1 - F_\lambda(T))p_\lambda(k_1)\\
P(\mathcal{H}_{1,4}, \lambda, \mu) &= p_{B1}(1 - F_\lambda(T))(1 - F_\mu(T))\\
P(\mathcal{H}_{1,5}, \lambda, \mu, k_1) &= 2p_I(1 - F_\mu(T))p_\mu(k_1)\\
P(\mathcal{H}_{1,6}, \lambda, \mu, k_1, k_2) &= p_I p_\mu(k_1)p_\mu(k_2)\\
P(\mathcal{H}_{1,7}, \lambda, \mu, k_1, k_2) &= p_{B1} p_\mu(k_1)p_\lambda(k_2)\\
P(\mathcal{H}_{0,1}, \lambda, \mu) &= p_I(1 - F_\mu(T))(1 - F_\mu(T))\\
P(\mathcal{H}_{0,2}, \lambda, \mu, k_1) &= p_{B1} p_\lambda(k_1)(1 - F_\mu(T))\\
P(\mathcal{H}_{0,3}, \lambda, \mu, k_1, k_2) &= p_{B2} p_\lambda(k_1)p_\lambda(k_2).
\end{aligned}$$

Similar to [5], chi-square distribution is used to model the output of the energy detector $\mathcal{Y}$. Using this distribution, the probability of detection under different cases can be derived as

$$P(\mathcal{H}_1|\mathcal{H}_{1,1}) = \mathcal{Q}_m(\sqrt{4m\gamma}, \sqrt{\eta}), P(\mathcal{H}_1|\mathcal{H}_{1,2}, k_1) = \mathcal{Q}_m(\sqrt{(2m-k_1)\gamma + 2m\gamma}, \sqrt{\eta}) \quad (15)$$

$$P(\mathcal{H}_1|\mathcal{H}_{1,3}, k_1) = \mathcal{Q}_m(\sqrt{k_1\gamma + 2m\gamma}, \sqrt{\eta}), P(\mathcal{H}_1|\mathcal{H}_{1,4}) = \mathcal{Q}_m(\sqrt{2m\gamma}, \sqrt{\eta})$$

$$P(\mathcal{H}_1|\mathcal{H}_{1,5}, k_1) = \mathcal{Q}_m(\sqrt{(2m-k_1)\gamma}, \sqrt{\eta})$$

$$P(\mathcal{H}_1|\mathcal{H}_{1,6}, k_1, k_2) = \mathcal{Q}_m(\sqrt{(2m-k_1)\gamma + (2m-k_2)\gamma}, \sqrt{\eta})$$

$$P(\mathcal{H}_1|\mathcal{H}_{1,7}, k_1, k_2) = \mathcal{Q}_m(\sqrt{k_1\gamma + (2m-k_2)\gamma}, \sqrt{\eta})$$

where $\mathcal{Q}_m(a,b) = \int_b^\infty \frac{x^m}{a^{m-1}} e^{-\frac{x^2+a^2}{2}} I_{m-1}(ax)dx$ is the generalized Marcum Q-function [16] with $I_{m-1}(\cdot)$ being the modified Bessel function of the $(m-1)$th order, and η is the detection threshold for the energy detector. The probability of false alarm under different cases are given as

$$P(\mathcal{H}_1|\mathcal{H}_{0,1}) = 1 - \frac{\Gamma(m, \eta/2)}{\Gamma(m)} \quad (16)$$

$$P(\mathcal{H}_1|\mathcal{H}_{0,2}, k_1) = \mathcal{Q}_m(\sqrt{k_1\gamma}, \sqrt{\eta})$$

$$P(\mathcal{H}_1|\mathcal{H}_{0,3}, k_1, k_2) = \mathcal{Q}_m(\sqrt{k_1\gamma + k_2\gamma}, \sqrt{\eta}),$$

where $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$ and $\Gamma(z,x) = \int_0^x t^{z-1}e^{-t}dt$ are the complete and lower incomplete Gamma functions [17], respectively. Note that the probabilities of false alarm and detection given in (15) and (16), respectively, are conditional probabilities, conditioned on k_1 and k_2. By averaging the conditional probabilities of detection in (15) and the conditional probabilities of false alarm in (16) over k_1 and k_2, the overall unconditional probabilities of detection and false alarm can be calculated as (4) and (5), respectively.

APPENDIX B

DERIVATIONS OF (9) AND (10)

Using the chi-square distribution for $\mathcal{Y}$ in (8), by inspection, $\mathcal{Y}$ in $\mathcal{H}_{1,i}$ has freedom $2m$ and non-centrality parameter $(2m-k_1)\gamma + ...(2m-k_{N-i+1})\gamma + k_{N-i+2}\gamma + ... + k_N\gamma$. Therefore, the probability of detection for different cases in (8) can be calculated as

$$P(\mathcal{H}_1|\mathcal{H}_{1,i},k_1,...,k_N) =$$
$$\mathcal{Q}_m(\sqrt{(2m-k_1)\gamma+...(2m-k_{N-i+1})\gamma+k_{N-i+2}\gamma+...+k_N\gamma},\sqrt{\eta})$$

for $i=1,\cdots,N$. Similarly, the probability of false alarm can be calculated as

$$P(\mathcal{H}_1|\mathcal{H}_0,k_1,\cdots,k_N) = 1-\frac{\Gamma(m,\eta/2)}{\Gamma(m)}, \quad when\ k_1=k_2=\cdots=k_N=0 \tag{17}$$
$$P(\mathcal{H}_1|\mathcal{H}_0,k_1,\cdots,k_N) = \mathcal{Q}_m(\sqrt{k_1\gamma+k_2\gamma+\cdots+k_N\gamma},\sqrt{\eta}) \quad otherwise.$$

Next, we calculate the probabilities of the $N+1$ channel states. When there are N primary users, at the beginning of the sensing, the channel has $N+1$ possible states with probabilities

$$p_{Bi} = \binom{N}{i} p_b^i(\lambda,\mu)p_i^{N-i}(\lambda,\mu) \tag{18}$$
$$p_I = \binom{N}{N} p_i^N(\lambda,\mu)$$

where p_{Bi} is the probability that i primary users are busy and other primary users are idle, $i=1,\cdots,N$, and p_I is the probability that all the primary users are idle. Since each primary user's status changes at most once during the whole sensing period, the channel status changes up to N times when there are N primary users. Then, one as

$$P(\mathcal{H}_{1,i},\lambda,\mu,k_1,...,k_N) = p_{B(i-1)}\prod_{n_1=1}^{N-i+1}p_\mu(k_{n_1})\cdot\prod_{n_2=1}^{i-1}p_\lambda(k_{n_2})$$
$$P(\mathcal{H}_0,\lambda,\mu,k_1,...,k_N) = p_{BN}\prod_{n=1}^{N}p_\lambda(k_n),$$

where $P(\mathcal{H}_{1,i},\lambda,\mu,k_1,...,k_N)$ is the probability that $i-1$ primary users are idle and the rest $N-(i-1)$ primary users are busy at the end of the sensing period, $P(\mathcal{H}_0,\lambda,\mu,k_1,...,k_N)$ is the probability of that all the N primary users are idle at the end of the sensing period. Finally, using the above results, the overall unconditional probabilities of detection and false alarm can be calculated as (9) and (10), respectively.

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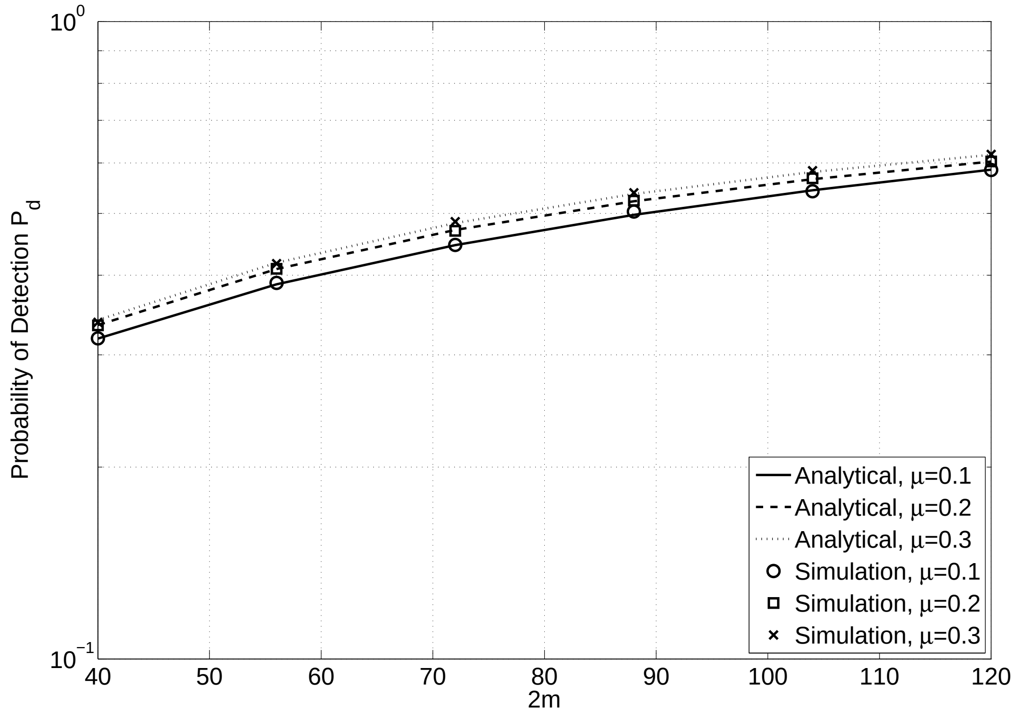

Fig. 1. Probability of detection P_d versus the number of samples $2m$ based on the NP criterion. Two primary users are considered, and the holding time is exponentially distributed.

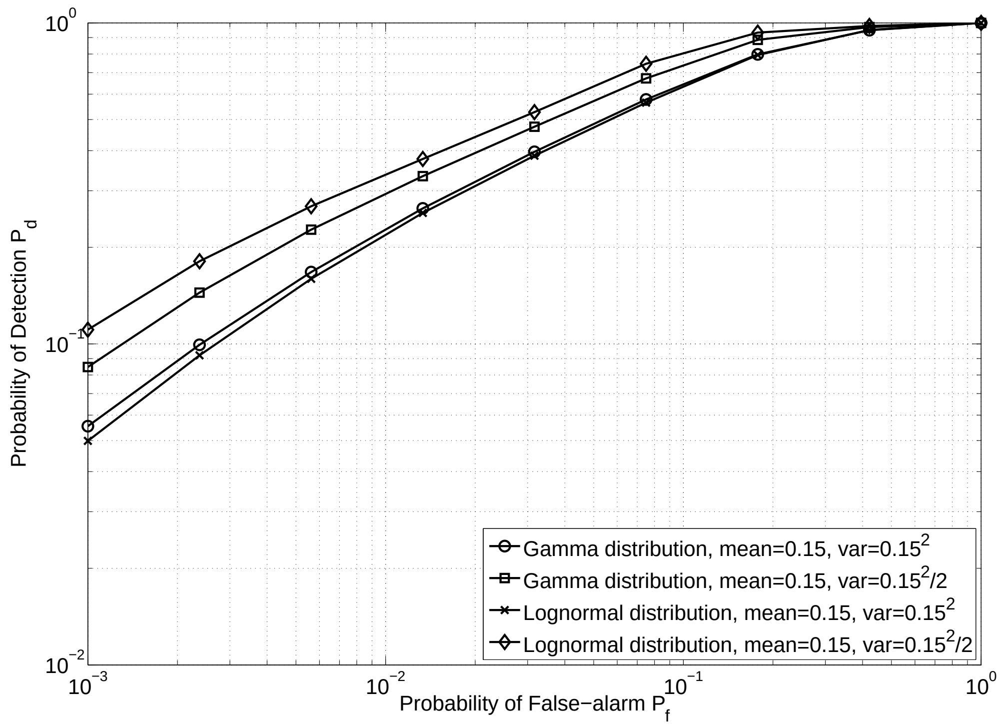

Fig. 2. The ROC curves for different models of the primary user traffic based on the NP criterion. Two primary users are considered.

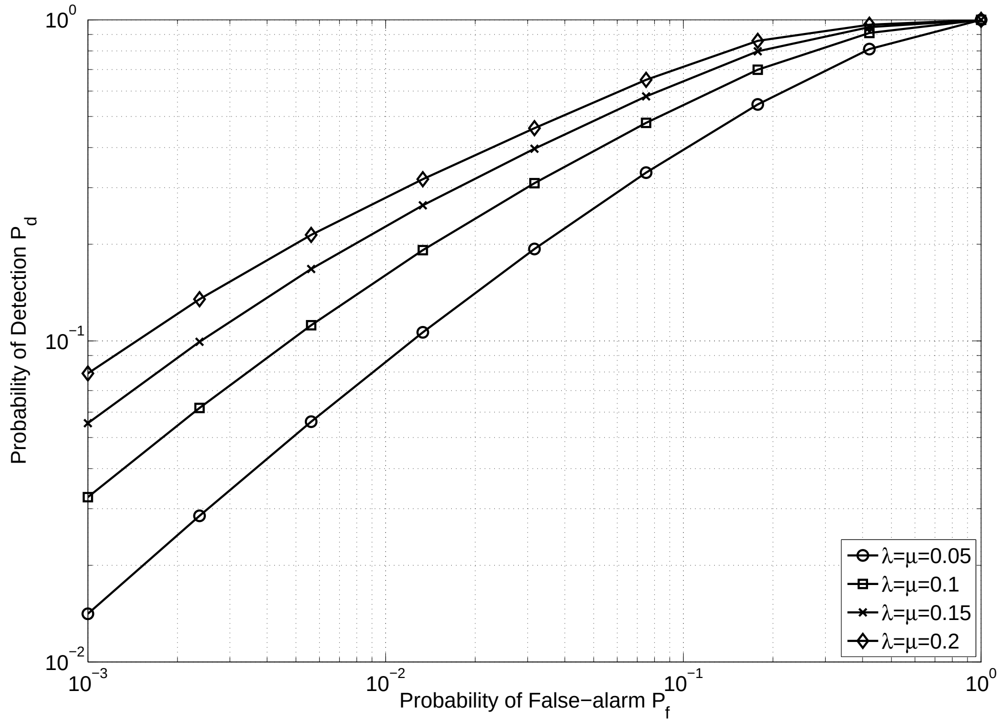

Fig. 3. The ROC curves for different values of mean holding time of the primary user when the holding time is exponentially distributed based on the NP criterion. Two primary users are considered.

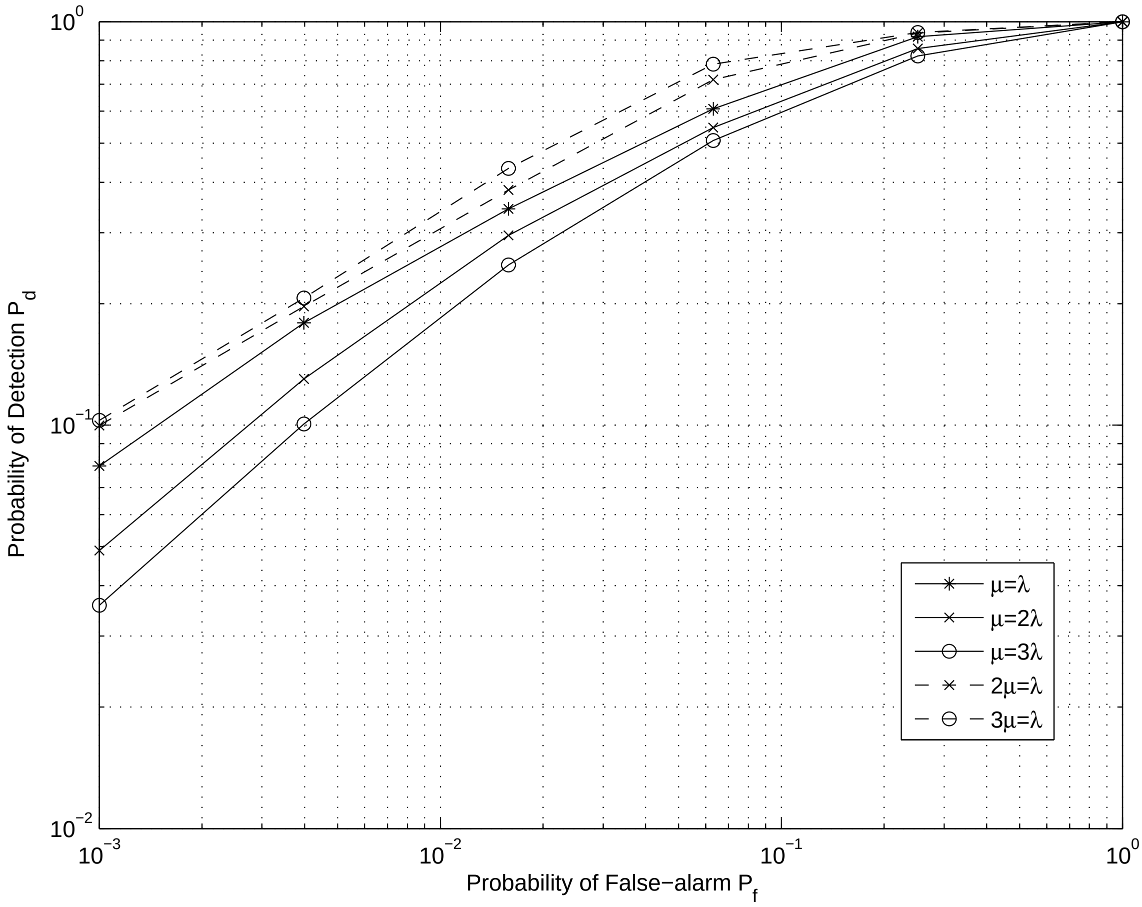

Fig. 4. The ROC curves for different relationships between μ and λ when the holding time is exponentially distributed based on the NP criterion with $\lambda = 0.2$. Two primary users are considered.

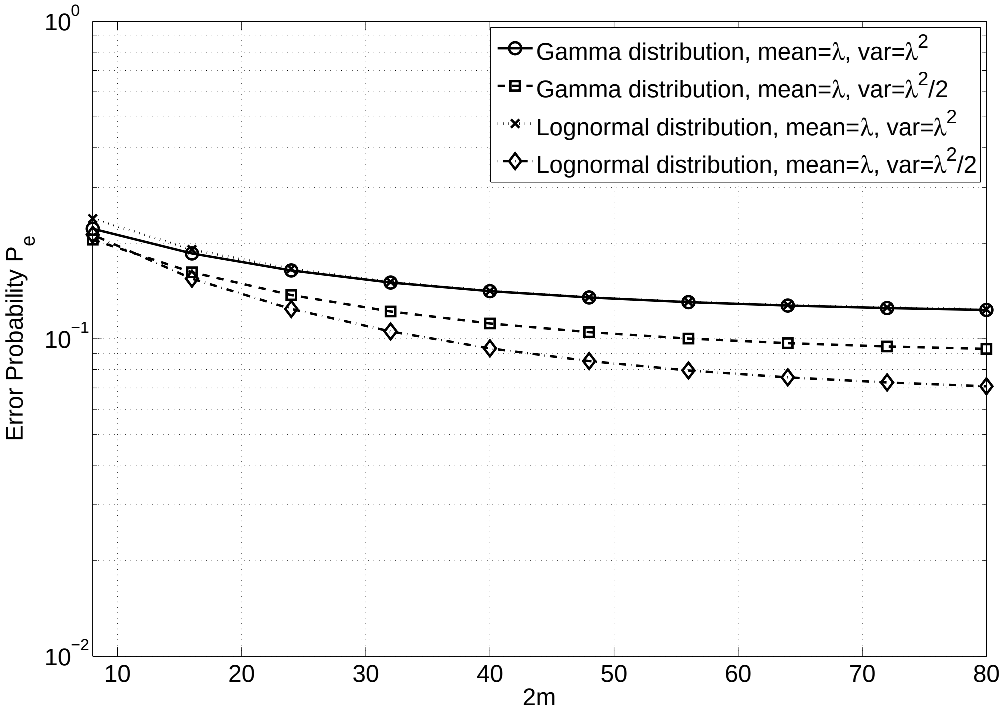

Fig. 5. Error probability versus the number of samples $2m$ based on the ME criterion for different models of the primary user traffic. Two primary users are considered.

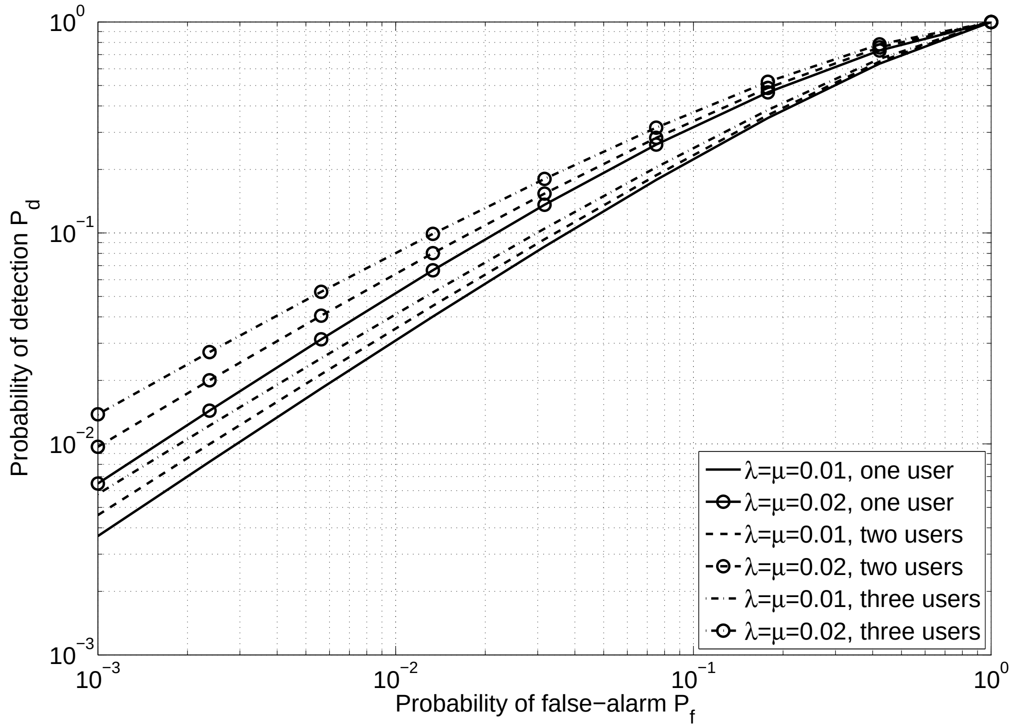

Fig. 6. The ROC curves for different numbers of primary users and different values of mean holding time of the primary user when the holding time is exponentially distributed based on the NP criterion.

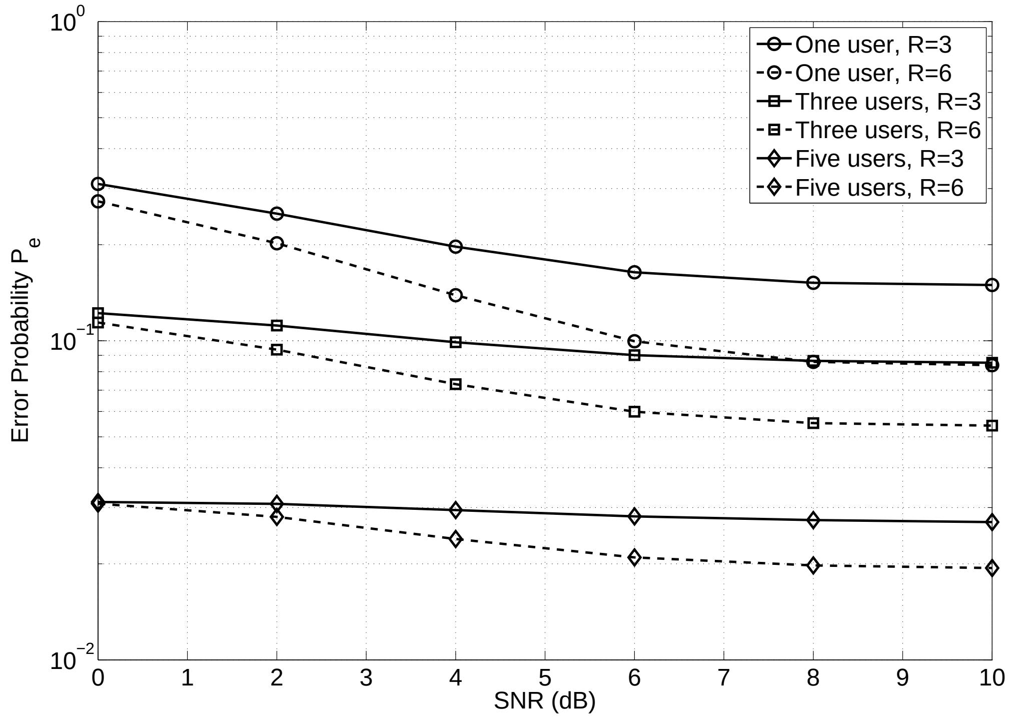

Fig. 7. Error probability versus SNR based on the ME criterion for different numbers of primary users and different values of R when the holding time is exponentially distributed.