

University of Warwick institutional repository: <http://go.warwick.ac.uk/wrap>

A Thesis Submitted for the Degree of PhD at the University of Warwick

<http://go.warwick.ac.uk/wrap/4421>

This thesis is made available online and is protected by original copyright.

Please scroll down to view the document itself.

Please refer to the repository record for this item for information to help you to cite it. Our policy information is available from the repository home page.

ESSAYS IN MARKET MICROSTRUCTURE

By

Hao Lin

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
AT
UNIVERSITY OF WARWICK, WARWICK BUSINESS SCHOOL
COVENTRY, UNITED KINGDOM
JULY 2006

To my parents and my wife.

Table of Contents

List of Tables	vi
List of Figures	vii
Acknowledgements	viii
Declaration	ix
Abstract	x
1 Introduction	1
1.1 The scope of the thesis	2
1.2 The organization of the thesis	8
2 Literature review	10
2.1 Literature on equity market making	11
2.1.1 Inventory models	12
2.1.2 Asymmetric information models	16
2.2 Literature on option market making	19
2.3 Literature on horse race bookmaking	25
3 Market making with inventory uncertainty and information asymmetry	29
3.1 Introduction	29
3.2 The model	34
3.2.1 Assumptions	34
3.2.2 The Kalman filter updating	38
3.2.3 The market maker's optimization problem	40
3.3 The solutions	42

3.3.1	The optimal bid and ask prices	42
3.3.2	The market maker's pricing bias	45
3.3.3	The market maker's pricing bias when the spread is constant	47
3.4	Numerical experiments	48
3.4.1	Parameter values	48
3.4.2	Simulated prices and quotes	50
3.4.3	The effect of risk aversion on the pricing bias	51
3.4.4	The market maker's cumulative profit	52
3.5	The continuous time extension of the monopolistic model	55
3.6	The extension to competitive market making	58
3.7	Conclusion	60
4	Market making with return commonality	63
4.1	Introduction	63
4.2	The model	69
4.2.1	Assumptions	69
4.2.2	The Kalman filter updating	72
4.2.3	The market maker's optimization problem	74
4.2.4	The results	75
4.3	The benefit of knowing return covariance	77
4.3.1	Parameter values and simulation design	77
4.3.2	The simulation results	79
4.4	Conclusion	83
5	The role of market makers' hedging on the spreads in options and stock markets	86
5.1	Introduction	86
5.2	The model	93
5.2.1	Assumptions	93
5.2.2	Bid and ask prices in the stock market	96
5.2.3	The informed traders' trading strategy	100
5.2.4	The OMM's hedging strategy	102
5.2.5	The general problem	105
5.3	When the OMM does not hedge: the benchmark case	106
5.3.1	The bid and ask prices in the underlying market	107
5.3.2	The option bid and ask prices	107
5.3.3	The informed trading in equilibrium	108
5.4	When the OMM hedges: the regular case	108
5.4.1	Parameter values	108

5.4.2	What happens when the OMM hedges?	110
5.4.3	Informed traders response to OMM's hedging	112
5.4.4	The effects of α , K and μ on stock and option spreads	117
5.4.5	The expected return on investment in stocks and options	122
5.5	A market for several options when hedging is done in the underlying stock	126
5.6	A market for several options when hedging is done with options	133
5.7	Empirical implications	140
5.8	Conclusion	142
6	A simple model of horse race bookmaking	145
6.1	Introduction	145
6.2	A simplified model	148
6.2.1	Assumptions	148
6.2.2	A numerical solution	152
6.3	Stochastic liabilities	155
6.3.1	First stage: constant σ	158
6.3.2	First stage: proportional σ	160
6.4	Subsequent stages odds setting	162
6.4.1	Expected demands	162
6.4.2	Noisy demands	167
6.5	Conclusion	170
7	Conclusion	174
7.1	Summary and conclusions	174
7.2	Directions for future research	181
A	Proof of results in Chapter 3	184
A.1	Proof of Proposition 3.1	184
A.2	Proof of Corollary 3.2	185
A.3	Proof of Proposition 3.3	186
A.4	Proof of Corollary 3.4	186
B	Simulation of random variables that has a covariance matrix V in Chapter 4	187
C	Simulation of shocks in Chapter 6	191
Bibliography		192

List of Tables

3.1	Parameter and initial values used in the simulation	49
5.1	Parameter values and variables' ranges used in the numerical calculations.	109
5.2	The effect of μ , α , and K on the informed trading strategy	113
5.3	Informed trading intensity in stocks in the cases of OMM's hedging and not hedging in the stock market.	114
5.4	Stock spreads in the cases of OMM's hedging and not hedging in the underlying market	118
5.5	Option spreads in the cases of OMM's hedging and not hedging in the underlying market	119
5.6	Summary of the effects of μ , α and K on stock and option spreads in the cases of the OMM's hedging and not hedging	120
5.7	The effect of μ on option spreads when the OMM hedges	122
5.8	The expected returns on investment in stocks and options	125
6.1	Parameter values used in the numerical solution	153
6.2	Bookmaker's quotes: deterministic demands	154
6.3	Bookmaker's quotes: stochastic demands with constant σ	158
6.4	Bookmaker's quotes: stochastic demands with proportional σ	160
6.5	Multistage odds setting: expected demands	164
6.6	Multistage odds setting: noisy demands	169

List of Figures

3.1	Market demand functions: the model of monopolistic market making	38
3.2	Simulated prices and quotes	50
3.3	The effect of risk aversion on the pricing bias	52
3.4	Mean vs. standard deviation of the cumulative profits of 25 and 100 time steps for different λ	54
4.1	The time series of prices	80
4.2	The benefit of knowing return covariance	81
4.3	The contribution of correlated order flow information in price estimation	82
5.1	Information structure of the model	97
5.2	Stock and option spreads when an OMM does and does not hedge in the underlying market	111
5.3	Stock and option spreads when the OMM makes a market for two options and hedges in the underlying market	131
5.4	Information structure when OMMs hedges using options	135
5.5	Stock and option spreads when OMMs hedge using the underlying and the other options	139
6.1	Distributions of books and normalized prices: expected demand functions	166
6.2	Distributions of books and normalized prices: noisy demand functions	168

Acknowledgements

I am extremely indebted to my advisor, Professor Stewart Hodges. He has been a great source of intellectual inspiration during the entire period of my doctoral study. His patient guidance and invaluable comments are indispensable to the completion of this thesis. I hope we will continue to co-operate in the future as we have done in these rewarding years.

I am also grateful to Professor Antonio Mello at the University of Wisconsin-Madison for his insights, motivation and hospitality. Chapter 5 of the thesis is completed under his supervision when I visited UW-Madison.

A special thank goes to the examiners of my thesis, Professor Anthony Neuberger and Professor Ian Tonks, for their helpful comments and suggestions.

I am thankful to my friends and colleagues at the Warwick Business School and School of Business at UW-Madison for their friendship over the years.

I would like to dedicate this thesis to my parents and my wife Lan. Without their continuous support, this work cannot be finished.

Finally I thank the financial support from the Warwick Postgraduate Research Fellowship, the University of Wisconsin-Madison Fellowship, and the Warwick Business School Studentship.

Coventry, United Kingdom
July, 2006

Hao Lin

Declaration

I declare that this thesis is submitted to the University of Warwick for the degree of Doctor of Philosophy in 2006. Except where acknowledged, the material contained in this thesis is my own work and that it has neither been previously published nor submitted elsewhere for the purpose of obtaining an academic degree.

Coventry, United Kingdom
July, 2006

Hao Lin

Abstract

Market making is central to the study of market microstructure. Market makers stand ready to provide liquidity, market stability and price discovery, issues of great importance to regulators, practitioners and academics. This thesis contributes to the literature by studying four topical issues related to market making.

The thesis consists of four essays. In the first essay we develop a simple multi-period model of market making for a monopolistic stock market maker. The market maker tries to solve simultaneously the problems of managing his inventory and trading with informed traders. He uses a Kalman filter to update his estimates of the unknown market prices through his noisy order flow observation. We analytically characterize the optimal bid and ask prices and find that they depend on the beginning inventory, the estimated price, and the market maker's prior estimation error of the price process for each time period. We obtain desirable numerical results by using properly chosen parameters. The extensions to the continuous time and a competitive market making environment are also discussed.

The second essay extends the model in the first essay to consider the market making of multiple stocks. The market maker still does not know the true prices but is assumed to know the return covariance structure of these stocks. When the market maker considers the correlated order flow information, his knowledge of the return covariance improves his estimation of the unknown price processes, resulting in higher cumulative profits and lower risks of the profits.

The third essay analyzes the effect of option market makers' hedging on the informed trading strategy and the subsequent changes in the costs of liquidity provision in both stock and option markets. In a sequential trading framework, an option market maker uses the stock market to hedge his option position. His hedging trade affects the way that informed traders submit their orders in both the stock and the option market, which in turn changes the informed trading pressure faced by the market makers in each market. Furthermore, information in the option trading is passed to the stock market through the hedging trade. Both stock and option spreads are wider with option market maker's hedging. The increase in the spreads is more significant when the option market maker hedges in the underlying market than when it hedges with different options.

The fourth essay provides a model of bookmaking in a horse race betting market. The bookmaker observes the noisy public betting flow and faces the risk of trading with possible informed traders, as well as the risk of his unbalanced liability exposures. Even the noisy demand can unbalance the bookmaker's book. In our model, the bookmaker revises his odds to mitigate the risk. Allowing the bookmaker to set odds over several rounds of betting gives a clear view of the bookmaker's price setting strategy and its impact on the public betting flow over time. The study of horse race bookmaking provides useful insights into the market making of state contingent claims such as options.

Chapter 1

Introduction

Over the past 30 years, market microstructure has become an important research area in financial economics. It studies the process by which investors' latent demands are ultimately translated into transactions (Madhavan (2000)). Of particular interest in the market microstructure research is the study of market makers. Market makers are a special group of dealers and they play a central role in financial markets. They stand ready to buy and sell securities on a regular and continuous basis at a publicly quoted price.¹ Perhaps the most prominent example of market makers are the specialists at New York Stock Exchange (NYSE). A specialist performs five essential functions in the specific securities allocated to him at NYSE. He manages the auction process by establishing the opening price for his security every day and executes orders for floor brokers. He also serves as catalysts by bringing buyers and sellers together, enabling a transaction to take place that otherwise would not have occurred. He provides capital by trading against the trend of the market to minimize the order imbalance.

¹<http://www.sec.gov/answers/mktmakers.htm>.

and stabilizes prices to ensure smooth trading.² Market makers therefore play an important role in price discovery and market stability, issues of great interests to regulators, practitioners and academics.

This thesis is differentiated from the previous studies on market microstructure in two aspects. First, we are mainly interested in the *normative* problem of how a market maker *should* behave; the answers are important guides to public and private decision makings. Second, we are particularly interested in how an option market maker should behave in setting his bid and ask prices. Options at different strike prices are a parameterized family of substitutable securities and the study of the relation among closely related option contracts poses challenges to researchers. The models in this thesis provide some foundations to understand the complexity of option market making in practice.

1.1 The scope of the thesis

This thesis contributes to the literature by investigating four issues related to the market making in various financial markets. The first issue is related to the well known literature on market makers' price setting strategies. Previous literature has documented that market makers generally face two problems: the inventory problem and the asymmetric information problem. Market makers face the risk of building up risky inventories in the market making business. They change their bid and ask

²<http://www.nyse.com/about/members/>.

prices to elicit the unexpected imbalance of buy and sell orders, aiming to restore inventories to a preferred position. In a market with asymmetric information, the bid-ask spread arises from the existence of informed traders, who have better knowledge of the stock's future value. Market makers lose on average to informed traders but recoup their information loss from trading with uninformed traders.

The literature however focuses on these two problems separately. In practice, market makers hardly face only one problem and not the other. It is of interest to integrate the two problems into one setting under which both problems can be analyzed simultaneously. In particular, a market maker's inventory positions should enter into the model explicitly so that his price setting strategy directly affects his inventory exposures. In addition, a market maker must be able to observe the noisy signal about the stock's true value, which only informed traders know exactly.

These two ingredients are incorporated into our model in Chapter 3. In a Bayesian updating framework, a monopolistic market maker uses the order flow information to estimate the true value of a stock. The order flow conveys two pieces of information: first, the net order flow contributes to the net changes in his inventory, and second, the part of informed trading in the overall order flow provides a noisy signal of the true prices. Using Bayes rule, the market maker obtains optimal estimates of the true prices and sets optimal bid and ask prices to influence the public order flow so as to manage his inventory positions.

The second issue of this thesis concerns the market making of multiple stocks. The traditional focus of market making research has been on the liquidity provision of an individual security. In practice, a market maker normally makes markets for more than one security. At NYSE, for example, there are currently 7 specialist firms making markets for approximately 2672 listed companies. It is therefore of interest to examine the implication of the multiple stocks market making on the market maker's price setting strategies and its impact on order flows.

In Chapter 4, we extend our single stock market making model in Chapter 3 to incorporate multiple stocks. We assume that the market maker, still a monopolist, does not know the true prices but knows the return covariance structure of these stocks. He still uses order flow information to update his estimates of the true prices. His knowledge of the return covariance structure in this case affects order flows. We find that, by considering the correlated order flow information in his price estimation, the market maker obtains better estimates of the stocks' true values, which improves his market making profitability.

A direct implication from the market maker's knowledge of the return covariance is that order flows exhibit liquidity commonality. Liquidity commonality is important because systematic liquidity is most likely a priced source of risk (Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), Pastor and Stambaugh

(2003)). Consistent with empirical findings of Coughenour and Saad (2004), our results provide another support to the claim that common market makers are a possible reason for liquidity commonality.

Chapter 4 also serves as a first step for our study of option market making. An option market maker makes markets for a number of option contracts, in a much similar way as market making multiple stocks. Our assumption of the stocks' return covariance structure is also particularly suitable for analyzing option market making. Our model in Chapter 4 therefore provides insights into the market making of the parameterized family of substitutable securities such as options with different strike prices.

The third issue in this thesis concerns the informed trading across stock and its option markets. For stocks with their options traded, informed trading can take place in either the underlying stock market or the option market. There is a growing body of literature that provides evidence that informed traders do trade in option markets and the option market makers' hedging activities affect the costs of liquidity provision in both stock and option markets. Analyzing the option market maker's hedging therefore provides an important link of the liquidity in these two markets.

Chapter 5 provides the first theoretical model that explicitly considers the option market makers' hedging activity and analyzes its effect on the informed trading strategies and the changes in the costs of liquidity provision in both the stock and the

option markets. We show that the hedging by market makers has a significant impact on the choice of the market that informed traders submit their orders to. Since information that is channeled by options trading is passed on to the stock market through the hedging trade of the market maker, both stock and option spreads are wider with option market maker's hedging. The model is able to explain the recent empirical results of Kaul, Nimalendran, and Zhang (2004) and de Fontnouvelle, Fisher, and Harris (2003), that suggest that the cost of hedging has an important effect on both stock and option spreads.

Chapter 5 also contributes to the literature that investigates how informed trading in the option market is distributed across strike prices. We study this problem by extending the model to include multiple options on the same underlying asset. We show that stock spreads are smaller when more options are traded. Compared to the case in which each option is the only traded option, the spreads of each option are smaller when multiple options are traded simultaneously. For the stock, we show that when informed traders have the choice of trading multiple options, there is less informational content in the option market maker's hedging trade. This eases the informed trading threat in the stock market and allows the stock market maker to narrow the stock spreads. Also, since options have a convex payoff structure, the combined hedge ratio of multiple options is smaller than the hedge ratio for each individual option. This reduces the option market maker's hedging cost, and in turn

narrows the option spreads.

The fourth issue of this thesis concerns the bookmaking in a horse race betting market. Studying horse race bookmaking provides a way to understand the market making of state contingent claims such as options since a bookmaker and an option market maker share many similarities. For example, an option market maker's profits are contingent on the final value of the stock. Suppose an option market maker deals with butterfly spreads instead of a single option, there is one and only one option of the spreads that will pay off at expiry. It is exactly the same in a horse race: there is one and only one horse that will win the race eventually. Furthermore, an option market maker wants to have flat and positive positions over all option exposures, so that he could always have positive profits and avoid negative ones. Similarly in a horse betting market, a bookmaker sells liabilities over all horses and wishes to avoid large liability exposures. Since a horse betting market is an especially simple financial market, in which the complexity of the pricing problem is reduced, it provides a clear view of pricing issues which are more complicated elsewhere.

Chapter 6 provides a simple model of bookmaking in a horse race betting market. The book is liable to become unbalanced because the betting demands are noisy and the bookmaker may not know the correct odds to quote. He has to worry about trading with informed traders. Furthermore, whenever his book becomes unbalanced, the bookmaker wants to re-balance it so that the problem of having extremely high

liability exposures can be alleviated. Even random shocks from noise traders are costly to the bookmaker since his book could become less balanced.

In our model, the bookmaker revises his odds to mitigate the risk. He influences the public betting flow by raising the normalized prices for horses with high initial liabilities and lowering the normalized prices for horses with low initial liabilities. Allowing the bookmaker to set odds several rounds before the race starts gives us a clear view of the bookmaker's price setting strategy and its impact on the public betting flow over time. Our model helps to understand the complexity of managing a series of state contingent exposures such as options for a single expiry date.

1.2 The organization of the thesis

Following the introduction, the thesis is organized in the following way. In Chapter 2, we review the literature that is related to our work in this thesis, namely, the literature on equity market making, the literature on option market making, and the literature on horse race bookmaking. Chapter 3 develops a simple multi-period model for a monopolistic market maker who tries to solve simultaneously the problems of managing his inventory and trading with informed traders. The market maker uses a Kalman filter to update his estimates of the true prices through his order flow observation. Chapter 4 extends the single stock model in Chapter 3 to consider the market making of multiple stocks. Given the market maker's knowledge of the return covariance structure, the market maker improves his market making profitability by

considering the correlated order flow information in his price estimation. Chapter 5 studies the informed trading in the option market and the effect of the option market maker's hedging on the costs of liquidity provision in both stock and option markets. We find the option market maker's hedging trade conveys information and affects the informed traders' order placement strategy, which in turn changes the informed trading pressure faced by the market makers in each market. Chapter 6 provides a simple model of bookmaking in a horse race betting market. A bookmaker influences the public betting flow by setting appropriate odds for all horses to mitigate the risk of an unbalanced book. The bookmaker's odds setting strategy provides useful insights into the management of state contingent exposures such as options. Finally, Chapter 7 provides concluding remarks and indicates directions for future research.

Chapter 2

Literature review

In the frictionless Walrasian models of trading behavior, perfect competition and free entry are typically assumed. Market microstructure studies what Walrasian models have ignored, yet an important aspect of financial markets: frictions (O'Hara (1995)). It examines the process of price formation in the presence of risks, costs and asymmetric information.¹

In this chapter, we discuss the literature that is relevant to our work in the thesis. We divide this chapter into three sections. The first section reviews the literature on equity market making, the traditional focus of the market microstructure study. Given its uniqueness, we review the literature on option market making separately in the second section. The last section reviews the literature on horse race bookmaking since we will develop some strong parallels between this and option market making.

¹For a comprehensive review of the theoretical literature on market microstructure, see O'Hara (1995). Madhavan (2000), Stoll (2003) and Biais, Glosten and Spatt (2005) provide more recent surveys.

2.1 Literature on equity market making

Early literature on market microstructure focuses on the operations of agents called market makers. They stand ready to buy and sell securities on a regular and continuous basis at a publicly quoted price. Given the central role of market makers, it is a natural starting point to study how prices are determined inside the so-called *black box* of a security market (Stoll (1976) and O'Hara (1995)).

Market makers quote bid and ask prices at which they buy and sell securities. The difference between the bid and ask prices is called the bid-ask spread. The analysis of how bid-ask spreads are determined provides insights of how market makers set prices in the market.

The bid-ask spread reflects the difference between what active buyers must pay and what active sellers receive. It is an estimator of the cost of liquidity provision and the illiquidity of a market (Stoll (2003)). Demsetz (1968) observes that buyers and sellers may enter the market at different times and the bid-ask spread arises as the size of price concession needed for immediate transaction. Demsetz (1968) is the first to describe the specialists at NYSE as *suppliers of immediacy*: they passively provide liquidity to accommodate the transitory order imbalance by adjusting bid-ask spreads, and thereby stabilize prices. The bid-ask spread provides the specialists with the appropriate return under competition.

Many empirical researchers analyze the cross-sectional variation in spreads and

find it can be explained by economic variables.² In fact, the relation between the spread of a security and the trading characteristics of that security is one of the strongest and most robust relations in finance (Stoll (2003)). In a more recent study, Stoll (2000) finds over 79% of the cross-sectional variation in proportional spreads is explained by stock characteristics. The well-known key results are: spreads are lower for stocks with greater trading volume, with lower return volatility, with higher price, and with smaller trading imbalance.

In the following sections, we discuss two general theoretical frameworks that determine the bid-ask spread of a security, namely, inventory risk models and asymmetric information models.

2.1.1 Inventory models

Inventory models focus on the issue that market makers must carry unwanted inventories in order to perform their dealership functions. When there is an order imbalance that moves the market maker away from his desired inventory position, he adjusts the bid and ask prices to attract orders and re-optimizes his inventory position. Important papers of inventory models include Garman (1976), Amuhud and Mendelson (1980), Stoll (1978), Ho and Stoll (1981, 1983), O'Hara and Oldfield (1986) and Madhavan and Smidt (1993).

²See, for example, Demsetz(1968), Stoll (1978), Branch and Freed (1977), Tinic (1972), Tinic and West (1974).

Garman (1976) is perhaps the earliest paper focusing on the market maker's inventory risk. While Demsetz (1968) studies the trading desire of an individual trader, Garman (1976) focuses on the stochastic arrivals of order flows and how the market itself works to solve the clearing problem. He characterizes an exchange market by a flow of stochastic orders to buy and sell and the imbalance between supply and demand could temporarily arise. This imbalance gives an importance to the *temporal microstructure*, i.e., how the exchange between buy and sell actually occurs at any point in time.³ In his dealership structure, the single, monopolistic market maker's objective is to maximize his expected profit per unit of time, subject to the avoidance of failure (i.e., running out of cash or inventory). Central to the dealer's problem is therefore the asynchronism between the flow of buys and sells. Garman's analysis demonstrates that the inventory determines the dealer's viability.

Garman's (1976) work stimulates the subsequent research in the inventory models. For example, Amihud and Mendelson (1980) explicitly incorporate inventory into the dealer's pricing problem and show that the dealer has a preferred inventory level because of the nature of the order arrival process. Optimal bid and ask prices exhibit a positive spread. Stoll (1978) examines the dealer's role as the supplier of immediacy. Different from the order-based analysis in Garman (1976) and Amihud and Mendelson (1980), Stoll's focus is on the portfolio risk that the dealer faces. The dealer is willing

³Garman (1976) is the first to use the term *microstructure*, defined as the *moment to moment trading activities*.

to alter his desired portfolio position to accommodate other traders' trading needs. In the same way that any intermediary must be compensated, risk averse dealers⁴ must be rewarded for the costs of providing their services. The market spreads reflect the cost of bearing the risk associated with the unwanted inventory.

The inventory model in Stoll (1978) has been extended to account for multiple stocks, multiple time periods (Ho and Stoll (1981)) and multiple market makers (Ho and Stoll (1983)). The intertemporal model of Ho and Stoll (1981) differs from the risk neural intertemporal models of Garman (1976) and Amihud and Mendelson (1980) in that the market maker's attitude toward risk affects his optimal pricing policy. In solving the maximization problem of the market maker's expected utility of the terminal wealth, the authors find the spread is largely independent of the market maker's inventory position but depends on the fundamental characteristics of the stock (such as the risk of the stock) and the market maker (such as his risk aversion and the time horizon). The market maker affects the order arrival processes by moving the *placement* of the spread rather than adjusting the *size* of the spread.

The literature that we have reviewed so far focuses on a single, monopolistic dealer. Ho and Stoll (1983) examine the pricing setting strategy in a model of competitive dealers. They analyze two competing market makers each trading two stocks and choosing bid and ask prices to maximize his own expected utility. However their

⁴In Garman (1976) and Amihud and Mendelson (1980), the dealer is assumed to be a risk neutral monopolist whose prices reflect largely his market power.

simple one-period model does not explicitly incorporate the intertemporal inventory position of the competing market maker, so the strategic element of market maker's price setting strategies is not considered. Since traders could trade with the market maker who has the best price, each market maker's pricing problem should depend on the actions of every other market maker (O'Hara (1995)). Modeling the competition among market makers is therefore more complicated. The formal treatment of the competitive dealership would require a careful game-theoretical analysis.

In summary, central to inventory models are the uncertainties in order flows, which can result in inventory problems for market makers. Since unbalanced inventory carries risk, market makers have to moderate the random order flows according to their respective risk structure. The bid-ask spread arises either to reflect market makers' market power (Garman (1976) and Amihud and Mendelson (1980)) or to compensate market makers for bearing the risk of holding undesired inventories (Stoll (1978) and Ho and Stoll (1981, 1983)). In the long run order flows are balanced by assumption and they are irrelevant in determining security prices. But in the short run, they do affect the fine price behavior. Important empirical questions arise from inventory models include what a market maker's preferred inventory position is and whether the inventory level induces mean-reverting in security prices⁵. These studies advance our knowledge of how prices are determined in the *black box* of a security

⁵See, for example, Madhavan and Smidt (1991), Manaster and Mann (1996), Lyons (1995) and Laux (1993).

market.

2.1.2 Asymmetric information models

Recent literature on market microstructure applies the insights from information economics to study the dealer's behavior. The origin of asymmetric information models is usually attributed to an influential paper by Jack Treynor (writing under the pseudonym of Bagehot (1971)). Treynor suggests the presence in the market of traders who have superior information (informed traders) and who are liquidity motivated (uninformed traders). In an anonymous market, dealers must lose to informed traders, for the informed traders are not identified.⁶ He notes that the losses to the informed must be offset by the profits from the uninformed traders if dealers are to stay in business. The spread therefore reflects a balancing of losses to the informed with gains from the uninformed. That is, adverse selection imposes a cost, which must be made up by a spread, even for a risk neutral, competitive market maker (Copeland and Galai (1983)). This provides a way to explain the market bid-ask spreads without relying on exogenous technological specifications of the transaction cost. Important theoretical papers that build on the adverse-selection cost of the spread include Glosten and Milgrom (1985), Easley and O'Hara (1987), and Kyle (1985).

Since informed traders only buy when the price is low and sell when the price

⁶Bagehot (1971) also argues that informed traders not only possess informational advantages, but also the option not to trade.

is high, the trades themselves could reveal the underlying information and therefore affect the price process. This is the insight of the *sequential* trading models of Glosten and Milgrom (1985) and Easley and O'Hara (1987) in which a rational market maker gradually *learns* this underlying information and sets *ex post* regret-free prices that converge to the expected value of the underlying asset. In Glosten and Milgrom (1985), the bid-ask spread is increasing in the information asymmetry (as measured by the percentage of informed traders in total trading population) and in the degree of asset value uncertainty (as measured by the range of possible values of the asset). Easley and O'Hara (1987) allow for trades of different sizes and assume that traders with superior information prefer larger size transactions. Adverse selection arises because a rational market maker interprets large orders as a signal of informational trading and adjusts the price and spread accordingly.

Kyle first characterizes the *strategic* behavior of informed traders who maximize the value of their private information before the information becomes common knowledge. In a *batch* trading model, the market maker aggregates the orders and sets one price (instead of bid and ask prices) to clear the market. Kyle demonstrates the existence of a rational expectation equilibrium in his model in which the informed traders' conjecture of the market maker's pricing policy and the market maker's inference about the informed traders' information are both correct in the equilibrium. He shows that information is gradually incorporated into prices across time and market

prices eventually incorporate all available information.

Kyle's (1985) model was further extended by Holden and Subrahmanyam (1992) and Admati and Pfleiderer (1988). Holden and Subrahmanyam (1992) incorporate competition among multiple risk-averse insiders with long-lived private information. They demonstrate the existence of a unique linear equilibrium in which competition among insiders is associated with high trading volumes and rapid revelation of private information. In the limiting case in which the number of informed traders goes to infinity, all information is revealed in the first trading interval and the price equals the true value instantaneously. Admati and Pfleiderer (1988) develop a model of strategic play by informed and uninformed traders. They allow some uninformed traders (called discretionary uninformed traders) to have discretion over which time period they want to trade in.⁷ The optimal behavior for an uninformed discretionary trader is determined by solving for the minimum cost trading period in which to transact. Admati and Pfleiderer show that in the equilibrium the discretionary traders choose the lowest-cost period in which the variance of the uninformed trade is highest. It follows that to maximize the variance of the uninformed trade, discretionary traders all select the same period to transact, inducing the observed patterns⁸ that trades are concentrated in particular time periods within the trading day and the periods of high trading volume also tend to be the periods of higher return variability.

⁷Other papers that consider the uninformed strategic behavior include Foster and Viswanathan (1990), Seppi (1990), and Spiegel and Subrahmanyam (1992).

⁸See, for example, Jain and Joh (1988) and French and Roll (1986).

In summary, the key element of asymmetric information models is that trades convey information. A market maker, by observing trading activities, gradually learns the information held by informed traders. The market maker adjusts prices so that, at any point in time, the prices reflect the expectation of the security's value, conditional on all public information, including the prior trades. The price dynamics are therefore derived from the mechanism of the market maker's learning process. Most asymmetric information models basically solve this dynamic learning problem via an application of Bayes rule.⁹ The strategic traders models of Kyle (1985) and subsequent papers (e.g., Holden and Subrahmanyam (1992) and Admati and Pfleiderer (1988)) link the market microstructure research to the rational expectation literature to allow better characterization of the trading behavior of different market participants.

2.2 Literature on option market making

Literature on option market making focuses on the informational role of option trading. In a complete market, an option is a redundant asset: its prices are determined unilaterally by the price of its underlying asset and do not convey information (Black and Scholes (1973)). If, however, the market is not complete and market frictions do exist, an option is not redundant and an option trading process contains information about the underlying asset prices. Back (1993) shows that, with continuous trading and asymmetric information, an option can no longer be priced via simple

⁹Notable exceptions are the earlier papers such as Bagehot (1971) and Copeland and Galai (1983).

arbitrage. The option market therefore provides another channel through which informed traders can profit from their information. Black (1975) argues that the option market is an ideal venue for informed trading because of its leverage advantages, low transaction costs, less stringent margin conditions, and the absence of the up tick rule for shorting.

One way to study the informational effect of option trading on the underlying market is to examine which market leads the other in information discovery (the lead-lag effect). The empirical findings are however inconclusive. The results of Manaster and Rendleman (1982), and Anthony (1988) suggest that the price changes in option markets lead the price changes in stock markets. However, Stephan and Whaley (1990), Chan, Chung and Johnson (1993) and others find no evidence that supports such lead. These conflicting results may be due to the particular time frame or the market structure that the data set has been drawn from.

With the increasing availability of data sources, researchers are able to make better inference of informed trading in option markets. For example, Chakravarty, Gulen, and Mayhew (2004) find direct evidence that the option market contributes about 17% on average in price discovery; indirectly, Mayhew, Sarin, and Shastri (1995) find that informed traders migrate between stock and option markets in response to the changes in the option's margin requirements. Easley, O'Hara and Srinivas (1998) and Pan and Potoshman (2005) find signed option trading volume helps to forecast stock

returns. Cherian and Weng (1999) investigate the presence of volatility informational trading in the option market and its implications for option bid-ask implied volatility spread. They find a positive correlation between option volume and implied volatility spread given the presence of directional and volatility informational traders, a result consistent with that of Easley, O'Hara and Srinivas (1998). Cao, Chen, and Griffin (2005) observe abnormal option trading volume prior to takeover announcements.

Another line of empirical research examines the factors that are significant in determining the option spread. Vijh (1990) finds that the adverse-selection component of the option spread on Chicago Board Options Exchange (CBOE) is very small. Lee and Yi (2001) investigate the extent of information-motivated trading conditional on the trade size in the option and stock market. They show that the adverse selection component of the bid-ask spread decreases with option delta, implying that options with greater financial leverage attract more informed investors. Jameson and Wilhelm (1992) provide empirical evidences that option market makers face risks in managing inventory that is unique to option markets. They show that risks associated with the inability to rebalance an option position continuously and the uncertainty about the return volatility of the underlying stock each accounts for a statistically and economically significant proportion of the bid-ask spreads quoted for a sample of CBOE options. Mayhew (2002) examines the effects of competition and market structure

on equity option spreads and find that options listed on multiple exchanges have narrower spreads than those listed on a single exchange, but the difference diminishes as the option's volume increases.

Compared to many empirical studies, theoretical works on this area are however surprisingly sparse. Papers by Biais and Hillion (1994), Easley, O'Hara, and Srinivas (1998), and John, Koticha, Narayanan, and Subrahmanyam (2003) suggest that the amount of informed trading in the option market depends on the relative liquidity in the option and its underlying market. Biais and Hillion (1994) analyze the effects of introducing options when information may be asymmetric. Although the option mitigates the market breakdown problem created by market incompleteness and asymmetric information, the market maker finds more difficult to interpret the informational content of trades and consequently the introduction of options reduces the information efficiency of the market.

Both Easley, O'Hara, and Srinivas (1998) and John, Koticha, Narayanan, and Subrahmanyam (2003) use a sequential trading approach to address the similar question. Easley, O'Hara, and Srinivas (1998) specify the importance of the volume in the price discovery process and the information transmission between cash and option markets. They develop a model of the informational role of the signed options trading volume in predicting future stock prices. John, Koticha, Narayanan, and Subrahmanyam (2003) consider a one-trade model in which traders can trade either a stock or a put

option and they allow a single wealth-constrained informed trader. They analyze the impact of option trading and margin rules on the behavior of informed trading and on the market microstructure of both stock and option markets.

Analyzing option market maker's hedging activities provides another important way to understand the informational linkage between the option and its underlying market. Cho and Engle (1999) propose a *derivative hedge theory* and argue that in a perfect hedge world, option spreads arise from the illiquidity of the underlying market, rather than from inventory risk or informed trading in the options itself. Spreads in the derivative markets exist because market makers in those markets find it difficult to hedge their position given an illiquid underlying market. Their paper provides insights that adverse selection in the underlying market arises from the possibility of the option market maker's inability of executing his initial delta hedge at the level he wants to.

This insight has been investigated by an increasing number of empirical researchers recently. Kaul, Nimalendran, and Zhang (2004) find that the underlying stock's spread has an important impact on the option spread due to option market maker's hedging activities. de Fontnouvelle, Fishe, and Harris (2003) show that the option's delta and the underlying stock's effective spread are significantly related to the size of option spreads. Petrella (2001) proposes an empirical model that explicitly considers the hedging costs faced by option market makers to minimize delta risk exposure,

the order processing costs and other factors that affect the market makers' gross profit. His model shows that the option spread is positively related to the spread of the underlying assets. The reservation spread, computed as a linear combination of the option delta and the underlying asset's tick size, plays a very important role in explaining the option spread.

In summary, an option market provides another channel through which informed traders can profit from their informational advantage. The unique features of the option market, in particular, issues like optimal hedging, risk exposure, and the linkage between derivatives and underlying markets give rise to much interesting research recently. However how exactly an option market maker should behave in setting bid and ask prices remains unclear and represents challenges to researchers, partly due to the complexity of the close relation among substitutable securities that options pose. Options with the same underlying asset but different strikes and maturities are some examples. Market making of these options is affected both by the market maker's information set and by his inventory positions. The market maker's information set is enriched by more information generated from his order flow observations on different strikes / maturities. The inventory problem becomes more complex because the market maker's risk exposure of substitutable options may change depending on the correlations among these options. Addressing option market making therefore requires a careful formulation and analysis of option market makers' information set

and risk exposure.

2.3 Literature on horse race bookmaking

There are some important parallels between how a horse race bookmaker in the UK manages his book and how an option market maker does. For this reason, we include a chapter on the former even though extending it to a comprehensive model of option market making would be rather difficult.

The British horse betting system provides a convenient way to study the market making of state contingent claims such as options. In a n horse race, each horse corresponds to each future state of the world. When the i th horse wins the race, the i th state obtains. Furthermore, the odds determine the prices of the basic Arrow-Debreu securities that pay a dollar if a particular state obtains and nothing otherwise. Since a bookmaker sets odds for each horse, all basic securities are priced and traded (Shin (1992)). A horse race betting market also has a well-defined termination point at which each asset (bet) possesses a definite value. Horse betting markets are especially simple financial markets, in which the complexity of the pricing problems is reduced.

In addition, a horse race bookmaker and an option market maker share many other similarities. In each case there is a clear maturity date (end of race vs. expiry of options) at which the profits can be unambiguously counted. In each case too, the payoff at maturity is known beforehand as a outcome which is contingent on a single observation (name of the winning horse vs. expiry date value of the underlying).

The analogy would be particularly perfect if an option market maker simply accepts orders from butterfly spreads: there is one and only one option of the spread that will pay off at expiry. It is exactly the same as in a horse race: there is one and only one horse that will win the race. Moreover, an option market maker wants to have some flat, positive positions over all option exposures, so that he could always have positive profits and avoid negative ones. Similarly, a bookmaker sells liabilities over all horses and wishes to avoid large liabilities. Both an option market maker and a bookmaker therefore wish to obtain a more balanced book in which large exposures can be avoided. The bookmaker's strategies can therefore be represented as one of meeting this contingent profile as satisfactorily as possible.

Wagering markets have long been a focus of economic research.¹⁰ The early work applies the advances in utility theory to examine investors' risk-taking behavior in gambling markets.¹¹ Recent focus of the academic research is on the testing of information efficiency in betting markets.¹² Much of the academic attention has been devoted to examine whether there are significant differences in the expected return to wagers placed on those possible outcomes judged by the market to possess a high likelihood of occurring compared with those judged to possess a low likelihood of occurring. Empirical evidence tends to suggest that betting level stakes in the lower

¹⁰Sauer (1998) is a recent survey of the economics literature on gambling market.

¹¹See, for example, Friedman and Savage (1948), Hirshleifer (1966), Markowitz (1952), Samuelson (1952), among many others.

¹²See, for example, Hausch, Ziembra and Rubinstein (1981), Hausch and Ziembra (1994). Vaughan Williams (1999) surveys information efficiency in betting markets.

range of odds produce a significant higher return than betting the same stake in the higher ranges of odds, a phenomenon usually known as the favorite-longshot bias (see, for example, Dowie (1976), as presented by Crafts (1985)). When a favorite-longshot bias exists, the percentage mark-ups in the prices over the true liabilities are generally not uniform. In particular, normalized prices on the favorites of the race understate the winning chances of these horses, while the normalized prices on the longshots exaggerate their winning chances.

In a series of papers, Shin (1991, 1992, 1993) uses game theoretical models to provide an explanation of the favorite-longshot bias from the supply side of the market, i.e., the bookmaker. He studies how the bookmaker sets prices against punters who may have insider information. In his model, asymmetric information creates the favorite-longshot bias through the optimal pricing response by the bookmaker. In order to recover sufficient revenue from outsiders to pay the insider their winnings, the bookmaker deliberately raises the odds offered against favorites winning the race and lowers those on longshots, whilst ensuring at the same time that their profit margin is sufficient to protect them against the expected proportion of insiders amongst their customers.¹³ Subsequent empirical works find evidence in support of Shin's models.¹⁴

There has been rather limited academic attention devoted to the study of bookmaking. Broadly speaking, the bookmaker's problem is analogous to that faced by

¹³Shin (1993) empirically estimates the probability that the bookmaker encounters an insider to be a statistically significant 0.02.

¹⁴See, for example, Vaughan Williams and Paton (1997) and Gabriel and Marsden (1990, 1991).

the market makers in setting bid and ask prices in financial markets. Most literature on horse race betting markets deals with the parimutuel system where the bookmaker's strategy is defined for him and is trivial. Shin's work is based on the British betting system and can be seen as a model of bookmaking where the bookmaker faces asymmetric information.

In summary, bookmaking in a betting market provides an useful channel to study market making of state contingent claims such as options. A bookmaker faces problems of managing his odds across different horses and trading with possible insiders, in a much similar way as those faced by an option market maker. A betting market however is a simple financial market in which the complexity of the pricing problem is reduced. It allows us to focus on the essential features of the market making problem and provides insights into the market maker's price setting strategy.

Chapter 3

Market making with inventory uncertainty and information asymmetry

3.1 Introduction

Ever since the study by Demsetz (1968), market making has been an important area of market microstructure research. In short, market makers are a special group of dealers responsible for providing a continuous market by quoting bid and ask prices for investors to buy and sell securities. Previous research has documented that market makers generally face two problems: the problem of accumulating unwanted risky inventories (the inventory problem), and the problem of trading with informed traders (the asymmetric information problem). This chapter provides a normative analysis of a monopolistic market maker's strategy when he considers these two problems simultaneously.¹

¹The assumption of monopolistic market making can be justified by institutional features in some major exchanges such as New York Stock Exchange where there is currently one specialist, or market maker, per stock. The specialist's main responsibility is to maintain a *fair and orderly market*. For

Previous literature focuses on the inventory problem and the asymmetric information problem separately. It is well known that market makers change their bid and ask prices in order to elicit unexpected imbalance of the buy and sell orders, in order to restore their inventories towards a preferred position. In a market with asymmetric information, the bid-ask spread arises from the existence of informed traders, who have better knowledge of the future value of the stock. The market makers lose on average to informed traders but recoup their information loss from trading with uninformed traders.

In our model we put both the inventory and asymmetric information problems into a single framework and analyze the market maker's pricing strategy under such a framework. We model a monopolistic market maker similar to a specialist at NYSE. He has the perfect knowledge of his inventories and the order flow information, but does not know the true value of the stock. Given a constant absolute risk aversion (CARA) utility function, the market maker maximizes his expected utility of per period profits, which comprise two parts: the incremental value of his inventories, and the instantaneous cash flow from his trading with the public buy and sell demands. Neither the order flow nor the inventory alone provides a sufficient statistic for him to set optimal bid and ask prices. He must use these two pieces of information together to estimate the true price in order to set the optimal prices. We obtain the closed

a detailed description of the background on NYSE, see Hasbrouck, Sofianos and Sosebee (1993) and Teweles and Bradley (1998). The specialist participation rate, (specialist purchase + specialist sales)/(2 × total volume), is about 15% (NYSE Fact Book).

form solutions of the optimal bid and ask prices that the market maker sets of every time period.

Our model is related to the seminal paper of Kyle (1985). Kyle considers the problem of trading with informed traders *explicitly* and managing the order imbalance *implicitly*. The net order flow from both informed and noise traders affects the market maker's inventory position in the Kyle's model. Kyle derives *one* price that clears all market orders. In contrast, our model considers the trading with informed traders *implicitly* and the inventory problem *explicitly*. We also take Kyle's analysis one step further by characterizing the optimal *bid and ask prices* set by the market maker to manage the problems of inventory and asymmetric information simultaneously.

Central to our model is a Bayesian updating framework that the market maker uses to update his estimates of true prices over time. The framework can be summarized in a particular state-space form called the Kalman filter.² The market maker uses the Kalman filter to continuously update his belief of the true market prices through the noisy order flow observation. The Kalman filter has been widely used in economics and finance studies.³ In the related market microstructure literature, Madhavan and Smidt (1991) apply the Kalman filter algorithm to develop a model of intra-day price

²The Kalman filter is described in Kalman (1960) and Anderson and Moore (1979). Harvey (1989) and Hamilton (1994) provide nice treatment in the context of econometrics and time series analysis.

³See, for example, Fama and Gibbons (1982) on the study of the relationship between expected real rates of return and expected inflation, Hsieh and Kulatilaka (1982) on the modeling of primary metals markets, and Wolff (1987)'s work on the foreign exchange behavior.

movements. In their model, a representative market maker uses Bayes rule to update his belief. The purpose of their paper is to find an econometric model to test the empirical data. Our model has a simpler construction compared to theirs. More recently, Koopman and Lai (1999) use the Kalman filter to smooth the estimates of the fundamental prices and spreads of three liquid stocks on London Stock Exchange.

As in many market microstructure models, order flows play an important role in this model.⁴ All information is aggregated in the order flow and the market maker processes this information through the Kalman filter so that he learns the non-public information. In particular, the order flow in this model conveys two pieces of information. First, the net order flow is the net change in the market maker's inventory levels; and second, the part of informed trading in overall order flow provides a noisy signal of the true prices.

In this chapter, we first find that the optimal bid and ask prices are linear functions of the beginning inventory of each period. The parameters of the price functions are themselves functions of known variables generated by the Kalman filter algorithm. Of particular importance is that some parameters contain the market maker's estimation variance from his noisy order flow observations. We then define the pricing bias (PB) as the difference between the market maker's mid-quote and his prior estimate of the

⁴In inventory models, order flow alters equilibrium risk premia (Stoll (1978) and Ho and Stoll (1981)). In asymmetric information models, order flow observation provides information about payoffs (Glosten and Milgrom (1985) and Kyle (1985)). In foreign exchange market, order flow also conveys information of short term variation of exchange rates. Evans and Lyons (2002) find that macroeconomic variables and order flow together significantly improve the explaining power.

true price. The PB measures the deviation of the quotes from the market maker's prior belief of the price. We find that the PB is linearly related to the beginning inventory of each period and is negatively correlated with the market maker's relative risk aversion parameter. Intuitively the more risk averse the market maker is, the stronger mean reverting effects of the inventory on positioning the quotes are. We also consider a special case of the PB when the total spread is constant.

In the numerical work that follows, we simulate a sample price process together with the optimal bid and ask prices for 100 time steps. We are interested in the market making profitability in our model. We calculate the summary statistics of the cumulative profits of the market making over a number of time periods. In general, our model gives reasonably good numerical results using properly chosen model parameters and initial values.

We then consider the extension of the monopolistic model to the continuous time. We summarize the model into a system of three state variables, namely, the market maker's estimation error, the change in the market maker's inventory, and the change in the market maker's cumulative profit. We show that the system has nice economic interpretations and provides further insights into the model. Finally, we discuss the extension to the competitive market making in which market makers compete for order flows. We argue that the equilibrium bid-ask spread is the smallest possible one when each market maker has a zero expected utility.

This chapter is organized as follows. Section 3.2 describes our basic model of market making. Section 3.3 provides the main solutions, followed by numerical results in Section 3.4. Section 3.5 extends the model to the continuous time. Section 3.6 discusses the competitive market making and Section 3.7 concludes this chapter. All proofs are presented in Appendix A.

3.2 The model

3.2.1 Assumptions

The market for a risky security operates over an indefinite number of short time periods. Prices are set at points in time t , $t = 1, 2, \dots, T$. We refer to the time interval between time points t and $t + 1$ as period t . Order flow \tilde{Q}_t occurs during period t .

There are three types of market participants: a risk-averse monopolistic market maker, many informed traders and many uninformed, liquidity traders. The market maker knows neither whom he is trading with, nor any trader's trading activities. The only information that the market maker has is his order book. We assume that all orders are market orders and submitted to the market maker so that he has perfect knowledge of the order flow information. We do not explicitly characterize the trading behavior of informed and uninformed traders. Their existence however gives rise to the market maker's noisy order flow observation of the stock's true value, which we define as the price at which the expected net order flow is zero.

The timing of the events and the information structure are as follows. At the

beginning of period t , the market maker has a prior belief, denoted as \hat{p}_t , of the true price p_t , based on his information up to and including period $t - 1$. The noise of his belief is $S_t = \text{var}_{t-1}[p_t]$. He sets the bid (b_t) and ask (a_t) prices for traders to sell and buy securities. Traders submit orders during period t at the bid and ask prices and the net order flow observed by the market maker is \tilde{Q}_t . With the net order flow information, the market maker updates his belief through the Kalman filter and obtains a posterior belief \hat{p}_{t+1} with noise S_{t+1} at the beginning of next period $t + 1$.

The public demand for buying (\tilde{Q}_{B_t}) and selling securities (\tilde{Q}_{S_t}) are represented by the following two equations:

$$\tilde{Q}_{B_t} = Q_0 - \alpha(a_t - p_t) + \sigma_B \tilde{\varepsilon}_B \quad (3.2.1)$$

$$\tilde{Q}_{S_t} = Q_0 - \beta(p_t - b_t) + \sigma_S \tilde{\varepsilon}_S \quad (3.2.2)$$

where α and β are positive constants. Note that when $a_t = b_t = p_t$, $E[\tilde{Q}_B - \tilde{Q}_S] = 0$, i.e., the expected net order flow is zero with cleared demands Q_0 . The noise of the market demands are $\sigma_B \tilde{\varepsilon}_B$ and $\sigma_S \tilde{\varepsilon}_S$, where σ_B and σ_S are the standard deviations of the public buy and sell orders, respectively.⁵ We assume that noise represented by $\tilde{\varepsilon}$ with various subscripts is normal i.i.d. with zero mean and unit variance.

There are two themes from the market demand functions (3.2.1) and (3.2.2). First, the average level of orders depends on the difference between the market maker's bid

⁵One can also think that the demands for order flows come from two sources. The orders submitted by informed traders are linear in mispricing ($a_t - p_t$ and $p_t - b_t$). Uninformed traders create noise in the order flow ($Q_0 + \sigma_B \tilde{\varepsilon}_B$ and $Q_0 + \sigma_S \tilde{\varepsilon}_S$).

(ask) price and the true market price. Second, the market maker sets his bid and ask prices using his information set, which includes both his estimate \hat{p}_t and his beginning inventory level I_t . Write $a_t = a_t(\hat{p}_t, I_t)$ and $b_t = b_t(\hat{p}_t, I_t)$. Now we assume that the market maker follows a linear pricing rule according to $a_t(\hat{p}_t, I_t) = \hat{p}_t + \delta_{a_t}(I_t)$ and $b_t(\hat{p}_t, I_t) = \hat{p}_t - \delta_{b_t}(I_t)$, with δ_{a_t} (δ_{b_t}) being the ask (bid) component of the spread.⁶

The market demand functions now become:

$$\tilde{Q}_{B_t} = Q_0 - \alpha(\hat{p}_t + \delta_{a_t} - p_t) + \sigma_B \tilde{\varepsilon}_B \quad (3.2.3)$$

$$\tilde{Q}_{S_t} = Q_0 + \beta(\hat{p}_t - \delta_{b_t} - p_t) + \sigma_S \tilde{\varepsilon}_S \quad (3.2.4)$$

From the market maker's viewpoint, the sell orders (\tilde{Q}_{S_t}) he receives increase his inventory and buy orders (\tilde{Q}_{B_t}) reduce it. Given the market demand functions (3.2.3) and (3.2.4), the net orders are given by:

$$\Delta I_t = \tilde{Q}_t = \tilde{Q}_{S_t} - \tilde{Q}_{B_t} = (\alpha + \beta)\hat{p}_t + \alpha\delta_{a_t} - \beta\delta_{b_t} - (\alpha + \beta)p_t + \sigma_Q \tilde{\varepsilon}_Q \quad (3.2.5)$$

where $\sigma_Q \tilde{\varepsilon}_Q = \sigma_S \tilde{\varepsilon}_S - \sigma_B \tilde{\varepsilon}_B$ and σ_Q is the standard deviation of the net order flow. Note that a positive value of \tilde{Q}_t indicates more public sell orders than public buy orders, which contributes to an increase in the market maker's inventory level ($\Delta I > 0$). The net order flow (\tilde{Q}_t) is the only information that the market maker observes from the market.⁷

⁶In fact, we will solve a quadratic optimization problem for the market maker and the optimal solution is linear.

⁷Clearly our model is a batch trading model similar to Kyle (1985), in which informed and uninformed orders are lumped together.

We note that the buy and sell orders are crossed first before they are submitted to the market maker, in a much similar way as in Kyle (1985). So the market maker only knows the net order flow information but not the order flow information on each side of the market. The simulations in Section 3.4 are consistent since the market maker only uses the net order flow information when he updates his beliefs.

Finally we assume that the true value of the stock follows a simple random walk, defined as:

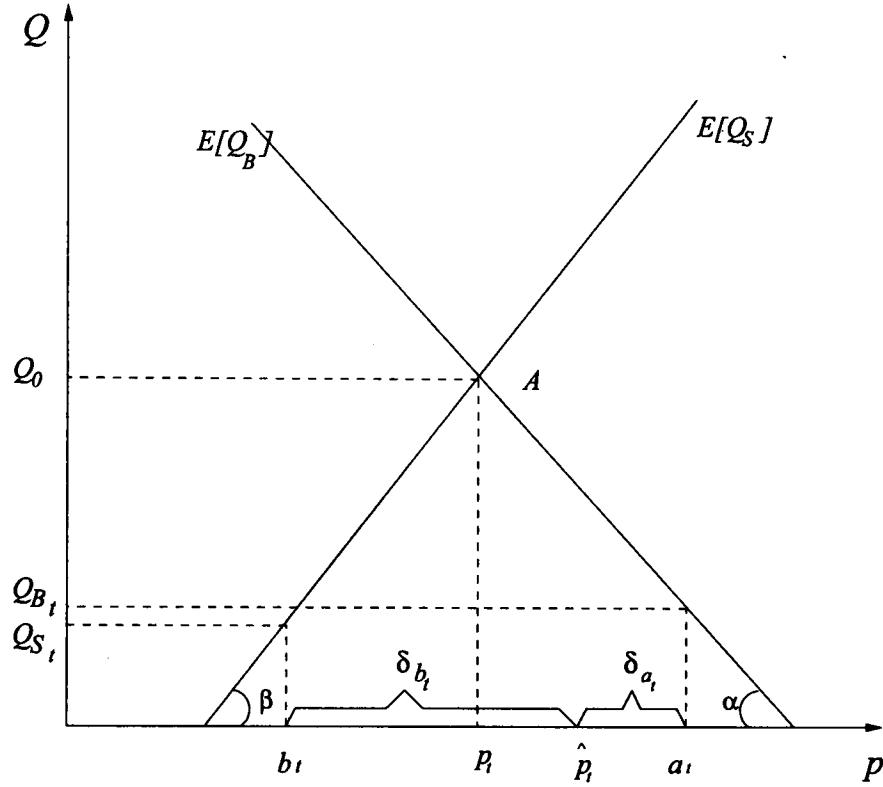
$$p_{t+1} = p_t + \sigma_\mu \tilde{\varepsilon}_\mu \quad (3.2.6)$$

where σ_μ is the standard deviation of the true price.

Figure 3.1 shows the assumptions of the market demand functions. $E[\tilde{Q}_B]$ and $E[\tilde{Q}_S]$ are the expected public buy and sell demands which intersect at point A , at which the market clears at price p_t , with the cleared demand of Q_0 . The slopes of $E[\tilde{Q}_B]$ and $E[\tilde{Q}_S]$ are α and β , respectively and \hat{p}_t is the market maker's prior estimate of p_t before he observes the order flow during period t . The market maker's bid and ask prices are b_t and a_t and $E[Q_{S_t}]$ and $E[Q_{B_t}]$ are the corresponding demands for public sell and buy orders at time t . Note that in this particular case, $E[Q_{B_t}]$ exceeds $E[Q_{S_t}]$. We call the difference between a_t (b_t) and \hat{p}_t as the ask (bid) component δ_{a_t} (δ_{b_t}) of the spread.

It is worth noting that we put the price on the horizontal axis and the quantity on the vertical axis in Figure 3.1, different from the conventional way of drawing figures

Figure 3.1: Market demand functions: the model of monopolistic market making. The expected buy and sell order flows are $E[\tilde{Q}_B]$ and $E[\tilde{Q}_S]$. The true market price is p_t , and \hat{p}_t is the market maker's prior estimate (somewhere different from p_t). Note b_t (a_t) is bid (ask) price quoted by the market maker and δ_{b_t} (δ_{a_t}) is the bid (ask) component of the spread.



in economics. The reason is the following. In this model, the public demands for orders depend on the prices set by the market maker. Traders choose the sizes of their buy and sell demands in responds to the perceived difference between the true price p_t and market maker's quotes. Figure 3.1 demonstrates this relation.

3.2.2 The Kalman filter updating

In this model the market maker updates his estimates of the true value of the stock under a Bayesian framework. Given the market maker's prior estimate \hat{p}_t , he observes

the order flow information and updates his belief to \hat{p}_{t+1} . Specifically, this Bayesian framework can be summarized in the state-space form of the Kalman filter. The Kalman filter extracts the information of the state variables from the noisy observation and provides the optimal estimates of the state variables.

According to Harvey (1989), equation (3.2.5) is the measurement equation and equation (3.2.6) is the transition equation of the Kalman filter in our model. The transition equation describes how the system evolves over time. Since the state variables are not directly observable, the measurement equation provides the link between the state variables and the observations.

Define \hat{p}_{t+1} as the market maker's optimal estimate after observing order flows \tilde{Q}_t during time period t . Applying the Kalman filter algorithm, the market maker's belief is updated through the following equation:

$$\hat{p}_{t+1} \equiv E_t[p_{t+1}] = \hat{p}_t + \frac{(\alpha + \beta)S_t}{(\alpha + \beta)S_t + \sigma_Q^2} (\alpha\delta_{a_t} - \beta\delta_{b_t} - \tilde{Q}_t) \quad (3.2.7)$$

It follows that with the information from the new net order flow observation \tilde{Q}_t , the market maker updates his estimate for true price p_{t+1} in the next period $t + 1$. The noise of the optimal estimation in equation (3.2.7) comes from the order flow observation \tilde{Q}_t , which contains the noisy signals from traders, and the previous estimation variance S_t . The estimation variance itself is updated through the following Riccati equation:

$$S_{t+1} \equiv var_t[p_{t+1}] = \frac{\sigma_Q^2 S_t}{(\alpha + \beta)^2 S_t + \sigma_Q^2} + \sigma_\mu^2 \quad (3.2.8)$$

One important feature of the Kalman filter algorithm is its convergence to the steady state where the estimation variance S_t becomes time-invariant. From equation (3.2.8), the speed of convergence depends on the parameters we choose, in particular, σ_μ and σ_Q . In the numerical experiments reported in Section 3.4, the Riccati equation actually converges to the steady state very quickly, in just a few time steps.

3.2.3 The market maker's optimization problem

We consider a myopic market maker who maximizes his expected utility of per period profit.⁸ The market maker's objective function is given by:

$$\max_{a_t, b_t} E[U(\tilde{\pi}_t)] \quad (3.2.9)$$

where $\tilde{\pi}_t$ is the profit during time period t , defined as:

$$\tilde{\pi}_t = \mathbb{I}_t + \mathbb{T}_t = (I_{t+1}p_{t+1} - I_t p_t) + (a_t \tilde{Q}_{B_t} - b_t \tilde{Q}_{S_t}) \quad (3.2.10)$$

Equation (3.2.10) shows that, for each trading period t , the market maker's profit comes from two sources: the incremental value of his inventory (\mathbb{I}_t) and the cash flow (\mathbb{T}_t). The incremental value of his inventory is defined as the difference between the value of the ending inventory, $I_{t+1} \cdot p_{t+1}$, and the value of the beginning inventory, $I_t \cdot p_t$. The ending inventory (I_{t+1}) is defined as the sum of the beginning inventory I_t plus the net order flow \tilde{Q}_t that the market maker obtains during the period, i.e., $I_{t+1} = I_t + \tilde{Q}_t = I_t + \Delta I_t$. The cash flow part is simply obtained from the marker

⁸Similar to our model, Blume, Easley and O'Hara (1994) and Brown and Jennings (1989) also consider a myopic market maker who maximizes per period expected utility.

maker's trading activity with the public buy and sell demands, \tilde{Q}_{B_t} and \tilde{Q}_{S_t} , evaluated at respective ask and bid prices.⁹

We assume that the market maker has a negative exponential utility function of the form

$$U(\tilde{\pi}_t) = -e^{-\lambda \tilde{\pi}_t}$$

where λ is his risk aversion parameter. Under the assumption of normality, $\tilde{\pi}_t$ is also normally distributed so the maximization of the above utility function is equivalent to the usual mean-variance optimization problem after some monotonic transformation.

The objective function, therefore, can be written as:

$$\max_{a_t, b_t} [E[\tilde{\pi}_t] - \frac{\lambda}{2} var[\tilde{\pi}_t]] \quad (3.2.11)$$

Equation (3.2.11) shows that the market maker's objective is to maximize his expected profit ($E[\tilde{\pi}_t]$), taking into account of the risk of obtaining this profit ($var[\tilde{\pi}_t]$), which is adjusted by his risk aversion parameter λ . That is, the market maker maximizes his *risk adjusted* expected profit.

It is worth mentioning that although the market maker could not observe the profit $\tilde{\pi}_t$ directly (since p_t is unobservable), he still has sufficient knowledge to optimize his expected utility. It is because that given the negative exponential utility function, all

⁹We assume that although the market maker calculates his cash flows on both buy and sell side of the market, he chooses to only update his belief from the net order flow. Of course, it will be a nicer model if the market maker could update his information on both sides of the market. But in our model, we want to have simple and closed-form solutions to the market maker's pricing problem and therefore we assume that the market maker only uses the net order flow information in his Kalman filter updating.

that matters are basically the mean and the variance of the profit, which are functions of the market maker's estimates \hat{p}_t and the estimation variance S_t .

3.3 The solutions

3.3.1 The optimal bid and ask prices

We first calculate the expectation and the variance of the market maker's profit $\tilde{\pi}_t$, using equation (3.2.10) and obtain:

$$E[\tilde{\pi}_t] = (\delta_{a_t} + \delta_{b_t})Q_0 - (\alpha\delta_{a_t}^2 + \beta\delta_{b_t}^2) - (\alpha + \beta)S_t \quad (3.3.1)$$

$$\text{var}[\tilde{\pi}_t] = f_{1_t}(\delta_{a_t}, \delta_{b_t}, \hat{p}_t)S_t + f_{2_t}(\delta_{a_t}, \delta_{b_t}, \hat{p}_t)\sigma_\mu^2 + \delta_{a_t}^2\sigma_B^2 + \delta_{b_t}^2\sigma_S^2 \quad (3.3.2)$$

where $f_{1_t, 2_t}(\delta_{a_t}, \delta_{b_t}, \hat{p}_t)$ are time varying functions of δ_{a_t} , δ_{b_t} , \hat{p}_t and other parameters such as α , β and the state variable I_t .

Equation (3.3.1) has a straightforward economic interpretation. It shows that the expected per period profit is reduced $(-(\alpha + \beta)S_t)$ by the market maker's prior estimation variance S_t , further adjusted by a scalar $(\alpha + \beta)$. This is, in fact, the amount of money that the market maker loses due to the presence of market asymmetric information. Notice that the noise is S_t , the square of the standard deviation of prior estimation error. It follows that the market maker has enormous incentive to process information correctly to reduce this estimation error. If the market maker uses some non-optimal methods to process information, or if he incorrectly knows the S_t , the market making business would be much less profitable. Under the assumptions of

this model, the Kalman filter is the optimal method for the market maker to process his information.

We substitute equations (3.3.1) and (3.3.2) back into the market maker's optimization equation (3.2.11). We then have a quadratic optimization problem with δ_{a_t} and δ_{b_t} as decision variables. The optimal values of δ_{a_t} and δ_{b_t} are obtained by taking the first order conditions. We then have a system of linear equations with unknown variables δ_{a_t} and δ_{b_t} , which can be solved through substitution. The following proposition gives the optimal bid and ask prices.

Proposition 3.1. *The optimal bid and ask prices that a monopolistic market maker sets at time t are given by:*

$$a_t = \hat{p}_t + A_{1,t}Q_0 - A_{2,t}I_t \quad (3.3.3)$$

$$b_t = \hat{p}_t - B_{1,t}Q_0 - B_{2,t}I_t \quad (3.3.4)$$

where $A_{1,t,2,t}$ and $B_{1,t,2,t}$ are strictly positive time varying functions obtained from the Kalman filter and they depend on the model parameters and in particular, the prior estimation variance S_t .

Proposition 3.1 shows that the optimal bid and ask prices depend on the market maker's prior estimate \hat{p}_t and the amount of the cleared demand, Q_0 , which is the expected order flow when the bid and ask prices equal to p_t . More importantly, the prices explicitly depend on the beginning inventory level I_t of the time period t . The bid and ask prices are both monotonically decreasing functions of I_t . This relation agrees with one's intuition. For example, if the market maker has a higher beginning inventory holding I_t , he would like to sell more of his inventory and buy fewer securities. According to Proposition 3.1, the market maker lowers both bid and

ask prices by lowering the position of his spread. The traders observe more favorable price for buying securities and less favorable price for selling securities. Through this, the market maker is able to get rid of his excessive inventories.

It is common in the inventory models of the market microstructure literature that bid and ask prices explicitly depend on the inventory levels. Inventory is generally mean reverting, reflecting the fact that the market maker wishes to maintain it at some preferred level. It can also be interpreted as that the market maker *hedges* his inventory through setting appropriate bid and ask prices. What is special of this model is that our optimal bid and ask prices depend on two things explicitly. First, as in most inventory models, they depend on the market maker's inventory. Second, as a result of the market maker's learning of the true prices, the bid and ask prices depend on his prior estimation variance (S_t).

As a special case of Proposition 3.1, if the market maker is risk neutral, i.e., he is only concerned about the expected profit, then his risk aversion parameter, λ , becomes zero. Setting $\lambda = 0$ in equations (3.3.3) and (3.3.4), we have $a_t = \frac{Q_0}{2\alpha}$ and $b_t = \frac{Q_0}{2\beta}$. In this case, the bid and ask prices are constant and independent of inventory fluctuations. Risk neutrality can be found in most asymmetric information models where the inventory problem is generally ignored. However, by considering the inventory, a very important source of risk for the market maker, our model gives a richer characterization of his pricing strategy.

The bid-ask spread is given by the following Corollary 3.2.

Corollary 3.2. *The spread that the market maker sets at time t is given by:*

$$f(S_t)Q_0 + \lambda^2 \sigma_\mu^2 (\beta \sigma_B^2 - \alpha \sigma_S^2) I_t$$

where $f(S_t)$ is a strictly positive time varying function obtained from the Kalman filter and depends on the model parameters and in particular the prior estimation variance, S_t .

As we expect, the spread is related to S_t and I_t . Furthermore, the total spread is a monotonic function of I_t given the parameters of the model. However, it is not obvious whether it is a monotonically increasing or decreasing function and it depends on the parameter values. When we have $\alpha \sigma_S^2 = \beta \sigma_B^2$, the total spread is independent of the inventory levels.

3.3.2 The market maker's pricing bias

We define the market maker's pricing bias (PB) as the difference between the quoted mid-price, $\frac{1}{2}(a_t + b_t)$, and his prior estimate \hat{p}_t of the true value of the stock (p_t). To focus on the economic intuition, we make further assumptions that the buy and sell demands are perfectly symmetric, i.e., $\alpha = \beta$, $\sigma_B^2 = \sigma_S^2$ and $\varepsilon_B = \varepsilon_S$. Proposition 3.3 summarizes the market maker's pricing bias.

Proposition 3.3. *When the buy and sell orders are perfectly symmetric, we denote $\alpha = \beta = \theta$, $\sigma_B^2 = \sigma_S^2 = \sigma^2$ and $\varepsilon_B = \varepsilon_S$. The market maker's pricing bias is given by:*

$$PB \equiv \frac{1}{2}(a_t + b_t) - \hat{p}_t = -f(\lambda)I_t \quad (3.3.5)$$

where $f(\lambda)$ is a strictly positive and monotonically decreasing function in λ .

The pricing bias basically measures the deviation of the mid-quote from the market maker's prior estimate of the true price. The market maker concerns the PB because it tells the position of his quotes relative to his prior belief. A big PB indicates that either there is a high concentration of orders on one side of the market, or there is a big movement in the true market price. In both cases, the market maker's prior estimates might be so wrong.

Equation (3.3.5) shows that this pricing bias depends on the market maker's inventory positions through a function of λ . The higher the inventory level, the bigger the absolute value of the bias is and the higher incentive that market maker has to revise his price to restore his inventory to the desired level. Furthermore, this bias is related to the market maker's risk preference λ . When the market maker is risk neutral ($\lambda = 0$) and does not care about the risk of his inventory, he has no incentive to deviate his price from his prior estimate \hat{p}_t so the pricing bias is zero. For a risk averse market maker, $f(\lambda)$ is a monotonically decreasing function in λ . As the market maker becomes more risk averse with increasing λ , he puts more bias on positioning his quotes and has stronger incentive to revise his quotes to restore his inventory to the preferred position.

3.3.3 The market maker's pricing bias when the spread is constant

Now let's look at a special case of the pricing bias when the spread is constant.

Constant spreads have been frequently observed in practice. Although the market maker could adjust bid and ask prices continuously, he often tends to quote a relatively stable or even constant spread. There are at least two reasons for this. First, it is the direct result of the competition among market makers. Competition reduces the spread to a particular level that every market maker has to follow. Second, by quoting a constant spread, the market maker is able to hide certain information since traders could infer the information when they observe the changing spread sizes. For example, the widening (narrowing) of spreads probably means high (low) market demands for the security. By quoting a relatively constant spread, the market maker does not reveal such information to traders and other competing market makers.

We assume that the constant spread is exogenously given as $2m$ where m is a positive real number. The bid and ask prices therefore have the relation of $a_t - b_t = 2m$. The mid-price is given by $a_t - m$ or $b_t + m$. Following the same line of calculation, we obtain the market maker's PB as stated in the following Corollary 3.4.

Corollary 3.4. *Given a constant spread of $2m$, the market maker's pricing bias is given by the following expression:*

$$PB' = -M_t I_t - N_t m \quad (3.3.6)$$

where M_t and N_t are the time-varying functions of model parameters and the prior estimation variance, S_t . Moreover, M_t is strictly positive.

When a constant spread is used by all market makers, the only important thing for individual market maker is how to position his quotes. The PB' in Corollary 3.4 gives such a position. Again, the PB' is a monotonically decreasing function of I_t , as is in the more general case described in Proposition 3.3. It is also linearly related to m , the half spread.

The equation (3.3.6) of PB' is clearly mean reverting and tends to reduce the inventory position given different beginning inventory levels. For example, a high beginning inventory tends to decrease the market maker's PB' (note the negative sign) by lowering the position of the whole spread. As a result, the market maker encourages traders to buy securities and discourages them from selling securities. In this way, the market maker is able to reduce his inventory.

3.4 Numerical experiments

3.4.1 Parameter values

There are seven parameters and four initial values in this model. The seven parameters are Q_0 , α , β , λ , σ_μ , σ_S and σ_B . The four initial values are p_0 , \hat{p}_0 , I_0 , and S_0 . Let's discuss their values separately.

We first consider the seven parameters in the model. Without loss of generality, we assume $\alpha = \beta = 1$. We assume $Q_0 = 4$, so that the bid and ask spread is around 1 to 2. We choose $\sigma_B = \sigma_S = 2.5$, and $\sigma_\mu = 0.03$. The reason for choosing this particular value for σ_μ is the following. We would like to treat each time step in our

Table 3.1: Parameter and initial values used in the simulation. This table reports the parameter and initial values used in the simulation. Q_0 is the expected order flow when $a_t = b_t = p_t$. α and β are positive constants in the public demand functions (3.2.1) and (3.2.2). λ is the market maker's risk aversion parameter. σ_μ is the standard deviation of the true price process. σ_B and σ_S are the standard deviations of public buy and sell demands. p_0 is the initial price and \hat{p}_0 is the market maker's initial estimate of p_0 . I_0 is the initial inventory and S_0 is the market maker's initial estimation variance.

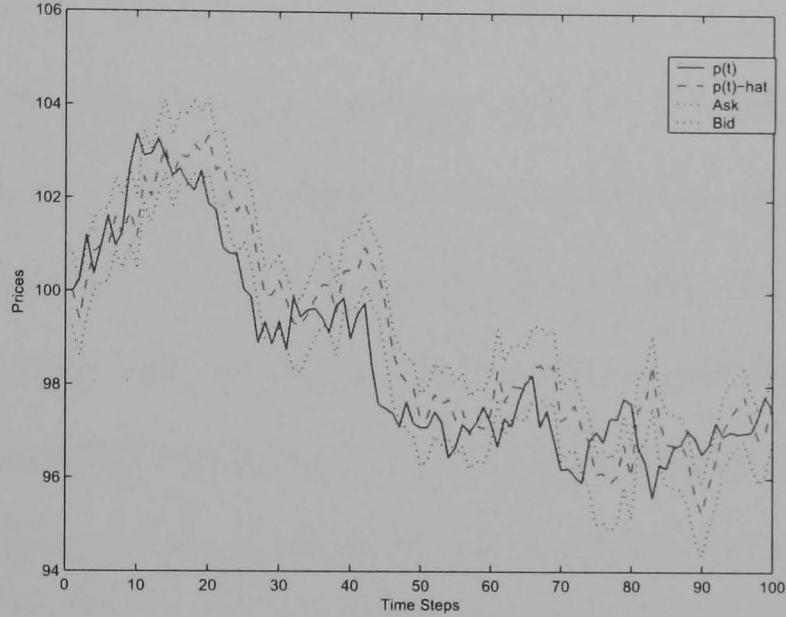
Q_0	α	β	λ	σ_μ	σ_S	σ_B	p_0	\hat{p}_0	I_0	S_0
4	1	1	0.5	0.03	2.5	2.5	100	100	0	steady state value

simulation as one trading day. Since the standard deviation of a typical stock price is about 40% per year, the corresponding daily standard deviation is about $\frac{40\%}{\sqrt{252}}$ if the number of trading days per year is 252. This makes the daily standard deviation of the stock price process, σ_μ , of about 0.03. We assume the risk aversion parameter λ equals 0.5.

For the four initial values, we assume that the true value of the stock starts from 100 at time 0 ($p_0 = 100$) and the market maker has zero initial inventory ($I_0 = 0$). At the beginning of the first period, the market maker's estimate (\hat{p}_0) of the stock's true value p_0 is normally distributed with mean $\hat{p}_0 = p_0 = 100$ and some noise S_0 . We use the steady state estimation variance for this S_0 . As we state in Section 3.2.2, the Kalman filter in our model converges to the steady state fairly quickly.

Table 3.1 summarizes the parameter and initial values that we use in the numerical experiments.

Figure 3.2: Simulated prices and quotes The figure shows one simulation of the processes of the true prices and the bid/ask prices for 100 time periods. The solid line is the true price process p_t and the dashed line is the market maker's estimates \hat{p}_t . Two boundary dotted lines are the bid (b_t) and ask (a_t) prices set by the market maker.



3.4.2 Simulated prices and quotes

Figure 3.2 shows one simulation of the processes of the true price and bid and ask prices for 100 time steps ($t = 100$).

Overall the model gives reasonable bid and ask prices using the parameters and initial values discussed in Section 3.4.1. The solid line is the true price process p_t . It starts from 100 with the highest value at about 104 and the lowest value at about 96. The dashed line is the market maker's estimates \hat{p}_t . Except at $t = 0$ at which $p_t = \hat{p}_t$ by assumption, the estimates \hat{p}_t generally lack the true price p_t by one time step. This agrees with one's intuition because the market maker can only update his estimate

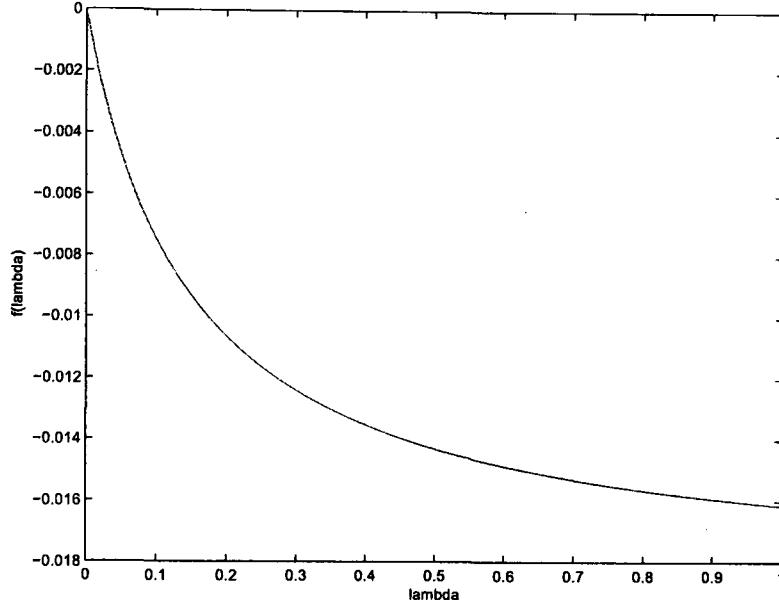
after observing \tilde{Q}_t during time period t , and the new information is only reflected in the market maker's next estimate \hat{p}_{t+1} . The effect of this timing difference is not very damaging as long as the true price moves in the same direction. But when there is a sharp turn of the direction, the market maker's estimate could be very wrong, resulting in bid and ask prices not containing the true price and in that case, the market maker would lose money to informed traders. The two boundary dotted lines are the market maker's bid and ask prices. In this particular simulation, true price process generally runs within the bid and ask bounds so overall the market maker makes profits. Occasionally the prices fluctuate outside the bid-ask bounds, as during the time periods 68 to 73. These are the situations when the market maker is likely to lose to informed traders.

3.4.3 The effect of risk aversion on the pricing bias

In Proposition 3.3, we state that under perfect symmetry, the market maker's pricing bias (PB), which is the difference between his mid-quote and the prior estimate \hat{p}_t , is a linear function of the beginning inventory I_t of period t . The coefficient $f(\lambda)$ is a monotonically decreasing function of the market maker's risk aversion parameter λ . Figure 3.3 shows this function $f(\lambda)$ for different values of λ ranging between 0.001 and 1, with each increment being 0.001.

At least two points are clearly shown from Figure 3.3. First, the function $f(\lambda)$ is indeed a monotonically decreasing function of λ . With increasing λ , $f(\lambda)$ decreases

Figure 3.3: The effect of risk aversion on the pricing bias The figure shows the coefficient of inventory $f(\lambda)$ for different λ values. As predicted in Proposition 3.3, $f(\lambda)$ is a monotonically decreasing function in λ and it starts from the origin.



at a decreasing rate. Second, the function $f(\lambda)$ starts from the origin: when λ is zero, $f(\lambda)$ equals zero. Since our model considers the risk of holding inventory, the risk aversion parameter λ directly affects the market maker's pricing bias. If the risk aversion parameter is zero, the inventory risk is not reflected in the model.

3.4.4 The market maker's cumulative profit

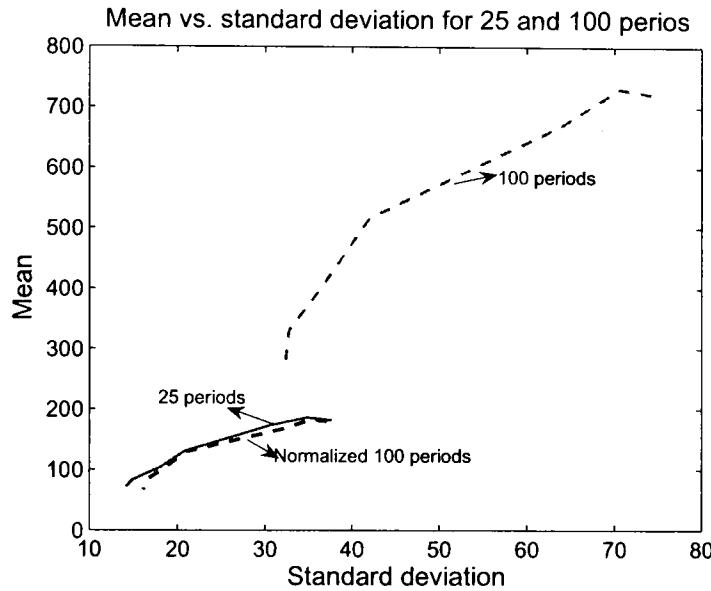
Here we consider the market maker's cumulative profits over time since they represent the market maker's profitability in our model. We do the calculation in three steps. First, for each simulation of the true price and the bid/ask price processes, we calculate the cumulative profits for 25 and 100 periods. Second, we repeat this calculation for 1000 simulations. We then have 1000 cumulative profits for 25 time

periods and for 100 time periods, respectively. To obtain the statistical properties, we calculate the mean and the standard deviation of these two groups of cumulative profits. Finally, to show the importance of the risk aversion parameter λ in our model, we repeat the first two steps for different values of λ ranging from 0.001 to 0.9. We then plot these means against standard deviations of the cumulative profits for different values of λ .

We expect two properties from the figure. First, we expect that, for each λ , the means of the cumulative profits of 100 time steps are about the 4 times of those of 25 time steps. Similarly, the standard deviations of the cumulative profits of 100 time steps are about the twice of those of 25 time steps. Second, we expect that, for respective 25 and 100 time steps, the means and standard deviations are both monotonically decreasing in the risk aversion parameter λ . These two properties are clearly shown in Figure 3.4. Note also that the smaller dashed line is the normalized mean and standard deviation of 100 time periods when we take the $\frac{1}{4}$ of the mean and the $\frac{1}{2}$ of the standard deviation of the cumulative profits of 100 periods. Obviously the smaller dashed line and the solid line (for 25 periods) are independent of but close to each other.

We can also observe from Figure 3.4 that the market maker in our model generally makes profits over both 25 and 100 time steps. For 25 time steps, the market maker's cumulative profits range between 100 and 200, and the standard deviations range

Figure 3.4: Means and standard deviations of the cumulative profits of 25 and 100 time steps for different λ over 1000 simulations. Both means and standard deviations are decreasing in λ . The solid (dashed) line is for the means and the standard deviations of 25 (100) time steps. The smaller dashed line is the normalized mean and standard deviation of 100 time steps.



between 15 and 45. For 100 time steps, his cumulative profits range between 300 and 750, and the standard deviations range between 35 and 80. Of course, these particular numbers depend on the parameter and initial values we choose. But the clear property that both means and standard deviations decrease with increasing λ is exactly as we have expected. This in fact explains how much profits that the market maker prepares to give up in order to control his acceptable level of risk. For example, at high level of risk aversion, the market maker is willing to accept a dramatically reduced level of expected profits in an effort to achieve a lower level of the standard deviation of the expected profits.

3.5 The continuous time extension of the monopolistic model

In this section, we consider the continuous time extension of the monopolistic model of market making that we have analyzed so far. When the time between trading intervals becomes very small, our model can be generalized into a system of three state variables, namely, the market maker's estimation error, the change in the market maker's inventory, and finally, the change in the market maker's cumulative profit. The system of these three variables provides nice economic intuitions and insights.

Again, we assume that the demands for buying and selling securities are perfectly symmetric. The results of the asymmetric case, although more algebraically complicated, are found to be qualitatively the same as the symmetric case. As in Proposition 3.3, we assume:

$$\alpha = \beta = \theta, \sigma_B^2 = \sigma_S^2 = \sigma^2, \varepsilon_B = \varepsilon_S = \varepsilon \quad (3.5.1)$$

Also, we define $f_t = p_t - \hat{p}_t$. Recall that p_t is the true value of the stock at time t , and \hat{p}_t is the market maker's prior estimate of p_t before observing the new order flow during time period t . f_t therefore measures the difference between the true price and the market maker's estimate of this price. We call f_t the market maker's estimation error. Furthermore, between time t and $t - 1$, let dI_t be the change in the market maker's inventory and $d\pi_t$ be the change in the market maker's cumulative profit.

The following proposition summarizes the continuous time system of three state variables:

Proposition 3.5. *When the buy and sell demands are perfectly symmetric as defined by equation (3.5.1), the continuous time system of our monopolistic model of market making can be summarized in the following three state variables:*

$$df_t = -\omega_t f_t dt + \sigma_\mu dW_t \quad (3.5.2)$$

$$dI_t = -2\theta(f_t + B_t I_t)dt \quad (3.5.3)$$

$$\begin{aligned} d\pi_t = & 2[A_t(1 - \theta A_t)Q_0^2 - \theta(f_t + B_t I_t)^2]dt \\ & + \{[1 - 2\theta(f_t + B_t I_t)]\sigma_\mu + 2A_t\sigma\}dW_t \end{aligned} \quad (3.5.4)$$

where A_t and B_t are non-negative time varying functions generated from the Kalman filter and depend on the model parameters and estimation variance S_t . Furthermore, $\omega_t = \frac{2\theta^2 S_t}{(2\theta^2 S_t + \sigma^2)}$, and dW_t is the standard Brownian motion with unit variance.

Proposition 3.5 has nice economic interpretations. First, equation (3.5.2) describes the evolution of the market maker's pricing error, i.e., the difference between the true price and the market maker's estimate of this price. It is clearly a mean reverting Ornstein-Uhlenbeck process with zero mean. As the market maker observes more information from order flows, his estimates are more close to the true value and his estimation error approaches zero. Note that df_t is normally distributed with zero mean and variance σ_μ^2 .

Equation (3.5.3) describes the evolution of the market maker's inventory positions. It is useful to state the following relationship:

$$f_t + B_t I_t \equiv \frac{1}{2}(a_t + b_t) - \hat{p}_t$$

The right hand side of the equation ($(\frac{1}{2}(a_t + b_t) - \hat{p}_t)$) is our definition of the pricing bias (PB) in equation (3.3.5). Therefore, the market maker's pricing bias directly affects

his inventory positions and drives his inventory to its long run mean of zero. Note that dI_t is a mean reverting process which itself is driven by the Ornstein-Uhlenbeck process of df_t .

The last equation (3.5.4) is more complicated. We can show that the first dt term, $2A_t(1 - \theta A_t)Q_0^2 dt$, is strictly positive. It comes from the steady liquidity trading component or the pure noise trading. It is the positive profit that the market maker can make for sure. The second dt term, $-2\theta(f_t + B_t I_t)^2 dt$, can be thought of as the market maker's expected loss due to his mis-pricing. Recall that $(f_t + B_t I_t)$ is equivalent to the market maker's pricing bias. This is the part that the market maker loses to informed traders because informed traders have better knowledge of the true price than he does. Note that the market maker's loss is quadratic in this pricing bias. Therefore the market maker has great incentives to estimate the price as accurate as possible. The last term is the noise in the market maker's profit. Although the noise has a zero expectation, it certainly complicates the market maker's profit estimation. In summary, the change in the cumulative profit comprises three parts: a sure profit part due to steady liquidity trading, a sure loss part due to the market maker's pricing error, and finally the noise due to the random components in the order flows and the price process.

3.6 The extension to competitive market making

In this section we consider the extension of the original monopolistic market making model into the competitive environment in which market makers compete for order flows. The examples of the competitive dealership markets include NASDAQ and the London Stock Exchange, among many others. Competition can be in the form of real quotes competition among market makers.

In competitive dealership markets, traders transact with dealers who set the best prices, and each dealer's pricing problem in principle depends on the actions of every other dealers (O'Hara (1995)). Therefore, to solve a general case of competitive market making is complicated since each dealer's optimal strategy depends on his expectations of other dealers' actions.

In the early inventory literature, Ho and Stoll (1983) provide a simple one period model of competitive market making. They analyze two competing market makers each trading two stocks and choosing bid and ask prices to maximize his respective expected utility. Ho and Stoll analytically characterize the pricing strategies for each market maker under competition. However, their model does not explicitly consider the inventory of the competing market maker, so the strategic component of the market makers' price setting strategies is not considered.

In the asymmetric information literature, competitive market making equilibrium is generally obtained by the market maker's zero expected profit condition under a

rational expectations framework (Glosten and Milgrom (1985)). The reason for this zero-profit condition is that competition and risk neutrality remove all economic rent earned by any competitive market maker. If each market maker starts with the same prior belief and trading information is common knowledge, all competing market makers would quote the same bid and ask prices.

In our model of monopolistic market making, the spreads result from the market maker's market power of observing both the market buy and sell demands. However, competition among market makers forces the bid-ask spread to be the smallest possible since otherwise one market maker could undercut the other market makers' quotes to obtain order flows. This will give each market maker the highest trading volume. Unlike previous models with a zero expected profit condition for the competitive equilibrium, a market maker in our model is affected by his respective inventory risk, adjusted by his risk aversion parameter. Therefore, we require a zero expected utility condition as the necessary condition for our competitive equilibrium.

More formally, the objective function for the market maker i is given by:

$$\begin{aligned} & \min_{a_t^i, b_t^i} (a_t^i - b_t^i) \\ & s.t. \mathcal{Z}^i = E[U(\tilde{\pi}_t^i)] = 0 \end{aligned} \tag{3.6.1}$$

where b_t^i and a_t^i are the bid and ask prices for market maker i .¹⁰

¹⁰We note that the risk aversion of each individual competitive market maker has not been specified. We would like to treat that the aggregate risk aversion of the multiple competitive market makers is the same as the risk aversion of the monopolistic market maker. It is not the case that the

This constrained optimization problem can be solved by the Lagrange method. By forming the Lagrangian and taking the first order derivatives with respect to a_t^i and b_t^i , we have:

$$\frac{\partial \mathcal{Z}^i}{\partial a_t^i} = \frac{\partial \mathcal{Z}^i}{\partial b_t^i} \quad (3.6.2)$$

which gives a linear relation in a_t^i and b_t^i . Note also that the constrain

$$E[U(\tilde{\pi}_t^i)] = 0 \quad (3.6.3)$$

is a quadratic function in a_t^i and b_t^i .¹¹ By simultaneously solving equations (3.6.2) and (3.6.3), we obtain two pairs of optimal a_t^i and b_t^i . The optimal spread is given by the pair with the smallest difference.

The following proposition summarizes the competitive bid-ask spread for market maker i :

Proposition 3.6. *The competitive bid-ask spread is the solution to the constrained optimization problem (3.6.1), which is obtained by simultaneously solving the equations (3.6.2) and (3.6.3) and taking the pair of a_t^i and b_t^i which gives the smallest difference.*

3.7 Conclusion

Market microstructure has recognized the inventory risk and asymmetric information as two important factors affecting a market maker's price setting strategy. In

each competitive market maker has the same risk aversion as the monopolistic market maker since in that case, the overall risk aversion of the market makers would be different.

¹¹This is clearly true by observing the equations of the expected profit (equation (3.3.1)) and the variance of the profit (equation (3.3.2))

this chapter, we develop and analyze a simple model of market making in which a monopolistic market maker considers these two problems simultaneously.

There are two distinctive features in this model. First, the market maker in this model is exposed to the inventory risk. He tries to manage his inventory to an acceptable level. Second, the market maker is exposed to market asymmetric information. Without knowing whom he is trading with, the market maker updates his estimates of the stock's true value through his order flow observation. In a Bayesian updating framework, our market maker's estimation problem can be summarized in a particular state-space form called the Kalman filter. The Kalman filter describes how the information in the market maker's order flow observation is impounded into his price setting process. We contribute to the literature by combining the inventory risk and asymmetric information into a simple dynamic setting and analyzing the properties under such a setting.

We analytically characterize the optimal bid and ask prices that the market maker sets over time. We also find that the market maker deliberately biases his quotes to manage his inventory. Of particular importance is our numerical results of the effects of risk aversion on the market making profitability. We show that, with increasing risk aversion parameters, both the means and the standard deviations of the market maker's expected cumulative profits decrease. Therefore the market maker in our model prepares to give up much of his profits to control the risk of his inventory.

Inventory is risky and consumes capital. Our model provides a simple way for market makers to optimally process information to control inventory.

The extension of the model to the continuous time provides useful insights. The system of three state variables describes the process of how the market maker's estimation error changes his inventory positions and reduces his cumulative profits. In particular, the market maker deliberately biases his bid and ask prices to move his inventory to the long run mean of zero. His pricing bias also greatly reduces the market maker's profitability. The market maker therefore has enormous interests to process information correctly, and under the assumptions in this model, the Kalman filter provides the optimal method to estimate the true price.

The Kalman filter in this chapter has a nice structure capable of analyzing more complicated questions. Of particular interest is to apply the technique to model option market making. Options are closely related securities. In an option market, how information is generated and disseminated at different strike levels and how market makers manage the inventory of portfolios of different option contracts remain important research topics. In the next chapter, we will extend the model to examine the market making of multiple stocks. The correlation among stocks would provide some useful insights into the market making of the closely related securities such as options.

Chapter 4

Market making with return commonality

4.1 Introduction

The traditional focus of the research on market making has been on the liquidity provision for an individual security. In practice, however, a market maker hardly only makes a market for one security. At NYSE, for example, there are currently 7 specialist firms that make markets for approximately 2672 listed companies.¹ It is therefore of interest to examine the implications of the market making of multiple stocks on the market maker's price setting strategies and its impact on order flows. In this chapter, we consider a monopolistic stock market maker who sets optimal bid and ask prices for multiple stocks while these stocks' price processes have the known variance-covariance structure. We argue that the market maker's knowledge of the stocks' return structure improves his estimation of the unknown price process for each

¹The data is as the end of December 31, 2005 (<http://www.nyse.com/about>).

stock and his overall market making profitability.

Admati (1985) is an early analysis of an equilibrium market with multiple risky securities. In her model, the information possessed by heterogeneous agents is partially revealed by the equilibrium prices. The rich correlation structures of payoffs, supplies and the error terms in each information signal give various interactions between assets and consequently create phenomena that are impossible in models with a single risky asset.

Other early theoretical literature on the market making of multiple stocks includes Subrahmanyam (1991), Gorton and Pennacchi (1993) and Caballe and Krishnan (1994). These models extend the ideas of Kyle (1984 and 1985) and Admati and Pfleiderer (1988) to analyze the strategic trading behavior of informed or uninformed traders. Subrahmanyam (1991) observes that information asymmetry is mainly a problem of an individual stock and hence provides a rational for trading a stock index. Both Subrahmanyam (1991) and Gorton and Pennacchi (1993) argue that the welfare of uninformed traders improves when they trade the composite of securities since their trading losses are reduced.

Examining the market making of multiple stocks, Caballe and Krishnan (1994) suggest that the specialist learns from order flows in other stocks, and in particular, the incremental explanatory power of the aggregate order flow affects his price setting strategy. Under their setting, a market maker can potentially learn about

every security from each order flow information given correlated fundamentals. As an example, if a market maker can observe the order flow of a market index, he can infer the informativeness of the order flows of index constituent stocks.

In this chapter, we consider how a monopolistic market maker sets prices for multiple securities. The market maker faces two risks. The first risk comes from the inventories of stocks that he makes the market for. His price setting strategies affect the market demands and hence the order imbalance of every stock, contributing to his inventory management problem. The market maker also faces the risk of trading with informed traders. He does not know the true price process for each stock but is assumed to know the variance-covariance structure of these stocks' returns.² His knowledge of the stocks' return covariance structure affects the order flow of all stocks through his optimal price setting strategy. Compared to only using the individual order flow information to update his estimation of the true value of that stock, when the market maker considers the correlated order flow information in his price estimation, he obtains better estimates for every stock. The improved estimates help to mitigate the problem of information asymmetry.

One interesting phenomena arising from the model is that the market maker's

²This assumption is not unrealistic in practice. For example, a specialist firm often employs a number of specialists each making a market for a stock. The specialists know the historical return covariance of the stocks that the firm makes markets for, but each specialist does not know the true value of the stock allocated to him. We will show that when the specialists consider the correlated order flow information of every stock from their knowledge of the return covariance structure of these stocks, the specialist firm improve its overall market making profitability.

knowledge of the return covariance structure gives rise to liquidity commonality of stocks' order flows. Liquidity commonality is important because systematic liquidity is most likely a priced source of risk (Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), Pastor and Stambaugh (2003)). This topic has been studied extensively in recent years. Chordia, Roll and Subrahmanyam (2000) first use the term *commonality* in describing the common underlying determinants of liquidity, trading costs, and other individual microstructure phenomena. They find that individual liquidity co-moves with market liquidity, and that liquidity commonality remains significant even after controlling for individual liquidity determinants. Using a Principle Component Analysis approach, Hasbrouck and Seppi (2001) document the existence of a single common liquidity factor of Dow 30 stocks, though the commonality is not very strong. Coughenour and Saad (2004) analyze the cause of liquidity commonality and argue from a liquidity supply perspective that common market makers are one reason for liquidity commonality. For markets without any designated liquidity suppliers, Brockman and Chung (2002) and Bauer (2004) find that liquidity commonality also exists in the purely order-driven settings of the Stock Exchange of Hong Kong and the Swiss Stock Exchange respectively. Domowitz, Hansch and Wang (2005) further examine the linkage between liquidity commonality that is due to cross-sectional correlation in order types (market and limit orders), and return commonality that is caused by correlation in order flows (order direction and

size). These empirical works demonstrate the increasing importance of understanding common liquidity effects in financial markets.

We set up our model in a Kalman filter updating framework. The state vector is the stocks' unknown price processes which evolve linearly. The market maker can only observe the order flows of all stocks and use this noisy observation to estimate the true price processes. By setting the optimal bid and ask prices for each stock, the risk averse market maker maximizes his expected profit per time period, subject to the risk of the profit. We obtain a closed form solution of the optimal prices that the market maker sets. Two factors affect these prices in particular: the covariance structure of the stocks' returns, and the market maker's inventory positions. Since inventory carries risk, the market maker generally keeps minimum inventory levels of all stocks.

With the help of the analytical solution, we ask the question: how much does the market maker benefit from his knowledge of the return covariance structure? We analyze two different cases. In the first case, the make maker treats the individual stock's order flow information on its own and does not consider the correlation in order flow information when he updates his beliefs of the true price processes of every stock. In the second case, the market maker learns from order flows in other stocks and updates his estimates using the order flow information of all stocks together. We simulate the market maker's estimated price processes under these two cases

and calculate his cumulative profits over some time periods in each case. Numerical results show that the market maker obtains higher expected cumulative profits and lower standard deviations of profits in the second case compared to those in the first case. These results demonstrate the benefit of integrating order flow information of all stocks in price estimations.

The model in this chapter serves as a first step of our study on option market making. An option market maker makes markets for a number of option contracts, in a much similar way as market making multiple stocks. Furthermore, our assumption of the stocks' return covariance structure in this chapter is intended to represent the correlation in volatilities. It is a common view that an option is a claim on its underlying asset and an option market maker uses the underlying market to hedge his risk exposure. In fact, at least from our conversation with some practitioners, in market making options, where the underlying or its futures is liquid, option market makers consider themselves to be trading in implied volatilities on different strike prices of options, rather than the underlying, which is hedged virtually automatically. Our model therefore provides insights into the market making of the parameterized family of substitutable securities such as options.

This chapter is organized as follows. In Section 4.2, we describe our model of the market making of multiple stocks. In Section 4.3 we show numerically the benefits of taking the return covariance into price estimation. Section 4.4 concludes the chapter.

4.2 The model

4.2.1 Assumptions

We follow the similar model settings as in Chapter 3 except that we introduce stocks with the known covariance structure.³ We analyze the general case with n stocks and obtain the optimal bid and ask prices that the market maker sets.

We consider a risk-averse, monopolistic market maker who is trading with both informed and uninformed traders. The market maker does not know whom he is trading with, nor the traders' trading activities. The only information he has is from his order book, and in this case, the order flow information of all stocks. We assume that all orders are submitted to him so that he has the perfect knowledge of all order flow information.

The market demands for buying and selling n stocks, $\tilde{\mathbf{q}}_B$ and $\tilde{\mathbf{q}}_S$, respectively, are $(n \times 1)$ vectors and given by:

$$\tilde{\mathbf{q}}_B = \mathbf{q}_0 - \boldsymbol{\alpha}(\mathbf{a}_t - \mathbf{p}_t) + \Sigma_B \tilde{\boldsymbol{\varepsilon}}_B = \mathbf{q}_0 - \boldsymbol{\alpha}(\hat{\mathbf{p}}_t + \boldsymbol{\delta}_{at} - \mathbf{p}_t) + \Sigma_B \tilde{\boldsymbol{\varepsilon}}_B \quad (4.2.1)$$

$$\tilde{\mathbf{q}}_S = \mathbf{q}_0 - \boldsymbol{\beta}(\mathbf{p}_t - \mathbf{b}_t) + \Sigma_S \tilde{\boldsymbol{\varepsilon}}_S = \mathbf{q}_0 + \boldsymbol{\beta}(\hat{\mathbf{p}}_t - \boldsymbol{\delta}_{bt} - \mathbf{p}_t) + \Sigma_S \tilde{\boldsymbol{\varepsilon}}_S \quad (4.2.2)$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are positive definite $(n \times n)$ diagonal matrices. The market maker sets the bid (\mathbf{b}_t) and ask (\mathbf{a}_t) prices (both $n \times 1$ vectors) for n stocks according to $\mathbf{a}_t = \hat{\mathbf{p}}_t + \boldsymbol{\delta}_{at}$ and $\mathbf{b}_t = \hat{\mathbf{p}}_t - \boldsymbol{\delta}_{bt}$. $\boldsymbol{\delta}_{at}$ and $\boldsymbol{\delta}_{bt}$ are $(n \times 1)$ vectors of ask and

³To avoid duplications, we briefly discuss the model assumptions in this section. Most assumptions are generally the multivariate extensions of the assumptions of the single stock market making model in Chapter 3.

bid components of the spreads and $\hat{\mathbf{p}}_t$ is the market maker's prior estimate at the beginning of time period t .

Clearly the market demands depend on the difference between the prices quoted by the market maker and the unobservable *true* price vector \mathbf{p}_t , which we define as the price at which $E[\tilde{\mathbf{q}}_B - \tilde{\mathbf{q}}_S] = 0$ if $\mathbf{a}_t = \mathbf{b}_t = \mathbf{p}_t$. Note that \mathbf{q}_0 is a $(n \times 1)$ vector of the expected order flows when $\mathbf{a}_t = \mathbf{b}_t = \mathbf{p}_t$; also $\Sigma_B \Sigma'_B$ and $\Sigma_S \Sigma'_S$ give $(n \times n)$ covariance matrices of the public buy and sell order flows. We assume that $\tilde{\boldsymbol{\varepsilon}}_B$ and $\tilde{\boldsymbol{\varepsilon}}_S$ are $(n \times 1)$ normally distributed vectors with zero mean vectors and identity variance matrices.

Given the equations (4.2.1) and (4.2.2), the net orders from the market maker's viewpoint are given by:

$$\tilde{\mathbf{q}}_t = \tilde{\mathbf{q}}_S - \tilde{\mathbf{q}}_B = \alpha \delta_{at} - \beta \delta_{bt} - (\alpha + \beta)(\mathbf{p}_t - \hat{\mathbf{p}}_t) + \Sigma_q \tilde{\boldsymbol{\varepsilon}}_q \quad (4.2.3)$$

where $\Sigma_q \tilde{\boldsymbol{\varepsilon}}_q = \Sigma_S \tilde{\boldsymbol{\varepsilon}}_S - \Sigma_B \tilde{\boldsymbol{\varepsilon}}_B$ and $\Sigma_q \Sigma'_q$ is a $(n \times n)$ covariance matrix of the net order flows. Likewise, we assume $\tilde{\boldsymbol{\varepsilon}}_q$ is a $(n \times 1)$ normally distributed noise vector with a zero mean vector and an identity covariance matrix.

We assume that the true price processes of n stocks are given by:

$$\mathbf{p}_t = \mathbf{p}_{t-1} + \mathbf{C}\boldsymbol{\mu} \quad (4.2.4)$$

where $\mathbf{C}\mathbf{C}' = \mathbf{V}$ is and \mathbf{V} is a $(n \times n)$ positive definite covariance matrix of n stocks. $\boldsymbol{\mu}$ is the standard system noise, normally distributed with zero mean vector and identity covariance matrix.

The entry on the i th row and the j th column of matrix \mathbf{V} , $v_{i,j}$, is given by $v_{i,j} = a\delta_{i,j} + b\theta^{|i-j|}$ where $\delta_{i,j}$ is the Kronecker delta ($\delta_{i,j} = 1$ for $i = j$, else 0), and a, b and θ are positive constants. More specifically, the covariance matrix \mathbf{V} is given by:⁴

$$\mathbf{V} = \begin{bmatrix} a+b & b\theta & b\theta^2 & \dots & b\theta^{n-2} & b\theta^{n-1} \\ b\theta & a+b & b\theta & \dots & b\theta^{n-3} & b\theta^{n-2} \\ b\theta^2 & b\theta & a+b & \dots & b\theta^{n-4} & b\theta^{n-3} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ b\theta^{n-2} & b\theta^{n-3} & b\theta^{n-4} & \dots & a+b & b\theta \\ b\theta^{n-1} & b\theta^{n-2} & b\theta^{n-3} & \dots & b\theta & a+b \end{bmatrix}$$

It is worth noting that the assumption of the structure of \mathbf{V} is intended to represent the correlation in volatilities. It is known that the correlation of the implied volatilities between options with different strikes is related to the difference between

⁴In Appendix B, we show how to simulate random variables that has a covariance in the form of matrix \mathbf{V} . The assumption of the structure of matrix \mathbf{V} is intended as appropriate for options where the correlation between option i and j depends on $|i - j|$. For equities, we could use the Sharpe diagonal model (or single index model) where matrix \mathbf{V} is generated by

$$\mathbf{V} = \sigma_m^2 \begin{bmatrix} \beta_1 \\ \vdots \\ \vdots \\ \beta_N \end{bmatrix} \begin{bmatrix} \beta_1 & \dots & \dots & \beta_N \end{bmatrix} + \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \sigma_N^2 \end{bmatrix}$$

where σ_m^2 is the market variance, $\sigma_1^2, \dots, \sigma_N^2$ are residual variances, and β_1, \dots, β_N are market factor loadings. In this case, matrix \mathbf{C} can be generated from the following $N \times (N + 1)$ matrix:

$$\mathbf{C} = \begin{bmatrix} \beta_1\sigma_m & \sigma_1 & \dots & \dots & 0 \\ \beta_2\sigma_m & 0 & \sigma_2 & \dots & 0 \\ \vdots & 0 & \vdots & \ddots & \vdots \\ \beta_N\sigma_m & 0 & \dots & \dots & \sigma_N \end{bmatrix}.$$

Our results are qualitative the same given this alternative assumption of \mathbf{V} .

their exercise prices. This correlation is captured by the assumption of $v_{i,j}$, where the relation between option i and j is related to $\theta^{|i-j|}$. When $\theta = 1$, \mathbf{V} breaks down to the correlation structure in a Sharpe model where the covariance among stocks is a constant b .

4.2.2 The Kalman filter updating

We set up our model under the standard Kalman filter framework. Following Harvey (1989), equation (4.2.4) is our stochastic system's *transition* equation which describes how the state vector (\mathbf{p}_t) evolves. Equation (4.2.3) shows how the state vector is observed with noise and is our *measurement* equation. Applying the Kalman filter algorithm, we obtain the following equation that describes how the market maker updates his estimates ($\hat{\mathbf{p}}_t$) of the true price vector (\mathbf{p}_t):

$$\hat{\mathbf{p}}_{t+1} = \hat{\mathbf{p}}_t + \Sigma_t(\boldsymbol{\alpha} + \boldsymbol{\beta})'[(\boldsymbol{\alpha} + \boldsymbol{\beta})\Sigma_t(\boldsymbol{\alpha} + \boldsymbol{\beta})' + \Sigma_q\Sigma_q']^{-1}(\boldsymbol{\alpha}\delta_{at} - \boldsymbol{\beta}\delta_{bt} - \tilde{\mathbf{q}}_t) \quad (4.2.5)$$

where Σ_t is the prior estimation variance at the beginning of time t and is updated through the following Riccati equation:

$$\Sigma_{t+1} = \Sigma_t - \Sigma_t(\boldsymbol{\alpha} + \boldsymbol{\beta})'[(\boldsymbol{\alpha} + \boldsymbol{\beta})\Sigma_t(\boldsymbol{\alpha} + \boldsymbol{\beta})' + \Sigma_q\Sigma_q']^{-1}(\boldsymbol{\alpha} + \boldsymbol{\beta})\Sigma_t + \mathbf{V} \quad (4.2.6)$$

Note that in equation (4.2.5), the market maker's next period price estimates, $\hat{\mathbf{p}}_{t+1}$, include his noisy order flow observation $\tilde{\mathbf{q}}_t$. He puts a weight (given by $\Sigma_t(\boldsymbol{\alpha} + \boldsymbol{\beta})'[(\boldsymbol{\alpha} + \boldsymbol{\beta})\Sigma_t(\boldsymbol{\alpha} + \boldsymbol{\beta})' + \Sigma_q\Sigma_q']^{-1}$, denoted as \mathbf{W}_t) on this observation in updating his belief of the true price \mathbf{p}_t . This weight \mathbf{W}_t essentially involves the market maker's estimation

variance Σ_t , which itself evolves under equation (4.2.6) through time. Examining equation (4.2.6) shows that the return covariance matrix \mathbf{V} plays an important role in updating the precision of the market maker's forecast of $\hat{\mathbf{p}}_t$.

Since the return covariance matrix \mathbf{V} affects the market maker's estimate $\hat{\mathbf{p}}_t$ through Σ_t , \mathbf{V} also affects the market order flows $\tilde{\mathbf{q}}_t$ (equation (4.2.3)), i.e., the net order flows exhibit some common effects due to the structure of \mathbf{V} . It is therefore important for the market maker to recognize the correlation among the order flows of all stocks. If he considers this correlated order flow information, his price estimation will be more accurate than if he only uses individual order flow information in estimating the prices of the specific stock.

We make further assumptions here to simplify the calculation. We assume that over the time we use the steady state estimation variance instead of the time-varying Σ_t since the Kalman filter converges to the steady state quite quickly. Denote the steady state estimation variance as Σ with $\Sigma = \Sigma_t = \Sigma_{t+1}$, Σ is obtained by solving the Riccati equation (4.2.6) and is the solution of the following equation:

$$\Sigma(\alpha + \beta)'[(\alpha + \beta)\Sigma(\alpha + \beta)' + \Sigma_q\Sigma_q']^{-1}(\alpha + \beta)\Sigma = \mathbf{V} \quad (4.2.7)$$

The Kalman filter updating equation can now be written as:

$$\hat{\mathbf{p}}_{t+1} = \hat{\mathbf{p}}_t + \mathbf{V}\Sigma^{-1}(\alpha + \beta)^{-1}(\alpha\delta_{at} - \beta\delta_{bt} - \mathbf{q}_t) \quad (4.2.8)$$

Furthermore, we can write the true price vector \mathbf{p}_t as

$$\mathbf{p}_t = \hat{\mathbf{p}}_t + \mathbf{M}_t \tilde{\mathbf{e}} \quad (4.2.9)$$

where $\tilde{\mathbf{e}}$ is a $(n \times 1)$ vector, normally distributed with a zero mean vector and an identity covariance matrix and \mathbf{M}_t is a $(n \times n)$ matrix that summarizes the estimation error of $\hat{\mathbf{p}}_t$. It can be shown that $\mathbf{M}_t \mathbf{M}'_t = \Sigma_t$. Given the steady state Σ , we have $\mathbf{M} \mathbf{M}' = \Sigma$. Equation (4.2.9) will be useful in the market maker's optimization problem in the next section.

4.2.3 The market maker's optimization problem

The market maker is assumed to set his optimal bid and ask prices to maximize his expected utility of per period profit. His objective function is given by:

$$\max_{\mathbf{a}_t, \mathbf{b}_t} E[\mathbf{U}(\tilde{\pi}_t)]$$

where $\tilde{\pi}_t$ is his profit at time t and given by

$$\tilde{\pi}_t = \mathbb{I}_t + \mathbb{T}_t = (\mathbf{p}'_{t+1} \mathbf{I}_{t+1} - \mathbf{p}'_t \mathbf{I}_t) + (\mathbf{a}'_t \tilde{\mathbf{q}}_B - \mathbf{b}'_t \tilde{\mathbf{q}}_S) \quad (4.2.10)$$

Equation (4.2.10) is our definition of the market maker's profit at time t . His profit comes from two sources: the incremental value of his inventory (\mathbb{I}_t) and the cash flow (\mathbb{T}_t). The end of the period inventory (\mathbf{I}_{t+1}) is the sum of the beginning inventory (\mathbf{I}_t) plus the net order flow that the market maker obtains during the period ($\tilde{\mathbf{q}}_t$). The incremental value of the inventory is defined as the difference between the

value of the end of the period inventory ($\mathbf{p}'_{t+1} \cdot \mathbf{I}_{t+1}$), and the value of the inventory at the beginning of the period ($\mathbf{p}'_t \cdot \mathbf{I}_t$). The cash flow is simply obtained from the market maker's trading activities with the public buy and sell demands, $\tilde{\mathbf{q}}_B$ and $\tilde{\mathbf{q}}_S$, evaluated at respective ask and bid prices.

We assume that the market maker has a CARA utility function with risk aversion parameter λ . Since $\tilde{\boldsymbol{\pi}}_t$ is multivariate normal, the objective function is equivalent to:

$$\max_{\mathbf{a}_t, \mathbf{b}_t} E[\tilde{\boldsymbol{\pi}}_t] - \frac{\lambda}{2} \text{var}[\tilde{\boldsymbol{\pi}}_t] \quad (4.2.11)$$

That is, the market maker's objective is to maximize his expected profit ($E[\tilde{\boldsymbol{\pi}}_t]$), taking into account of the risk of obtaining this profit ($\text{var}[\tilde{\boldsymbol{\pi}}_t]$), which is evaluated according to his risk aversion parameter λ .

Note that given unobservable \mathbf{p}_t , the market maker can not directly observe his profit from equation (4.2.10). He however is still able to maximize the profit $\tilde{\boldsymbol{\pi}}_t$ since the profit is normally distributed and given his CARA utility function, he only needs to compute the mean and the variance of the profit, which the market maker has sufficient knowledge to calculate.

4.2.4 The results

Our main result is summarized in the following theorem.

Theorem 4.1. *The market maker's optimal bid (\mathbf{b}_t) and ask (\mathbf{a}_t) prices for multiple stocks with known covariance structure are given by:*

$$\begin{aligned}\mathbf{b}_t &= \hat{\mathbf{p}}_t - \boldsymbol{\delta}_{b_t} \\ \mathbf{a}_t &= \hat{\mathbf{p}}_t + \boldsymbol{\delta}_{a_t}\end{aligned}$$

where $\hat{\mathbf{p}}_t$ is the market maker's estimate of the unknown true price \mathbf{p}_t at the beginning of time period t . Let $\mathbf{x} = [\delta_{at} \ \delta_{bt}]'$ and it is given by

$$\mathbf{x}_t = [\mathbf{J} - \lambda(\mathbf{A} + \mathbf{F})]^{-1}(-\mathbf{b} + \frac{\lambda}{2}\mathbf{EI}_t) \quad (4.2.12)$$

where \mathbf{A} , \mathbf{E} , \mathbf{F} , \mathbf{J} , and \mathbf{b} are vectors and matrices obtained from the Kalman filter and they depend on the parameters of the model and in particular, the return covariance matrix \mathbf{V} . Specifically, we have $\mathbf{b} = [\mathbf{Q}_0 \ \mathbf{Q}_0]', \mathbf{J} = \begin{bmatrix} -2\alpha & 0 \\ 0 & -2\beta \end{bmatrix}, \mathbf{E} = \begin{bmatrix} \alpha' \mathbf{V} \\ -\beta' \mathbf{V} \end{bmatrix},$

$$\mathbf{A} = \begin{bmatrix} \alpha' \mathbf{M} \mathbf{M}' \alpha & -\alpha' \mathbf{M} \mathbf{M}' \beta \\ -\beta' \mathbf{M} \mathbf{M}' \alpha & \beta' \mathbf{M} \mathbf{M}' \beta \end{bmatrix},$$

and

$$\mathbf{F} = \begin{bmatrix} \alpha' \mathbf{V} \alpha + \Sigma_B \Sigma'_B & -\alpha' \mathbf{V} \beta \\ -\beta' \mathbf{V} \alpha & \beta' \mathbf{V} \beta + \Sigma_S \Sigma'_S \end{bmatrix}.$$

Theorem 4.1 gives the optimal bid and ask prices that the market maker sets at each time t . The return covariance matrix \mathbf{V} of the system transition equation (4.2.4) gives the market maker extra information, which is embedded in the Kalman filter updating equation from $\hat{\mathbf{p}}_t$ to $\hat{\mathbf{p}}_{t+1}$ and in the steady state estimation variance Σ . Matrix \mathbf{V} affects the prices directly through matrices \mathbf{E} and \mathbf{F} . It also affects the market maker's estimation error \mathbf{M} , which enters into the pricing strategy through matrix \mathbf{A} . Note that \mathbf{A} is time invariant under the steady state Σ . In fact, under the steady state, the optimal prices \mathbf{a}_t and \mathbf{b}_t only change with the market maker's estimated prices $\hat{\mathbf{p}}_t$ and his inventory \mathbf{I}_t .

The market maker's optimal bid and ask prices are linear in his beginning inventory level \mathbf{I}_t . Since the inventory carries risk, the risk averse market maker in this

model tries to keep minimum levels of inventory positions. Consistent with the literature on inventory models, he influences the market demands by adjusting his bid and ask prices.

4.3 The benefit of knowing return covariance

We are interested in how much the market maker can benefit from considering the correlated order flow information when he updates his estimates of the unknown price processes for all stocks. In this section, we show numerically how the market maker's knowledge of return covariance improves his overall market making profitability.

4.3.1 Parameter values and simulation design

We follow the similar assumptions of parameter values as in Chapter 3 except now these parameters are vectors and matrices.⁵ First we assume the number of stocks that the market maker makes markets for is n . Let \mathbf{q}_0 be a $(n \times 1)$ vector of 4's so that the bid-ask spread for each stock is around 1 to 2. $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $(n \times n)$ identity matrices, and $\boldsymbol{\Sigma}_B$ and $\boldsymbol{\Sigma}_S$ are also $(n \times n)$ diagonal matrices with diagonal entries of 2.5.

For all stocks, the initial values and the market maker's initial estimates of the true prices are assumed to be the same and equal to 100's, so \mathbf{p}_0 and $\hat{\mathbf{p}}_0$ are both $(n \times 1)$ vectors of 100's. We assume that the market maker does not have any initial

⁵The assumptions of parameter values here are intended to be consistent with those in the single stock market making model in Chapter 3.

inventories for all stocks, so \mathbf{I}_0 is a $(n \times 1)$ vector of 0's.

The variance of the market maker's forecast, Σ_t , converges to the steady state quite quickly, only after a few time steps. So we use the steady state Σ for all calculations. The matrix \mathbf{C} in equation (4.2.4) is calculated from our assumption of the return covariance matrix \mathbf{V} . Appendix B describes a method to simulate matrix \mathbf{V} .

To study the effects of the market maker's knowledge of the return covariance structure on his market making profitability, we perform the calculation under two different cases. In Case 1, although the market maker observes the order flows of all stocks, he does not update his estimates using the correlated order flow information. He simply treats the order flow information of each individual stock on its own and does not consider any correlation among the order flows of different stocks. In this case, the covariance structure of \mathbf{V} in the Kalman filter updating equation (4.2.8) and Riccati equation (4.2.6) is diagonal. In Case 2, the market maker recognizes that, for a specific stock, the order flow information of other stocks provides useful information for him to update his estimates of the value of that stock. In this case, the return covariance matrix \mathbf{V} has a full structure as we have defined and the market maker uses this matrix \mathbf{V} in his price estimation.

We note that the market maker in this model also has a problem of portfolio maximization in which the structure of \mathbf{V} is an important factor. When we do the

comparison, however, we are mostly interested in how the structure of \mathbf{V} affects the market maker's price estimation. In Case 1, the market maker only uses the diagonal entries of matrix \mathbf{V} in price estimation while in Case 2, he uses the full structure of matrix \mathbf{V} .

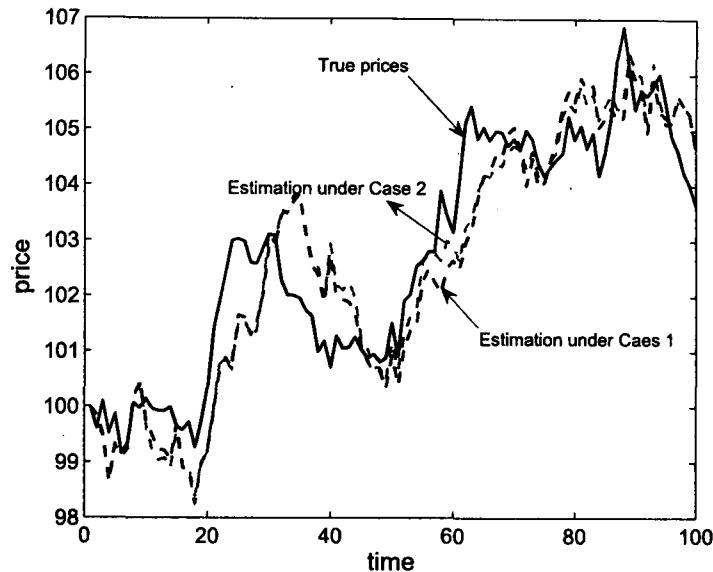
Our calculation follows the following three steps:

- (1) For each simulation, we assume there are 100 time steps, i.e., the market maker has 100 chances to set bid and ask prices. At each time t , we calculate the market maker's estimated prices ($\hat{\mathbf{p}}_t$) as well as the bid and ask prices he sets for each stock. Given these prices, we calculate his cumulative profits as given by equation (4.2.10) over these 100 time steps.
- (2) We repeat the simulation described in step (1) for 1000 times and calculate the means and standard deviations for these cumulative profits.
- (3) Since the market maker is risk averse, we repeat the calculation in step (1) - (2) for a range of risk aversion parameters to examine the effects of risk aversion on the market maker's cumulative profits.

4.3.2 The simulation results

Figure 4.1 gives a time series plot of one stock price process and the market maker's estimates under Cases 1 and Case 2. The solid line corresponds to the true prices. The dashed line under Case 2 is for the case when the market maker uses the correlated order flow information in his price estimation. When the market maker only uses the

Figure 4.1: The time series of prices. This figure shows the time series plot of the true price (the solid line) and the market maker's estimates under Case 1 and Case 2 (dashed lines as indicated).

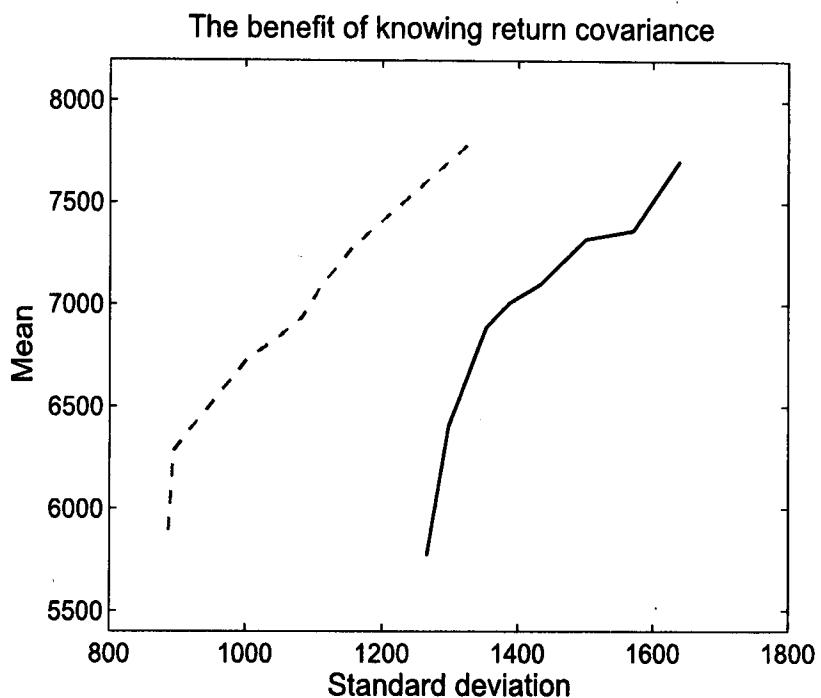


order flow information of an individual stock to estimate the prices of that stock, his estimates correspond to the dashed line under Case 1. Generally, the market maker's estimates are more close to the true prices under Case 2 than those under Case 1. Therefore when the market maker considers the correlated order flow information in his price estimation for every stock, he obtains better estimates.

Figure 4.2 shows the result of the means and the standard deviations of market maker's cumulative profits for 100 time steps, with increasing risk aversion parameters. Here we assume that the market maker makes markets for 10 stocks, so $n = 10$. The solid line is the result for Case 1 and the dashed line is the result for Case 2.

First we observe that, with increasing risk aversion parameters, both the means

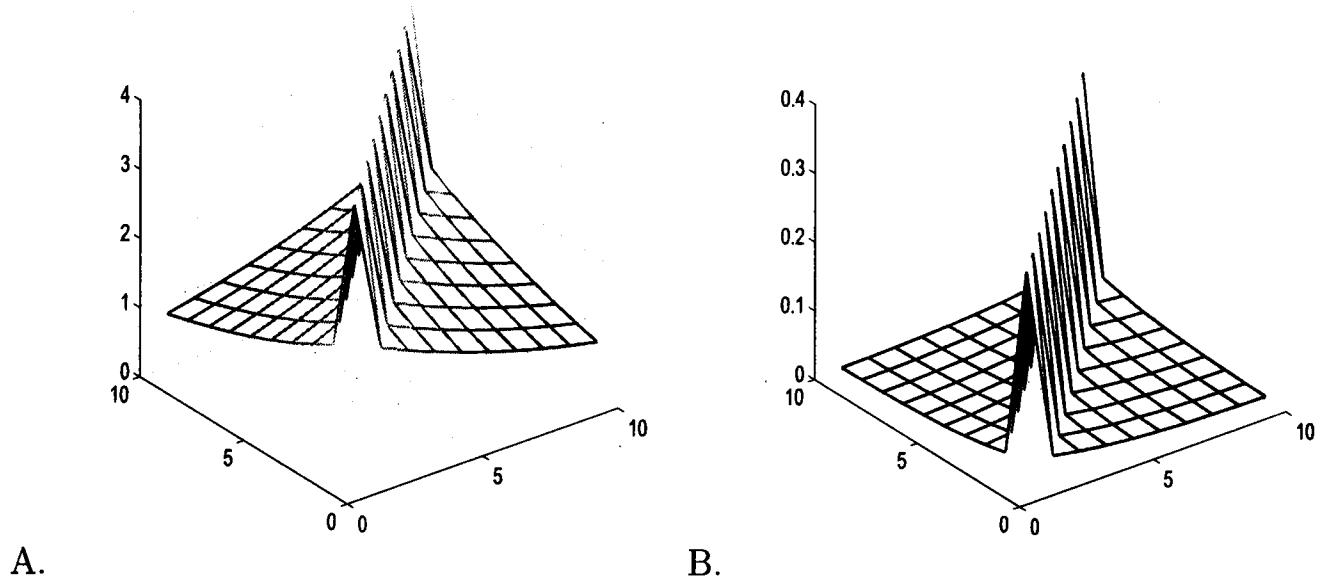
Figure 4.2: The benefit of knowing return covariance. This figure shows the benefit of the market maker's knowledge of return covariance structure in price estimation. The solid line is the means and standard deviations of market maker's cumulative profits when he only uses the order flow information of an individual stock to estimate the prices of that stock (Case 1). The dashed line is the result when he uses the correlated order flow information in his price updating (Case 2). The market maker's knowledge of the return covariance increases the means of cumulative profits and reduces the standard deviations of these profits.



and the standard deviations of profits for the two cases decrease monotonically. It is not surprising since the more risk averse the market maker is, the more he cares about the risk of his profits. Given the market maker's mean-variance utility function (equation (4.2.11)), increasing risk aversion parameters reduce his expected utility, so both the means and the risks of the profits are smaller.

It is interesting to compare these two lines. When the market maker considers correlated order flow information in his price estimation, there are small increases

Figure 4.3: The contribution of correlated order flow information in price estimation Panel A shows the 3-D plot of the return covariance matrix \mathbf{V} . Panel B shows the weight \mathbf{W}_t that the market maker puts on his price updating equation when using correlated order flow information.



in the average levels of these cumulative profits, but relatively big reductions in the standard deviations of these cumulative profits. Intuitively, in this case, his estimates are more accurate and close to the true prices. He cannot achieve much higher expected profits since both the incremental value of his inventory and the cash flow of his profits do not change much. He can however estimate his inventory positions more accurately, which contributes to the relatively big reductions in the risks of his cumulative profits. So by considering the correlated order flow information in his price estimation, the market maker improves his market making profitability.

Examining the numbers in Figure 4.2 shows that the average of the cumulative profits of 10 stocks over 100 time periods are around 6000 to 8000, with the standard

deviations around 1000 to 2000. Considering the number of stocks (10), the contribution of each stock in reducing the risk between the two cases is not that great. In fact, in the price updating equation (4.2.5), the weight (\mathbf{W}_t) that the market maker puts on his observation of the order flow $\tilde{\mathbf{q}}_t$ (given by $\Sigma_t(\alpha+\beta)'[(\alpha+\beta)\Sigma_t(\alpha+\beta)'+\Sigma_{\mathbf{q}}\Sigma_{\mathbf{q}}']^{-1}$) has strong diagonal terms. Figure 4.3 shows the contribution of the correlated order flow information in the market maker's price estimation. Panel A is the 3-D plot of the return covariance matrix \mathbf{V} . Panel B is the plot of the weight \mathbf{W}_t that the market maker puts on his price estimation when using correlated order flow information. Clearly, similar to the structure of matrix \mathbf{V} , the weight also has strong diagonal terms. The off-diagonal terms are relatively small and this is shown by the clear discrete jump from the diagonal terms. It follows that although the market maker considers the correlated order flow information in his price estimation, the improvement on his market making profitability is limited.

4.4 Conclusion

In this chapter we consider a model of the market making of multiple stocks by a monopolistic market maker. The market maker does not know the true prices for each individual stock but knows the return covariance structure of these stocks. His knowledge of the return covariance affects his price estimation, which in turn affects the order flows. When the market maker does not isolate the individual stock's order flow information but integrates the correlated order flow information of all stocks, his

knowledge of the return covariance improves his price estimation.

The market maker in this model faces both the problems of managing his inventory and trading with informed traders. By considering correlated order flow information, the market maker achieves better estimation of the true prices. This allows him to obtain better estimates of his future inventory positions, which helps to mitigate the inventory management problem. Better estimation of the true price processes also reduces the information asymmetry. The market maker's knowledge of the return covariance structure therefore increases his expected cumulative profits and reduces the variance of these profits, as shown by our numerical results.

Our model also leads to liquidity commonality, an important topic on its own. Liquidity commonality arises from the market demand functions which depend on the market maker's estimation of the true price processes. Since the market maker's knowledge of the return covariance structure affects his price estimation, it in turn affects the market demands. The commonality in market demands further improves the market maker's price estimation when he integrates full order flow information. Consistent with the empirical findings of Coughenour and Saad (2004), this chapter provides another support to the claim that common market makers are one reason for liquidity commonality.

This chapter provides insights into the market making of state contingent claims such as options. In essence, options are a parameterized family of substitutable

securities. In particular, an option market maker's information is enriched by the correlation among option contracts. We enjoy some success in this model by showing that the market maker's knowledge of the correlation structure improves his price estimation. This chapter therefore offers a useful structure and indicates a promising direction for future study on option market making.

Chapter 5

The role of market makers' hedging on the spreads in options and stock markets

5.1 Introduction

Market making in derivative securities raises important considerations that are absent in market making of cash securities. By virtue of the replicating argument, which establishes a no arbitrage relationship between the value of the derivative and the value of its underlying asset, liquidity in the underlying market must be linked to the liquidity in the derivative asset. Cho and Engle (1999), Kaul, Nimalendran, and Zhang (2004) and de Fontnouvelle, Fishe, and Harris (2003) have all found this to be the case in option markets.

One aspect that has been ignored in the academic literature but is part of the daily routine of market makers in derivative assets is hedging. An option market maker (OMM), for example, as the implicit counter-party in a transaction involving

the option, must decide whether a delta hedge needs to be set up to reduce the risk of an unfavorable price movement. Since hedging involves trading, how easy and cheap it is for the market maker to carry out his hedging trades must impact the bid-ask prices posted by the OMM. Also, if the option market maker quotes prices for a range of options, cumulative hedging combined with the no arbitrage pricing relationships must affect the relative spreads in different option contracts.

In this chapter we analyze these issues in a model where some traders have the advantage of possessing private information and trade either in the stock or in the option market. In doing so, informed traders face market makers who in the course of their activities explicitly take into consideration the effects of hedging. We show that hedging by market makers has a significant influence on the magnitude of the bid-ask prices in the various markets - option and the underlying stock -, and because of that drastically affect the actions of superiorly informed traders. The result that an important source of liquidity in the option market is strongly linked to the liquidity in the underlying cash market through the hedging of market makers is acknowledged by Cho and Engle (1999), who aptly put it: "The traditional focus of most liquidity-building activity at derivatives exchanges has been directed at locals: the traditional market makers and the source of capital in derivatives markets. However, there is another source of liquidity for derivatives markets: the already existing liquidity of other deeper markets that can be tapped through hedging." We corroborate their

conjecture by finding that hedging by market makers is an important factor in explaining the provision of liquidity in the various markets. In addition, we show that because hedging by market makers is informative, hedging contributes decisively to explain why all spreads, but especially those in options, are in reality so high.

Much academic literature has suggested that informed trading takes place in the option market. Theoretical works by Biais and Hillion (1994), Easley, O'Hara, and Srinivas (1998), and John, Koticha, Narayanan, and Subrahmanyam (2003) suggest that the amount of informed trading in the option market depends on the relative liquidity in the option and its underlying markets. More recently, there have been several empirical studies that uncovered findings related to the hedging activities of option market makers. For example, Cho and Engle (1999) argue that if an option can be perfectly hedged, the option spreads arise from the illiquidity of the underlying market, rather than from inventory risk or informed trading in the option market itself. The underlying spreads and hedging parameters, such as delta and gamma from the option's pricing model, are expected to affect the option spreads. We corroborate the conjecture of Cho and Engle. Kaul, Nimalendran, and Zhang (2004) find that the underlying stock's spread has an important impact on the option spreads and that this is due to the hedging activities of OMMs. We show that this adverse selection that results from private information is a component that explains an important fraction of the spreads. Finally, our results are consistent with the work of de Fontnouvelle,

Fishe, and Harris (2003), who find that the option's delta is significantly related to the size of the option spreads, suggesting that the cost of hedging plays a role in setting the spreads.

Despite the mounting evidence that the option market maker's hedging trades affect both stock and option spreads, there is no theoretical work that explicitly models how option market makers' hedging strategies affect the actions of informed traders, and the costs of providing liquidity. Our model builds on the sequential trading framework of Easley, O'Hara and Srinivas (1998) and John, Koticha, Narayanan and Subrahmanyam (2003)¹. In these sequential trading models, informed and uninformed traders interact through risk neutral competitive market makers, that stand ready to trade. What distinguishes our model from the previous literature is that the option market maker actively hedges his option position. We consider two cases: hedging in the underlying stock and hedging with another option written on the same underlying. The OMM forms this hedge ratio based on his perception of the expected stock and option payoffs after observing the option's order flow. In equilibrium, competition forces the OMM to zero expected profits both from market making and from trading for hedging reasons.

¹Similar to our work, a number of authors have studied option market making with a sequential trading approach. For example, Biais and Hillion (1994) analyze the pricing of state contingent claims in option and stock markets and show that the option mitigates the market breakdown problem created by the combination of market incompleteness and asymmetric information; Easley, O'Hara and Srinivas (1998) develop and test a model of the informational role of the signed options' transactions volume in predicting future stock prices; and John, Koticha, Narayanan and Subrahmanyam (2003) analyze the impact of option trading and margin rules on the behavior of informed traders and on the microstructure of stock and option markets.

This chapter contributes to the literature in several ways. First, we add to the surprisingly sparse theoretical literature² on option market making by adding delta hedging by OMMs. If an option's payoff can be replicated by continuously rebalancing a portfolio composed of the underlying stock and the risk free asset (Black and Scholes (1973)), an option is a redundant asset and therefore its price conveys no additional information. Since market frictions exist and in reality continuous replication is not feasible, replication is not redundant and the option price conveys valuable information. Theoretical studies on option market making try to prove this point and to provide insights about the informational link between the various securities markets.

Using the case where the option market maker does not hedge as a benchmark, we find that the option market maker's hedging trade contains relevant information. When the informational content is high, the option market maker's hedging trade constitutes an additional important source of information trading in the underlying market. The increased threat of informed trading from the option market maker's hedging leads the stock market maker to respond by setting wider spreads for the stock. This, in turn, widens the spreads in the option, simply because of the additional transaction costs that hedging imposes. Interestingly, we find the benefits of imitating an informed trade through hedging do not compensate the additional costs of hedging,

²See, for example, Back (1993), Brennan and Cao (1996), Biais and Hillion (1994), Easley, O'Hara and Srinivas (1998), and John, Koticha, Narayanan and Subrahmanyam (2003).

thus the result of larger option spreads when the OMM hedges. That is, hedging by OMMs raise the costs of trading both in stocks and in options.

The model in this chapter also shows what OMM hedging does to the distribution of informed trading across options that differ in their strike prices. In general, spreads in the underlying stock go down when a second option is introduced. Also, compared to the case of a single option, the spreads for each option go down when there are several options trading simultaneously. Why? Because when informed traders have the choice to trade in several options, there is less informational content in the hedging trades of each option market maker. If the delta hedge is carried out in the underlying stock, this eases the information threat to the stock and allows the stock market maker to narrow the spreads. Moreover, since options have convex payoff structures, the effective combined hedge ratio is smaller when there are several options. This helps to reduce the option market maker's hedging cost and contributes to narrow the option spreads.

We also consider the alternative case of OMM hedging using options offered by other OMMs. When this is the case, we find that, in comparison to the case of hedging using the underlying stock, the spreads for both the stock and the option that has lower exercise price are further reduced, while the spread of the option with the higher exercise price widens. This finding helps to understand why option market makers often prefer hedges that use other options.

Our model is also related to the literature on market fragmentation (see, for example, Pagano (1989) for a model without information asymmetry and Chowdhry and Nanda (1991) for a model in the presence of asymmetric information). We basically study the question of price setting in two related markets in the presence of asymmetric information. Although we explicitly model the option market and its underlying market, the idea can be extended to the cases where two closely related assets are traded in different and imperfectly integrated markets. In fact, in our model, a trader can get a delta exposure by trading in the primary market (the underlying market) or by buying a call in the option market with the OMM hedging in the underlying market. What makes our model interesting is that we explicitly analyze the OMM's hedging activities which link the liquidities of these two imperfectly integrated markets.

Finally, the issue of arbitrage arises if there are multiple trading venues. Interestingly, Kumar and Seppi (1994) show that while arbitrage would ultimately draw futures and its underlying markets together, its immediate impact is to increase the bid-ask spread in both markets, thus reducing the liquidity in each market, a result similar to ours.

The rest of the chapter is organized as follows. Section 5.2 presents the model. Section 5.3 discusses the benchmark results when an OMM does not hedge. Numerical solutions are discussed in Section 5.4. Section 5.5 extends the basic model to consider

a market for several options when hedging is done in the underlying market. In Section 5.6, the OMM hedges with options offered by other OMMs. Empirical implications are discussed in Section 5.7. Section 5.8 concludes the chapter.

5.2 The model

5.2.1 Assumptions

Consider an economy with two financial markets: an option market where an European call option is traded, and a stock market where its underlying asset is traded. Later we will consider the case of several options traded on the same underlying asset. The state of nature is denoted by θ , $\theta \in \{H, L\}$ with equal probability. The true value of the underlying asset is \tilde{v} , with $\tilde{v} \in \{v_H, v_L\}$ corresponding to the two states of nature. The exercise price of the call option is $K \in [v_L, v_H]$. There are two types of traders: informed traders and uninformed traders. Informed traders comprise α % of the total trading population and the rest $(1 - \alpha)\%$ are uninformed traders. Uninformed traders trade in both stock and option markets for unspecified, exogenous reasons, such as portfolio rebalancing, hedging, etc. Specifically, $\beta\%$ of them trade stocks and the rest $(1 - \beta)\%$ trade options. Informed traders privately observe an *identical* informative signal of the final value of the underlying asset. They randomize their trades in stock and option markets to exploit their information advantage. The signal they receive is denoted by S where $S = G(B)$ indicates a good (bad) signal. The precision of the signal is measured by μ , the probability that the signal is correct

about the state of nature, i.e., $Pr(S = G|\theta = H) = Pr(S = B|\theta = L) = \mu$ and $Pr(S = B|\theta = H) = Pr(S = G|\theta = L) = 1 - \mu$. We assume $\mu > 0.5$, so that the signal is informative.

There is one representative, competitive market maker in the underlying market (SMM) and one representative, competitive market maker in the option market (OMM). Each market maker observes the order flow in his own market. They do not know the realization of the underlying asset \tilde{v} but use Bayes rule to update their beliefs of \tilde{v} . Each market maker randomly selects a single order from the pool of orders submitted to him.³ All agents are risk neutral and there is no discounting.

There are four time points ($t = 0, 1, 2, 3$). At time 0, informed traders observe a private signal. At time 1, market makers post prices for traders to buy or sell one unit of stock or call option. Informed traders submit orders according to the signal they observe. If the signal is good ($S = G$), they buy either one stock, with probability ν , or one call, with probability $(1 - \nu)$. If the signal is bad ($S = B$), they sell either one stock, with probability ω , or one call, with probability $(1 - \omega)$. Both ν and ω are endogenous variables and will be determined in equilibrium. It is reasonable to assume that uninformed traders buy and sell securities with equal probability in each

³It is, therefore, possible that an order submitted by traders might not be filled in this round of trading. In particular, if the OMM chooses to hedge in the spot market, the chance that his order is selected is the same as other traders' orders. The OMM's order does not have any preference over other orders. Therefore there is a finite probability that the OMM's hedging trade may not be filled. However since the OMM calculates his option prices based on expected zero profit condition (Section 5.2.4), this should not affect the equilibrium prices he sets.

market.

At time 2, the OMM observes the state of nature. What distinguishes our model from the previous literature, in particular the work of Easley, O'Hara and Srinivas (1998) and John et al. (2003), is that, at time 2, the OMM trades in the underlying market at the stock's bid and ask prices set by the SMM at time 1 to hedge his option position.⁴ The OMM's hedging position depends on the hedge ratio, which is based on his observation of the option order flow (to be discussed in section 5.2.4).⁵ We note that in this sequential trading model, the OMM's hedging trade might not be executed by the stock market maker. However, the OMM has taken this possibility into account when he sets his optimal options prices. That is, the optimal options prices correctly reflect the expected hedging costs incurred by the OMM. It is also worth noting that although the OMM knows the state of nature before the SMM, he is only allowed to do a hedging trade at time 2; he could not trade more in the underlying market.

⁴In section 5.6 we analyze hedging with another option.

⁵In this model, traders are only able to submit an order of unit size, in both stock and option markets. The OMM, however, will submit orders *exactly* according to his hedge ratio. The OMM's hedging trade could be easily spotted by the SMM since an order of a size different from one could only be the OMM's hedging trade.

One question, therefore, that needs to be addressed is whether the SMM charges different stock spreads to the OMM's hedging trade. If the SMM believes that the OMM is uninformed and OMM's hedging trade contains no information, the SMM would charge an arbitrarily small spread, say ϵ to the OMM's hedging trade. It is because the competition in stock market making implies that each SMM is always willing to trade with uninformed traders and earn the spread. In the limit, this ϵ could be very small and approach zero. However, if the SMM believes that the OMM's hedging trade contains information, he would charge the same spreads as to the orders submitted by informed or uninformed traders. As we show in Section 5.4, the OMM's hedging trade does contain information, which constitutes an additional source of informed trading threat. We therefore argue that, in this model, the SMM quotes the same spreads to every stock order.

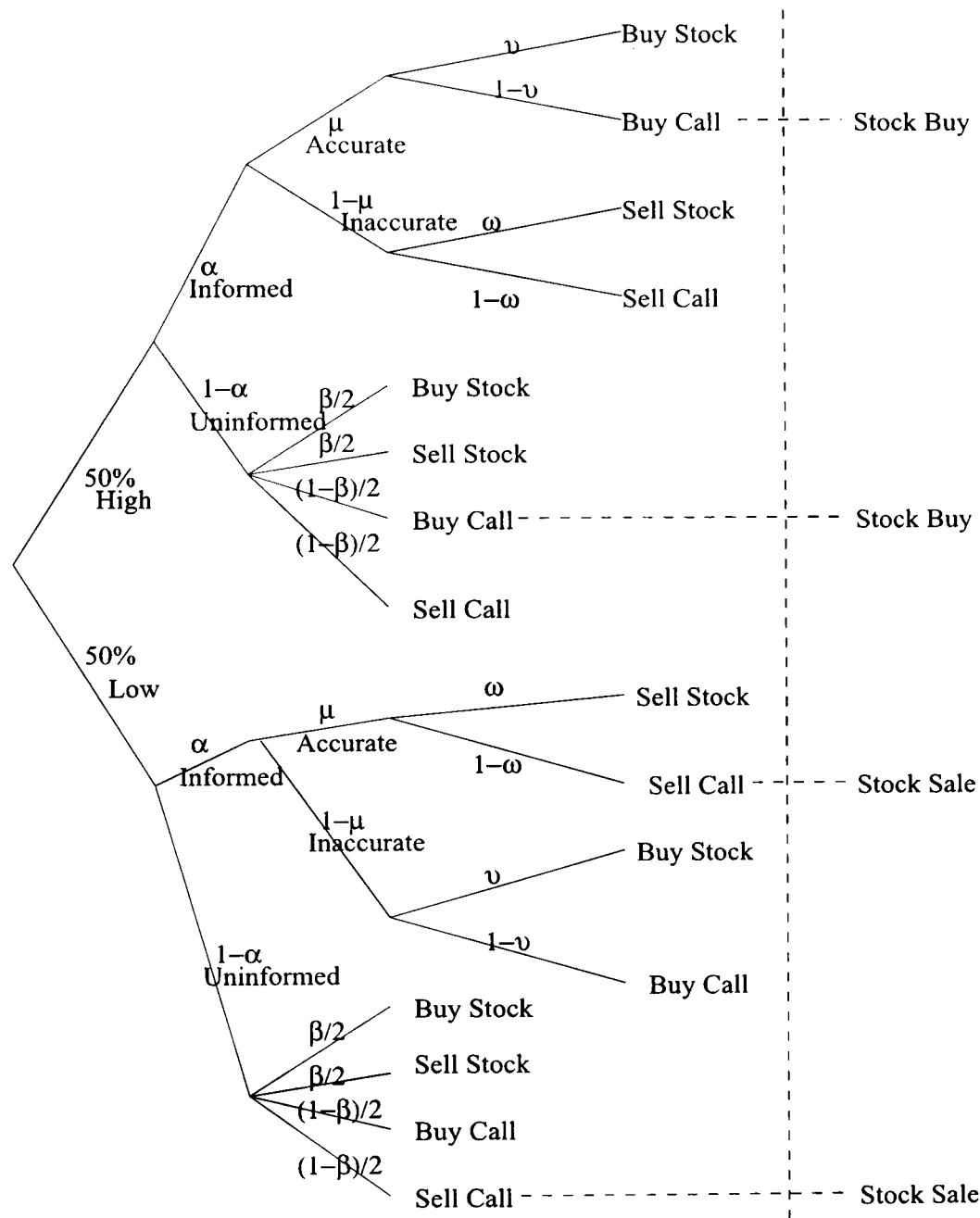
Finally, at time 3, all other market participants observe the state. The call option expires at time 3 when the option's payoff is realized. If the OMM hedges at time 2, his hedging position is liquidated at time 3.

We assume that there is competition in market making for both the stock and the options. Therefore, at time 1, the SMM makes zero expected profits in equilibrium in his stock market making. Also, at time 1, the OMM makes zero expected profits when he takes into account his option market making and his trading for hedging reasons (at time 2). Figure 5.1 shows the information structure of the model. The probabilities of informed and uninformed trading in both stock and option markets are shown to the left of the vertical dashed line. This part of the figure corresponds to the scenario where the OMM does not hedge in the underlying market. Special in our model, are the OMM's hedging activities, shown to the right of the vertical dashed line. These nodes correspond to the cases where the OMM hedges in the underlying market.

5.2.2 Bid and ask prices in the stock market

We first derive the bid and ask prices when the SMM makes zero expected profits in making a competitive market for the stock. The bid and ask prices are equal to the expected values of the underlying stock, conditional on the SMM observed order flow. Denote $S_{b(a)}$ as the stock bid (ask) price, we have $S_b = E[\tilde{v}|\text{Stock Sale}]$ and $S_a = E[\tilde{v}|\text{Stock Buy}]$, respectively. Expanding the expectations and taking the two

Figure 5.1: Information Structure of the model. The information tree shows the probabilities of informed and uninformed trading in both stock and option markets. The probabilities shown to the left of the vertical dashed line correspond to the case of OMM not hedging. The probabilities from the OMM's hedging trades are shown to the right of the vertical dashed line.



possible states into account, we have:

$$S_b = E[\tilde{v}|\text{Stock Sale}] = v_H Pr(\theta = H|\text{Stock Sale}) + v_L Pr(\theta = L|\text{Stock Sale})$$

$$S_a = E[\tilde{v}|\text{Stock Buy}] = v_H Pr(\theta = H|\text{Stock Buy}) + v_L Pr(\theta = L|\text{Stock Buy})$$

Note that the OMM's hedging trade affects the above conditional probabilities and hence the stock bid and ask prices.

Let's consider the bid price here (the ask price can be obtained in a similar fashion).

Using Bayes rule, we have:

$$Pr(\theta = L|\text{Stock Sale}) = \frac{Pr(\theta = L)Pr(\text{Stock Sale}|\theta = L)}{Pr(\theta = L)Pr(\text{Stock Sale}|\theta = L) + Pr(\theta = H)Pr(\text{Stock Sale}|\theta = H)}$$

We first look at $Pr(\text{Stock Sale}|\theta = L)$, the probability of a stock sale, given that the state of nature is low. This probability may come from three possible sources: an informed stock sale, an uninformed stock sale, and an OMM's hedging sale. From Figure 5.1, when the state is low, the probability of an informed sale is $\alpha\mu\omega$, and an uninformed sale is $(1 - \alpha)\beta_2^1$. The OMM's hedging sale depends on how he perceives the moneyness of his position in the option, given the current state. When the state is low ($\theta = L$), his long call position is always out of the money and therefore needs to be hedged. The probability that the OMM's sells in hedging is equivalent to the probability that the OMM has a long call position.⁶ This probability equals the sum

⁶Note that we only say that the OMM would sell stocks in the underlying market if he knows that the state is low. We do not say anything about how much he would sell. In fact, the OMM would hedge according to his optimal hedge ratio, which is less than 1 (to be discussed in Section 5.2.4).

of the probabilities of an informed call sale ($\alpha\mu(1 - \omega)$) and an uninformed call sale ($((1 - \alpha)(1 - \beta)\frac{1}{2})$). To summarize, we have:

$$\begin{aligned}
 Pr(\text{Stock Sale}|\theta = L) &= Pr(\text{Informed Stock Sale}|\theta = L) \\
 &\quad + Pr(\text{Uninformed Stock Sale}|\theta = L) \\
 &\quad + Pr(\text{OMM Hedge Sale}|\theta = L) \\
 &= \alpha\mu\omega + (1 - \alpha)\beta\frac{1}{2} + Pr(\text{Informed Call Sale}|\theta = L) \\
 &\quad + Pr(\text{Uninformed Call Sale}|\theta = L) \\
 &= \alpha\mu\omega + (1 - \alpha)\beta\frac{1}{2} + \alpha\mu(1 - \omega) + (1 - \alpha)(1 - \beta)\frac{1}{2} \\
 &= \alpha\mu + (1 - \alpha)\frac{1}{2} \tag{5.2.1}
 \end{aligned}$$

When the OMM observes a sale and the state is high ($\theta = H$), the OMM's long position in the call is in the money and he does not need to hedge. Hence, the probability in a sale that the OMM would hedge when the state is high is zero. We have:

$$\begin{aligned}
 Pr(\text{Stock Sale}|\theta = H) &= Pr(\text{Informed Stock Sale}|\theta = H) \\
 &\quad + Pr(\text{Uninformed Stock Sale}|\theta = H) \\
 &\quad + Pr(\text{OMM Hedge Sale}|\theta = H) \\
 &= \alpha(1 - \mu)\omega + (1 - \alpha)\beta\frac{1}{2} + 0 \\
 &= \alpha(1 - \mu)\omega + (1 - \alpha)\beta\frac{1}{2} \tag{5.2.2}
 \end{aligned}$$

Since the OMM only hedges when he perceives that he might lose money in his position in the option, the probabilities that he would hedge in different states are therefore different. This difference directly impacts the stock bid and ask prices.

From equations (5.2.1) and (5.2.2), we obtain

$$\Pr(\theta = L | \text{Stock Sale}) = \frac{\alpha\mu + (1 - \alpha)\frac{1}{2}}{\alpha\mu + \alpha(1 - \mu)\omega + (1 - \alpha)(1 + \beta)\frac{1}{2}}$$

$$\Pr(\theta = H | \text{Stock Sale}) = \frac{\alpha(1 - \mu)\omega + (1 - \alpha)\beta\frac{1}{2}}{\alpha\mu + \alpha(1 - \mu)\omega + (1 - \alpha)(1 + \beta)\frac{1}{2}}$$

The following proposition summarizes the bid and ask prices in the stock market when the OMM hedges.

Proposition 5.1. *When an OMM hedges in the underlying market, the stock's bid and ask prices are given by:*

$$S_b = \frac{v_H[\alpha(1 - \mu)\omega + (1 - \alpha)\beta\frac{1}{2}] + v_L[\alpha\mu + (1 - \alpha)\frac{1}{2}]}{\alpha\mu + \alpha(1 - \mu)\omega + (1 - \alpha)(1 + \beta)\frac{1}{2}} \quad (5.2.3)$$

$$S_a = \frac{v_H[\alpha\mu + (1 - \alpha)\frac{1}{2}] + v_L[\alpha(1 - \mu)\nu + (1 - \alpha)\beta\frac{1}{2}]}{\alpha\mu + \alpha(1 - \mu)\nu + (1 - \alpha)(1 + \beta)\frac{1}{2}} \quad (5.2.4)$$

5.2.3 The informed traders' trading strategy

Informed traders attempt to exploit their information advantage by trading either in the stock, in the option, or in both markets. If they concentrate on trading in one market, they will be more likely identified by market makers in that market. Market makers would then set the widest possible spreads to protect themselves against trading by informed traders. Therefore, we conjecture that informed traders choose a mixed strategy by randomizing their trades in both stock and option markets, while

the pure strategies of trading in a single market are also feasible. Informed traders will mix their trades in two markets to maximize their total expected profit. In equilibrium, they are indifferent between the two pure strategies of trading in either market that they mix.

Recall that if informed traders receive a good signal, they buy either a stock with probability ν , or a call option with probability $(1 - \nu)$; if they receive a bad signal, they sell either a stock with probability ω , or a call option with probability $(1 - \omega)$. Their expected profit (π) from the two strategies of trading only in the stock or the option market, conditional on receiving a good or bad signal, is given by

$$\pi(S = G) = \begin{cases} E[\tilde{v}|S = G] - S_a & : \text{If buying a stock} \\ E[(\tilde{v} - K)^+|S = G] - C_a & : \text{If buying a call} \end{cases}$$

and

$$\pi(S = B) = \begin{cases} S_b - E[\tilde{v}|S = B] & : \text{If selling a stock} \\ C_b - E[(\tilde{v} - K)^+|S = B] & : \text{If selling a call} \end{cases}$$

where $C_{b(a)}$ is the bid (ask) price of the call option.

Expanding the conditional expectations, we have $E[\tilde{v}|S = B] = v_H(1 - \mu) + v_L\mu$, $E[\tilde{v}|S = G] = v_H\mu + (1 - \mu)v_L$ and $E[(\tilde{v} - K)^+|S = B] = (v_H - K)(1 - \mu)$, $E[(\tilde{v} - K)^+|S = G] = (v_H - K)\mu$. The indifference condition of trading in either the stock or the option market yields that

$$v_H\mu + (1 - \mu)v_L - S_a = (v_H - K)\mu - C_a \quad (5.2.5)$$

$$\underbrace{S_b - (v_H(1 - \mu) + v_L\mu)}_{\text{Trading Stocks}} = \underbrace{C_b - (v_H - K)(1 - \mu)}_{\text{Trading Options}} \quad (5.2.6)$$

The next proposition states the bid and ask prices in the option market when the OMM hedges in the underlying market.

Proposition 5.2. *From equations (5.2.5) and (5.2.6), we obtain the call option's bid and ask prices:*

$$C_b = S_b - v_L \mu - K(1 - \mu) \quad (5.2.7)$$

$$C_a = S_a - K\mu - v_L(1 - \mu) \quad (5.2.8)$$

Equations (5.2.7) and (5.2.8) also imply that

$$C_a - C_b = (S_a - S_b) - (K - v_L)(2\mu - 1) \quad (5.2.9)$$

Since $\mu > 0.5$ and $K > v_L$, we have $(K - v_L)(2\mu - 1) > 0$. Consistent with the derivative hedge hypothesis presented in Cho and Engle (1999), equation (5.2.9) shows that an option spread is positively related to a stock spread.

Finally, informed traders maximize their total expected profit in both stock and option markets by choosing the probabilities (ν and ω) of trading in each market. The informed traders' maximization problem can be expressed as:

$$\begin{aligned} \max_{\nu, \omega} & \{ \nu [E(\tilde{v}|S = G) - S_a] + (1 - \nu) [E((\tilde{v} - K)^+|S = G) - C_a] \\ & + \omega [S_b - E(\tilde{v}|S = B)] + (1 - \omega) [C_b - E((\tilde{v} - K)^+|S = B)] \} \end{aligned} \quad (5.2.10)$$

5.2.4 The OMM's hedging strategy

The OMM provides a market for options and also trades for hedging reasons. In this section we construct a delta hedging strategy using the stock. Later, we analyze the case of hedging with different options.

The market maker's delta hedge is given by:

$$\Delta \equiv \frac{\partial C}{\partial S} = \frac{Pr(\text{Call Buy})E[(\tilde{v} - K)^+|\text{Call Buy}] - Pr(\text{Call Sell})E[(\tilde{v} - K)^+|\text{Call Sell}]}{Pr(\text{Call Buy})E[\tilde{v}|\text{Call Buy}] - Pr(\text{Call Sell})E[\tilde{v}|\text{Call Sale}]} \quad (5.2.11)$$

Equation (5.2.11) conveys two pieces of information. First, the OMM estimates the stock and option payoffs by looking at the direction of the orders he receives. He is able to calculate the expected stock and option payoff, conditional on the probabilities associated with the occurrence of different option order flows. Second, the OMM estimates the ratio of the expected option payoff to the expected stock payoff. This is how much the OMM thinks his option's position changes given a small, instantaneous change in the stock. We note that this hedge ratio is based on the OMM's estimates, which might be different from the hedge ratio used by other traders.

As usual, we expand the expectation functions and have $E[(\tilde{v} - K)^+|\text{Call Buy}] = (\tilde{v} - K)Pr(\theta = H|\text{Call Buy})$ and $E[(\tilde{v} - K)^+|\text{Call Sale}] = (\tilde{v} - K)Pr(\theta = H|\text{Call Sale})$. Similarly, $E[\tilde{v}|\text{Call Buy}] = v_H Pr(\theta = H|\text{Call Buy}) + v_L Pr(\theta = L|\text{Call Buy})$ and $E[\tilde{v}|\text{Call Sell}] = v_H Pr(\theta = H|\text{Call Sale}) + v_L Pr(\theta = L|\text{Call Sale})$. Substituting these conditional probabilities and applying Bayes rule, we obtain

$$\Delta = \frac{(v_H - K)[(2 - \nu - \omega)\mu - (1 - \omega)]}{(v_H - v_L)[(2 - \nu - \omega)\mu - 1] + v_H\omega - v_L\nu} \quad (5.2.12)$$

Next, competition imposes that the OMM must expect to make zero profits both from making a market for options and from trading for hedging reasons. Competition

in the business of option market making reduces any positive expected profit to zero, including the results from trading that occur because of market making, for otherwise if an OMM had zero expected profit in making a market for options, but expected positive profits from hedging, he could narrow the options' spreads and attract additional options' orders. Conversely, if he expected losses from hedging, he would be forced to widen his options' spreads to stay in business.

We can now calculate the OMM's expected profit in making a market for options and in trading for hedging reasons, over three periods. At time 1, his expected profit comes from his option market making. At time 2, his expected profit comes from the construction of his hedging position in the stock market. At time 3, it comes from his expected option payoff and the stock value when the hedging position terminates.

More specifically, we have:

$$\begin{aligned}
 E[\pi_{t=1}^{OMM}] &= -Pr(\text{Call Sale})C_b + Pr(\text{Call Buy})C_a \\
 E[\pi_{t=2}^{OMM}] &= -Pr(\text{Call Buy})Pr(\theta = H|\text{Call Buy})\Delta S_a \\
 &\quad + Pr(\text{Call Sale})Pr(\theta = L|\text{Call Sale})\Delta S_b \\
 E[\pi_{t=3}^{OMM}] &= -Pr(\text{Call Buy})E((\tilde{v} - K)^+|\text{Call Buy}) \\
 &\quad + Pr(\text{Call Sale})E((\tilde{v} - K)^+|\text{Call Sale}) \\
 &\quad + Pr(\text{Call Buy})E(\tilde{v}|\text{Call Buy})\Delta \\
 &\quad - Pr(\text{Call Sale})E(\tilde{v}|\text{Call Sale})\Delta
 \end{aligned}$$

where Δ is given by equation (5.2.12), S_a and S_b are given by equations (5.2.4) and (5.2.3), C_a and C_b are given by equations (5.2.7) and (5.2.8).

The OMM's zero expected profit condition implies that $E[\pi_{t=1}^{OMM}] + E[\pi_{t=2}^{OMM}] + E[\pi_{t=3}^{OMM}] = 0$. Calculating the probabilities using Bayes rule, we have:

$$\begin{aligned}
 E[\pi_{t=1,2,3}^{OMM}] &= -\frac{1}{2}(\alpha(1-\omega) + 2\rho)C_b + \frac{1}{2}(\alpha(1-\nu) + 2\rho)C_a \\
 &\quad - \frac{1}{2}(v_H - K)[\alpha(1-\mu)(1-\omega) - \alpha\mu(1-\nu)] \\
 &\quad + \frac{1}{2}\Delta[\alpha\mu(1-\omega) + \rho]S_b - \frac{1}{2}\Delta[\alpha\mu(1-\nu) + \rho]S_a \\
 &\quad + \frac{1}{2}\Delta[\alpha\mu(1-\nu) - \alpha(1-\mu)(1-\omega)]v_H \\
 &\quad - \frac{1}{2}\Delta[\alpha\mu(1-\omega) - \alpha(1-\mu)(1-\nu)]v_L \\
 &= 0
 \end{aligned} \tag{5.2.13}$$

where $\rho = (1-\alpha)(1-\beta)\frac{1}{2}$.

5.2.5 The general problem

The general problem in our setting consists of a system of three conditions that can be solved using the Lagrange method:

1. Informed traders choose the probabilities of trading in either the stock or the option market to maximize their total expected profit (equation (5.2.10));
2. In equilibrium, informed traders are indifferent between the pure strategies of trading the stock or the options, conditional on receiving a good or bad signal (equations (5.2.5) and (5.2.6));

3. The OMM earns zero combined expected profits (equation (5.2.13)).

To focus on the economic intuition rather than algebraic manipulations, we present the numerical solutions in Section 5.4. Before that, in Section 5.3, we analyze, as a benchmark case, the situation where the OMM does not hedge.

5.3 When the OMM does not hedge: the benchmark case

For the purposes of comparison, we briefly present here the bid and ask prices in both the stock and the option markets when the OMM does not hedge his market making trades in the underlying market. The results are similar to those in John et al. (2003), except that we consider a call option instead of a put option. These benchmark results are then compared with the results when the OMM hedges, allowing us to understand the effects of market maker's hedging strategies both on informed trading, as well as on the costs of providing liquidity in the two markets.

When the OMM does not hedge, there is no trade by the option market maker in the underlying stock market. Informed traders continue to randomize their trades in the two markets to exploit their information advantage. In Figure 5.1, the information structure to the left of the vertical dashed line corresponds to the case when the OMM does not hedge.

5.3.1 The bid and ask prices in the underlying market

The SMM makes a zero expected profit in the underlying market. This implies that the stock bid and ask prices are the expected values of the underlying asset, conditional on the stock order flow that observed by the SMM.

Denote $S'_{a(b)}$ as the stock ask (bid) price when the OMM does not hedge. We have $S'_b = E[\tilde{v}|\text{Stock Sale}]$ and $S'_a = E[\tilde{v}|\text{Stock Buy}]$. Note that now in the underlying market, there is no order flow caused by the OMM's hedging trade. In other words, the probability that a stock order comes from the OMM is simply zero. Using Bayes rule, we can show that the stock bid and ask prices when the OMM does not hedge are given by the following equations:

$$S'_b = \frac{v_H[\alpha(1-\mu)\omega + (1-\alpha)\beta\frac{1}{2}] + v_L[\alpha\mu\omega + (1-\alpha)\beta\frac{1}{2}]}{\alpha\omega + (1-\alpha)\beta} \quad (5.3.1)$$

$$S'_a = \frac{v_H[\alpha\mu\nu + (1-\alpha)\beta\frac{1}{2}] + v_L[\alpha(1-\mu)\nu + (1-\alpha)\beta\frac{1}{2}]}{\alpha\nu + (1-\alpha)\beta} \quad (5.3.2)$$

5.3.2 The option bid and ask prices

Since the OMM makes the option market and does not hedge the incoming order flow, competition in the option market making implies that he makes a zero expected profit. The option bid and ask prices are then equal to the option's expected payoffs, conditional on the option order flow.

Denote $C'_{a(b)}$ as the option ask (bid) price when the OMM does not hedge, we have $C'_b = E[(\tilde{v} - K)^+|\text{Call Sale}]$ and $C'_a = E[(\tilde{v} - K)^+|\text{Call Buy}]$. Simple derivations

using Bayes rule give the option bid and ask prices as follows:

$$C'_b = (v_H - K) \frac{\alpha(1 - \mu)(1 - \omega) + (1 - \alpha)(1 - \beta)^{\frac{1}{2}}}{\alpha(1 - \omega) + (1 - \alpha)(1 - \beta)} \quad (5.3.3)$$

$$C'_a = (v_H - K) \frac{\alpha\mu(1 - \nu) + (1 - \alpha)(1 - \beta)^{\frac{1}{2}}}{\alpha(1 - \nu) + (1 - \alpha)(1 - \beta)} \quad (5.3.4)$$

5.3.3 The informed trading in equilibrium

Informed traders randomize their trades in both the option and the underlying stock markets. Their expected profits from trading only in the stock or the option market is that already described in Section 5.2.3.

Substituting the bid and ask prices of the stock and the option (equations (5.3.1) - (5.3.4)) into the expected profits indifference condition (5.2.5) and (5.2.6), we are able to calculate the equilibrium probabilities that informed traders randomize their orders between the two markets. Specifically, the equilibrium probabilities (denoted by ν' and ω') that informed traders trade in the underlying market, conditional on receiving a good or a bad signal, are the same and given by:

$$\nu' = \omega' = \frac{\beta[\alpha(v_H - v_L) + (1 - \alpha)(1 - \beta)(K - v_L)]}{\alpha[v_H - K + \beta(K - v_L)]}$$

5.4 When the OMM hedges: the regular case

5.4.1 Parameter values

When the OMM hedges - we call this the regular case to emphasize that this is the most natural situation - the positions that result from making a market, there are no

Table 5.1: Parameter values and variables' ranges used in the numerical calculations. The parameter values and variables' ranges are shown in Table 5.1. The high and low prices of the stock in the future are denoted by v_H and v_L . β indicates the percentage of uninformed traders who trade in the stock market. μ indicates the precision of the informed traders' private signal. α is the percentage of informed traders in the trading population. K is the call option's exercise price.

Parameters	Values	Variables	Ranges
v_H	50	μ	0.85-0.99
v_L	30	α	0.25, 0.50, 0.75
β	0.5	K	35, 40, 45

closed form solutions and the problem must be solved numerically. In the numerical calculations, we will show the effects of OMM's hedging on the informed traders trading strategies, as well as the effects on the costs of providing liquidity in both the stock and the option markets. We first fix the option's exercise price (K) and present simulations using different values for the precision of the signal observed by informed traders (μ), and for the percentage of informed traders (α). We then repeat the calculations for in the money, at the money and out of the money options with different exercise prices.

Table 5.1 summarizes the parameter values and the ranges of variables used in the calculations.

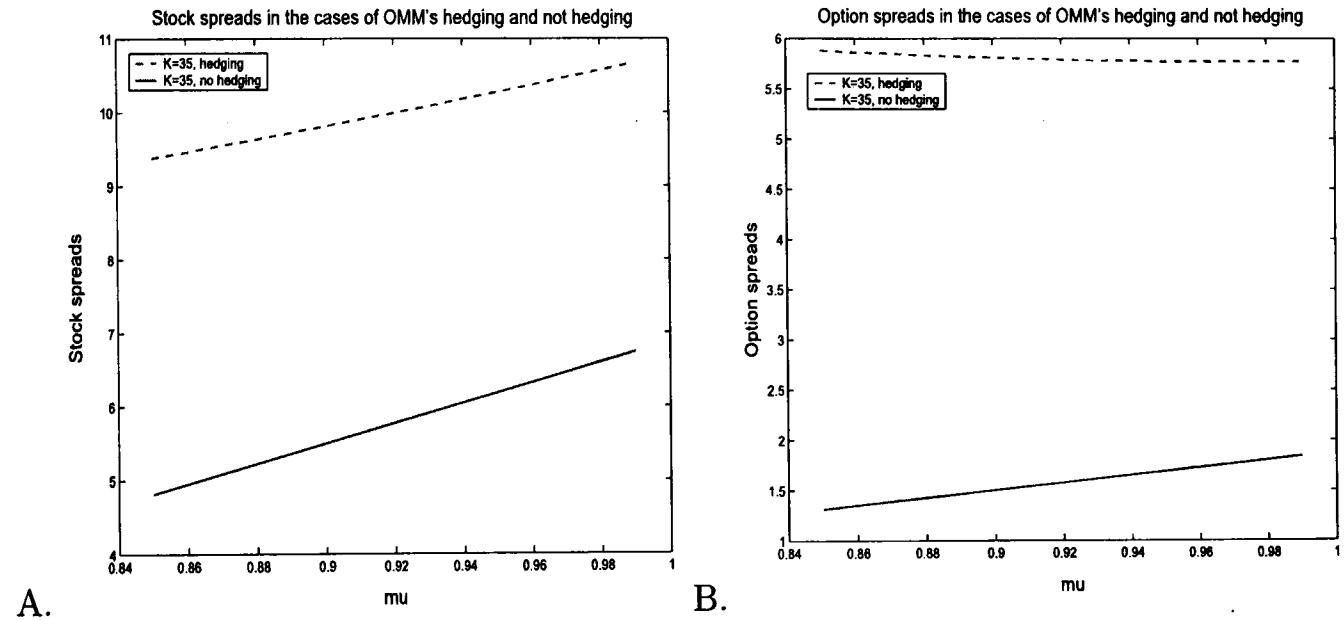
5.4.2 What happens when the OMM hedges?

We first ask how, in general, stock and option spreads⁷ change when the OMM hedges, relative to the case of no hedging. What we find is that both spreads become wider with hedging. Therefore, hedging by the OMM results in higher transaction costs for traders in the cash and in the option markets. Figure 5.2, Panel A(B) graphically presents the stock (option) spreads when the percentage of informed traders is $\alpha = 0.25$, and the exercise price of the option is $K = 35$. The vertical distance between the base line and the solid lines corresponds to the spreads without hedging, and the vertical distance between the base line and the dashed lines corresponds to the spreads with hedging. Both spreads are clearly wider when the OMM decides to hedge. Comparatively, option spreads widen by more than stock spreads.

Why do the spreads widen significantly when the OMM hedges? When the OMM hedges, he must consider the following three effects in setting the option spreads: First, due to bid-ask spreads in the underlying stock market, hedging incurs transaction costs. To cover this cost the OMM must set wider spreads in the option market. Second, trading for hedging reasons unintendedly conveys private information to the stock market. This information spillover effect occurs only when the OMM hedges. To see this, suppose that the OMM faces an option sell order. If this order comes from

⁷We use quoted spreads as the measure of liquidity cost. Cho and Engle (1999) use percentage spreads, defined as the spread divided by the midpoint of the bid and ask prices (spread/mid-quote) in their paper. Our qualitative findings do not change if we use different measures.

Figure 5.2: Stock and option spreads when an OMM does and does not hedge in the underlying market Panel A shows stock spreads in the cases of OMM's hedging (the dashed line) and not hedging (the solid line). Panel B shows options spreads in the cases of OMM's hedging (the dashed line) and not hedging (the solid line). Panel A and B are both for option with $K = 35$ and $\alpha = 0.25$.



an informed trader and the market maker decides to hedge his resulting long position in the option with the underlying stock, the hedge requires him to sell stocks. Doing so is precisely what the informed trader would have done if he traded in the stock market. In other words, by hedging, the OMM trades exactly in the same direction as an informed trader. Therefore, hedging allows the OMM to recover some of his information loss in trading with informed traders, and this narrows the option spreads. Third, if this option order comes from an uninformed trader and the OMM already makes money by trading with the uninformed trader, he incurs additional transaction costs in hedging in the stock market. The net overall effect of wider option spreads

in the case of hedging means that the transaction costs incurred in hedging are more important than the informational spillover gains.

In the underlying market, the OMM's hedging trade constitutes an additional stock order flow that, as discussed, might contain information. Thus, the SMM faces a higher probability that a stock order might contain superior information. Not knowing the informational content of the OMM's hedging trade, the SMM protects himself by setting wider stock spreads.

The reason that option spreads widen comparatively more is because the OMM's hedging cost is of the same magnitude as stock spreads, and, therefore, affects option spreads disproportionately.

In summary, the OMM's hedging imposes higher transaction costs in both the stock and the option markets. As a result, both informed and uninformed traders are worse off with OMM's hedging. For informed traders, hedging makes their private information less profitable. For uninformed traders, hedging puts a higher constraint for liquidity trading. Hedging has negative welfare implications on traders.

5.4.3 Informed traders response to OMM's hedging

Next we look at how informed traders respond to the OMM's hedging by changing the probability of stock buying (ν) (selling (ω)) when they receive a good (bad) signal. Table 5.2 presents the comparative statics of signal precision (μ), the percentage of informed traders (α), and exercise price (K) on ν and ω , in the cases of hedging and

Table 5.2: **The effect of μ , α , and K on the informed trading strategy** This table shows how informed traders would change their trading intensity in the stock market (ν and ω) in response to the changes in the parameter values of μ , α , and K , in both cases of OMM's hedging and not hedging. The $+(-)$ sign indicates a positive (negative) relationship between ν and ω and parameter μ , α , and K .

	OMM hedging $\nu \neq \omega$	OMM not hedging $\nu = \omega$
μ	+	independent
α	+	-
K	+	+

not hedging, respectively. Table 5.3 reports these probabilities in more detail, with panel A for the case of hedging, and panel B for the case of not hedging.

When the OMM does not hedge

In section 5.3.3 we have concluded that when the OMM does not hedge, the probabilities of stock buying (ν) and selling (ω) are the same. However, looking more closely, it appears that informed traders favor trading more in stocks than in options.⁸ This is because the range of possible values that give informed traders a profit is larger in the case of the stock ($v_H - v_L$) than in the case of the option ($v_H - K$). In the words of John et al. (2003) stocks are more information-sensitive than options. The equality between ν and ω comes from the assumption that high and low states occur equally likely, and also the precision of the private signal is the same in both states.

⁸This may seem in contradiction to the conventional wisdom that options provide leverage, see Black (1975). In this section, we are comparing unit trades (one stock or one call). In section 5.4.5, we will discuss expected returns on trading stocks or options and show that option trading provides better returns.

Table 5.3: Informed trading intensity in stocks in the cases of OMM's hedging and not hedging in the stock market. Panel A reports the informed trading intensity in stocks (ν and ω) when the OMM hedges in the stock market. We divide the table for different K . For the same K , we then calculate ν and ω for different α , in the increasing order of signal precision μ . Panel B reports the similar results when the OMM does not hedge in the stock market.

Panel A: Informed trading intensity in stocks when the OMM hedges

μ	$K = 35$			$K = 40$			$K = 45$			
	$\frac{\alpha}{\omega}$	$\frac{\alpha}{\nu}$	$\frac{\omega}{\nu}$	$\frac{\alpha}{\omega}$	$\frac{\alpha}{\nu}$	$\frac{\omega}{\nu}$	$\frac{\alpha}{\omega}$	$\frac{\alpha}{\nu}$	$\frac{\omega}{\nu}$	
0.85	1.00	0.33	1.00	0.71	1.00	0.88	1.00	0.72	1.00	0.89
0.86	1.00	0.35	1.00	0.71	1.00	0.88	1.00	0.75	1.00	0.89
0.87	1.00	0.37	1.00	0.71	1.00	0.88	1.00	0.78	1.00	0.89
0.88	1.00	0.39	1.00	0.71	1.00	0.88	1.00	0.81	1.00	0.90
0.89	1.00	0.40	1.00	0.72	1.00	0.88	1.00	0.84	1.00	0.90
0.90	1.00	0.42	1.00	0.72	1.00	0.88	1.00	0.86	1.00	0.90
0.91	1.00	0.43	1.00	0.72	1.00	0.88	1.00	0.88	1.00	0.90
0.92	1.00	0.45	1.00	0.73	1.00	0.89	1.00	0.91	1.00	0.90
0.93	1.00	0.46	1.00	0.73	1.00	0.89	1.00	1.00	1.00	0.90
0.94	1.00	0.47	1.00	0.73	1.00	0.89	1.00	1.00	1.00	0.90
0.95	1.00	0.49	1.00	0.73	1.00	0.89	1.00	1.00	1.00	0.90
0.96	1.00	0.50	1.00	0.74	1.00	0.89	1.00	1.00	1.00	0.90
0.97	1.00	0.51	1.00	0.74	1.00	0.89	1.00	1.00	1.00	0.90
0.98	1.00	0.52	1.00	0.74	1.00	0.89	1.00	1.00	1.00	0.90
0.99	1.00	0.53	1.00	0.75	1.00	0.89	1.00	1.00	1.00	0.90

Panel B: Informed trading intensity in stocks when the OMM does not hedge

μ	$K = 35$			$K = 40$			$K = 45$											
	$\frac{\alpha}{\omega}$	$\frac{\alpha}{\nu}$	$\frac{\omega}{\nu}$	$\frac{\alpha}{\omega}$	$\frac{\alpha}{\nu}$	$\frac{\omega}{\nu}$	$\frac{\alpha}{\omega}$	$\frac{\alpha}{\nu}$	$\frac{\omega}{\nu}$									
0.85	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.86	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.87	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.88	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.89	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.90	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.91	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.92	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.93	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.94	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.95	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.96	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.97	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.98	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90
0.99	0.79	0.79	0.64	0.64	0.60	0.60	1.00	1.00	0.83	0.83	0.72	0.72	1.00	1.00	1.00	1.00	0.90	0.90

Comparative statics shows that these probabilities are both decreasing in the number of informed traders, α , are increasing in the exercise price of the option, K , and independent of the signal precision, μ .

A higher α increases the threat of informed trading in both the stock and the option markets, which reduces the expected profit to informed traders. Since the information is more sensitive in the stock market, this reduction is also larger in that market. Consequently, informed traders migrate to the option market, so both ν and ω decrease.

Also, as K increases, the option becomes less information sensitive ($v_H - K$). As a result, informed traders increase their trading in the stock market, and both ν and ω increase.

The independence of ν and ω from μ comes from two offsetting effects: first, because of the stock's higher information sensitivity, a more precise information gives informed traders greater incentive to trade stocks, which increases ν and ω ; and, second, the relatively higher stock spreads compared to option spreads reduce such incentives, thus decreasing ν and ω .

When the OMM hedges

When the OMM hedges, ν and ω are no longer the same. Recall that the OMM optimally hedges according to his perceived moneyness of the option's position, and this means that he does not hedge every time. This asymmetry in the OMM's hedging

behavior breaks down the equality between ν and ω .

The comparative statics also changes. Now both probabilities ν and ω are increasing in α and K , and non-decreasing in μ . Since the effect of K is the same with and without hedging, the previous intuition still applies. In what follows, we discuss the intuition for the different effects of α and μ when the OMM hedges.

First recall that hedging increases spreads in both markets but option spreads go up significantly more than stock spreads (Section 5.4.2). Informed traders, therefore, find it relatively cheaper to trade stocks, so both ν and ω increase with hedging.

As the proportion of insiders, α , increases, the OMM will more likely trade with informed traders using the option market. Hence, hedging trades contain more information. Consequently, informed traders find it easier to hide their identities in the stock market and as a result increase their probability of trading the stock. In fact, in the stock market, the marginal contribution of an OMM's hedging trade in helping informed traders disguise their identity goes up with a higher α . To see this, let us consider two possible compositions of the trading population. (i) There is 1 OMM, 1 informed trader, and 100 uninformed traders. (ii) There is 1 OMM, 99 informed traders and 100 uninformed traders. In case (i), the probability that a stock order might come from an informed trader is $\frac{1}{101}$ when the OMM does not hedge, and $\frac{2}{102}$ when he hedges. In case (ii), these two probabilities are $\frac{99}{199}$ and $\frac{100}{200}$, respectively. Clearly in case (i), the informed trading pressure faced by the SMM almost doubles

$(\frac{1}{101} \text{ to } \frac{2}{102})$, while in case (ii), it only increases by merely 0.5% ($\frac{99}{199} \text{ to } \frac{100}{200}$). So when α is high, the OMM's hedging trade does not change much the pressure on SMM by informed trading, and therefore the stock spread is also not adjusted by much. Informed traders take this opportunity by increasing the intensity of stock trading as α increases.

Both ν and ω are non-decreasing in the precision of the signal, μ . This is because, with hedging, the percentage increase in option spreads is much higher than that in stock spreads. Together with the fact that the higher information sensitivity in the stock attracts informed traders with higher μ , both ν and ω are non-decreasing in μ when the OMM hedges.

5.4.4 The effects of α , K and μ on stock and option spreads

After reviewing the results of the informed traders' actions from the option market maker's decision to hedge, we are now in a position to compare the effects of the various parameters - the signal precision (μ), the percentage of informed traders (α), and the option exercise price (K) - on the stock and on the option spreads, with and without hedging by the OMM. Tables 5.4 and 5.5 report the results for different K and α with increasing μ . In both tables, Panel A reports the results in the case of the OMM hedging, and Panel B the results in the case of the OMM not hedging. Table 5.6 provides the summary of the comparative statics, with + (-) sign indicating a positive (negative) relationship.

Table 5.4: Stock spreads in the cases of OMM's hedging and not hedging in the underlying market Panel A reports the stock spreads when the OMM hedges in the stock market. We divide the table for different K . For the same K , we then calculate the stock spreads for different α , in the increasing order of signal precision μ . Panel B reports the similar results when the OMM does not hedge in the stock market.

Panel A: Stock spreads when the OMM hedges												
μ	$K = 35$			$K = 40$			$K = 45$					
	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$
0.85	9.38	11.26	12.77	9.11	11.16	12.75	8.92	10.86	12.53			
0.86	9.46	11.46	13.07	9.20	11.36	13.05	9.05	11.09	12.85			
0.87	9.55	11.66	13.38	9.30	11.56	13.36	9.17	11.31	13.17			
0.88	9.63	11.86	13.68	9.40	11.77	13.66	9.29	11.54	13.49			
0.89	9.72	12.06	13.99	9.50	11.97	13.97	9.42	11.77	13.81			
0.90	9.81	12.26	14.29	9.60	12.18	14.28	9.54	12.00	14.13			
0.91	9.90	12.46	14.60	9.71	12.39	14.58	9.66	12.23	14.45			
0.92	9.99	12.66	14.90	9.82	12.60	14.89	9.78	12.46	14.77			
0.93	10.08	12.86	15.21	9.91	12.80	15.19	9.91	12.69	15.09			
0.94	10.18	13.07	15.51	10.03	13.01	15.50	10.03	12.91	15.41			
0.95	10.27	13.27	15.81	10.15	13.22	15.80	10.15	13.14	15.73			
0.96	10.37	13.47	16.12	10.28	13.44	16.11	10.28	13.37	16.05			
0.97	10.47	13.68	16.42	10.40	13.65	16.42	10.42	13.60	16.37			
0.98	10.57	13.88	16.73	10.52	13.86	16.72	10.55	13.83	16.69			
0.99	10.67	14.08	17.03	10.65	14.07	17.03	10.66	14.06	17.01			

Panel B: Stock spreads when the OMM does not hedge												
μ	$K = 35$			$K = 40$			$K = 45$					
	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$
0.85	4.81	7.88	10.94	5.60	8.75	11.38	5.60	9.33	11.81			
0.86	4.95	8.10	11.25	5.76	9.00	11.70	5.76	9.60	12.15			
0.87	5.09	8.33	11.56	5.92	9.25	12.03	5.92	9.87	12.49			
0.88	5.23	8.55	11.88	6.08	9.50	12.35	6.08	10.13	12.83			
0.89	5.36	8.78	12.19	6.24	9.75	12.68	6.24	10.40	13.16			
0.90	5.50	9.00	12.50	6.40	10.00	13.00	6.40	10.67	13.50			
0.91	5.64	9.23	12.81	6.56	10.25	13.33	6.56	10.93	13.84			
0.92	5.78	9.45	13.13	6.72	10.50	13.65	6.72	11.20	14.18			
0.93	5.91	9.68	13.44	6.88	10.75	13.98	6.88	11.47	14.51			
0.94	6.05	9.90	13.75	7.04	11.00	14.30	7.04	11.73	14.85			
0.95	6.19	10.13	14.06	7.20	11.25	14.63	7.20	12.00	15.19			
0.96	6.33	10.35	14.38	7.36	11.50	14.95	7.36	12.27	15.53			
0.97	6.46	10.58	14.69	7.52	11.75	15.28	7.52	12.53	15.86			
0.98	6.60	10.80	15.00	7.68	12.00	15.60	7.68	12.80	16.20			
0.99	6.74	11.03	15.31	7.84	12.25	15.93	7.84	13.07	16.54			

The tables show that when the OMM does not hedge, spreads in the stock increase with μ , α , and K , while option spreads increase with μ and α , but decrease with K . On the other hand, when the OMM hedges, stock spreads increase with μ and α , but decrease with K ; option spreads increase with α , decrease in K , and have an ambiguous relationship with μ . In the remaining part of this subsection, we discuss these effects in more detail. For each effect, we separate the discussion in the stock

Table 5.5: Option spreads in the cases of OMM's hedging and not hedging in the underlying market Panel A reports the option spreads when the OMM hedges in the stock market. We divide the table for different K . For the same K , we then calculate the option spreads for different α , in the increasing order of signal precision μ . Panel B reports the similar results when the OMM does not hedge in the stock market.

Panel A: Option spreads when the OMM hedges									
	$K = 35$			$K = 40$			$K = 45$		
μ	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$
0.85	5.88	7.76	9.27	2.11	4.16	5.75	0.02	0.36	2.03
0.86	5.86	7.86	9.47	2.00	4.16	5.85	0.01	0.29	2.05
0.87	5.85	7.96	9.68	1.90	4.16	5.96	0.01	0.21	2.07
0.88	5.83	8.06	9.88	1.80	4.17	6.06	0.01	0.14	2.09
0.89	5.82	8.16	10.09	1.70	4.17	6.17	0.01	0.07	2.11
0.90	5.81	8.26	10.29	1.60	4.18	6.28	0.01	0.02	2.13
0.91	5.80	8.36	10.50	1.51	4.19	6.38	0.01	0.01	2.15
0.92	5.79	8.46	10.70	1.42	4.20	6.49	0.01	0.01	2.17
0.93	5.78	8.56	10.91	1.31	4.20	6.59	0.01	0.01	2.19
0.94	5.78	8.67	11.11	1.23	4.21	6.70	0.01	0.01	2.21
0.95	5.77	8.77	11.31	1.15	4.22	6.80	0.01	0.01	2.23
0.96	5.77	8.87	11.52	1.08	4.24	6.91	0.01	0.01	2.25
0.97	5.77	8.98	11.72	1.00	4.25	7.02	0.01	0.01	2.27
0.98	5.77	9.08	11.93	0.92	4.26	7.12	0.01	0.01	2.29
0.99	5.77	9.18	12.13	0.84	4.27	7.23	0.01	0.01	2.31

Panel B: Option spreads when the OMM does not hedge									
	$K = 35$			$K = 40$			$K = 45$		
μ	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$
0.85	1.31	4.38	7.44	0.00	1.75	4.38	0.00	0.00	1.31
0.86	1.35	4.50	7.65	0.00	1.80	4.50	0.00	0.00	1.35
0.87	1.39	4.63	7.86	0.00	1.85	4.63	0.00	0.00	1.39
0.88	1.43	4.75	8.08	0.00	1.90	4.75	0.00	0.00	1.43
0.89	1.46	4.88	8.29	0.00	1.95	4.88	0.00	0.00	1.46
0.90	1.50	5.00	8.50	0.00	2.00	5.00	0.00	0.00	1.50
0.91	1.54	5.13	8.71	0.00	2.05	5.13	0.00	0.00	1.54
0.92	1.58	5.25	8.93	0.00	2.10	5.25	0.00	0.00	1.58
0.93	1.61	5.38	9.14	0.00	2.15	5.38	0.00	0.00	1.61
0.94	1.65	5.50	9.35	0.00	2.20	5.50	0.00	0.00	1.65
0.95	1.69	5.63	9.56	0.00	2.25	5.63	0.00	0.00	1.69
0.96	1.73	5.75	9.78	0.00	2.30	5.75	0.00	0.00	1.73
0.97	1.76	5.88	9.99	0.00	2.35	5.88	0.00	0.00	1.76
0.98	1.80	6.00	10.20	0.00	2.40	6.00	0.00	0.00	1.80
0.99	1.84	6.13	10.41	0.00	2.45	6.13	0.00	0.00	1.84

and in the option market, both when the OMM hedges and does not hedge.

The effect of an increase in α

This effect is pretty obvious. Stock and option spreads always widen when the proportion of insiders increases. When the OMM hedges, the higher the α , the more information contain his hedging trades, and therefore the SMM adjusts even more upwards the spread in the stock.

Table 5.6: **Summary of the effects of μ , α and K on stock and option spreads in the cases of the OMM's hedging and not hedging.** This table summarizes the effects of μ , α and K on stock and option spreads. The $+$ ($-$) sign indicates a positive (negative) relationship. The ambiguous relationship between μ and option spreads when the OMM hedges is indicated by a $?$ sign where Table 5.7 provides detailed analysis.

Stock spreads		Option spreads	
OMM not hedge	OMM hedges	OMM not hedge	OMM hedges
μ	+	+	+
α	+	+	+
K	+	-	-

The effect of an increase in K

When the OMM does not hedge, an increase in K reduces the information sensitivity of the options, causing informed traders to migrate to the stock market, leading to wider stock spreads. When the OMM hedges, however, spreads in option markets widen more than spreads in stocks, and there is less informed trading in options with a higher K (section 5.4.3). Proportionally more trades in the option market come from uninformed traders, and consequently the OMM's hedging trades contain less information. In equilibrium, the spreads in the stock market decrease as K increases, when the OMM hedges. This also explains why the difference in stock spreads narrows between the case of the OMM hedging and the case of the OMM not hedging as K increases. A higher K means less informed trading in the call option, so lower information content in the OMM's hedging trade, moving the stock spreads in the case of hedging closer to the spreads when the OMM does not hedge. It should be obvious

that as K increases, option spreads decrease regardless of whether the OMM hedges or not. When options are deeply out of the money, both the stock and option spreads decline and the cases of the OMM's hedging and not hedging tend to converge.⁹

The effect of an increase in μ

A higher μ means a more accurate signal and the higher threat of losses from informed trading. The SMM protects himself by setting wider stock spreads, in both cases of hedging and not hedging.

In the option market, the effect of μ is ambiguous, and depends on the interaction of several factors. An increased signal precision increases the expected losses from informed trading, and therefore to a response by the OMM of widening the option spreads. However, a higher μ translates into higher gains from hedging to the OMM, since a hedging trade contains more information. Since hedging helps the OMM to recover some of the losses to informed traders, the OMM sets a narrower spread relative to the case of no hedging. Finally, in Section 5.4.3 we saw that informed traders migrate to the stock market as μ increases, and this also helps to reduce the option spreads. Table 5.7 summarizes the effect of μ on option spreads: the +(-) sign corresponds to a positive (negative) relation between the option spread and μ . When the threat of informed trading is less significant (relatively small α) option spreads decrease as μ increases. On the other hand, when the threat of informed trading is

⁹Our results that out of the money options have narrower spreads are consistent with the empirical findings of Kaul et al. (2004).

Table 5.7: **The effect of μ on option spreads when the OMM hedges** This table summarize the effect of μ on option spreads when the OMM hedges. We divide the table into ITM, ATM, and OTM options. In each category, for different levels of α , the $+(-)$ sign indicates a positive (negative) relationship.

In the money			At the money			Out of the money		
$\alpha =$			$\alpha =$			$\alpha =$		
0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
-	+	+	-	+	+	-	-	+

more significant (relatively high α), option spreads increase as μ increases.

5.4.5 The expected return on investment in stocks and options

In this section, we calculate the returns that informed traders expect when they trade in stocks and in options. We then compare the effects of OMM's hedging on these expected returns.

The expected returns in stocks and options are defined as the expected profit of trading in each market, divided by the mid-quote of the bid and ask prices in that market. Specifically, we have

$$\text{Stock return} = \frac{\nu[E(\tilde{v}|S = G) - S_a] + \omega[S_b - E(\tilde{v}|S = B)]}{(S_a + S_b)/2} \quad (5.4.1)$$

$$\text{Option return} = \frac{(1 - \nu)[E((\tilde{v} - K)^+|S = G) - C_a] + (1 - \omega)[C_b - E((\tilde{v} - K)^+|S = B)]}{(C_a + C_b)/2} \quad (5.4.2)$$

First, we discuss the returns when the OMM does not hedge. Examination of Table

5.8-Panel A shows a few unsurprising facts. Both stock and option returns are increasing in μ and decreasing in α : a higher μ gives informed traders higher expected returns; a higher α leads market makers in both stock and option markets to set wider spreads, resulting in smaller returns from trading.

Perhaps more important, option returns are in general higher than stock returns. This is consistent with Black's (1975) intuition that informed traders prefer to trade in the option market because of the leverage effect. The fact that options have relatively smaller spreads gives informed traders a form of implicit edge, which helps to increase the returns from trading.

Note that in certain cases, e.g., when $K = 40$, $\alpha = 0.25$; and $K = 45$, $\alpha = 0.25$ and 0.50, the returns from trading options are actually zero. At first this may seem strange, but a closer examination explains the reason. The returns are directly related to the probability of informed trading in one market. The cases of zero returns from trading options correspond to precisely those situations when informed traders prefer to pursue a (pure) strategy of only trading stocks (i.e., $\nu = \omega = 1$).

A natural question that arises is why, for at the money ($K = 40$) and out of the money ($K = 45$) options, informed traders do not trade options when α is low, and as α increases, they then start to trade options. This is related to the information sensitivity of stocks over options that we discussed earlier. Informed traders only

choose a mixed strategy of trading both stocks and options when the stock's information advantage is not too large. When the stock's information advantage exceeds a certain threshold, informed traders switch to a pure strategy of only trading stocks. This threshold increases with α . Why? A higher proportion of informed traders results in wider stock and option spreads, with stock spreads widening more due to the stock's higher information sensitivity (see Section 5.4.3). Consequently, at a higher α , informed traders are more likely to trade options, leading to positive option returns at higher levels of α and zero option returns at lower levels of α .

The returns for stock and option trading when the OMM's hedges are reported in Table 5.8-Panel B. First, returns are generally lower when compared to the case of not hedging, since hedging by the OMM raises the spreads in both markets.

Perhaps more striking is that now option trading returns are not always greater than stock trading returns. Does Black's intuition no longer hold here? In what follows, we will argue that his intuition still holds. Recall that returns explicitly depend on the probability of informed trading in one market. Close examination of informed trading intensity (ν and ω) with hedging (Table 5.3) reveals that when informed traders receive bad news they always sell stocks and never trade call options (i.e., $\omega = 1$). This is why informed traders' returns from trading option are significantly lower when compared to the no hedging case. In other words, because there is no profit on the sale of the option, this drives down the returns from trading options.

Table 5.8: The expected returns on investment in stocks and options This table reports the returns on investing in stocks and options for informed traders. The returns are defined by equations (5.4.1) and (5.4.2). Panel A reports the results when the OMM does not hedge in the underlying market. We divide the table for different K and for each K , we further divide the column for different levels of α . We report the stock returns and option returns under column marked by S and O, respectively, in the increasing order of signal precision μ . Panel B reports the similar results when the OMM hedges in the underlying market.

Panel B: Returns on investment when the OMM does not hedge

μ	$K = 35$						$K = 40$						$K = 45$					
	$\alpha = 0.25$		$\alpha = 0.50$		$\alpha = 0.75$		$\alpha = 0.25$		$\alpha = 0.50$		$\alpha = 0.75$		$\alpha = 0.25$		$\alpha = 0.50$		$\alpha = 0.75$	
	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O
0.85	0.18	0.26	0.10	0.29	0.05	0.17	0.21	0.00	0.11	0.18	0.05	0.15	0.21	0.00	0.12	0.00	0.05	0.09
0.86	0.19	0.27	0.10	0.30	0.05	0.17	0.22	0.00	0.11	0.18	0.05	0.15	0.22	0.00	0.12	0.00	0.05	0.09
0.87	0.19	0.28	0.10	0.31	0.05	0.17	0.22	0.00	0.12	0.19	0.05	0.15	0.22	0.00	0.12	0.00	0.05	0.09
0.88	0.20	0.29	0.11	0.32	0.05	0.18	0.23	0.00	0.12	0.19	0.05	0.16	0.23	0.00	0.13	0.00	0.05	0.10
0.89	0.20	0.29	0.11	0.32	0.05	0.18	0.23	0.00	0.12	0.20	0.05	0.16	0.23	0.00	0.13	0.00	0.05	0.10
0.90	0.21	0.30	0.11	0.33	0.05	0.19	0.24	0.00	0.12	0.20	0.05	0.17	0.24	0.00	0.13	0.00	0.06	0.10
0.91	0.21	0.31	0.12	0.34	0.05	0.19	0.25	0.00	0.13	0.21	0.06	0.17	0.25	0.00	0.14	0.00	0.06	0.10
0.92	0.22	0.32	0.12	0.35	0.05	0.20	0.25	0.00	0.13	0.21	0.06	0.18	0.25	0.00	0.14	0.00	0.06	0.11
0.93	0.22	0.32	0.12	0.36	0.06	0.20	0.26	0.00	0.13	0.22	0.06	0.18	0.26	0.00	0.14	0.00	0.06	0.11
0.94	0.23	0.33	0.12	0.37	0.06	0.21	0.26	0.00	0.14	0.22	0.06	0.18	0.26	0.00	0.15	0.00	0.06	0.11
0.95	0.22	0.34	0.13	0.37	0.06	0.21	0.27	0.00	0.14	0.23	0.06	0.19	0.27	0.00	0.15	0.00	0.06	0.11
0.96	0.24	0.35	0.13	0.38	0.06	0.22	0.28	0.00	0.14	0.23	0.06	0.19	0.28	0.00	0.15	0.00	0.06	0.12
0.97	0.24	0.35	0.13	0.39	0.06	0.22	0.28	0.00	0.15	0.24	0.06	0.20	0.28	0.00	0.16	0.00	0.06	0.12
0.98	0.25	0.36	0.14	0.40	0.06	0.23	0.29	0.00	0.15	0.24	0.06	0.20	0.29	0.00	0.16	0.00	0.06	0.12
0.99	0.25	0.37	0.14	0.41	0.06	0.23	0.29	0.00	0.15	0.25	0.07	0.20	0.29	0.00	0.16	0.00	0.07	0.12

Panel A: Returns on investment when the OMM hedges

μ	$K = 35$						$K = 40$						$K = 45$					
	$\alpha = 0.25$		$\alpha = 0.50$		$\alpha = 0.75$		$\alpha = 0.25$		$\alpha = 0.50$		$\alpha = 0.75$		$\alpha = 0.25$		$\alpha = 0.50$		$\alpha = 0.75$	
	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O	S	O
0.85	0.08	0.18	0.06	0.04	0.03	0.01	0.11	0.13	0.06	0.05	0.03	0.01	0.13	0.00	0.08	0.00	0.03	0.02
0.86	0.09	0.19	0.06	0.05	0.03	0.01	0.11	0.12	0.07	0.06	0.03	0.01	0.13	0.00	0.08	0.00	0.03	0.02
0.87	0.09	0.20	0.07	0.05	0.03	0.01	0.12	0.12	0.07	0.06	0.03	0.01	0.14	0.00	0.09	0.00	0.04	0.02
0.88	0.10	0.21	0.07	0.06	0.04	0.01	0.13	0.11	0.08	0.07	0.04	0.01	0.15	0.00	0.09	0.00	0.04	0.02
0.89	0.10	0.22	0.08	0.06	0.04	0.01	0.14	0.10	0.08	0.07	0.04	0.01	0.15	0.00	0.10	0.00	0.04	0.02
0.90	0.11	0.22	0.08	0.06	0.04	0.01	0.15	0.09	0.09	0.07	0.04	0.02	0.16	0.00	0.10	0.00	0.04	0.02
0.91	0.12	0.23	0.09	0.07	0.04	0.01	0.16	0.08	0.09	0.07	0.04	0.02	0.17	0.00	0.10	0.00	0.05	0.02
0.92	0.12	0.24	0.09	0.07	0.04	0.01	0.17	0.06	0.10	0.07	0.05	0.02	0.18	0.00	0.11	0.00	0.05	0.02
0.93	0.13	0.25	0.09	0.07	0.05	0.01	0.18	0.00	0.10	0.08	0.05	0.02	0.18	0.00	0.11	0.00	0.05	0.02
0.94	0.14	0.25	0.10	0.08	0.05	0.01	0.19	0.00	0.10	0.08	0.05	0.02	0.19	0.00	0.12	0.00	0.05	0.02
0.95	0.14	0.26	0.10	0.08	0.05	0.02	0.20	0.00	0.11	0.08	0.05	0.02	0.20	0.00	0.12	0.00	0.05	0.03
0.96	0.15	0.26	0.11	0.08	0.05	0.02	0.20	0.00	0.11	0.08	0.05	0.02	0.20	0.00	0.13	0.00	0.06	0.03
0.97	0.16	0.27	0.11	0.09	0.06	0.02	0.21	0.00	0.12	0.08	0.06	0.02	0.20	0.00	0.13	0.00	0.06	0.03
0.98	0.16	0.27	0.12	0.09	0.06	0.02	0.22	0.00	0.12	0.08	0.06	0.02	0.20	0.00	0.13	0.00	0.06	0.03
0.99	0.17	0.28	0.12	0.09	0.06	0.02	0.22	0.00	0.13	0.09	0.06	0.03	0.20	0.00	0.14	0.00	0.06	0.03

The question is why with the OMM hedging, informed traders always decide to sell stocks when they receive bad news and never trade the option? Call options are good in a positive signal case (bull market). When the news are bad, however, informed traders' expected profits are limited by the call's bid price. If there is no hedging, the bid price is still somewhat high, making it profitable for informed traders to sell the options and obtain the bid prices. However, hedging by the OMM makes the option spreads significantly wider (see Section 5.4.2), to the point where an option sale is not anymore profitable for informed traders.

Now turn back to Black's point. If we only focus on the buy side (i.e., buy call vs. buy stock), returns from trading options are higher than from trading stocks, reflecting the leverage advantage of options. However, since we are dealing with call options and the probability of selling these by informed traders is zero, the returns from trading options on the sell side are smaller than the returns from trading stocks. Obviously, the right comparison should be between trading stocks versus trading call options when the signal is good and trading puts when the signal is bad. In this case, Black's argument remains valid, whether the OMM hedges or not.

5.5 A market for several options when hedging is done in the underlying stock

Until now, we have considered the case of a call option and the underlying stock. In reality, there is a range of option contracts and not just a single contract. Various

options provide greater flexibility and more opportunities for traders to meet their trading needs. In this and the next section, we extend the basic model to consider market making two related options.¹⁰ First, we consider the case of one OMM who makes markets for two options written on the same stock but with different exercise prices, and hedges his option trades in the underlying market. In the next section, there are two OMMs, each making a market for one option on the same stock, and each hedging using the other option.

Suppose there are two call options, denoted by C^1 and C^2 with exercise prices K^1 and K^2 where $K^1 < K^2$. Informed traders can trade C^1 , C^2 , or both. In particular, if informed traders receive a good signal, they could buy C^1 with probability p or C^2 with probability $(1 - p)$; if they receive a bad signal, they could sell C^1 with probability q or C^2 with probability $(1 - q)$, where p and q are determined in equilibrium. Uninformed traders buy and sell C^1 and C^2 with equal probabilities for exogenous reasons. The OMM continues to hedge in the underlying stock market when he receives an option order and perceives that he might lose money in his option position.

¹⁰We also consider the extension where a monopolistic OMM hedges in the underlying market and compare it to the case of a competitive OMM. We calculate the stock and option spreads and analyze how informed traders would adjust their trading strategy. We find that option spreads are generally wider, but the stock spreads are wider or narrower depending on the parameter values. In addition, when the OMM is monopolistic, informed traders continue to trade stocks and options at relatively low level of signal precision (μ), but leave the option market and concentrate on trading stocks as μ increases. Furthermore, informed traders are more likely to trade stocks when μ is high. Intuitively, the monopolistic option market reduces the incentive for informed traders to trade options. Although the OMM can now command higher option spreads, he loses the order flow. Overall, our findings remain qualitatively the same compared to the case of competitive option market making.

Examining the stock bid and ask prices, we have the following proposition:

Proposition 5.3. *The stock bid and ask prices have the same functional forms as in equations (5.2.3) and (5.2.4), whether there is one or more call options and the option market maker hedges in the stock market.*

Intuitively, the introduction of the second call option does not change the overall probability structure of the OMM's hedging trade. A second option allocates different probabilities of trading C^1 and C^2 (i.e., p and $(1-p)$ in the case of option buy, or q and $(1-q)$ in the case of option sale), but the combined probability of the OMM's hedging trade remains the same, regardless of the number of available options. Consequently, the stock bid and ask prices have the same functional form. Of course, the exact stock bid and ask prices are different since the probabilities that informed traders allocate their trades between stocks and options (i.e., ν and ω) change.

Informed traders have an additional choice of trading the second option. Following the same rationale as in Section 5.2.4, informed traders, in trying to disguise themselves from the market makers, choose a mixed strategy of trading either the stock, option C^1 , or option C^2 . This means that they expect to be indifferent between

trading either the stock or options. This gives the following equations:

$$E[\tilde{v}|S = G] - S_a =$$

$$p(E[(\tilde{v} - K^1)^+|S = G] - C_a^1) + (1 - p)(E[(\tilde{v} - K^2)^+|S = G] - C_a^2) \quad (5.5.1)$$

$$S_b - E[\tilde{v}|S = B] =$$

$$q(C_b^1 - E[(\tilde{v} - K^1)^+|S = B]) + (1 - q)(C_b^2 - E[(\tilde{v} - K^2)^+|S = B]) \quad (5.5.2)$$

where subscript a (b) indicates the ask (bid) price. The left of both equations are the expected profit of trading stocks and the right are the expected profits of trading options.

In addition, informed traders are indifferent between trading option C^1 or option C^2 :

$$E[(\tilde{v} - K^1)^+|S = G] - C_a^1 = E[(\tilde{v} - K^2)^+|S = G] - C_a^2 \quad (5.5.3)$$

$$\underbrace{C_b^1 - E[(\tilde{v} - K^1)^+|S = B]}_{\text{Trading Option } C^1} = \underbrace{C_b^2 - E[(\tilde{v} - K^2)^+|S = B]}_{\text{Trading Option } C^2} \quad (5.5.4)$$

Collectively, equations (5.5.1)-(5.5.4) give the following proposition:

Proposition 5.4. *The bid-ask spreads of two call options C^1 and C^2 written on the same asset but with different exercise prices, satisfy the following condition:*

$$C_{\text{spread}}^1 - C_{\text{spread}}^2 = (K^2 - K^1)(2\mu - 1)$$

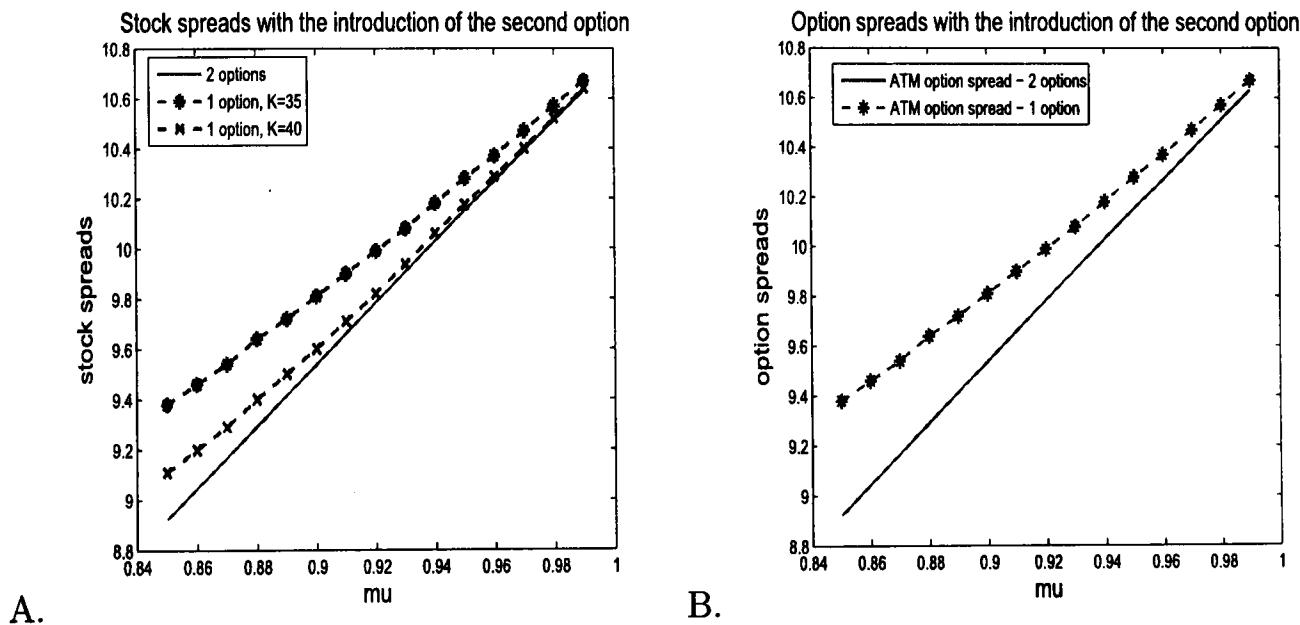
Proposition 5.4 states that the difference in the spreads of two call options with different strike prices and written on the same asset is proportional to the difference in

the strikes prices K^1 and K^2 times a factor that is a function of the informed traders' private signal precision μ . Since $\mu > 0.5$ and $K^1 < K^2$, we have $C^1 > C^2$, i.e., the relatively more in the money call option has wider spreads than the relatively less in the money call option. Intuitively, a call option with a lower strike price K is more sensitive to private information, and therefore it attracts more informed trading. This is consistent with the findings of Kaul et al.(2004) that informed traders participate in more liquid contracts that have a high level of noise and liquidity trading, in contrast to the conjecture that informed traders prefer OTM options (Black (1975)).

Consider that when informed traders observe a good signal, they buy more C^1 and consequently push up C_a^1 . If there is a bad signal, informed traders sell the underlying stock. This means that $C^1 > C^2$. As an extreme example, when the informed traders' signal precision is 100% ($\mu = 1$), $C_a^1 = S_a - K^1$, $C_a^2 = S_a - K^2$ and $C_b^1 = C_b^2 = S_b - v_L$. It is easy to see that when informed traders receive a good signal, they buy more of option C^1 and push up C_a^1 ; when they receive a bad signal, they sell the stock (since $C_b^1 = C_b^2 < S_b$). Furthermore, the relation $C^1 - C^2 = K^2 - K^1$ indicates that the difference in exercise prices, K^1 and K^2 , serves to compensate the OMM's expected loss from trading with more informed traders in the market for C^1 than in the market for C^2 .

Next, we calculate the equilibrium spreads for the stock and for each option when two options are traded. We consider that $K^1 = 35$ (the option is in the money)

Figure 5.3: Stock and option spreads when the OMM makes a market for two options and hedges in the underlying market Panel A shows stock spreads with the introduction of the second option. The solid line for stock spreads when two options ($K^1 = 35$ and $K^2 = 40$) are traded. The two dot lines are for stock spreads when each option is the only traded option. Panel B shows option spreads with the introduction of the second option. The solid line are for ATM option spreads when two options are traded. The dot line are for ATM option spreads when it is the only traded option.



and $K^2 = 40$ (the option is at the money). We then compare the stock and option spreads in this section to the case where each option is the only traded option, as in the previous sections. Figure 5.3, Panel A shows the stock spreads when two options are traded and when each option is the only traded option, respectively. Panel B shows the at the money option spreads when the option is one of the two options being traded, and when there is only one option traded.

An interesting aspect revealed by these figures is that the introduction of the second option causes both the stock and the option spreads to narrow. Why? The

reason for the reduction in the spreads of the options is because of the convexity property of an option payoff. The OMM's hedging costs come from his hedge ratio. When the OMM trades two options, his cumulative hedge ratio, weighted by the probability of hedging each position (i.e., Δ^1 and Δ^2), can be calculated as

$$\Delta^{\text{effective}} = \Delta^1 \times Pr(\text{Net Long } \Delta^1) + \Delta^2 \times Pr(\text{Net Long } \Delta^2)$$

where $Pr(\text{Net Long } \Delta^{1(2)})$ is the probability of a net long position in Δ^1 or Δ^2 . We can then compare it to the hedge ratios if the OMM only trades either option C^1 or option C^2 . The results show that this cumulative hedge ratio is smaller than the hedge ratio of either of the options, suggesting that the OMM has a smaller position to hedge. If we calculate the hedging costs explicitly by taking the spreads of the underlying stock, we can show that the OMM incurs smaller costs in hedging, and this translates into narrower spreads when the OMM trades the two options.

The reduction in the stock spreads is due to less informational content in the OMM's hedging trade. As we have discussed previously, the OMM's hedging trade contains information. With certain probability, the OMM trades in the same direction as informed traders do if they trade in the underlying market. We can calculate the probability of the informed trading in the option market as the proxy for informational content in the OMM's hedging trade, which can be shown to be

$$\alpha\mu(\omega - \nu).$$

Numerical results suggest that there is less informational content in the hedging trade

when two options are traded than when there is only one option traded. The less threat of informed trading in the underlying market narrows the stock spreads.

Finally, we can ask what happens to the expected cost of trading options when the second option is introduced. We define the expected cost of trading options as

$$\sum_{i=1,2} Pr(\text{trading option } i) \times C_{\text{spread}}^i$$

where $Pr(\text{trading option } i)$ is the probability of trading option i , i.e., the net probability of a long position in option i ; and C_{spread}^i are the spreads of option C^i . Numerical calculations show that the expected cost of trading options is smaller when the second option is introduced. Obviously, the reduction in option spreads helps to reduce this cost. In sum, the introduction of the second call option helps to reduce the cost of trading options for all market participants.

5.6 A market for several options when hedging is done with options

In this section there are two OMMs, each making a market for one option, and each hedging using the other option. For convenience, we denote Options Market Maker 1(2) as OMM1 (OMM2) who makes market for option C^1 (C^2). Recall that $K^1 < K^2$, so $C^1 > C^2$.

Let us focus on the informed trading strategy and the OMM's hedging strategy in the option market. Since OMMs do not trade in the underlying market, the functional

forms of stock bid and ask prices are the same as in the case where there is no hedging at all, as described by equations (5.3.1) and (5.3.2).

We conjecture that in equilibrium, for ($\mu < 1$), informed traders choose the mixed strategy of trading two options simultaneously:

- (1) On receiving good news, informed traders either buy stocks, or simultaneously buy C^1 and sell C^2 ;
- (2) On receiving bad news, informed traders either sell stocks, or simultaneously sell C^1 and buy C^2 .

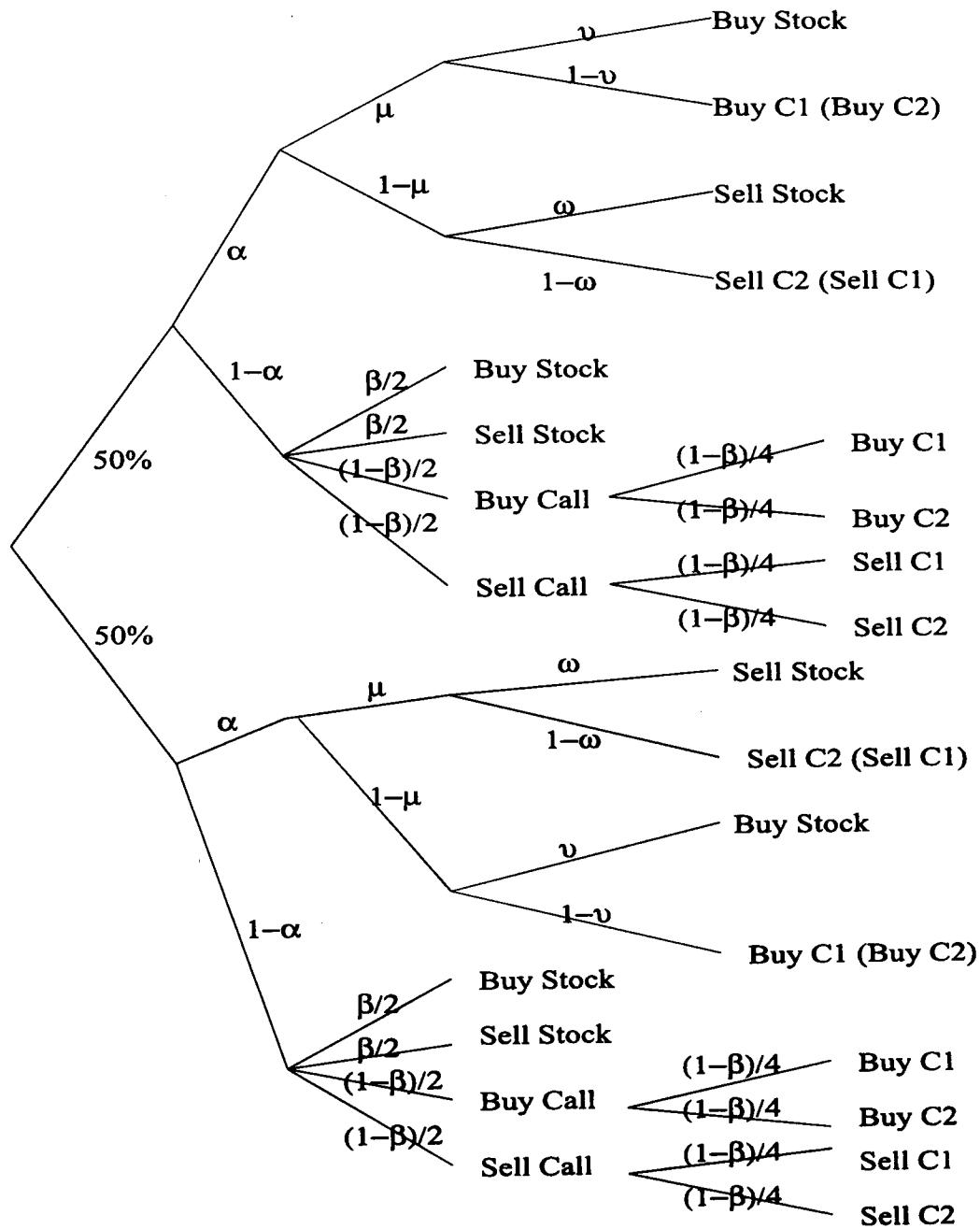
Therefore, it is possible that both OMMs receive orders from an informed trader.

Figure 5.4 shows the information structure when the OMMs hedge using options.

Informed traders are indifferent between the two choices of strategies (1) and (2). For strategy (1), they are indifferent between buying stocks, and simultaneously buy C^1 and sell C^2 . For strategy (2), they are indifferent between selling stocks, and simultaneously sell C^1 and buy C^2 . We have:

$$\begin{aligned}
 & Pr(\text{Informed buy stock})\{E[\tilde{v}|S = G] - S_a\} \\
 &= Pr(\text{Informed buy } C^1)\{E[(\tilde{v} - K^1)^+|S = G] - C_a^1\} \\
 &\quad + Pr(\text{Informed sell } C^2)\{C_b^2 - E[(\tilde{v} - K^2)^+|S = G]\}
 \end{aligned} \tag{5.6.1}$$

Figure 5.4: Information structure when OMMs hedges using options The figure shows the information structure when two OMMs hedge using options. The two strategies are: (1) On receiving good news, informed traders either buy stocks, or buy C^1 and sell C^2 ; (2) On receiving bad news, informed traders either sell stocks, or sell C^1 and buy C^2 . The options trading strategy in (2) is shown in brackets.



and

$$\begin{aligned}
 & Pr(\text{Informed sell stock})\{S_b - E[\tilde{v}|S = B]\} \\
 &= Pr(\text{Informed sell } C^1)\{C_b^1 - E[(\tilde{v} - K^1)^+|S = B]\} \\
 &\quad + Pr(\text{Informed buy } C^2)\{E[(\tilde{v} - K^2)^+|S = B] - C_a^2\} \tag{5.6.2}
 \end{aligned}$$

Equation (5.6.1) gives:

$$\begin{aligned}
 \mu\nu\{E[\tilde{v}|S = G] - S_a\} &= \mu(1 - \nu)\{E[(\tilde{v} - K^1)^+|S = G] - C_a^1\} \\
 &\quad + (1 - \mu)(1 - \omega)\{C_b^2 - E[(\tilde{v} - K^2)^+|S = G]\} \tag{5.6.3}
 \end{aligned}$$

The last term in equation (5.6.3) describes the informed traders' short position in option C^2 . One can see that when the precision of the signal is perfect ($\mu = 1$), this term disappears and the indifference condition reduces to buying stocks and buying only call option C^1 . It is precisely because the signal is imperfect, that informed traders choose a mixed strategy in the option market by trading the two options simultaneously. Going long one option and shorting the other gives them more power and also allows them to hide their identity in the option market more effectively.

Each OMMs hedges with the other, so each OMM receives orders from the other OMM, and in turn places orders with it. We conjecture that, in equilibrium, strategy (1) implies that OMM2 has a net short C^2 position and sells C^1 to hedge. After receiving OMM2's short C^1 orders, OMM1 still has net long C^1 position and therefore buys C^2 to hedge.

Denote OMM2's net short position in C^1 as x . Market making competition implies that OMM2's zero expected profit condition is given by:

$$\begin{aligned} & Pr(C^2 \text{ Sell})\{E[(\tilde{v} - K^2)^+ | C^2 \text{ Sell}] - C_b^2\} \\ & + Pr(C^2 \text{ Buy})\{C_a^2 - E[(\tilde{v} - K^2)^+ | C^2 \text{ Buy}]\} \\ & + x\Delta^2\{C_b^1 - E[(\tilde{v} - K^1)^+ | x]\} = 0 \end{aligned} \quad (5.6.4)$$

where Δ^2 is OMM2's hedge ratio and is defined as:

$$\Delta^2 \equiv \frac{\frac{\partial C^2}{\partial S}}{\frac{\partial C^1}{\partial S}} = \frac{Pr(C^2 \text{ Sell})E[(\tilde{v} - K^2)^+ | C^2 \text{ Sell}] - Pr(C^2 \text{ Buy})E[(\tilde{v} - K^2)^+ | C^2 \text{ Buy}]}{Pr(C^2 \text{ Sell})E[(\tilde{v} - K^1)^+ | C^2 \text{ Sell}] - Pr(C^2 \text{ Buy})E[(\tilde{v} - K^1)^+ | C^2 \text{ Buy}]} \quad (5.6.5)$$

We have $Pr(C^2 \text{ sell}) = Pr(C^2 \text{ Informed sell}) + Pr(C^2 \text{ Uninformed sell}) = \frac{1}{2}[\alpha\mu(1 - \omega) + \alpha(1 - \mu)(1 - \omega) + \frac{2}{4}(1 - \alpha)(1 - \beta)]$ and $Pr(C^2 \text{ buy}) = Pr(C^2 \text{ Informed buy}) + Pr(C^2 \text{ Uninformed buy}) = 0 + \frac{1}{2}[\frac{2}{4}(1 - \alpha)(1 - \beta)]$.

Similarly, OMM1 has expected profits from the trading of C^1 and his net hedging in C^2 , that is,

$$\begin{aligned} & Pr(C^1 \text{ Buy})\{C_a^1 - E[(\tilde{v} - K^1)^+ | C^1 \text{ Buy}]\} \\ & + Pr(C^1 \text{ sell})\{E[(\tilde{v} - K^1)^+ | C^1 \text{ sell}] - C_b^1\} \\ & + x\Delta^2\{E[(\tilde{v} - K^1)^+ | x] - C_b^1\} \\ & + Pr(C^2 \text{ net hedging buy})\Delta^1\{E[(\tilde{v} - K^2)^+ | C^2 \text{ net hedging buy}] - C_a^2\} \\ & = 0 \end{aligned} \quad (5.6.6)$$

We have $Pr(C^1 \text{ buy}) = Pr(C^1 \text{ Informed buy}) + Pr(C^1 \text{ Uninformed buy}) = \frac{1}{2}[\alpha\mu(1 - \nu) + \alpha(1 - \mu)(1 - \nu) + \frac{2}{4}(1 - \alpha)(1 - \beta)]$, $Pr(C^1 \text{ sell}) = Pr(C^1 \text{ Informed sell}) + Pr(C^1 \text{ Uninformed sell}) = 0 + \frac{1}{2}[\frac{2}{4}(1 - \alpha)(1 - \beta)]$, and the net hedging probability is given by $Pr(C^2 \text{ net hedging buy}) = \frac{1}{2}[\alpha(1 - \nu) - x]$.

The equilibrium condition of strategy (2) also gives the zero expected profit conditions for OMM1 and OMM2, denoted, respectively as (A) and (B).

The informed traders problem is to maximize their expected profits by mixing between strategies (1) and (2). Specifically, we have

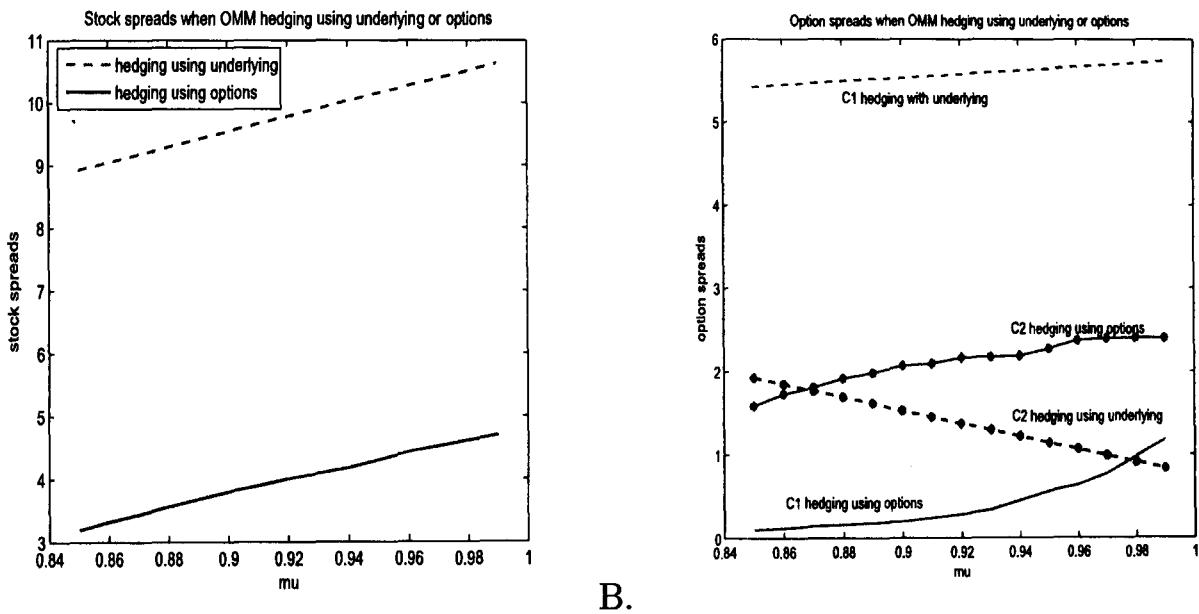
$$\max_{\nu, \omega} \nu[E(\tilde{v}|S = G) - S_a] + \omega[S_b - E(\tilde{v}|S = B)] \quad (5.6.7)$$

Subject to (5.6.1), (5.6.2), (5.6.4), (5.6.6), (A) and (B)

Figure 5.5 compares the equilibrium spreads of the stock, call option C^1 and call option C^2 , when the OMM hedges in the option market versus when the OMM hedges in the underlying market, as in the previous section. Panel A shows the stock spreads when an OMM hedges using the underlying stock versus when it uses the option market. Panel B shows the option spreads in the two different hedging cases. The figure reveals several interesting facts. First, when an OMM hedges with an option, the stock spreads are smaller than when the OMM hedges using the underlying. When the OMMs hedge with each other, they do not trade in the stock market and hence there is less informed trading in this market.

Second, the spreads of the in-the-money option C^1 are greatly reduced when the

Figure 5.5: Stock and option spreads when OMMs hedge using the underlying and the other options Panel A shows the stock spreads when an OMM hedges using the underlying market (dashed line) and when two OMMs hedge with each other using the other option (the solid line). Panel B shows the option spreads for in-the-money option (C^1) and at-the-money option (C^2). The dotted line is the spreads of C^1 when an OMM hedges using the underlying market and the solid line is the spreads of C^1 when two OMMs hedge with each other using the other option. The dotted line with asterisk is the spreads of C^2 when an OMM hedges using the underlying market and the solid line with asterisk is the spreads of C^2 when two OMMs hedge with each other using the other option.



OMM hedges with options compared to when the OMM hedges with the underlying stock. C^1 is the most information sensitive option, and attracts more informed traders. The lower spreads reflects the benefit of hedging using options rather than the underlying for OMMs. OMM1 better hedges himself and transfers more hedging risk to C^2 .

Third, the spreads of option C^2 are generally wider at high levels of signal precision (μ) when the OMMs hedge with options than when they hedge using the stock. For

OMM2, when both OMMs hedge, the incremental informed trading threat that results from OMM1's hedging trades dominates the gains he gets from also hedging, since option C^1 is the most information sensitive. Note also that the spreads of C^2 are increasing in the signal precision μ . Intuitively, the more precise the signal is, the greater the informed trading threat from the OMM1's hedging trades.

Finally, the spreads of C^1 are smaller than the spreads of C^2 . One would expect the opposite in our model, since the amount of informed trading is larger in the more information sensitive option C^1 . When both OMMs hedge with each other, OMM1 transfers much of the informed trading threat to OMM2 and achieves even smaller spreads.

5.7 Empirical implications

In the previous models of stock and option market making, market makers in each market set prices equal to the conditional expectations of the value of the security, given the order flow they observe. In contrast, our model focuses on the OMMs' hedging strategies and asks what is the effect of hedging in both markets, as well as on how informed traders place their orders. We show that the OMM's hedging contains information, which forces the SMM to set wider spreads in the underlying stock. In addition, the OMM sets wider option spreads to cover his hedging cost. As a result, the OMM's hedging increases the costs of liquidity provision in two markets. Thus, our model provides several testable implications regarding the spreads and

market participants' trading behavior:

Implication 1. The OMM might hedge in the same direction that informed traders trade in the underlying market. Therefore, the OMM's hedging trades contain private information.

Implication 2. Both stock and option spreads are significantly wider due to OMM's hedging.

Implication 3. Hedging by OMMs affects the trading strategy of informed traders. In particular, informed traders trade more in the stock market as the percentage of informed traders in the total trading population increases. In addition, the precision of informed traders' private signal affects the informed trading intensity in the underlying market.

Implication 4. Stock spreads decrease as the option's exercise price increase, when the OMM hedges. The precision of the informed traders' private signal affects the option spreads in an ambiguous way, and the final result depends on the option's exercise prices, as well as on the percentage of informed traders in the total trading population.

Implication 5. When the OMM makes a market for several options with different exercise prices, the spreads for each individual option go down relative to the case of a single traded option. The stock spreads are also smaller when the OMM makes a market for several options.

Recently there is growing body of empirical evidence that seems to support several of the implications in our model. For example, the study by de Fontnouvelle et al. (2003) of the behavior of bid-ask spreads and volume in option markets, during the competition for listing, finds that the option's delta and the underlying stock spread are significantly related to the size of the option spreads. The authors conjecture that the cost of hedging plays a role in setting option spreads. Analysis of informed trading in option markets by Kaul et al. (2004) shows that the underlying stock's spread has an important impact on the option spreads due to the hedging activities of OMMs. Adverse selection in the underlying stock spreads seems to explain a significant fraction of the option spreads. Both studies find evidence consistent with Implication 2 above. Finally, experimental research by de Jong, Koedijk, and Schnitzlein (2001) suggests that informed traders may favor ITM options.

5.8 Conclusion

In this chapter, we develop a model of stock and option market making to evaluate the effects of OMM's hedging on informed trading strategies and the subsequent change in the costs of liquidity provision. Informed traders strategically randomize their trades in both markets to exploit their private information. The OMM updates his estimates of future states from his observation of the option order flow and hedges in the underlying market, or in the option market. The competition in option market making implies that he makes a zero combined expected profit in both option market

making and hedging.

Our model sheds some light on the complex practice of option market making. We find that with hedging, the OMM might inadvertently trade in the same direction as informed traders would do. Hence the hedging trades contain information. The SMM (or other OMMs) responds to this increasing threat of informed trading by setting wider stock (option) spreads. As the OMM takes stock spreads as given when he hedges he pays additional transaction costs when hedging that outweigh some recovery of information loss in making a market for informed traders, and this results in wider option spreads. In summary, the costs of liquidity provision in the two markets increase and both informed and uninformed traders face higher transaction costs. Although informed traders find it might be easier to hide their identity in the underlying market with OMM's hedging, the overwhelmingly increased transaction costs in the two markets make their private signal less profitable when compared to the case where the OMM does not hedge.

With the introduction of several options, informed traders have additional choices to exploit their informational advantage. We consider two different hedging cases: (1) the OMM hedges in the underlying stock market, and (2) all OMMs hedge using the option market. When the OMM hedges using the underlying, we find that the OMM's hedging trade is less informative and both stock and option spreads are narrower. The reduction in option spreads is due to the convex structure of the option's payoff,

which helps to reduce the hedging costs. In the case of stocks, the less informative hedging trade eases the informational threat from the OMM's hedging trade, and this helps reduce the stock spreads. We then compare the equilibrium spreads between the second and the first cases. We find that when OMMs hedge using the option markets, the spreads of both stock and in-the-money options are greatly reduced, while spreads of at-the-money options widen slightly. Less informed trading in stocks and no informed trading threat from the OMMs' hedging trade help to reduce the stock spreads. For OMM1, hedging using options is more effective in transferring the threat of informed trading to the other option, leading to an increase in the spreads of that option.

Chapter 6

A simple model of horse race bookmaking

6.1 Introduction

This chapter provides a simple model of bookmaking in a horse race betting market.

We are motivated to study horse race bookmaking as a way to understand the market making of state contingent claims such as options. Horse race bookmaking and option market making share many similarities. Shin (1992, p426-427) argues that:

...In its simplest formulation, the market for bets in an n -horse race corresponds to a market for contingent claims with n states of the world, where the i th state corresponds to the outcome that the i th horse wins the race.

Moreover, the basic securities (Arrow-Debreu securities) which pay a dollar if a particular state obtains and nothing otherwise, have their prices determined by the betting odds. Since odds are offered on each horse, all basic securities are traded...

In addition, an option market maker's profits are contingent on the stock's final value. Suppose the option market maker deals with butterfly spreads instead of a single option, there is one and only one option of the spreads that will pay off at expiry. It is exactly the same in a horse race: there is one and only one horse that will win the race eventually. Furthermore, an option market maker wants to have some flat, positive positions over all option exposures, so that he could always have positive profits and avoid negative ones. In a horse betting market, a bookmaker sells liabilities over all horses and wishes to avoid large liability exposures. Since a horse betting market is an especially simple financial market, in which the complexity of the pricing problem is reduced, it provides a clear view of pricing issues which are more complicated elsewhere. It is perhaps surprising that the market making literature has not previously taken up this natural approach to modeling market making state contingent claims.

A horse race betting market is one form of wagering markets, which are particularly simple financial markets in which many important economic issues have been analyzed¹. Investors' risk preference and market information efficiency are some examples of these issues. There is however a limited literature on horse race bookmaking. In a series of papers, Shin (1991, 1992 and 1993) analyzes the price setting strategy when a bookmaker faces asymmetric information. Shin's analysis attributes to the well-known favorite-longshot bias. The favorite-longshot bias means that the

¹See Sauer (1998) for a recent survey on the economics of wagering markets.

normalized prices on the favorites of the race understate the winning chances of these horses, while the normalized prices on the longshots exaggerate their winning chances.

In our model, the bookmaker faces the risk of trading with possible informed traders as well as the risk of an unbalanced book. The book is liable to become unbalanced because the betting demands are noisy and the bookmaker may not know the correct odds to quote. There is a sizeable body of evidence that points to the prevalence of insider trading in the market for bets (Crafts (1985), also in Shin (1993)). As an uninformed trader, a bookmaker therefore has an important job to extract information from insiders from his betting flow observations. Furthermore, whenever his book becomes unbalanced, the bookmaker wants to re-balance it so that the problem of having extremely high liability exposures can be alleviated. Even random shocks from noisy traders are costly to the bookmaker since his book could become less balanced.

The bookmaker in this model revises his odds to mitigate the risk. He influences the public betting flow by raising the normalized prices for horses with high initial liabilities and lowering the normalized prices for horses with low initial liabilities. Interestingly, the normalized prices exhibit the favorite-longshot bias under our assumption of the constant standard deviation of the individual revenue.

Allowing the bookmaker to set odds several rounds before the race starts gives us a clear view of the bookmaker's price setting strategy and its impact on the public

betting flow over time. The odds revision reflects the way that a bookmaker learns the noisy market demand. We first analyze the multistage odds setting under expected demands and then allow the stochastic demands affect the subsequent liabilities. In both cases of expected demands and stochastic demands, the bookmaker gets rid of the positive liability exposures over a few rounds of odds setting. The book generally gets more balanced over time and the normalized prices approach to the roundness of the book. Our model helps to understand the complexity of managing a series of state contingent exposures such as options for a single expiry date.

It is worth noting that the horse race in our model is the British system where a bookmaker sets odds for difference horses prior to the race starts. It is not the system in the North America where odds are determined by the parimutuel method in which prices are proportional to amounts wagered.

This chapter is organized in the following way. For ease of exposition, we first analyze the case where the noise in the betting demands is zero in Section 6.2. Section 6.3 considers the first stage odds setting of the model with stochastic noise. In Section 6.4 the odds in the subsequent stages are analyzed. Section 6.5 concludes the chapter.

6.2 A simplified model

6.2.1 Assumptions

For ease of exposition, we first look at the case where the public betting demands are deterministic functions of the odds. This is relaxed in the following section.

We consider an N -horse race in which each horse is indexed as $i = 1, \dots, N$. Denote the bookmaker's subjective probability that the i th horse wins the race as p_i and the odds that the bookmaker quotes as q_i , $0 \leq p_i \leq 1$ and $0 \leq q_i \leq 1$ for all i . There is a one to one correspondence between the quoted odds and the prices of bets. For example, odds of k to l correspond to the price of $l/(k + l)$.

The bookmaker faces two kinds of risks. The first risk is the uncertainty about his wealth on each horse i , and the second is the uncertainty about which horse wins the race eventually. In this simplified model, we consider a simpler formulation in which the bookmaker's wealth is deterministic.

We assume that the bookmaker instantaneously knows the total money he has already received, denoted as M , and his existing liabilities, denoted as L_i on horse i . Denote W_i as the bookmaker's current wealth on horse i , we have:

$$W_i = M - L_i \quad (6.2.1)$$

When horse i wins (state i occurs) eventually, the bookmaker's wealth on this horse is the difference between the money he has already collected and his liability on this horse.

Given the bookmaker's subjective probability p_i and quote q_i , we assume that the market demand function for horse i , denoted as Q_i , is given by:

$$Q_i = c \frac{p_i}{q_i} + b \quad (6.2.2)$$

where b and c are some positive constants. Clearly, the higher the quote q_i , the lower the demand Q_i . Note that Q_i is also the bookmaker's current liability on horse i in this round of betting.

Denote R_i as the bookmaker's revenue on horse i , we have $R_i = Q_i q_i$. The bookmaker's total revenue is the sum of his revenues over all horses. Using equation (6.2.2), we have:

$$\begin{aligned}
 \sum_{i=1}^N R_i &= \sum_{i=1}^N Q_i q_i \\
 &= \sum_{i=1}^N \left(c \frac{p_i}{q_i} + b \right) q_i \\
 &= \sum_{i=1}^N (c p_i + b q_i) \\
 &= c + ab
 \end{aligned} \tag{6.2.3}$$

where $\sum_{i=1}^N p_i = 1$ and $\sum_{i=1}^N q_i = a > 0$. By construction, the total revenue is constant and independent of q_i , as given by equation (6.2.3). The bookmaker's decision of different quotes q_i affects his current liabilities on different horses through equation (6.2.2), it however does not affect his total revenue. Assuming a constant total revenue is the major simplification we make in this model. With this, we are able to focus on the bookmaker's liability management problem, our primary purpose in this chapter. However, the assumption of the constant revenue is not unrealistic. It is understood that a bookmaker's total revenue is relatively less volatile than his liability exposures over different horses. One can also think of this assumption from the

prospect of bettors rather than the bookmaker. We can imagine that overall, bettors put the same amount of money every round of betting.

The sum of the odds on all horses $\sum_{i=1}^N q_i$ requires some explanation. In a betting market, *Dutch books* generally refer to the portfolios that guarantee a payoff of one but whose price is less than one. To avoid the existence of Dutch books, the summation of all odds $\sum_{i=1}^N q_i$ normally exceeds one and the difference between the sum and one is often called the over-roundness of the book. The over-roundness of the book gives the bookmaker some positive returns for providing the service. We simply assume $\sum_{i=1}^N q_i = a$ where $a > 1$.² The value of a however is restricted by the competition among bookmakers. Our analysis is a partial equilibrium in which the competition results in the return of $(a - 1)$ for each bookmaker for providing the service. One could also imagine that this constrain is set by regulators that requires each bookmaker only has the specified return.

Assuming the bookmaker has a negative exponential utility function, his expected utility of wealth over all horses is given by

$$E[U] = -\frac{1}{\lambda} \sum_{i=1}^N p_i e^{-\lambda w_i} \quad (6.2.4)$$

where λ is the bookmaker's risk aversion parameter.

²Shin (1992, 1993) also requires the similar assumption in his models. He interprets a as the bids posted by different bookmakers and each one submits his bids for monopoly rights to the betting market. The bookmaker who sets the lowest bids wins.

The bookmaker's maximization problem is given by:

$$\begin{aligned} & \max_{q_1, \dots, q_N} E[U] \\ & \text{s.t. } \sum_{i=1}^N q_i = a \end{aligned} \quad (6.2.5)$$

The bookmaker sets odds (q_1, \dots, q_N) for different horses to maximize his expected utility of wealth over all horses, subject to his requirement of the roundness of his book.

The bookmaker's maximization problem can be theoretically solved using the Lagrange method. The analytical solution is however difficult to obtain, partly because the bookmaker's subjective probabilities (p_1, \dots, p_N) enter into the maximization problem, and because his utility function has an exponential form. To focus on the economic intuitions, in the next section, we solve the problem numerically for a small number of horses.

6.2.2 A numerical solution

In this section, we solve the bookmaker's maximization problem (6.2.5) numerically. We first make assumptions of the parameter values in the model. Let's consider a 6-horse race, so $N = 6$. We let the bookmaker's total revenue be 100. Consistent with empirical results, the roundness of the book is assumed to be 1.15, so the bookmaker gets 15% return on the odds he quotes.³ Let c , the constant part of the bookmaker's

³Kuypers (2000) reports that, in football fixed odds betting the over-roundness of the book is remarkably constant at around 11.5% for all the major bookmakers. The average over-roundness in

Table 6.1: **Parameter values used in the numerical solution** This table reports the parameter values used in the numerical solution. N is the number of horses. $\sum_{i=1}^N R_i$ is the total revenue. a is the roundness of the book. c is the constant part of the total revenue. λ is the bookmaker's risk aversion parameter. $p_{1,\dots,6}$ is the bookmaker's subjective probability of each horse winning the race.

N	$\sum_{i=1}^N R_i$	a	c	λ	p_1	p_2	p_3	p_4	p_5	p_6
6	100	1.15	50	0.1	0.30	0.25	0.20	0.15	0.08	0.02

total revenue in equation (6.2.3), be 50. The value of b therefore equals $b = \frac{100-c}{a} = 43.48$. Let the bookmaker's risk aversion parameter be 0.1. Table 6.1 summarizes our assumptions of parameter values. We also need to specify the bookmaker's subjective probabilities p_i for all horses. The sum of all these probabilities needs to be 1. These probabilities are also reported in Table 6.1.

We are interested in how imbalances in his book are corrected. We assume the bookmaker inherits some initial liability positions across different horses and he wishes, by setting appropriate odds, to have a more balanced book in which extremely high liability exposures can be avoided. In this simplified model, the bookmaker only has one chance to re-balance his book. It will be, of course, more interesting to have a multi-stage framework in which the bookmaker has several rounds of opportunities to re-balance his book. It is, however, only interesting if we introduce stochastic components in the bookmaker's wealth levels, which is the focus of the next two sections.

the sample of 3382 games is 11.5% with a standard deviation of only 0.34. In horse races, the over-roundness varies considerably among bookmaker's notional implied profit margin (average 25.63%) and internet betting exchanges (with a maximum of 5%) (Smith, Paton and Vaughan Williams (2006)).

Table 6.2: Bookmaker's quotes: deterministic demands This table reports the bookmaker's odds for different initial liabilities when the individual demands are deterministic. For horse i , p_i is the bookmaker's subjective probability of horse i winning the race; L_i is the initial liability that the bookmaker's inherits; q_i is the bookmaker's odds set for this round of betting; q_i/p_i is the normalized price. Panel A reports the results when the bookmaker's initial liabilities are the same. In Panel B the initial liabilities are highly imbalanced.

		Panel A: Same Initial Liability			Panel B: Different Initial Liabilities		
Horse i	p_i	L_i	q_i	q_i/p_i	L_i	q_i	q_i/p_i
1	0.30	0	0.35	1.15	-20	0.24	0.81
2	0.25	0	0.29	1.15	0	0.26	1.05
3	0.20	0	0.23	1.15	30	0.36	1.79
4	0.15	0	0.17	1.15	10	0.18	1.23
5	0.08	0	0.09	1.15	0	0.08	1.05
6	0.02	0	0.02	1.15	-10	0.02	0.92

Since our simple one-shot model offers useful intuitions, we first analyze its results and use them as the basis for comparison with more complicated formulations.

Here we consider two cases of the bookmaker's initial liability positions. In the first case, the bookmaker inherits a flat book in which his initial liabilities are the same across all horses. In the second case, he inherits different liabilities across different horses. Table 6.2 reports the odds (q_i) and the normalized prices (q_i/p_i) for two initial liability (L_i) distributions.

First, when the initial liabilities are the same, as shown in Table 6.2-Panel A, the odds that the bookmaker quotes are simply the products of his subjective probability (p_i) and the roundness of his book (a). That is, the normalized prices are constant

across all horses and equal to 1.15. Intuitively, since the bookmaker already has a balanced book, his best option is to keep the same liability distribution and simply set odds to satisfy his required over-roundness of his book.

Perhaps the more interesting case is the second one when the bookmaker inherits an imbalanced book, as showed in the Table 6.2-Panel B. Here the bookmaker inherits a highly unbalanced book. The initial liabilities range from a positive 30 to a negative -20. The optimal odds reflect the bookmaker's effort to re-balance his book, as shown by the ratios of q_i/p_i , his normalized prices. Observe that the normalized prices are higher for horses with large initial liabilities (horse 3 and 4), and lower for horses with small initial liabilities (horse 1 and 6). That is, the bookmaker raises the prices for horses with larger initial liabilities and lowers the prices for horses with smaller initial liabilities. Since bets contribute to new liabilities on different horses, he effectively encourages more bets on horses 1 and 6 and less bets on horses 3 and 4. In this way, the bookmaker is able to achieve a more balanced book in which extreme liability exposures are avoided.

6.3 Stochastic liabilities

In this section, we introduce noise into the demand function (6.2.2). We assume that the bookmaker has stochastic demands for each individual horse but still has a constant overall revenue. That is, the bookmaker knows how much he would collect from all horses but is not sure about the exact amount from each horse.

For each horse i , we write the individual revenue \tilde{R}_i as:

$$\tilde{R}_i = cp_i + bq_i + \sigma_i \varepsilon_i \quad (6.3.1)$$

where σ_i is the standard deviation of the noise and ε_i is normally distributed standard random noise with zero mean and unit variance. Note the tilde sign indicates a random variable. The market demand function for each horse is given by:

$$\tilde{Q}_i = \frac{\tilde{R}_i}{q_i} = c\frac{p_i}{q_i} + b + \frac{\sigma_i}{q_i} \varepsilon_i \quad (6.3.2)$$

In the numerical solutions, we examine the effect of two different structures of σ 's on the current liability positions.

We assume that the bookmaker still has a constant overall revenue, the same as before. That is, $\sum_{i=1}^N R_i = c + ab$. Equation (6.3.1) effectively requires $\sum_{i=1}^N \sigma_i \varepsilon_i = 0$. Therefore, for any two different horses i and j , we note that the random noises ε_i and ε_j are not necessarily independently distributed.

The bookmaker's wealth on each horse, \tilde{W}_i , is given by:

$$\tilde{W}_i = M - L_i - \tilde{Q}_i \quad (6.3.3)$$

where M is the total money he has already collected, L_i is the existing liability on horse i and \tilde{Q}_i is the current liability. Clearly here \tilde{W}_i is now no longer deterministic but normally distributed. Let the mean of W_i be M_i and the variance be V_i . The

bookmaker's expected utility is given by:

$$\begin{aligned}
 E[\tilde{U}] &= E\left[-\frac{1}{\lambda} \sum_{i=1}^N p_i e^{-\lambda \tilde{W}_i}\right] \\
 &= -\frac{1}{\lambda} \sum_{i=1}^N p_i E[e^{-\lambda \tilde{W}_i}] \\
 &= -\frac{1}{\lambda} \sum_{i=1}^N p_i e^{-\lambda(M_i - \frac{1}{2}\lambda V_i)}
 \end{aligned}$$

since $e^{-\lambda \tilde{W}_i}$ is lognormally distributed. Computationally it is as easy to optimize this as it was for the deterministic case in the previous section.

The bookmaker's maximization problem is given by:

$$\begin{aligned}
 &\max_{q_1, \dots, q_N} E[\tilde{U}] \\
 &s.t. \sum_{i=1}^N q_i = a
 \end{aligned} \tag{6.3.4}$$

Similarly as in the deterministic case, the bookmaker chooses odds q_1, \dots, q_N for different horses to maximize his expected utility of wealth, subject to his required roundness of the book.

In what follows, we solve the bookmaker's maximization problem for two different σ structures numerically.

Table 6.3: Bookmaker's quotes: stochastic demands with constant σ This table reports the bookmaker's odds for different initial liabilities when the individual demands are stochastic. We assume here the standard deviation σ_i of the individual revenue R_i is constant. For horse i , p_i is the bookmaker's subjective probability of horse i winning the race; L_i is the initial liability that the bookmaker's inherits; q_i is the bookmaker's odds set for this round of betting; q_i/p_i is the normalized prices. Panel A reports the results when the bookmaker's initial liabilities are the same. In Panel B the initial liabilities are highly imbalanced.

		Panel A: Same Initial Liability			Panel B: Different Initial Liabilities		
Horse i	p_i	L_i	q_i	q_i/p_i	L_i	q_i	q_i/p_i
1	0.30	0	0.33	1.11	-20	0.23	0.79
2	0.25	0	0.28	1.11	0	0.26	1.03
3	0.20	0	0.22	1.12	30	0.34	1.71
4	0.15	0	0.17	1.14	10	0.18	1.22
5	0.08	0	0.10	1.23	0	0.09	1.15
6	0.02	0	0.04	2.22	-10	0.04	1.93

6.3.1 First stage: constant σ

Here we assume the standard deviations (σ 's) of the revenue R_i of each individual horse (equation (6.3.1)) are the same and equal to 1.⁴ Table 6.3 reports the bookmaker's quotes when all σ 's are the same. Panel A reports the quotes when the bookmaker inherits the same initial liabilities and Panel B reports the quotes with different initial liabilities.

First notice that in Panel A, when the bookmaker inherits the same initial liabilities, q_i/p_i 's are no longer the same as in the deterministic case (Table 6.2-Panel A).

⁴If we have constant σ 's, they must be quite small to avoid real possibility of negative revenue for some horses.

In particular, when the demand function becomes stochastic, the bookmaker's odds exhibit the favorite-longshot bias. That is, the bookmaker reduces the normalized prices for horses with the high chance of winning (horse 1 and 2) and increases the normalized prices for horses with the low chance of winning (horse 5 and 6). The favorite-longshot bias arises from this model because the constant σ has a disproportional effect on the current liability for each horse. The effect is much stronger for the longshots (e.g., horse 6) than for the favorites (e.g., horse 1) since the noise is more significant in affecting the current liability of longshots. Recall that we have constant σ 's for the revenues and their effect on current liabilities is given by $\frac{\sigma_i}{q_i}$ (equation (6.3.2)). Since the longshots have smaller q_i 's, they have relatively big $\frac{\sigma_i}{q_i}$'s. The longshots therefore get penalized and the normalized prices (q_i/p_i) increase.

Table 6.3-Panel B reports the bookmaker's quotes when he inherits different initial liabilities. Comparing to Table 6.2-Panel B, the normalized prices for horses 1, 2, 3 and 4 are reduced and for horses 5 and 6 are increased. These changes partly reflect the favorite-longshot bias that we have just discussed. Notice that the normalized prices for horses 3 and 4 are still relatively high, reflecting the way that the bookmaker manages his liabilities is that he raises the relative prices for horses with high initial liabilities in an effort to reduce his liabilities over these horses. Of particular interest is that the normalized price for the longshot (horse 6) increases a lot, reflecting the stronger effect of the constant standard deviation of revenue on the least preferable

Table 6.4: Bookmaker's quotes: stochastic demands with proportional σ
This table reports the bookmaker's odds for different initial liabilities when the individual demands are stochastic. We assume here the standard deviation σ_i of individual revenue R_i is proportional to the bookmaker's subjective probability p_i of horse i winning the race. For horse i , p_i is the bookmaker's subjective probability of horse i winning the race; L_i is the initial liability that the bookmaker's inherits; q_i is the bookmaker's odds set for this round of betting; q_i/p_i is the normalized prices. Panel A reports the results when the bookmaker's initial liabilities are the same. In Panel B the initial liabilities are highly imbalanced.

		Panel A: Same Initial Liability			Panel B: Different Initial Liabilities		
Horse i	p_i	L_i	q_i	q_i/p_i	L_i	q_i	q_i/p_i
1	0.30	0	0.35	1.15	-20	0.25	0.82
2	0.25	0	0.29	1.15	0	0.26	1.06
3	0.20	0	0.23	1.15	30	0.35	1.75
4	0.15	0	0.17	1.15	10	0.18	1.23
5	0.08	0	0.09	1.15	0	0.08	1.06
6	0.02	0	0.02	1.15	-10	0.02	0.93

horse.

6.3.2 First stage: proportional σ

Now we assume that σ_i is proportional to the bookmaker's subjective probability p_i . More specifically, we let $\sigma_i = m \times p_i$ and $m = 6$. Table 6.4 reports the bookmaker's quotes when σ_i is proportional to the bookmaker's subjective probability p_i . Panel A reports the quotes when the bookmaker inherits constant initial liabilities and Panel B reports the quotes with different initial liabilities.

- Table 6.4-Panel A shows that when the bookmaker inherits the same initial liabilities, the bookmaker quotes the same odds as in the deterministic case (Table

6.2-Panel A). The normalized prices for all horses are the same and equal to the roundness of his book (1.15). Intuitively, when σ_i is proportional to p_i , the standard deviation of the current liability \tilde{Q}_i is proportional to p_i/q_i (equation (6.3.2)). Therefore, the variance of liability \tilde{Q}_i is constant if the ratio of p_i/q_i is constant. The ratio of p_i/q_i is constant in this formulation, the same as in the deterministic case. The noise in fact does not affect the bookmaker's decision and he quotes the same odds as in the deterministic case. The normalized prices satisfy the bookmaker's required roundness of his book.

Table 6.4-Panel B shows the odds when the bookmaker inherits different initial liabilities. Since in this formulation the standard deviation of the current liabilities \tilde{Q}_i is proportional to the ratio of p_i/q_i , the horses with high p_i/q_i (or low q_i/p_i , the normalized prices) get penalized. Comparing to the deterministic case in Table 6.2-Panel B, clearly, previous bargains (low q_i/p_i , the normalized prices) get penalized and their normalized prices increase (horses 1, 2, 5 and 6). Horses previously with high normalized prices (horses 3 and 4) see improvement in their prices and their prices decrease. We find that horses with high initial liabilities (horses 3 and 4) still have relatively high prices. Therefore the bookmaker still uses odds to influence the public betting flow to manage his book.

6.4 Subsequent stages odds setting

Since a bookmaker typically sets odds for several rounds before the race starts, in this section, we examine the effect of this multistage bets setting in the context of stochastic demands that we have introduced in Section 6.3. It is worth noting that in the subsequent stages the bookmaker continues to maximize the expected utility of terminal wealth, without taking account of probable future trading opportunities. That is, the bookmaker is myopic. This section is divided into two parts. The first subsection deals with the subsequent stages' odds in terms of expected demand functions. The second part introduces noise in calculating the demands. In each subsection, we proceed under our assumptions of the two σ structures, i.e., the constant σ 's and the proportional σ 's.

6.4.1 Expected demands

Here we calculate the multistage odds in terms of the expected market demands. That is, we calculate the expected wealth from equation (6.3.3), which is effectively obtained by taking the expectation of the market demand function (6.3.2). In any rounds of bets setting, given his initial liability positions, the bookmaker sets current odds and obtains his current wealth on each horse. His current wealth is essentially the negative value of his liability for each horse and we take the negative value of the current wealth as the the initial liability in the next round. Since the noise has a

zero mean, when we take the expectation of the demand function (equation (6.3.2)), the random noise itself does not affect the wealth, although the variance of noise affects the odds through the bookmaker's maximization problem (6.3.4). Table 6.5 reports the multistage odds under our assumption of two σ structures. In each case, we let the bookmaker start with the same unbalanced book and see how his book evolves as he has five (5) chances to set his odds (from L_0 to L_4). We also report the normalized prices q_i/p_i for every horse in each round of odds setting. Figure 6.1 shows the distributions of the book and the normalized prices. Numbers 1 - 5 indicate the first round to the fifth round of bets setting. Note that the bookmaking in this model is quite profitable. The bookmaker obtains positive wealth (negative liabilities) over all horses after two rounds of bets setting.⁵

Constant σ

Table 6.5-Panel A reports the odds that the bookmaker sets when σ 's are constant and equal to 1. Note that negative liabilities correspond to positive wealth. So it is beneficial to the bookmaker to have only negative liabilities (positive wealth). We observe that after the first round of betting, the bookmaker's liabilities on all horses except horse 3 are negative. After the second round of betting, all his liabilities are negative.

As we have discussed, constant σ 's introduce the favorite-longshot bias and it is

⁵There is however no clear relationship between the normalized prices and the liabilities for different horses, possibly because that the risk component (V_i) of the bookmaker's utility function has a stronger effect than the endowment component (M_i) in his maximization problem.

Table 6.5: Multistage odds setting: expected demands This table reports the odds that the bookmaker sets for five rounds of betting. L indicates the bookmaker's liability positions over 6 horses. For horse i , $L0_i$ is the initial liability and $L1_i - L4_i$ are subsequent liabilities; q_i/p_i is the normalized price. SD is the standard deviation of respective liabilities position. Panel A reports the odds that the bookmaker sets when σ 's are constant. Panel B reports the odds when σ_i is proportional to p_i .

Panel A: Odds with constant σ

Horse	p_i	$L0_i$	q_i/p_i	$L1_i$	q_i/p_i	$L2_i$	q_i/p_i	$L3_i$	q_i/p_i	$L4_i$	q_i/p_i
1	0.30	-20.00	0.79	-13.24	1.02	-20.80	1.10	-31.98	1.13	-44.29	1.13
2	0.25	0.00	1.03	-7.88	1.11	-19.33	1.13	-31.70	1.14	-44.38	1.14
3	0.20	30.00	1.71	2.68	1.32	-15.99	1.20	-30.89	1.16	-44.44	1.16
4	0.15	10.00	1.22	-5.57	1.18	-19.57	1.16	-32.80	1.15	-45.79	1.15
5	0.08	0.00	1.15	-12.95	1.15	-25.99	1.15	-38.90	1.15	-51.77	1.15
6	0.02	-10.00	1.93	-40.60	1.69	-67.53	1.54	-91.63	1.45	-113.55	1.45
SD		17.22		14.76		19.54		20.10		27.67	

Panel B: Odds with proportional σ

Horse	p_i	$L0_i$	q_i/p_i	$L1_i$	q_i/p_i	$L2_i$	q_i/p_i	$L3_i$	q_i/p_i	$L4_i$	q_i/p_i
1	0.30	-20.00	0.82	-15.68	1.03	-23.65	1.11	-35.03	1.14	-47.50	1.14
2	0.25	0.00	1.06	-9.35	1.13	-21.51	1.14	-34.29	1.15	-47.25	1.15
3	0.20	30.00	1.75	1.99	1.35	-17.41	1.22	-32.84	1.17	-46.74	1.16
4	0.15	10.00	1.23	-5.87	1.19	-20.29	1.16	-33.86	1.16	-47.10	1.15
5	0.08	0.00	1.06	-9.35	1.13	-21.51	1.14	-34.29	1.15	-47.25	1.15
6	0.02	-10.00	0.93	-12.60	1.08	-22.62	1.12	-34.67	1.14	-47.38	1.15
SD		17.22		6.11		2.16		0.76		0.27	

clearly shown by the normalized prices of q_i/p_i in Table 6.5-Panel A. The normalized

prices are generally higher for the longshot, horse 6, and lower for the favorites, horse

- After five rounds of betting, the normalized prices are close to the roundness of the book (1.15) for all horses except horse 6.

As the bookmaker continues to re-balance his book over time, his book becomes less volatile in general. The standard deviation of his book decreases from 17.22 of

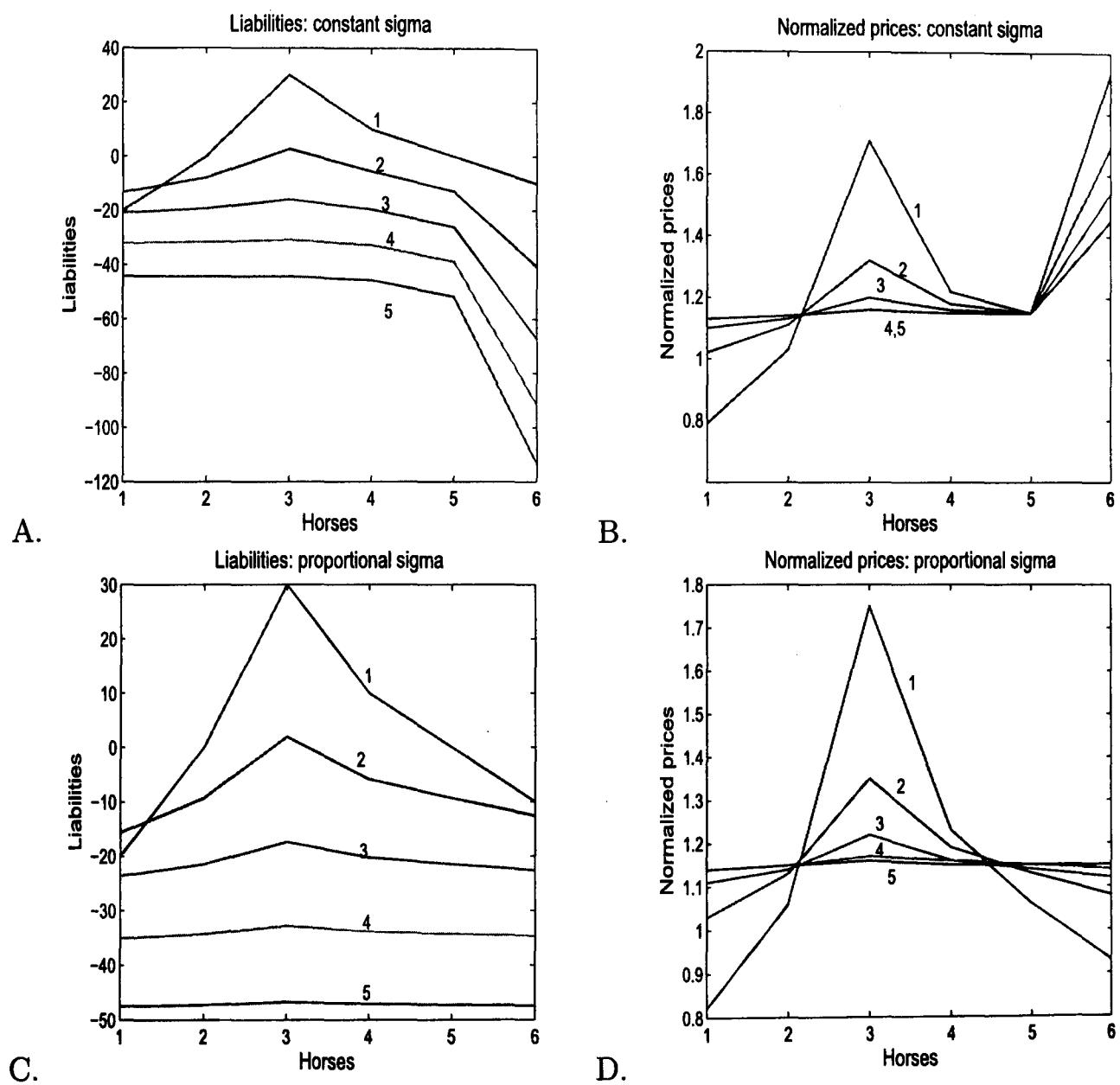
the initial book to 14.76 in the second round. After that, the standard deviation of his book gradually increases slightly. Close observation reveals that the increase in the standard deviations is due to the strong negative liability of horse 6 (the standard deviation of the liabilities on horse 1 - 5 is only 3.21 in the fifth round), which comes from the negative initial liability and more importantly the huge variance of the current liability on horse 6.⁶

Proportional σ

Table 6.5-Panel B reports the bookmaker's odds when σ_i is proportional to his subjective probability p_i . As we have discussed previously, compared to the deterministic case, proportional σ 's of stochastic demands penalize the previous bargains and raise their prices but do not introduce the favorite-longshot bias. As shown by the results in Panel B, over several rounds, the bargains in the previous rounds always get penalized in the next round and as a consequence, all normalized prices are *pulled* towards the roundness of the book (1.15). Furthermore, the absence of the favorite-longshot bias helps the bookmaker to balance his book more quickly than in the case when σ 's are constant. The standard deviation decreases from 17.22 of the initial book to 0.27 in the fifth round.

⁶Recall that the standard deviation of the current liability in equation (6.3.2) is $\frac{\sigma_i}{q_i}$. Since horse 6 has the smallest odds (q_6), given the constant σ 's, it has the biggest standard deviation.

Figure 6.1: Distributions of books and normalized prices: expected demand functions These figures show the distributions of liability positions and normalized prices over 5 rounds of bets setting for 6 horses. Panel A, B (C, D) reports the distributions of liabilities and normalized prices when σ 's are constant (proportional to p_i 's). Numbers 1-5 indicate the 1st to 5th round of odds setting.



6.4.2 Noisy demands

Now we calculate the true stochastic demands using equation (6.3.2). The noise $\sigma_i \varepsilon_i$ in the equation of individual revenue (6.3.1) now affects the bookmaker's current liabilities as well as the remaining liability in each round of bets setting. In Appendix C, we simulate the noise $\sigma_i \varepsilon_i$ that satisfies $\sum_{i=1}^N \sigma_i \varepsilon_i = 0$. Table 6.6 reports the odds that the bookmaker sets with noisy demands under our assumption of two σ structures. Figure 6.2 illustrates the distributions of the book and the normalized prices. Numbers 1 - 5 indicate the first round to the fifth round of bets setting. Similarly to the case of expected demands, the bookmaker makes quite a lot of profits in this model. After two rounds of bets setting, the bookmaker obtains positive wealth (negative liabilities) over all horses.

Constant σ

Table 6.6-Panel A reports the odds that the bookmaker sets with stochastic demands when σ 's are constant. The same as in the case of expected demands, after two rounds of bets setting, the bookmaker gets rid of all positive liability exposures. Comparing to the case of the expected demands, the noise in the demands makes the overall book more volatile. For example, in the last round, the standard deviation of the liabilities over horse 1-5 is now 10.36, compared to 3.21 in the case of the expected demands. The strong negative liability on horse 6 increases the overall standard deviations of the book over the last few rounds.

Figure 6.2: Distributions of books and normalized prices: noisy demand functions This figure shows the distributions of liability positions and normalized prices over 5 rounds of bets setting for 6 horses. Panel A, B (C, D) reports the distributions of liabilities and normalized prices when σ 's are constant (proportional to p_i 's). Numbers 1-5 indicate the 1st to 5th round of odds setting.

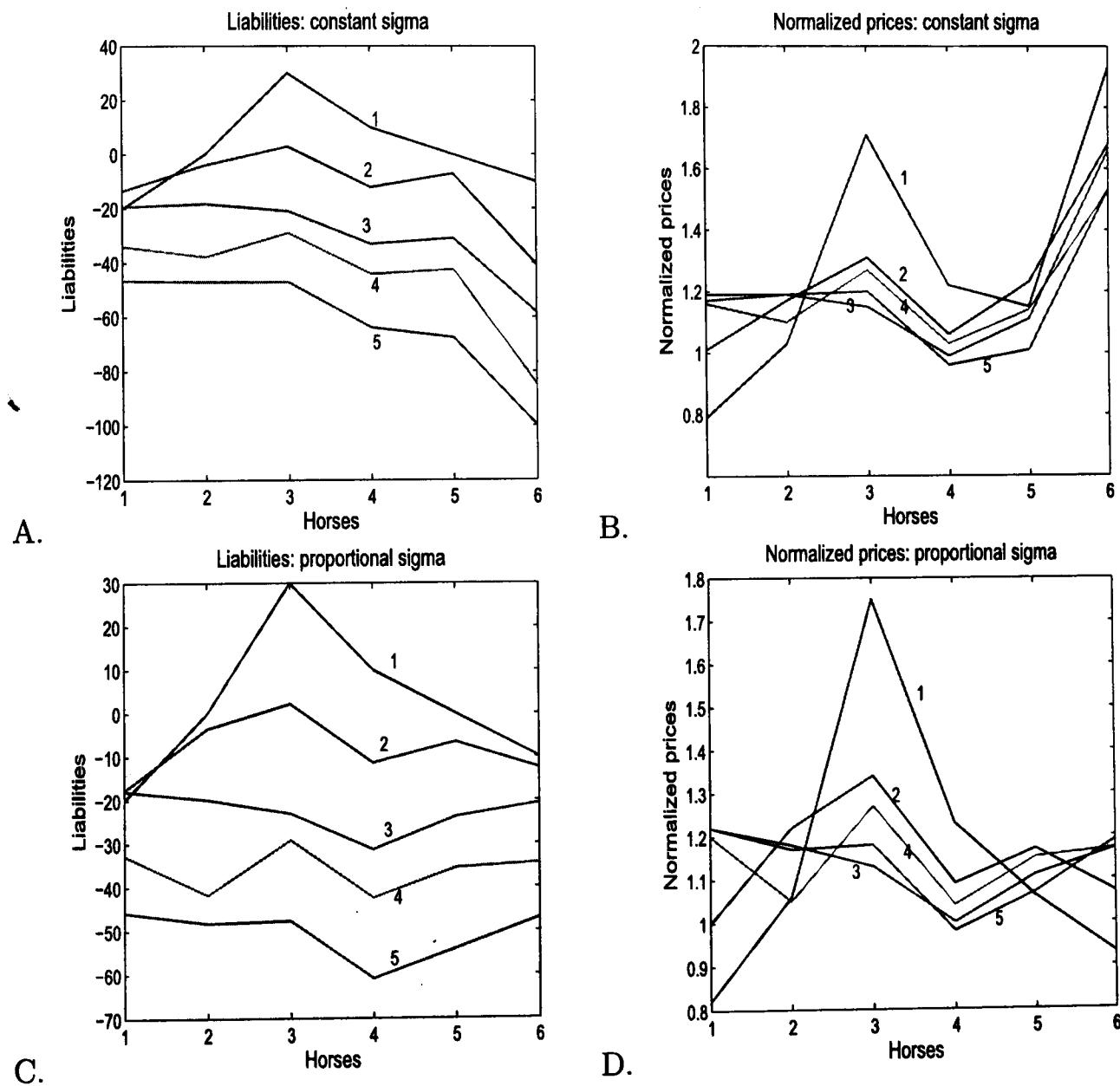


Table 6.6: Multistage odds setting: noisy demands This table reports the odds that the bookmaker sets for five rounds of betting. L indicates the bookmaker's liability positions over 6 horses. For horse i , $L0_i$ is the initial liability and $L1_i - L4_i$ are subsequent liabilities; q_i/p_i is the normalized price. SD is the standard deviation of respective liabilities position. Panel A reports the odds that the bookmaker sets when σ 's are constant. Panel B reports the odds when σ_i is proportional to p_i .

Panel A: Odds with constant σ

Horse	p_i	$L0_i$	q_i/p_i	$L1_i$	q_i/p_i	$L2_i$	q_i/p_i	$L3_i$	q_i/p_i	$L4_i$	q_i/p_i
1	0.30	-20.00	0.79	-13.55	1.01	-19.30	1.17	-33.96	1.16	-44.69	1.19
2	0.25	0.00	1.03	-3.99	1.17	-18.25	1.19	-37.78	1.10	-47.10	1.19
3	0.20	30.00	1.71	2.86	1.31	-20.93	1.15	-29.01	1.27	-47.16	1.20
4	0.15	10.00	1.22	-12.19	1.06	-33.05	0.99	-44.24	1.03	-63.85	0.96
5	0.08	0.00	1.15	-7.16	1.23	-30.99	1.11	-42.53	1.14	-67.62	1.01
6	0.02	-10.00	1.93	-40.74	1.68	-58.41	1.66	-84.82	1.53	-100.03	1.53
SD		17.22		15.07		15.18		20.10		20.78	

Panel B: Odds with proportional σ

Horse	p_i	$L0_i$	q_i/p_i	$L1_i$	q_i/p_i	$L2_i$	q_i/p_i	$L3_i$	q_i/p_i	$L4_i$	q_i/p_i
1	0.30	-20.00	0.82	-17.66	1.00	-17.90	1.22	-32.72	1.20	-45.78	1.22
2	0.25	0.00	1.06	-3.51	1.22	-19.87	1.18	-41.77	1.05	-48.23	1.17
3	0.20	30.00	1.75	2.20	1.34	-23.11	1.13	-29.16	1.27	-47.70	1.18
4	0.15	10.00	1.23	-14.45	1.09	-31.42	1.00	-42.47	1.04	-60.92	0.98
5	0.08	0.00	1.06	-6.57	1.17	-23.87	1.11	-35.59	1.15	-54.08	1.07
6	0.02	-10.00	0.93	-12.63	1.07	-20.72	1.17	-34.44	1.17	-47.02	1.19
SD		17.22		7.11		4.74		5.20		5.81	

The noise also makes the normalized prices more volatile, as shown by comparing

Figure 6.2-Panel B with Figure 6.1-Panel B. The favorite-longshot bias still exists in the constant σ case. The normalized prices are higher for the longshot (horse 6) and lower for the favorite (horse 1).

With the stochastic demands, the bookmaker has a more difficult job of making his book. In the deterministic case, the bookmaker worries about the balance of his

book and needs to deliberately set odds in such a way to influence the public betting flow to achieve a more balanced book. The bookmaker also worries about trading with informed traders since his subjective prediction of the winner of the race might be wrong. With the stochastic demand functions, even the random shocks in the demand functions are costly to the bookmaker, as these shocks complicate the signal in the betting flow information and and more importantly, they make the book less balanced.

Proportional σ

Table 6.6-Panel B reports the multistage odds with proportional σ 's. As we have discussed, proportional σ 's penalize the previous bargains and raise their prices. The normalized prices are generally pulled towards the roundness of the book (1.15).

Without the favorite-longshot bias, the liability positions are less volatile compared to the case of constant σ 's. The standard deviations are reduced to around 5 to 6 after five rounds of betting as compared to around 20 in the case of constant σ 's. Of course, the noise in demand functions makes both the book and the normalized prices more volatile compared to the case of expected demands (Figure 6.1, Panel C, D and Figure 6.2, Panel C, D).

6.5 Conclusion

This chapter provides a simple model of horse race bookmaking. The bookmaker worries about trading with informed traders since the betting demands are noisy and

he may not know the correct odds to quote. He also worries about his unbalanced book since he may have high liability exposures over some horses. The noisy demands complicate the signal in the bookmaker's betting flow observation and make his book less balanced. Even random shocks from noisy traders are costly to the bookmaker since his book could become unbalanced.

In our model, the bookmaker revises his odds to mitigate the risk. He influences the public betting flow by raising the normalized prices for horses with high initial liabilities and lowering the normalized prices for horses with low initial liabilities. The bettors find less attractive prices for horses with larger initial liabilities and therefore place smaller bets on these horses. Through this new round of betting, the bookmaker achieves a more balanced book. Allowing the bookmaker to set several rounds of odds before the race starts gives a clear view of the bookmaker's odds setting strategy and its impact on the public betting flow over time.

Our model gives other interesting results. In particular, the favorite-longshot bias arises in our model naturally as a consequence of our assumption of the constant standard deviation of the revenue on each horse. Shin (1991, 1992, 1993) argues that the favorite-longshot bias arises from information asymmetry in betting markets. It is therefore not obvious whether the favorite-longshot bias could arise from specific model formulations or necessarily from information asymmetry as in Shin's model.

More importantly, our model helps to understand the complexity of managing a

series of state contingent exposures such as options. A bookmaker in a horse betting market and an option market maker face similar problems of managing their state contingent exposures. In this model, the bookmaker has a clear idea of the expiry date of his state contingent claims (when the race finishes and the winner is declared). At any point in time before the race starts, the bookmaker knows exactly what his losses would be for every horses. This clear expiry date makes the bookmaker's operation considerably easier as compared to the problem faced by an option market maker.

In reality, an option market maker faces a complicated problem of market making. He must use the order flow to estimate the true value of the underlying asset (or, more precisely, the true implied volatility of the options). He also tries to use the order flow on some options to influence the order flow of other options since option contracts are closely related securities. Furthermore, he must set bid and ask prices to flat his book (the portfolio of his option contracts) to avoid any large option exposures.

Clearly, an option market maker faces a combination of the problems of our models in Chapter 4 and Chapter 6. In Chapter 4, we provides a model of how a market maker uses his knowledge of the correlated order flow information to improve his estimation of the unknown price process. In Chapter 6, we provides a model of how a bookmaker manages his state contingent claims which have a clear and fixed expiry date. In both models, the market maker and the bookmaker face noisy order flows or betting flows. Furthermore, the market maker in Chapter 4 has a learning problem

to solve while the bookmaker in Chapter 6 knows a clear expiry date of his exposures. Of course, our model in this chapter is not a complete one for modeling option market making. But the combination of the insights from both chapters gives a more clearer picture of the complex market making problem that an option market maker faces.

It will be interesting to extend the bookmaking model in this chapter to include the bookmaker's learning problem. The bookmaker must be able to learn the true probabilities of the horses winning the race from his noisy betting flow observation. Furthermore, it will be nice to have a model in which odds themselves change stochastically. By incorporating these two issues, our model would provide a more complete model of market making state contingent claims.

Chapter 7

Conclusion

This thesis has studied four related topics on market making, an area of great importance to regulators, practitioners and academics. We are interested in the normative analysis of market making and in particular, market making options. Models in this thesis provide important insights of what option market makers are juggling with, especially the complicated problems they face and the nature of the risk they are exposed to. In this chapter, we briefly summarize the key findings of our work and indicates potential directions for future research.

7.1 Summary and conclusions

In Chapter 3 we develop and analyze a simple model of stock market making in which a monopolistic market maker solves both the problems of inventory management and asymmetric information. There are two distinctive features in this model. First, the market maker in this model is exposed to the inventory risk. He tries to manage

his inventory to an acceptable level. Second, the market maker is exposed to market asymmetric information. Without knowing whom he is trading with, the market maker updates his estimates of the stock's true value through his order flow observation. In a Bayesian updating framework, our market maker's estimation problem can be summarized into a particular state-space form called the Kalman filter. The Kalman filter describes how the information in the market maker's order flow observation is impounded into his price setting process. We contribute to the literature by combining the inventory risk and asymmetric information into a simple dynamic setting and analyzing the properties under such a setting.

In this model, the risk aversion of the market maker has an important effect on the market making profitability. With increasing risk aversion, both the means and the standard deviations of the market maker's expected cumulative profits decrease. Therefore the market maker in our model is willing to give up much of his profits to control the risk of his inventory positions. Inventory is risky and consumes capital. Our model provides a simple way for market makers to optimally process information to control inventory.

The extension of the model to the continuous time provides useful insights into our stochastic system. The system of three state variables describes the process of how the market maker's estimation error changes his inventory positions and reduces his cumulative profits. In particular, the market maker deliberately biases his bid

and ask prices to move his inventory to the long run mean of zero. His pricing bias also greatly reduces the market maker's profitability. The market maker therefore has enormous interests to process information correctly, and under the assumptions in this model, the Kalman filter provides the optimal method to estimate the true price.

Motivated by the problem of market making options at several different strikes, where the commonality can be quite extreme, in Chapter 4 we extend the single stock market making model in Chapter 3 to study the market making of multiple stocks. The market maker still does not know the each stock's true value but knows the return covariance structure of these stocks. In particular, our assumption of the stocks' return covariance structure in this chapter is intended to represent the correlation in volatilities, an important feature in option contracts. The market maker's knowledge of the return covariance affects his price estimations, which in turn affects the market demands. If the market maker learns information form the order flow of other stocks, his estimation of the prices of every stock improves. In particular, the better estimates help the market maker value his inventory positions more accurately, which reduces the risk of his inventories. Better estimates also help mitigate information asymmetry. The market maker's knowledge of the return covariance structure, together with his ability to use the correlated order flow information in his price estimation, improves his market making profitability. An interesting result from the model is that the

correlated market demands give rise to liquidity commonality, an important topic itself. Our paper provides another support to the empirical findings by Coughenour and Saad (2004) who claim that common market makers are one reason for liquidity commonality.

In Chapter 5 we move our attention to the informational link between financial markets. We analyze the effect of an option market maker's hedging on the informed trading strategy and the subsequent changes in the costs of liquidity provision in stock and option markets. We find that with hedging, the OMM might inadvertently trade in the same direction as informed traders would do. Hence the hedging trades contain information. The SMM (or other OMMs) responds to this increasing threat of informed trading by setting wider stock (option) spreads. As the OMM takes stock spreads as given when he hedges he pays additional transaction costs when hedging that overweigh some recovery of information loss in making a market for informed traders, and this results in wider option spreads. In summary, the costs of liquidity provision in the two markets increase and both informed and uninformed traders face higher transaction costs. Although informed traders find it might be easier to hide their identity in the underlying market with OMM's hedging, the overwhelmingly increased transaction costs in the two markets make their private signal less profitable when compared to the case where the OMM does not hedge.

With the introduction of several options, informed traders have additional choices

to exploit their informational advantage. We consider two different hedging cases: (1) the OMM hedges in the underlying stock market, and (2) all OMMs hedge using the option market. When the OMM hedges using the underlying, we find that the OMM's hedging trade is less informative and both stock and option spreads are narrower. The reduction in option spreads is due to the convex structure of the option's payoff, which helps to reduce the hedging costs. In the case of stocks, the less informative hedging trade eases the informational threat from the OMM's hedging trade, and this helps reduce the stock spreads. We then compare the equilibrium spreads between the second and the first cases. We find that when OMMs hedge using the option markets, the spreads of both stock and in-the-money options are greatly reduced, while spreads of at-the-money options widen slightly. Less informed trading in stocks and no informed trading threat from the OMMs' hedging trade help to reduce the stock spreads. For OMM1, hedging using options is more effective in transferring the threat of informed trading to the other option, leading to an increase in the spreads of that option.

Motivated by the problems of market making state contingent claims, in Chapter 6, we develop a simple model of horse race bookmaking. The bookmaker worries about trading with informed traders since the betting demands are noisy and he may not know the correct odds to quote. He also worries about his unbalanced book since he may expose to high liability positions over some horses. The noisy demands

complicate the signal in the bookmaker's betting flow observation and make his book less balanced. Even random shocks from noisy traders are costly to the bookmaker since his book could become unbalanced.

In our model, the bookmaker revises his odds to mitigate the risk. He influences the public betting flow by raising the normalized prices for horses with high initial liabilities and lowering the normalized prices for horses with low initial liabilities. The bettors find less attractive prices for horses with larger initial liabilities and therefore place smaller bets on these horses. Through this new round of betting, the bookmaker achieves a more balanced book. Allowing the bookmaker to set several rounds of odds before the race starts gives a clear view of the bookmaker's odds setting strategy and its impact on the public betting flow over time.

Our model gives other interesting results. In particular, the favorite-longshot bias arises in our model naturally as a consequence of our assumption of the constant standard deviation of the revenue on each horse. Shin (1991, 1992, 1993) argues that the favorite-longshot bias arises from information asymmetry in betting markets. It is therefore not obvious whether the favorite-longshot bias could arise from specific model formulations or necessarily from information asymmetry as in Shin's model.

More importantly, our model helps to understand the complexity of managing a series of state contingent exposures such as options. A bookmaker in a horse betting market and an option market maker face similar problems of managing their state

contingent exposures. In this model, the bookmaker has a clear idea of the expiry date of his state contingent claims (when the race finishes and the winner is declared). At any point in time before the race starts, the bookmaker knows exactly what his losses would be for every horses. This clear expiry date makes the bookmaker's operation considerably easier as compared to the problem faced by an option market maker.

In reality, an option market maker faces a complicated problem of market making. He must use the order flow to estimate the true value of the underlying asset (or, more precisely, the true implied volatility of the options). He also tries to use the order flow on some options to influence the order flow of other options since option contracts are closely related securities. Furthermore, he must set bid and ask prices to flat his book (the portfolio of his option contracts) to avoid any large option exposures.

Clearly, an option market maker faces a combination of the problems of our models in Chapter 4 and Chapter 6. In Chapter 4, we provides a model of how a market maker uses his knowledge of the correlated order flow information to improve his estimation of the unknown price process. In Chapter 6, we provides a model of how a bookmaker manages his state contingent claims which have a clear and fixed expiry date. In both models, the market maker and the bookmaker face noisy order flows or betting flows. Furthermore, the market maker in Chapter 4 has a learning problem to solve while the bookmaker in Chapter 6 knows a clear expiry date of his exposures. The combination of the insights from both chapters gives a more clearer picture of

the complex market making problem that an option market maker faces.

7.2 Directions for future research

Our objective in this thesis is to provide a normative analysis of how a market maker should behave in financial markets. In particular, we have developed different approaches to study the market making in an option market. Compared to stocks, the most distinctive feature of options is that they are closely related securities, i.e., a parameterized family of substitutable securities. Options with the same underlying asset but different strikes are some examples. It will be of interest to extend some of the modeling techniques used in this thesis to better characterize the complexity of option market making in practice.

There are a number of ways in which our study can be extended. First, the Kalman filter algorithm that we have used in Chapter 3 and 4 has a nice structure capable of analyzing more complicated problems. Of particular interest is to apply the technique to study option market marking. How information is generated and disseminated at different strikes and how market makers manage the inventory of portfolios of different option contracts are interesting research topics. The Kalman filter provides a constructive way of analyzing these questions. In fact, our result in Chapter 4 provides the first step towards this direction. We have shown that the market maker obtains better estimates of the unknown price processes of multiple securities by considering the correlated order flow information through his Kalman

filter updating.

Second, in Chapter 3, we assume that the market maker is myopic by maximizing his expected utility of per period of profit. This assumption simplifies the calculation since at any point in time, the market maker only cares about his profit and risk in the current period. An alternative formulation is that the market maker maximizes his overall expected utility of per period profit over the entire time horizon. Comparing to equation (3.2.9), the alternative maximization problem is given by:

$$\max_{a_t, b_t} E \left[\sum_{t=1}^{\infty} U(\tilde{\pi}_t) \right]$$

The market maker may have interesting intertemporal strategic behavior under this alternative formulation since the optimal bid and ask prices of the current period may affect his expected utility of the later periods. It is therefore worth investigating the market maker's optimal price setting strategy under this alternative formulation of his maximization problem.

Third, in Chapter 5 we only consider the simplest hedging by an option market maker: the delta hedging. This hedging strategy only provides a temporary protection against small price movements in the underlying market. In practice, more complicated hedging strategies such as gamma and vega hedging have been widely used. It is of interest to study the effects of these hedging strategies on the informed trading behavior and how information is transmitted between the related markets, both theoretically and empirically. Furthermore, we only consider European options

which can be exercised at maturity. How the early exercise option of American options affects informed trading strategies remains to be analyzed.

Finally, our model in Chapter 6 provides a clear view of how a bookmaker sets odds over time to balance his state contingent obligations. Given the similarities between option market making and horse race bookmaking, a horse race betting market provides a convenient channel to study option market making. It will be interesting to extend the bookmaking model in Chapter 6 to include the bookmaker's learning problem. The bookmaker must be able to learn the true probabilities of the horses winning the race from his noisy betting flow observation. Furthermore, it will be nice to have a model in which odds themselves change stochastically. Incorporating these issues would contribute to our understanding of how an option market maker should behave in the complex market making environment.

Appendix A

Proof of results in Chapter 3

A.1 Proof of Proposition 3.1

From equations (3.2.10), we obtain the expectation and the variance of the profits as in equations (3.3.1) and (3.3.2). The coefficients $f_{1t}(\delta_{a_t}, \delta_{b_t}, \hat{p}_t)$ and $f_{2t}(\delta_{a_t}, \delta_{b_t}, \hat{p}_t)$ are as follows:

$$\begin{aligned} f_{1t}(\delta_{a_t}, \delta_{b_t}, \hat{p}_t) &= 4[(\alpha + \beta)\hat{p}_t + \alpha\delta_{a_t} - \beta\delta_{b_t}]^2 - 8(\alpha + \beta)[(\alpha + \beta)\hat{p}_t + \alpha\delta_{a_t} - \beta\delta_{b_t}]\hat{p}_t \\ f_{2t}(\delta_{a_t}, \delta_{b_t}, \hat{p}_t) &= [I_t + (\alpha + \beta)\hat{p}_t + \alpha\delta_{a_t} - \beta\delta_{b_t}]^2 - 2(\alpha + \beta)[I_t + (\alpha + \beta)\hat{p}_t + \alpha\delta_{a_t} - \beta\delta_{b_t}]\hat{p}_t \end{aligned}$$

Substituting equations (3.3.1) and (3.3.2) back into the maximization problem (3.2.11), and taking first order conditions with respect to δ_{a_t} and δ_{b_t} , we have a system of linear equations as follows:

$$\begin{aligned} Q_0 - [2\alpha + \lambda(4\alpha^2 S_t + \alpha^2 \sigma_\mu^2 + \sigma_B^2)]\delta_{a_t} + \lambda\alpha\beta(4S_t + \sigma_\mu^2)\delta_{b_t} - \lambda\alpha\sigma_\mu^2 I_t &= 0 \\ Q_0 - [2\beta + \lambda(4\beta^2 S_t + \beta^2 \sigma_\mu^2 + \sigma_S^2)]\delta_{b_t} + \lambda\alpha\beta(4S_t + \sigma_\mu^2)\delta_{a_t} - \lambda\beta\sigma_\mu^2 I_t &= 0 \end{aligned}$$

Solving the above equations through substation, we obtain

$$\begin{aligned}\delta_{a_t} &= K\{[2\beta + \lambda\beta(\alpha + \beta)(4S_t + \sigma_\mu^2) + \lambda\sigma_S^2]Q_0 - \lambda\alpha\sigma_\mu^2(2\beta + \lambda\sigma_S^2)I_t\} \\ &= A_{1_t}Q_0 - A_{2_t}I_t \\ \delta_{b_t} &= K\{[2\alpha + \lambda\alpha(\alpha + \beta)(4S_t + \sigma_\mu^2) + \lambda\sigma_S^2]Q_0 + \lambda\beta\sigma_\mu^2(2\alpha + \lambda\sigma_B^2)I_t\} \\ &= B_{1_t}Q_0 + B_{2_t}I_t\end{aligned}$$

where $K = [4\alpha\beta + 2\lambda(\alpha\sigma_S^2 + \beta\sigma_B^2) + \lambda^2\sigma_S^2\sigma_B^2 + (4S_t + \sigma_\mu^2)(2\lambda\alpha\beta^2 + 2\lambda\alpha^2\beta + \lambda^2\beta^2\sigma_B^2 + \lambda^2\alpha^2\sigma_S^2)]^{-1}$ is a positive function of S_t .

The bid and ask prices are given by

$$a_t = \hat{p}_t + \delta_{a_t}$$

$$b_t = \hat{p}_t - \delta_{b_t}$$

Substituting δ_{a_t} and δ_{b_t} gives the results. \square

A.2 Proof of Corollary 3.2

The total bid-ask spread is the difference between the bid and and ask prices and has the following expression:

$$\delta_{a_t} + \delta_{b_t} = K\{[2(\alpha + \beta) + \lambda(\alpha + \beta)^2(4S_t + \sigma_\mu^2) + \lambda(\sigma_S^2 + \sigma_B^2)]Q_0 + \lambda^2\sigma_\mu^2(\beta\sigma_B^2 - \alpha\sigma_S^2)I_t\}$$

where K has the same expression as in the Proof of Proposition 3.1 and is a positive function of S_t . \square

A.3 Proof of Proposition 3.3

The pricing bias (PB) is obtained as follows:

$$PB \equiv \frac{1}{2}(a_t + b_t) - \hat{p}_t = \frac{1}{2}(\delta_{a_t} - \delta_{b_t}) = -k_t \lambda \theta \sigma_\mu^2 (2\theta + \lambda \sigma^2) I_t$$

where we denote $f(\lambda) = k_t \lambda \theta \sigma_\mu^2 (2\theta + \lambda \sigma^2)$ and $k_t = [4\theta^2 + 4\lambda \theta \sigma^2 + \lambda^2 \sigma^4 + (4S_t + \sigma_\mu^2)(4\lambda \theta^3 + 2\lambda^2 \theta^2 \sigma^2)]$ is a positive function of S_t and λ . \square

A.4 Proof of Corollary 3.4

Denote the constant spread as $2m$ where m is a positive number, then bid and ask prices has relationship as $a_t - b_t = 2m$. The pricing bias therefore is $b_t + m - \hat{p}_t$.

Following the same line of calculations to maximize the expected utility of per period profit, we obtain the optimal PB' as

$$\begin{aligned} PB' &= \kappa_t \{-\lambda(\alpha + \beta)\sigma_\mu^2 I_t - [2(\alpha - \beta) + \lambda(\alpha^2 - \beta^2)(4S_t + \sigma_\mu^2) + \lambda(\sigma_B^2 - \sigma_S^2)]m\} \\ &= -M_t I_t - N_t m \end{aligned}$$

where $\kappa_t = [2(\alpha + \beta) + \lambda(4(\alpha + \beta)^2(S_t + \sigma_\mu^2) + \sigma_B^2 + \sigma_S^2)]$ is a positive function of S_t . \square

Appendix B

Simulation of random variables that has a covariance matrix V in Chapter 4

In this section, we show how to simulate n random variables \tilde{r}_j with the following covariance matrix:

$$\mathbf{V} = \begin{bmatrix} a+b & b\theta & b\theta^2 & \dots & b\theta^{n-2} & b\theta^{n-1} \\ b\theta & a+b & b\theta & \dots & b\theta^{n-3} & b\theta^{n-2} \\ b\theta^2 & b\theta & a+b & \dots & b\theta^{n-4} & b\theta^{n-3} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ b\theta^{n-2} & b\theta^{n-3} & b\theta^{n-4} & \dots & a+b & b\theta \\ b\theta^{n-1} & b\theta^{n-2} & b\theta^{n-3} & \dots & b\theta & a+b \end{bmatrix}$$

i.e., where $v_{i,j} = a\delta_{i,j} + b\theta^{|i-j|}$ (using Kronecker delta notation: $\delta_{i,j} = 1$ for $i = j$, else 0), and a, b are positive.

Consider variables generated as:

$$\tilde{r}_j = \tilde{v}_j + k \sum_{s=0}^{\infty} \theta^s \tilde{u}_{j-s}$$

where $\tilde{v}_j \sim N(0, a)$, $\tilde{u}_i \sim N(0, 1)$ are all independent, and \tilde{v}_i, \tilde{v}_j are also independent.

Then,

$$\tilde{r}_i \tilde{r}_j = \tilde{v}^2 + k^2 \sum_{s=0}^{\infty} \theta^{2s+|i-j|} \tilde{u}_{\min(i,j)-s}^2 + \text{cross terms}$$

leading to

$$\begin{aligned} E[\tilde{r}_i \tilde{r}_j] &= a\delta_{ij} + k^2 \sum_{s=0}^{\infty} \theta^{2s+|i-j|} \\ &= a\delta_{ij} + \frac{k^2}{1-\theta^2} \theta^{|i-j|} \\ &= a\delta_{ij} + b\theta^{|i-j|} \end{aligned}$$

by choosing $k = \sqrt{b(1-\theta^2)}$

At first it appears that we need to generate an infinite number of \tilde{u}_i 's (for $i = n, n-1, \dots, 2, 1, 0, -1, -2, \dots, -\infty$) in order to do this.

Fortunately closer inspection reveals that it is not so bad. We generate the sum required for the first random variable as:

$$\tilde{w}_1 = \sum_{s=0}^{\infty} \theta^s \tilde{u}_{1-s}$$

as a single drawing from $N(0, \frac{1}{1-\theta^2})$.

The variables can therefore be generated as:

$$\begin{aligned}
 \tilde{r}_1 &= \tilde{v}_1 + k\tilde{w}_1 \\
 \tilde{r}_2 &= \tilde{v}_2 + k(\theta\tilde{w}_1 + \tilde{u}_2) \\
 \tilde{r}_3 &= \tilde{v}_3 + k(\theta^2\tilde{w}_1 + \theta\tilde{u}_2 + \tilde{u}_3) \\
 &\vdots \\
 \tilde{r}_j &= \tilde{v}_j + k(\theta^{j-1}\tilde{w}_1 + \sum_{s=0}^{j-2} \theta^s \tilde{u}_{j-s})
 \end{aligned}$$

Taking into account the standard deviations of \tilde{v}_j and \tilde{w}_1 , the general form of variables \tilde{r}_j is:

$$\tilde{r}_j = \sqrt{a}\tilde{v}_j + k\left(\frac{1}{\sqrt{1-\theta^2}}\theta^{j-1}\tilde{u}_1 + \sum_{s=0}^{j-2} \theta^s \tilde{u}_{j-s}\right)$$

where $\tilde{v}_{1,\dots,n}$ and $\tilde{u}_{1,\dots,n}$ are all independent $N(0, 1)$. We generate $2n$ random variables to do this.

This can be interpreted as a decomposition of the covariance matrix \mathbf{V} into a square root form $\mathbf{V} = \mathcal{C}\mathcal{C}' + \mathbf{D}$, where $\mathbf{D} = a\mathbf{I}$ and \mathcal{C} is a $(n \times n)$ matrix in the form of:

$$\begin{bmatrix}
 \frac{k}{\sqrt{1-\theta^2}} & 0 & 0 & \cdots & 0 \\
 \frac{k\theta}{\sqrt{1-\theta^2}} & k & 0 & \cdots & 0 \\
 \frac{k\theta^2}{\sqrt{1-\theta^2}} & k\theta & k & 0 & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 \frac{k\theta^{n-1}}{\sqrt{1-\theta^2}} & k\theta^{n-2} & \cdots & k\theta & k
 \end{bmatrix}$$

Alternatively, let $\mathbf{V} = \mathbf{CC}'$ where \mathbf{C} is a $(n \times 2n)$ matrix in the form of:

$$\begin{bmatrix} \frac{k}{\sqrt{1-\theta^2}} & 0 & 0 & \cdots & 0 & \sqrt{a} & 0 & 0 & \cdots & 0 \\ \frac{k\theta}{\sqrt{1-\theta^2}} & k & 0 & \cdots & 0 & 0 & \sqrt{a} & 0 & \cdots & 0 \\ \frac{k\theta^2}{\sqrt{1-\theta^2}} & k\theta & k & \cdots & 0 & 0 & 0 & \sqrt{a} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & 0 & \vdots & \vdots & \ddots & \vdots \\ \frac{k\theta^{n-1}}{\sqrt{1-\theta^2}} & k\theta^{n-2} & \cdots & k\theta & k & 0 & 0 & 0 & \cdots & \sqrt{a} \end{bmatrix}$$

Appendix C

Simulation of shocks in Chapter 6

In this section, we show how to simulate the shocks in the individual revenue function (6.3.2) that satisfy $\sum_{i=1}^N \sigma_i \varepsilon_i = 0$.

We write $\tilde{v}_i = \sigma_i \varepsilon_i$. We wish to simulate $\tilde{v}_1, \dots, \tilde{v}_N$ with the standard deviations $\sigma_1, \dots, \sigma_N$ such that $\sum_{i=1}^N \tilde{v}_i = 0$.

We simulate:

$$\tilde{v}_i = \sqrt{r_i} \epsilon_i - r_i \sum_{j=1}^N \sqrt{r_j} \epsilon_j \quad (\text{C.0.1})$$

where r_i is a constant and ϵ_i is the normally distributed noise with mean zero and variance V . Equation (C.0.1) gives $\sum_{i=1}^N \tilde{v}_i = 0$ as long as $\sum_{i=1}^N r_i = 1$. The variance of \tilde{v}_i is given by:

$$var(\tilde{v}_i) = (r_i - r_i^2 \sum_{j=1}^N r_j) V = r_i(1 - r_i)V$$

We require $r_i - r_i^2 = \frac{\sigma_i^2}{V}$ for $i = 1, \dots, N$ and $\sum_{i=1}^N r_i = 1$.

Given the assumption of σ_i 's, we can solve for V and r_i . The shocks in equation (6.3.2) can be simulated by using equation (C.0.1).

Bibliography

- [1] A. Admati. A noisy rational expectations equilibrium for multi-asset securities markets. *Econometrica*, 53:629–658, 1985.
- [2] A. Admati and P. Pfleiderer. A theory of intraday patterns: volume and price variability. *Review of Financial Studies*, 1:3 40, 1988.
- [3] Y. Amihud and H. Mendelson. Dealership market: market making with inventory. *Journal of Financial Economics*, 8:31–53, 1980.
- [4] Y. Amihud and H. Mendelson. Asset pricing and the bid-ask spread. *Journal of Financial Economics*, 17:223–249, 1986.
- [5] B. Anderson and J. Moore. *Optimal Filtering*. Prentice-Hall, 1979.
- [6] J. Anthony. The interrelation of stock and option market trading-volume data. *Journal of Finance*, 43:949–961, 1988.
- [7] K. Back. Asymmetric information and options. *Review of Financial Studies*, 6:435–472, 1993.

- [8] W. Bagehot. The only game in town. *Financial Analysts Journal*, 27:12–14, 1971.
- [9] W. Bauer. Commonality in liquidity in pure order-driven markets. Working paper, Wharton School, University of Pennsylvania, 2002.
- [10] B. Biais, L. Glosten, and C. Spatt. Market microstructure: a survey of microfoundations, empirical results and policy implications. *Journal of Financial Markets*, 8:217–264, 2005.
- [11] B. Biais and P. Hillion. Insider and liquidity trading in stock and options markets. *Review of Financial Studies*, 7:743–780, 1994.
- [12] F. Black. Facts and fantasy in use of options. *Financial Analysts Journal*, 31:36–41, 1975.
- [13] F. Black and M. Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81:637–659, 1973.
- [14] L. Blume, D. Easley, and M. O’Hara. Market statistics and technical analysis: the role of volume. *Journal of Finance*, 49:153–182, 1994.
- [15] B. Branch and W. Freed. Bid-ask spreads on the AMEX and the big board. *Journal of Finance*, 32:159–163, 1974.

- [16] M. Brennan and H. Cao. Information, trade, and derivative securities. *Review of Financial Studies*, 9:163–208, 1996.
- [17] M. Brennan and A. Subrahmanyam. Market microstructure and asset pricing: on the compensation for illiquidity in stock returns. *Journal of Financial Economics*, 41:441–464, 1996.
- [18] P. Brockman and D. Chung. Commonality in liquidity: evidence from the order-driven market structure. *Journal of Financial Research*, 25:521–539, 2002.
- [19] D. Brown and R. Jennings. On technical analysis. *Review of Financial Studies*, 2:527–552, 1989.
- [20] J. Caballe and M. Krishnan. Imperfect competition in a multi-security market with risk neutrality. *Econometrica*, 62:695–704, 1994.
- [21] C. Cao, Z. Chen, and J. Griffin. The informational content of option volume prior to takeovers. *Journal of Business*, 78:1073–1109, 2005.
- [22] S. Chakravarty, H. Gulen, and S. Mayhew. Informed trading in stock and option markets. *Journal of Finance*, 59:1235–1257, 2004.
- [23] K. Chan, Y. Chung, and H. Johnson. Why option prices lag stock prices: A trading based explanation. *Journal of Finance*, 48:1049–1075, 1993.

- [24] J. Cherian and W. Weng. An empirical analysis of directional and volatility trading in options markets. *Journal of Derivatives*, 7:53–65, 1999.
- [25] Y. Cho and R. Engle. Modelling the impact of market activity on bid-ask spreads in the options market. NBER Working paper 7331, 1999.
- [26] T. Chordia, R. Roll, and A. Subrahmanyam. Commonality in liquidity. *Journal of Financial Economics*, 56:3–28, 2000.
- [27] B. Chowdhry and V. Nanda. Multi-market trading and market liquidity. *Review of Financial Studies*, 4:483–511, 1991.
- [28] T. Copeland and D. Galai. Information effects and the bid-ask spread. *Journal of Finance*, 38:1457–1469, 1983.
- [29] J. Coughenour and M. Saad. Common market makers and commonality in liquidity. *Journal of Financial Economics*, 73:37–70, 2004.
- [30] N. Crafts. Some evidence of insider knowledge in horse race betting in Britain. *Economica*, 52:295–304, 1985.
- [31] P. de Fontnouvelle, R. Fishe, and J. Harris. The behavior of bid-ask spreads and volume in options markets during the competition for listing in 1999. *Journal of Finance*, 58:2437–2463, 2003.

- [32] C. de Jong, K. Koedijk, and C. Schnitzlein. Stock market quality in the presence of a traded option. Working paper, Erasmus University Rotterdam, 2001.
- [33] H. Demsetz. The cost of transacting. *Quarterly Journal of Economics*, 82:33–53, 1968.
- [34] I. Domowitz, O. Hansch, and X. Wang. Liquidity commonality and return co-movement. *Journal of Financial Market*, 8:351–376, 2005.
- [35] J. Dowie. On the efficiency and equity of betting markets. *Economica*, 43:139–150, 1976.
- [36] D. Easley and M. O’Hara. Price, trade size and information in securities markets. *Journal of Financial Economics*, 19:69–90, 1987.
- [37] D. Easley, M. O’Hara, and P. S. Srinivas. Option volume and stock prices: evidence on where informed traders trade. *Journal of Finance*, 53:431–465, 1998.
- [38] M. Evans and R. Lyons. Order flow and exchange rate dynamics. *Journal of Political Economy*, 110:170–180, 2002.
- [39] E. Fama and M. Gibbons. Inflation, real returns and captial investment. *Journal of Monetary Economics*, 9:297–323, 1982.

- [40] F. Foster and S. Viswanathan. A theory of interday variations in volume, variance, and trading costs in securities markets. *Review of Financial Studies*, 3:593–624, 1990.
- [41] K. French and R. Roll. Stock return variance: the arrival of information and the reaction of traders. *Journal of Financial Economics*, 17:5–26, 1986.
- [42] M. Friedman and L. Savage. The utility analysis of choice involving risk. *Journal of Political Economics*, 56:279–304, 1948.
- [43] P. Gabriel and J. Marsden. An examination of market efficiency in British racetrack betting. *Journal of Political Economics*, 98:874–875, 1990.
- [44] P. Gabriel and J. Marsden. An examination of market efficiency in British race-track betting: errata and corrections. *Journal of Political Economics*, 99:657–659, 1991.
- [45] M. Garman. Market microstructure. *Journal of Financial Economics*, 3:257–275, 1976.
- [46] L. Glosten and P. Milgrom. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics*, 14:71–100, 1985.

- [47] G. Gorton and G. Pennacchi. Security baskets and index-linked securities. *Journal of Business*, 66:1–27, 1993.
- [48] J. Hamilton. *Time series analysis*. Princeton, N.J.: Princeton University Press, 1994.
- [49] A. Harvey. *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge University Press, 1989.
- [50] J. Hasbrouck and D. Seppi. Common factors in prices, order flows and liquidity. *Journal of Financial Economics*, 59:383–411, 2001.
- [51] J. Hasbrouck, G. Sofianos, and D. Sosebee. New York Stock Exchange trading system and procedures. *Working paper, New York Stock Exchange*, 1993.
- [52] D. Hausch and W. Ziemba. *Finance Handbook*, chapter Efficiency of sports and lottery betting markets. Elsevier-North Holland, 1994.
- [53] D. Hausch, W. Ziemba, and M. Rubinstein. Efficiency of the market for race-track betting. *Management Science*, 27:1435–1452, 1981.
- [54] J. Hirshleifer. Investment decision under uncertainty: application of the state-preference approach. *Quarterly Journal of Economics*, 66:252–277, 1966.
- [55] T. Ho and H. Stoll. Optimal dealer pricing under transactions and return unvertainty. *Journal of Financial Economics*, 9:47–73, 1981.

- [56] T. Ho and H. Stoll. The dynamics of dealer markets under competition. *Journal of Finance*, 38:1053–1074, 1983.
- [57] C. Holden and A. Subrahmanyam. Long-lived private information and imperfect competition. *Journal of Finance*, 47:247–270, 1992.
- [58] D. Hsieh and N. Kulatilaka. Rational expectation and risk premia in forward markets: primary metals at the London Metals Exchange. *Journal of Finance*, 37:1199–1207, 1982.
- [59] P. Jain and G. Joh. The dependence between hourly prices and trading volume. *Journal of Financial and Quantitative Analysis*, 23:269–284, 1988.
- [60] M. Jameson and W. Wilhelm. Market making in the options markets and the costs of discrete hedge rebalancing. *Journal of Finance*, 47:765–779, 1992.
- [61] K. John, A. Koticha, R. Narayanan, and M. Subrahmanyam. Margin rules, informed trading in derivatives, and price dynamics. Working paper, New York University, 2003.
- [62] R. Kalman. A new approach to linear filtering and prediction problems. *ASME Journal of Basic Engineering*, 82:35–45, 1960.
- [63] G. Kaul, M. Nimalendran, and D. Zhang. Informed trading and option spreads. Working paper, University of Florida, 2004.

- [64] S. Koopman and H. Lai. Modelling bid-ask spreads in competitive dealership markets. *Discussion paper, Financial Markets Group, London School of Economics*, 1999.
- [65] P. Kumar and D. Seppi. Information and index arbitrage. *Journal of Business*, 67:481–509, 1994.
- [66] T. Kuypers. Information and efficiency: an empirical study of a fixed odds betting market. *Applied Economics*, 32:1353–1363, 2000.
- [67] A. Kyle. Market structure, information, futures markets, and price formation. in G. Storey, A. Schmitz, and A. Sarris (eds), *International Agricultural Trade: Advanced Readings in Price Formation, Market Structure and Price Instability*, pages 45–64, 1984.
- [68] A. Kyle. Continuous auctions and insider trading. *Econometrica*, 53:1315–1335, 1985.
- [69] P. Laux. Dealer market structure, outside competition, and the bid-ask spread. *Journal of Economic Dynamics and Control*, 19:683–710, 1995.
- [70] J. Lee and C. Yi. Trade size and information-motivated trading in the options and stock markets. *Journal of Financial and Quantitative Analysis*, 36:485–501, 2001.

- [71] R. Lyons. Test of microstructural hypotheses in the foreign exchange market. *Journal of Financial Economics*, 39:321–351, 1995.
- [72] A. Madhavan. Market microstructure: a survey. *Journal of Financial Markets*, 3:205–258, 2000.
- [73] A. Madhavan and S. Smidt. A Bayesian model of intraday specialist pricing. *Journal of Financial Economics*, 30:99–134, 1991.
- [74] A. Madhavan and S. Smidt. An analysis of changes in specialist quotes and inventories. *Journal of Finance*, 48:1595–1628, 1993.
- [75] S. Manaster and R. Rendelman Jr. Option prices as predictors of equilibrium stock prices. *Journal of Finance*, 37:1043–1057, 1992.
- [76] S. Manaster and S. Mann. Life in the pits: competitive market making and inventory control. *Review of Financial Studies*, 9:953–976, 1996.
- [77] H. Markowitz. The utility of wealth. *Journal of Political Economics*, 56:151–158, 1952.
- [78] S. Mayhew. Competition, market structure, and bid-ask spreads in stock option market. *Journal of Finance*, 57:931–958, 2002.

- [79] S. Mayhew, A. Sarin, and K. Shastri. The allocation of informed trading across related markets: an analysis of the impact of changes in equity-option margin requirements. *Journal of Business*, 55:1635–1654, 1995.
- [80] M. O’Hara. *Market Microstructure Theory*. Blackwell, 1995.
- [81] M. O’Hara and G. Oldfield. The microeconomics of market making. *Journal of Financial and Quantitative Analysis*, 21:361–376, 1986.
- [82] M. Pagano. Trading volume and asset liquidity. *Quarterly Journal of Economics*, 104:225–274, 1989.
- [83] J. Pan and A. Pотeshman. The information in option volume for future stock prices. Forthcoming, *Review of Financial Studies*, 2005.
- [84] L. Pastor and R. Stambaugh. Liquidity risk and expected stock returns. *Journal of Political Economy*, 111:642–685, 2003.
- [85] G. Petrella. Options market making: theory and evidence. Working paper, Universita Cattolica del Sacro Cuore, 2001.
- [86] P. Samuelson. Probability, utility, and the independence axiom. *Econometrica*, 20:670–678, 1952.
- [87] R. Sauer. The economics of wagering markets. *Journal of Economic Literature*, 36:2021–2064, 1998.

- [88] D. Seppi. Equilibrium block trading and asymmetric information. *Journal of Finance*, 45:73–94, 1990.
- [89] H. Shin. Optimal betting odds against insider traders. *Economic Journal*, 101:1179–1185, 1991.
- [90] H. Shin. Prices of state contingent claims with insider traders, and the favourite-longshot bias. *Economic Journal*, 102:426–435, 1992.
- [91] H. Shin. Measuring the incidence of insider trading in a market for state-contingent claims. *Economic Journal*, 103:1141–1153, 1993.
- [92] M. Smith, D. Paton, and L. Vaughan Williams. Market efficiency in person-to-person betting. Working paper, 2006.
- [93] M. Spiegel and A. Subrahmanyam. Informed speculation and hedging in a non-competitive securities market. *Review of Financial Studies*, 5:307–330, 1992.
- [94] J. Stephan and R. Whaley. Intraday price change and trading volume relations in the stock and stock option markets. *Journal of Finance*, 45:191–220, 1990.
- [95] H. Stoll. Dealer inventory behavior: an empirical investigation of NASDAQ stocks. *Journal of Financial and Quantitative Analysis*, 11:359–380, 1976.
- [96] H. Stoll. The supply of dealer services in securities markets. *Journal of Finance*, 33:1133–1151, 1978.

- [97] H. Stoll. Friction. *Journal of Finance*, 55:1479–1514, 2000.
- [98] H. Stoll. *Handbook of the Economics of Finance*, volume 1A, chapter Market microstructure, pages 553–604. Elsevier, 2003.
- [99] A. Subrahmanyam. A theory of trading in stock index future. *Review of Financial Studies*, 4:17–51, 1991.
- [100] R. Teweles and E. Bradley. *The Stock Market, 7th Edition*. New York: John Wiley, 1998.
- [101] S. Tinic. The economics of liquidity services. *Quarterly Journal of Economics*, 86:79–93, 1972.
- [102] S. Tinic and R. West. Marketability of common stocks in Canada and the USA: a comparison of agents vs. dealer dominated markets. *Journal of Finance*, 29:729–746, 1974.
- [103] L. Vaughan Williams. Information efficiency in betting markets: a survey. *Bulletin of Economic Research*, 51:1–30, 1999.
- [104] L. Vaughan Williams and D. Paton. Why is there a favorite long shot bias in british racetrack betting markets. *Economic Journal*, 107:150–158, 1997.
- [105] A. Vijh. Liquidity of the CBOE equity options. *Journal of Finance*, 45:1157–1179, 1990.

- [106] C. Wolff. Forward foreign exchange rates, expected spot rates, and premia: A signal-extraction approach. *Journal of Finance*, 42:395–406, 1987.