Building and Assessing Subject Knowledge in Mathematics for Pre-Service Students

Janet Ainley and Mary Briggs

Mathematics Education Research Centre, Institute of Education, University of Warwick, Coventry CV4 7AL

Background

In planning and teaching curriculum courses for pre-service primary teachers, both within a one-year Post Graduate Certificate of Education (PGCE) programme, and in a four-year undergraduate degree leading to Qualified Teacher Status (QTS), we have always been aware that mathematics presents particular problems because of the experiences and attitudes students bring to the subject. We have always tried to balance students' learning about how children learn mathematics in school, with reflection on their own experiences as learners, and with understanding of the mathematical content of the curriculum. In the past, mathematical content has generally been approached indirectly through discussion of activities and materials appropriate for the primary classroom. However the recent introduction in the U.K. of a National Curriculum for Primary Mathematics in Initial Teacher Training (ITT) which places considerable emphasis on students' subject knowledge, has meant that we have had to rethink the balance within courses, and to place much more overt emphasis on developing students' mathematical knowledge.

In this paper we describe the way in which we have approached building and assessing mathematical knowledge during the pilot phase of the ITT National Curriculum, examine some of the students' responses to our approach, and discuss the issues this has raised.

Context: the students and the courses

All our students have a mathematics qualification at 16+ (GCSE), and a few have specialised in mathematics beyond this level. However, even with this requirement for a Grade C pass at GCSE, there are still wide variations in students' previous experiences in mathematics;

- some have barely reached the pass level,
- some, regardless of their level of qualification, have low levels of confidence and /or bad experiences of learning mathematics, and
- some have only procedural knowledge of many areas.

The ITT National Curriculum requires knowledge beyond GCSE in some areas, and a further problem is that some of the content does not have clear links with the current National Curriculum for schools that the students are expected to teach. This raises concerns from the students about why they need to 'know' this particular selection from the domain of mathematics.

From previous studies (Crook and Briggs 1991, Briggs, 1992, 1993) it is clear that pre-service students come with varying degrees of confidence with mathematics. One area that stands out from this research is the influence confidence with mathematics has on the students' choice of age group they finally wish to teach. Thus the problems of confidence are often most acute amongst students wishing to specialise in Key Stage 1 (4 - 7 year olds); the very students for whom the gap between the mathematics they are expected to know, and what they will need to teach, is widest. It is also clear that those with limited personal confidence value an approach that leads them through developing their own understanding of mathematics they can already 'do', rather than presenting them predominantly with mathematics at their own level, particularly at the start of the course.

The mathematics curriculum courses we teach are planned around both topics within the school curriculum (e.g. understanding patterns, multiplication, measurement, probability) and more general topics and issues, (e.g. assessment, lesson planning, language and mathematics, questioning techniques, children's informal methods). The courses are taught through a mixture of lead sessions, which introduce and give an overview of the topic, and smaller group sessions in which students have more opportunity for discussion and 'hands-on' activity. At an early stage in all courses, students are introduced to Skemp's (1976) work on instrumental and relational understanding, and the vocabulary established here is used throughout the course. There is a strong emphasis on making links within mathematics and across the primary curriculum. This is an important issue in relation to the ITT National Curriculum since within the document itself, such links are not immediately obvious to the students. This approach matches closely the findings of Askew et al. (1997 p. 93):

> *What would appear to matter in relation to the effectiveness of teachers is not formal qualifications or the amount of formal subject knowledge, but the* nature of the knowledge about the subject *that teachers have. The connectedness of teachers' mathematical knowledge in terms of their appreciation of the multi-faceted nature of mathematical meanings does appear to be a factor associated with greater pupil learning gains.*

In discussions on course design, particularly in the light of current external pressures, we have been aware of various tensions in trying to balance

- developing competence in mathematics and developing confidence with mathematics
- valuing subject knowledge and valuing pedagogic knowledge
- valuing procedural knowledge and valuing conceptual understanding
- supporting students and assessing students.

Within the courses, students are set study tasks and school-based tasks, which are designed to link closely to taught sessions. For example, following a lead session on mathematical language, students are set the task of observing the language used by children and their teacher within a mathematics lesson. They then use their observations as a basis for discussion of potential difficulties and effective questioning techniques in a subsequent group session.

Auditing subject knowledge

Since institutions providing ITT courses are responsible for students meeting the subject knowledge requirements of the ITT National Curriculum, one option open to them is to test students on entry using either written tests or computer programmes. Testing like this tells the students and their tutors what they don't know and can damage fragile confidence. It also means that the students need to be tested again to demonstrate that they now 'know' the areas covered. The tests that are used often give access to the instrumental understanding that students have and do not allow access to their relational understanding. This is clearly dependent on the questions set, but for those compiling the tests, questions which allow insights into relational understanding are more difficult and time consuming to write and mark.

A Self-assessment Approach

Because of our concerns about students' confidence, we have adopted a different approach. At various stages in the courses, we have used study tasks for students to make self-assessments of their subject knowledge in mathematics, using the grid shown in Figure 1. At the beginning of a course, this is done against the school National Curriculum. The work described in this paper focuses on a group of third year students who were involved in the pilot phase of the ITT National Curriculum for Primary Mathematics. These students were asked to complete a self-assessment against the section of the ITT National Curriculum which deals with subject knowledge. The wording of the task emphasised that this was a personal assessment, to which no judgements would be attached, and that there was no point in ‘cheating’ by claiming more confidence than they really felt. It was also left open for students to include as much or as little detail as they felt was appropriate. For a number of students this emphasised their confusion over why they thought they needed to ‘know’ aspects of the ITT National Curriculum to teach in a primary school. The area of proof was a particular example.

INSERT FIGURE 1 ABOUT HERE

Having completed this self-assessment, students were asked to set themselves some targets for the entries in the centre or right-hand columns which they were going to work on. They already had the course programme, which made it clear which topics would be addressed in taught sessions, and they were also offered the following suggestions for ways in which they might work on their subject knowledge:

- looking at children’s understanding and classroom activities in workshop sessions,
- looking at school text books,
- using reference books (e.g. Duncan (1993), Haylock (1995)) and trying out activities,
- discussing ideas with friends,
- asking for help from a student taking mathematics as their subject specialism,
- asking for help from a tutor.

The following are examples of some items that two students identified as areas of difficulty as part of their self assessment. They were specifically asked about how they had undertaken the task of filling in the grid.

Sandra

I'm confident I can ...	I think I can ...	I have trouble with ...
Number and algebra *the real number system* *indices* *number operations and algebra* *equations, functions and graphs* *measures* *shape and space*	*using the multiplicative structure of ratio and % to solve problems* *understanding of gradients and intercepts* *calculating the length of arcs* *observed relative frequency*	*Probability* *Mathematical reasoning and proof*

Bill

I'm confident I can ...	I think I can ...	I have trouble with ...
Number and algebra (most) *Proof* *measures (most)* *shape and space* *Probability and statistics*	*distributive laws* *follow rigorous mathematical argument* *Cartesian co-ordinates in 2D* *geometrical construction* *calculate area of circle* *identify 3D shapes, properties etc.*	*Interpreting functions* *Compound measures* *volume and capacity* *formulas for surface area and volume of prisms* *continuous and discrete*

Both Sandra and Bill talked about the process, beginning with the ITT National Curriculum document. Sandra described getting started, *"I read carefully the statements on the left hand side to see if I understand what this means. The language is the major difficulty with these documents but once people have explained it or I have read an explanation for the statement, there is an 'oh yes, is that what that means, I understand that!'"* Both students found that they had to read and re-read the statements in the document and spend time trying to work out what they meant. Sandra explained her decisions related to one of the statements, *"With the real number system I looked at the explanation in the right hand column to see if helped. The order and size of number is given as* *is place value. That was enough for me to make a decision about which column to place it in.*" Other decisions were not as straight forward, and they again centred round concerns about understanding the language. Some topics they placed in the 'have trouble' with column in order to work on this later, or said they thought they understood what is meant by this statement but wanted to check it out so they put it in the 'I think I can' column, but still checked out the language. Both students felt it was a lengthy process as they were doing the task but it made it easier to focus on those areas on which they really needed to work.

What is interesting when you look at the two examples is the range of items they felt confident about and the variation in those they identified that they needed to work on. This reflects the range of previous experiences of the students, and indicates the difficulties of planning a course to meet their varying needs.

As well as focusing students' attention on areas of strength and weakness in their own knowledge, we hoped that this task would serve two other purposes:

- to improve students' familiarity with the contents and vocabulary of the curriculum documents, (the vocabulary used in the ITT National Curriculum caused difficulty especially as the examples given in the document are not helpful in establishing a relational understanding of the specific wording),

- to help students to feel that learning mathematics was something which was within their own control, and which they could undertake independently, (in the same way in which they might research another area of the curriculum which they were required to teach), and to offer them strategies for doing this.

The mathematics curriculum course for third year students ran during the Autumn and Spring terms, after which they began a long school placement. The first self-assessment task was set at the beginning of the Autumn term. Three weeks into the Spring term, the second task was set. This required students to review the progress they had made on the targets they had set, and to identify specific area in which they felt they still needed help to improve their confidence and understanding (Figure 2). Students were aware that Tasks 1 and 2 would be collected in by their tutors at this point for two purposes: tutors would add comments and give specific feedback, and the needs students identified would be used to put together a programme for ‘drop-in surgeries’, which would be held in the second half of term. In practice these surgery sessions were not used by many students, but those who did use them were very enthusiastic about the opportunity to work at their own pace on tasks in a non-threatening environment.

INSERT FIGURE 2 ABOUT HERE

Assessing Competence

Because of the need to assess students against new Standards for the award of QTS, introduced by the Department for Education and Employment (DfEE) we felt it was necessary to have some more formal assessment of subject knowledge at the end of this process. Having put the emphasis on developing student confidence and independence, setting a formal test seemed an inappropriate and counter-productive way of doing this. We decided instead to use the idea of testing in a different way, and ask students to write test questions for each other[1]. At the end of the Spring term students were set Task 3 as shown in Figure 3. The critical issue about the task is the students have to choose questions that will enable them to gain access to their fellow students’ knowledge. This is transferable to the classroom, as a page of algorithms ticked or crossed will tell the teacher little about understanding whereas more open-ended task will offer the opportunity of access to learners understanding.

INSERT FIGURE 3 ABOUT HERE

[1] We are indebted to Dr Anne Watson for this idea.

We hoped that this assignment would fulfil a number of objectives:

- it would give students a purpose and context for consolidating their own subject knowledge in particular areas,
- it would give tutors insight into the extent of their confidence in these areas,
- it would offer an opportunity for students to practise assessment skills which had been emphasised in the course,
- it would raise issues about assessment, and particularly about the limitations of written assessment which might relate to and inform students' work in the classroom,
- it would give tutors insight into the quality of students' thinking on these issues, and an opportunity for feedback.

This task was treated as a formal assignment, which was marked on a pass/fail basis, with students being given brief written feedback. Despite the heavy workload which students had at this time, and a feeling amongst some that this was a 'silly' assignment, the professionalism with which most of them undertook this task was impressive. Although a few, understandably, took a minimalist approach, many have produced word-processed sheets with their questions and appropriate illustrations. Some took a lot of trouble to set their questions in elaborate, and sometimes entertaining, contexts. They had also clearly taken the task of answering each other's questions seriously, and in some cases produced very detailed written answers. Comments in many of the assignments also indicated that a lot of discussion, and in some cases some effective teaching, had gone on around answering the questions that were set.

All the assignments (with Tasks 1 and 2 included for reference) were marked by one tutor. Of 139 assignments submitted, only five were failed, either because the student had not completed what was required, or because there was a serious mathematical error in the work which the student had not recognised. These five students were given the opportunity to discuss their work with a mathematics tutor, and re-submit it during the following year.

As the assignments were being marked, 15 scripts were selected, not as a representative sample, but because they contained interesting examples relating to objectives we had in setting the assignment. All of the students involved have given permission for examples from their work to be used in the following discussion. They are identified by aliases which give a rough indication of their initial level of confidence in mathematics (based on the distribution of items across the three columns in their original self-assessment); confident students have been assigned names beginning with C, those not confident, names beginning with N. Most, but not all, of the ‘C’ students are taking mathematics as their subject specialism.

Outcomes and discussion

The common areas of concern

We were not surprised to find that the areas most often identified in the ‘I have trouble with ...’ column of the self-assessment sheets, and targeted as areas for improvement, included aspects of algebra (particularly functions, graphs, intercepts and gradients), simultaneous equations, probability, statistics, fractions and methods of proof. Formulae for area and volume also concerned many students, and calculation with decimals and ratios were also common topics for their questions in task 3.

In their initial self-assessments, a number of students found difficulty understanding some of the terms and symbols used in the curriculum documents, and many of the queries to tutors were about this. These problems were generally quickly resolved, and so did not appear in the topics chosen for task 3. In fact a very common response from students was that the self-assessment helped them to realise how much they *did* know, once they sorted out the unfamiliar vocabulary. The opinion of many students was that although examples are provided within the ITT National Curriculum, these were not helpful in making the vocabulary clear.

A number of students also identified aspects of mental arithmetic as areas of concern, either referring to specific things, such as multiplication tables, or more generally to developing more flexible strategies. This may have been stimulated by the emphasis placed on mental arithmetic and on a variety of methods during the course, but may also indicate students' lack of confidence in their own informal methods.

In sessions during the course students had raised the issue of understanding different ways of working. For some non-specialists the issue was evident when working on percentages. They has only been taught to work these out using a unitary method and found following other students explanations difficult. The concerns centred on two things, understanding children's methods, (e.g. if they focused on factors as their fellow students had) and understanding other students methods for the assessment task.

<u>Setting 'hard' questions</u>

Deciding on an appropriate level of difficulty for their questions was an understandable concern for many students, particularly those more confident in mathematics. One of the mathematics subject specialists complained that there was no point in setting questions for her colleagues as they 'would obviously get them all right'. In response to this (a response which students probably found very frustrating) we pointed out that deciding on how to make the questions challenging was part of the task, but also reminded them of the distinction between instrumental and relational understanding, and suggested that they should try to test for the later. Getting an appropriate level of difficulty seemed to be much less of a concern for the students less confident in mathematics, presumably because they had a clearer sense of what their colleagues would find difficult. There was also the problem of the potential lack of detail in the answer that they might get with possible short cuts and no working out with which to make sense of the answer.

Despite our emphasis on the distinction between different kinds of understanding, the questions posed by students were almost exclusively instrumental in nature, and those who wanted to set ‘hard’ questions generally made them procedurally difficult by choosing awkward numbers, or problems with many stages. Setting word problems, often containing irrelevant information, was another strategy used to make questions more difficult.

In considering her Pythagoras question (Figure 4)[2], Nell had two suggestions for making it more challenging. The first involved setting the question in a ‘real life’ context.

> *It would perhaps have been challenging had I set the question with a context, for example - a cat is stuck 5 metres up a tree. A fireman brings his ladder and places it 12 metres away from the tree. How long would the ladder need to be? This would have added the challenge of having to visualise the scene for themselves, requiring them to realise that the ladder would have formed the hypotenuse of the triangle. (Nell)*

INSERT FIGURE 4 ABOUT HERE

This contrasts with Nira’s intention in designing a question, by coincidence also about Pythagoras. The first part of her question is ‘abstract’ while the second part is set in the context of measurements of a building (Figure 5). She anticipated that the context would make the question easier, but her analysis of the students’ responses surprised her.

INSERT FIGURE 5 ABOUT HERE

> *I was surprised that both students found the second part of the question harder. I thought that applying the rule (Pythagoras) to a real life situation would actually clarify it, showing its practical application and therefore demanding common sense rather than*

---

[2] In reproducing the students’ questions we have tried as far as possible to retain the original formats. Items which were originally hand-written are indicated by a change of font.

sound theory, perhaps, while the first part of the question was, I thought, more difficult because it asked for direct application of the rule not in any context. (Nira)

Nell and Nira's contrasting views of the effects of setting problems in context mirror differing purposes for which contextualised problems are commonly used in mathematics classrooms. As a teaching devise, mathematical ideas are often set within contexts to make them more accessible to children. However, in assessment contextualised problems are commonly used to provide the more difficult elements, designed to test whether mathematical ideas have been understood sufficiently well to be applied (Ainley, 1997). This is an issue which we would in future want to discuss explicitly with students arising from their experiences in completing this task, particularly in the light of recent research in this area in the context of national testing in schools (e.g. Cooper and Dunne (1998)).

Nell's second suggestion for increasing the difficulty of her question was to offer the problem in a different form from the standard presentation.

Similarly an added assessment of how well the students had grasped this topic would have been to choose a question which involved working out the length of one of the other sides rather than the hypotenuse. This would have involved more than a simple use of the equation. It would have required a manipulation of it, in order to allow the correct calculation. This would have shown that the students had a much deeper knowledge and understanding than is evident from my original question.

A similar approach was used by a few other students in setting their questions. Norman posed a question about the capacity of a cylinder (Figure 6), requiring the student to find the height of the cylinder, given the radius and capacity. He commented on his reasons for the choice of this question, and the difficulties the students experienced in answering it.

The formula was provided which showed that I was not testing memory skills, but wanted a more in depth look at whether, or not, they could manipulate the formula to find the height of the tank.. []

The main weaknesses were found in my second question - calculating the height of the tank. Both respondents had difficulty in identifying what they had to manipulate in order to get the answer. Neither felt that the phrasing of the question was particularly clear, although largely due to the fact that they did not realise, instantly, that they had to rearrange the formula.

INSET FIGURE 6 ABOUT HERE

<u>The instrumental nature of questions</u>

Although the overwhelmingly instrumental nature of the questions set was disappointing, this was to some extent offset by the growing awareness of the limitations of this form of assessment which emerged in the students' analysis of the strengths and weaknesses of their questions. Both Nell and Neil used this vocabulary explicitly in discussing their questions.

Both questions really only give an indication of the students' instrumental understanding. They are able to apply known rules and carry out the ensuing calculations but had no relational understanding - they are not able to explain why the Pythagoras rule worked, for example. (Nell)

My rationale was to test instrumental knowledge, and this was broadly successful. In order to test understanding and knowledge of why fractions behave in the way they do, it would be necessary to use more examples of fractions in differing contexts, .. (Neil)

Cassie set a straightforward question, asking for the volume of a cylinder, given the radius and length (sic), and anticipated how this might be answered:

> *When answering this question the students would fall into one of three distinct groups. These are those who knew the rule instrumentally and would apply it to this situation, those who didn't know the rule but could work out the answer relationally using logic, and those who couldn't answer the question at all. (Cassie)*

She did not discuss how the marker might be able to distinguish between the first and second categories just from the written response, and neither did she comment explicitly on how she judged the understanding of one of students who wrote:

> *Not sure of the equation*
> *is it* $^{1}/_{2}$ *diameter x length?*
> *Diameter =* πr^2 *[faint question mark] = .314 [faint question mark] x* 5^2
> $\therefore$ *[crossing out]* $^{1}/_{2}$ *(.314 x* 5^2*) x 7 = volume* *(Cassie: response from student B)*

However with hindsight Cassie acknowledged a different weakness of this question.

> *A weakness, however, is that if you could not do the question then you would have to leave it blank and this can be quite disheartening. (Cassie)*

Claire was one of the very few students who used an open question in her assessment. The whole of her question sheet on 'Representing functions algebraically' is reproduced (Figure 7) as it illustrates the care and insight with which her assessment was constructed. She commented that the introductory section

> *achieved its purpose of reminding students how to set functions out formally without influencing the knowledge being tested in the questions. (Claire)*

INSERT FIGURE 7 ABOUT HERE

She is equally clear about her intentions for the second (open) question.

> *Q2 is designed not only to test the student's ability to represent functions algebraically, but also to consider the process of forming functions given inputs and outputs. This question was particularly successful, and both students commented that this question was harder and they had to really think.[ ] On reflection it may have been better to ask for 3, 4 or 5 functions for Q2 as this would have stretched their minds further and perhaps encouraged the students to use square roots (not just the four operations) and represent these algebraically. (Claire)*

Although Conny set fairly 'traditional' questions on intercepts and gradients, she was also critical of the effectiveness of her questions, and considered a more open approach.

> *I think the questions I wrote enabled me to assess their knowledge and understanding at a basic level but failed to give an indication of the exact breadth and depth of their knowledge. If I wished to obtain more accurate information [...] I could have attempted to assess their understanding through careful questioning. [ ] If I were to do this exercise again I would have tried to leave the questions on algebra more open ended, enabling students to show me just how much they know and understand about gradients and intercepts. For example I may have asked the question 'Is it possible to tell what the gradient and intercept would be just by looking at the equation, and if so, how?' (Conny)*

Claire's willingness to use open questions is perhaps an indication of her confidence in being able to cope with whatever responses she received. Many students less confident in their own mathematics were clearly more disconcerted by getting responses from students which contradicted their expectations, and this in turn raised some issues about the nature of assessment. In the classroom a teacher expects a range of responses to a task but unexpected responses can occur because of the phrasing of the question. What one student may have felt was clear, another may have found ambiguous. Other responses may have more to do with what the task was intended to assess, and what it actually assessed. Question design for assessment purposes is a complex area. Apple (1989) raises some general assessment questions in relation to equity but also the relationship between assessment and the curriculum. We focus on two of his questions and the links to the task given to the students. His first question is *Whose knowledge is taught?* : clearly in this case this is an imposed selection. The related assessment question is, *What knowledge is assessed and equated with achievement?* In mathematics the right answer is clearly a major focus of what is equated with achievement, particularly in summative assessment. The knowledge assessed is more instrumental with marks equated to achievement. The focus in the formative assessment would be looking for ways of working that give the assessor more information about how the questions had been tackled and assist planning for future teaching. The knowledge being assessed is more relational with an emphasis on progress as well as achievement.

Apples' second curriculum question is Why *is it taught in a particular way to this particular group?* Again this is related to the imposition of the curriculum by external bodies. The associated assessment question is*, Are the form, content and mode of assessment appropriate for different groups and individuals?* It is this final question which can perhaps shed light on some of the difficulties students encountered. The match of form, content and mode of the assessment are all aspects of the assessment process that can affect the assessment outcomes. The learner's response to the task may have influenced their ways of working; knowing that some else would be doing the same to their questions, and knowing the purpose behind the task, all affected the final outcomes.

We had expected perhaps more relationally focused questions, and yet if we think about the students' own experience of assessment and their knowledge of national testing in school, the focus is overwhelmingly instrumental. The students were also being asked to focus on a task that would give them information about a fellow student's achievement at a specific point in time, not to build up a clear and holistic picture of an individual achievement over time, using a variety of methods of collecting information upon which to base their judgements. This raises the issue of students' confidence not just in their own abilities with mathematics, but as in previous examples, the issue of understanding someone else's methods of working something out. Many of these students still rely on instrumental understanding of specific areas of mathematics and following one method or algorithm. We had not given students examples of what relational testing might look like. Indeed, we had not given students *any* examples of the kinds of questions they might ask, but the books they may have turned to for help in working on their own understanding contain many such examples, which inevitably tend to be largely procedural.

Assessing relational understanding is obviously more difficult than assessing instrumental understanding. In schools, some of the attempts to assess relational understanding have focused on Attainment Target 1 of the schools' National Curriculum, which addresses *Using and Applying Mathematics*. Assessment tasks in the form of games were used in an early phase of national testing in the U.K., which caused difficulties for teachers in assessing achievement in the given context. We might reflect on whether we were actually asking too much of students, given the development of their pedagogic skills and the agreed problems that experienced teachers have with assessment issues. The task, however, demonstrated to students the very real issues of the limitations of data available for teachers when they need to make judgements about pupils attainment, therefore adding greatly to the further development of their pedagogic skills.

<u>What is being assessed?</u>

Nina set a fairly straightforward three-part question on simultaneous equations, nicely chosen so that each part involved a little more difficulty than the previous one. She had anticipated that this would allow her to see the techniques the students used to solve them, and their competence in symbol manipulation. However, she had not anticipated that one student would use a 'trial and error' approach to the more difficult questions, and she found it difficult to analyse his response.

Similarly, she had set another question, requiring the students to draw a shape from given co-ordinates, reflect it in the x-axis, and then give it's area. However, since the drawing was done on squared paper, Nina found that her question did not test the skills she had intended.

> *This resulted in them just counting squares which meant that they did not demonstrate that they knew and could use a formulae for area. (Nina)*

Carol's difficulties in understanding one of the responses she received emerged in a group discussion while students were working on their assignments. Carol had set a three part question (shown in Figure 8), designed to test knowledge of forming algebraic expressions. She described her intentions as follows:

> *Part a uses easy numbers, enabling the students to get a 'feel' for what the answer should be and therefore prompting the method if unsure. The second part repeats this completely but using letters. So the student is logically lead (sic) forward. [ ] Part c stretches the student, increasing the number of aspects to the expression. (Carol)*

INSERT FIGURE 8 ABOUT HERE

The first two parts of the question presented no problems, but for part c), one student gave the expected response of $\frac{a}{u}+\frac{a}{v}$ while the other gave $a+a\div(\frac{u+v}{2})$. Carol was initially very puzzled by this response, and her first thought was to see whether this was the expected response in a different form. She was competent enough with symbol manipulation to realise that it wasn't, and so raised the problem with her tutor, and other students. The group (all mathematics specialists) eventually worked out that what the student was trying to express was the total distance travelled ($a + a$) divided by the average time for the journey ($(u + v) \div 2$). Focusing as Carol was on expressing statements algebraically, it was hard to see what was wrong with this. It took a lot more discussion before they realised that the student's problem lay elsewhere: she didn't fully understand the nature of compound units.

Beyond Assessment

Carol's written version of the incident described above is typical of the responses of many students when analysing errors made by their subjects. They clearly put themselves in the role of a teacher, including suggestions about how they might obtain a better assessment of their subjects through questioning them about their responses, and also offered ideas for future work.

> *In the light of this [the] student needs to revise compound measure. Furthermore it would be interesting to ask [the other] student to talk through her response to ensure that she really understands 'why' she's doing what she's doing and that she's not just mechanically following a process. (Carol)*

A number of students went beyond this to make more general points about the implications of their assessment for the classroom. For example, Cathy worked with two students of very differing abilities in mathematics, and made some general points about how she might have assessed them more effectively.

> *If I was to reset the two questions for student B I would need to write down the theories , and explain what was required before setting the questions. Student B needed more help with the questions, which means that I had set the questions at a higher level than her ability allowed. Unlike student A, who needed more challenging questions, student B needed less challenging questions. I noticed that student B failed to read the question thoroughly, before answering. This led to simple mistakes being made and the student asking me questions that had no relevance. (Cathy)*

Nancy noticed differences in the attitudes taken by the students she tested, which she felt were significant for teaching.

After asking for some help, student A managed to work out section (c). Student B, however, wasn't interested in finding out how to do it, she had a different attitude and was happy to leave the sum unsure how to do it. These attitudes, evident on a one-to -one basis would be useful to the teacher in seeing the attitude towards work of their pupils and would enable them to plan work to encourage those who needed to ask for help, but didn't, and so on. (Nancy)

Some students were prompted to reflect on what they felt they had learnt about their own approaches to mathematics through trying to assess others.

These questions and the answers of both students have caused me to re-evaluate my own approach to the topics of fractions and probability. I have since concluded that in all my own mathematics I err on the side of caution choosing logical systematic and 'simpler' ways of carrying out tasks. I believe with mathematics that 'slick' methods such as those demonstrated by student A in particular are super if you feel confident enough to believe them. I think that personally I would always turn to my trusted reliable basic methods as a way of checking a more condensed approach is correct. (Natasha)

I was sceptical about this manner of assessing subject knowledge because it is ultimately dependent upon us telling you what we can do. However, having seen the process in operation I can fully appreciate its merits. I was pleased to find that I could competently answer all the questions that I was posed by my fellow students in order for them to fully understand my written questions. These questions would not have been asked of me had I sat a formal exam. It showed me that I needed to have a deep knowledge of the subject to be able to be put on the spot and ***explain*** *the process rather than just manipulate the formulae to find the answer. (Norman)*

In retrospect I think that this was a very useful exercise to carry out because while trying to answer my own questions on algebra I was forced to refer back to my notes to check I was doing it right. This meant that the task required me to really consolidate my own understanding. I feel confident with both these areas of mathematics and I also feel confident that I have developed the necessary skills enabling me to strengthen my knowledge and understanding of any other subject in mathematics if need be. (Conny)

Overall setting these questions has helped me to review my subject knowledge. I am now more confident that throughout my career I will be able to review my subject knowledge as and when I come to be required to teach something I have forgotten how to do (Nell)

From our point of view as tutors, Conny's and Nell's comments are particularly pleasing, as they indicate that the three tasks we set have been effective not just in helping students to develop their competence in areas of mathematics, but also their confidence in their ability to research and revise an area for themselves. We were also gratified by comments such as those from Natalie and Cathy below, which indicate that some students at least had appreciated our objectives in choosing at assess subject knowledge as we had done.

I feel that I have learnt a lot in this process and I am now in the position where I feel confident that I can assess other people's knowledge and understanding in these areas of mathematics. (Natalie)

I have learnt a great deal about assessing work from this part of the assignment. (Cathy).

Conclusions

The exercise has raised a number of issues and questions about students' subject knowledge, and our assessment of it. In particular, was this an effective way for us to get an insight into the subject knowledge of a large year group of students? In some cases, gaps in knowledge were apparent in either the questions they set, or their analysis of them, and work has been returned for re-submission after a consultation with a mathematics tutor. In other cases, it was less clear cut where the problems lay. For example Neil set a question asking for the **area** of a cylinder (Figure 9), but in his written commentary wrote the heading **Capacity** and described how he had needed to give both students the formula (for capacity) before they could answer the question. His written account showed no indication that he was aware of the confusion, or that either of the students had mentioned it. However, when his script was returned with a comment about this, he was mortified, and came to explain that he couldn't believe his own mistake. It clearly wasn't the case that he did not know the difference between area and capacity, or the structure of the formulae for each.

INSERT FIGURE 9 ABOUT HERE

Another possible criticism of the structure of the task was, as Norman pointed out, that it depended on the students telling us what they could do. They were allowed to choose the two topics on which they set questions, and although we specified that these were not to be from areas in which they were initially confident, there was still scope for students to avoid those areas they were least confident with. Though this would also depend upon the areas covered by questions they were answering for someone else.

In the overall evaluation of the course, many students said that they found this method of assessment reasonable and unthreatening, though often they did resent the time it took (not surprising given the pressure they are already under). A few made comments about the time factor particularly in relation to the initial audit, which may be eased by presenting the audit in the form of a 'tick list' rather than requiring them to copy out sections of the curriculum by hand. A smaller number of students, mainly those taking mathematics as a subject specialism, were very resentful of having to undertake any form of assessment of their subject knowledge, a view with which it is difficult not to sympathise. The very small number of students who felt the whole process was a complete waste of time, and said they would rather have just been set a test, were all mathematics specialists.

Our intention that the task of setting test questions was one which had a value in itself for the students learning about teaching as well as about mathematics was broadly successful, as some of the comments reported above indicate. However, it could undoubtedly been more effective if the course programme had allowed time for discussion and comparison of the outcomes amongst groups of students.

One very worrying aspect of the whole exercise, and an important lesson for us as tutors, was the very low level of mathematical content which some students felt they needed to address for their own learning, and on which they set questions which clearly challenged some of their colleagues. For example, Nancy set a four part question on multiplying and dividing decimals (Figure 10) at a level which might have appeared in an assessment for the end of Key Stage 2 (i.e. at age 11). More worryingly, both the students she gave it to had difficulties in answering at least one part of the question, and one clearly used tally marks on her paper, arranged in rows of four, to answer part d). We have to be prepared to take very seriously the issues this raises, particularly when place value and number work generally already play such a major role in our courses.

INSERT FIGURE 10 ABOUT HERE

Further developments

The developments of this approach to building and assessing are already underway for the next cohorts of students. The next group of undergraduates have had the benefit of many of the difficult areas of mathematics being addressed directly over a longer period (two years), with the development of additional teaching materials and specific sessions. The intention behind the teaching sessions is to give a rationale for the benefits of the additional mathematical knowledge required to their classroom practice. For example, one group of sessions aims to link the forms of proof to activities that might be seen in any primary classroom. Emphasis on the language used in the National Curriculum documents, and how to unpack the knowledge students already have, are clear lessons from this trial. The other major change is to push the students to focusing on at least one of the most problematic areas by setting the wording for the assessment task so that one of the areas covered should come from the following; proof, algebra, fractions, statistics and probability. The assignment brief also offers some guidance on the kinds of questions to asked.

How do you decide upon the questions to set?

The questions cannot be ones that can be solved without some working out as these will not give you any insights into your fellow students ways of working. If you think about the parallels in teaching young children it is difficult to unpack methods from a formal algorithm or to easily spot what kind of difficulties the children have had. So asking a question like add 1/2 and 1/4 is clearly too simple and doesn't allow you a dialogue with the person answering the question. The questions don't have to be something you have made up scratch, a useful source here would be secondary text books for ideas, but be careful that you understand the questions and answers in order to be able to discuss the issues involved in setting the questions and analysing the responses. This exercise will also assist you in thinking about the appropriateness of some assessment tasks/questions and the implications for this in your own teaching of mathematics.

For the PGCE students we are trying to use the assessment task within teaching sessions supported by tutorial times focusing partly on their audits of their subject knowledge and how they might work on developing this area within a much more pressurised course timetable.

Although we feel reasonably confident that our approach to developing students' subject knowledge in mathematics has been successful in enabling students to tackle a wider range of mathematical topics confidently, and so to meet aspects of the DfEE Standards for the Award of Qualified Teacher Status, we are concerned that there may be a cost. As more time and attention are focused on developing skills in higher levels of mathematics, we have less time to look in depth at the mathematics curriculum of the primary school. We feel strongly that what is most important is not how much mathematics students know, but the nature of that knowledge and the ways in which they are able to apply it within their teaching. In trying to find ways to build and assess subject knowledge which value these aspects of mathematical knowledge, we are encouraged by the comments of Askew et al (1997 p. 93) that *'more' is not necessarily 'better' in terms of the mathematical subject knowledge that teachers need to help pupils develop their understanding of mathematics.*

Ainley, J. (1997) Constructing Purpose in Mathematical Activity, in E. Pehkonen (Ed.) *Proceedings of the Twenty First Conference of the International Group for the Psychology of Mathematics Education*. Lahti: University of Helsinki.

Apple, M. W. (1989) How equality has been redefined in the conservative restoration. In W. Secada, (Ed.) *Equity and Education.* New York: Falmer Press.

Askew, M., Brown, M., Rhodes, V. Johnson, D. and William, D.(1997) *Effective Teachers of Numeracy* (a report for the Teacher Training Agency). London: King's College.

Briggs, M. (1993) Bags and Baggage revisited (or another suitcase on the rack) *Mathematics Education Review*, No 2, pp. 16-20.

Briggs, M. (1994) Unpacking your bags, *Mathematics Education Review*, No 4, pp. 15-24.

Cooper, B. and Dunne, M. (1998) Social Class, gender, equity and National Curriculum tests in mathematics, in P. Gates (Ed.) *Proceedings of the First International Mathematics Education and Society Conference*. Nottingham: University of Nottingham.

Crook, J and Briggs, M. (1991) Bags and Baggage, in D. Pimm and E. Love (Eds.) *Teaching and Learning School Mathematics.* London: Hodder and Stoughton.

Duncan, A. (1993) *What Primary Teachers Should Know About Maths*. London: Hodder and Stoughton.

Haylock, D. (1995) *Mathematics Explained for Primary Teachers*. London: Paul Chapman Publishing.

Skemp, R. (1976),Relational Understanding and Instrumental Understanding, *Mathematics Teaching* 77, pp 20-26